

Camada equivalente aplicada ao processamento e interpretação de dados de campos potenciais

Vanderlei C. Oliveira Jr.

2016



**Observatório
Nacional**



Generalização da integral de continuação para cima

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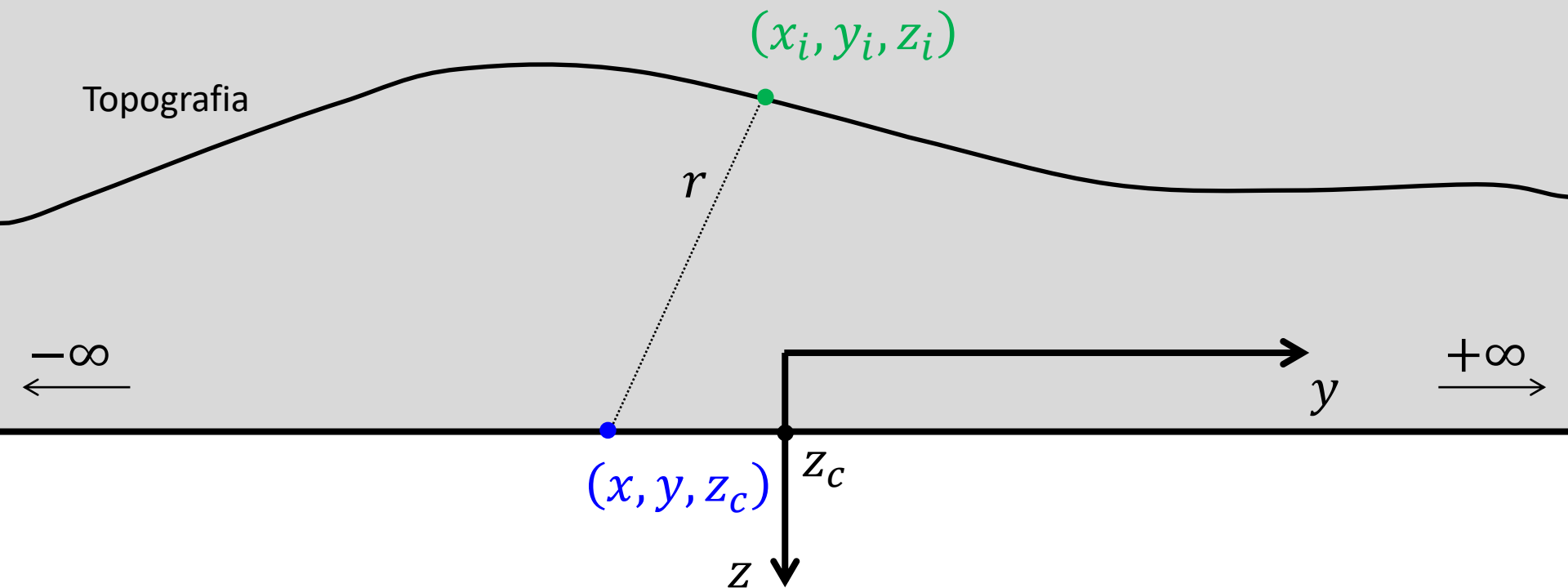
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Tal como mencionado anteriormente, a técnica da camada equivalente é baseada em uma equação integral chamada **integral de continuação para cima** (Skeels, 1947; Henderson and Zietz, 1949; Henderson, 1960; Roy, 1962; Bhattacharyya, 1967; Henderson, 1970; Twomey, 1977; Blakely, 1996)



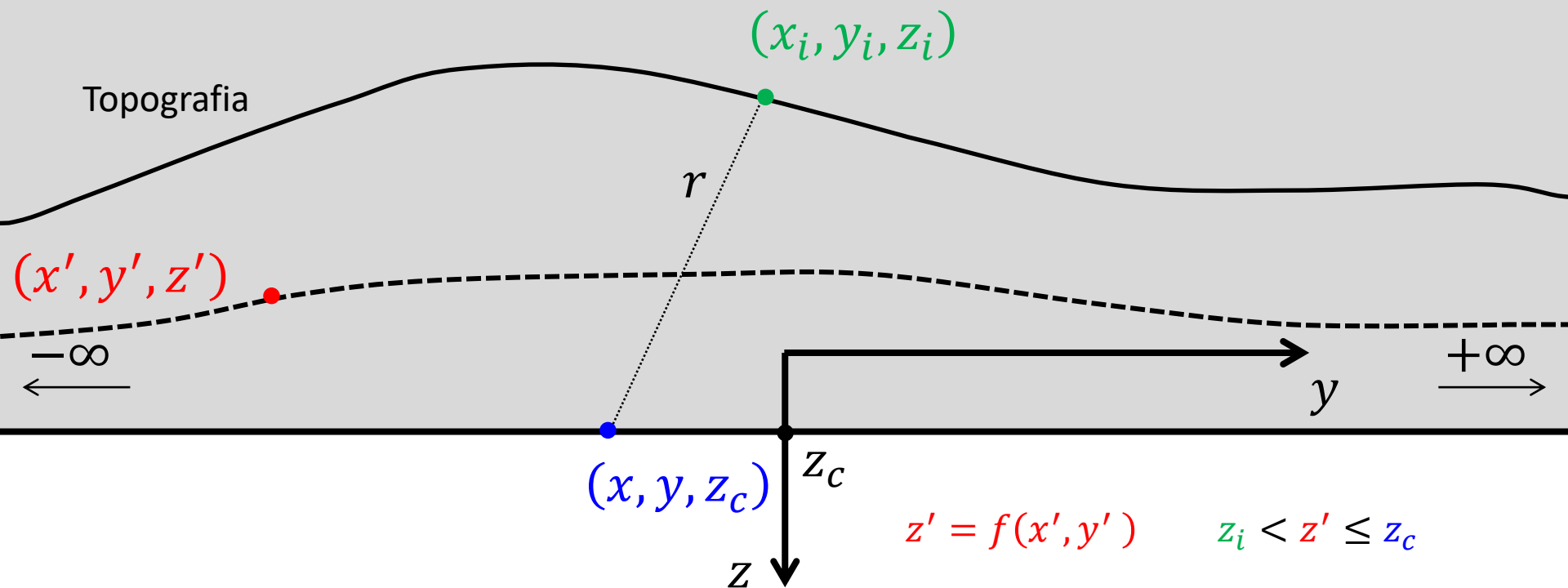
$$U_i = \frac{z_c - z_i}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{U(x, y, z_c)}{r^3} dx dy$$

$$r = [(x_i - x)^2 + (y_i - y)^2 + (z_i - z_c)^2]^{\frac{1}{2}}$$

A integral de continuação para cima, em sua forma clássica, restringe a aplicação da técnica da camada equivalente para o caso em que: **1)** as fontes equivalentes estão localizadas sobre um plano horizontal e **2)** as funções harmônicas representam o efeito gravitacional das fontes equivalentes

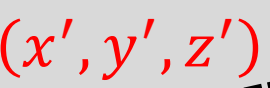
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Nesta parte do curso, vamos generalizar a integral de continuação para cima com o intuito de remover estas duas restrições



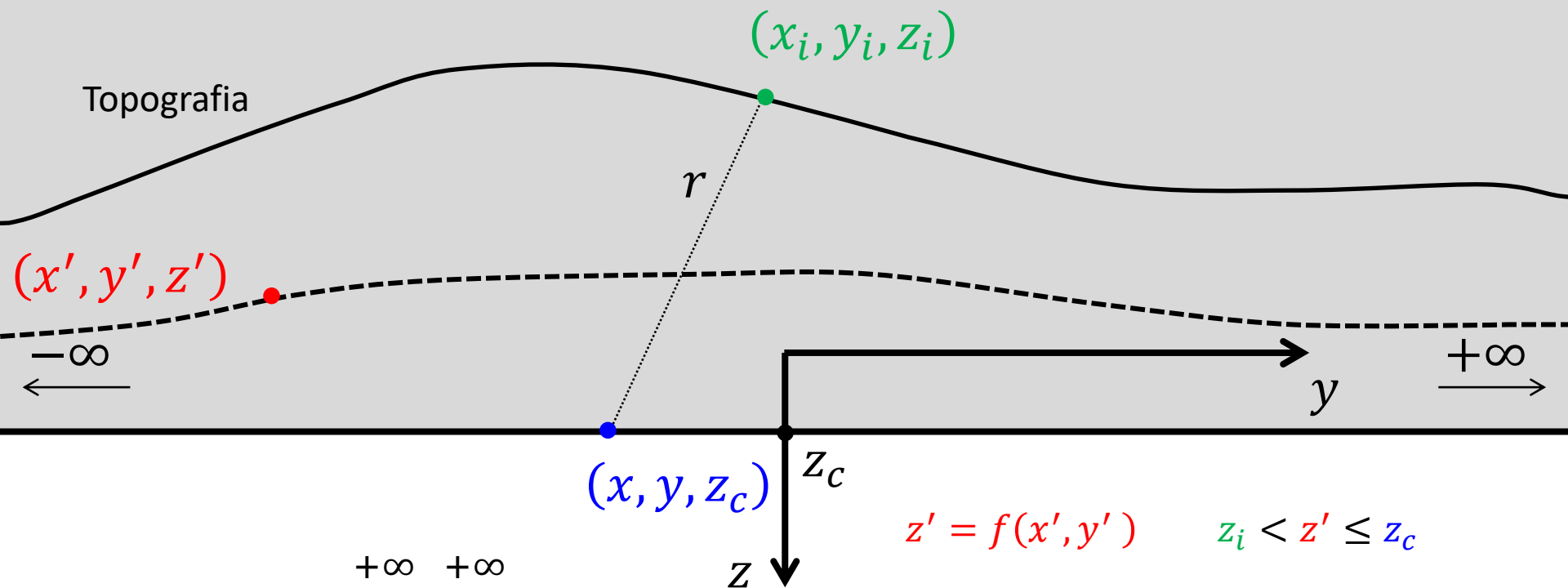
$$U_i = \frac{z_c - z_i}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{U(x, y, z_c)}{r^3} dx dy$$

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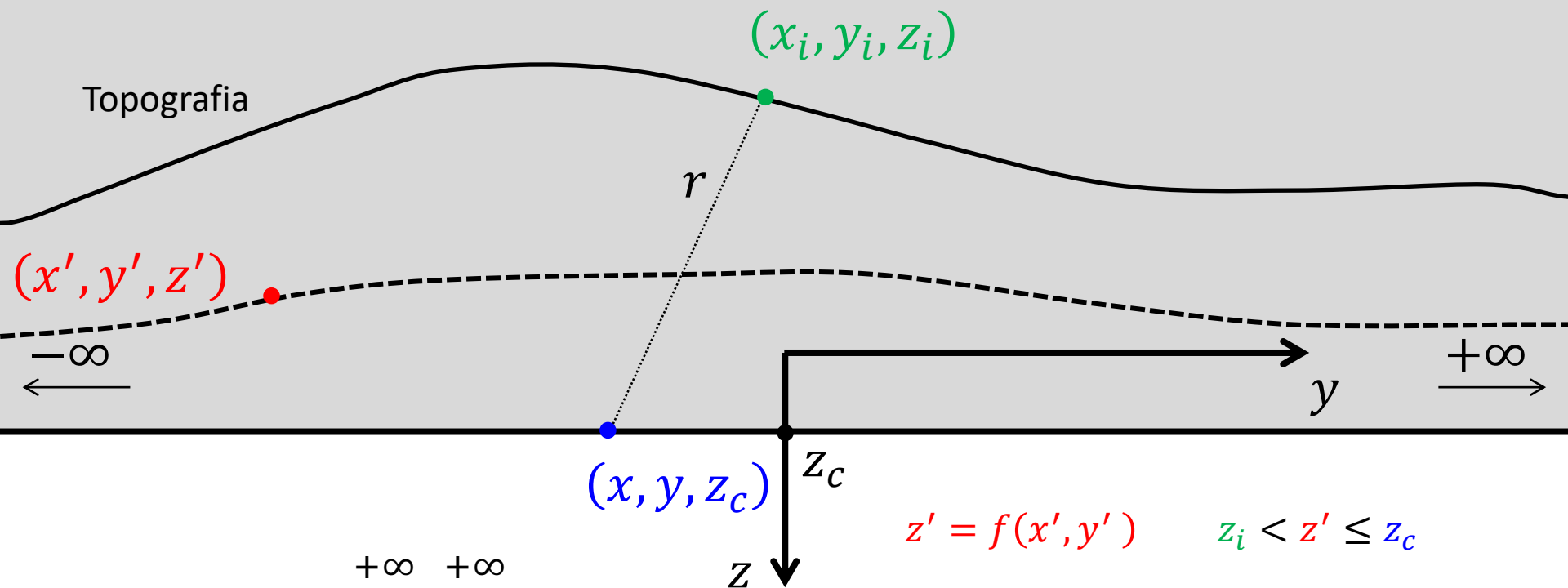
$$r = [(x_i - x)^2 + (y_i - y)^2 + (z_i - z_c)^2]^{\frac{1}{2}}$$



$$U(x, y, z_c) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(x', y', z') \underbrace{g(x - x', y - y', z_c - z')}_{\text{Função harmônica que tende a zero à medida em que os pontos } (x', y', z') \text{ e } (x, y, z_c) \text{ tende ao infinito}} dx' dy'$$

$$U_i = \frac{z_c - z_i}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{U(x, y, z_c)}{r^3} dx dy$$

$$r = [(x_i - x)^2 + (y_i - y)^2 + (z_i - z_c)^2]^{\frac{1}{2}}$$

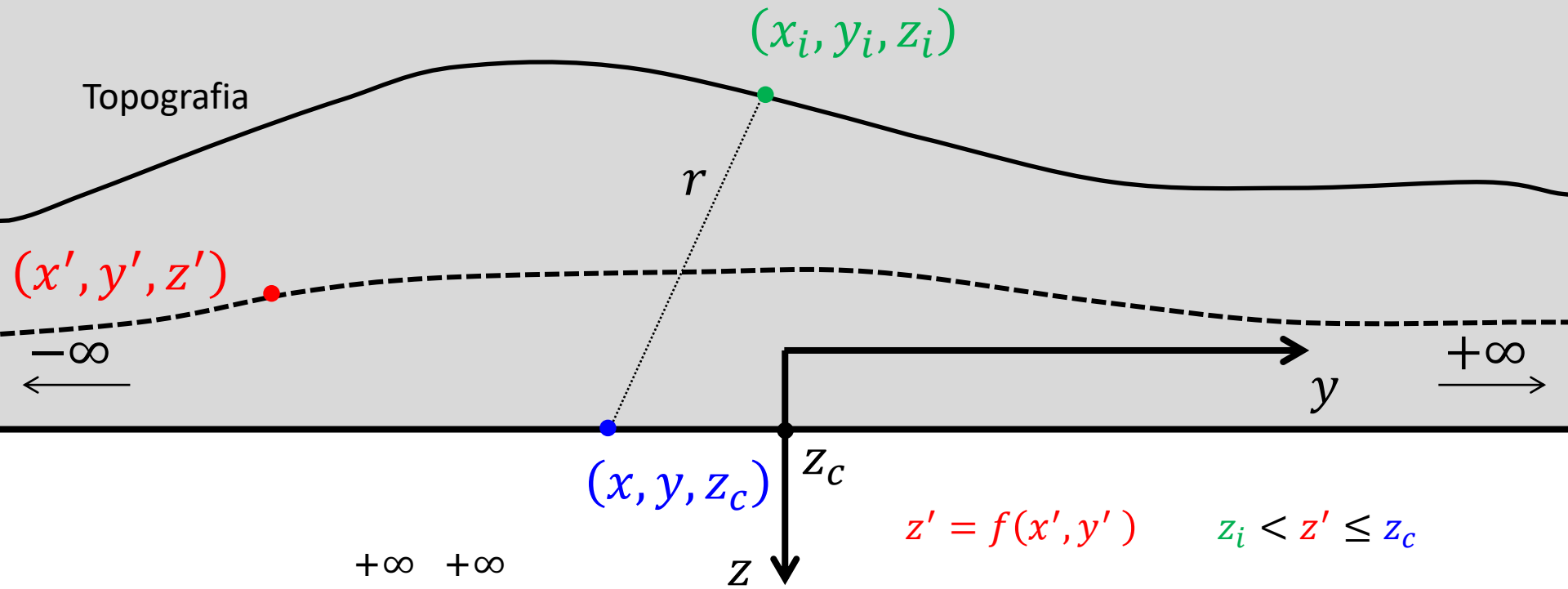


$$U(x, y, z_c) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \underbrace{p(x', y', z')} g(x - x', y - y', z_c - z') dx' dy'$$

$$U_i = \frac{z_c - z_i}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{U(x, y, z_c)}{r^3} dx dy$$

Função que satisfaz as condições necessárias para que esta integral seja convergente

$$r = [(x_i - x)^2 + (y_i - y)^2 + (z_i - z_c)^2]^{\frac{1}{2}}$$

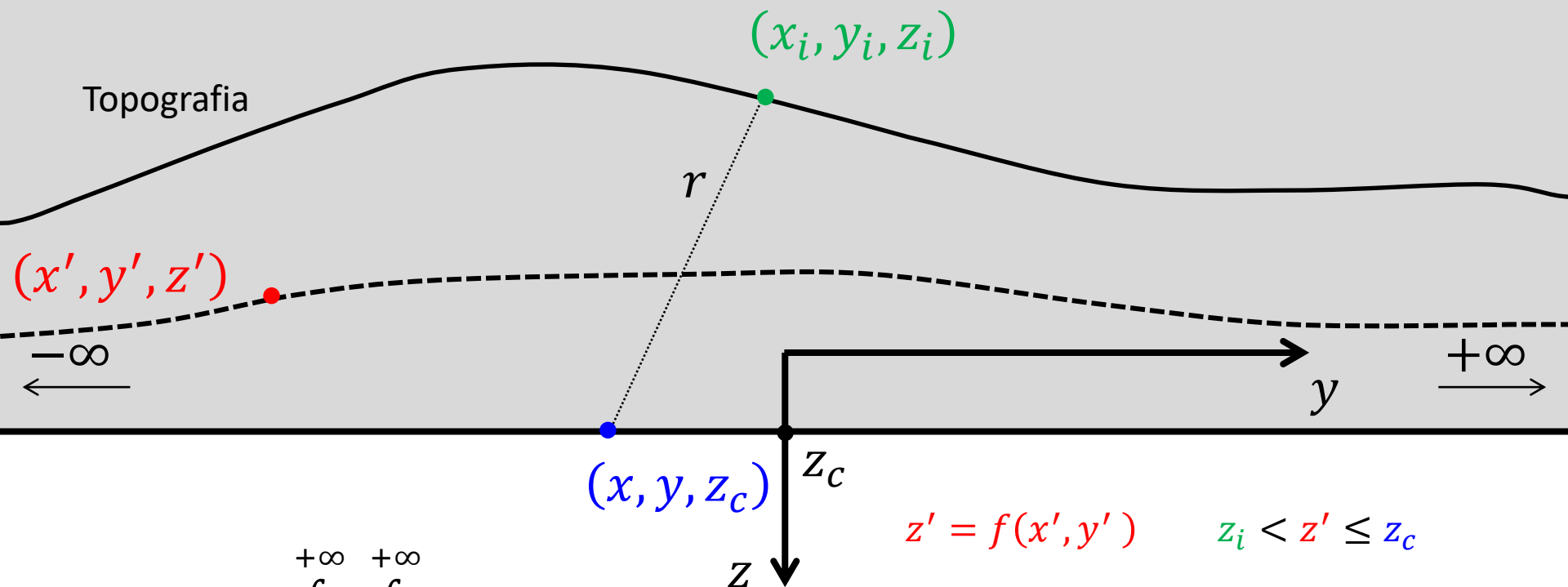


$$U(x, y, z_c) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(x', y', z') g(x - x', y - y', z_c - z') dx' dy'$$

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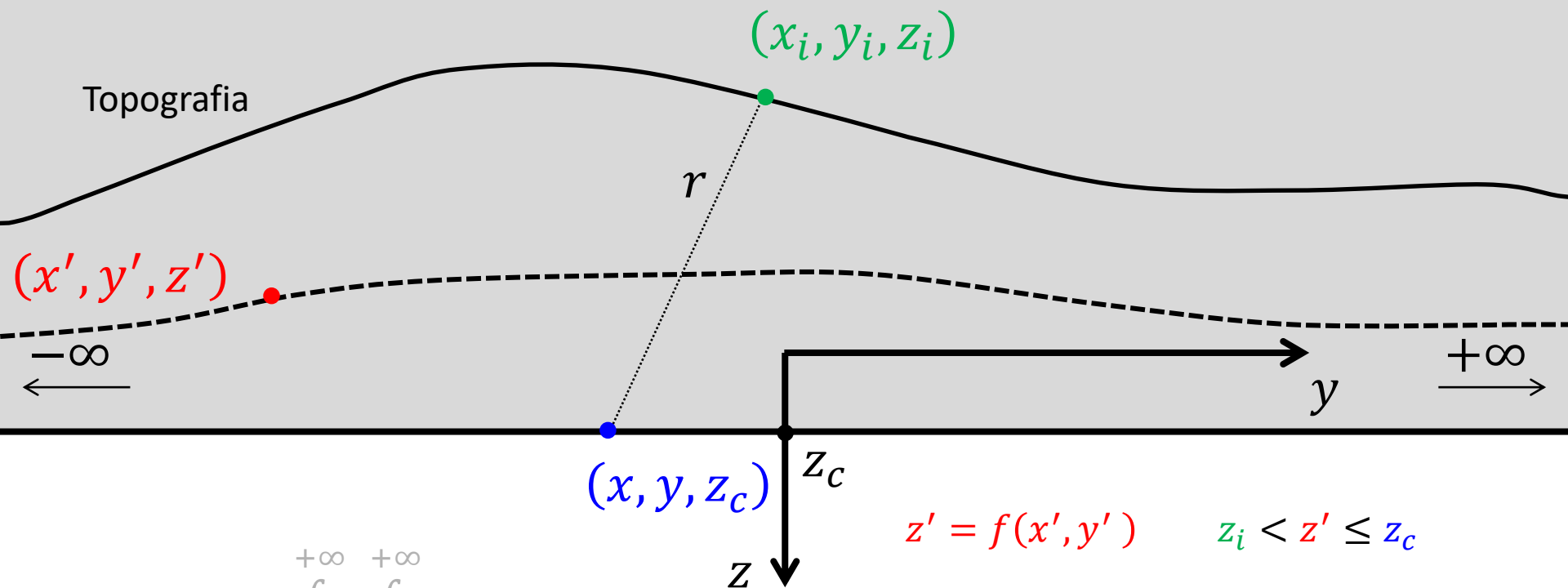
Substituindo esta nova integral na integral de continuação para cima e mudando a ordem da integração...



$$U(x, y, z_c) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(x', y', z') g(x - x', y - y', z_c - z') dx' dy'$$

$$U_i = \frac{z_c - z_i}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{U(x, y, z_c)}{r^3} dx dy \quad r = [(x_i - x)^2 + (y_i - y)^2 + (z_i - z_c)^2]^{\frac{1}{2}}$$

$$U_i = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(x', y', z') \left\{ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{g(x - x', y - y', z_c - z') (z_c - z_i)}{2\pi r^3} dx dy \right\} dx' dy'$$



$$U(x, y, z_c) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(x', y', z') g(x - x', y - y', z_c - z') dx' dy'$$

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$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{g(\textcolor{blue}{x} - \textcolor{red}{x}', \textcolor{blue}{y} - \textcolor{red}{y}', \textcolor{blue}{z}_c - \textcolor{red}{z}')}{2\pi} \frac{(\textcolor{blue}{z}_c - \textcolor{green}{z}_i)}{r^3} d\textcolor{blue}{x} d\textcolor{blue}{y}$$

$$r = [(\textcolor{green}{x}_i - \textcolor{blue}{x})^2 + (\textcolor{green}{y}_i - \textcolor{blue}{y})^2 + (\textcolor{green}{z}_i - \textcolor{blue}{z}_c)^2]^{\frac{1}{2}}$$

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{g(x - x', y - y', z_c - z')}{2\pi} \frac{(z_c - z_i)}{r^3} dx dy$$

$$r = [(x_i - x)^2 + (y_i - y)^2 + (z_i - z_c)^2]^{\frac{1}{2}}$$

$$\epsilon = x - x' \quad \eta = y - y' \quad \varsigma = z_c - z'$$

Mudança de variáveis

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{g(x - x', y - y', z_c - z')}{2\pi} \frac{(z_c - z_i)}{r^3} dx dy$$

$$r = [(x_i - x)^2 + (y_i - y)^2 + (z_i - z_c)^2]^{\frac{1}{2}}$$

$$\epsilon = x - x' \quad \eta = y - y' \quad \varsigma = z_c - z'$$

$$\begin{aligned} z_i &< z' \leq z_c \\ z_i - z' &< 0 \leq \underbrace{z_c - z'}_{\varsigma} \\ z_i - z' &< \varsigma \end{aligned}$$

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{g(x - x', y - y', z_c - z')}{2\pi} \frac{(z_c - z_i)}{r^3} dx dy$$

$$r = [(x_i - x)^2 + (y_i - y)^2 + (z_i - z_c)^2]^{\frac{1}{2}}$$

$$\epsilon = x - x' \quad \eta = y - y' \quad \varsigma = z_c - z'$$

$$z_i < z' \leq z_c$$

$$z_i - z' < 0 \leq \underbrace{z_c - z'}_{\varsigma}$$

$$z_i - z' < \varsigma$$

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{g(\epsilon, \eta, \varsigma) [\varsigma - (z_i - z')]}{\{[(x_i - x') - \epsilon]^2 + [(y_i - y') - \eta]^2 + [(z_i - z') - \varsigma]^2\}^{\frac{3}{2}}} d\epsilon d\eta =$$

$$= g(x_i - x', y_i - y', z_i - z')$$

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{g(x - x', y - y', z_c - z')}{2\pi} \frac{(z_c - z_i)}{r^3} dx dy$$

$$r = [(x_i - x)^2 + (y_i - y)^2 + (z_i - z_c)^2]^{\frac{1}{2}}$$

$$\epsilon = x - x' \quad \eta = y - y' \quad \varsigma = z_c - z'$$

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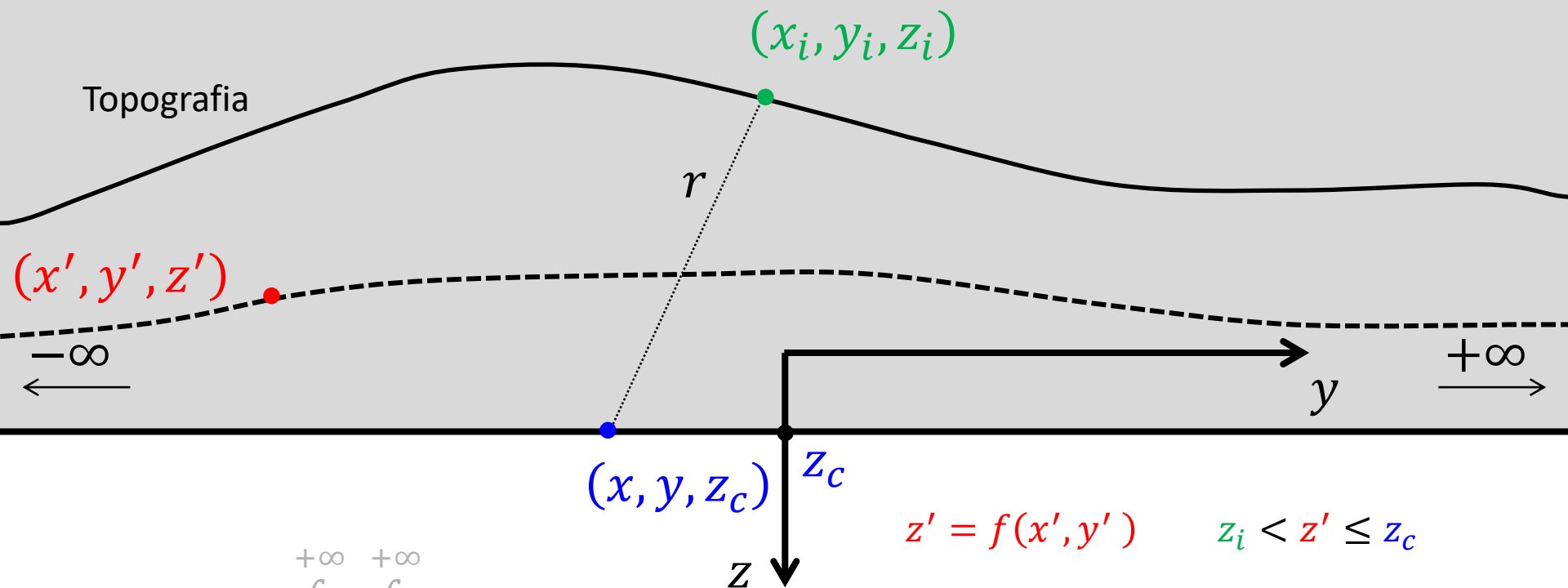
$$z_i - z' < 0 \leq \underbrace{z_c - z'}_{\varsigma}$$

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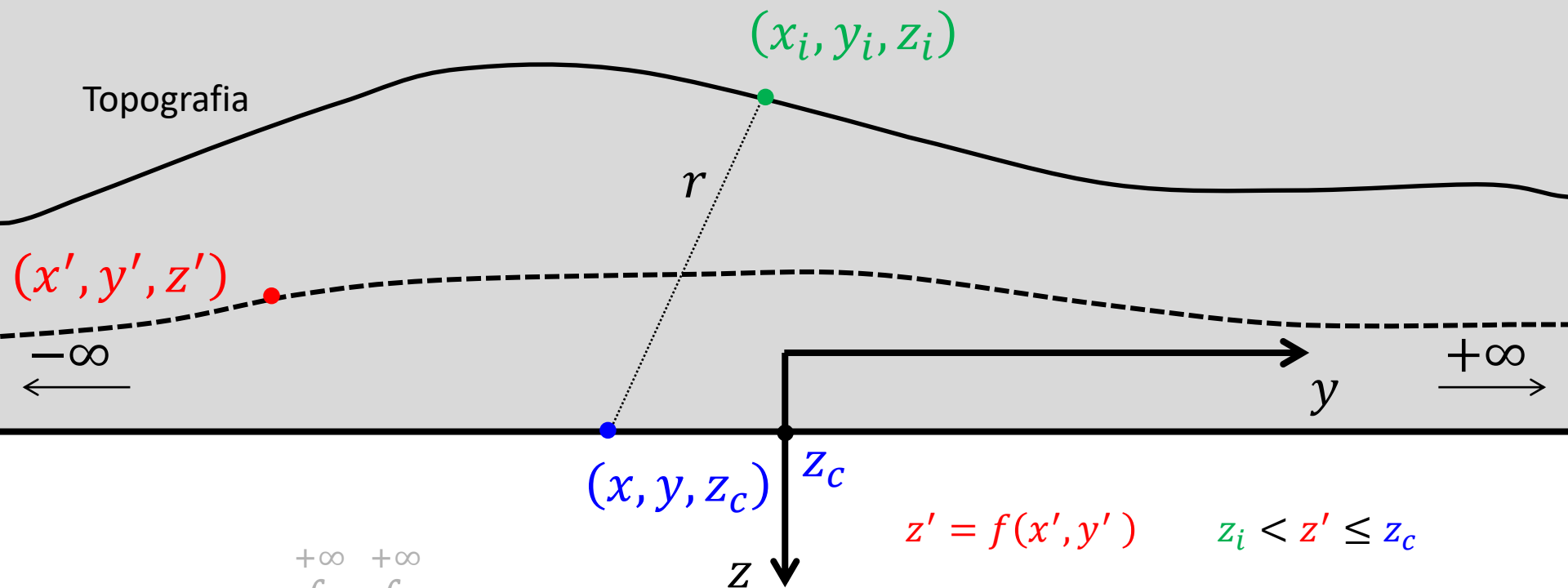
$$U_i = \frac{z_c - z_i}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{U(x, y, z_c)}{[(x_i - x)^2 + (y_i - y)^2 + (z_i - z_c)^2]^{\frac{3}{2}}} dx dy$$



$$U(x, y, z_c) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(x', y', z') g(x - x', y - y', z_c - z') dx' dy'$$

$$U_i = \frac{z_c - z_i}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{U(x, y, z_c)}{r^3} dx dy \quad r = [(x_i - x)^2 + (y_i - y)^2 + (z_i - z_c)^2]^{\frac{1}{2}}$$

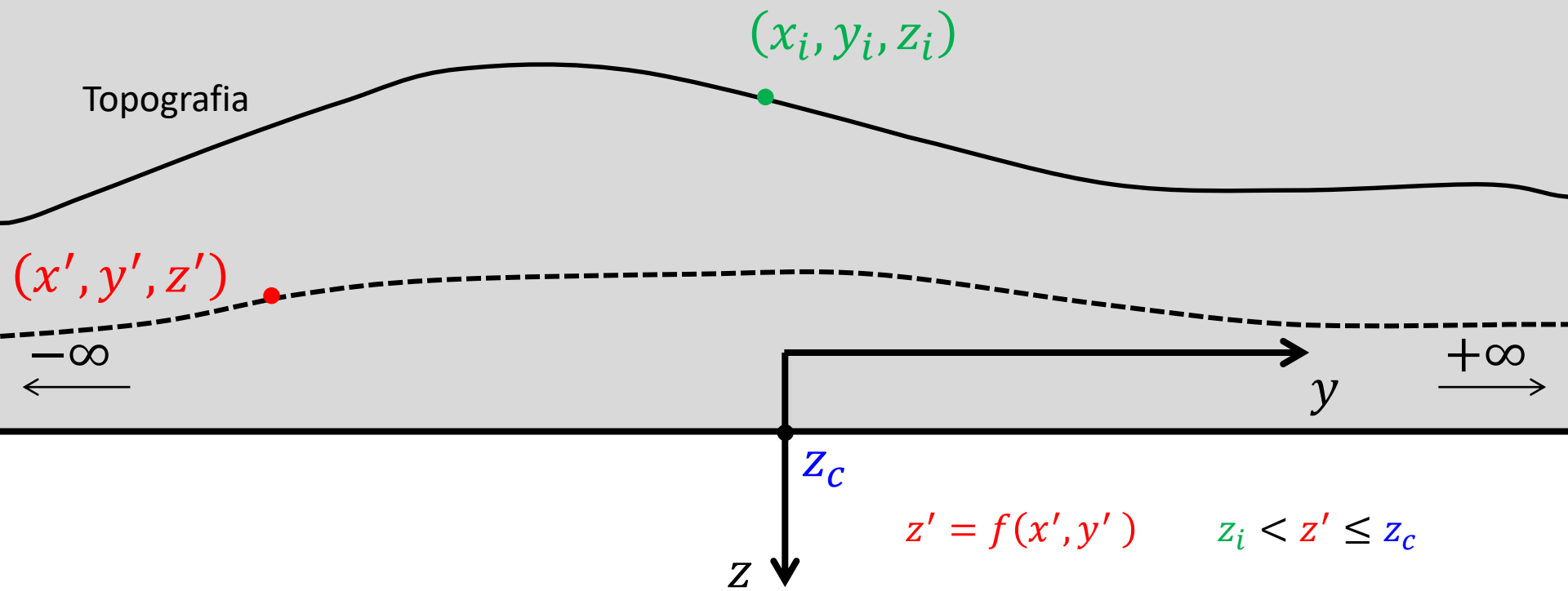
$$U_i = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(x', y', z') \left\{ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{g(x - x', y - y', z_c - z') (z_c - z_i)}{2\pi r^3} dx dy \right\} dx' dy'$$



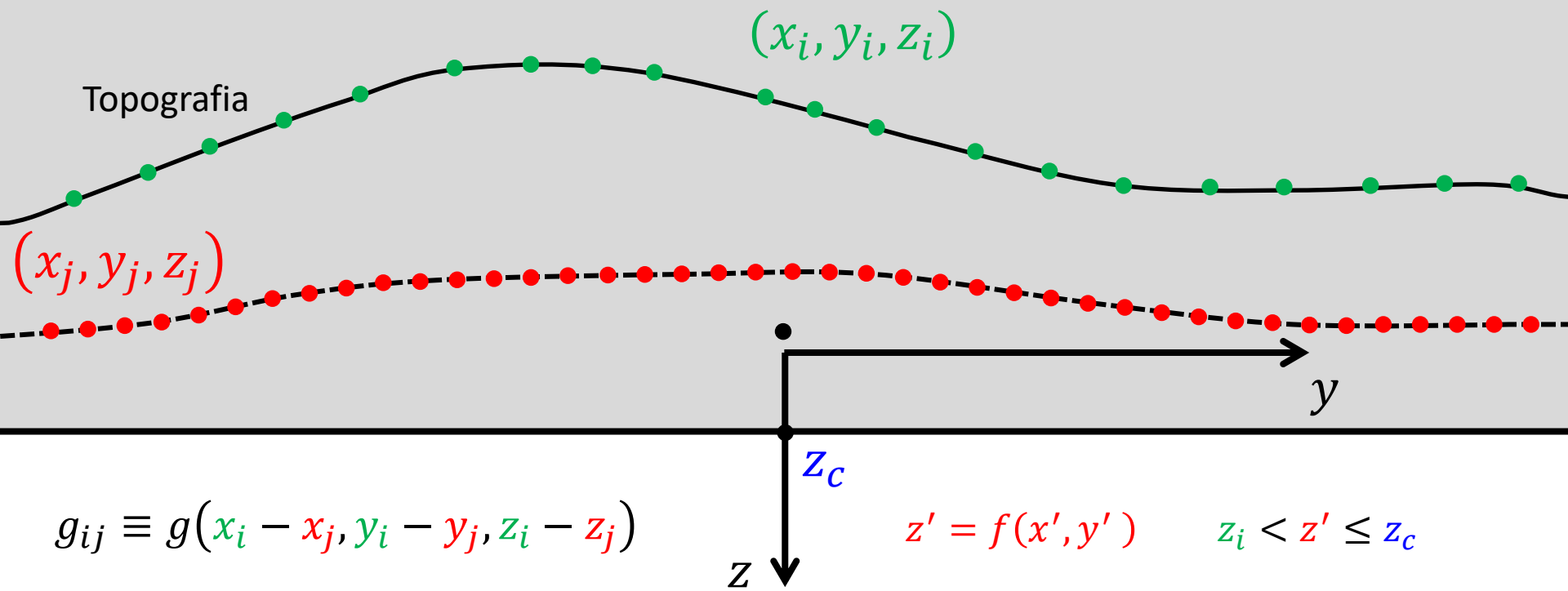
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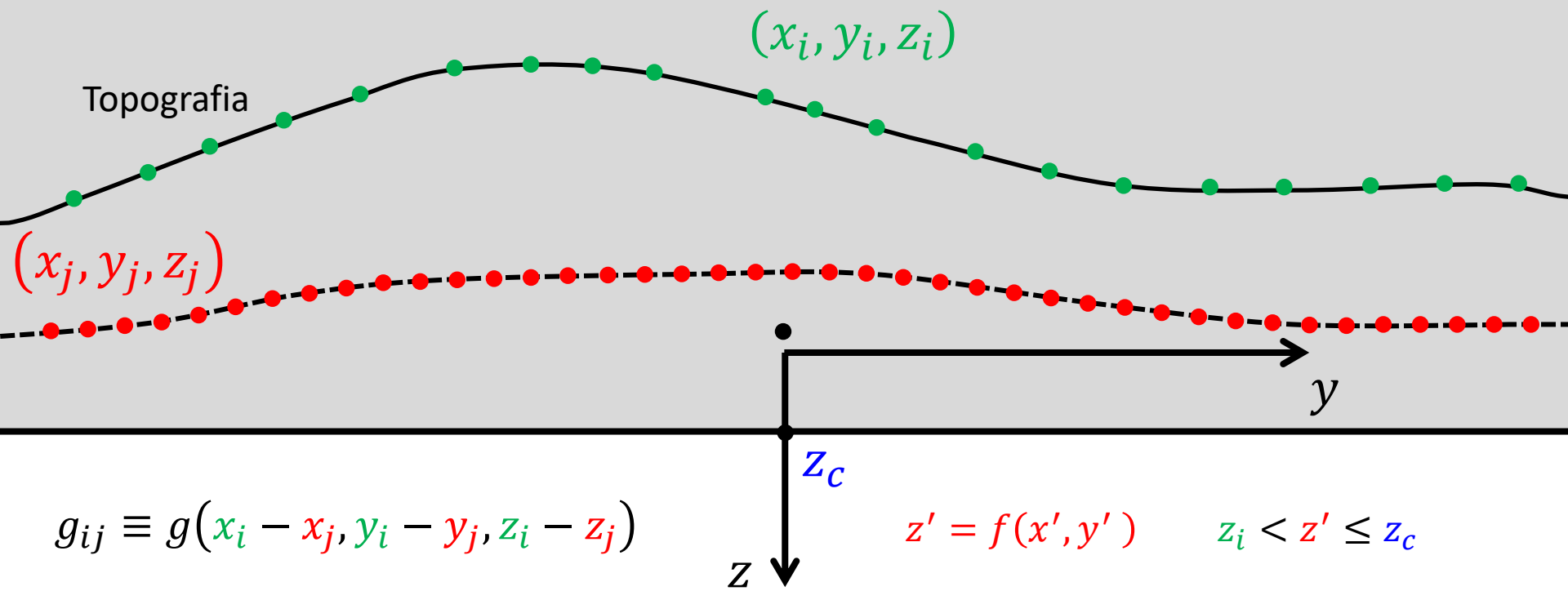
$$g_{ij} \equiv g(x_i - x_j, y_i - y_j, z_i - z_j)$$

$$p_j \equiv p(x_j, y_j, z_j)$$

$$z' = f(x', y') \quad z_i < z' \leq z_c$$

$$U_i \approx \sum_{j=1}^M p_j g_{ij}$$

$$U_i = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(x', y', z') g(x_i - x', y_i - y', z_i - z') dx' dy'$$



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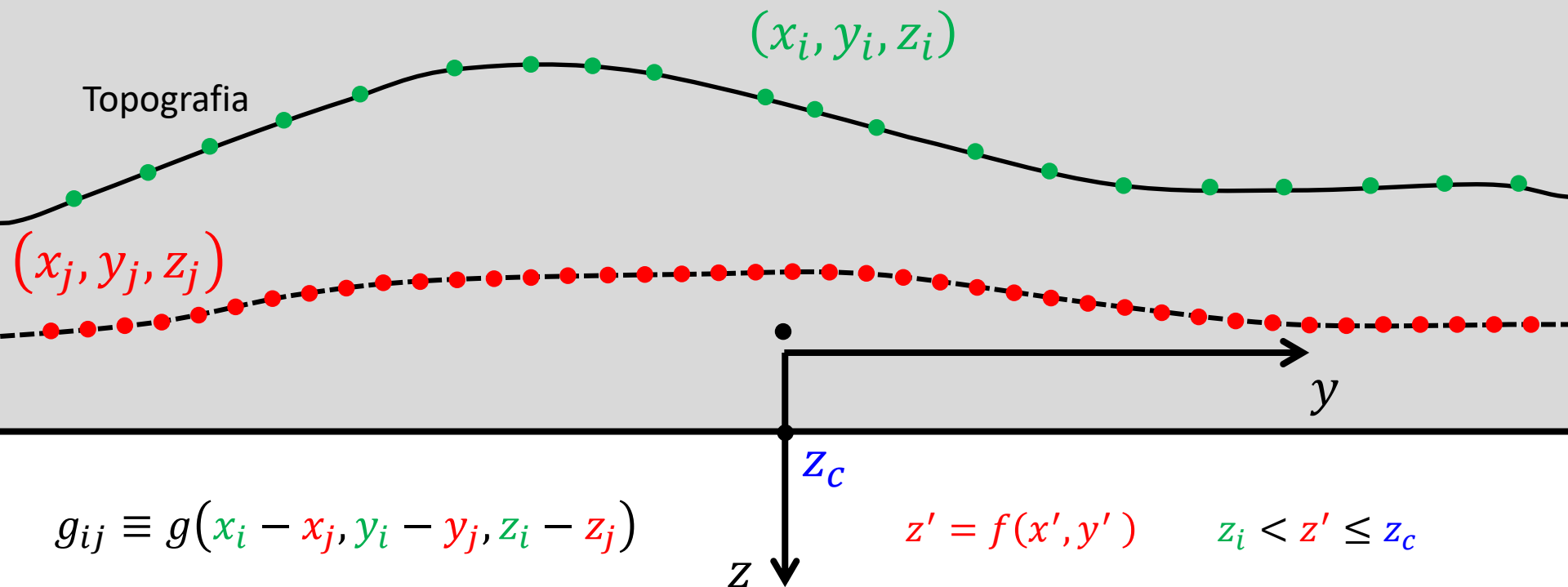
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$$z' = f(x', y') \quad z_i < z' \leq z_c$$

$$[\mathbf{U}]_i = U_i \quad [\mathbf{A}]_{ij} = g_{ij}$$

$$[\mathbf{p}]_j = p_j$$

$$\mathbf{U}_{N \times 1} = \mathbf{A}_{N \times M} \mathbf{p}_{M \times 1}$$



$$g_{ij} \equiv g(x_i - x_j, y_i - y_j, z_i - z_j)$$

$$p_j \equiv p(x_j, y_j, z_j)$$

Note que esta abordagem
não impõe que as fontes
equivalentes estejam sobre
um plano, nem restringe o
tipo de função harmônica a
ser utilizada

$$z' = f(x', y')$$

$$z_i < z' \leq z_c$$

$$[\mathbf{U}]_i = U_i$$

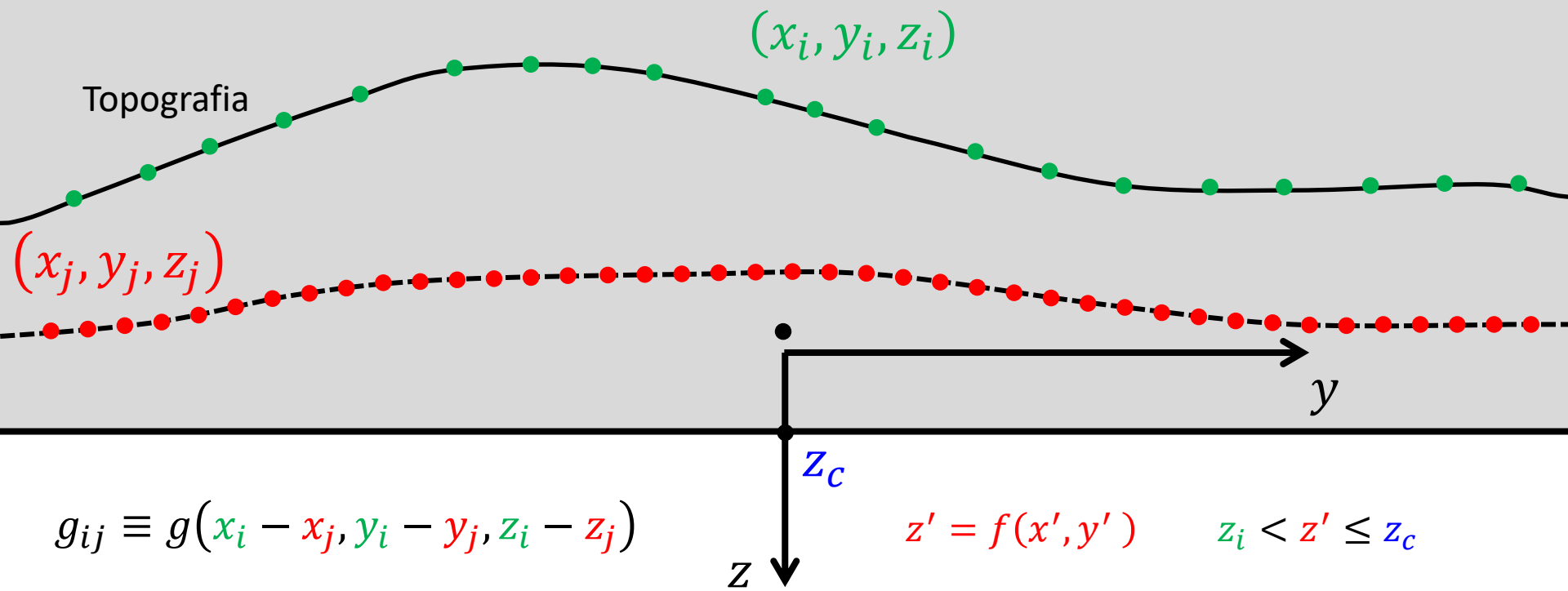
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$$\mathbf{U}_{N \times 1} = \mathbf{A}_{N \times M} \mathbf{p}_{M \times 1}$$

$$U_i \approx \sum_{j=1}^M p_j g_{ij}$$

$$U_i = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(x', y', z') g(x_i - x', y_i - y', z_i - z') dx' dy'$$



$$g_{ij} \equiv g(x_i - x_j, y_i - y_j, z_i - z_j)$$

$$p_j \equiv p(x_j, y_j, z_j)$$

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$$\mathbf{U}_{N \times 1} = \mathbf{A}_{N \times M} \mathbf{p}_{M \times 1}$$

$$U_i \approx \sum_{j=1}^M p_j g_{ij} \leftarrow \text{Como escolher estas funções harmônicas?}$$

$$U_i = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(x', y', z') g(x_i - x', y_i - y', z_i - z') dx' dy'$$

$$\theta_{ij} = \frac{1}{\left[(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2 \right]^{\frac{1}{2}}}$$

Em geral, as função harmônicas g_{ij} envolvem derivas da função inverso da distância entre o i-ésimo ponto de observação (x_i, y_i, z_i) e a posição (x_j, y_j, z_j) da j-ésima fonte equivalente

$$\theta_{ij} = \frac{1}{\left[(\textcolor{teal}{x}_i - \textcolor{red}{x}_j)^2 + (\textcolor{teal}{y}_i - \textcolor{red}{y}_j)^2 + (\textcolor{teal}{z}_i - \textcolor{red}{z}_j)^2 \right]^{\frac{1}{2}}}$$

Geralmente utilizada com dados
gravimétricos

$$g_{ij} = \kappa_g \partial_{\textcolor{teal}{z}} \theta_{ij} \quad \kappa_g = 10^5 G$$

Em geral, as função
harmônicas g_{ij} envolvem
derivadas da função inverso da
distância entre o i-ésimo
ponto de observação
 $(\textcolor{teal}{x}_i, \textcolor{teal}{y}_i, \textcolor{teal}{z}_i)$ e a posição
 $(\textcolor{red}{x}_j, \textcolor{red}{y}_j, \textcolor{red}{z}_j)$ da j-ésima fonte
equivalente

$$\theta_{ij} = \frac{1}{\left[(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2 \right]^{\frac{1}{2}}}$$

Geralmente utilizada com dados gravimétricos

$$g_{ij} = \kappa_g \partial_z \theta_{ij} \quad \kappa_g = 10^5 G$$

Geralmente utilizada com dados magnetométricos

$$g_{ij} = \kappa_m m_j \hat{\mathbf{F}}_0^T \mathbf{H}_{ij} \hat{\mathbf{m}} \quad \kappa_m = 10^9 \frac{\mu_0}{4\pi}$$

$$\mathbf{H}_{ij} = \begin{bmatrix} \partial_{xx} \theta_{ij} & \partial_{xy} \theta_{ij} & \partial_{xz} \theta_{ij} \\ \partial_{xy} \theta_{ij} & \partial_{yy} \theta_{ij} & \partial_{yz} \theta_{ij} \\ \partial_{xz} \theta_{ij} & \partial_{yz} \theta_{ij} & \partial_{zz} \theta_{ij} \end{bmatrix}$$

Em geral, as funções harmônicas g_{ij} envolvem derivadas da função inverso da distância entre o i -ésimo ponto de observação (x_i, y_i, z_i) e a posição (x_j, y_j, z_j) da j -ésima fonte equivalente

Campo geomagnético local

$$\hat{\mathbf{F}}_0 = \begin{bmatrix} \cos I \cos D \\ \cos I \sin D \\ \sin I \end{bmatrix}$$

Magnetização da fonte equivalente

$$\hat{\mathbf{m}} = \begin{bmatrix} \cos i \cos d \\ \cos i \sin d \\ \sin i \end{bmatrix}$$

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