



Camada equivalente aplicada ao processamento e interpretação de dados de campos potenciais

Vanderlei C. Oliveira Jr.



2016







Distúrbio de gravidade (parte B)

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2016



$$\delta \mathbf{g}_P = \mathbf{g}_P - \mathbf{\gamma}_I$$

$$\delta \mathbf{g}_P = \mathbf{g}_P - \mathbf{\gamma}_P \quad \Delta \mathbf{g}_P = \mathbf{g}_0 - \mathbf{\gamma}_Q$$

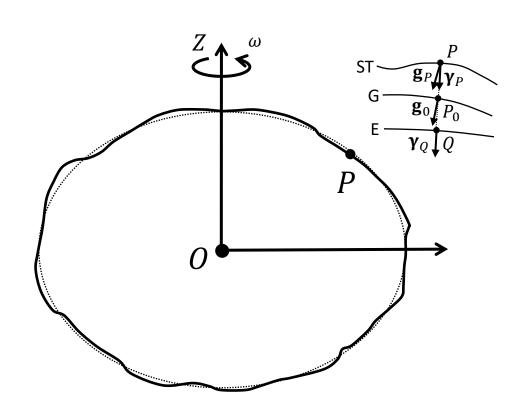
Vetor anomalia de gravidade

$$\delta g_P = g_P - \gamma_P \quad \Delta g_P = g_0 - \gamma_Q$$

Distúrbio de gravidade

$$\Delta g_P = g_0 - \gamma_Q$$

Anomalia de gravidade



$$\mathbf{\delta g}_P = \mathbf{g}_P - \mathbf{\gamma}_P$$

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Distúrbio de gravidade

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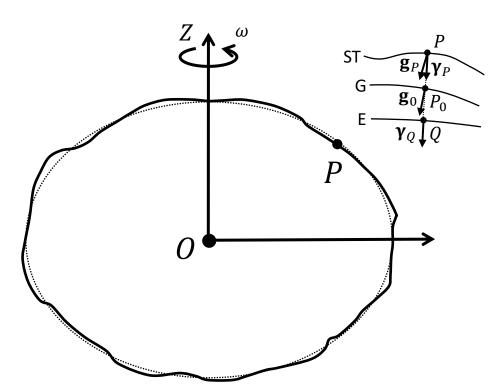
Anomalia de gravidade

Vetor gravidade normal

$$\mathbf{\gamma}_P = \nabla \widetilde{W}_P$$
$$= \nabla U_P + \nabla \Phi_P$$

Vetor gravidade

$$\mathbf{g}_P = \nabla W_P$$
$$= \nabla V_P + \nabla \Phi_P$$



$$\mathbf{\delta g}_P = \mathbf{g}_P - \mathbf{\gamma}_P \quad \mathbf{\Delta g}$$

$$\Delta \mathbf{g}_P = \mathbf{g}_0 - \mathbf{\gamma}_Q$$

Vetor anomalia de gravidade

$$\delta g_P = g_P - \gamma_P$$

Distúrbio de gravidade

$$\delta g_P = g_P - \gamma_P \quad \Delta g_P = g_0 - \gamma_Q$$

Anomalia de gravidade

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Potencial de gravidade normal

$$\widetilde{W}_P = U_P + \Phi_P$$

Potencial de gravidade

$$W_P = V_P + \Phi_P$$

Potencial centrífugo

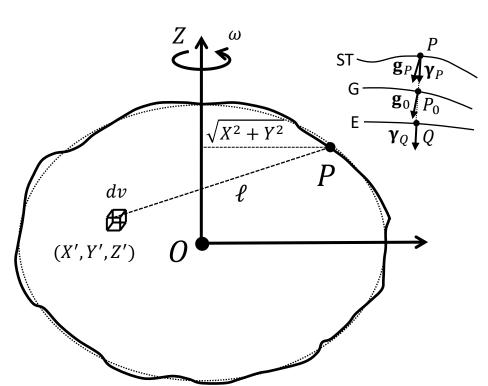
$$\Phi_P = \frac{1}{2}\omega^2(X^2 + Y^2)$$

Potencial gravitacional normal

$$U_P = G \iiint \frac{\tilde{\rho}}{\ell} dv$$

Potencial gravitacional

$$V_P = G \iiint \frac{\rho}{\ell} dv$$



$$\Delta g_P = g_0 - \gamma_Q$$

Anomalia de gravidade

$$U_P = G \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\tilde{\rho}}{\ell} dv$$

Considere que ρ se anula fora do volume da Terra

Considere que $\tilde{\rho}$ se anula fora do volume da Terra Normal

$$V_P = G \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\rho}{\ell} dv$$

Vetor gravidade normal

$$\mathbf{\gamma}_P = \nabla \widetilde{W}_P$$
$$= \nabla U_P + \nabla \Phi_P$$

gravidade

 $\delta g_P = g_P - \gamma_P$

Distúrbio de

gravidade

Vetor gravidade

$$= \nabla V_P + \nabla \Phi_P$$

 $\mathbf{g}_P = \nabla W_P$

Potencial de gravidade normal

$$\widetilde{W}_P = U_P + \Phi_P \qquad W_P = V_P + \Phi_P$$

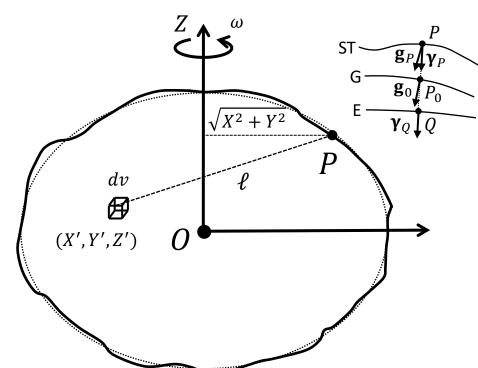
Potencial centrífugo

$$\Phi_P = \frac{1}{2}\omega^2(X^2 + Y^2)$$

Potencial gravitacional Potencial gravitacional normal

$$U_P = G \iiint rac{\widetilde{
ho}}{\ell} dv \qquad \qquad V_P = G$$

 $V_P = G \iiint \frac{\rho}{\rho} dv$



$$\delta \mathbf{g}_P = \mathbf{g}_P - \mathbf{\gamma}_P$$

$$\Delta \mathbf{g}_P = \mathbf{g}_0 - \mathbf{\gamma}_Q$$

Vetor anomalia de gravidade

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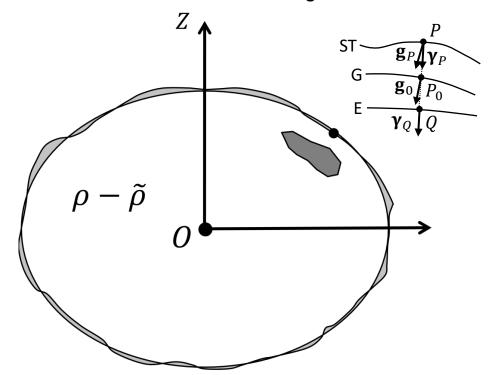
$$U_P = G \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\tilde{\rho}}{\ell} dv$$

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Considere que $\tilde{\rho}$ se anula fora do volume da Terra Normal

$$V_P = G \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\rho}{\ell} dv$$

$$\mathbf{\delta g}_{P} = G \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (\rho - \tilde{\rho}) \nabla \frac{1}{\ell} d\nu$$



$$\delta \mathbf{g}_P = \mathbf{g}_P - \mathbf{\gamma}_P$$

$$\Delta \mathbf{g}_P = \mathbf{g}_0 - \mathbf{\gamma}_Q$$

Vetor anomalia de gravidade

$$\delta g_P = g_P - \gamma_P$$

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$$\Phi_P = \frac{1}{2}\omega^2(X^2 + Y^2)$$

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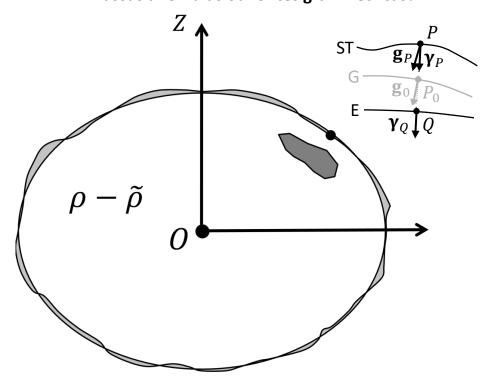
$$U_P = G \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\tilde{\rho}}{\ell} dv$$

Considere que ρ se anula fora do volume da Terra

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$$V_P = G \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\rho}{\ell} dv$$

$$\mathbf{\delta g}_{P} = G \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (\rho - \tilde{\rho}) \nabla \frac{1}{\ell} d\nu$$



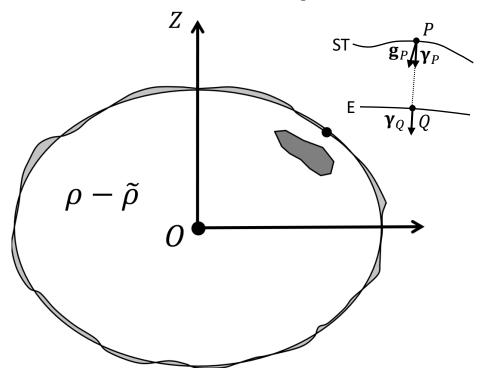
$$\delta \mathbf{g}_P = \mathbf{g}_P - \mathbf{\gamma}_P$$

$$\mathbf{g}_P = \mathbf{\gamma}_P + \mathbf{\delta}\mathbf{g}_P$$

$$\delta g_P = g_P - \gamma_P$$
Distúrbio de gravidade

$$\gamma_P\gg \|\mathbf{\delta g}_P\|$$
 Condição observada na prática

$$\mathbf{\delta g}_{P} = G \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (\rho - \tilde{\rho}) \nabla \frac{1}{\ell} dv$$



$$\delta \mathbf{g}_P = \mathbf{g}_P - \mathbf{\gamma}_P$$

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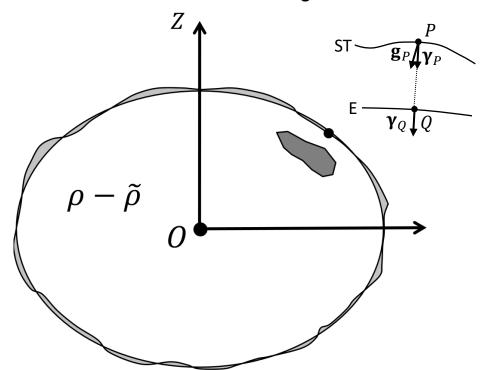
$$\delta g_P = g_P - \gamma_P$$
Distúrbio de gravidade

$$\gamma_P\gg \|oldsymbol{\delta g}_P\|$$

Condição observada
na prática

Esta integral pode ser reescrita de tal forma que represente o efeito de cada fonte, separadamente

$$\mathbf{\delta g}_{P} = G \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (\rho - \tilde{\rho}) \nabla \frac{1}{\ell} dv$$



$$\delta \mathbf{g}_P = \mathbf{g}_P - \mathbf{\gamma}_P$$

$$\mathbf{g}_P = \mathbf{\gamma}_P + \mathbf{\delta}\mathbf{g}_P$$

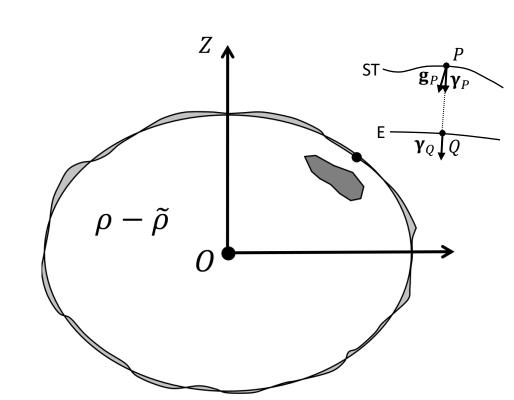
$$\delta g_P = g_P - \gamma_P$$
Distúrbio de gravidade

$$\gamma_P\gg \|oldsymbol{\delta g}_P\|$$

Condição observada
na prática

Neste expressão, considerou-se que cada fonte possui um contraste de densidade $\Delta \rho = \rho - \tilde{\rho}$ constante

$$\delta \mathbf{g}_{P} = G \sum_{j} \Delta \rho_{j} \iiint_{v_{j}} \nabla \frac{1}{\ell} dv$$



$$\delta \mathbf{g}_P = \mathbf{g}_P - \mathbf{\gamma}_P$$

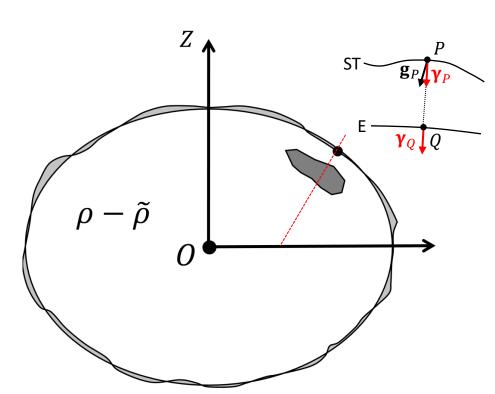
$$\mathbf{g}_P = \mathbf{\gamma}_P + \mathbf{\delta} \mathbf{g}_P$$

$$\delta g_P = g_P - \gamma_P$$
Distúrbio de gravidade

$$\gamma_P\gg \|\mathbf{\delta g}_P\|$$
 Condição observada na prática

Em geral, considera-se que a direção do vetor gravidade normal no ponto
$$P$$
 é igual a direção do vetor

gravidade normal no ponto Q. No sistema de coordenadas geodésicas, esta direção é constante ao longo da normal ao elipsoide



 $\mathbf{\delta g}_{P} = G \sum_{j} \Delta \rho_{j} \iiint_{v_{j}} \nabla \frac{1}{\ell} dv$

$$\delta \mathbf{g}_P = \mathbf{g}_P - \mathbf{\gamma}_P$$

$$\mathbf{g}_P = \mathbf{\gamma}_P + \mathbf{\delta} \mathbf{g}_P$$

$$\delta g_P = g_P - \gamma_P$$
Distúrbio de gravidade

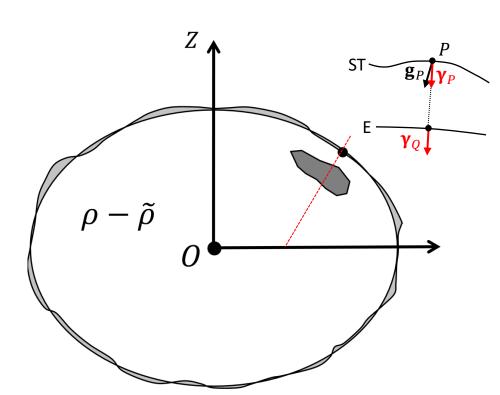
$$\gamma_P\gg \|oldsymbol{\delta g}_P\|$$
 Condição observada na prática

$$\mathbf{\gamma}_P = \gamma_P \hat{\mathbf{\gamma}}_0$$

Direção constante normal ao elipsoide

Em geral, considera-se que a direção do vetor gravidade normal no ponto P é igual a direção do vetor gravidade normal no ponto Q. No sistema de coordenadas geodésicas, esta direção é constante ao longo da normal ao elipsoide

$$\delta \mathbf{g}_{P} = G \sum_{j} \Delta \rho_{j} \iiint_{v_{j}} \nabla \frac{1}{\ell} dv$$



$$\delta \mathbf{g}_P = \mathbf{g}_P - \mathbf{\gamma}_P$$

$$\mathbf{g}_P = \mathbf{\gamma}_P + \mathbf{\delta}\mathbf{g}_P$$

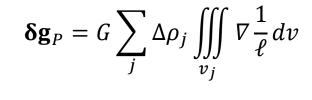
$$\delta g_P = g_P - \gamma_P$$

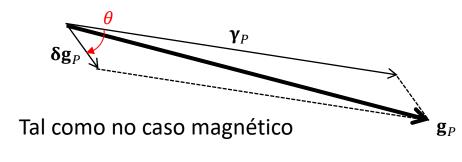
Distúrbio de gravidade

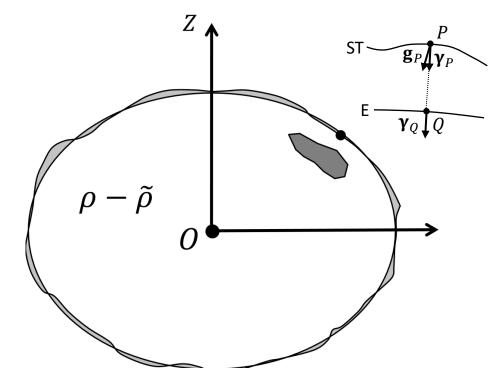
$$\gamma_P\gg \|\mathbf{\delta}\mathbf{g}_P\|$$

Condição observada na prática

$$\mathbf{\gamma}_P = \gamma_P \hat{\mathbf{\gamma}}_0$$







$$\delta \mathbf{g}_P = \mathbf{g}_P - \mathbf{\gamma}_P$$

$$\mathbf{g}_P = \mathbf{\gamma}_P + \mathbf{\delta}\mathbf{g}_P$$

$$\delta g_P = g_P - \gamma_P$$

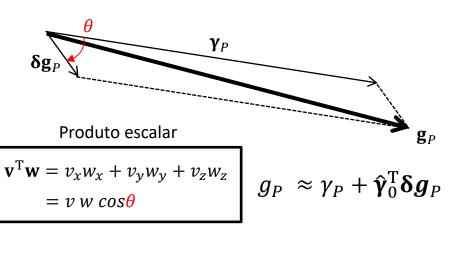
Distúrbio de gravidade

$$\gamma_P \gg \|\mathbf{\delta}\mathbf{g}_P\|$$

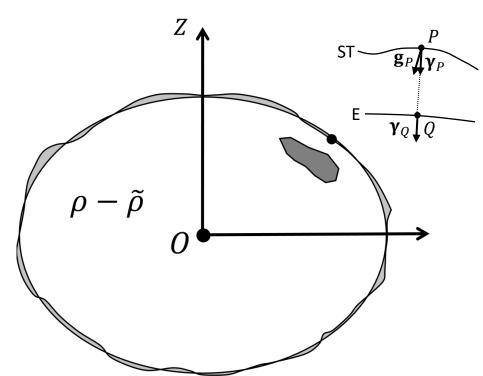
Condição observada na prática

$$\mathbf{\gamma}_P = \gamma_P \hat{\mathbf{\gamma}}_0$$

$$\delta \mathbf{g}_P = G \sum_j \Delta \rho_j \iiint_{v_j} \nabla \frac{1}{\ell} dv$$



$$\delta g_P \approx \hat{\boldsymbol{\gamma}}_0^{\mathrm{T}} \boldsymbol{\delta} \mathbf{g}_P$$



$$\delta \mathbf{g}_P = \mathbf{g}_P - \mathbf{\gamma}_P$$

$$\mathbf{g}_P = \mathbf{\gamma}_P + \mathbf{\delta}\mathbf{g}_P$$

$$\delta g_P = g_P - \gamma_P$$

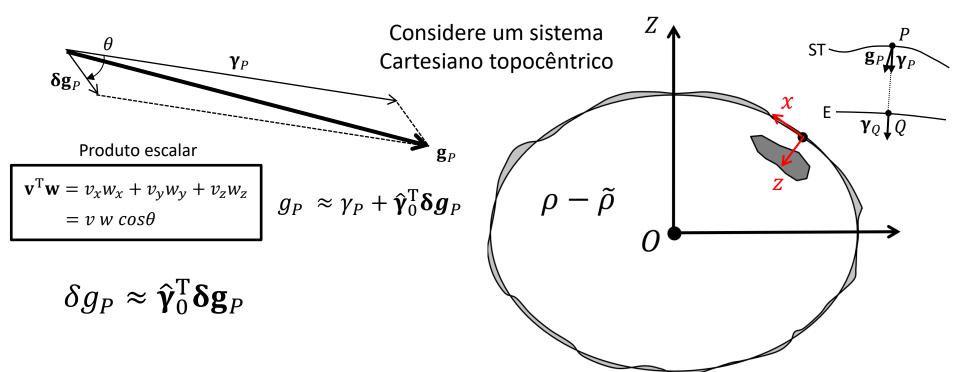
Distúrbio de gravidade

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Condição observada na prática

$$\delta \mathbf{g}_{P} = G \sum_{j} \Delta \rho_{j} \iiint_{v_{j}} \nabla \frac{1}{\ell} dv$$

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$$\delta \mathbf{g}_P = \mathbf{g}_P - \mathbf{\gamma}_P$$

$$\mathbf{g}_{P} = \mathbf{\gamma}_{P} + \mathbf{\delta}\mathbf{g}_{P}$$

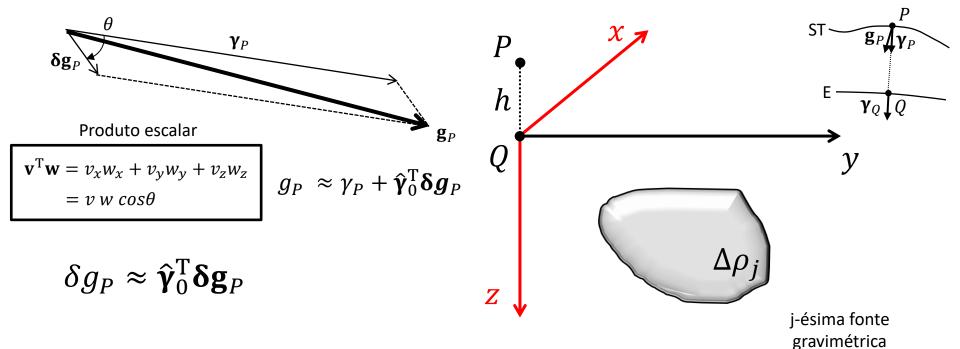
$$\delta g_P = g_P - \gamma_P$$

Distúrbio de gravidade

$$\gamma_P \gg \|\mathbf{\delta}\mathbf{g}_P\|$$

Condição observada na prática

$$\mathbf{\gamma}_P = \gamma_P \hat{\mathbf{\gamma}}_0$$



 $\mathbf{\delta g}_{P} = G \sum_{j} \Delta \rho_{j} \iiint_{v_{j}} \nabla \frac{1}{\ell} dv$

$$\delta \mathbf{g}_i = \mathbf{g}_i - \mathbf{\gamma}_i$$

$$g_{\underline{i}} = \gamma_{\underline{i}} + \delta g_{\underline{i}}$$

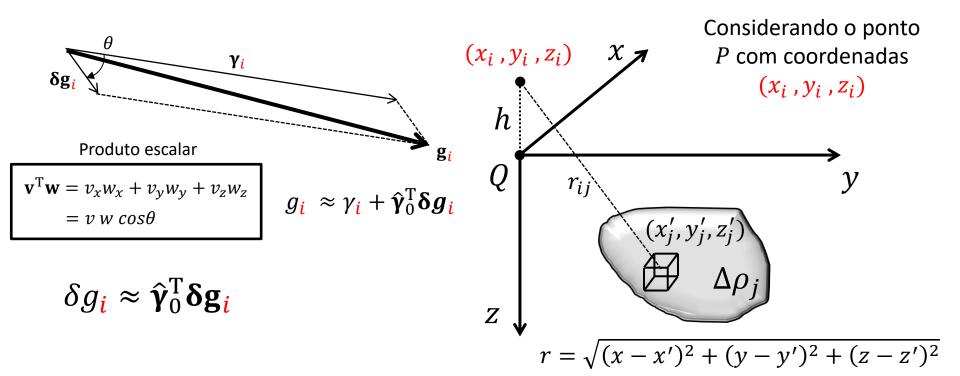
$$\delta g_{\mathbf{i}} = g_{\mathbf{i}} - \gamma_{\mathbf{i}}$$

Distúrbio de gravidade

$$\gamma_i \gg \|\delta \mathbf{g}_i\|$$

Condição observada na prática

$$\mathbf{\gamma}_i = \gamma_i \hat{\mathbf{\gamma}}_0$$



 $\delta \mathbf{g}_{i} = G \sum_{j} \Delta \rho_{j} \iiint_{v_{i}} \nabla \frac{1}{r_{ij}} dv$

$$\delta \mathbf{g}_{i} = \mathbf{g}_{i} - \mathbf{\gamma}_{i}$$

$$\mathbf{g}_{i} = \mathbf{\gamma}_{i} + \mathbf{\delta}\mathbf{g}_{i}$$

$$\delta g_{\mathbf{i}} = g_{\mathbf{i}} - \gamma_{\mathbf{i}}$$

Distúrbio de gravidade

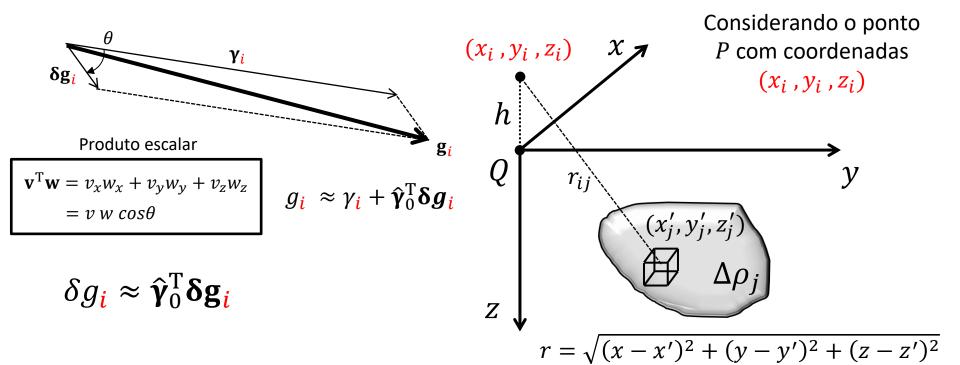
$$\gamma_i \gg \|\delta \mathbf{g}_i\|$$

Condição observada na prática

Embora tudo esteja calculado na posição
$$(x_i, y_i, z_i)$$
, as equações também podem ser avaliadas em outros pontos próximos referidos a este mesmo sistema de coordenadas

$$\delta \mathbf{g}_{i} = G \sum_{j} \Delta \rho_{j} \iiint_{v_{j}} \nabla \frac{1}{r_{ij}} dv$$

$$\mathbf{\gamma_i} = \gamma_i \hat{\mathbf{\gamma}}_0$$



$$\delta \mathbf{g}_i = \mathbf{g}_i - \mathbf{\gamma}_i$$

$$\mathbf{g}_i = \mathbf{\gamma}_i + \mathbf{\delta} \mathbf{g}_i$$

$$\delta g_i = g_i - \gamma_i$$

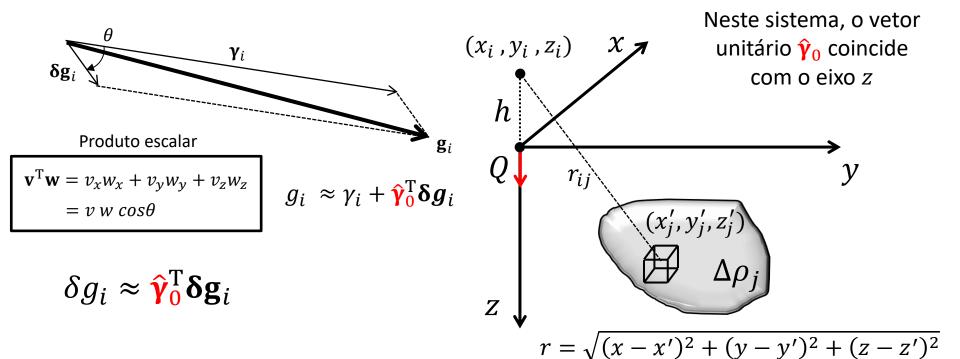
Distúrbio de gravidade

$$\gamma_i \gg \|\mathbf{\delta}\mathbf{g}_i\|$$

Condição observada na prática

$$\delta \mathbf{g}_i = G \sum_j \Delta \rho_j \iiint_{v_j} \nabla \frac{1}{r_{ij}} dv$$

$$\mathbf{\gamma}_i = \gamma_i \hat{\mathbf{\gamma}}_0$$



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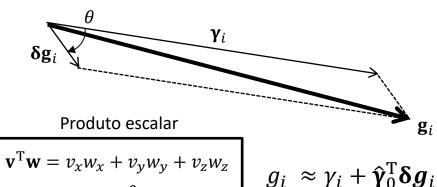
$$\delta g_i = g_i - \gamma_i$$

Distúrbio de gravidade

$$\gamma_i \gg \|\mathbf{\delta}\mathbf{g}_i\|$$

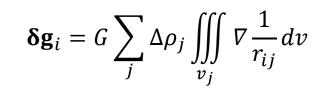
Condição observada na prática

$$\mathbf{\gamma}_i = \gamma_i \hat{\mathbf{\gamma}}_0$$

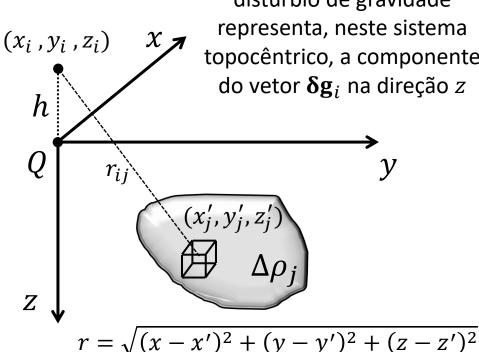


$$= v w cos\theta$$

$$\delta g_i \approx \hat{\mathbf{\gamma}}_0^{\mathrm{T}} \mathbf{\delta} \mathbf{g}_i$$



Consequentemente, o distúrbio de gravidade representa, neste sistema topocêntrico, a componente do vetor $\delta \mathbf{g}_i$ na direção z



$$\delta \mathbf{g}_i = \mathbf{g}_i - \mathbf{\gamma}_i$$

$$\mathbf{g}_i = \mathbf{\gamma}_i + \mathbf{\delta}\mathbf{g}_i$$

$$\delta g_i = g_i - \gamma_i$$

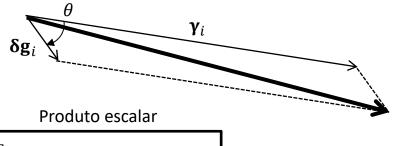
Distúrbio de gravidade

$$\gamma_i \gg \|\mathbf{\delta}\mathbf{g}_i\|$$

Condição observada na prática

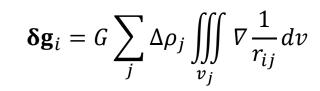
 $g_i \approx \gamma_i + \hat{\mathbf{\gamma}}_0^{\mathrm{T}} \mathbf{\delta} \mathbf{g}_i$

$$\mathbf{\gamma}_i = \gamma_i \hat{\mathbf{\gamma}}_0$$

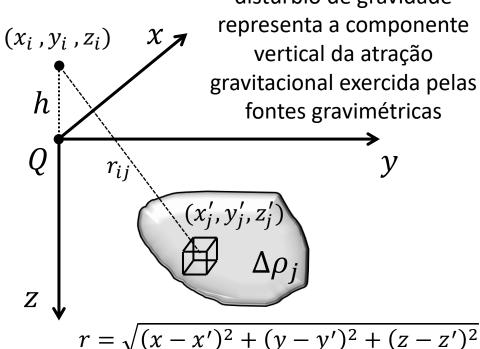


$$\mathbf{v}^{\mathrm{T}}\mathbf{w} = v_{x}w_{x} + v_{y}w_{y} + v_{z}w_{z}$$
$$= v \ w \ cos\theta$$

$$\delta g_i \approx \hat{\mathbf{\gamma}}_0^{\mathrm{T}} \mathbf{\delta} \mathbf{g}_i$$



Ou, analogamente, o distúrbio de gravidade representa a componente vertical da atração gravitacional exercida pelas fontes gravimétricas



$$\delta \mathbf{g}_i = \mathbf{g}_i - \mathbf{\gamma}_i$$

$$\mathbf{g}_i = \mathbf{\gamma}_i + \mathbf{\delta} \mathbf{g}_i$$

$$\delta g_i = g_i - \gamma_i$$

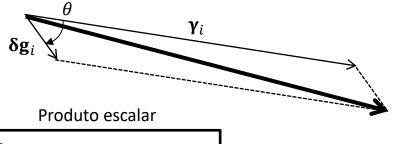
Distúrbio de gravidade

$$\gamma_i \gg \|\mathbf{\delta}\mathbf{g}_i\|$$

Condição observada na prática

 $g_i \approx \gamma_i + \hat{\boldsymbol{\gamma}}_0^{\mathrm{T}} \boldsymbol{\delta} \boldsymbol{g}_i$

$$\mathbf{\gamma}_i = \gamma_i \hat{\mathbf{\gamma}}_0$$



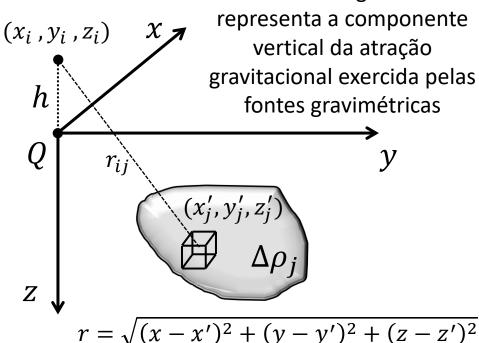
$$\mathbf{v}^{\mathrm{T}}\mathbf{w} = v_x w_x + v_y w_y + v_z w_z$$
$$= v w \cos\theta$$

$$\delta g_i \approx \hat{\mathbf{\gamma}}_0^{\mathrm{T}} \mathbf{\delta} \mathbf{g}_i$$

$$\delta g_i \approx G \sum_j \Delta \rho_j \iiint_{v_j} \frac{\partial}{\partial z} \frac{1}{r_{ij}} dv$$

$$\mathbf{\delta g}_i = G \sum_j \Delta \rho_j \iiint_{v_j} \nabla \frac{1}{r_{ij}} dv$$

Ou, analogamente, o distúrbio de gravidade representa a componente vertical da atração gravitacional exercida pelas fontes gravimétricas



Referências

 Blakely, R. J., 1996, Potential theory in gravity and magnetic applications: Cambridge University Press.