

Camada equivalente aplicada ao processamento e interpretação de dados de campos potenciais

Vanderlei C. Oliveira Jr.

2016



**Observatório
Nacional**



Anomalia de Campo Total (parte B)

Vanderlei C. Oliveira Jr.

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**Observatório
Nacional**



$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_i$$

Campo principal

$$\mathbf{B}_i = B_i \hat{\mathbf{B}}_i$$

Campo crustal

$$\mathbf{T}_i = \mathbf{F}_i + \mathbf{B}_i$$

Campo total

$$F_i \gg B_i$$

Condição observada
na prática

$$\Delta T_i = T_i - F_i$$

Anomalia de campo total

$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_i$$

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$$\mathbf{T}_i = \mathbf{F}_i + \mathbf{B}_i$$

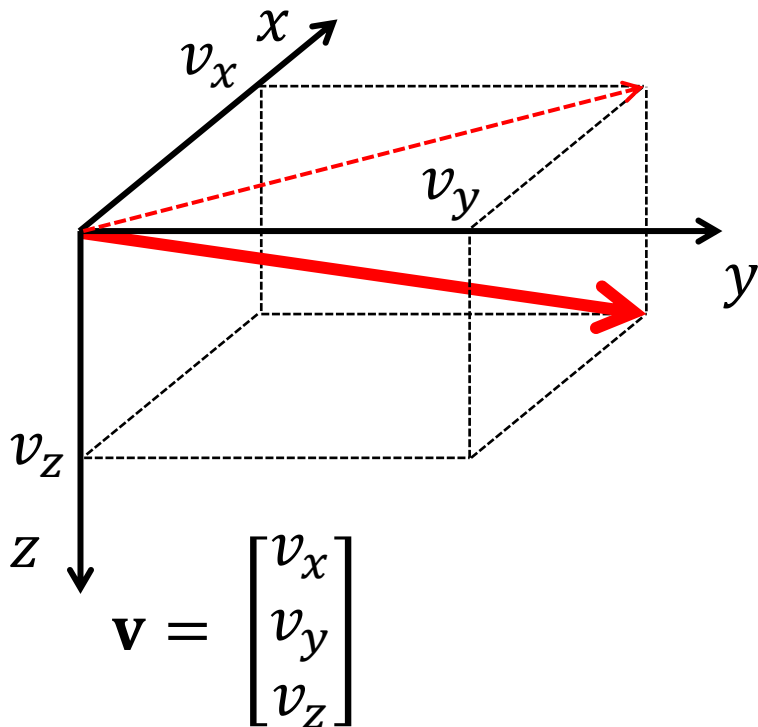
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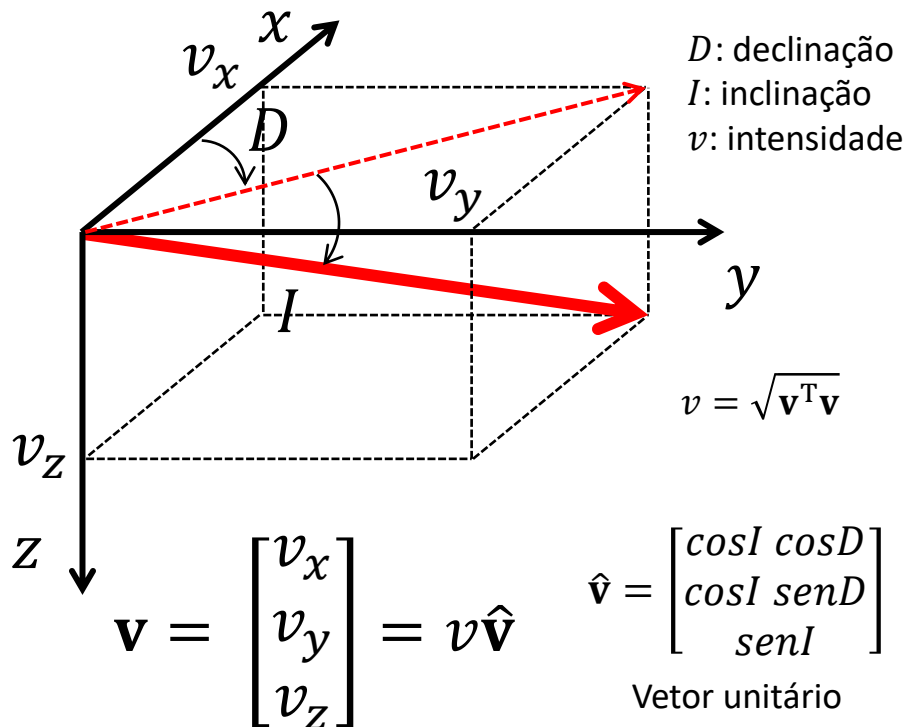
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(por exemplo, IGRF)

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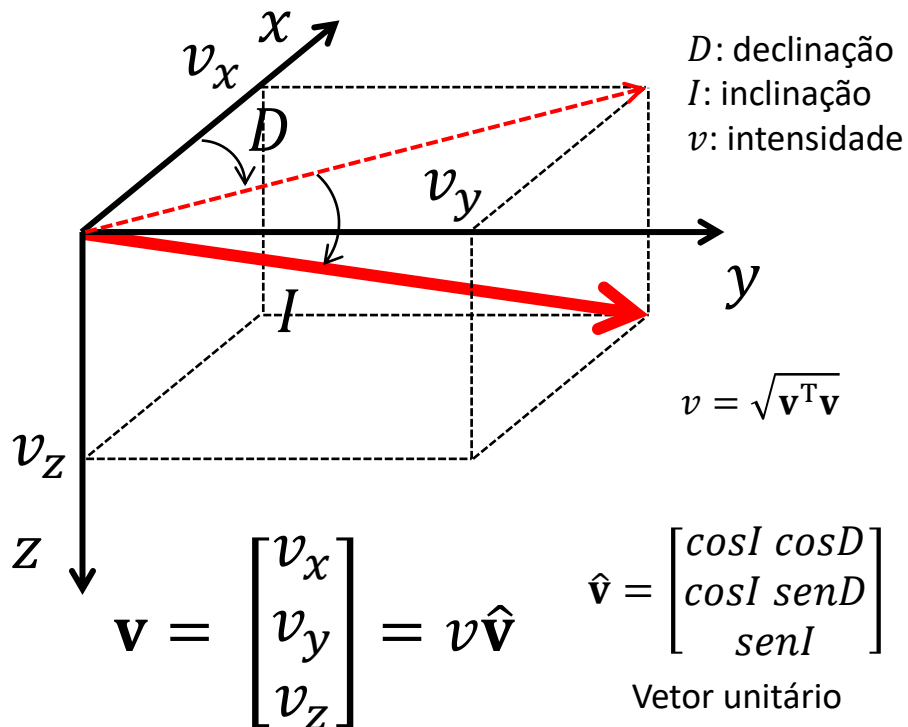
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$$\mathbf{B}_i = \sum_j \mathbf{b}_i^j$$

Soma do campo
produzido por todas as
rochas magnetizadas

$$\mathbf{T}_i = \mathbf{F}_i + \mathbf{B}_i$$

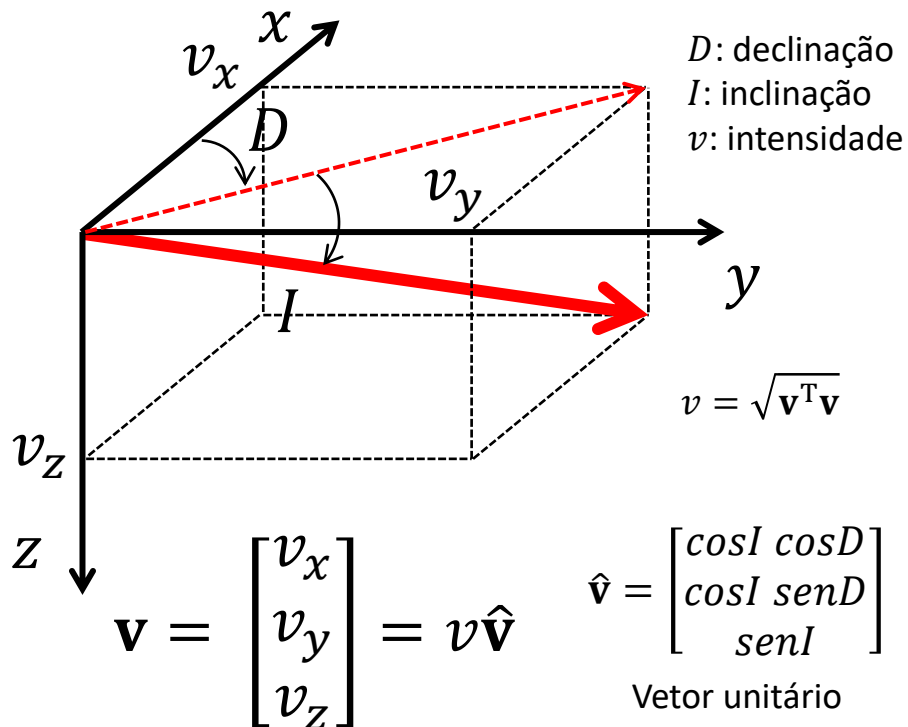
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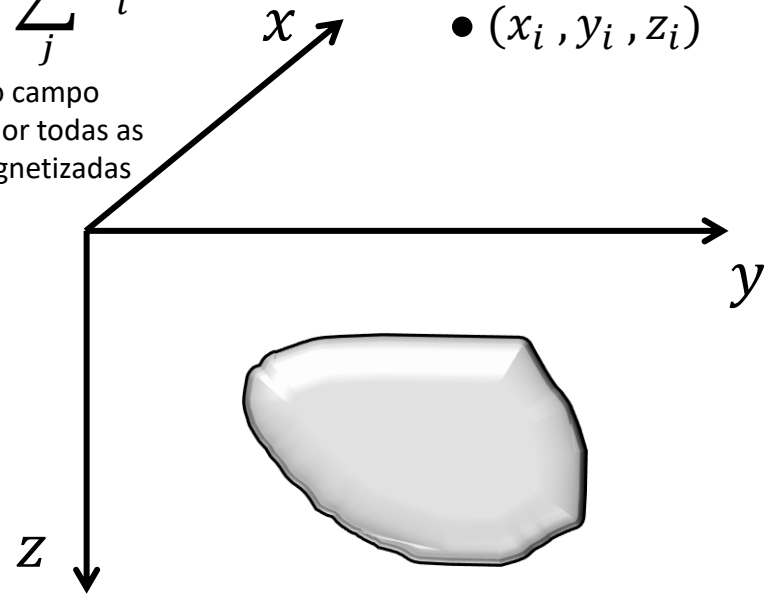
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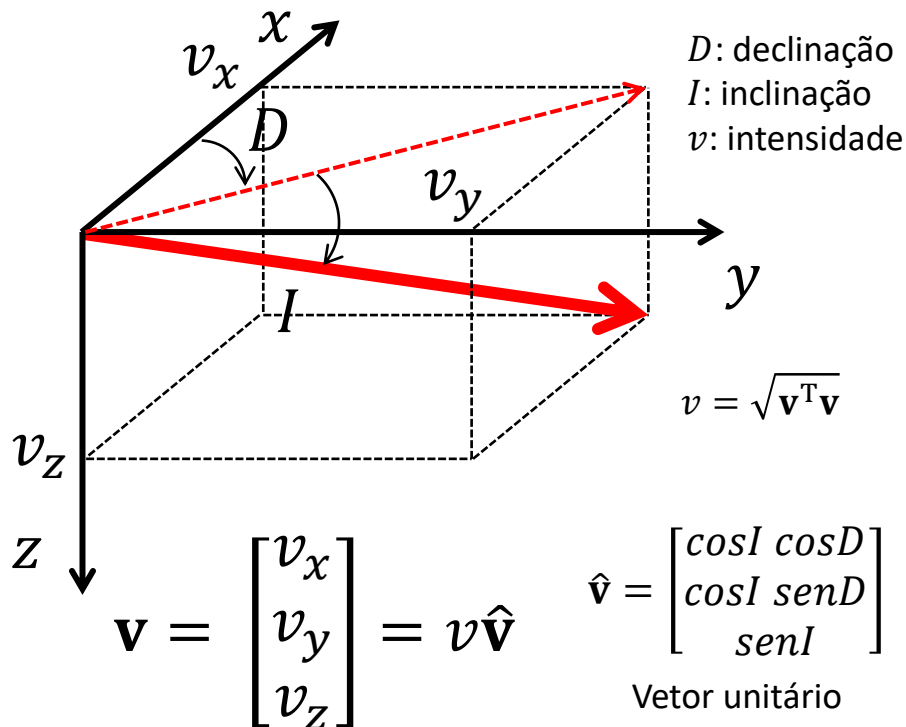
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Anomalia de campo total



Considere o campo produzido
pela j -ésima fonte magnética na
posição (x_i, y_i, z_i)



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(por exemplo, IGRF)

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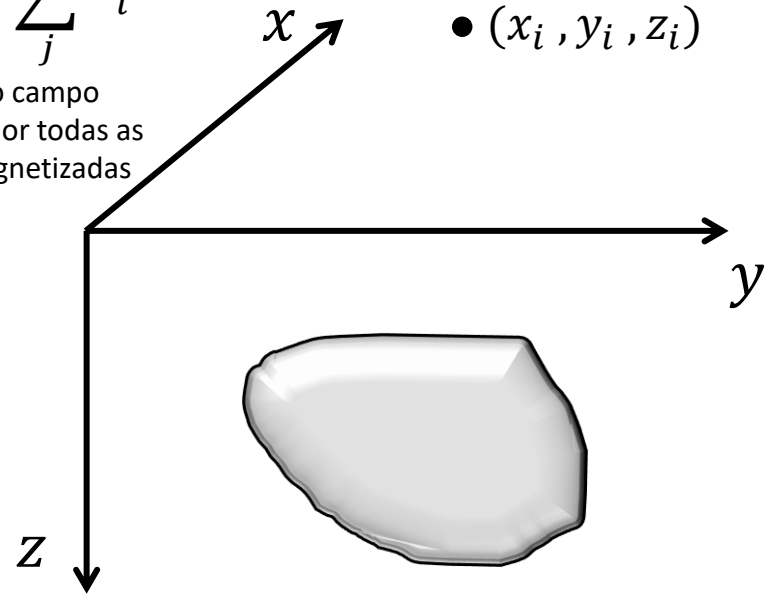
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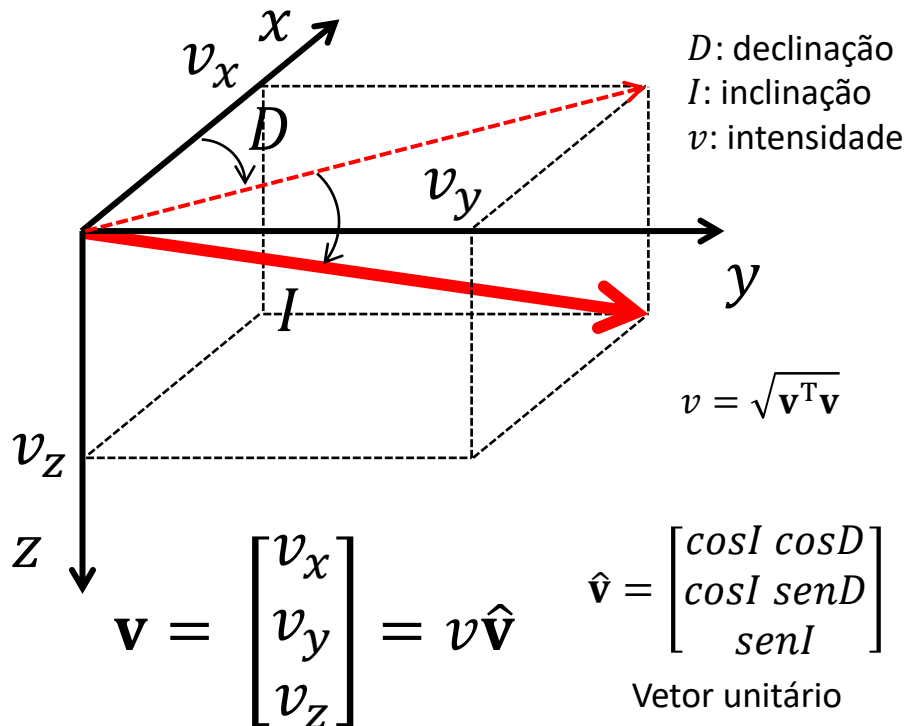
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Anomalia de campo total



A indução magnética \mathbf{b}_i^j é
uma integral avaliada no
volume da j-ésima fonte
magnética



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Campo principal
(por exemplo, IGRF)

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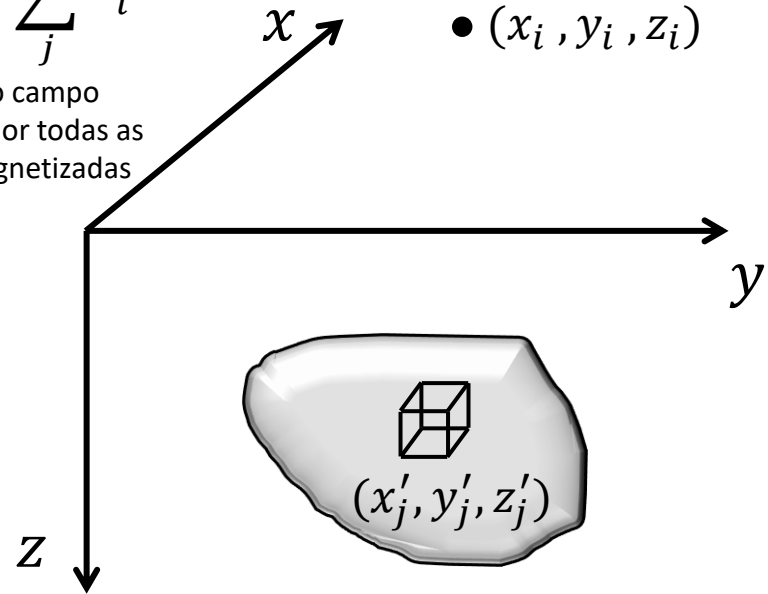
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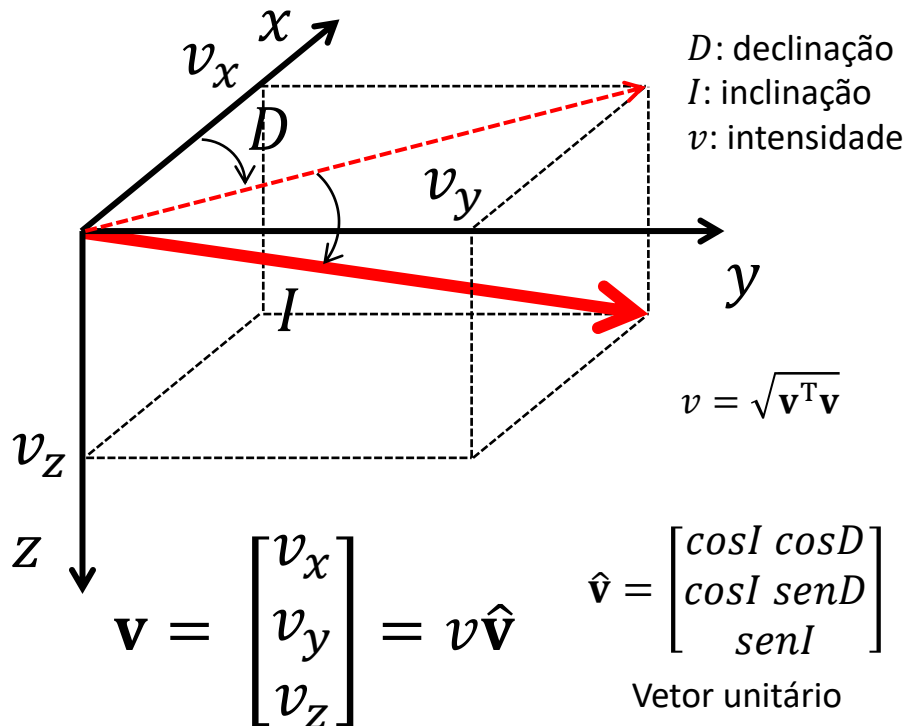
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Anomalia de campo total



(x'_j, y'_j, z'_j) são as coordenadas de
um elemento de volume $dx dy dz$
dentro da j -ésima fonte



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(por exemplo, IGRF)

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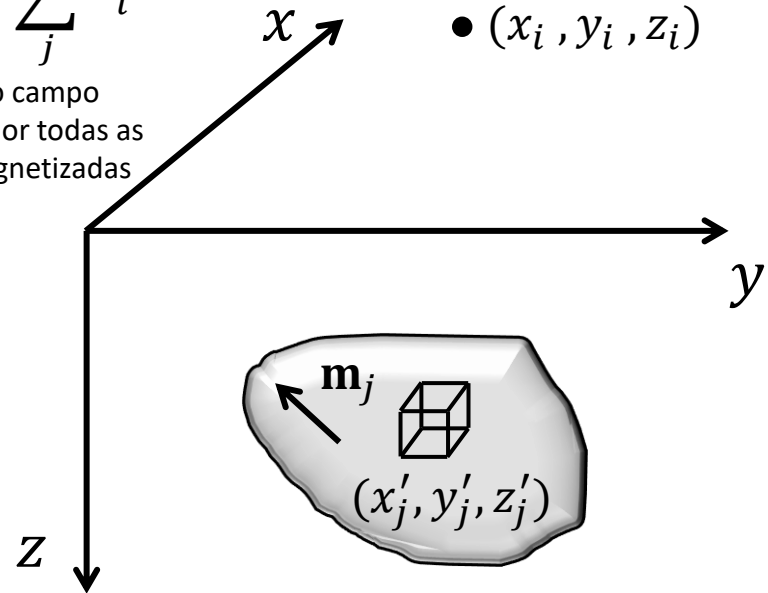
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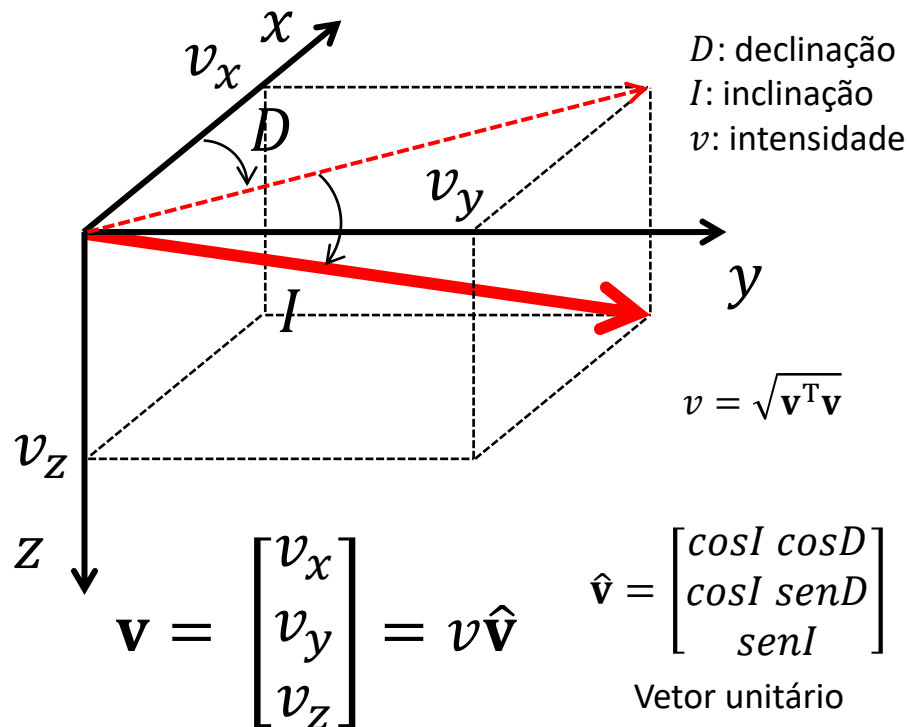
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(x'_j, y'_j, z'_j) são as coordenadas de
um elemento de volume $dx dy dz$
dentro da j -ésima fonte

Considere uma que a j -ésima
fonte tenha magnetização
com direção constante

$$\mathbf{m}_j = m_j \hat{\mathbf{m}}_j$$



$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_i$$

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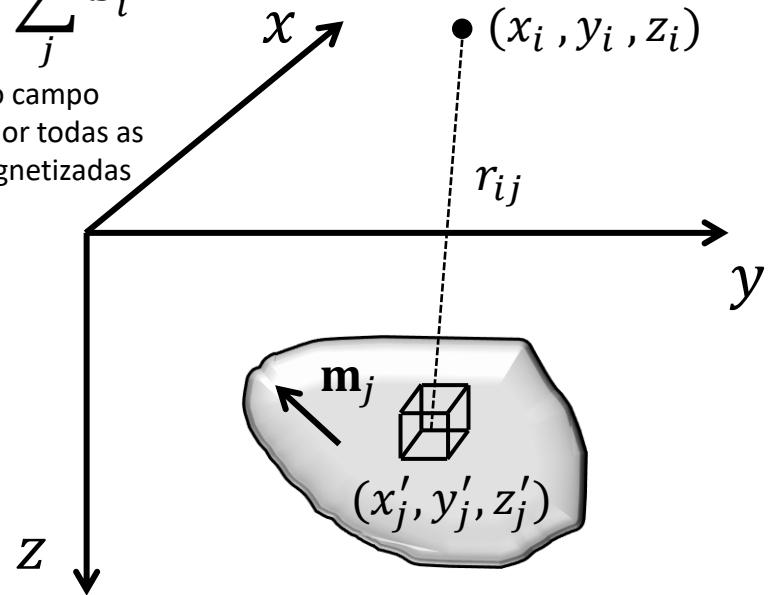
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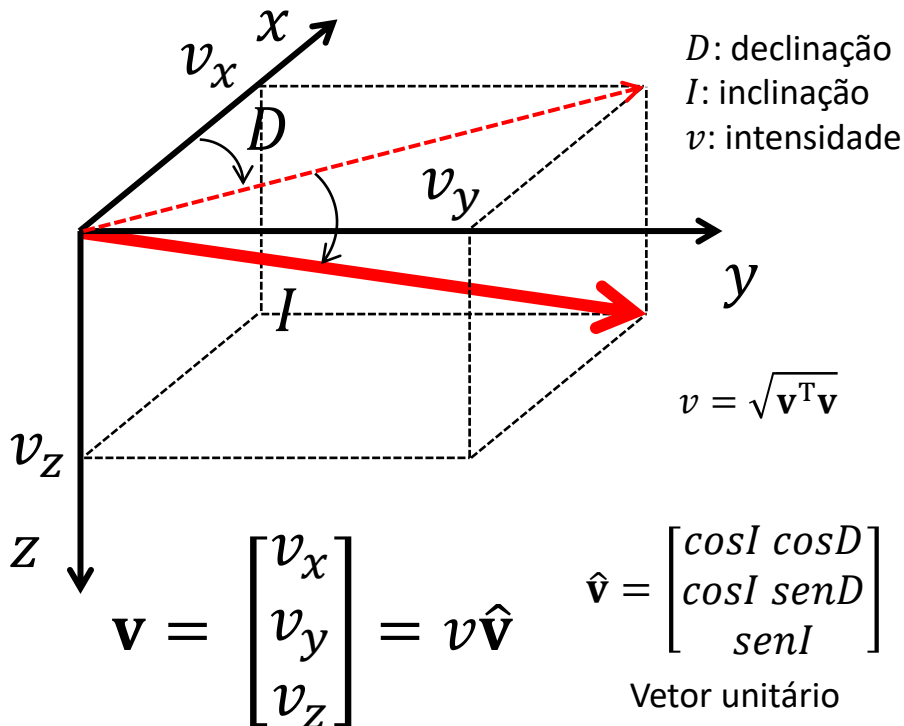
Soma do campo
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$$\Phi_i^j = \iiint_{v_j} m_j \frac{1}{r_{ij}} dv_j$$

A integral é avaliada
no volume da fonte

$$r_{ij} = \sqrt{(x_i - x'_j)^2 + (y_i - y'_j)^2 + (z_i - z'_j)^2}$$



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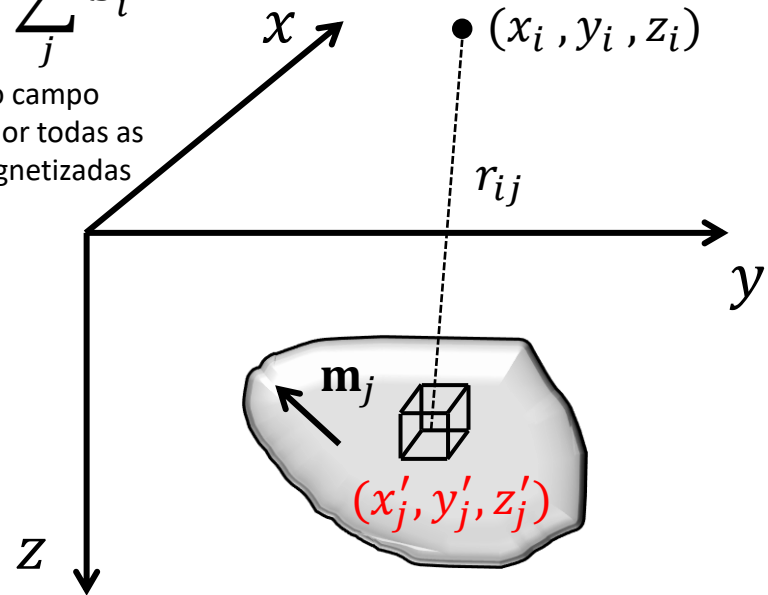
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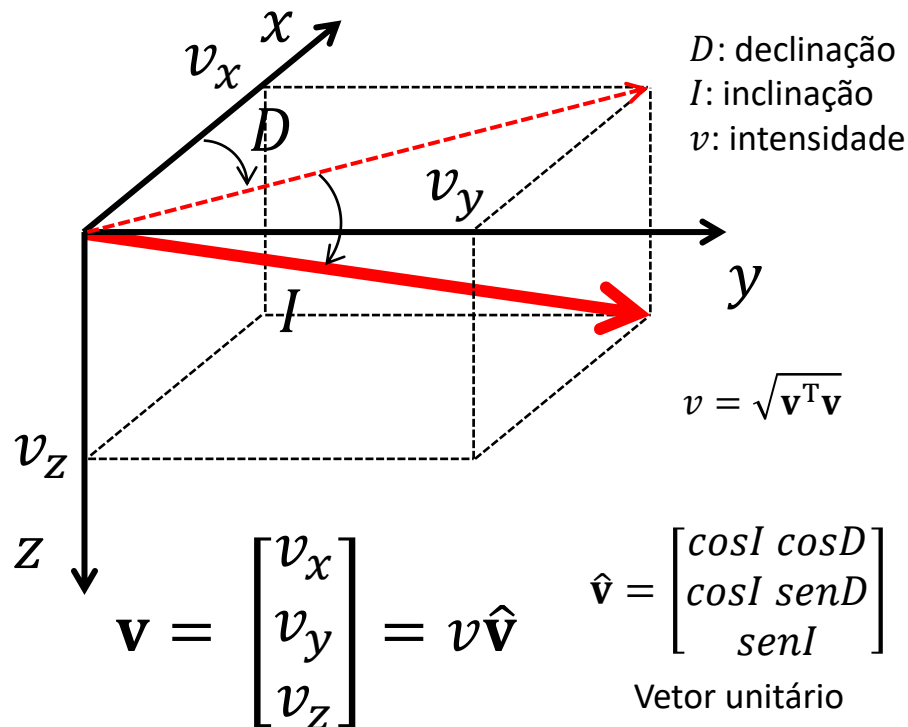


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Variáveis de integração



$$\mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = v \hat{\mathbf{v}}$$

$$\hat{\mathbf{v}} = \begin{bmatrix} \cos I \cos D \\ \cos I \sin D \\ \sin I \end{bmatrix}$$

Vetor unitário

$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_i$$

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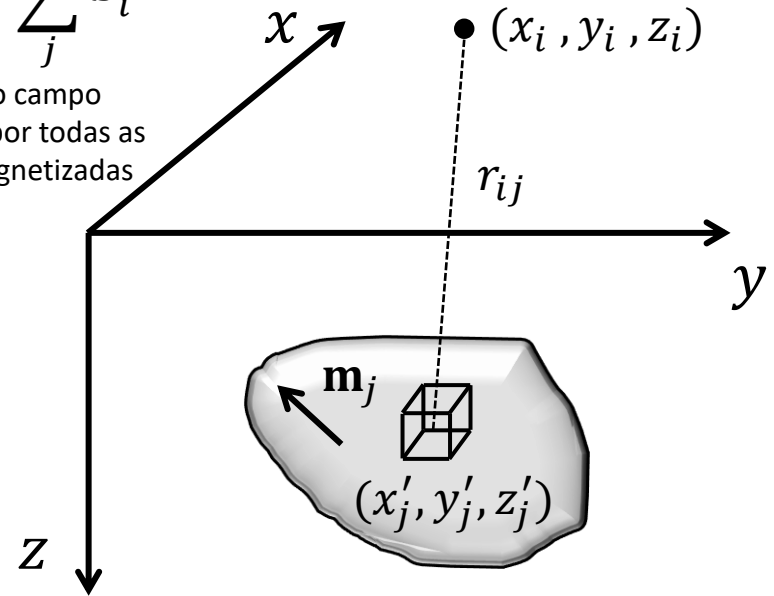
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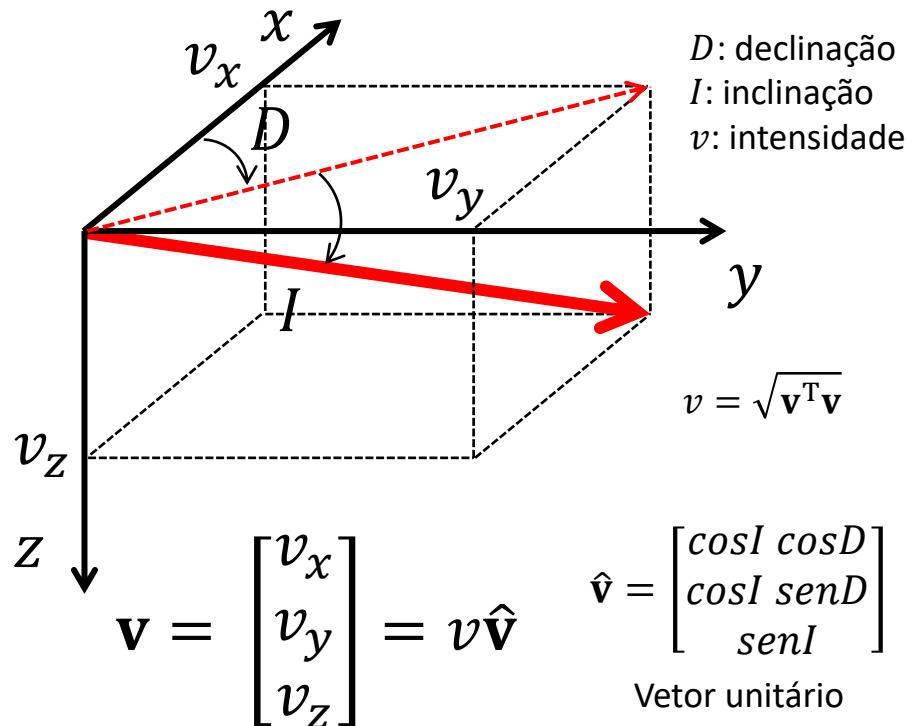


$$\Phi_i^j = \iiint_{v_j} m_j \frac{1}{r_{ij}} dv_j$$

A integral é avaliada
no volume da fonte

$$r_{ij} = \sqrt{(x_i - x_j')^2 + (y_i - y_j')^2 + (z_i - z_j')^2}$$

$$\mathbf{b}_i^j = \kappa_m \mathbf{M}_i^j \hat{\mathbf{m}}_j \quad \kappa_m = 10^9 \frac{\mu_0}{4\pi}$$



$$\mathbf{M}_i^j = \begin{bmatrix} \partial_{xx} \Phi_i^j & \partial_{xy} \Phi_i^j & \partial_{xz} \Phi_i^j \\ \partial_{xy} \Phi_i^j & \partial_{yy} \Phi_i^j & \partial_{yz} \Phi_i^j \\ \partial_{xz} \Phi_i^j & \partial_{yz} \Phi_i^j & \partial_{zz} \Phi_i^j \end{bmatrix}$$

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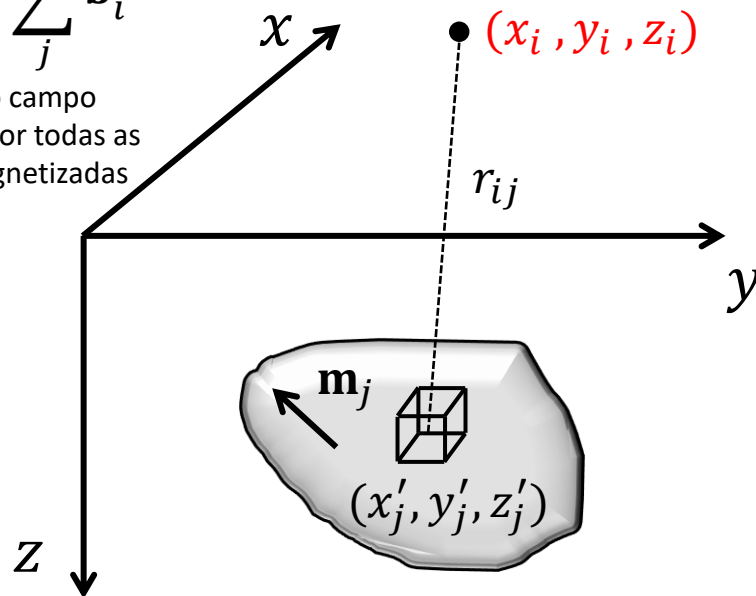
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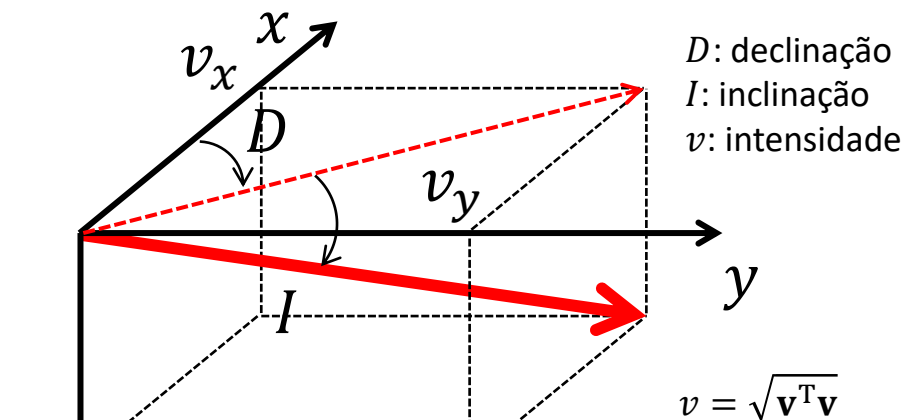
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$$\mathbf{b}_i^j = \kappa_m \mathbf{M}_i^j \hat{\mathbf{m}}_j$$

$$\kappa_m = 10^9 \frac{\mu_0}{4\pi}$$



D: declinação
I: inclinação
v: intensidade

$$v = \sqrt{\mathbf{v}^T \mathbf{v}}$$

$$\mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = v \hat{\mathbf{v}}$$

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$$\mathbf{M}_i^j = \begin{bmatrix} \partial_{xx} \Phi_i^j & \partial_{xy} \Phi_i^j & \partial_{xz} \Phi_i^j \\ \partial_{xy} \Phi_i^j & \partial_{yy} \Phi_i^j & \partial_{yz} \Phi_i^j \\ \partial_{xz} \Phi_i^j & \partial_{yz} \Phi_i^j & \partial_{zz} \Phi_i^j \end{bmatrix}$$

As derivadas são
calculadas em
relação às
coordenadas do
ponto de
observação

$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_i$$

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(por exemplo, IGRF)

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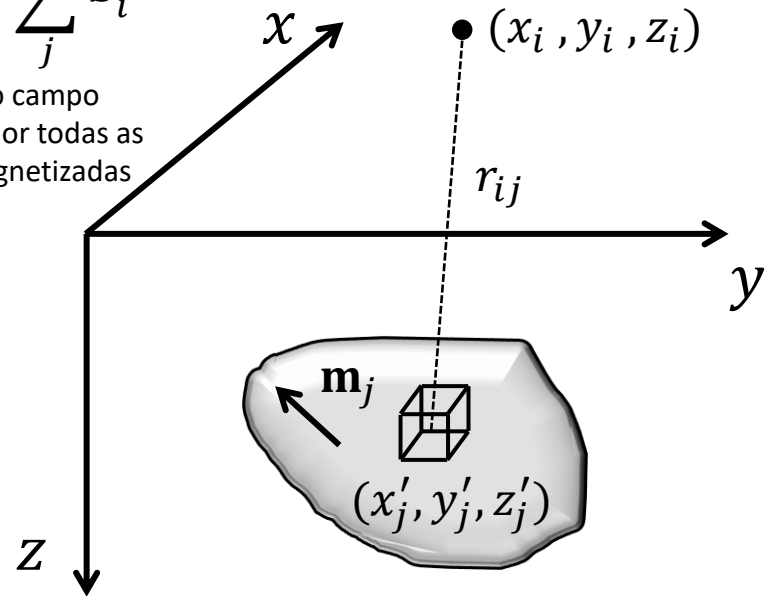
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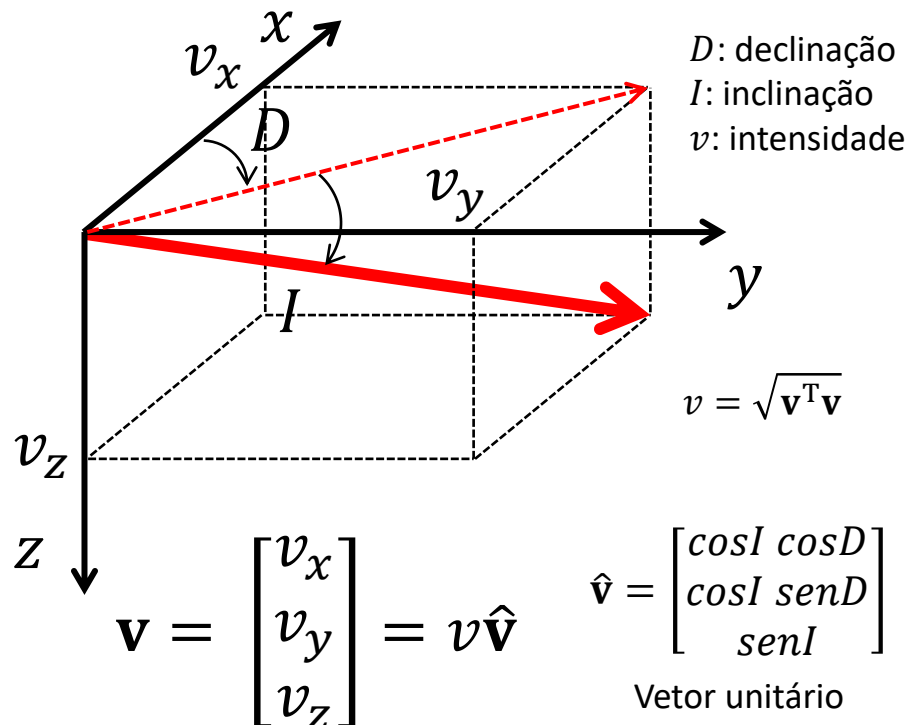
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$$r_{ij} = \sqrt{(x_i - x_j')^2 + (y_i - y_j')^2 + (z_i - z_j')^2}$$

$$\mathbf{b}_i^j = \kappa_m \mathbf{M}_i^j \hat{\mathbf{m}}_j \quad \kappa_m = 10^9 \frac{\mu_0}{4\pi}$$

Tente calcular os elementos desta matriz!

$$\mathbf{M}_i^j = \begin{bmatrix} \partial_{xx} \Phi_i^j & \partial_{xy} \Phi_i^j & \partial_{xz} \Phi_i^j \\ \partial_{xy} \Phi_i^j & \partial_{yy} \Phi_i^j & \partial_{yz} \Phi_i^j \\ \partial_{xz} \Phi_i^j & \partial_{yz} \Phi_i^j & \partial_{zz} \Phi_i^j \end{bmatrix}$$



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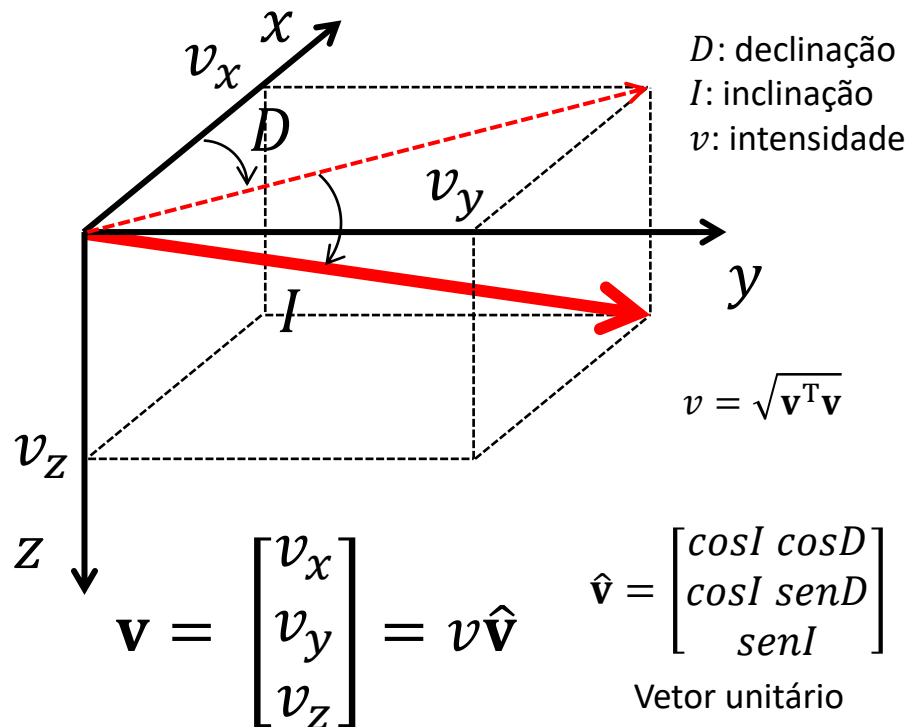
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$$\mathbf{T}_i = \mathbf{F}_i + \mathbf{B}_i$$

Campo total

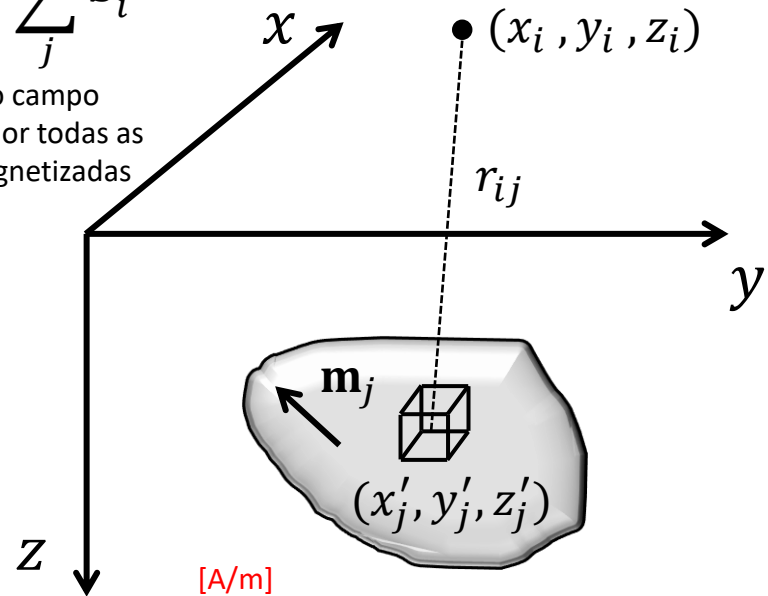
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Soma do campo
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$$\Phi_i^j = \iiint_{v_j} m_j \frac{1}{r_{ij}} dv_j$$

[A m] [A/m] [m⁻¹]

A integral é avaliada
no volume da fonte

$$r_{ij} = \sqrt{(x_i - x'_j)^2 + (y_i - y'_j)^2 + (z_i - z'_j)^2}$$

[m]

$$\mathbf{b}_i^j = \kappa_m \mathbf{M}_i^j \hat{\mathbf{m}}_j$$

[nT] [10⁹H/m][A/m] nT

$$\kappa_m = 10^9 \frac{\mu_0}{4\pi}$$

[10⁹H/m] [H/m]

Converte
T para nT

$$\mathbf{M}_i^j = \begin{bmatrix} \partial_{xx} \Phi_i^j & \partial_{xy} \Phi_i^j & \partial_{xz} \Phi_i^j \\ \partial_{xy} \Phi_i^j & \partial_{yy} \Phi_i^j & \partial_{yz} \Phi_i^j \\ \partial_{xz} \Phi_i^j & \partial_{yz} \Phi_i^j & \partial_{zz} \Phi_i^j \end{bmatrix}$$

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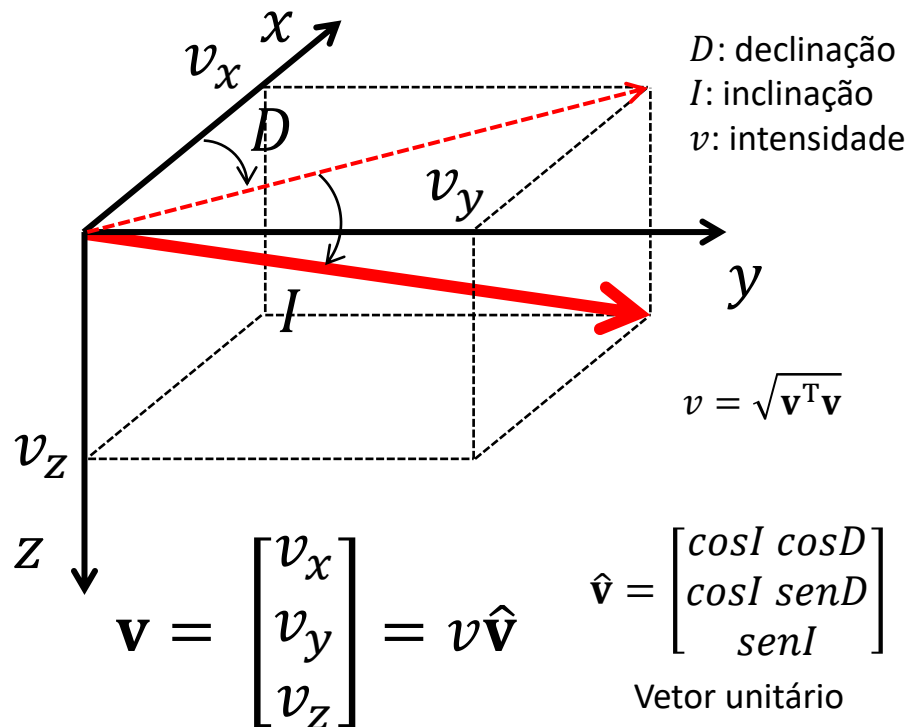
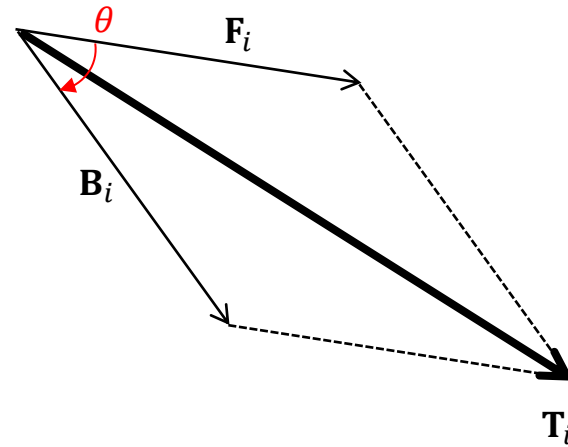
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$$\mathbf{T}_i = \mathbf{F}_i + \mathbf{B}_i$$

Campo total

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Anomalia de campo total



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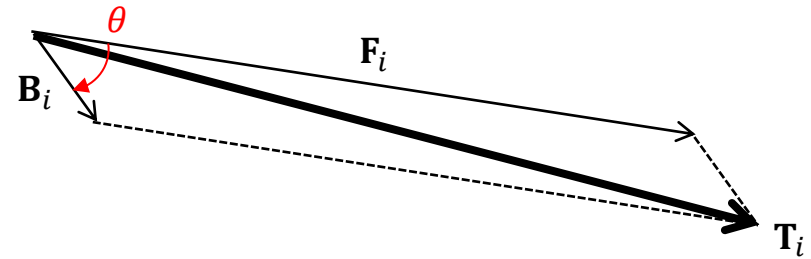
Campo crustal

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Campo total

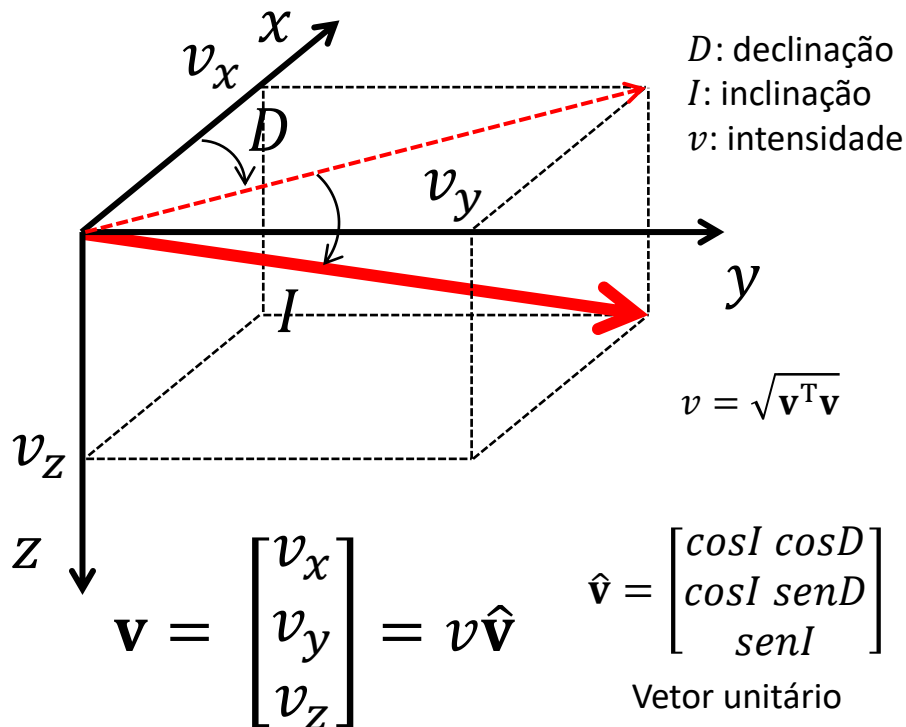
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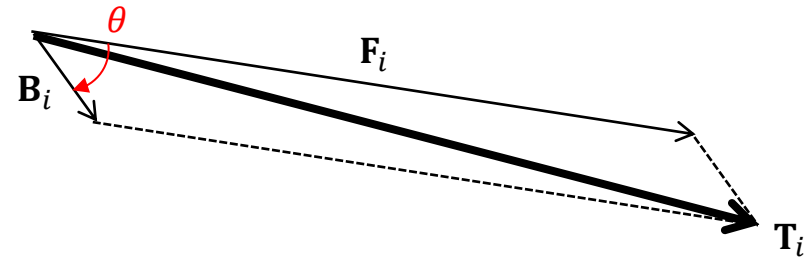
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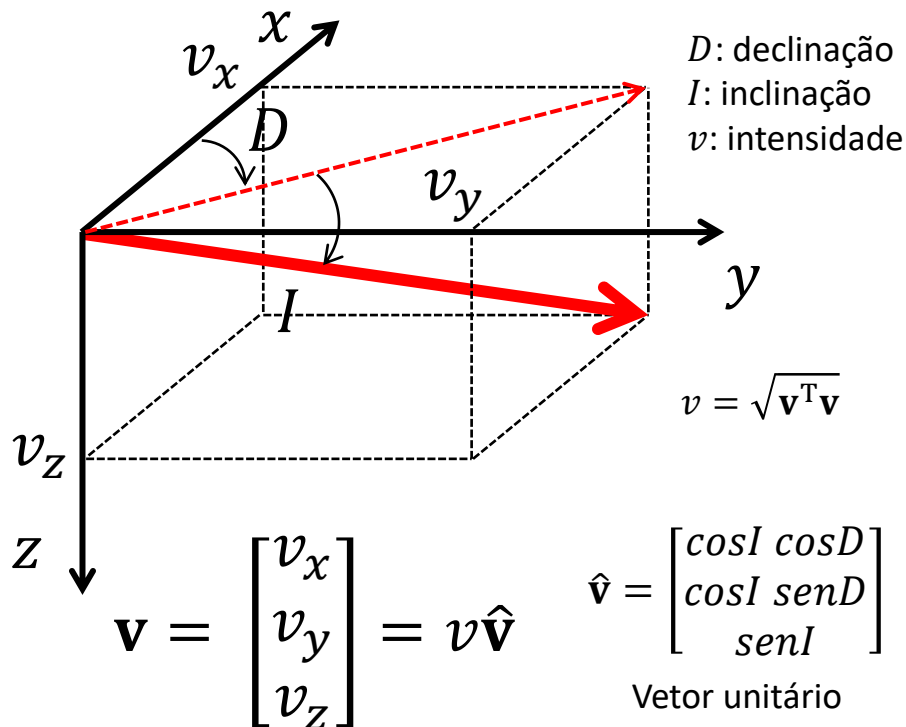
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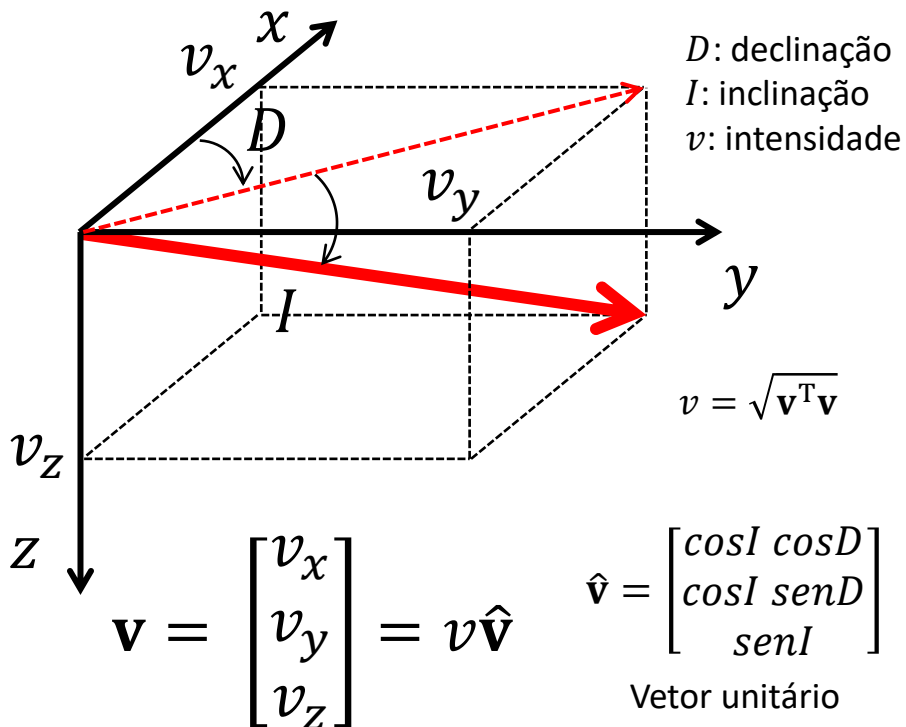
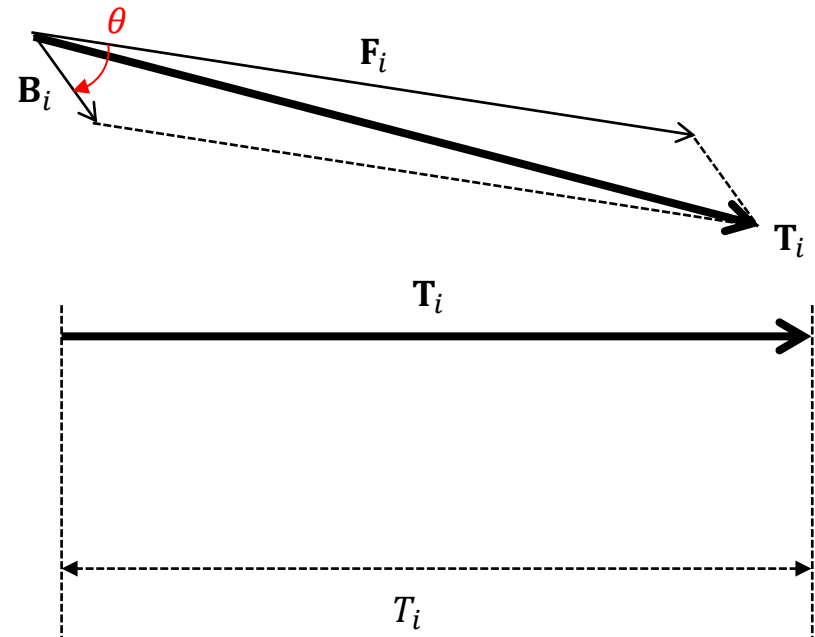
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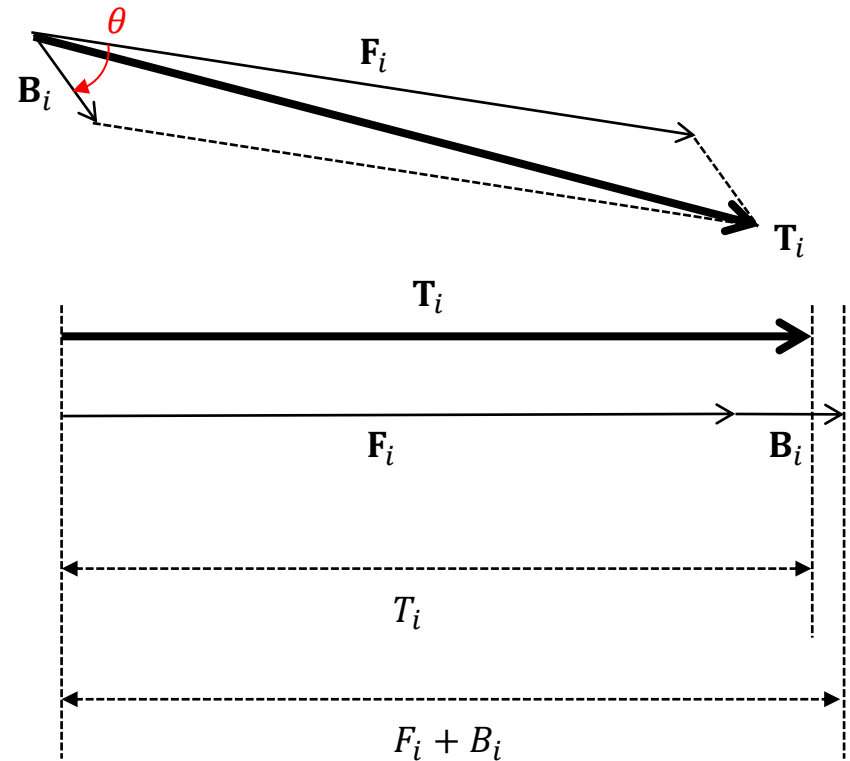
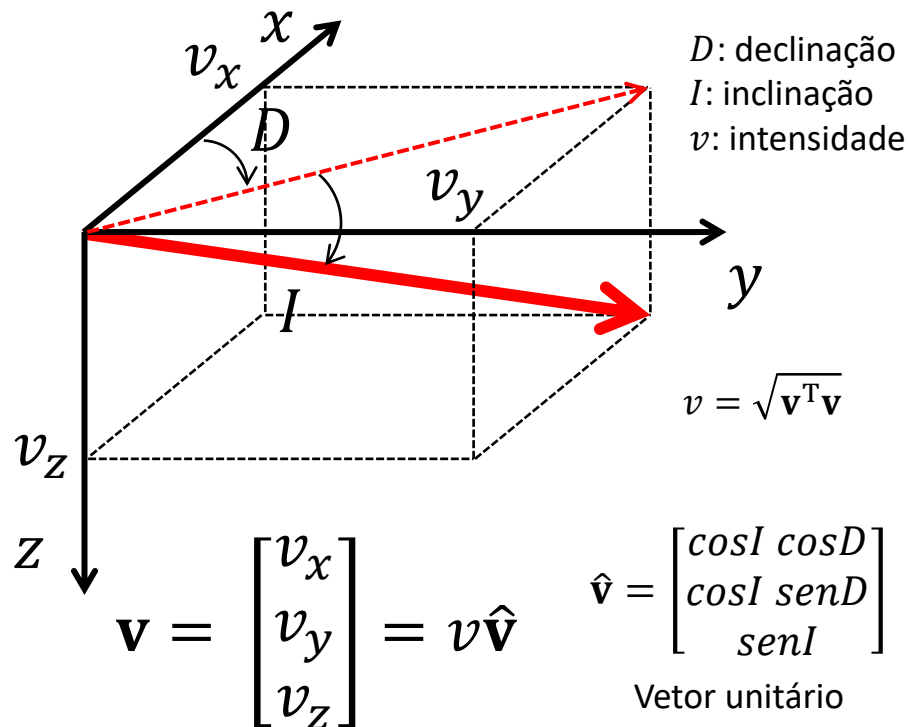
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Anomalia de campo total



$$T_i \neq F_i + B_i$$

Direção constante

$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_0$$

Campo principal
(por exemplo, IGRF)

$$\mathbf{B}_i = B_i \hat{\mathbf{B}}_i$$

Campo crustal

$$\mathbf{T}_i = \mathbf{F}_i + \mathbf{B}_i$$

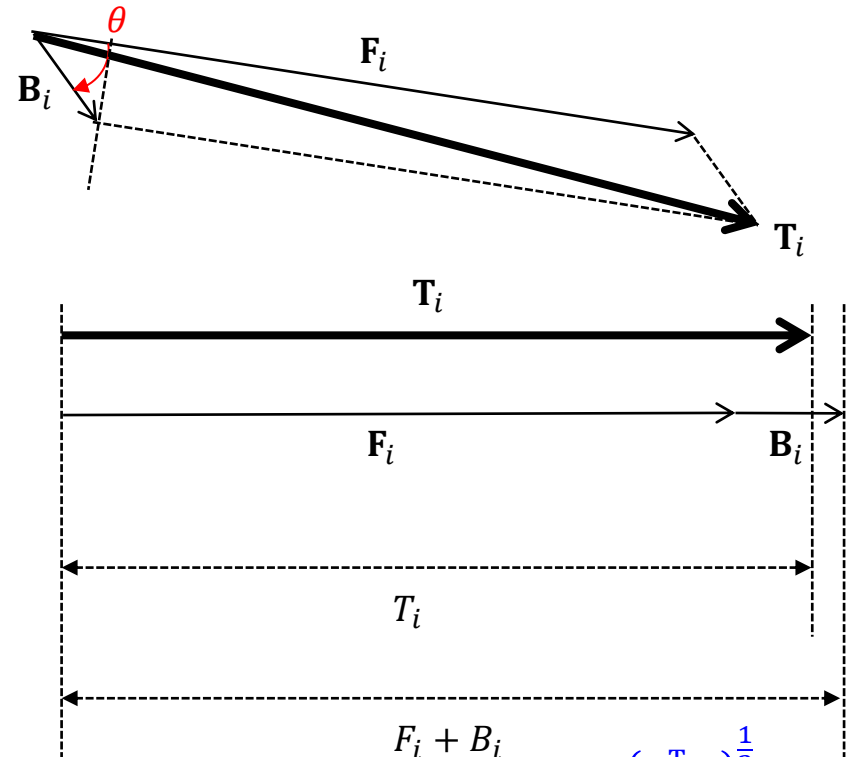
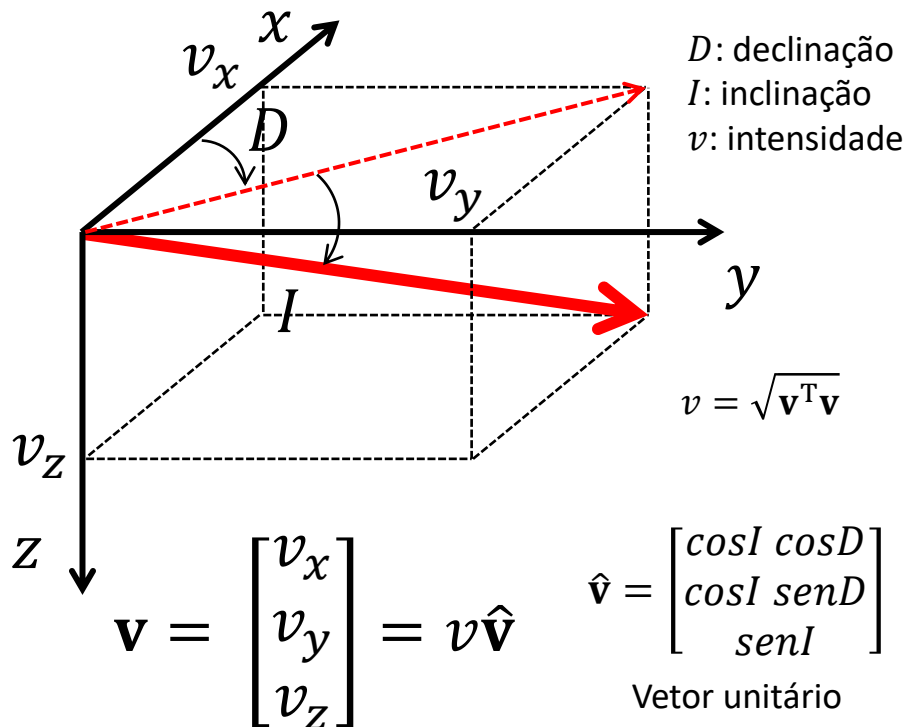
Campo total

$$F_i \gg B_i$$

Condição observada
na prática

$$\Delta T_i = T_i - F_i$$

Anomalia de campo total



$$T_i \neq F_i + B_i$$

$$T_i = [(\mathbf{F}_i + \mathbf{B}_i)^T (\mathbf{F}_i + \mathbf{B}_i)]^{\frac{1}{2}}$$

$$T_i \approx (\mathbf{F}_i^T \mathbf{F}_i)^{\frac{1}{2}} + \frac{\mathbf{F}_i^T \mathbf{B}_i}{(\mathbf{F}_i^T \mathbf{F}_i)^{\frac{1}{2}}}$$

Direção constante

$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_0$$

Campo principal
(por exemplo, IGRF)

$$\mathbf{B}_i = B_i \hat{\mathbf{B}}_i$$

Campo crustal

$$\mathbf{T}_i = \mathbf{F}_i + \mathbf{B}_i$$

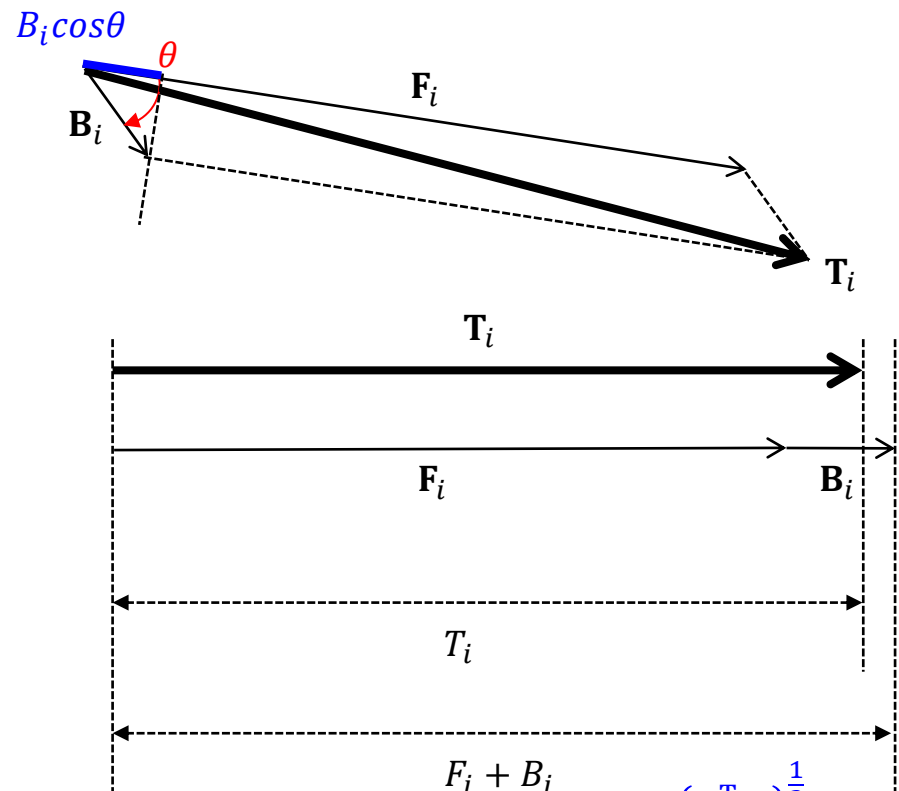
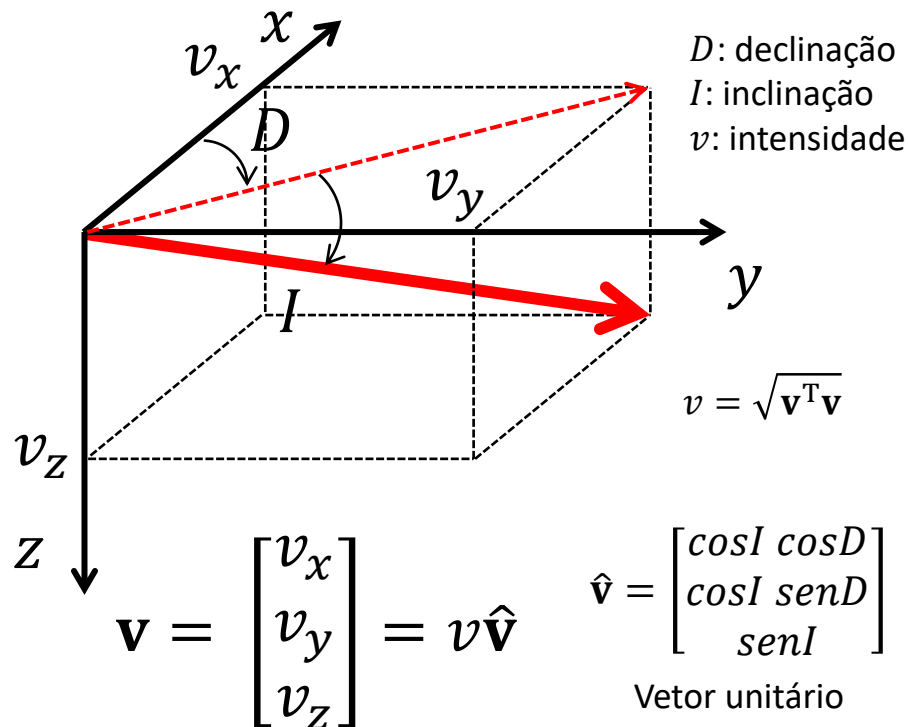
Campo total

$$F_i \gg B_i$$

Condição observada
na prática

$$\Delta T_i = T_i - F_i$$

Anomalia de campo total



Produto escalar

$$\mathbf{v}^T \mathbf{w} = v_x w_x + v_y w_y + v_z w_z$$

$$= v w \cos \theta$$

$$T_i \approx F_i + \hat{\mathbf{F}}_0^T \mathbf{B}_i$$

Direção constante

$$\mathbf{F}_i = F_i \hat{\mathbf{F}}_0$$

Campo principal
(por exemplo, IGRF)

$$\mathbf{B}_i = B_i \hat{\mathbf{B}}_i$$

Campo crustal

$$\mathbf{T}_i = \mathbf{F}_i + \mathbf{B}_i$$

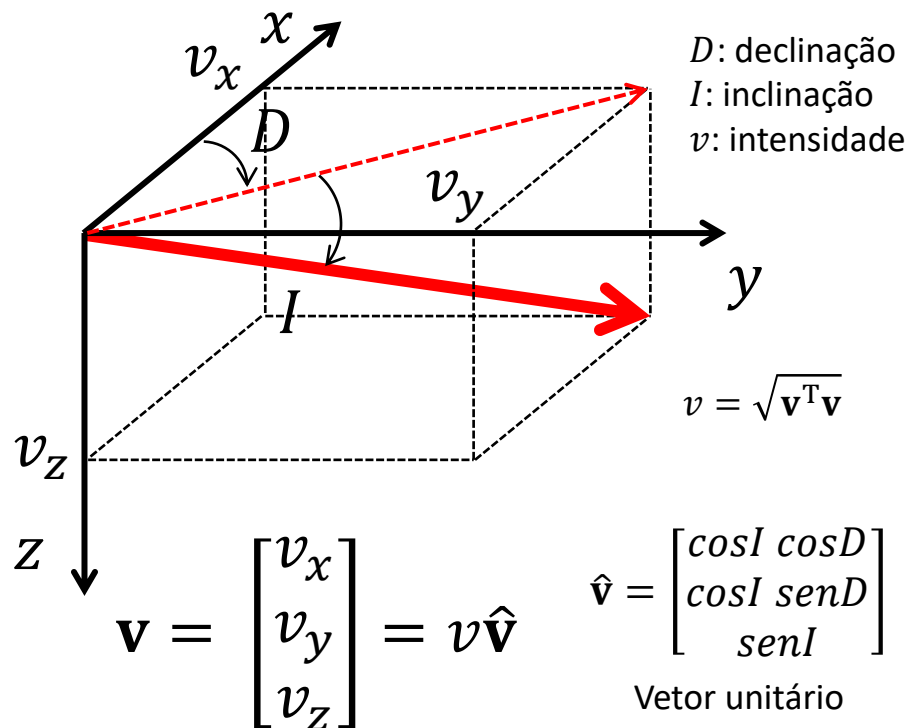
Campo total

$$F_i \gg B_i$$

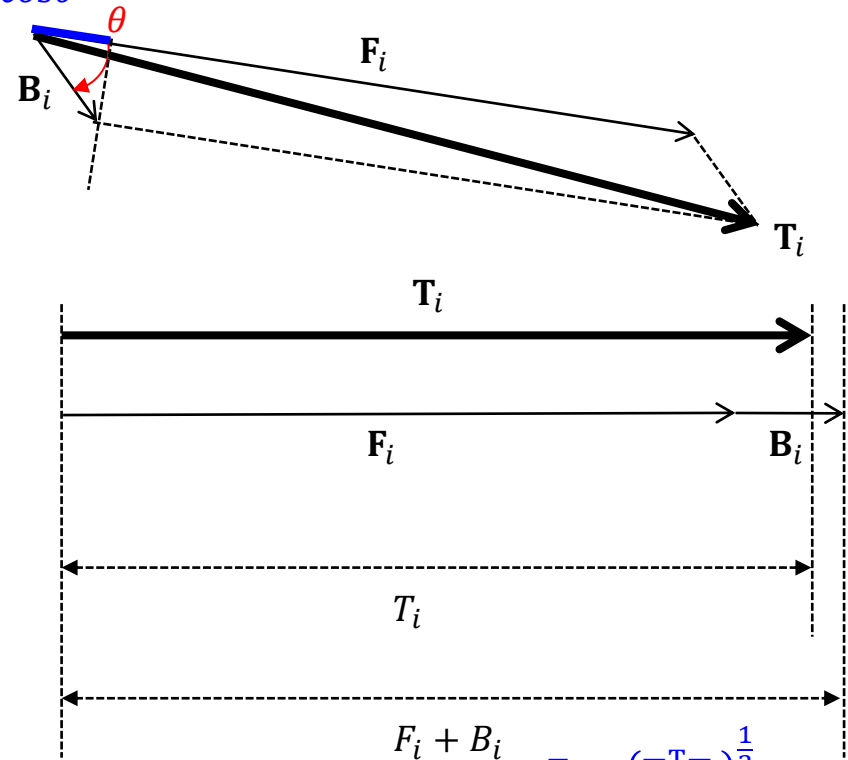
Condição observada
na prática

$$\Delta T_i = T_i - F_i$$

Anomalia de campo total



$B_i \cos \theta$



$$\Delta T_i \approx \hat{\mathbf{F}}_0^T \mathbf{B}_i$$

Blakely (1996)
Langel e Hinze (1998)

Referências

- Blakely, R. J., 1996, Potential theory in gravity and magnetic applications: Cambridge University Press.
- Langel, R. A., e Hinze, W. J., 1998, The magnetic field of the earth's lithosphere: The satellite perspective: Cambridge University Press.