2.3.2 Example 2.3

The next example is a complex one from structural engineering design that is relevant in civil/mechanical/aerospace engineering applications. It appeared as a problem in Reference 4. It is developed in detail here. The problem is to redesign the basic tall flagpole in view of the phenomenal increase in wind speeds during extreme weather conditions. In recent catastrophic events, the wind speeds in tornadoes have been measured at over 350 miles per hour. These high speeds appear to be the norm rather than an unusual event.

Design Problem: Minimize the mass of a standard 10-m tubular flagpole to withstand wind gusts of 350 miles per hour. The flagpole will be made of structural steel. Use a factor of safety of 2.5 for the structural design. The deflection of the top of the flagpole should not exceed 5 cm. The problem is described in Figure 2.8.

Mathematical Model: The mathematical model is developed in detail for completeness and to provide a review of useful structural [5] and aerodynamic relations [6]. The relations are expressed in original symbols rather than in standard format of optimization problems to provide an insight into problem formulation.

Design Parameters: The structural steel [5] has the following material constants:

E (modulus of elasticity): 200 E+09 Pa
σ_{all} (allowable normal stress): 250 E+06 Pa
τ_{all} (allowable shear stress): 145 E+06 Pa

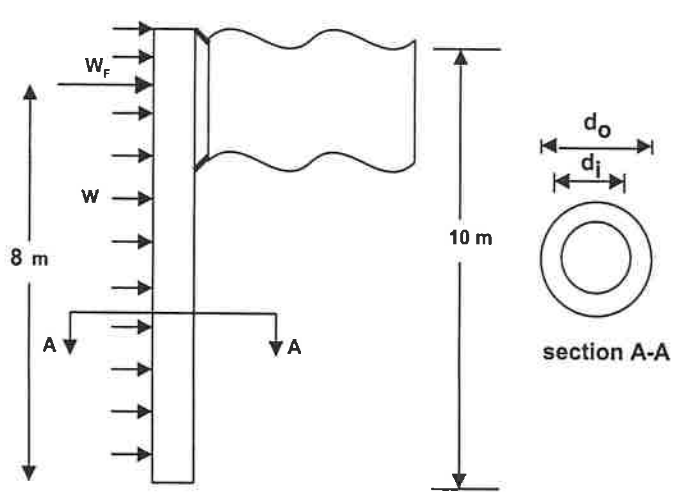

Figure 2.8 Flagpole design: Example 2.3.

γ (material density): 7860 kg/m^3
FS (factor of safety): 2.5
g (gravitational acceleration) = 9.81 m/s^2

For the aerodynamic calculations the following are considered:

ρ (standard air density): 1.225 kg/m^3
C_d (drag coefficient of cylinder): 1.0
W_F (flag wind load at 8 m): 5000 N
V_W (wind speed): 350 mph (156.46 m/s)

The geometric parameters are

L_p: the location of flag wind load (8 m)
L: length of the pole (10 m)
δ_{all}: permitted deflection (5 cm)

Design Variables: The design variables shown in Fig 2.8 are

d_o: outside diameter (x_1) [Note: x's are not used in the model]
d_i: inside diameter (x_2)

Geometric Relations: The following relations will be useful in later calculations:
A: area of cross-section = $0.25 * \pi * (d_o^2 - d_i^2)$
I: diametrical moment of inertia = $\pi * (d_o^4 - d_i^4)/64$
Q/t: first moment of area above the neutral axis divided by thickness
$= (d_o^2 + d_o d_i + d_i^2)/6$

Objective Function: The objective function is the weight of the 10-m uniform flagpole:

$$\text{Weight: } f(x_1, x_2): L * A * \gamma * g \tag{2.10}$$

Constraint Functions: The wind load per unit length (F_D) on the flagpole is calculated as

$$F_D = 0.5 * \rho * V_W^2 * C_d * d_o$$

The bending moment at the base of the pole due to this uniform wind load on the entire pole is

$$M_W = 0.5 * F_D * L * L$$

The bending moment due to the wind load on the flag is

$$M_F = W_F * L_p$$

Bending (normal) stress at the base of the pole is

$$\sigma_{bend} = 0.5 * (M_W + M_F) * d_o/I$$

Normal stress due to the weight is

$$\sigma_{weight} = \gamma * g * L$$

Total normal stresses to be resisted for design is the sum of the normal stresses computed above. Incorporating the factor of safety and the allowable stress from material values, the first inequality constraint can be set up as

$$g_1(x_1, x_2): \quad \sigma_{bend} + \sigma_{weight} \le \sigma_{all}/FS \tag{2.11}$$

The maximum shear load in the cross section is

$$S = W_F + F_D * L$$

The maximum shear stress in the pole is

$$\tau = S * Q/(I * t)$$

The second inequality constraint based on handling the shear stresses in the flagpole is

$$g_2(x_1, x_2): \quad \tau \le \tau_{all}/FS \tag{2.12}$$

The third practical constraint is based on the deflection of the top of the pole. This deflection due to a uniform wind load on the pole is

$$\delta_W = F_D * L^4/(8 * E * I)$$

The deflection at the top due to the flag wind load at L_p is

$$\delta_F = (2 * W_F * L^3 - W_F * L * L * L_p)/(E * I)$$

The third constraint translates to

$$g_3(x_1, x_2) \quad \delta_W + \delta_F \le \delta_{all} \tag{2.13}$$

To discourage solutions where $d_o < d_i$, we will include a geometric constraint:

$$g_4(x_1, x_2) \quad d_o - d_i \ge 0.001 \tag{2.14}$$

Side Constraints: This defines the design region for the search.

$$2\text{ cm} \le d_o \le 100\text{ cm}; \quad 2\text{ cm} \le d_i \le 100\text{ cm} \tag{2.15}$$

MATLAB Code: The m-files for this example are given below. An important observation in this problem, and structural engineering problems in particular, is the order of magnitude of the quantities in the constraining equations. The stress constraints are of the order of 10E+06, while the displacement terms are of the order of 10E-02. Most numerical techniques struggle to handle this range. Typically, such problems need to be normalized before being solved. It is essential in applying numerical techniques used in optimization.

This example is plotted in two parts. In the first part, each inequality constraint is investigated alone. Two curves are shown for each constraint. This avoids clutter. The side for the location of the hash mark on the constraint is determined by drawing these curves in color. The blue color indicates the feasible direction. Alternately, quiver plots can be used. The second part is the consolidated curve shown. The consolidated curve has to be drawn by removing the comments on some of the code and commenting the code that is not needed. Since the optimal solution cannot be clearly established, the zoom feature of MATLAB is used to narrow down the solution.

<u>ex2_3.m</u> (the main script file)

```
% Chapter 2: Optimization with MATLAB
% Dr. P.Venkataraman
% Example 2.3 Sec.2.3
%
% graphical solution using MATLAB (two design variables)
% Optimal design of a Flag Pole for high winds
% Ref. 2.4
%---------------------------------------------------
% global statement is used to share same information
% between various m-files
global ELAS SIGALL TAUALL GAM FS GRAV
global RHO CD FLAGW SPEED LP L DELT
%---------------------------------------------------
% Initialize values
ELAS = 200e+09;        % Modulus of elasticity -Pa
SIGALL = 250E+06;      % allowable normal stress -Pa
TAUALL = 145e+06;      % allowable shear stress - Pa
```

```
AREA = 0.25* pi*(X1.^2 - X2.^2);
INERTIA = pi*(X1.^4 - X2.^4)/64;
FD = 0.5*RHO*SPEED*SPEED*CD*X1;
dw = FD*L^4./(8*ELAS*INERTIA);
df = (2.0*FLAGW*L^3 - FLAGW*L*L*LP)./(ELAS*INERTIA);

retval = dw + df;
```

ineq4_ex3.m (the fourth constraint)

```
function retval = ineq4_ex3(X1,X2)
retval = X1 - X2;
```

Figure 2.9 displays the graphical or consolidated solution to the problem. The code is available in the m-file above (by commenting the codes that create the new figure windows and removing the comments on the code that is currently commented). The optimal solution is not very clear. Figure 2.10 is obtained by zooming in near the neighborhood of 0.6. See *help zoom* for instruction on its use. It can be achieved by

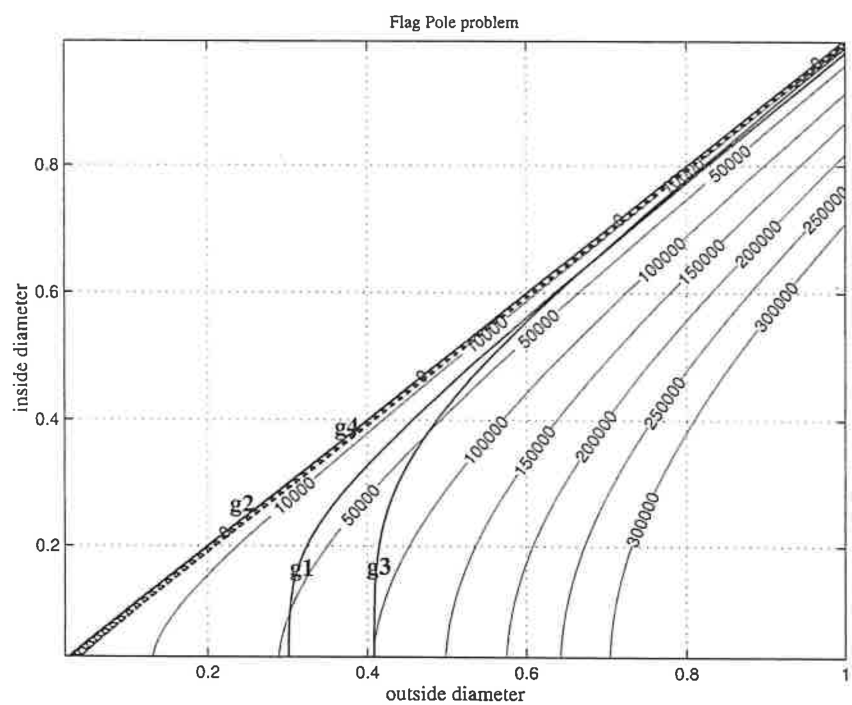

Figure 2.9 Graphical solution: Example 2.3.

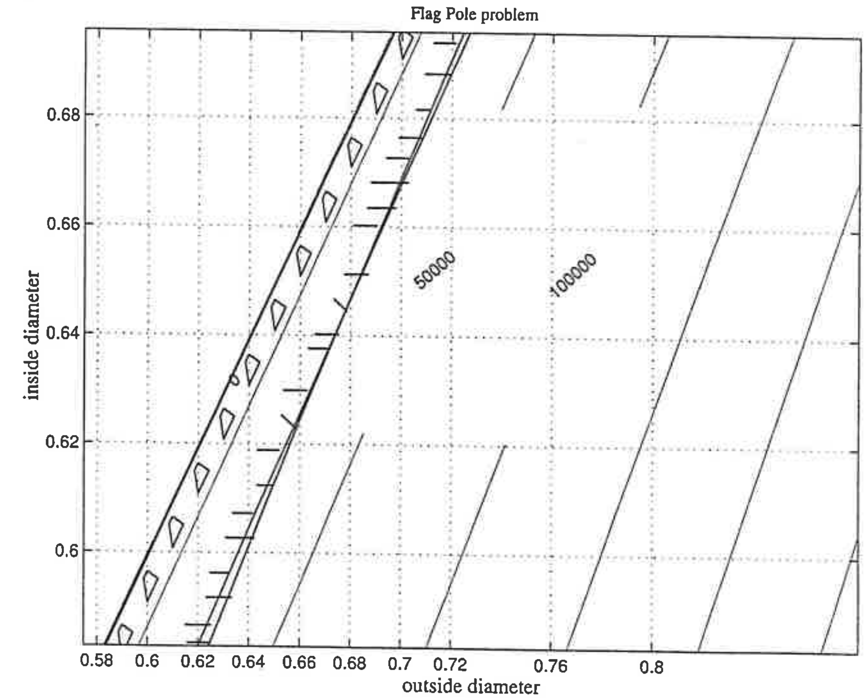

Figure 2.10 Graphical solution (zoomed): Example 2.3.

typing **zoom** at the workspace prompt and using the mouse to drag a rectangle around the region that needs to be enlarged. From Figure 2.10 the solution is around the outside diameter of 0.68 m and the inside diameter of 0.65 m. Typing **zoom** in the workspace again will toggle the figure back to normal state. In Figure 2.10, the tick marks are placed on the figure through the command window for better interpretation of the solution.

The graphics in the above example were created using the same statements encountered previously. Color contours were used to establish the feasible region. The zoom feature was employed to obtain a better estimate of the solution.

2.3.3 Example 2.4

This example is from the area of heat transfer. The problem is to design a triangular fin of the smallest volume that will at least deliver specified fin efficiencies. The graphical feature of this code is very similar to Example 2.3. In this example, the inequality constraints are computed and returned from a single function m-file rather than separate files considered in the previous example. Another new feature in this example is to invoke special mathematical functions, the Bessel functions that are