Processing and interpreting potential-field data via Equivalent-Layer Technique

Vanderlei C. Oliveira Jr.

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Observatório Nacional

PINGA

A research group for inverse problems in Geophysics

Summary

- Motivation
- Potential-field data
- The Equivalent-Layer (EqL) Technique
- Theoretical aspects
- Some open questions

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- **Motivation**
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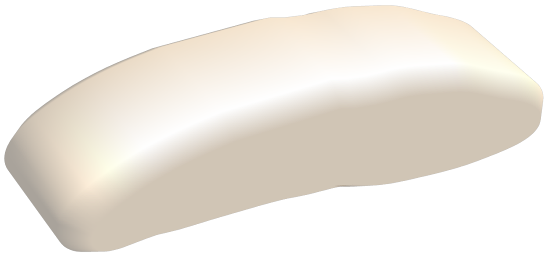
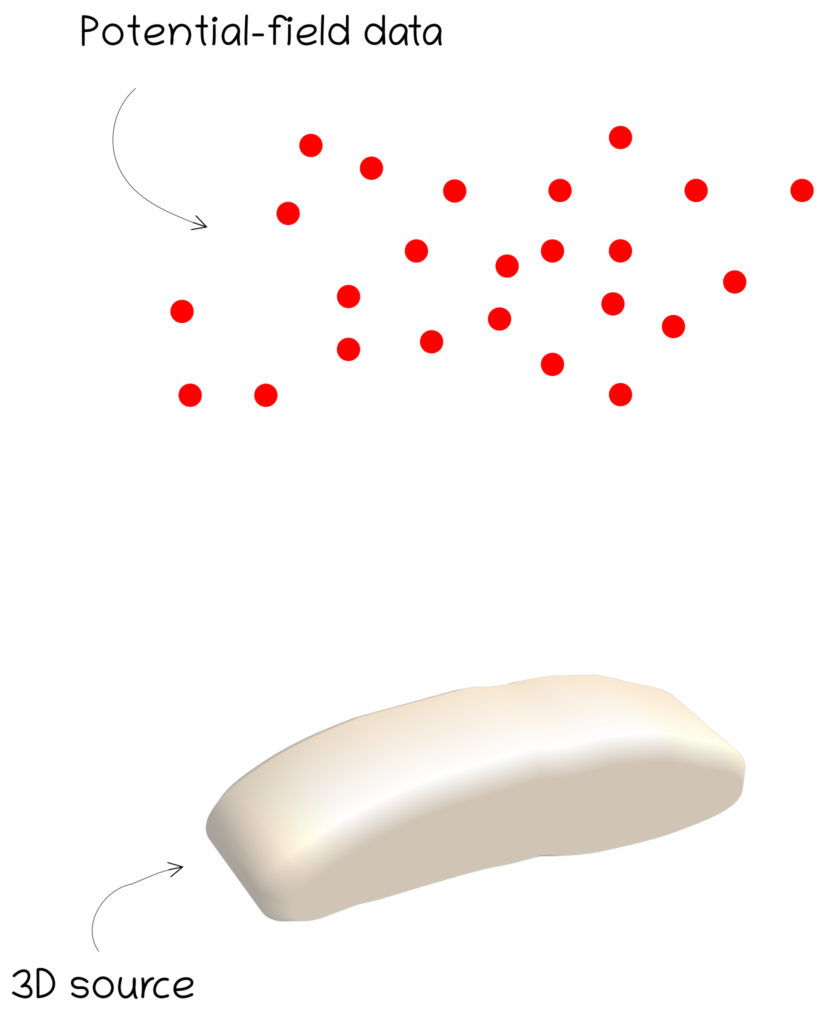
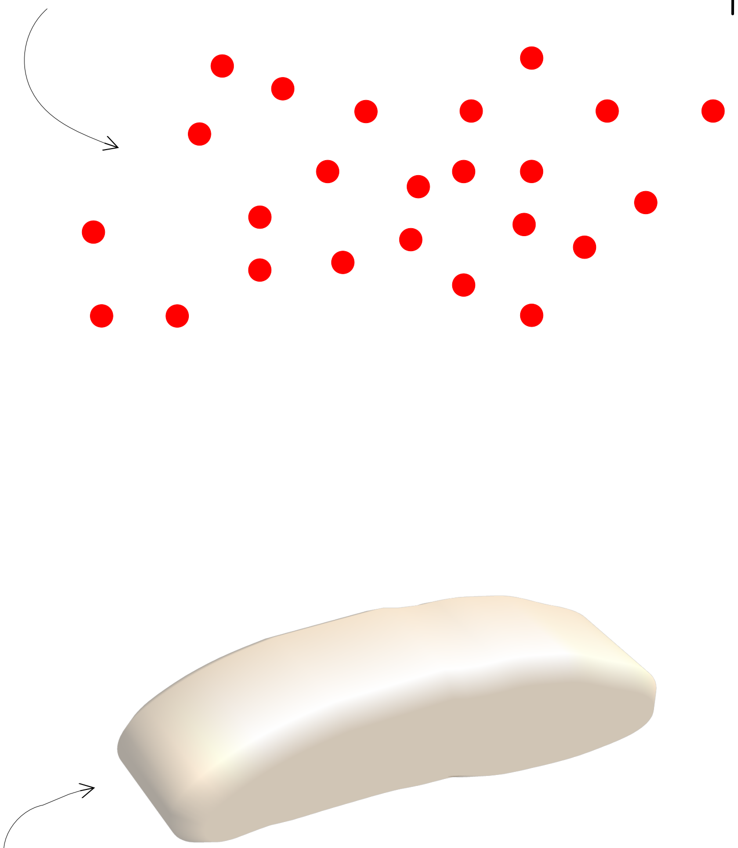


3D source

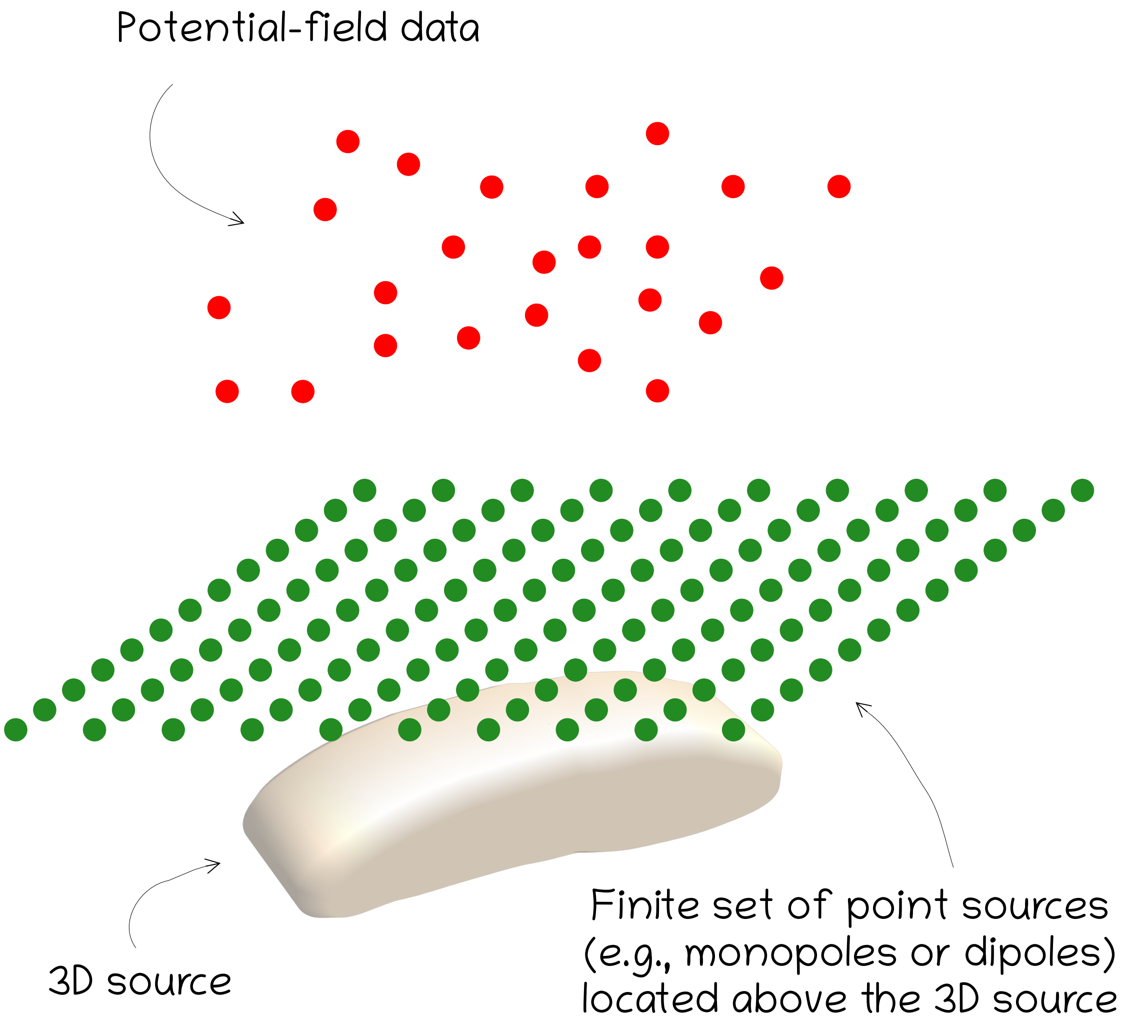

Potential-field data
3D source

Throughout this presentation, I consider that potential-field data can be represented by harmonic functions

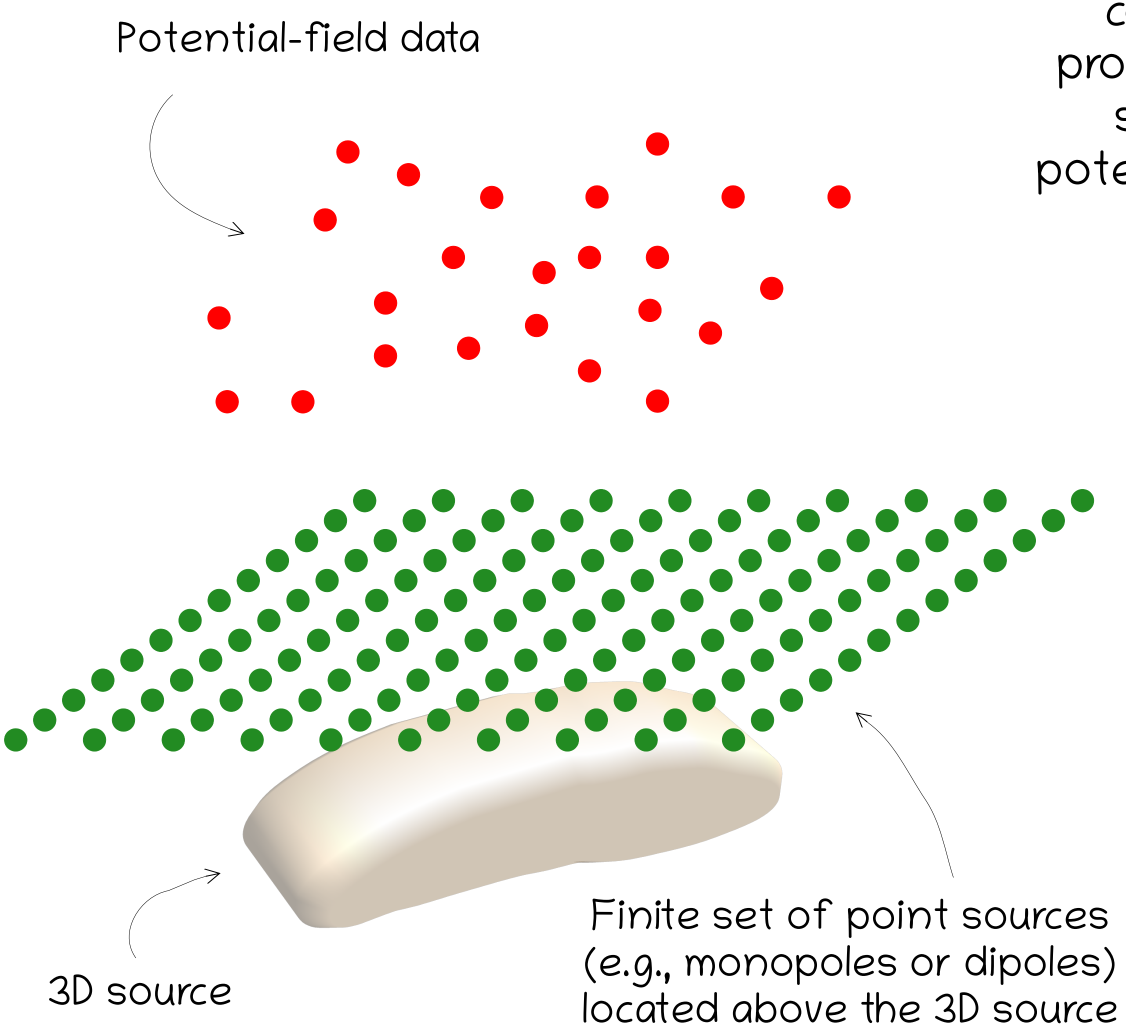
Potential-field data

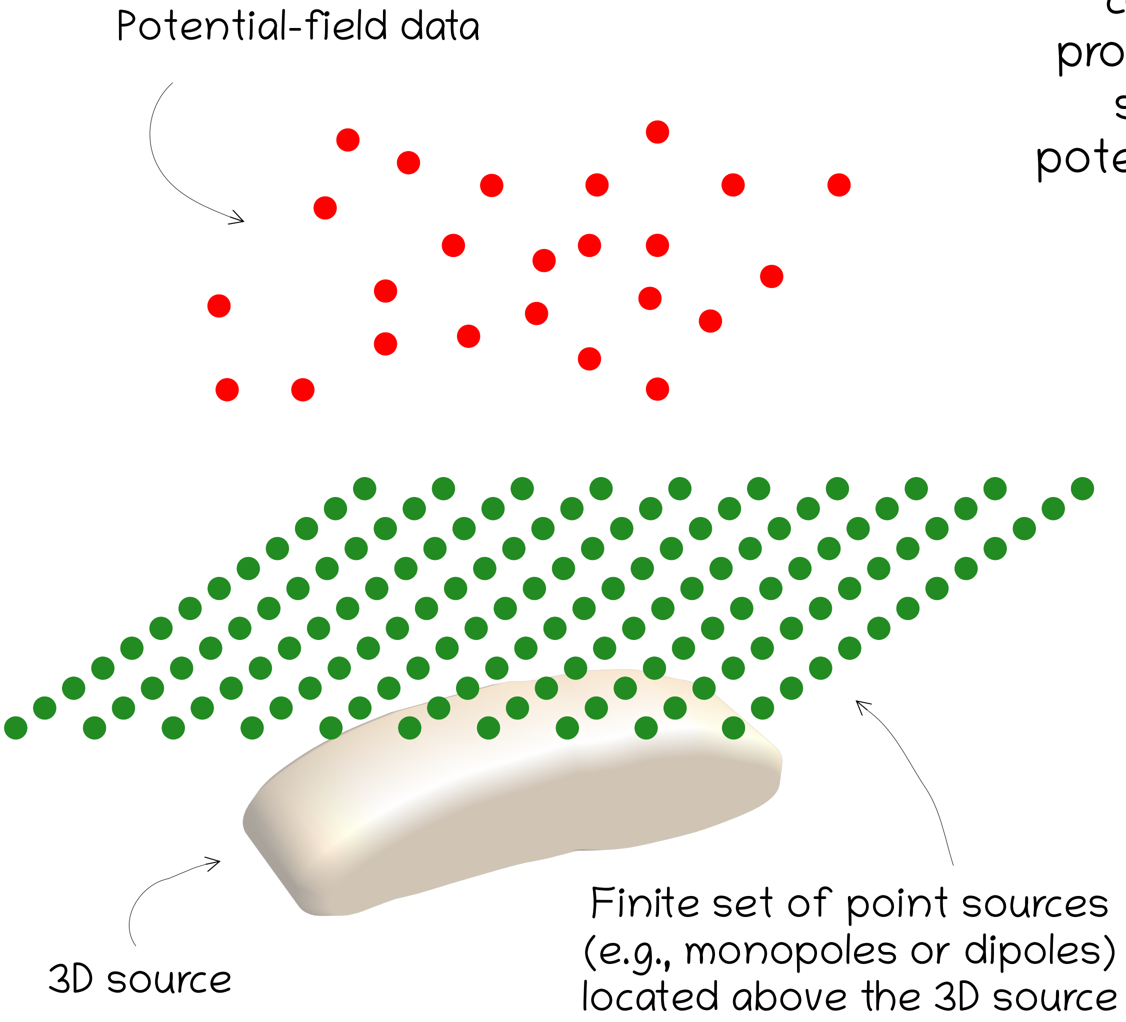


3D source


Potential-field data
3D source
Finite set of point sources
(e.g., monopoles or dipoles)
located above the 3D source

The Equivalent-Layer (EqL) Technique consists in first estimating the physical-property distribution of these **point sources** so that they satisfactorily reproduce the potential-field data caused by the 3D source at the **observation points**





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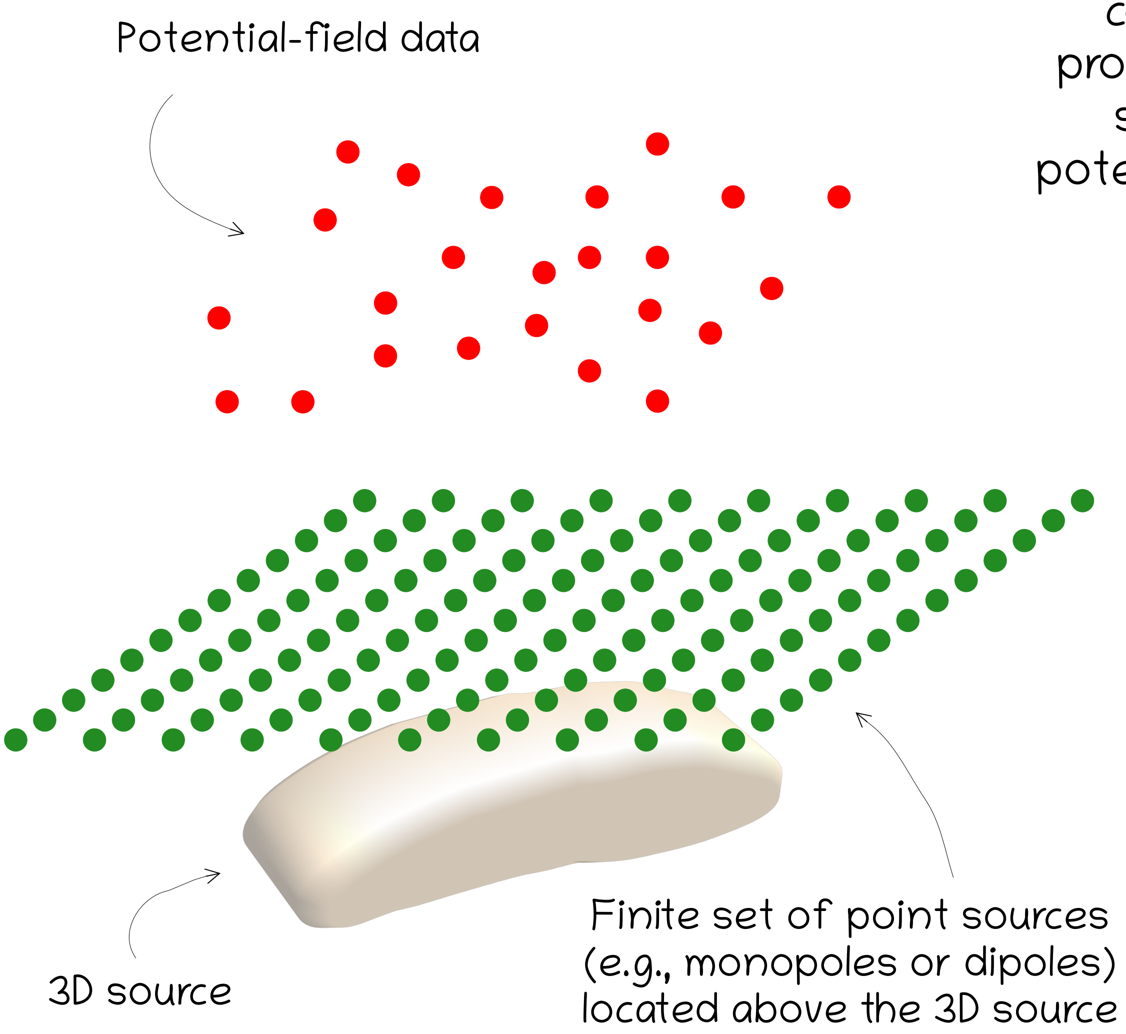
Then, the estimated physical property distribution can be used, for example, to:



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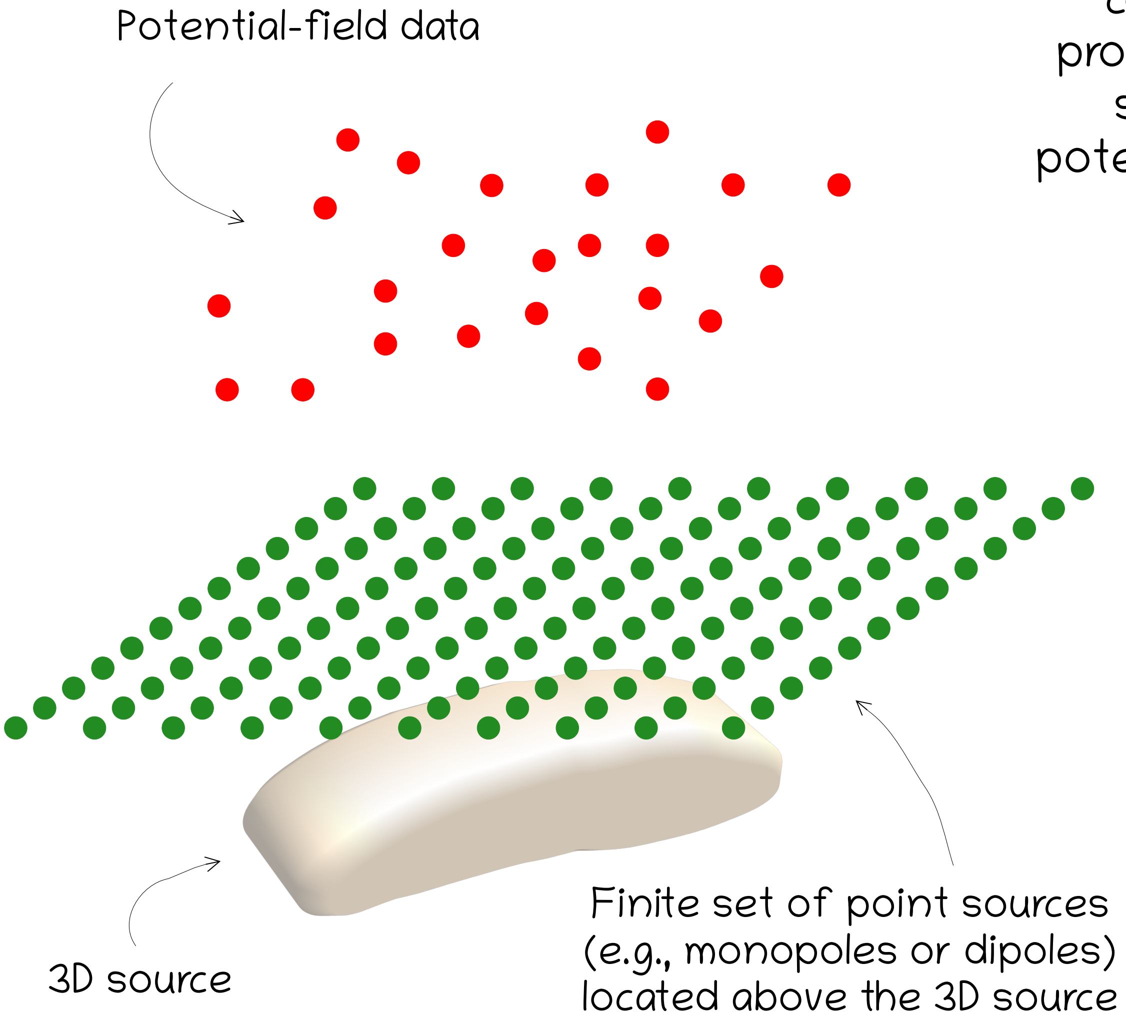
* predict the potential field at points without measurements (interpolation, upward/downward continuation)



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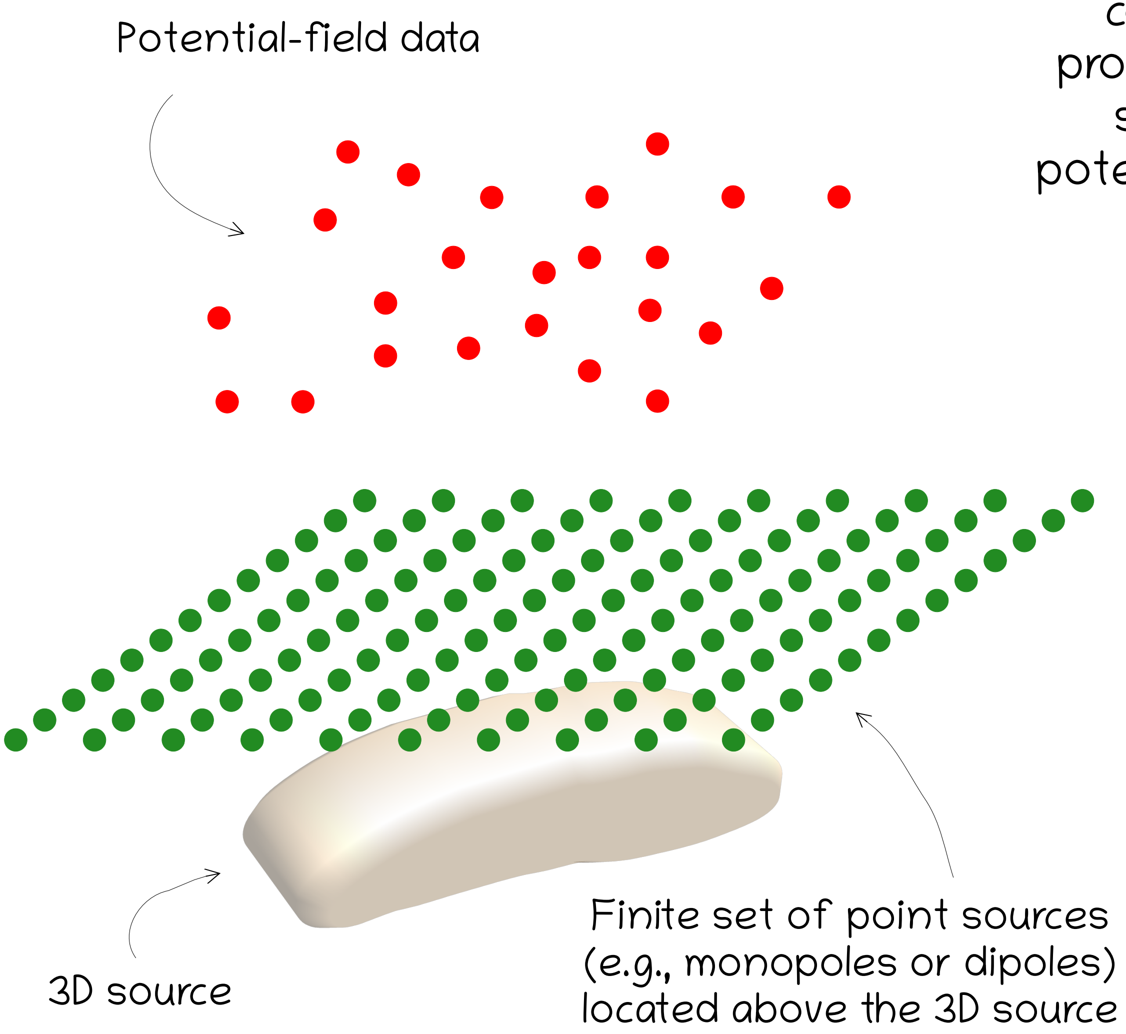
* predict the potential field at points without measurements (interpolation, upward/downward continuation)

* reduce total-field anomalies to the pole



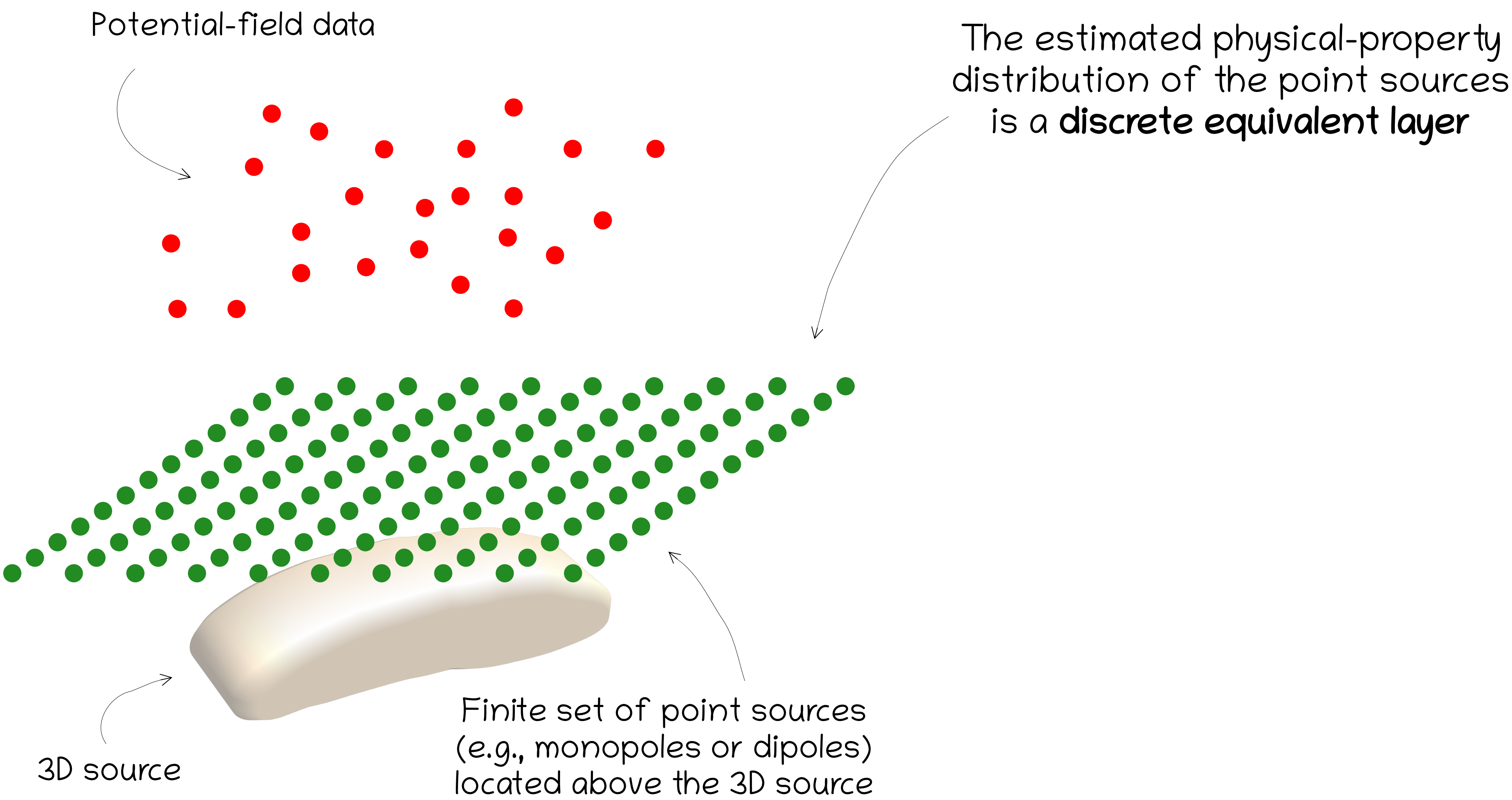
The Equivalent-Layer (EqL) Technique consists in first estimating the physical-property distribution of these point sources so that they satisfactorily reproduce the potential-field data caused by the 3D source at the observation points

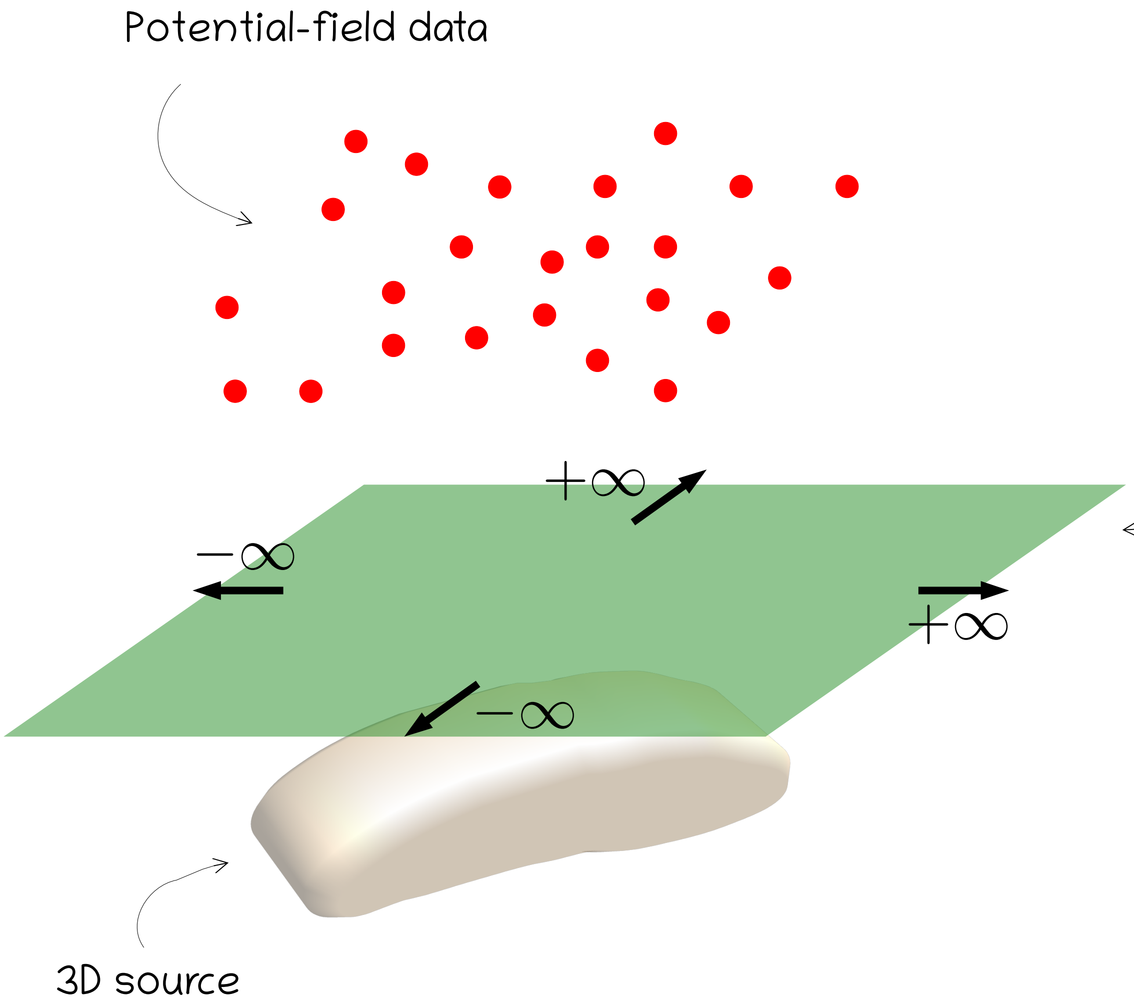
Then, the estimated physical property distribution can be used, for example, to:

* predict the potential field at points without measurements (interpolation, upward/downward continuation)

* reduce total-field anomalies to the pole

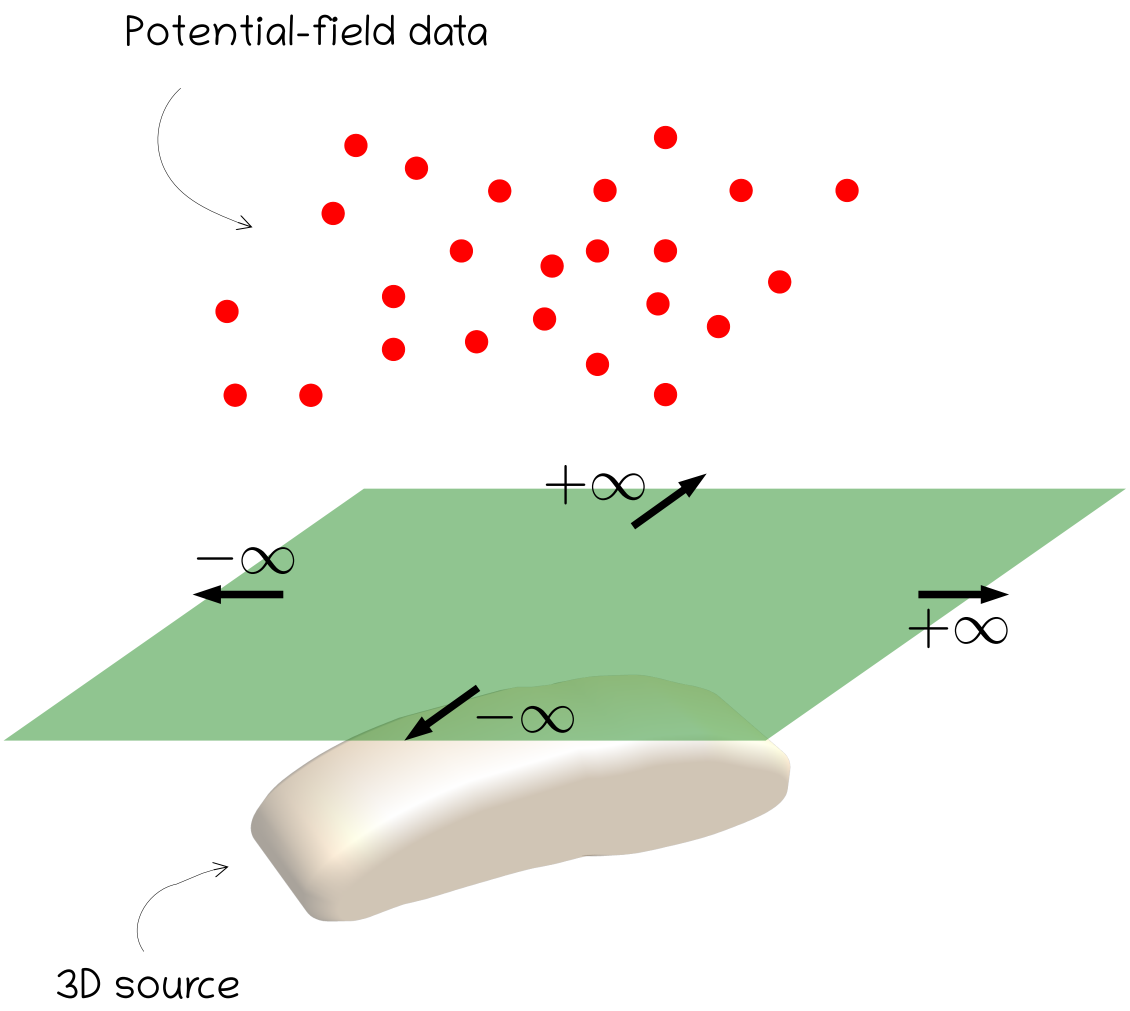
* convert gravity disturbance (g_z) into tensor components (g_{xx}, g_{xy}, g_{xz} ...)


Potential-field data
The estimated physical-property distribution of the point sources is a **discrete equivalent layer**
3D source
Finite set of point sources (e.g., monopoles or dipoles) located above the 3D source



The estimated physical-property distribution of the point sources is a discrete equivalent layer

A discrete equivalent layer, in turn, is a discrete representation for a **continuous equivalent layer**



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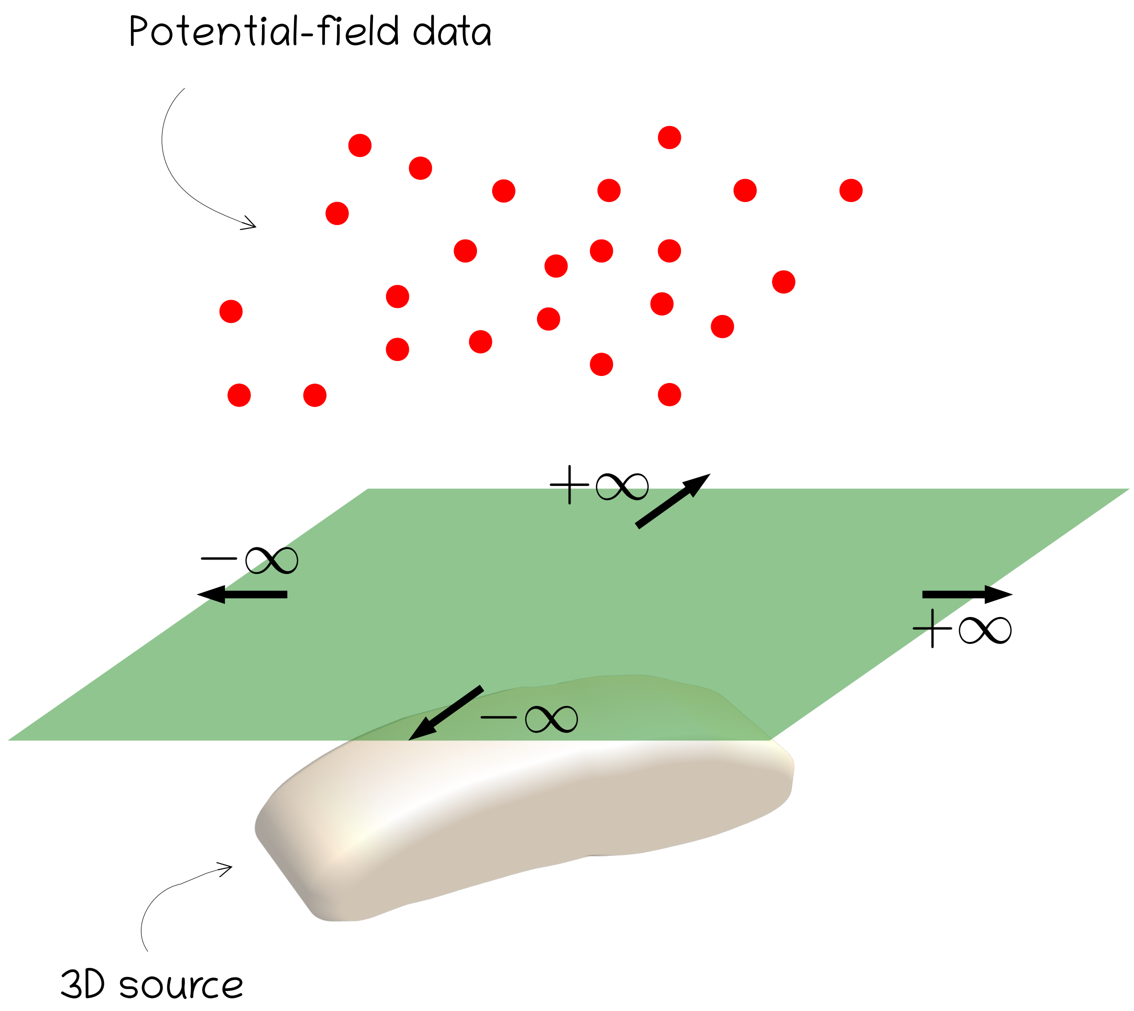
Continuous eq. layers can be obtained by solving integral equations for boundary value problems of potential theory, such as the Dirichlet and Neumann's problems



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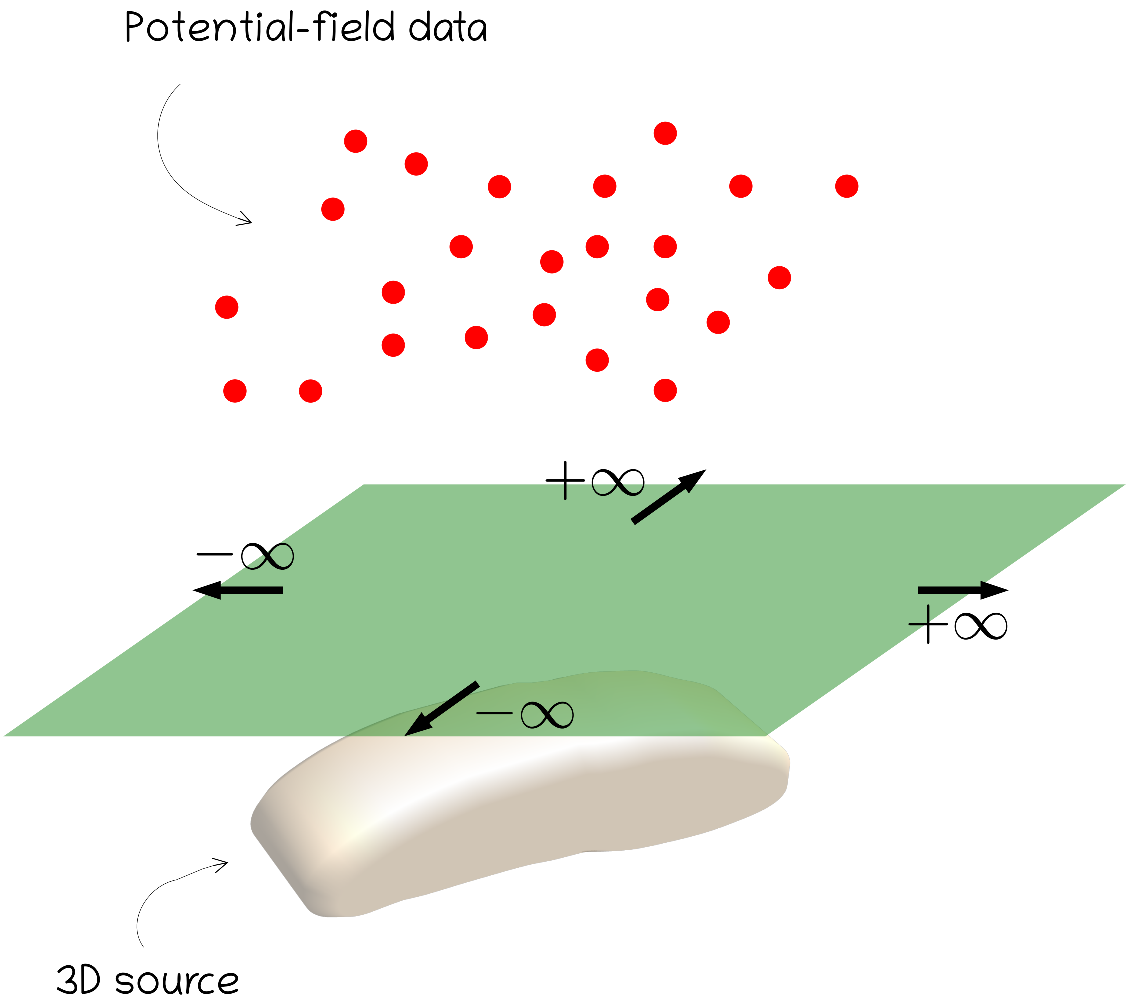
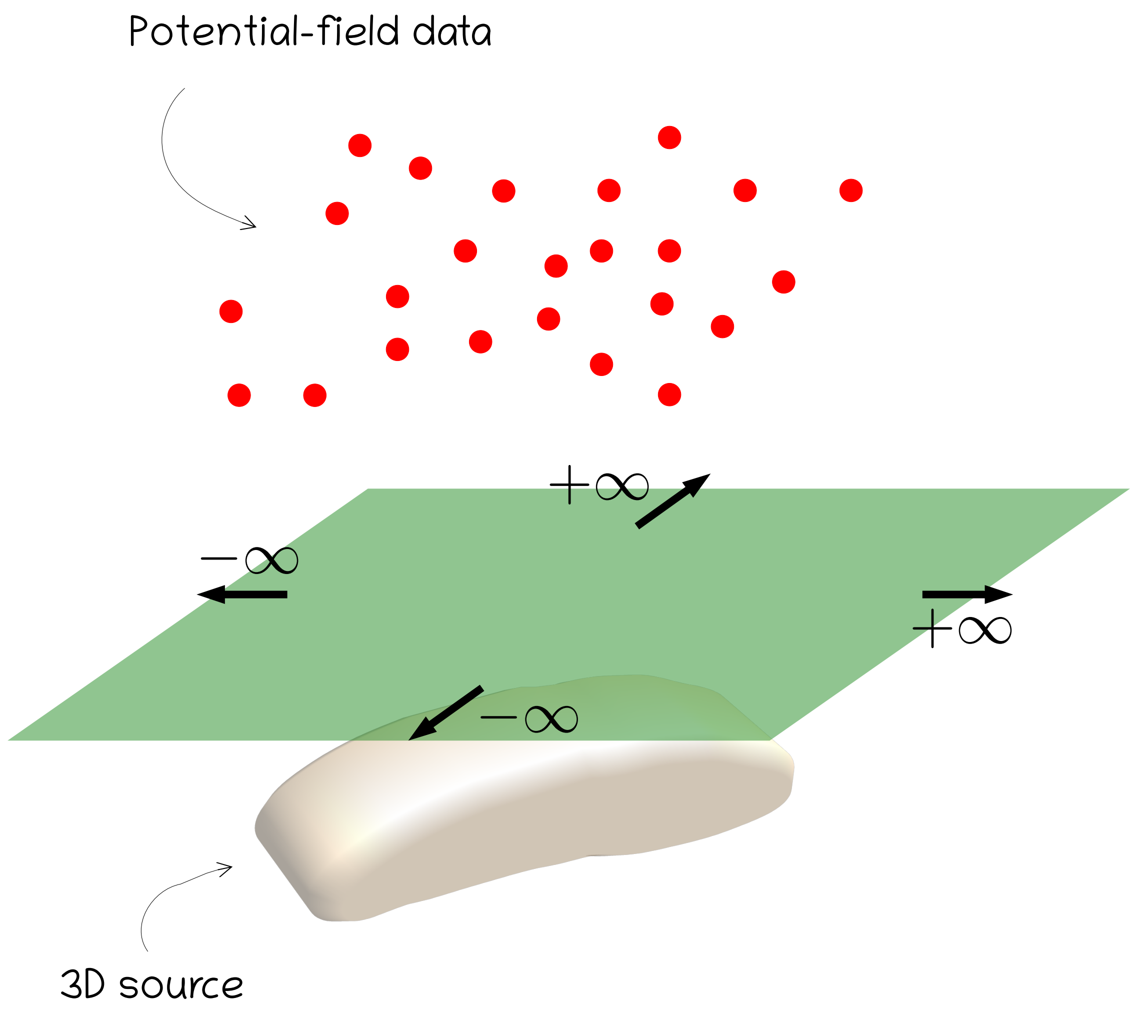
A discrete equivalent layer, in turn, is a discrete representation for a continuous equivalent layer

Continuous eq. layers can be obtained by solving integral equations for boundary value problems of potential theory, such as the Dirichlet and Neumann's problems

In this case, they are conveniently called here **analytical equivalent layers**

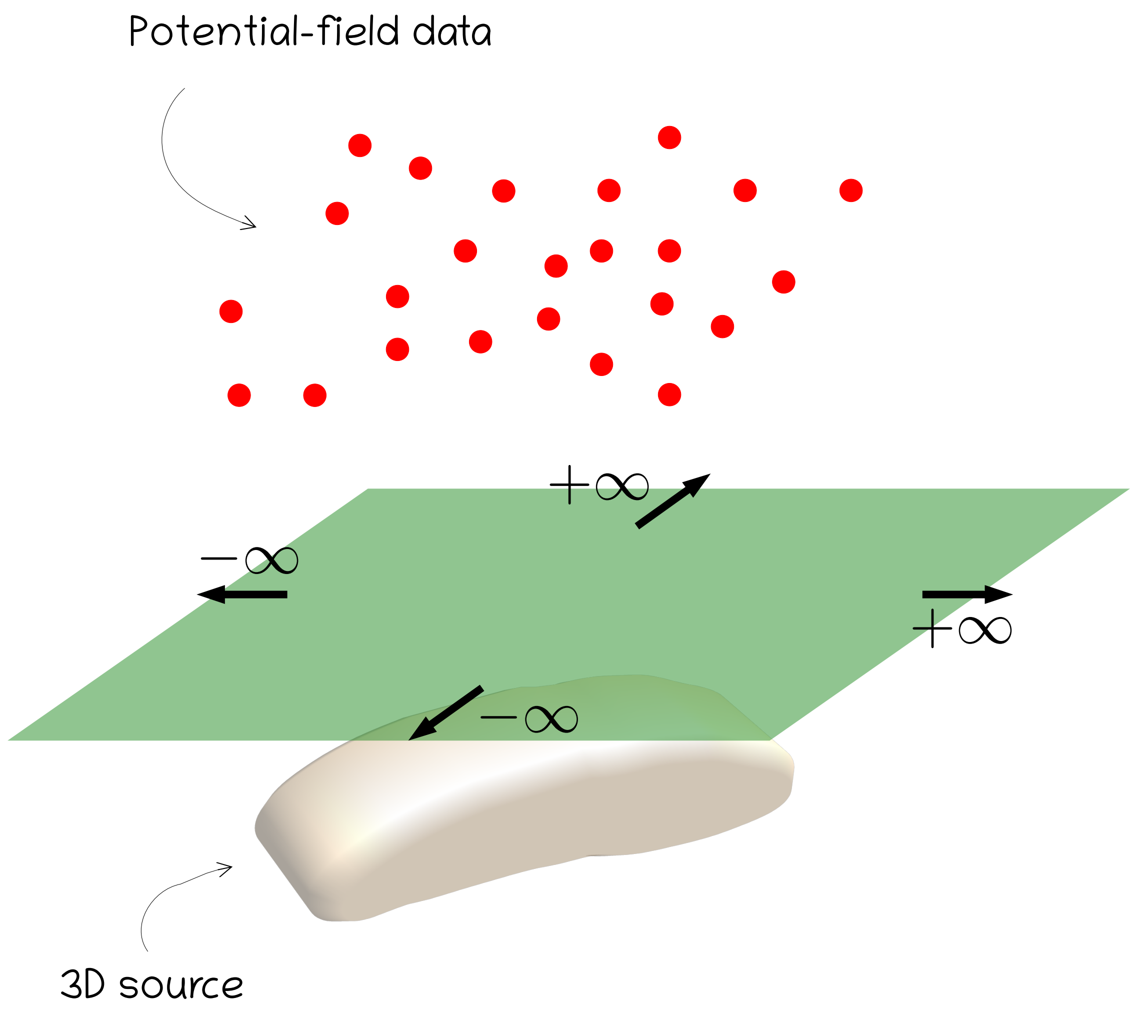
The main goals here are:





The main goals here are:

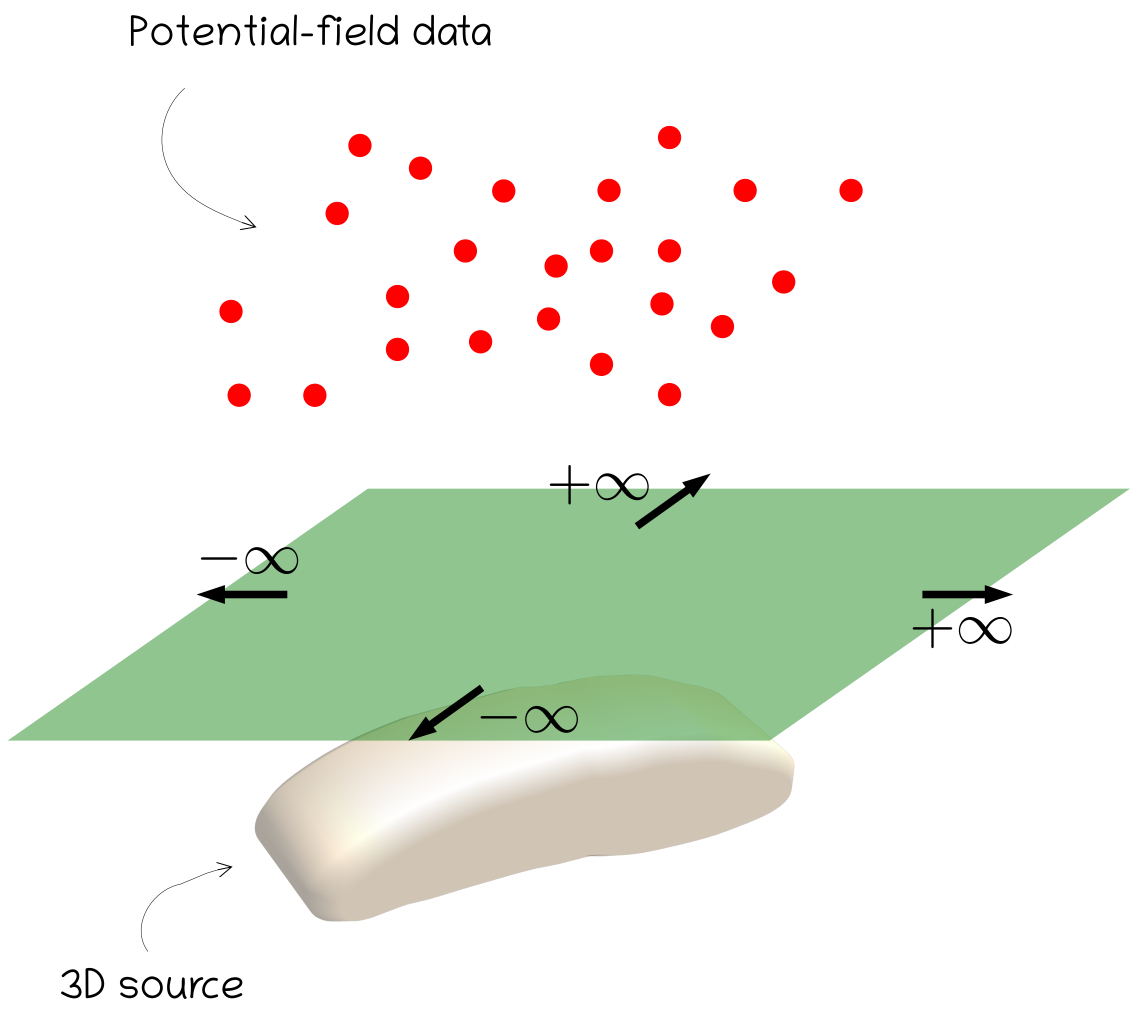
* deducing analytical equivalent layers of monopoles and dipoles for different potential-field data



The main goals here are:

* deducing analytical equivalent layers of monopoles and dipoles for different potential-field data

* show that they fill some theoretical gaps of EqL technique



The main goals here are:

* deducing analytical equivalent layers of monopoles and dipoles for different potential-field data

* show that they fill some theoretical gaps of EqL technique

* indicate some theoretical gaps that have not yet been solved

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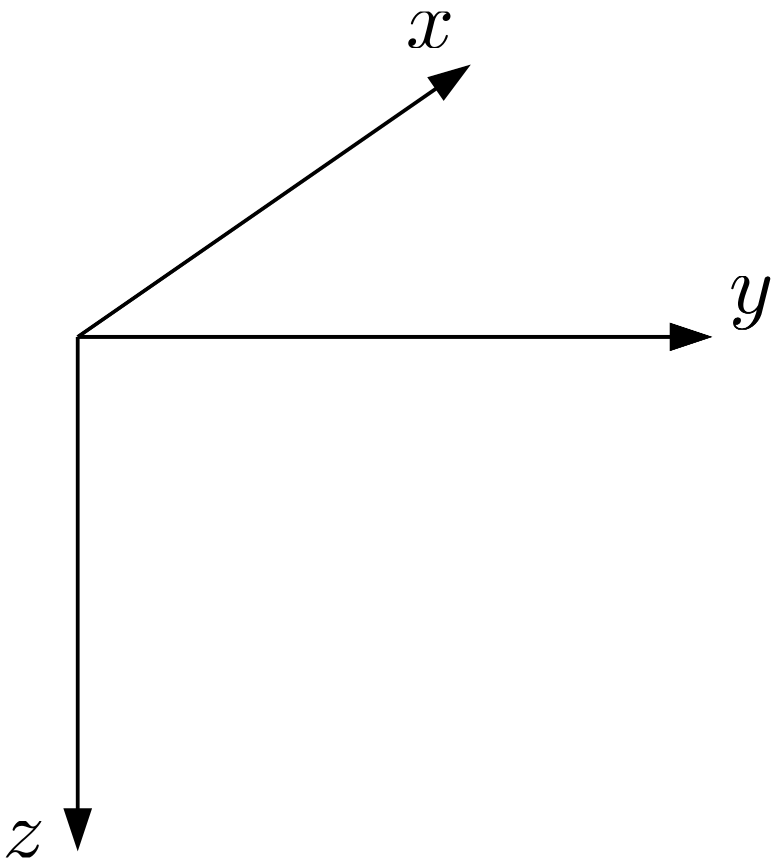
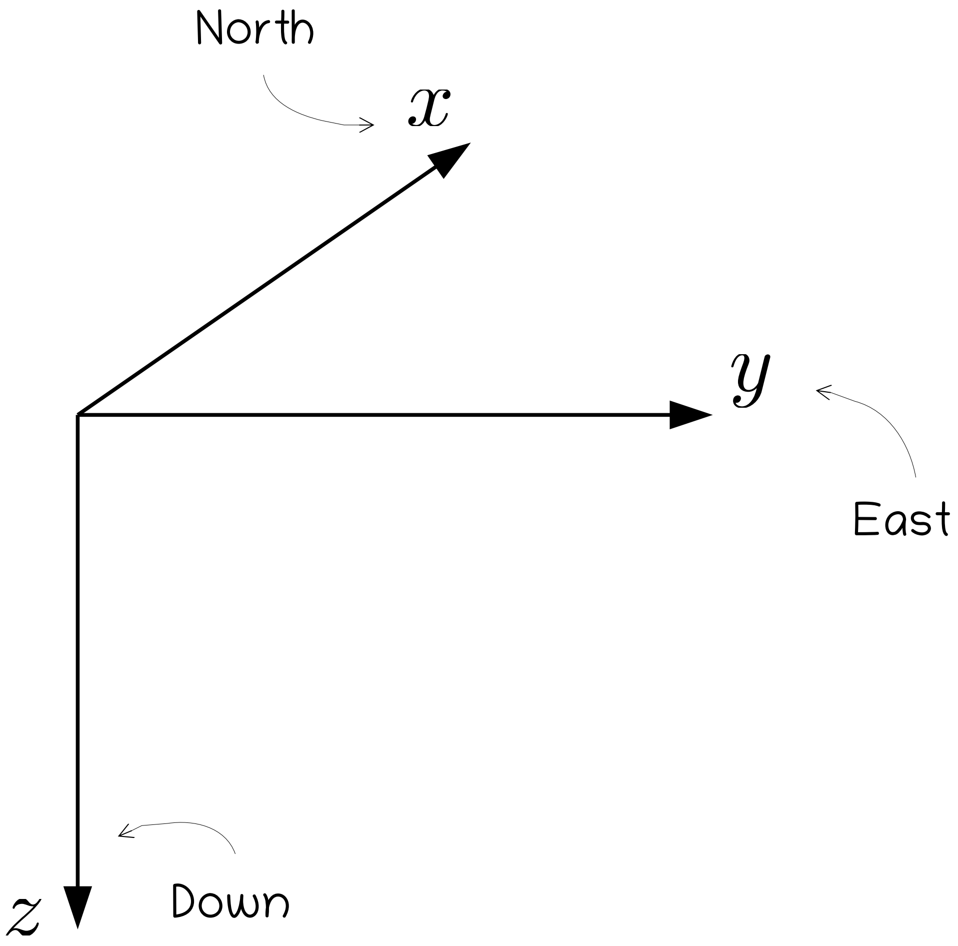
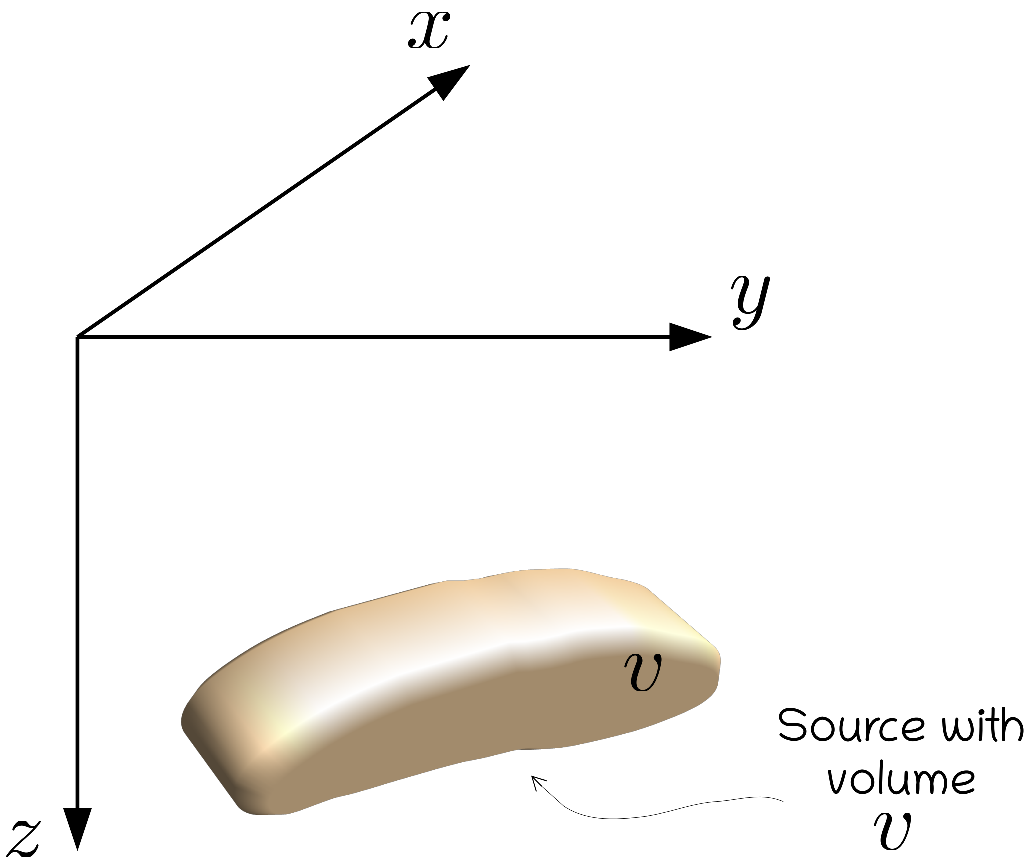
Topocentric Cartesian system

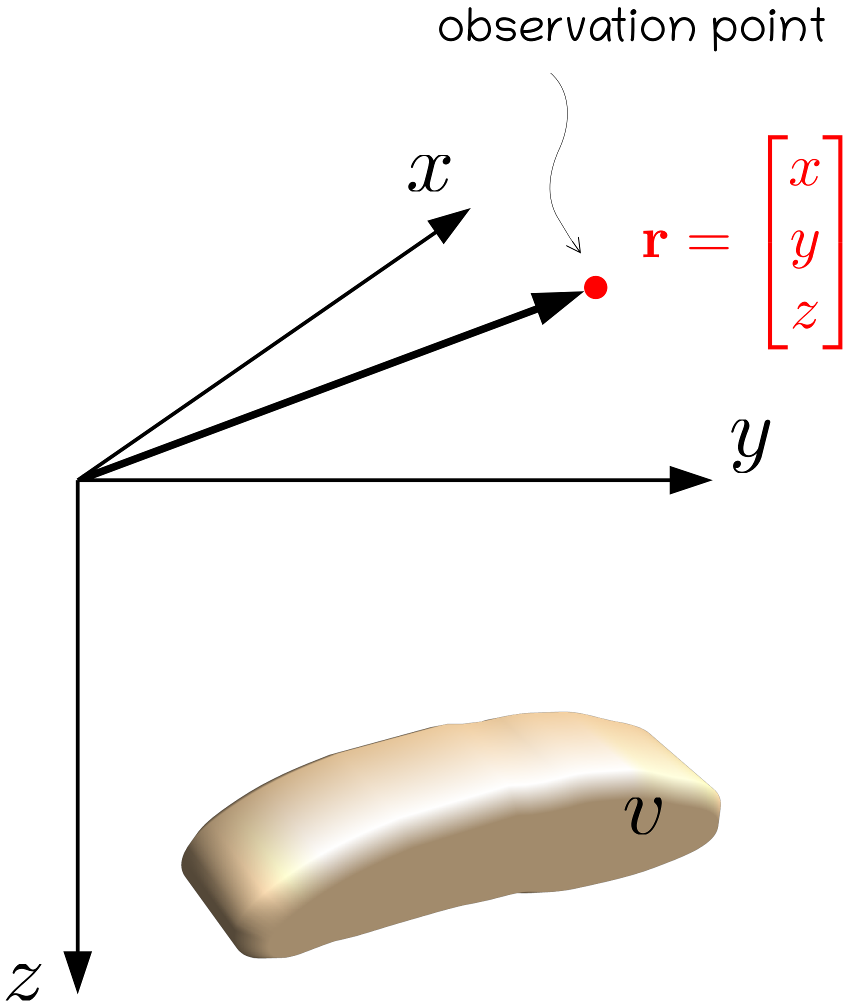


Topocentric Cartesian system

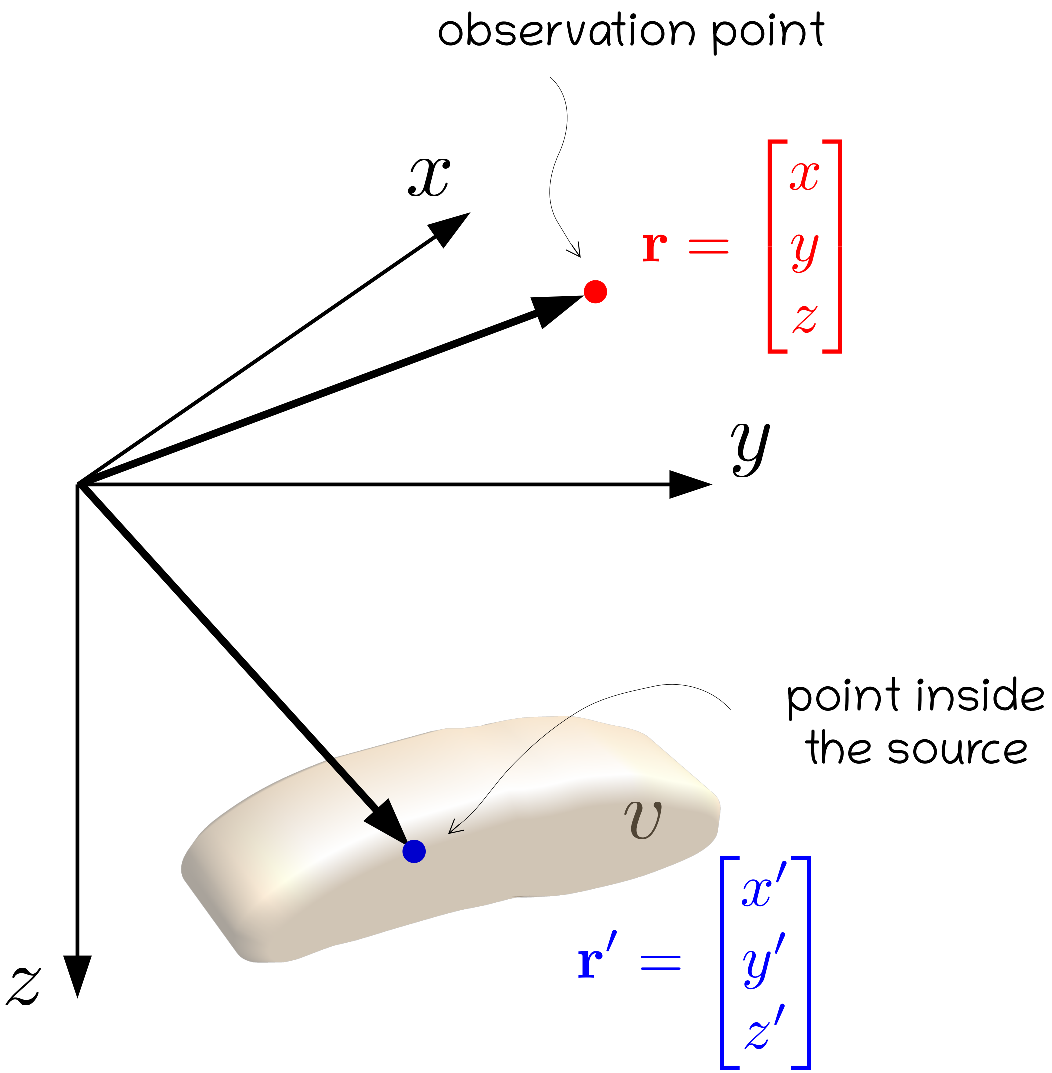




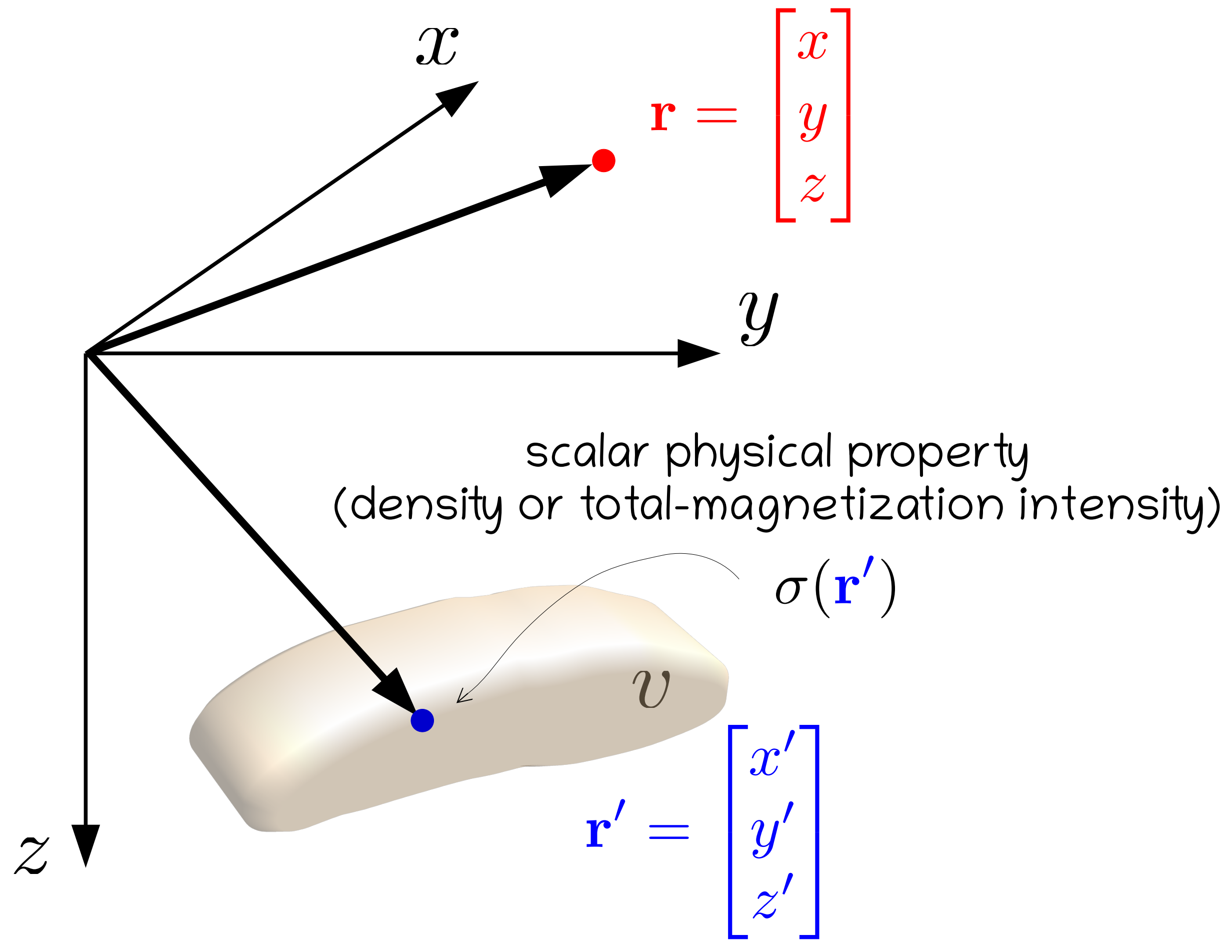
x
y
z
v
Source with
volume
v



observation point
$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
x
y
z
v

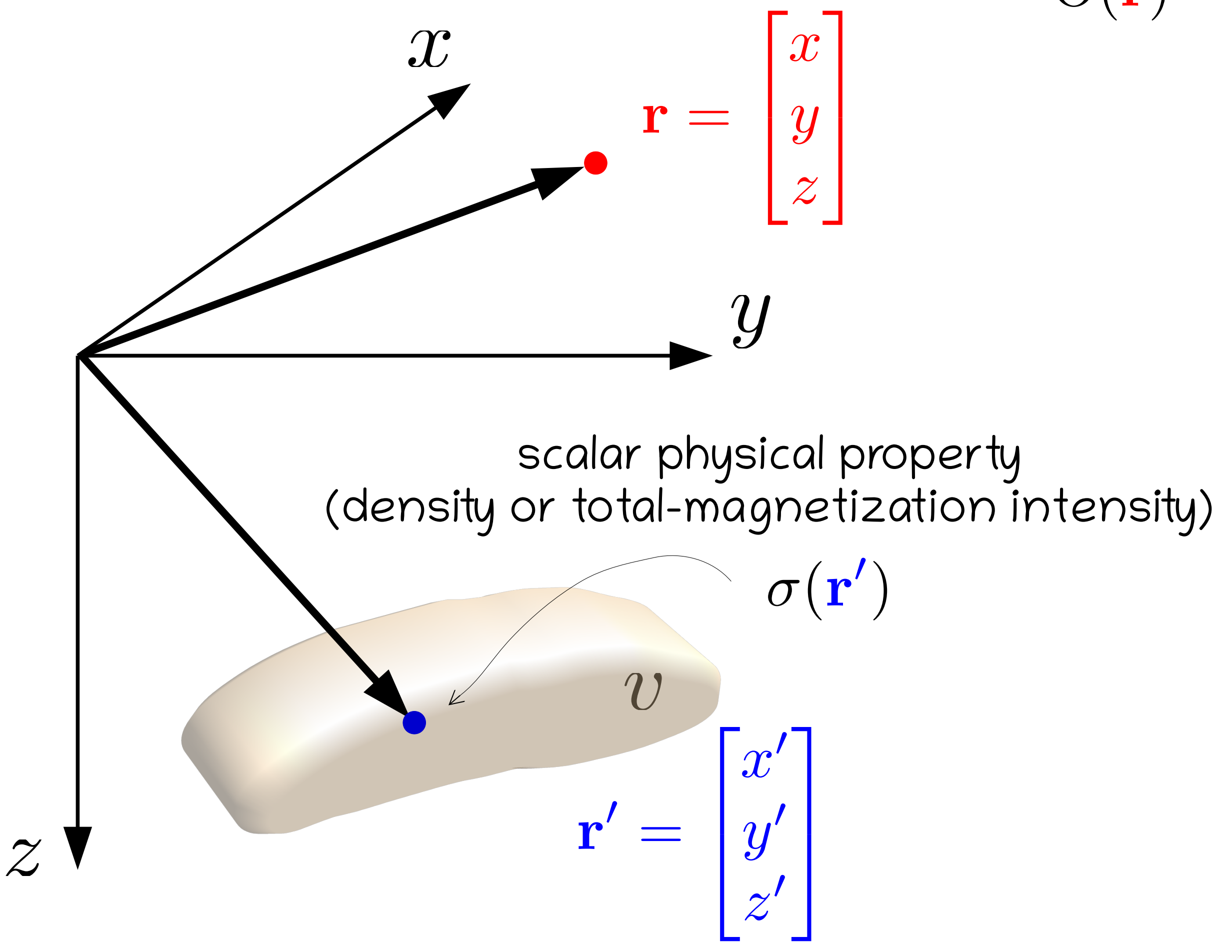


observation point
$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
x
y
z
point inside the source
v
$\mathbf{r}' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$



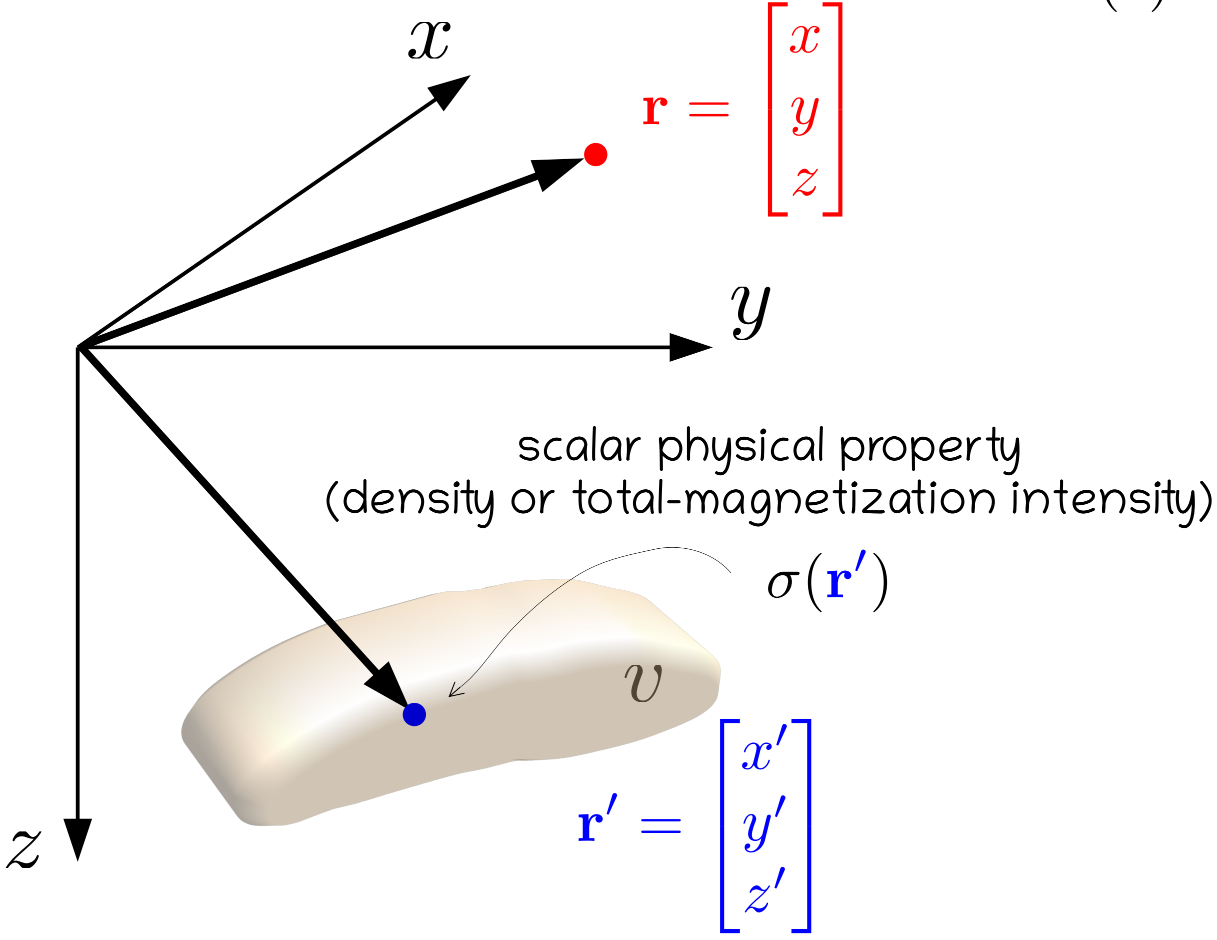
x
$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
y
scalar physical property
(density or total-magnetization intensity)
$\sigma(\mathbf{r}')$
v
$\mathbf{r}' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$
z

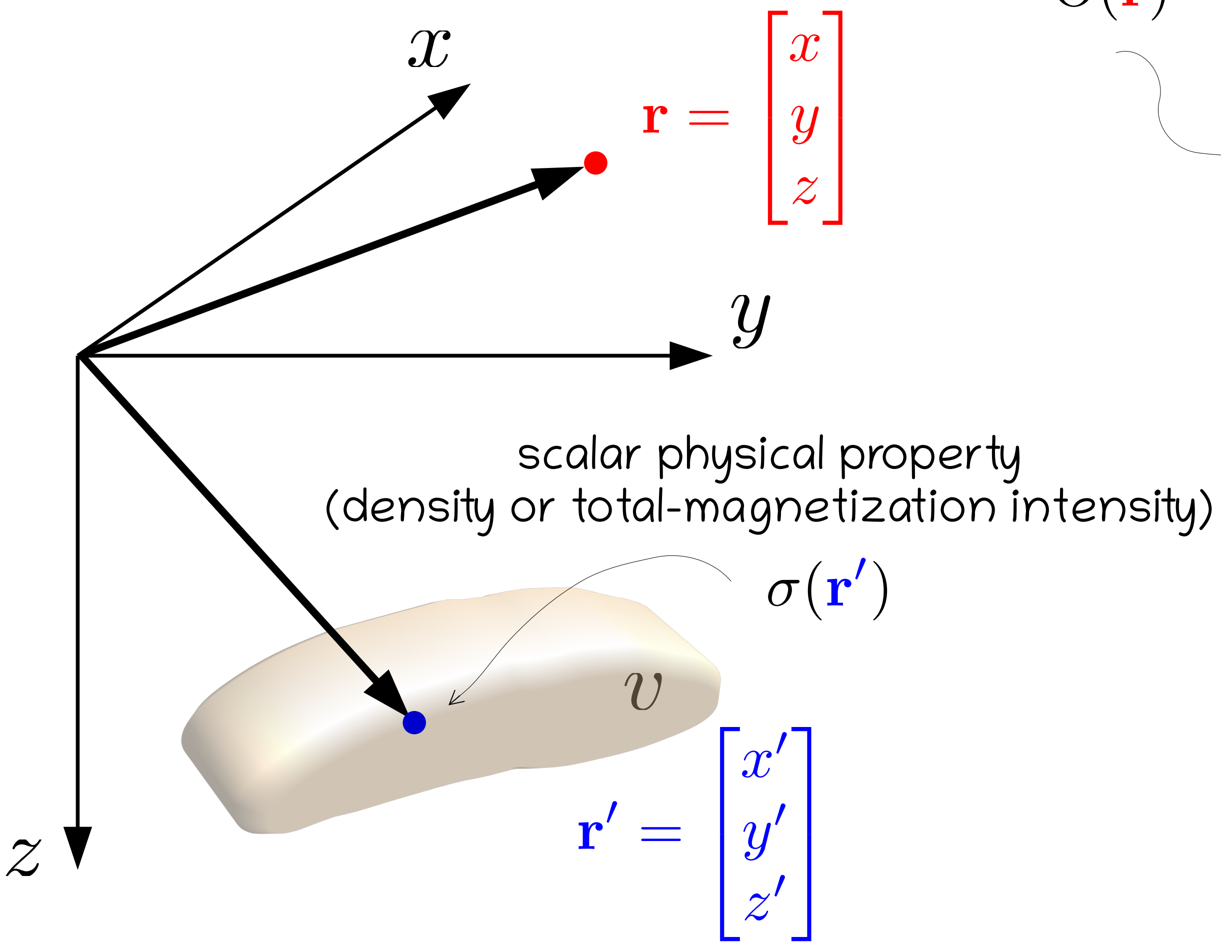
$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$dx' \, dy' \, dz'$



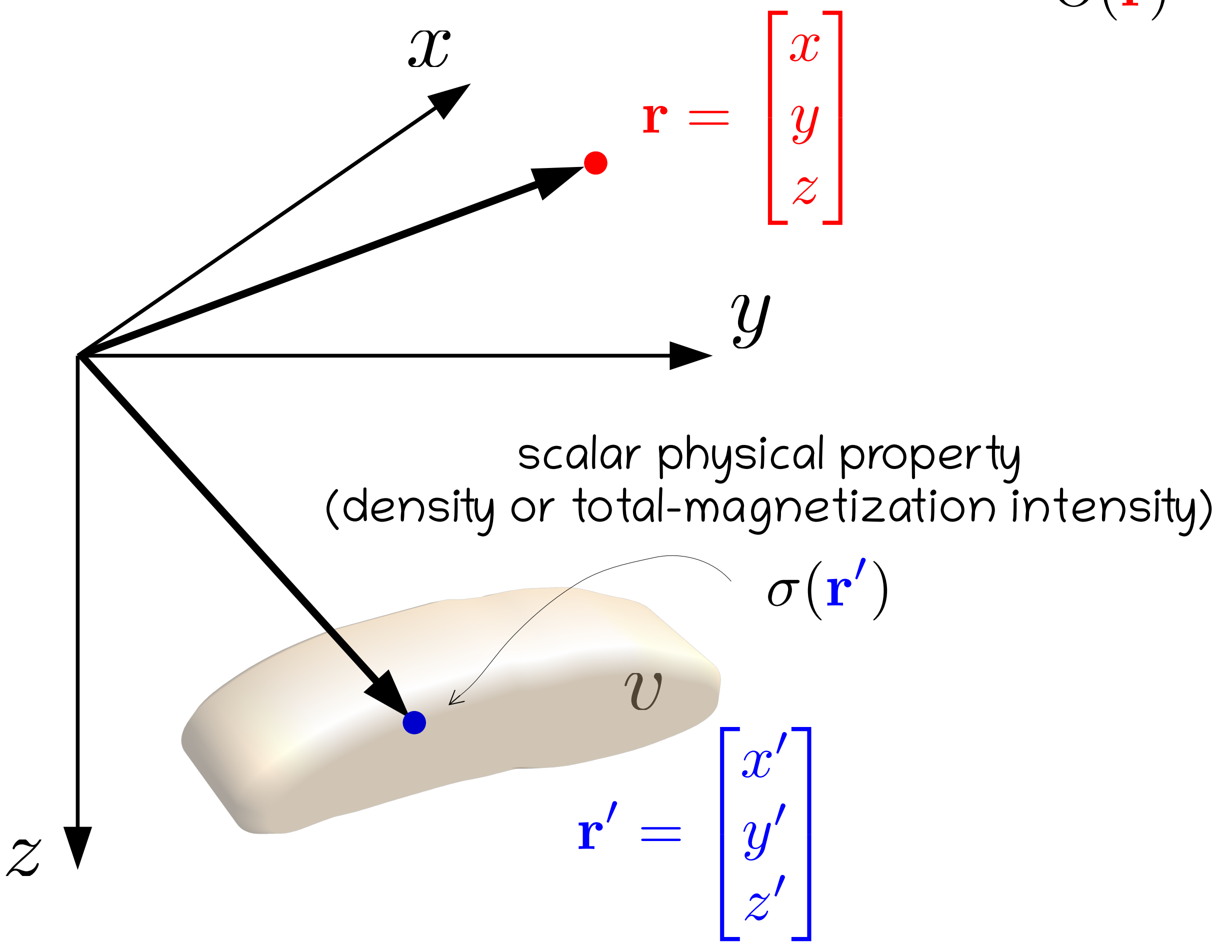


$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

This function approaches 0 as $\| \mathbf{r} - \mathbf{r}' \|$ becames infinite, for fixed $\mathbf{r}'$, and it is harmonic at points $\mathbf{r}$ outside v, i.e., it satisfies:

$$\partial_{xx}\Theta(\mathbf{r}) + \partial_{yy}\Theta(\mathbf{r}) + \partial_{zz}\Theta(\mathbf{r}) = 0$$

for $\mathbf{r} \notin v$

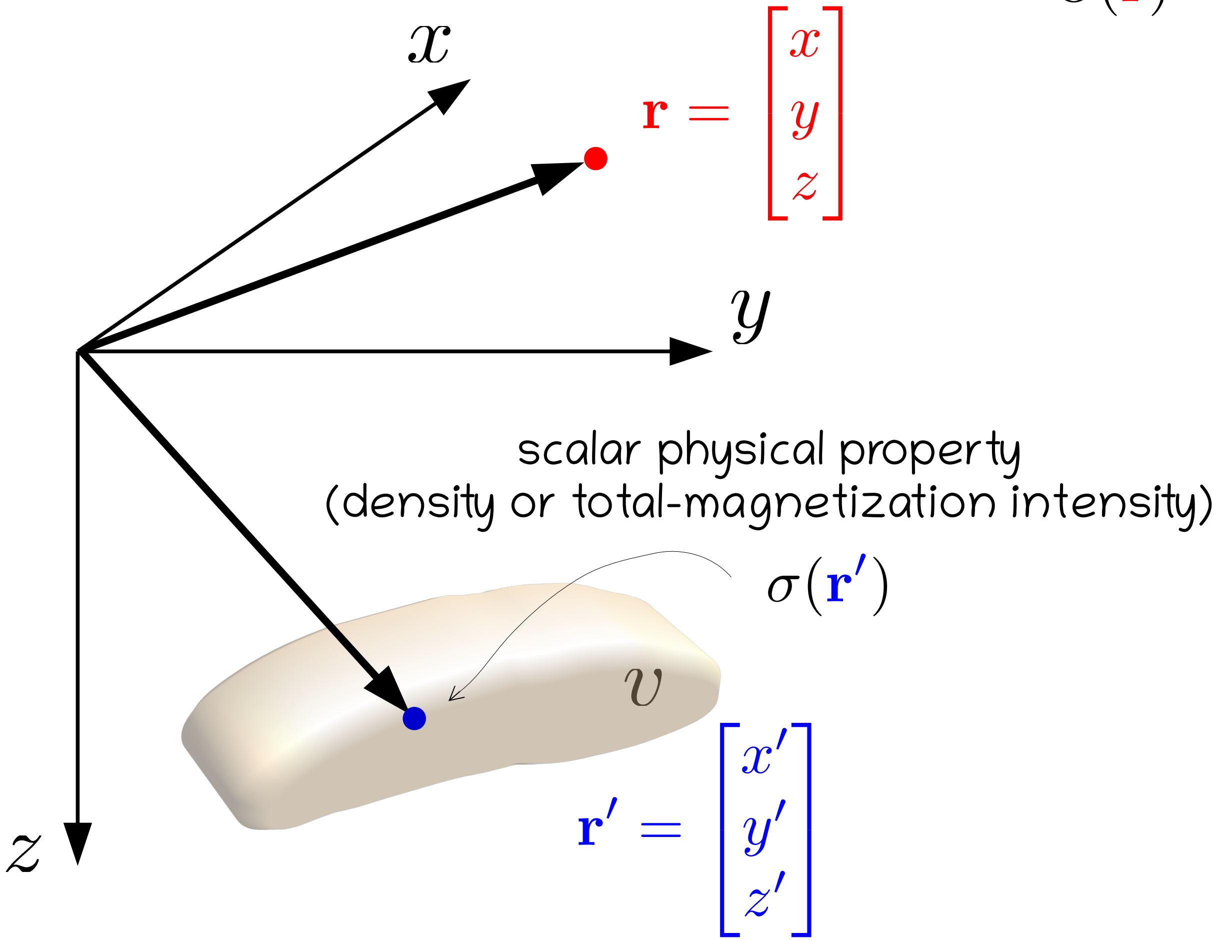


$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

Some conditions are imposed to the volume v and the scalar function $\sigma(\mathbf{r}')$ in order to guarantee the existence of the first and second derivatives of $\Theta(\mathbf{r})$. For details, see Kellogg (1967, Theorem III, p. 156)

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}')\, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|}\, dv'$$

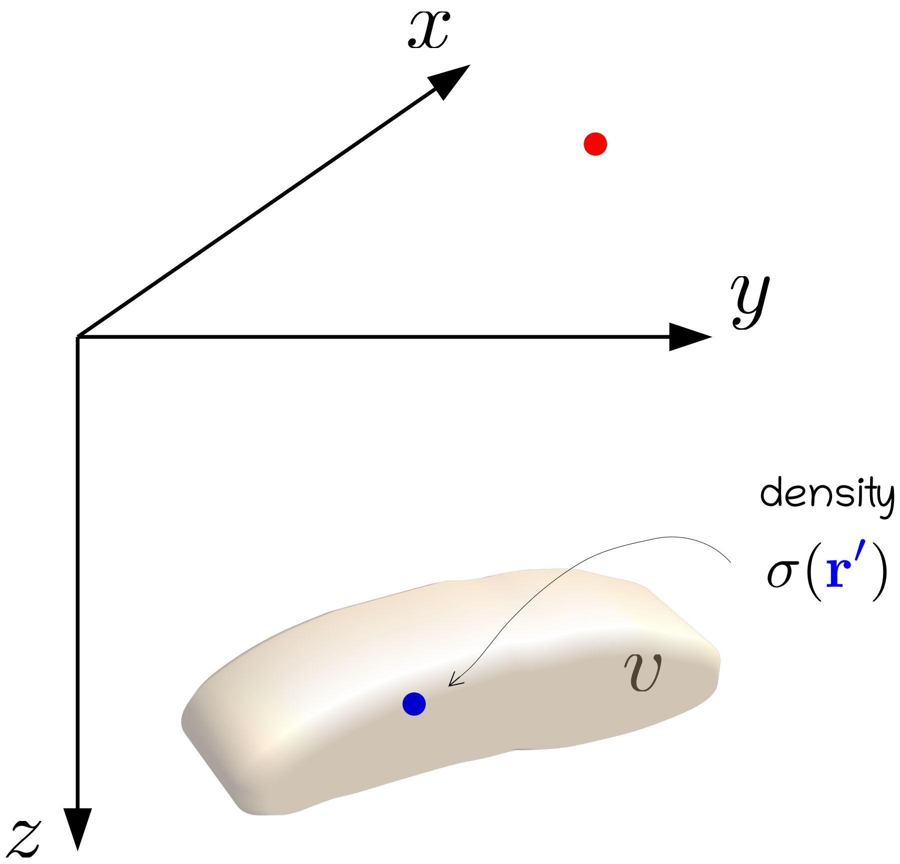
constant defining
gravitational or
magnetic field



Gravitational field

density

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



Gravitational field

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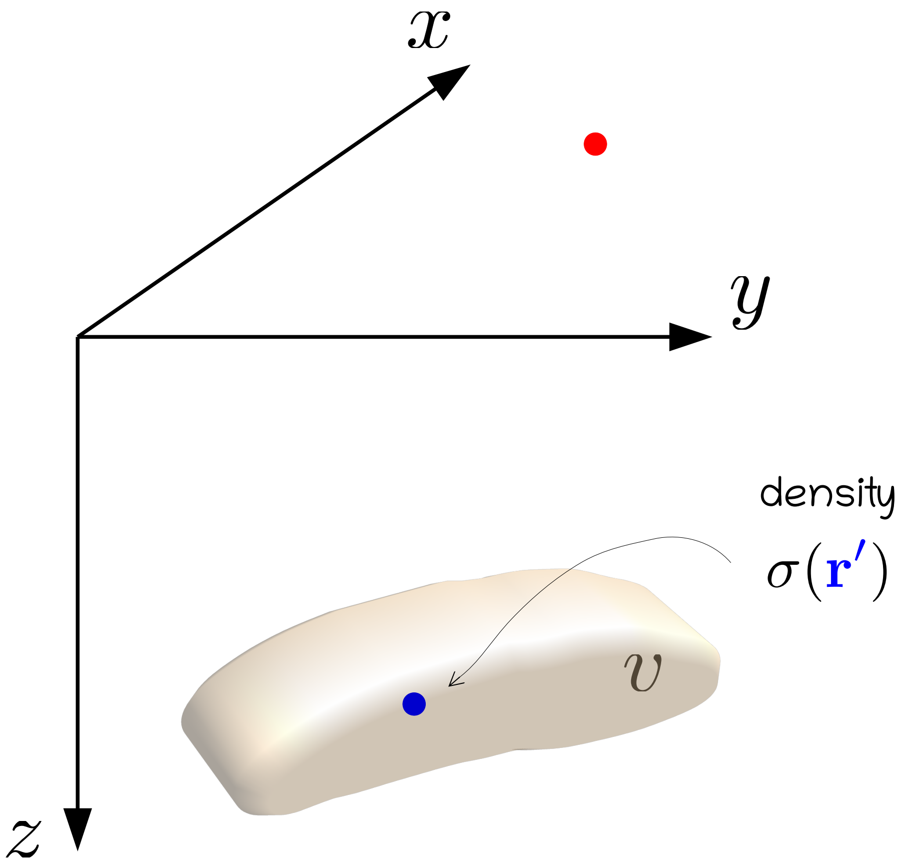
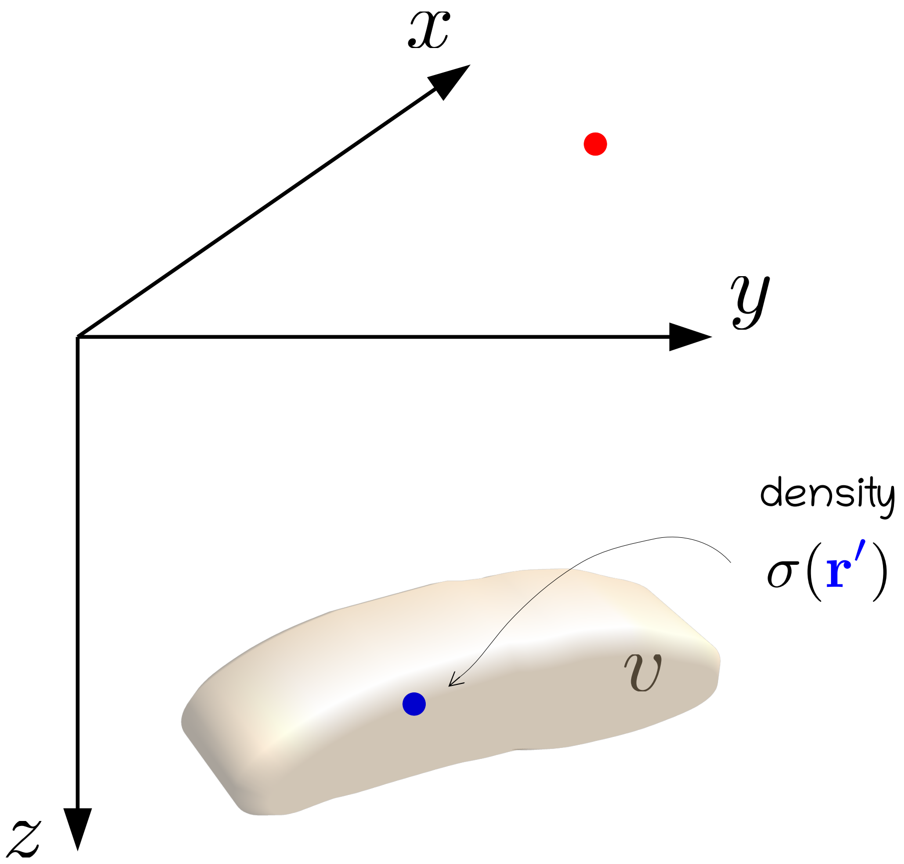


$\Theta(\mathbf{r})$ gravitational potential

Gravitational field

density

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



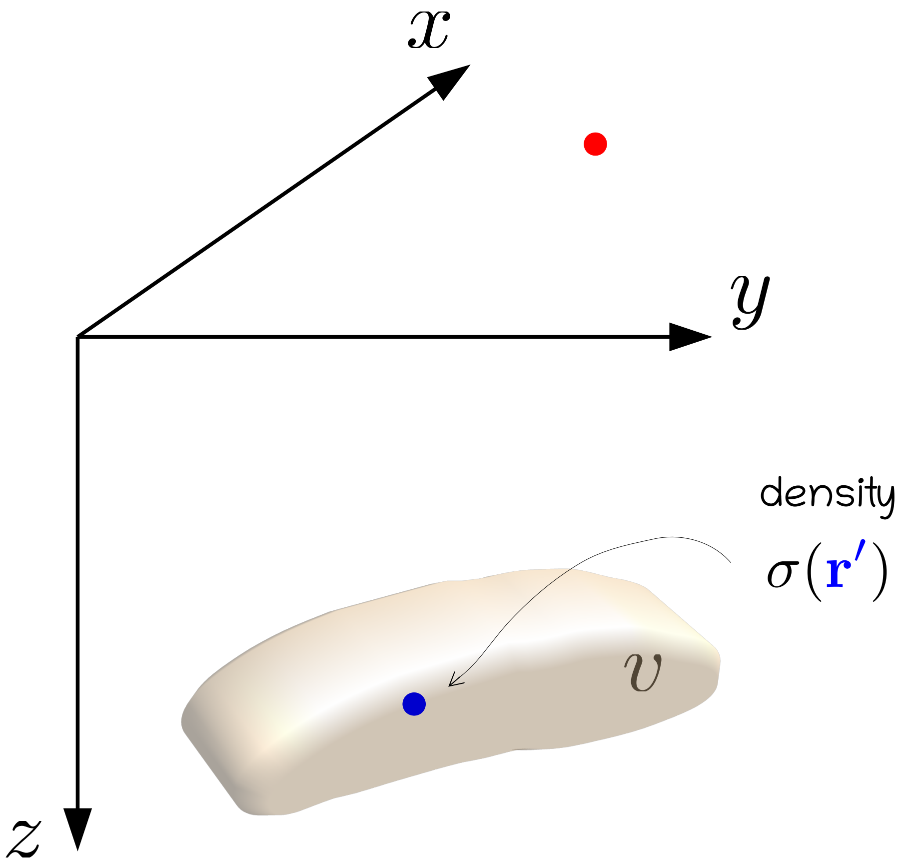
$\Theta(\mathbf{r})$ gravitational potential

$\partial_z \Theta(\mathbf{r})$ gravity disturbance

Gravitational field

density

$$\Theta(\mathbf{r}) = \kappa \iiint_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



$\Theta(\mathbf{r})$ gravitational potential

$\partial_z \Theta(\mathbf{r})$ gravity disturbance

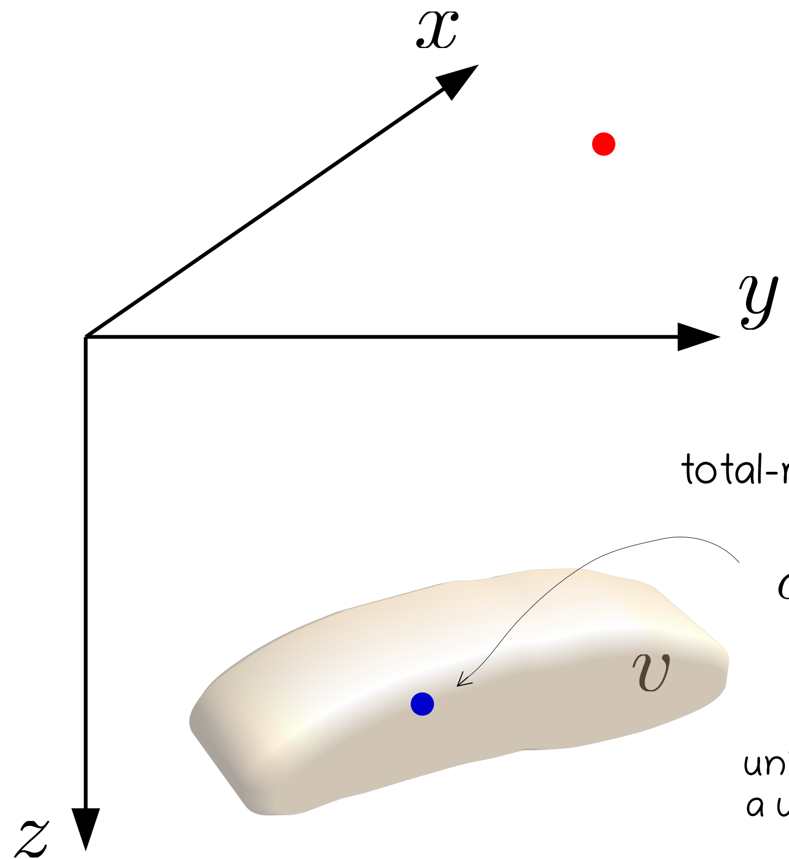
$\partial_{\alpha\beta} \Theta(\mathbf{r})$ gravity gradient tensor

x, y, z

Magnetic field

total-magnetization
intensity

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



total-magnetization
vector

$$\sigma(\mathbf{r}') \, \hat{\mathbf{h}}$$

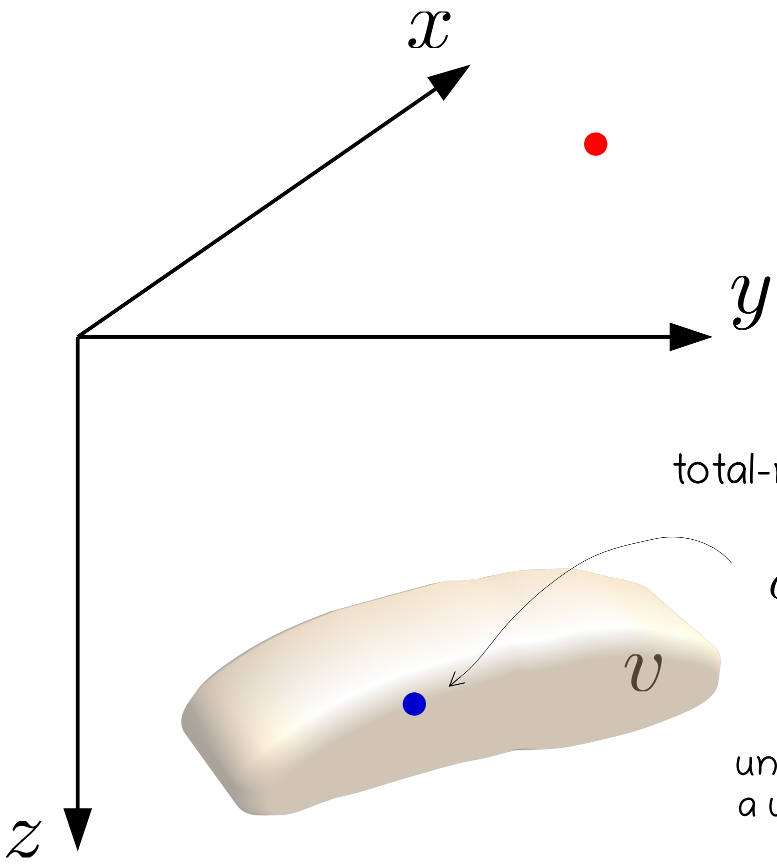
unit vector defining
a uniform direction

Magnetic field

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$-\partial_h \Theta(\mathbf{r})$ mag. scalar potential



total-magnetization vector

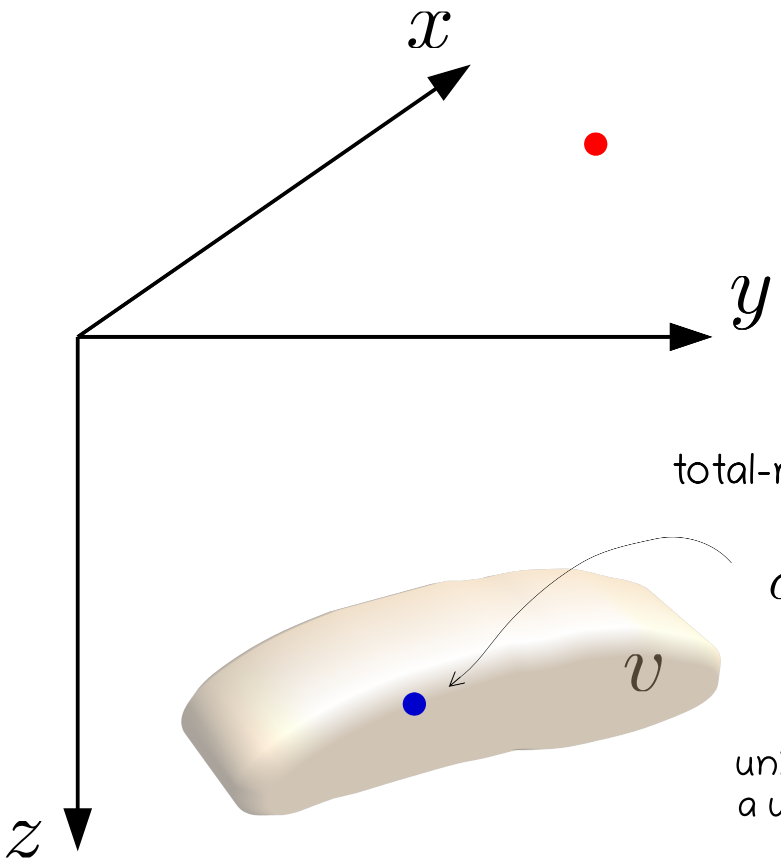
$\sigma(\mathbf{r}') \, \hat{\mathbf{h}}$

unit vector defining a uniform direction

Magnetic field

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$



$-\partial_h \Theta(\mathbf{r})$ mag. scalar potential

$$\partial_h \Theta(\mathbf{r}) = \nabla \Theta(\mathbf{r})^\top \hat{\mathbf{h}}$$

total-magnetization vector

$$\sigma(\mathbf{r}')\ \hat{\mathbf{h}}$$

unit vector defining a uniform direction

Magnetic field

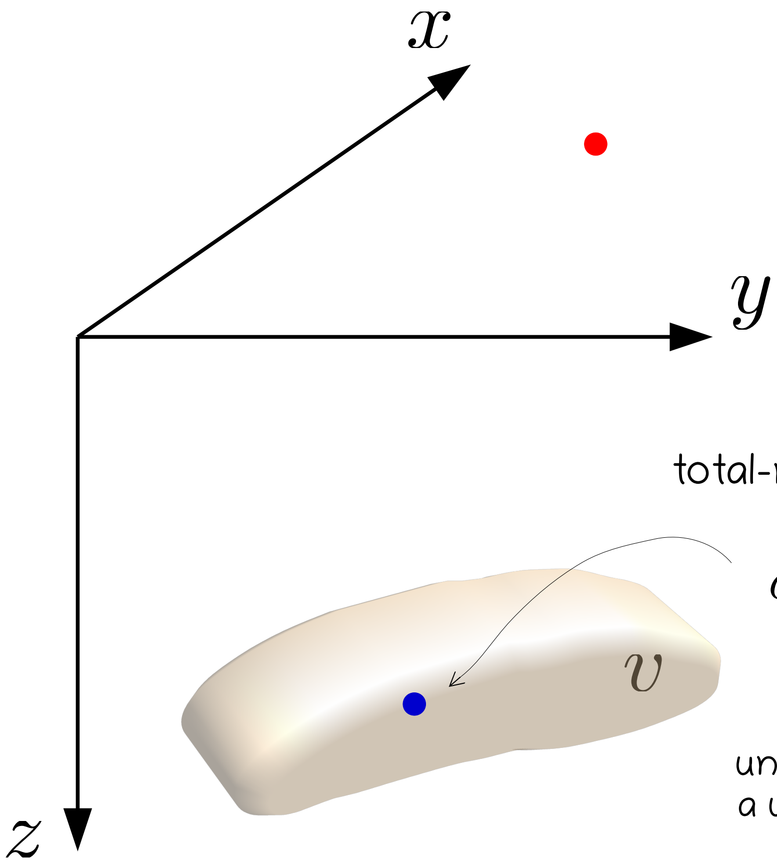
$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

total-magnetization intensity

$-\partial_h \Theta(\mathbf{r})$ mag. scalar potential

$\partial_{\alpha h} \Theta(\mathbf{r})$ mag. induction field

x, y, z



total-magnetization vector

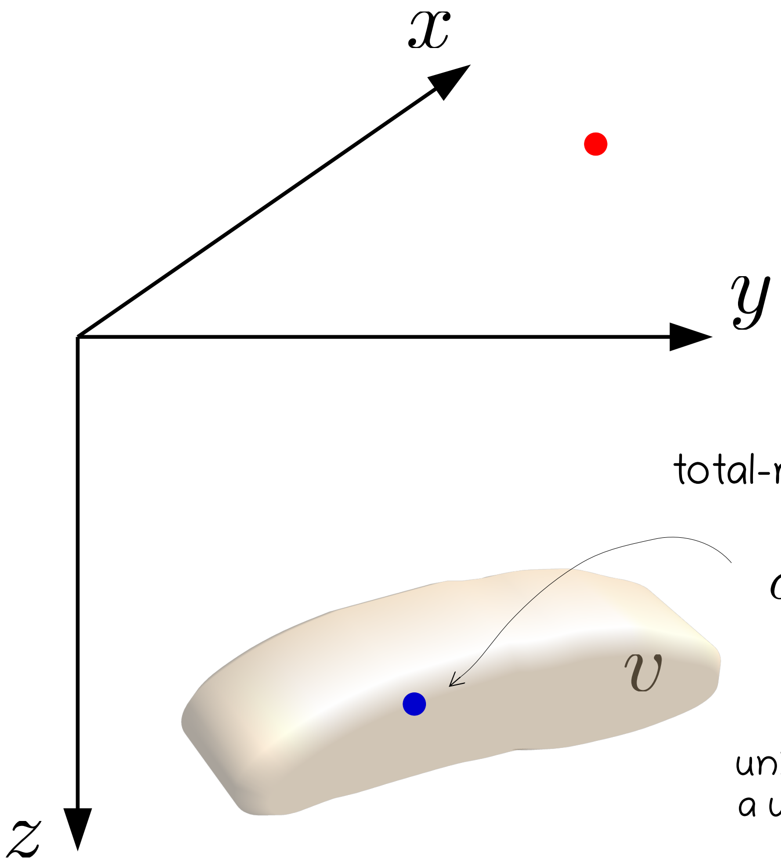
$\sigma(\mathbf{r}') \, \hat{\mathbf{h}}$

unit vector defining a uniform direction

Magnetic field

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



$-\partial_h \Theta(\mathbf{r})$ mag. scalar potential

$\partial_{\alpha h} \Theta(\mathbf{r})$ mag. induction field

x, y, z

$$\partial_{\alpha h} \Theta(\mathbf{r}) = \partial_{\alpha} \nabla \Theta(\mathbf{r})^{\top} \hat{\mathbf{h}}$$

total-magnetization vector

$\sigma(\mathbf{r}') \, \hat{\mathbf{h}}$

unit vector defining a uniform direction

Magnetic field

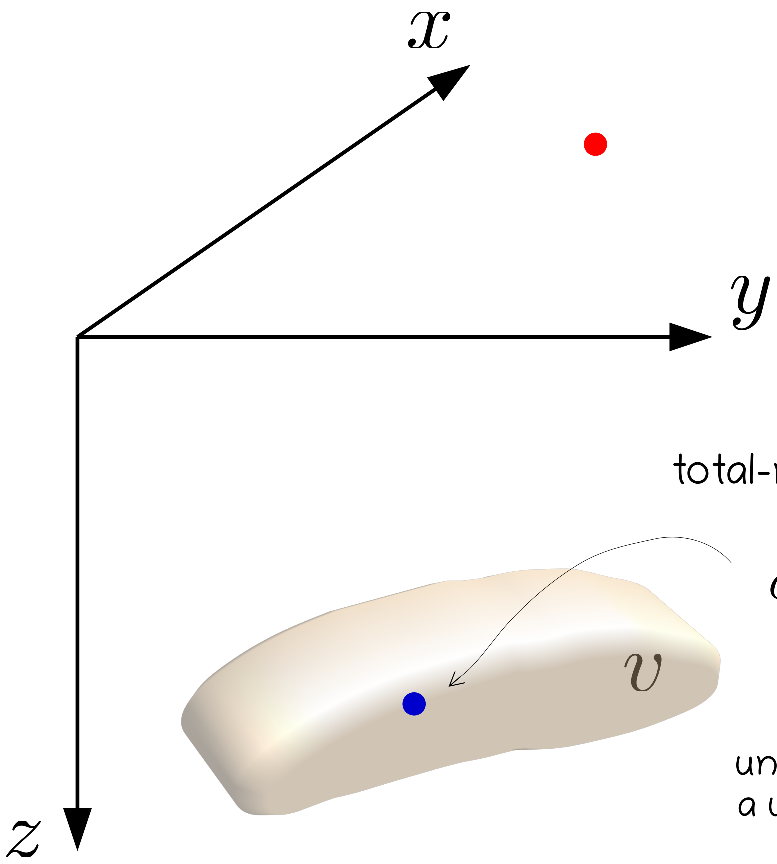
total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$-\partial_h \Theta(\mathbf{r})$ mag. scalar potential

$\partial_{\alpha h} \Theta(\mathbf{r})$ mag. induction field

$\partial_{th} \Theta(\mathbf{r})$ approx. total-field anomaly



total-magnetization vector

$\sigma(\mathbf{r}') \, \hat{\mathbf{h}}$

unit vector defining a uniform direction

Magnetic field

total-magnetization intensity

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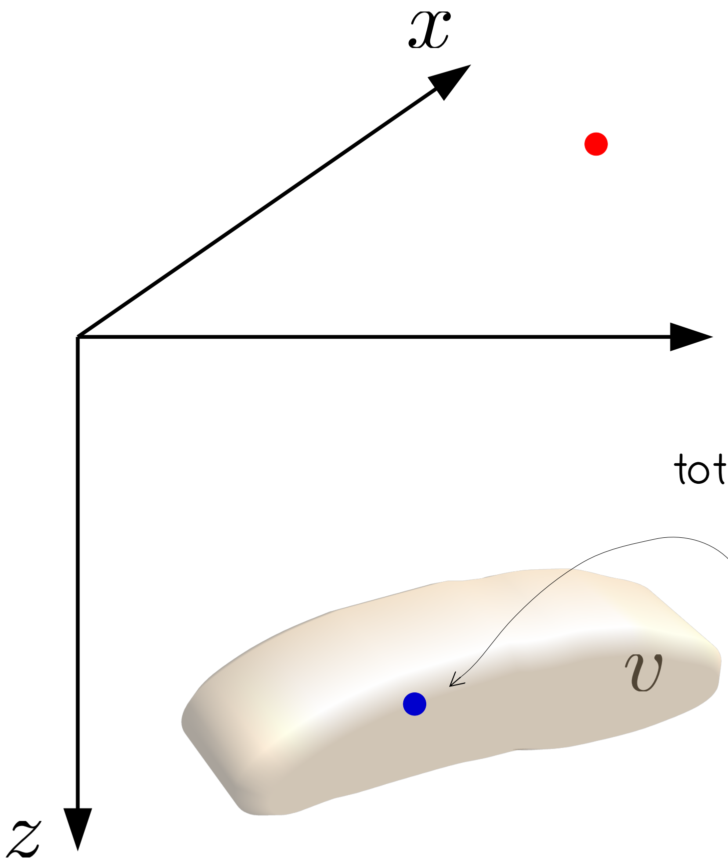


total-magnetization vector

$\sigma(\mathbf{r}') \; \hat{\mathbf{h}}$

unit vector defining a uniform direction

$-\partial_h \Theta(\mathbf{r})$ mag. scalar potential

$\partial_{\alpha h} \Theta(\mathbf{r})$ mag. induction field

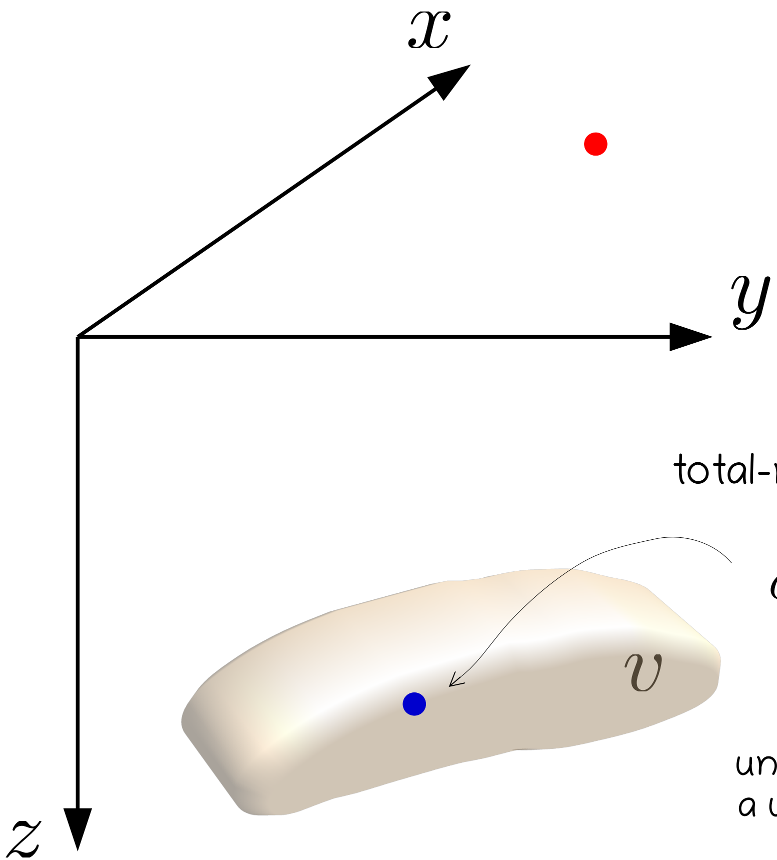
$\partial_{th} \Theta(\mathbf{r})$ approx. total-field anomaly

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Theta}(\mathbf{r}) \, \hat{\mathbf{h}}$$

Magnetic field

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



total-magnetization vector

$\sigma(\mathbf{r}') \, \hat{\mathbf{h}}$

unit vector defining a uniform direction

$-\partial_h \Theta(\mathbf{r})$ mag. scalar potential

$\partial_{\alpha h} \Theta(\mathbf{r})$ mag. induction field

$\partial_{th} \Theta(\mathbf{r})$ approx. total-field anomaly

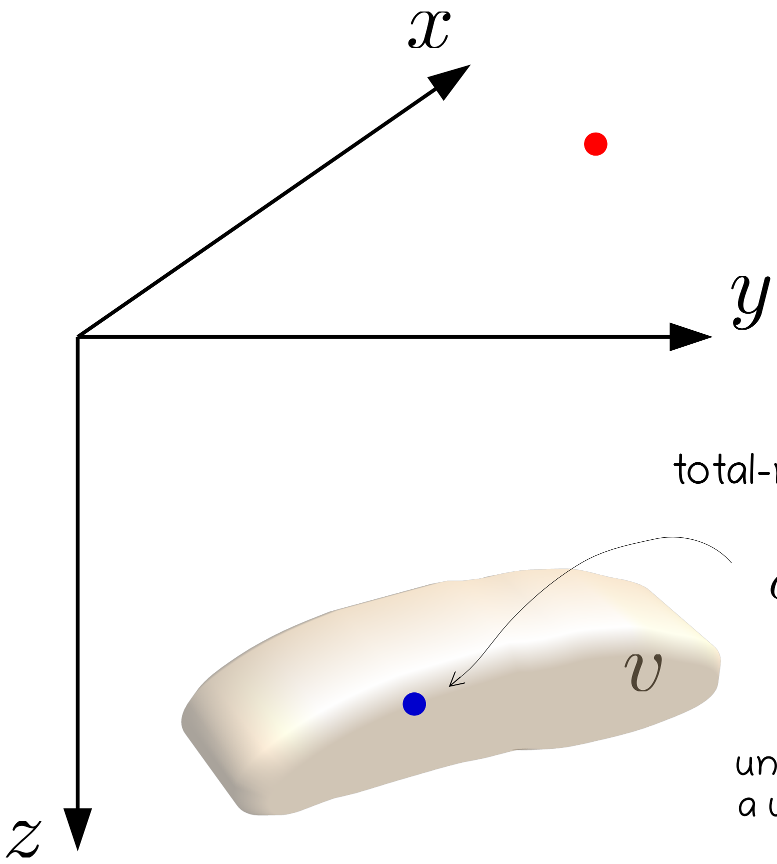
$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Theta}(\mathbf{r}) \, \hat{\mathbf{h}}$$

unit vector defining a constant direction for the main geomagnetic field

Magnetic field

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$



total-magnetization vector

$\sigma(\mathbf{r}')\,\hat{\mathbf{h}}$

unit vector defining a uniform direction

$-\partial_h \Theta(\mathbf{r})$ mag. scalar potential

$\partial_{\alpha h} \Theta(\mathbf{r})$ mag. induction field

$\partial_{th} \Theta(\mathbf{r})$ approx. total-field anomaly

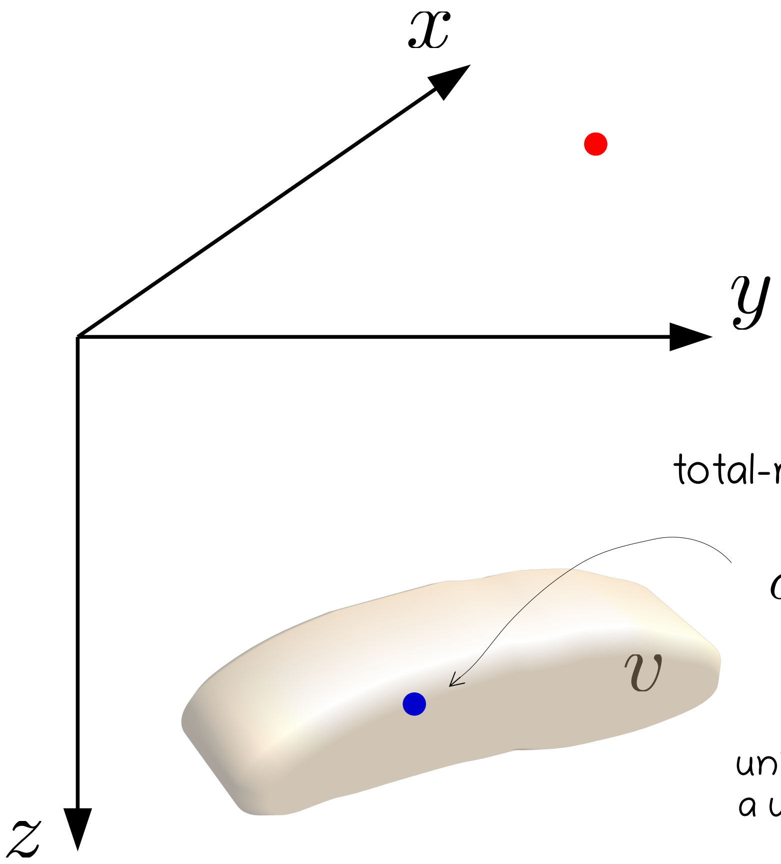
$$\partial_{th}\Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r})\,\hat{\mathbf{h}}$$

$$\mathbf{H}_\Theta(\mathbf{r}) = \begin{bmatrix} \partial_{xx}\Theta(\mathbf{r}) & \partial_{xy}\Theta(\mathbf{r}) & \partial_{xz}\Theta(\mathbf{r}) \\ \partial_{xy}\Theta(\mathbf{r}) & \partial_{yy}\Theta(\mathbf{r}) & \partial_{yz}\Theta(\mathbf{r}) \\ \partial_{xz}\Theta(\mathbf{r}) & \partial_{yz}\Theta(\mathbf{r}) & \partial_{zz}\Theta(\mathbf{r}) \end{bmatrix}$$

Magnetic field

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



total-magnetization vector

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unit vector defining a uniform direction

$-\partial_h \Theta(\mathbf{r})$ mag. scalar potential

$\partial_{\alpha h} \Theta(\mathbf{r})$ mag. induction field

$\partial_{th} \Theta(\mathbf{r})$ approx. total-field anomaly

$\partial_{zz} \Theta(\mathbf{r})$ RTP anomaly

reduced-to-the-pole

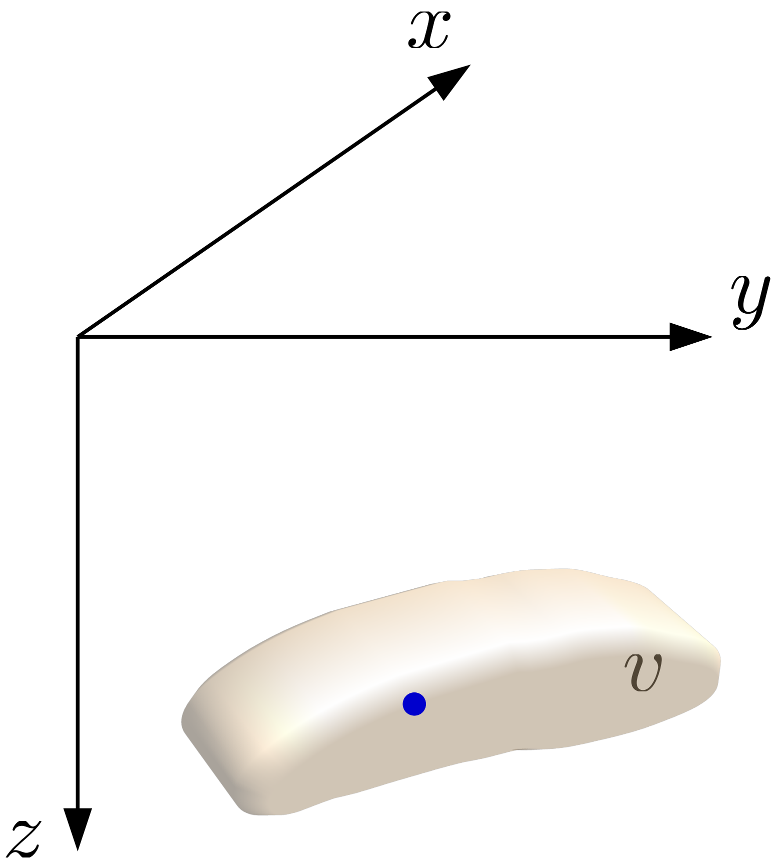
Summary

- Motivation
- **Potential-field data**
- The Equivalent-Layer (EqL) Technique
- Theoretical aspects
- Some open questions

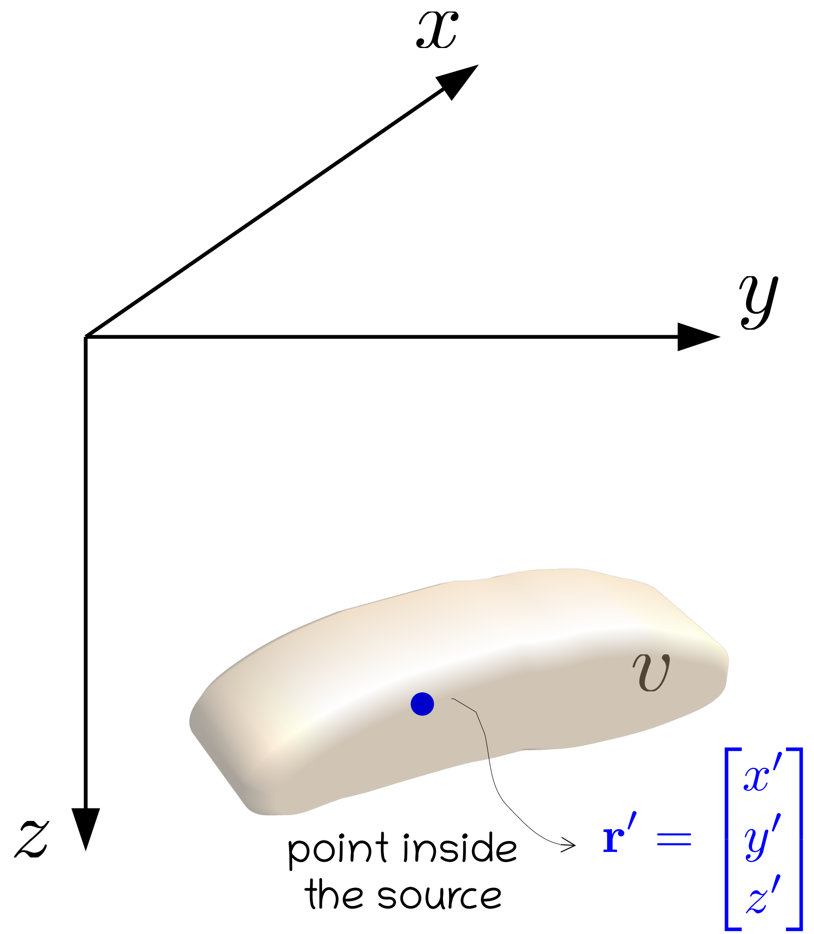
Summary

- Motivation
- Potential-field data
- **The Equivalent-Layer (EqL) Technique**
- Theoretical aspects
- Some open questions

Let us return to our source ...



Let us return to our source ...



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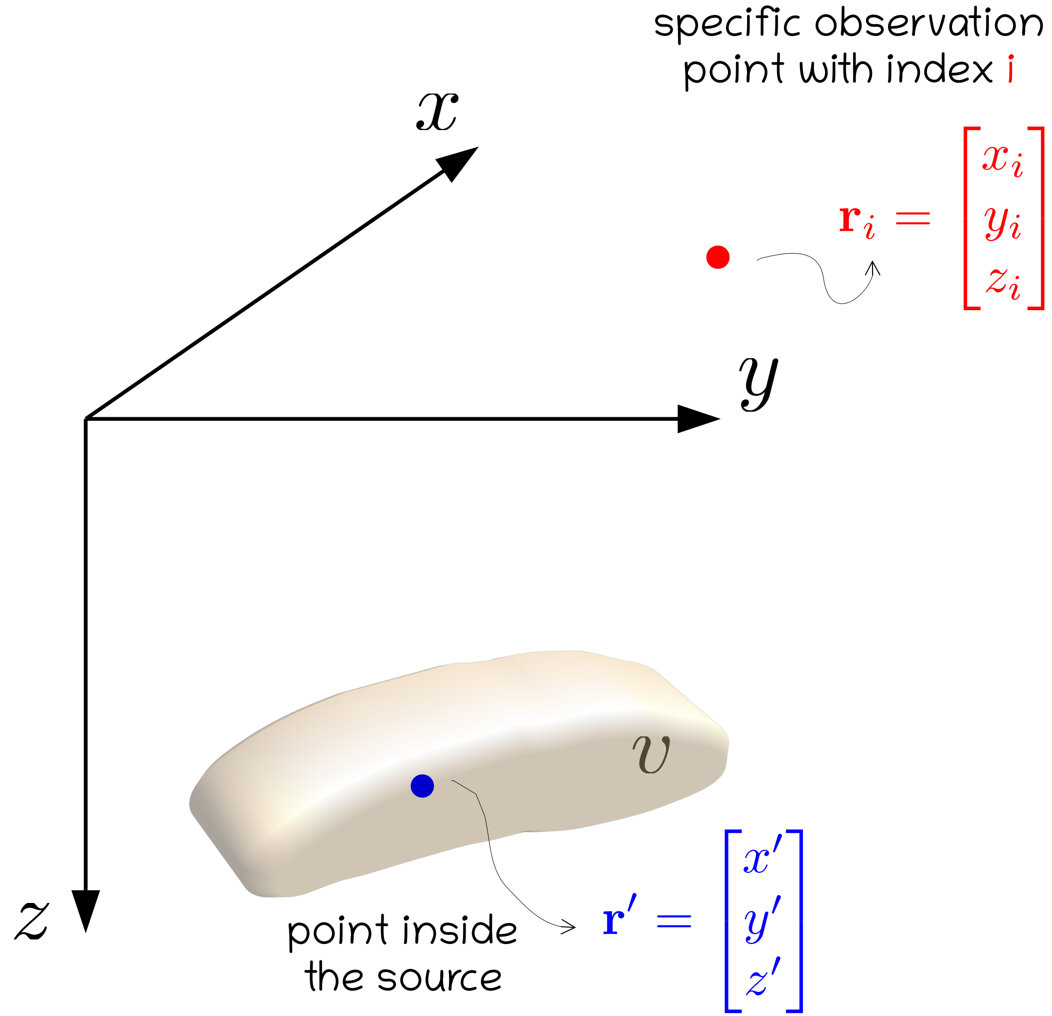
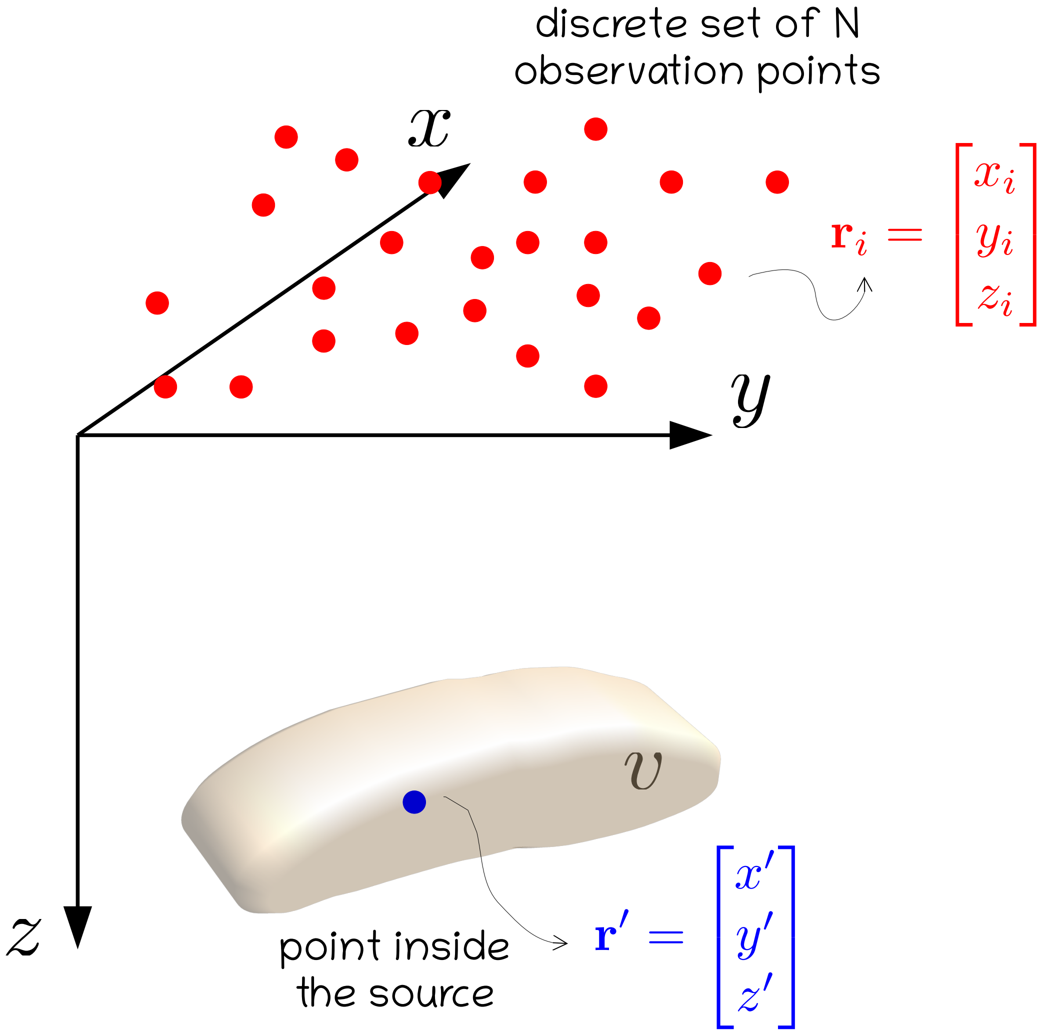
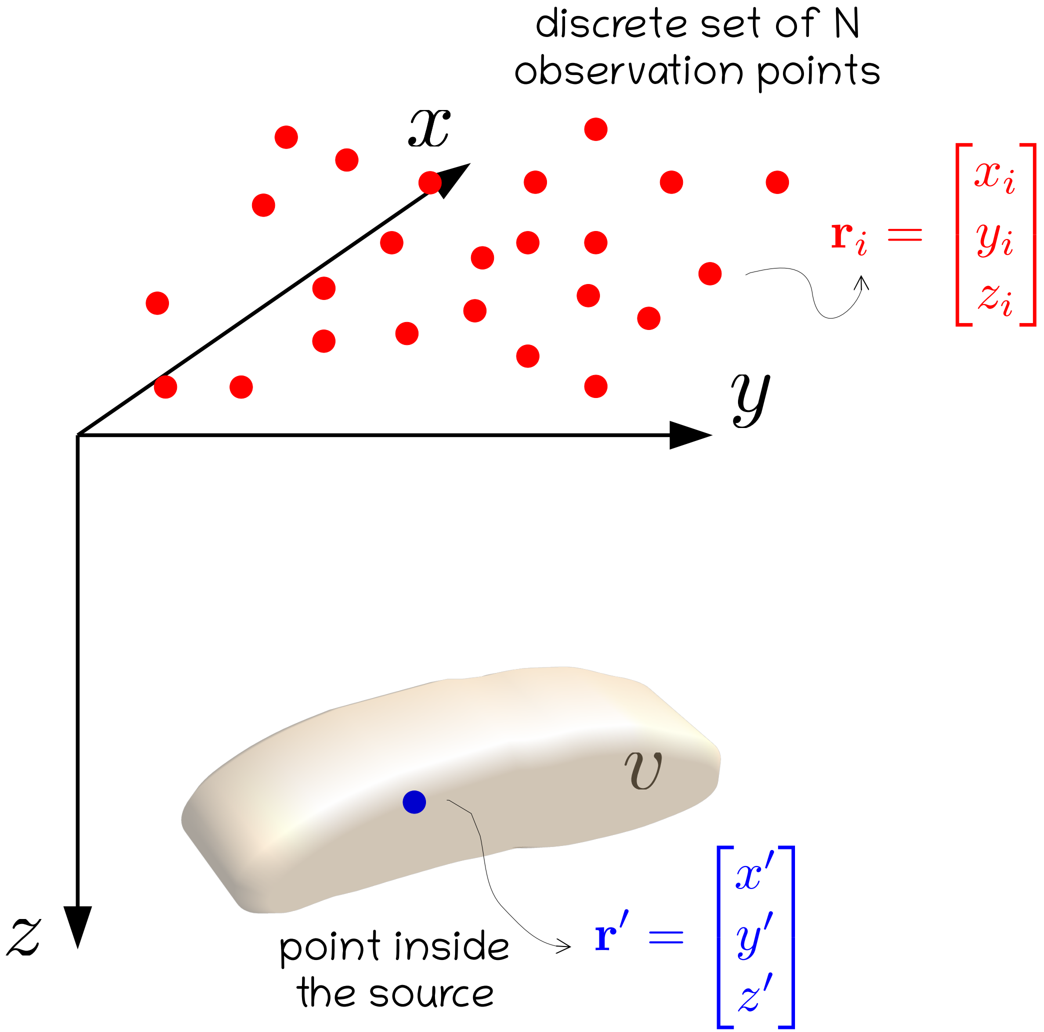




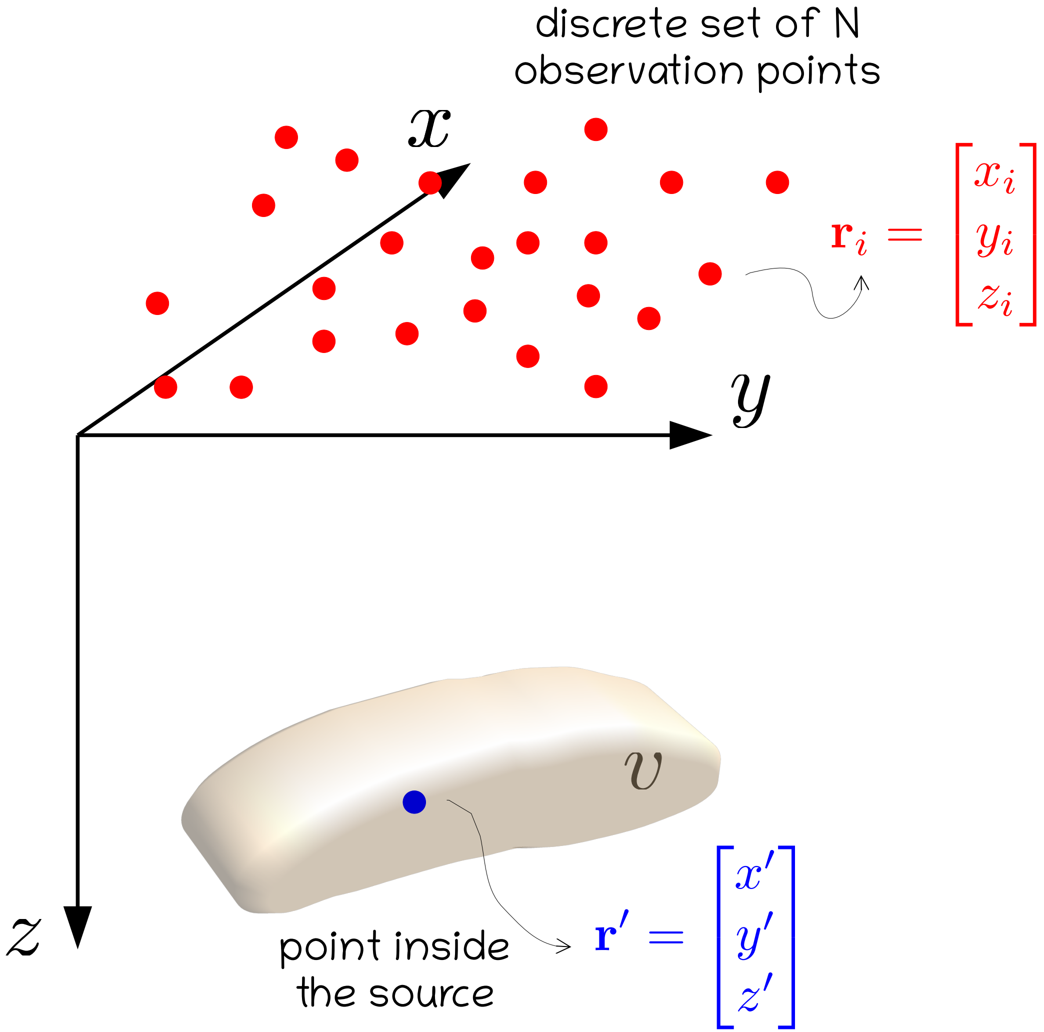
discrete set of N
observation points
x
y
z
$\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$
v
point inside
the source
$\mathbf{r}' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$



$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance

Gravitational field



density

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance

Gravitational field

density

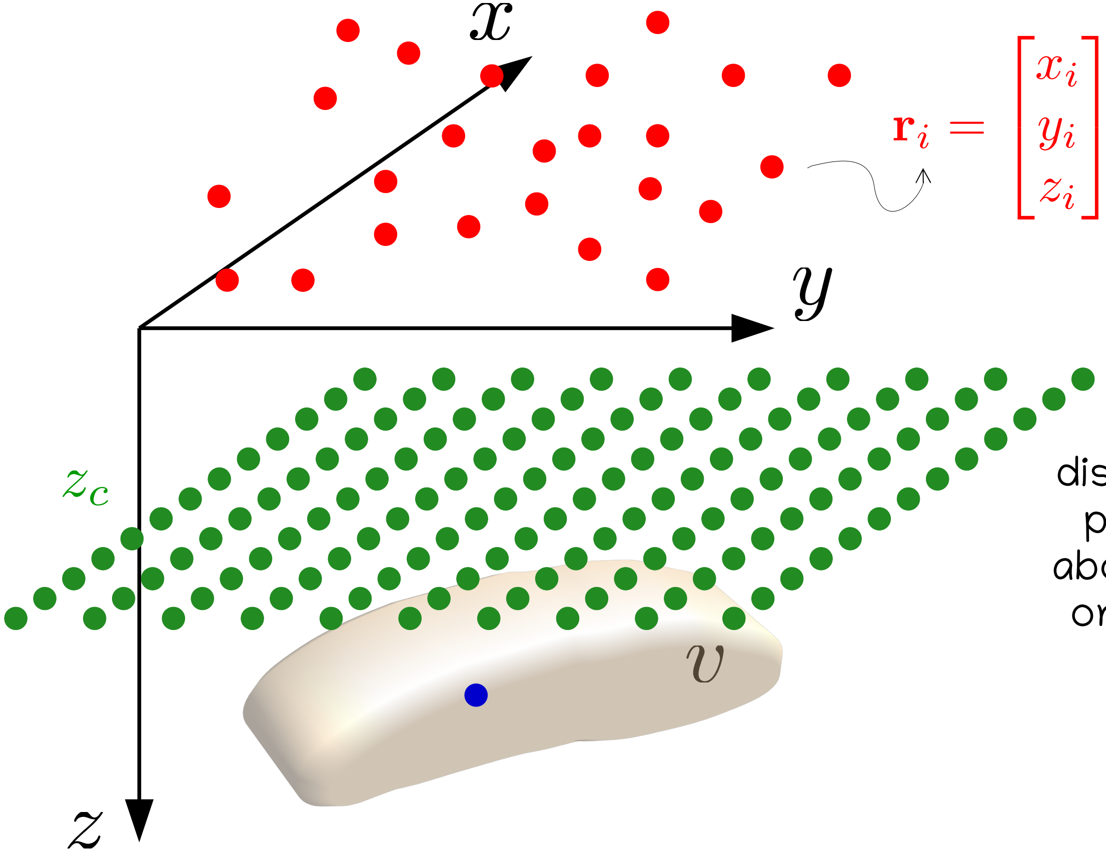
$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

discrete set of N observation points

$$\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance



discrete set of M points located above the source, on the plane z_c

Gravitational field

density

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

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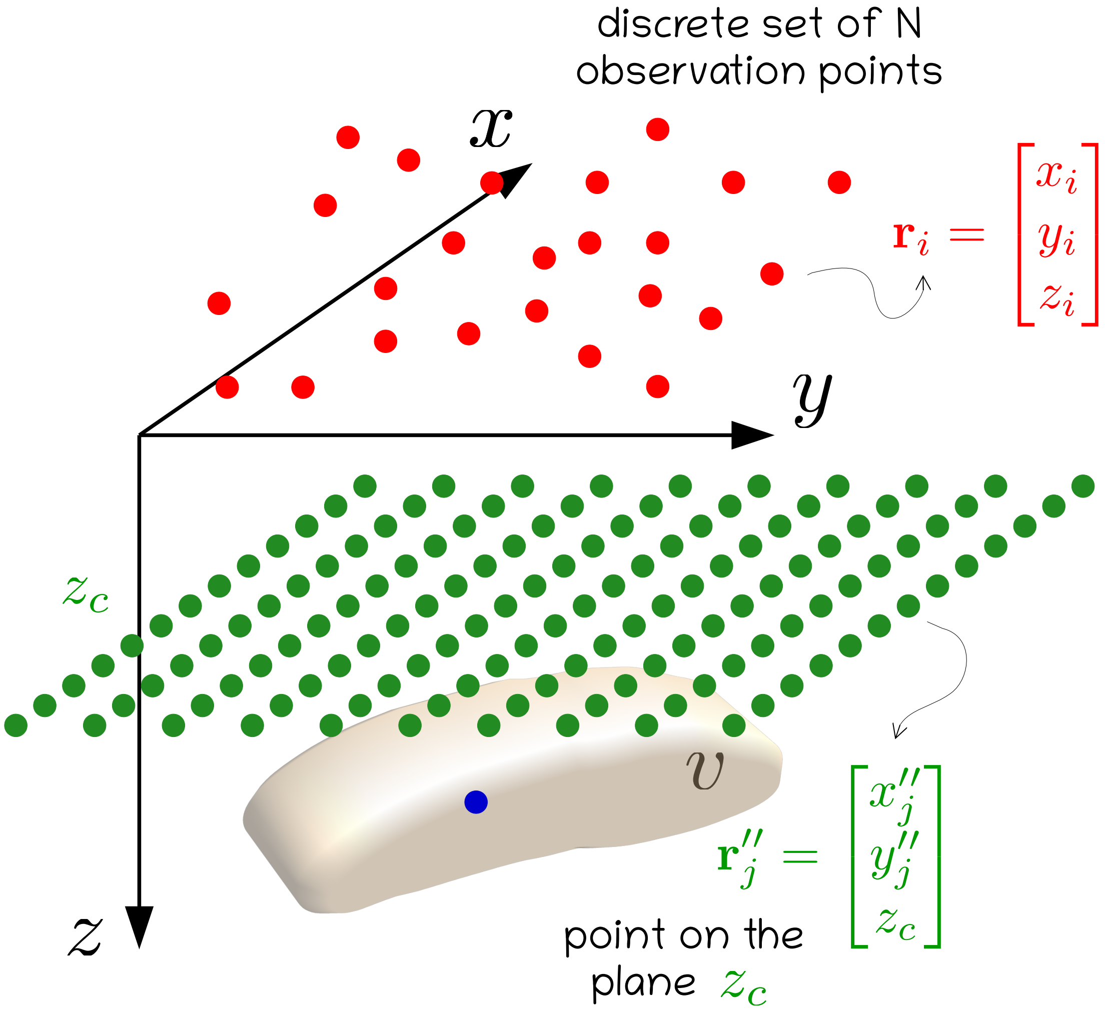
gravity disturbance



Gravitational field

density

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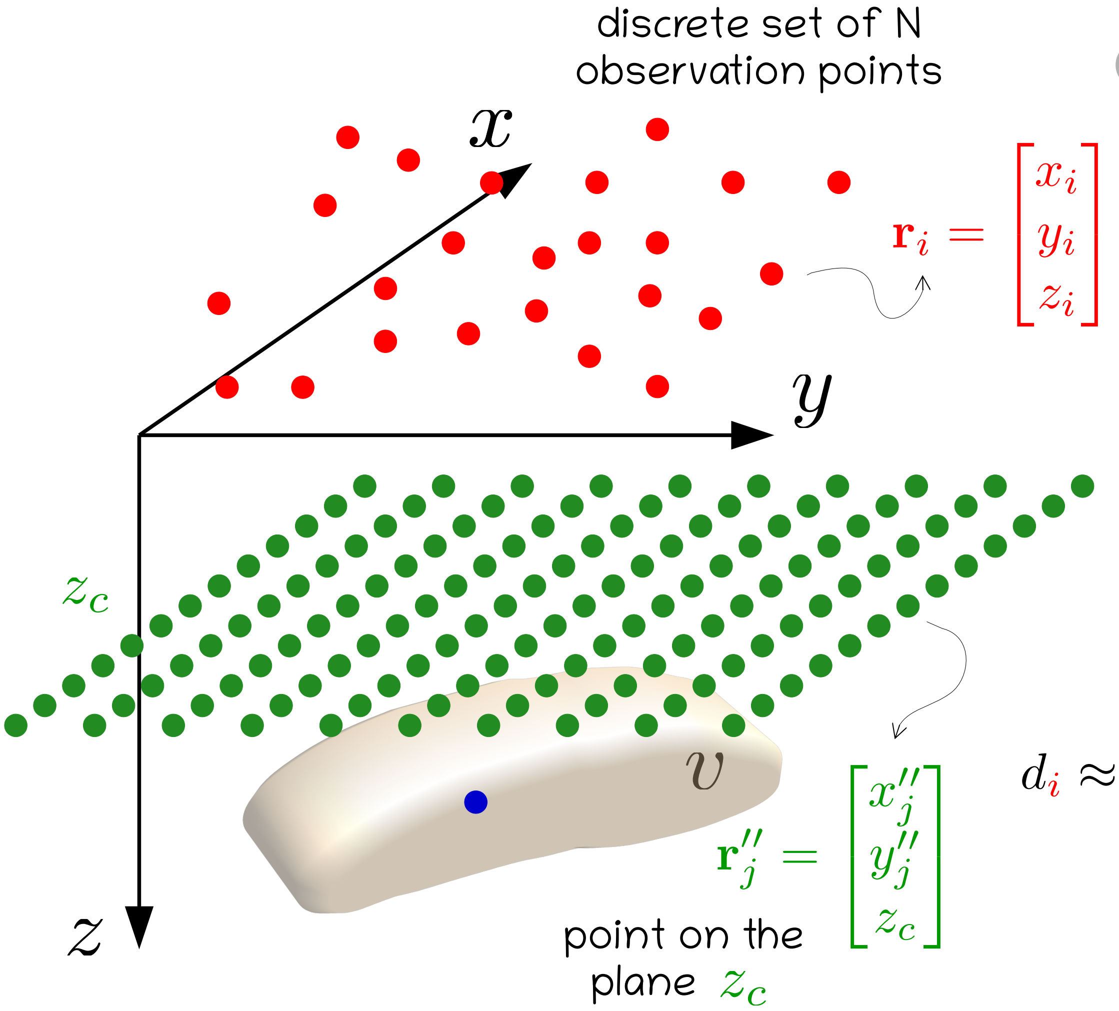
gravity disturbance



$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

Gravitational field

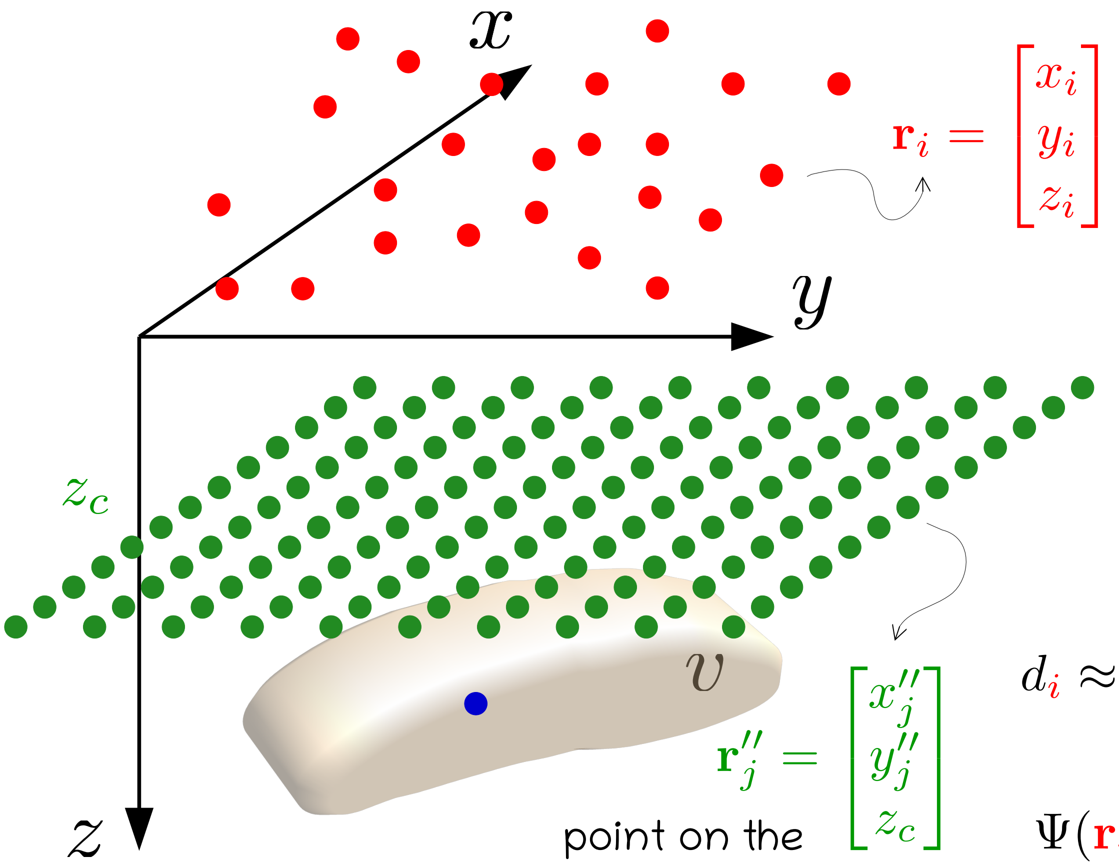
density

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance

discrete set of N observation points



$$\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

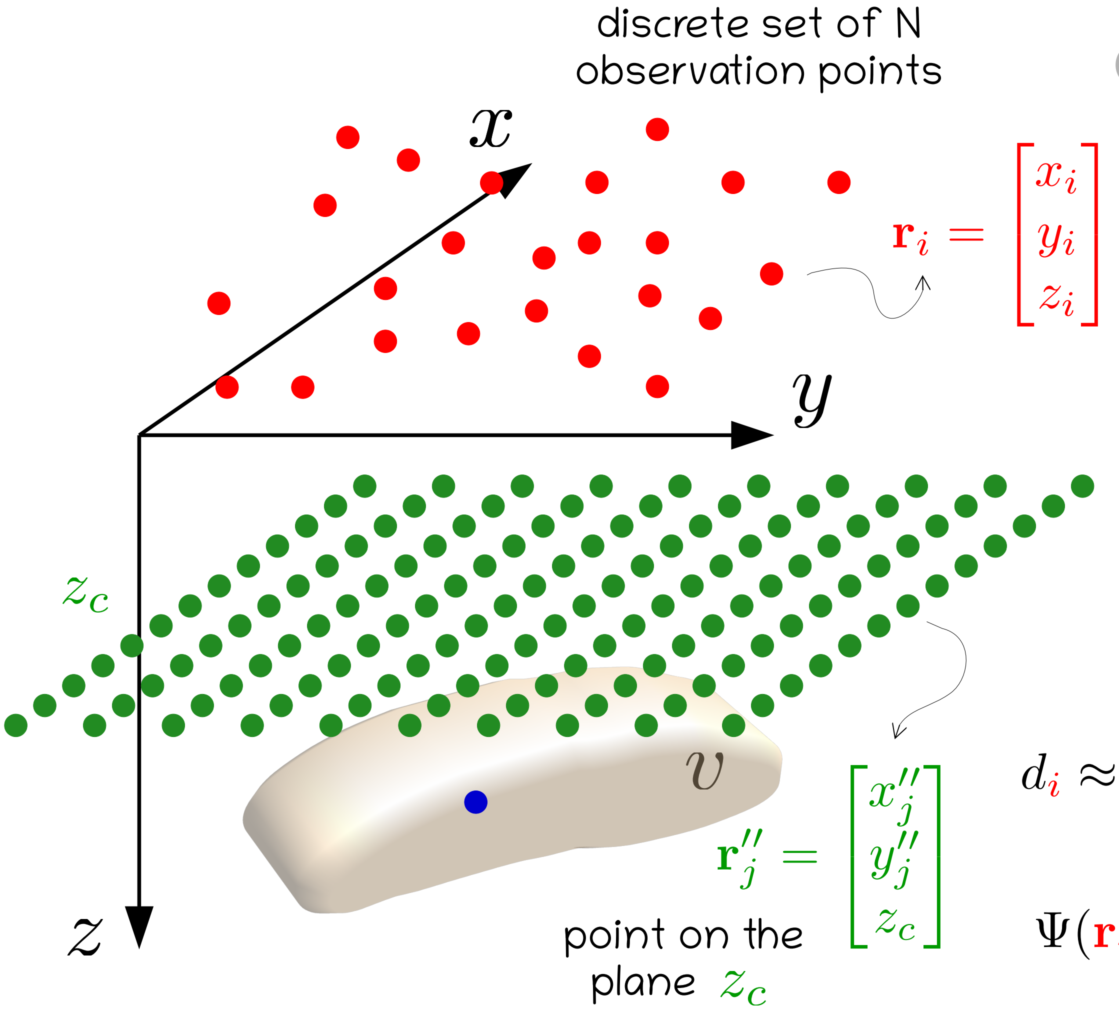
$$\mathbf{r}''_j = \begin{bmatrix} x''_j \\ y''_j \\ z_c \end{bmatrix}$$

point on the plane z_c

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

Gravitational field



density

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$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

$p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$

represents the gravity
disturbance produced
at the observation point
$\mathbf{r}_i$ by a monopole
located at $\mathbf{r}''_j$

Gravitational field

density

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

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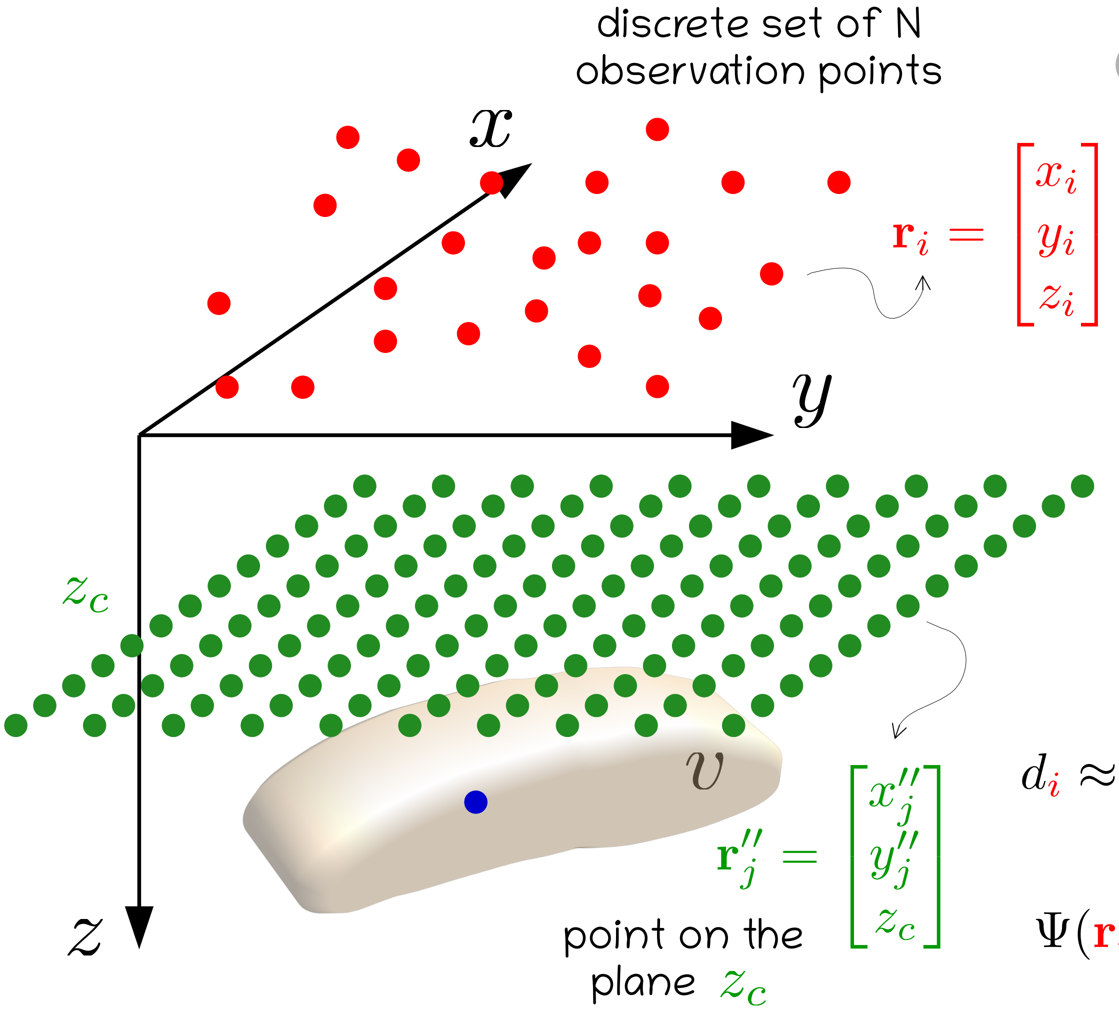
gravity disturbance



$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

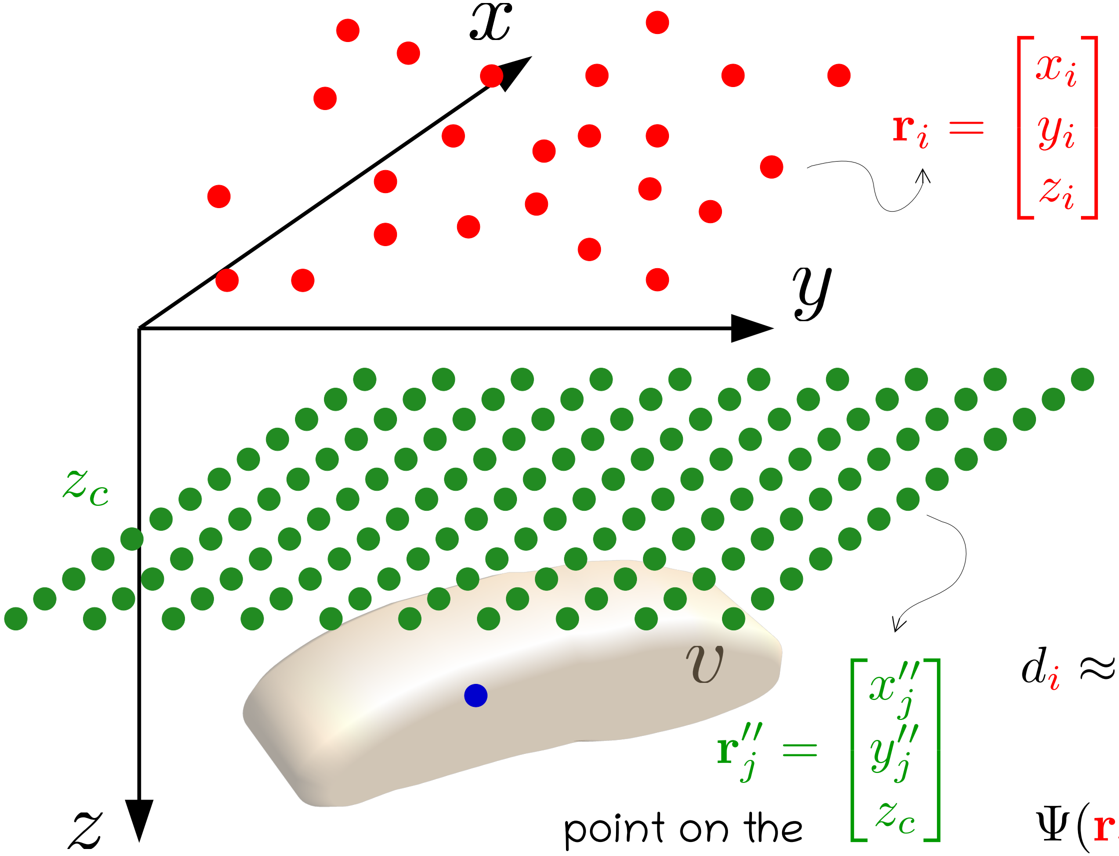
$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

$$\mathbf{d} \approx \mathbf{A} \, \mathbf{p}$$

In this case, the EqL Technique consists in solving this linear system for $\mathbf{p}$

Gravitational field

discrete set of N observation points



point on the plane z_c

density

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

parameter vector containing the physical-property distribution of the monopoles

$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

$$\mathbf{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}$$

Gravitational field

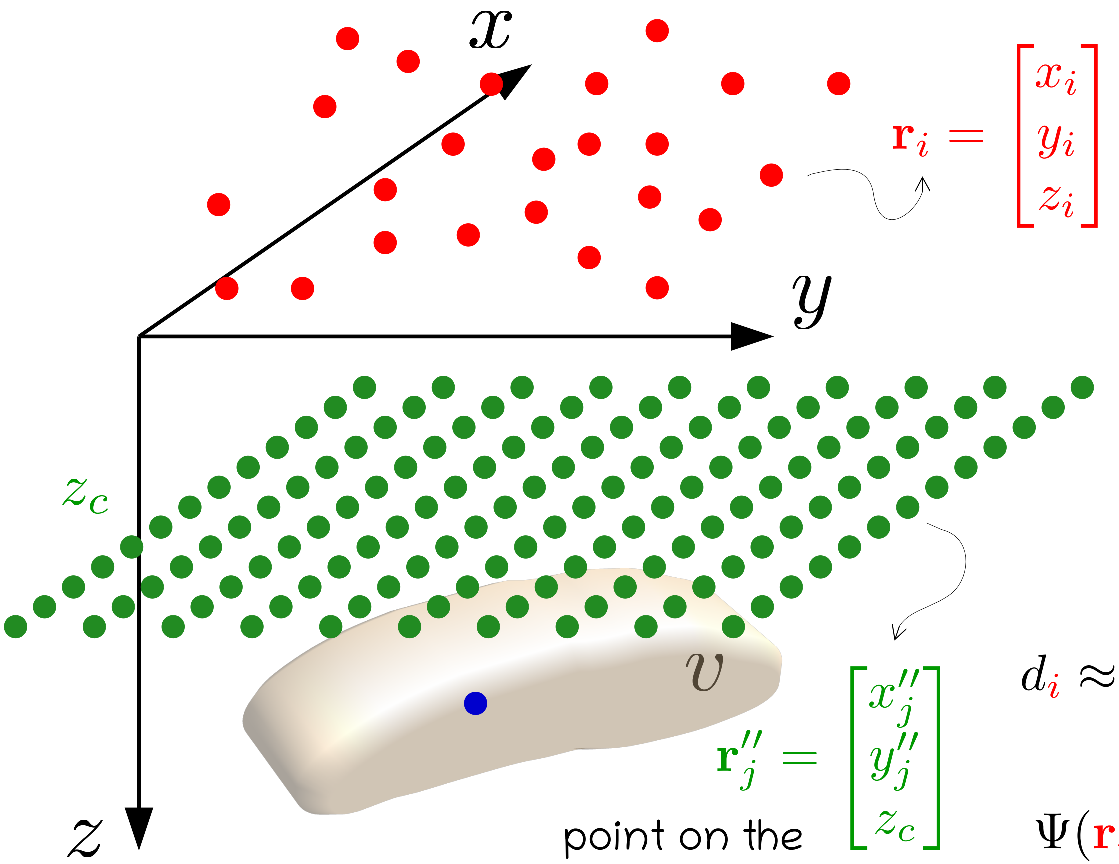
density

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance

discrete set of N observation points



$$\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

$$\mathbf{r}''_j = \begin{bmatrix} x''_j \\ y''_j \\ z_c \end{bmatrix}$$

point on the plane z_c

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

data vector containing the observations (in this case, gravity disturbance data)

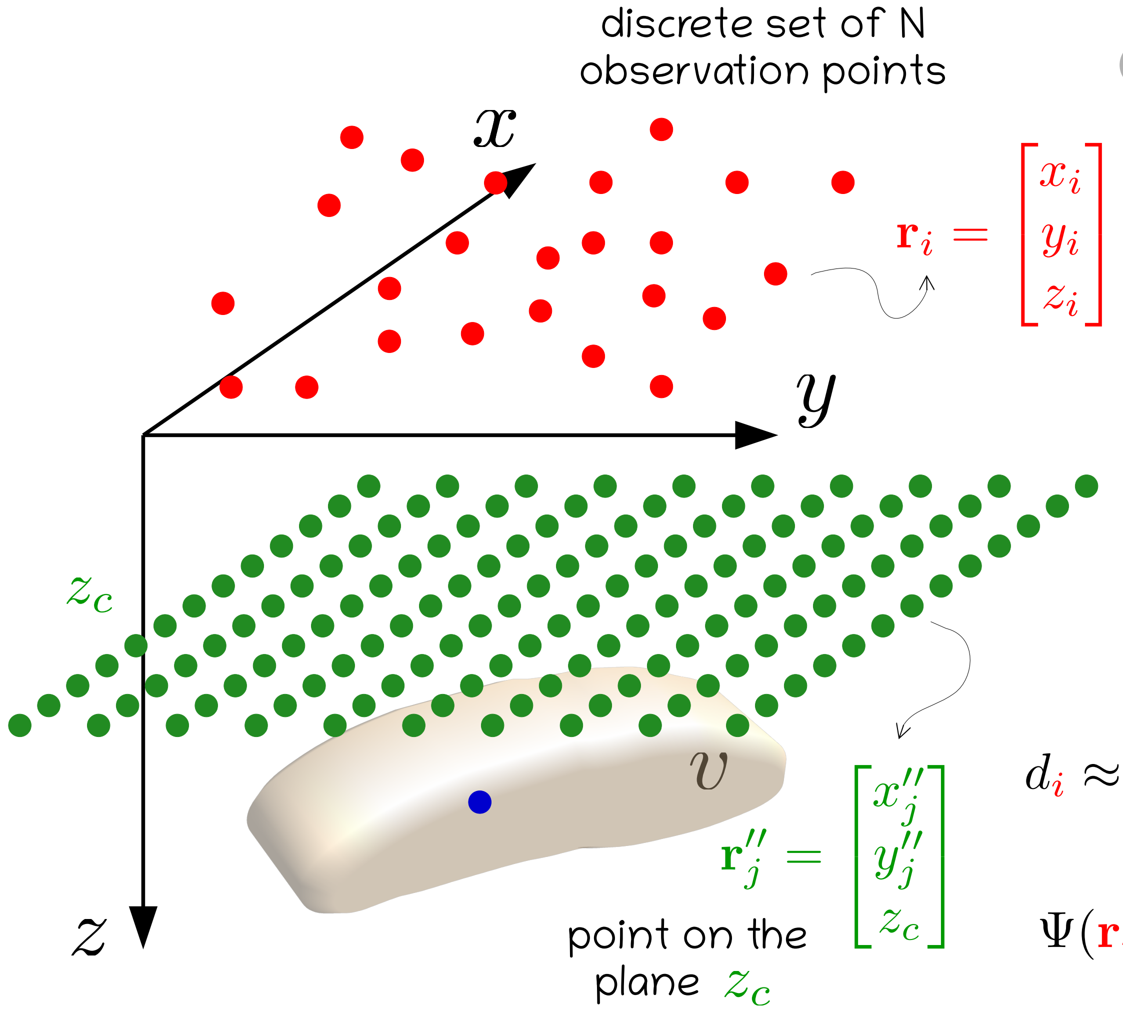
$$\mathbf{d} \approx \mathbf{A} \, \mathbf{p}$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}$$

Gravitational field

density

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r}-\mathbf{r}'\|}\, dv'$$



$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance

$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

$$d_i \approx \sum_{j=1}^{M} p_j\, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

$$a_{ij} = \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

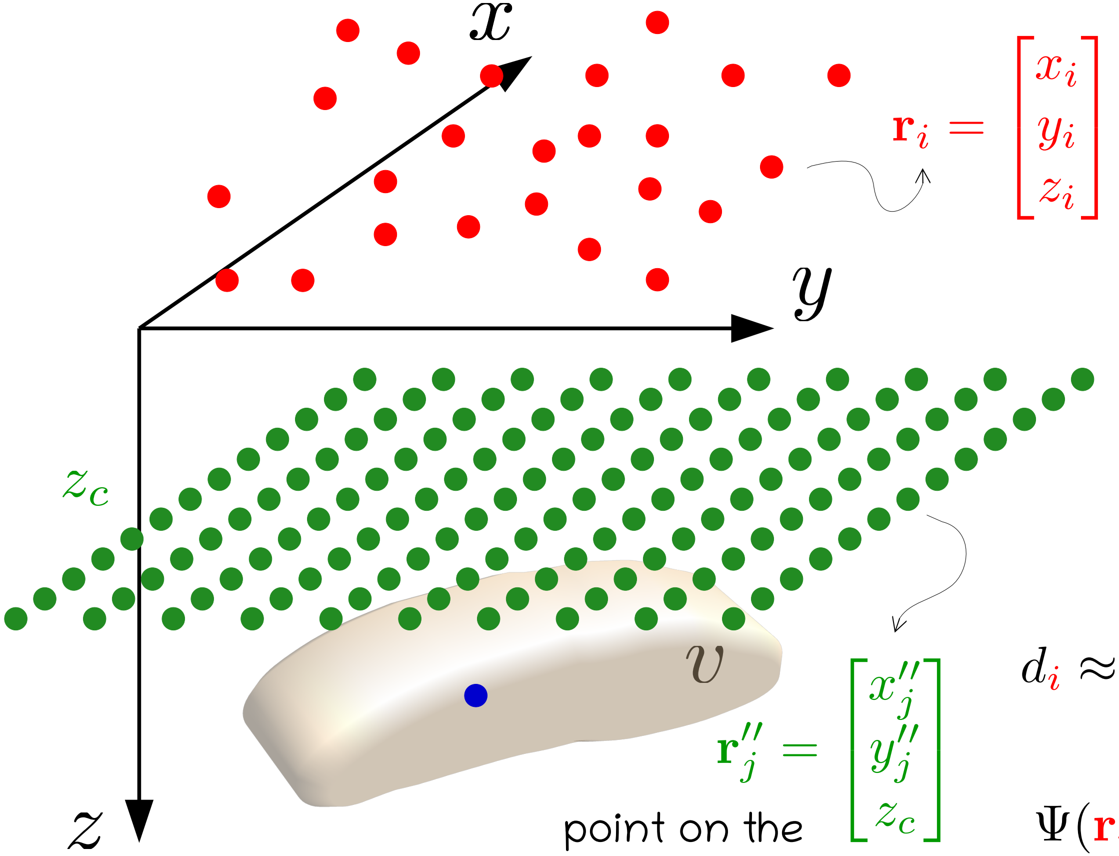
sensitivity matrix

Gravitational field

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

density

discrete set of N observation points



point on the plane z_c

$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance

The parameter vector **p** reproducing the gravity data defines a **discrete equivalent layer**

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

$$a_{ij} = \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

Gravitational field

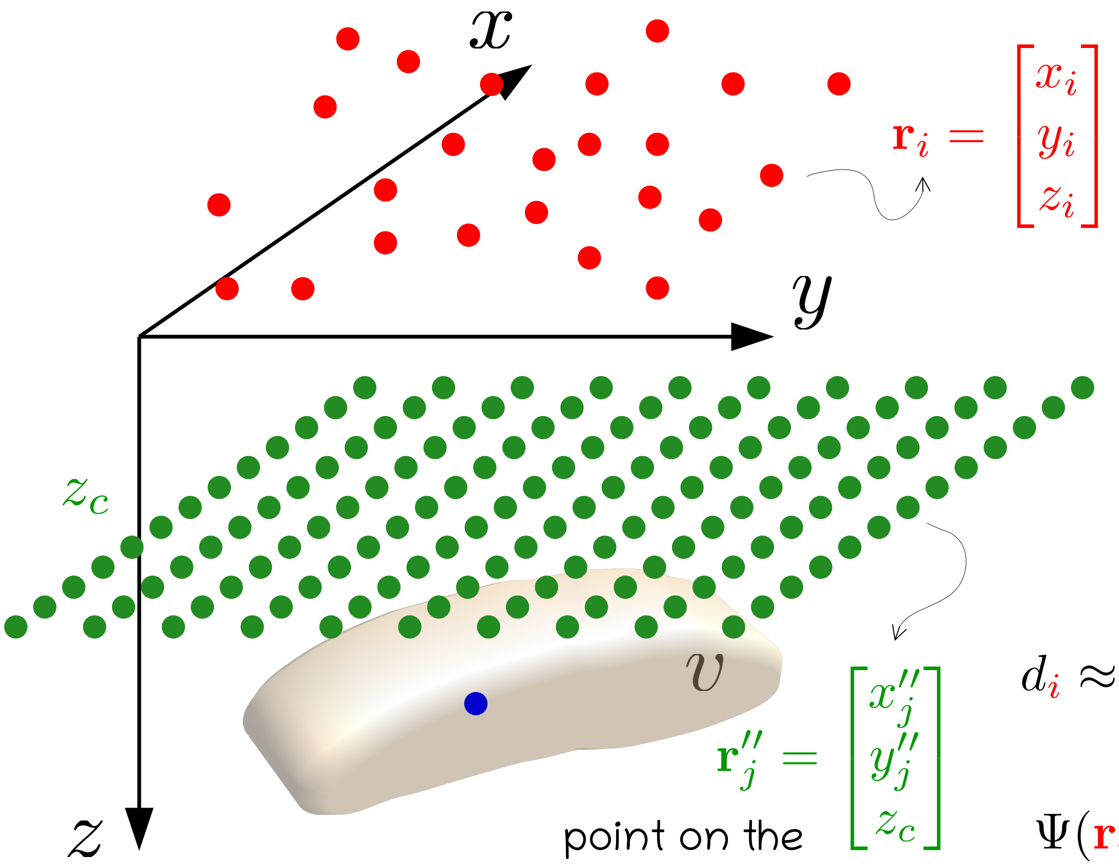
density

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

$$d_i \equiv \partial_z \Theta(\mathbf{r}_i)$$

gravity disturbance

discrete set of N observation points



$$\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

$$\mathbf{r}''_j = \begin{bmatrix} x''_j \\ y''_j \\ z_c \end{bmatrix}$$

point on the plane z_c

The physical property p_j of a single monopole located at $\mathbf{r}''_j$ is called **equivalent source**

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

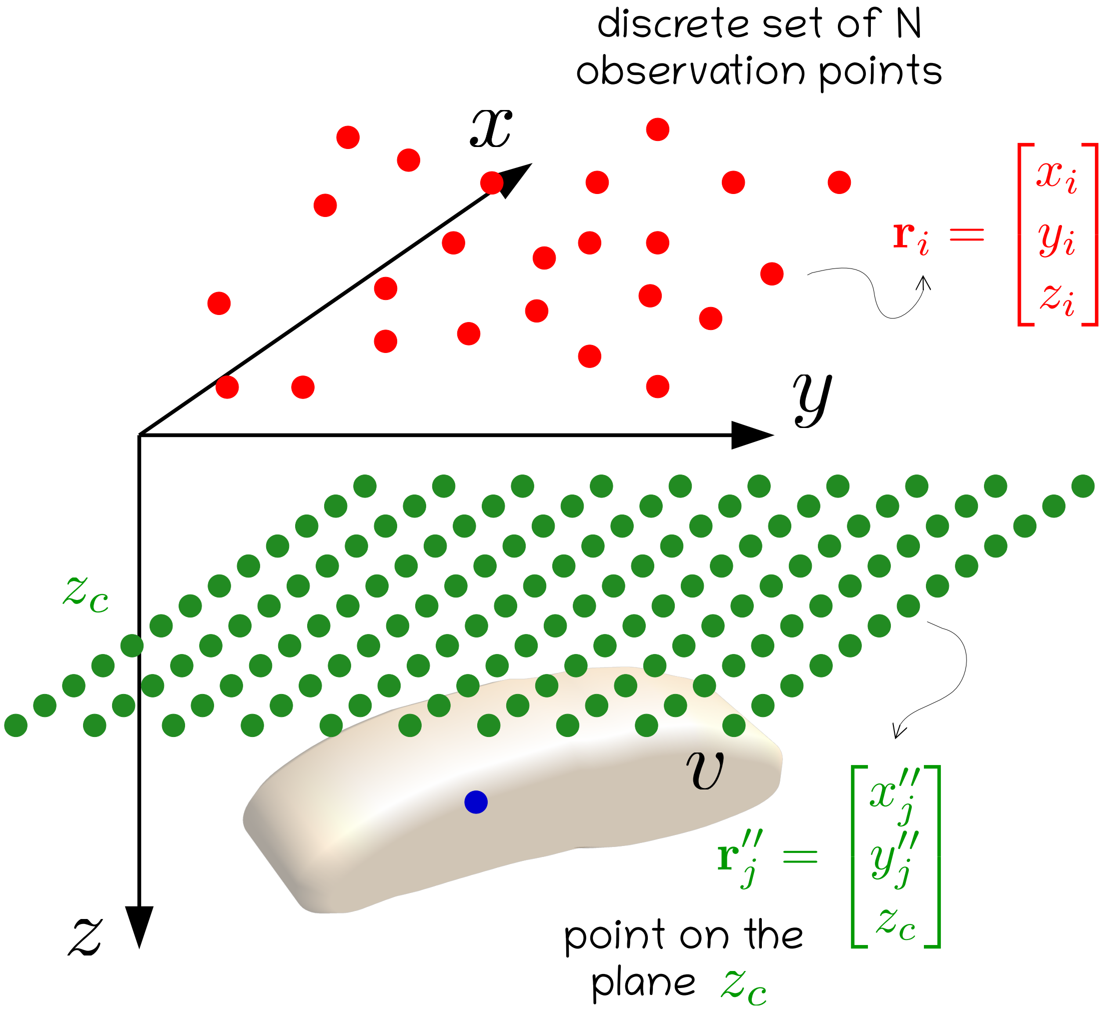
$$\mathbf{d} \approx \mathbf{A} \, \mathbf{p}$$

$$a_{ij} = \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

Gravitational field

density

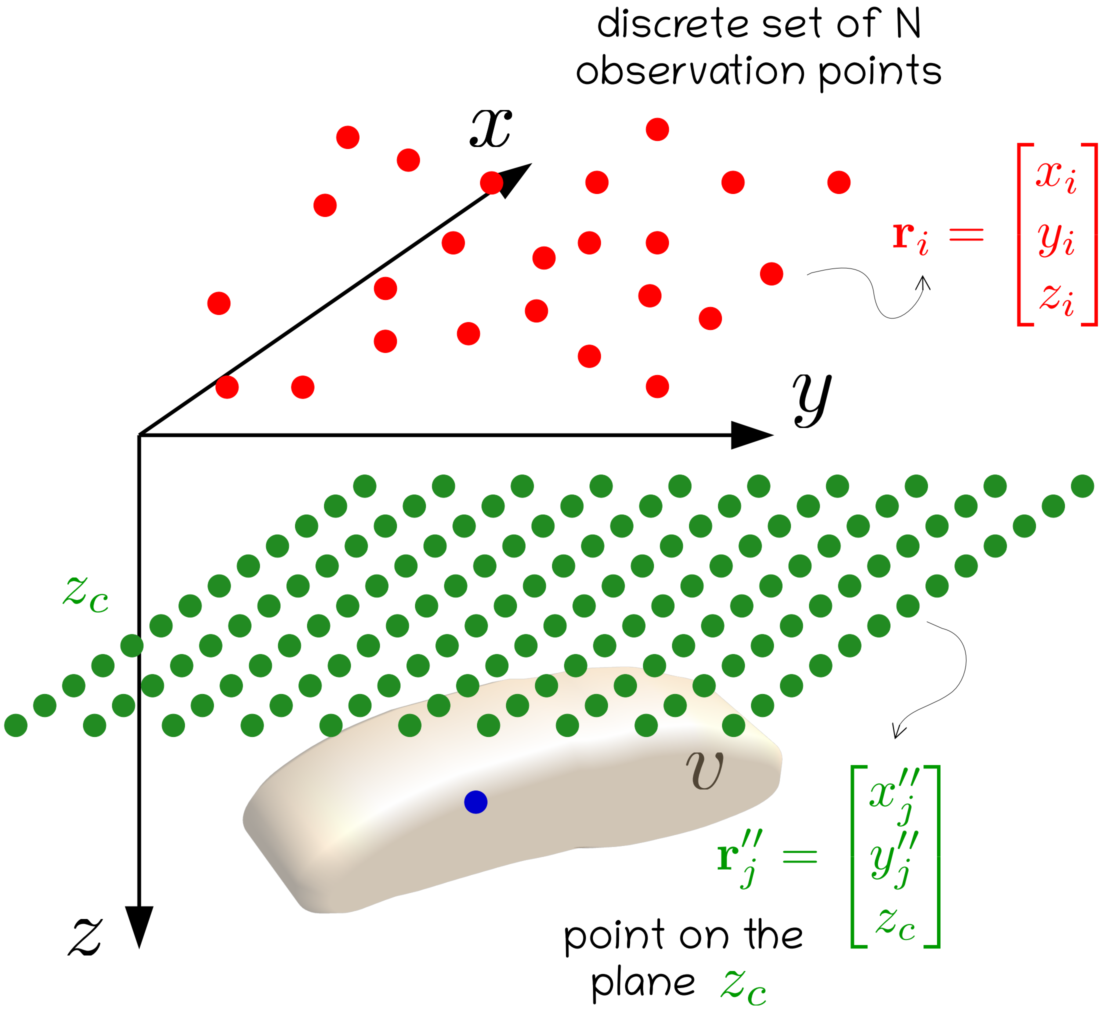
$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



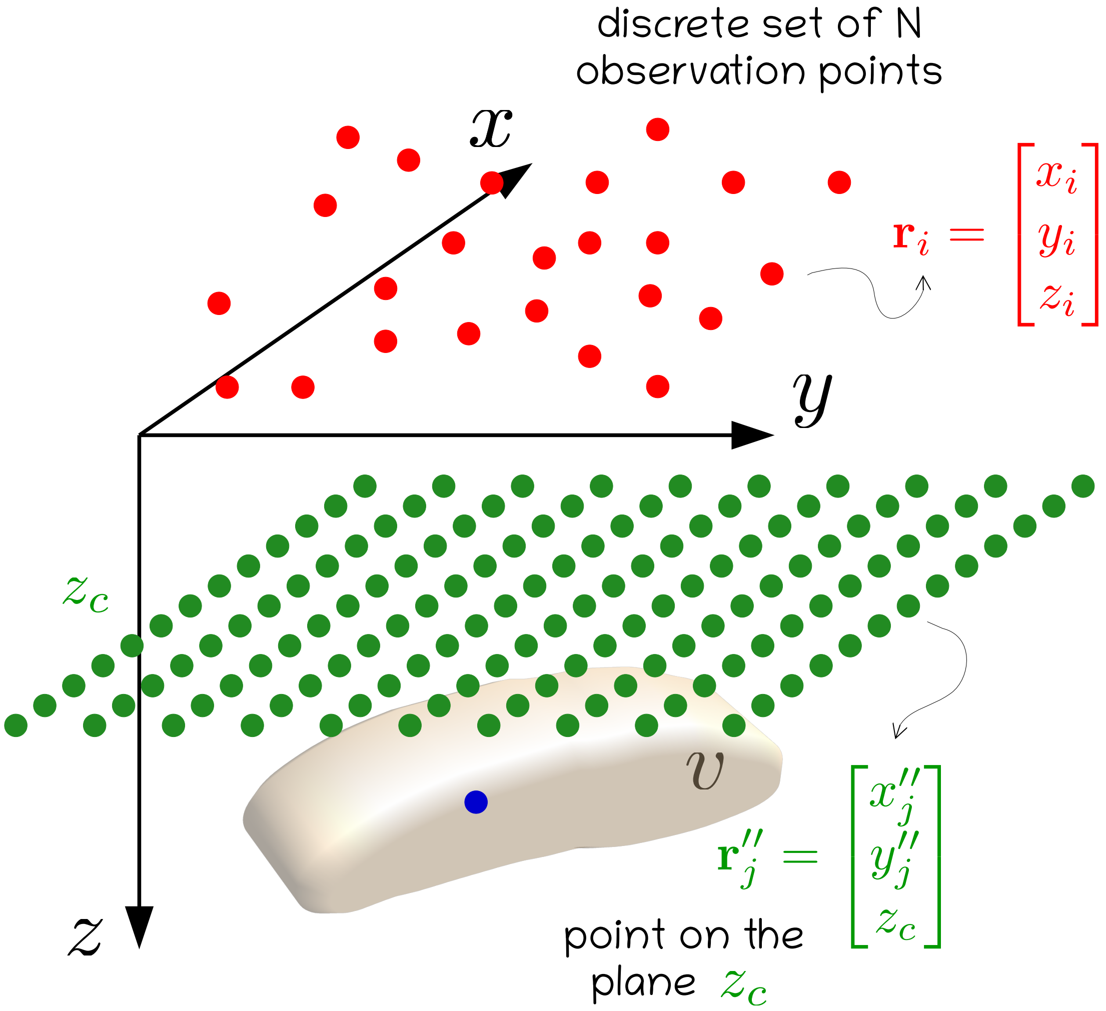
Magnetic field

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



Magnetic field



total-magnetization intensity

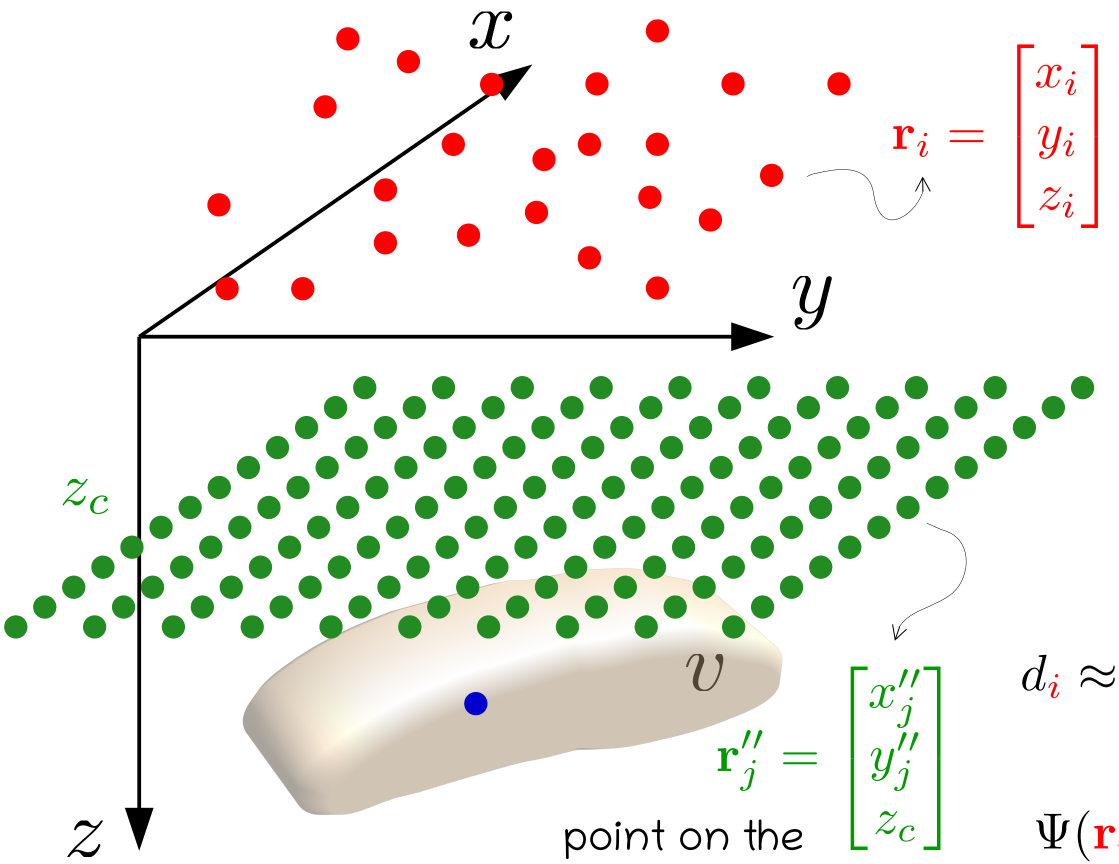
$$\Theta(\mathbf{r}) = \kappa \iiint\limits_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

$$d_i \equiv \partial_{th} \Theta(\mathbf{r}_i)$$

approx total-field anomaly

Magnetic field

discrete set of N observation points



$$\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

$$\mathbf{r}''_j = \begin{bmatrix} x''_j \\ y''_j \\ z_c \end{bmatrix}$$

point on the plane z_c

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

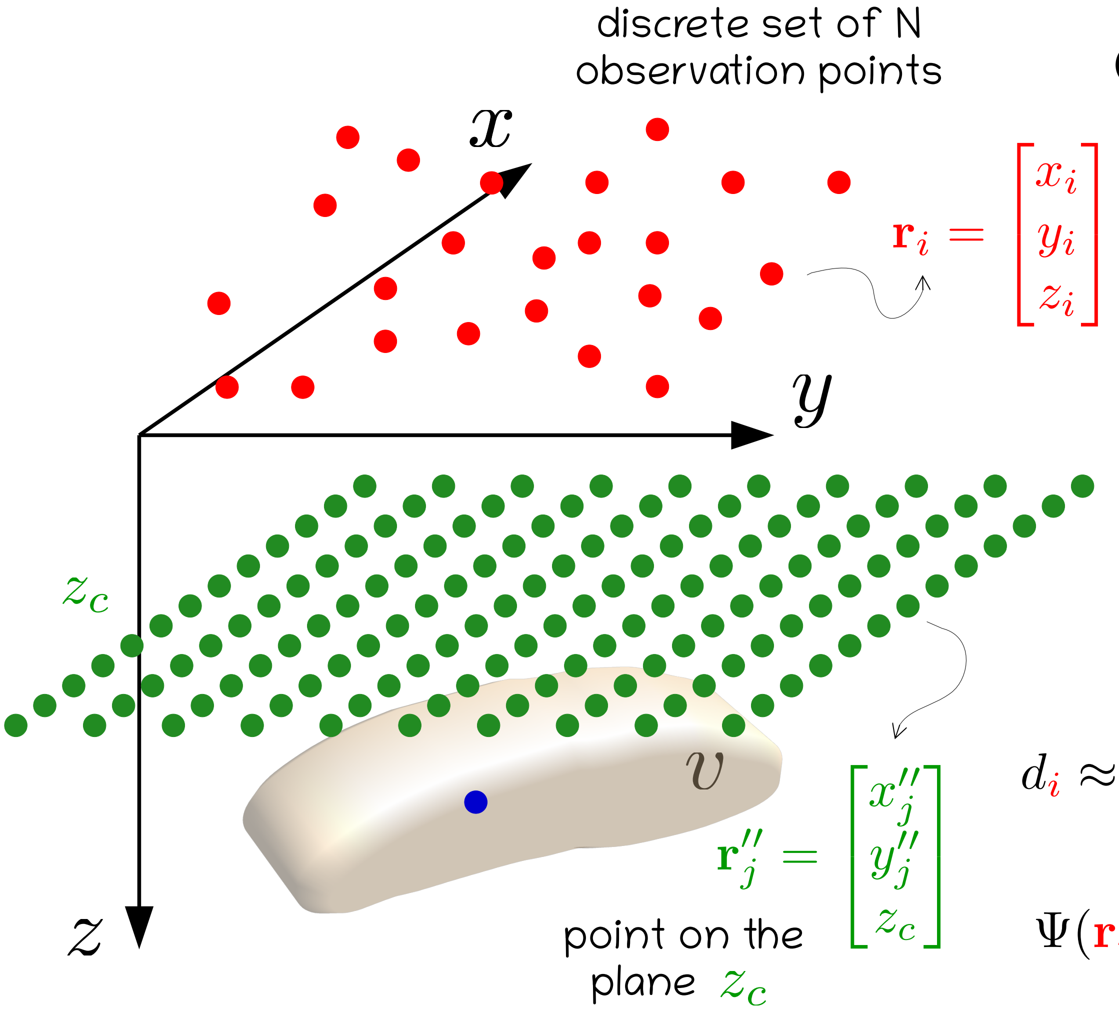
$$d_i \equiv \partial_{th} \Theta(\mathbf{r}_i)$$

approx total-field anomaly

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

Magnetic field



total-magnetization intensity

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$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

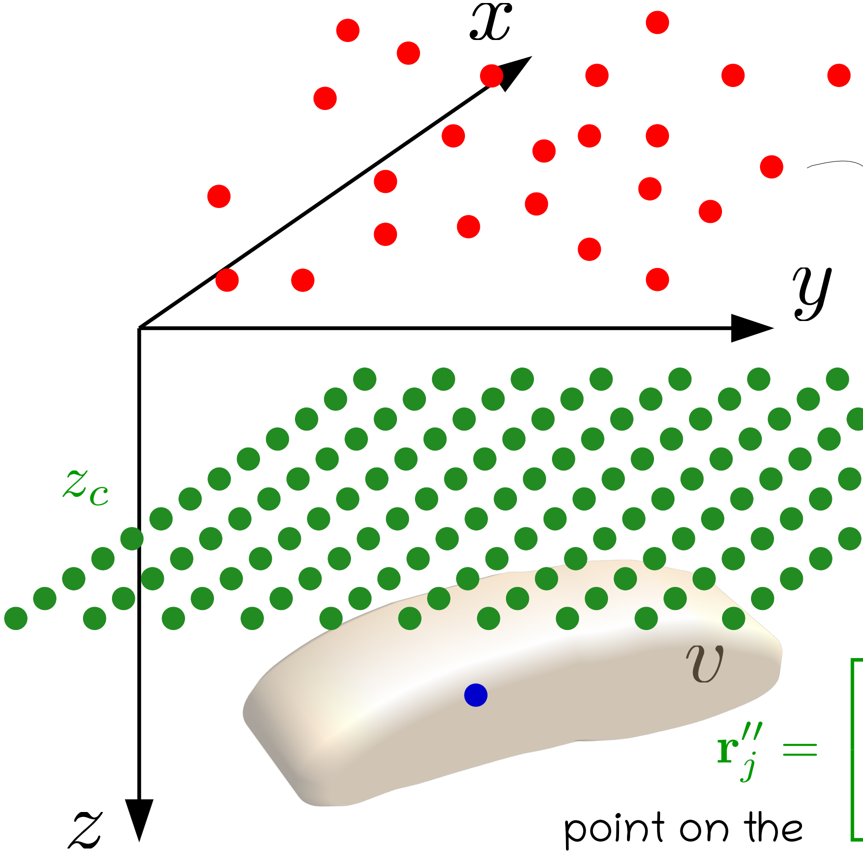
$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

$p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$

represents the approx total-field anomaly produced at the observation point $\mathbf{r}_i$ by a dipole located at $\mathbf{r}''_j$

Magnetic field

discrete set of N
observation points



$$\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

total-magnetization
intensity

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

$$d_i \equiv \partial_{th} \Theta(\mathbf{r}_i)$$

approx total-field anomaly

The physical property p_j of
a single dipole located at $\mathbf{r}_j''$
is an **equivalent source**

$$\mathbf{r}_j'' = \begin{bmatrix} x_j'' \\ y_j'' \\ z_c \end{bmatrix}$$

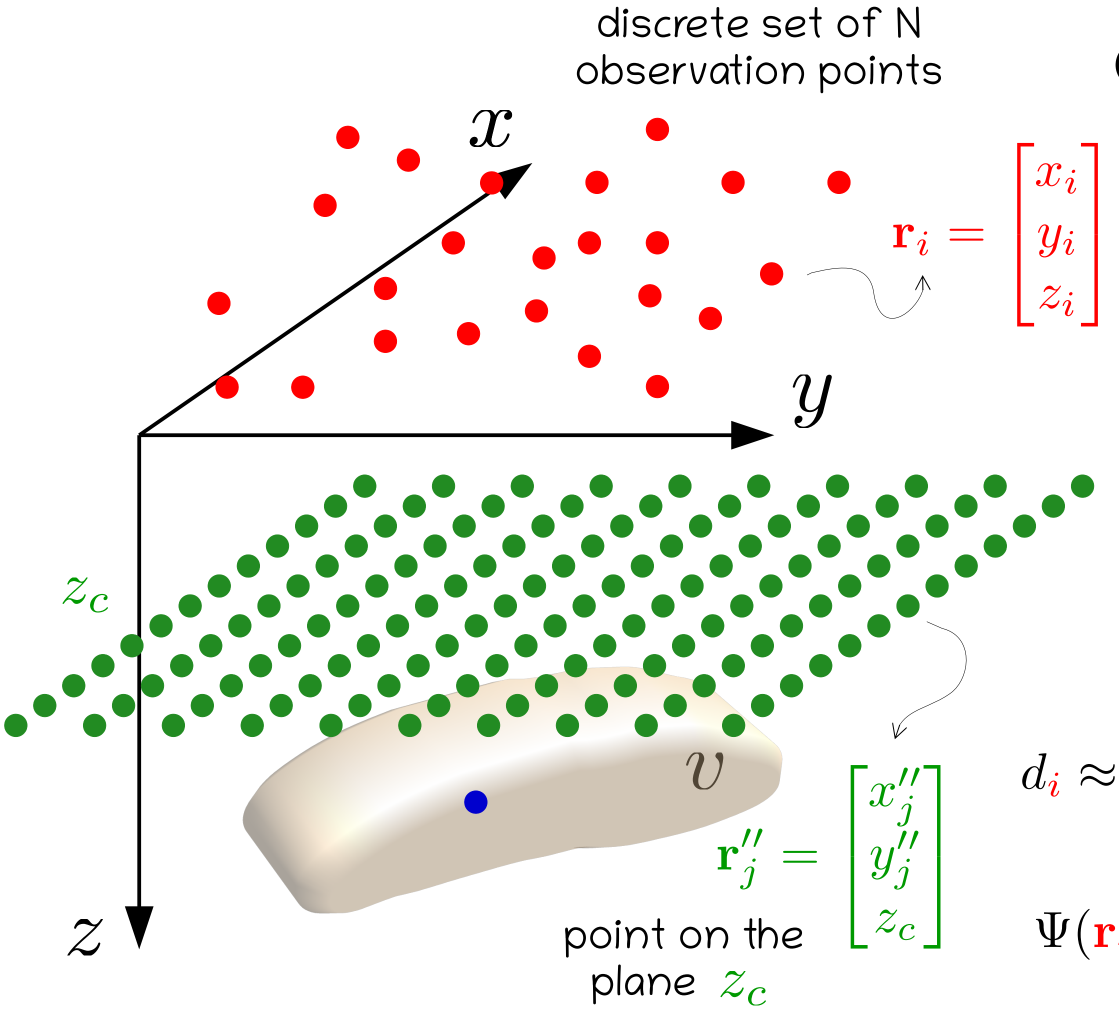
point on the
plane z_c

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}_j'')$$

$$\Psi(\mathbf{r}_i, \mathbf{r}_j'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}_j''\|}$$

$p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}_j'')$
represents the approx
total-field anomaly produced
at the observation point
$\mathbf{r}_i$ by a dipole
located at $\mathbf{r}_j''$

Magnetic field



total-magnetization intensity

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$$d_i \equiv \partial_{th} \Theta(\mathbf{r}_i)$$

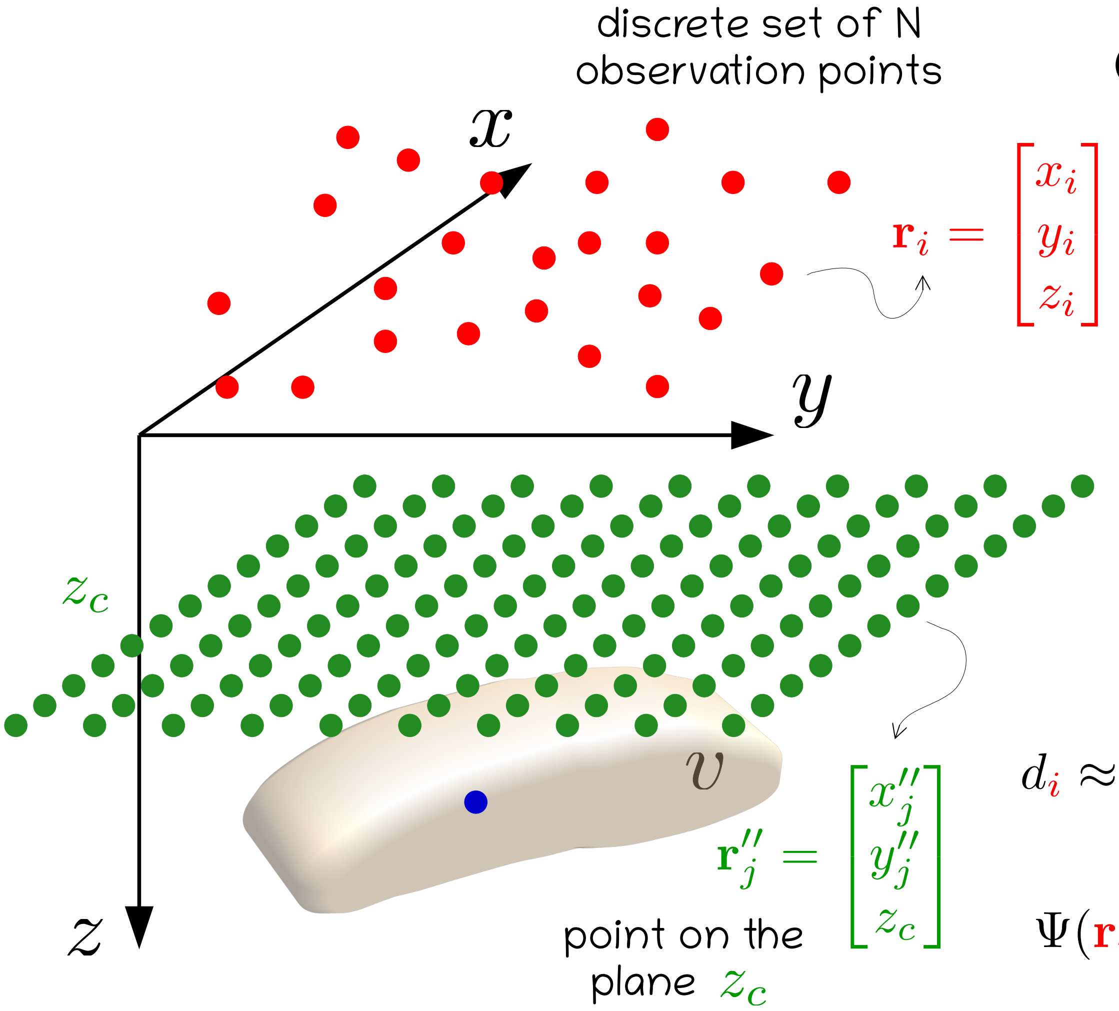
approx total-field anomaly

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\partial_{tv} \Psi(\mathbf{r}, \mathbf{r}'') = \hat{\mathbf{t}}^\top \mathbf{H}_\Psi(\mathbf{r}, \mathbf{r}'') \, \hat{\mathbf{v}}$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

Magnetic field



total-magnetization intensity

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$$d_i \equiv \partial_{th} \Theta(\mathbf{r}_i)$$

approx total-field anomaly

unit vector defining a constant direction for the main geomagnetic field

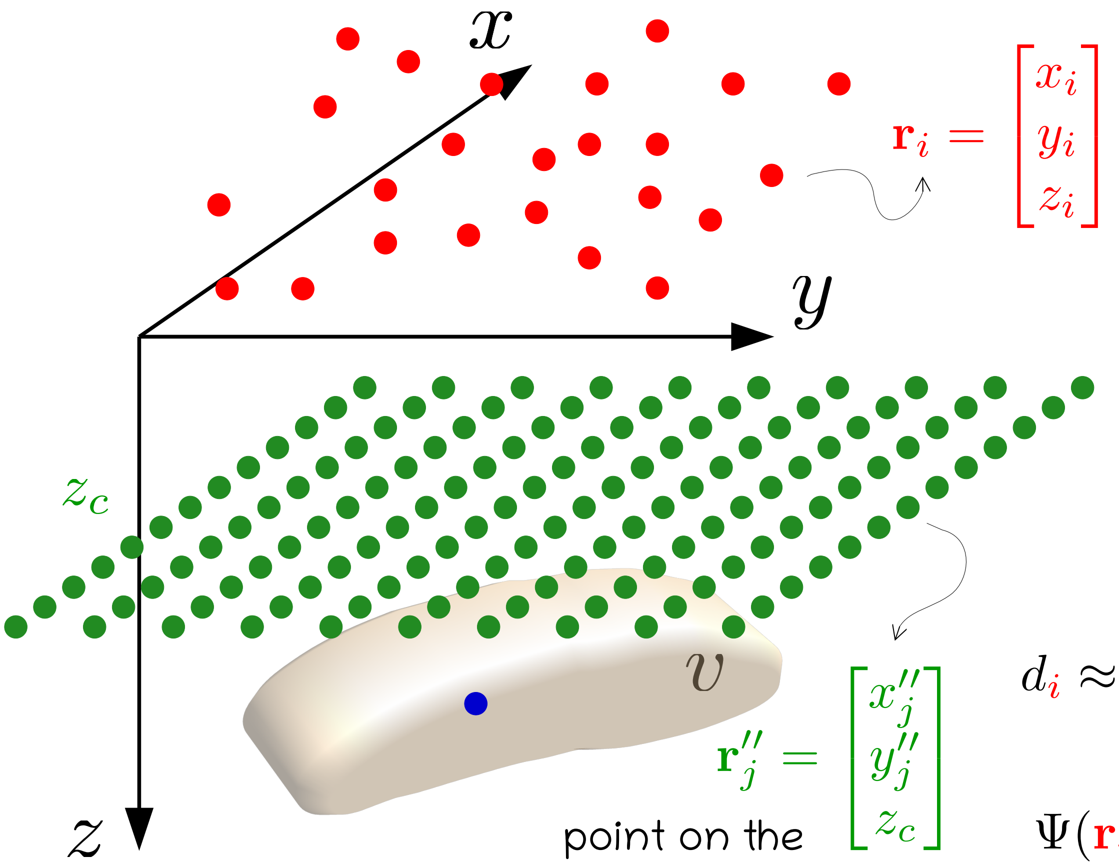
$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\partial_{tv} \Psi(\mathbf{r}, \mathbf{r}'') = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Psi}(\mathbf{r}, \mathbf{r}'') \, \hat{\mathbf{v}}$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

Magnetic field

discrete set of N observation points



$$\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$d_i \equiv \partial_{th} \Theta(\mathbf{r}_i)$$

approx total-field anomaly

$$\mathbf{r}''_j = \begin{bmatrix} x''_j \\ y''_j \\ z_c \end{bmatrix}$$

point on the plane z_c

unit vector defining a uniform magnetization direction for the dipoles

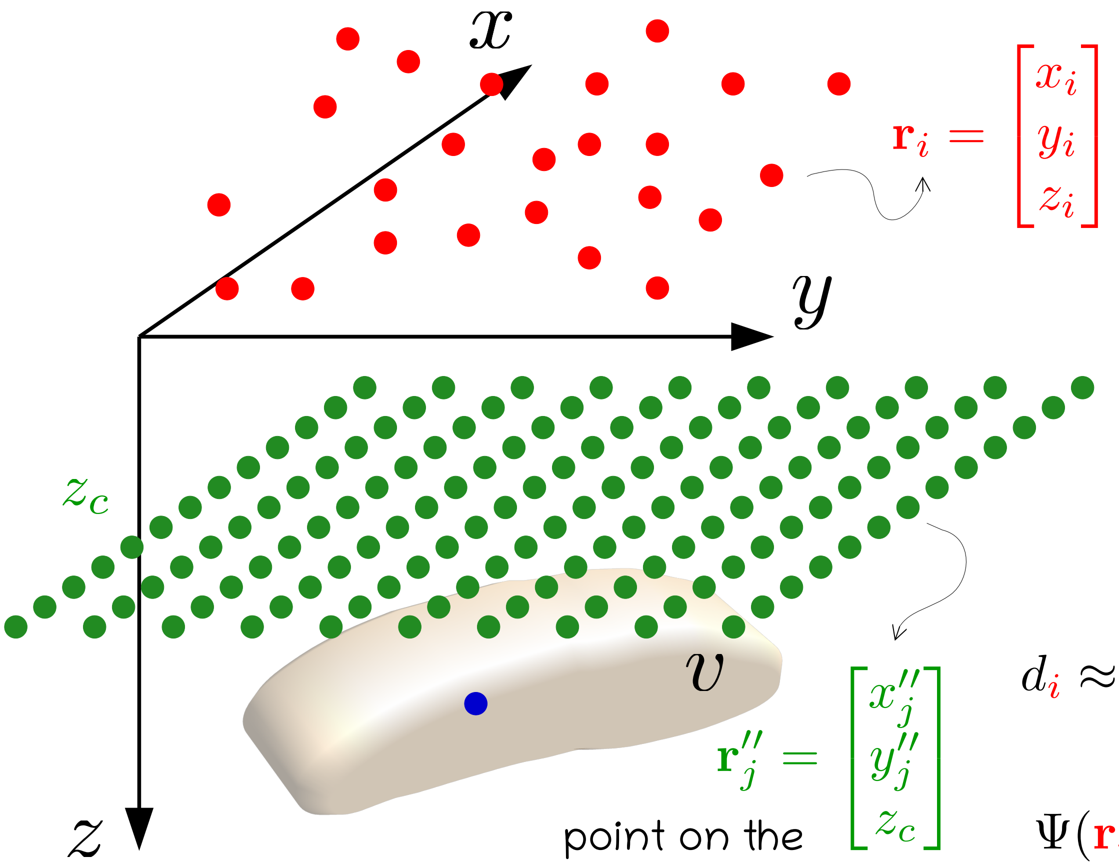
$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\partial_{tv} \Psi(\mathbf{r}, \mathbf{r}'') = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Psi}(\mathbf{r}, \mathbf{r}'') \, \hat{\mathbf{v}}$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

Magnetic field

discrete set of N observation points



point on the plane z_c

total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

$$d_i \equiv \partial_{th}\Theta(\mathbf{r}_i)$$

approx total-field anomaly

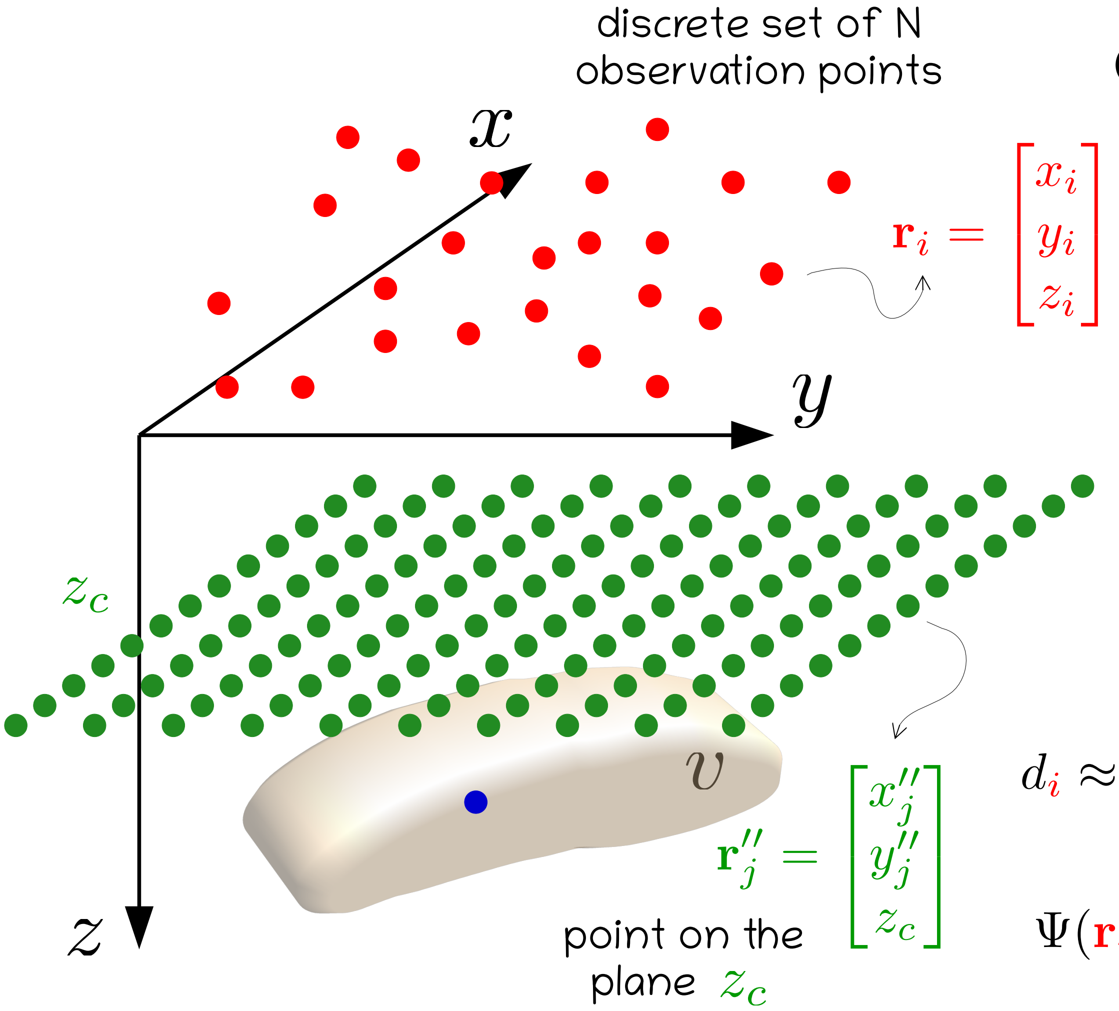
this direction may be arbitrary

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{tv}\Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\partial_{tv}\Psi(\mathbf{r}, \mathbf{r}'') = \hat{\mathbf{t}}^\top \mathbf{H}_\Psi(\mathbf{r}, \mathbf{r}'') \, \hat{\mathbf{v}}$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

Magnetic field



total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

$$d_i \equiv \partial_{th} \Theta(\mathbf{r}_i)$$

approx total-field anomaly

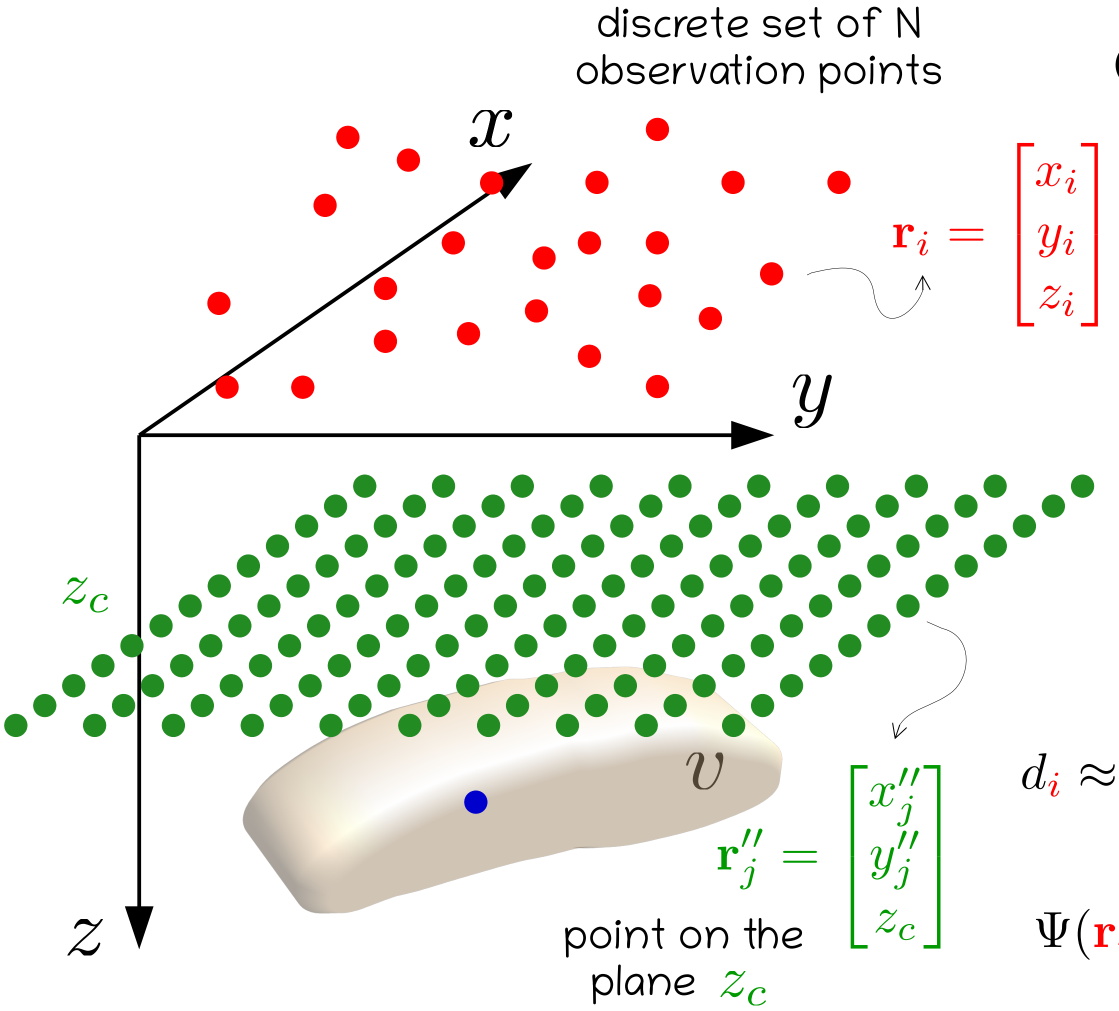
$$\mathbf{H}_\Psi(\mathbf{r}, \mathbf{r}'') = \begin{bmatrix} \partial_{xx}\Psi(\mathbf{r}, \mathbf{r}'') & \partial_{xy}\Psi(\mathbf{r}, \mathbf{r}'') & \partial_{xz}\Psi(\mathbf{r}, \mathbf{r}'') \\ \partial_{xy}\Psi(\mathbf{r}, \mathbf{r}'') & \partial_{yy}\Psi(\mathbf{r}, \mathbf{r}'') & \partial_{yz}\Psi(\mathbf{r}, \mathbf{r}'') \\ \partial_{xz}\Psi(\mathbf{r}, \mathbf{r}'') & \partial_{yz}\Psi(\mathbf{r}, \mathbf{r}'') & \partial_{zz}\Psi(\mathbf{r}, \mathbf{r}'') \end{bmatrix}$$

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\partial_{tv}\Psi(\mathbf{r}, \mathbf{r}'') = \hat{\mathbf{t}}^\top \mathbf{H}_\Psi(\mathbf{r}, \mathbf{r}'') \, \hat{\mathbf{v}}$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

Magnetic field



total-magnetization intensity

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$d_i \equiv \partial_{th} \Theta(\mathbf{r}_i)$$

approx total-field anomaly

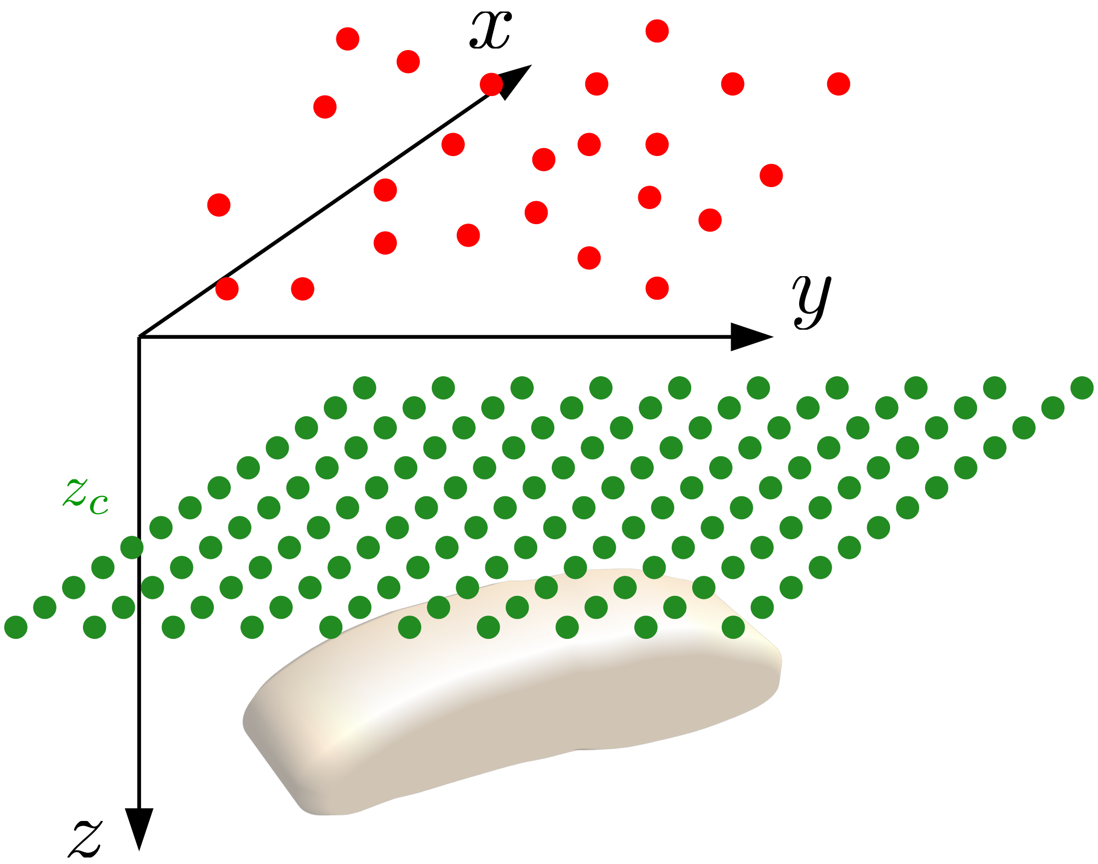
$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{tv} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\Psi(\mathbf{r}_i, \mathbf{r}''_j) = \frac{1}{2\pi} \frac{1}{\|\mathbf{r}_i - \mathbf{r}''_j\|}$$

$$\mathbf{d} \approx \mathbf{A} \, \mathbf{p}$$

Again, the EqL Technique consists in solving this linear system for $\mathbf{p}$

Classical EqL Technique



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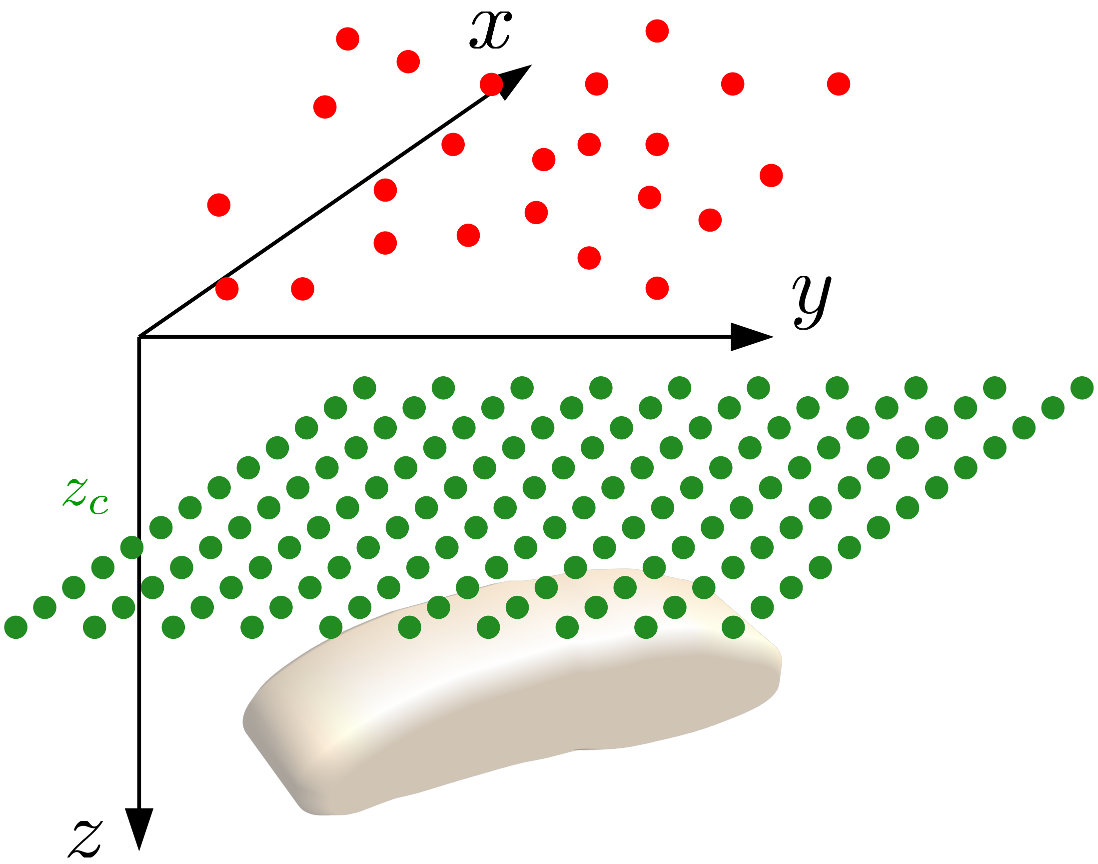
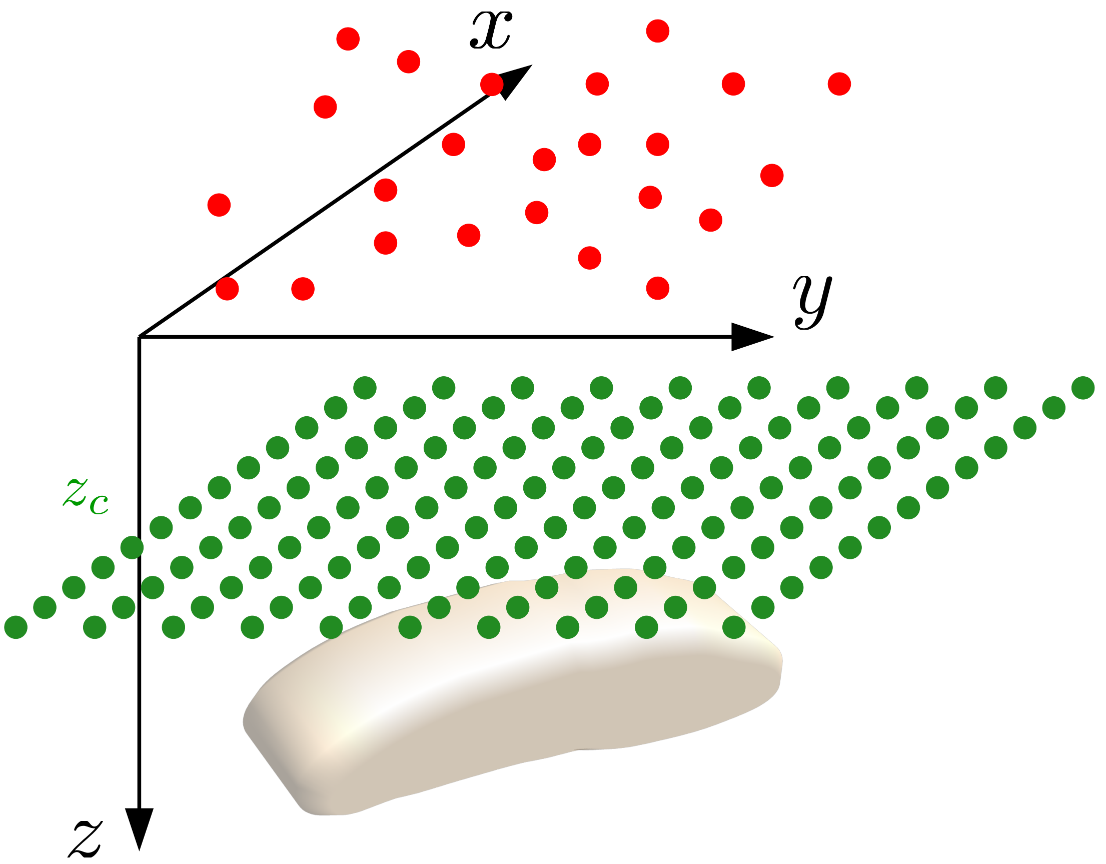
Classical EqL Technique



$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

solving a linear system for $\mathbf{p}$

Classical EqL Technique



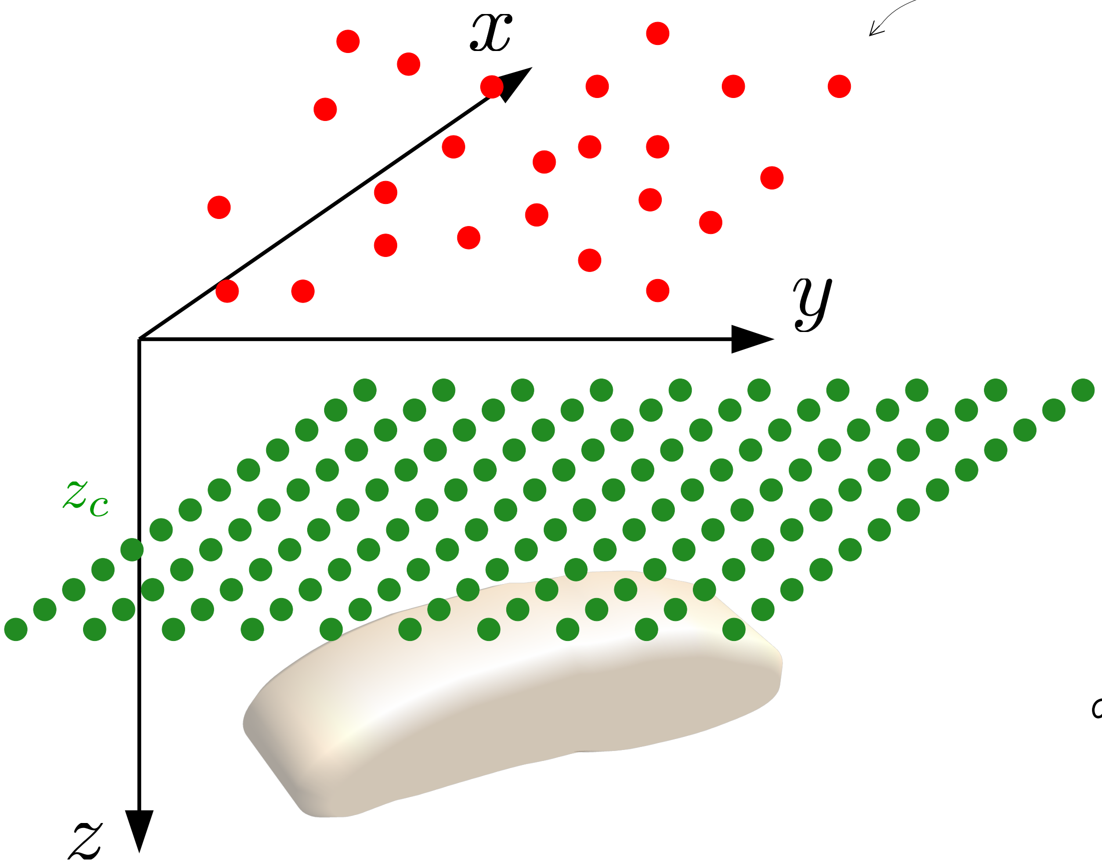
$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

$$\mathbf{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}$$

mass
(density)

moment
(magnetization)
intensity

Classical EqL Technique



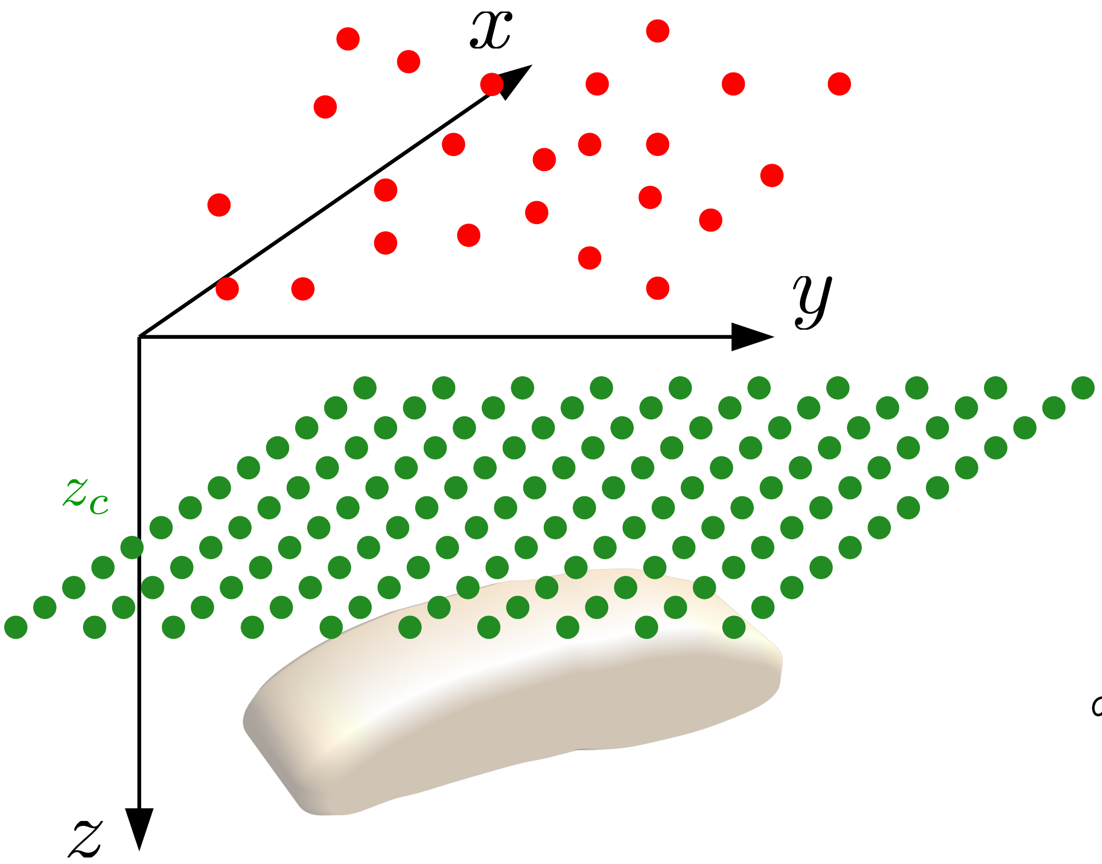
$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix} \qquad \mathbf{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}$$

gravity disturbance — mass (density)

approx total-field anomaly — moment (magnetization) intensity

Classical EqL Technique

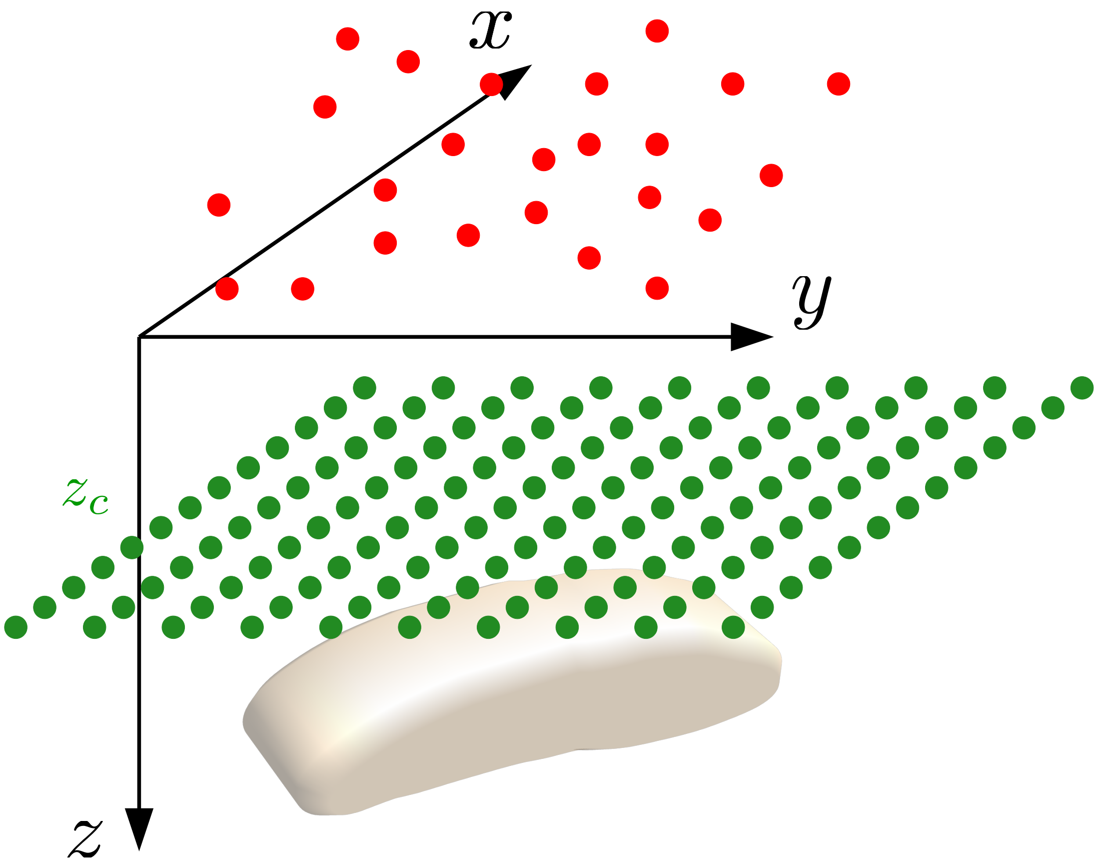


$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix} \qquad a_{ij} \qquad \mathbf{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}$$

gravity disturbance	monopole	mass (density)
approx total-field anomaly	dipole	moment (magnetization) intensity

Classical EqL Technique



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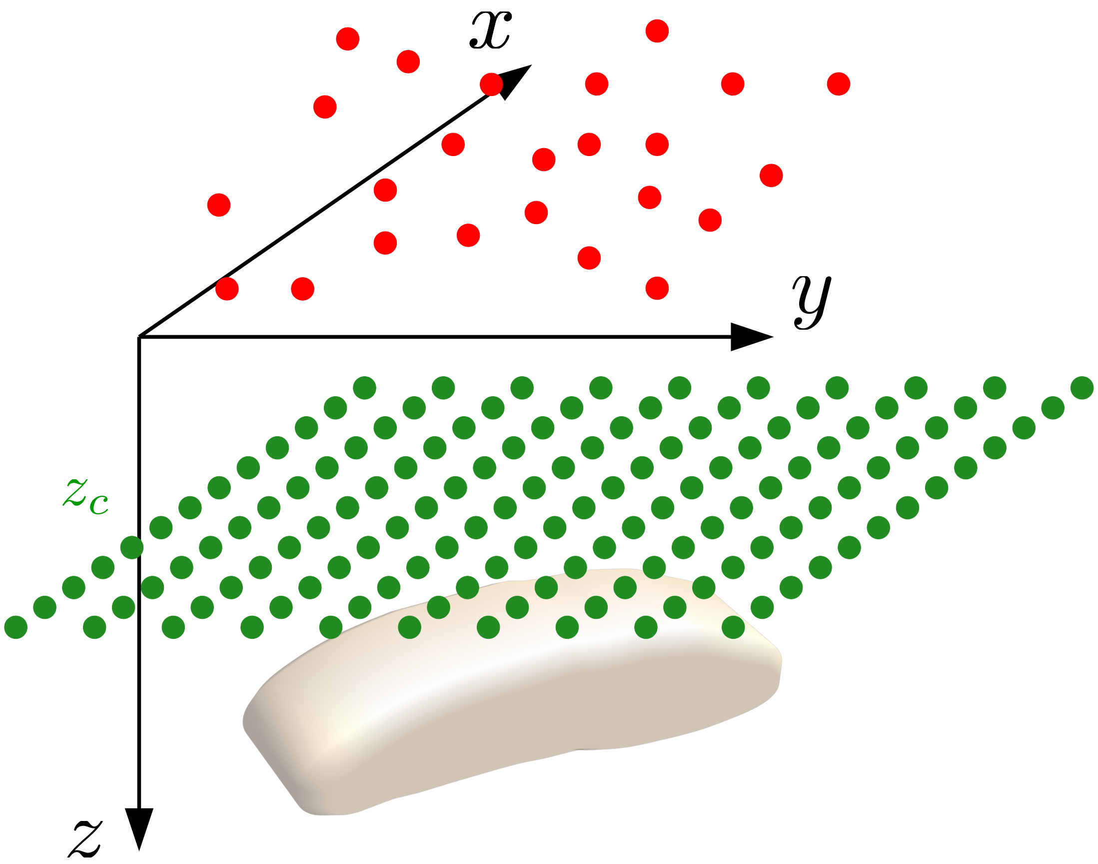
$$\mathbf{w} = \mathbf{T}\,\tilde{\mathbf{p}}$$

potential-field transformation

Classical EqL Technique



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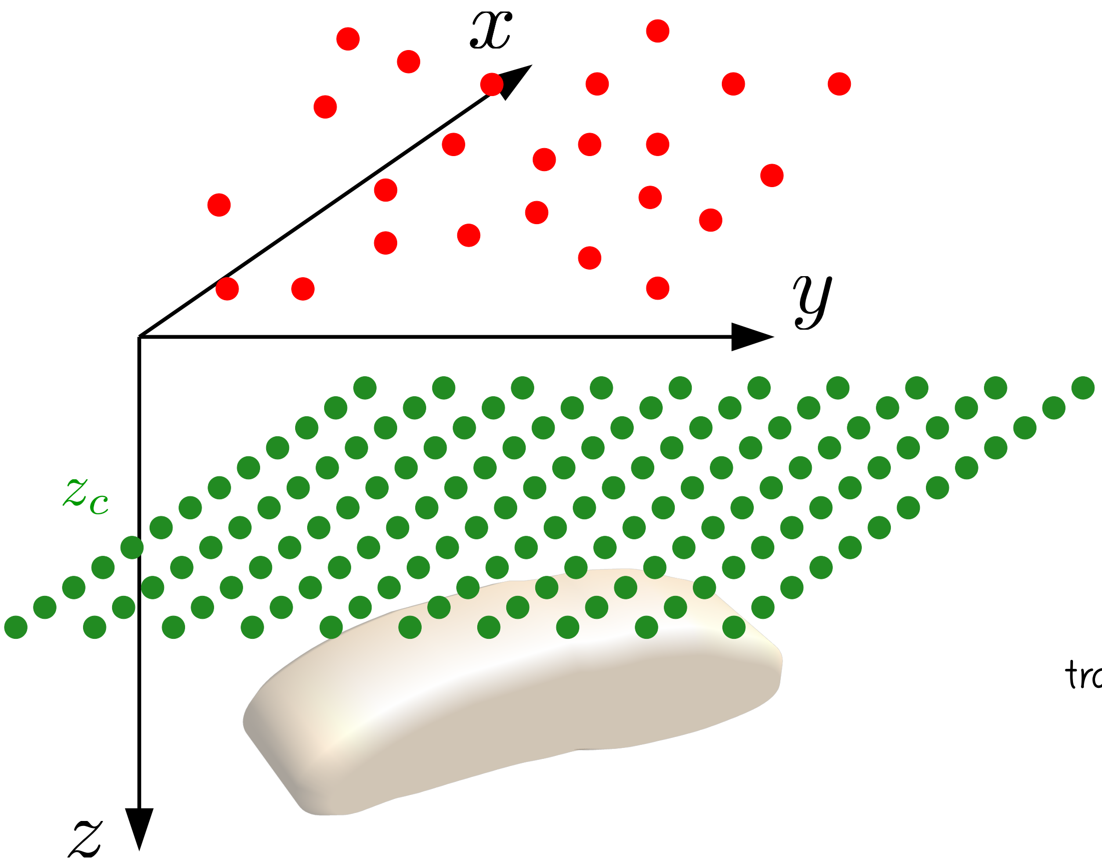
$$\mathbf{w} = \mathbf{T} \, \tilde{\mathbf{p}}$$

estimated
parameter vector
(equivalent layer)

Classical EqL Technique



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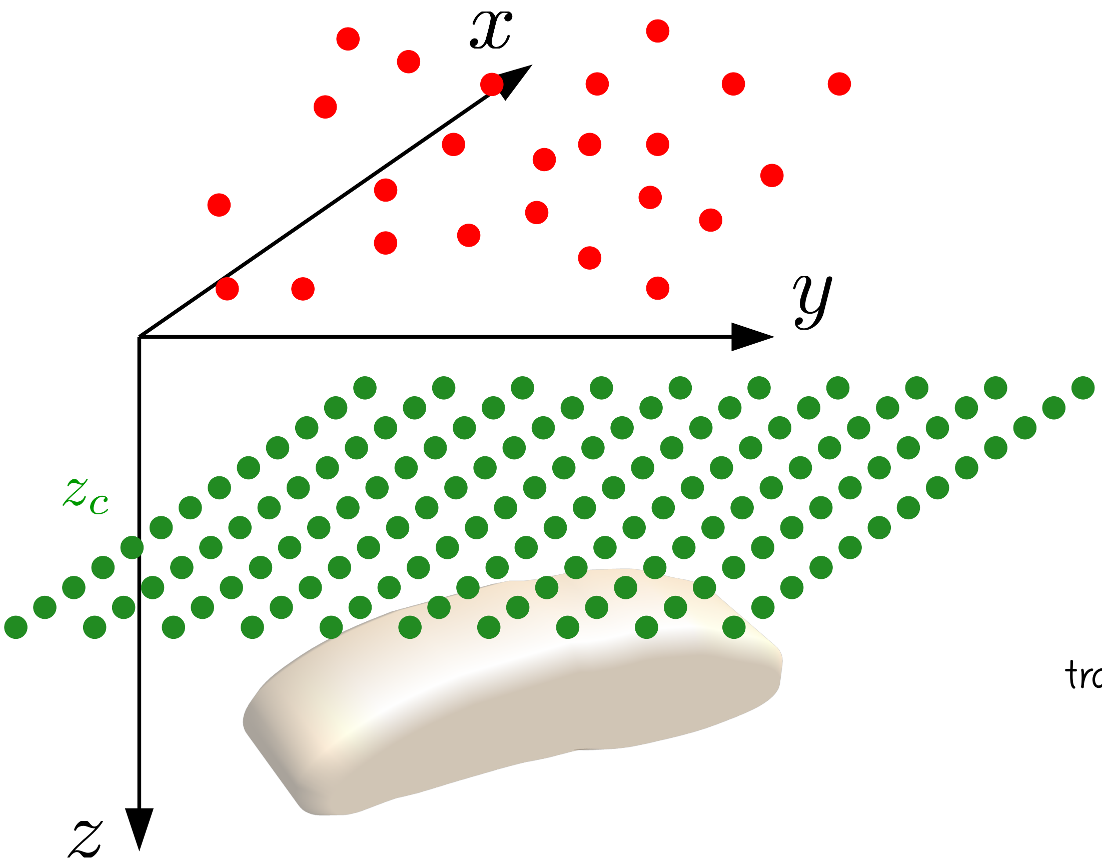
$$\mathbf{w} = \mathbf{T}\,\tilde{\mathbf{p}}$$

transformed data

estimated parameter vector (equivalent layer)

Classical EqL Technique



$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

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$$\mathbf{w} = \mathbf{T}\,\tilde{\mathbf{p}}$$

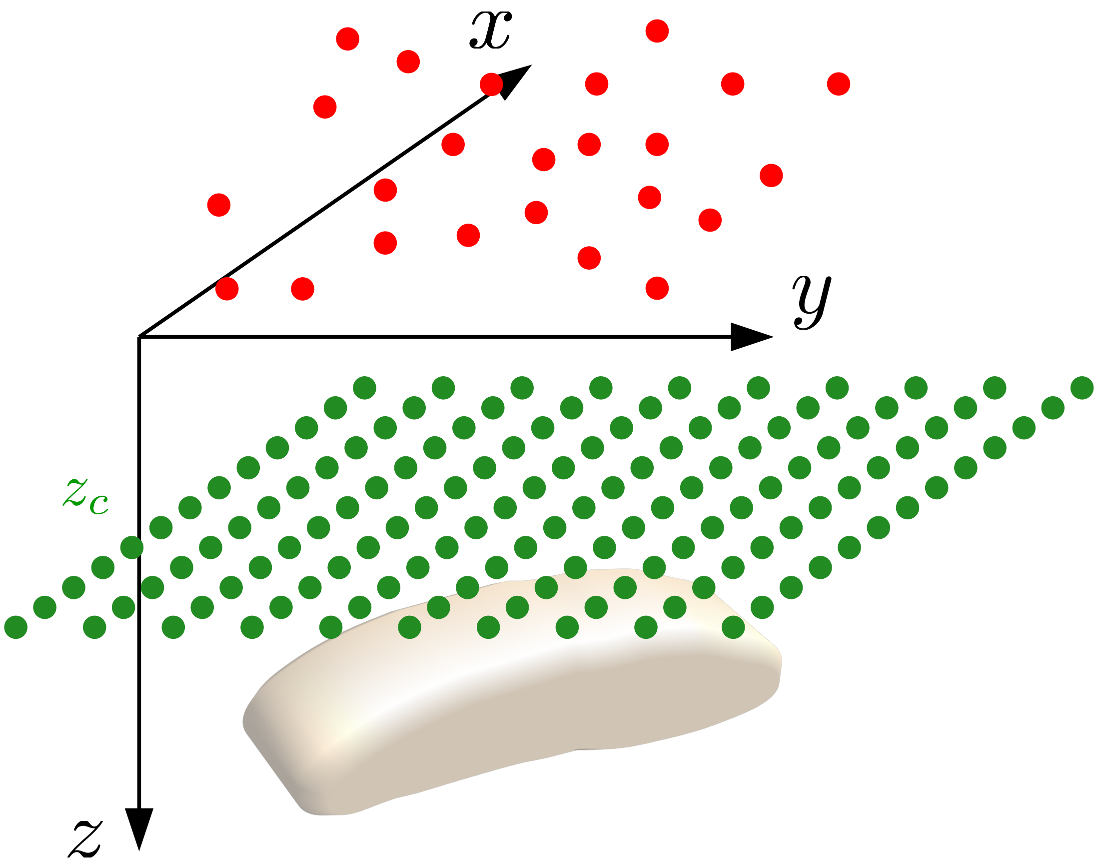
transformed data

t_{kj}

interpolation, upward continuation, RTP, field component conversion, ...

estimated parameter vector (equivalent layer)

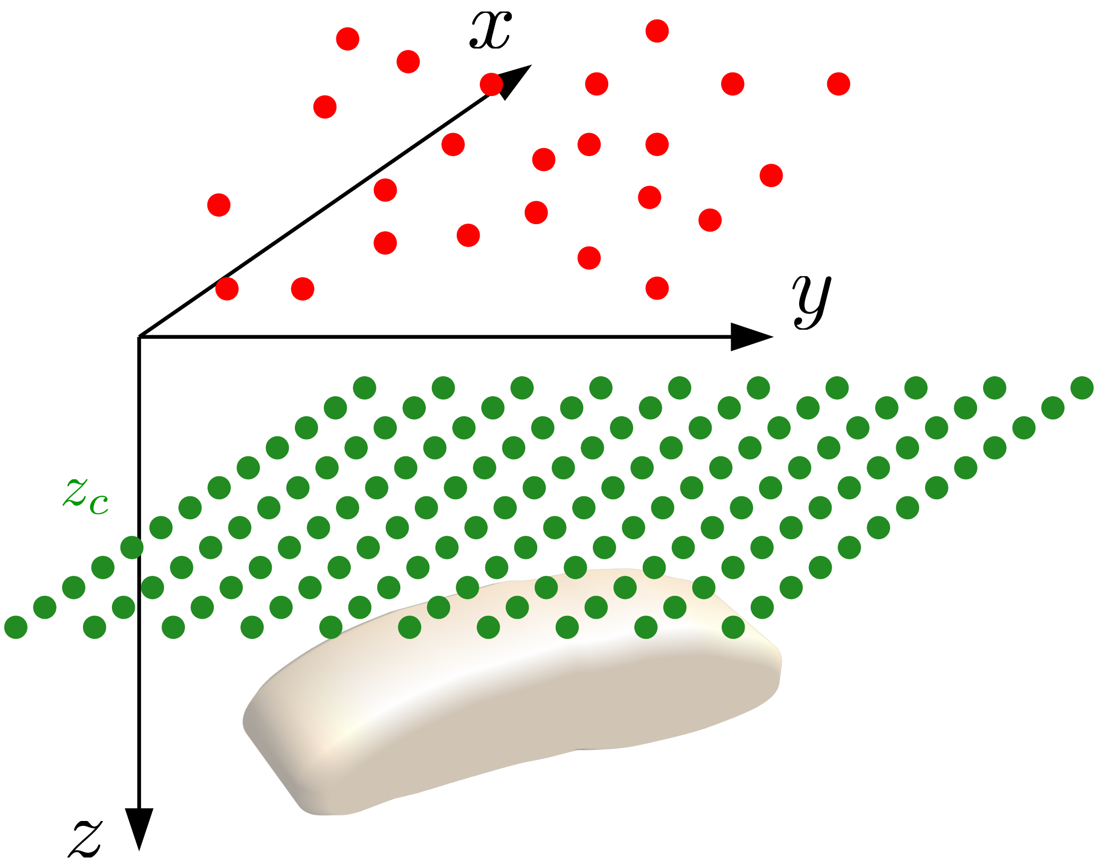
Classical EqL Technique



$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

A considerable amount of literature has been published on numerical methods for efficiently solving this linear system (e.g., Leão and Silva, 1989; Cordell, 1992; Mendonça and Silva, 1994; Guspí and Novara, 2009; Li and Oldenburg, 2010; Barnes and Lumley, 2011; Oliveira Jr. et al., 2013; Siqueira et al., 2017; Mendonça, 2020; Takahashi et al., 2020; Soler and Uieda, 2021)

Classical EqL Technique



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In this presentation, however, I will explore theoretical aspects of the EqL technique

Summary

- Motivation
- Potential-field data
- **The Equivalent-Layer (EqL) Technique**
- Theoretical aspects
- Some open questions

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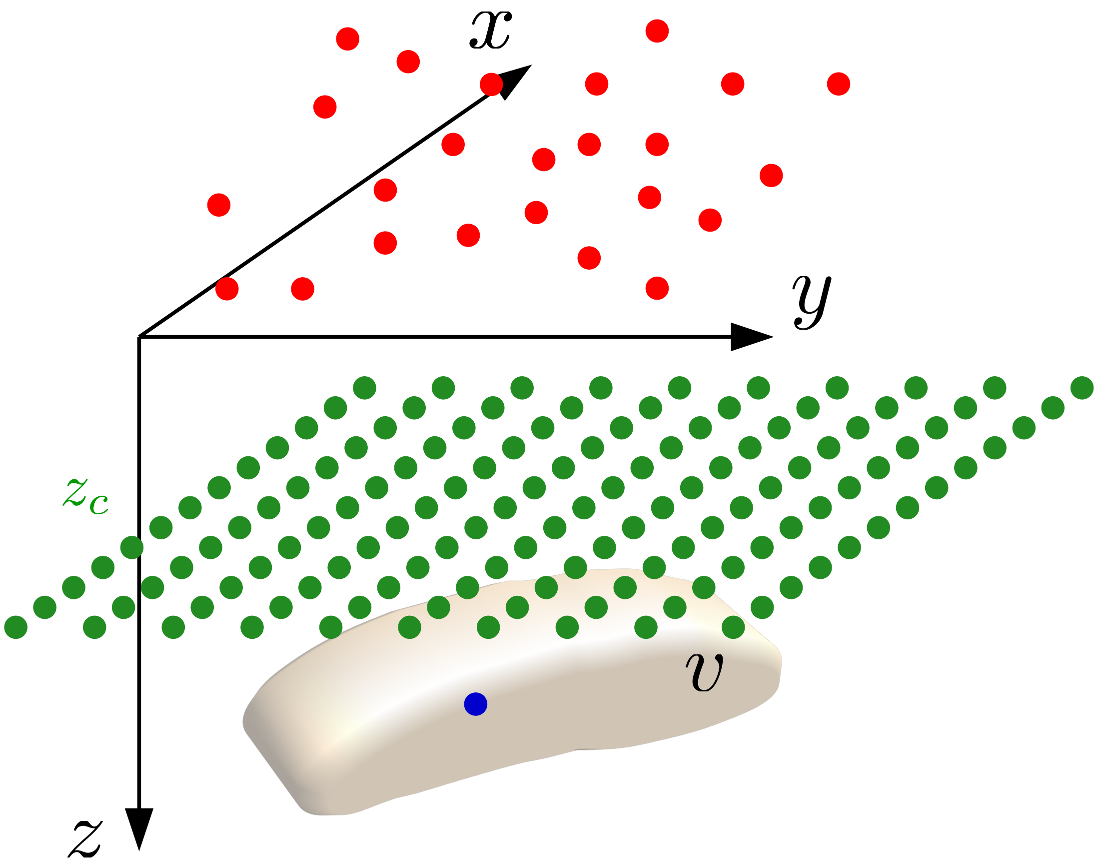
- Motivation
- Potential-field data
- The Equivalent-Layer (EqL) Technique
- **Theoretical aspects**
- Some open questions

We have briefly discussed **how** the EqL technique works, ...

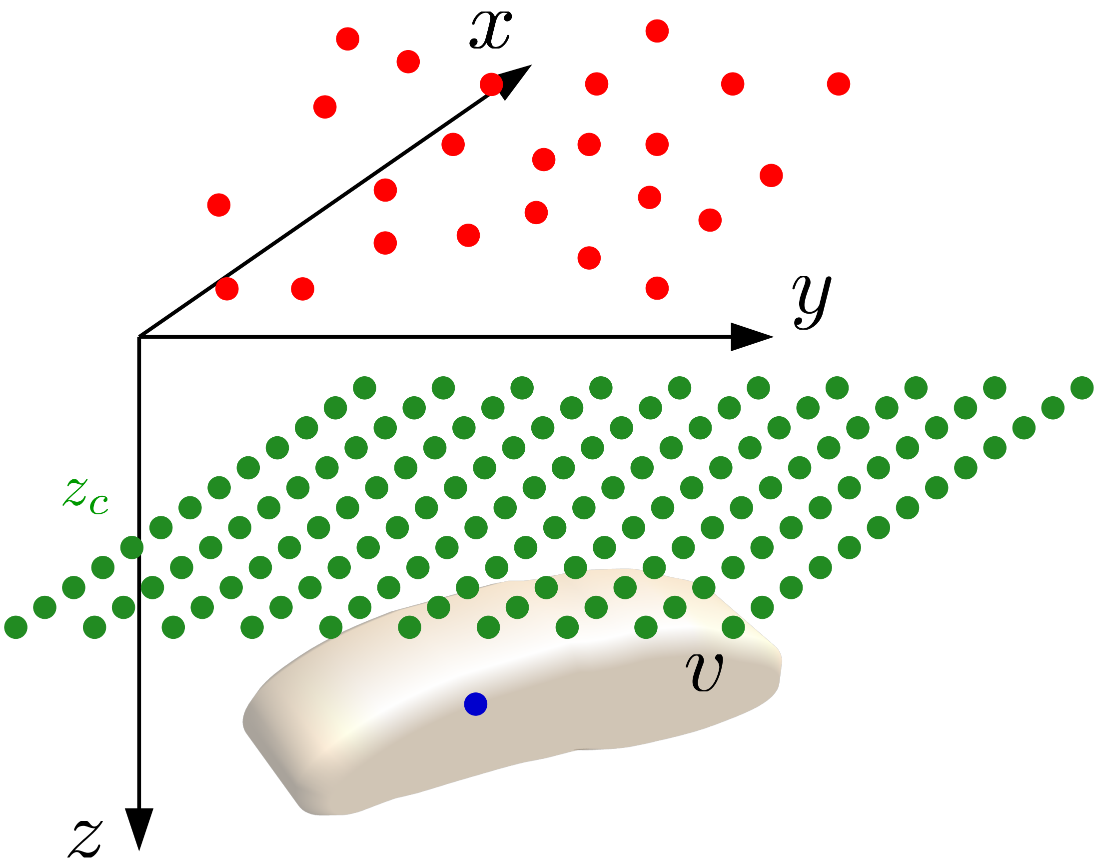
We have briefly
discussed **how** the EqL
technique works, ...

... now we need to
discuss **why**
the EqL technique works



$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

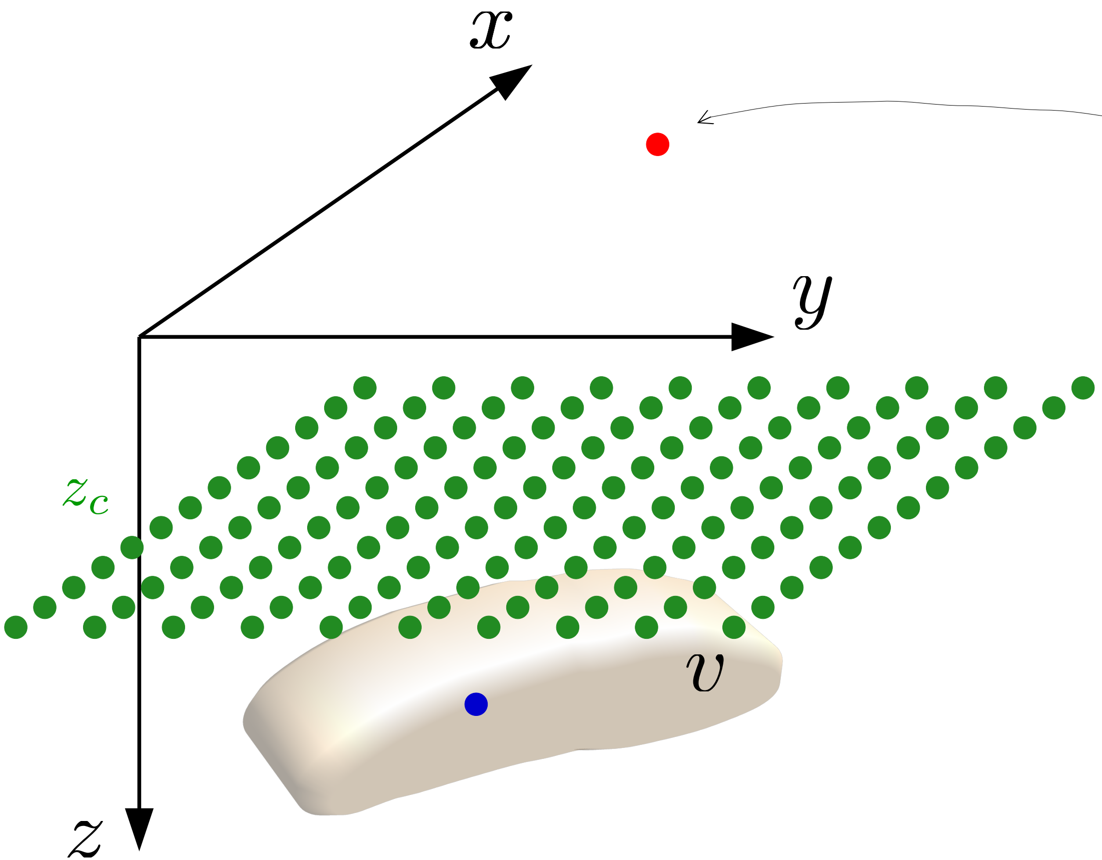
Let us return to the linear system for estimating the discrete equivalent layer



Discrete equivalent layer

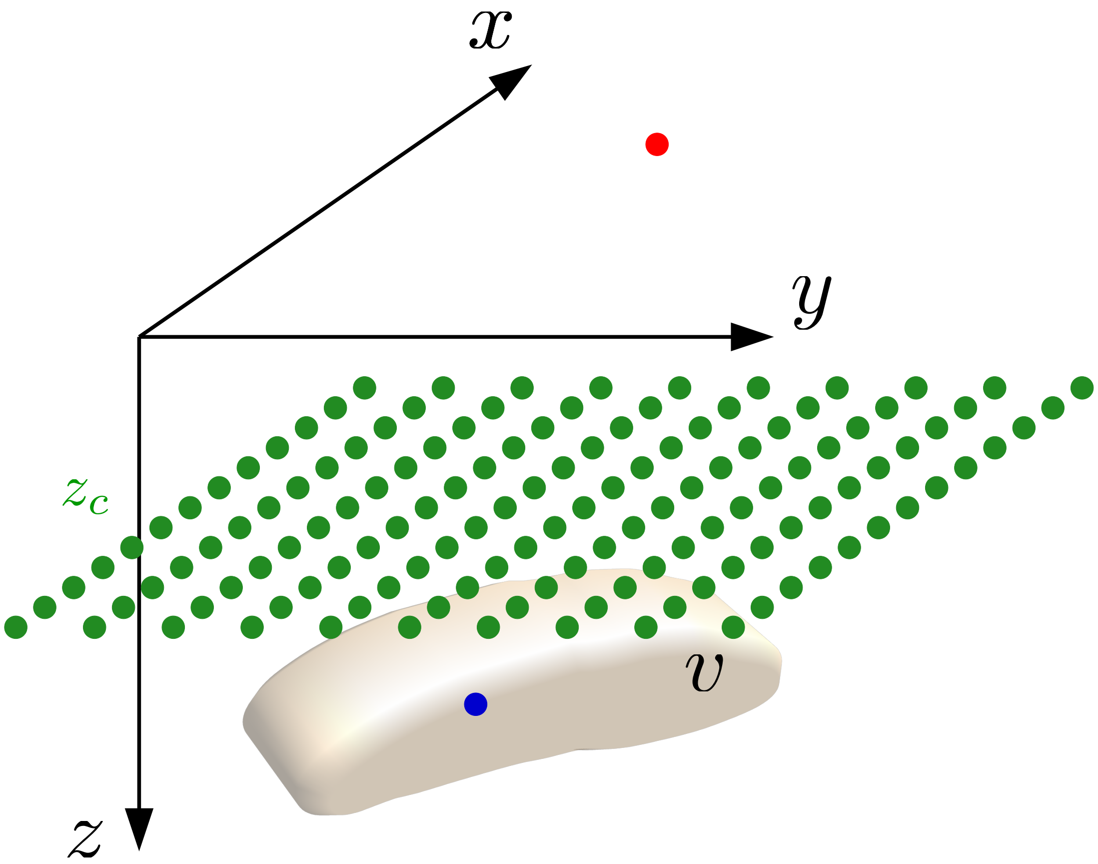
$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

Let us return to the linear system for estimating the discrete equivalent layer



$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

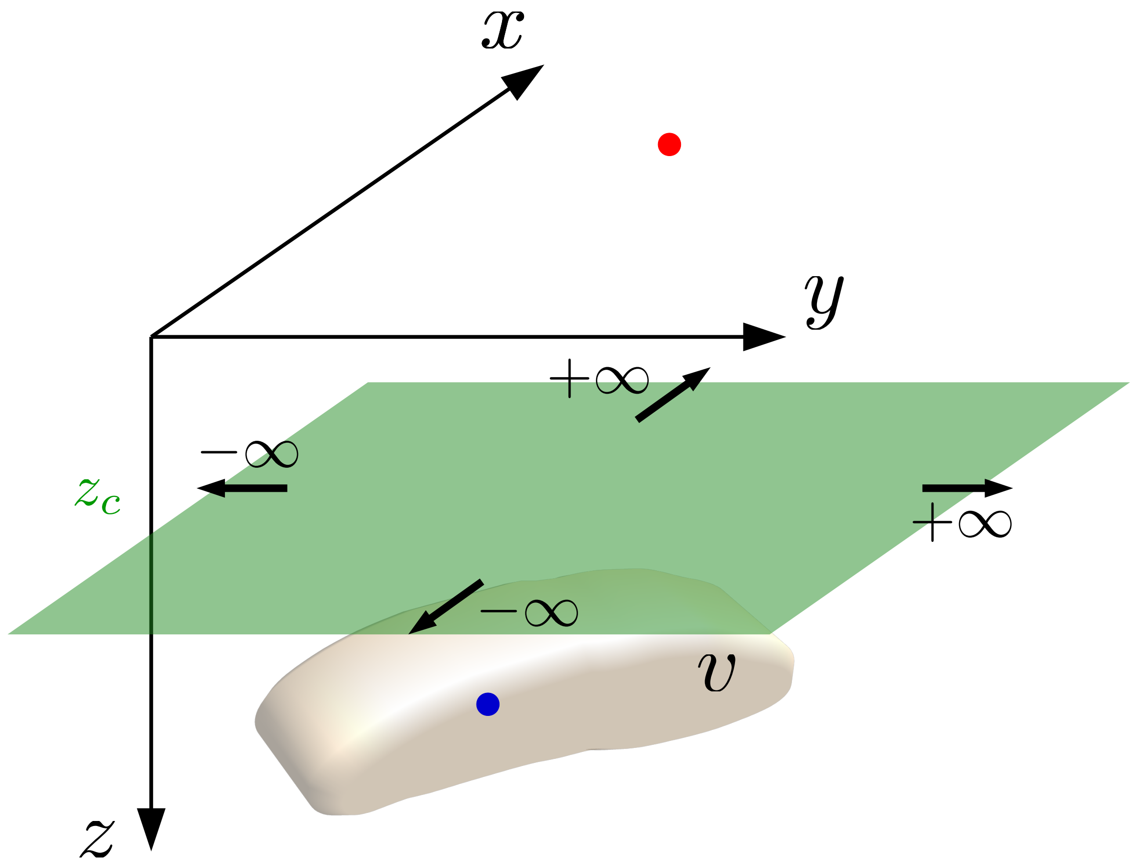
$$d_i \approx \sum_{j=1}^{M} p_j \, a_{ij}$$



$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

$$d_i \approx \sum_{j=1}^{M} p_j \, a_{ij}$$

Finite set of points defining
a **discrete equivalent layer**



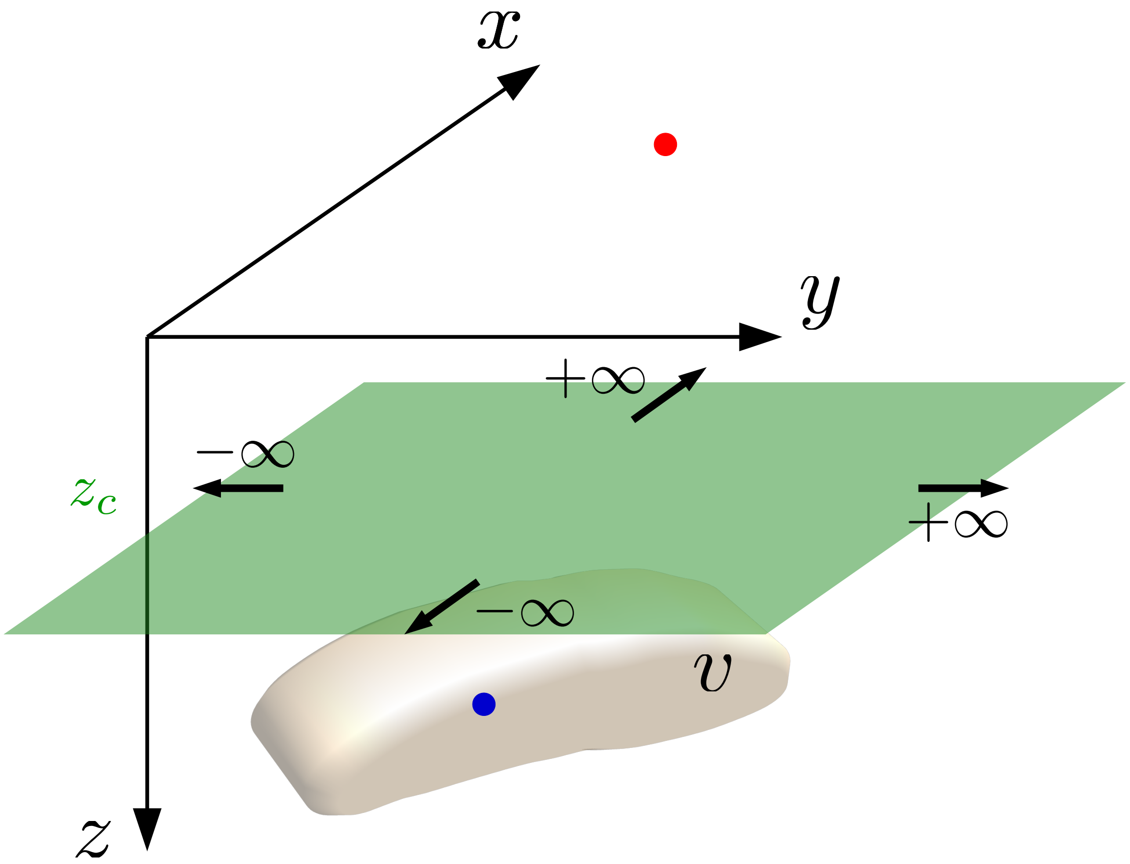
$$\mathbf{d} \approx \mathbf{A}\,\mathbf{p}$$

$$d_i \approx \sum_{j=1}^{M} p_j \, a_{ij}$$

Finite set of points defining
a **discrete equivalent layer**

$$d(\mathbf{r}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(\mathbf{r}'') \, a(\mathbf{r}, \mathbf{r}'') \, \overbrace{dS''}^{dx''\,dy''}$$

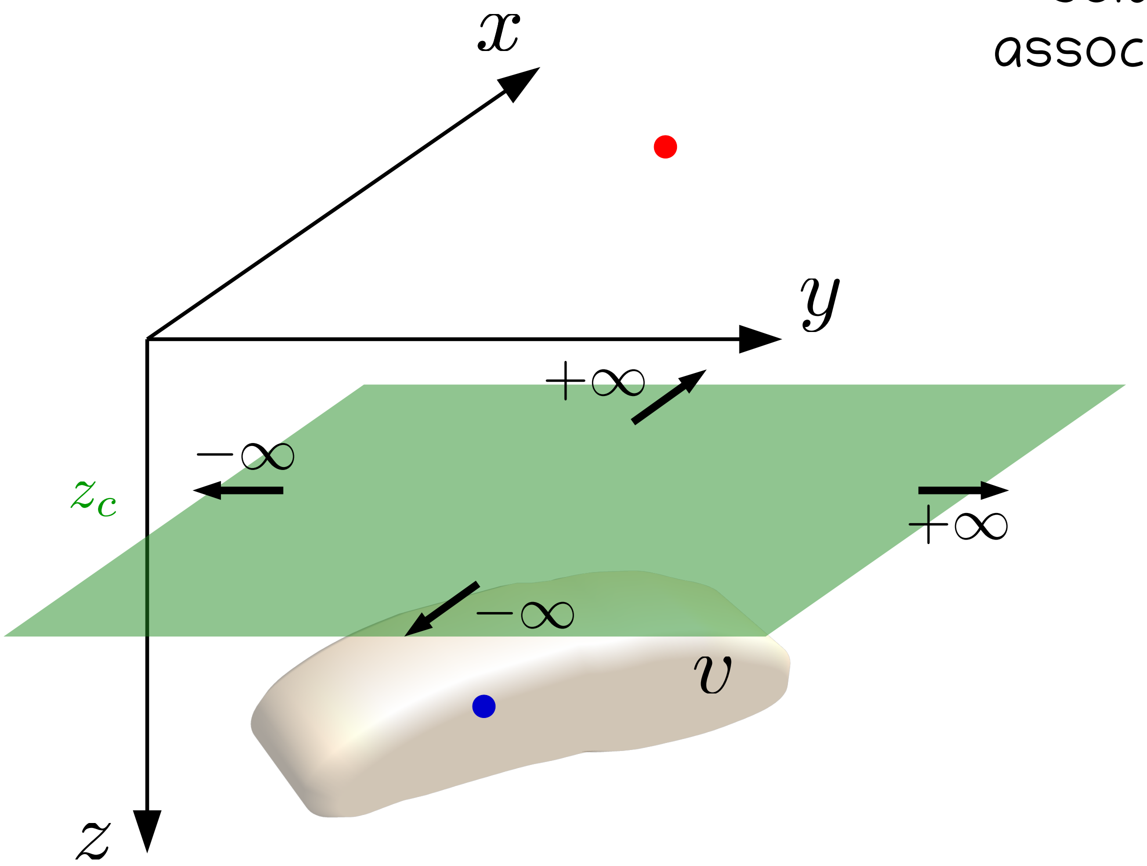
Infinite horizontal plane defining
a **continous equivalent layer**



Integral equation for the continuous physical-property distribution (equivalent layer) $p(\mathbf{r}'')$

$$d(\mathbf{r}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(\mathbf{r}'')\, a(\mathbf{r}, \mathbf{r}'')\, dS''$$

$dx''\, dy''$



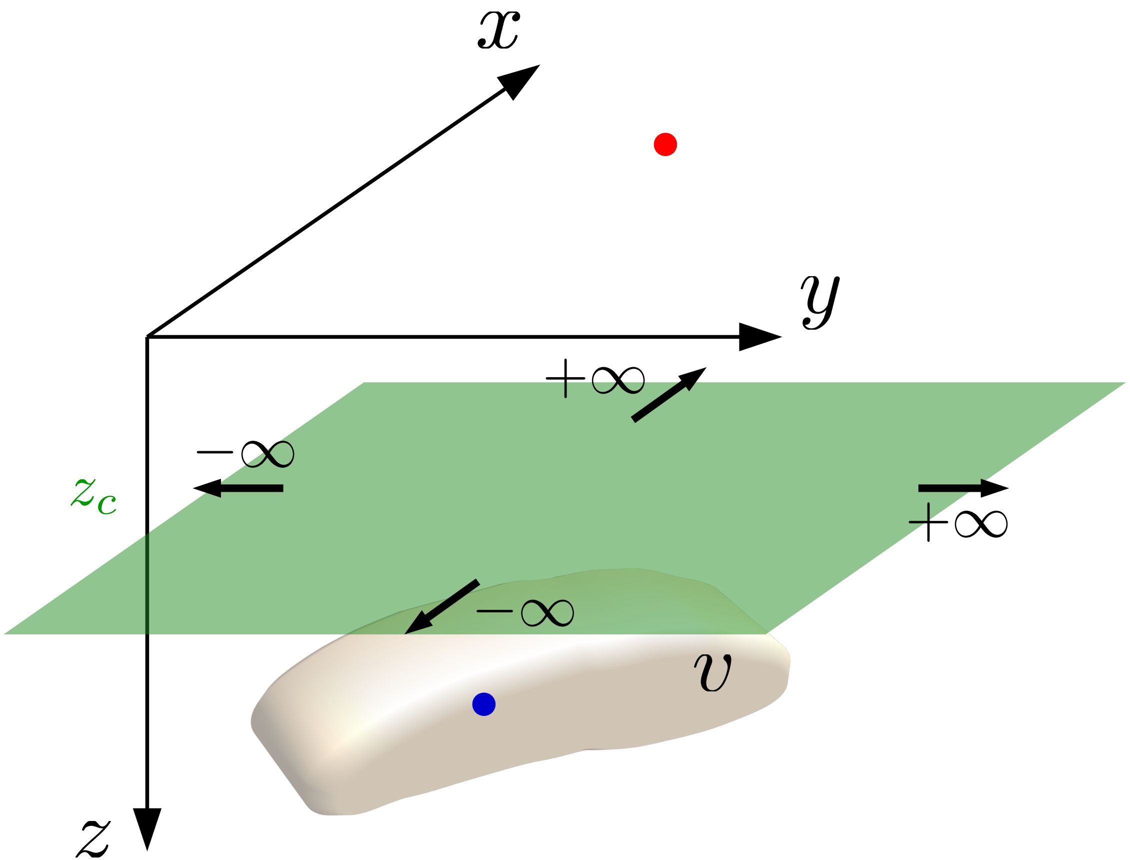
The following slides present analytical solutions for the integral equations associated with different potential-field data

Integral equation for the continuous physical-property distribution (equivalent layer) $p(\mathbf{r}'')$

$dx''\,dy''$

$$d(\mathbf{r}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(\mathbf{r}'')\, a(\mathbf{r}, \mathbf{r}'')\, dS''$$

These solutions are conveniently called here **analytical equivalent layers**



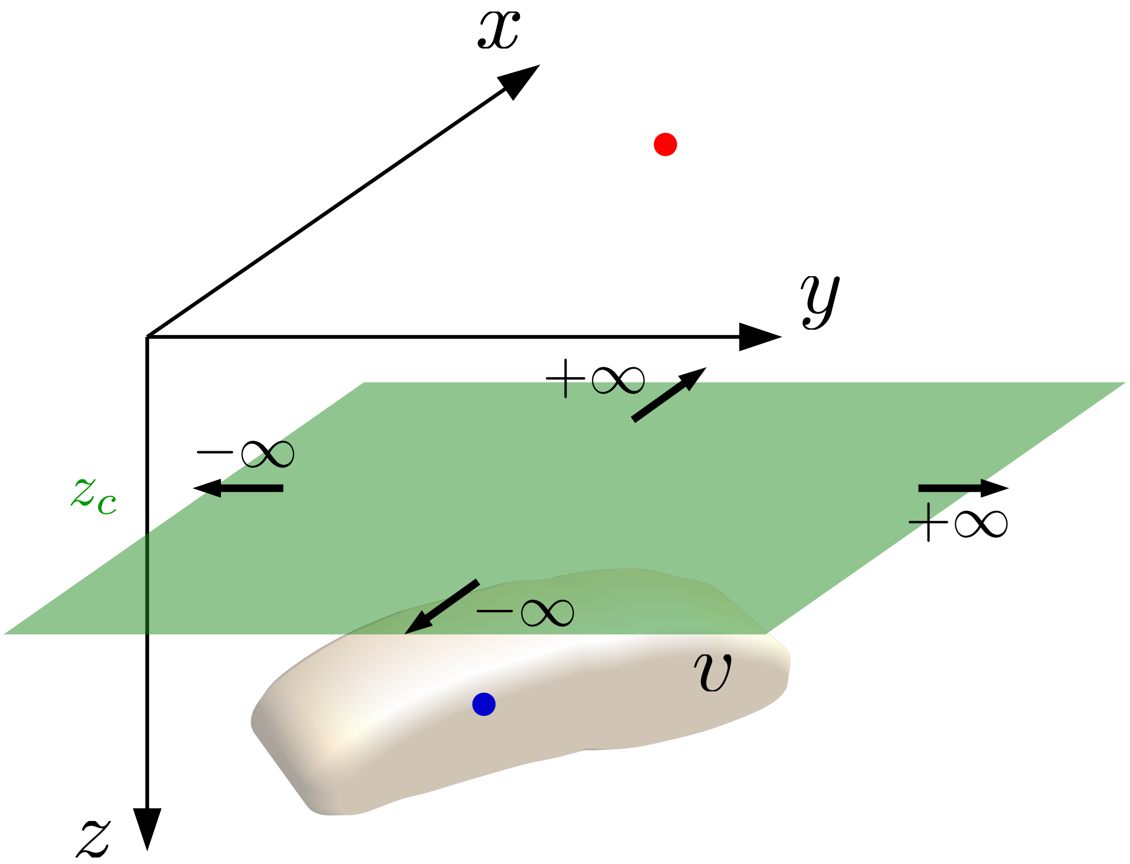
Integral equation for the continuous physical-property distribution (equivalent layer) $p(\mathbf{r}'')$

$dx''\,dy''$

$$d(\mathbf{r}) = \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} p(\mathbf{r}'')\, a(\mathbf{r}, \mathbf{r}'')\, dS''$$

Let us turn our attention back to this function

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$



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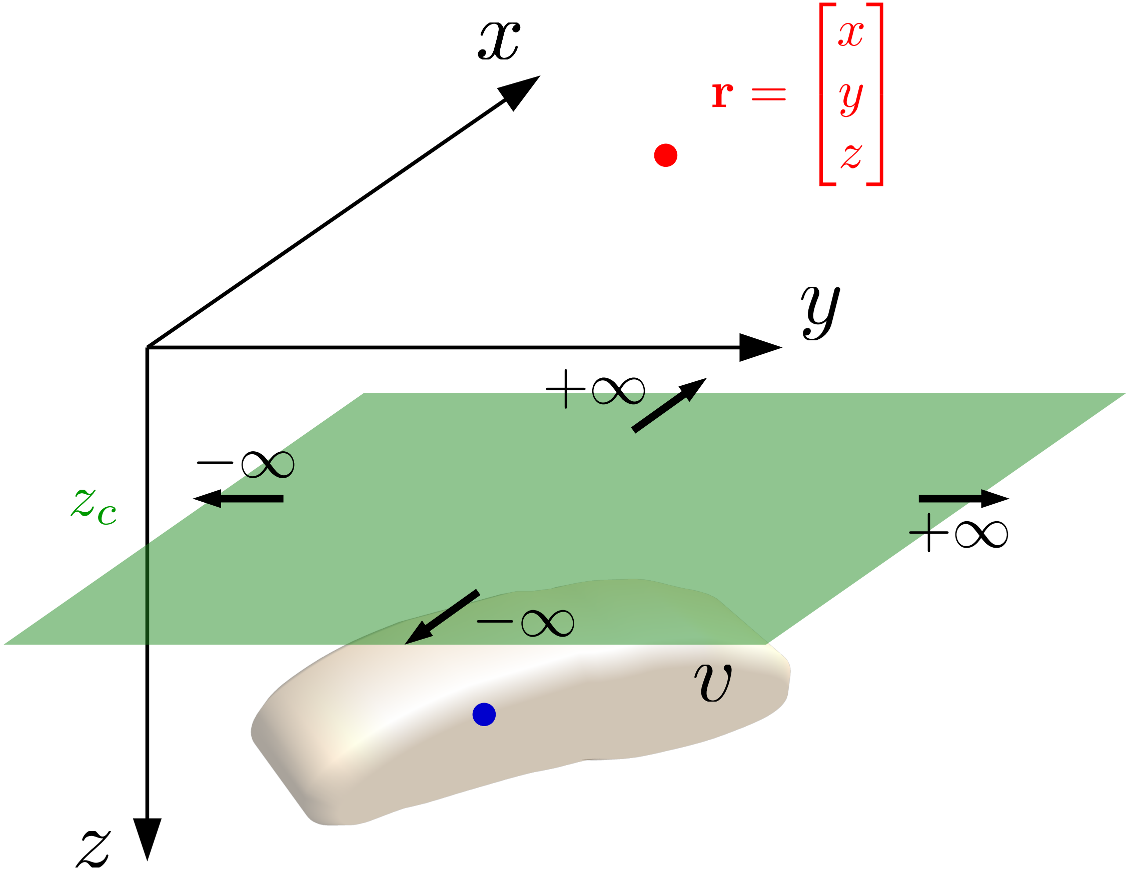

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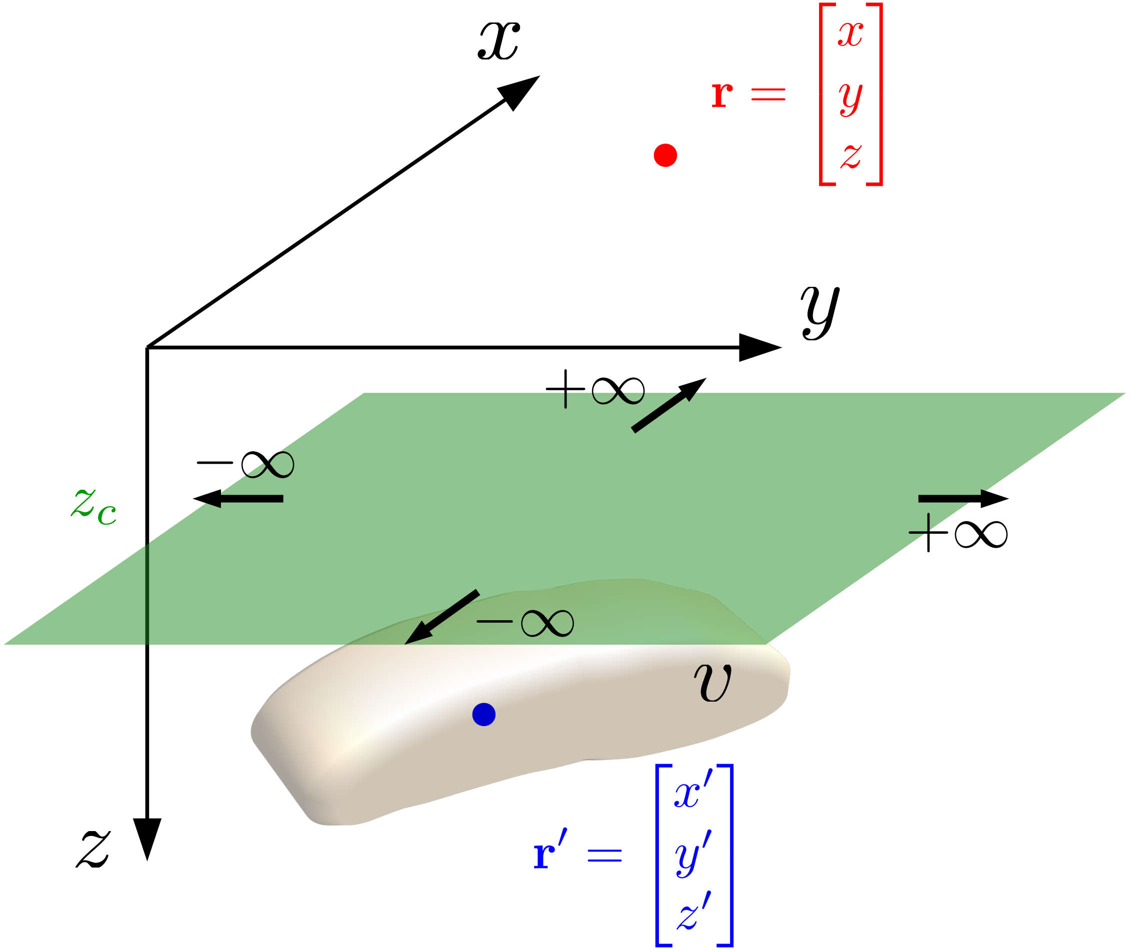
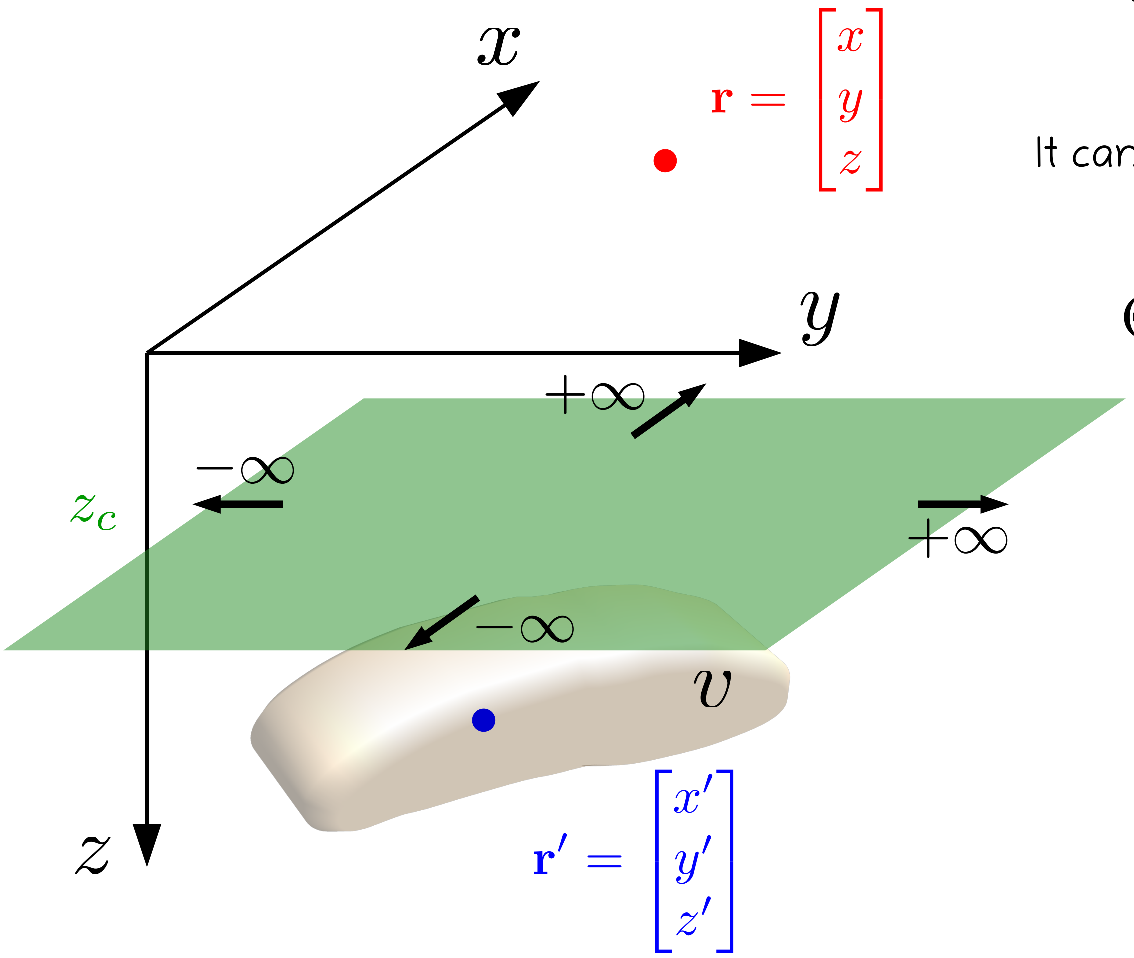


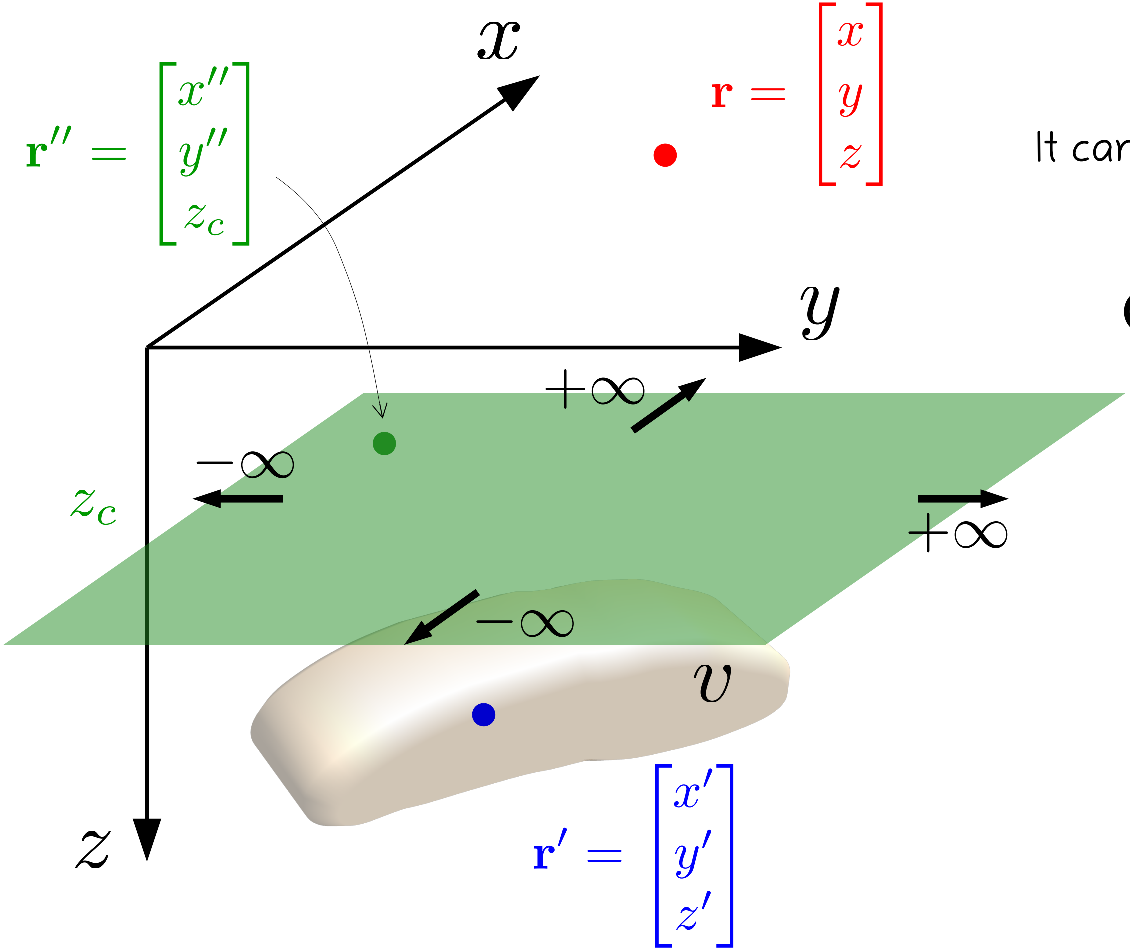


$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

It can be shown that (e.g., Roy, 1962; Reis et al., 2020):

$$\Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

which represents the solution of the **Neumann's problem** or the **second boundary value problem of potential theory** (Kellogg, 1967, p. 246) on a plane



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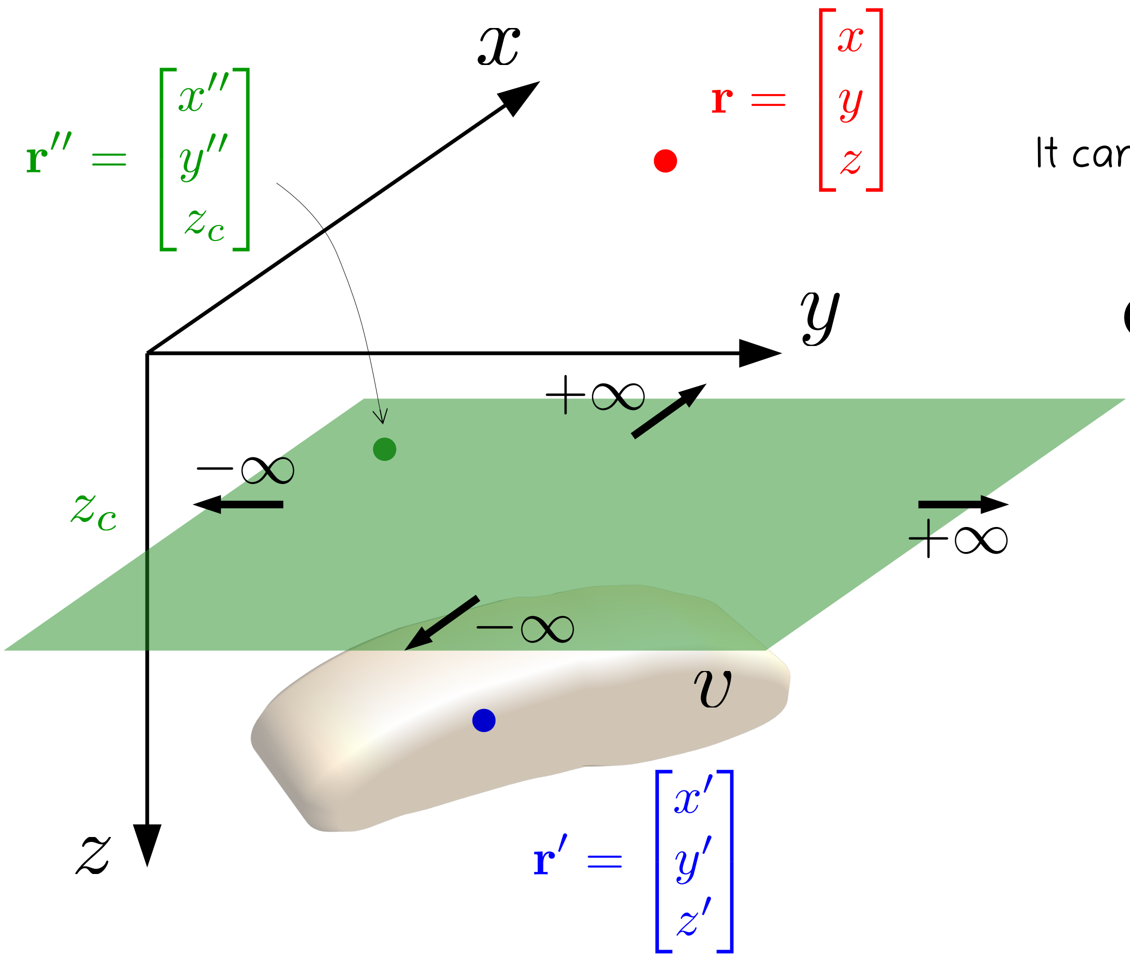

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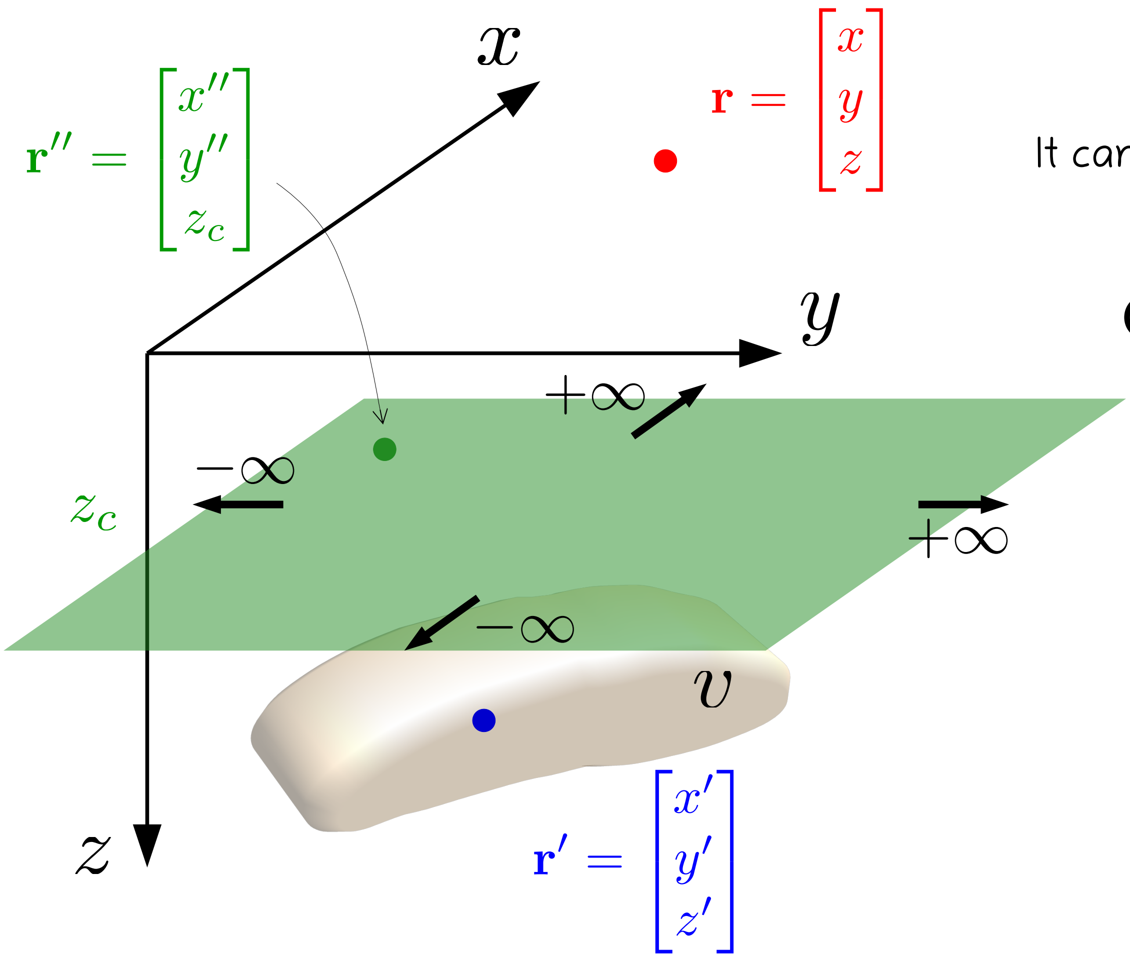

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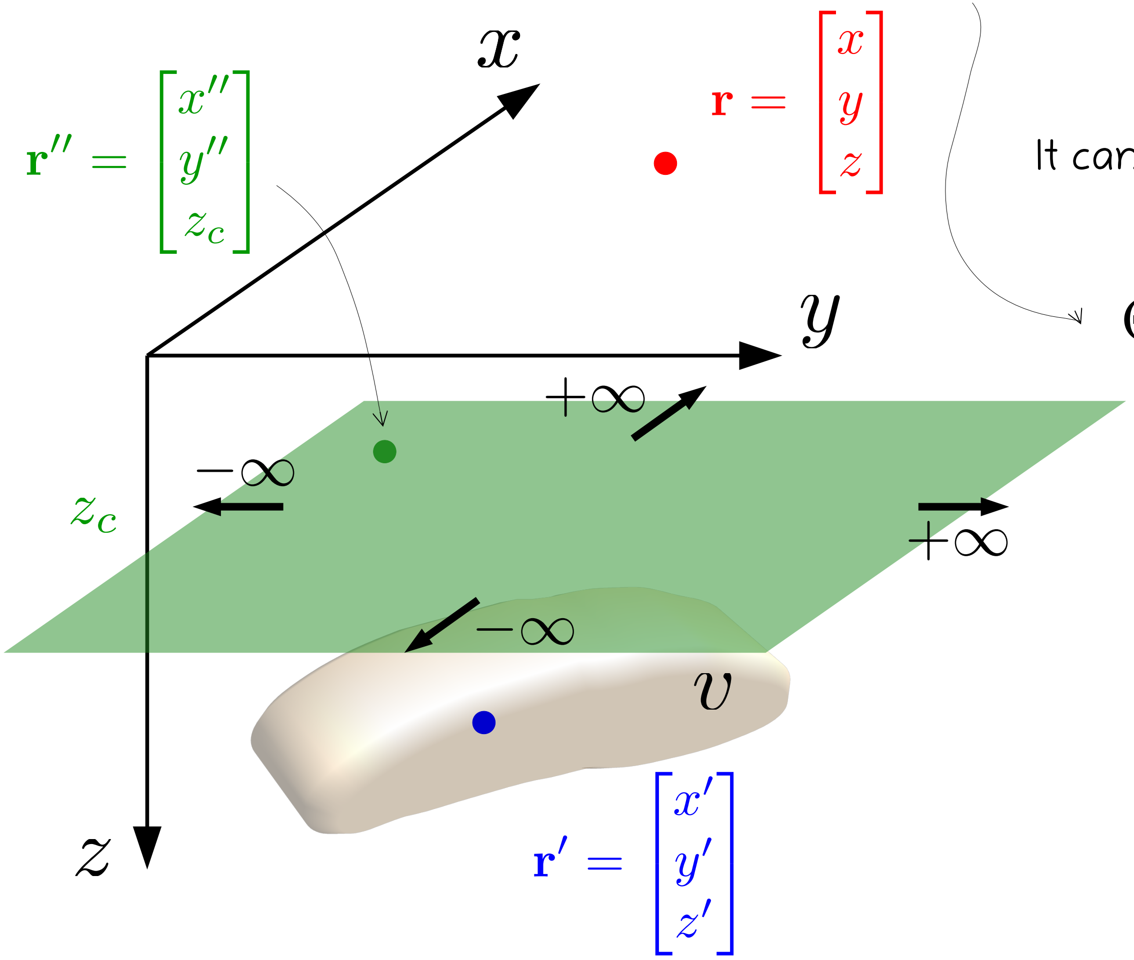
$$\partial_z \Theta(\mathbf{r}'') = \kappa \iiint_v \sigma(\mathbf{r}') \; \partial_z \frac{1}{\|\mathbf{r}'' - \mathbf{r}'\|} dv'$$

$$\Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$

Most of what is known or assumed to be true without any proof about the EqL technique can be deduced from this equation



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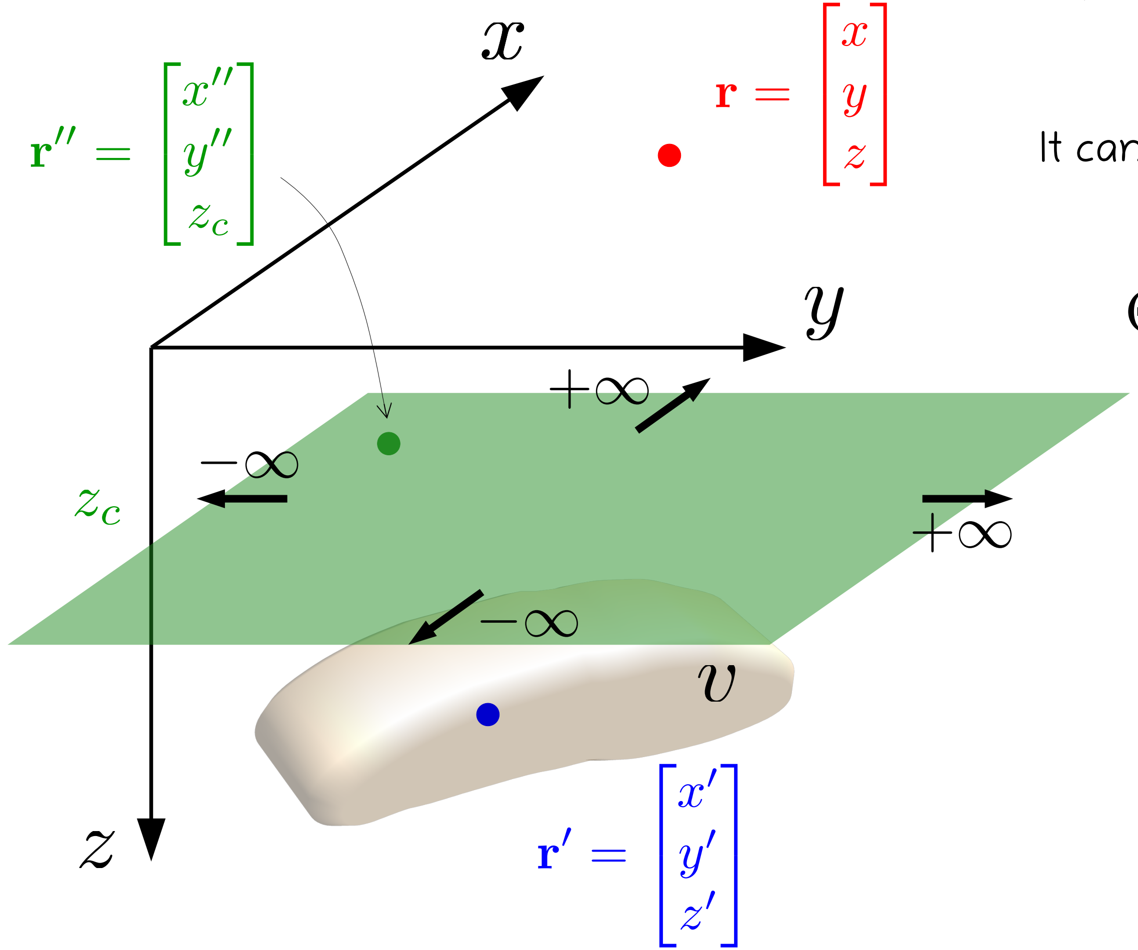

We are going to see four theoretical results that support most of the common potential-field transformations made via EqL technique

It can be shown that (e.g., Roy, 1962; Reis et al., 2020):

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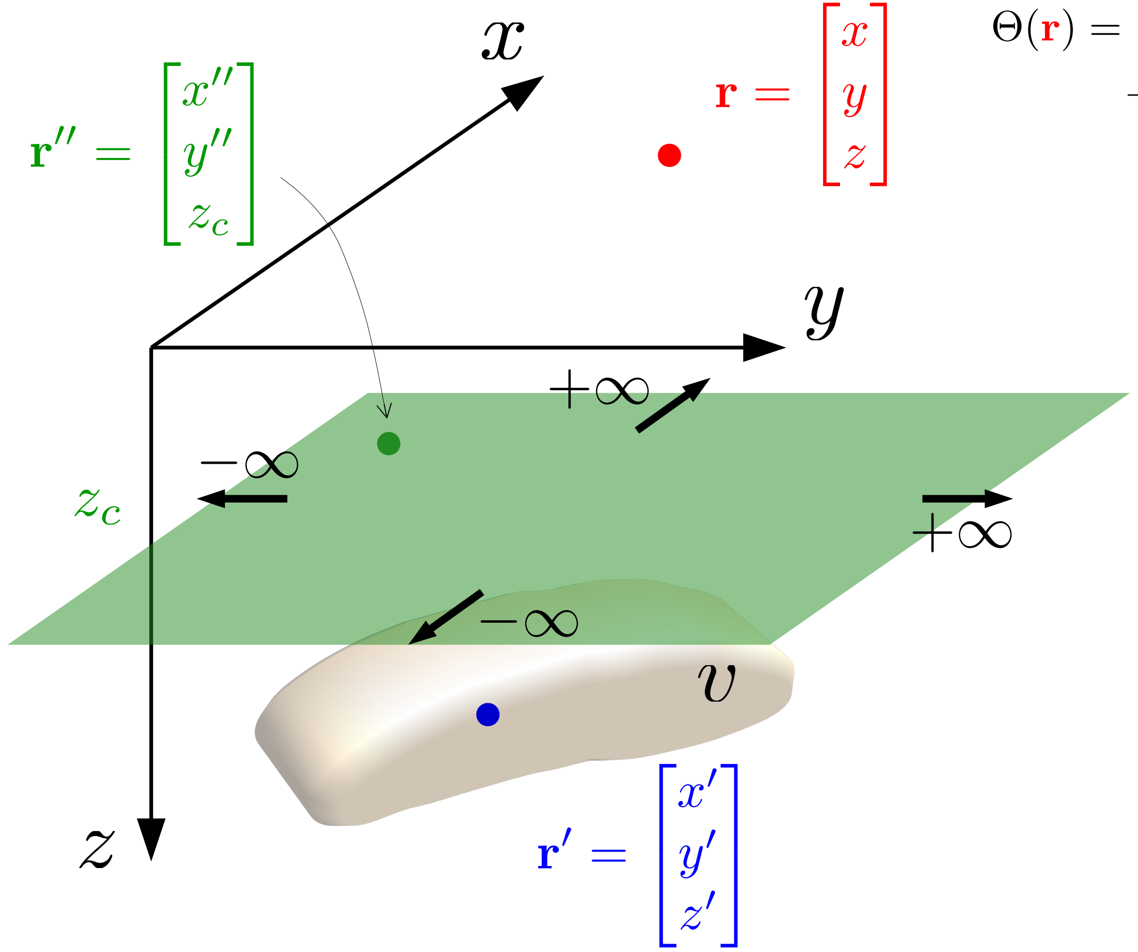

Result 1/4: The gravity disturbance can be exactly reproduced by an analytical eq. layer of monopoles

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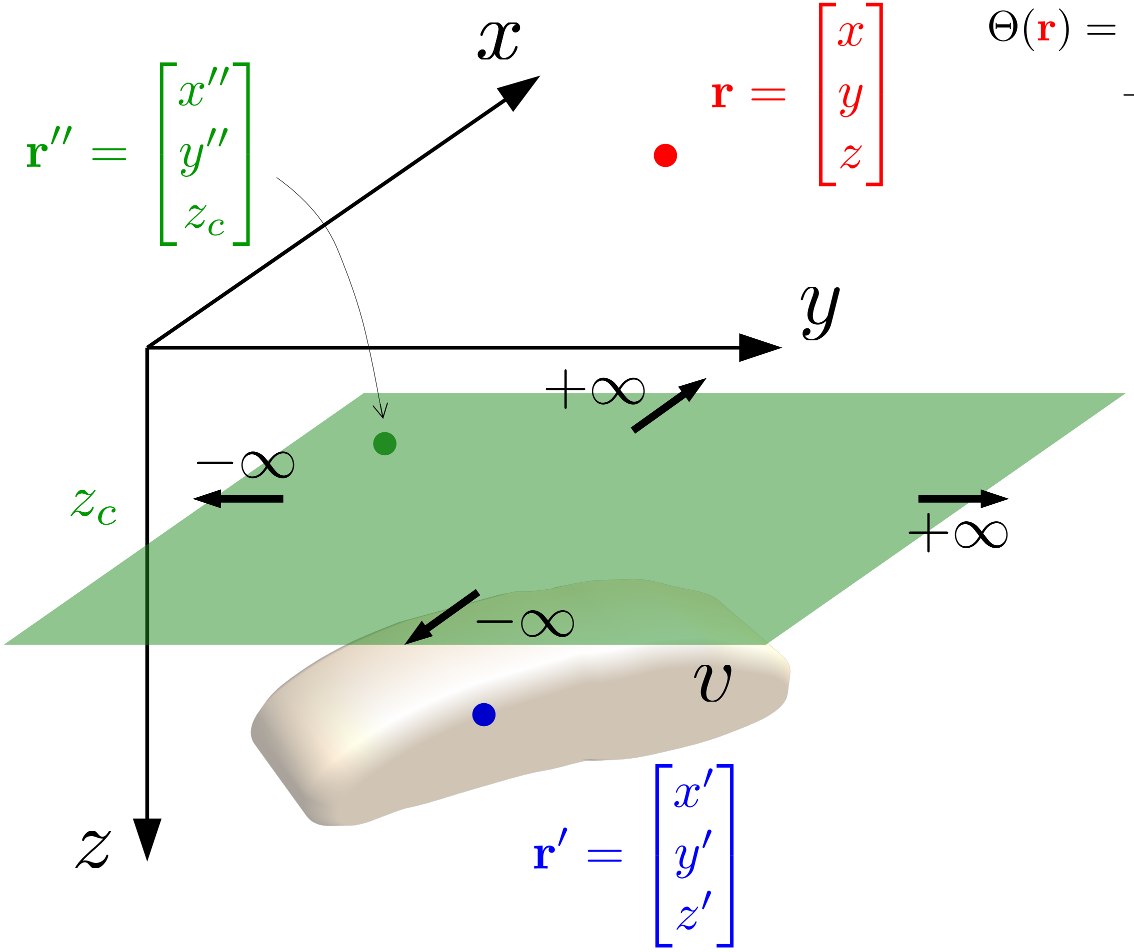

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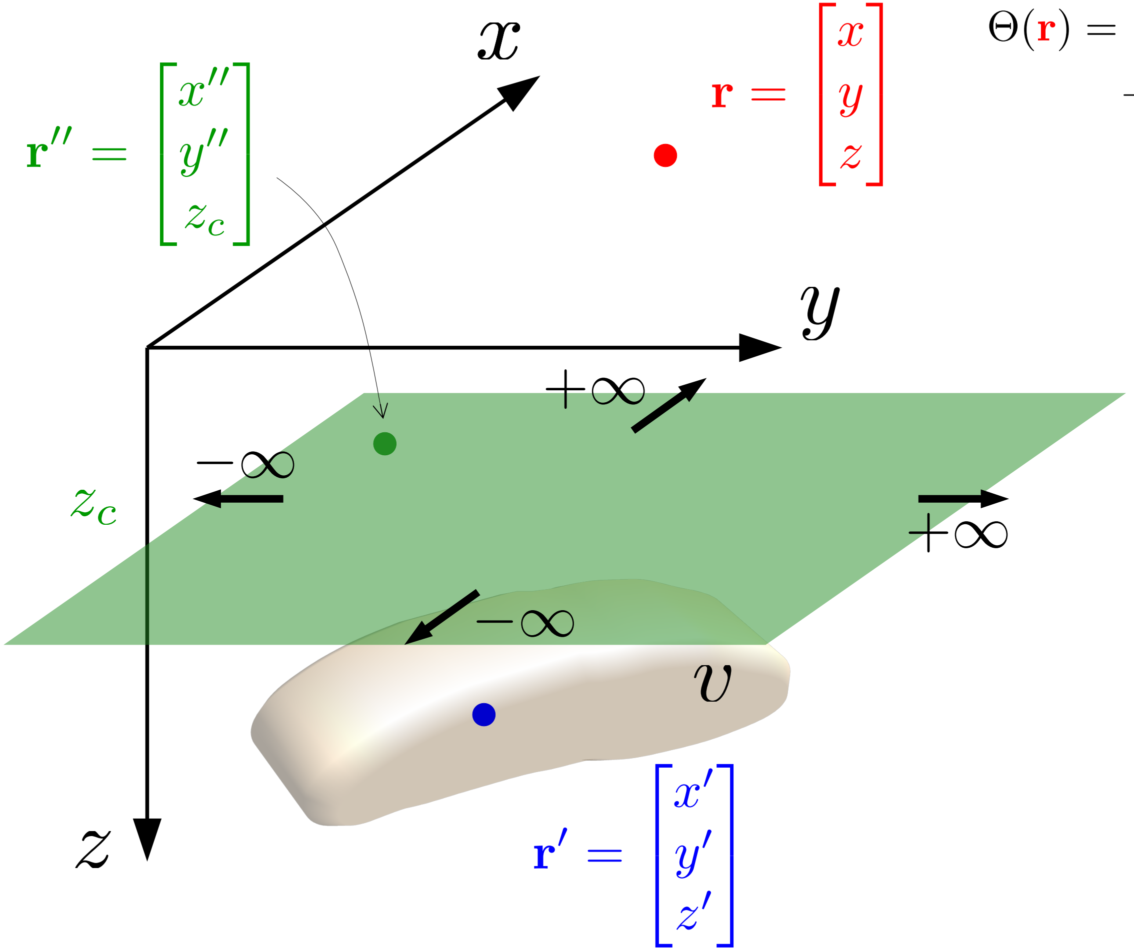

Dampney (1969) was the pioneer in using this result to interpolate gravity data.

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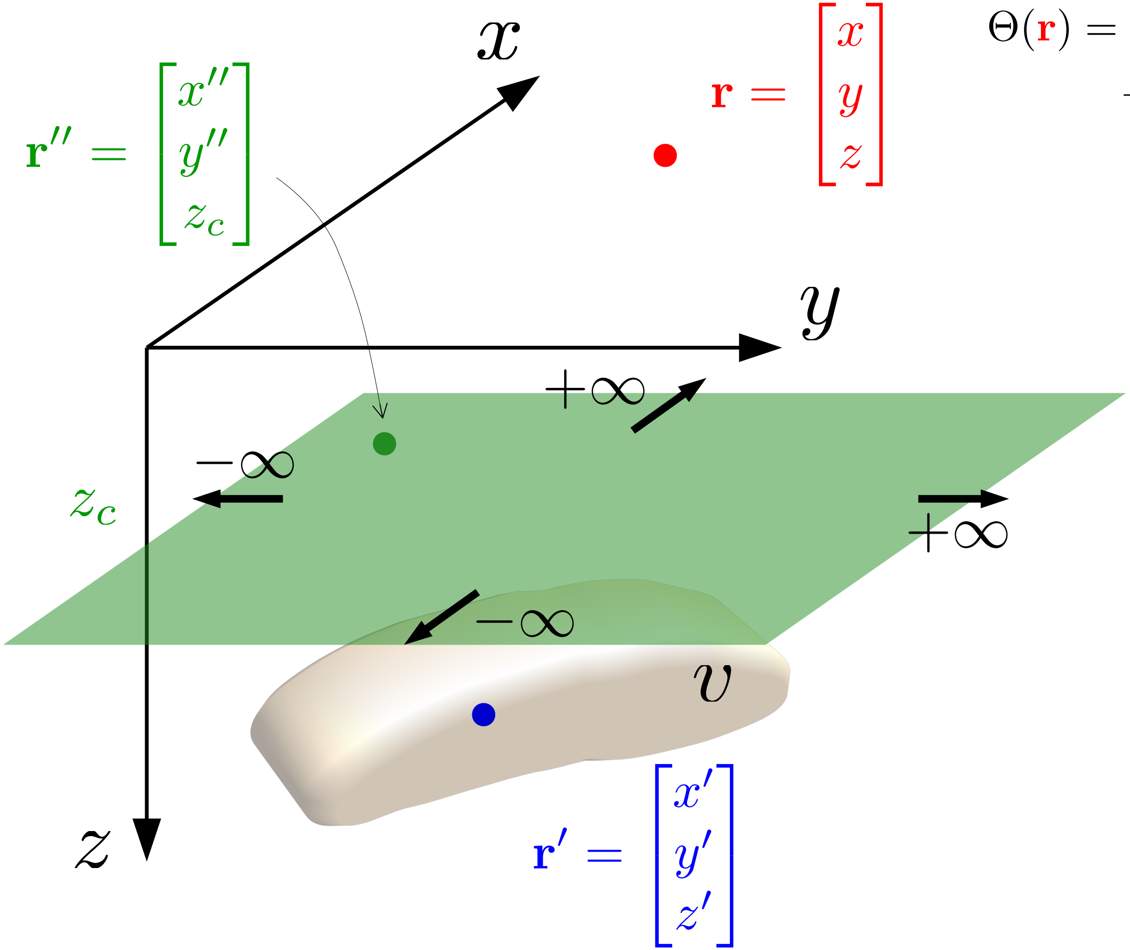


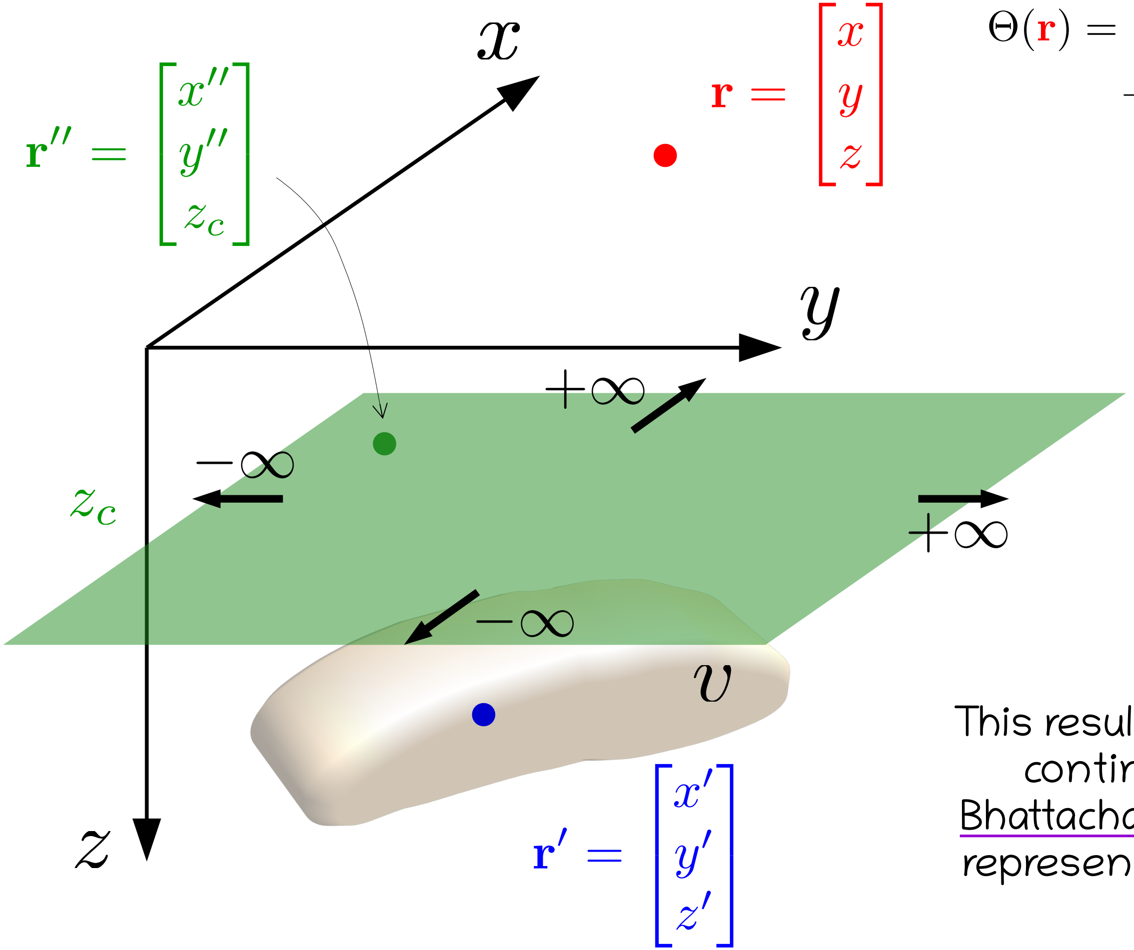
Dampney (1969) was the pioneer in using this result to interpolate gravity data.

A considerable amount of literature has been published on this topic since then

Result 1/4: The gravity disturbance can be exactly reproduced by an analytical eq. layer of monopoles

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$

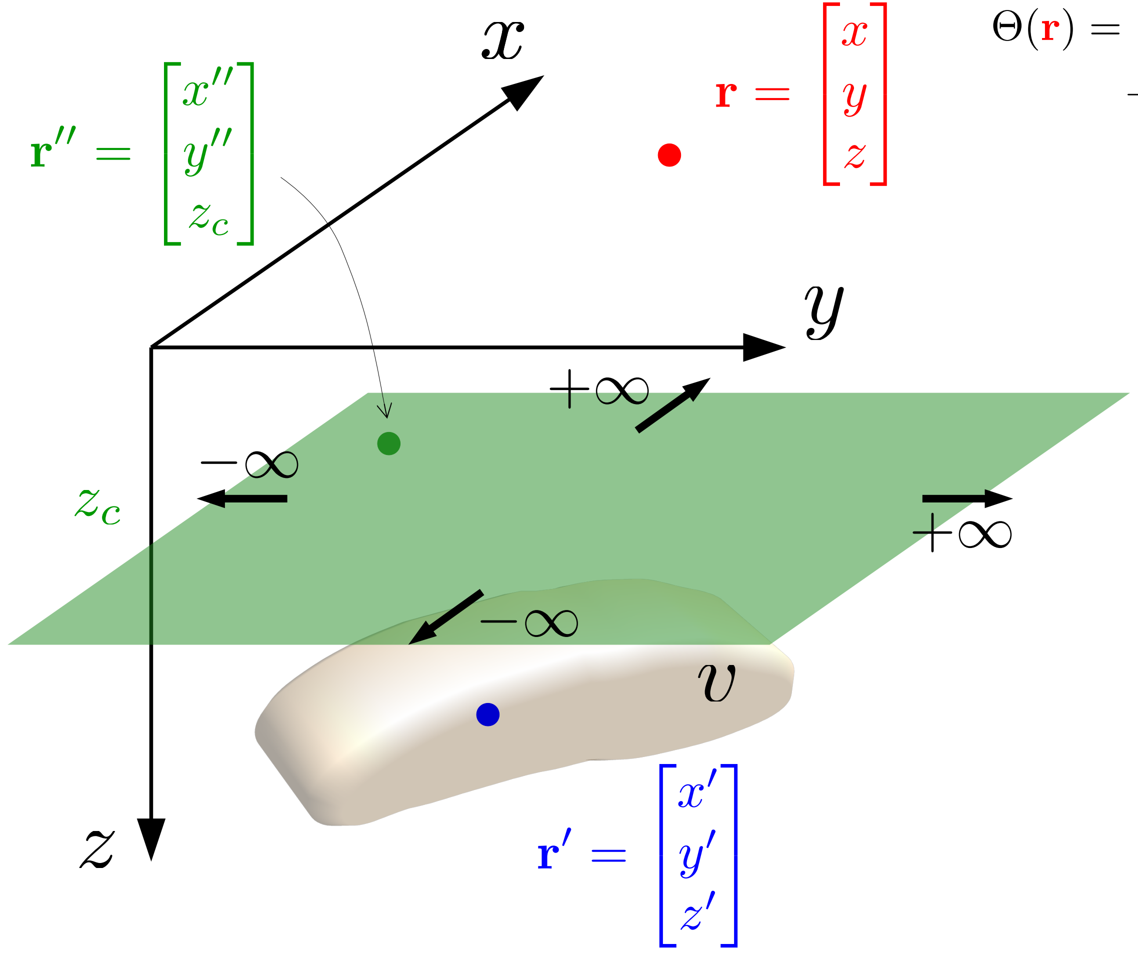


Dampney (1969) was the pioneer in using this result to interpolate gravity data.

A considerable amount of literature has been published on this topic since then

This result can also be directly deduced from the upward-continuation integral (e.g., Peters, 1949; Roy, 1962; Bhattacharyya, 1967; Henderson, 1970, Gunn, 1975), which represents the solution of Dirichlet's problem on a plane (Kellogg, 1967, p. 236)

Result 1/4: The gravity disturbance can be exactly reproduced by an analytical eq. layer of monopoles



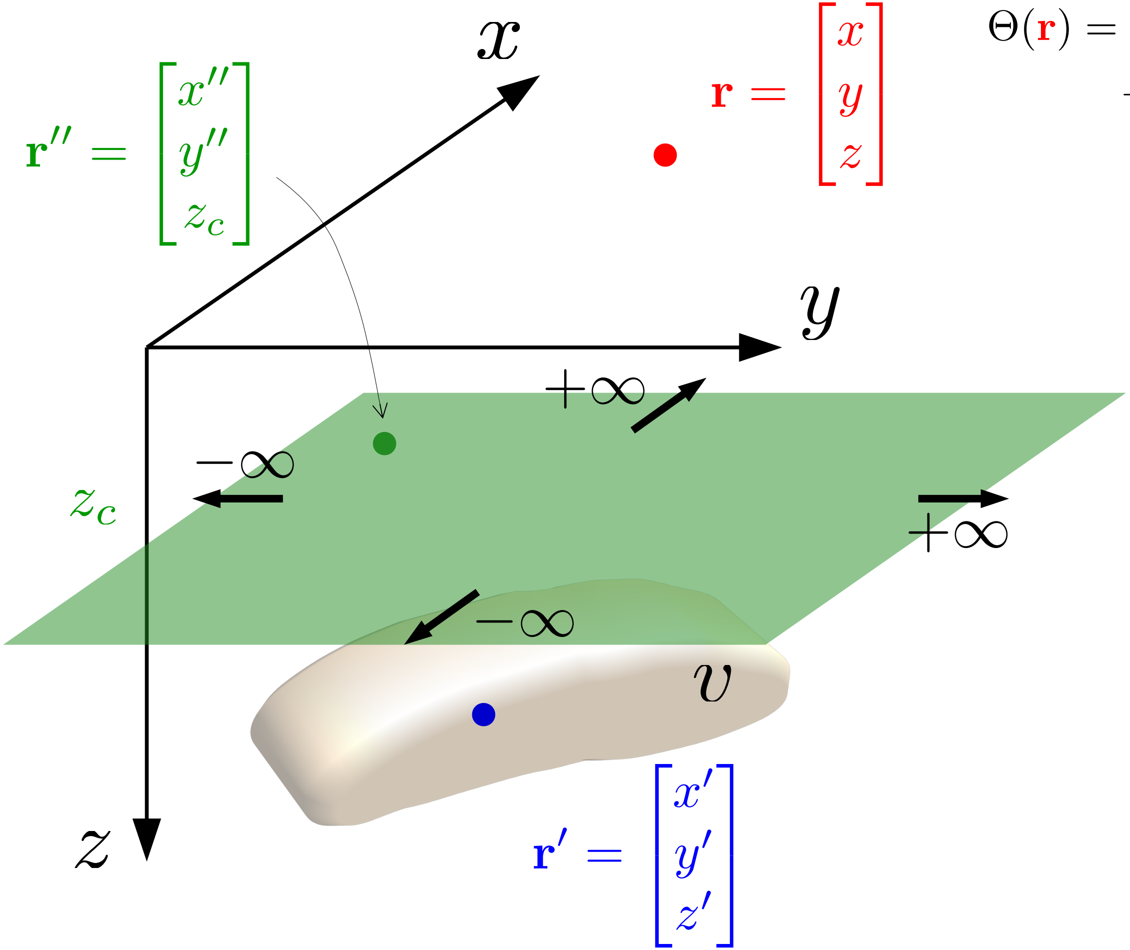
$$\Theta(\mathbf{r}) = \kappa \iiint\limits_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

$$\Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$

Deduction:

$\partial_z \Theta(\mathbf{r})$ gravity disturbance data

Result 1/4: The gravity disturbance can be exactly reproduced by an analytical eq. layer of monopoles



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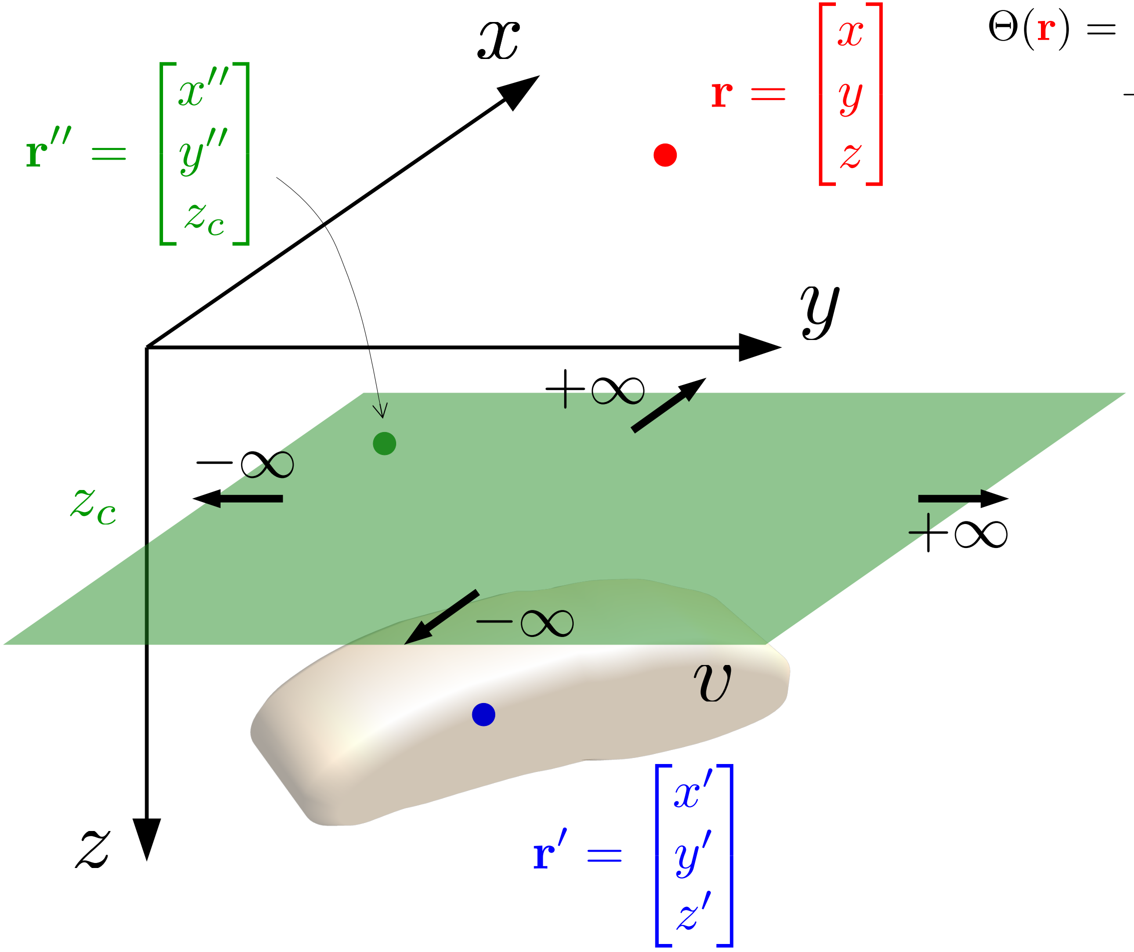
Deduction:

$\partial_z \Theta(\mathbf{r})$ gravity disturbance data

$$\partial_z \Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Result 1/4: The gravity disturbance can be exactly reproduced by an analytical eq. layer of monopoles



$$\Theta(\mathbf{r}) = \kappa \iiint\limits_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$

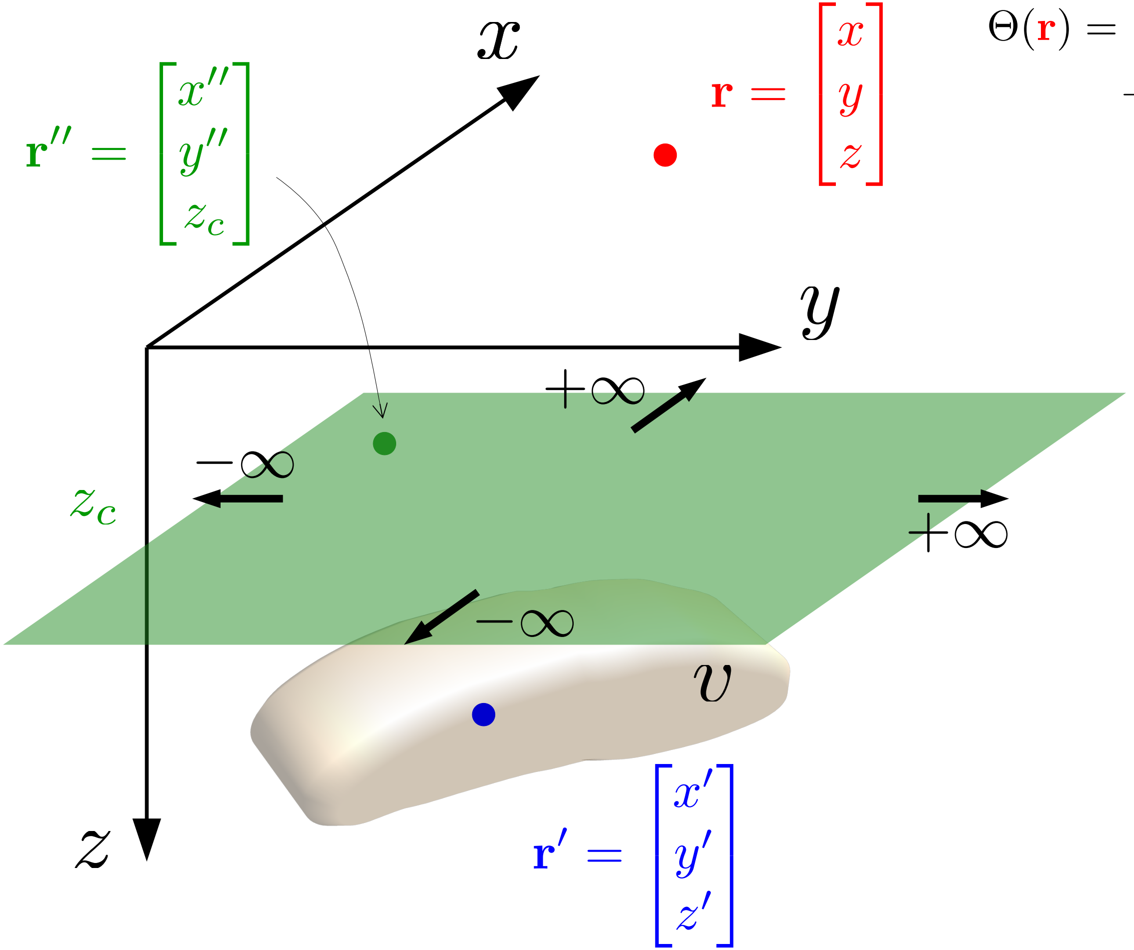
Deduction:

$\partial_z \Theta(\mathbf{r})$ gravity disturbance data

$$\partial_z \Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \underbrace{\partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r}, \mathbf{r}'') \, dS''}$$

This term represents the gravity disturbance produced at $\mathbf{r}$ by a monopole located at $\mathbf{r}''$

Result 1/4: The gravity disturbance can be exactly reproduced by an analytical eq. layer of monopoles



$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$

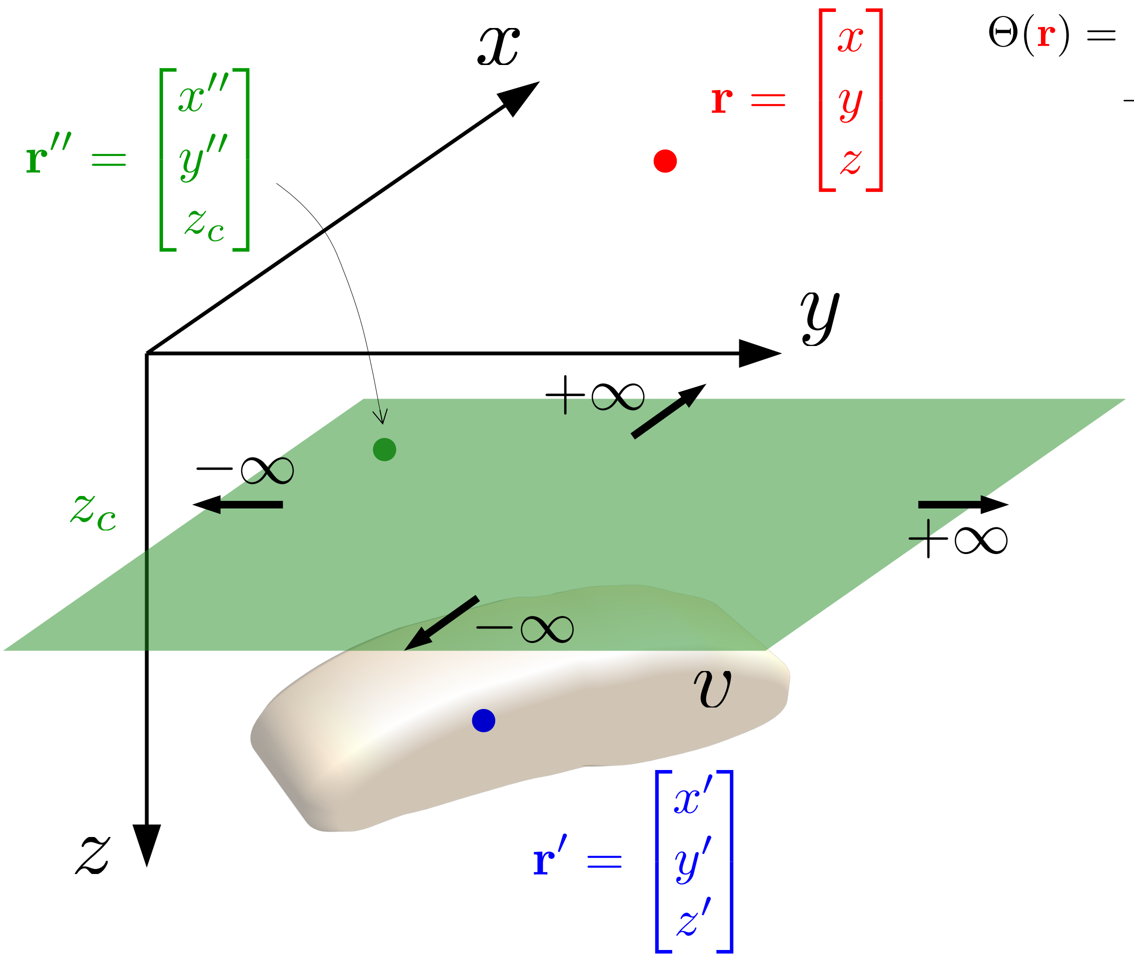
Deduction:

$\partial_z \Theta(\mathbf{r})$ gravity disturbance data

$$\partial_z \Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

This equation is the upward-continuation integral applied to the vertical derivative of $\Theta(\mathbf{r})$

Result 1/4: The gravity disturbance can be exactly reproduced by an analytical eq. layer of monopoles



$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

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Deduction:

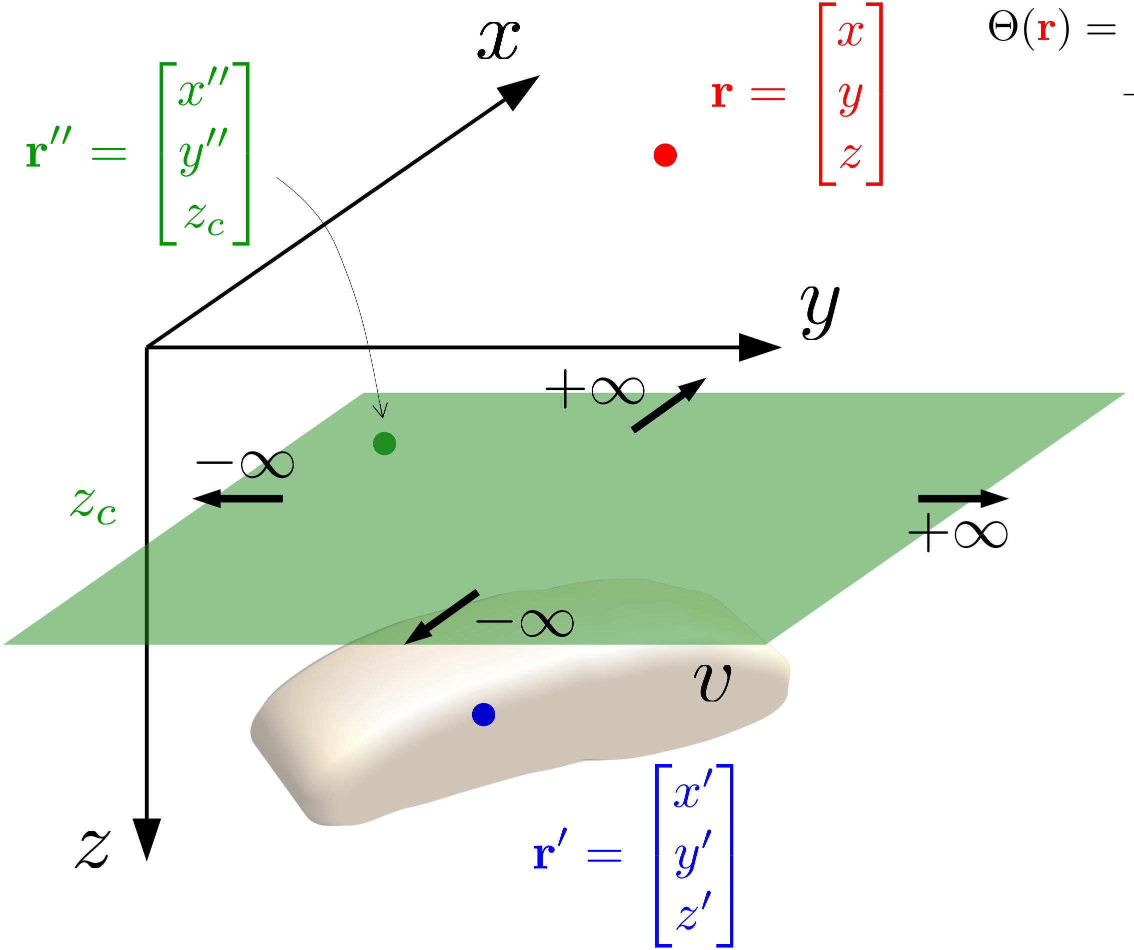
$\partial_z \Theta(\mathbf{r})$ gravity disturbance data

$$\partial_z \Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Analytical eq. layer given by:

$$\partial_z \Theta(\mathbf{r}'') = \kappa \iiint_v \sigma(\mathbf{r}') \, \partial_z \frac{1}{\|\mathbf{r}'' - \mathbf{r}'\|} dv'$$

Result 1/4: The gravity disturbance can be exactly reproduced by an analytical eq. layer of monopoles



$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

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Deduction:

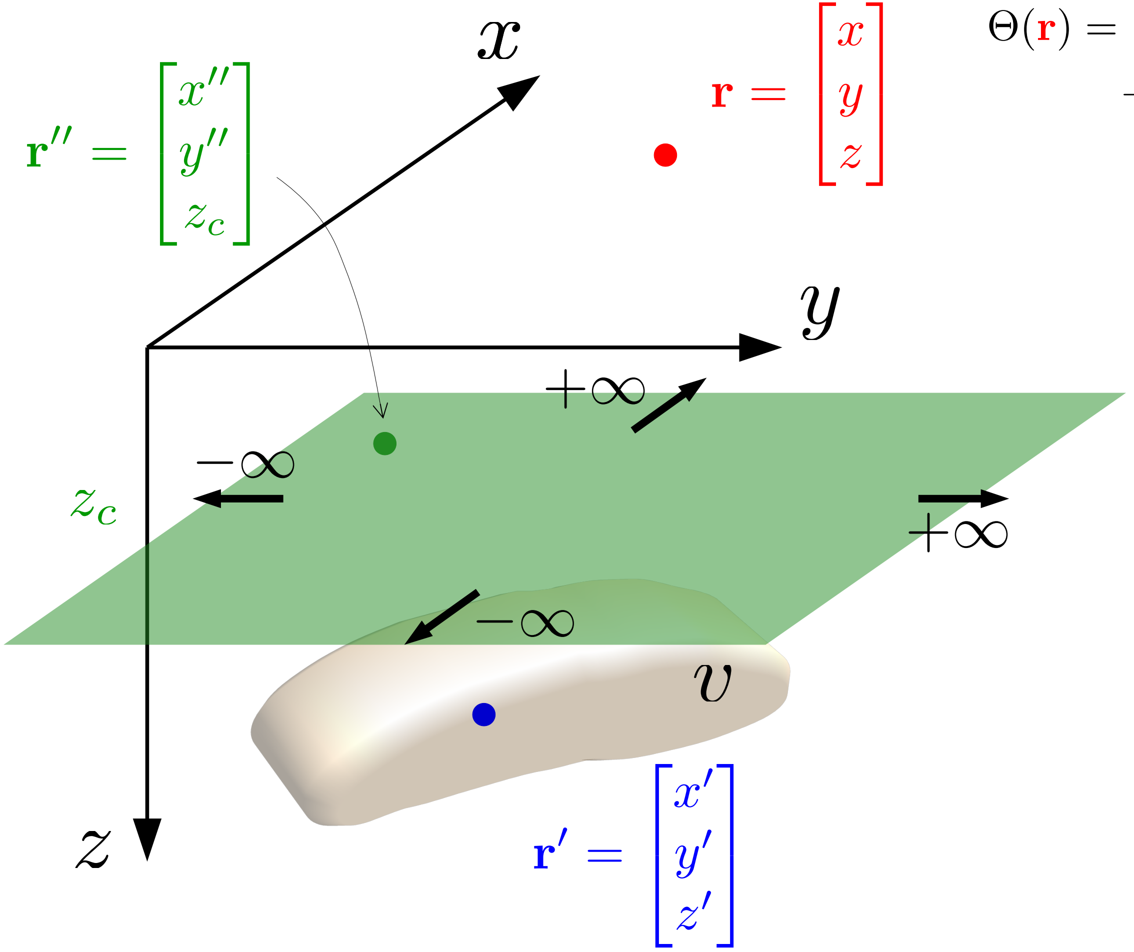
$\partial_z \Theta(\mathbf{r})$ gravity disturbance data

$$\partial_z \Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

This analytical eq. layer represents the grav. disturbance produced by the true sources on the plane z_c

$$\partial_z \Theta(\mathbf{r}'') = \kappa \iiint_v \sigma(\mathbf{r}') \; \partial_z \frac{1}{\|\mathbf{r}'' - \mathbf{r}'\|} \, dv'$$

Result 1/4: The gravity disturbance can be exactly reproduced by an analytical eq. layer of monopoles



$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

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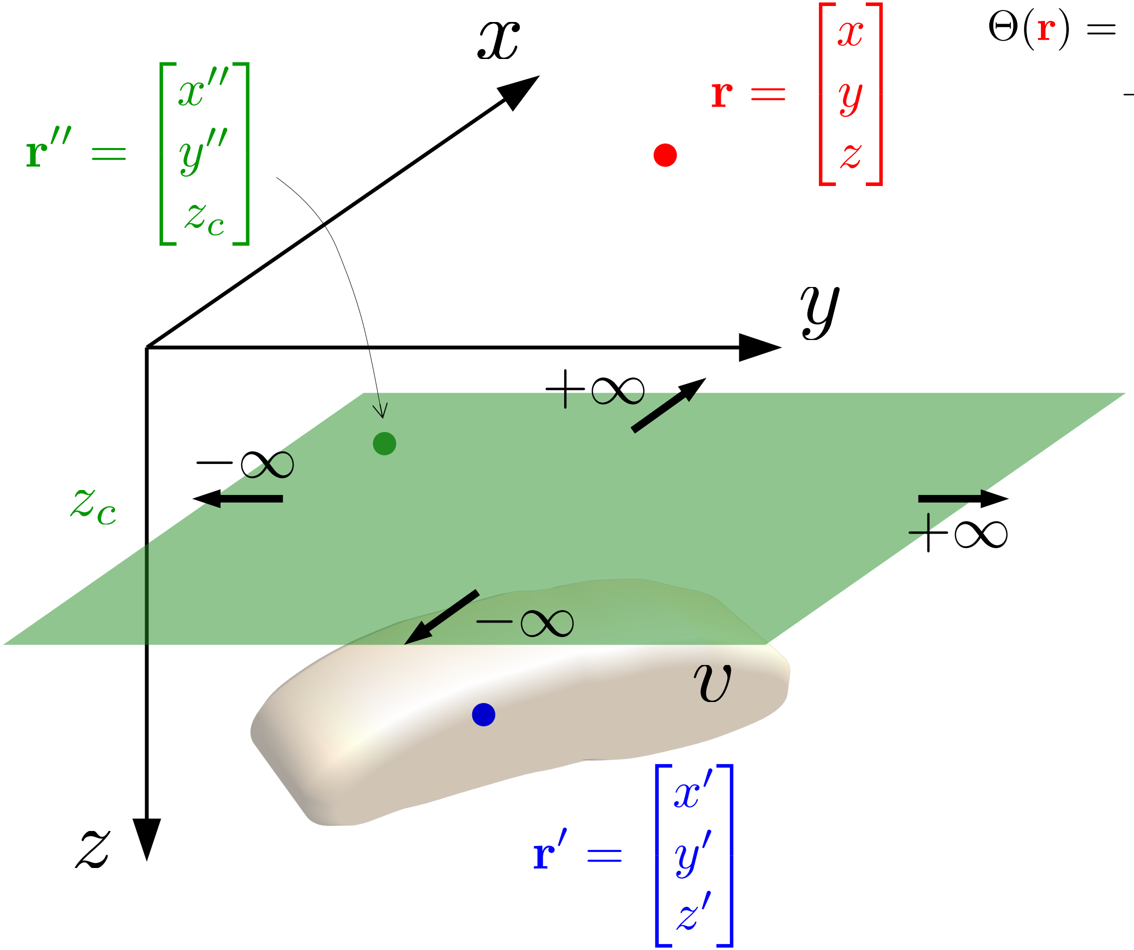
Deduction:

$\partial_z \Theta(\mathbf{r})$ gravity disturbance data

$$\partial_z \Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

Result 1/4: The gravity disturbance can be exactly reproduced by an analytical eq. layer of monopoles



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Deduction:

$\partial_z \Theta(\mathbf{r})$ gravity disturbance data

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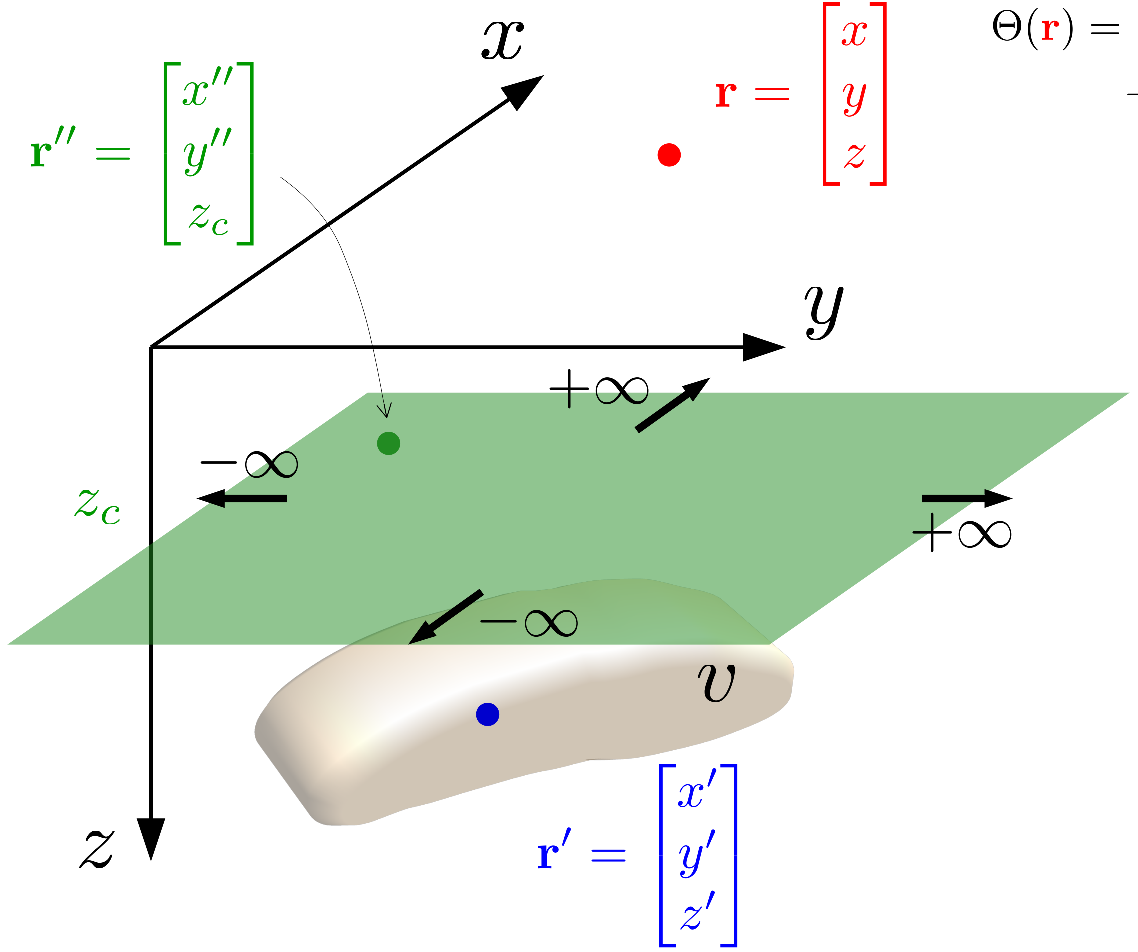
$\partial_z \Theta(\mathbf{r}_i)$

Result 1/4: The gravity disturbance can be exactly reproduced by an analytical eq. layer of monopoles



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Deduction:

$\partial_z \Theta(\mathbf{r})$ gravity disturbance data

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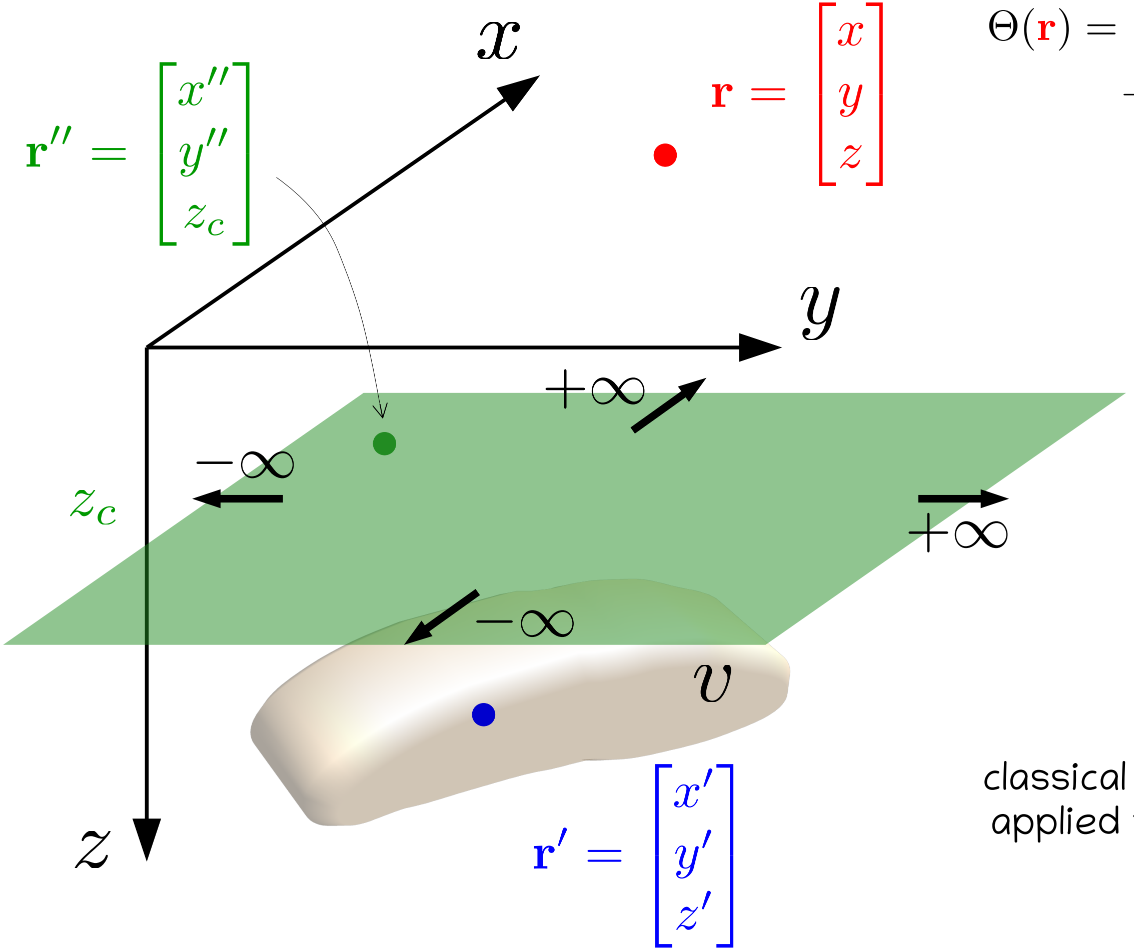
$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$\partial_z \Theta(\mathbf{r}_i)$

$\approx \partial_z \Theta(\mathbf{r}''_j)$

Result 1/4: The gravity disturbance can be exactly reproduced by an analytical eq. layer of monopoles



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Deduction:

$\partial_z \Theta(\mathbf{r})$ gravity disturbance data

$$\partial_z \Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

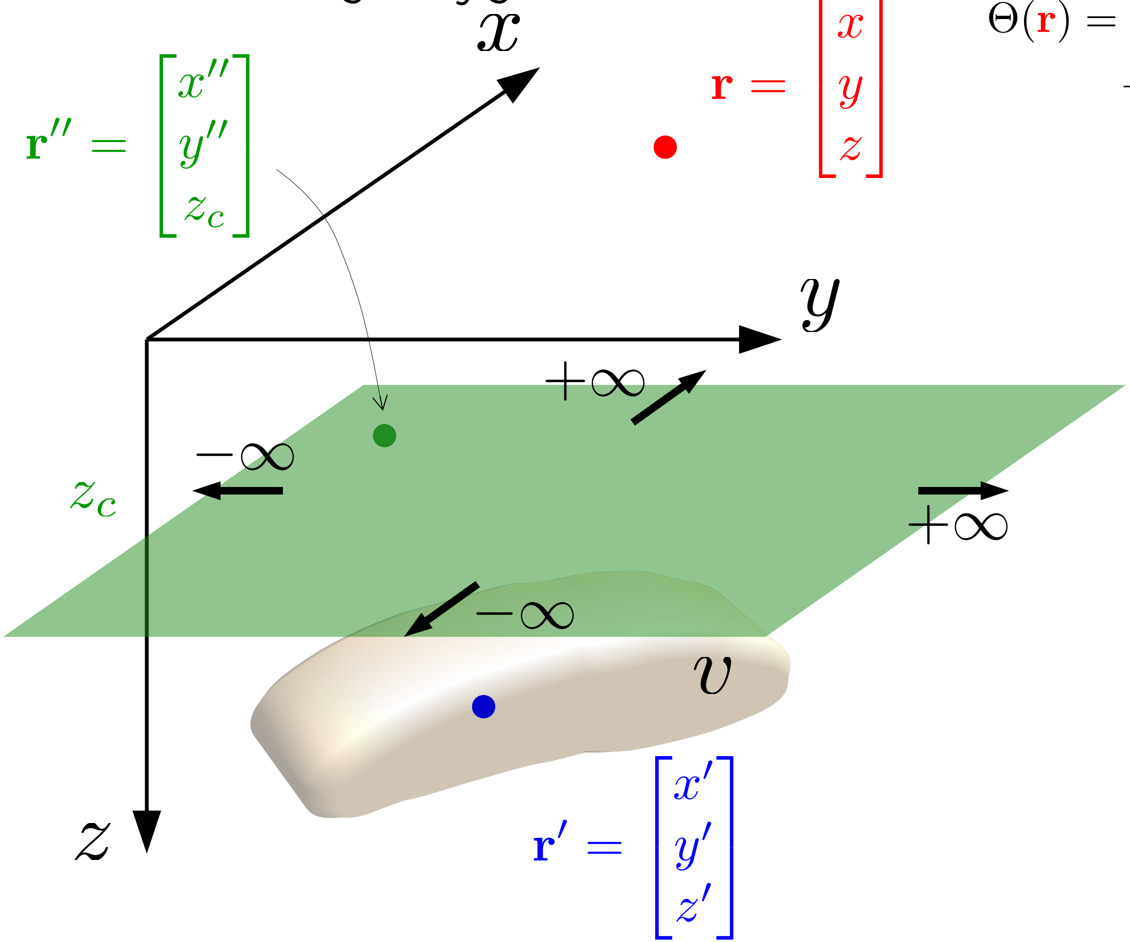
$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_z \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

classical EqL technique applied to gravity data

Result 2/4: The same analytical eq. layer of monopoles reproducing the gravity disturbance can also reproduce the gravitational potential and gravity gradient tensor

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$



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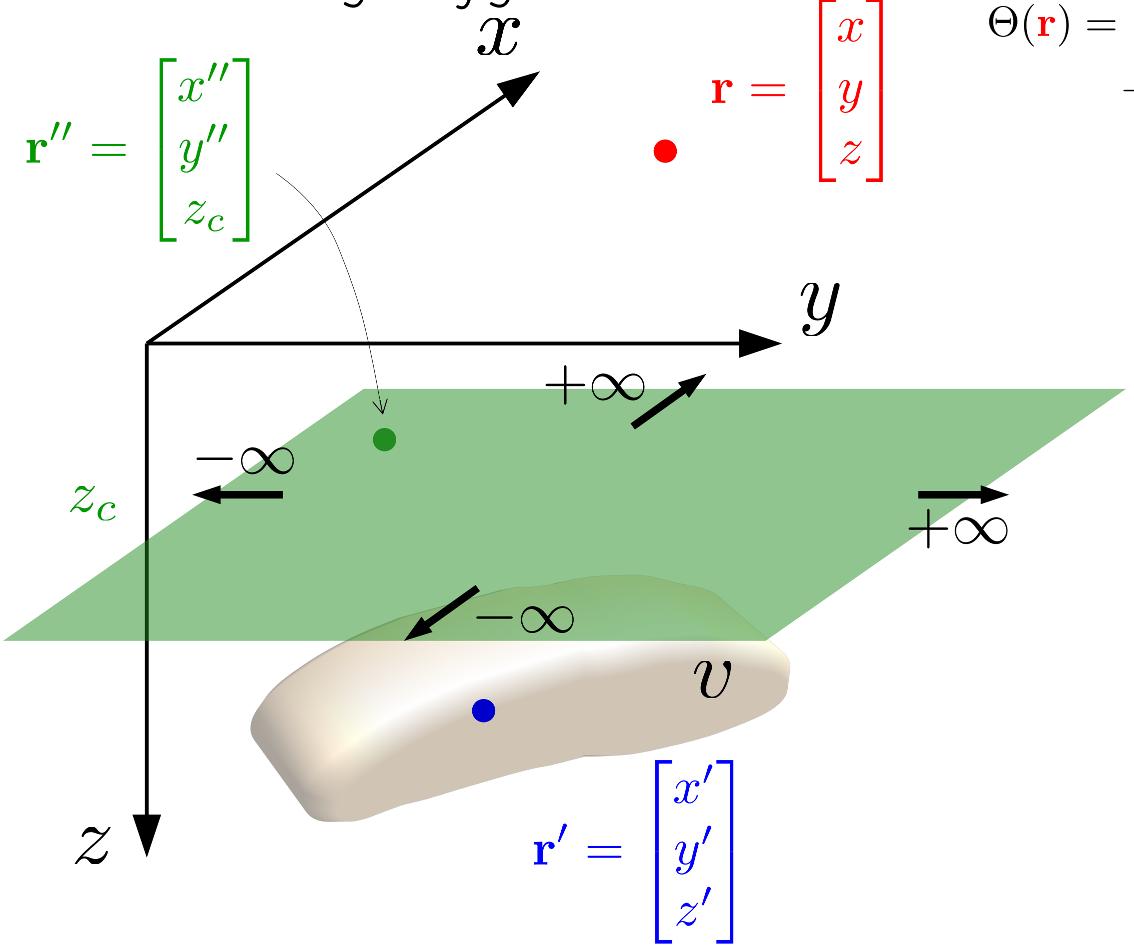

This result has been used by the scientific community for processing gravity-gradient data (e.g., Barnes and Lumley, 2011) or converting gravity disturbance into the gravity-gradient tensor (e.g., Piauilino et al., 2019)

Result 2/4: The same analytical eq. layer of monopoles reproducing the gravity disturbance can also reproduce the gravitational potential and gravity gradient tensor

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

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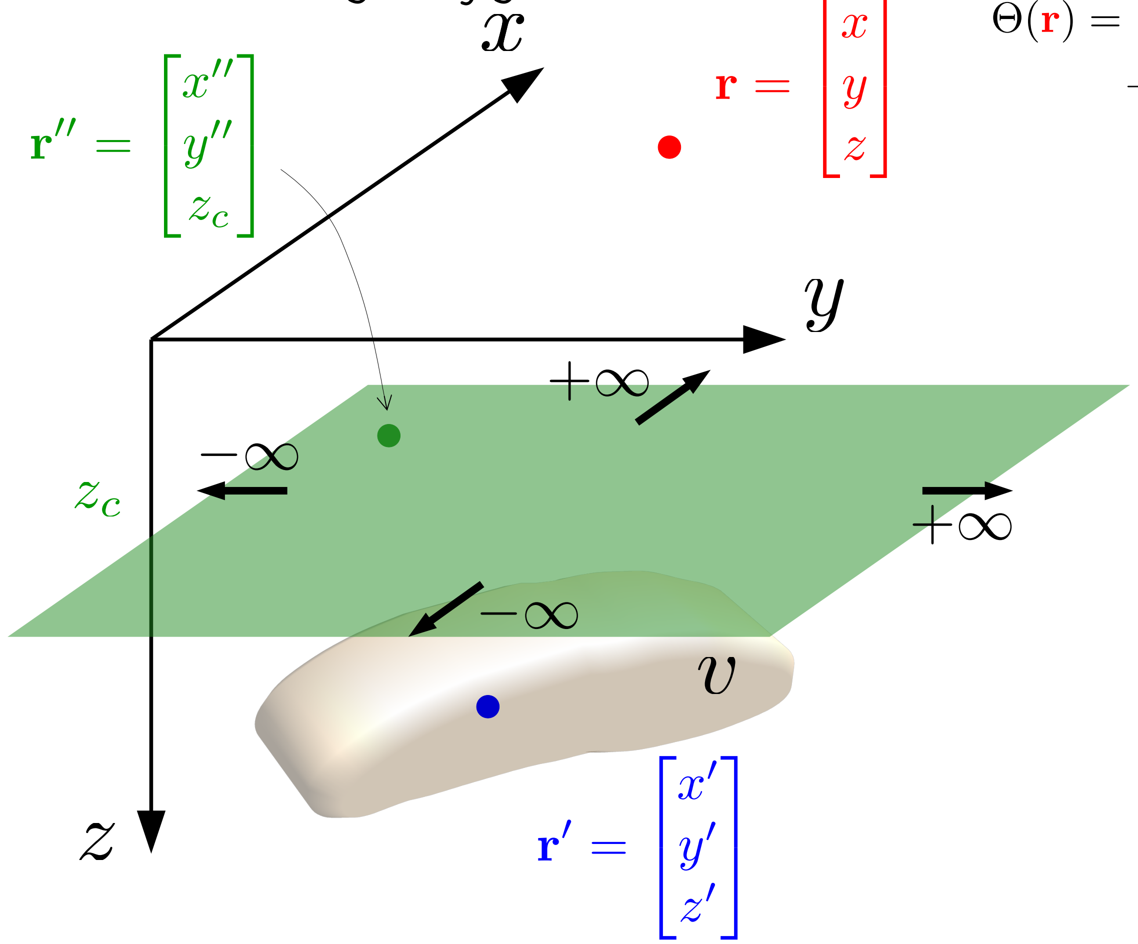
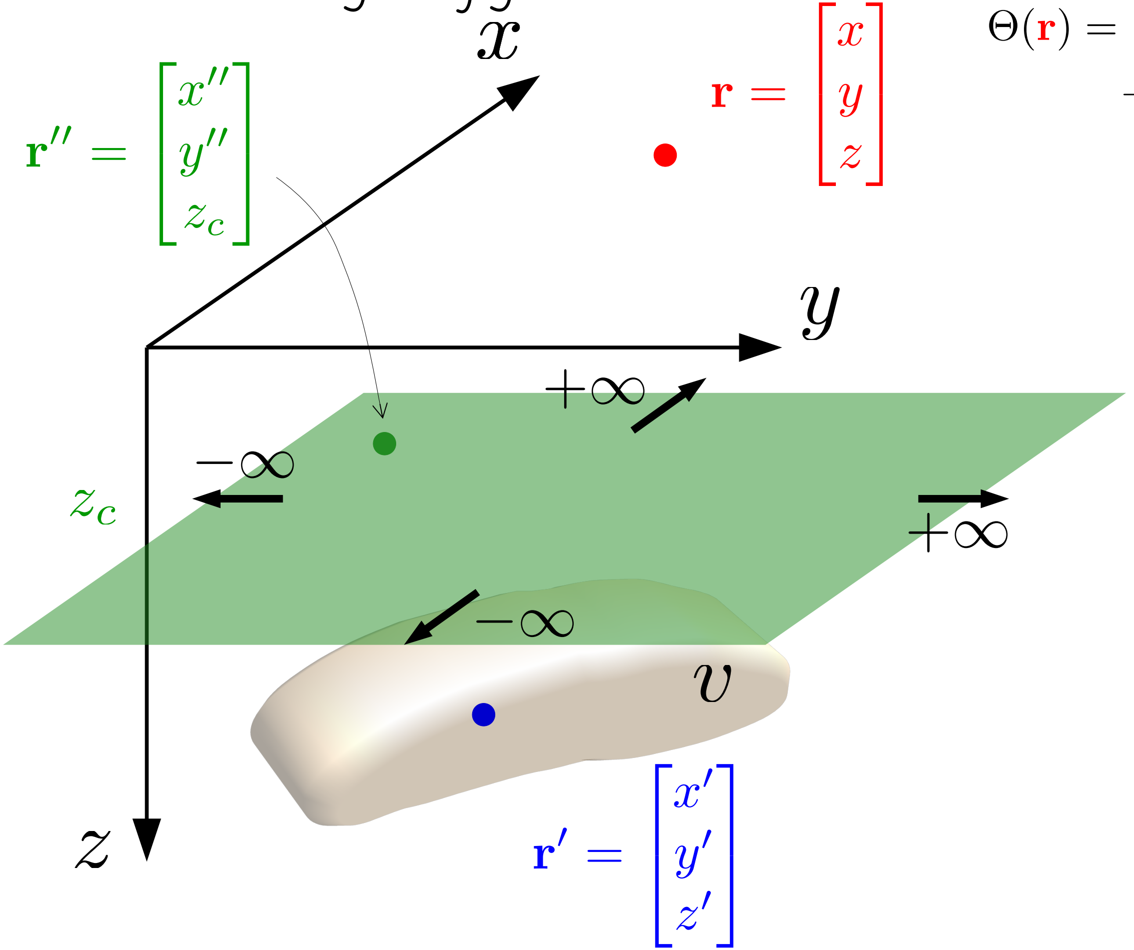


This result has been used by the scientific community for processing gravity-gradient data (e.g., Barnes and Lumley, 2011) or converting gravity disturbance into the gravity-gradient tensor (e.g., Piauilino et al., 2019)

However, researchers have not treated this topic in much detail and it is assumed to be true without any proof

Result 2/4: The same analytical eq. layer of monopoles reproducing the gravity disturbance can also reproduce the gravitational potential and gravity gradient tensor



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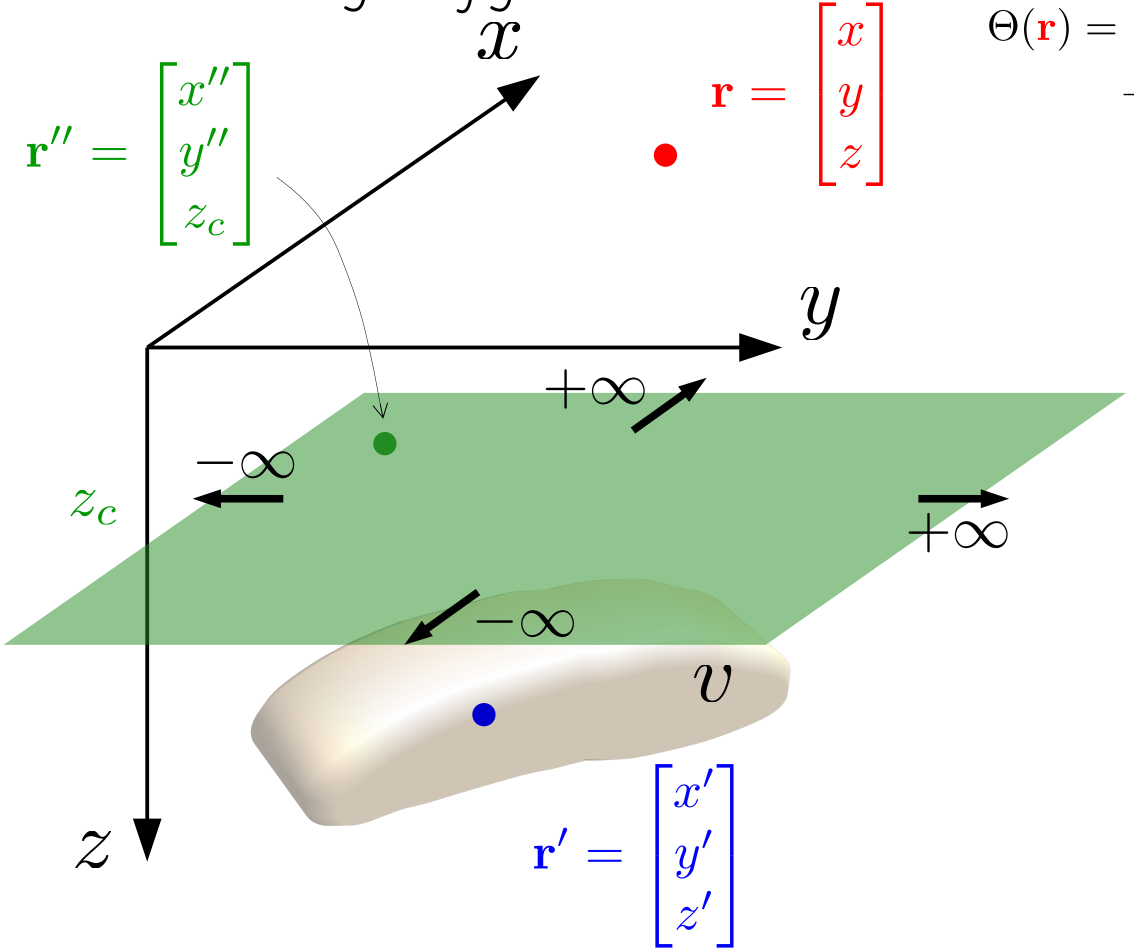
Deduction:

$\Theta(\mathbf{r})$ gravitational potential

Result 2/4: The same analytical eq. layer of monopoles reproducing the gravity disturbance can also reproduce the gravitational potential and gravity gradient tensor



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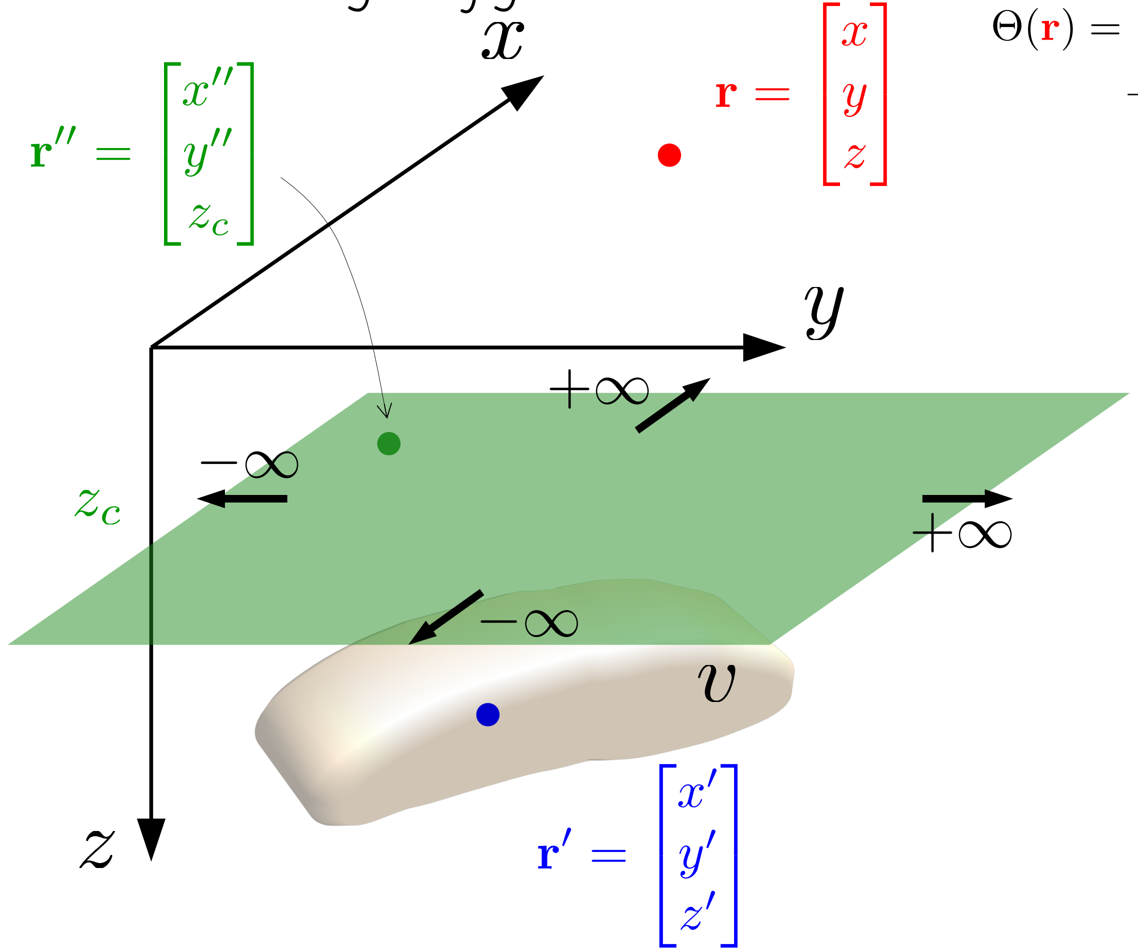
Deduction:

$\Theta(\mathbf{r})$ gravitational potential

$$\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Result 2/4: The same analytical eq. layer of monopoles reproducing the gravity disturbance can also reproduce the gravitational potential and gravity gradient tensor



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Deduction:

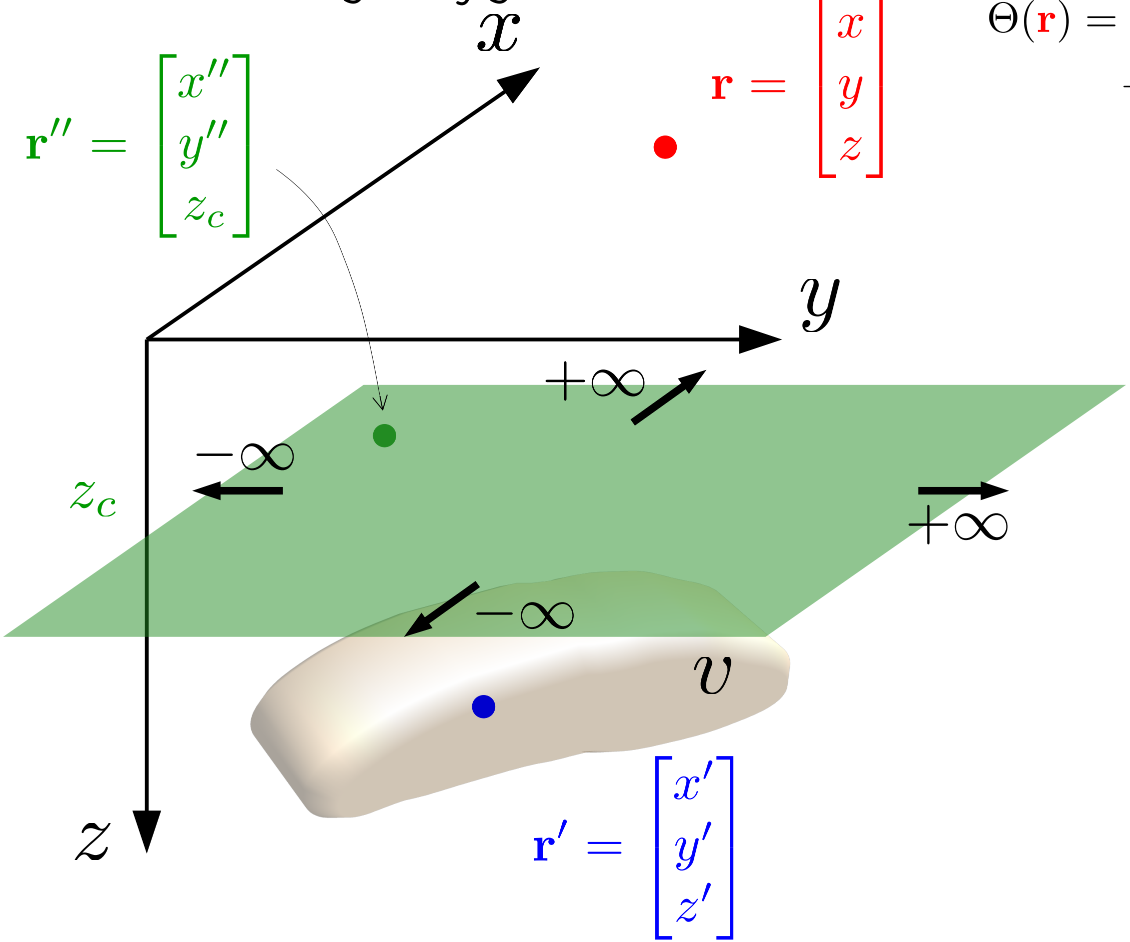
$\Theta(\mathbf{r})$ gravitational potential

$$\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$\partial_z \Theta(\mathbf{r})$ gravity disturbance

$$\partial_z \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Result 2/4: The same analytical eq. layer of monopoles reproducing the gravity disturbance can also reproduce the gravitational potential and gravity gradient tensor



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Deduction:

$\Theta(\mathbf{r})$ gravitational potential

$$\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$\partial_z \Theta(\mathbf{r})$ gravity disturbance

$$\partial_z \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Note that this term is not affected by the derivative

Result 2/4: The same analytical eq. layer of monopoles reproducing the gravity disturbance can also reproduce the gravitational potential and gravity gradient tensor

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

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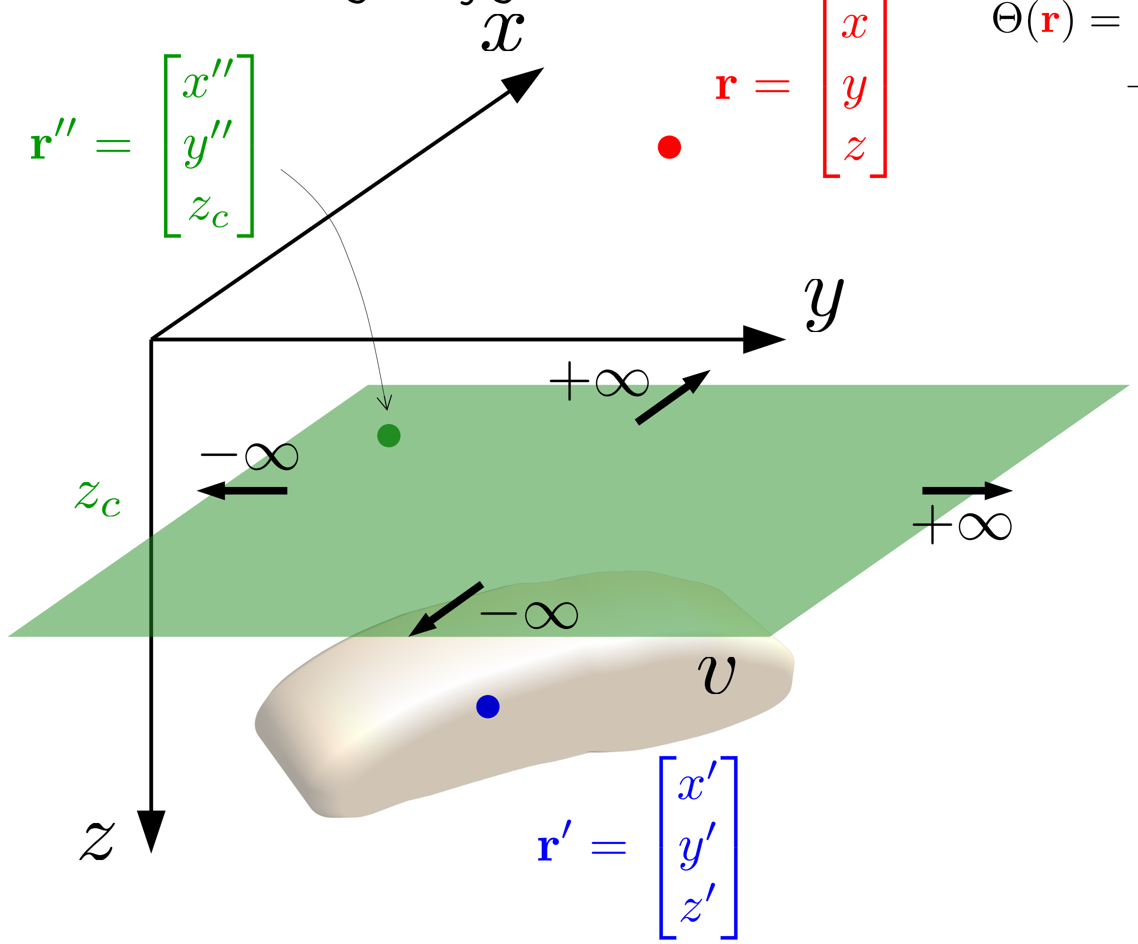


Deduction:

$\Theta(\mathbf{r})$ gravitational potential

$$\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

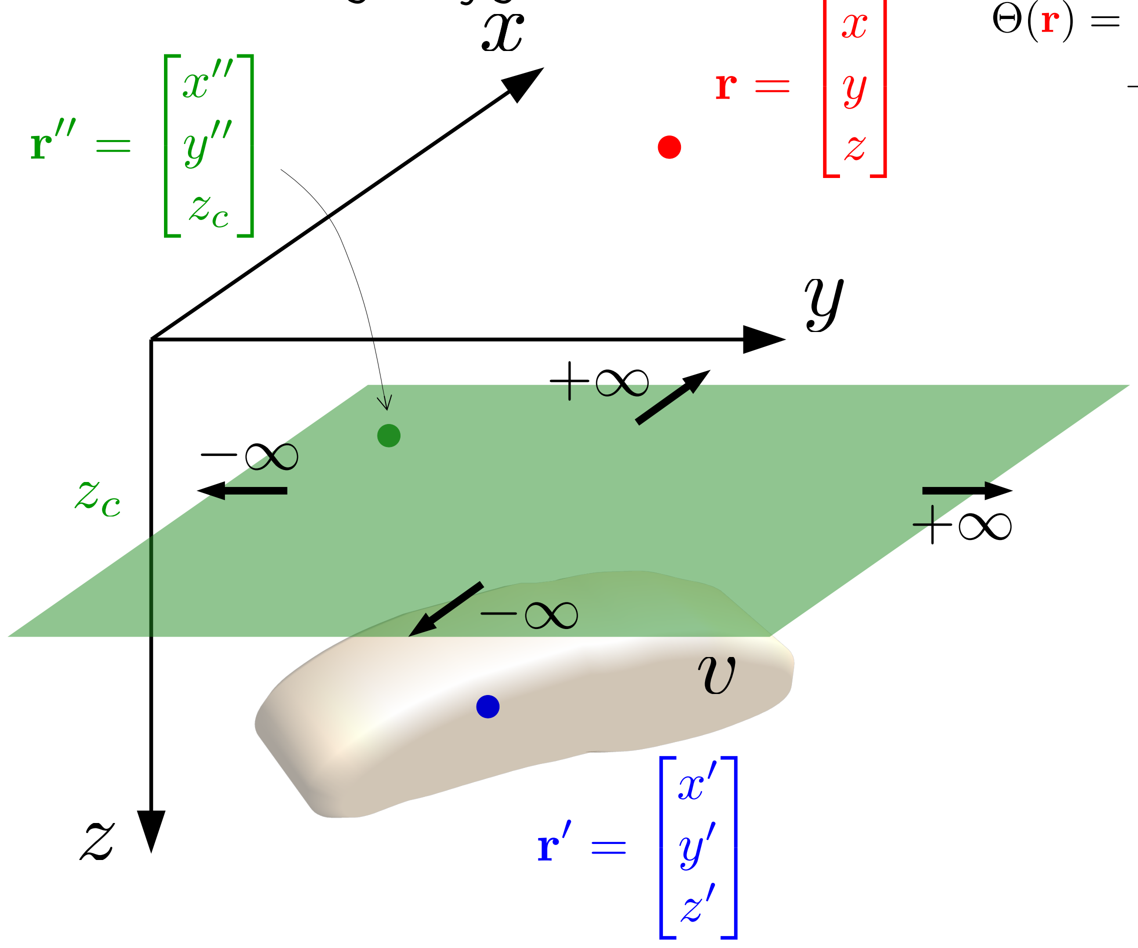
$\partial_z \Theta(\mathbf{r})$ gravity disturbance

$$\partial_z \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$\partial_{\alpha\beta} \Theta(\mathbf{r})$ gravity-gradient tensor component

$$\partial_{\alpha\beta} \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{\alpha\beta} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Result 2/4: The same analytical eq. layer of monopoles reproducing the gravity disturbance can also reproduce the gravitational potential and gravity gradient tensor



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Deduction:

$\Theta(\mathbf{r})$ gravitational potential

$$\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$\partial_z \Theta(\mathbf{r})$ gravity disturbance

$$\partial_z \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_z \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

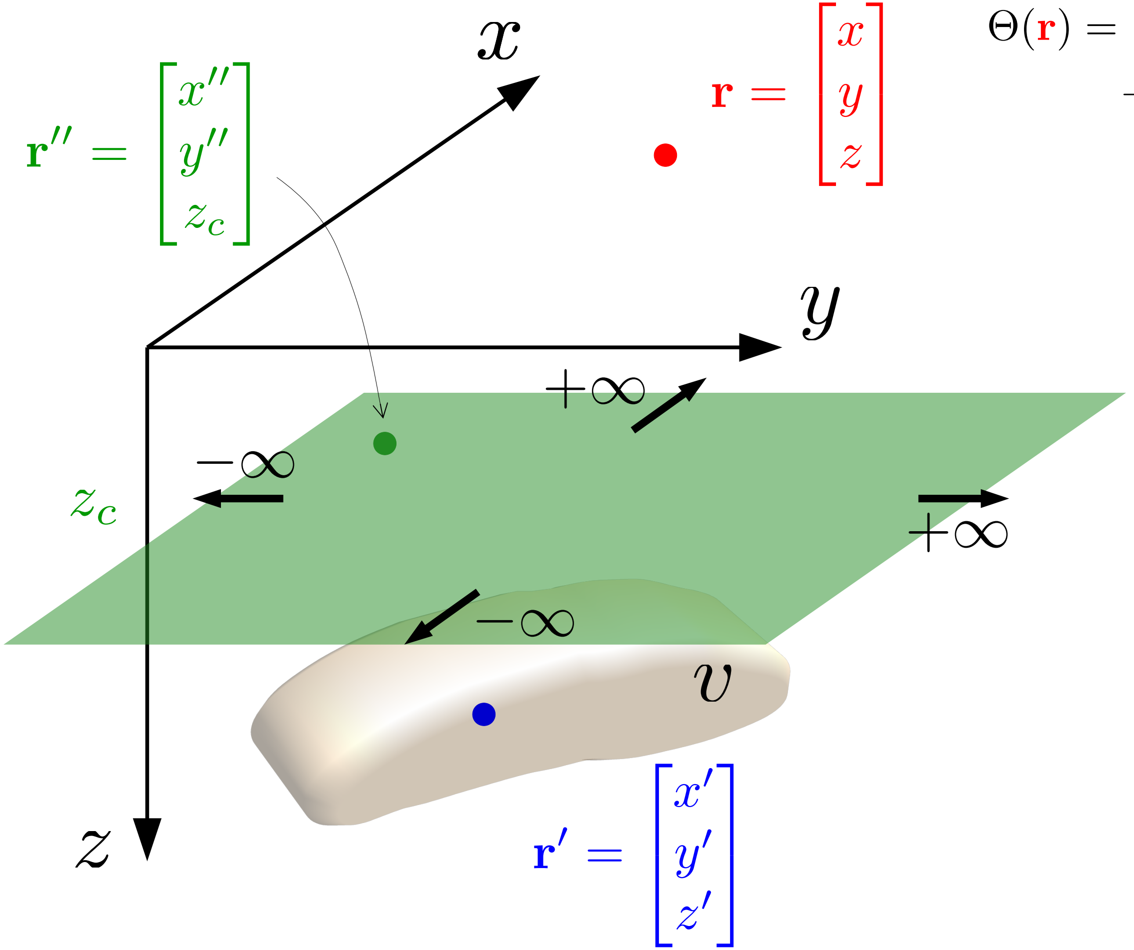
$\partial_{\alpha\beta} \Theta(\mathbf{r})$ gravity-gradient tensor component

$$\partial_{\alpha\beta} \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{\alpha\beta} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by an analytical eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

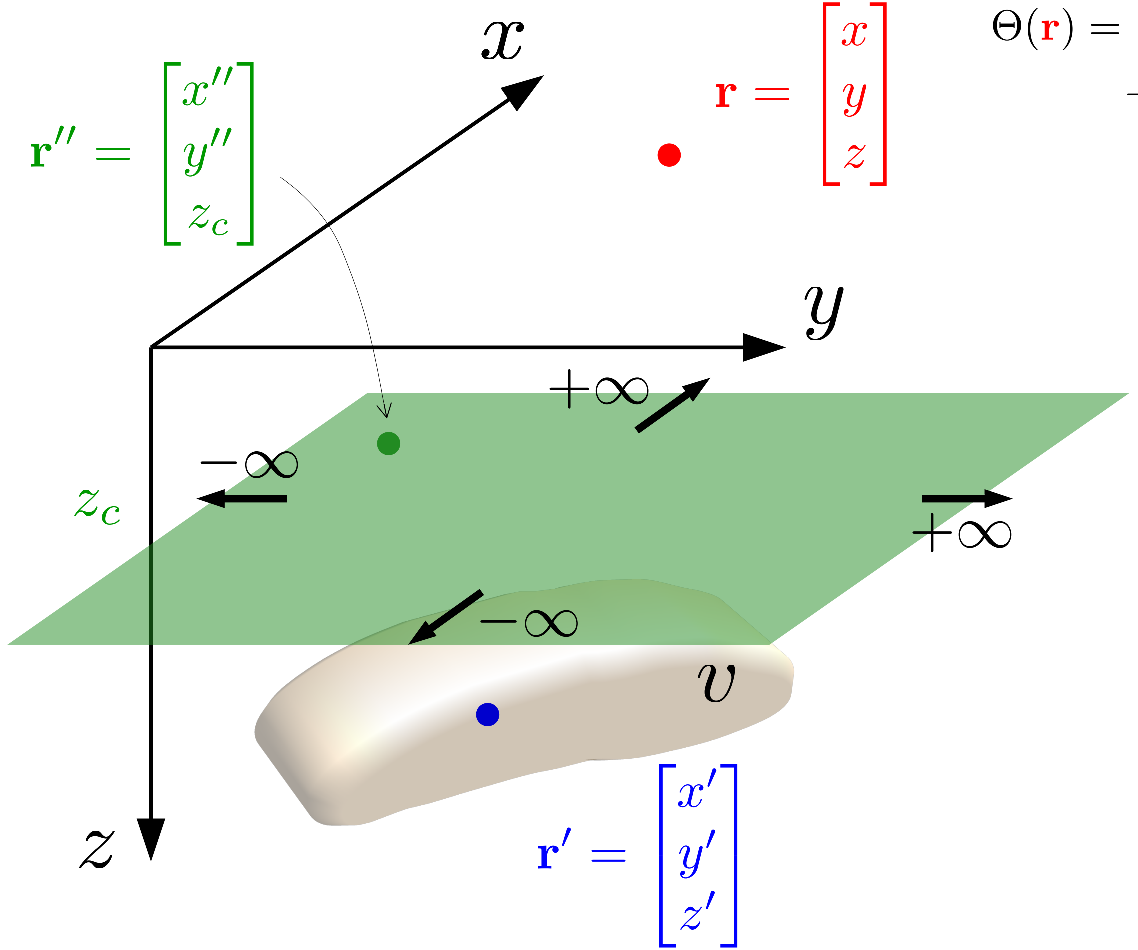
$$\Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$



Result 3/4: The approx total-field anomaly can be exactly reproduced by an analytical eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$



Most previous published studies are limited to the empirical use of a planar eq. layer of dipoles for processing total-field anomaly data

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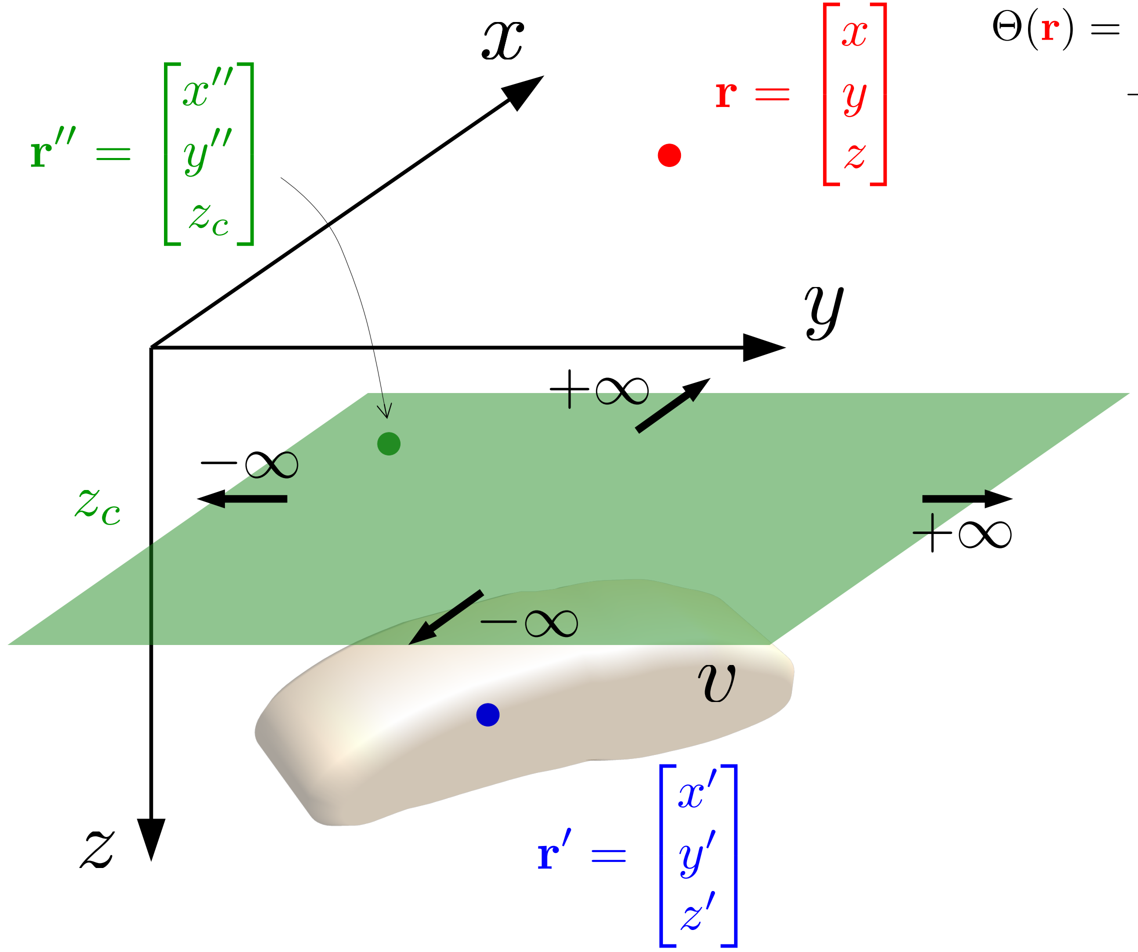

Most previous published studies are limited to the empirical use of a planar eq. layer of dipoles for processing total-field anomaly data

Few studies (e.g., Pedersen, 1991; Li et al., 2014; Reis et al., 2020) have drawn attention to the problem of proving the existence of a planar eq. layer of dipoles that exactly reproduces the approx total-field anomaly

Result 3/4: The approx total-field anomaly can be exactly reproduced by an analytical eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

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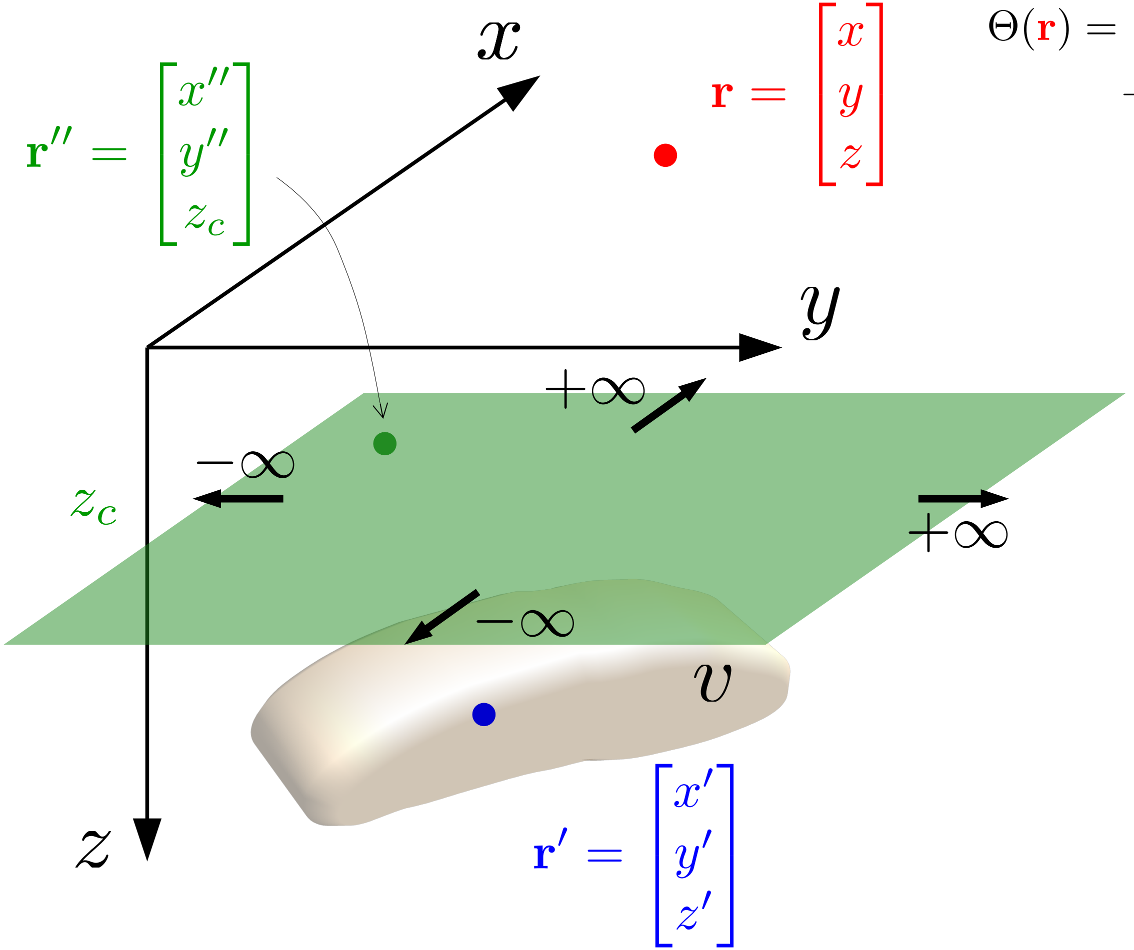

Deduction:

approx total-field anomaly

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by an analytical eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

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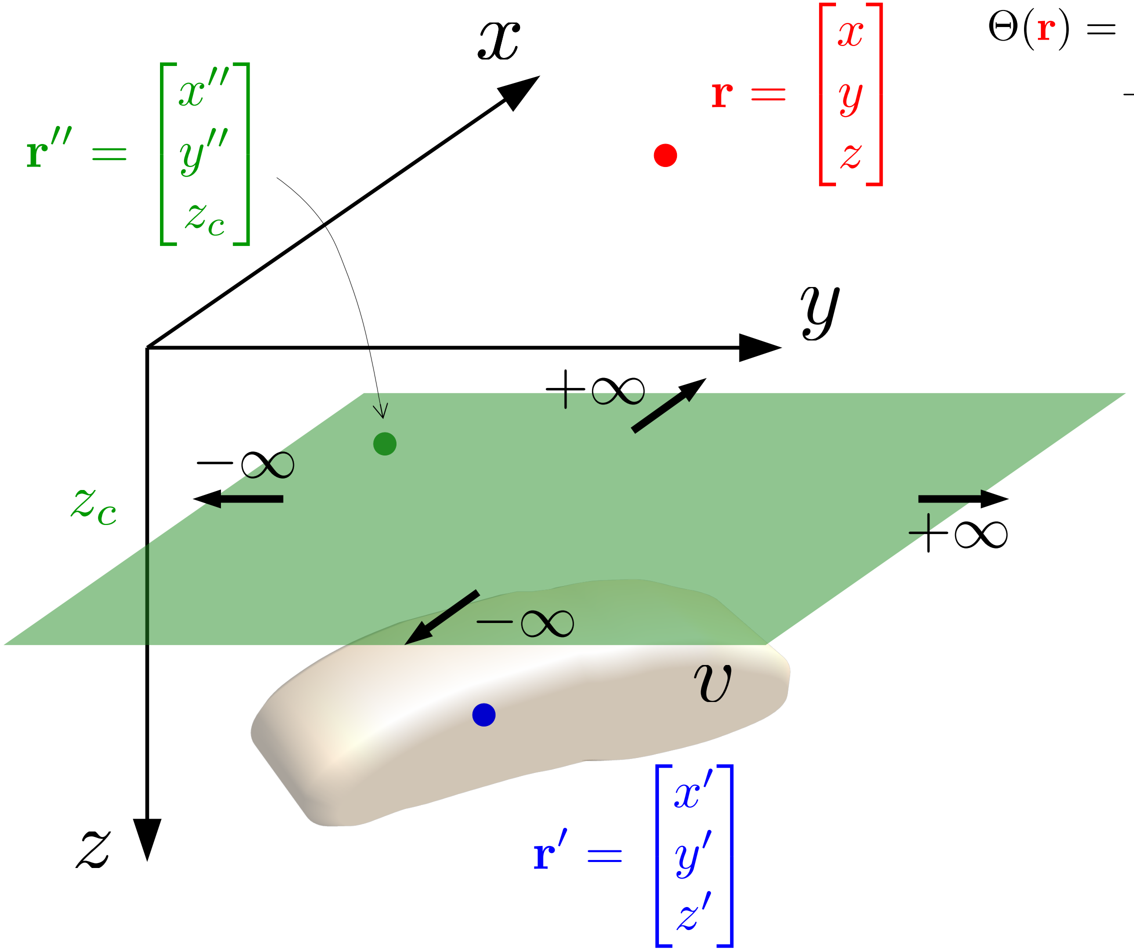


Deduction:

approx total-field anomaly

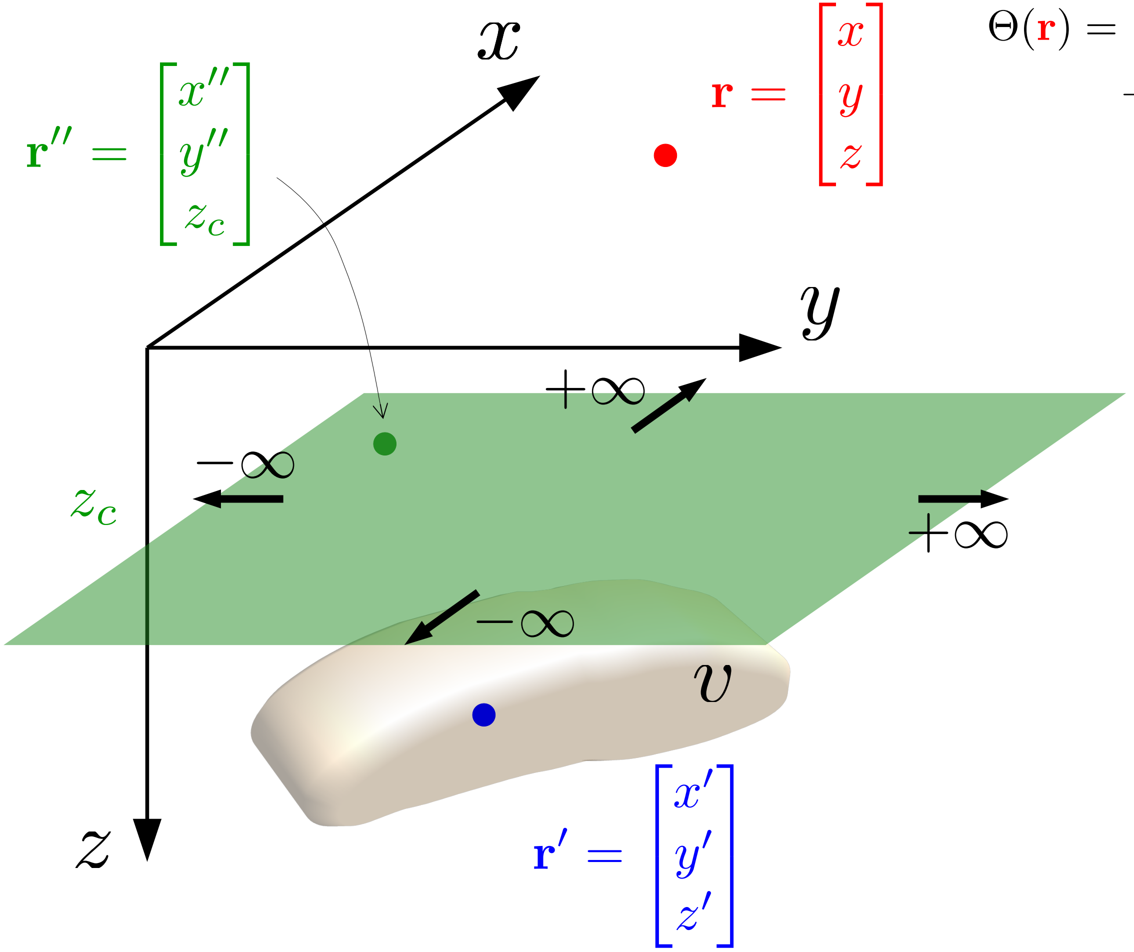
$$\partial_{th}\Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

Unit vector defining the constant direction of the main geomagnetic field

Result 3/4: The approx total-field anomaly can be exactly reproduced by an analytical eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$



Deduction:

approx total-field anomaly

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Theta}(\mathbf{r}) \, \hat{\mathbf{h}}$$

Unit vector defining the uniform total-magnetization direction of the true sources

Result 3/4: The approx total-field anomaly can be exactly reproduced by an analytical eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

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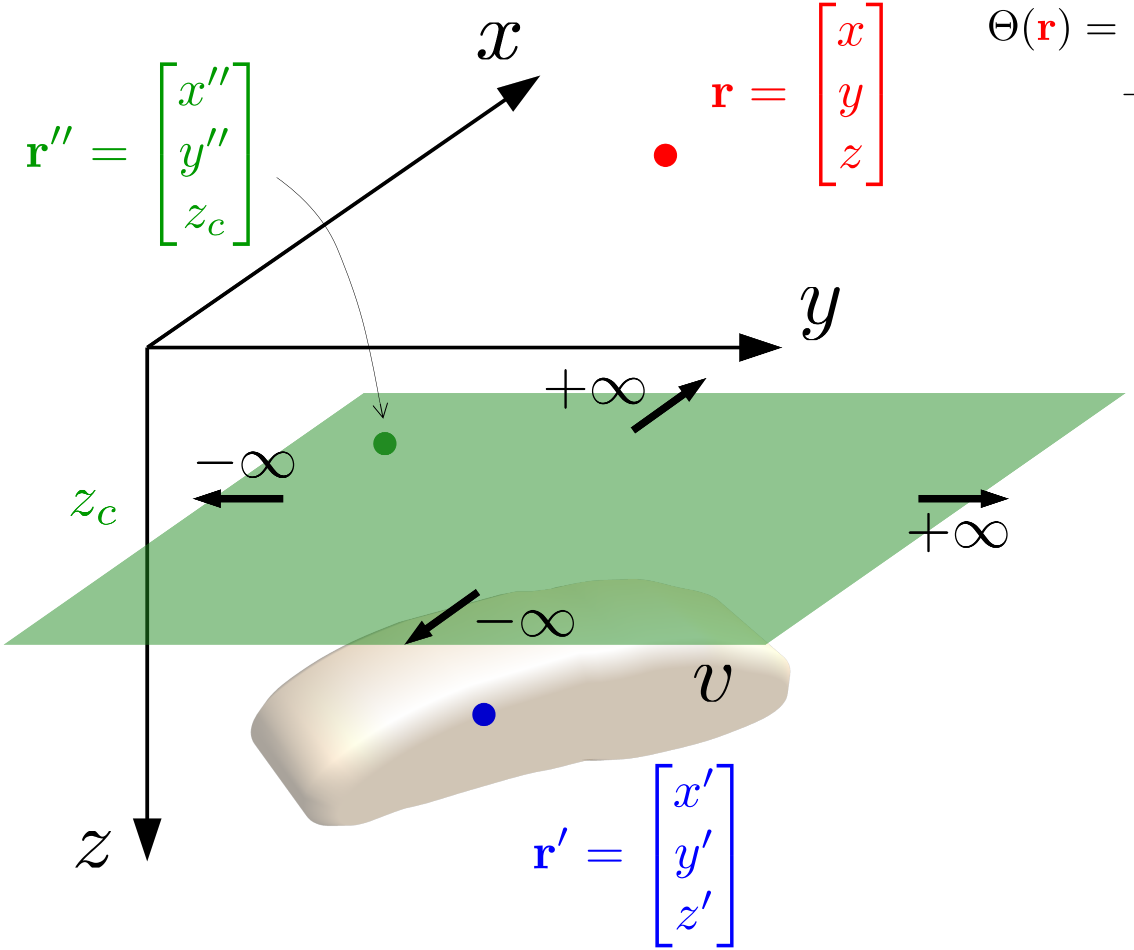


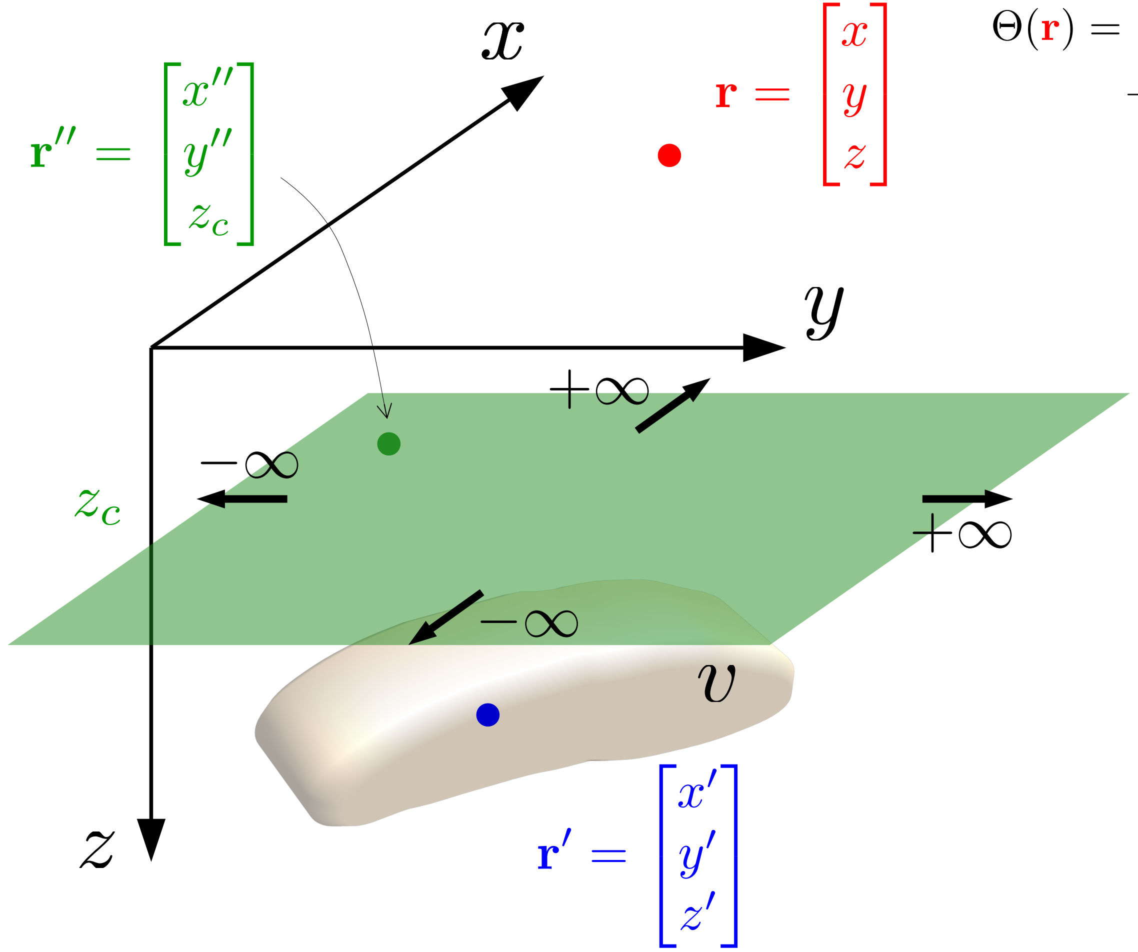
Deduction:

approx total-field anomaly

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\mathbf{H}_\Theta(\mathbf{r}) = \begin{bmatrix} \partial_{xx}\Theta(\mathbf{r}) & \partial_{xy}\Theta(\mathbf{r}) & \partial_{xz}\Theta(\mathbf{r}) \\ \partial_{xy}\Theta(\mathbf{r}) & \partial_{yy}\Theta(\mathbf{r}) & \partial_{yz}\Theta(\mathbf{r}) \\ \partial_{xz}\Theta(\mathbf{r}) & \partial_{yz}\Theta(\mathbf{r}) & \partial_{zz}\Theta(\mathbf{r}) \end{bmatrix}$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by an analytical eq. layer of dipoles having the same uniform total-magnetization direction of the true sources



$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$

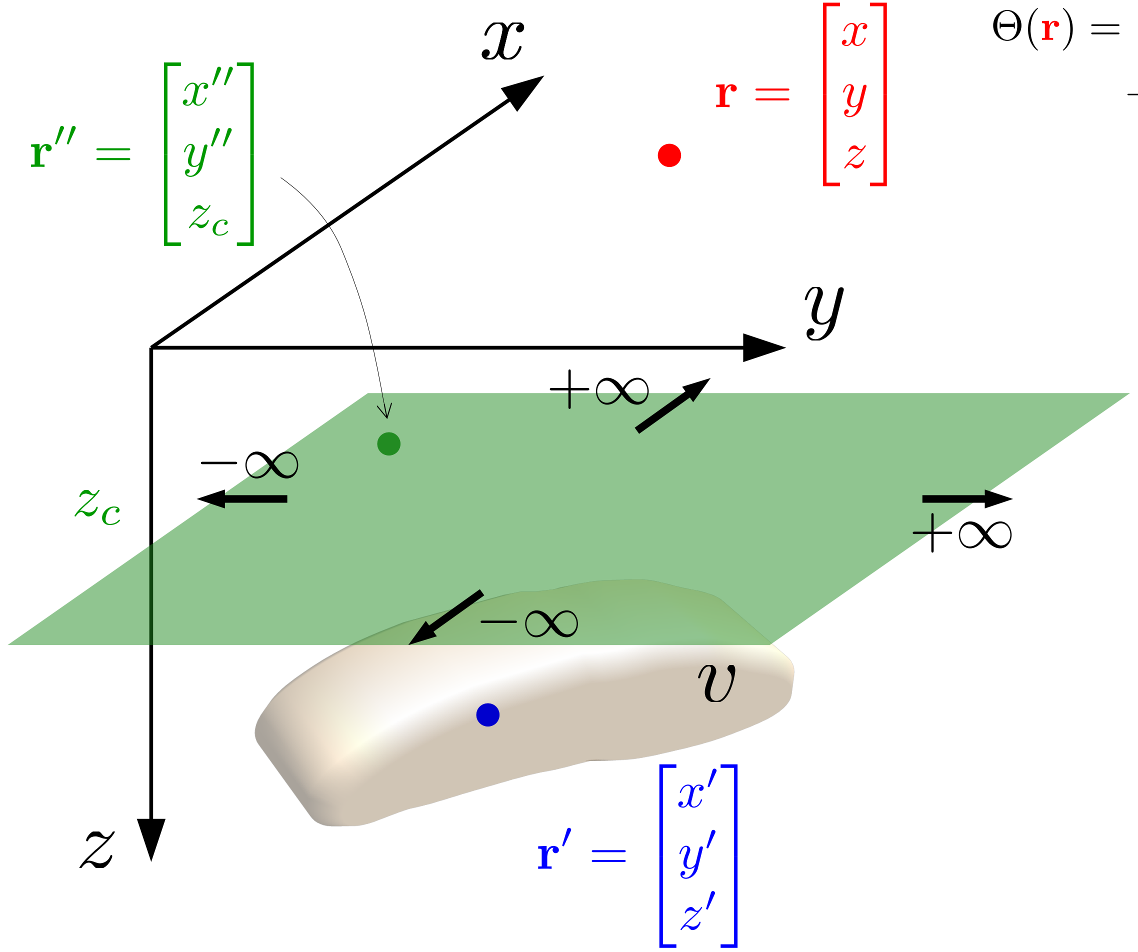
Deduction:

approx total-field anomaly

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th} \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by an analytical eq. layer of dipoles having the same uniform total-magnetization direction of the true sources



$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r}-\mathbf{r}'\|} \, dv'$$

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Deduction:

approx total-field anomaly

$$\partial_{th}\Theta(\mathbf{r}) = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Theta}(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th}\Theta(\mathbf{r}) = \iint \underbrace{\partial_z\Theta(\mathbf{r}'') \, \partial_{th}\Psi(\mathbf{r},\mathbf{r}'')}_{} \, dS''$$

This term represents the approx total-field anomaly produced at $\mathbf{r}$ by a dipole located at $\mathbf{r}''$

Result 3/4: The approx total-field anomaly can be exactly reproduced by an analytical eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

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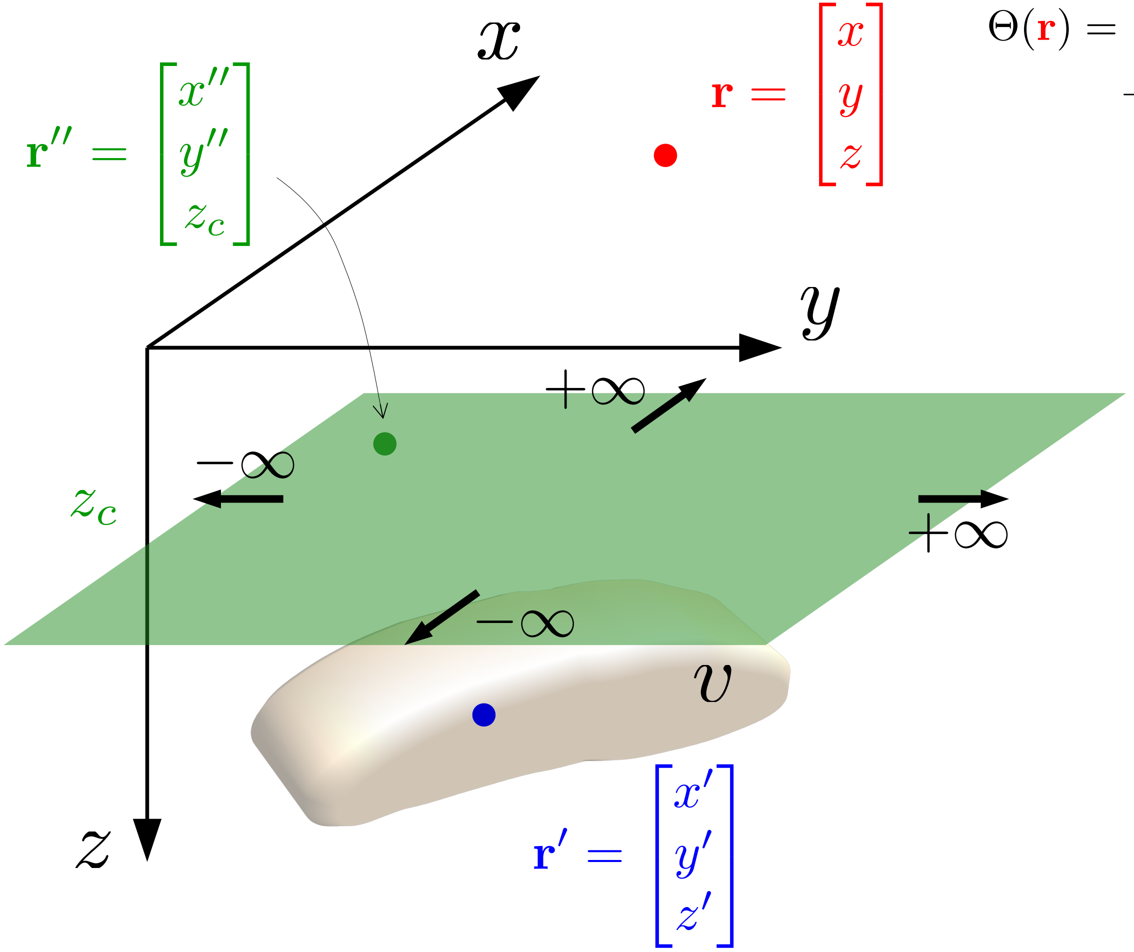

Deduction:

approx total-field anomaly

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Theta}(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th} \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$$\partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Psi}(\mathbf{r}, \mathbf{r}'') \, \hat{\mathbf{h}}$$

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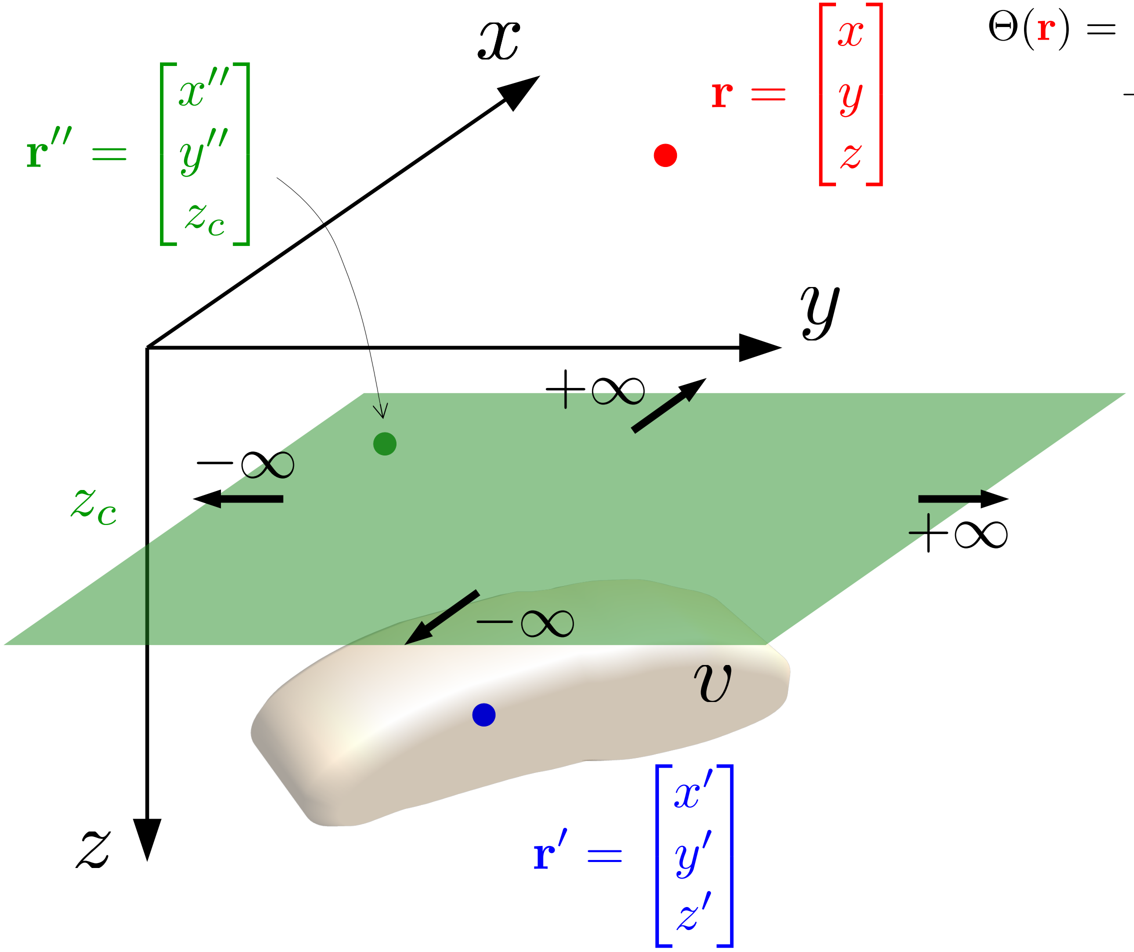


$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$

Deduction:

approx total-field anomaly

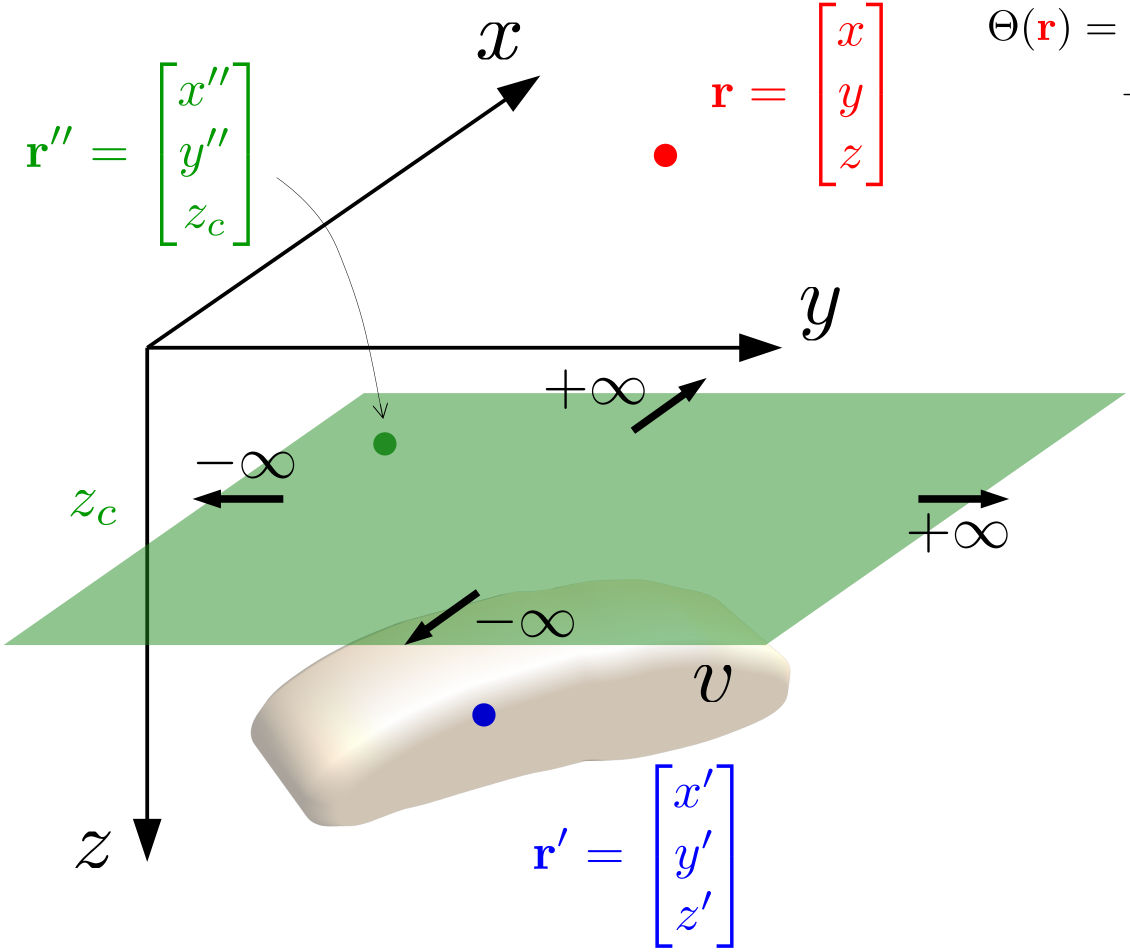
$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Theta}(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th} \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$$\partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Psi}(\mathbf{r}, \mathbf{r}'') \, \hat{\mathbf{h}}$$

Unit vector defining the same uniform total-magnetization direction of the true sources

Result 3/4: The approx total-field anomaly can be exactly reproduced by an analytical eq. layer of dipoles having the same uniform total-magnetization direction of the true sources



$$\Theta(\mathbf{r}) = \kappa \iiint\limits_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

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Deduction:

approx total-field anomaly

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th} \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

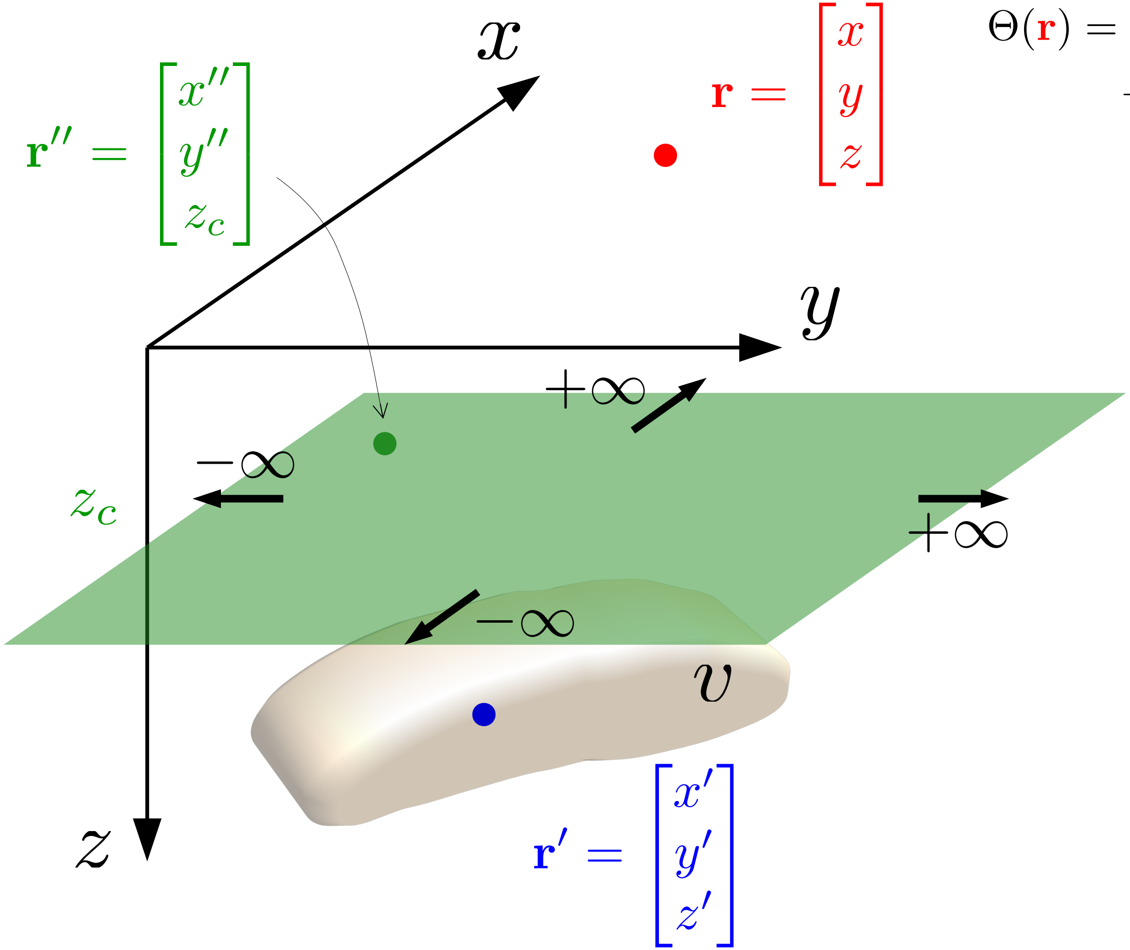
$$\partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') = \hat{\mathbf{t}}^\top \mathbf{H}_\Psi(\mathbf{r}, \mathbf{r}'') \, \hat{\mathbf{h}}$$

$$\mathbf{H}_\Psi(\mathbf{r}, \mathbf{r}'') = \begin{bmatrix} \partial_{xx} \Psi(\mathbf{r}, \mathbf{r}'') & \partial_{xy} \Psi(\mathbf{r}, \mathbf{r}'') & \partial_{xz} \Psi(\mathbf{r}, \mathbf{r}'') \\ \partial_{xy} \Psi(\mathbf{r}, \mathbf{r}'') & \partial_{yy} \Psi(\mathbf{r}, \mathbf{r}'') & \partial_{yz} \Psi(\mathbf{r}, \mathbf{r}'') \\ \partial_{xz} \Psi(\mathbf{r}, \mathbf{r}'') & \partial_{yz} \Psi(\mathbf{r}, \mathbf{r}'') & \partial_{zz} \Psi(\mathbf{r}, \mathbf{r}'') \end{bmatrix}$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by an analytical eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

$$\Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$



Deduction:

approx total-field anomaly

$$\partial_{th}\Theta(\mathbf{r}) = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Theta}(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th}\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Reis et al., 2020 were the first to deduce this analytical eq. layer by following a slightly different approach

Result 3/4: The approx total-field anomaly can be exactly reproduced by an analytical eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

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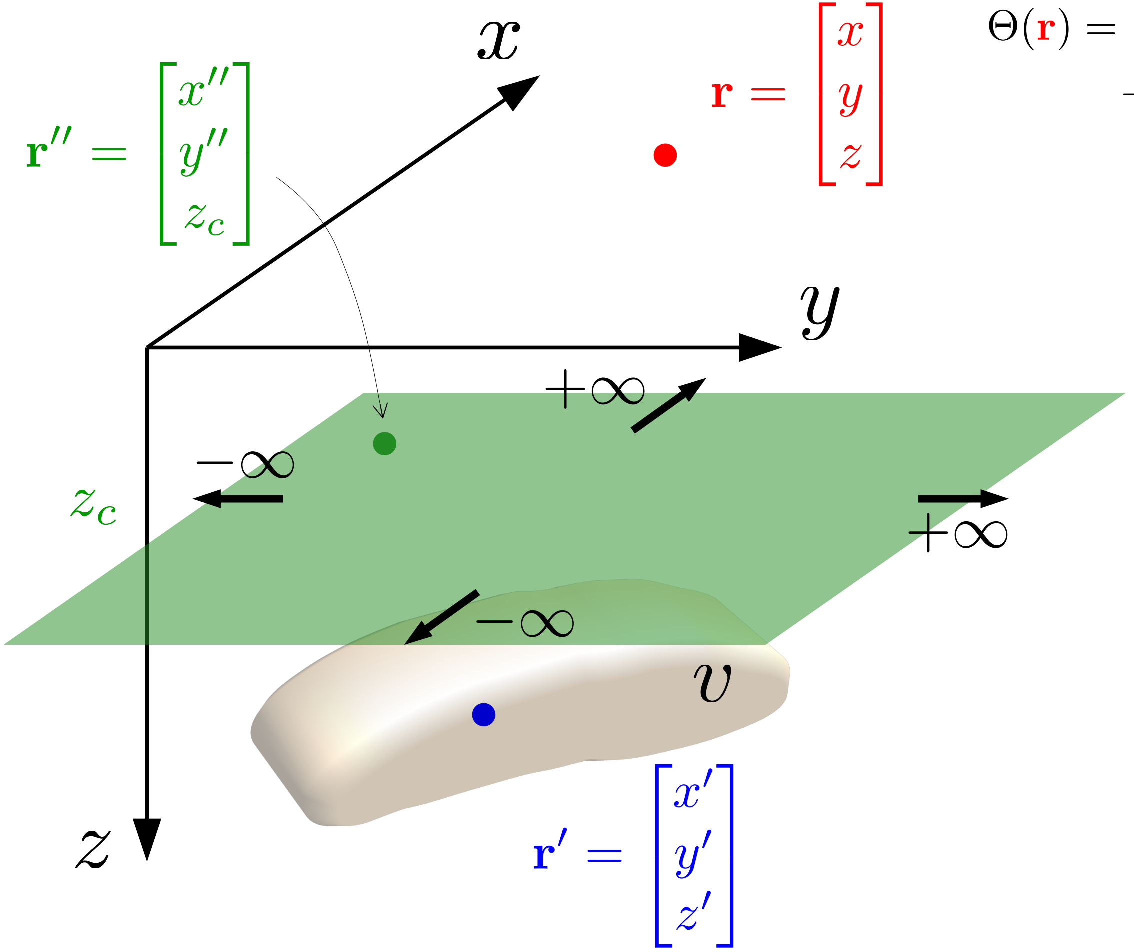


Deduction:

approx total-field anomaly

$$\partial_{th}\Theta(\mathbf{r}) = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Theta}(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th}\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

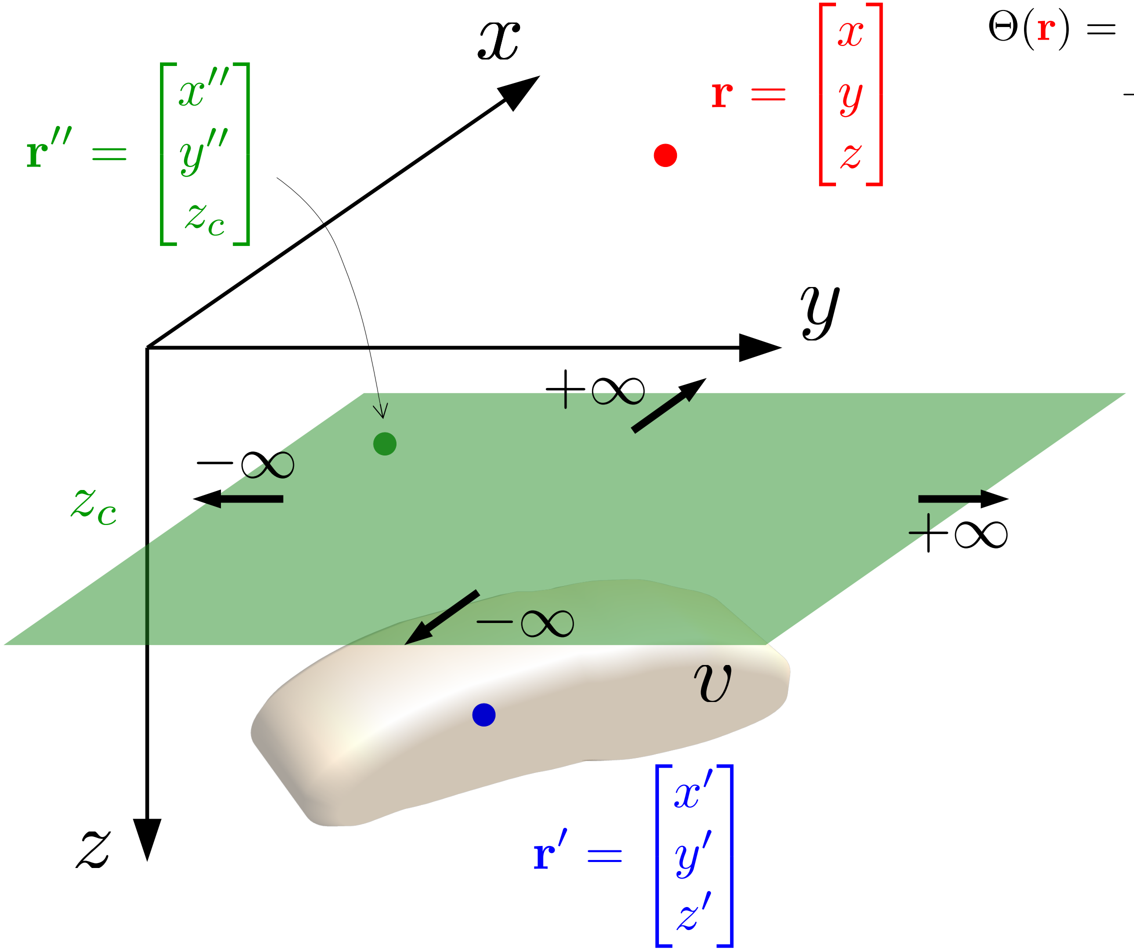
Total-magnetization intensity distribution within the true sources

$$\partial_z \Theta(\mathbf{r}'') = \kappa \iiint_v \sigma(\mathbf{r}') \; \partial_z \frac{1}{\|\mathbf{r}'' - \mathbf{r}'\|} dv'$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by an analytical eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

$$\Theta(\mathbf{r}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$



Deduction:

approx total-field anomaly

$$\partial_{th}\Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th}\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Note that this function is ≥ 0 at all points **r'** within the sources

$$\partial_z \Theta(\mathbf{r}'') = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \; \partial_z \frac{1}{\|\mathbf{r}'' - \mathbf{r}'\|} \, dv'$$

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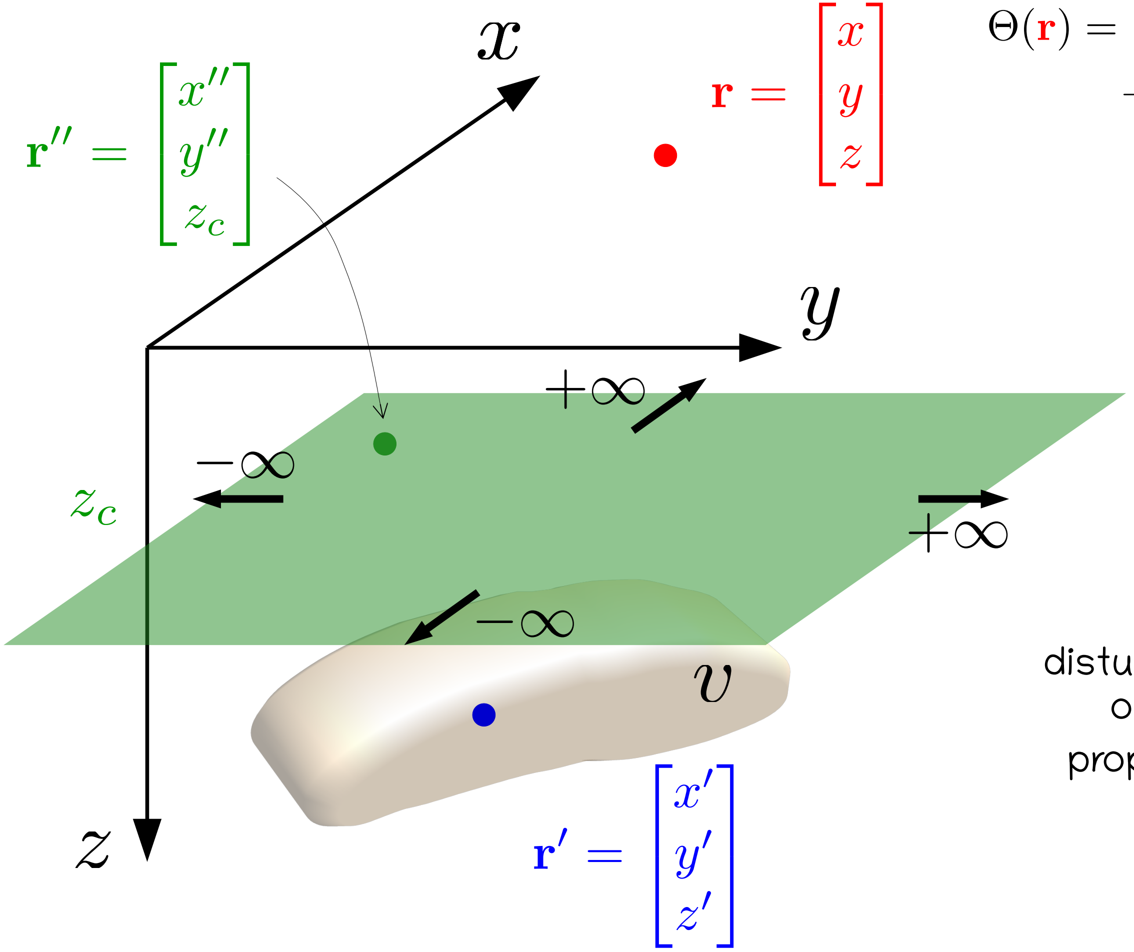


Deduction:

approx total-field anomaly

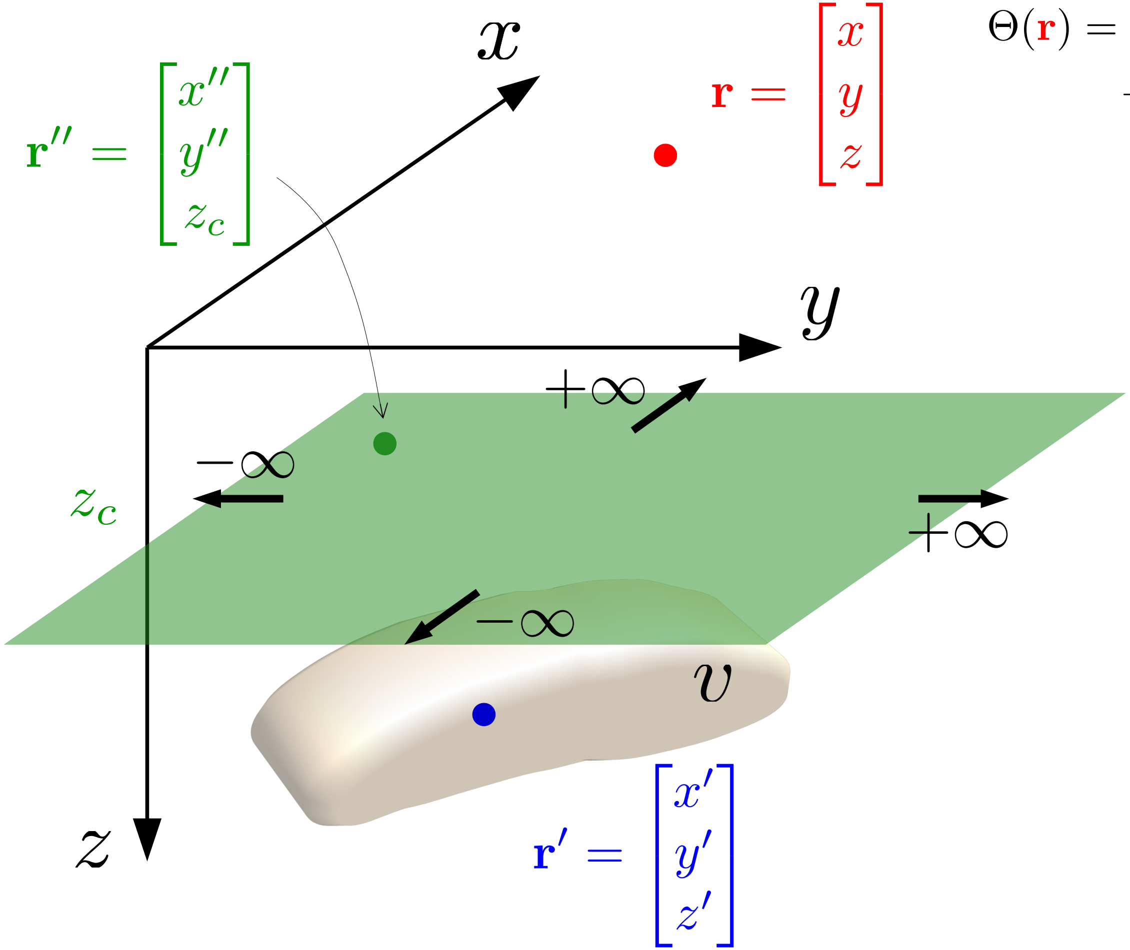
$$\partial_{th}\Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th}\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th}\Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

This analytical eq. layer represents the gravity disturbance that would be produced by the true sources on the plane z_c if they had a density distribution proportional to the total-magnetization intensity $\sigma(\mathbf{r}')$

$$\partial_z \Theta(\mathbf{r}'') = \kappa \iiint_v \sigma(\mathbf{r}') \; \partial_z \frac{1}{\|\mathbf{r}'' - \mathbf{r}'\|} \, dv'$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by an analytical eq. layer of dipoles having the same uniform total-magnetization direction of the true sources



$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

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Deduction:

approx total-field anomaly

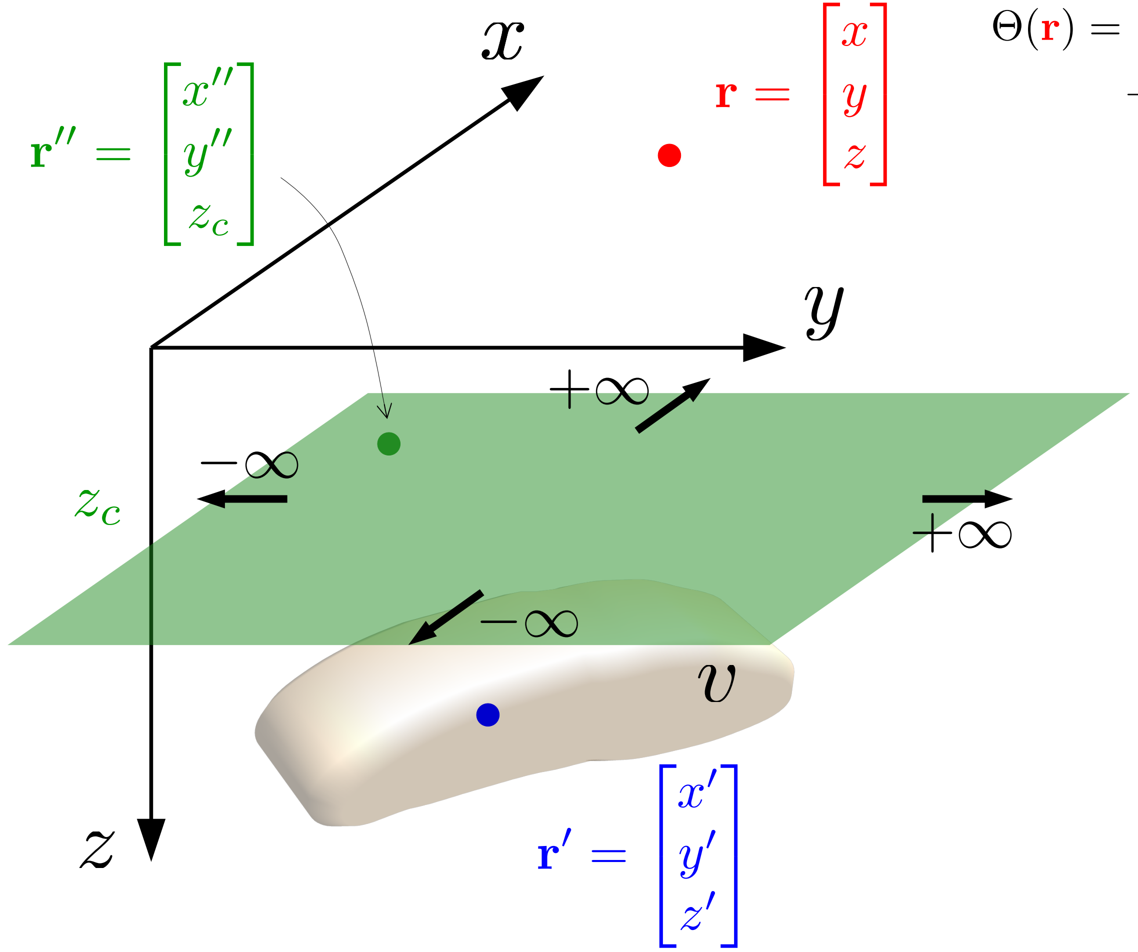
$$\partial_{th}\Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th}\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th}\Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{th}\Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by an analytical eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$



$$\Theta(\mathbf{r}) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$

Deduction:

approx total-field anomaly

$$\partial_{th}\Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th}\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th}\Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{th}\Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\approx \partial_z \Theta(\mathbf{r}''_j)$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by an analytical eq. layer of dipoles having the same uniform total-magnetization direction of the true sources

$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} dv'$$

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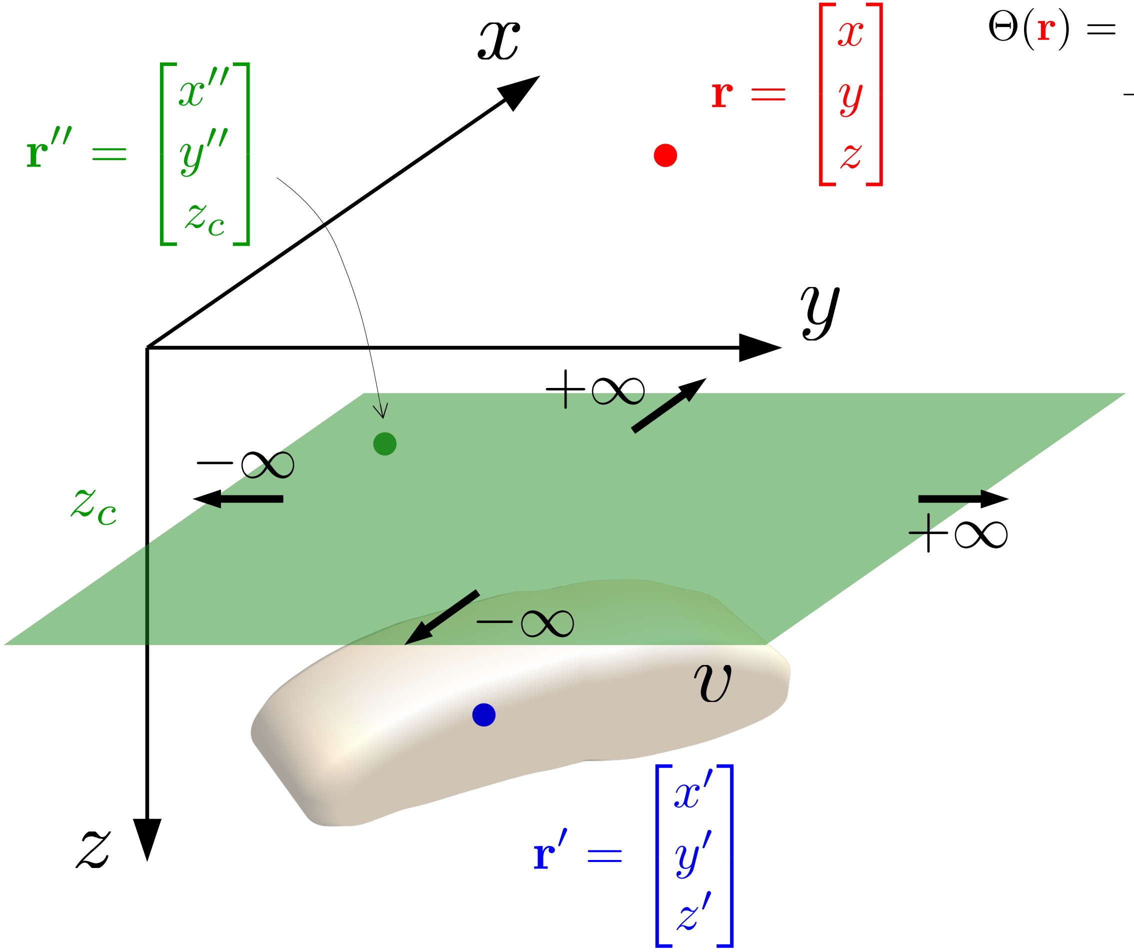


Deduction:

approx total-field anomaly

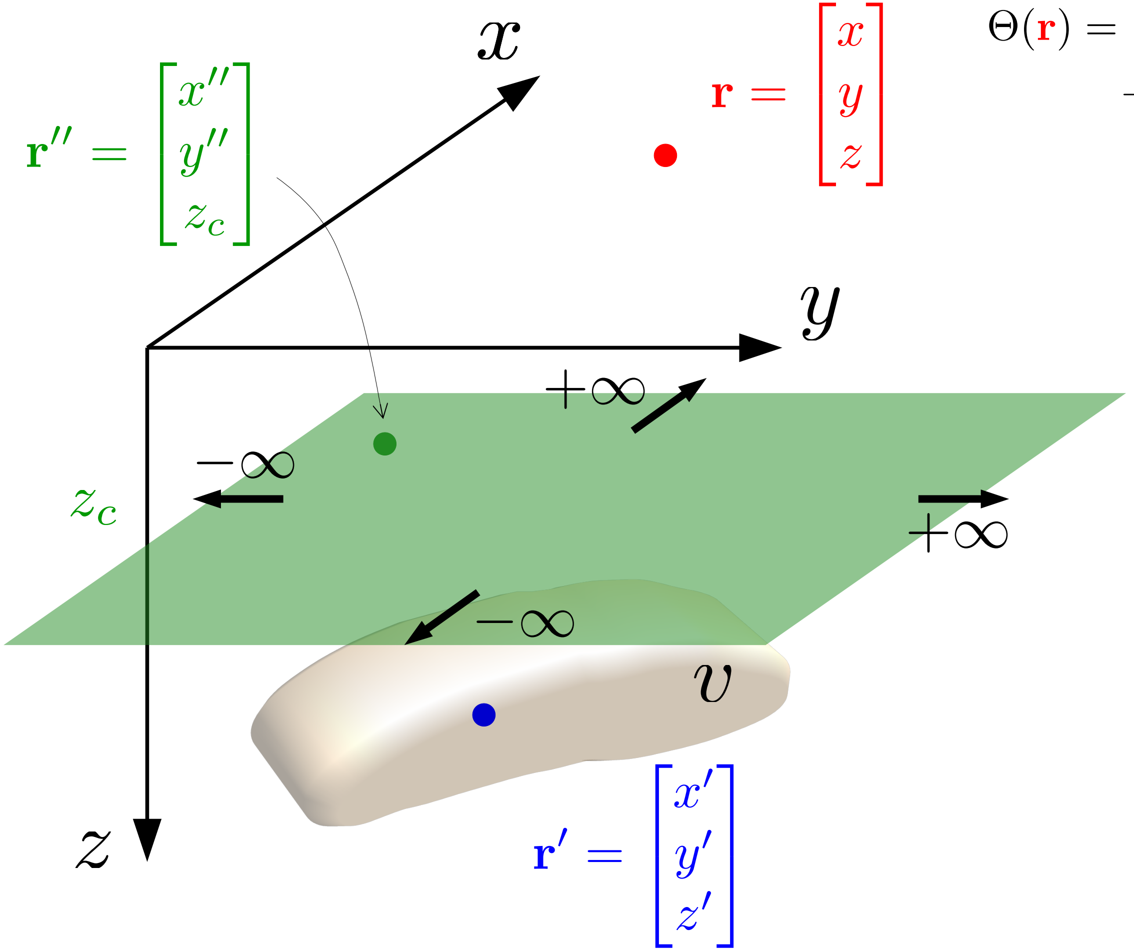
$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th} \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{th} \Psi(\mathbf{r}_i, \mathbf{r}''_j)$$

$$\partial_{th} \Theta(\mathbf{r}_i) \qquad \approx \partial_z \Theta(\mathbf{r}''_j)$$

Result 3/4: The approx total-field anomaly can be exactly reproduced by an analytical eq. layer of dipoles having the same uniform total-magnetization direction of the true sources



$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

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Deduction:

approx total-field anomaly

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^{\top} \mathbf{H}_{\Theta}(\mathbf{r}) \, \hat{\mathbf{h}}$$

$$\partial_{th} \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$$d_i \approx \sum_{j=1}^{M} p_j \, \partial_{th} \Psi(\mathbf{r}_i, \mathbf{r}_j'')$$

classical EqL technique
applied to magnetic data

Result 4/4: The same analytical eq. layer of dipoles reproducing the approx total-field anomaly with the same uniform total-mag. direction of the true sources can also reproduce the mag. scalar potential, field components and RTP anomaly

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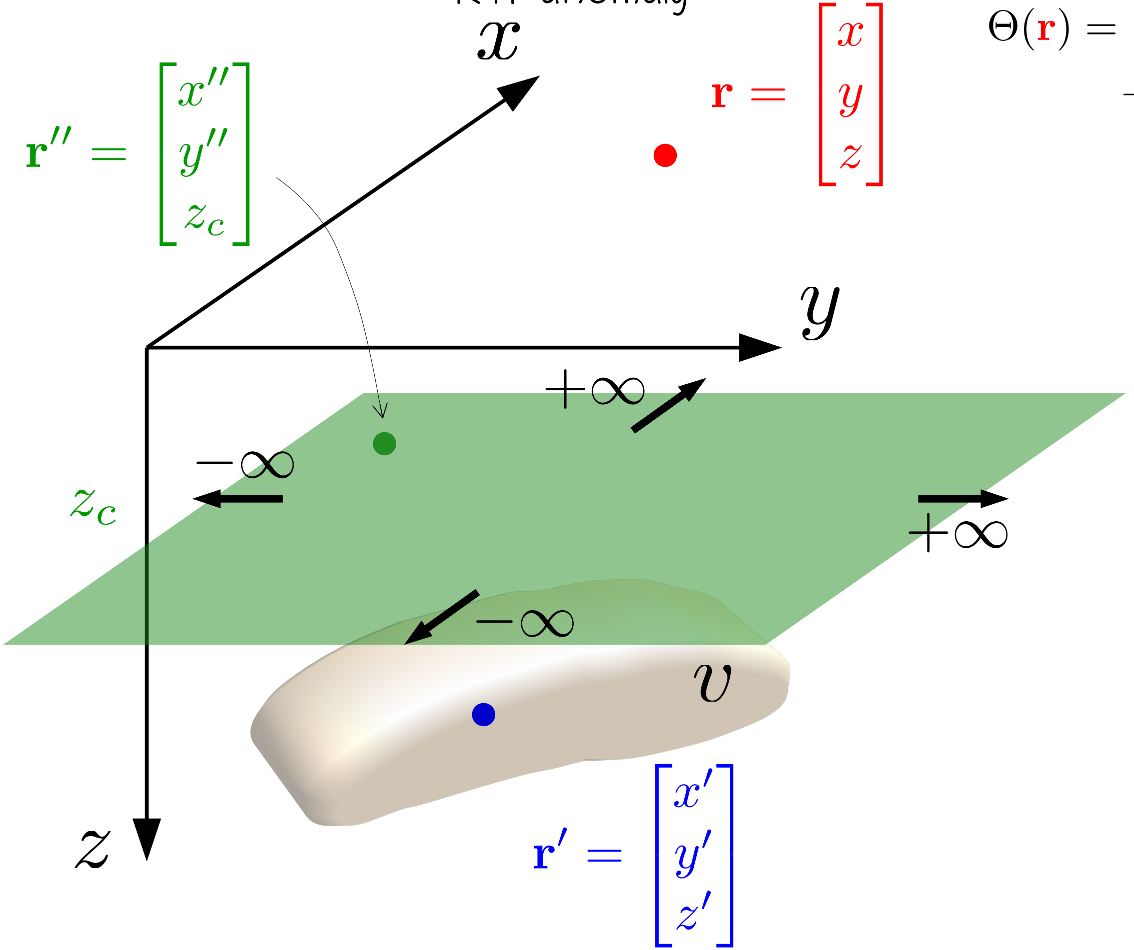

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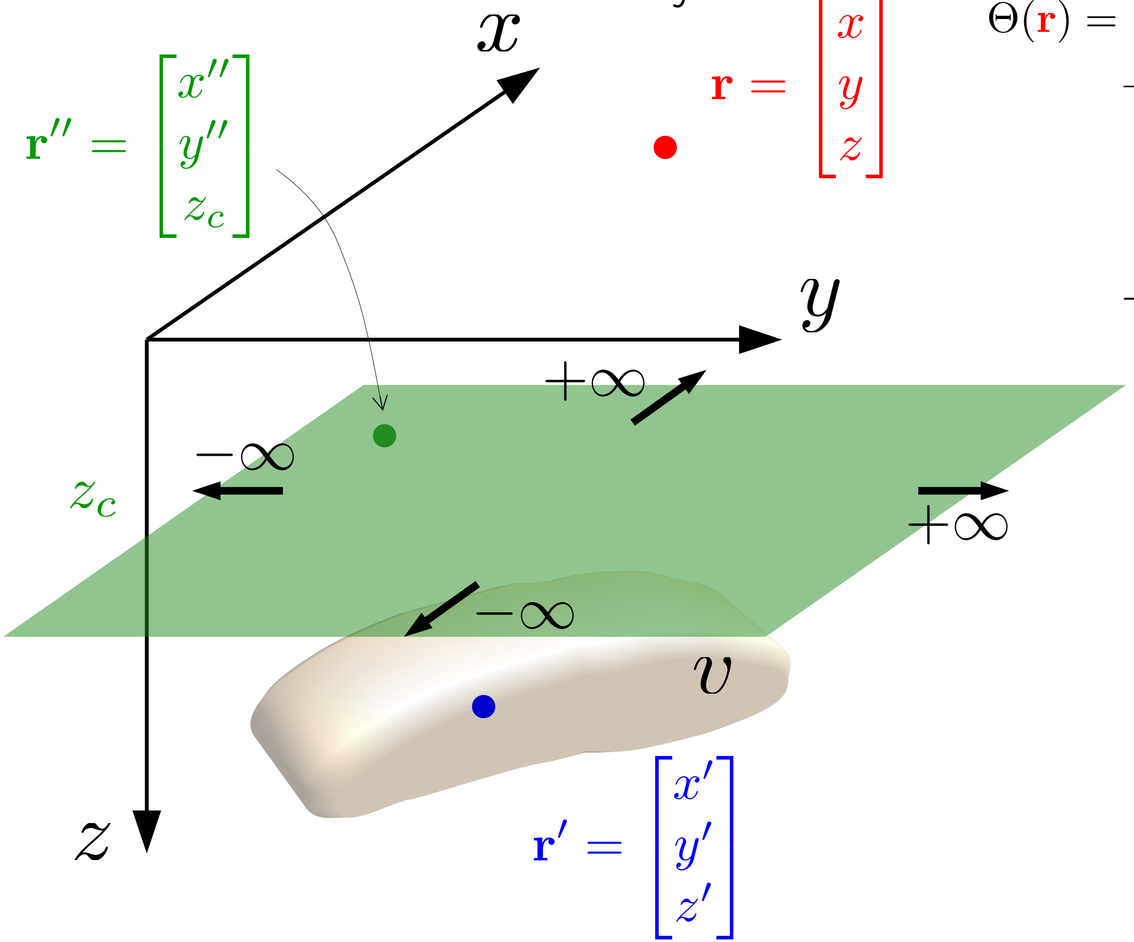

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$$-\partial_h \Theta(\mathbf{r}) = -\nabla \Theta(\mathbf{r})^\top \hat{\mathbf{h}}$$ mag scalar potential

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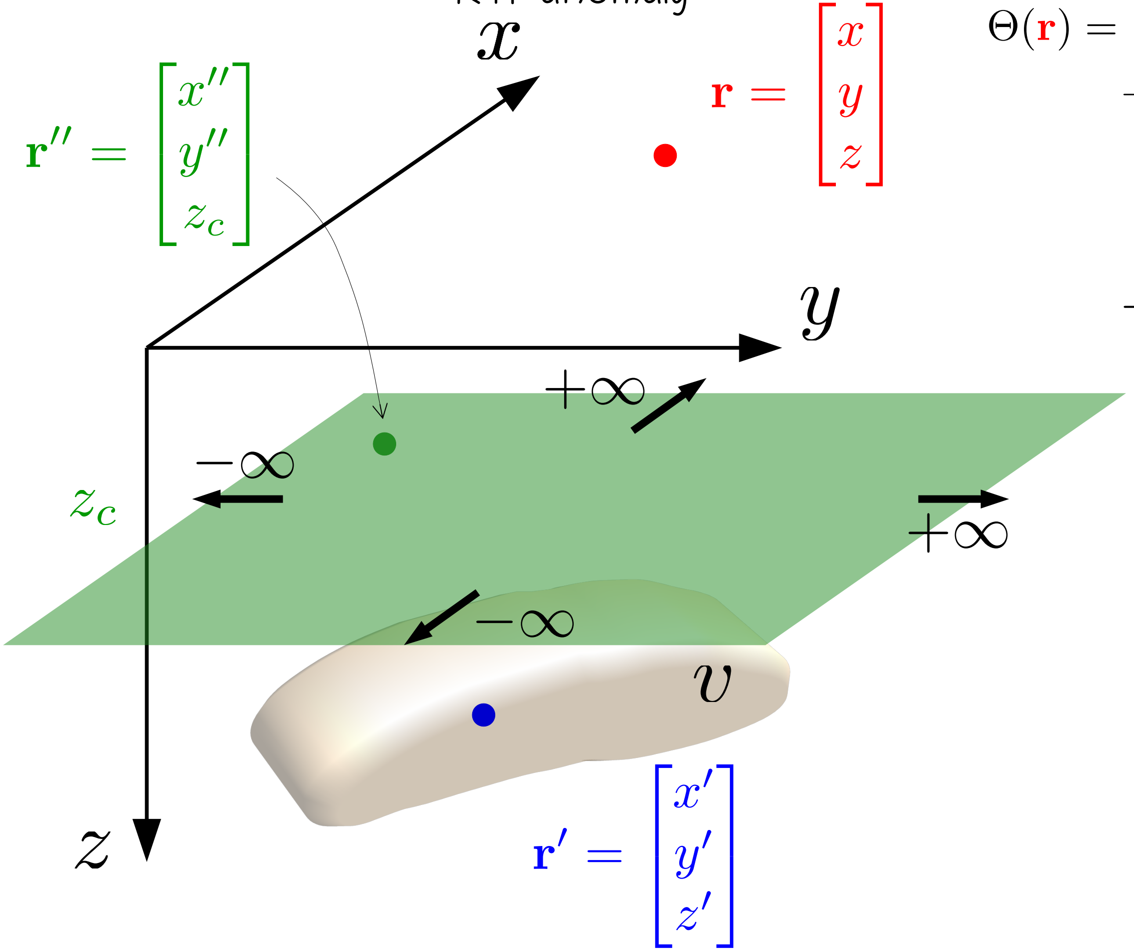

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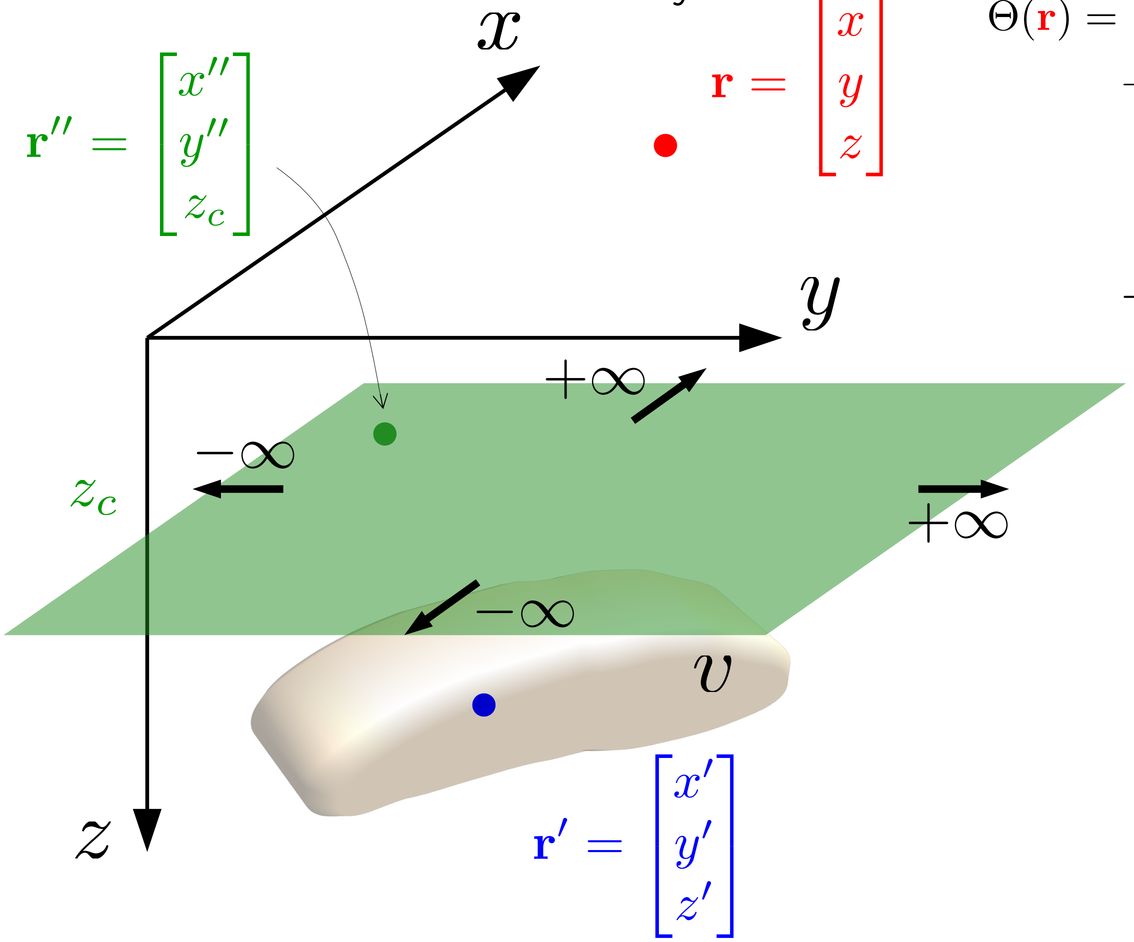

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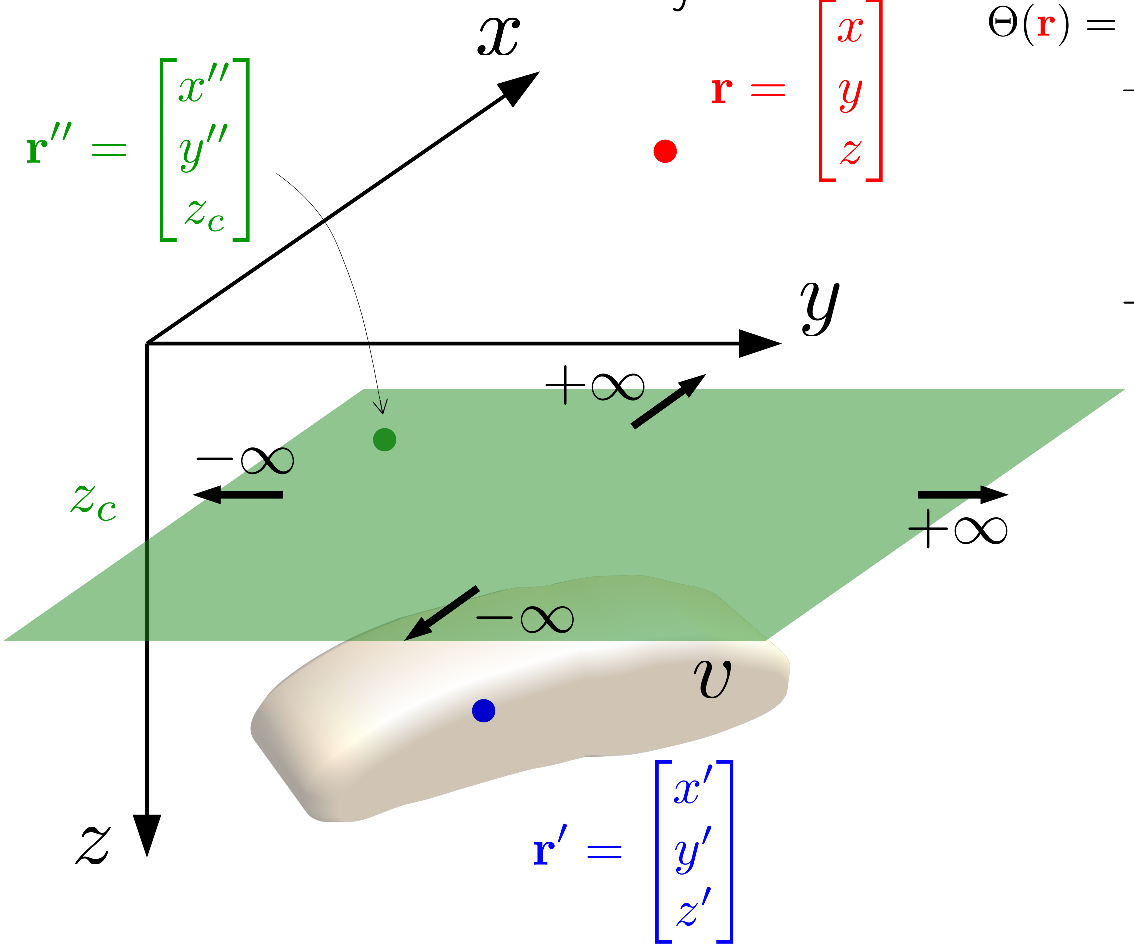

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$\partial_{zz} \Theta(\mathbf{r})$ RTP anomaly

Result 4/4: The same analytical eq. layer of dipoles reproducing the approx total-field anomaly with the same uniform total-mag. direction of the true sources can also reproduce the mag. scalar potential, field components and RTP anomaly

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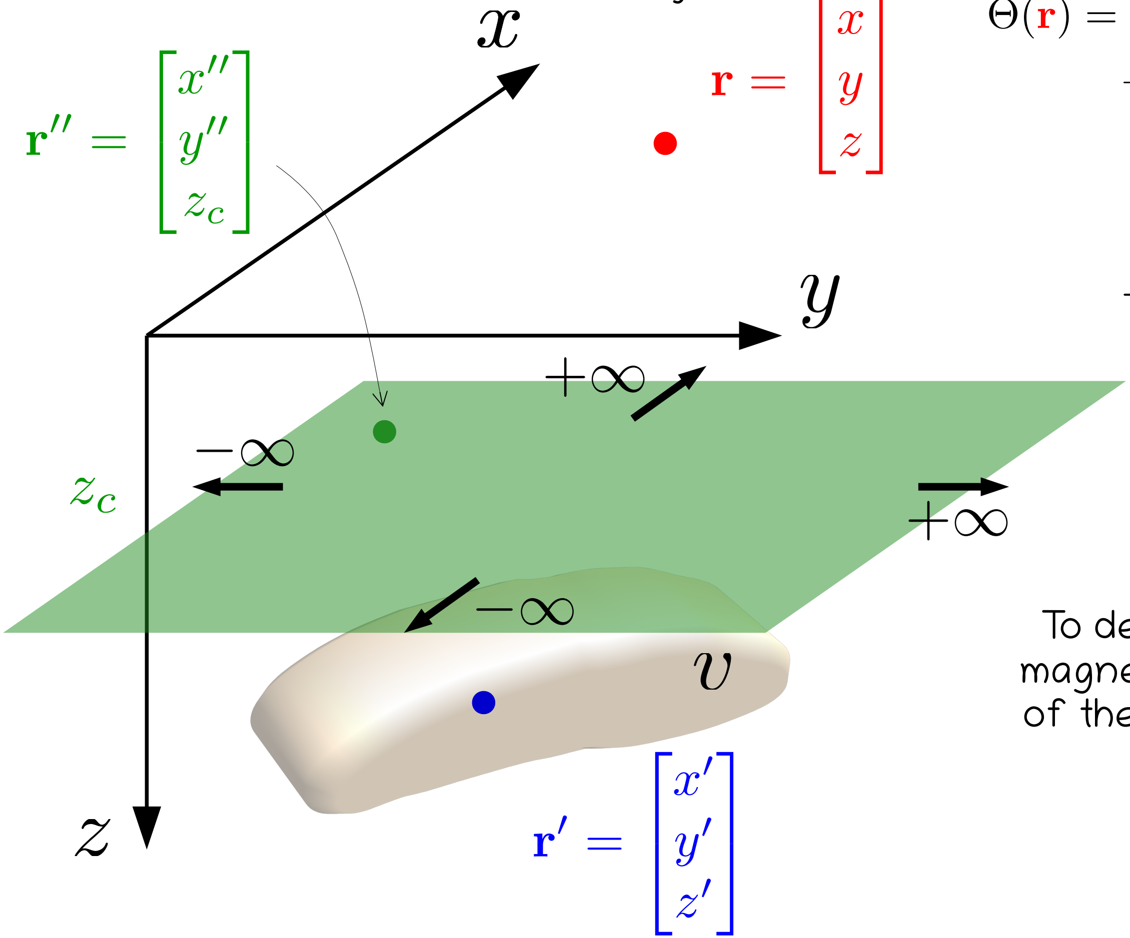

$$\Theta(\mathbf{r}) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS'' \qquad \Psi(\mathbf{r}, \mathbf{r}'') = \frac{1}{2\pi} \frac{1}{\|\mathbf{r} - \mathbf{r}''\|}$$

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$\partial_{zz} \Theta(\mathbf{r})$ RTP anomaly

To deduce the analytical eq. layer associated with these magnetic field quantities, we have to compute derivatives of the equation below with respect to the coordinates of the observation point $\mathbf{r}$

$$\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

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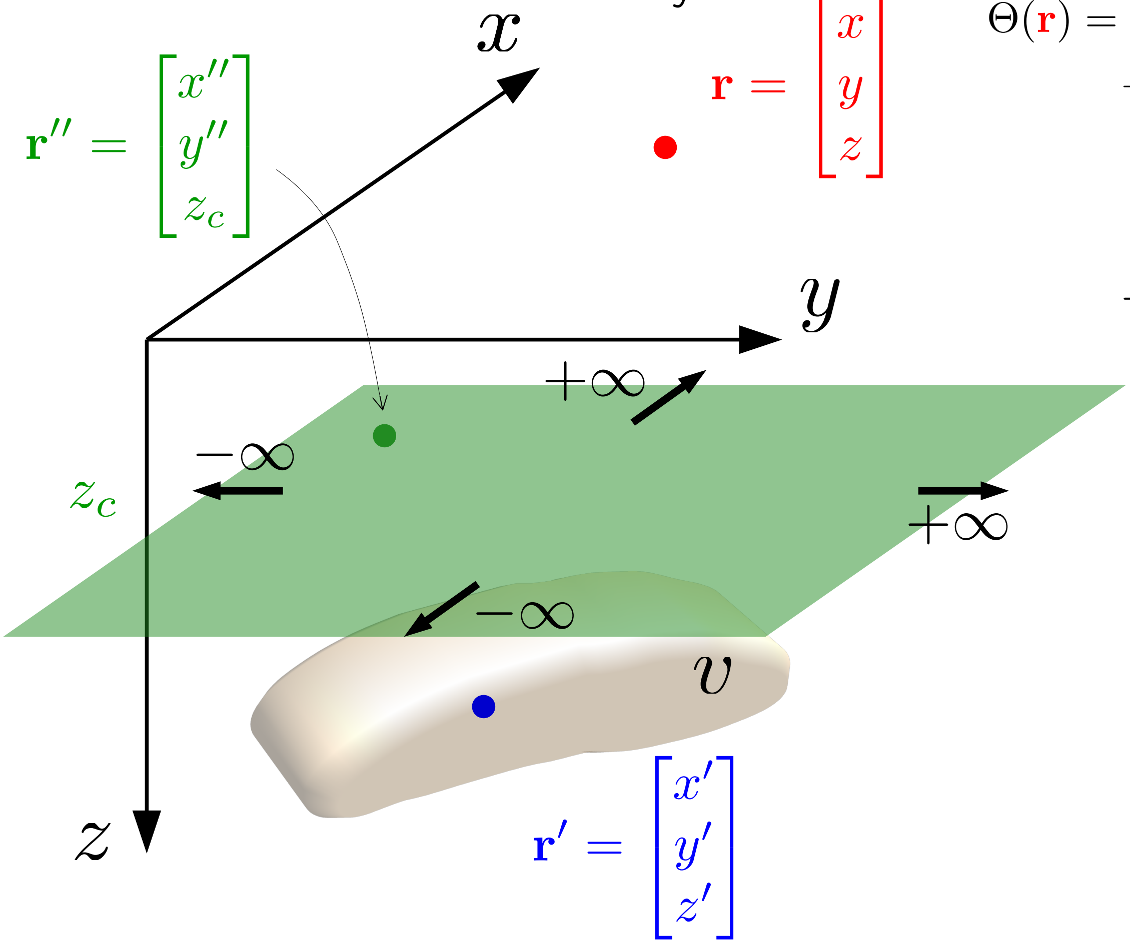

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$$\partial_{\alpha h} \Theta(\mathbf{r}) = \partial_\alpha \nabla \Theta(\mathbf{r})^\top \hat{\mathbf{h}}$$ mag field component

$$\partial_{zz} \Theta(\mathbf{r})$$ RTP anomaly

These derivatives, however, affect only this term

$$\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Result 4/4: The same analytical eq. layer of dipoles reproducing the approx total-field anomaly with the same uniform total-mag. direction of the true sources can also reproduce the mag. scalar potential, field components and RTP anomaly

$$\Theta(\mathbf{r}) = \kappa \iiint\limits_{v} \sigma(\mathbf{r}') \, \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

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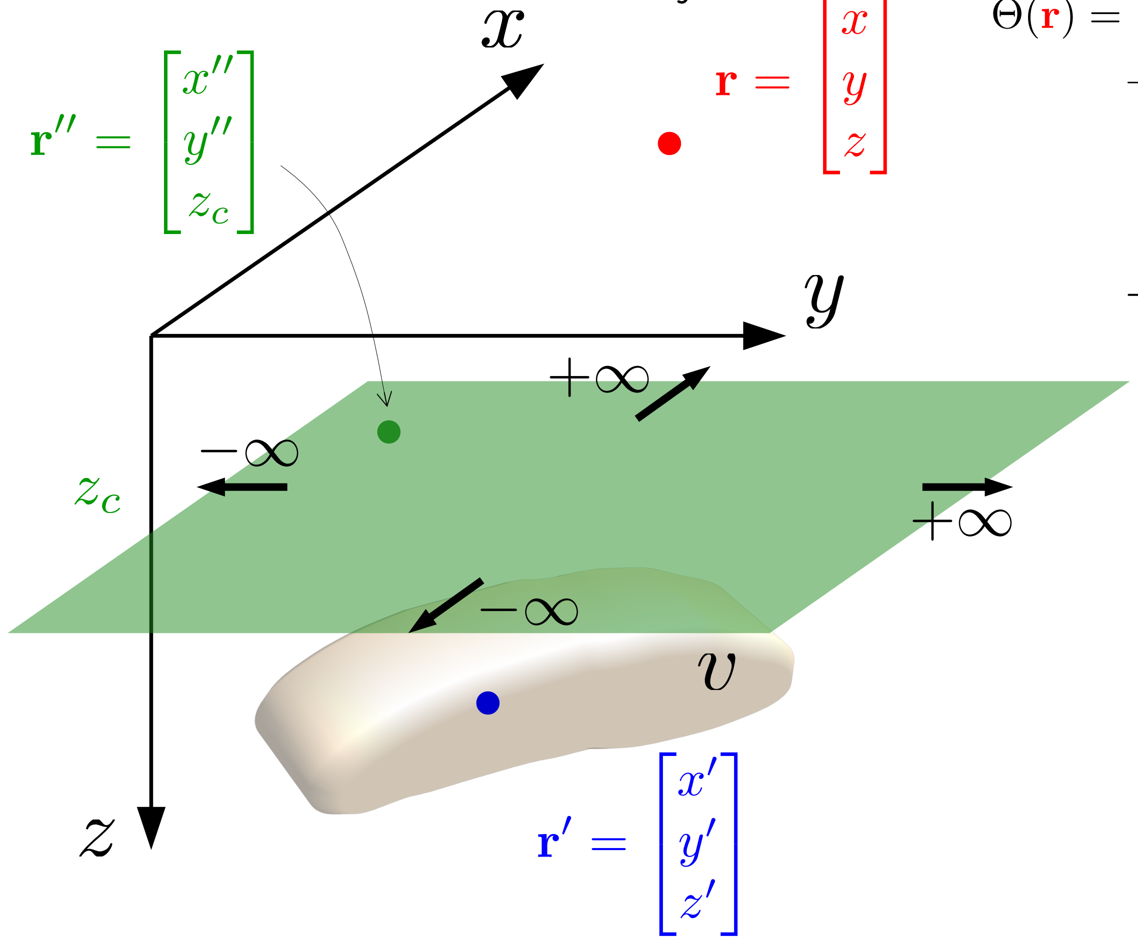


Deduction:

$$-\partial_h \Theta(\mathbf{r}) = -\nabla \Theta(\mathbf{r})^\top \hat{\mathbf{h}} \quad \text{mag scalar potential}$$

$$\partial_{th} \Theta(\mathbf{r}) = \hat{\mathbf{t}}^\top \mathbf{H}_\Theta(\mathbf{r}) \, \hat{\mathbf{h}} \quad \text{approx total-field an}$$

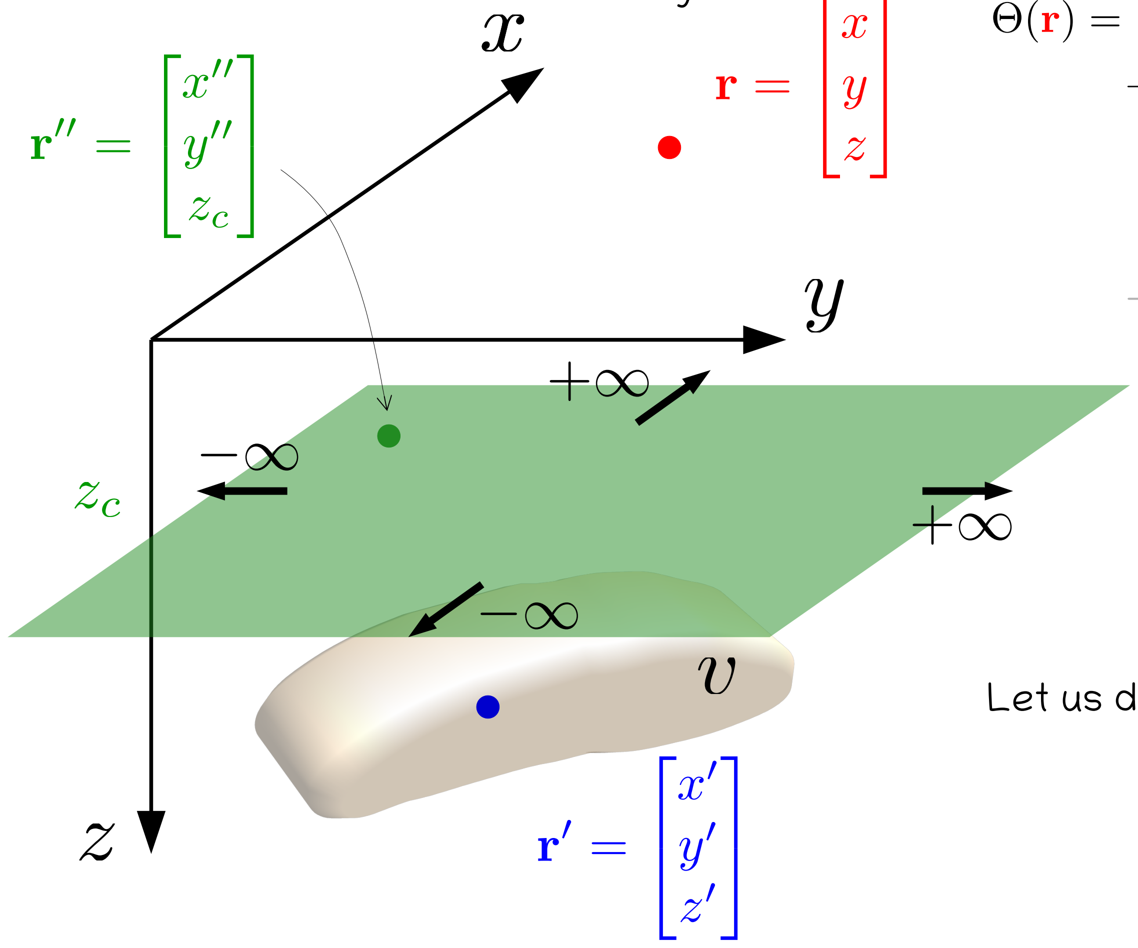
$$\partial_{\alpha h} \Theta(\mathbf{r}) = \partial_\alpha \nabla \Theta(\mathbf{r})^\top \hat{\mathbf{h}} \quad \text{mag field component}$$

$$\partial_{zz} \Theta(\mathbf{r}) \quad \text{RTP anomaly}$$

Hence, all these quantities are defined in terms of the same eq. layer

$$\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Result 4/4: The same analytical eq. layer of dipoles reproducing the approx total-field anomaly with the same uniform total-mag. direction of the true sources can also reproduce the mag. scalar potential, field components and RTP anomaly



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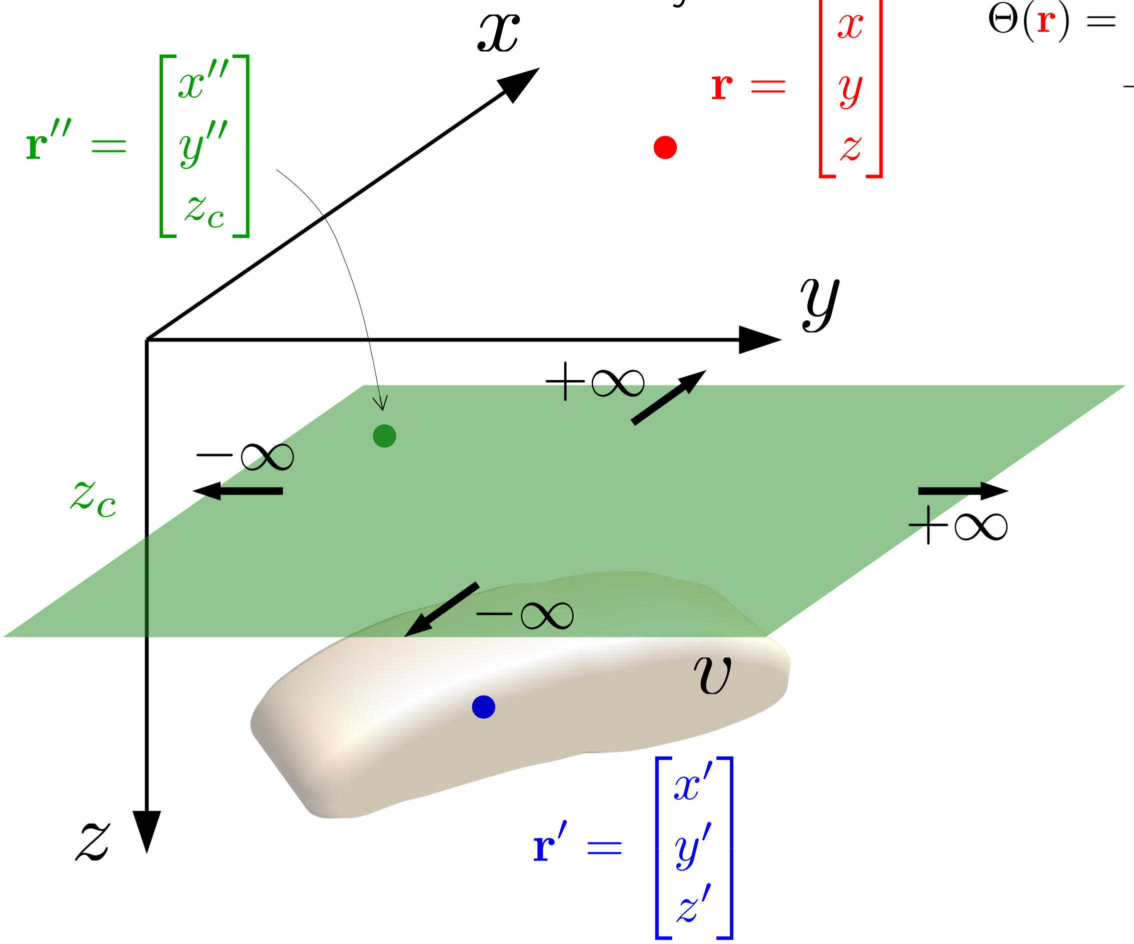
$\partial_{zz} \Theta(\mathbf{r})$ RTP anomaly

Let us draw our attention to these two field quantites

Result 4/4: The same analytical eq. layer of dipoles reproducing the approx total-field anomaly with the same uniform total-mag. direction of the true sources can also reproduce the mag. scalar potential, field components and RTP anomaly



$$\Theta(\mathbf{r}) = \kappa \iiint_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r} - \mathbf{r}'\|} \, dv'$$

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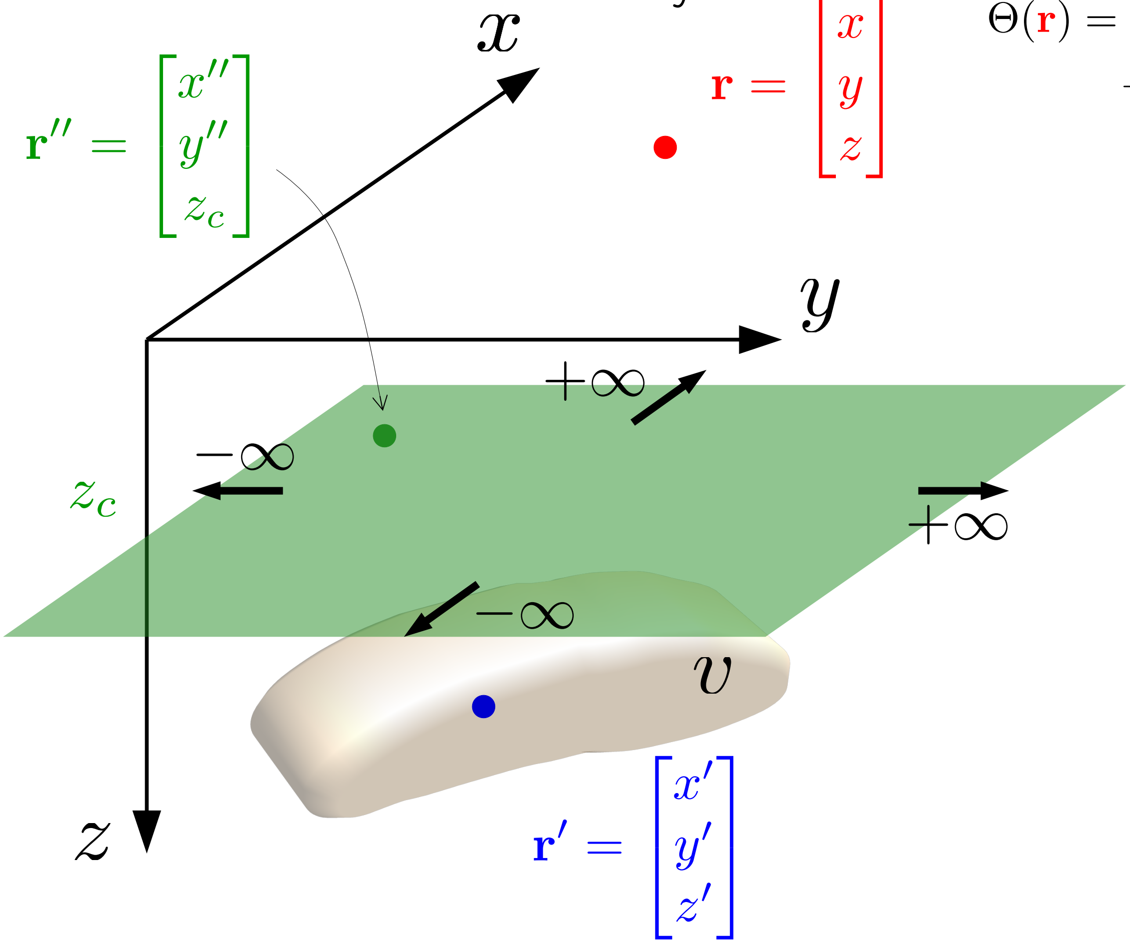
Deduction:

These equations represent the theoretical basis for computing the RTP in space domain, from total-field anomaly data, via EqL technique, according to the pioneer work of Silva (1986)

$$\partial_{th} \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$$\partial_{zz} \Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{zz} \Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

Result 4/4: The same analytical eq. layer of dipoles reproducing the approx total-field anomaly with the same uniform total-mag. direction of the true sources can also reproduce the mag. scalar potential, field components and RTP anomaly



$$\Theta(\mathbf{r}) = \kappa \iiint\limits_v \sigma(\mathbf{r}') \frac{1}{\|\mathbf{r}-\mathbf{r}'\|} \, dv'$$

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Deduction:

However, these equations have never been clearly defined in pioneer work of Silva (1986), nor in the succeeding studies (e.g., Leão and Silva, 1989; Guspí and Novara, 2009; Oliveira Jr. et al., 2013; Li et al., 2014; Reis et al., 2020; Takahashi et al., 2020) proposed to compute the RTP in space-domain, via EqL technique

$$\partial_{th}\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{th}\Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

$$\partial_{zz}\Theta(\mathbf{r}) = \iint \partial_z \Theta(\mathbf{r}'') \, \partial_{zz}\Psi(\mathbf{r}, \mathbf{r}'') \, dS''$$

List of theoretical results

Result 1/4: The gravity disturbance can be exactly reproduced by an analytical eq. layer of monopoles

Result 2/4: The same analytical eq. layer of monopoles reproducing the gravity disturbance can also reproduce the gravitational potential and gravity gradient tensor

Result 3/4: The approx total-field anomaly can be exactly reproduced by an analytical eq. layer of dipoles having the same uniform total-mag. direction of the true sources

Result 4/4: The same analytical eq. layer of dipoles reproducing the approx total-field anomaly with the same uniform total-mag. direction of the true sources can also reproduce the mag. scalar potential, field components and RTP anomaly

Summary

- Motivation
- Potential-field data
- The Equivalent-Layer (EqL) Technique
- **Theoretical aspects**
- Some open questions

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- Theoretical aspects
- **Some open questions**

- Is there an analytical eq. layer of dipoles with arbitrary uniform magnetization direction?

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approx total-field anomaly

total-magnetization direction
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?

arbitrary direction

- Is there an analytical eq. layer of dipoles with arbitrary uniform magnetization direction?

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- How to generalize the approach presented here to deduce analytical eq. layers in spherical coordinates?