Processing and interpreting potential-field data via Equivalent-Layer Technique

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Observatório Nacional

PINGA
A research group for inverse problems in Geophysics

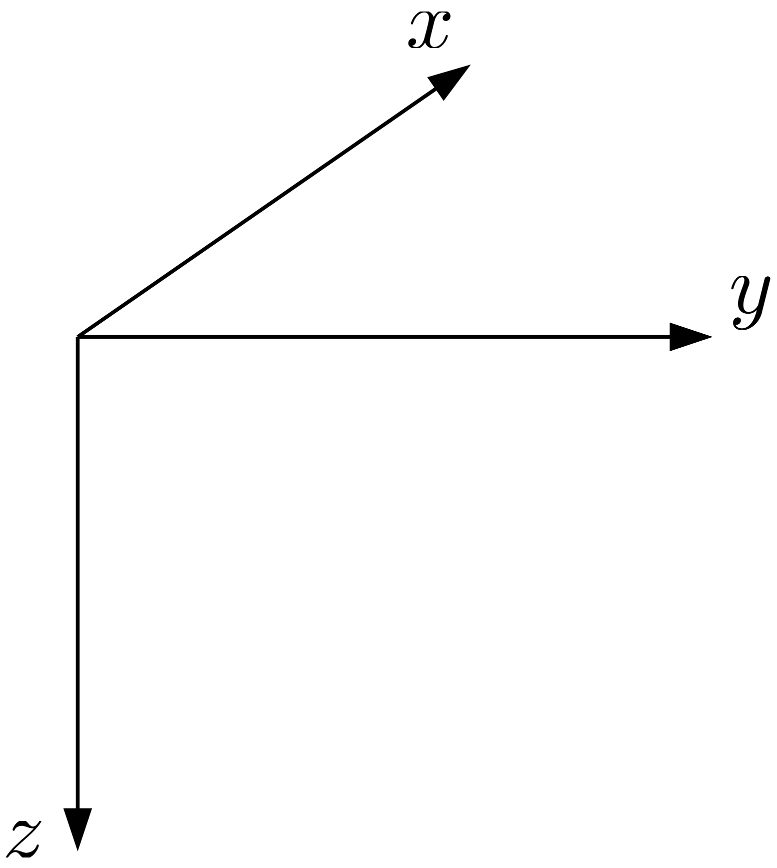
Summary

- Potential-field data
- The Equivalent-Layer (EqL) Technique
- Theoretical aspects
- Some open questions

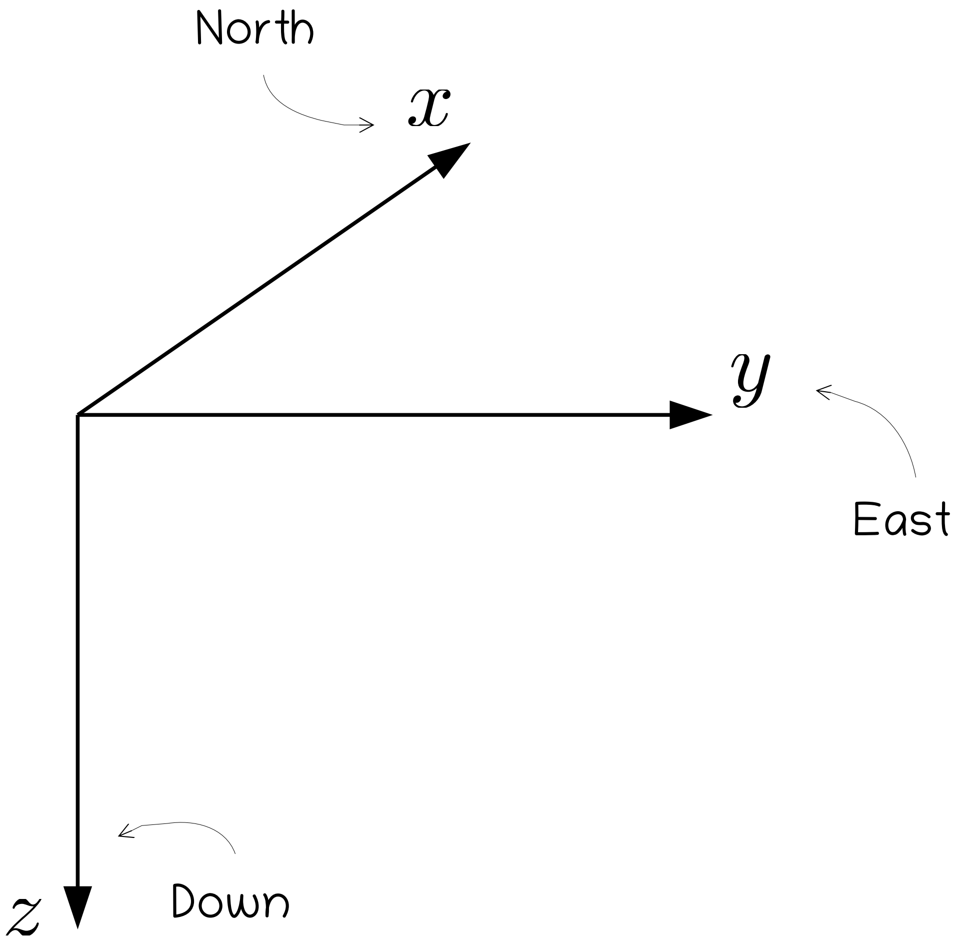
Summary

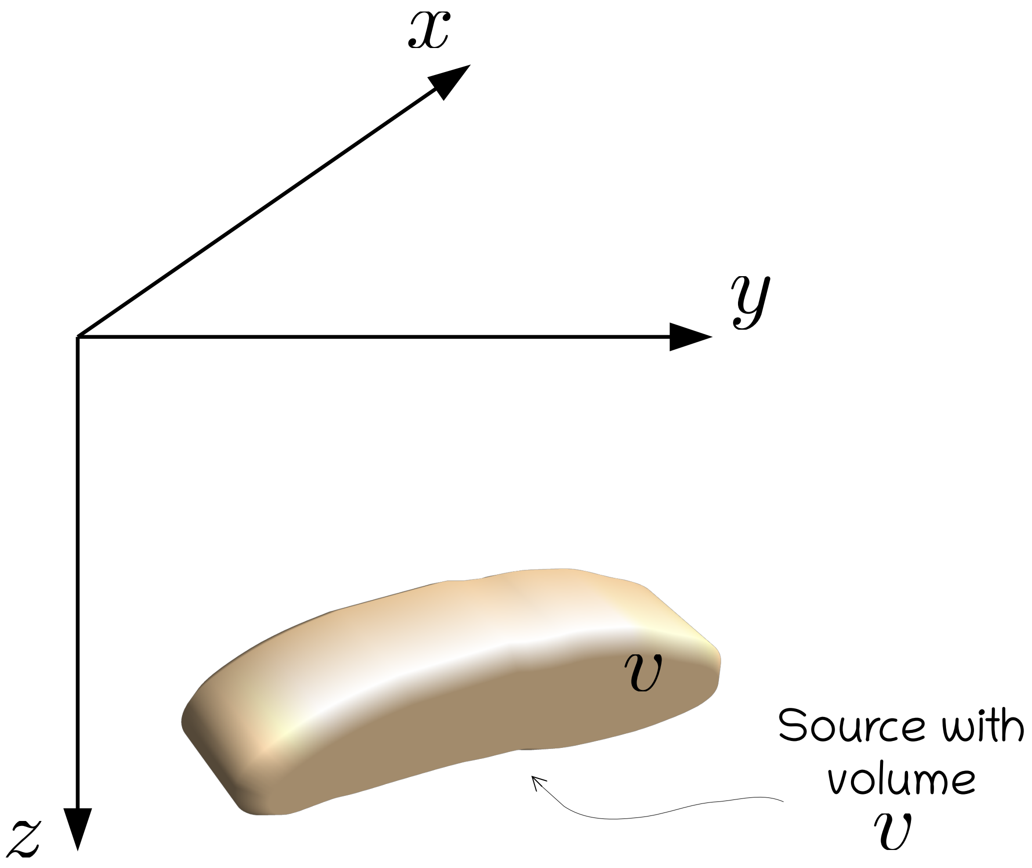
- **Potential-field data**
- The Equivalent-Layer (EqL) Technique
- Theoretical aspects
- Some open questions

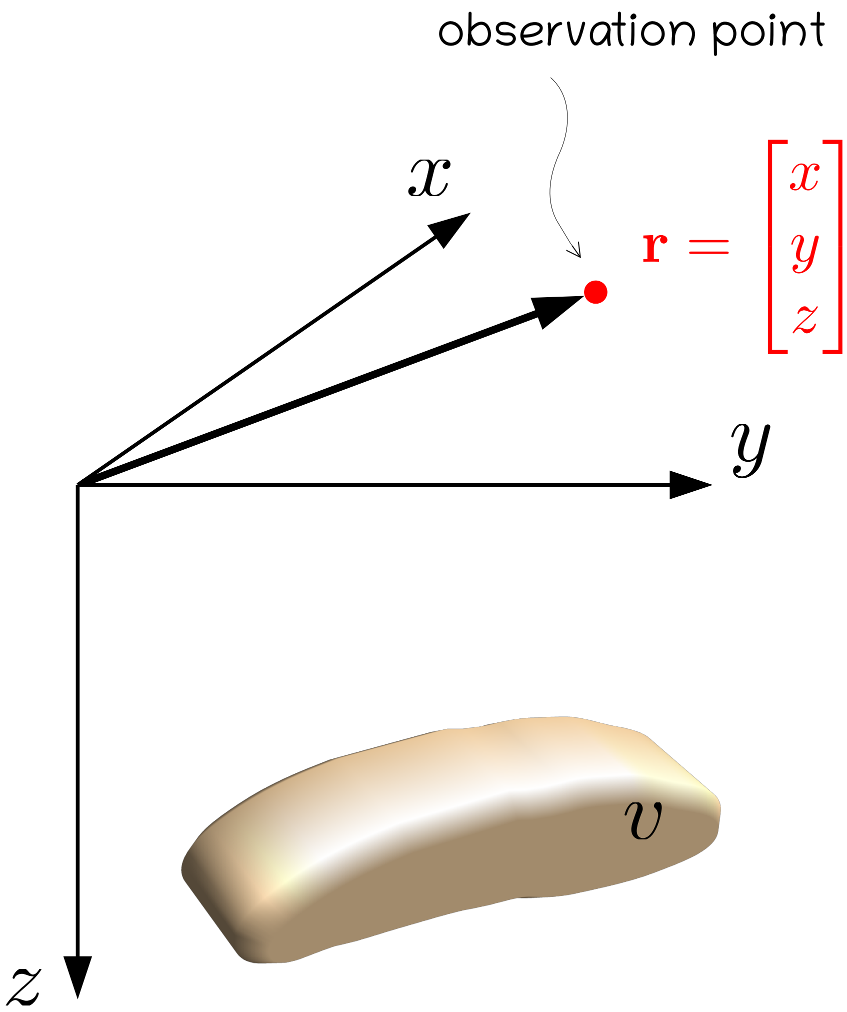
Topocentric Cartesian system



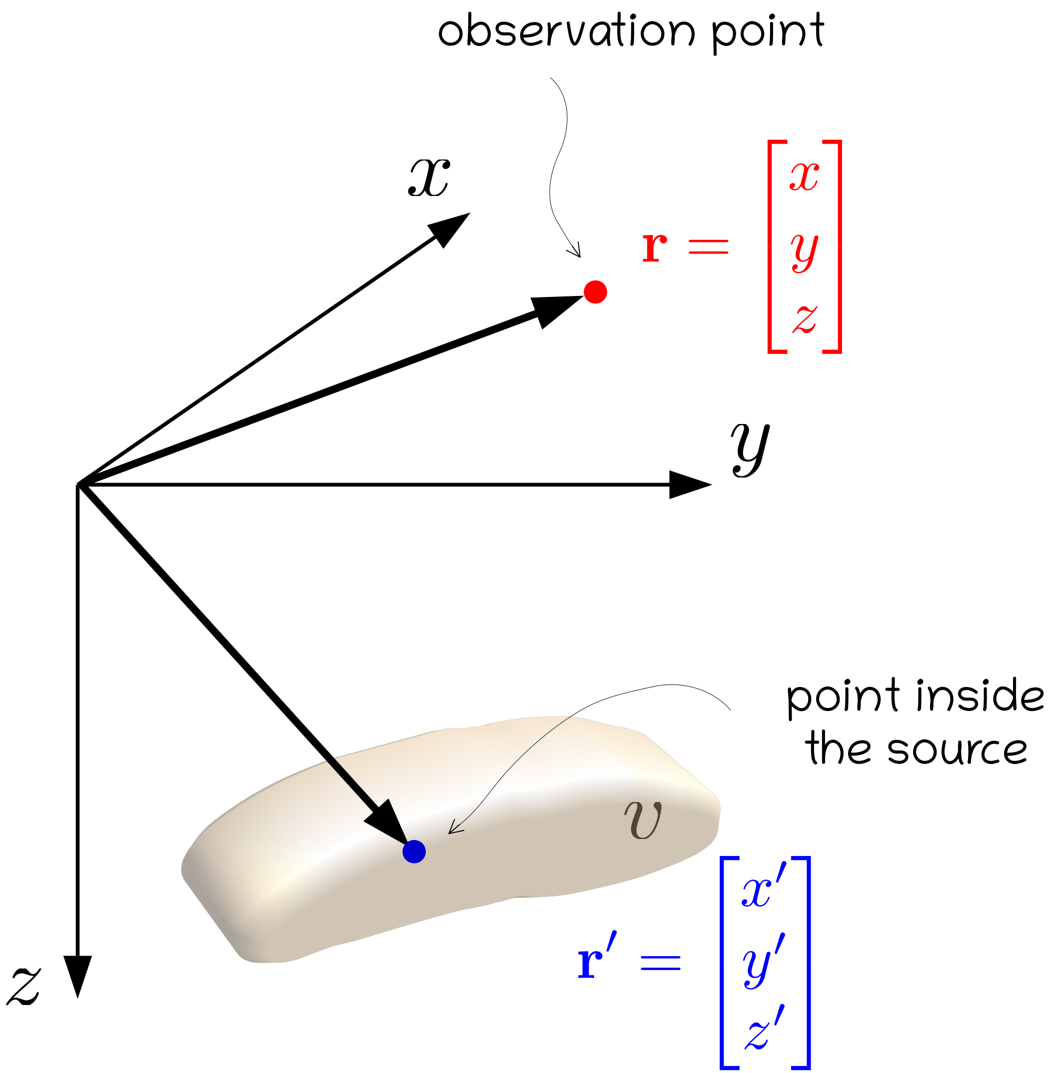
Topocentric Cartesian system



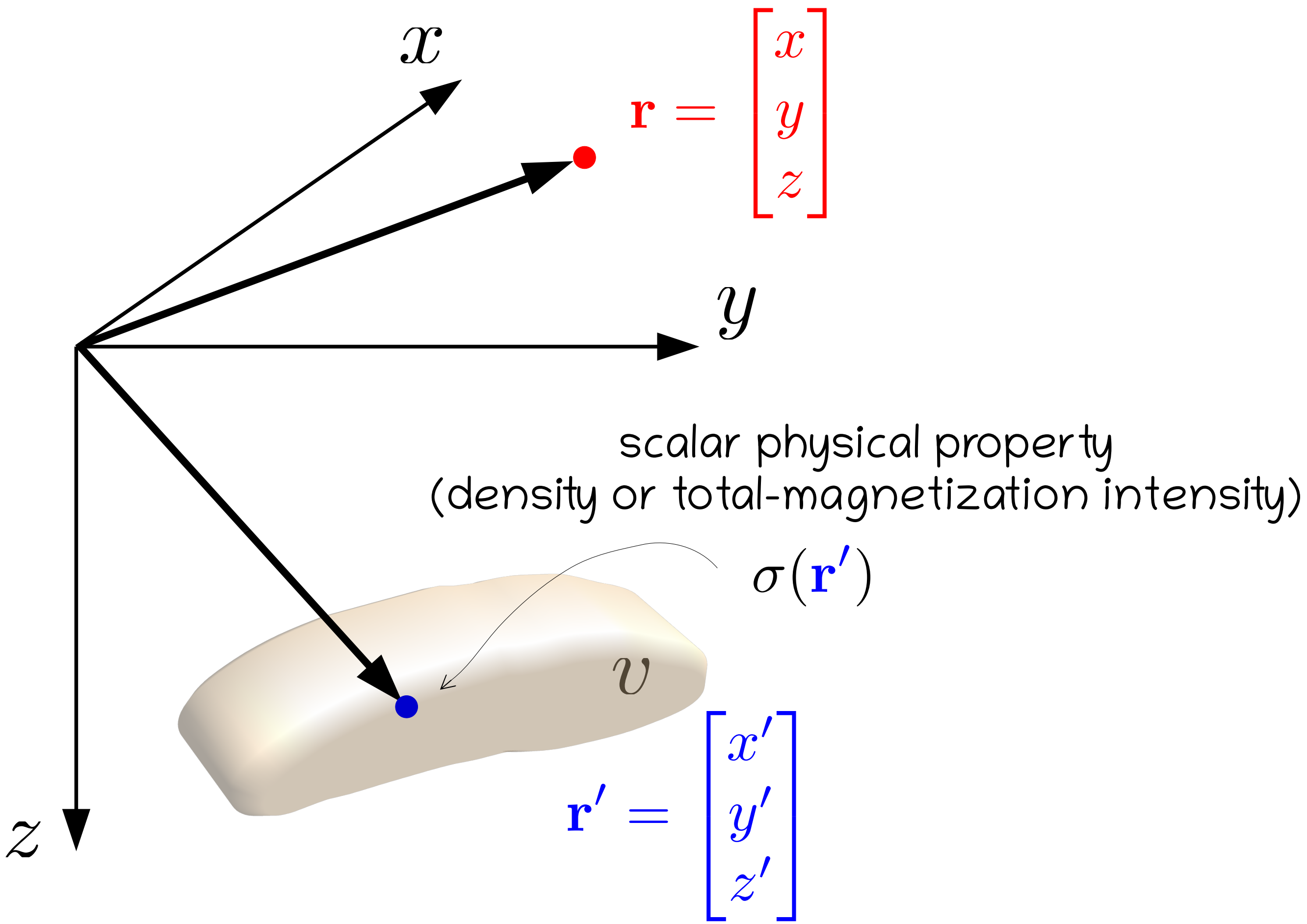

x
y
z
v
Source with
volume
v



observation point
x
$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
y
v
z

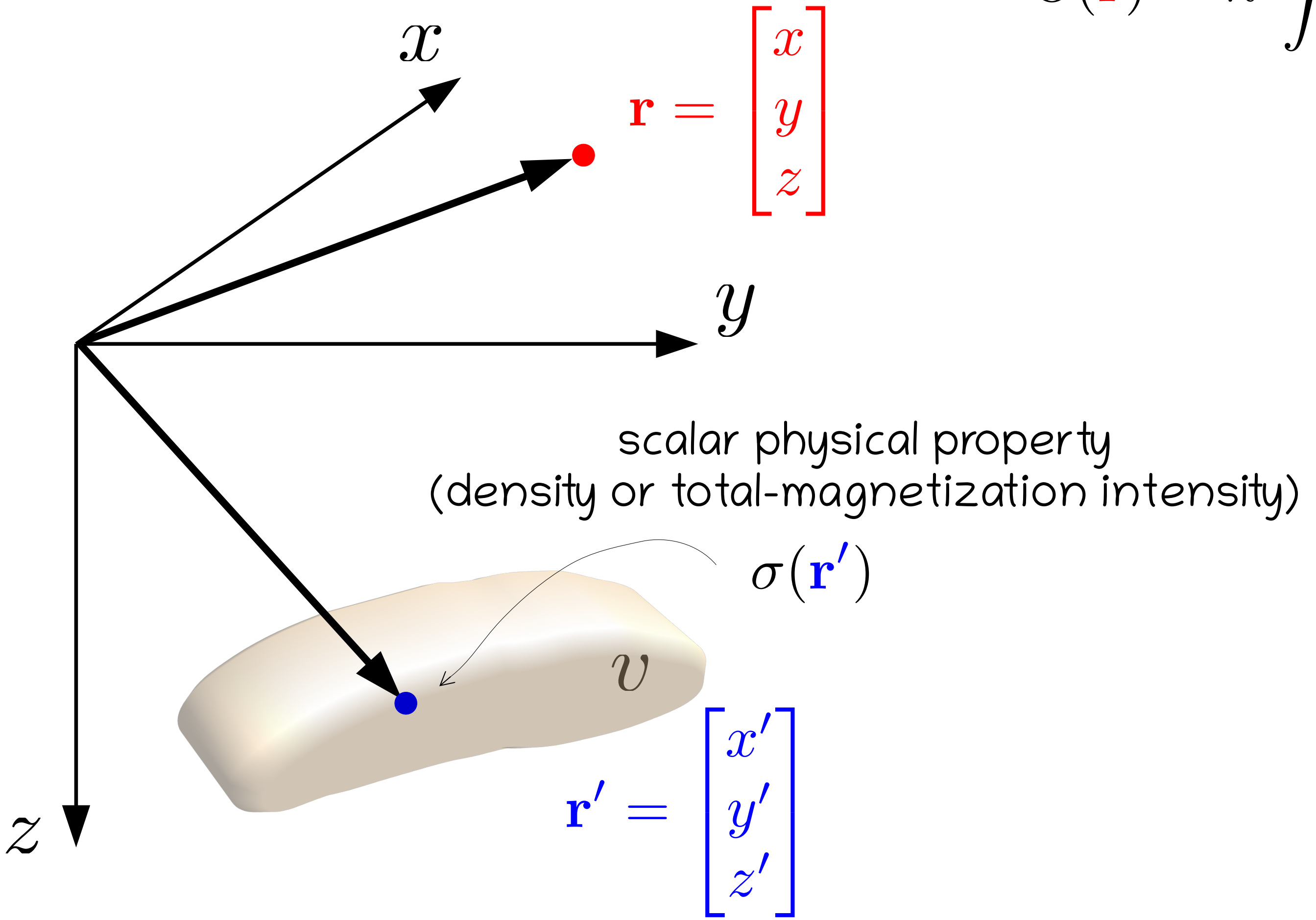


observation point
x
$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
y
point inside the source
v
$\mathbf{r}' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$
z



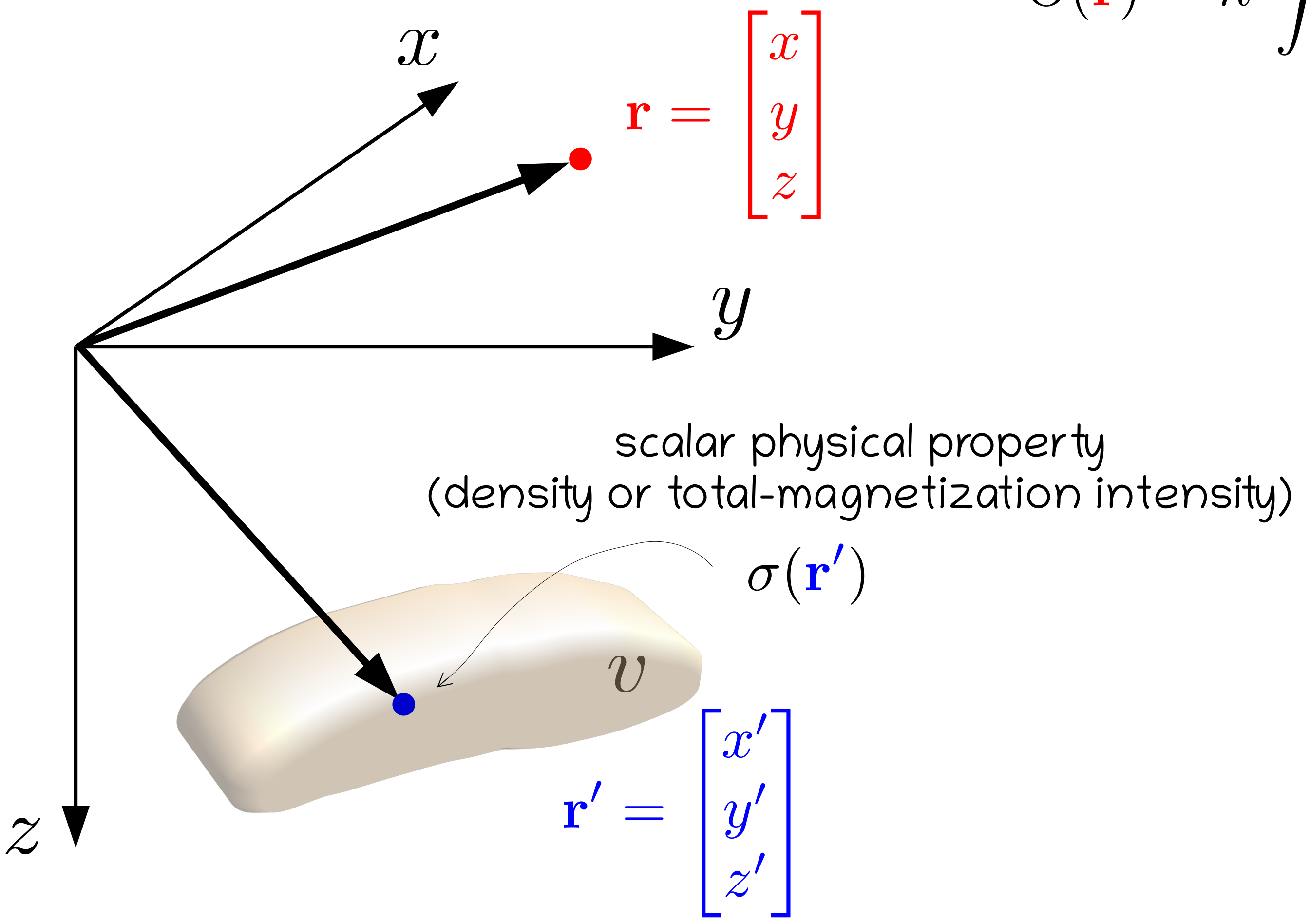
x
$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
y
scalar physical property
(density or total-magnetization intensity)
$\sigma(\mathbf{r}')$
v
$\mathbf{r}' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$
z

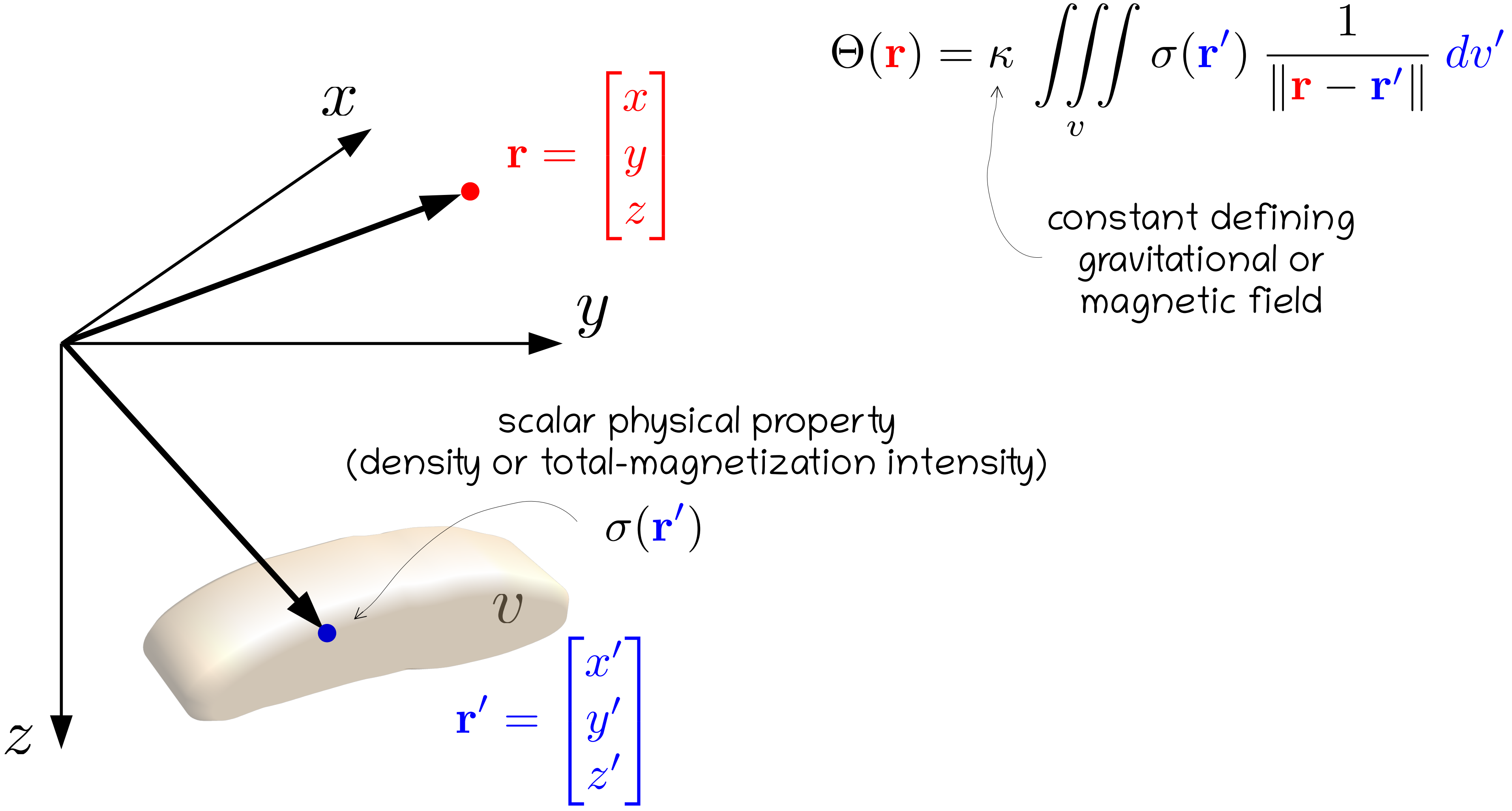
$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$dv' = dx' \, dy' \, dz'$



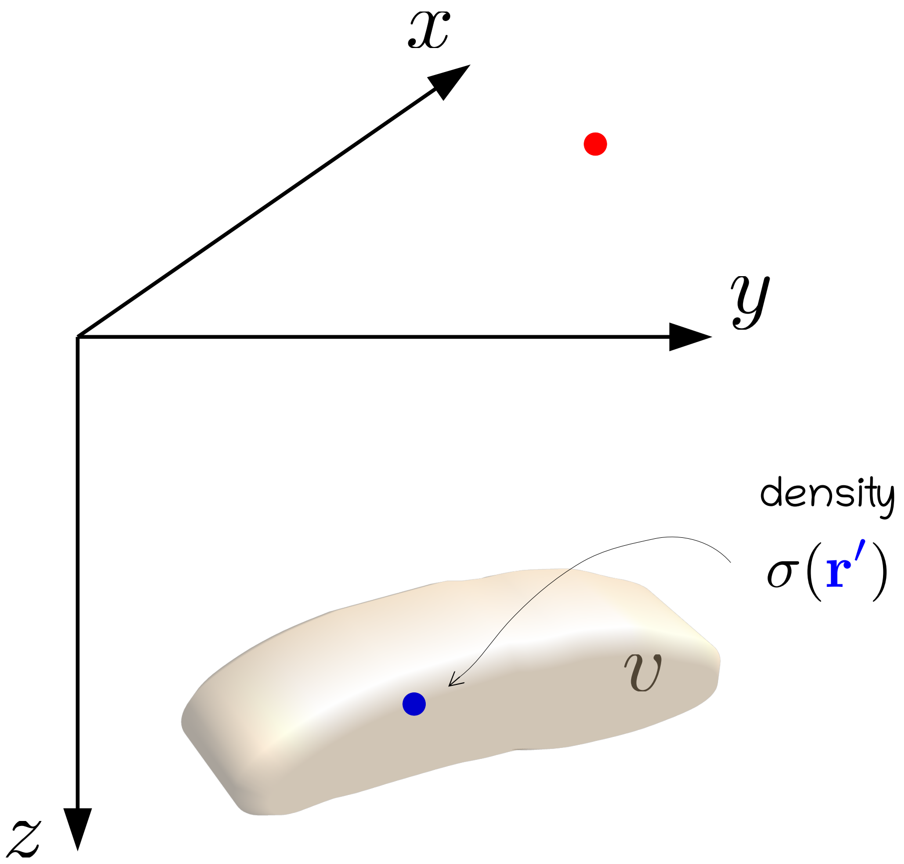


$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$
constant defining
gravitational or
magnetic field
x
y
z
$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
scalar physical property
(density or total-magnetization intensity)
$\sigma(\mathbf{r}')$
v
$\mathbf{r}' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$

Gravitational field

density

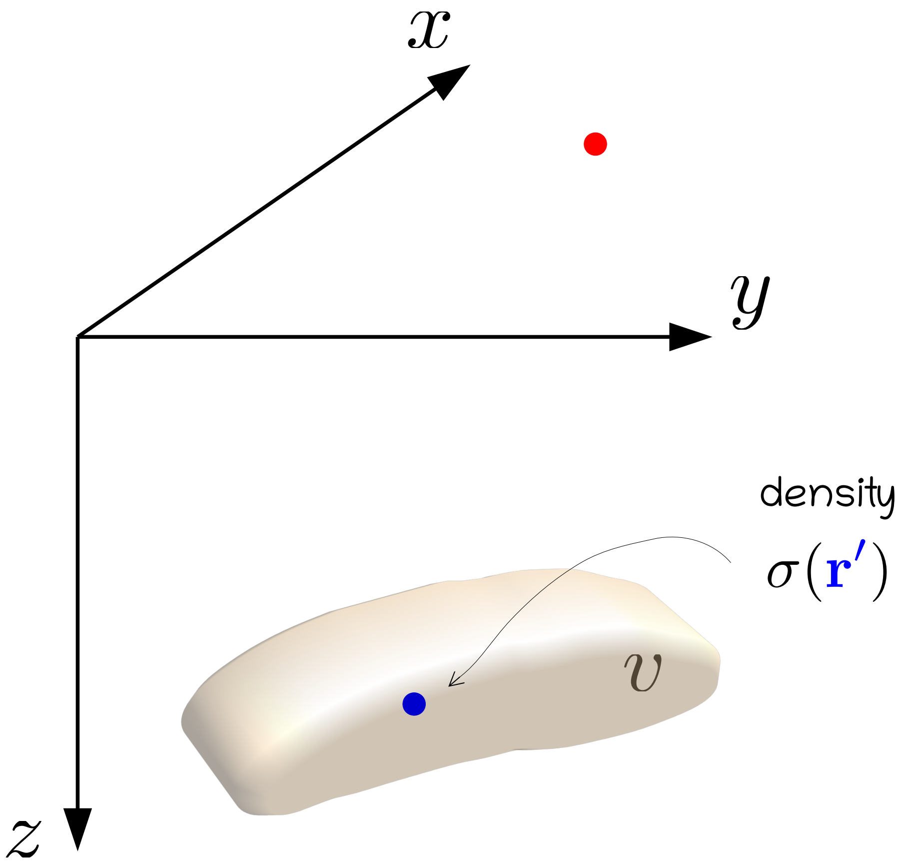
$$\Theta(\mathbf{r}) = \kappa \iiint_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



Gravitational field

density

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

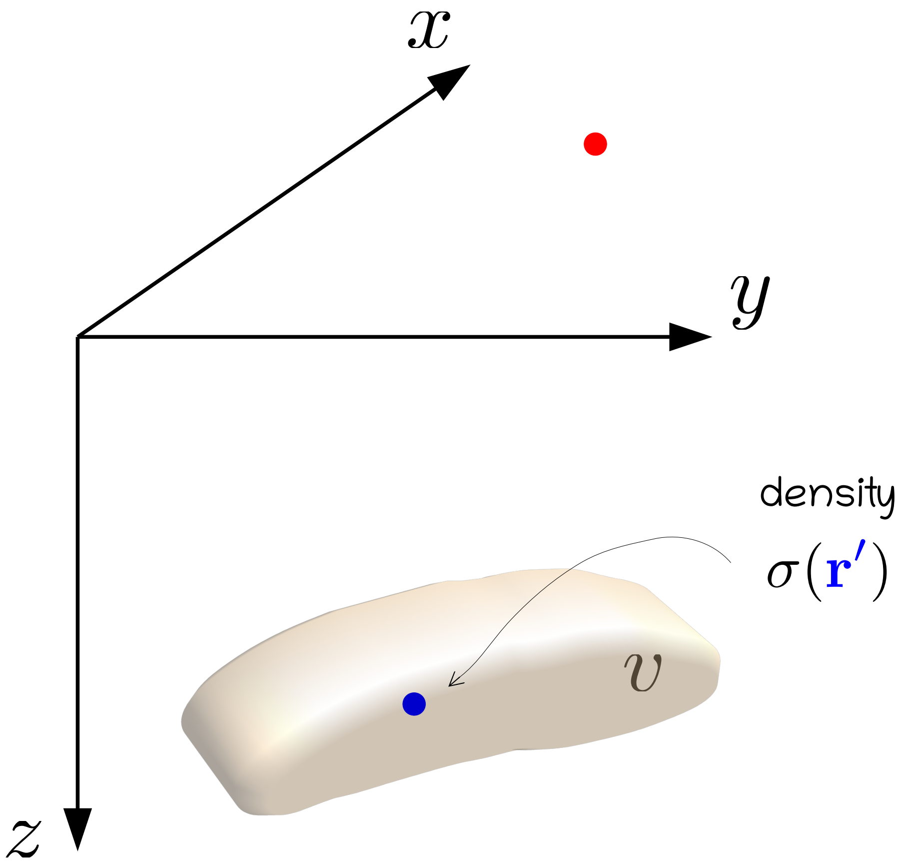


$\Theta(\mathbf{r})$ gravitational potential

Gravitational field

density

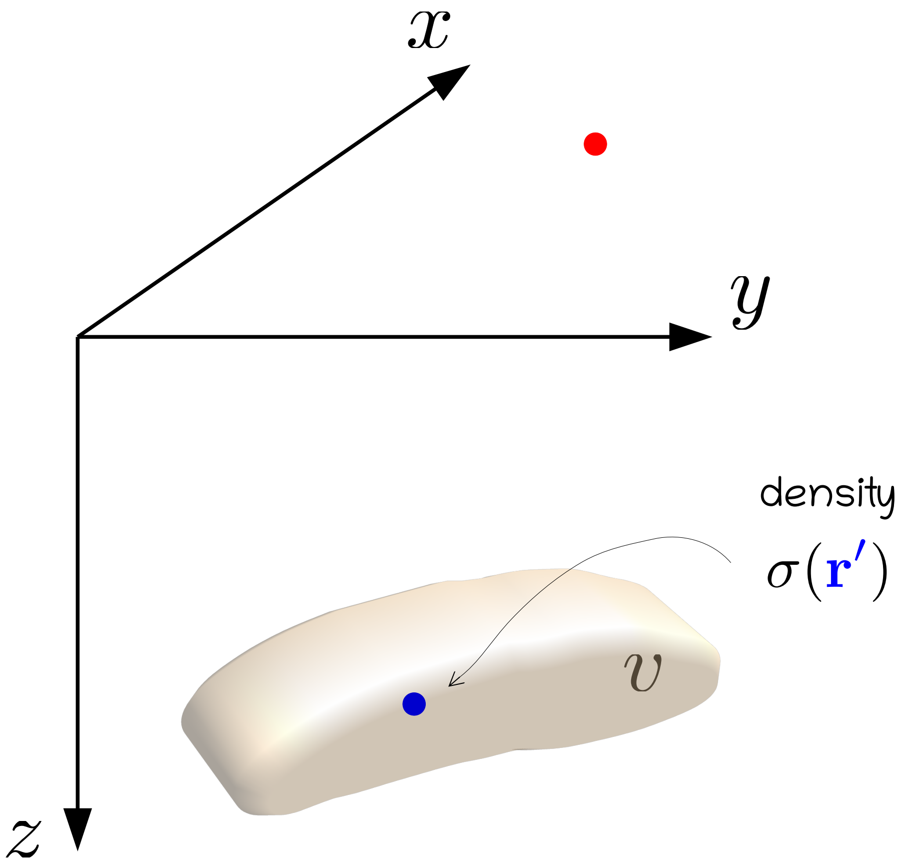
$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



$\Theta(\mathbf{r})$ gravitational potential

$\partial_z \Theta(\mathbf{r})$ gravity disturbance

Gravitational field



density

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$\Theta(\mathbf{r})$ gravitational potential

$\partial_z \Theta(\mathbf{r})$ gravity disturbance

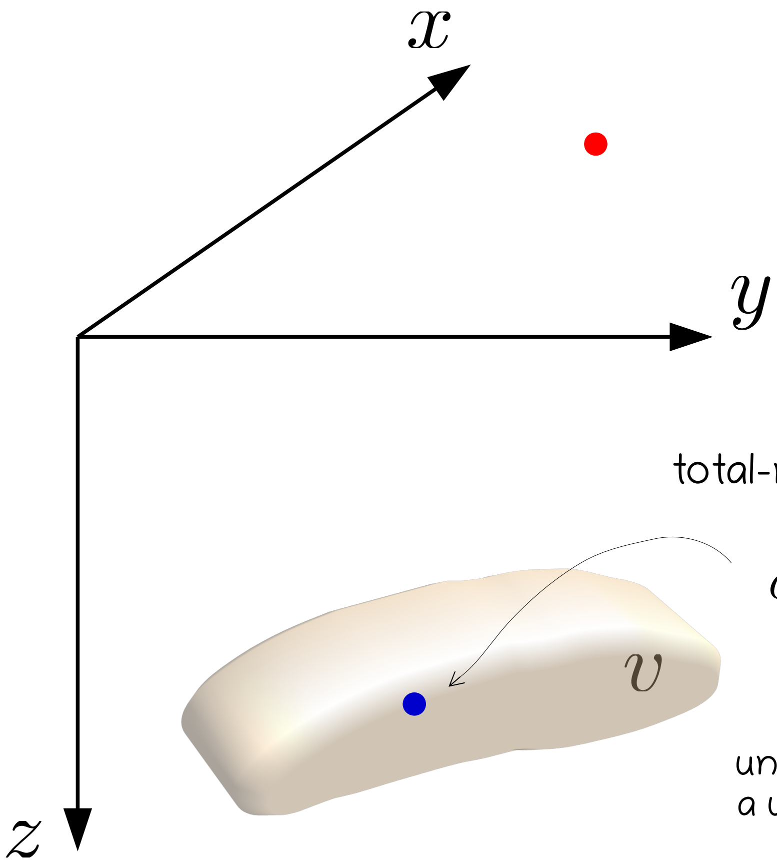
$\partial_{\alpha\beta} \Theta(\mathbf{r})$ gravity gradient tensor

x, y, z

Magnetic field

total-magnetization
intensity

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}')\, \frac{1}{\|\mathbf{r}-\mathbf{r}'\|}\, dv'$$



total-magnetization
vector

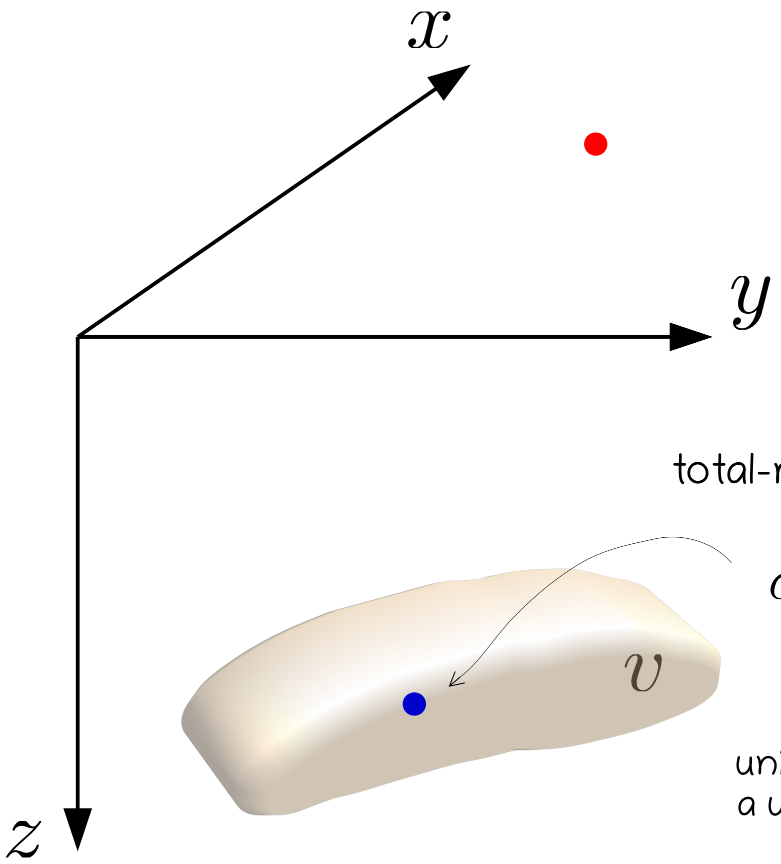
$$\sigma(\mathbf{r}')\ \hat{\mathbf{h}}$$

unit vector defining
a uniform direction

Magnetic field

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



$-\partial_h \Theta(\mathbf{r})$ mag. scalar potential

total-magnetization vector

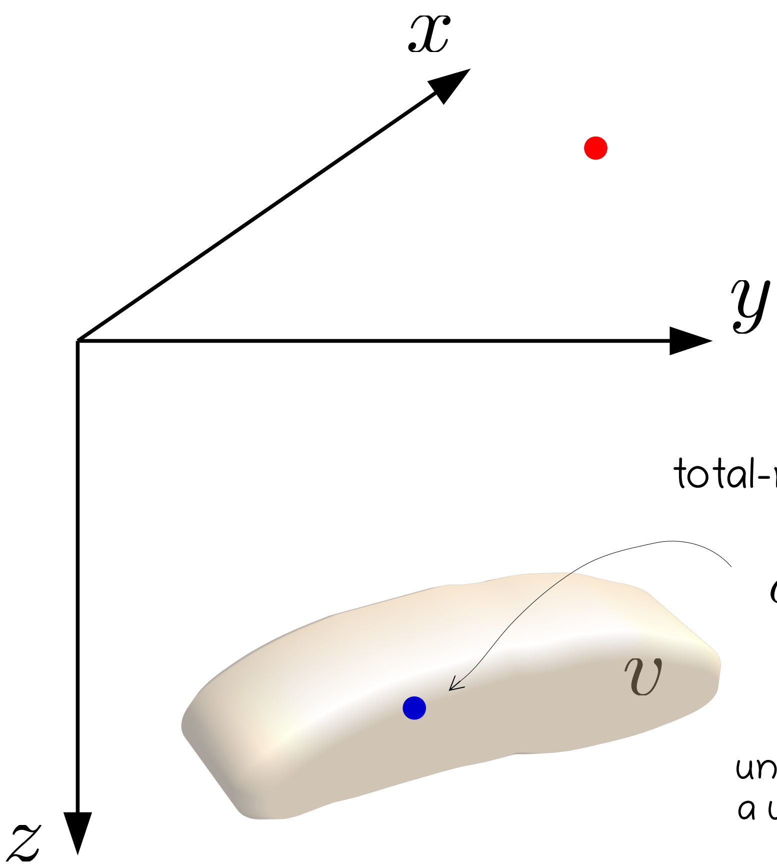
$\sigma(\mathbf{r}') \, \hat{\mathbf{h}}$

unit vector defining a uniform direction

Magnetic field

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



$-\partial_h \Theta(\mathbf{r})$ mag. scalar potential

$$\partial_h \Theta(\mathbf{r}) = \nabla \Theta(\mathbf{r})^\top \hat{\mathbf{h}}$$

total-magnetization vector

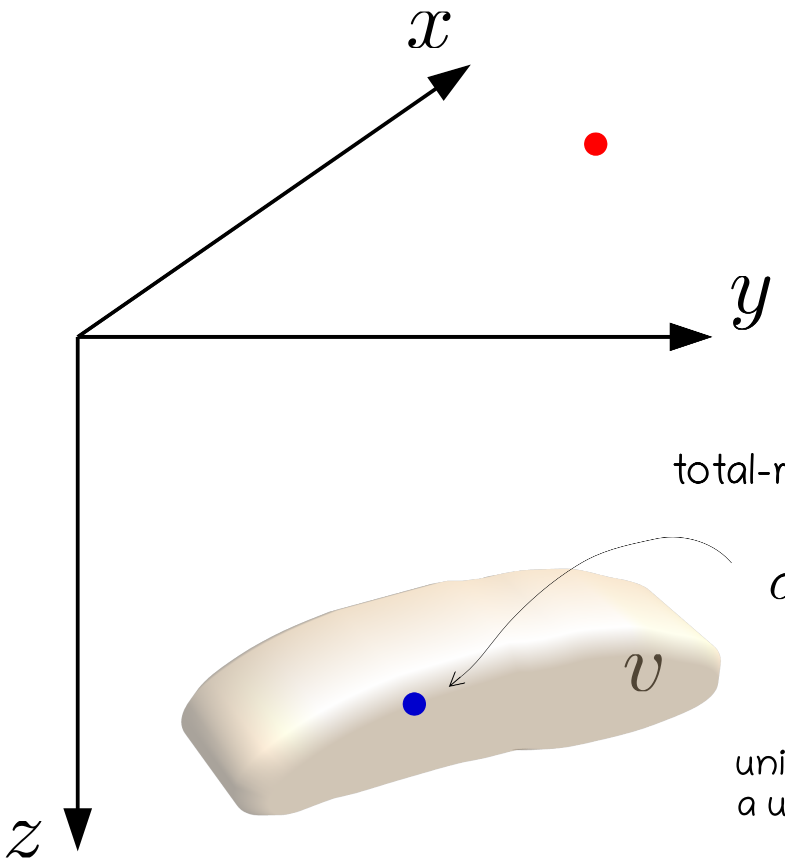
$$\sigma(\mathbf{r}') \, \hat{\mathbf{h}}$$

unit vector defining a uniform direction

Magnetic field

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$



$-\partial_h \Theta(\mathbf{r})$ mag. scalar potential

$\partial_{\alpha h} \Theta(\mathbf{r})$ mag. induction field

x, y, z

total-magnetization vector

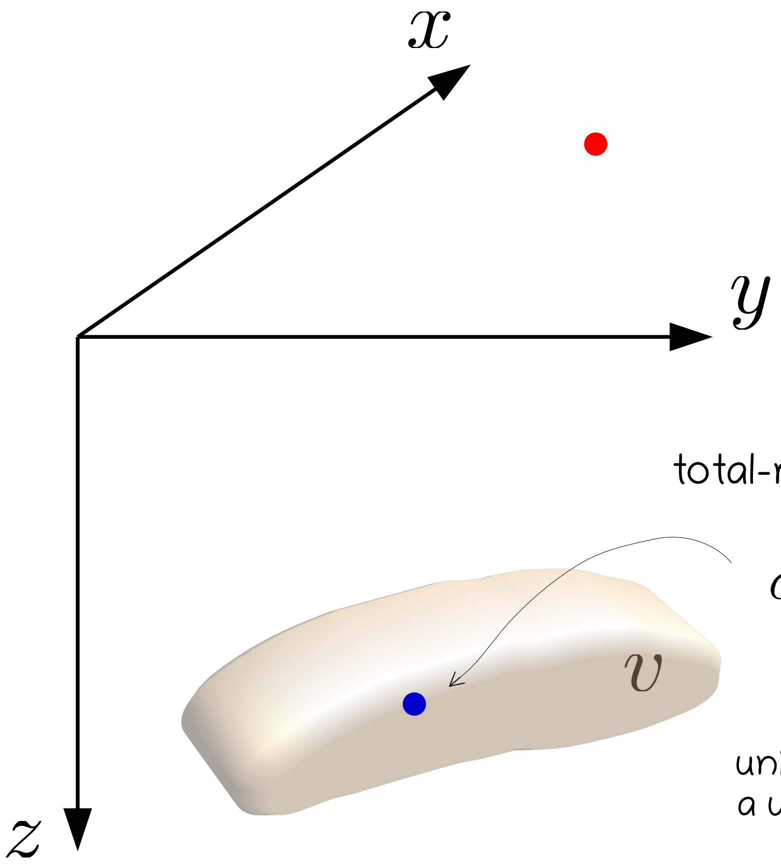
$\sigma(\mathbf{r}')\ \hat{\mathbf{h}}$

unit vector defining a uniform direction

Magnetic field

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



total-magnetization vector

$\sigma(\mathbf{r}') \, \hat{\mathbf{h}}$

unit vector defining a uniform direction

$-\partial_h \Theta(\mathbf{r})$ mag. scalar potential

$\partial_{\alpha h} \Theta(\mathbf{r})$ mag. induction field

x, y, z

$$\partial_{\alpha h} \Theta(\mathbf{r}) = \partial_{\alpha} \nabla \Theta(\mathbf{r})^{\top} \hat{\mathbf{h}}$$

Magnetic field

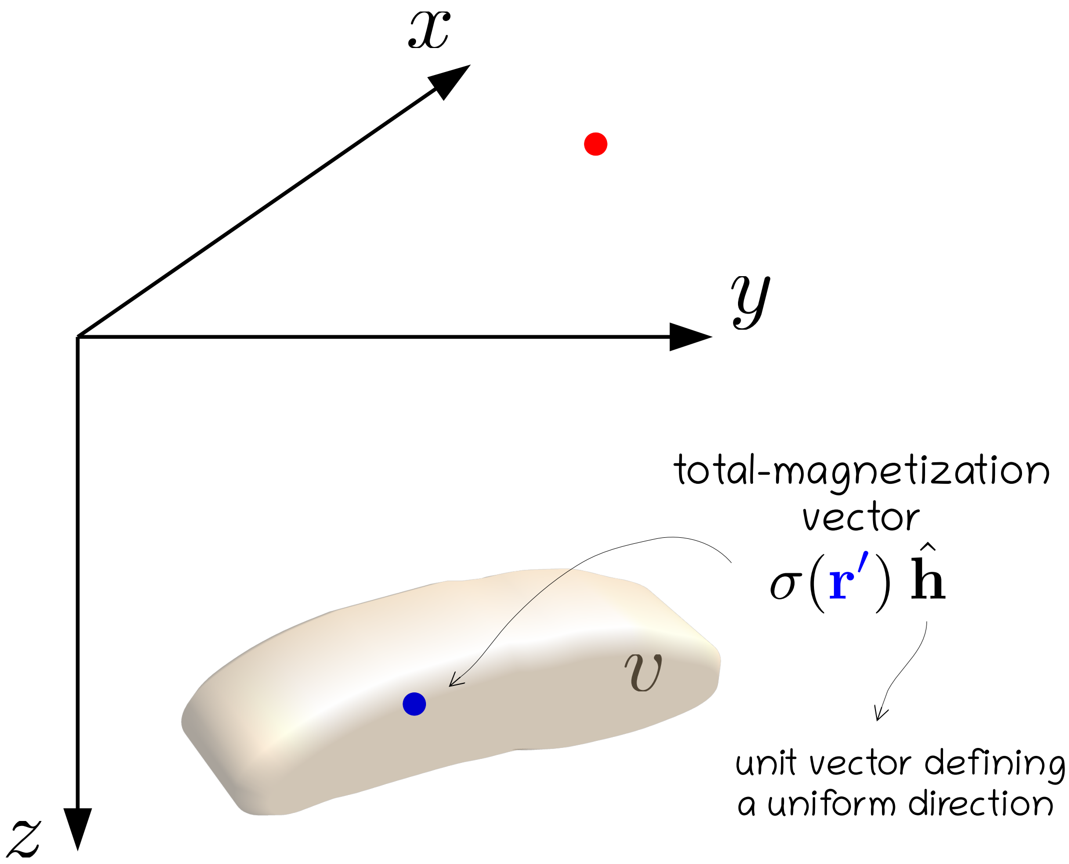
total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$-\partial_h \Theta(\mathbf{r})$ mag. scalar potential

$\partial_{\alpha h} \Theta(\mathbf{r})$ mag. induction field

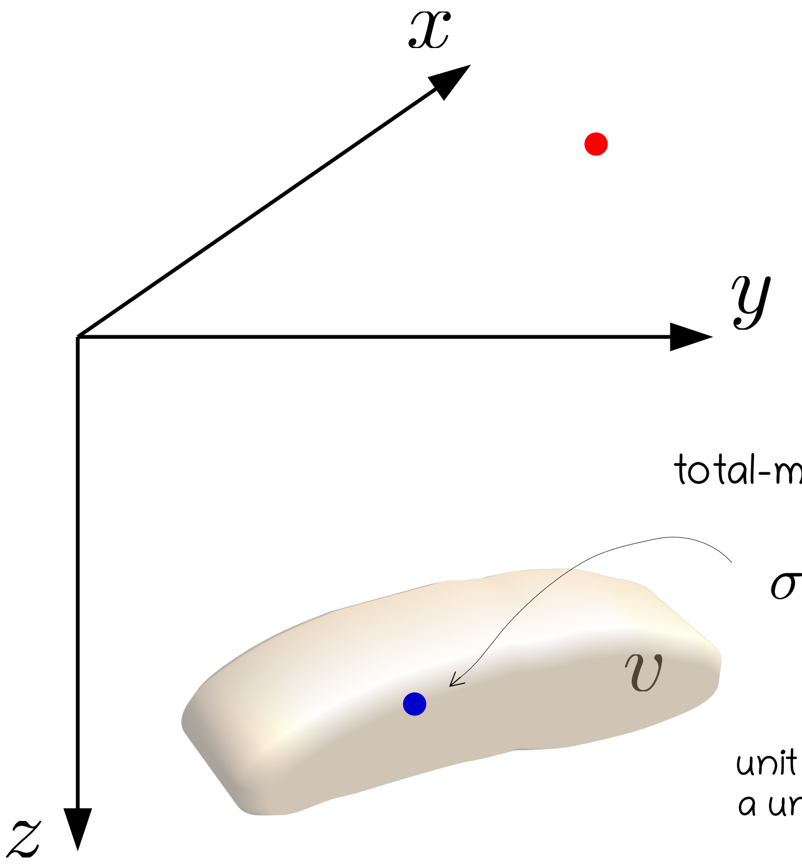
$\partial_{th} \Theta(\mathbf{r})$ approx. total-field anomaly



Magnetic field

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



total-magnetization vector

$\sigma(\mathbf{r}') \, \hat{\mathbf{h}}$

unit vector defining a uniform direction

$-\partial_{h}\Theta(\mathbf{r})$ mag. scalar potential

$\partial_{\alpha h}\Theta(\mathbf{r})$ mag. induction field

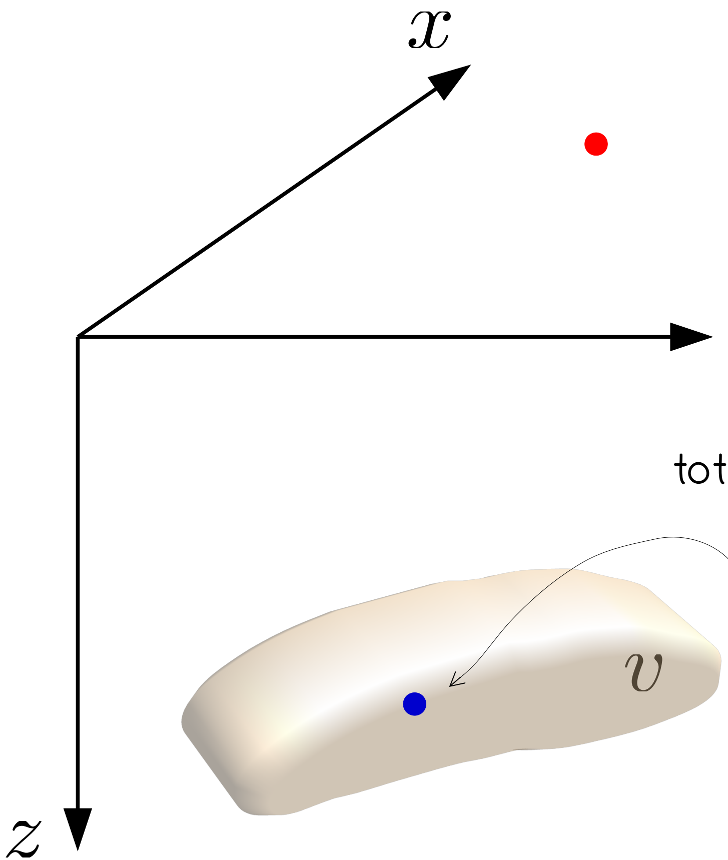
$\partial_{th}\Theta(\mathbf{r})$ approx. total-field anomaly

$$\partial_{th}\Theta(\mathbf{r}) = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Theta}(\mathbf{r}) \, \hat{\mathbf{h}}$$

Magnetic field

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



total-magnetization vector

$\sigma(\mathbf{r}') \, \hat{\mathbf{h}}$

unit vector defining a uniform direction

$-\partial_h \Theta(\mathbf{r})$ mag. scalar potential

$\partial_{\alpha h} \Theta(\mathbf{r})$ mag. induction field

$\partial_{th} \Theta(\mathbf{r})$ approx. total-field anomaly

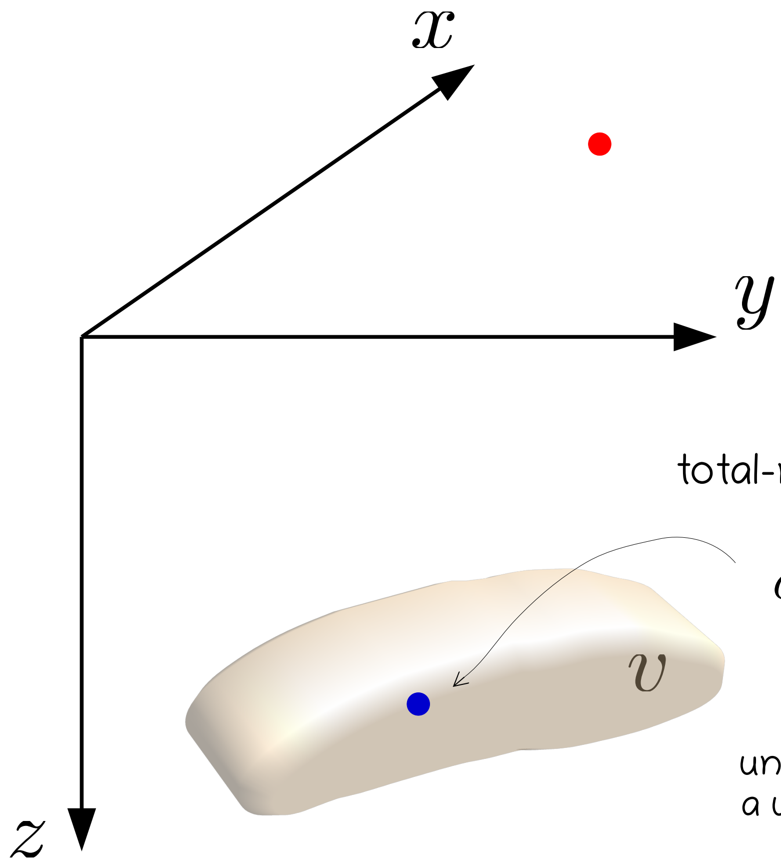
$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Theta}(\mathbf{r}) \, \hat{\mathbf{h}}$$

unit vector defining a constant direction for the main geomagnetic field

Magnetic field

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



total-magnetization vector

$\sigma(\mathbf{r}') \, \hat{\mathbf{h}}$

unit vector defining a uniform direction

$-\partial_h \Theta(\mathbf{r})$ mag. scalar potential

$\partial_{\alpha h} \Theta(\mathbf{r})$ mag. induction field

$\partial_{th} \Theta(\mathbf{r})$ approx. total-field anomaly

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Theta}(\mathbf{r}) \, \hat{\mathbf{h}}$$

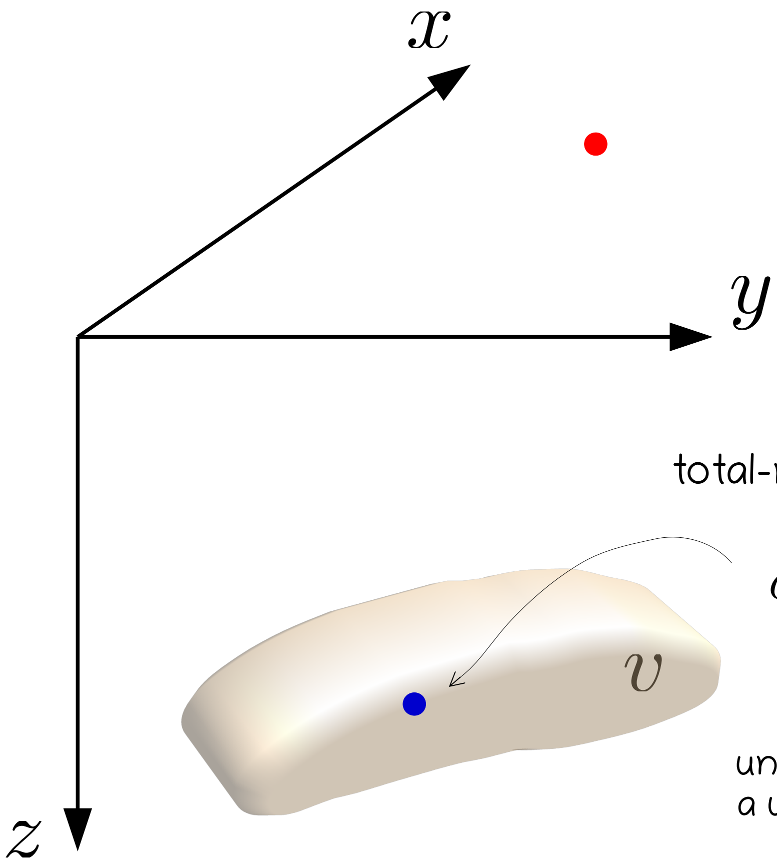
unit vector defining a constant direction for the main geomagnetic field

$$\mathbf{H}_{\Theta}(\mathbf{r}) = \begin{bmatrix} \partial_{xx}\Theta(\mathbf{r}) & \partial_{xy}\Theta(\mathbf{r}) & \partial_{xz}\Theta(\mathbf{r}) \\ \partial_{xy}\Theta(\mathbf{r}) & \partial_{yy}\Theta(\mathbf{r}) & \partial_{yz}\Theta(\mathbf{r}) \\ \partial_{xz}\Theta(\mathbf{r}) & \partial_{yz}\Theta(\mathbf{r}) & \partial_{zz}\Theta(\mathbf{r}) \end{bmatrix}$$

Magnetic field

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$



total-magnetization vector

$\sigma(\mathbf{r}')\,\hat{\mathbf{h}}$

unit vector defining a uniform direction

$-\partial_h \Theta(\mathbf{r})$ mag. scalar potential

$\partial_{\alpha h} \Theta(\mathbf{r})$ mag. induction field

$\partial_{th} \Theta(\mathbf{r})$ approx. total-field anomaly

$\partial_{zz} \Theta(\mathbf{r})$ RTP anomaly

reduced-to-the-pole

Summary

- **Potential-field data**
- The Equivalent-Layer (EqL) Technique
- Theoretical aspects
- Some open questions

Summary

- Potential-field data
- **The Equivalent-Layer (EqL) Technique**
- Theoretical aspects
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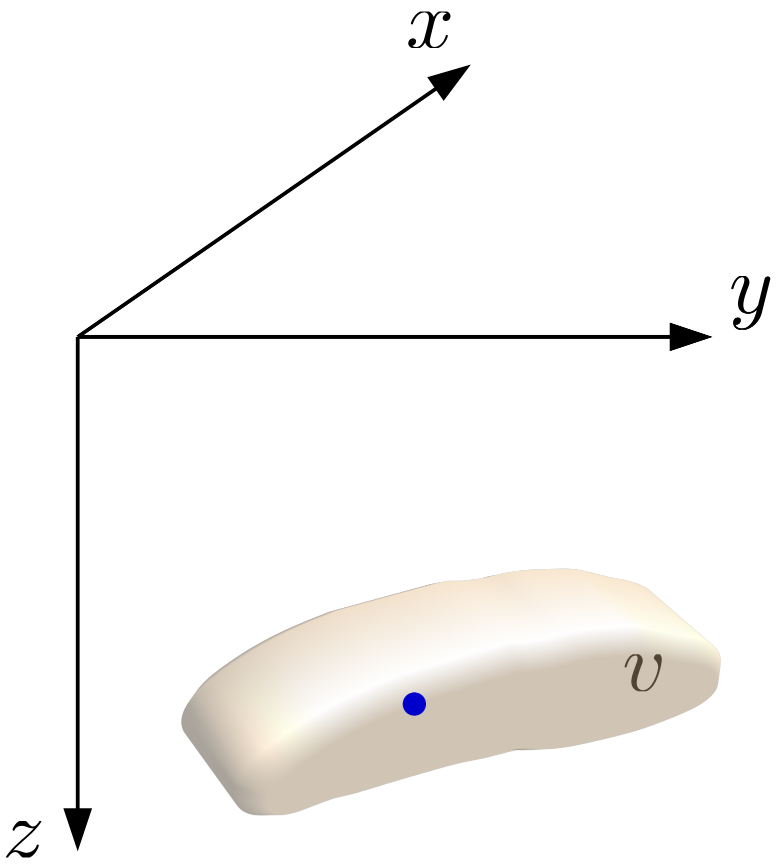
Let us return to our source ...



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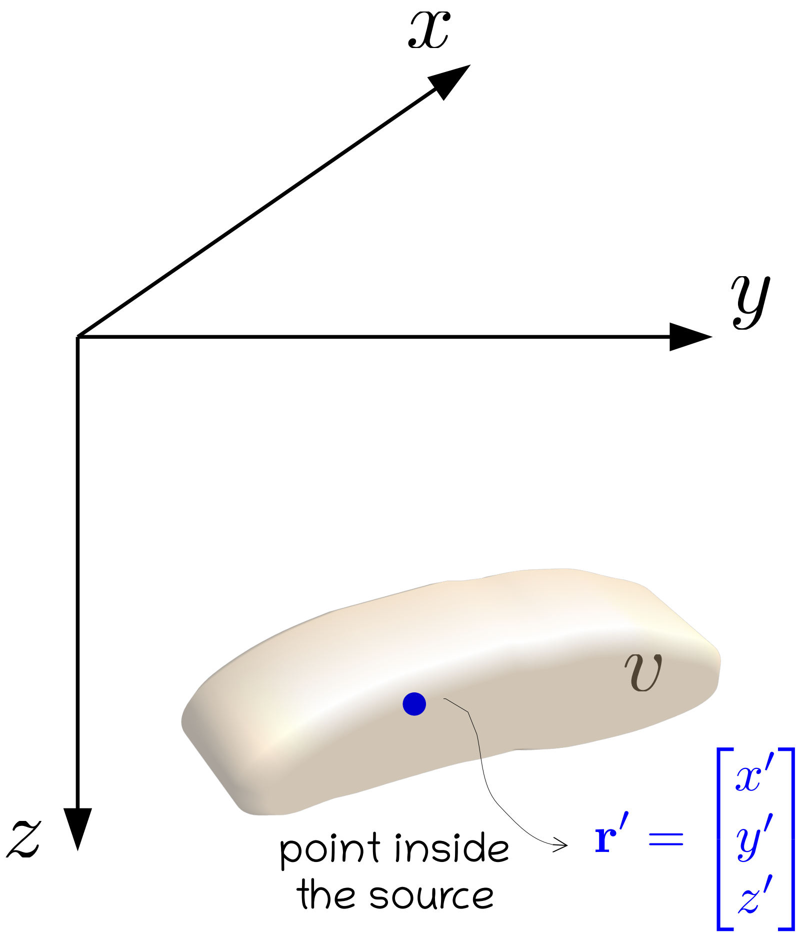

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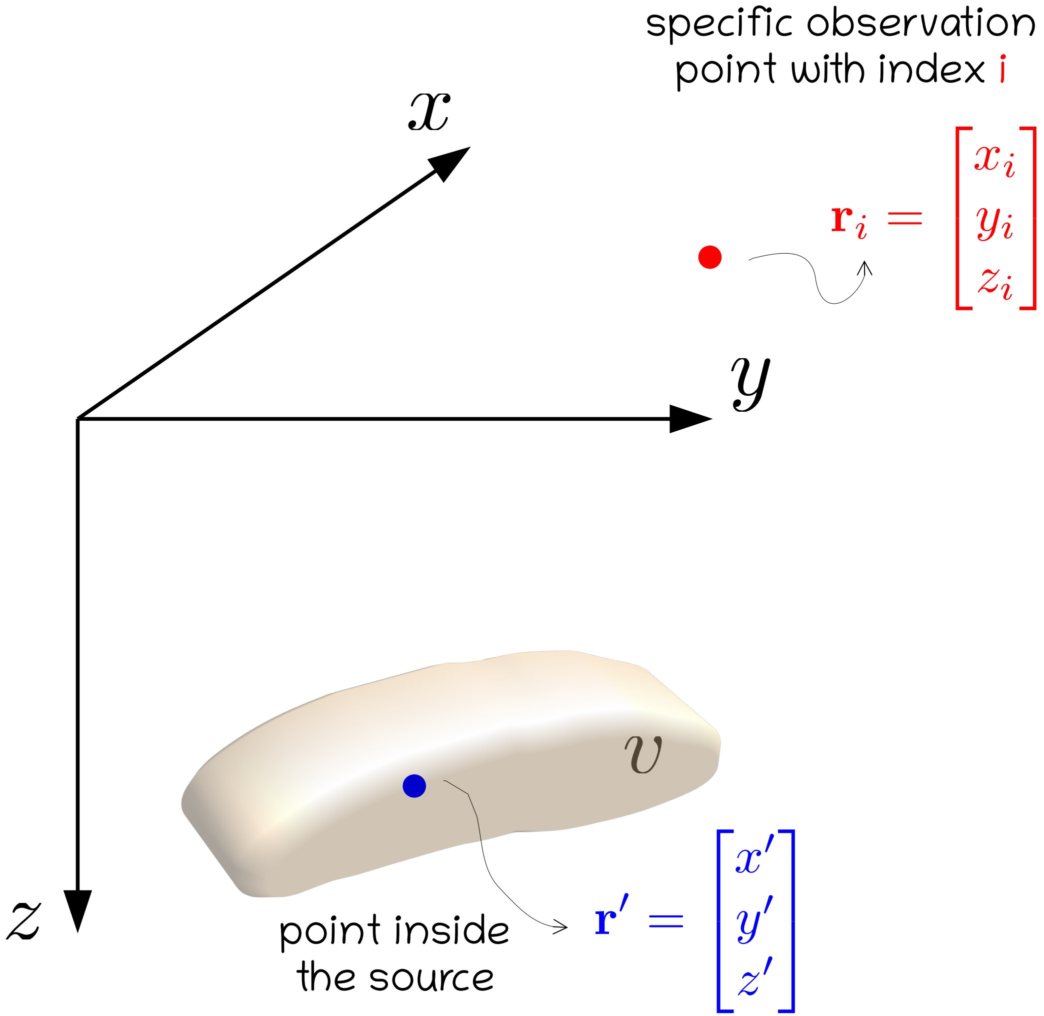
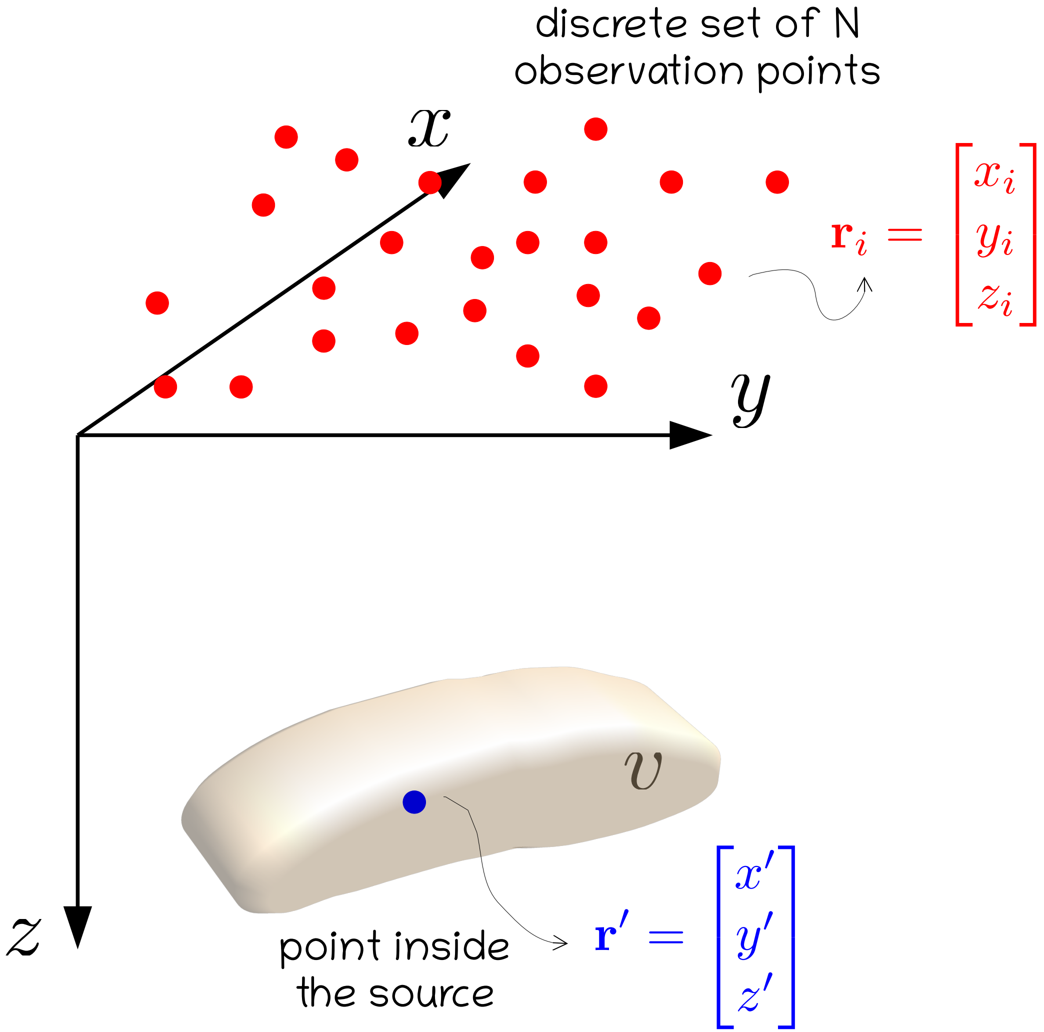
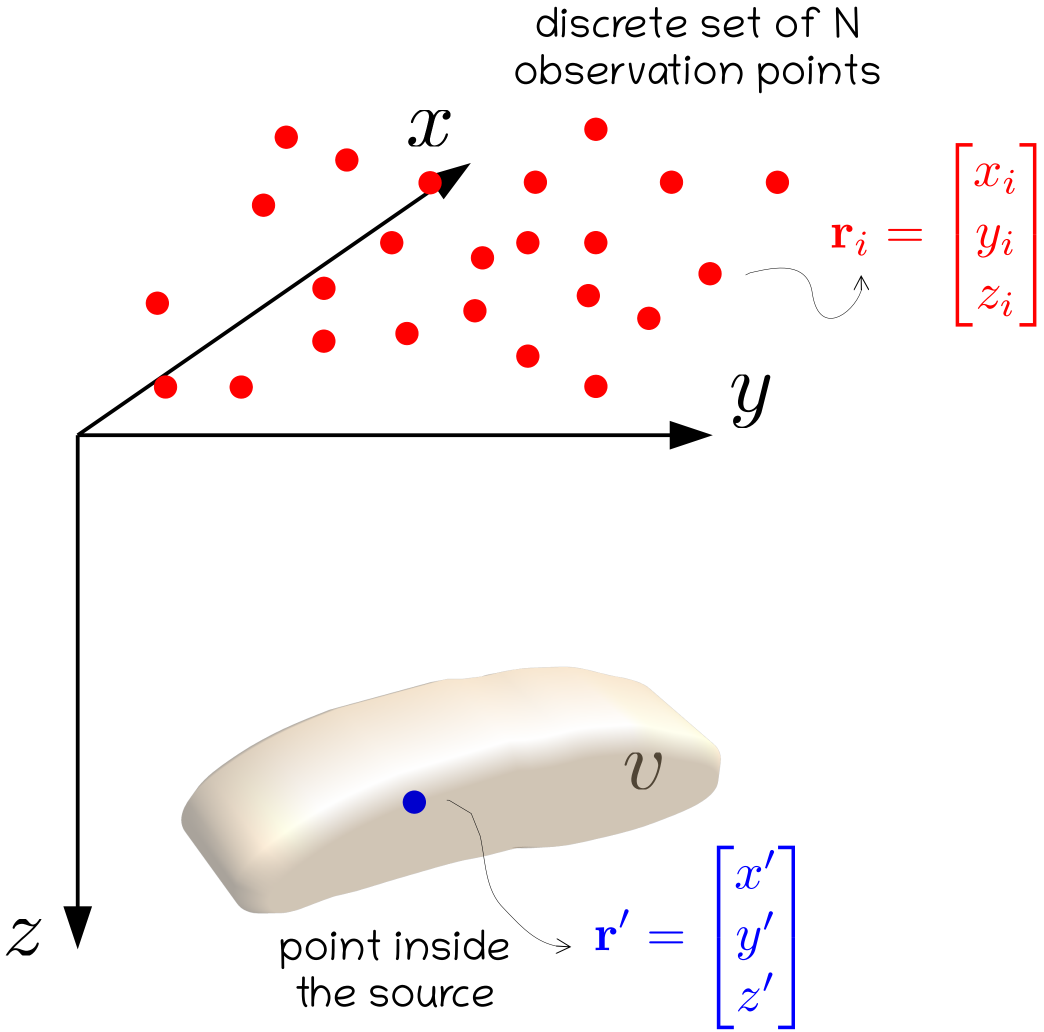




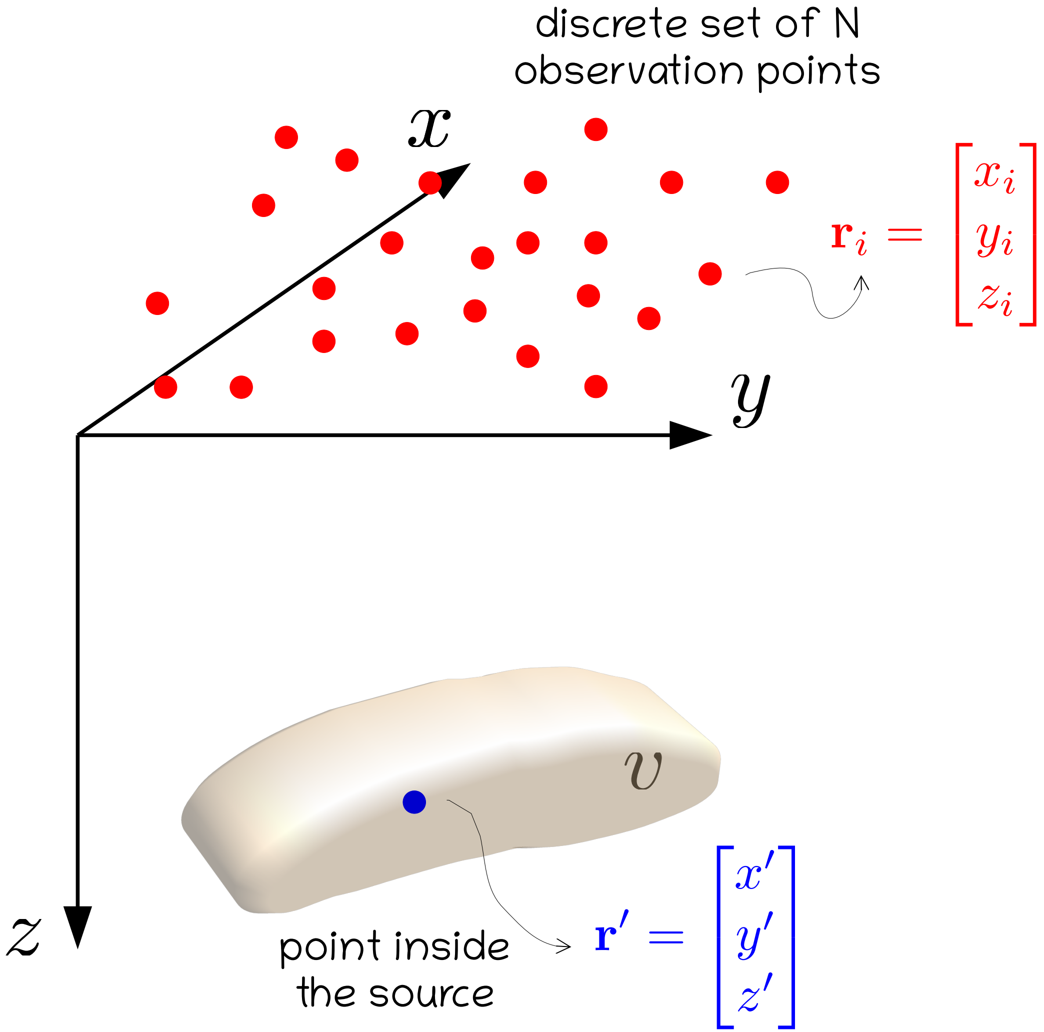
discrete set of N observation points
x
y
z
$\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$
v
point inside the source
$\mathbf{r}' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$



$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance

Gravitational field



density

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance

Gravitational field

density

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

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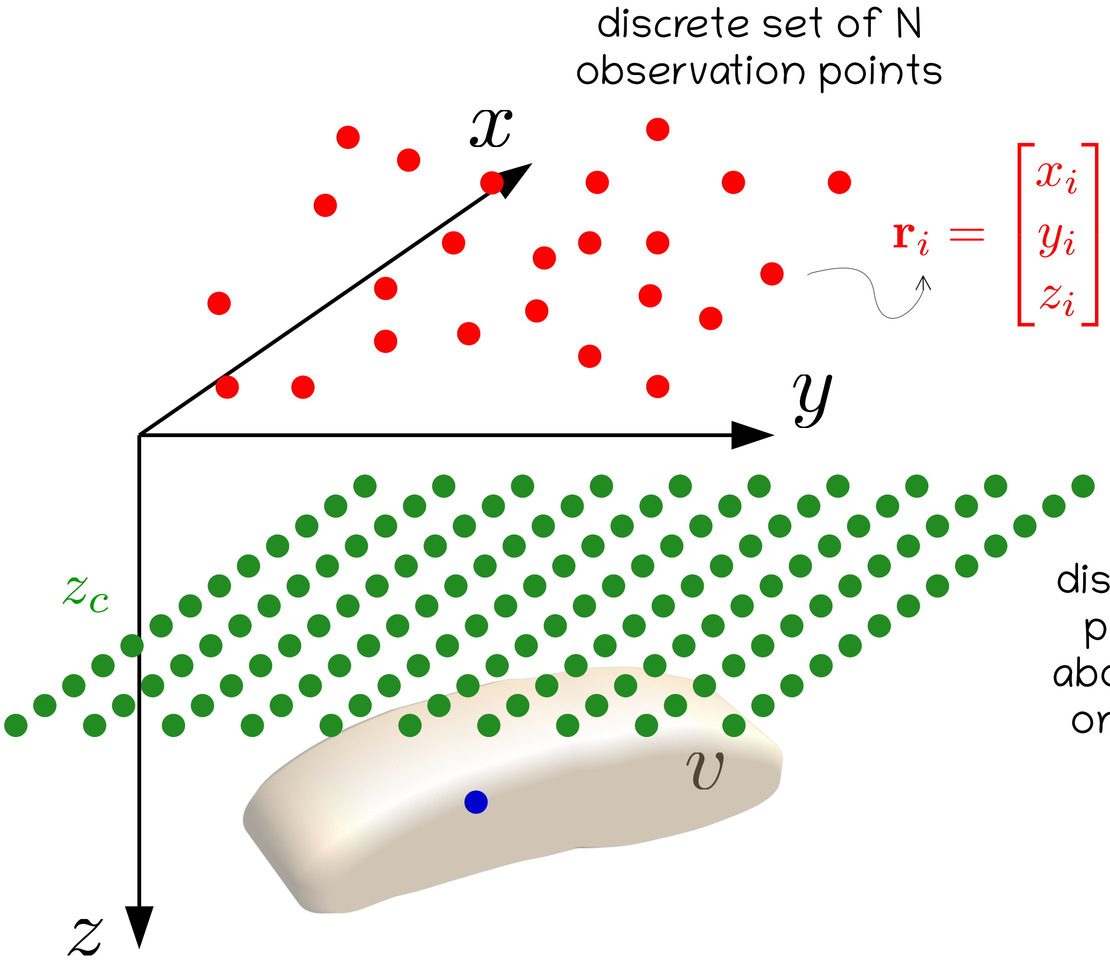
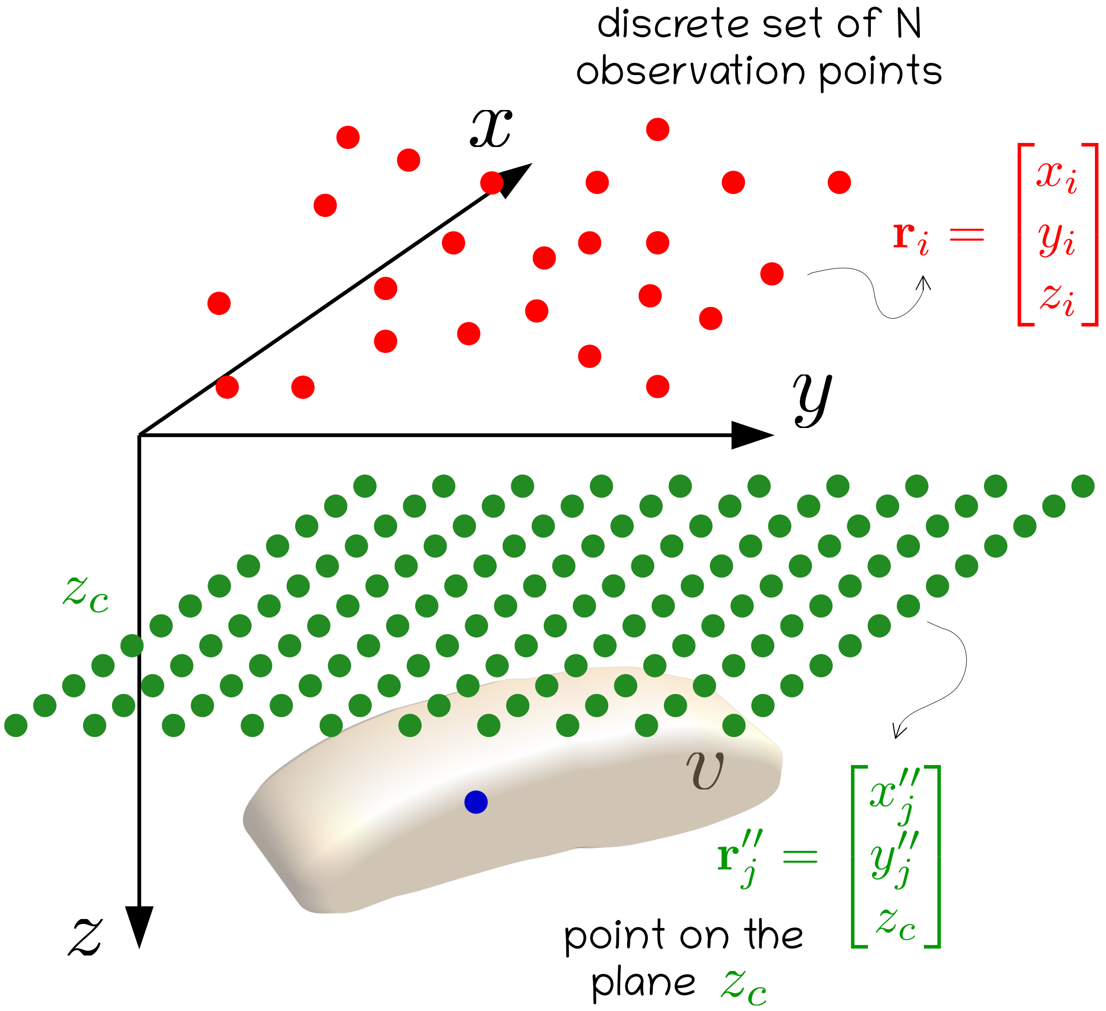


Gravitational field



density

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gravity disturbance

Gravitational field

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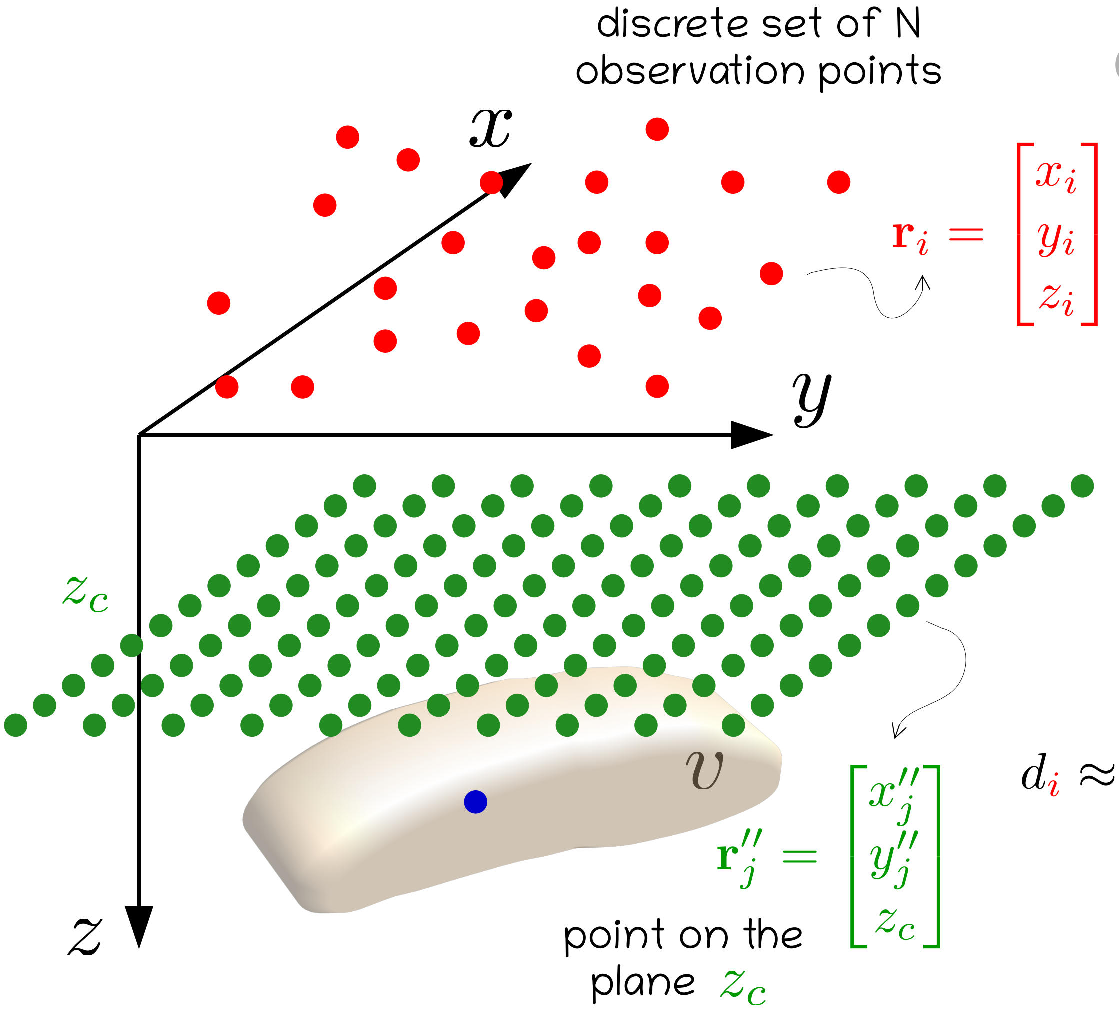


$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

Gravitational field

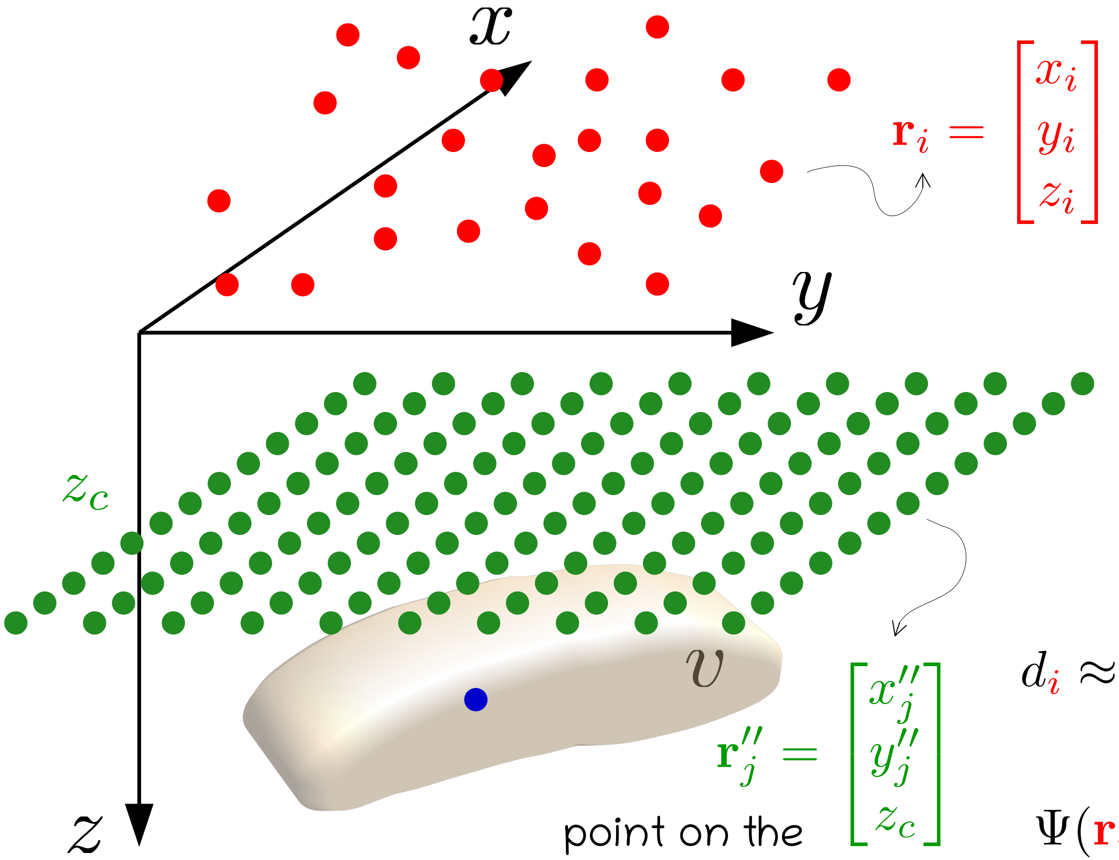
density

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance

discrete set of N
observation points



$$\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

$$\mathbf{r}''_j = \begin{bmatrix} x''_j \\ y''_j \\ z_c \end{bmatrix}$$

point on the
plane z_c

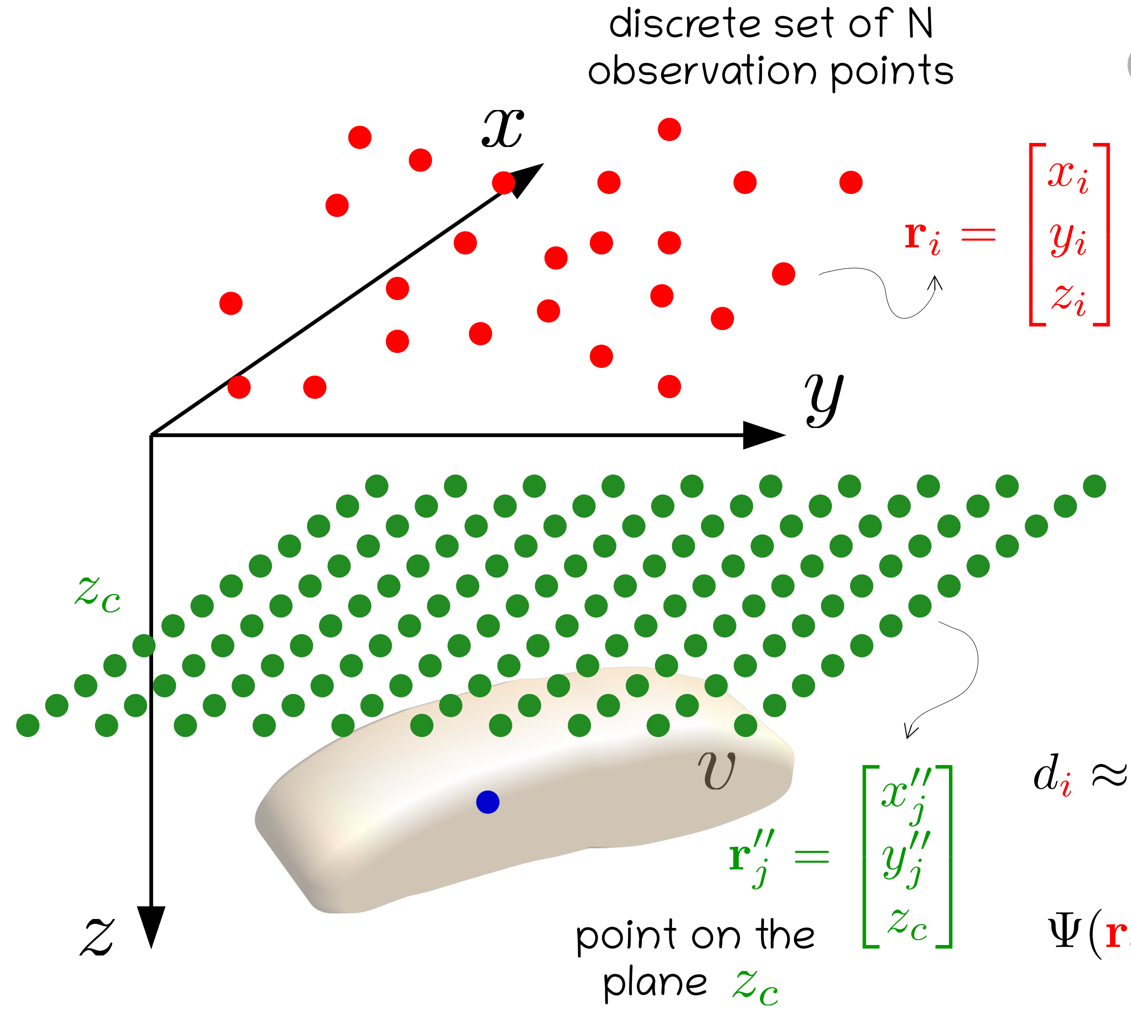
$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

Gravitational field

density

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}')\, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|}\, dv'$$



$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance

$$d_i \approx \sum_{j=1}^{M} p_j\, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

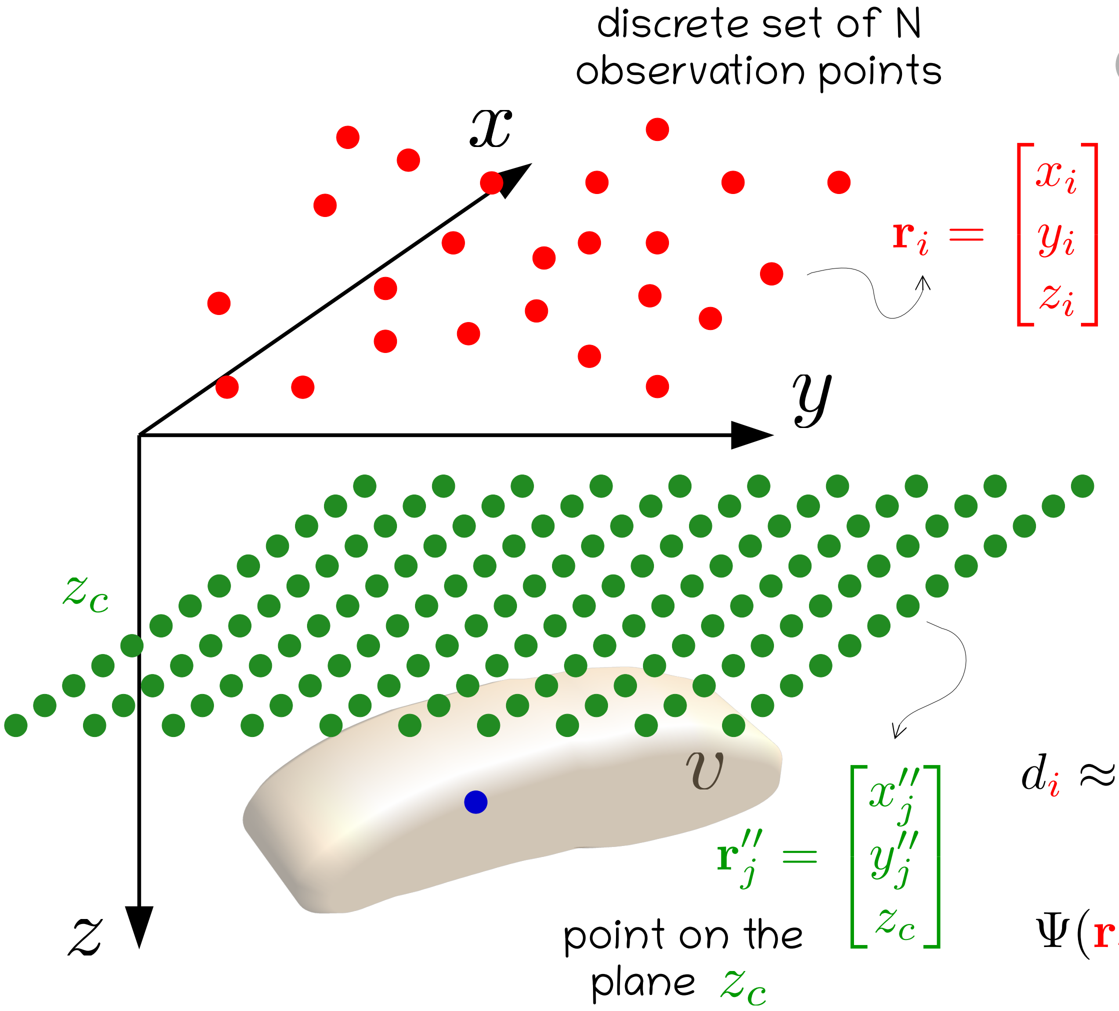
$p_j\, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$

represents the gravity disturbance produced at the observation point $\mathbf{r}_i$ by a monopole located at $\mathbf{r}''_j$

Gravitational field

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r}-\mathbf{r}'\|}\, dv'$$

density



$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance

$$d_i \approx \sum_{j=1}^{M} p_j\, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

$$\mathbf{d} \approx \mathbf{A}\, \mathbf{p}$$

In this case, the EqL Technique consists in solving this linear system for $\mathbf{p}$

Gravitational field

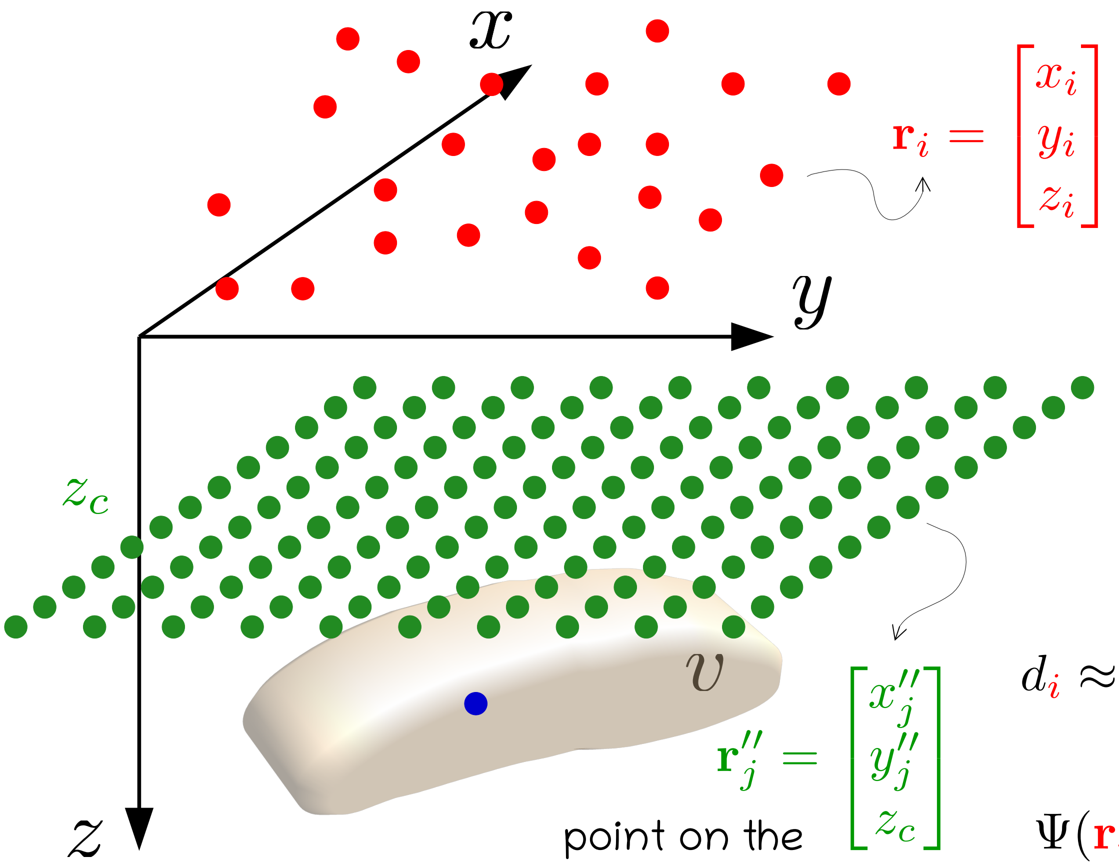
density

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r}-\mathbf{r}'\|} \, dv'$$

$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance

discrete set of N observation points



$$\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

$$\mathbf{r}''_j = \begin{bmatrix} x''_j \\ y''_j \\ z_c \end{bmatrix}$$

point on the plane z_c

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

parameter vector containing the physical-property distribution of the monopoles

$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

$$\mathbf{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}$$

Gravitational field

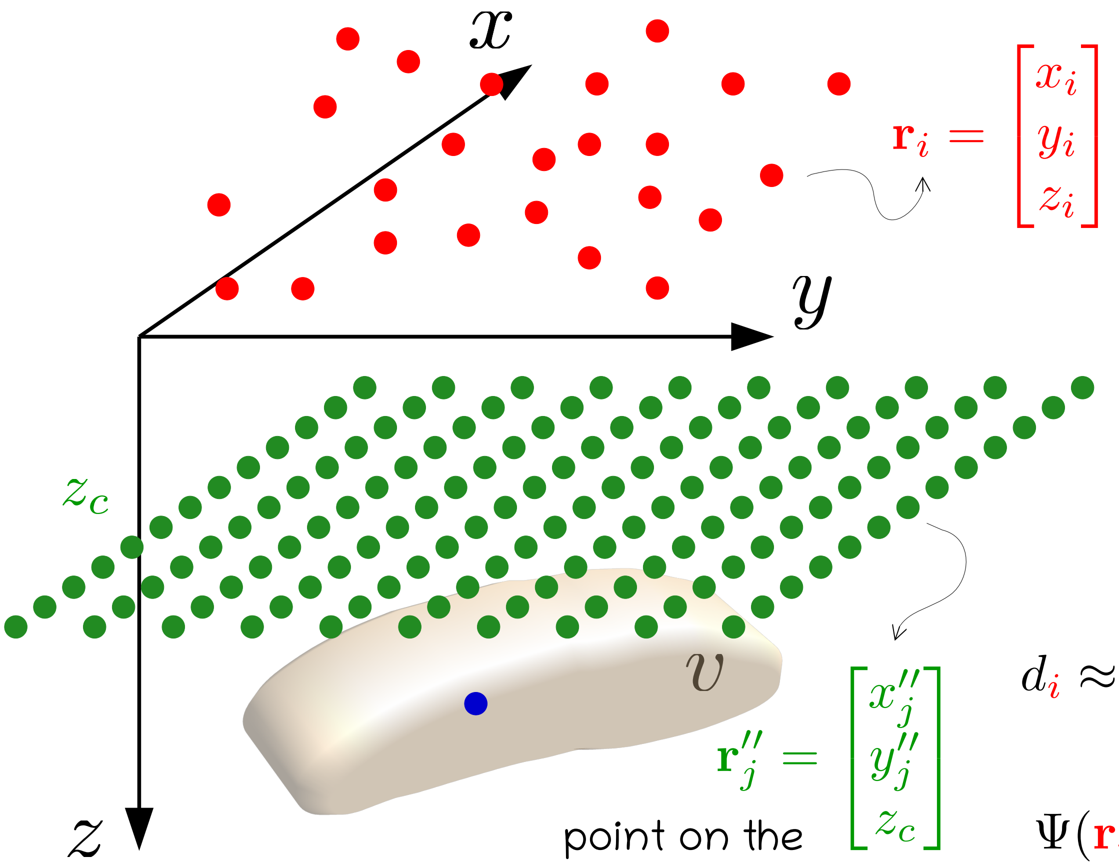
density

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance

discrete set of N observation points



point on the plane z_c

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

data vector containing the observations (in this case, gravity disturbance data)

$$\mathbf{d} \approx \mathbf{A} \, \mathbf{p}$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}$$

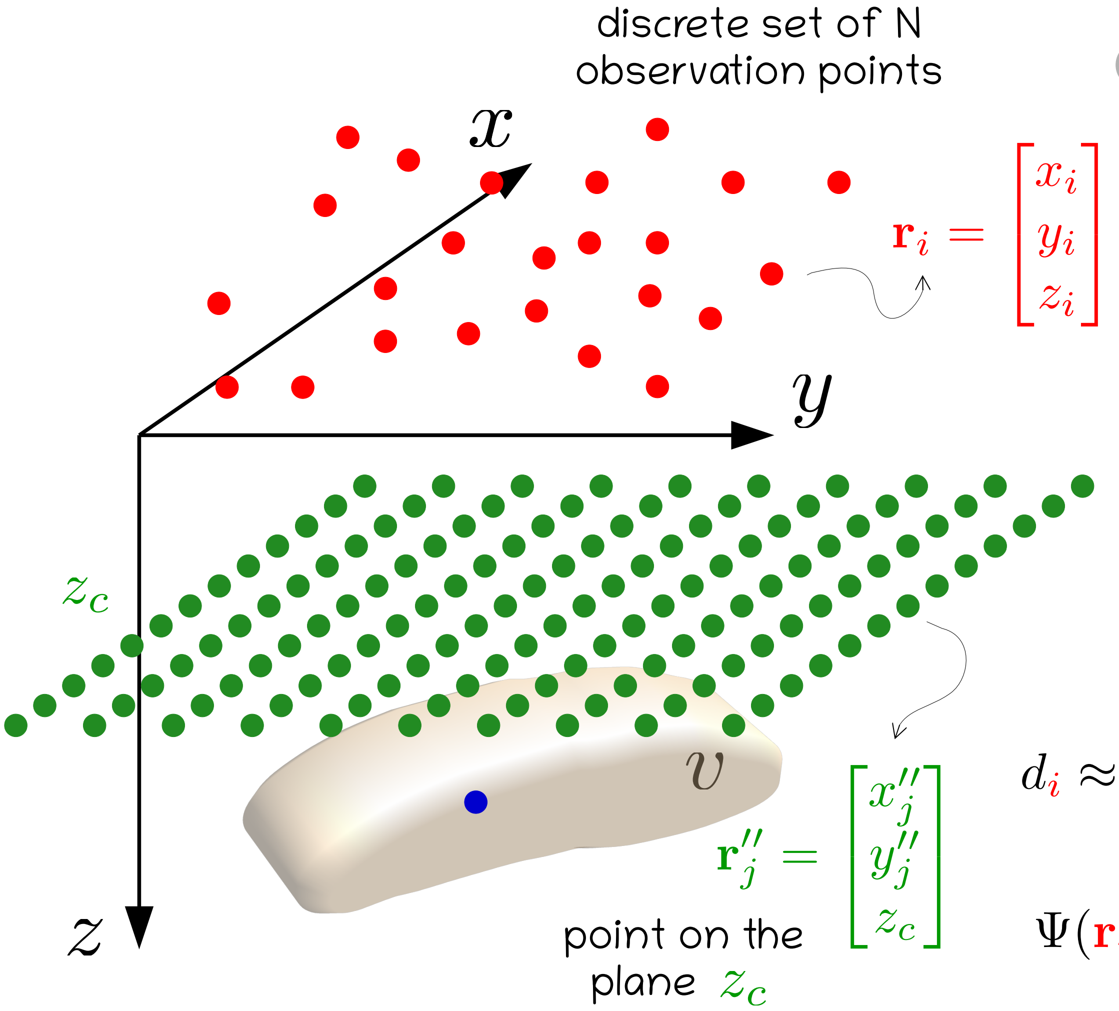
Gravitational field

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

density

$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance



$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

$$\mathbf{d} \approx \mathbf{A} \, \mathbf{p}$$

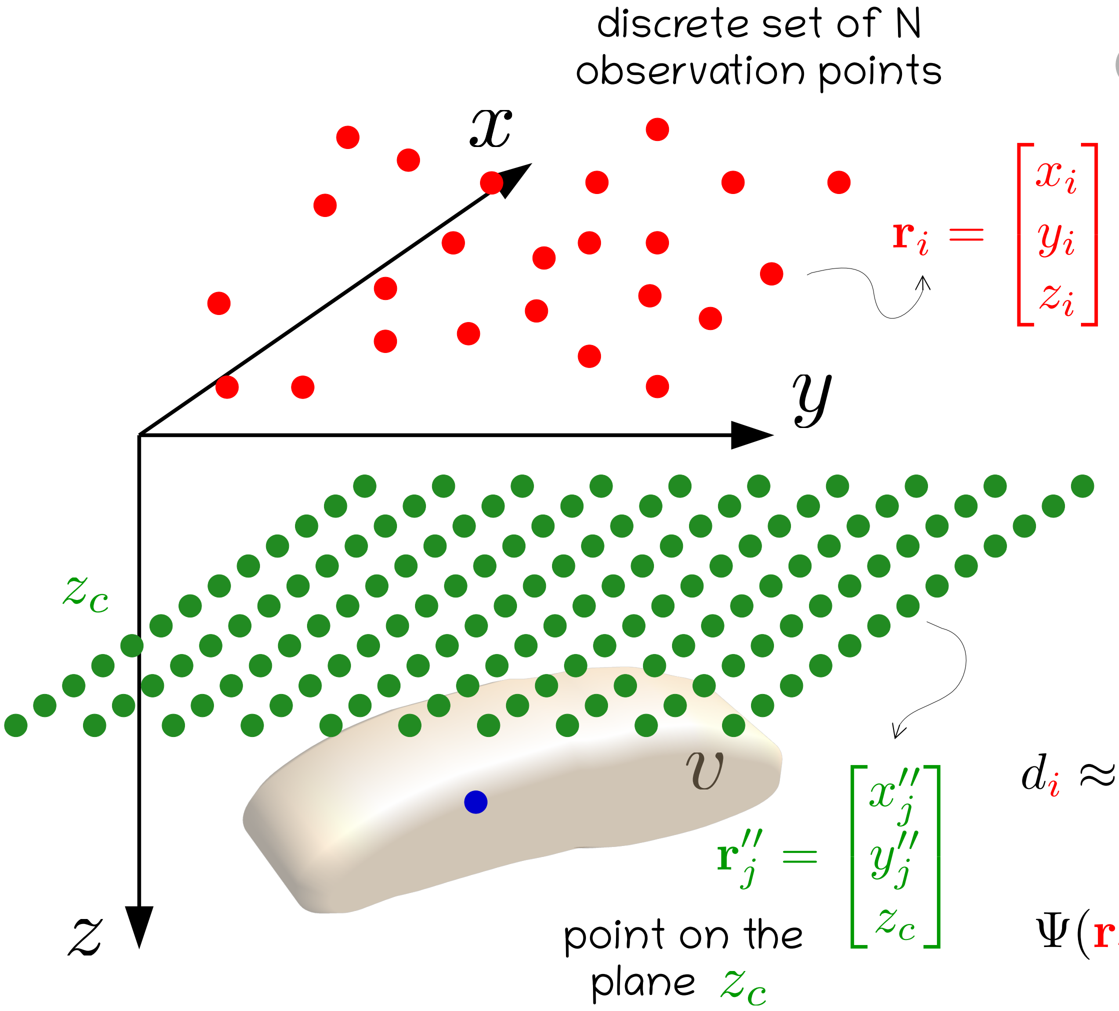
$$a_{ij} = \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

sensitivity matrix

Gravitational field

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

density



$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance

The parameter vector **p** reproducing the gravity data defines a **discrete equivalent layer**

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

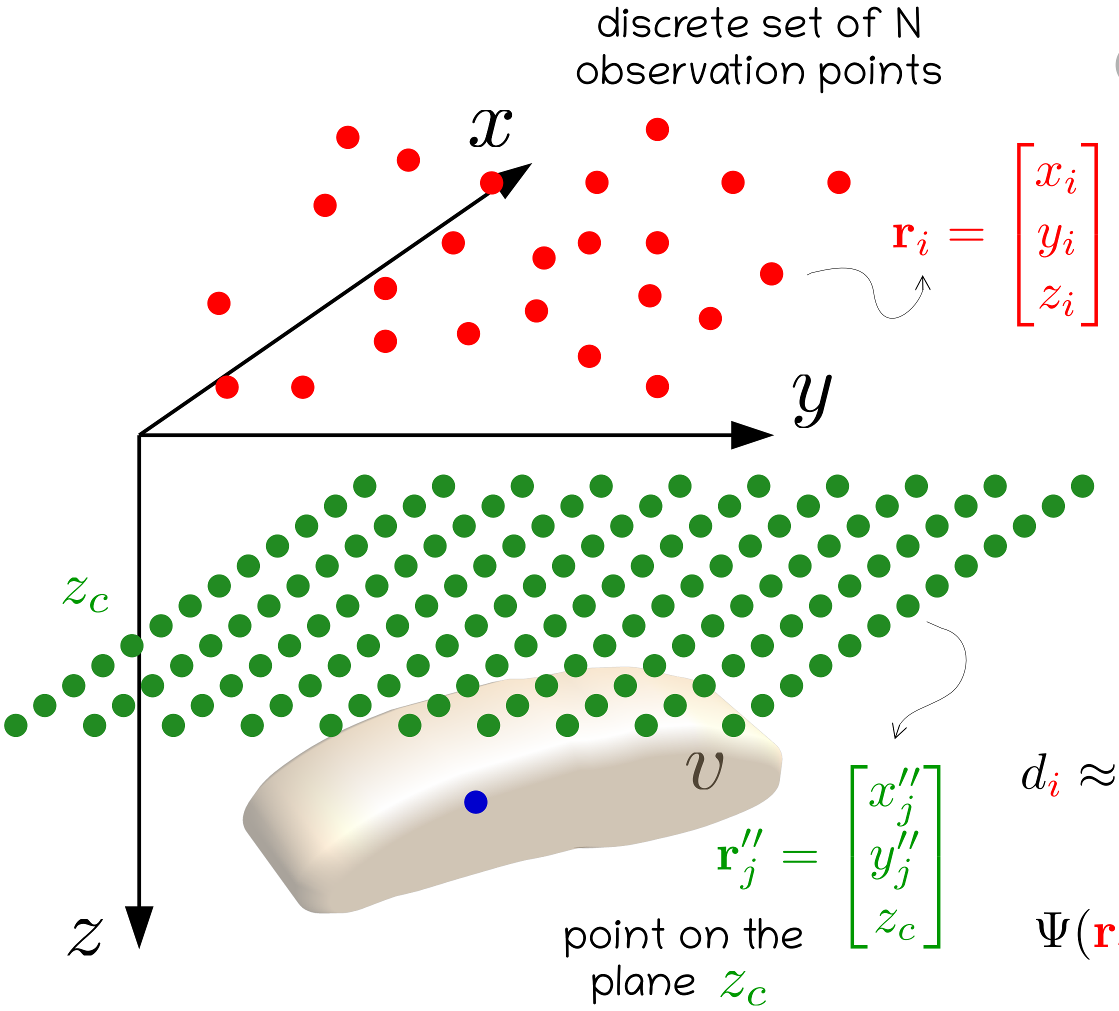
$$\mathbf{d} \approx \mathbf{A} \, \mathbf{p}$$

$$a_{ij} = \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

Gravitational field

density

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance

The physical property p_j of a single monopole located at $\mathbf{r}''_j$ is called **equivalent source**

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

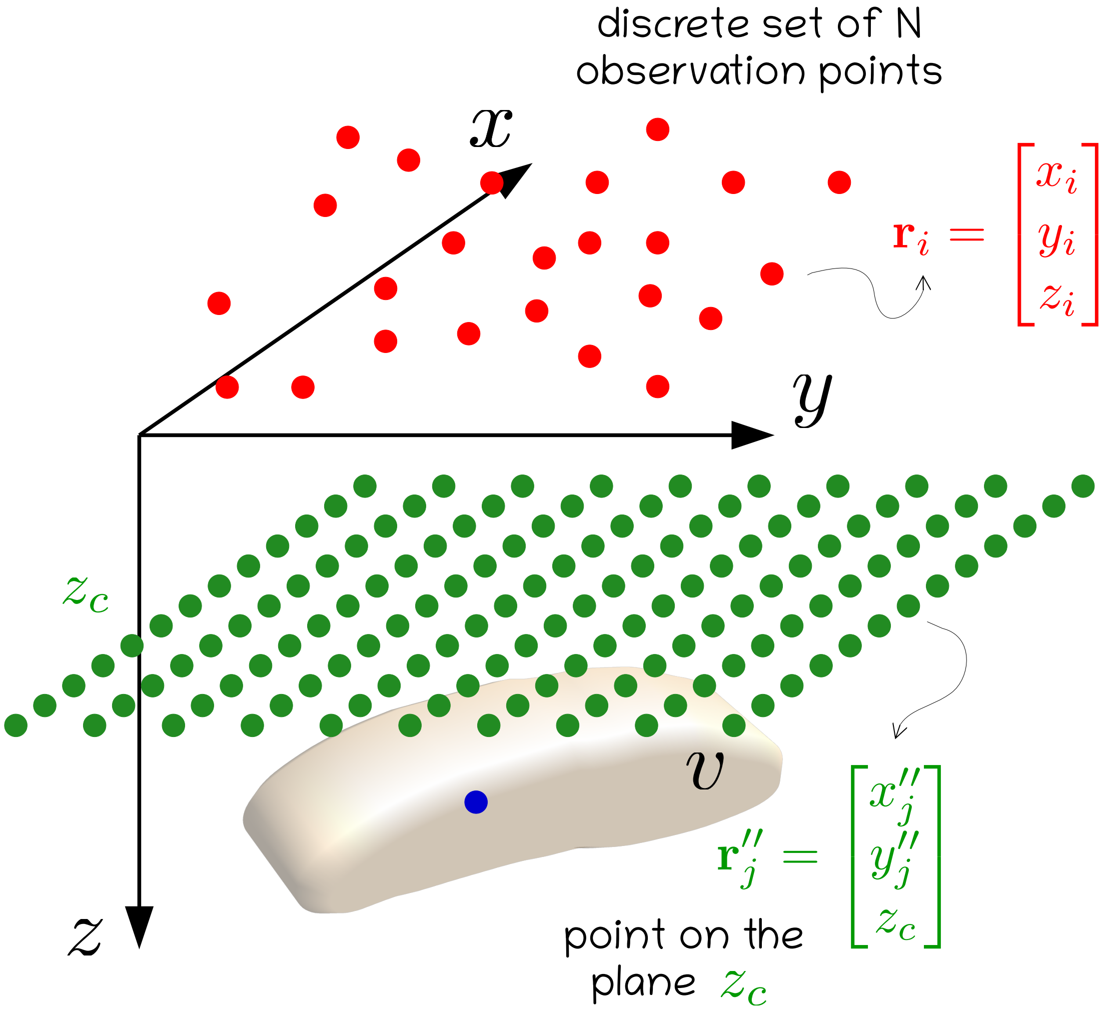
$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

$$\mathbf{d} \approx \mathbf{A} \, \mathbf{p}$$

$$a_{ij} = \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

Gravitational field

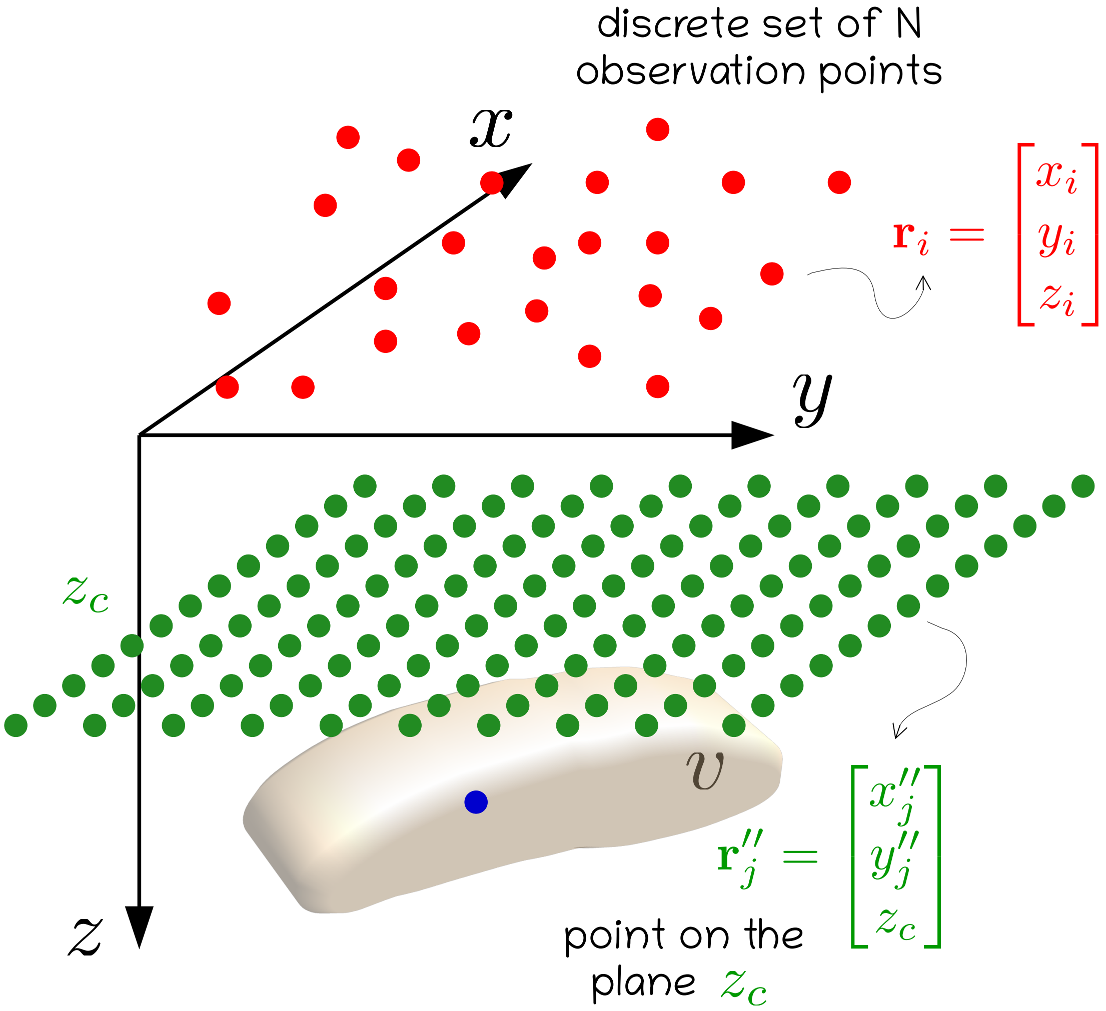
$$\Theta(\mathbf{r}) = \kappa \iiint_{v} \overset{\text{density}}{\sigma}(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



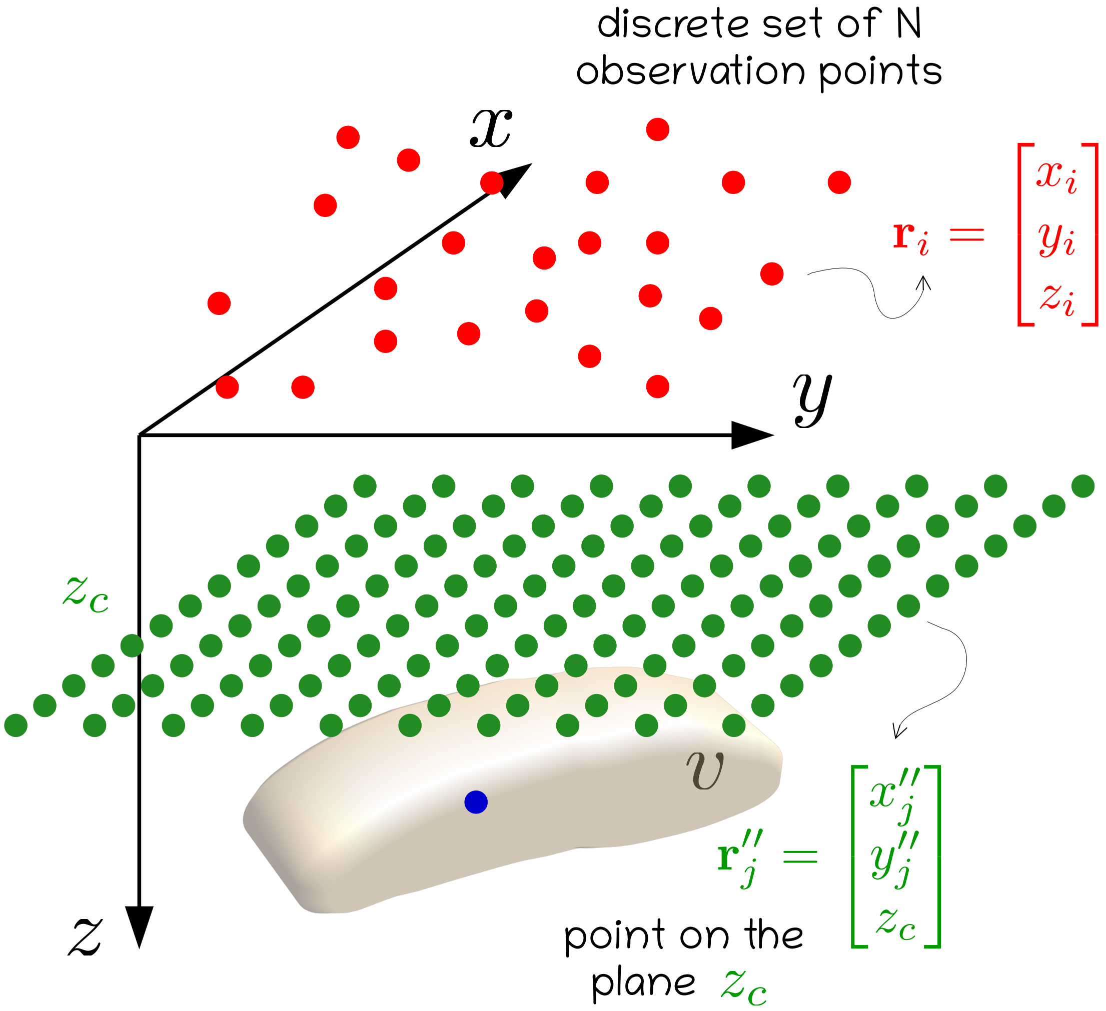
Magnetic field

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$



Magnetic field



total-magnetization intensity

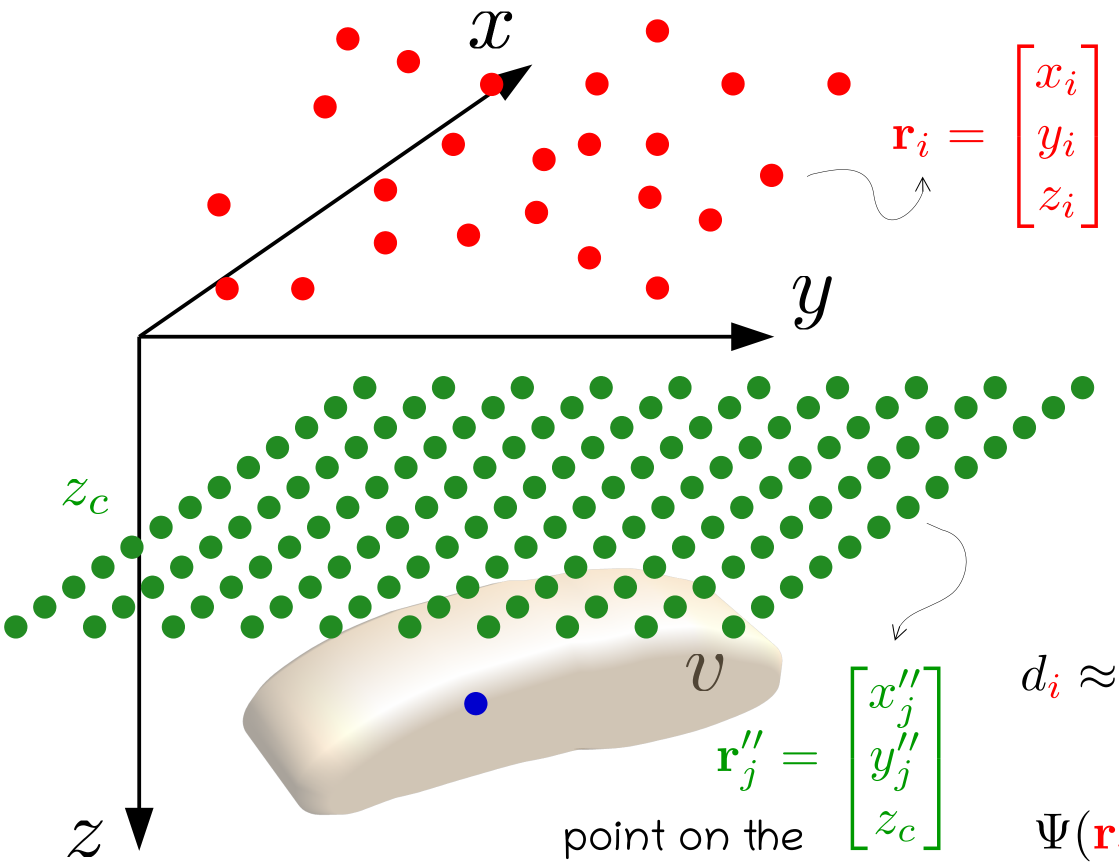
$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$d_i \equiv \partial_{th} \Theta(\mathbf{r}_i)$$

approx total-field anomaly

Magnetic field

discrete set of N observation points



$$\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$d_i \equiv \partial_{th} \Theta(\mathbf{r}_i)$$

approx total-field anomaly

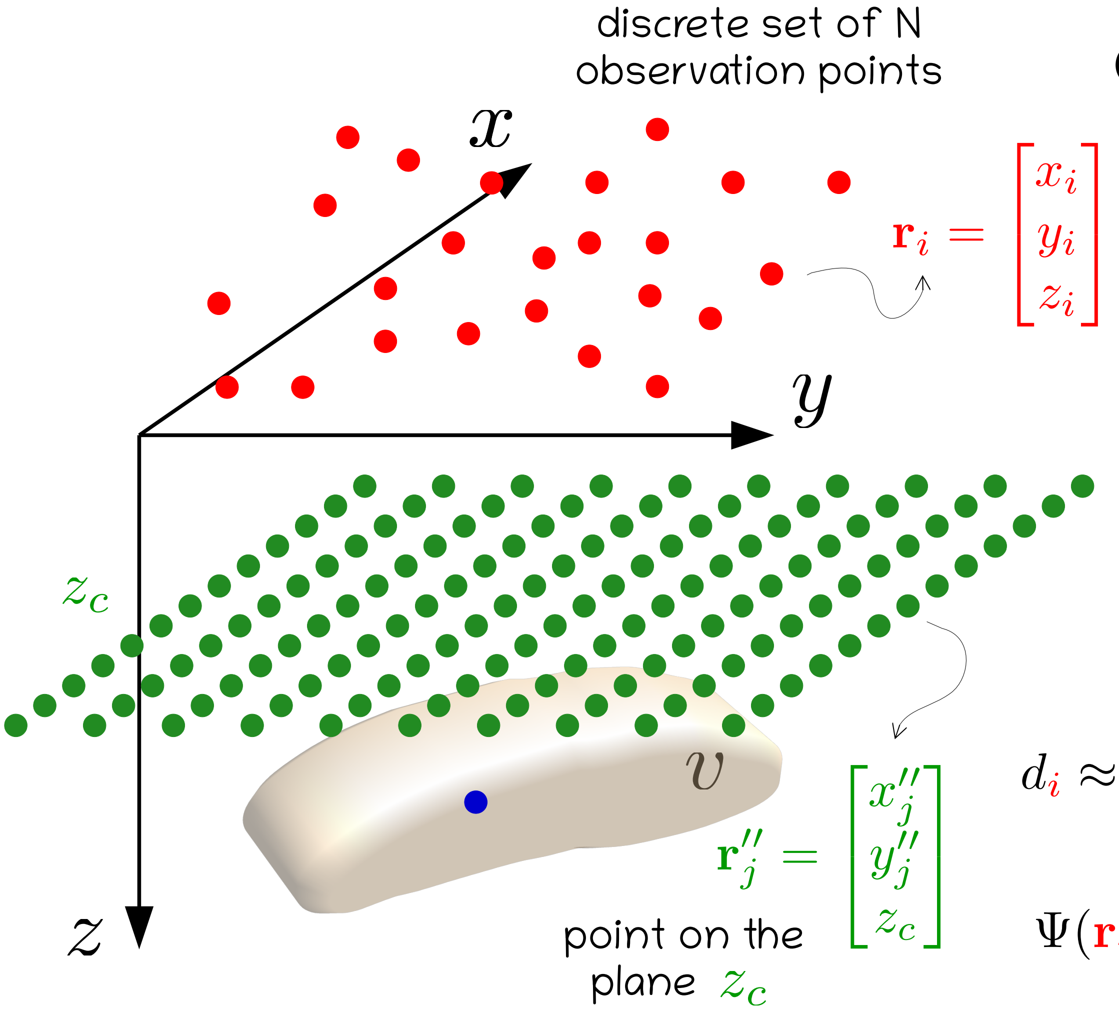
$$\mathbf{r}''_j = \begin{bmatrix} x''_j \\ y''_j \\ z_c \end{bmatrix}$$

point on the plane z_c

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

Magnetic field



total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$d_i \equiv \partial_{th} \Theta(\mathbf{r}_i)$$

approx total-field anomaly

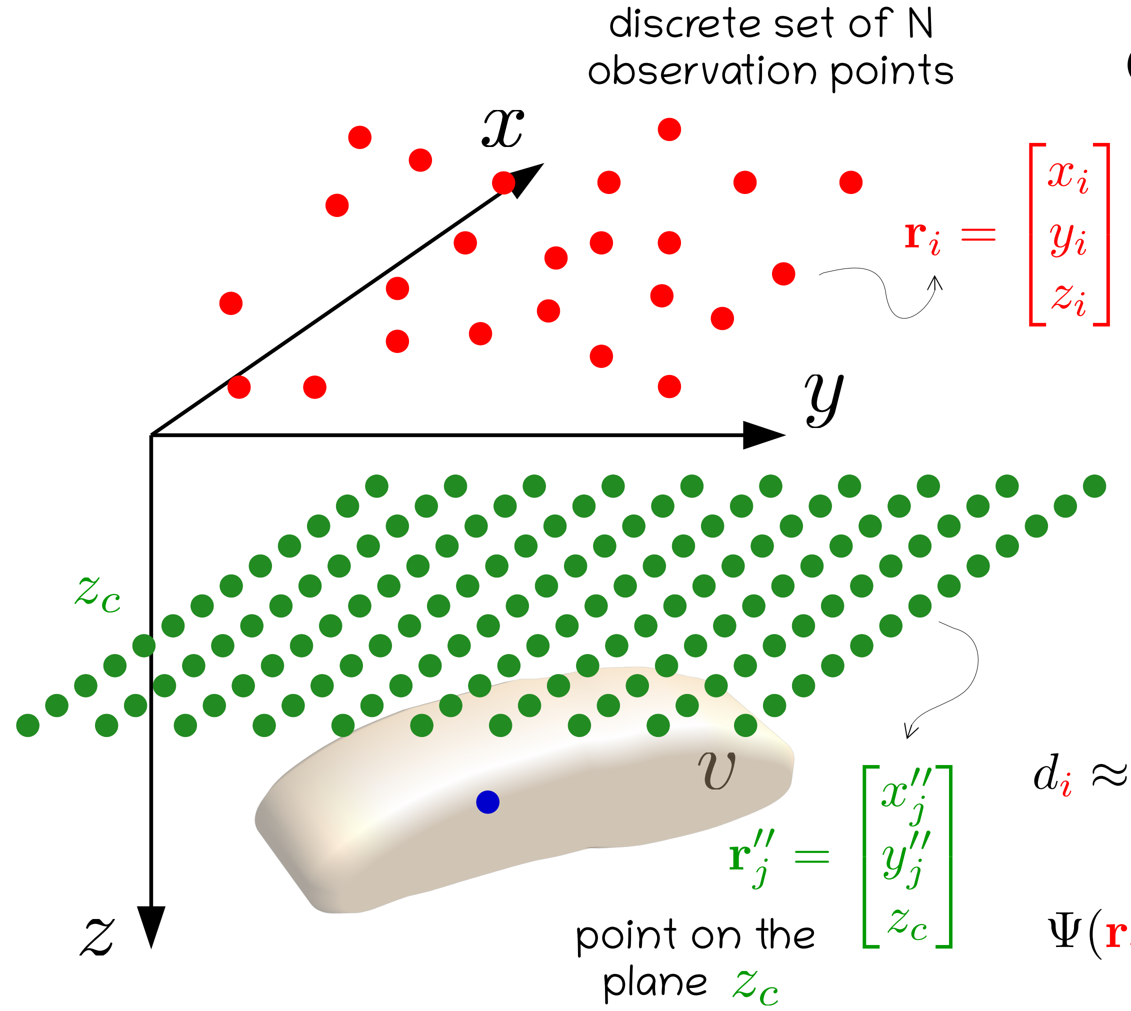
$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

$p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$

represents the approx total-field anomaly produced at the observation point $\mathbf{r}_i$ by a dipole located at $\mathbf{r}''_j$

Magnetic field



total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

$$d_i \equiv \partial_{th} \Theta(\mathbf{r}_i)$$

approx total-field anomaly

The physical property p_j of a single dipole located at $\mathbf{r}''_j$ is an **equivalent source**

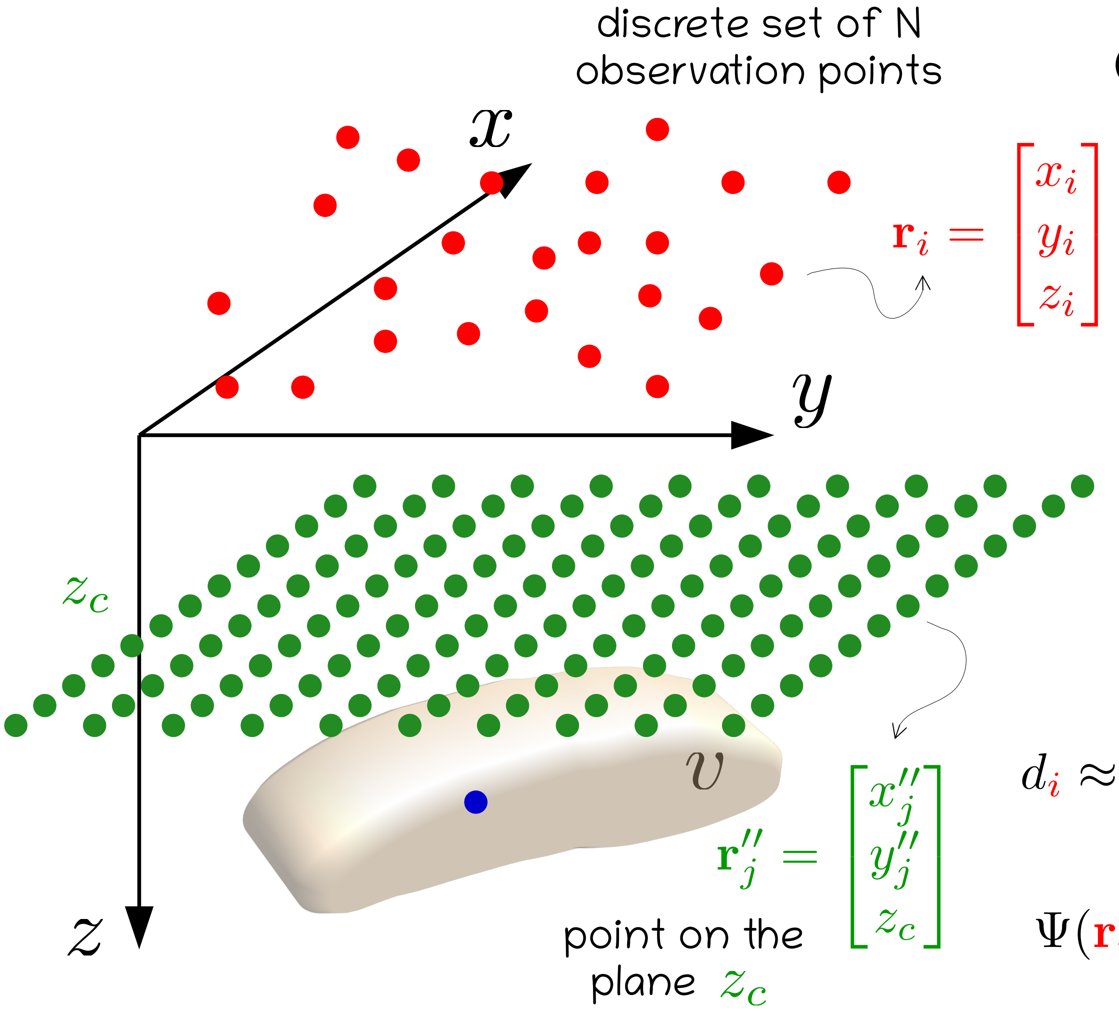
$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

represents the approx total-field anomaly produced at the observation point $\mathbf{r}_i$ by a dipole located at $\mathbf{r}''_j$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

Magnetic field



total-magnetization
intensity

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

$$d_i \equiv \partial_{th} \Theta(\mathbf{r}_i)$$

approx total-field anomaly

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\partial_{tv} \Psi(\mathbf{r}, \mathbf{r}'') = \hat{\mathbf{t}}^\top \mathbf{H}_\Psi(\mathbf{r}, \mathbf{r}'') \, \hat{\mathbf{v}}$$

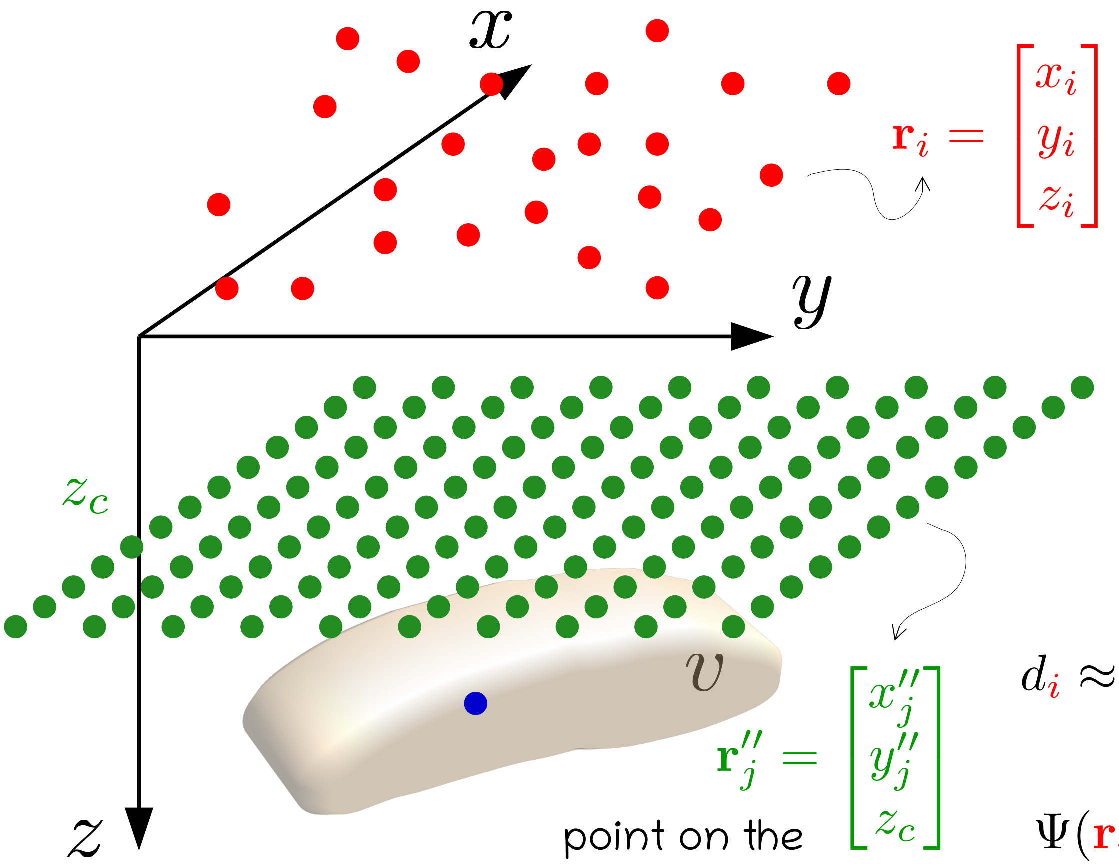
$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

Magnetic field

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

discrete set of N observation points



$$\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

$$d_i \equiv \partial_{th}\Theta(\mathbf{r}_i)$$

approx total-field anomaly

unit vector defining a constant direction for the main geomagnetic field

$$\mathbf{r}''_j = \begin{bmatrix} x''_j \\ y''_j \\ z_c \end{bmatrix}$$

point on the plane z_c

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{tv}\Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\partial_{tv}\Psi(\mathbf{r}, \mathbf{r}'') = \hat{\mathbf{t}}^\top \mathbf{H}_\Psi(\mathbf{r}, \mathbf{r}'')\, \hat{\mathbf{v}}$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

Magnetic field

total-magnetization intensity

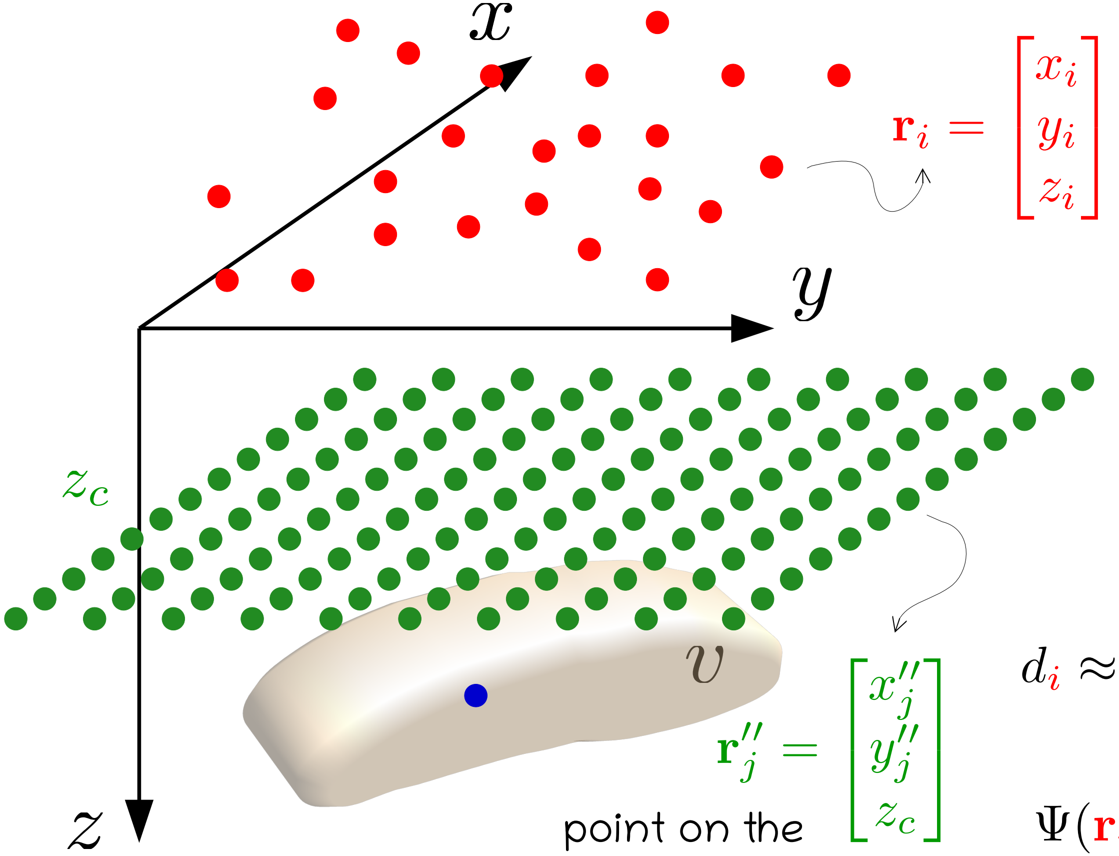
$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$d_i \equiv \partial_{th} \Theta(\mathbf{r}_i)$$

approx total-field anomaly

discrete set of N observation points

$$\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$



$$\mathbf{r}''_j = \begin{bmatrix} x''_j \\ y''_j \\ z_c \end{bmatrix}$$

point on the plane z_c

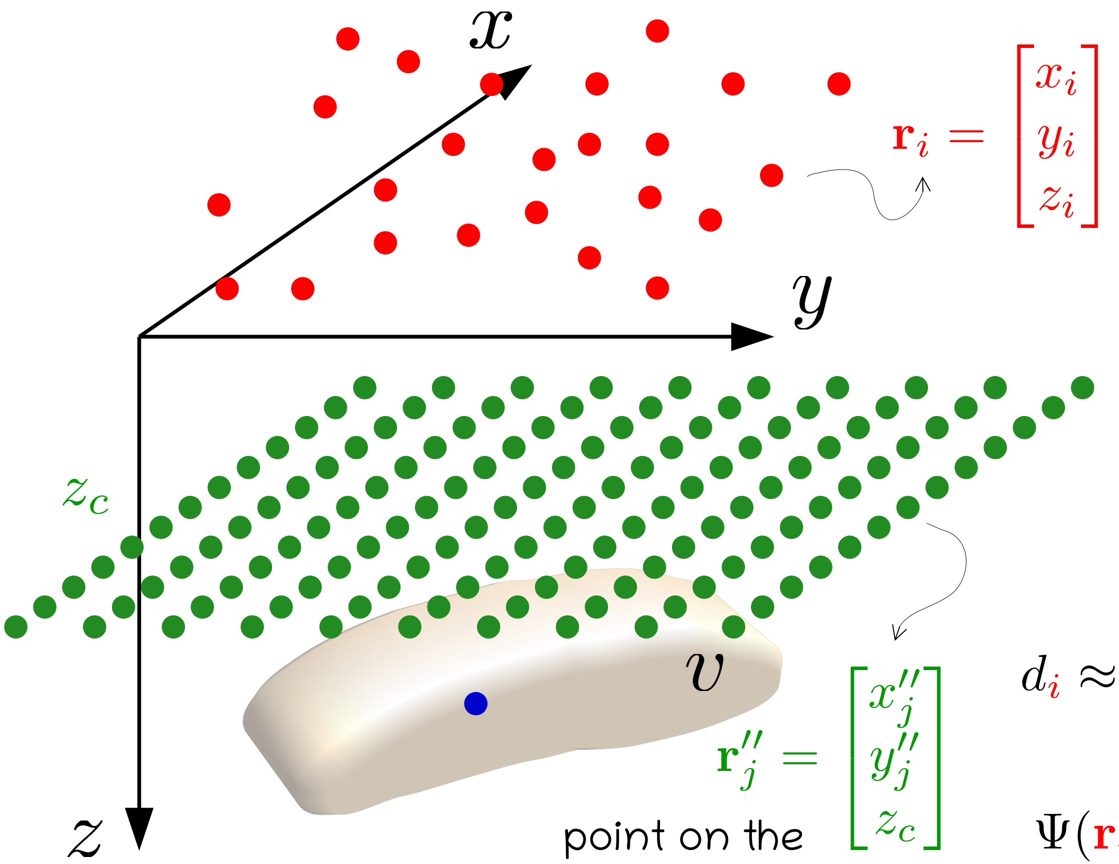
$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

unit vector defining a constant direction for the dipoles

$$\partial_{tv} \Psi(\mathbf{r}, \mathbf{r}'') = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Psi}(\mathbf{r}, \mathbf{r}'') \, \hat{\mathbf{v}}$$

Magnetic field



$$\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

$$\mathbf{r}''_j = \begin{bmatrix} x''_j \\ y''_j \\ z_c \end{bmatrix}$$

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$d_i \equiv \partial_{th} \Theta(\mathbf{r}_i)$$

approx total-field anomaly

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

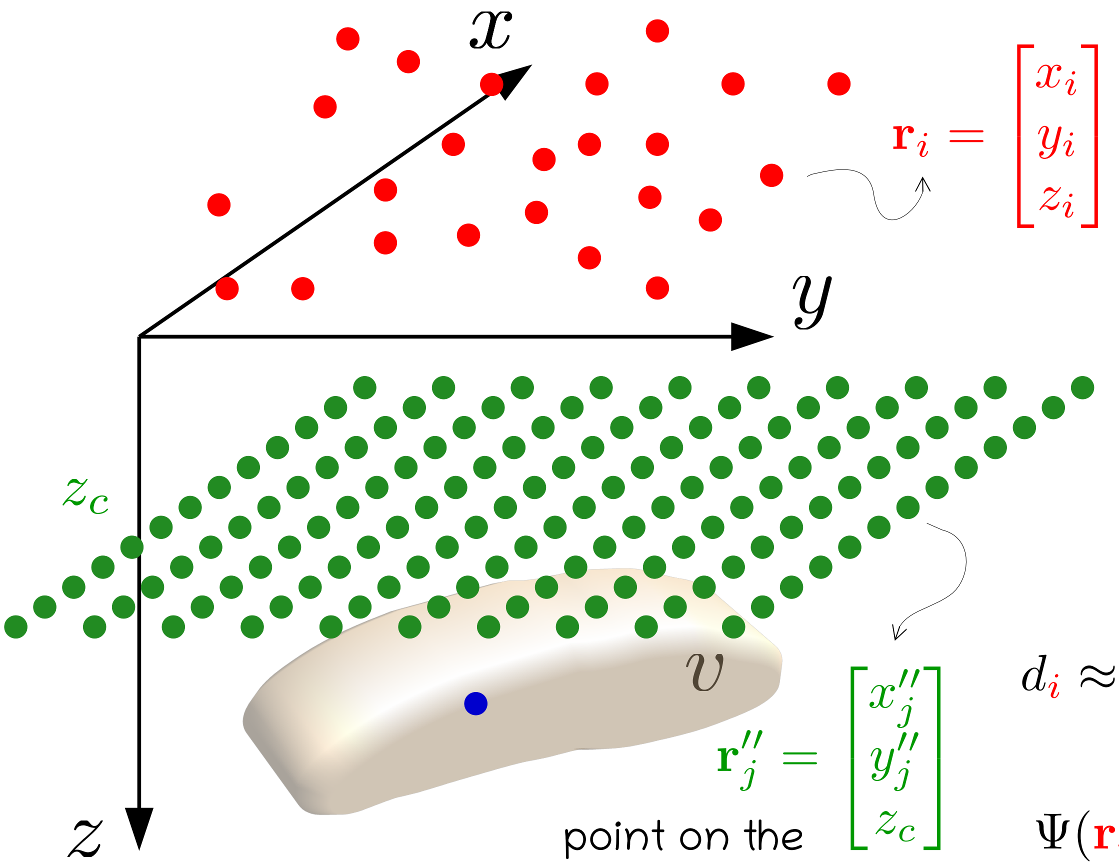
this direction may be arbitrary

$$\partial_{tv} \Psi(\mathbf{r}, \mathbf{r}'') = \hat{\mathbf{t}}^\top \mathbf{H}_\Psi(\mathbf{r}, \mathbf{r}'') \, \hat{\mathbf{v}}$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

Magnetic field

discrete set of N observation points



$$\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

$$\mathbf{r}''_j = \begin{bmatrix} x''_j \\ y''_j \\ z_c \end{bmatrix}$$

point on the plane z_c

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

$$d_i \equiv \partial_{th} \Theta(\mathbf{r}_i)$$

approx total-field anomaly

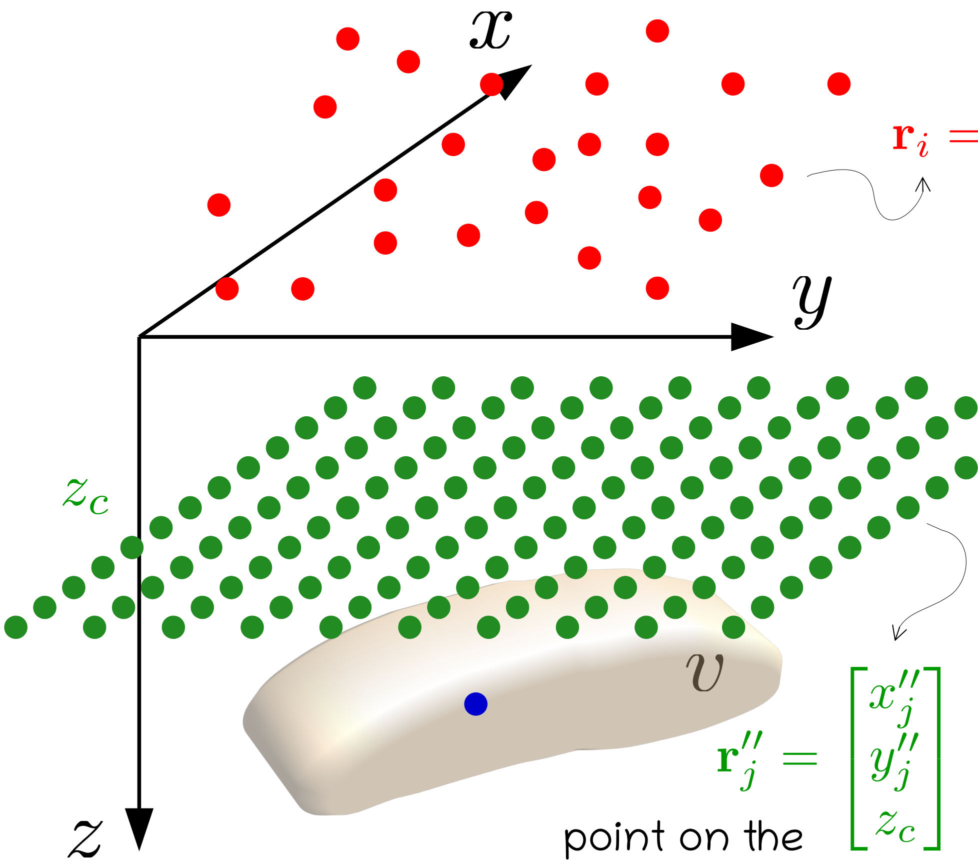
$$\mathbf{H}_\Psi(\mathbf{r}, \mathbf{r}'') = \begin{bmatrix} \partial_{xx}\Psi(\mathbf{r}, \mathbf{r}'') & \partial_{xy}\Psi(\mathbf{r}, \mathbf{r}'') & \partial_{xz}\Psi(\mathbf{r}, \mathbf{r}'') \\ \partial_{xy}\Psi(\mathbf{r}, \mathbf{r}'') & \partial_{yy}\Psi(\mathbf{r}, \mathbf{r}'') & \partial_{yz}\Psi(\mathbf{r}, \mathbf{r}'') \\ \partial_{xz}\Psi(\mathbf{r}, \mathbf{r}'') & \partial_{yz}\Psi(\mathbf{r}, \mathbf{r}'') & \partial_{zz}\Psi(\mathbf{r}, \mathbf{r}'') \end{bmatrix}$$

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\partial_{tv}\Psi(\mathbf{r}, \mathbf{r}'') = \hat{\mathbf{t}}^\top \mathbf{H}_\Psi(\mathbf{r}, \mathbf{r}'') \, \hat{\mathbf{v}}$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

Magnetic field



$$\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

$$\mathbf{r}''_j = \begin{bmatrix} x''_j \\ y''_j \\ z_c \end{bmatrix}$$

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

$$d_i \equiv \partial_{th} \Theta(\mathbf{r}_i)$$

approx total-field anomaly

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

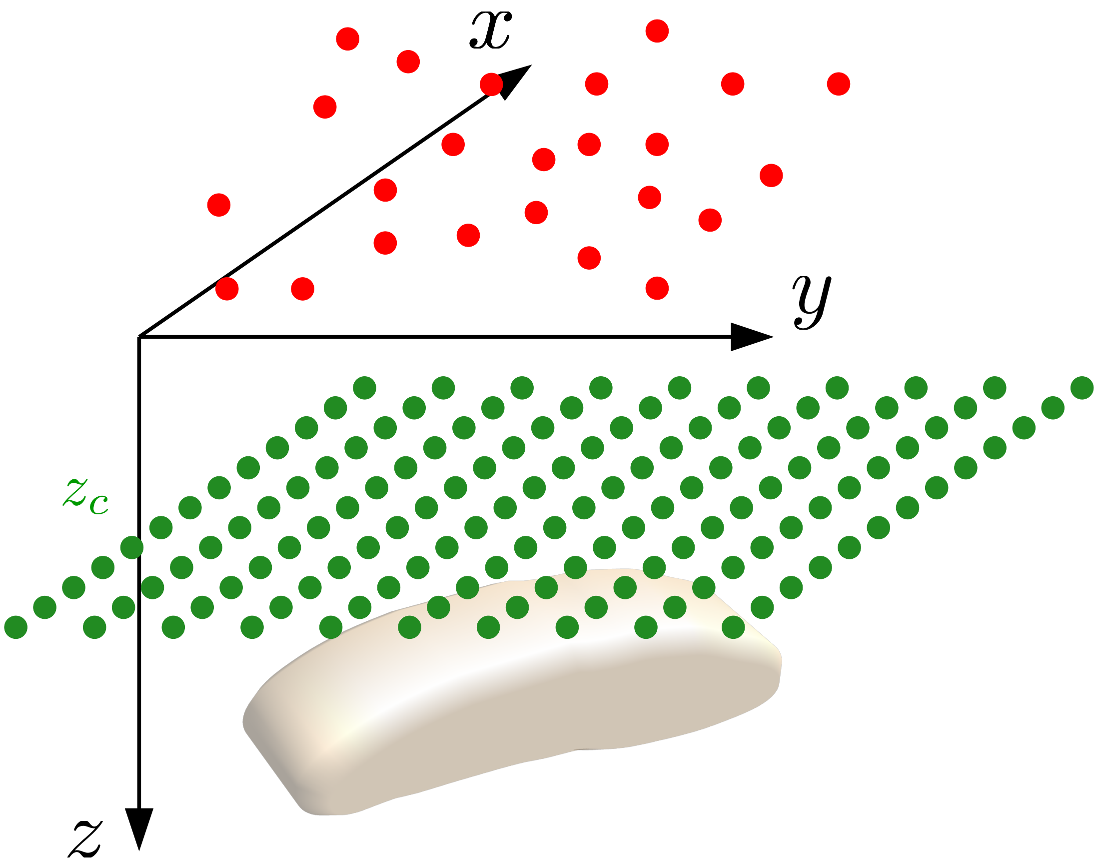
$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

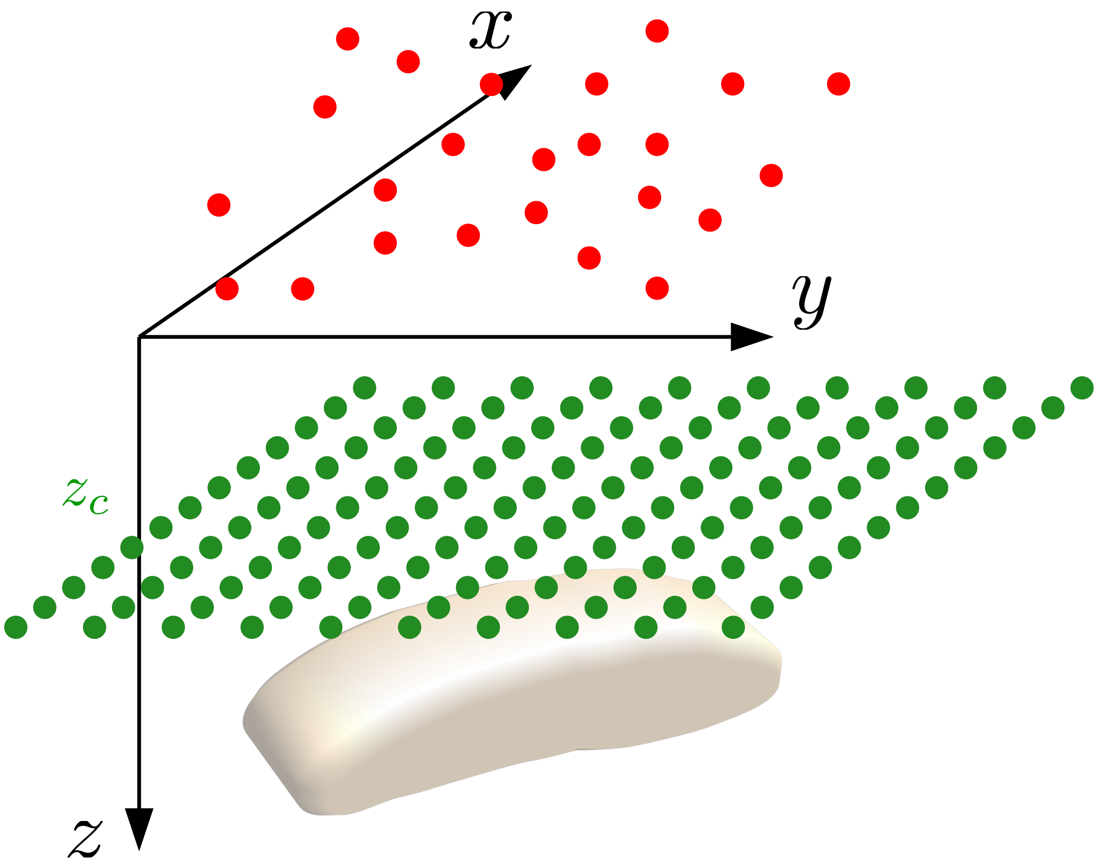
Again, the EqL Technique consists in solving this linear system for $\mathbf{p}$

Classical EqL Technique

$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$



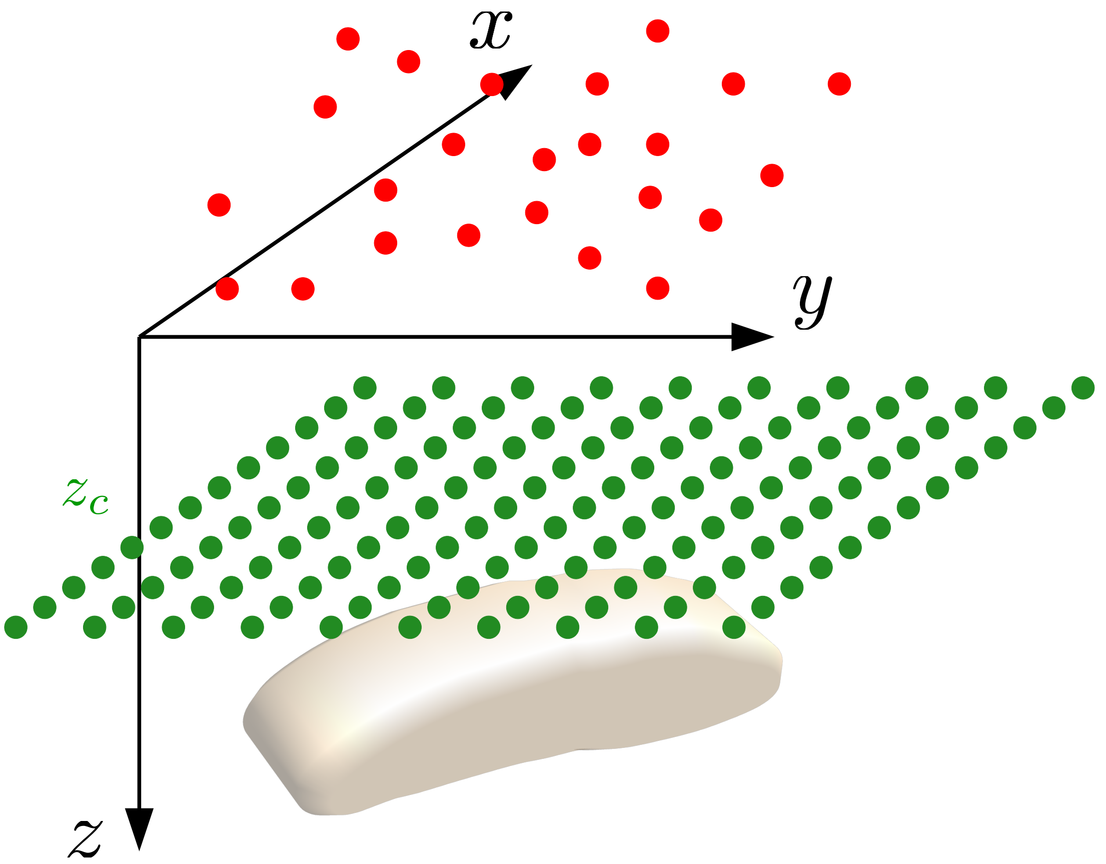
Classical EqL Technique



$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

solving a linear
system for $\mathbf{p}$

Classical EqL Technique



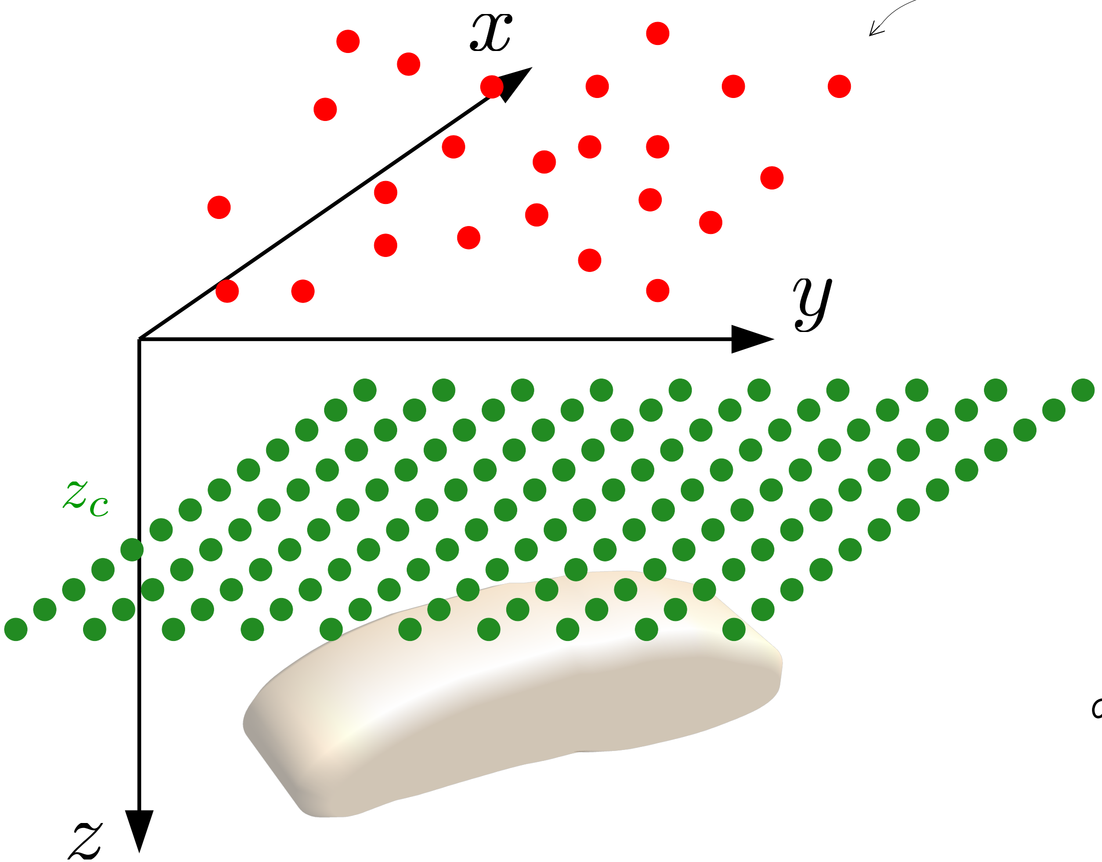
$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

$$\mathbf{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}$$

mass
(density)

moment
(magnetization)
intensity

Classical EqL Technique

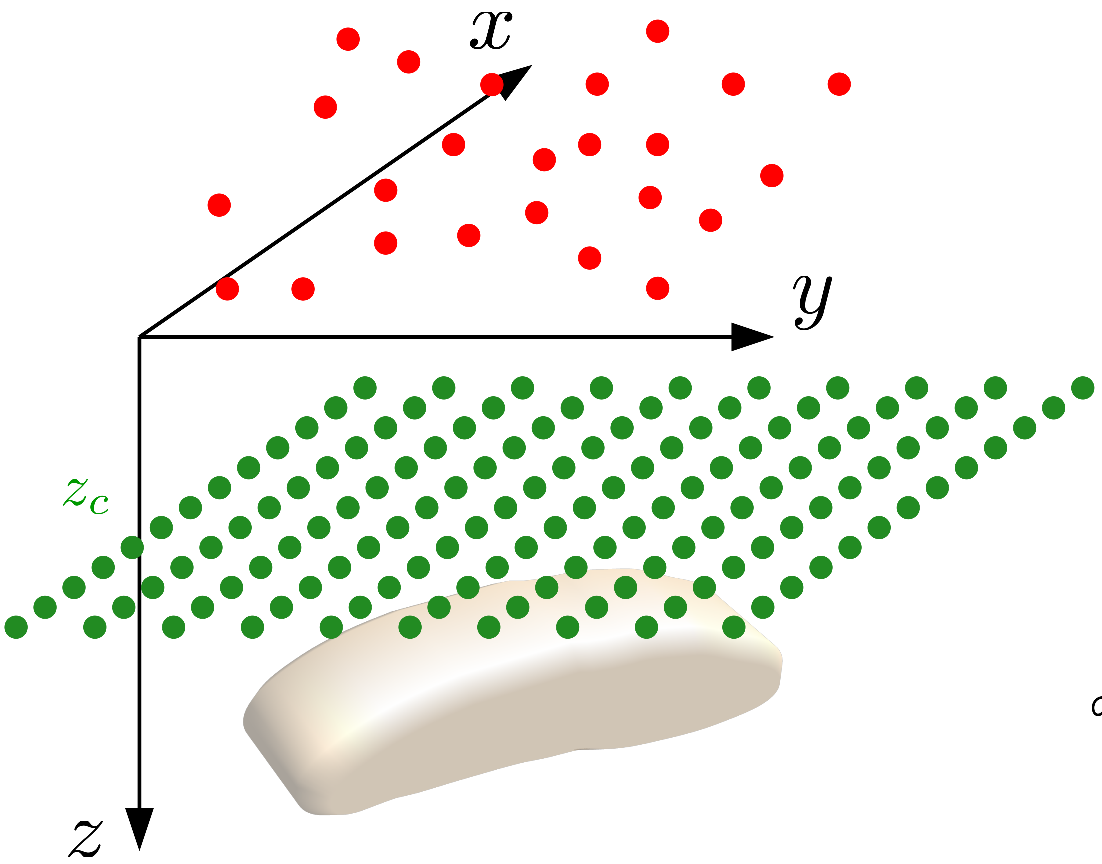


$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix} \qquad \mathbf{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}$$

$\mathbf{d}$	$\mathbf{p}$
gravity disturbance	mass (density)
approx total-field anomaly	moment (magnetization) intensity

Classical EqL Technique

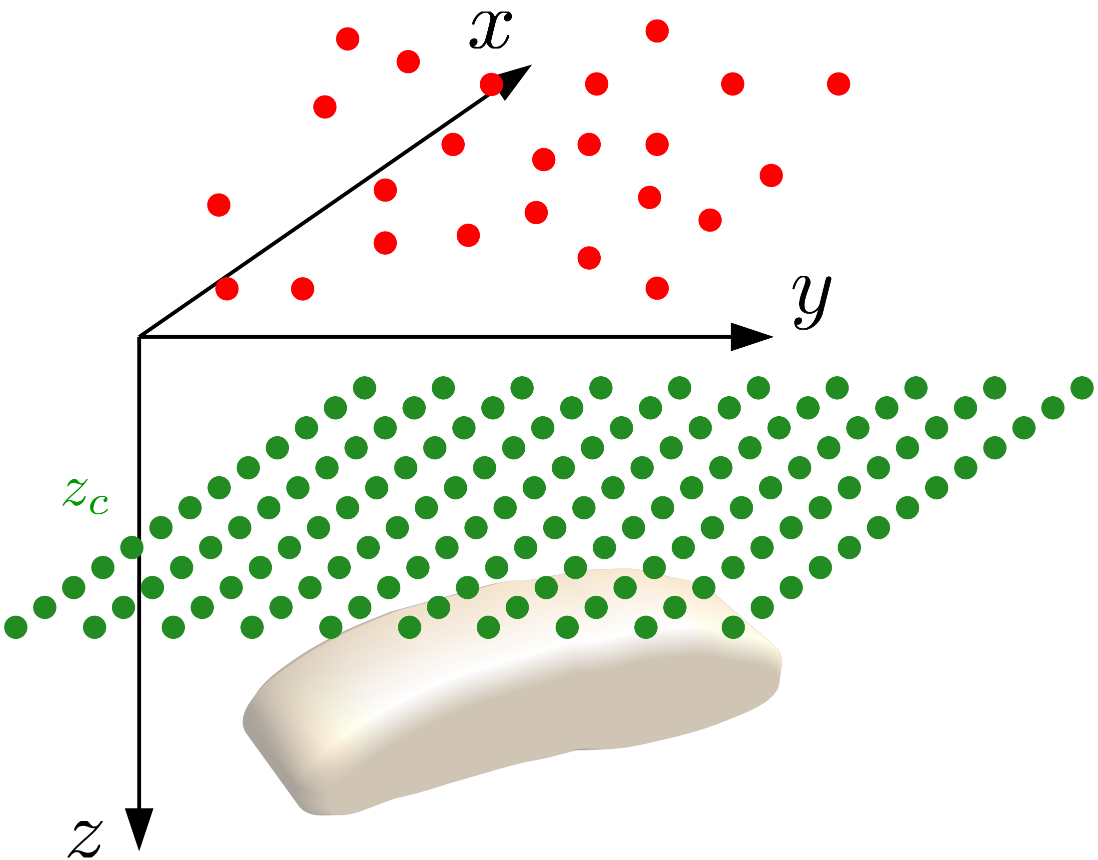


$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix} \qquad a_{ij} \qquad \mathbf{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}$$

gravity disturbance	monopole	mass (density)
approx total-field anomaly	dipole	moment (magnetization) intensity

Classical EqL Technique



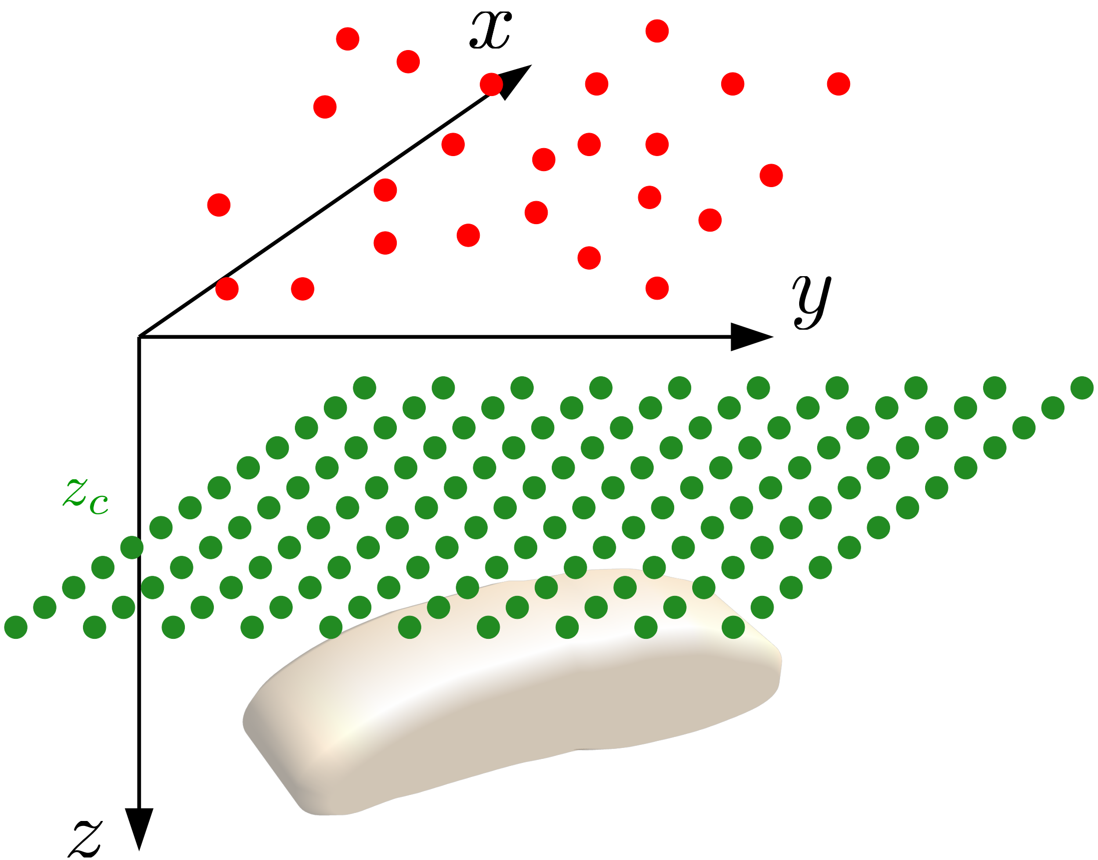
$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix} \qquad a_{ij} \qquad \mathbf{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}$$

$$\mathbf{w} = \mathbf{T}\,\tilde{\mathbf{p}}$$

potential-field transformation

Classical EqL Technique



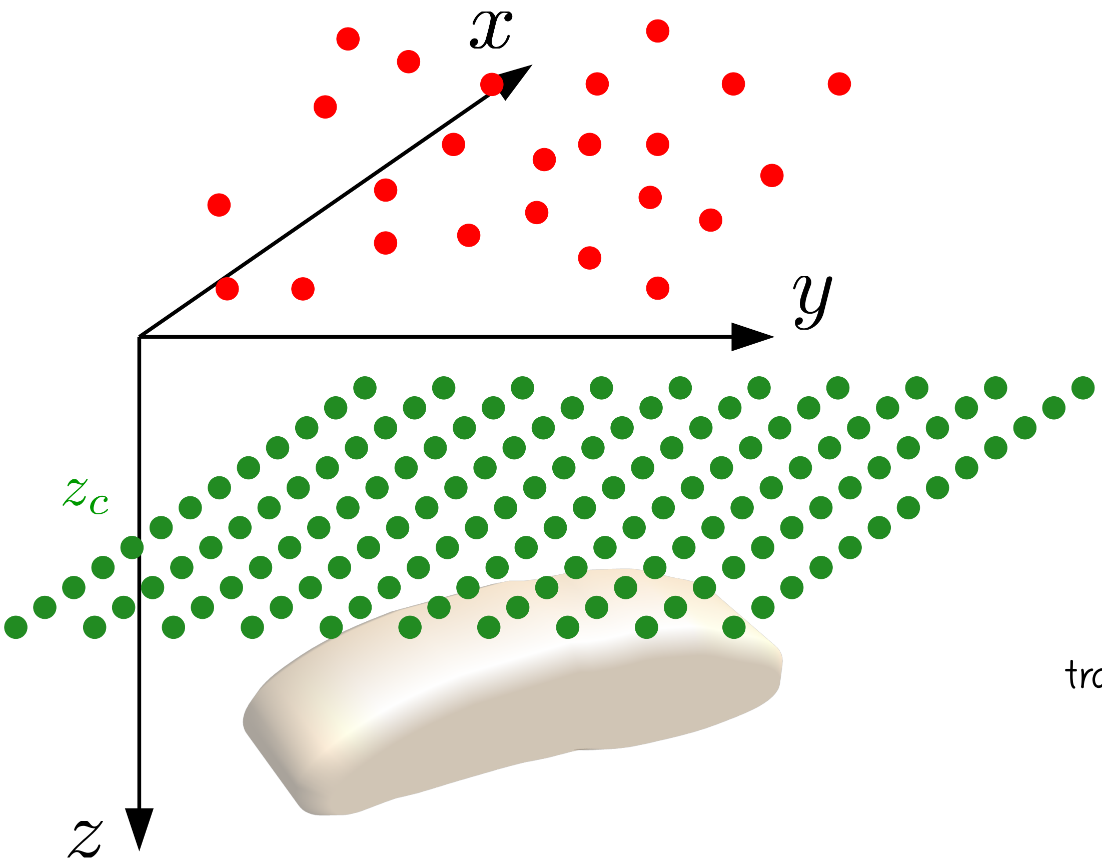
$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix} \qquad a_{ij} \qquad \mathbf{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}$$

$$\mathbf{w} = \mathbf{T}\,\tilde{\mathbf{p}}$$

estimated
parameter vector
(equivalent layer)

Classical EqL Technique



$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

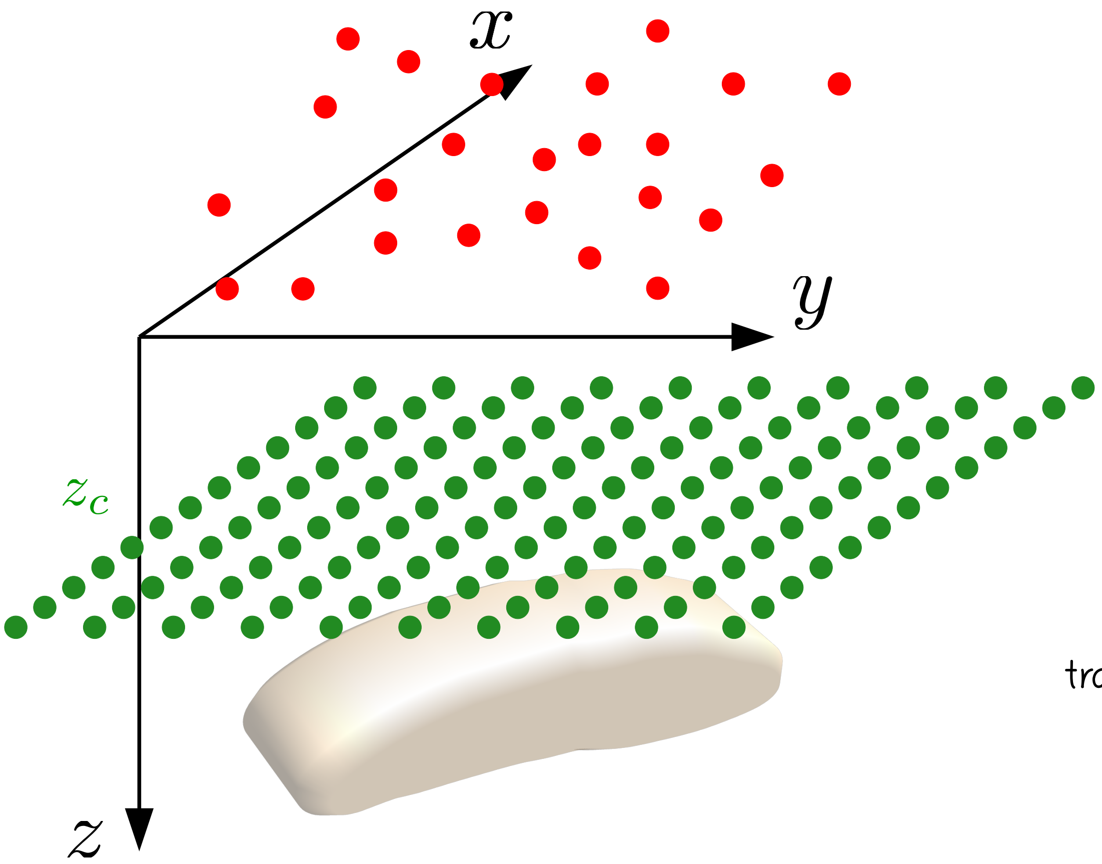
$$\mathbf{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix} \qquad a_{ij} \qquad \mathbf{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}$$

$$\mathbf{w} = \mathbf{T}\,\tilde{\mathbf{p}}$$

transformed data

estimated parameter vector (equivalent layer)

Classical EqL Technique



$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix} \qquad a_{ij} \qquad \mathbf{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}$$

$$\mathbf{w} = \mathbf{T}\,\tilde{\mathbf{p}}$$

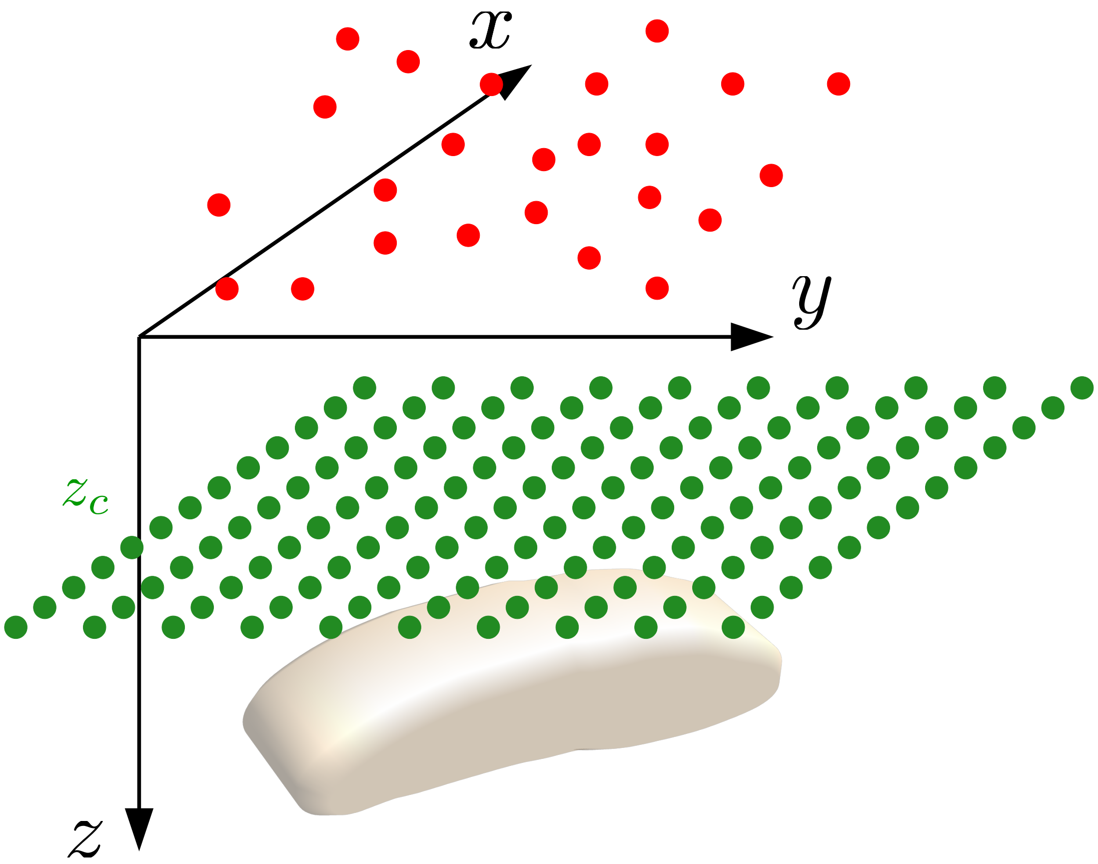
transformed data

t_{kj}

interpolation,
upward continuation, RTP,
field component conversion, ...

estimated
parameter vector
(equivalent layer)

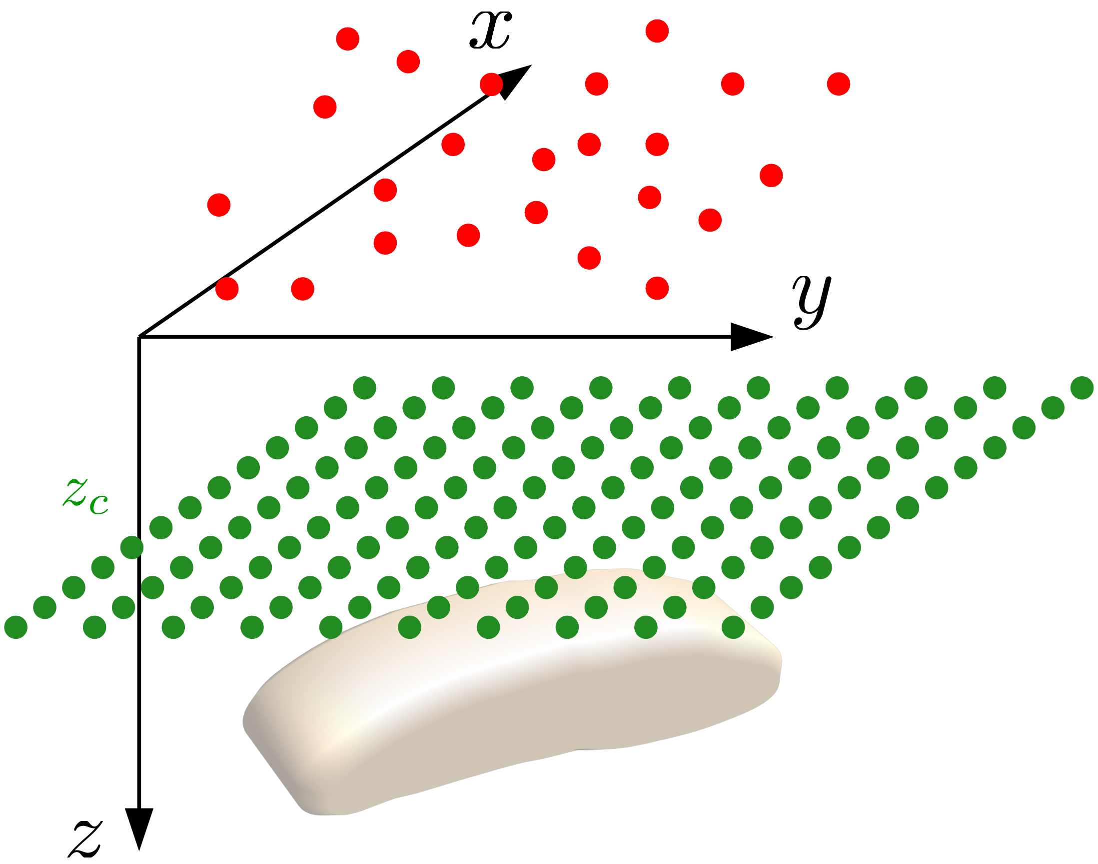
Classical EqL Technique



$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

A considerable amount of literature has been published on numerical methods for efficiently solving this linear system (e.g., Leão and Silva, 1989; Cordell, 1992; Mendonça and Silva, 1994; Guspí and Novara, 2009; Li and Oldenburg, 2010; Barnes and Lumley, 2011; Oliveira Jr. et al., 2013; Siqueira et al., 2017; Takahashi et al., 2020; Soler and Uieda, 2021)

Classical EqL Technique



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In this presentation, however, I will explore theoretical aspects of the EqL technique

Summary

- Potential-field data
- **The Equivalent-Layer (EqL) Technique**
- Theoretical aspects
- Some open questions

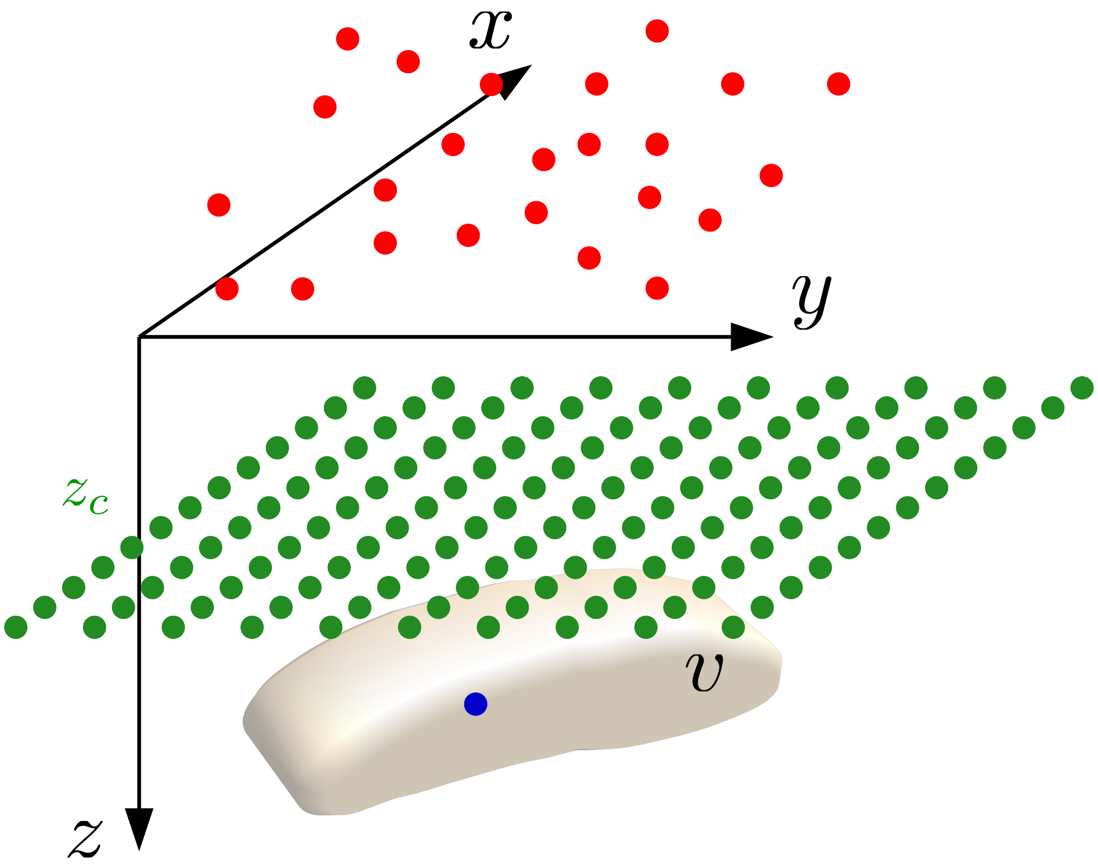
Summary

- Potential-field data
- The Equivalent-Layer (EqL) Technique
- **Theoretical aspects**
- Some open questions

We have briefly discussed **how** the EqL technique works, ...

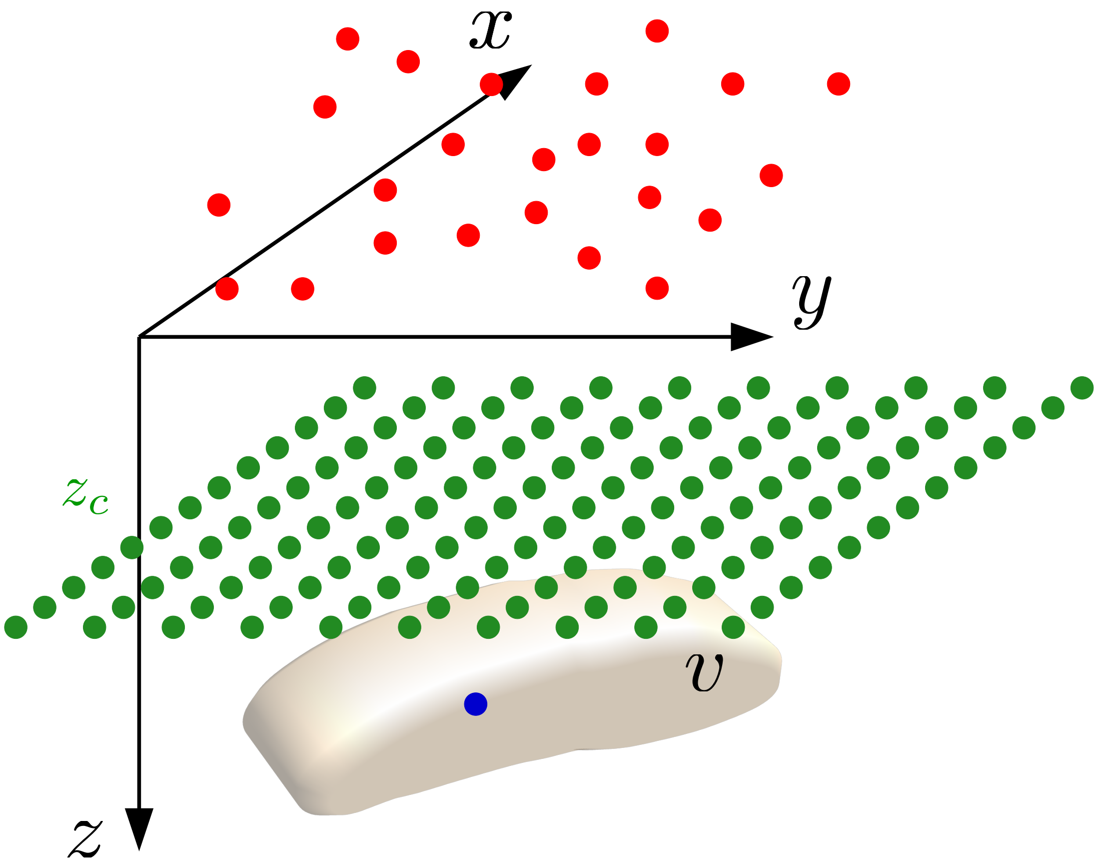
We have briefly discussed **how** the EqL technique works, ...

... now we need to discuss **why** the EqL technique works



$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

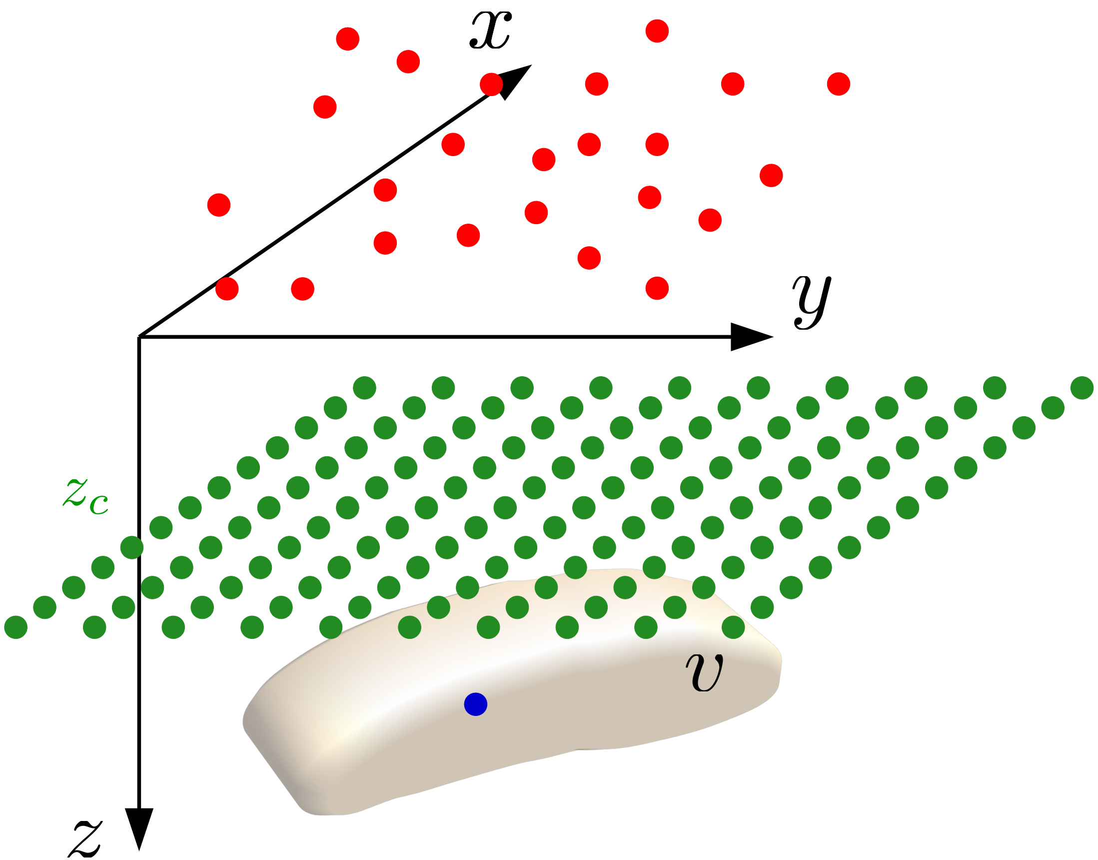
Let us return to the linear system for estimating the discrete equivalent layer



Discrete equivalent layer

$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

Let us return to the linear system for estimating the discrete equivalent layer

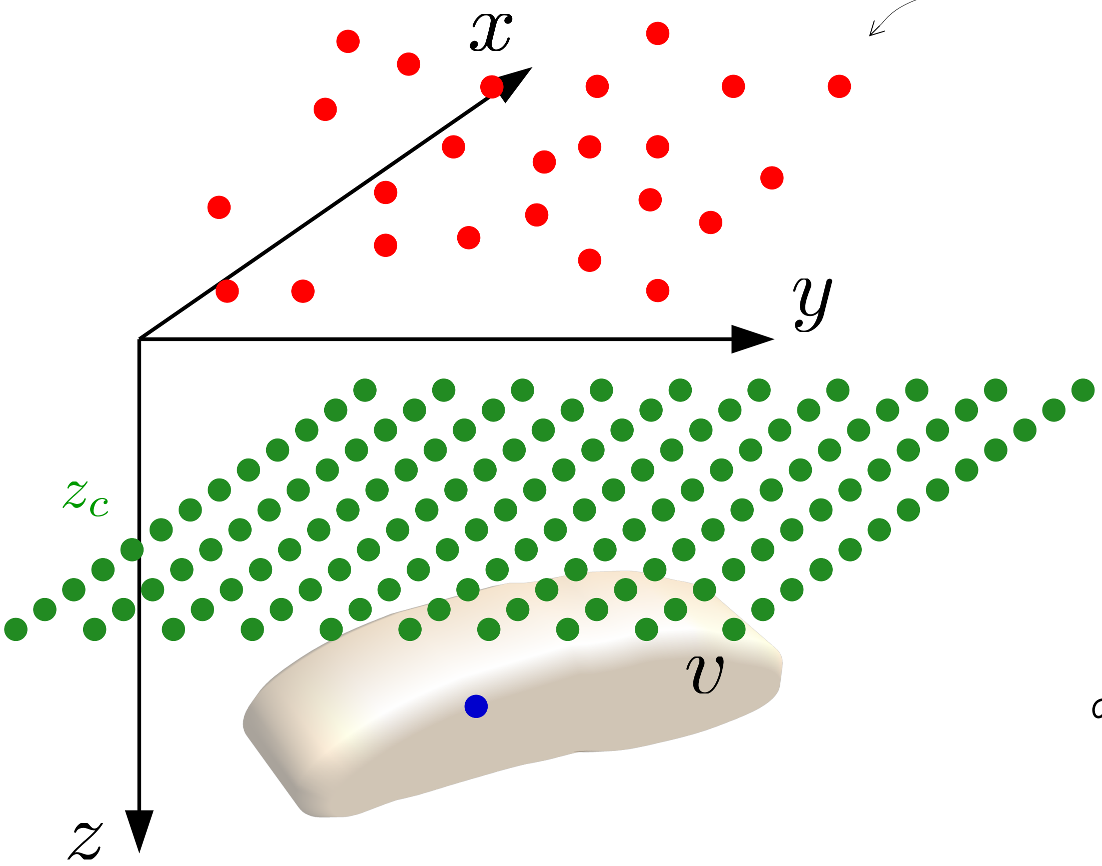


$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

$$\mathbf{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}$$

mass
(density)

moment
(magnetization)
intensity

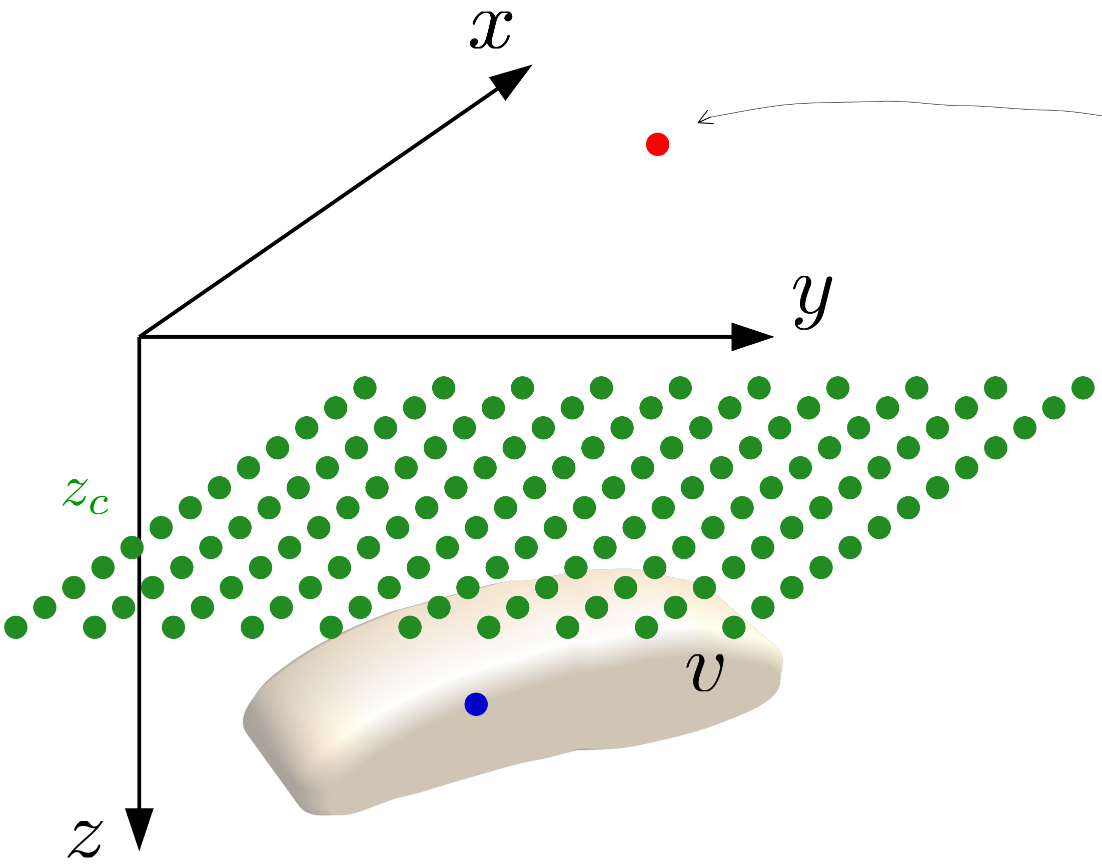


$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}$$

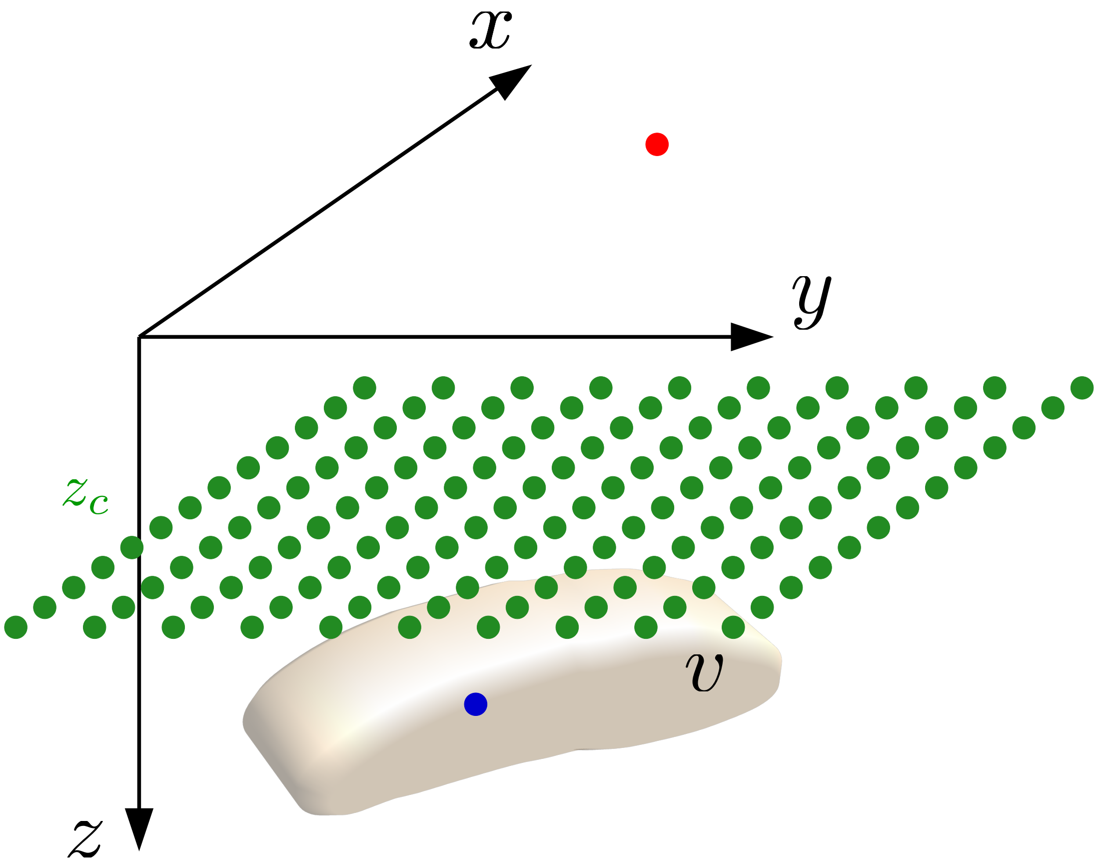
gravity
disturbance

approx total-field
anomaly



$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

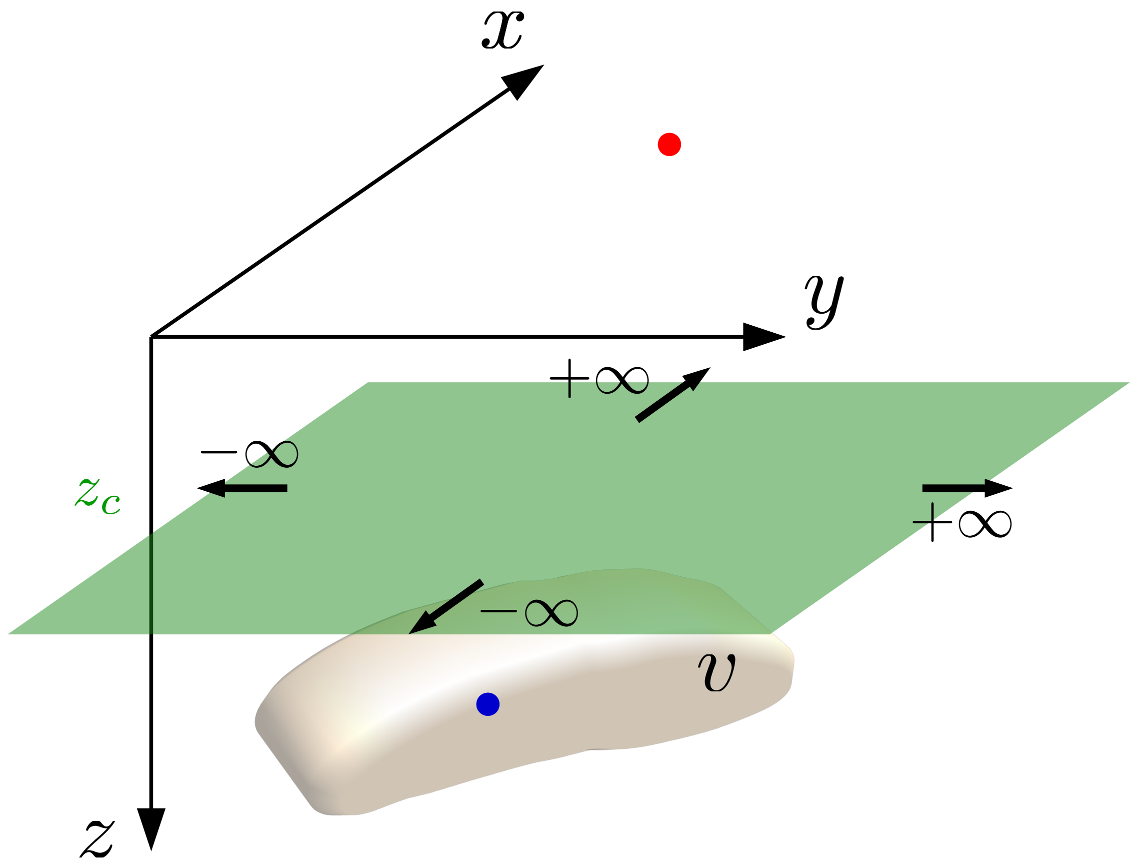
$$d_i \approx \sum_{j=1}^{M} p_j \, a_{ij}$$



$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

$$d_i \approx \sum_{j=1}^{M} p_j \, a_{ij}$$

Finite set of points defining
a **discrete equivalent layer**



$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

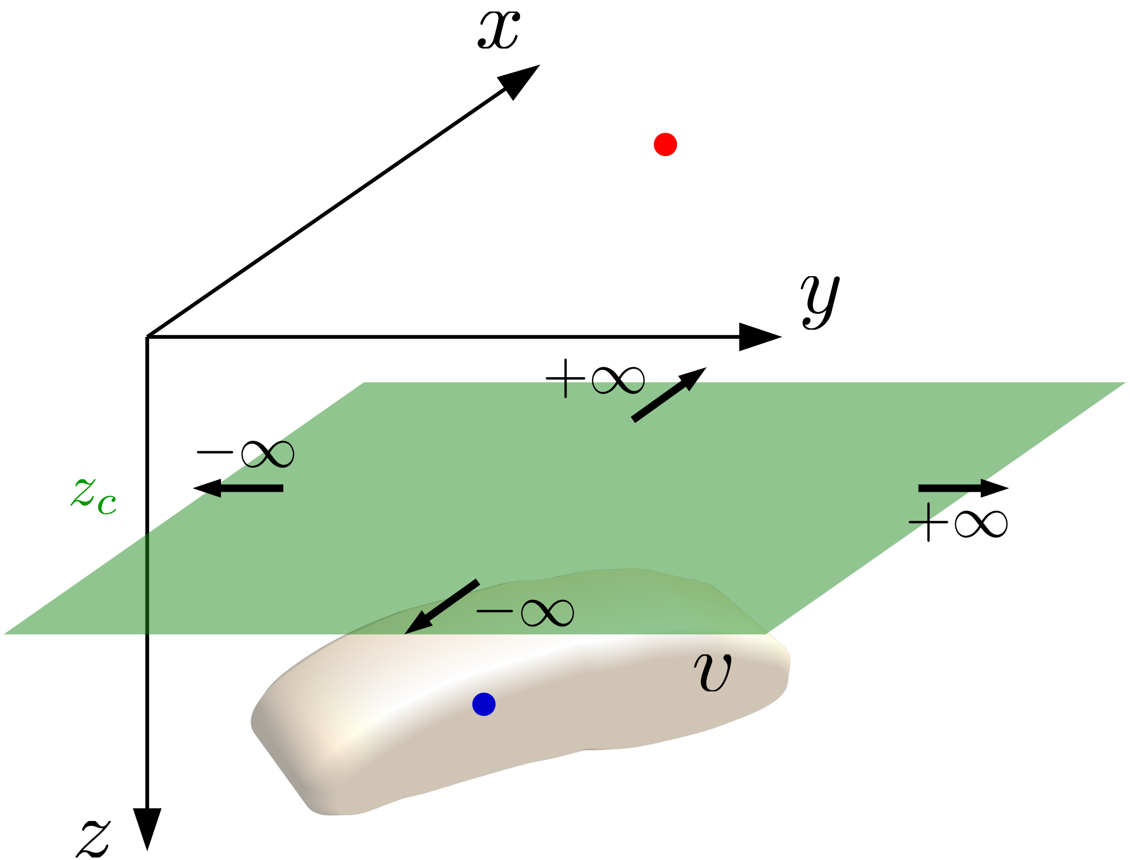
$$d_i \approx \sum_{j=1}^{M} p_j \, a_{ij}$$

Finite set of points defining
a **discrete equivalent layer**

$dx''\,dy''$

$$d(\mathbf{r}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(\mathbf{r}'')\, a(\mathbf{r}, \mathbf{r}'')\, dS''$$

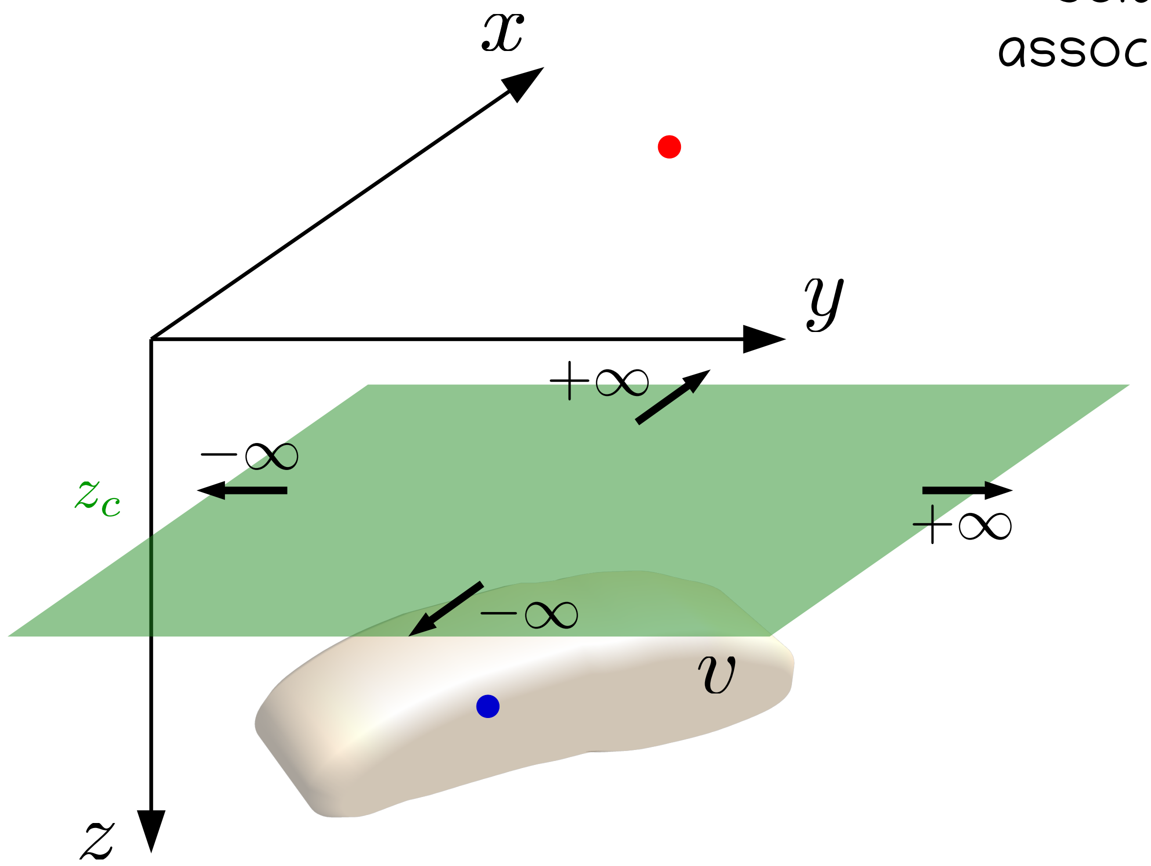
Infinite horizontal plane defining
a **continous equivalent layer**



Integral equation for the continuous physical-property distribution (equivalent layer) $p(\mathbf{r}'')$

$dx''\,dy''$

$$d(\mathbf{r}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(\mathbf{r}'')\, a(\mathbf{r}, \mathbf{r}'')\, dS''$$

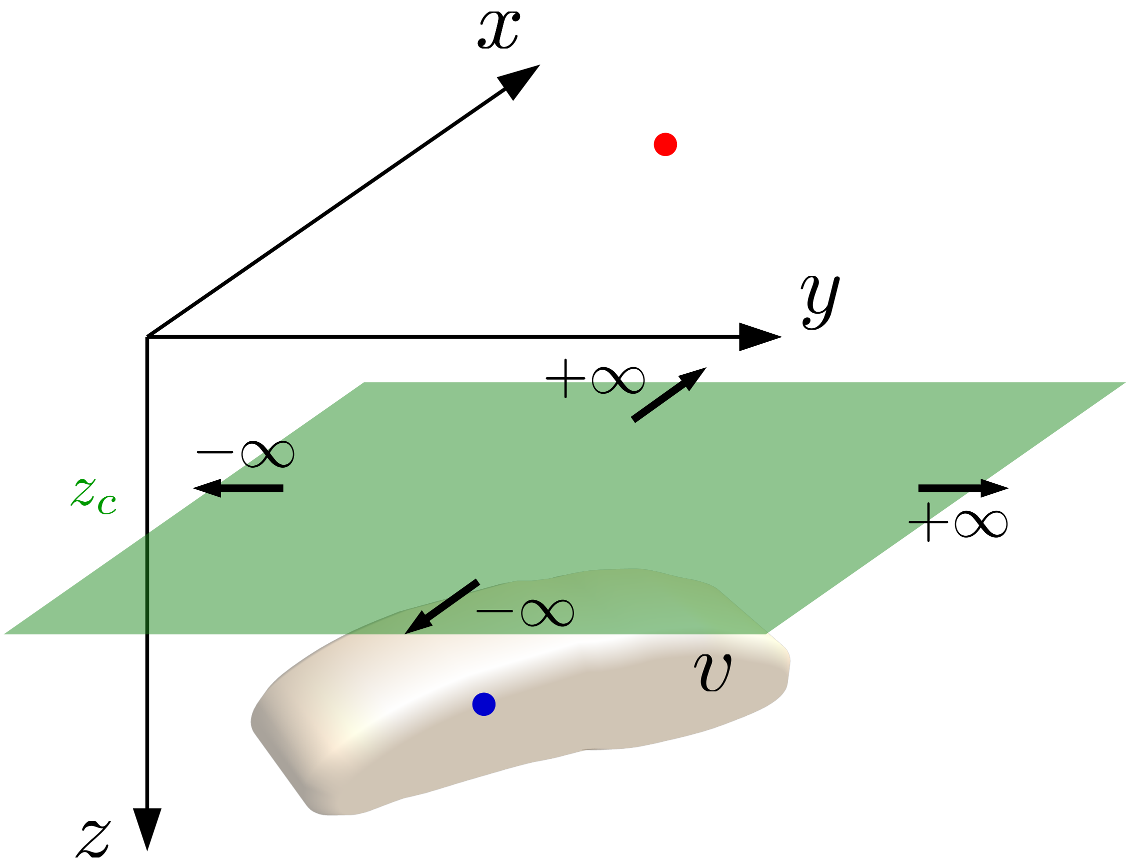


The following slides present analytical solutions for the integral equations associated with different potential-field data

Integral equation for the continuous physical-property distribution (equivalent layer) $p(\mathbf{r}'')$

$dx''\,dy''$

$$d(\mathbf{r}) = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} p(\mathbf{r}'')\, a(\mathbf{r}, \mathbf{r}'')\, dS''$$



These solutions define analytical equivalent layers

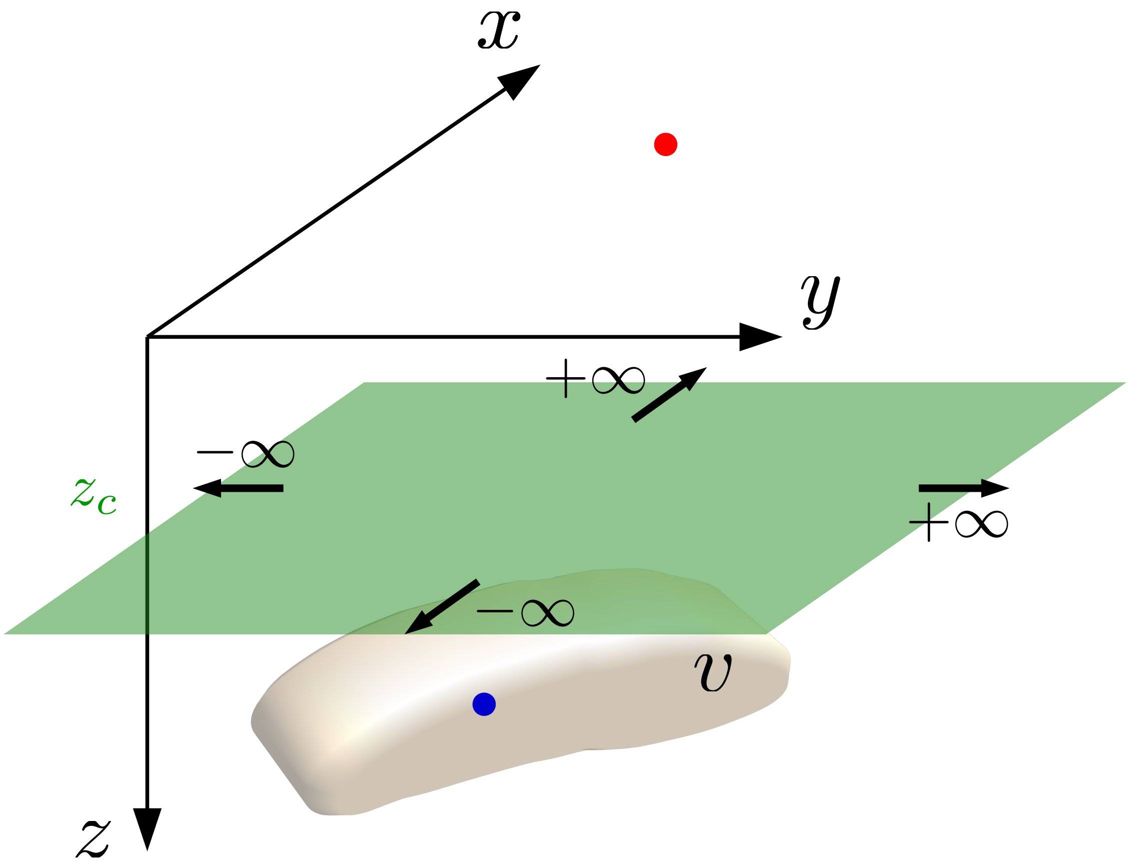
Integral equation for the continuous physical-property distribution (equivalent layer) $p(\mathbf{r}'')$

$dx''\,dy''$

$$d(\mathbf{r}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(\mathbf{r}'')\, a(\mathbf{r}, \mathbf{r}'')\, dS''$$

Let us turn our attention back to this function

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



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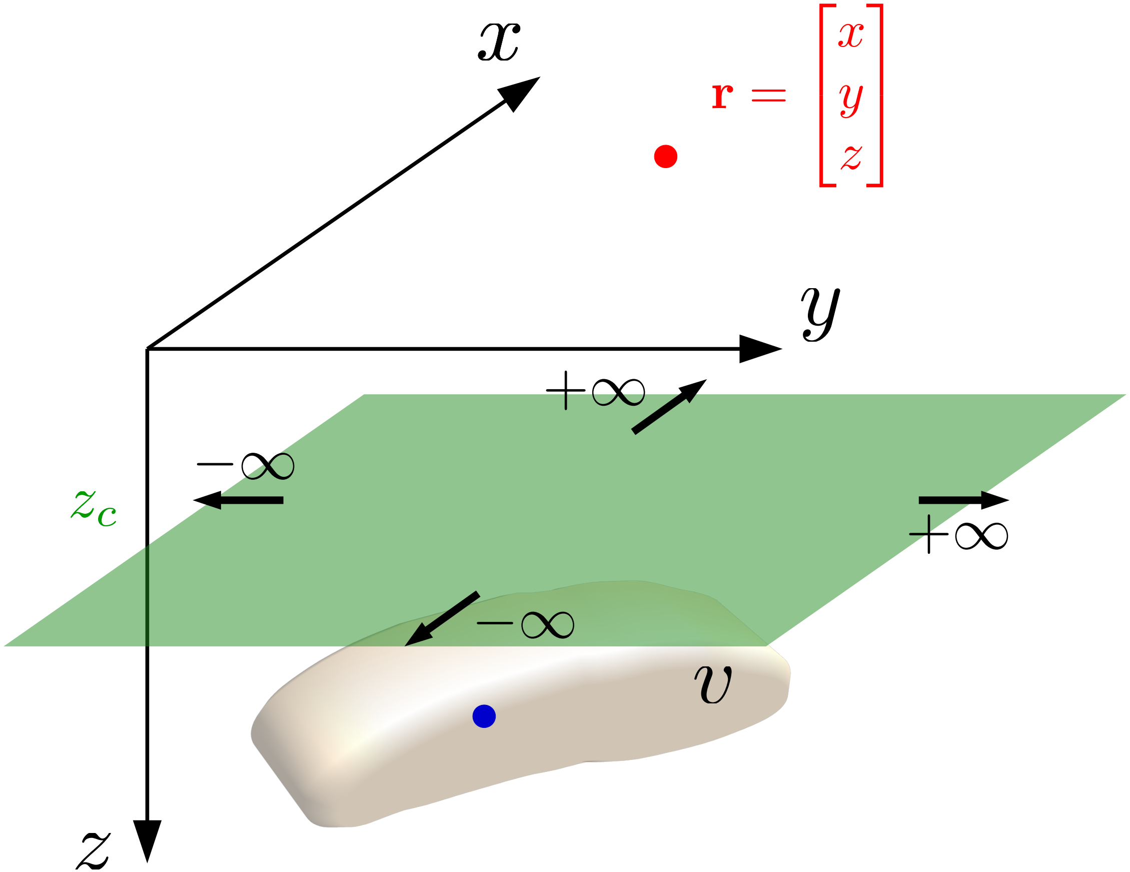

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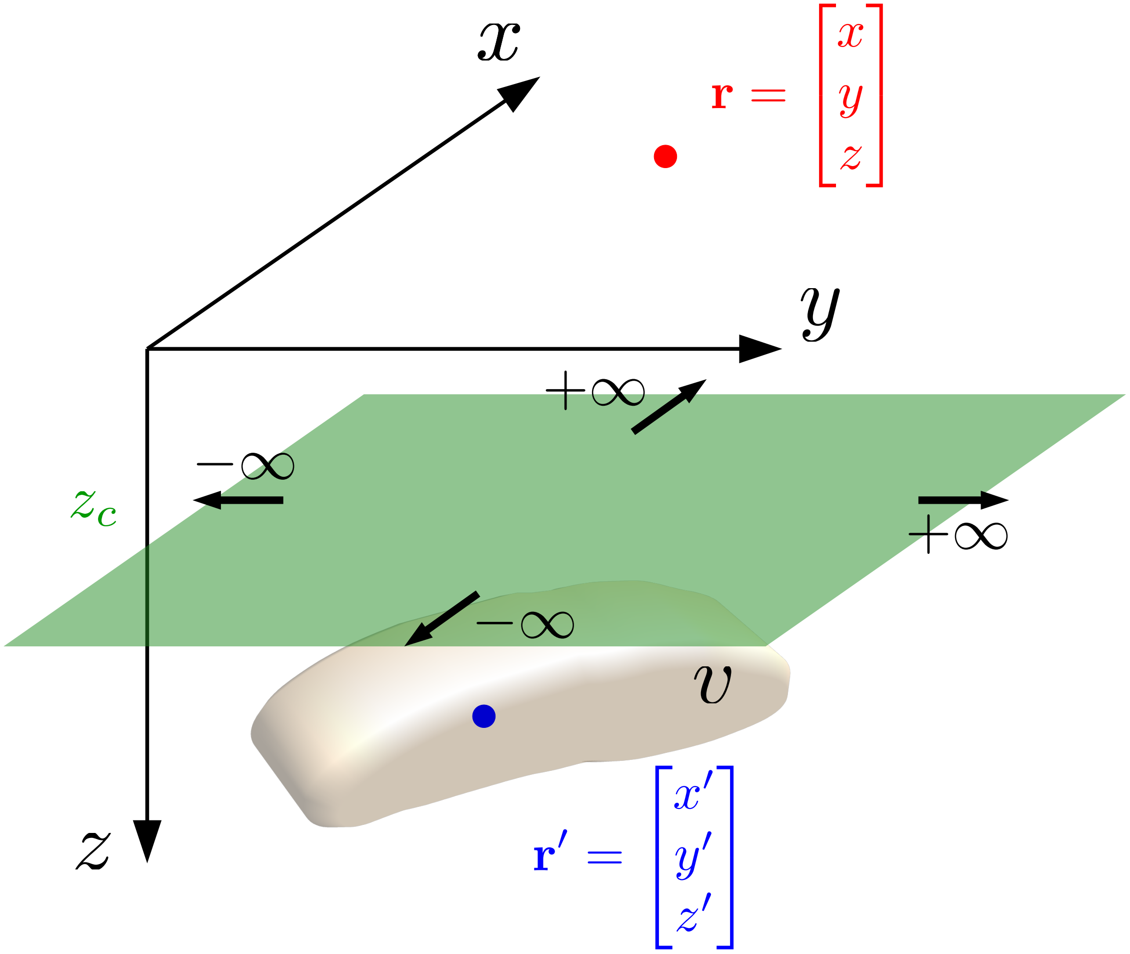
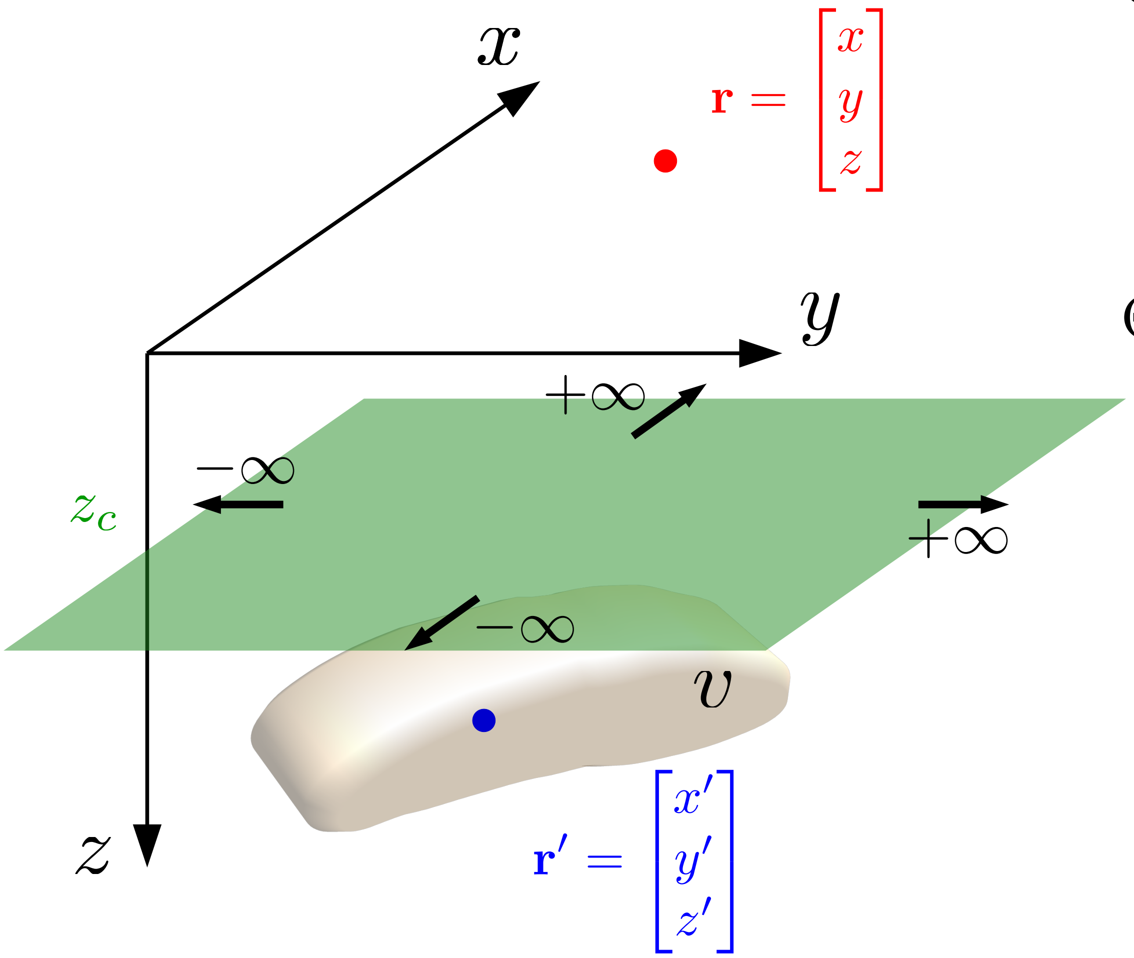


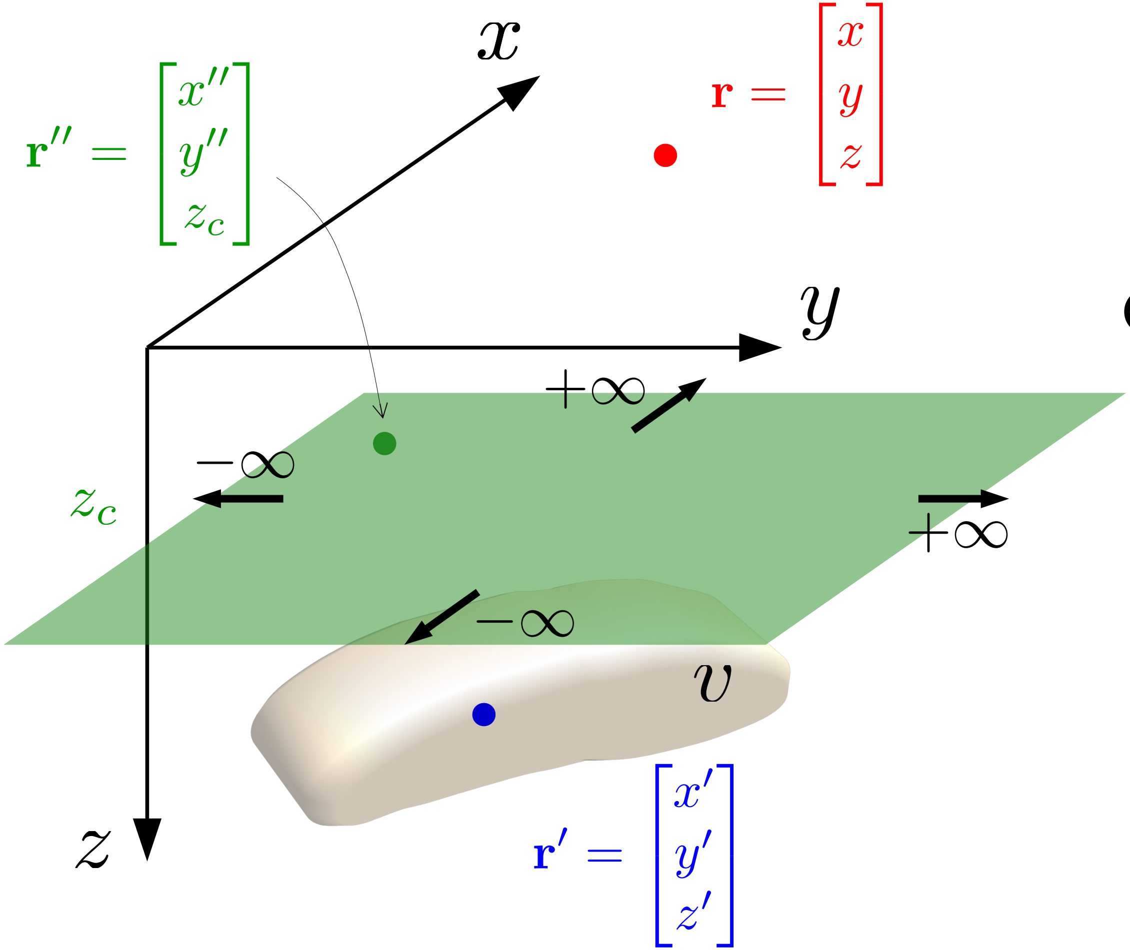


$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

It can be shown that

$$\Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

which represents the solution of the **Neumann's problem** or the **second boundary value problem of potential theory** (Kellogg, 1967, p. 246) on a plane



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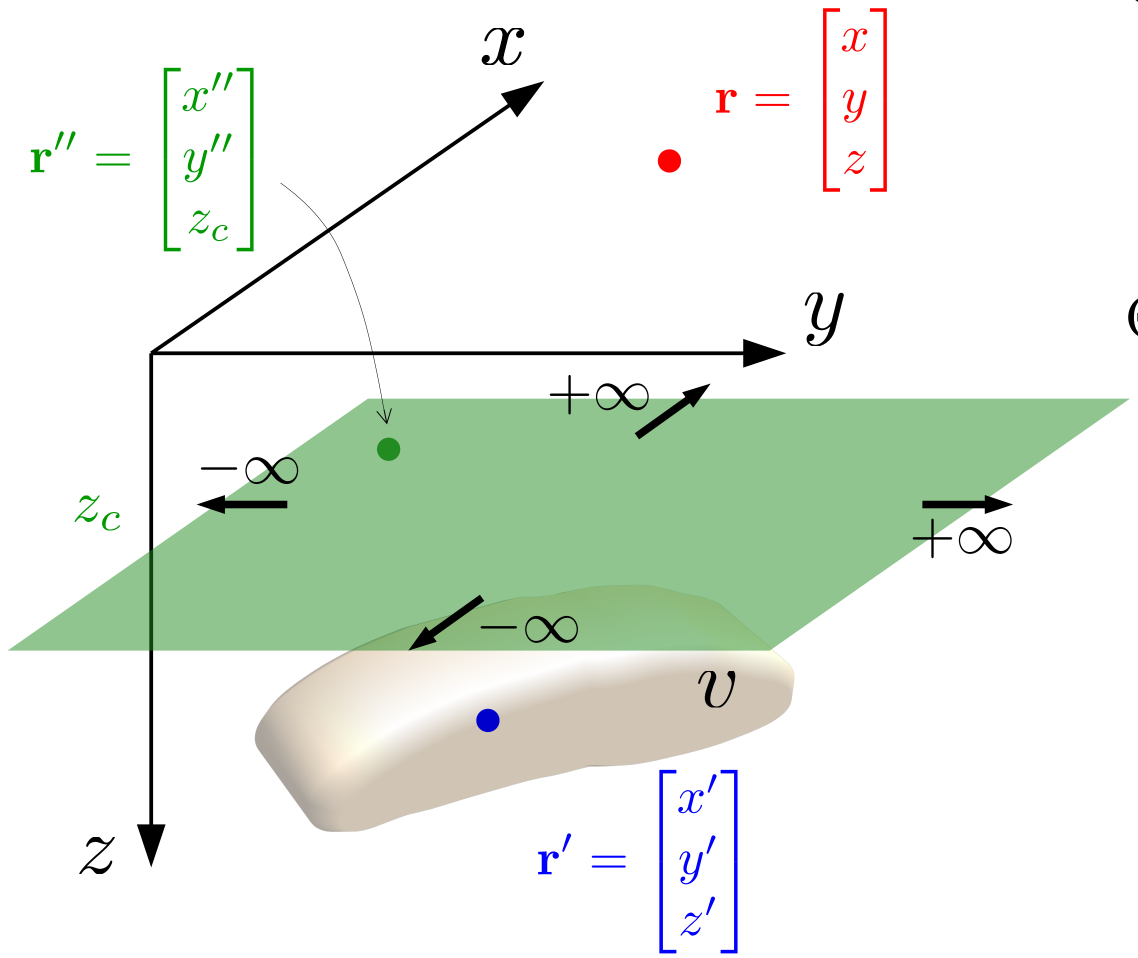

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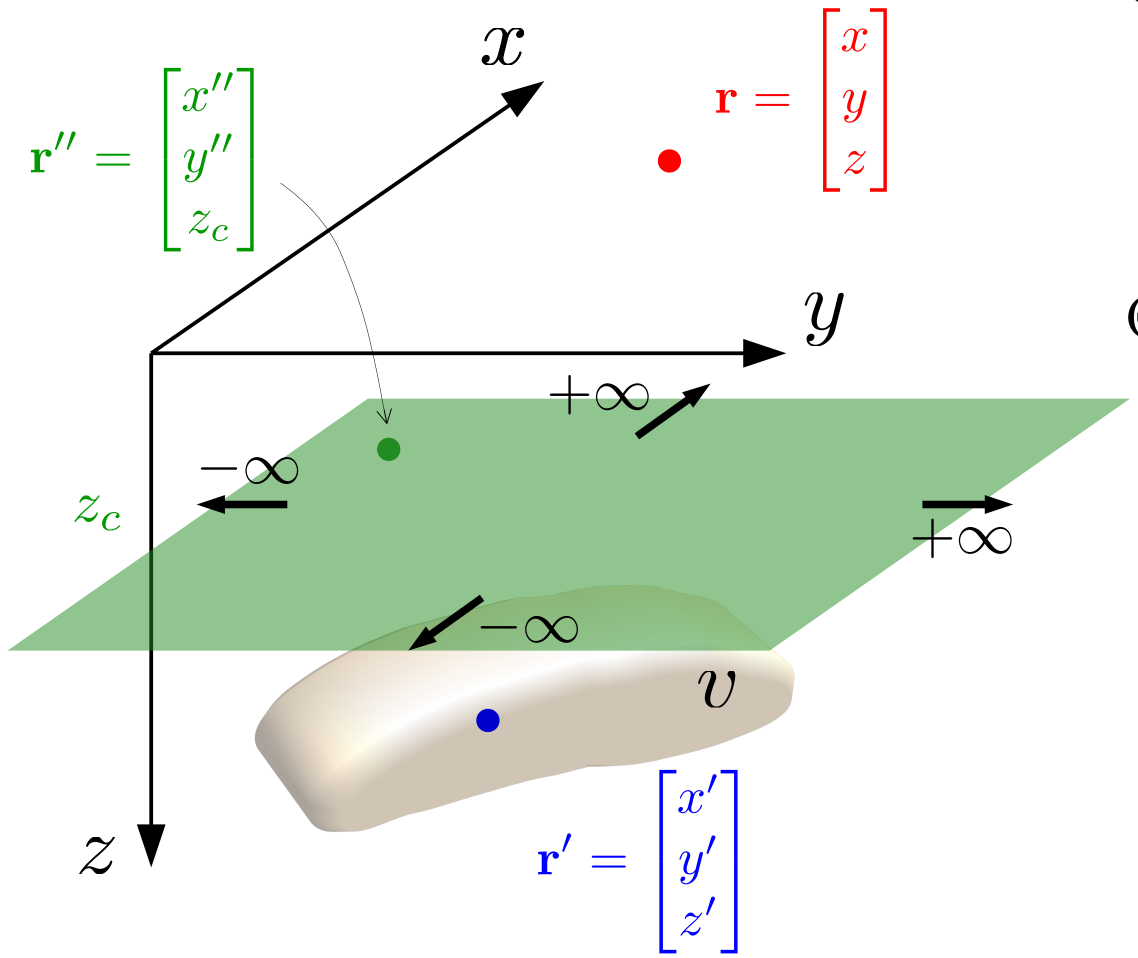

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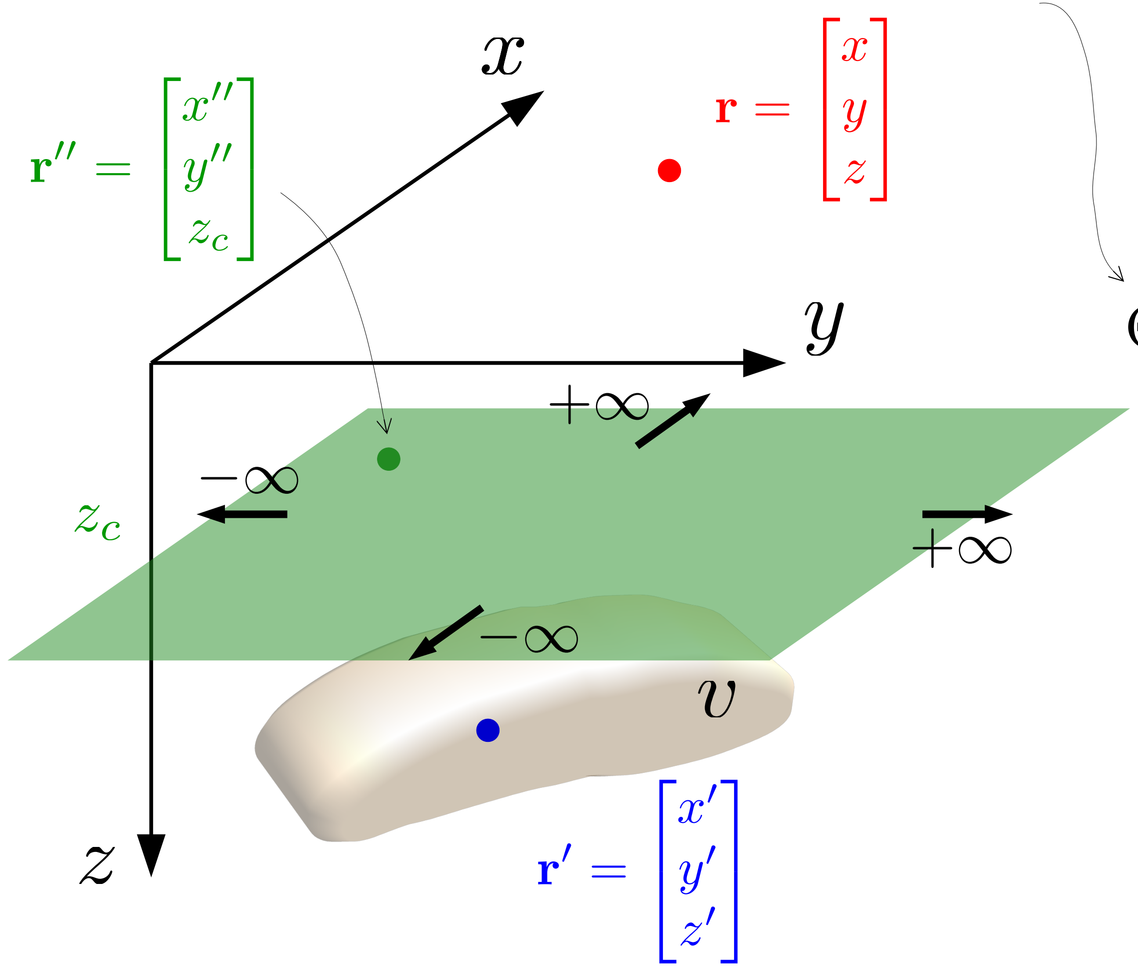
$$\partial_z \Theta(\mathbf{r}'') = \kappa \iiint\limits_v \sigma(\mathbf{r}') \; \partial_z \frac{1}{\|\mathbf{r}'' - \mathbf{r}'\|} dv'$$

$$\Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$

Most of what is known or assumed to be true without any proof about the EqL technique can be deduced from this equation



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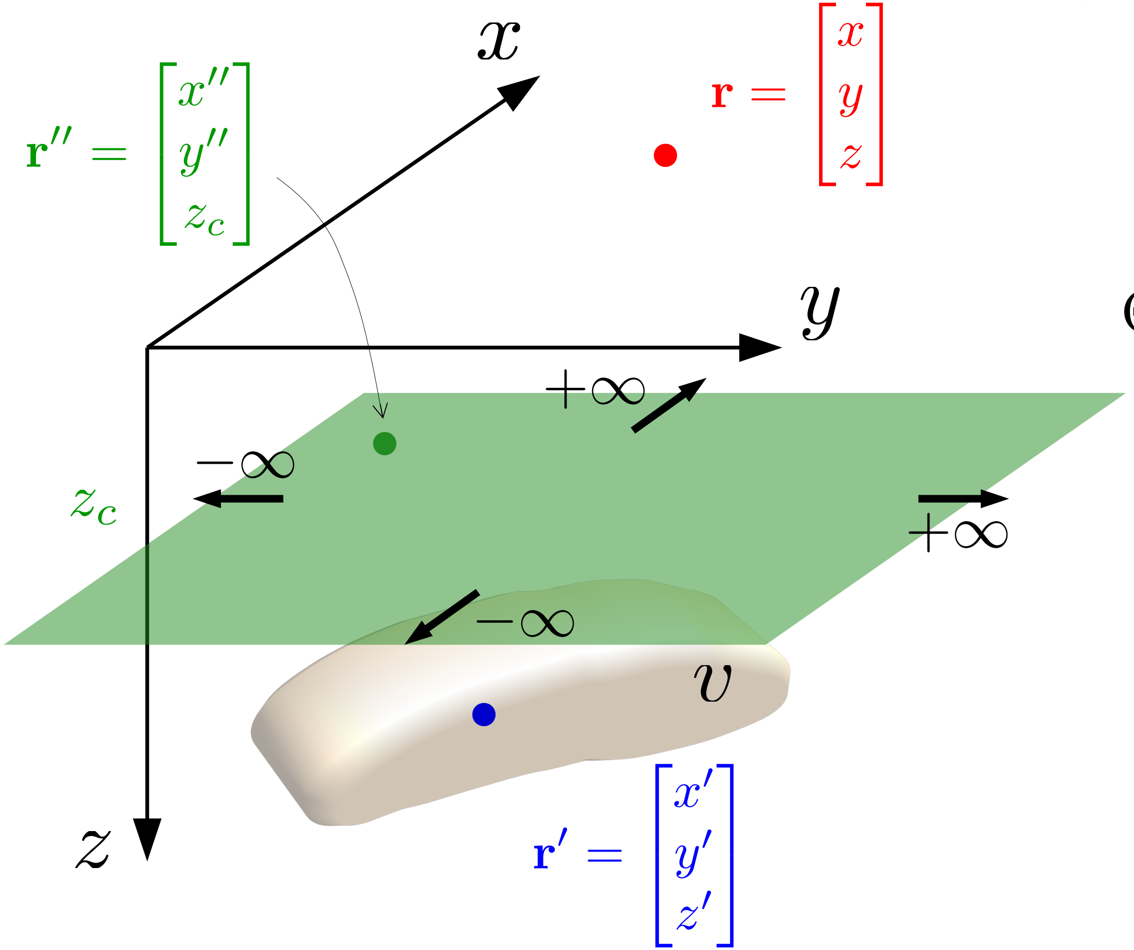
$$\Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$

We are going to see four theoretical results that support most of the commonly potential-field transformations made via EqL technique



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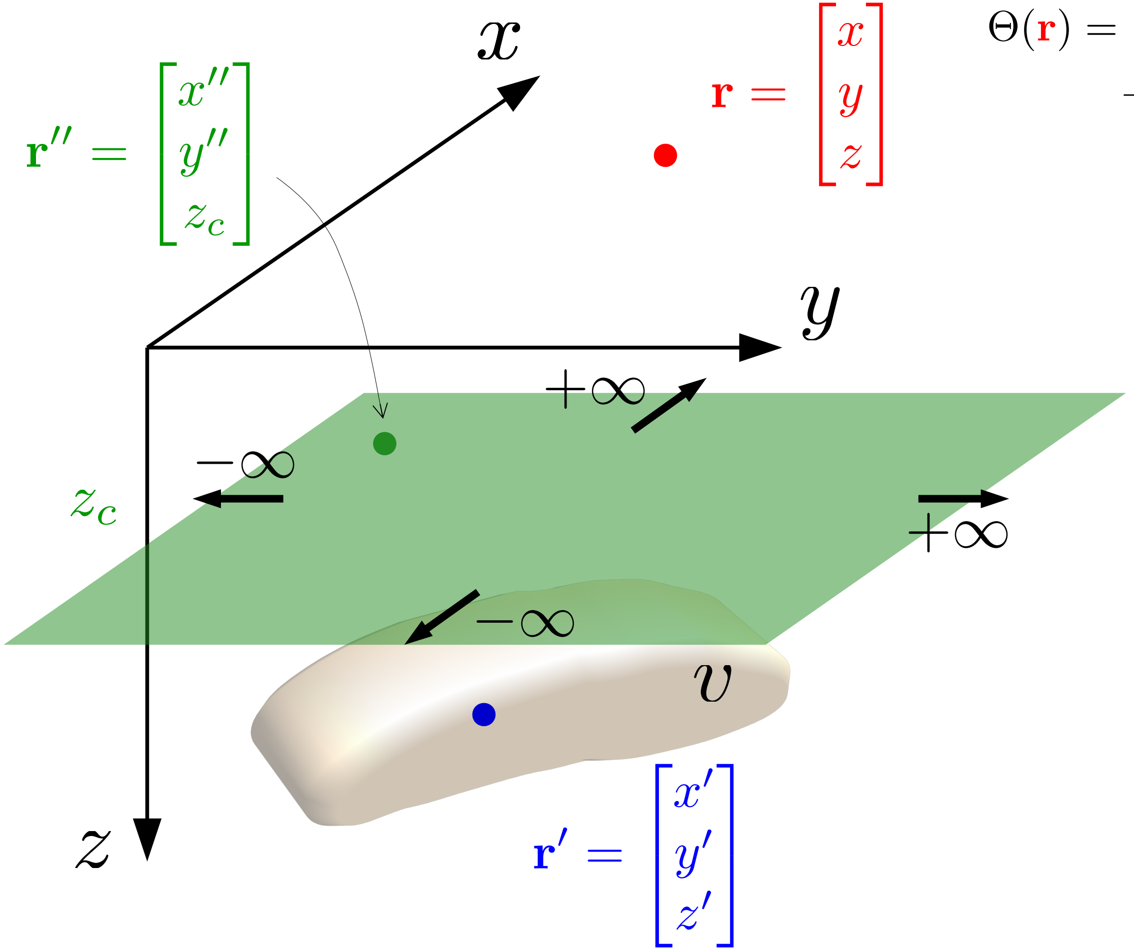
$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}')\, \frac{1}{\|\mathbf{r}-\mathbf{r}'\|}\, dv'$$

$$\Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'')\, \Psi(\mathbf{r},\mathbf{r}'')\, dS'' \qquad \Psi(\mathbf{r},\mathbf{r}'') = \frac{1}{2\pi}\frac{1}{\|\mathbf{r}-\mathbf{r}''\|}$$



Result 1/4: The gravity disturbance can be exactly reproduced by a planar eq. layer of monopoles

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

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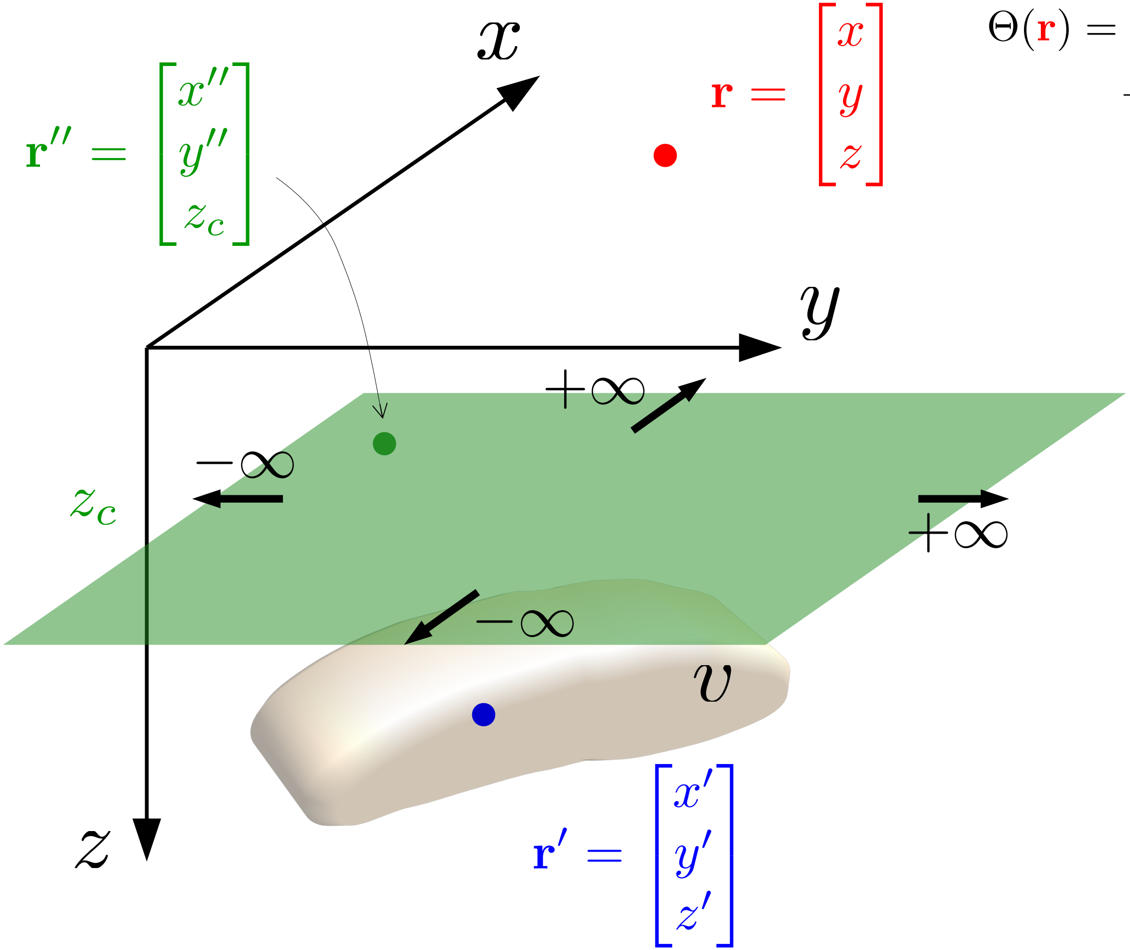

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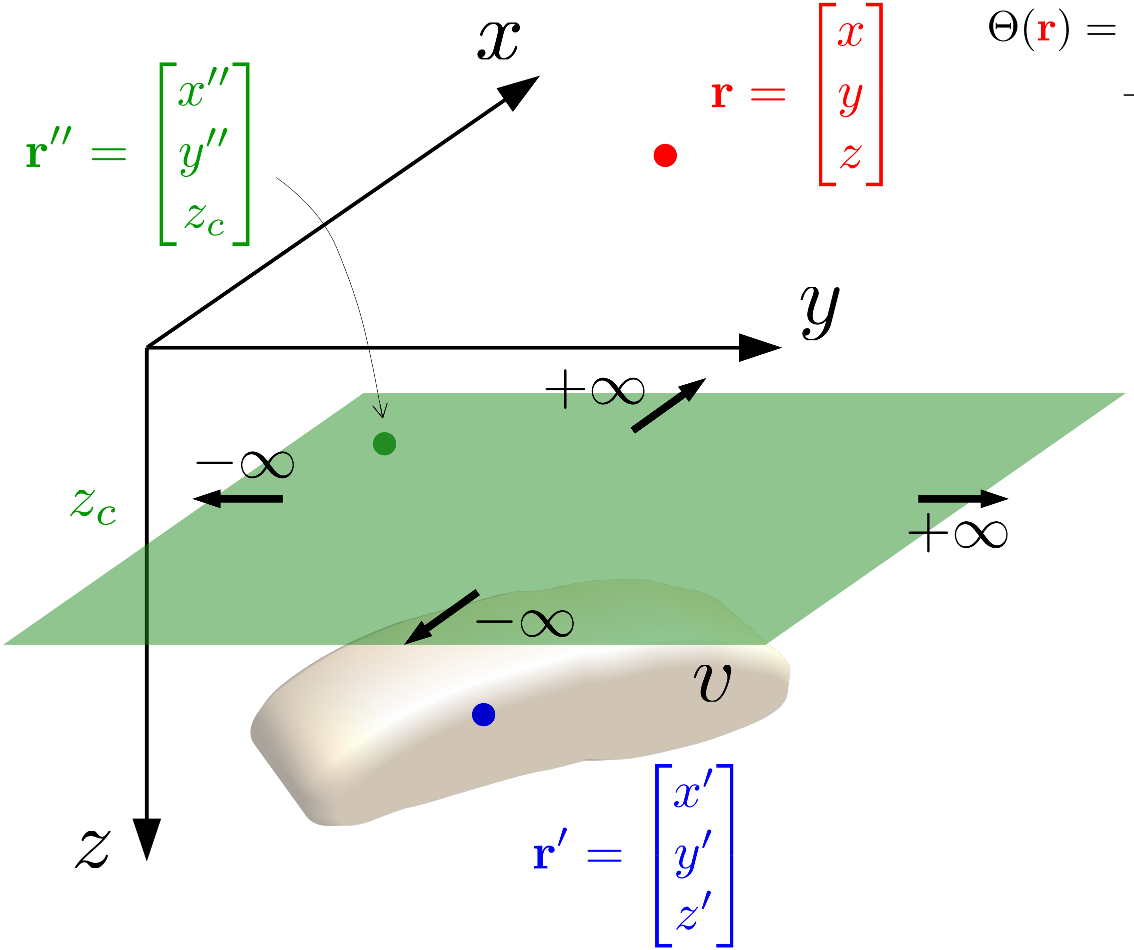

Dampney (1969) was the pioneer in using this result to interpolate gravity data.

Result 1/4: The gravity disturbance can be exactly reproduced by a planar eq. layer of monopoles

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

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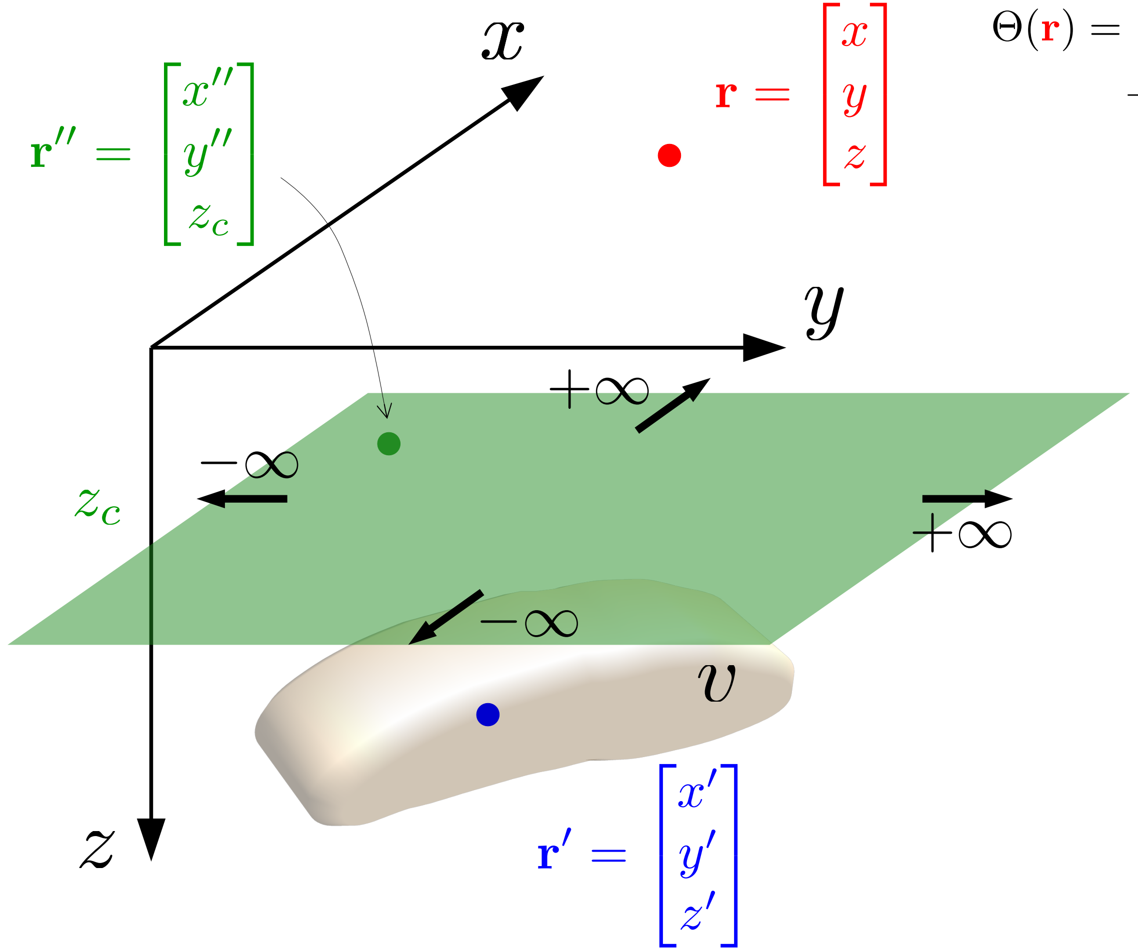

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A considerable amount of literature has been posteriorly published on this topic

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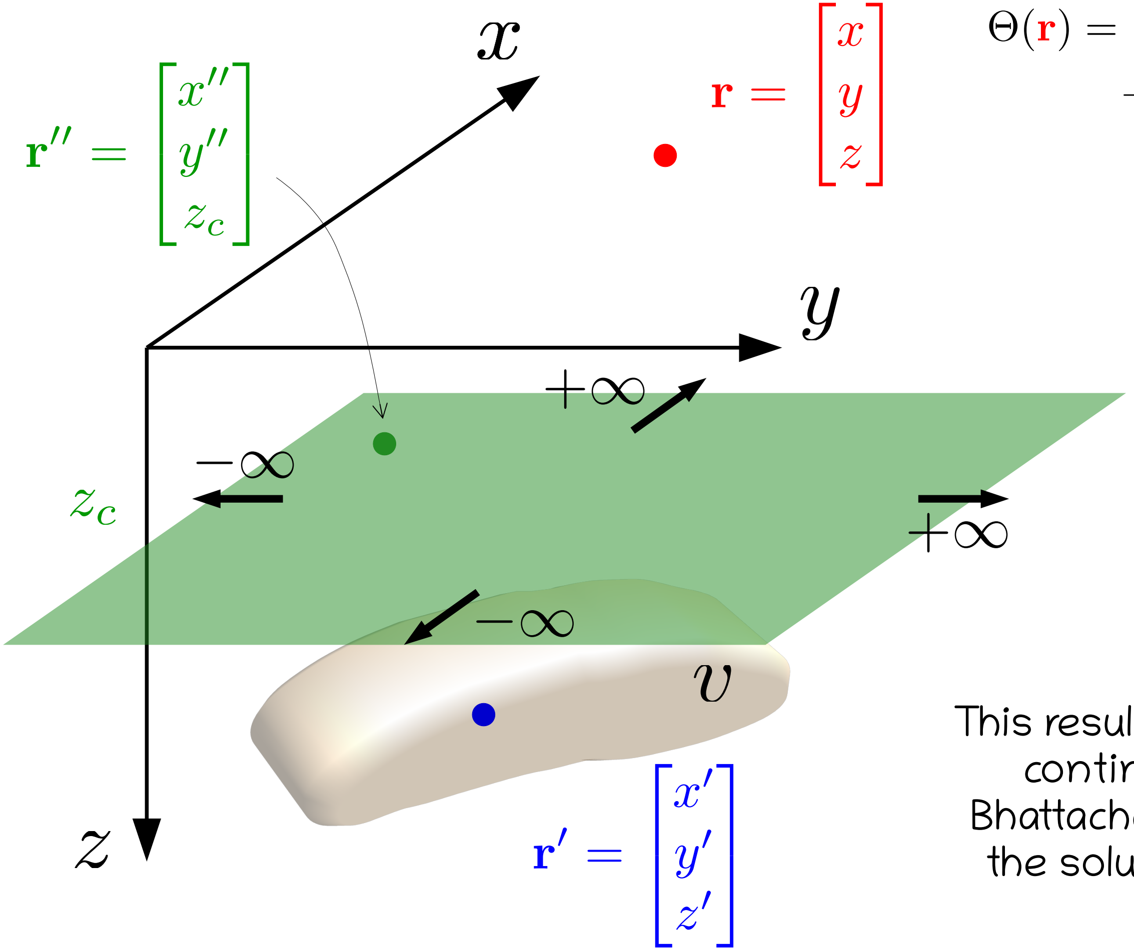

Dampney (1969) was the pioneer in using this result to interpolate gravity data.

A considerable amount of literature has been posteriorly published on this topic

This result can also be directly deduced from the upward-continuation integral (e.g., Peters, 1949; Roy, 1962; Bhattacharyya, 1967; Henderson, 1970), which represents the solution of Dirichlet's problem on a plane (Kellogg, 1967, p. 236)

Result 1/4: The gravity disturbance can be exactly reproduced by a planar eq. layer of monopoles

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

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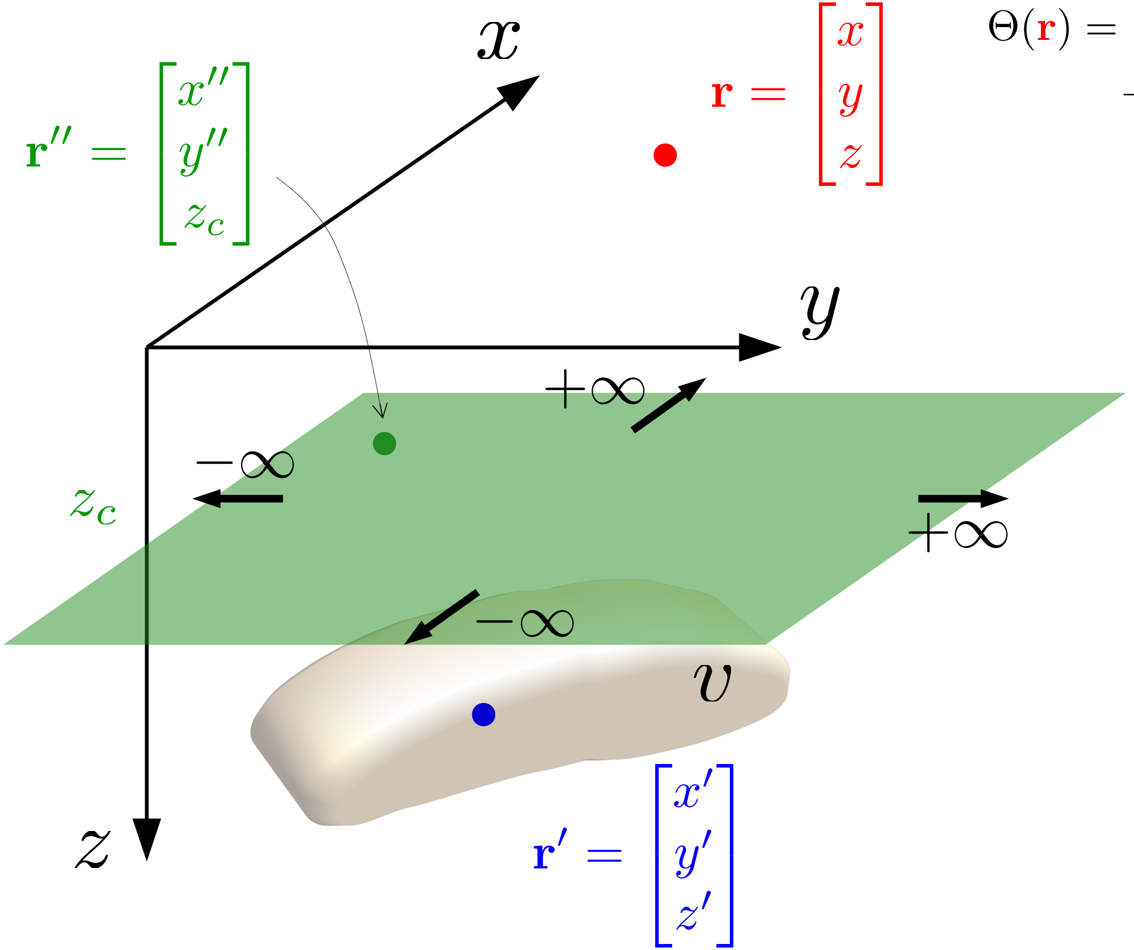


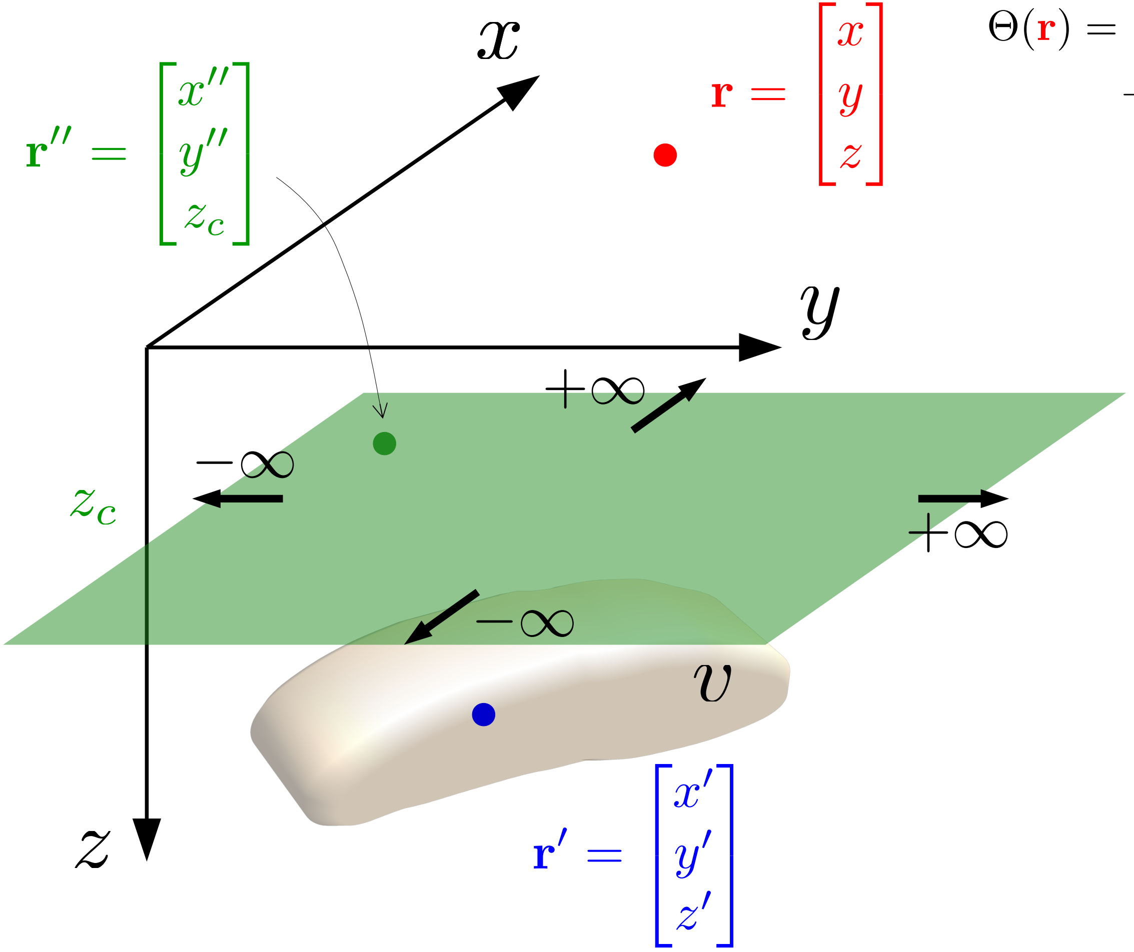
Deduction:

$\partial_z \Theta(\mathbf{r})$ gravity disturbance data

Result 1/4: The gravity disturbance can be exactly reproduced by a planar eq. layer of monopoles

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$

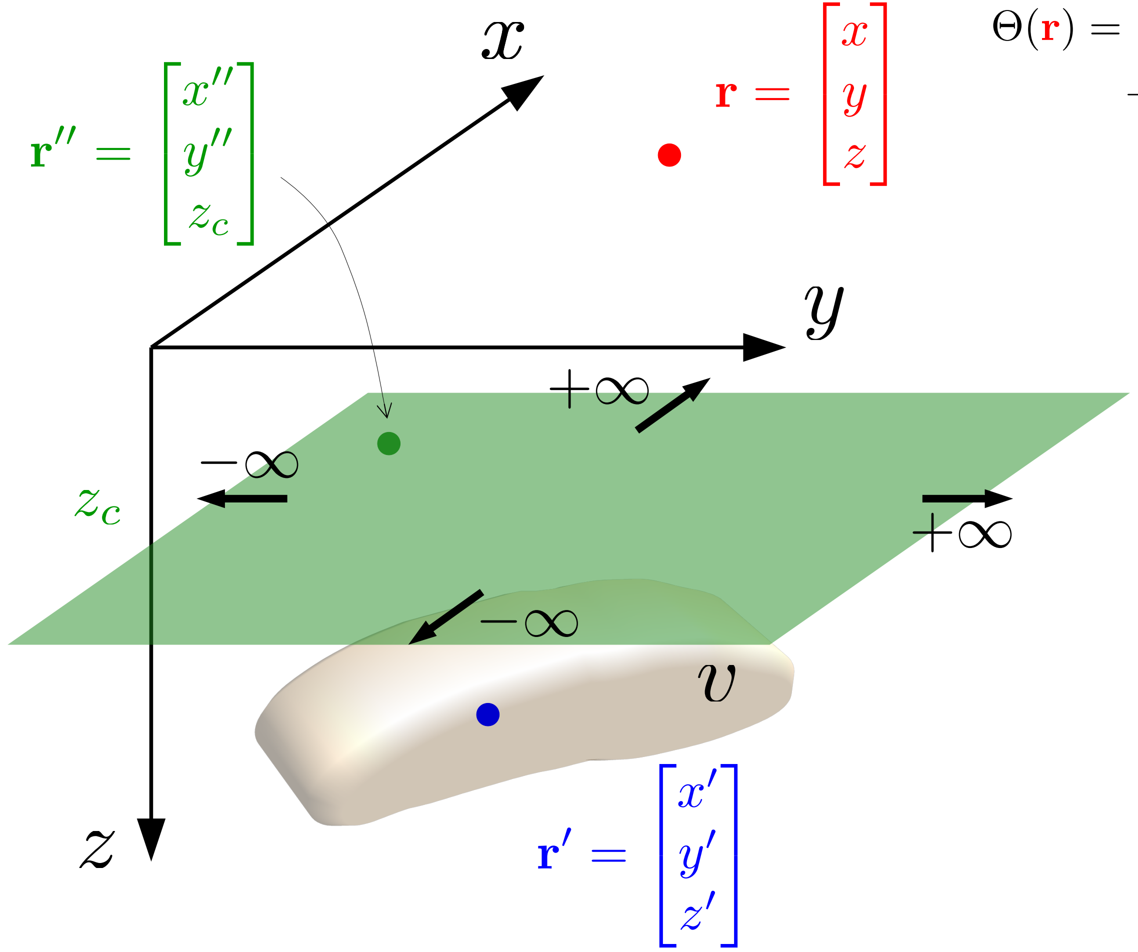


Deduction:

$\partial_z \Theta(\mathbf{r})$ gravity disturbance data

$$\partial_z \Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Result 1/4: The gravity disturbance can be exactly reproduced by a planar eq. layer of monopoles



$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

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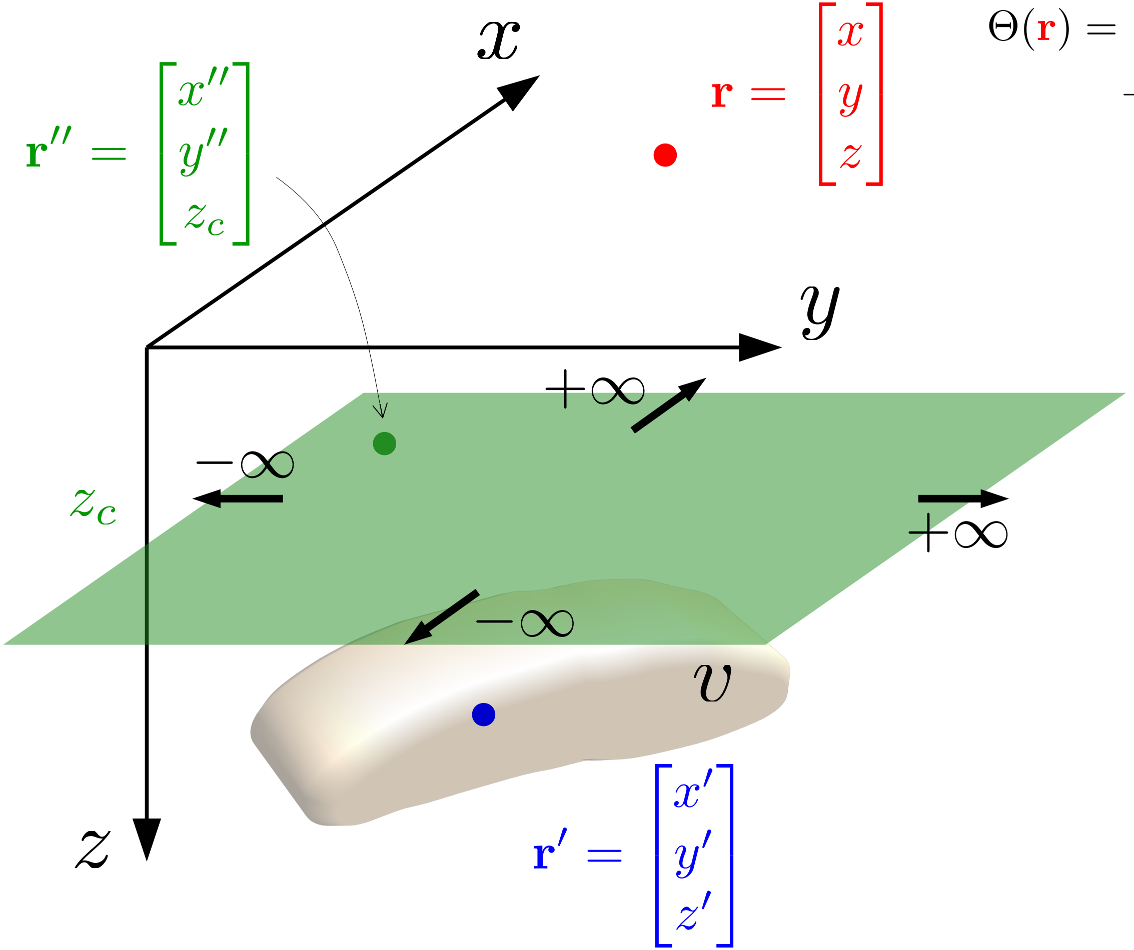
Deduction:

$\partial_z \Theta(\mathbf{r})$ gravity disturbance data

$$\partial_z \Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \underbrace{\partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r}, \mathbf{r}'')}_{} \, dS''$$

This term represents the gravity disturbance produced at $\mathbf{r}$ by a monopole located at $\mathbf{r}''$

Result 1/4: The gravity disturbance can be exactly reproduced by a planar eq. layer of monopoles



$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

$$\Theta(\mathbf{r}) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'')\, \Psi(\mathbf{r}, \mathbf{r}'')\, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$

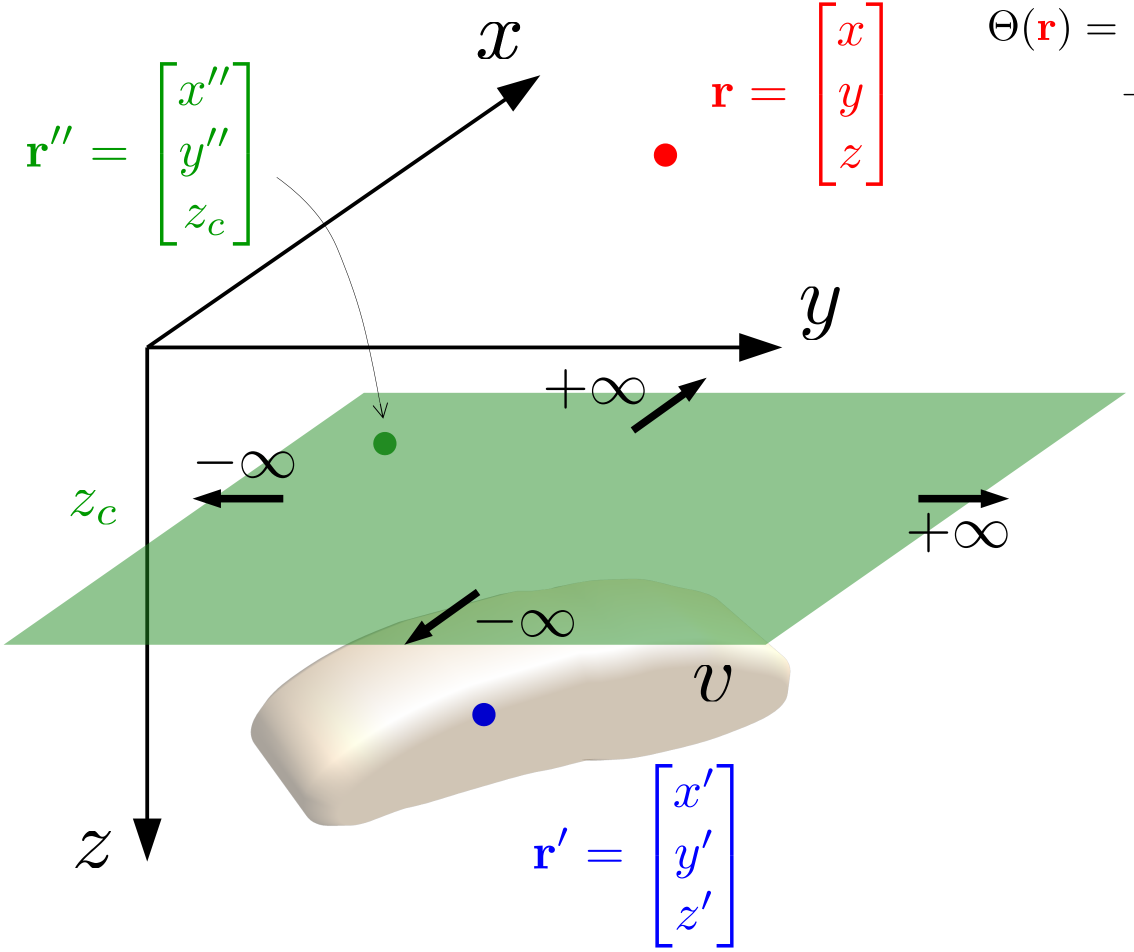
Deduction:

$\partial_z \Theta(\mathbf{r})$ gravity disturbance data

$$\partial_z \Theta(\mathbf{r}) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'')\, \partial_z \Psi(\mathbf{r}, \mathbf{r}'')\, dS''$$

This equation is the upward-continuation integral applied to the vertical derivative of $\Theta(\mathbf{r})$

Result 1/4: The gravity disturbance can be exactly reproduced by a planar eq. layer of monopoles



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Deduction:

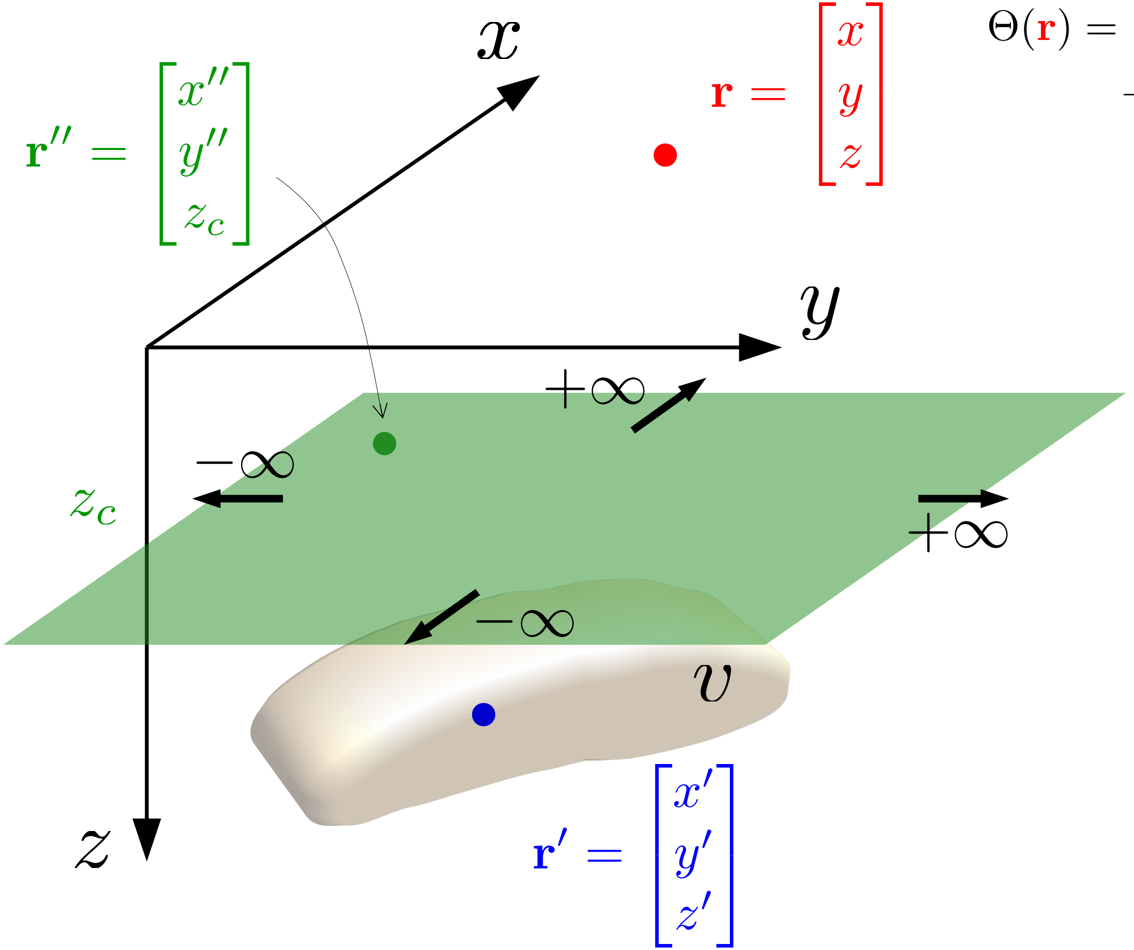
$\partial_z \Theta(\mathbf{r})$ gravity disturbance data

$$\partial_z \Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Analytical eq. layer given by:

$$\partial_z \Theta(\mathbf{r}'') = \kappa \iiint_v \sigma(\mathbf{r}') \, \partial_z \frac{1}{\|\mathbf{r}'' - \mathbf{r}'\|} \, dv'$$

Result 1/4: The gravity disturbance can be exactly reproduced by a planar eq. layer of monopoles



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Deduction:

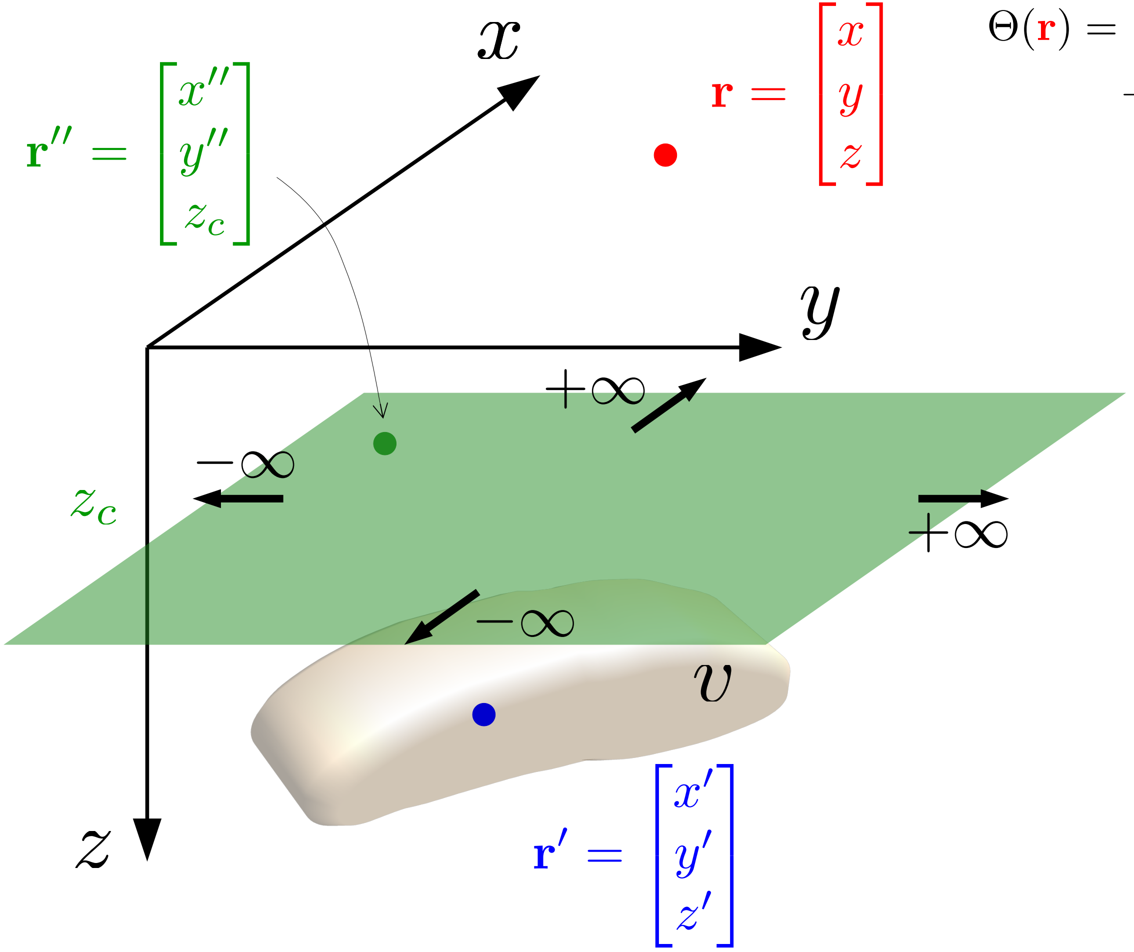
$\partial_z \Theta(\mathbf{r})$ gravity disturbance data

$$\partial_z \Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

This analytical eq. layer represents the grav. disturbance produced by the true sources on the plane z_c

$$\partial_z \Theta(\mathbf{r}'') = \kappa \iiint\limits_v \sigma(\mathbf{r}') \; \partial_z \frac{1}{\|\mathbf{r}'' - \mathbf{r}'\|} \, dv'$$

Result 1/4: The gravity disturbance can be exactly reproduced by a planar eq. layer of monopoles



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Deduction:

$\partial_z \Theta(\mathbf{r})$ gravity disturbance data

$$\partial_z \Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

Result 1/4: The gravity disturbance can be exactly reproduced by a planar eq. layer of monopoles

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

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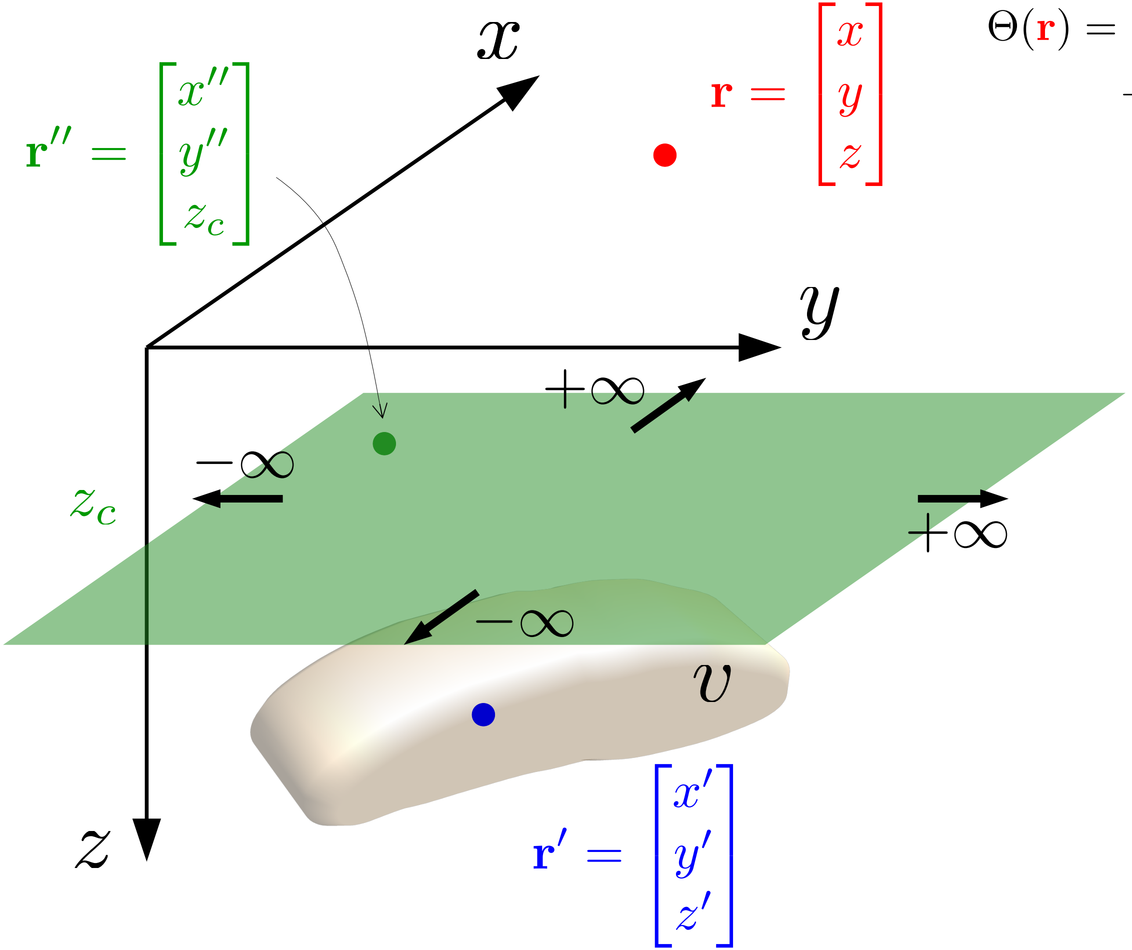

Deduction:

$\partial_z \Theta(\mathbf{r})$ gravity disturbance data

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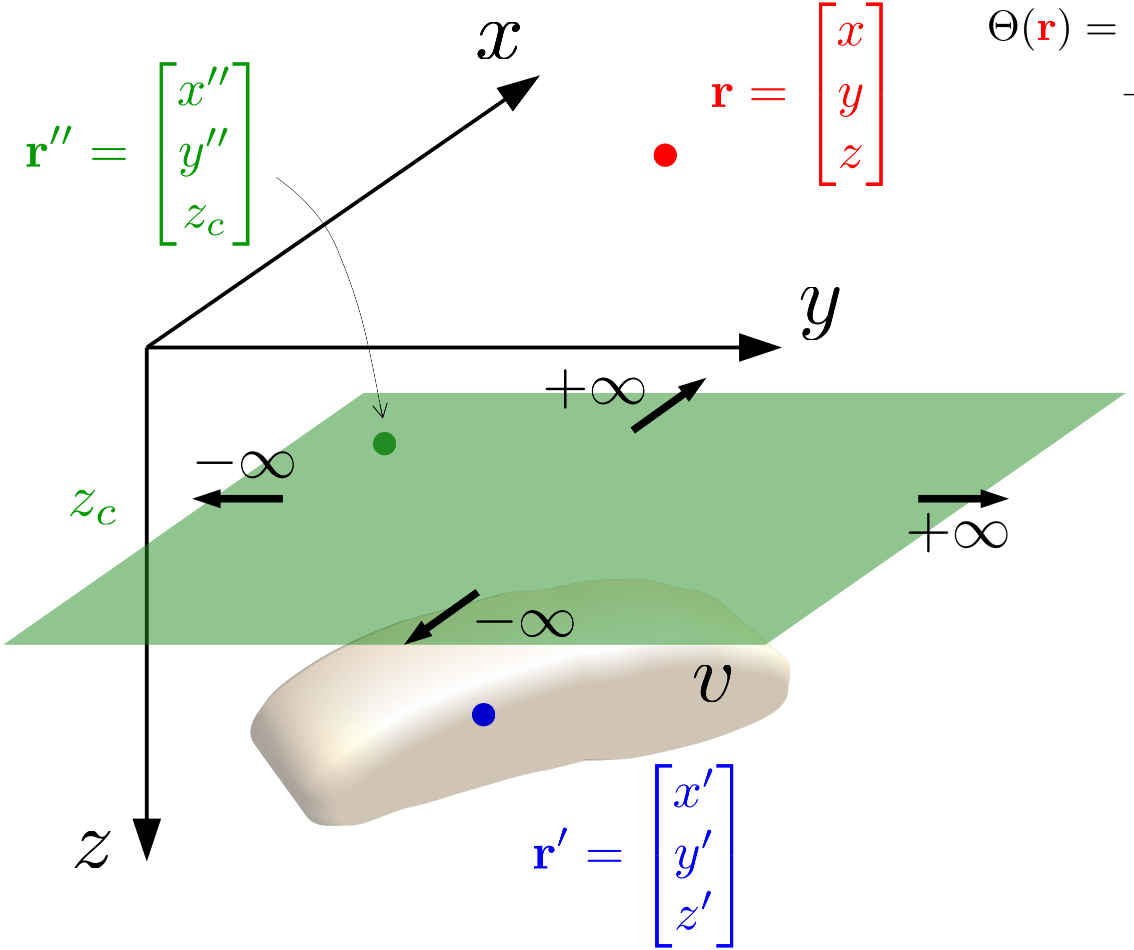
$\partial_z \Theta(\mathbf{r}_i)$

Result 1/4: The gravity disturbance can be exactly reproduced by a planar eq. layer of monopoles



$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

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$\partial_z \Theta(\mathbf{r})$ gravity disturbance data

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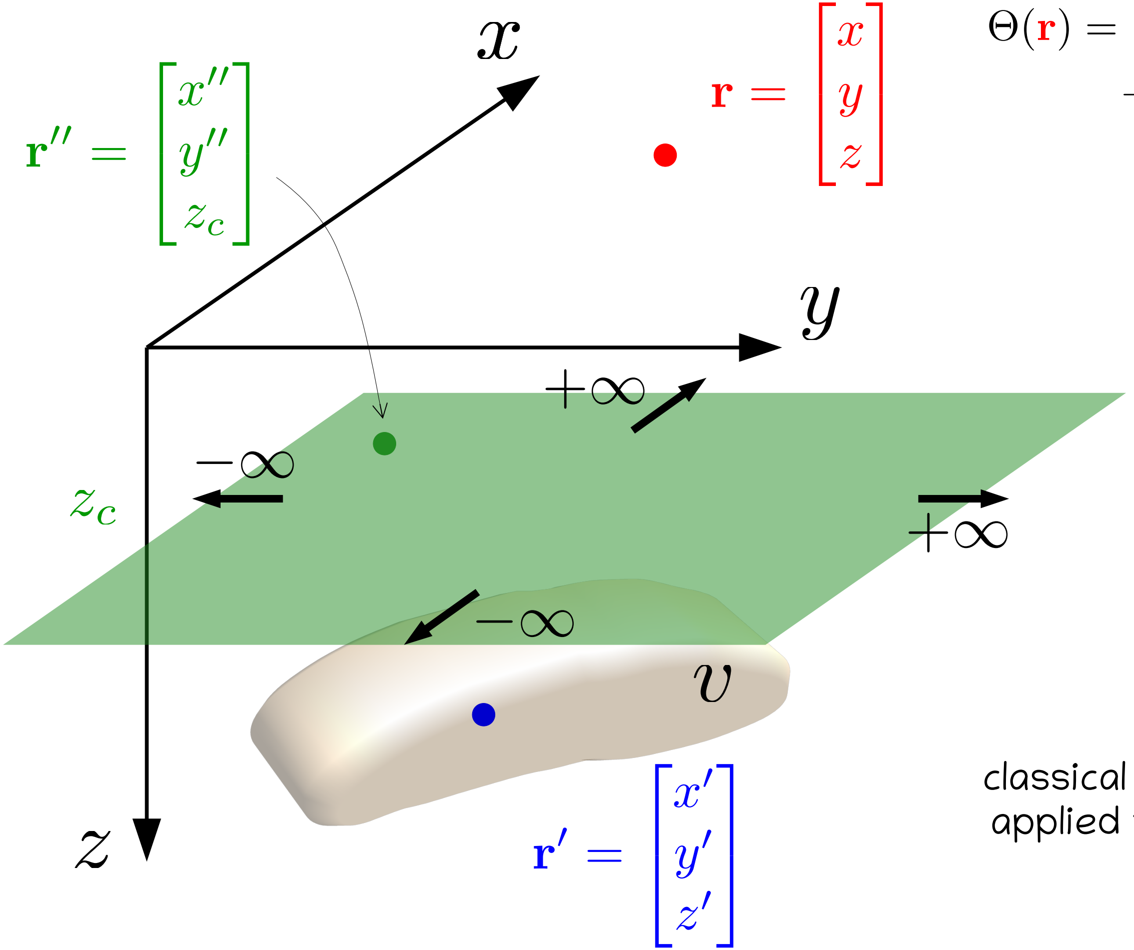
$\partial_z \Theta(\mathbf{r}_i)$

$\approx \partial_z \Theta(\mathbf{r}''_j)$

Result 1/4: The gravity disturbance can be exactly reproduced by a planar eq. layer of monopoles



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$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

classical EqL technique applied to gravity data

Result 2/4: The same eq. layer of monopoles reproducing the gravity disturbance can also reproduce the gravitational potential and gravity gradient tensor

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

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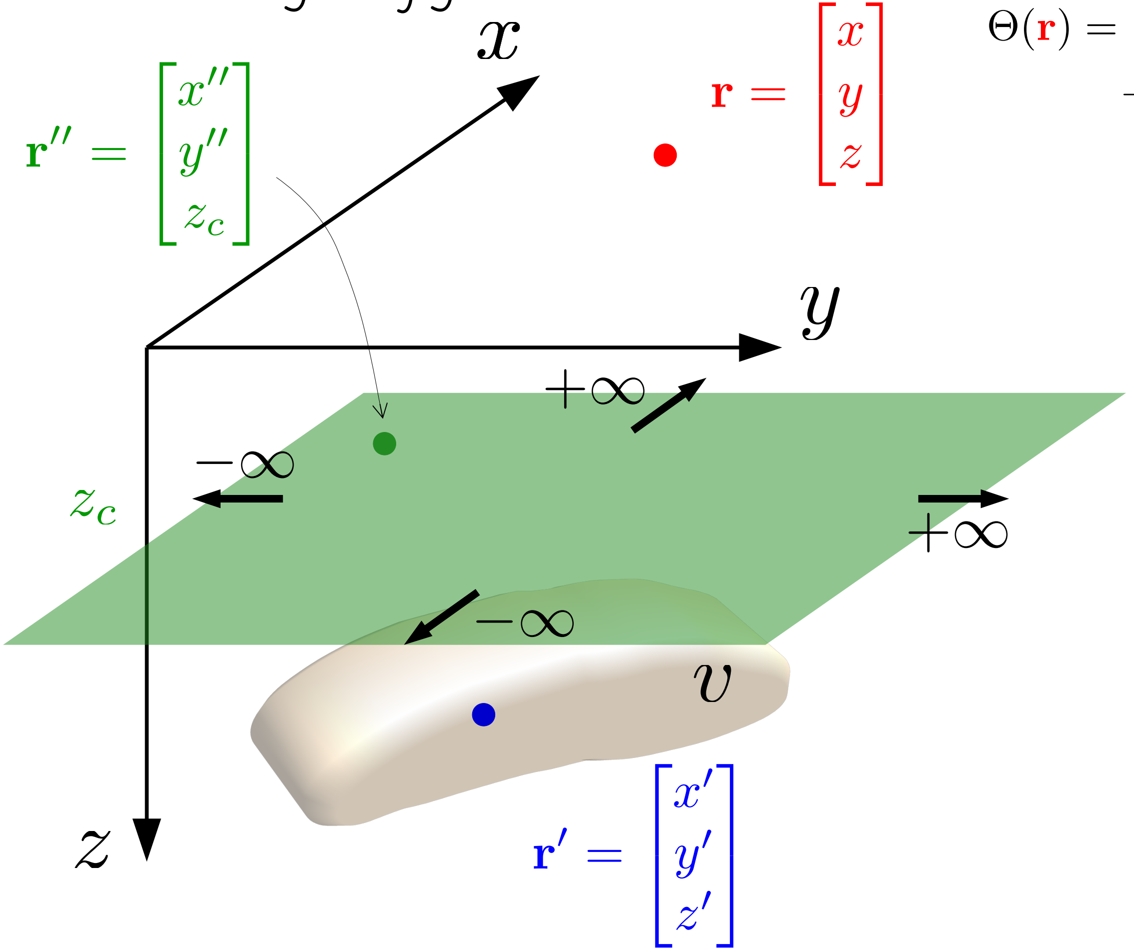

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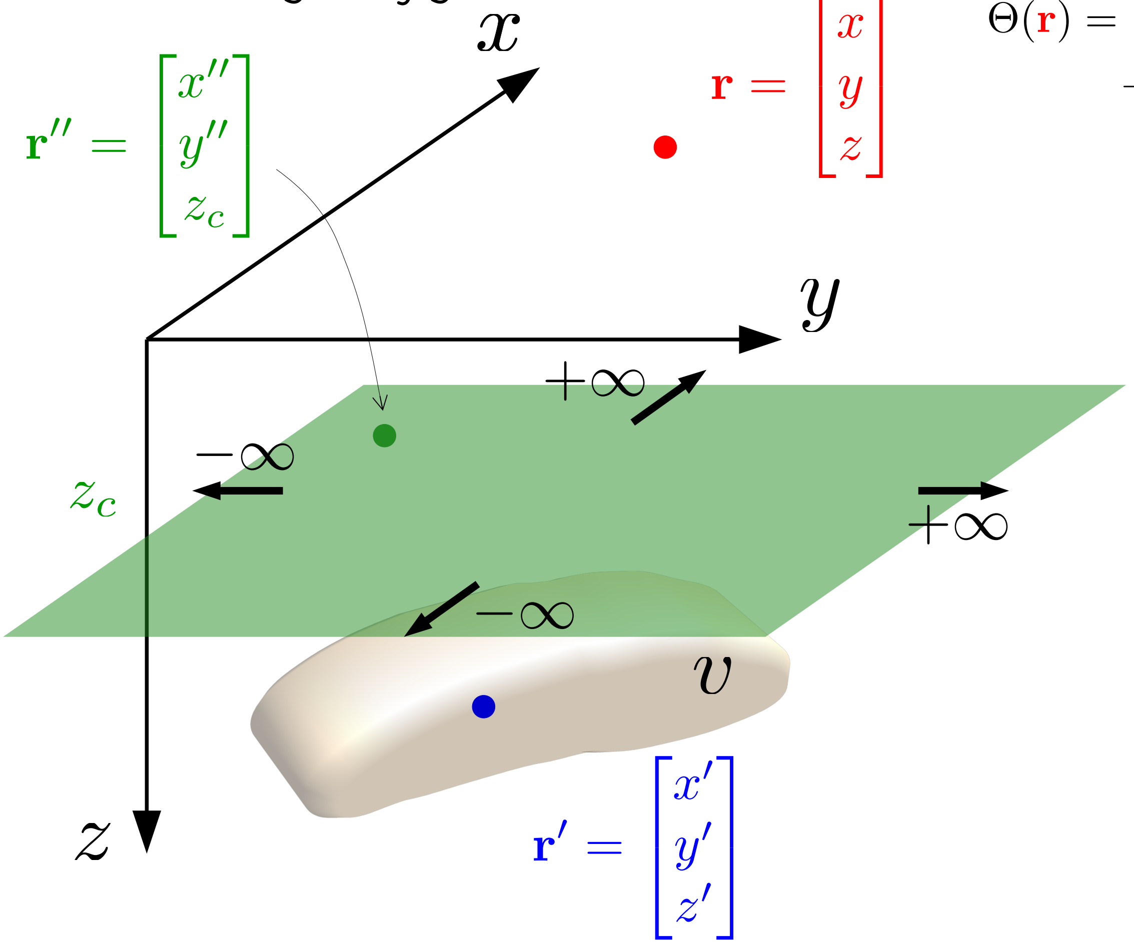

This result has been used by the scientific community for processing gravity-gradient data (e.g., Barnes and Lumley, 2011) or converting gravity disturbance (vertical component) into the gravity-gradient tensor (e.g., Piauilino et al., 2019)

Result 2/4: The same eq. layer of monopoles reproducing the gravity disturbance can also reproduce the gravitational potential and gravity gradient tensor

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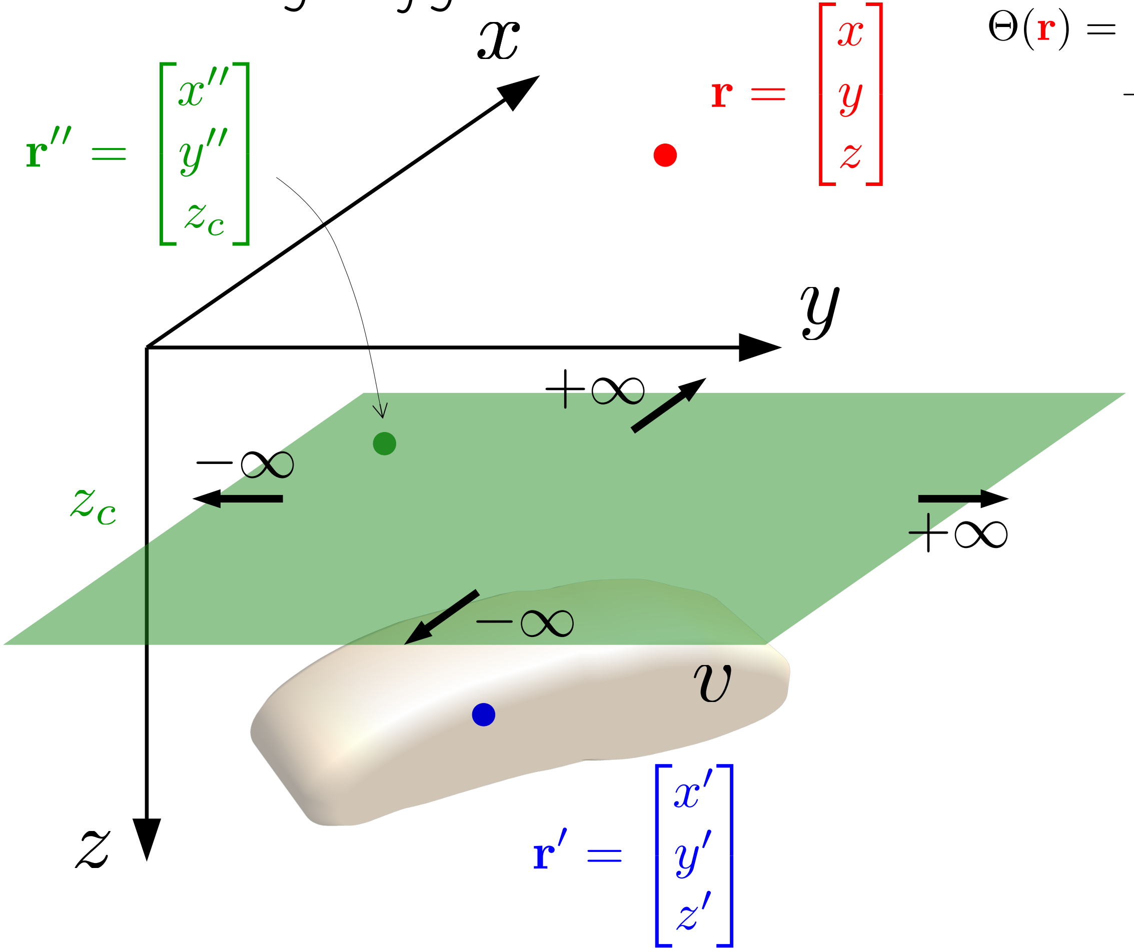
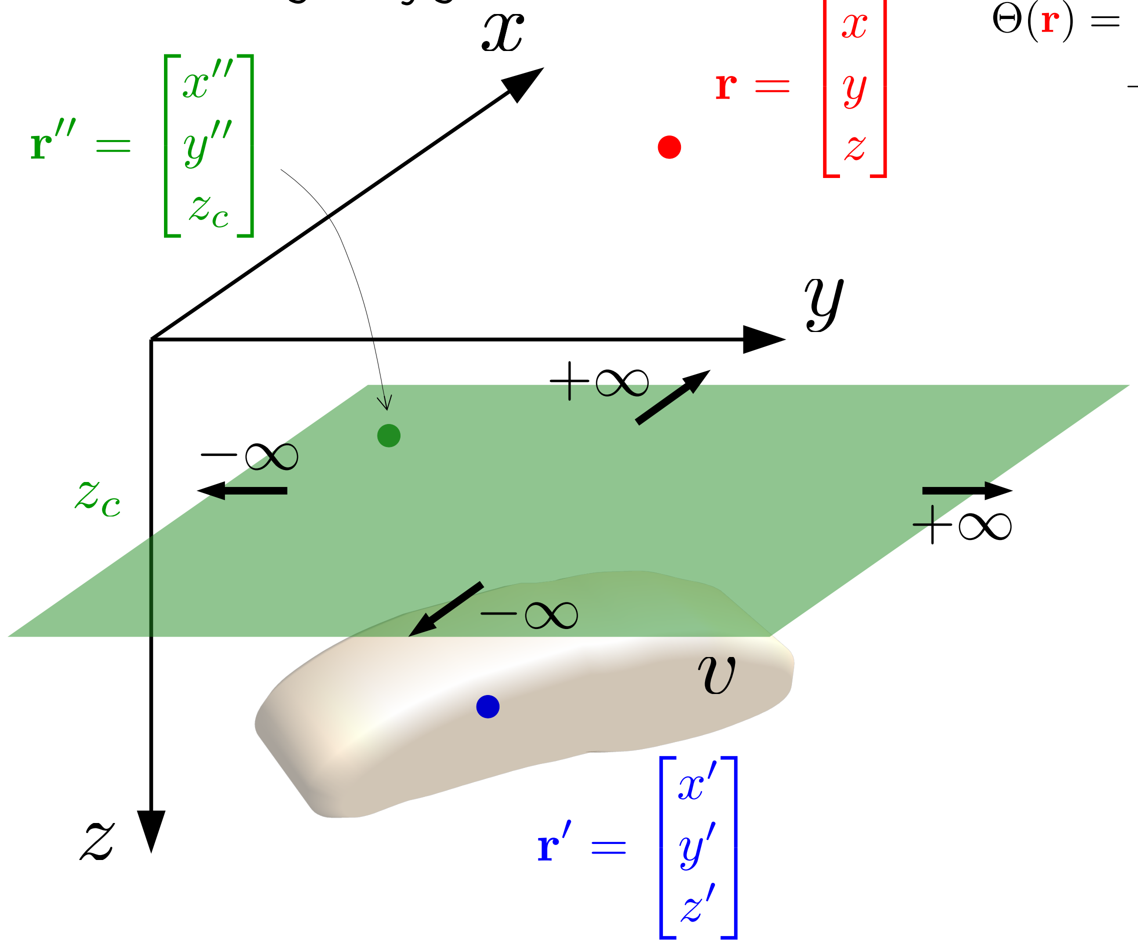


This result has been used by the scientific community for processing gravity-gradient data (e.g., Barnes and Lumley, 2011) or converting gravity disturbance (vertical component) into the gravity-gradient tensor (e.g., Piauilino et al., 2019)

However, researchers have not treated this topic in much detail and it is assumed to be true without any proof

Result 2/4: The same eq. layer of monopoles reproducing the gravity disturbance can also reproduce the gravitational potential and gravity gradient tensor



$$\Theta(\mathbf{r}) = \kappa \iiint\limits_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

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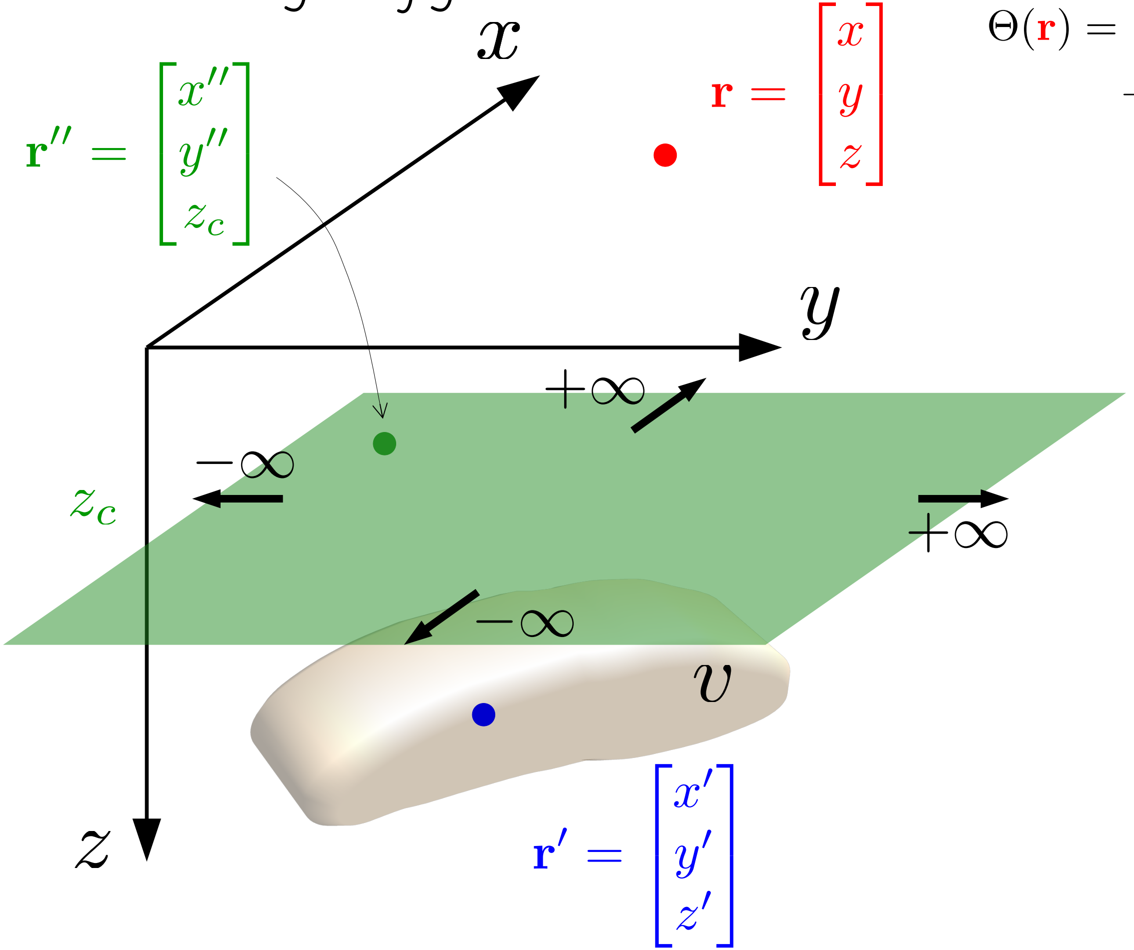
Deduction:

$\Theta(\mathbf{r})$ gravitational potential

Result 2/4: The same eq. layer of monopoles reproducing the gravity disturbance can also reproduce the gravitational potential and gravity gradient tensor



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$\Theta(\mathbf{r})$ gravitational potential

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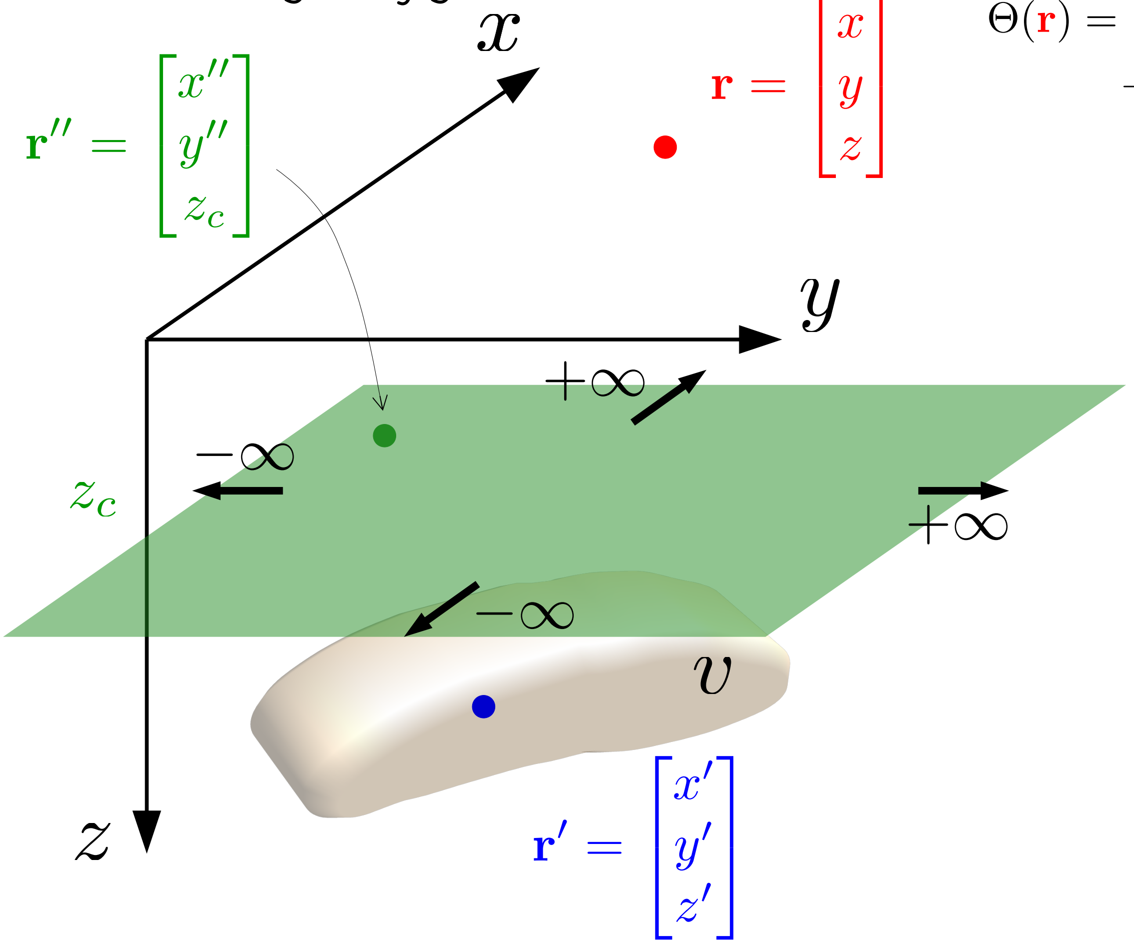

Deduction:

$\Theta(\mathbf{r})$ gravitational potential

$$\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r},\mathbf{r}'') \, dS''$$

$\partial_z \Theta(\mathbf{r})$ gravity disturbance

$$\partial_z \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r},\mathbf{r}'') \, dS''$$

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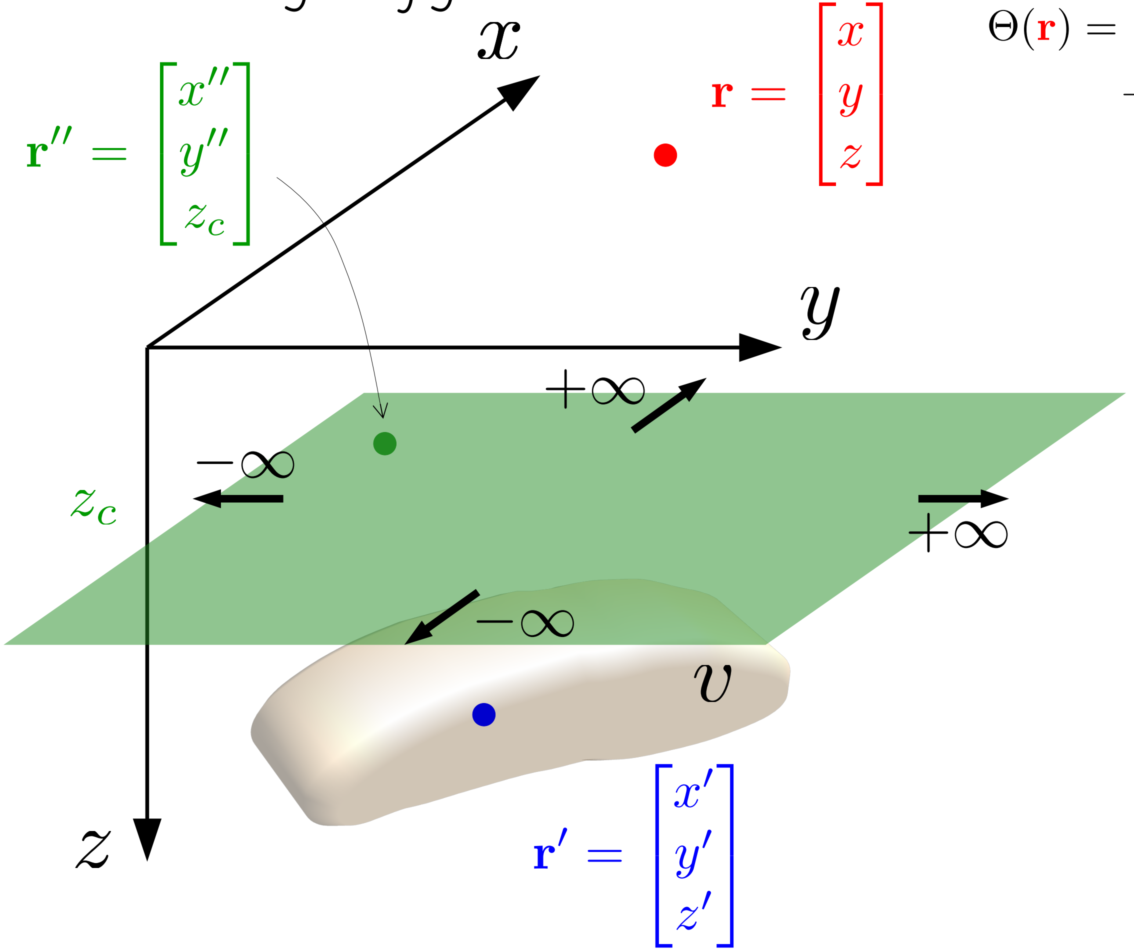


Deduction:

$\Theta(\mathbf{r})$ gravitational potential

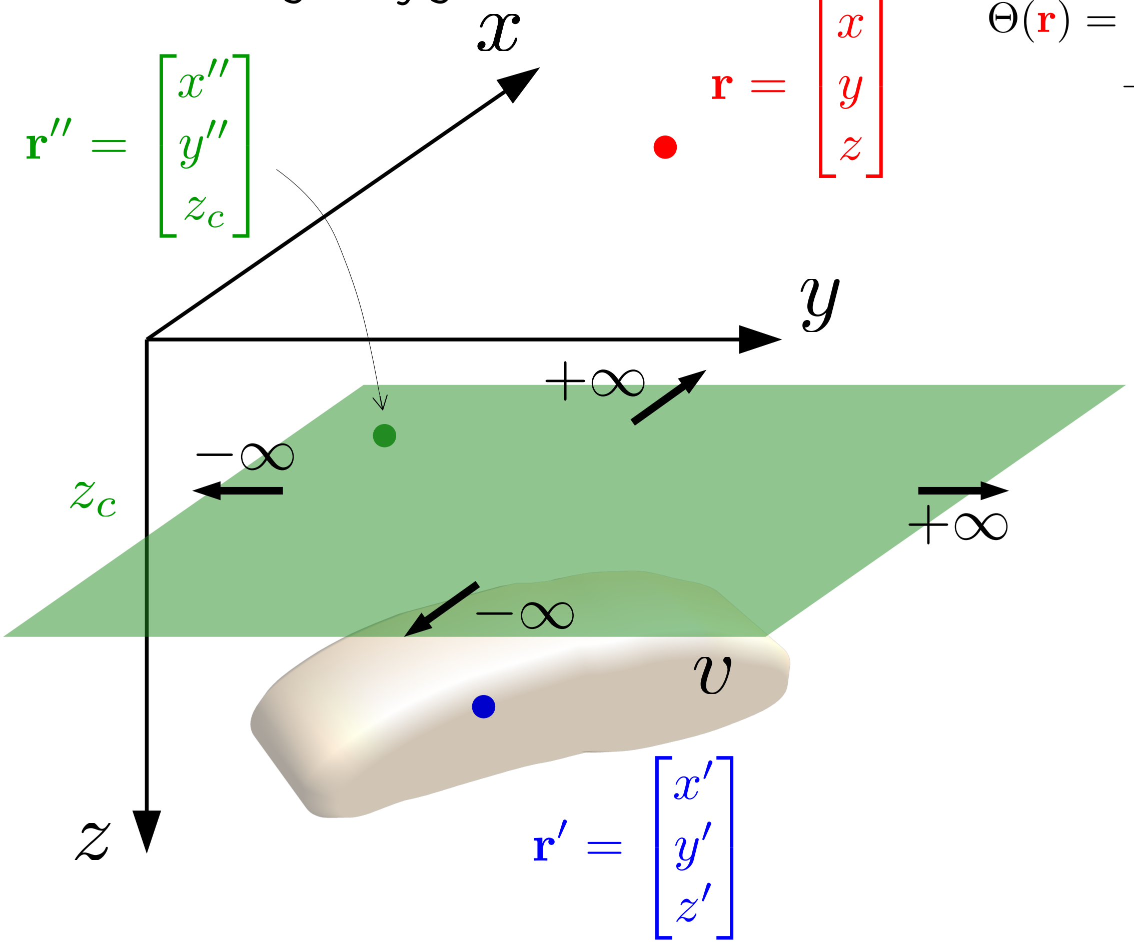
$$\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$\partial_z \Theta(\mathbf{r})$ gravity disturbance

$$\partial_z \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Note that this term is not affected by the derivative

Result 2/4: The same eq. layer of monopoles reproducing the gravity disturbance can also reproduce the gravitational potential and gravity gradient tensor



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Deduction:

$\Theta(\mathbf{r})$ gravitational potential

$$\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$\partial_z \Theta(\mathbf{r})$ gravity disturbance

$$\partial_z \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$\partial_{\alpha\beta} \Theta(\mathbf{r})$ gravity-gradient tensor component

$$\partial_{\alpha\beta} \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{\alpha\beta} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

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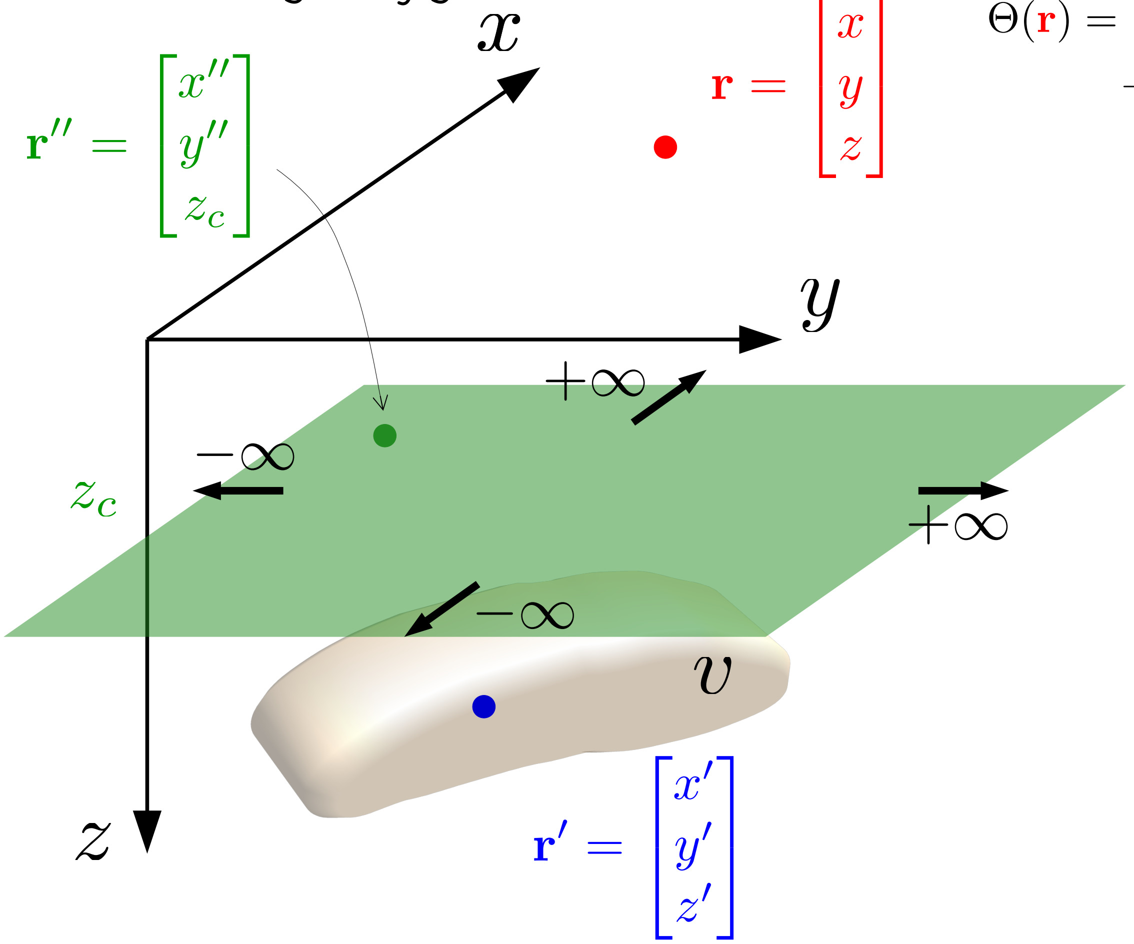

Deduction:

$\Theta(\mathbf{r})$ gravitational potential

$$\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$\partial_z \Theta(\mathbf{r})$ gravity disturbance

$$\partial_z \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$\partial_{\alpha\beta} \Theta(\mathbf{r})$ gravity-gradient tensor component

$$\partial_{\alpha\beta} \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{\alpha\beta} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by a planar eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

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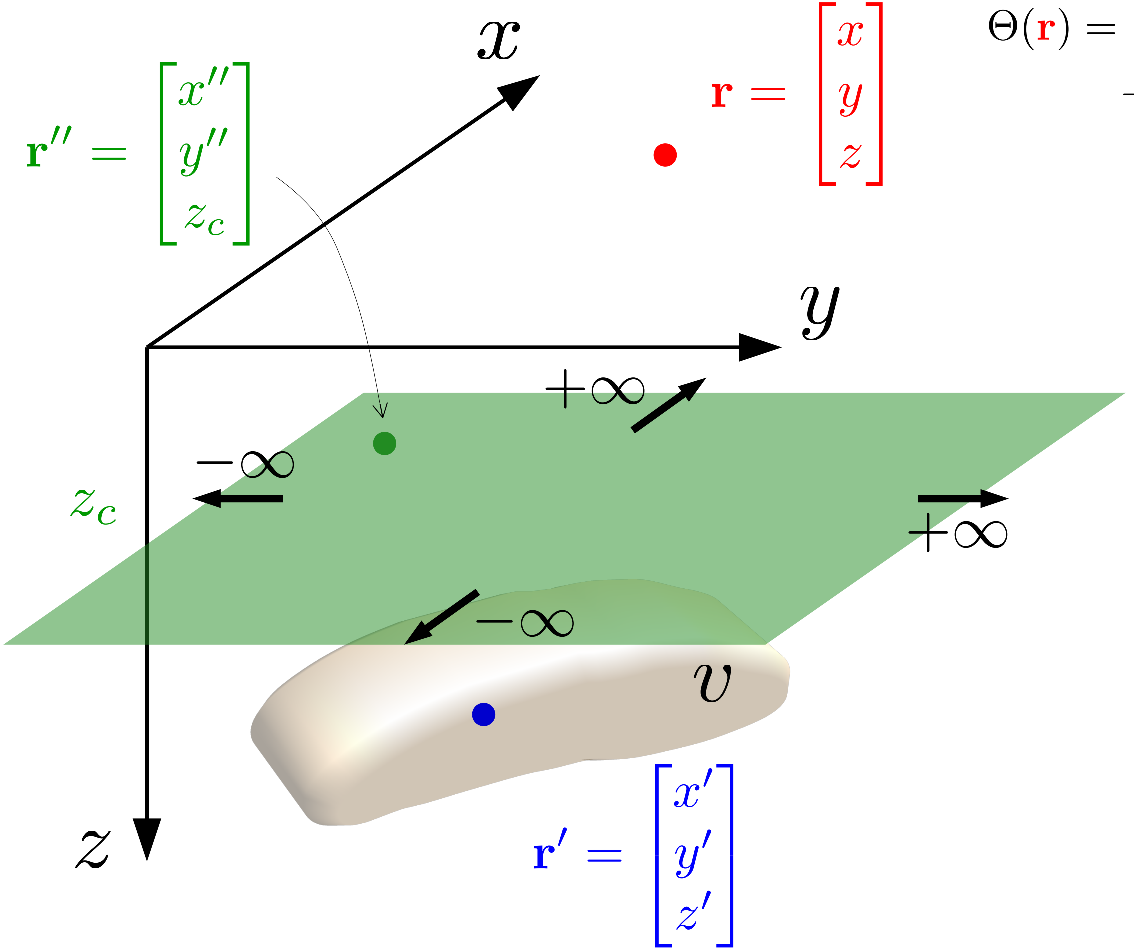

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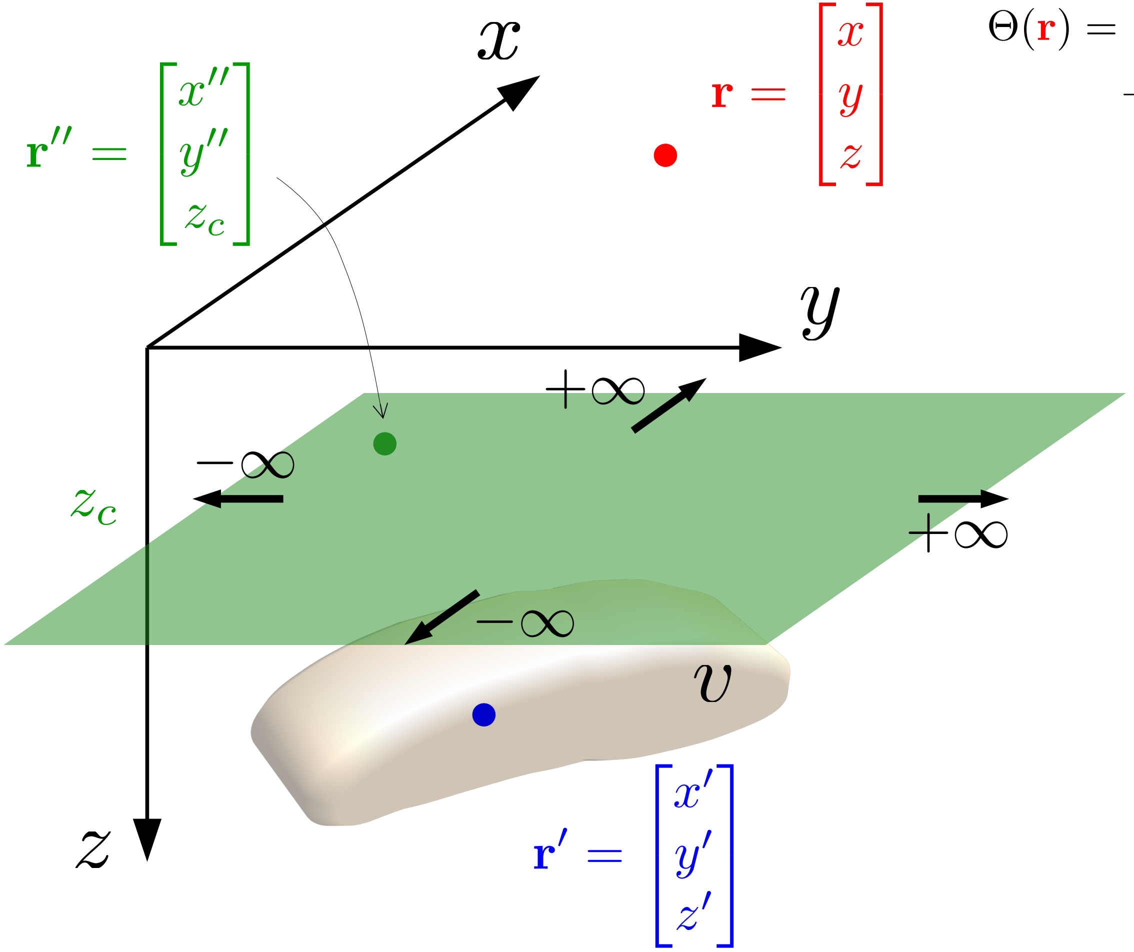

Most previous published studies are limited to the empirical use of a planar eq. layer of dipoles for processing total-field anomaly data

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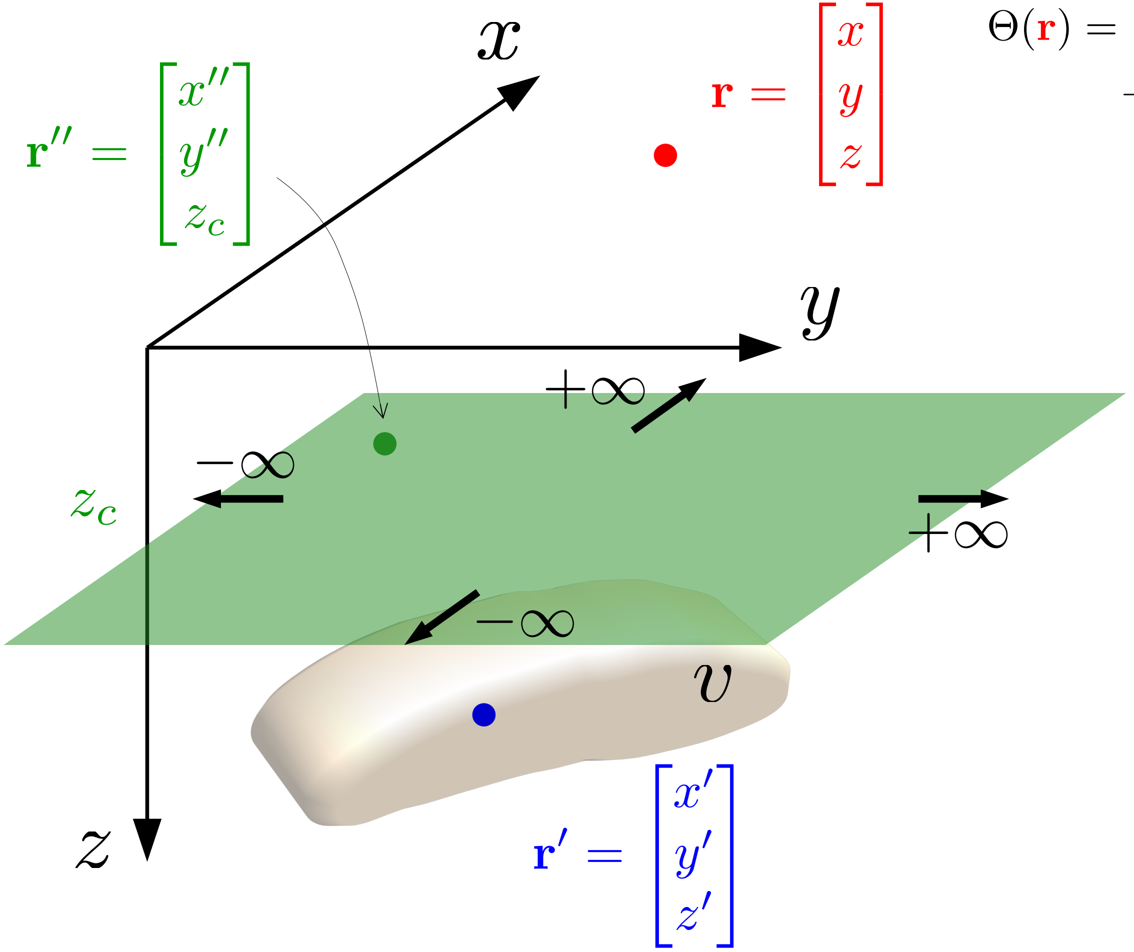

Most previous published studies are limited to the empirical use of a planar eq. layer of dipoles for processing total-field anomaly data

Few studies (e.g., Pedersen, 1991; Li et al., 2014; Reis et al., 2020) have drawn attention to the problem of proving the existence of a planar eq. layer of dipoles that exactly reproduces the approx total-field anomaly

Result 3/4: The approx total-field anomaly can be exactly reproduced by a planar eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r}-\mathbf{r}'\|} dv'$$

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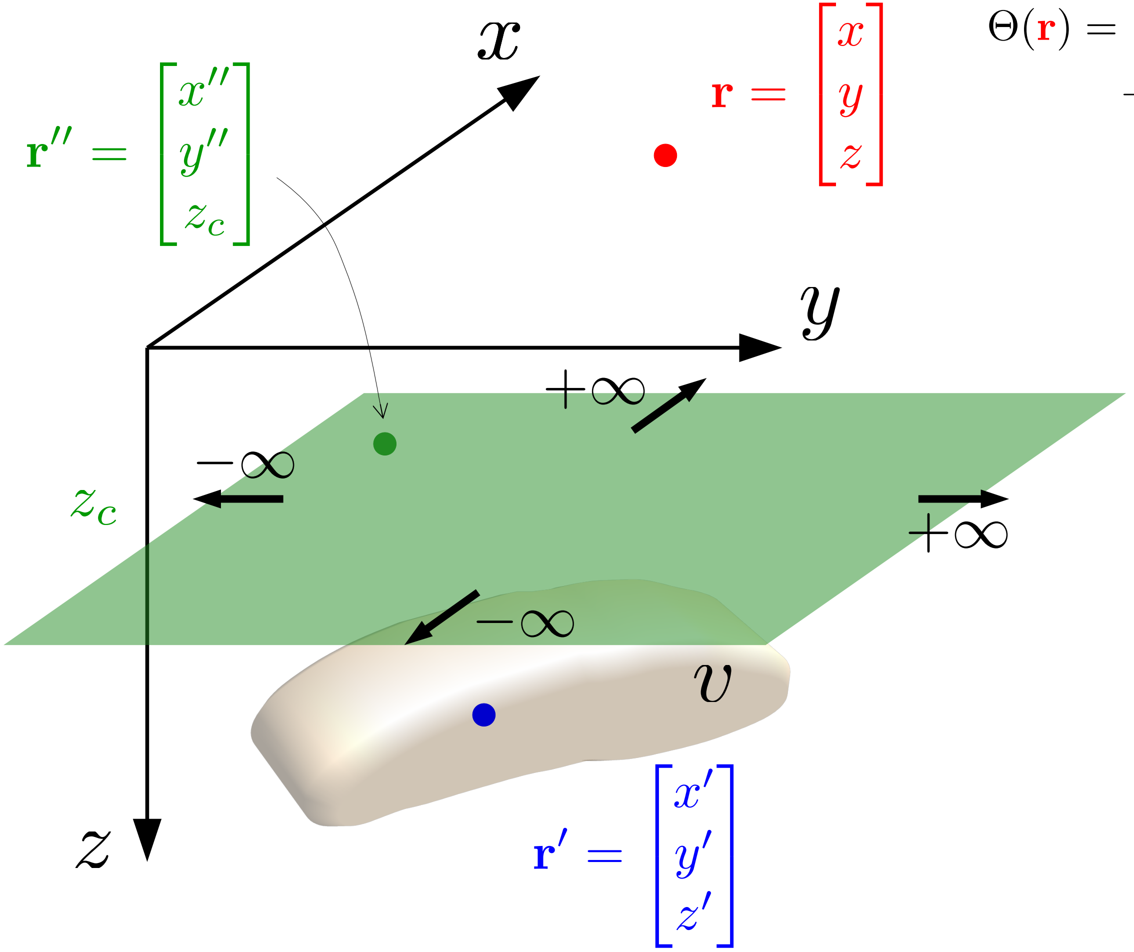

Deduction:

approx total-field anomaly

$$\partial_{th}\Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by a planar eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

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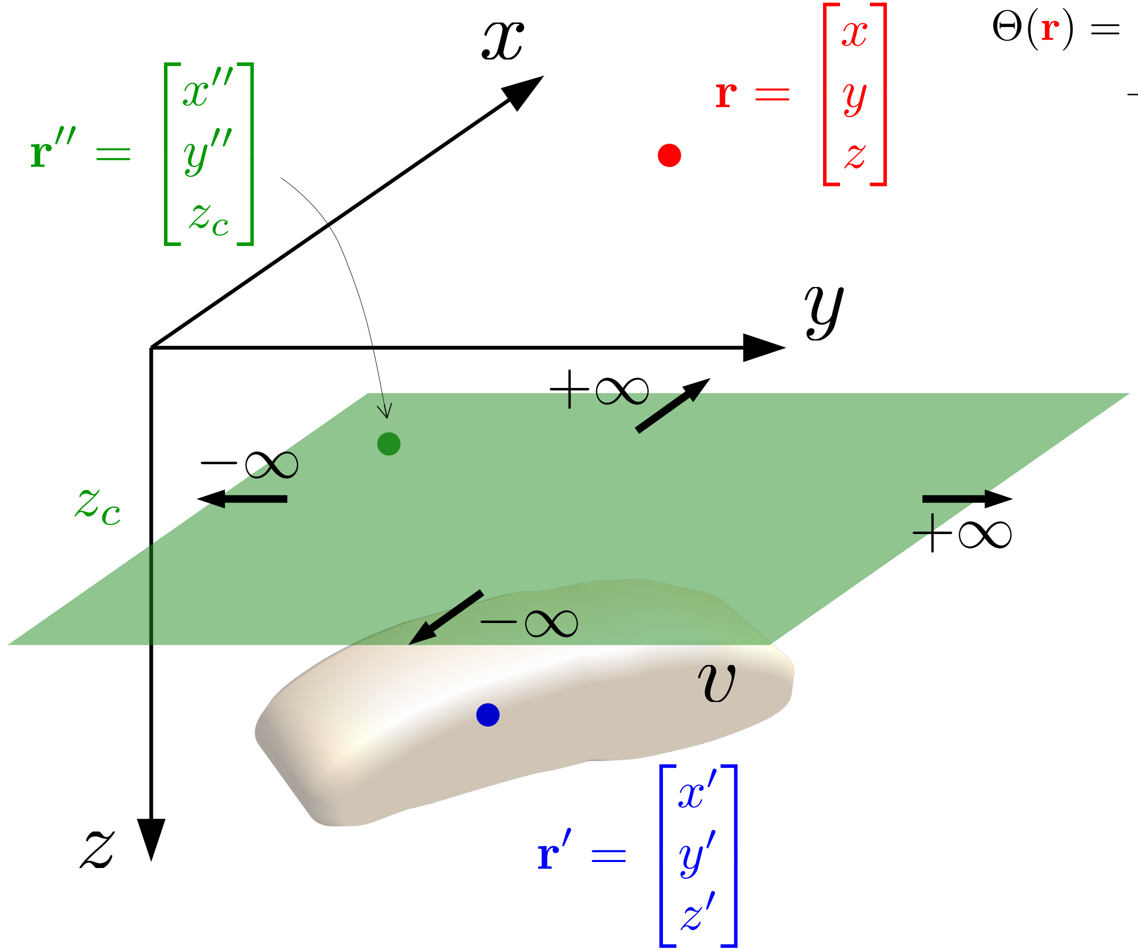

Deduction:

approx total-field anomaly

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Theta}(\mathbf{r}) \, \hat{\mathbf{h}}$$

Unit vector defining the constant direction of the main geomagnetic field

Result 3/4: The approx total-field anomaly can be exactly reproduced by a planar eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

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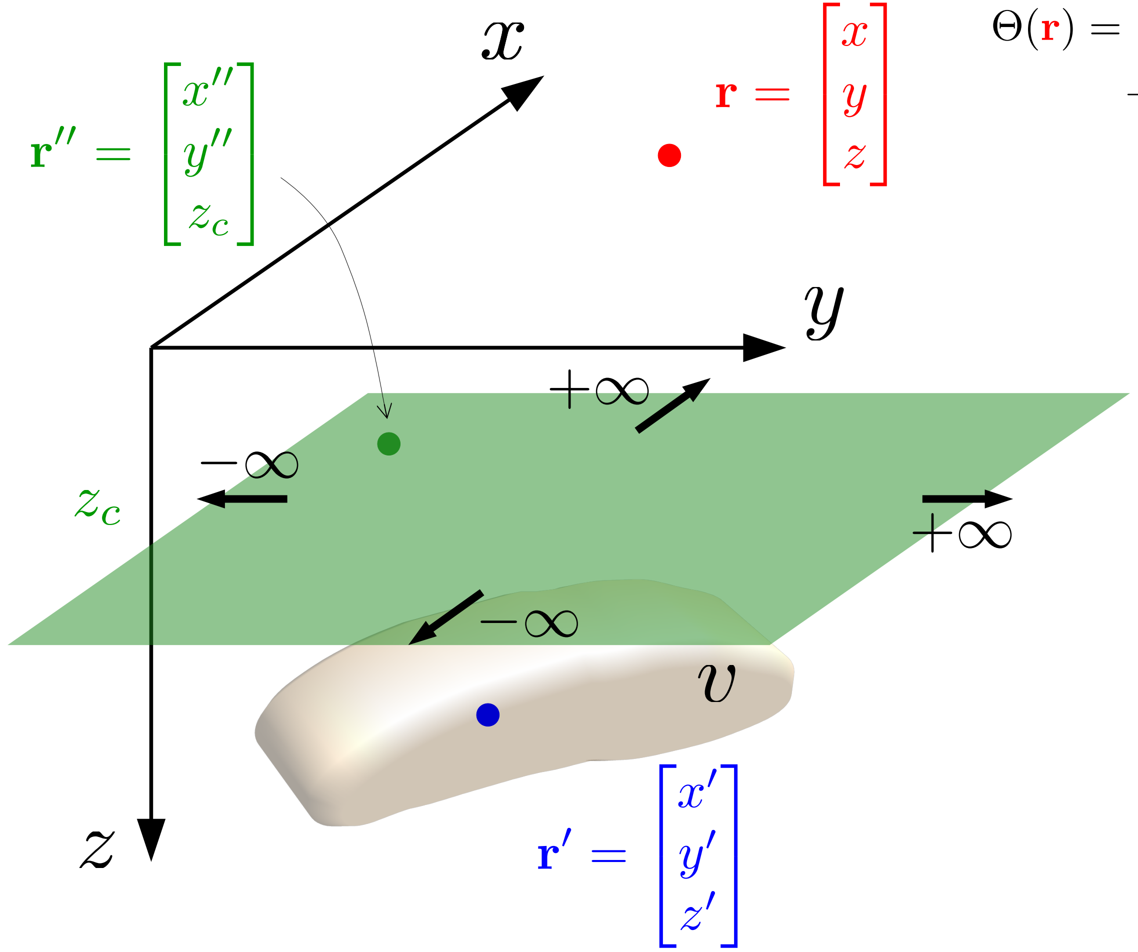


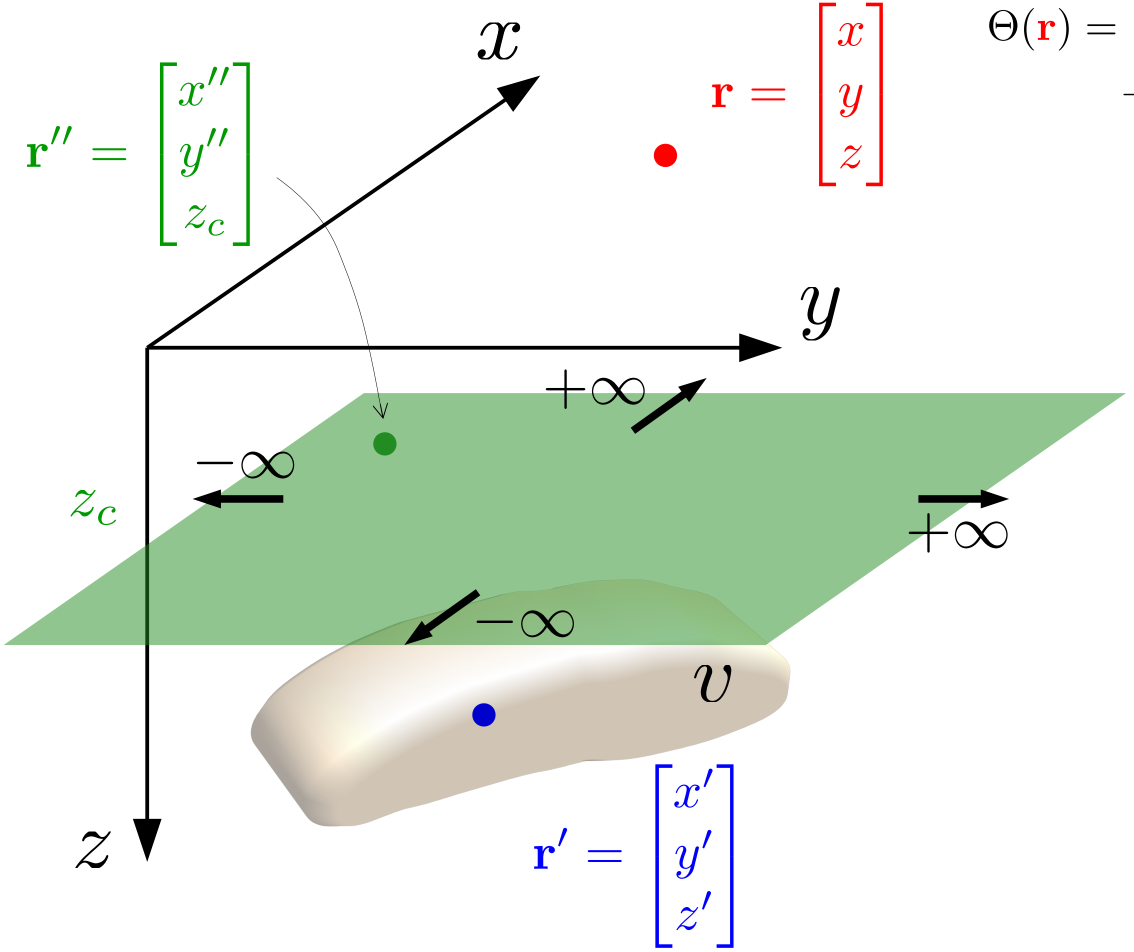
Deduction:

approx total-field anomaly

$$\partial_{th}\Theta(\mathbf{r}) = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Theta}(\mathbf{r}) \, \hat{\mathbf{h}}$$

Unit vector defining the uniform total-magnetization direction of the true sources

Result 3/4: The approx total-field anomaly can be exactly reproduced by a planar eq. layer of dipoles having the same uniform total-magnetization direction of the true sources



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Deduction:

approx total-field anomaly

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Theta}(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\mathbf{H}_{\Theta}(\mathbf{r}) = \begin{bmatrix} \partial_{xx}\Theta(\mathbf{r}) & \partial_{xy}\Theta(\mathbf{r}) & \partial_{xz}\Theta(\mathbf{r}) \\ \partial_{xy}\Theta(\mathbf{r}) & \partial_{yy}\Theta(\mathbf{r}) & \partial_{yz}\Theta(\mathbf{r}) \\ \partial_{xz}\Theta(\mathbf{r}) & \partial_{yz}\Theta(\mathbf{r}) & \partial_{zz}\Theta(\mathbf{r}) \end{bmatrix}$$

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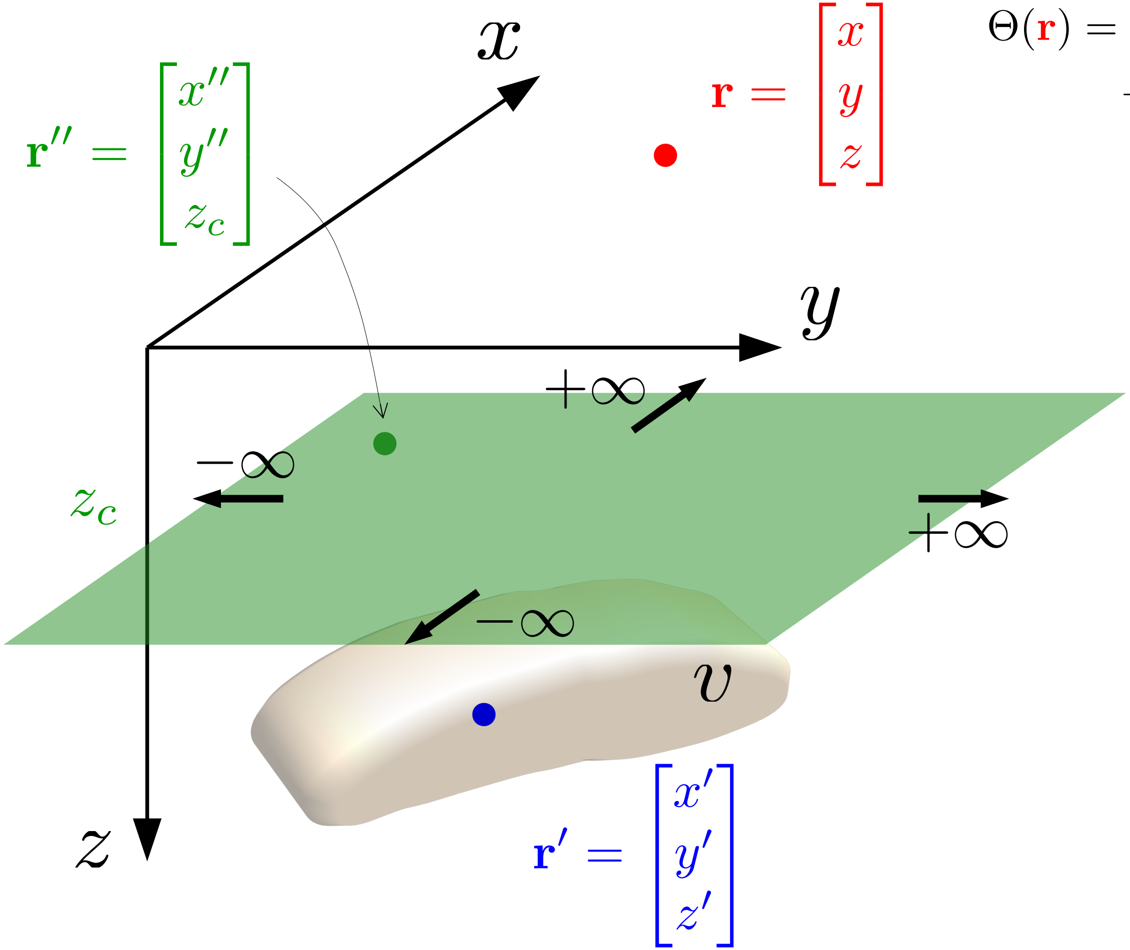


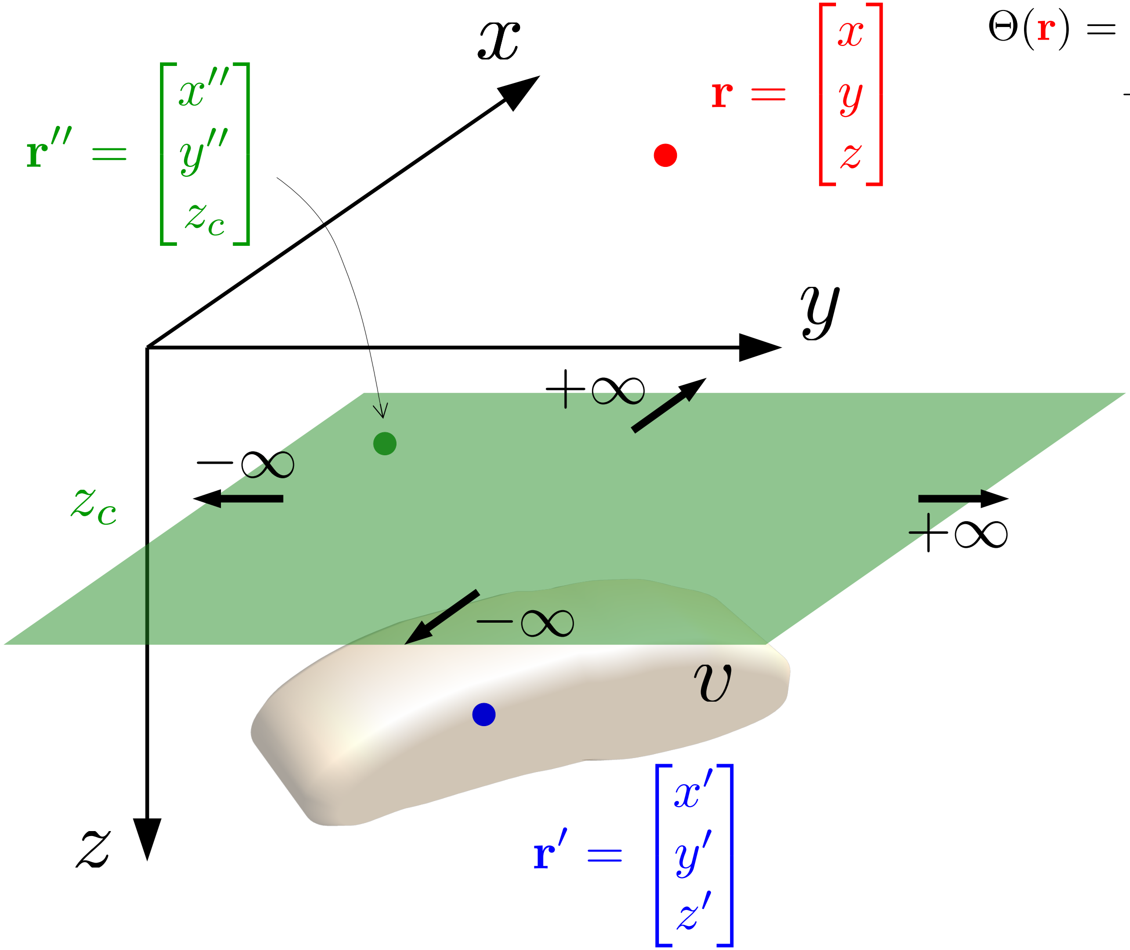
Deduction:

approx total-field anomaly

$$\partial_{th}\Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th}\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th}\Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by a planar eq. layer of dipoles having the same uniform total-magnetization direction of the true sources



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approx total-field anomaly

$$\partial_{th}\Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

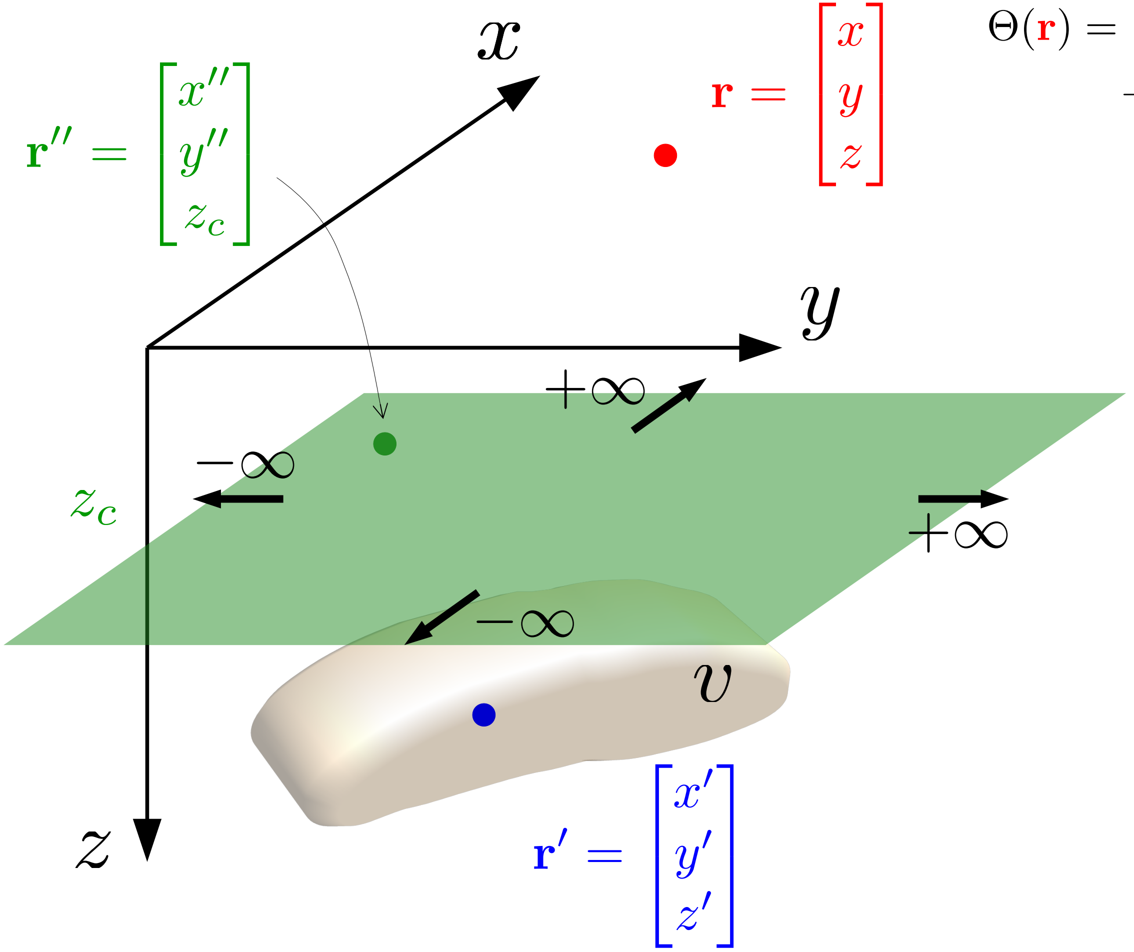
$$\partial_{th}\Theta(\mathbf{r}) = \iint \underbrace{\partial_z \Theta(\mathbf{r}'') \, \partial_{th} \Psi(\mathbf{r}, \mathbf{r}'')}_{} \, dS''$$

This term represents the approx total-field anomaly produced at $\mathbf{r}$ by a dipole located at $\mathbf{r}''$

Result 3/4: The approx total-field anomaly can be exactly reproduced by a planar eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$



Deduction:

approx total-field anomaly

$$\partial_{th}\Theta(\mathbf{r}) = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Theta}(\mathbf{r}) \, \hat{\mathbf{h}}$$

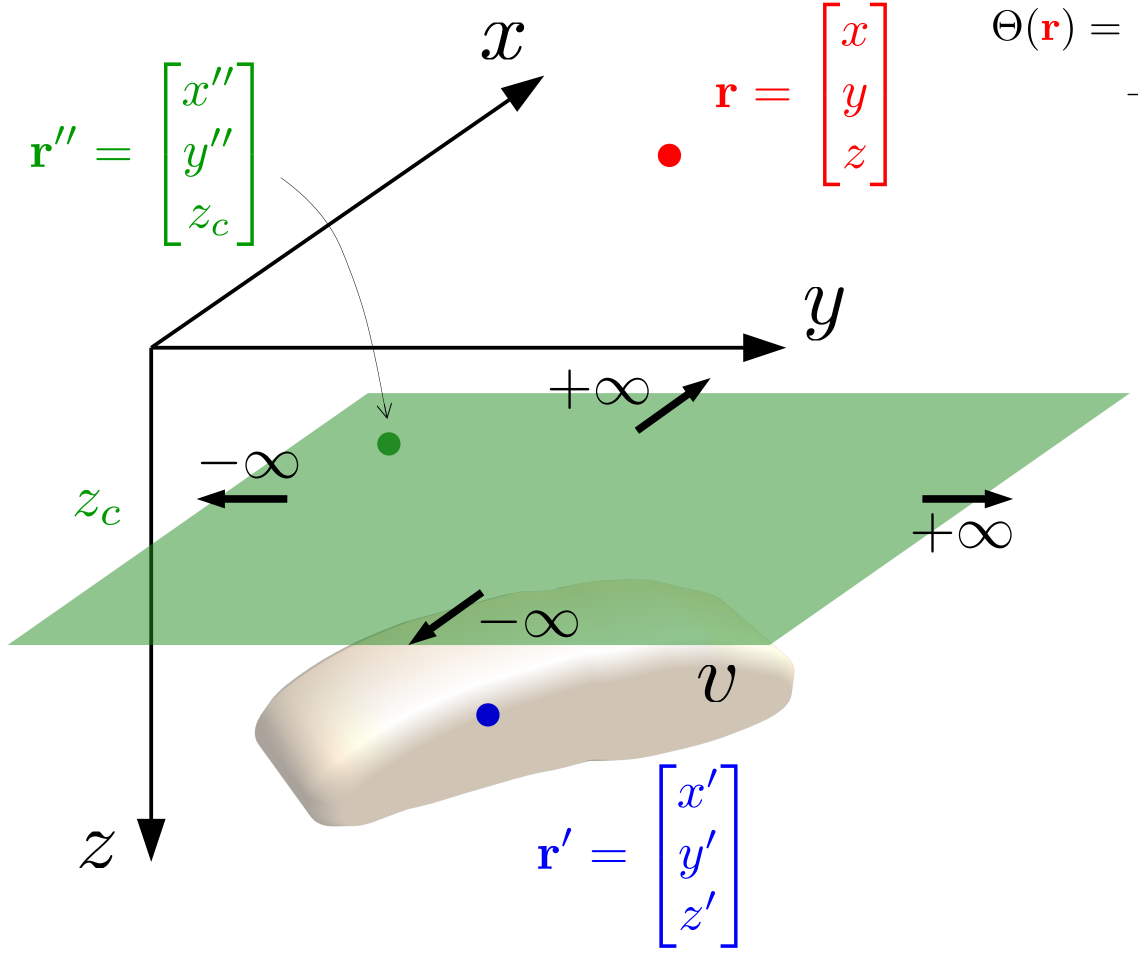
$$\partial_{th}\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$$\partial_{th}\Psi(\mathbf{r}, \mathbf{r}'') = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Psi}(\mathbf{r}, \mathbf{r}'') \, \hat{\mathbf{h}}$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by a planar eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$



Deduction:

approx total-field anomaly

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th} \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

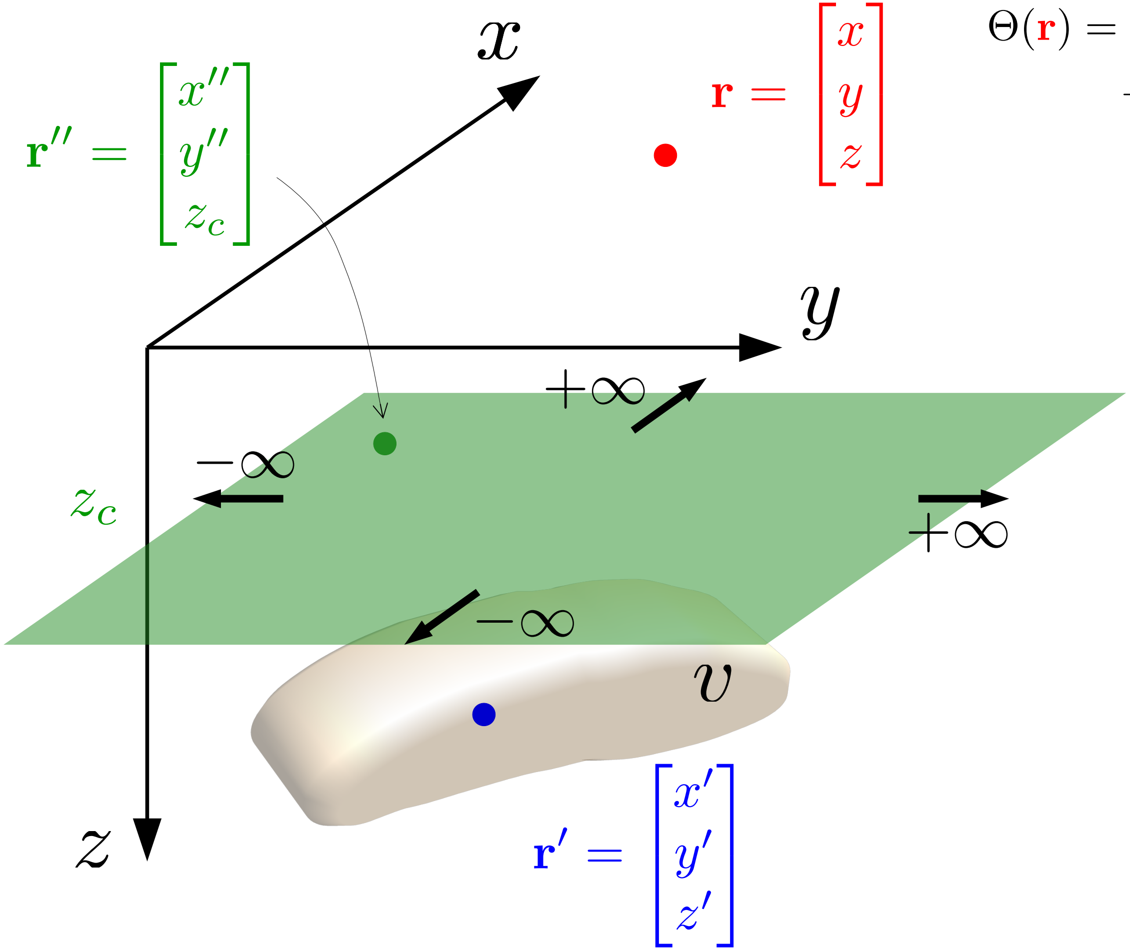
$$\partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') = \hat{\mathbf{t}}^\top \mathbf{H}_\Psi(\mathbf{r}, \mathbf{r}'') \, \hat{\mathbf{h}}$$

Unit vector defining the same uniform total-magnetization direction of the true sources

Result 3/4: The approx total-field anomaly can be exactly reproduced by a planar eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$



Deduction:

approx total-field anomaly

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th} \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

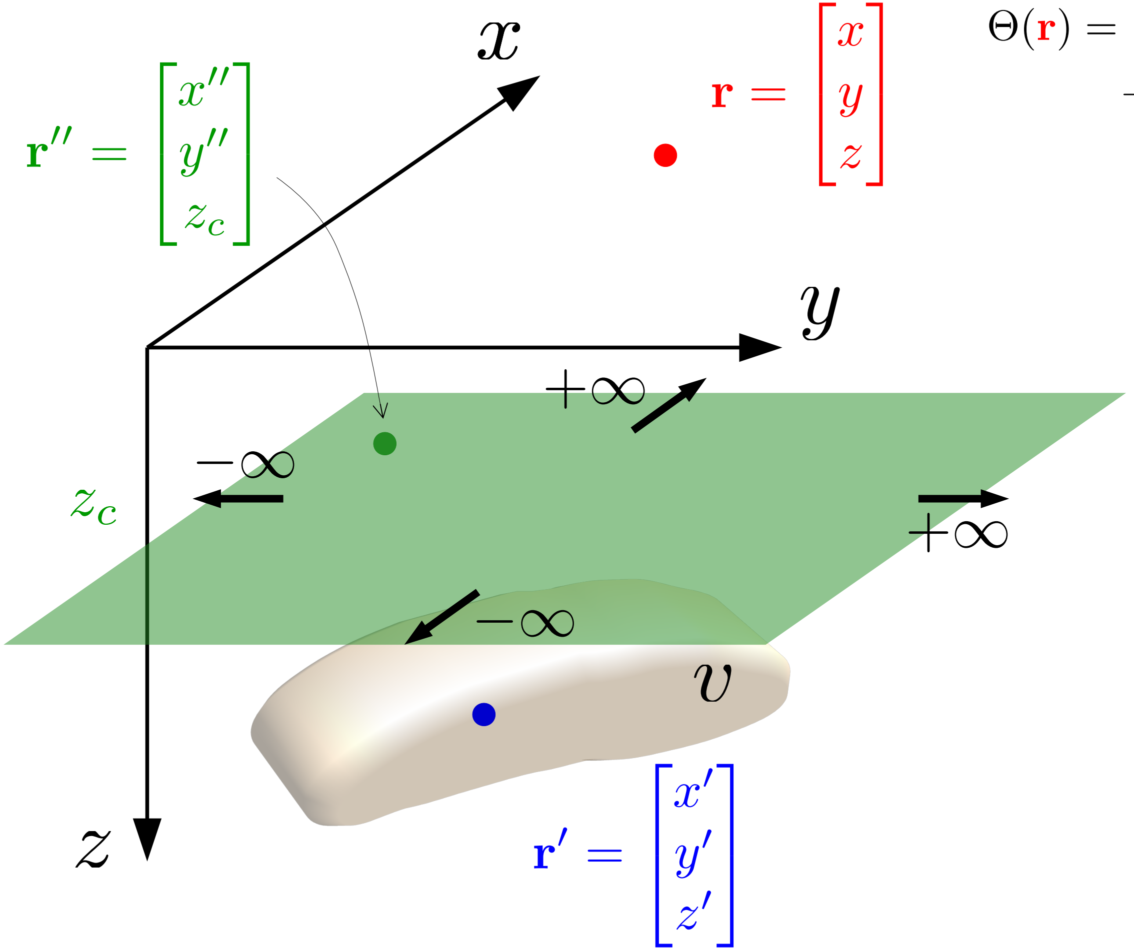
$$\partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') = \hat{\mathbf{t}}^\top \mathbf{H}_\Psi(\mathbf{r}, \mathbf{r}'') \, \hat{\mathbf{h}}$$

$$\mathbf{H}_\Psi(\mathbf{r}, \mathbf{r}'') = \begin{bmatrix} \partial_{xx} \Psi(\mathbf{r}, \mathbf{r}'') & \partial_{xy} \Psi(\mathbf{r}, \mathbf{r}'') & \partial_{xz} \Psi(\mathbf{r}, \mathbf{r}'') \\ \partial_{xy} \Psi(\mathbf{r}, \mathbf{r}'') & \partial_{yy} \Psi(\mathbf{r}, \mathbf{r}'') & \partial_{yz} \Psi(\mathbf{r}, \mathbf{r}'') \\ \partial_{xz} \Psi(\mathbf{r}, \mathbf{r}'') & \partial_{yz} \Psi(\mathbf{r}, \mathbf{r}'') & \partial_{zz} \Psi(\mathbf{r}, \mathbf{r}'') \end{bmatrix}$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by a planar eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$



Deduction:

approx total-field anomaly

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Theta}(\mathbf{r}) \, \hat{\mathbf{h}}$$

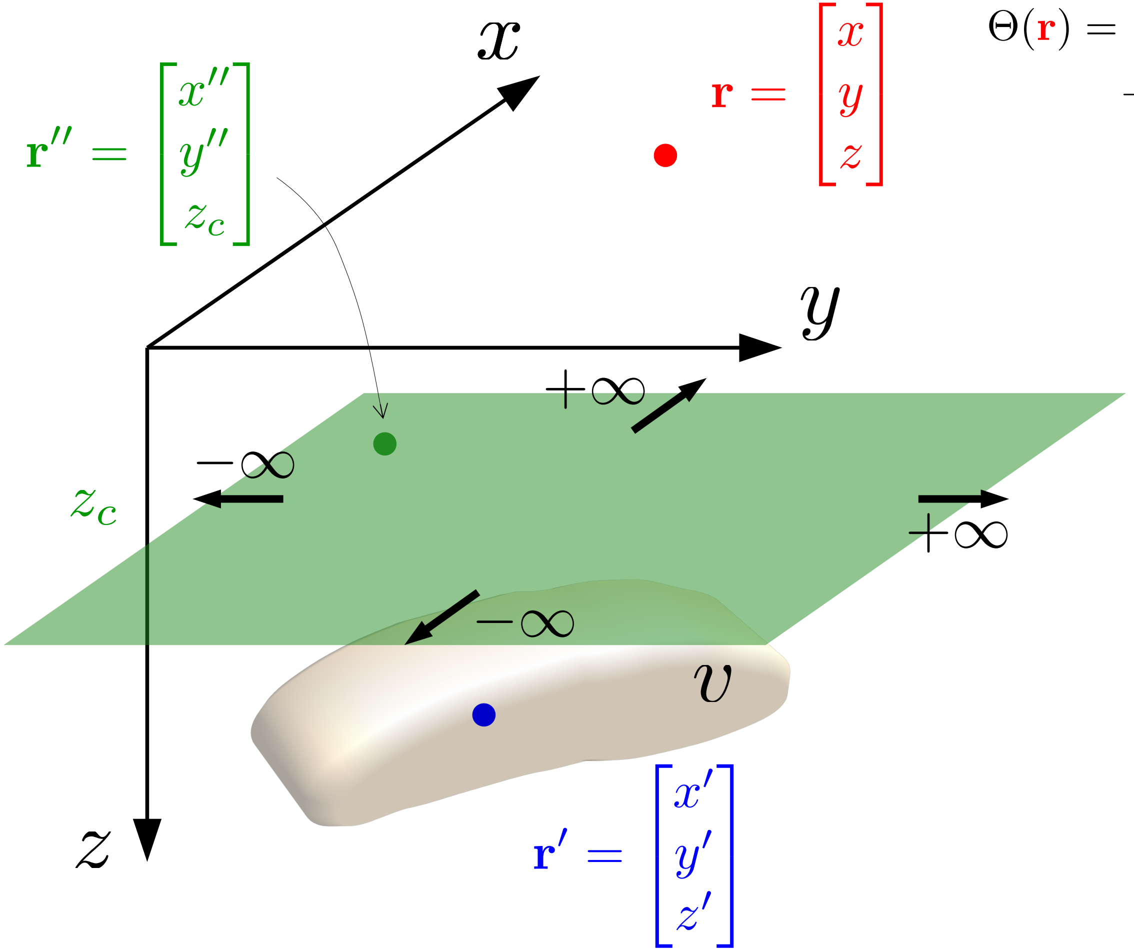
$$\partial_{th} \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Reis et al. (2020) were the first to deduce this analytical eq. layer by following a slightly different approach

Result 3/4: The approx total-field anomaly can be exactly reproduced by a planar eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_v \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$



Deduction:

approx total-field anomaly

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th} \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

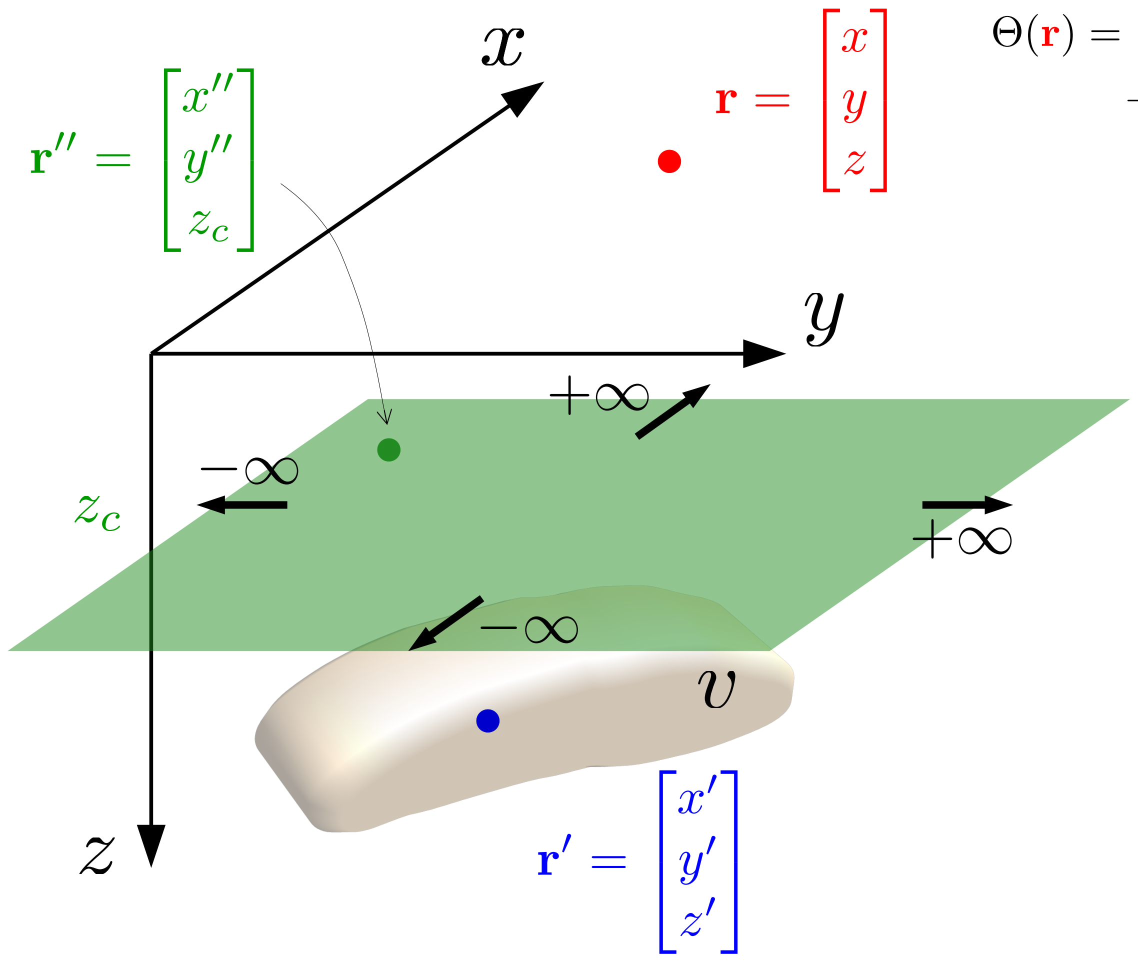
Total-magnetization intensity distribution within the true sources

$$\partial_z \Theta(\mathbf{r}'') = \kappa \iiint\limits_v \sigma(\mathbf{r}') \, \partial_z \frac{1}{\|\mathbf{r}'' - \mathbf{r}'\|} \, dv'$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by a planar eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

$$\Theta(\mathbf{r}) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$



Deduction:

approx total-field anomaly

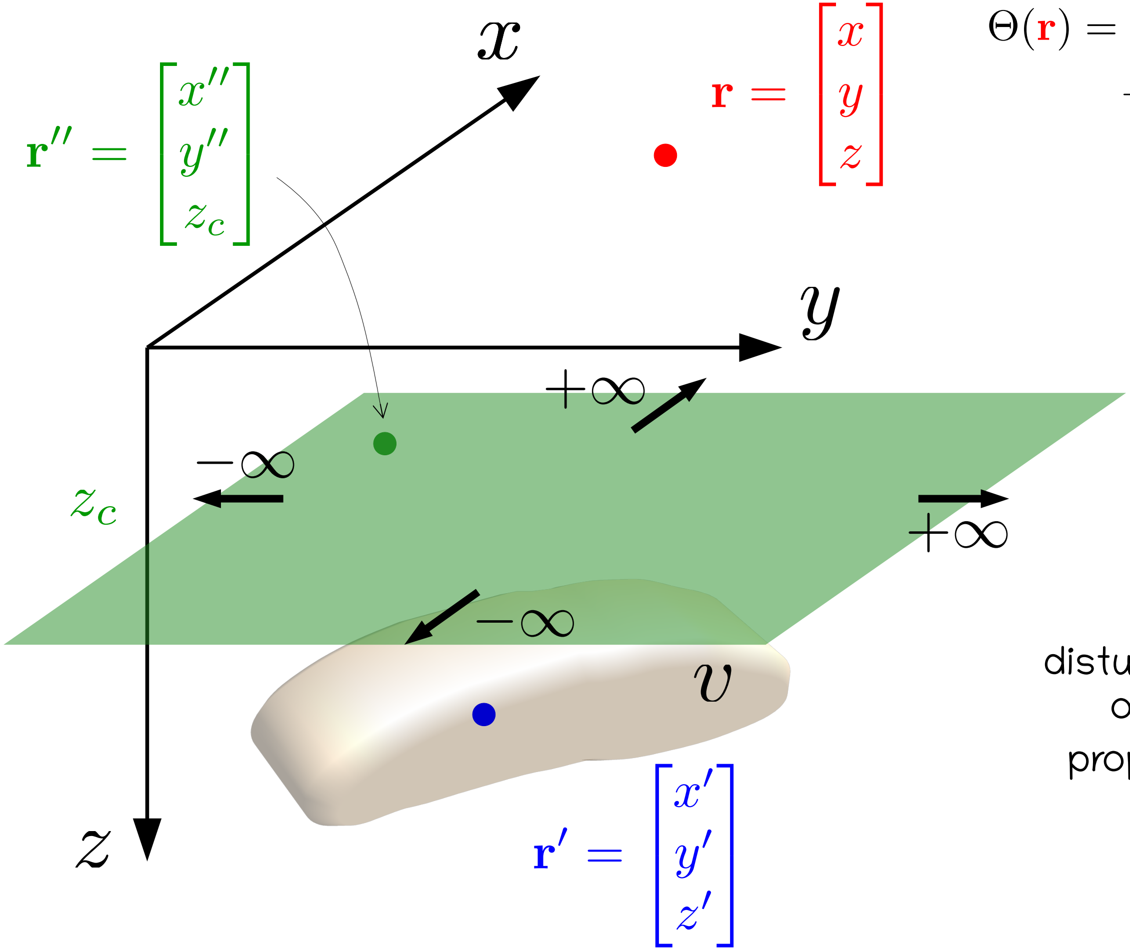
$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th} \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Note that this function is ≥ 0 at all points **r'** within the sources

$$\partial_z \Theta(\mathbf{r}'') = \kappa \iiint_v \sigma(\mathbf{r}') \; \partial_z \frac{1}{\|\mathbf{r}'' - \mathbf{r}'\|} dv'$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by a planar eq. layer of dipoles having the same uniform total-magnetization direction of the true sources



$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

$$\Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$

Deduction:

approx total-field anomaly

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th} \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

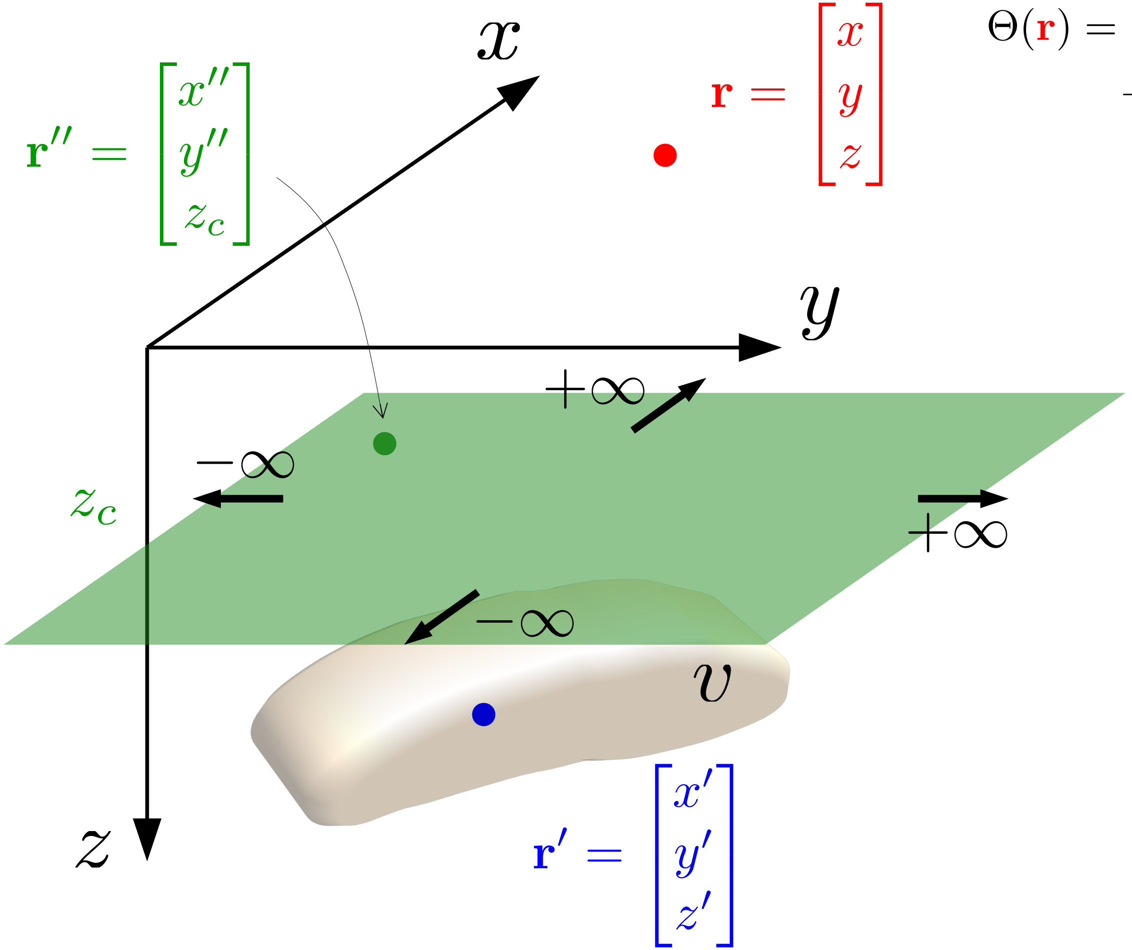
This analytical eq. layer represents the gravity disturbance that would be produced by the true sources on the plane z_c if they had a density distribution proportional to the total-magnetization intensity $\sigma(\mathbf{r}')$

$$\partial_z \Theta(\mathbf{r}'') = \kappa \iiint_v \sigma(\mathbf{r}') \; \partial_z \frac{1}{\|\mathbf{r}'' - \mathbf{r}'\|} dv'$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by a planar eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$



Deduction:

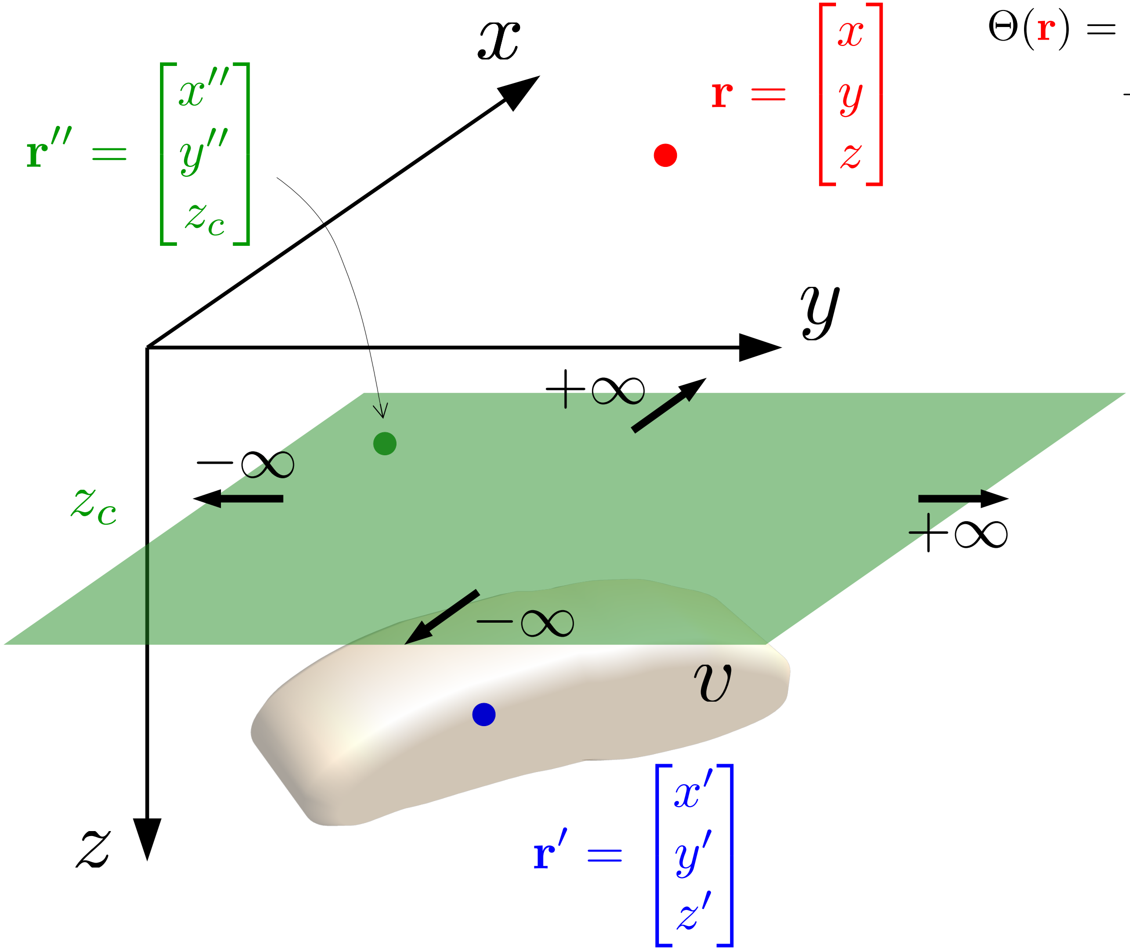
approx total-field anomaly

$$\partial_{th}\Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th}\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th}\Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{th}\Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by a planar eq. layer of dipoles having the same uniform total-magnetization direction of the true sources



$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

$$\Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$

Deduction:

approx total-field anomaly

$$\partial_{th}\Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

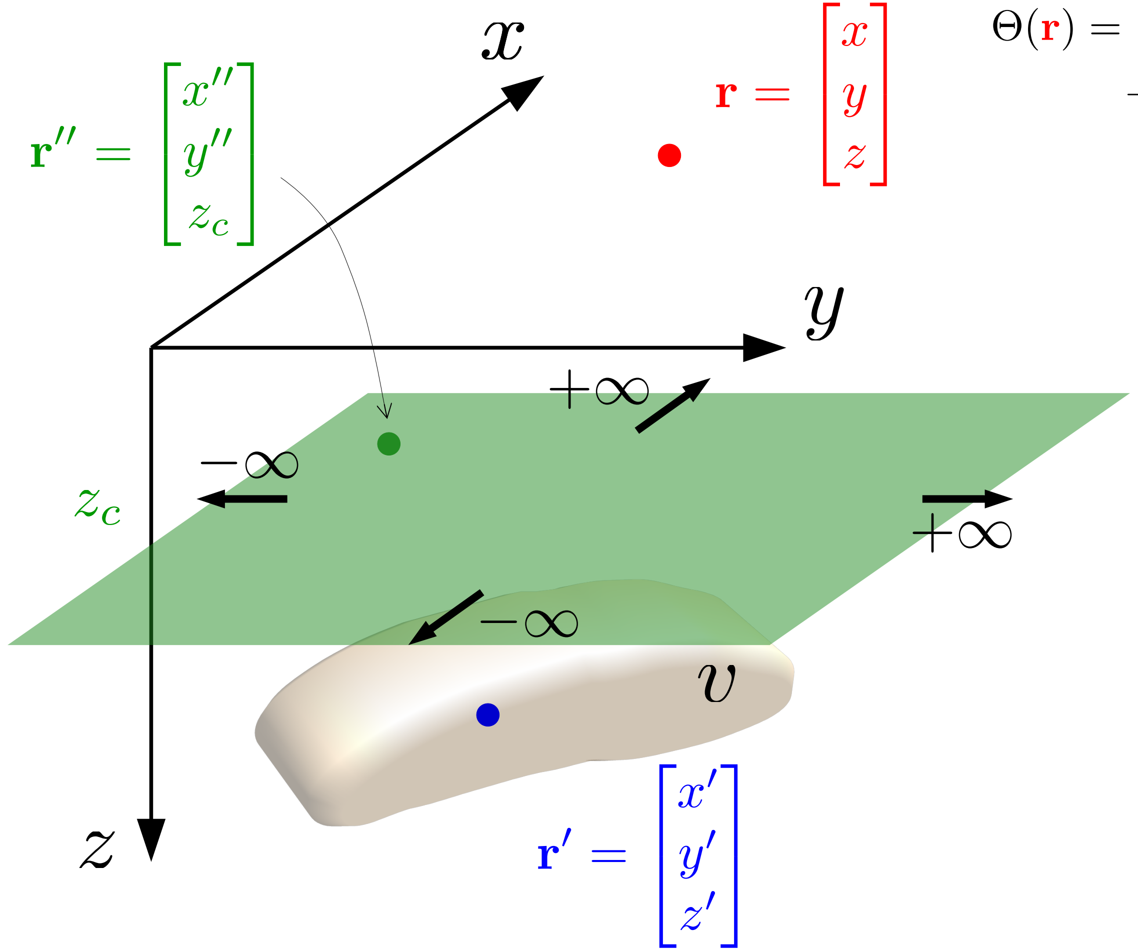
$$\partial_{th}\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th}\Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$$d_i \approx \sum_{j=1}^{M} \underbrace{p_j}_{\approx \partial_z \Theta(\mathbf{r}''_j)} \partial_{th}\Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by a planar eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$



Deduction:

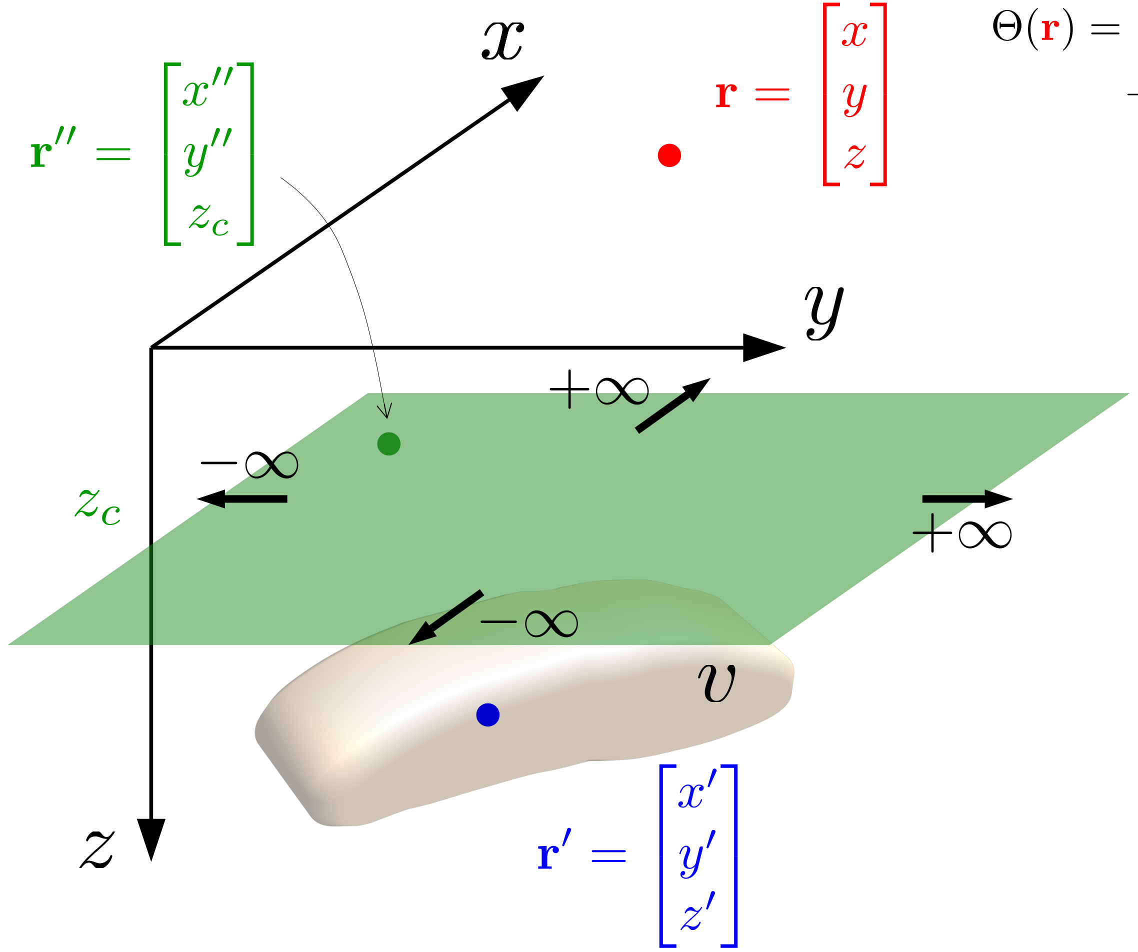
approx total-field anomaly

$$\partial_{th}\Theta(\mathbf{r}) = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Theta}(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th}\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th}\Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$$\underbrace{d_i}_{\partial_{th}\Theta(\mathbf{r}_i)} \approx \sum_{j=1}^{M} \underbrace{p_j}_{\approx \partial_z \Theta(\mathbf{r}''_j)} \, \partial_{th}\Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by a planar eq. layer of dipoles having the same uniform total-magnetization direction of the true sources



$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$

Deduction:

approx total-field anomaly

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th} \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

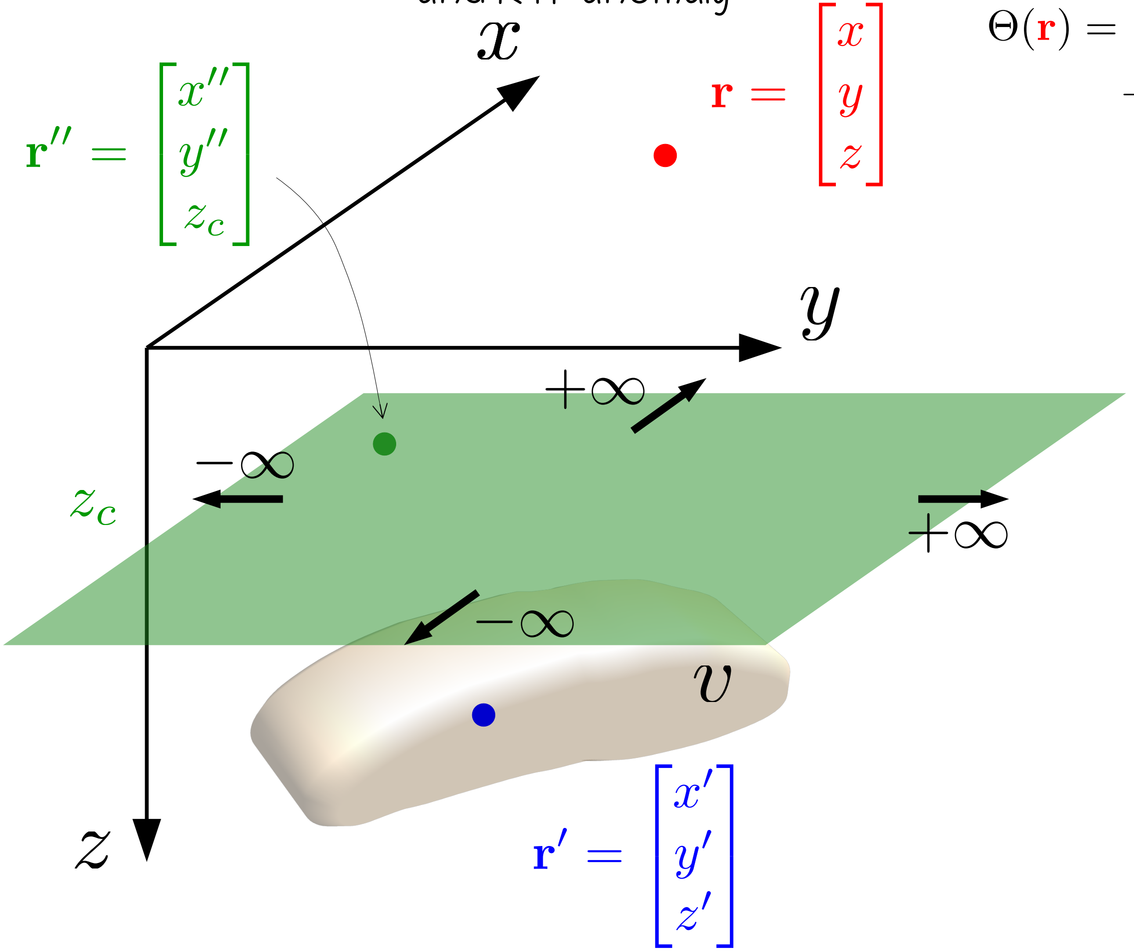
$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{th} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

classical EqL technique applied to magnetic data

Result 4/4: The same eq. layer of dipoles reproducing the approx total-field anomaly with the same uniform total-magnetization direction of the true sources can also reproduce the magnetic scalar potential, field components and RTP anomaly

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

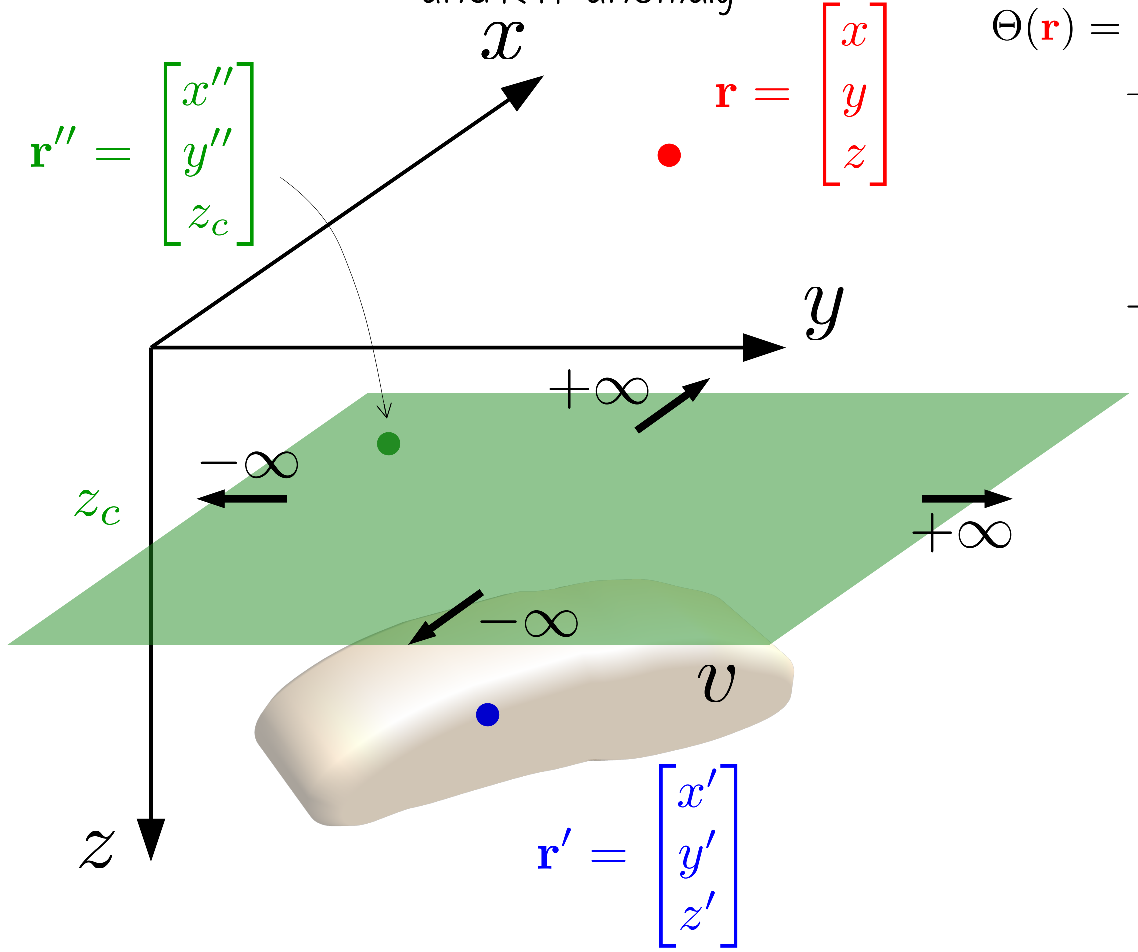
$$\Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$



Result 4/4: The same eq. layer of dipoles reproducing the approx total-field anomaly with the same uniform total-magnetization direction of the true sources can also reproduce the magnetic scalar potential, field components and RTP anomaly

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

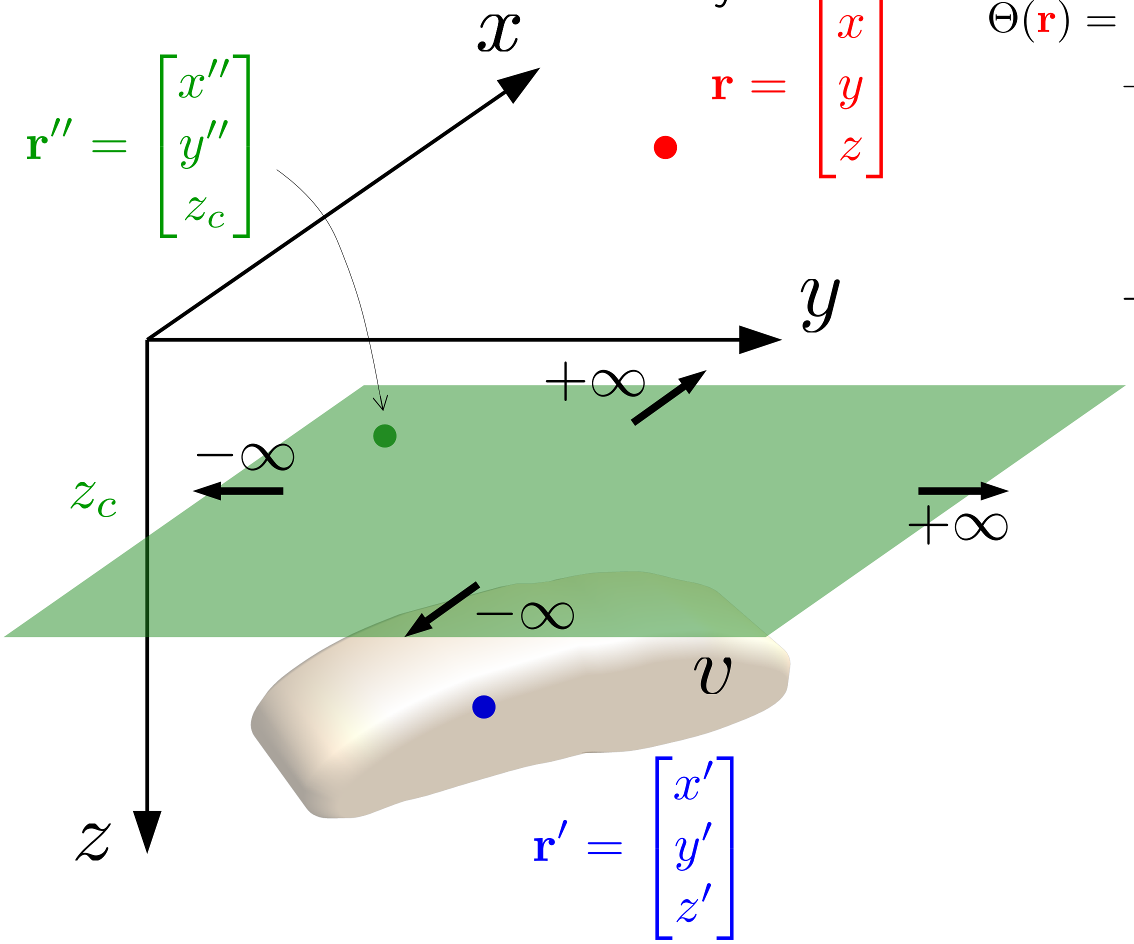
$$\Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$



Deduction:

$$-\partial_h \Theta(\mathbf{r}) = -\nabla \Theta(\mathbf{r})^\top \hat{\mathbf{h}}$$ mag scalar potential

Result 4/4: The same eq. layer of dipoles reproducing the approx total-field anomaly with the same uniform total-magnetization direction of the true sources can also reproduce the magnetic scalar potential, field components and RTP anomaly



$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

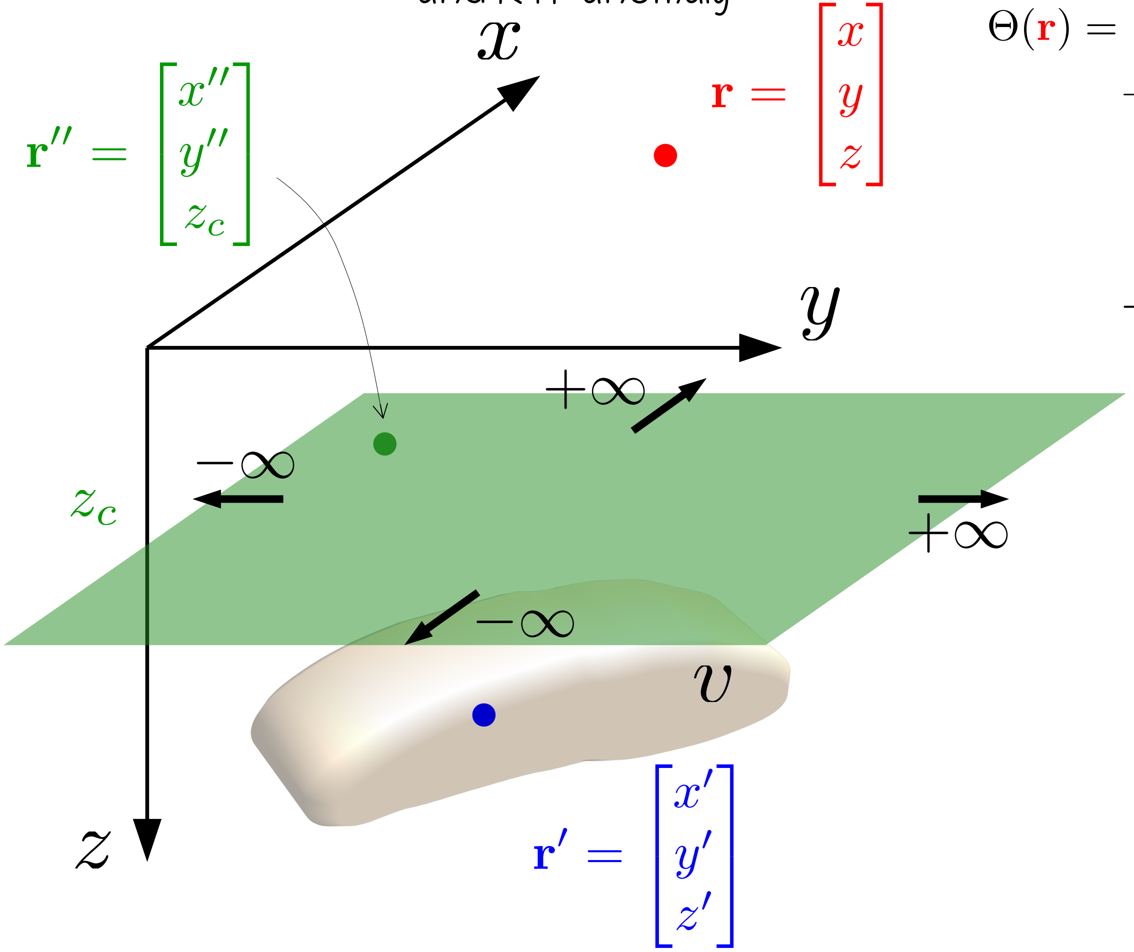
$$\Theta(\mathbf{r}) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}-\mathbf{r}''\|}$$

Deduction:

$$-\partial_h \Theta(\mathbf{r}) = -\nabla\Theta(\mathbf{r})^\top \hat{\mathbf{h}}$$ mag scalar potential

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$ approx total-field an

Result 4/4: The same eq. layer of dipoles reproducing the approx total-field anomaly with the same uniform total-magnetization direction of the true sources can also reproduce the magnetic scalar potential, field components and RTP anomaly



$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$

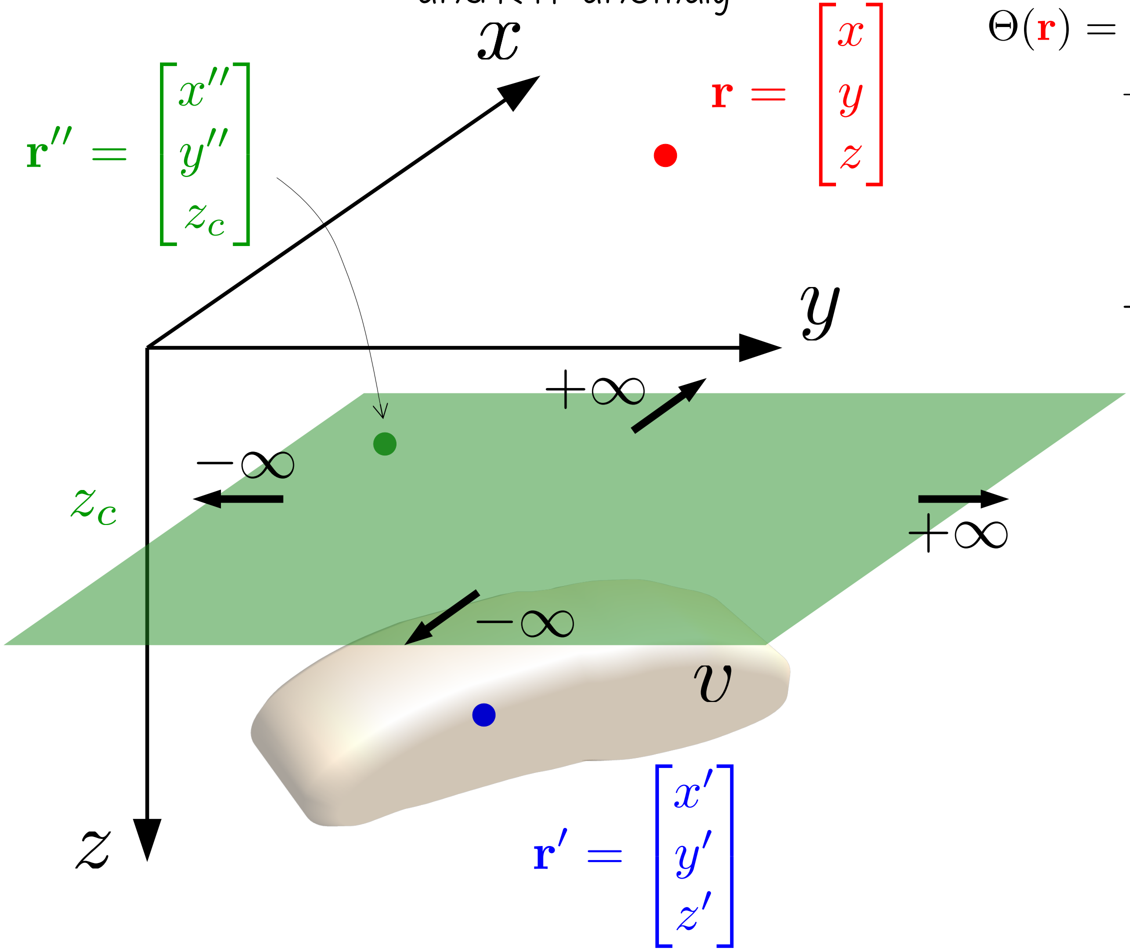
Deduction:

$-\partial_h \Theta(\mathbf{r}) = -\nabla \Theta(\mathbf{r})^\top \hat{\mathbf{h}}$ mag scalar potential

$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$ approx total-field an

$\partial_{\alpha h} \Theta(\mathbf{r}) = \partial_\alpha \nabla \Theta(\mathbf{r})^\top \hat{\mathbf{h}}$ mag field component

Result 4/4: The same eq. layer of dipoles reproducing the approx total-field anomaly with the same uniform total-magnetization direction of the true sources can also reproduce the magnetic scalar potential, field components and RTP anomaly



$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$

Deduction:

$-\partial_h \Theta(\mathbf{r}) = -\nabla \Theta(\mathbf{r})^\top \hat{\mathbf{h}}$ mag scalar potential

$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$ approx total-field an

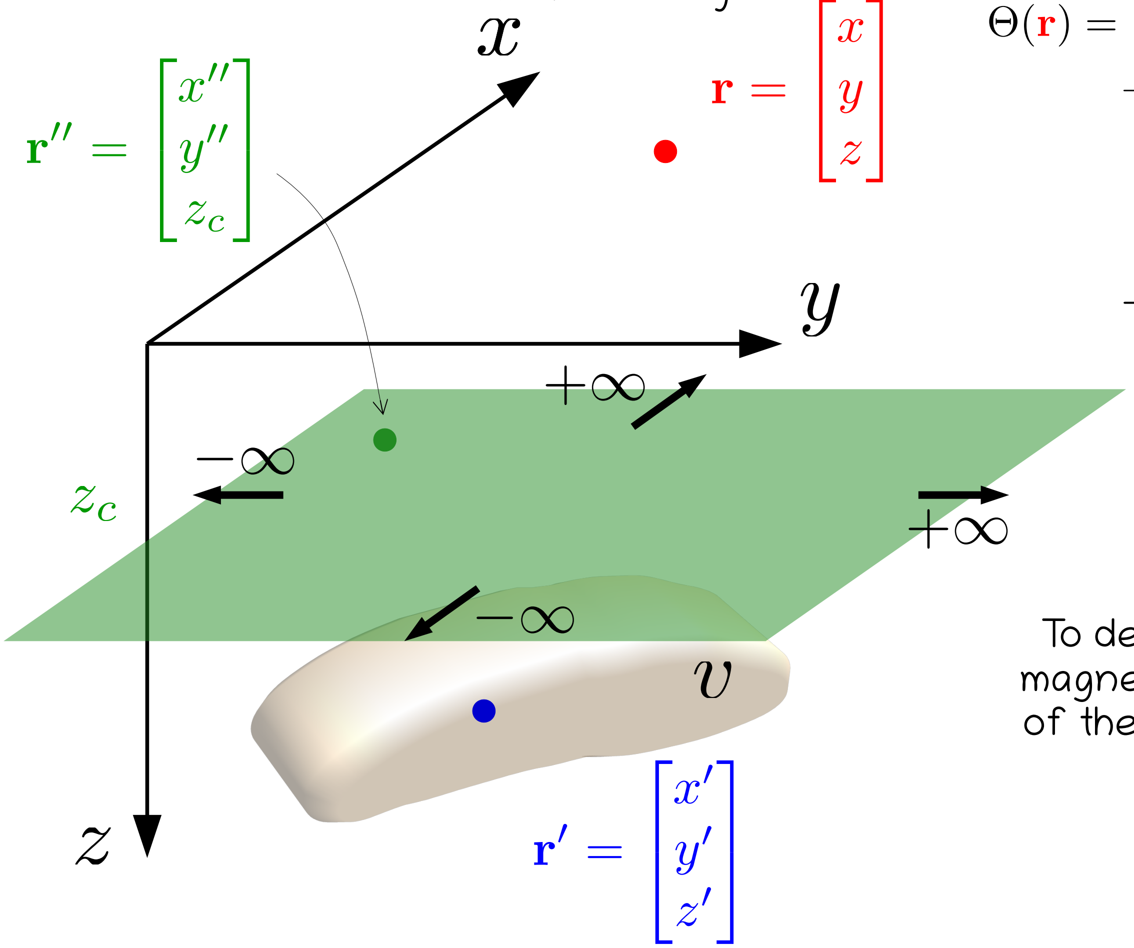
$\partial_{\alpha h} \Theta(\mathbf{r}) = \partial_\alpha \nabla \Theta(\mathbf{r})^\top \hat{\mathbf{h}}$ mag field component

$\partial_{zz} \Theta(\mathbf{r})$ RTP anomaly

Result 4/4: The same eq. layer of dipoles reproducing the approx total-field anomaly with the same uniform total-magnetization direction of the true sources can also reproduce the magnetic scalar potential, field components and RTP anomaly

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$



Deduction:

$$-\partial_h \Theta(\mathbf{r}) = -\nabla\Theta(\mathbf{r})^\top \hat{\mathbf{h}}$$ mag scalar potential

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$ approx total-field an

$$\partial_{\alpha h} \Theta(\mathbf{r}) = \partial_\alpha \nabla\Theta(\mathbf{r})^\top \hat{\mathbf{h}}$$ mag field component

$$\partial_{zz} \Theta(\mathbf{r})$$ RTP anomaly

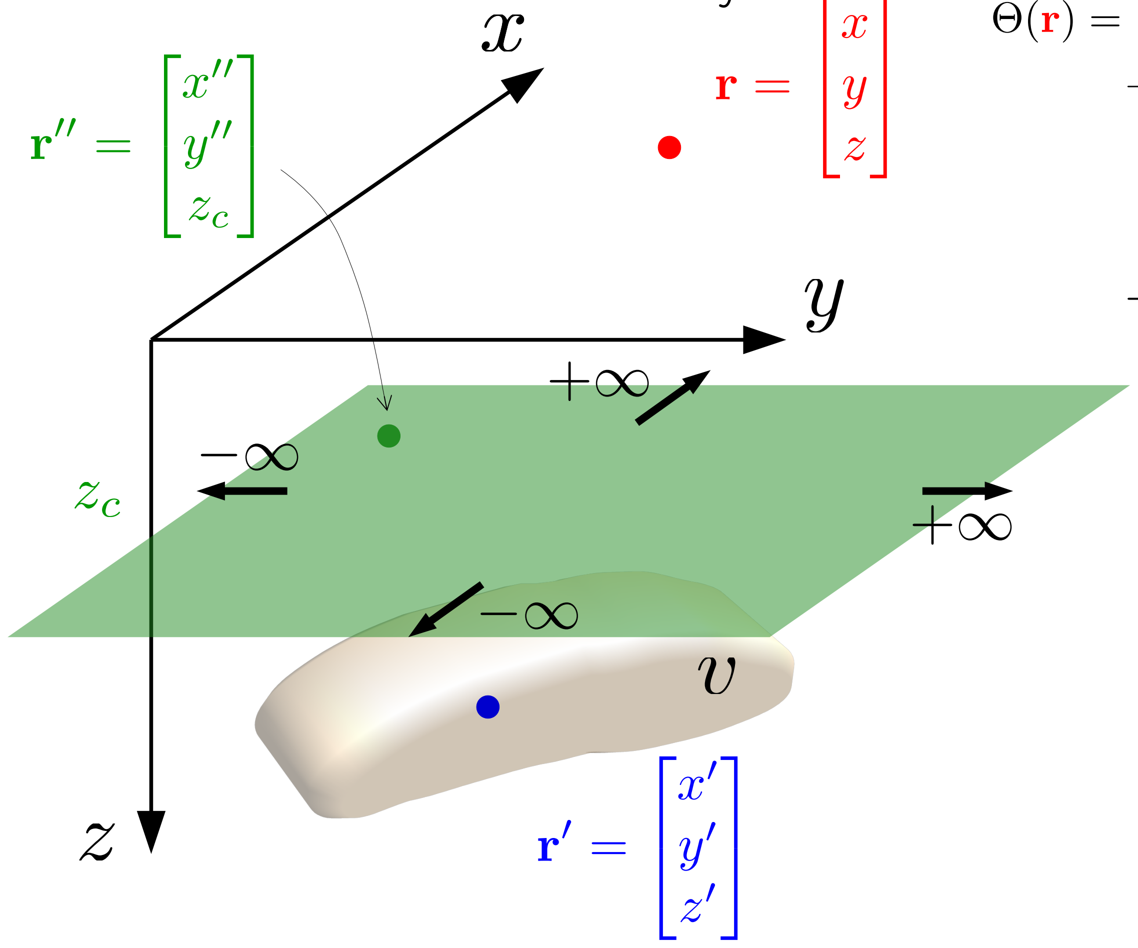
To deduce the analytical eq. layer associated with these magnetic field quantities, we have to compute derivatives of the equation below with respect to the coordinates of the observation point **r**

$$\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Result 4/4: The same eq. layer of dipoles reproducing the approx total-field anomaly with the same uniform total-magnetization direction of the true sources can also reproduce the magnetic scalar potential, field components and RTP anomaly

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$



Deduction:

$$-\partial_h \Theta(\mathbf{r}) = -\nabla \Theta(\mathbf{r})^\top \hat{\mathbf{h}} \quad \text{mag scalar potential}$$

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}} \quad \text{approx total-field an}$$

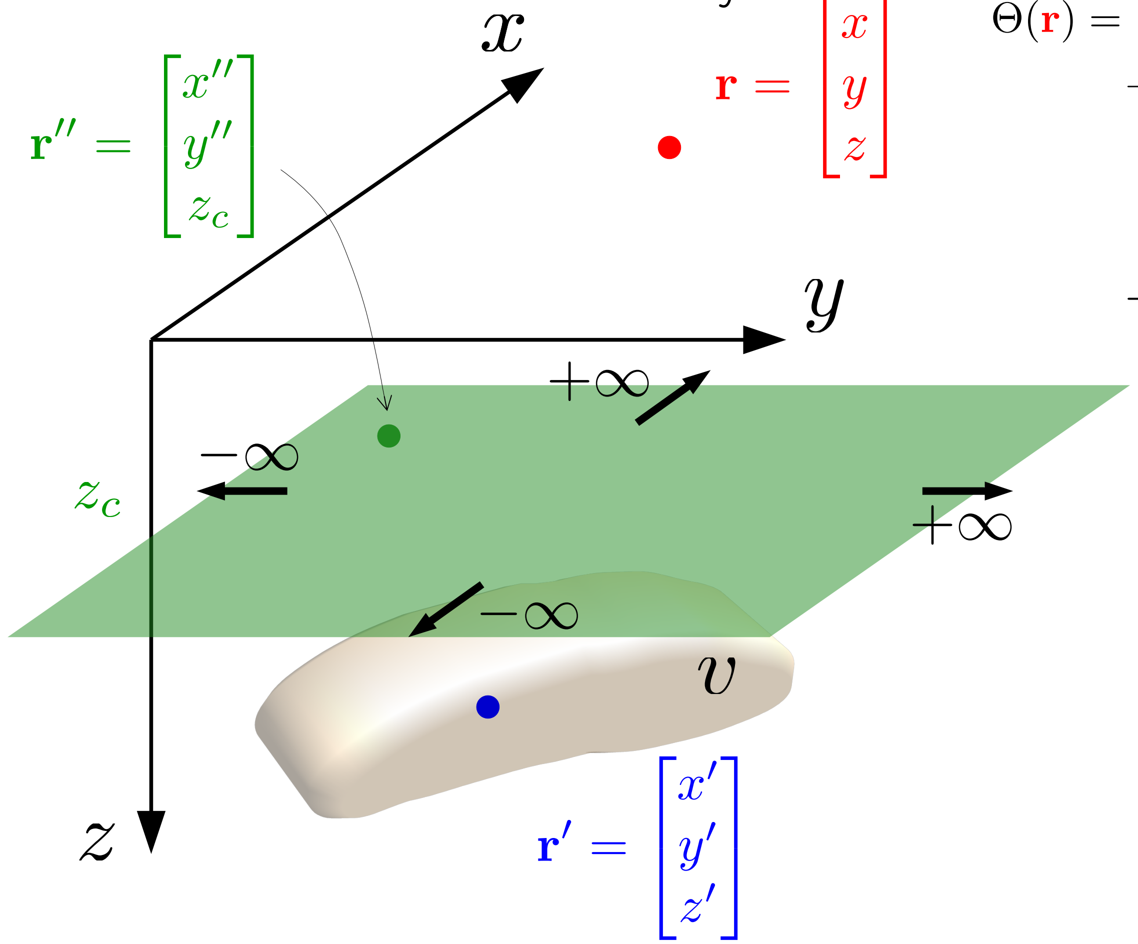
$$\partial_{\alpha h} \Theta(\mathbf{r}) = \partial_\alpha \nabla \Theta(\mathbf{r})^\top \hat{\mathbf{h}} \quad \text{mag field component}$$

$$\partial_{zz} \Theta(\mathbf{r}) \quad \text{RTP anomaly}$$

These derivatives, however, affect only this term

$$\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Result 4/4: The same eq. layer of dipoles reproducing the approx total-field anomaly with the same uniform total-magnetization direction of the true sources can also reproduce the magnetic scalar potential, field components and RTP anomaly



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$\partial_{\alpha h} \Theta(\mathbf{r}) = \partial_\alpha \nabla \Theta(\mathbf{r})^\top \hat{\mathbf{h}}$ mag field component

$\partial_{zz} \Theta(\mathbf{r})$ RTP anomaly

Hence, all these quantities are defined in terms of the same eq. layer

$$\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

List of theoretical results

Result 1/4: The gravity disturbance can be exactly reproduced by a planar eq. layer of monopoles

Result 2/4: The same eq. layer of monopoles reproducing the gravity disturbance can also reproduce the gravitational potential and gravity gradient tensor

Result 3/4: The approx total-field anomaly can be exactly reproduced by a planar eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

Result 4/4: The same eq. layer of dipoles reproducing the approx total-field anomaly with the same uniform total-magnetization direction of the true sources can also reproduce the magnetic scalar potential, field components and RTP anomaly

Summary

- Potential-field data
- The Equivalent-Layer (EqL) Technique
- **Theoretical aspects**
- Some open questions

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approx total-field anomaly

total-magnetization direction of the true sources

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analytical equivalent
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- Is there an analytical eq. layer of dipoles with arbitrary uniform magnetization direction?

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arbitrary direction

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- How to generalize the approach presented here to deduce analytical eq. layers in spherical coordinates?