

# **Camada equivalente aplicada ao processamento e interpretação de dados de campos potenciais**

Vanderlei C. Oliveira Jr.

2016



**Observatório  
Nacional**



# **Anomalia de Campo Total (parte B)**

Vanderlei C. Oliveira Jr.

2016



**Observatório  
Nacional**



$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_i$$

Campo principal

$$\mathbf{B}_i = B_i \hat{\mathbf{B}}_i$$

Campo crustal

$$\mathbf{T}_i = \mathbf{F}_i + \mathbf{B}_i$$

Campo total

$$F_i \gg B_i$$

Condição observada  
na prática

$$\Delta T_i = T_i - F_i$$

Anomalia de campo total

$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_i$$

Campo principal

$$\mathbf{B}_i = B_i \hat{\mathbf{B}}_i$$

Campo crustal

$$\mathbf{T}_i = \mathbf{F}_i + \mathbf{B}_i$$

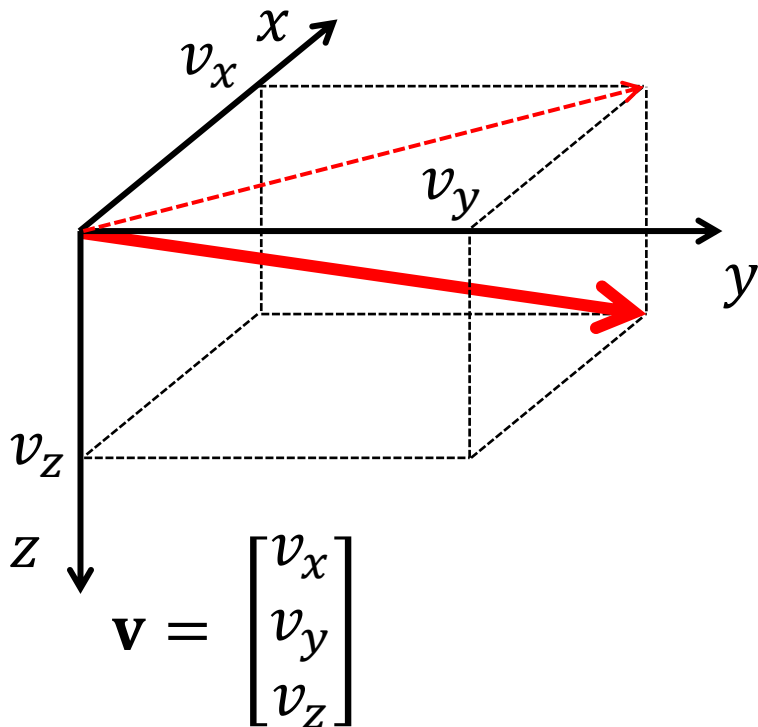
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Anomalia de campo total



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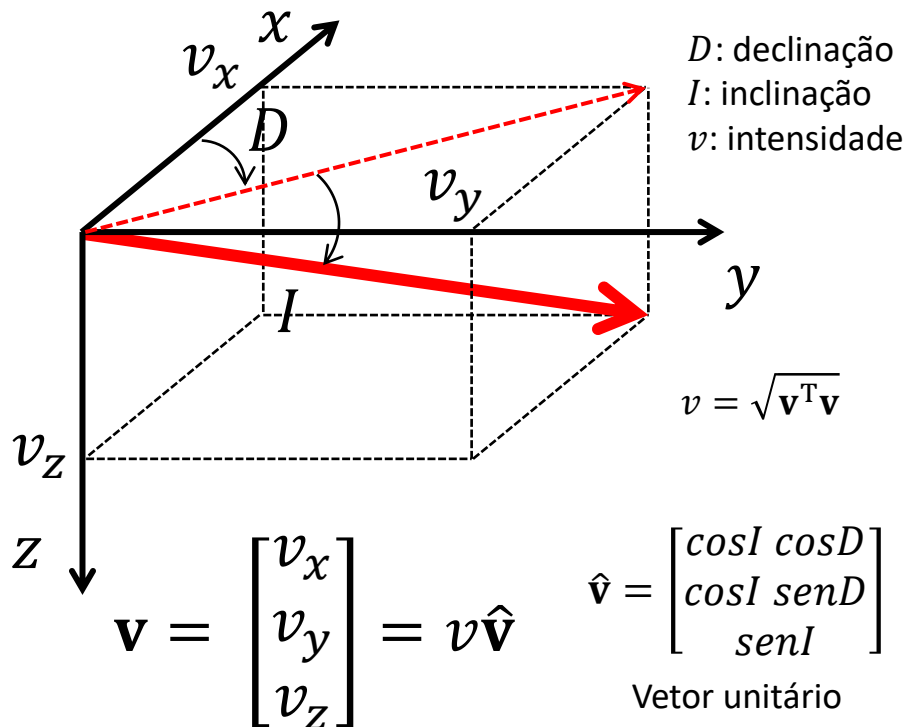
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Condição observada  
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Anomalia de campo total



$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_i$$

Campo principal  
(por exemplo, IGRF)

$$\mathbf{B}_i = B_i \hat{\mathbf{B}}_i$$

Campo crustal

$$\mathbf{T}_i = \mathbf{F}_i + \mathbf{B}_i$$

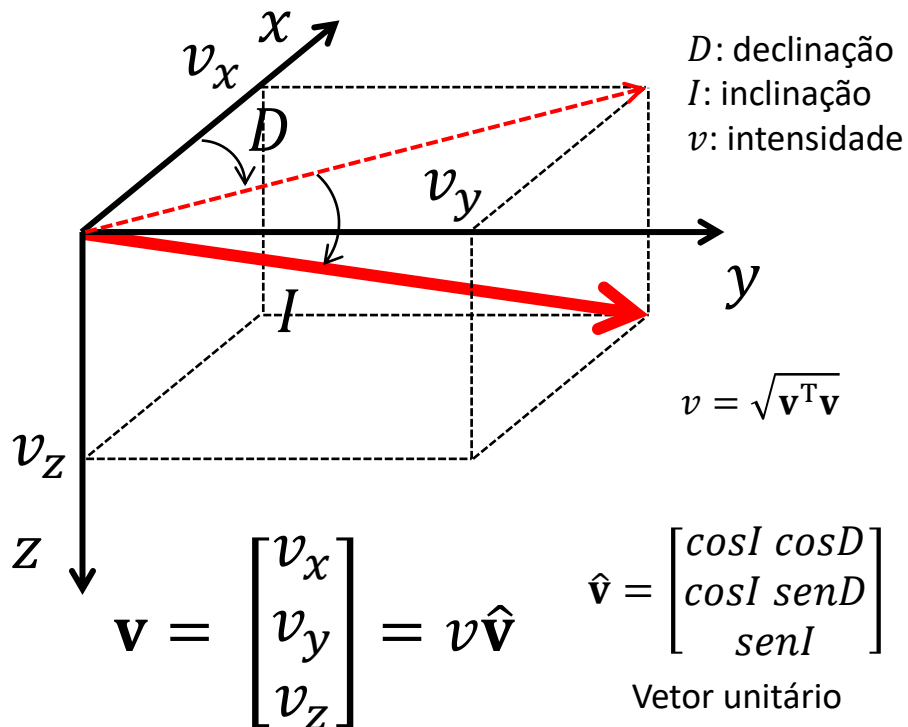
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Anomalia de campo total



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Campo principal  
(por exemplo, IGRF)

$$\mathbf{B}_i = B_i \hat{\mathbf{B}}_i$$

Campo crustal

$$\mathbf{B}_i = \sum_j \mathbf{b}_i^j$$

Soma do campo  
produzido por todas as  
rochas magnetizadas

$$\mathbf{T}_i = \mathbf{F}_i + \mathbf{B}_i$$

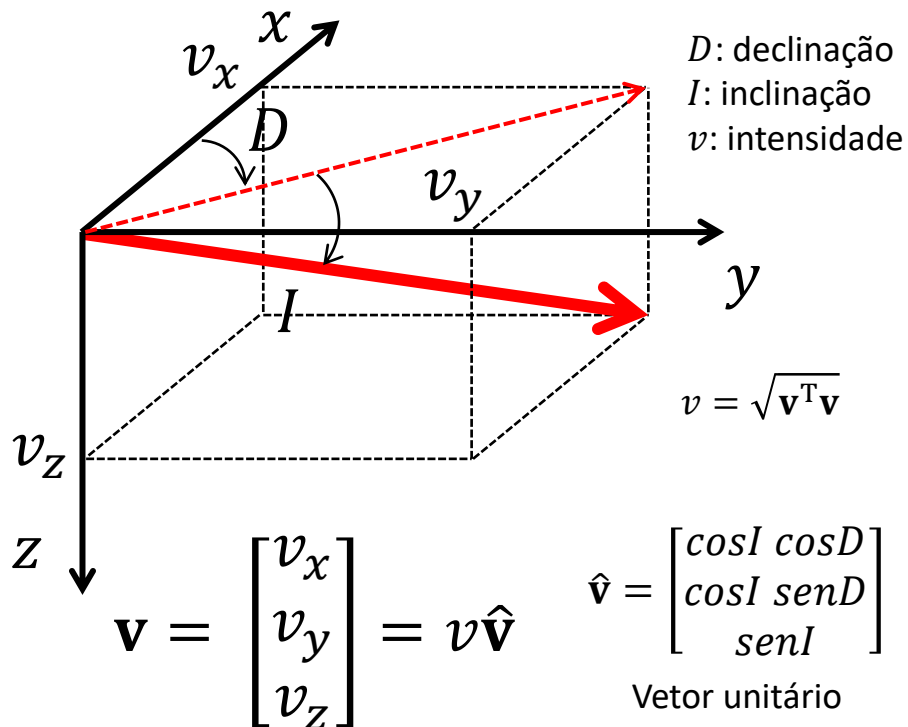
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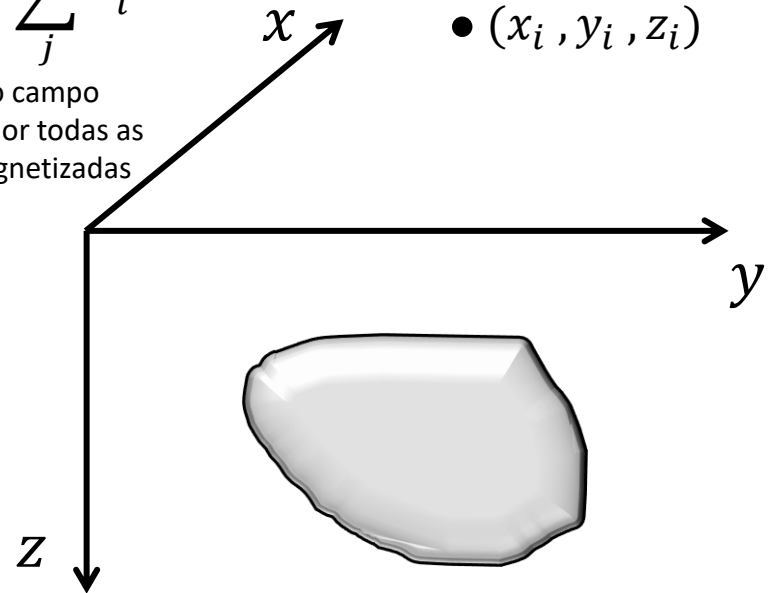
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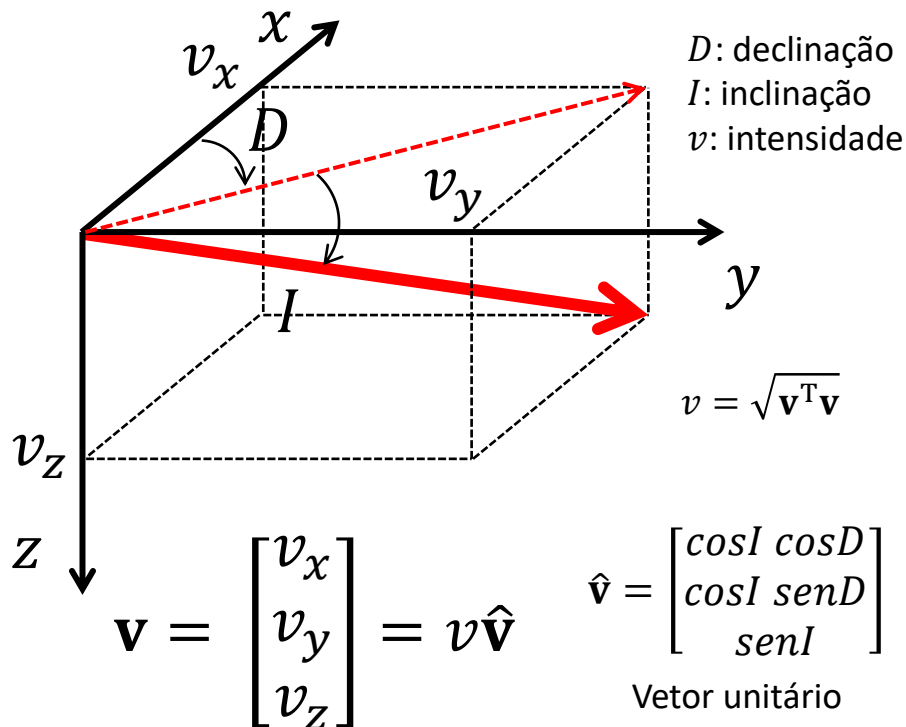
Condição observada  
na prática

$$\Delta T_i = T_i - F_i$$

Anomalia de campo total



Considere o campo produzido  
pela  $j$ -ésima fonte magnética na  
posição  $(x_i, y_i, z_i)$





$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_i$$

Campo principal  
(por exemplo, IGRF)

$$\mathbf{B}_i = B_i \hat{\mathbf{B}}_i$$

Campo crustal

$$\mathbf{B}_i = \sum_j \mathbf{b}_i^j$$

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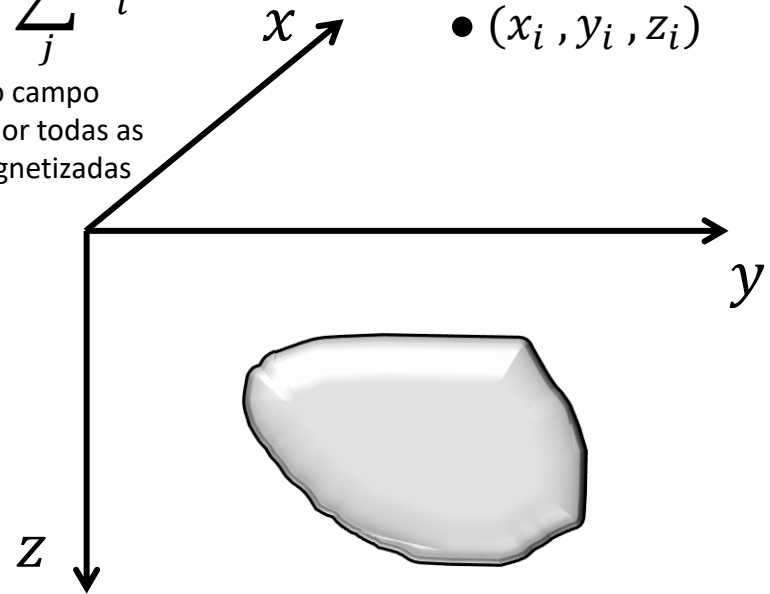
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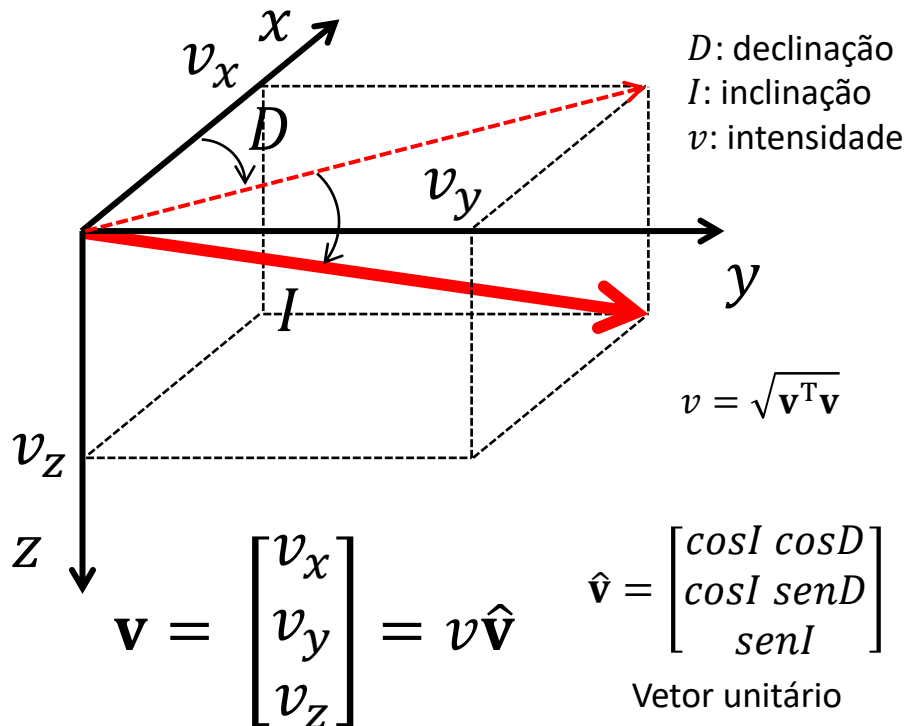
Condição observada  
na prática

$$\Delta T_i = T_i - F_i$$

Anomalia de campo total



A indução magnética  $\mathbf{b}_i^j$  é  
uma integral avaliada no  
volume da j-ésima fonte  
magnética



$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_i$$

Campo principal  
(por exemplo, IGRF)

$$\mathbf{B}_i = B_i \hat{\mathbf{B}}_i$$

Campo crustal

$$\mathbf{B}_i = \sum_j \mathbf{b}_i^j$$

Soma do campo  
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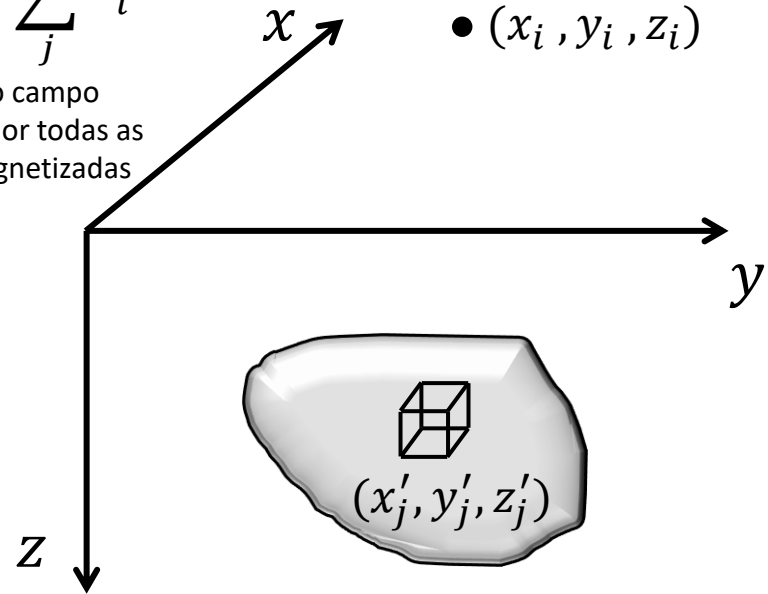
Campo total

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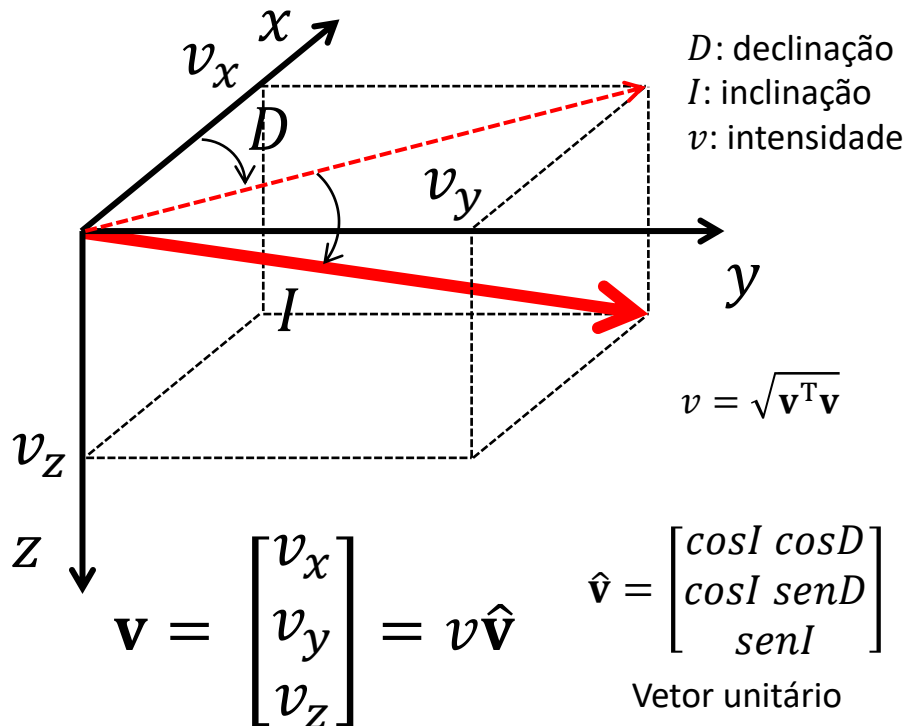
Condição observada  
na prática

$$\Delta T_i = T_i - F_i$$

Anomalia de campo total



$(x', y', z')$  são as coordenadas de  
um elemento de volume  $dx dy dz$   
dentro da  $j$ -ésima fonte



$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_i$$

Campo principal  
(por exemplo, IGRF)

$$\mathbf{B}_i = B_i \hat{\mathbf{B}}_i$$

Campo crustal

$$\mathbf{B}_i = \sum_j \mathbf{b}_i^j$$

Soma do campo  
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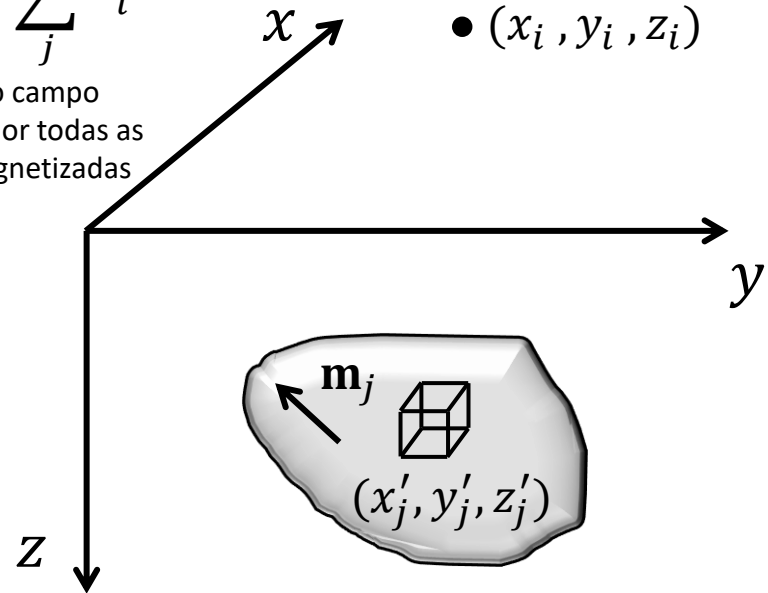
Campo total

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Condição observada  
na prática

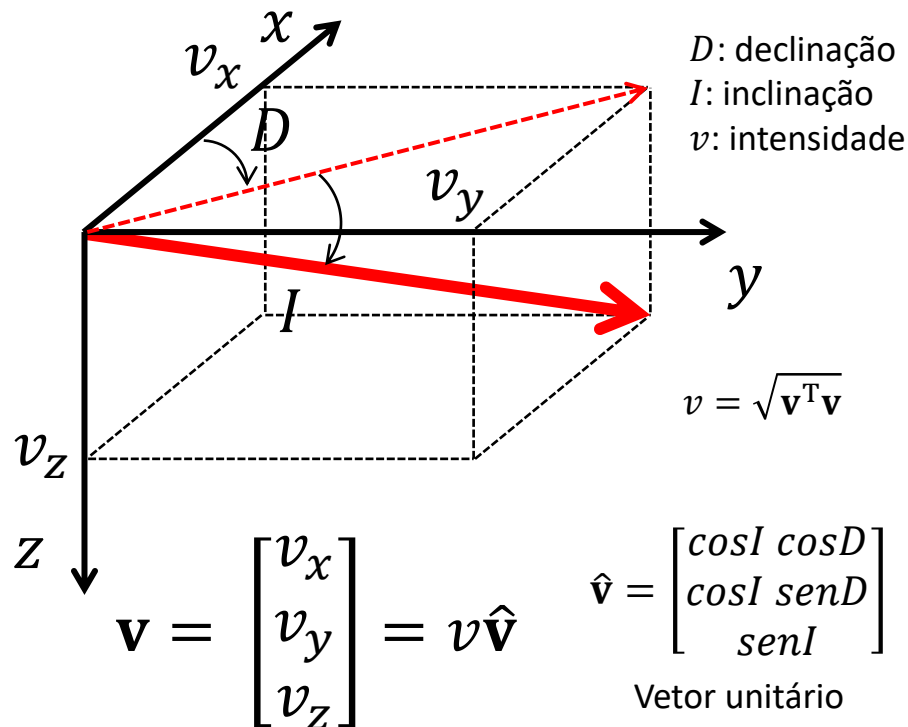
$$\Delta T_i = T_i - F_i$$

Anomalia de campo total



$(x', y', z')$  são as coordenadas de  
um elemento de volume  $dx dy dz$   
dentro da j-ésima fonte

Considere uma que a j-ésima  
fonte tenha magnetização  
constante  $\mathbf{m}_j = m_j \hat{\mathbf{m}}_j$



$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_i$$

Campo principal  
(por exemplo, IGRF)

$$\mathbf{B}_i = B_i \hat{\mathbf{B}}_i$$

Campo crustal

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$$\mathbf{T}_i = \mathbf{F}_i + \mathbf{B}_i$$

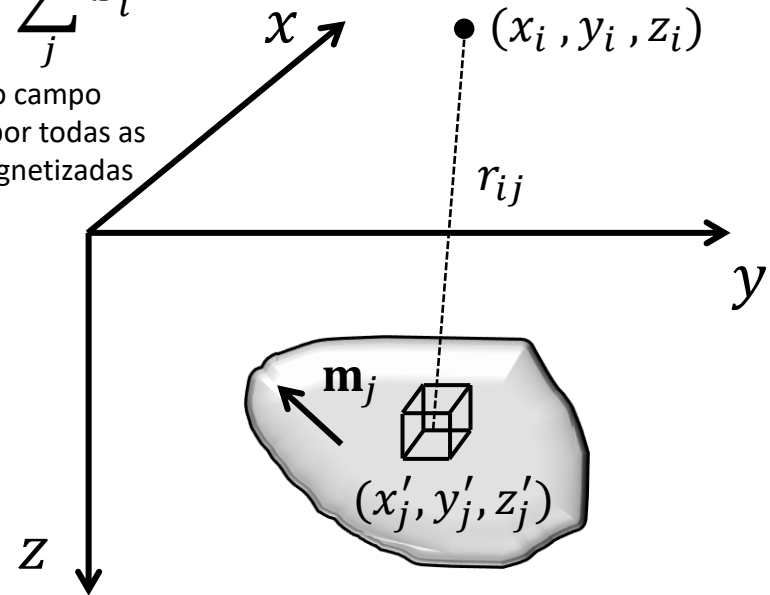
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$$\mathbf{B}_i = \sum_j \mathbf{b}_i^j$$

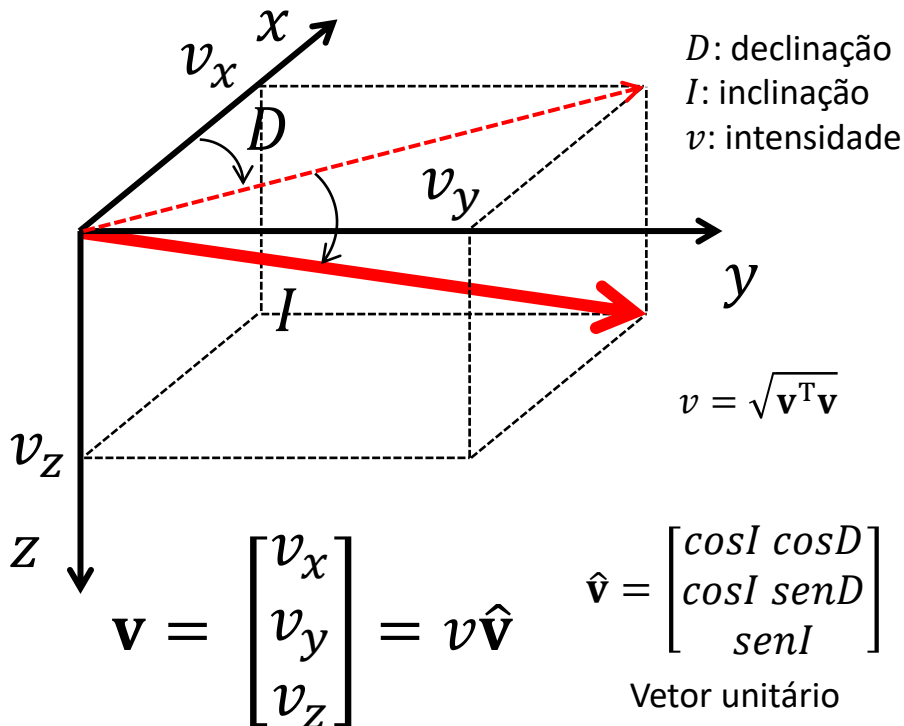
Soma do campo  
produzido por todas as  
rochas magnetizadas



$$\Phi = \iiint \frac{1}{r} dx dy dz$$

A integral é avaliada  
no volume da fonte

$$r = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$$



$$\hat{\mathbf{v}} = \begin{bmatrix} \cos I \cos D \\ \cos I \sin D \\ \sin I \end{bmatrix}$$

Vetor unitário

$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_i$$

Campo principal  
(por exemplo, IGRF)

$$\mathbf{B}_i = B_i \hat{\mathbf{B}}_i$$

Campo crustal

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Condição observada  
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$$\mathbf{T}_i = \mathbf{F}_i + \mathbf{B}_i$$

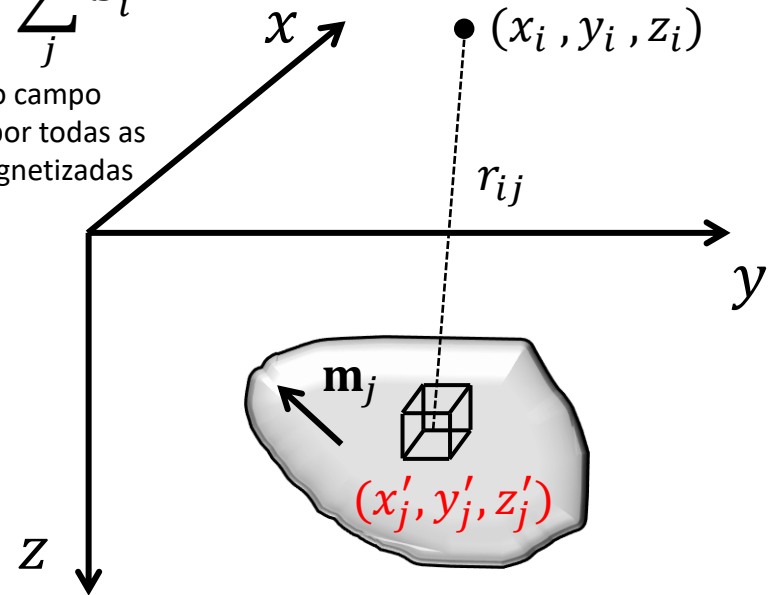
Campo total

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$$\mathbf{B}_i = \sum_j \mathbf{b}_i^j$$

Soma do campo  
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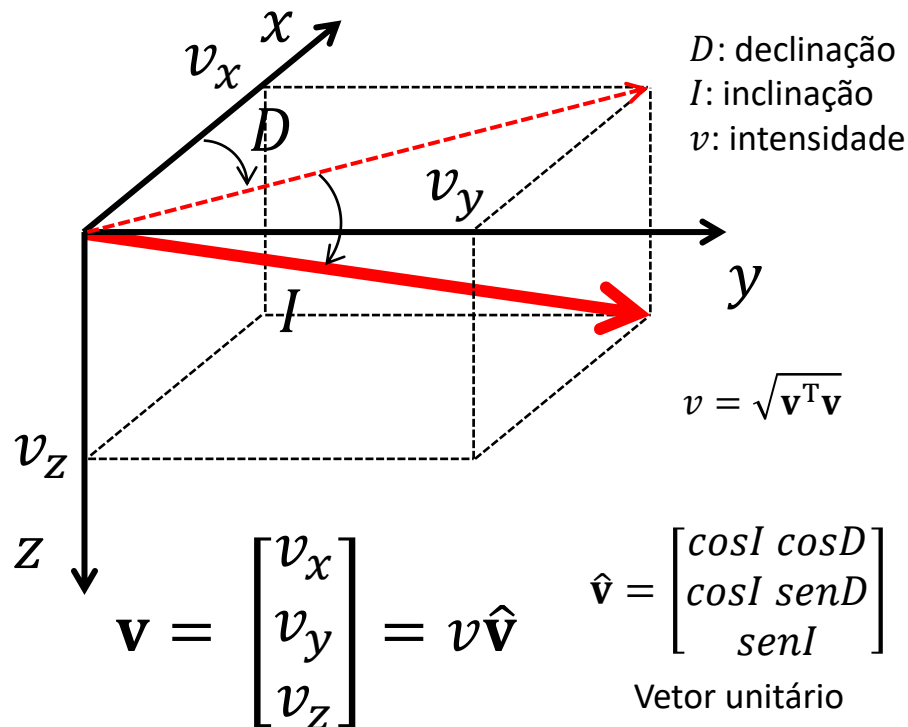


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Variáveis de integração



$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_i$$

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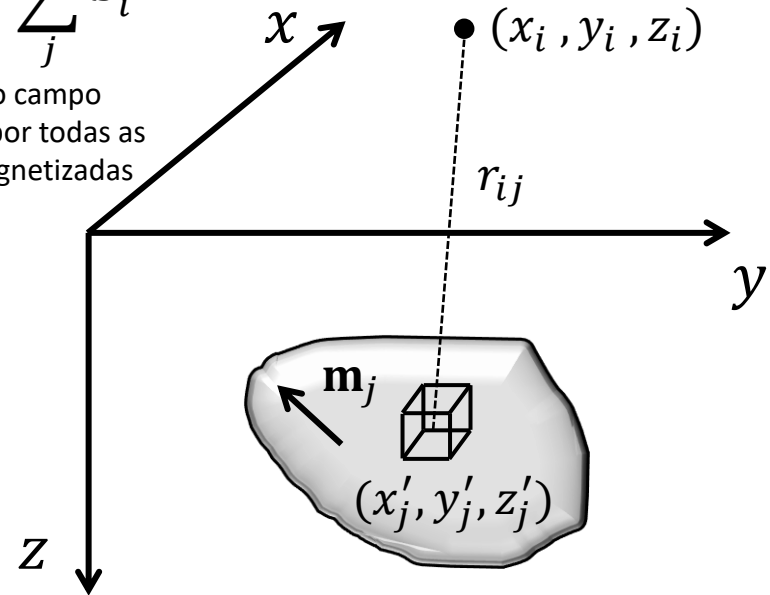
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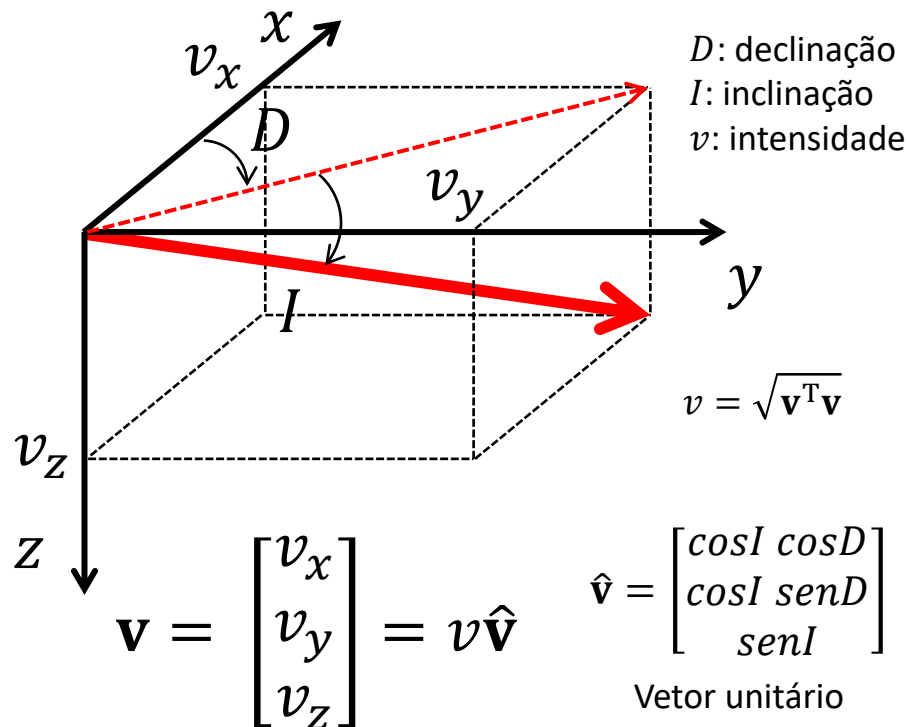


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$$\mathbf{b}_i^j = \kappa_m m_j \mathbf{M}_i^j \hat{\mathbf{m}}_j \quad \kappa_m = 10^9 \frac{\mu_0}{4\pi}$$



$$\mathbf{M}_i^j = \begin{bmatrix} \partial_{xx} \Phi_i^j & \partial_{xy} \Phi_i^j & \partial_{xz} \Phi_i^j \\ \partial_{xy} \Phi_i^j & \partial_{yy} \Phi_i^j & \partial_{yz} \Phi_i^j \\ \partial_{xz} \Phi_i^j & \partial_{yz} \Phi_i^j & \partial_{zz} \Phi_i^j \end{bmatrix}$$

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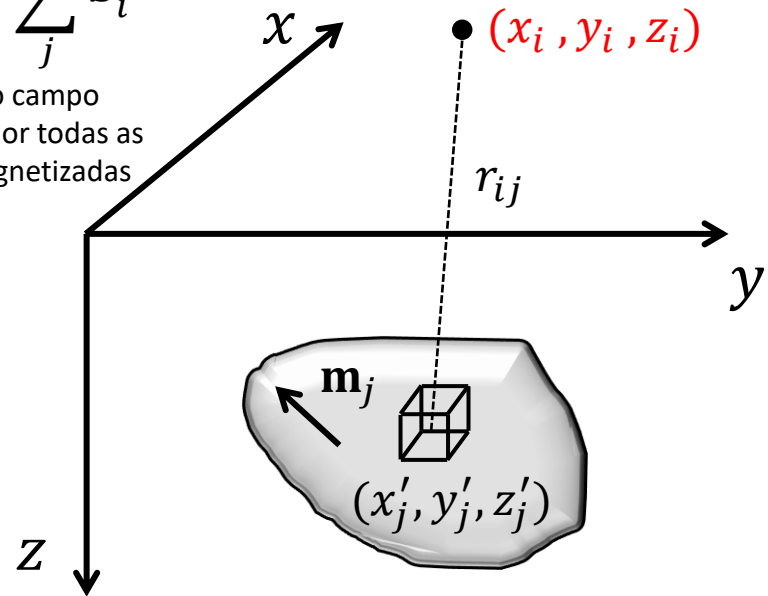
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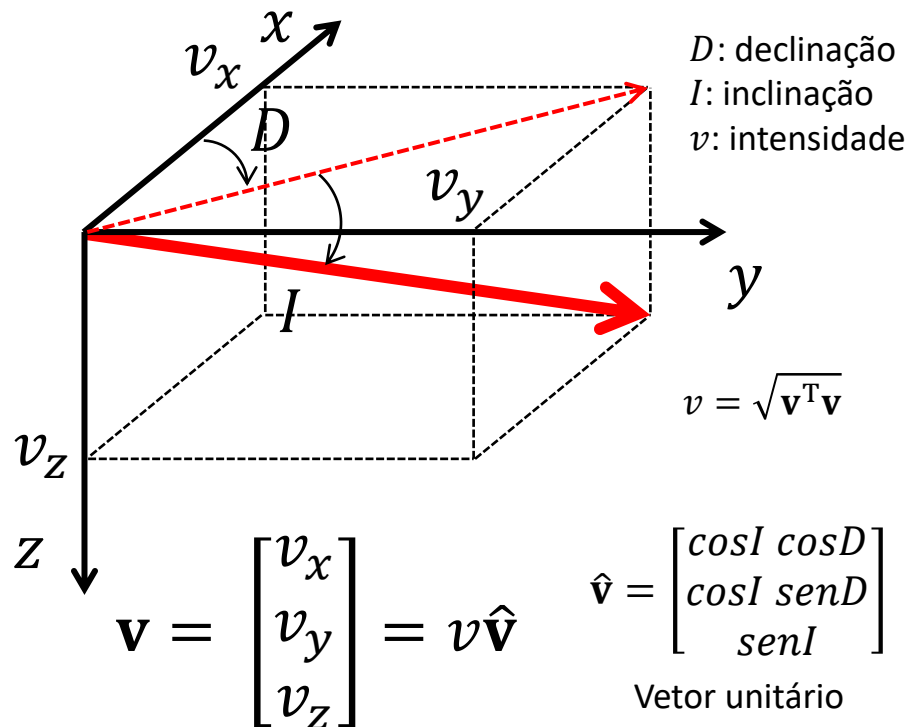


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As derivadas são  
calculadas em  
relação às  
coordenadas do  
ponto de  
observação

$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_i$$

Campo principal  
(por exemplo, IGRF)

$$\mathbf{B}_i = B_i \hat{\mathbf{B}}_i$$

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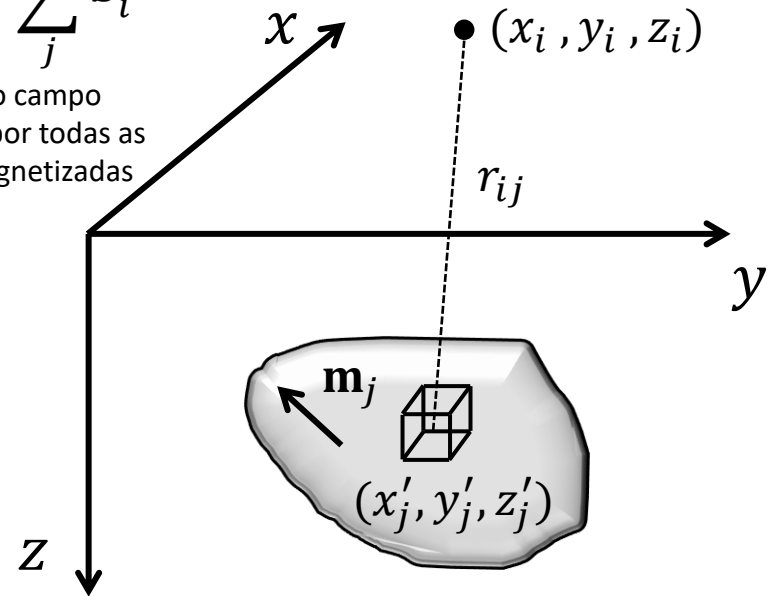
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Anomalia de campo total

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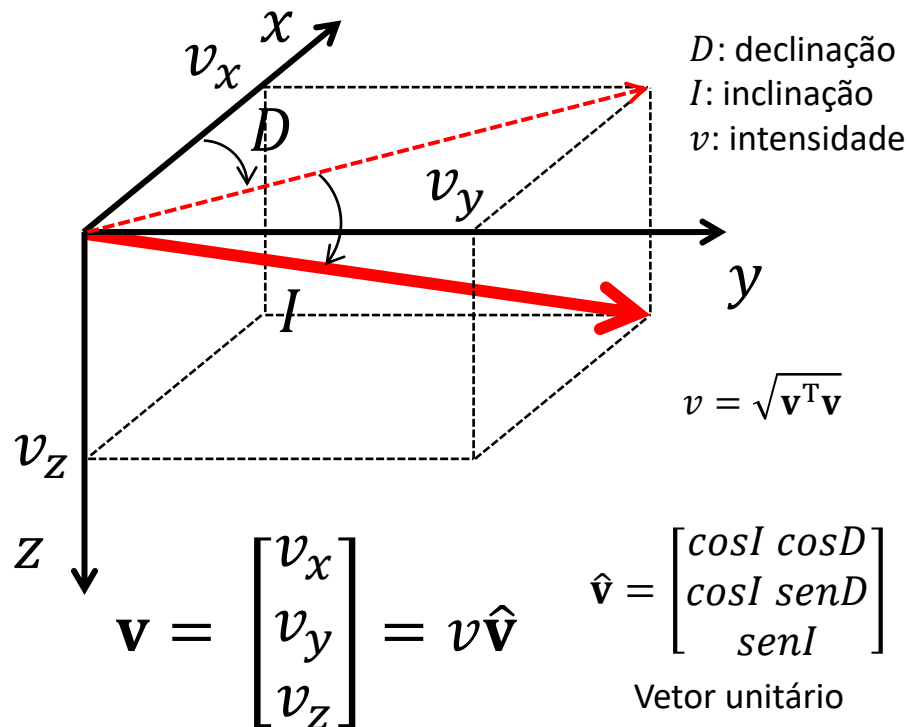
A integral é avaliada  
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$$r = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$$

$$\mathbf{b}_i^j = \kappa_m m_j \mathbf{M}_i^j \hat{\mathbf{m}}_j \quad \kappa_m = 10^9 \frac{\mu_0}{4\pi}$$

Calcule os elementos desta matriz

$$\mathbf{M}_i^j = \begin{bmatrix} \partial_{xx} \Phi_i^j & \partial_{xy} \Phi_i^j & \partial_{xz} \Phi_i^j \\ \partial_{xy} \Phi_i^j & \partial_{yy} \Phi_i^j & \partial_{yz} \Phi_i^j \\ \partial_{xz} \Phi_i^j & \partial_{yz} \Phi_i^j & \partial_{zz} \Phi_i^j \end{bmatrix}$$





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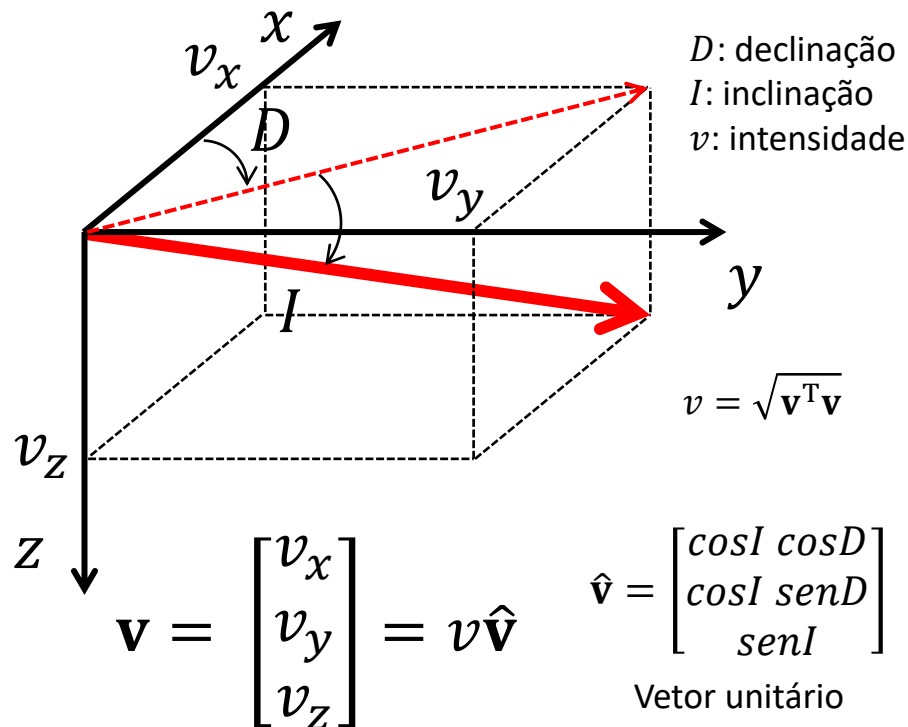
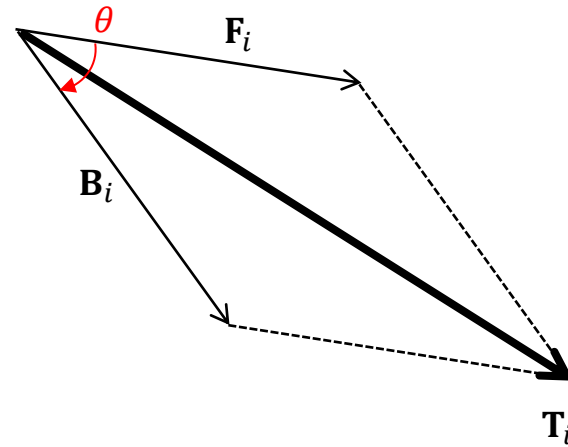
Condição observada  
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$$\mathbf{T}_i = \mathbf{F}_i + \mathbf{B}_i$$

Campo total

$$\Delta T_i = T_i - F_i$$

Anomalia de campo total



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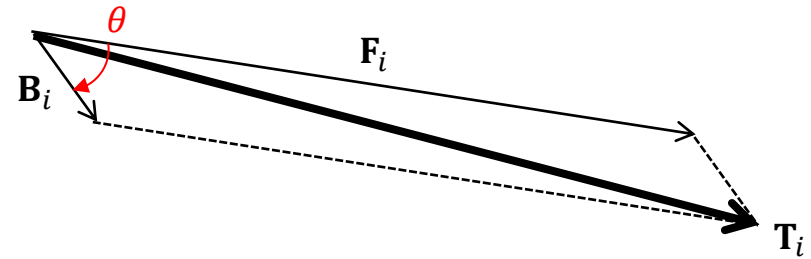
Campo crustal

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Campo total

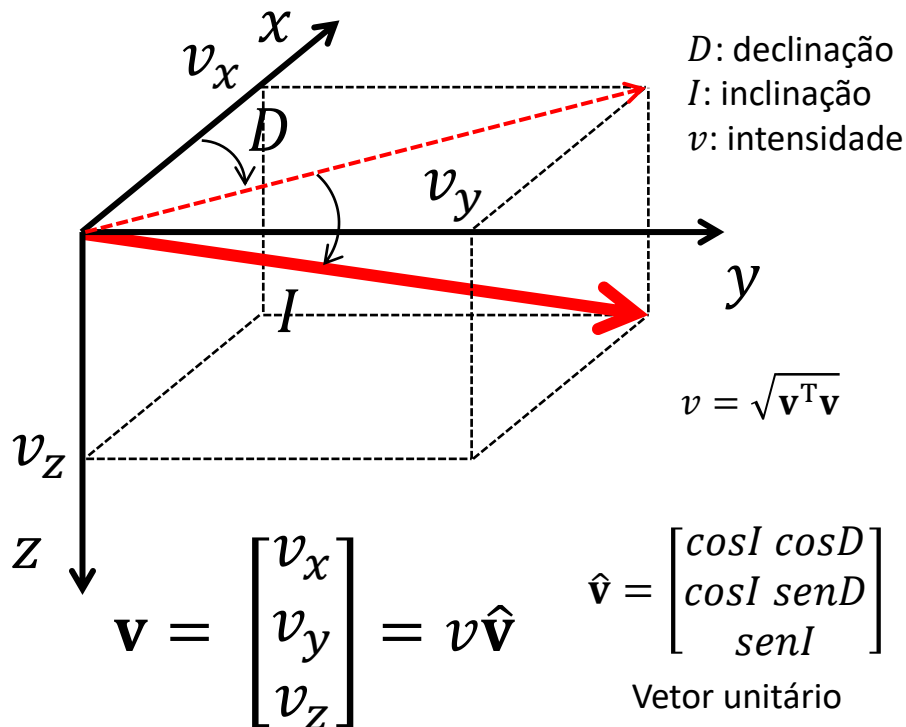
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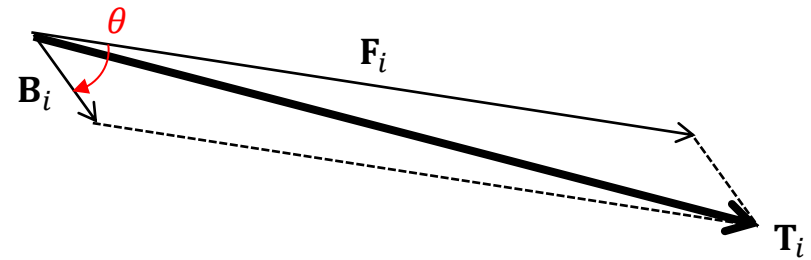
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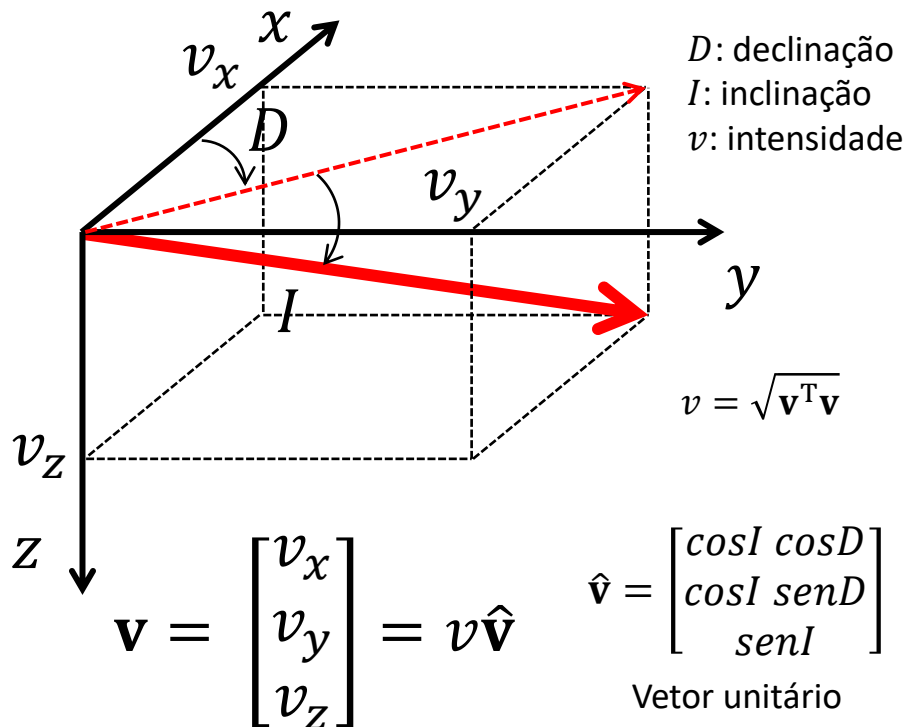
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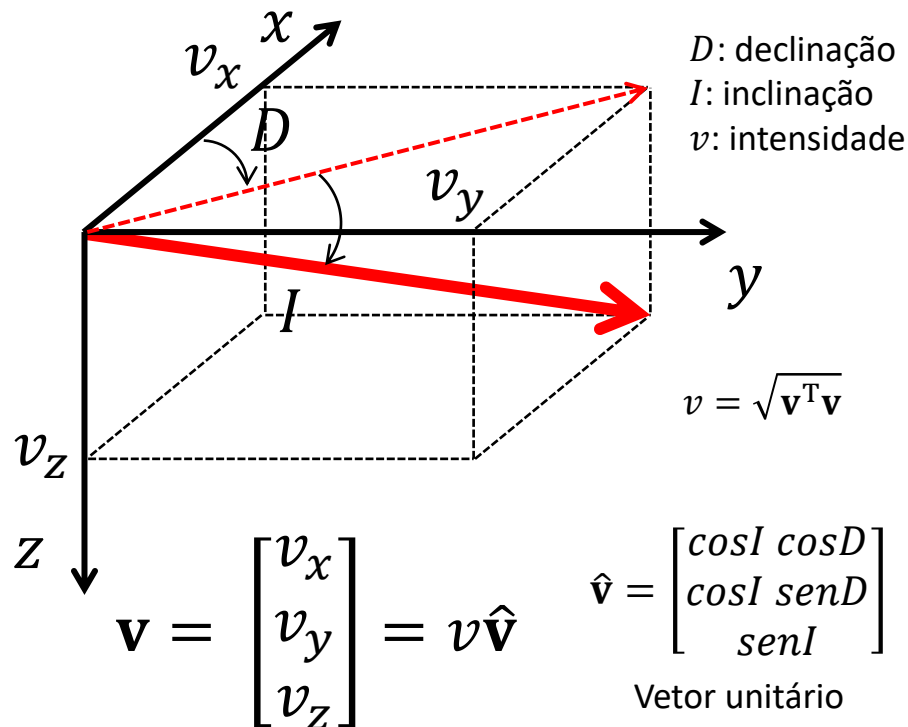
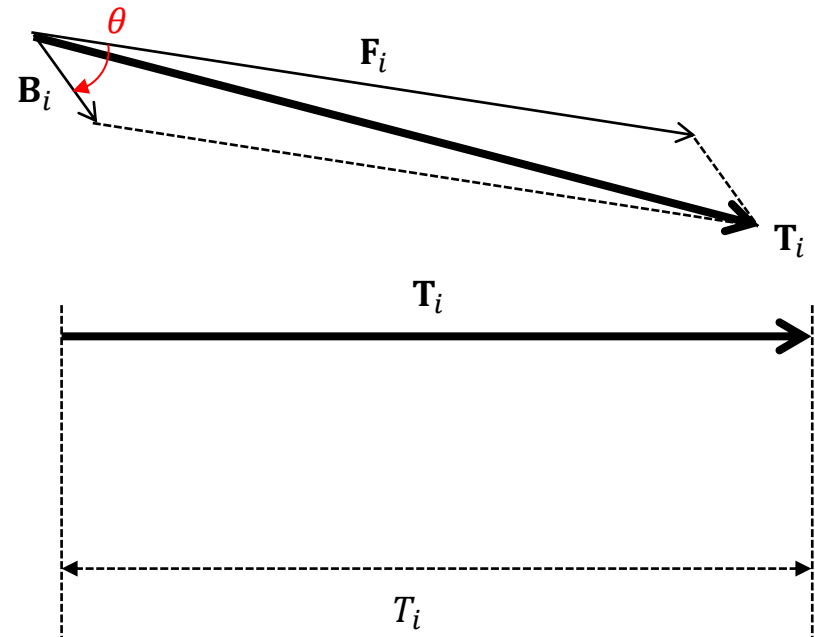
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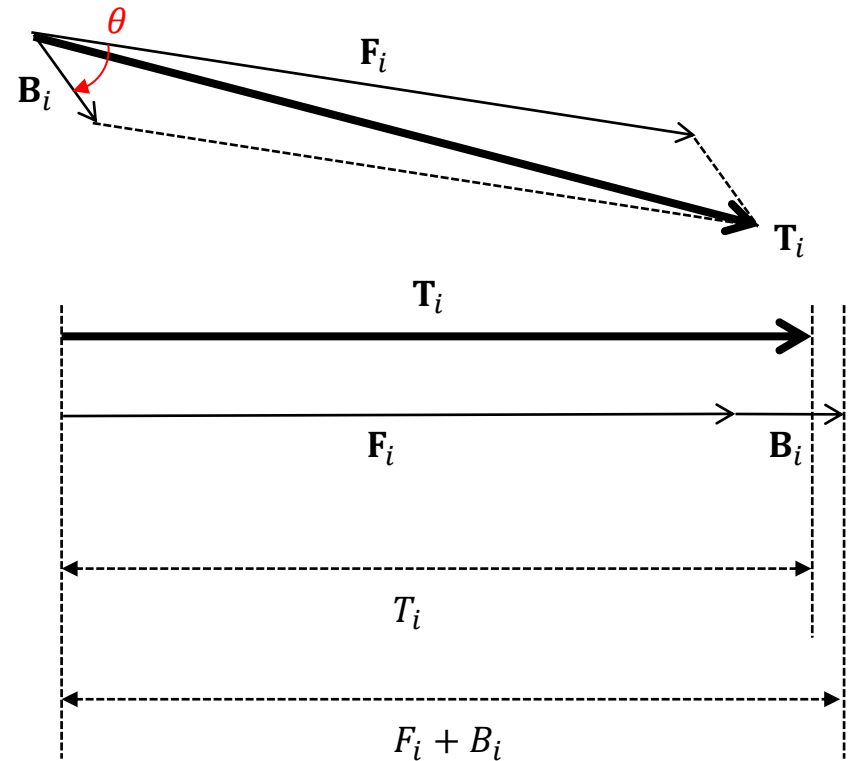
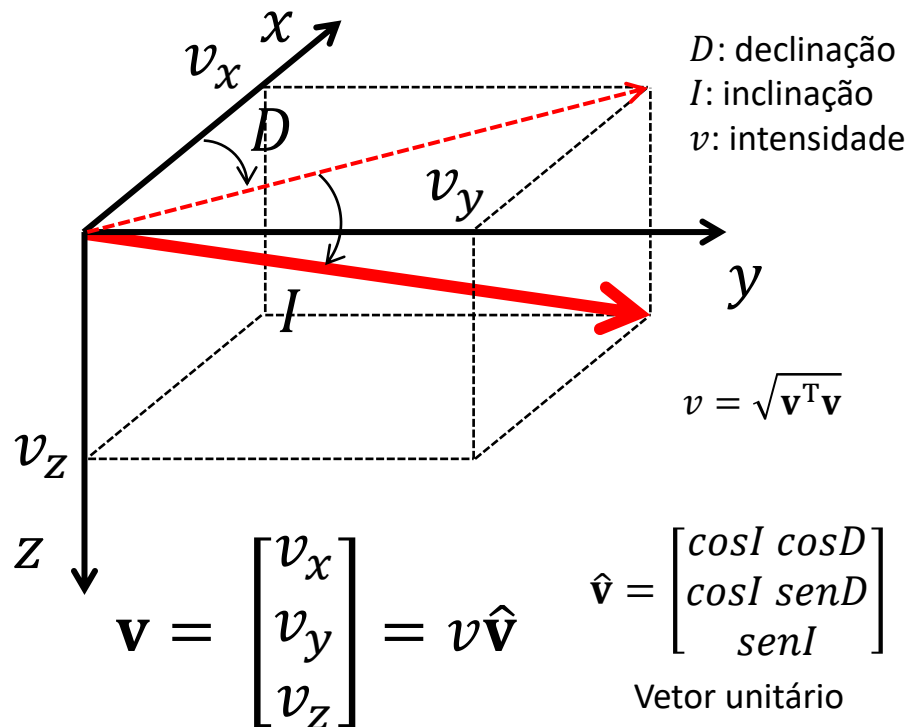
Campo total

$$F_i \gg B_i$$

Condição observada  
na prática

$$\Delta T_i = T_i - F_i$$

Anomalia de campo total



$$T_i \neq F_i + B_i$$

Direção constante

$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_0$$

Campo principal  
(por exemplo, IGRF)

$$\mathbf{B}_i = B_i \hat{\mathbf{B}}_i$$

Campo crustal

$$\mathbf{T}_i = \mathbf{F}_i + \mathbf{B}_i$$

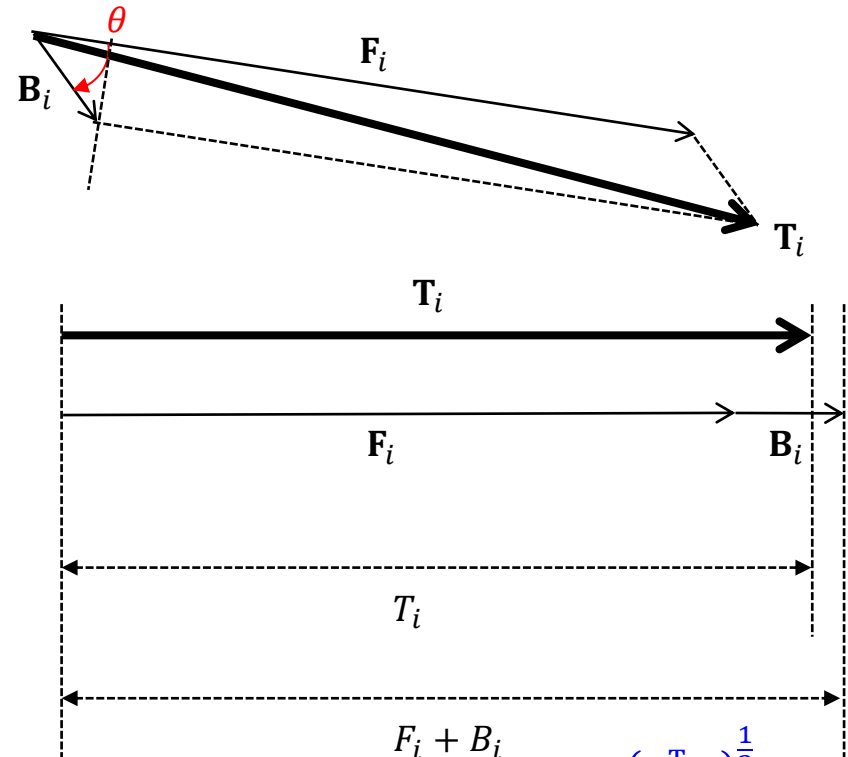
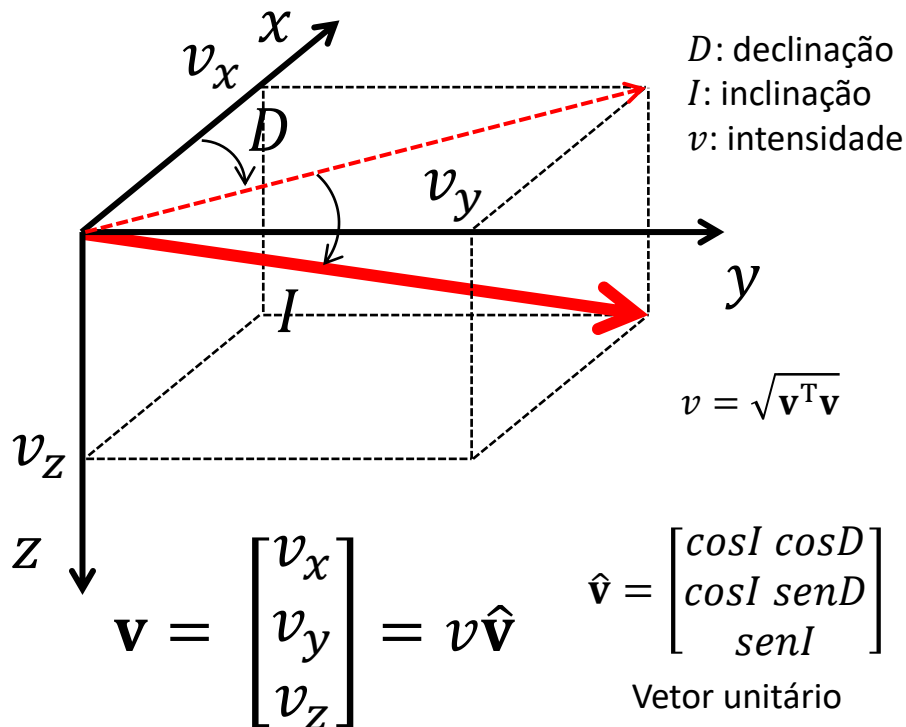
Campo total

$$F_i \gg B_i$$

Condição observada  
na prática

$$\Delta T_i = T_i - F_i$$

Anomalia de campo total



$$T_i \neq F_i + B_i$$

$$T_i = [(\mathbf{F}_i + \mathbf{B}_i)^T (\mathbf{F}_i + \mathbf{B}_i)]^{\frac{1}{2}}$$

$$T_i \approx (\mathbf{F}_i^T \mathbf{F}_i)^{\frac{1}{2}} + \frac{\mathbf{F}_i^T \mathbf{B}_i}{(\mathbf{F}_i^T \mathbf{F}_i)^{\frac{1}{2}}}$$

Direção constante

$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_0$$

Campo principal  
(por exemplo, IGRF)

$$\mathbf{B}_i = B_i \hat{\mathbf{B}}_i$$

Campo crustal

$$\mathbf{T}_i = \mathbf{F}_i + \mathbf{B}_i$$

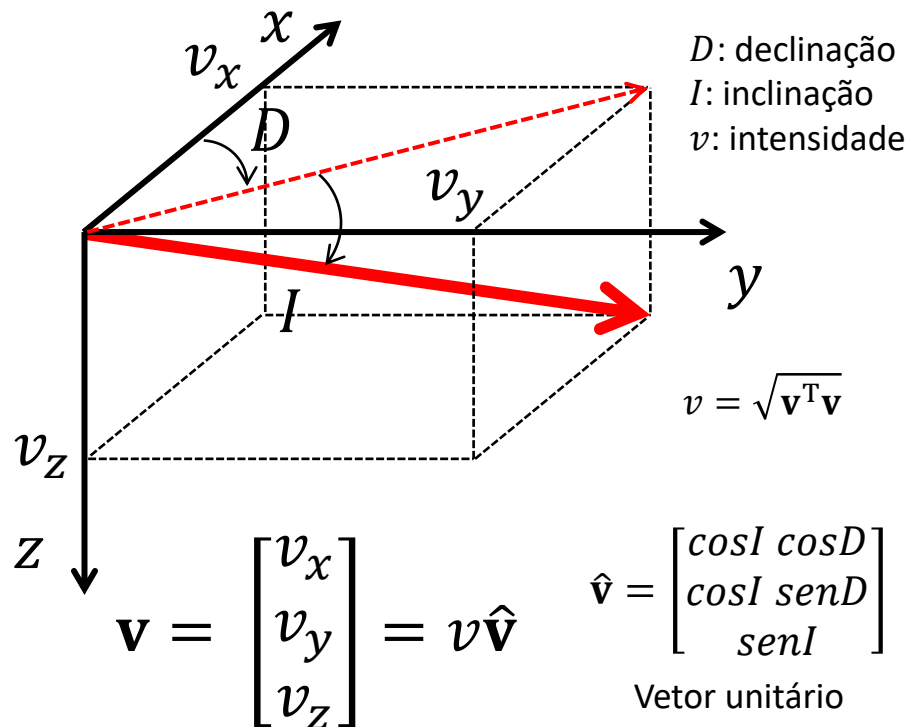
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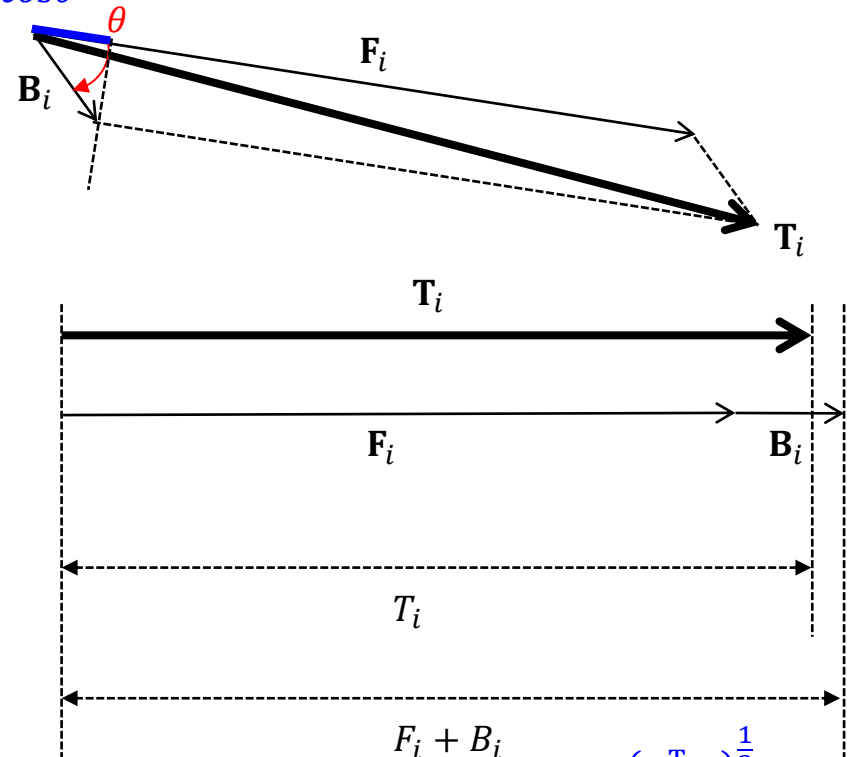
Condição observada  
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$$\Delta T_i = T_i - F_i$$

Anomalia de campo total



$B_i \cos \theta$



$$T_i \neq F_i + B_i$$

$$T_i = [(\mathbf{F}_i + \mathbf{B}_i)^T (\mathbf{F}_i + \mathbf{B}_i)]^{\frac{1}{2}}$$

$$T_i \approx (\mathbf{F}_i^T \mathbf{F}_i)^{\frac{1}{2}} + \frac{\mathbf{F}_i^T \mathbf{B}_i}{(\mathbf{F}_i^T \mathbf{F}_i)^{\frac{1}{2}}}$$

Produto escalar

$$\mathbf{v}^T \mathbf{w} = v_x w_x + v_y w_y + v_z w_z$$

$$= v w \cos \theta$$

$$T_i \approx F_i + \hat{\mathbf{F}}_0^T \mathbf{B}_i$$

Direção constante

$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_0$$

Campo principal  
(por exemplo, IGRF)

$$\mathbf{B}_i = B_i \hat{\mathbf{B}}_i$$

Campo crustal

$$\mathbf{T}_i = \mathbf{F}_i + \mathbf{B}_i$$

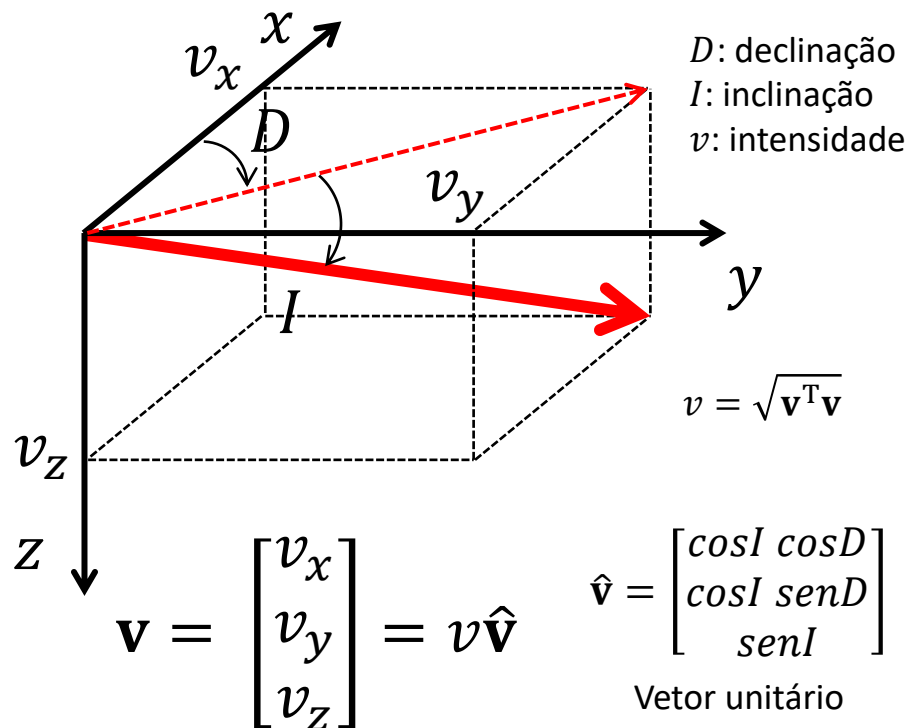
Campo total

$$F_i \gg B_i$$

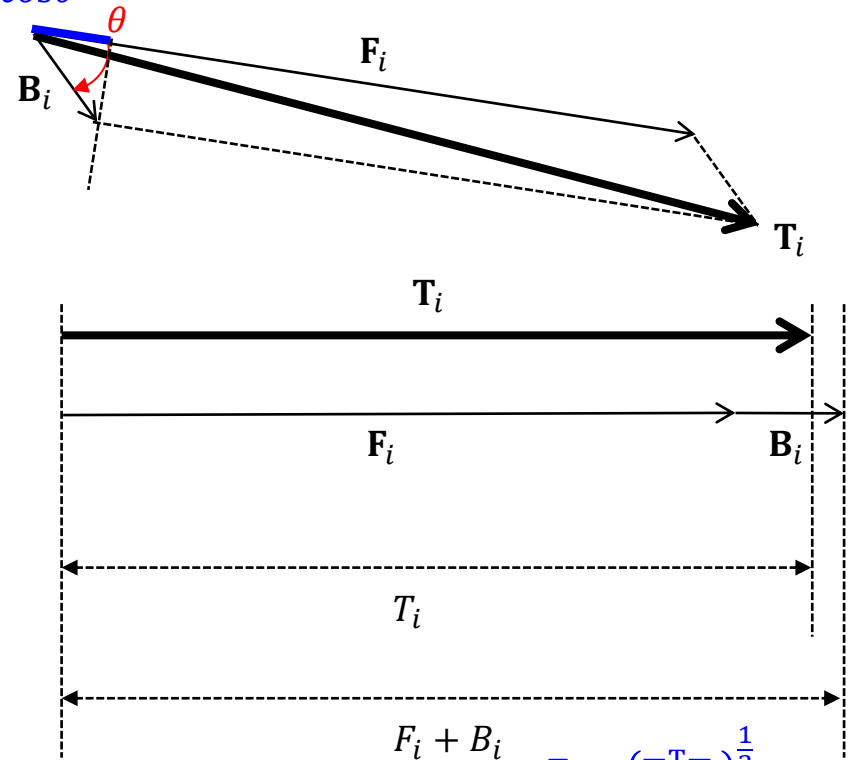
Condição observada  
na prática

$$\Delta T_i = T_i - F_i$$

Anomalia de campo total



$B_i \cos \theta$



Produto escalar

$$\mathbf{v}^T \mathbf{w} = v_x w_x + v_y w_y + v_z w_z$$

$$= v w \cos \theta$$

$$T_i \approx F_i + \hat{\mathbf{F}}_0^T \mathbf{B}_i$$

$$\Delta T_i \approx \hat{\mathbf{F}}_0^T \mathbf{B}_i$$

Blakely (1996)  
Langel e Hinze (1998)



# Referências

- Blakely, R. J., 1996, Potential theory in gravity and magnetic applications: Cambridge University Press.
- Langel, R. A., e Hinze, W. J., 1998, The magnetic field of the earth's lithosphere: The satellite perspective: Cambridge University Press.