

Regularização

Estrutura

- Problemas Inversos
 - Introdução
- Sistemas lineares
 - Determinante $\neq 0$
 - Determinante = 0
 - Determinante ≈ 0
- Problemas lineares
- Problemas não-lineares
- Regularização

Problemas Inversos

(Introdução)

$$\bar{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}_{M \times 1}$$

parâmetros

$$\bar{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}_{N \times 1} \quad \bar{g}(\bar{p}) = \begin{bmatrix} g_1(\bar{p}) \\ \vdots \\ g_N(\bar{p}) \end{bmatrix}_{N \times 1}$$

dados
observados dados
 preditos

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$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

norma L2
(função escalar)

Problemas Inversos

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$$\bar{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}_{M \times 1} \quad \nabla \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

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Problema linear



Problema não-linear



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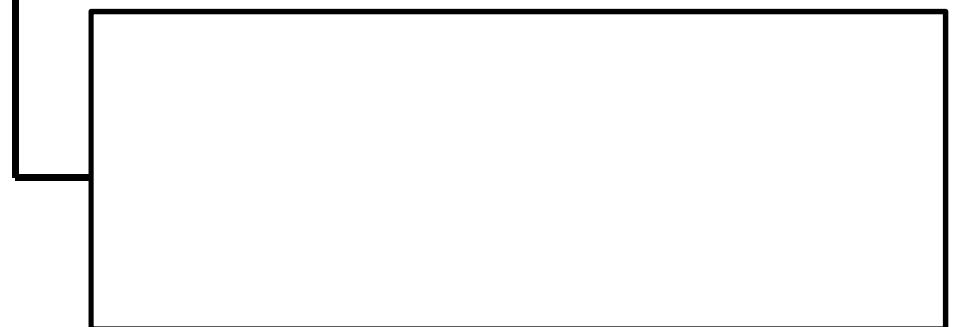
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Problemas Inversos

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norma L2
(função escalar)

$$\nabla \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

Problema linear

$$\bar{g}(\bar{p}) = \bar{B}\bar{p} + \bar{b}$$

$$\bar{p}^* = \left(\bar{B}^T \bar{B} \right)^{-1} \bar{B}^T [\bar{d} - \bar{b}]$$

Problema não-linear

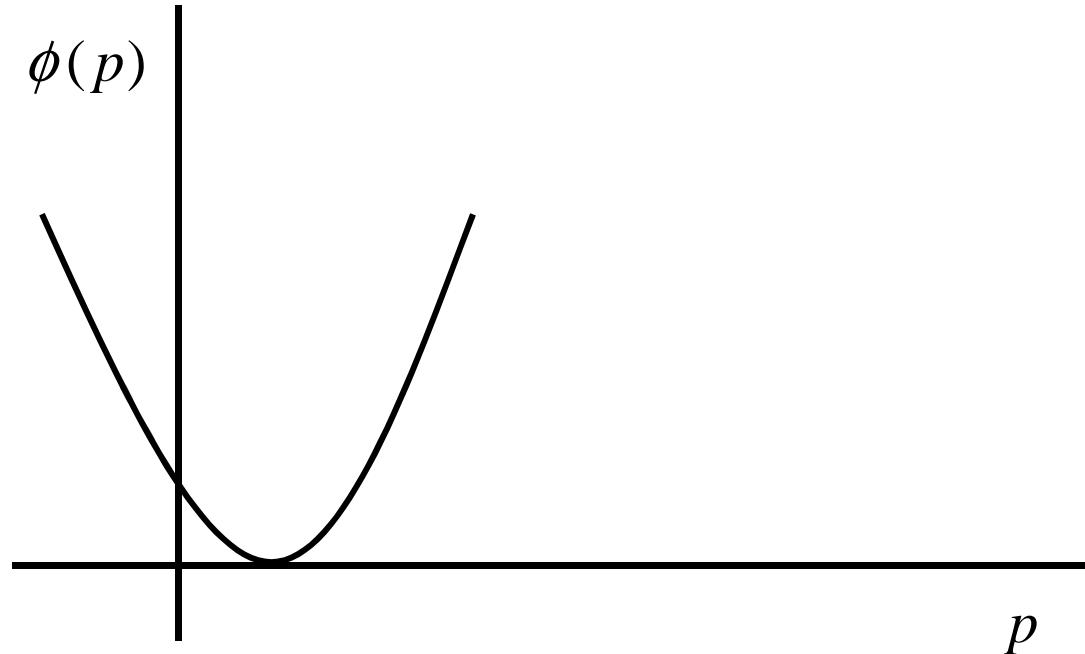
$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\Delta\bar{p} = \left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right)^{-1} \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Problemas Inversos

(Introdução)

Exemplo
linear 1D

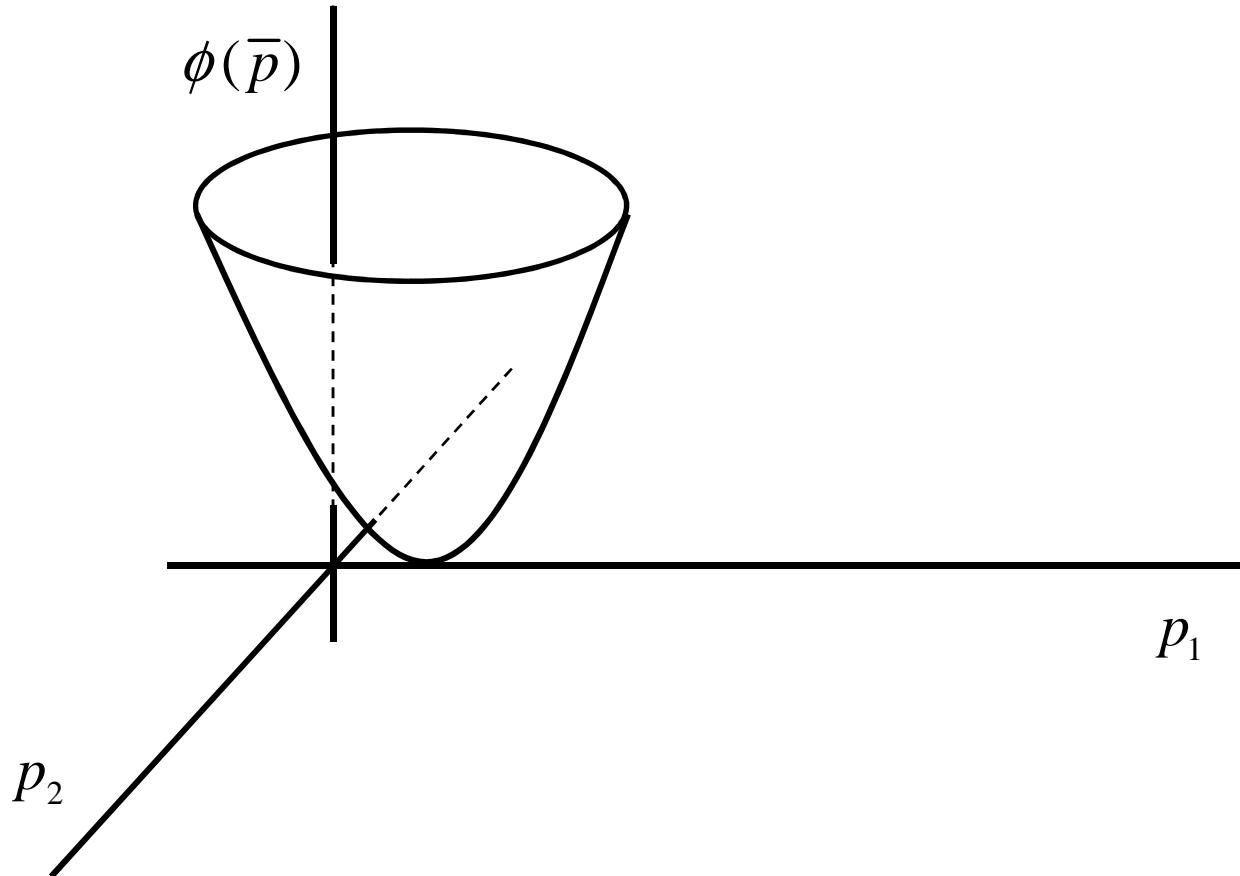


$$\phi(p) = [\bar{d} - \bar{g}(p)]^T [\bar{d} - \bar{g}(p)]$$

Problemas Inversos

(Introdução)

Exemplo
linear 2D

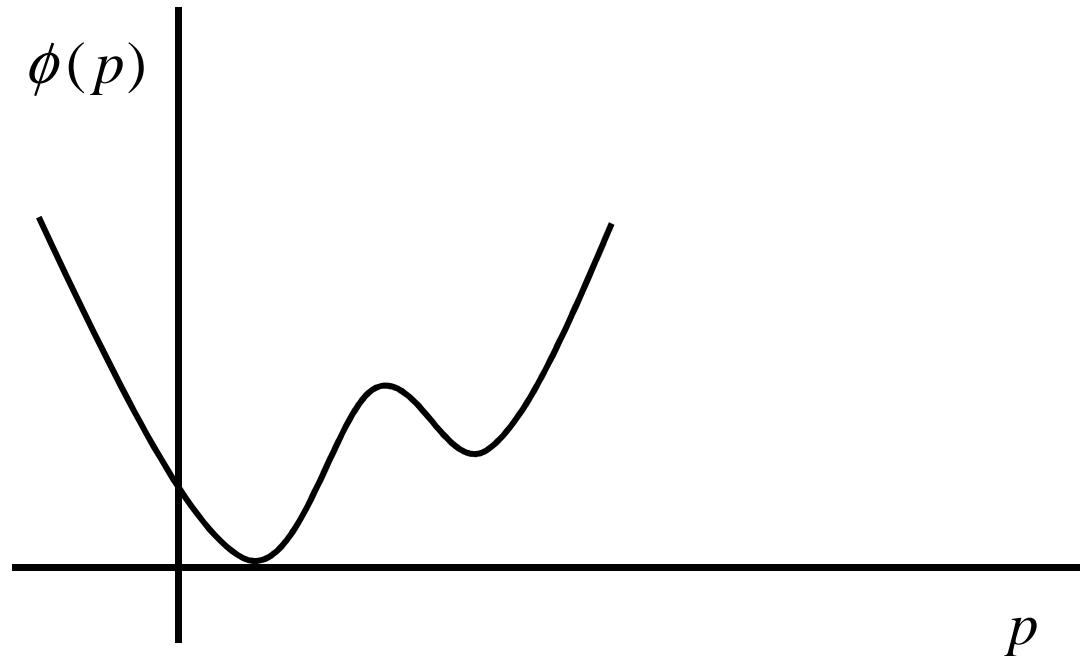


$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

Problemas Inversos

(Introdução)

Exemplo
não-linear 1D

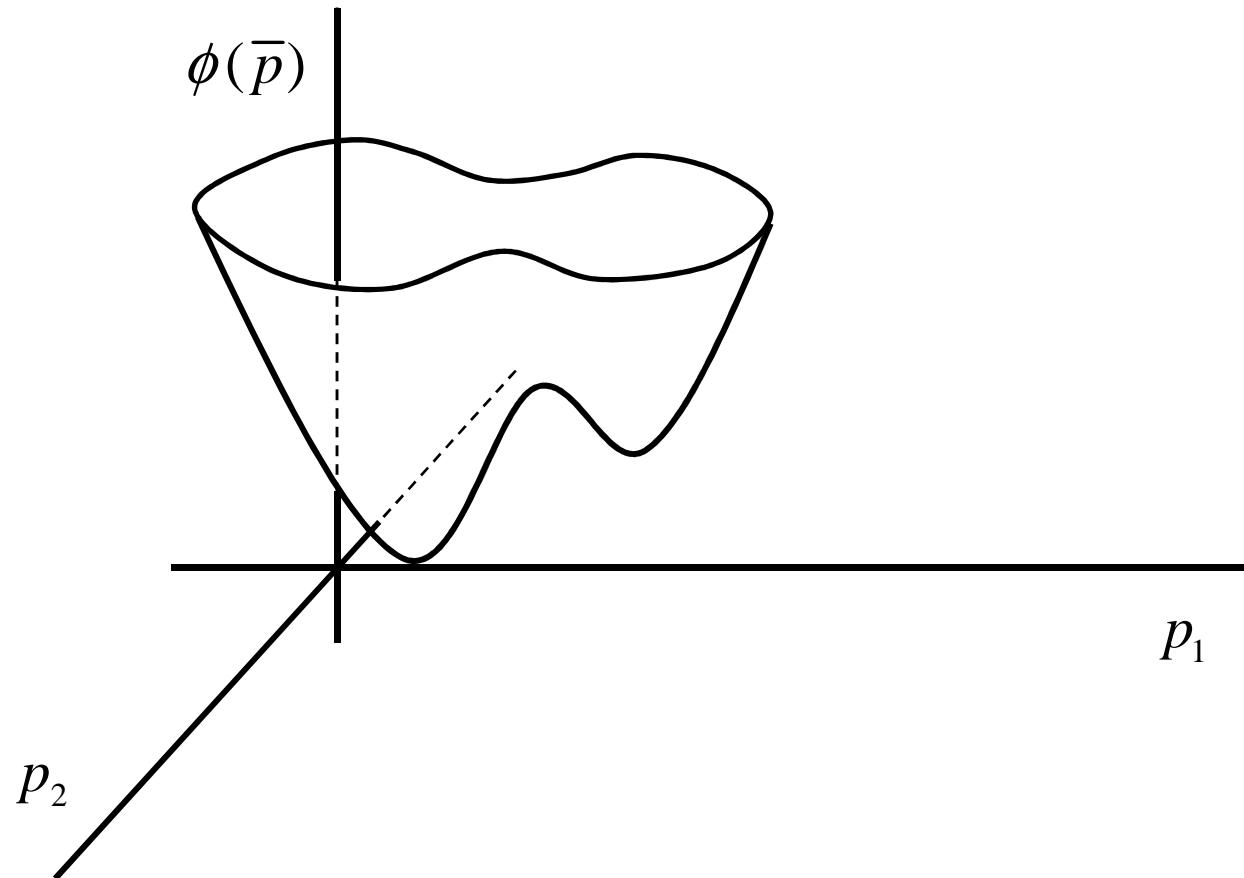


$$\phi(p) = [\bar{d} - \bar{g}(p)]^T [\bar{d} - \bar{g}(p)]$$

Problemas Inversos

(Introdução)

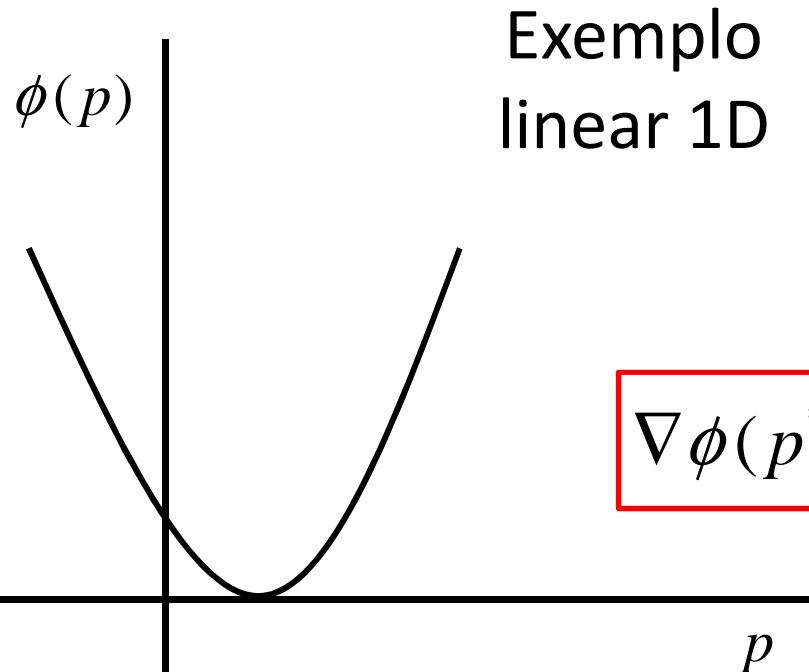
Exemplo
não-linear 2D



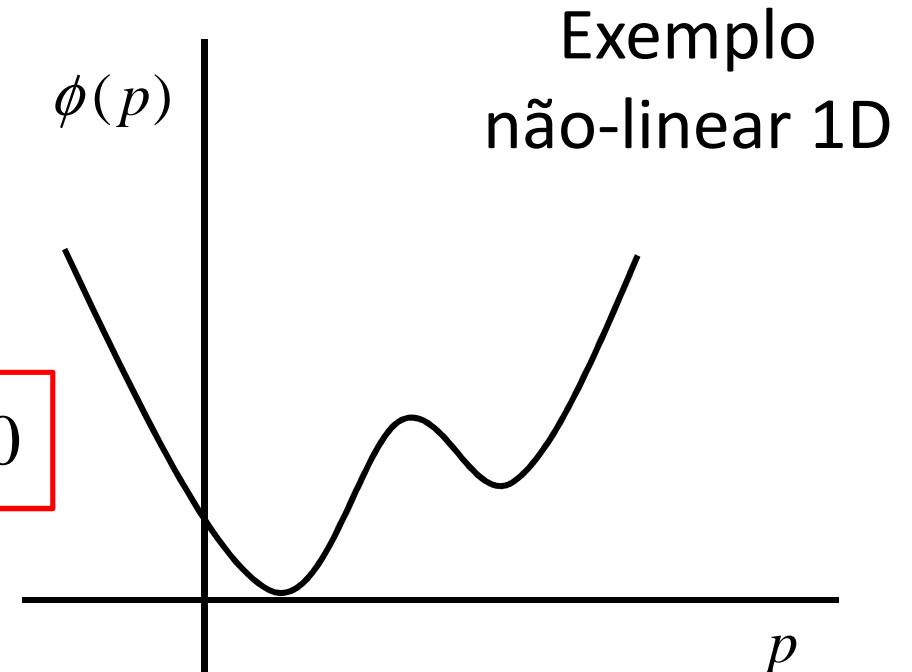
$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

Problemas Inversos

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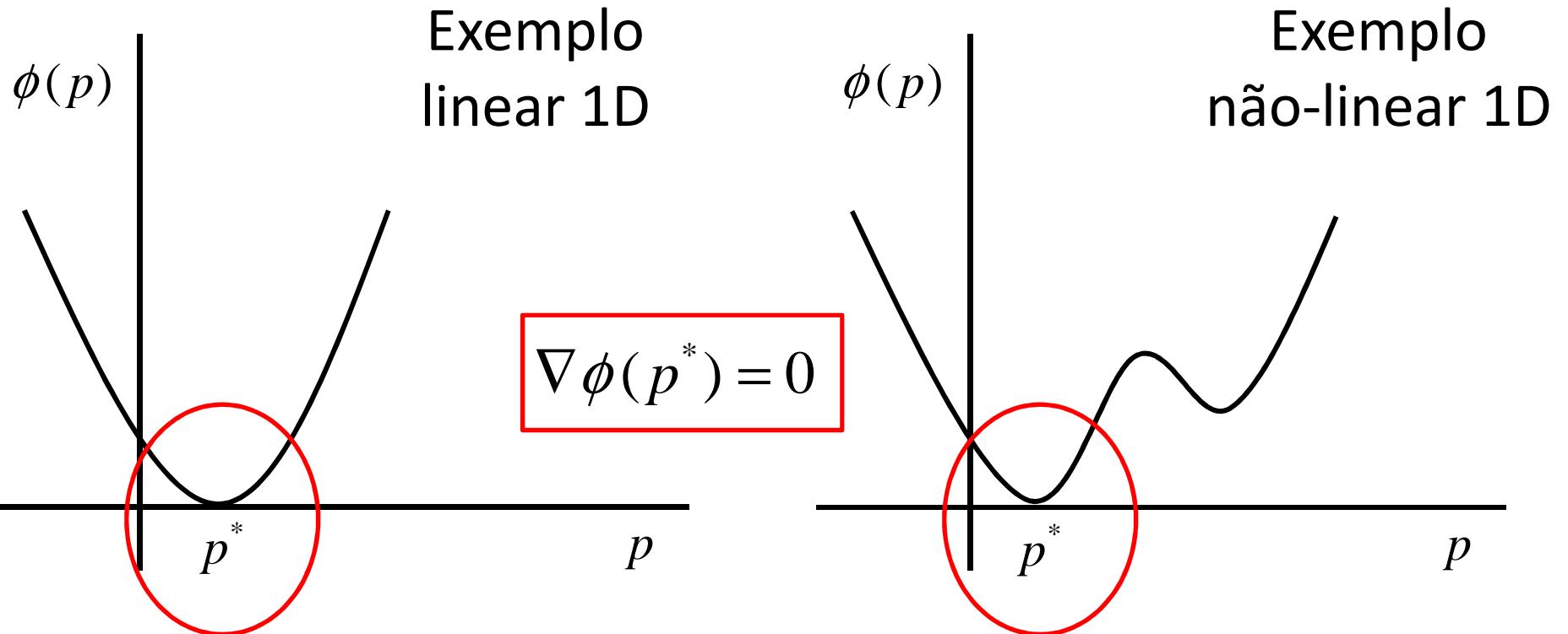


$$\nabla \phi(p^*) = 0$$



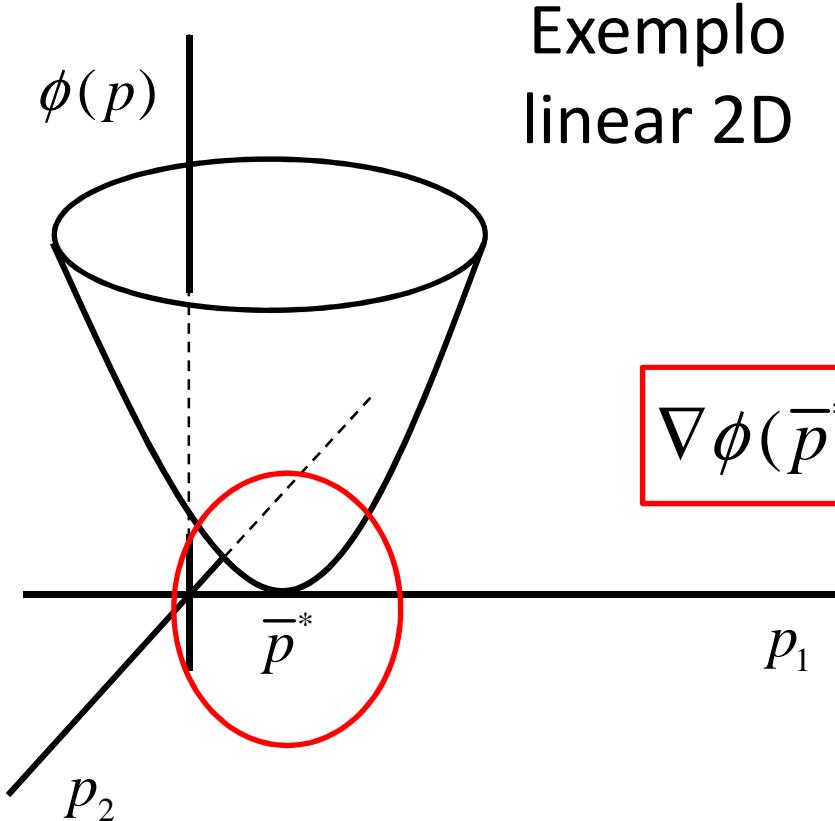
Problemas Inversos

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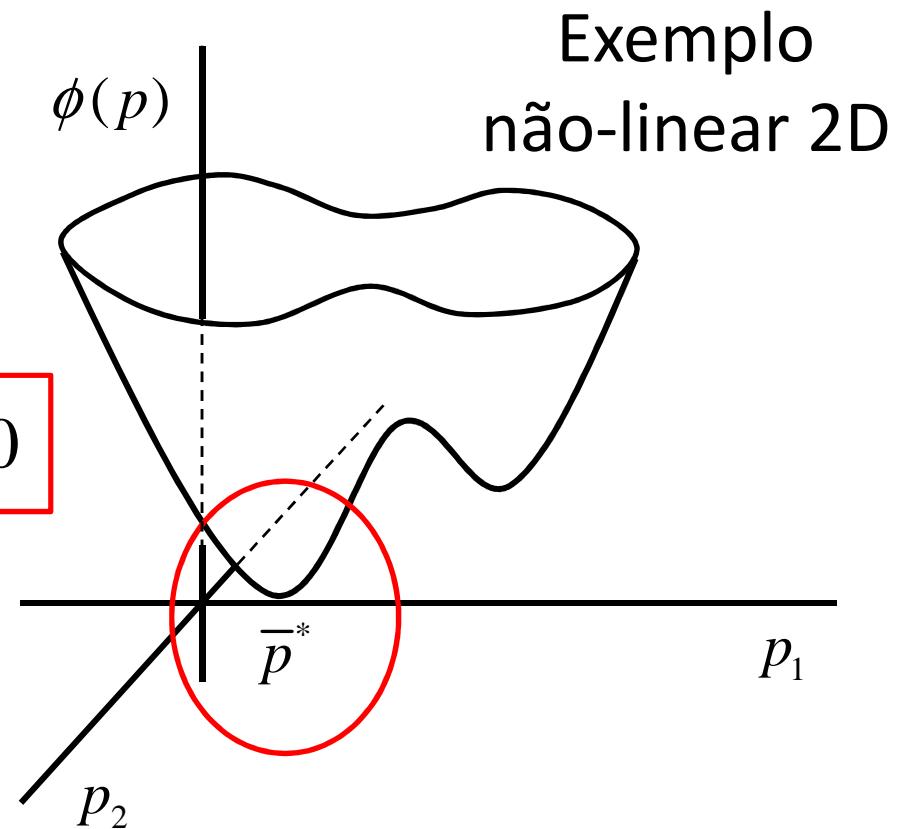


Problemas Inversos

(Introdução)

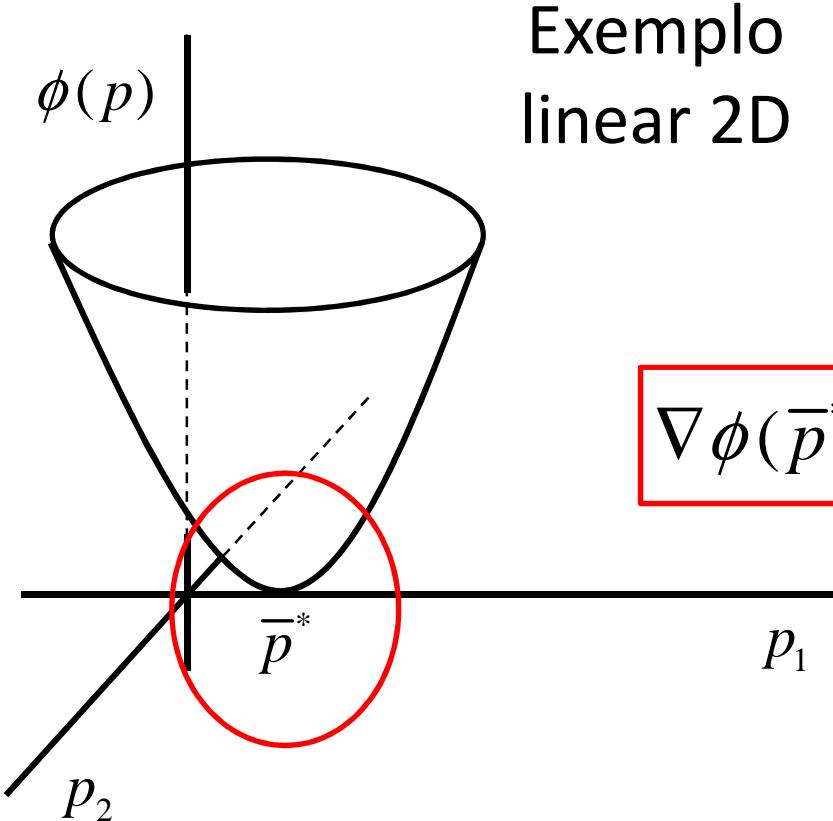


$$\nabla \phi(\bar{p}^*) = 0$$



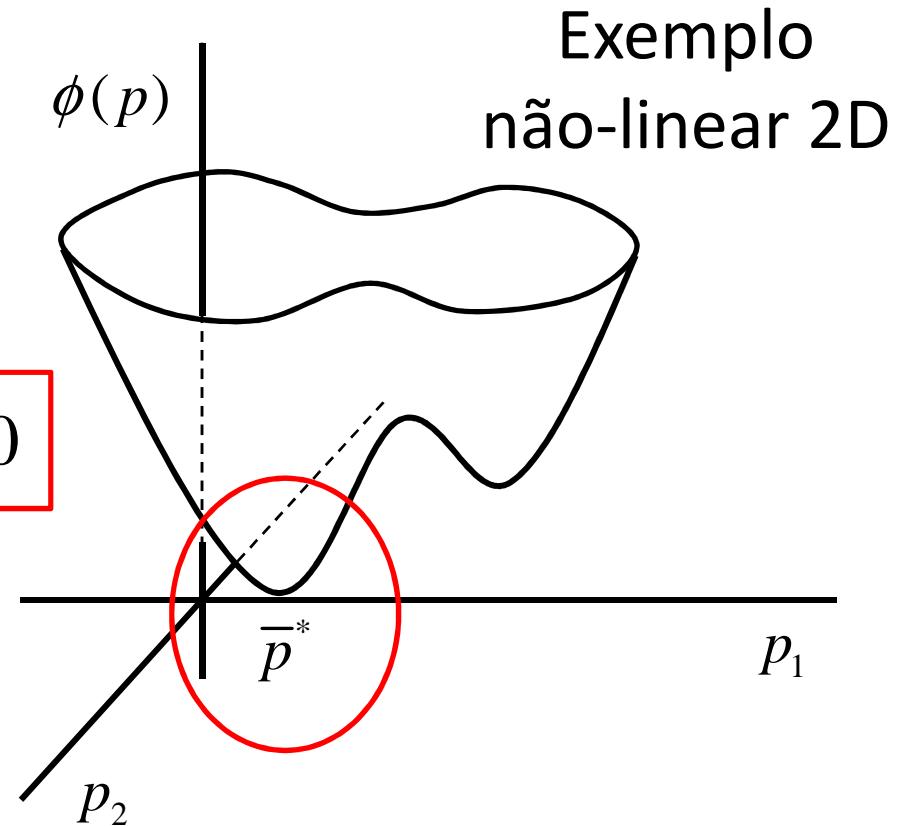
Problemas Inversos

(Introdução)



Exemplo
linear 2D

$$\nabla \phi(\bar{p}^*) = 0$$



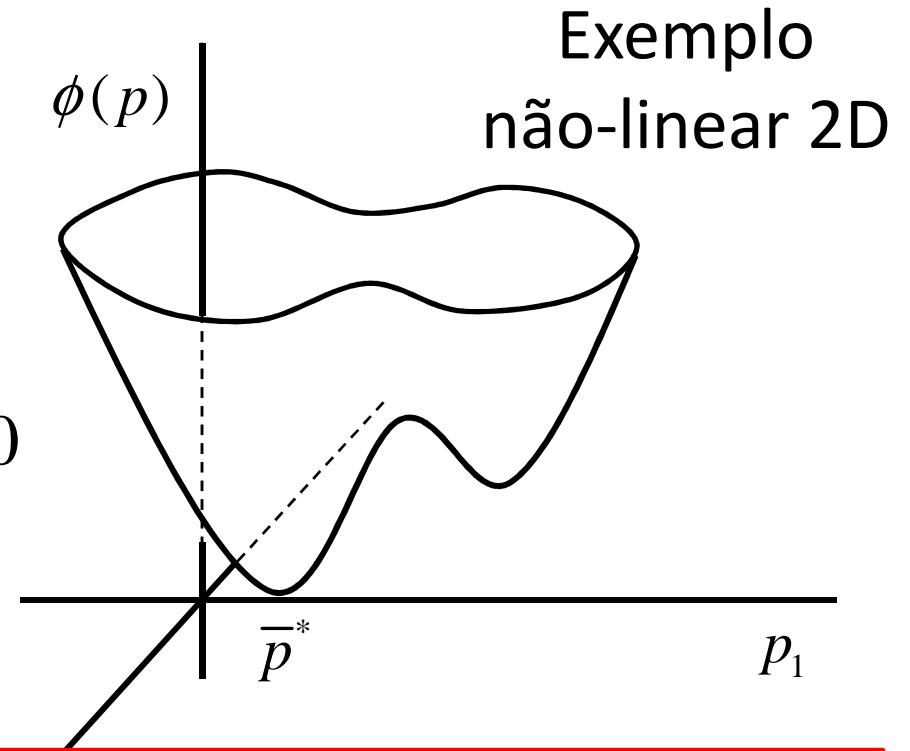
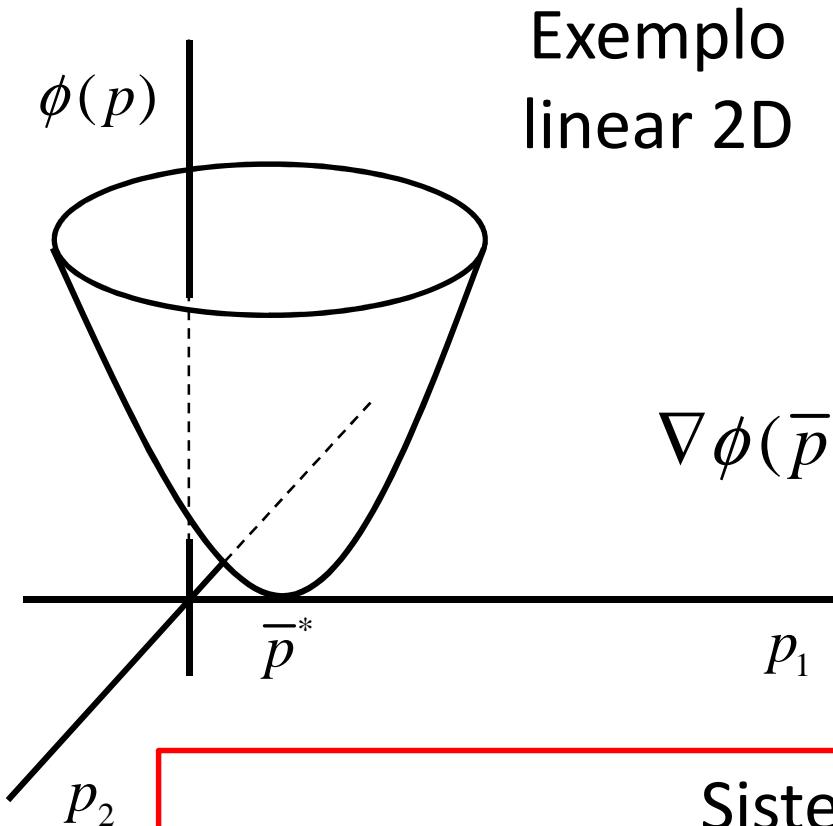
Exemplo
não-linear 2D

$$\bar{p}^* = \left(\begin{smallmatrix} \bar{B} & \bar{B} \end{smallmatrix} \right)^{-1} \bar{B}^T [\bar{d} - \bar{b}]$$

$$\Delta \bar{p} = \left(\begin{smallmatrix} \bar{G}(\bar{p}_0) & \bar{G}(\bar{p}_0) \end{smallmatrix} \right)^{-1} \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Problemas Inversos

(Introdução)



Sistemas lineares

$$\bar{p}^* = \left(\begin{array}{cc} \bar{B} & \bar{B} \end{array} \right)^{-1} \bar{B}^T [\bar{d} - \bar{b}]$$

$$\Delta \bar{p} = \left(\begin{array}{c} \bar{G}(\bar{p}_0) \\ \bar{G}(\bar{p}_0) \end{array} \right)^{-1} \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistemas lineares

$$\bar{b} = \begin{pmatrix} \bar{B}^T \\ \bar{B} \end{pmatrix}^{-1} \bar{B}^T \bar{r}$$

Sistemas lineares

$$\bar{b} = \begin{pmatrix} =^T = \\ B \quad B \end{pmatrix}^{-1} =^T B \bar{r}$$


Sistemas lineares

$$\begin{pmatrix} =^T = \\ \mathbf{B} \quad \mathbf{B} \end{pmatrix} \bar{\mathbf{b}} = \begin{pmatrix} =^T \\ \mathbf{B} \quad \bar{\mathbf{r}} \end{pmatrix}$$

Sistemas lineares

$$\underbrace{\begin{pmatrix} =^T = \\ \mathbf{B} & \mathbf{B} \end{pmatrix}}_{\mathbf{A}} \bar{\mathbf{b}} = \underbrace{\mathbf{B}^T}_{=^T} \bar{\mathbf{r}}$$
$$\mathbf{A} \bar{\mathbf{b}} = \bar{\mathbf{w}}$$

Sistemas lineares

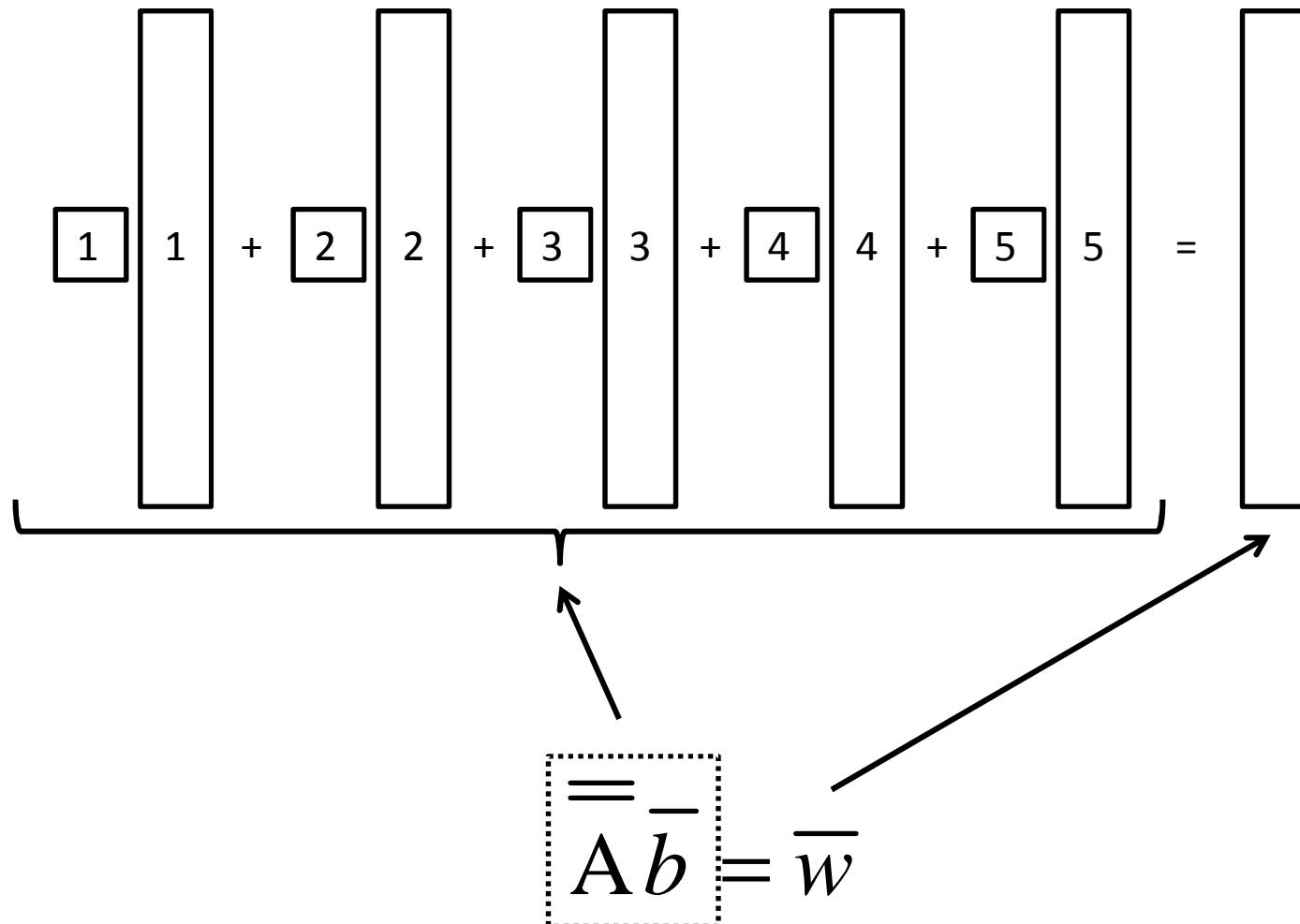
The diagram illustrates a linear system. It features three vertical rectangles of increasing height from left to right, representing vectors \vec{b} , \vec{w} , and $\vec{A}\vec{b}$. Three arrows point from the labels $=$, \vec{b} , and \vec{w} towards their respective vector representations.

$$\underline{\underline{A}} \underline{\underline{b}} = \underline{\underline{w}}$$

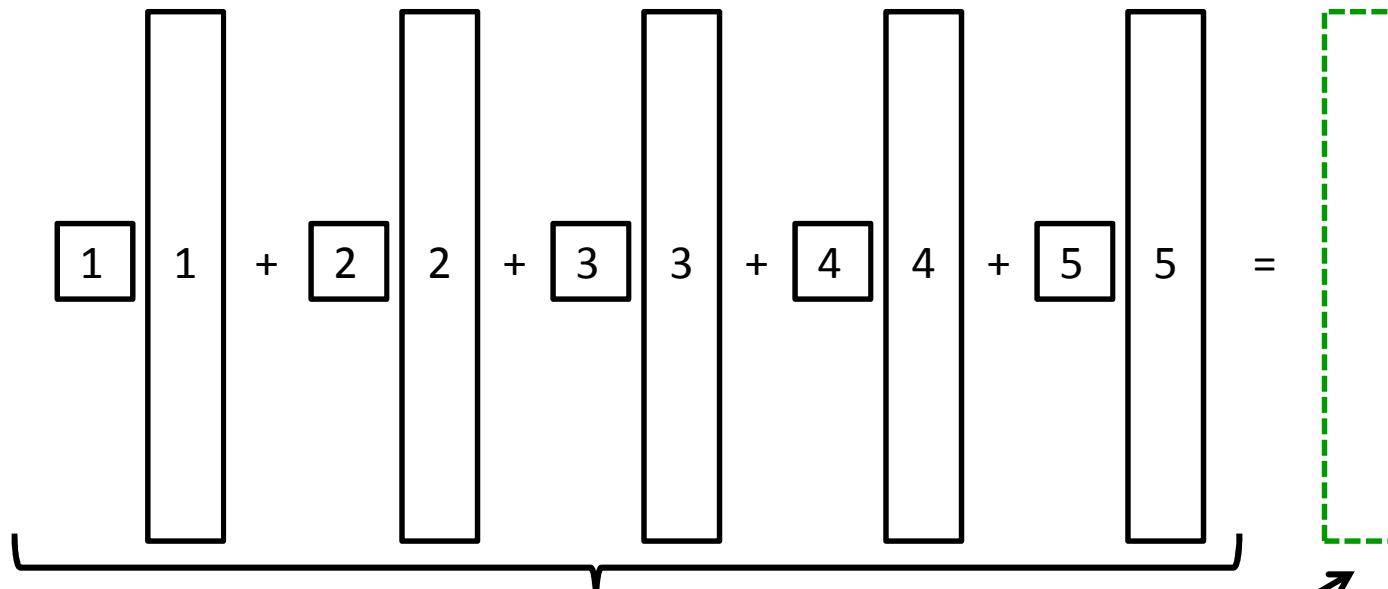
Sistemas lineares

$$\begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \xrightarrow{\quad} & \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \xrightarrow{\quad} & \begin{matrix} \end{matrix} \end{matrix}$$
$$= \underline{A} \underline{b} = \underline{w}$$

Sistemas lineares



Sistemas lineares

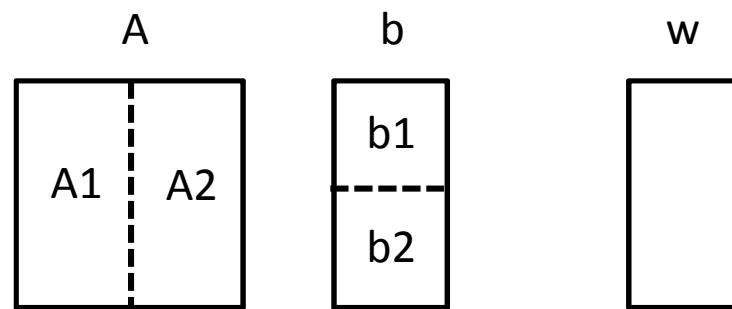


O vetor do lado direito é
uma *combinação linear* dos
vetores do lado esquerdo

$$\boxed{\bar{A}\bar{b} = \bar{w}}$$

Sistemas lineares

Exemplo 2D

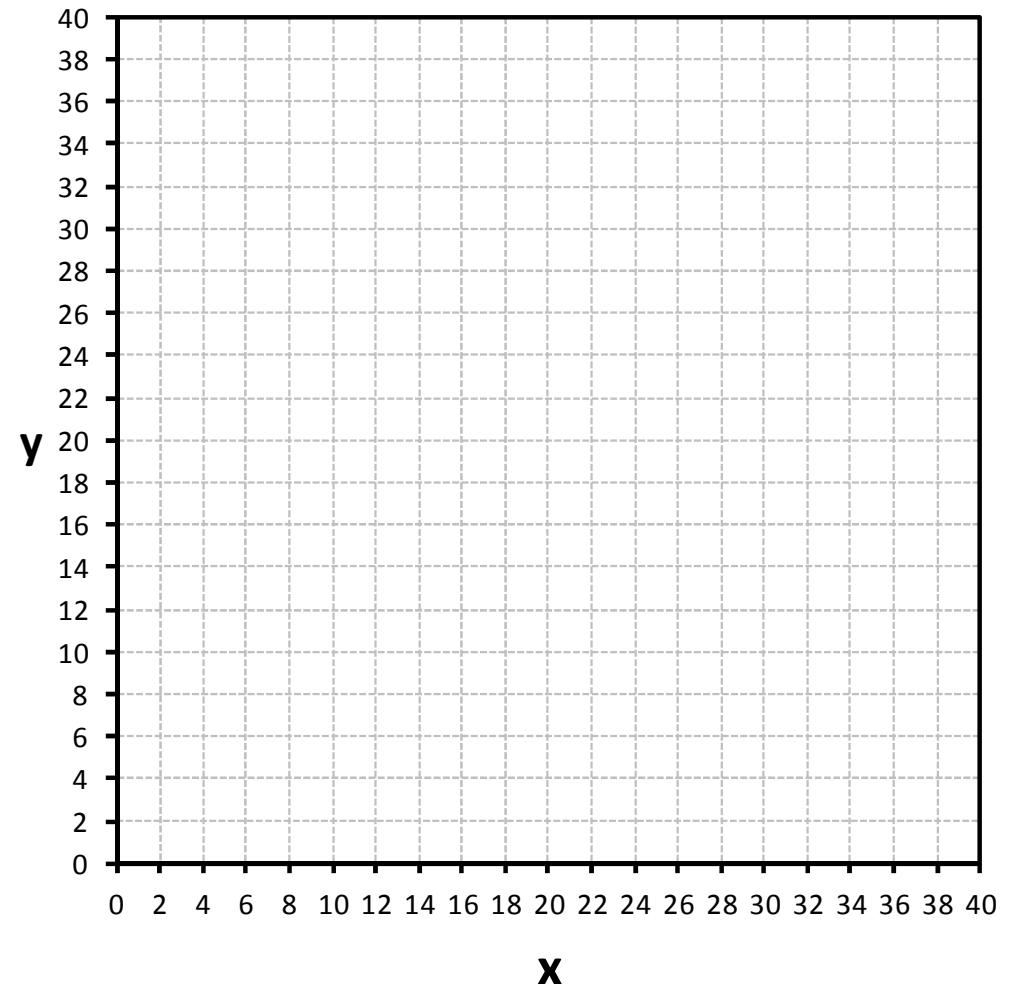


$$\boxed{b_1} \boxed{A_1} + \boxed{b_2} \boxed{A_2} = \boxed{}$$

Sistemas lineares

Exemplo 2D

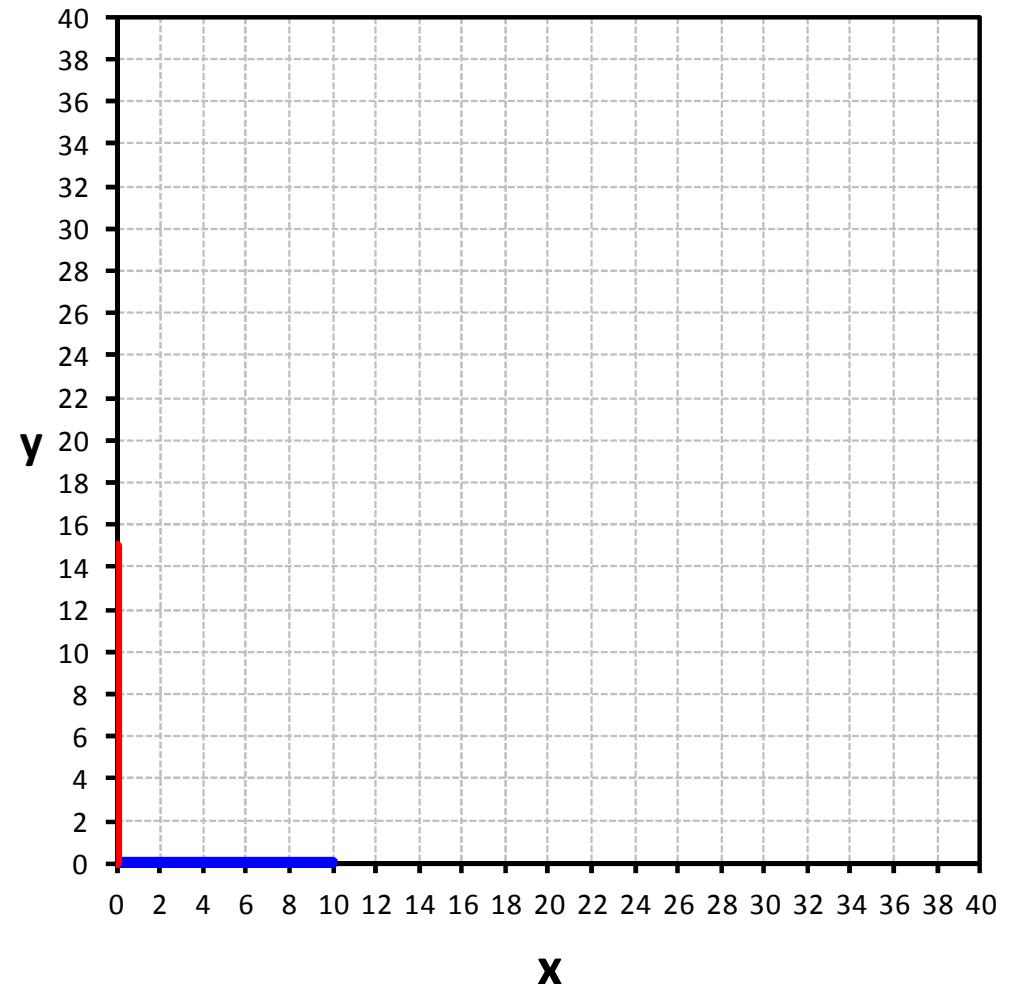
$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{}$$



Sistemas lineares

Exemplo 2D

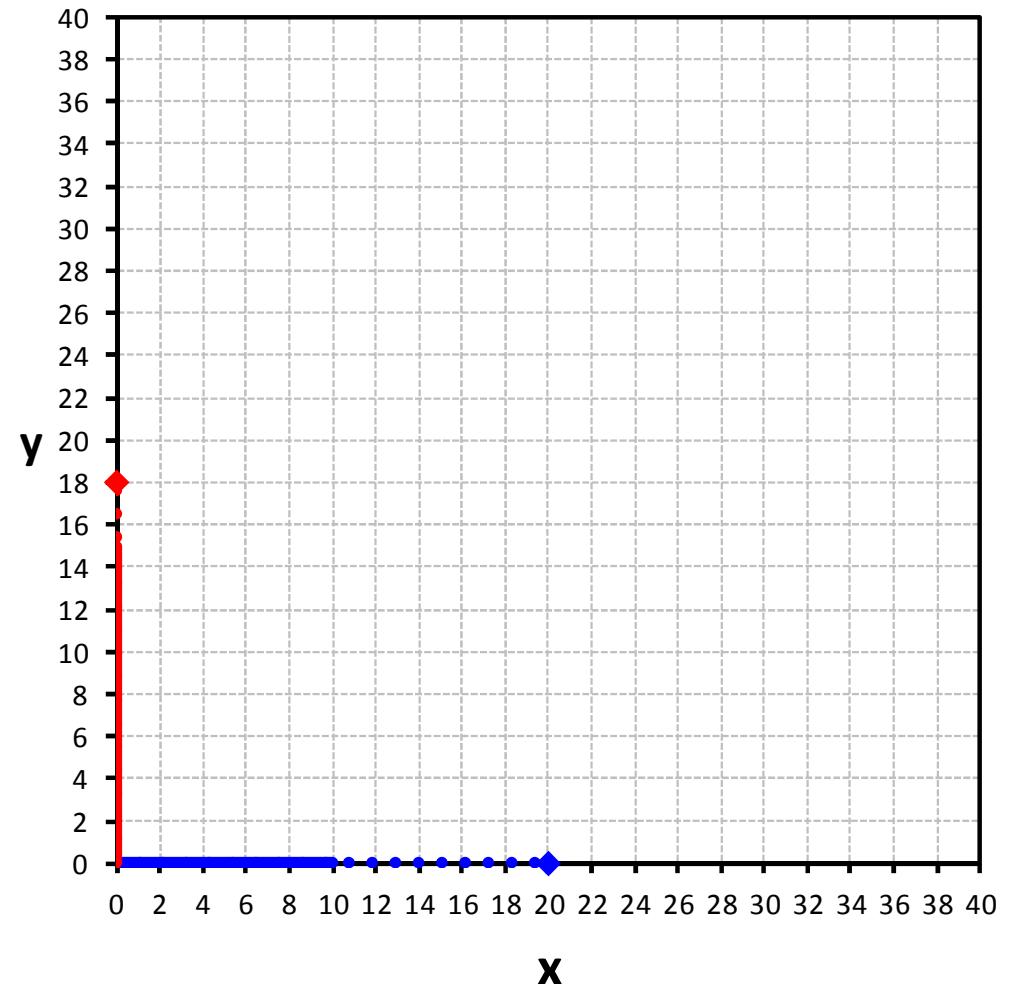
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Sistemas lineares

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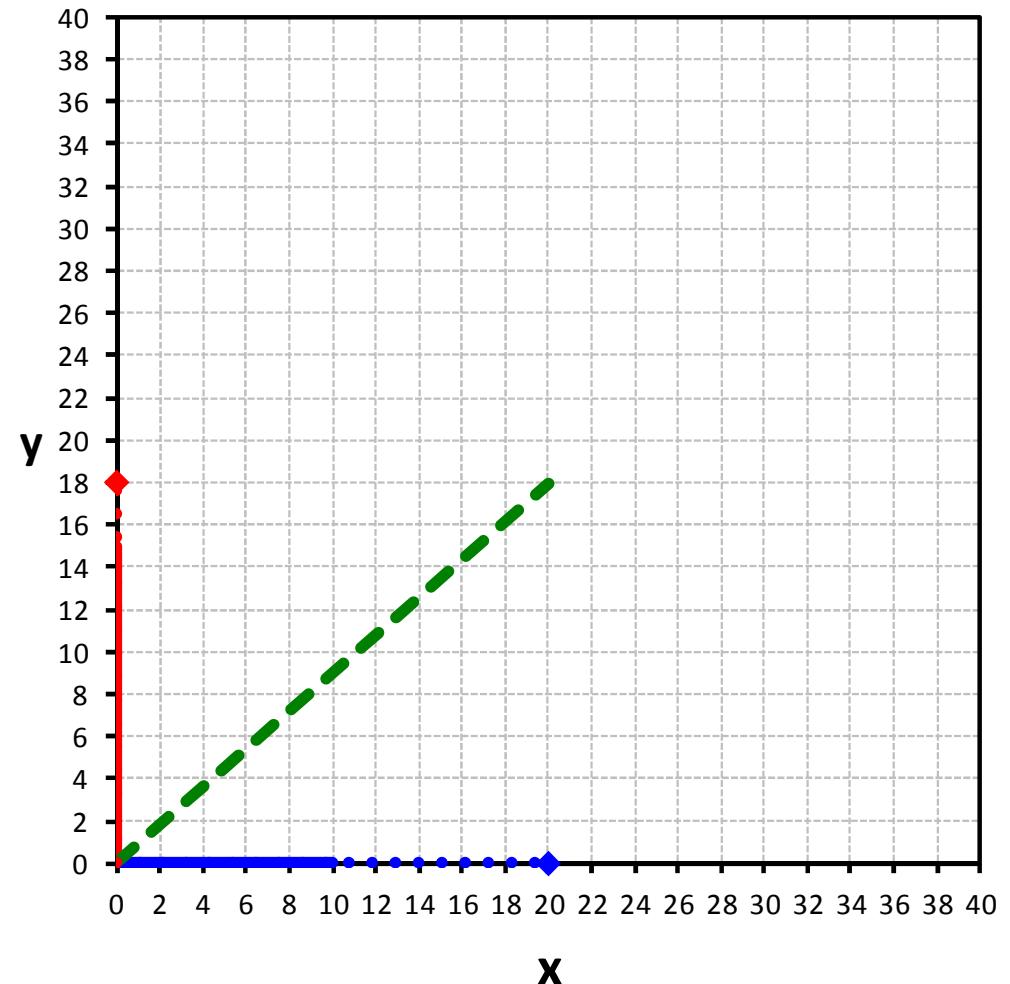
$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{}$$



Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{}$$

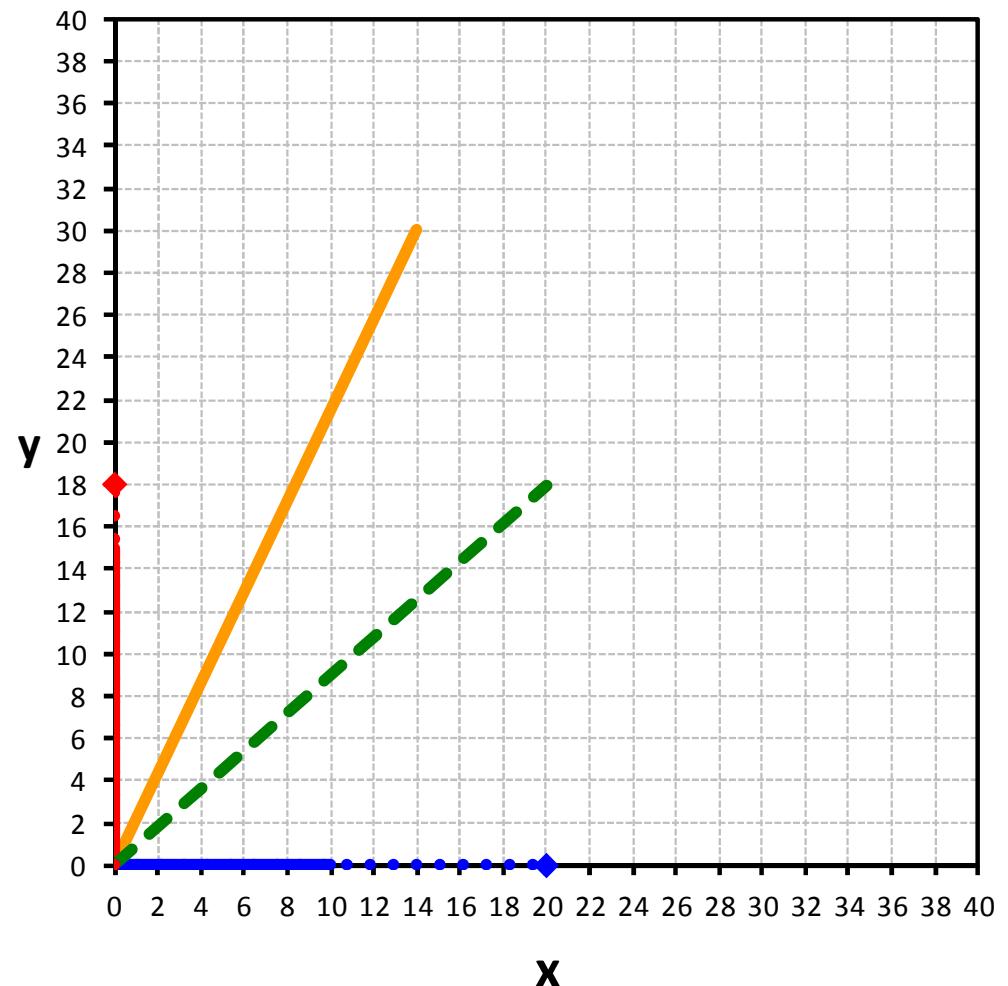


Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

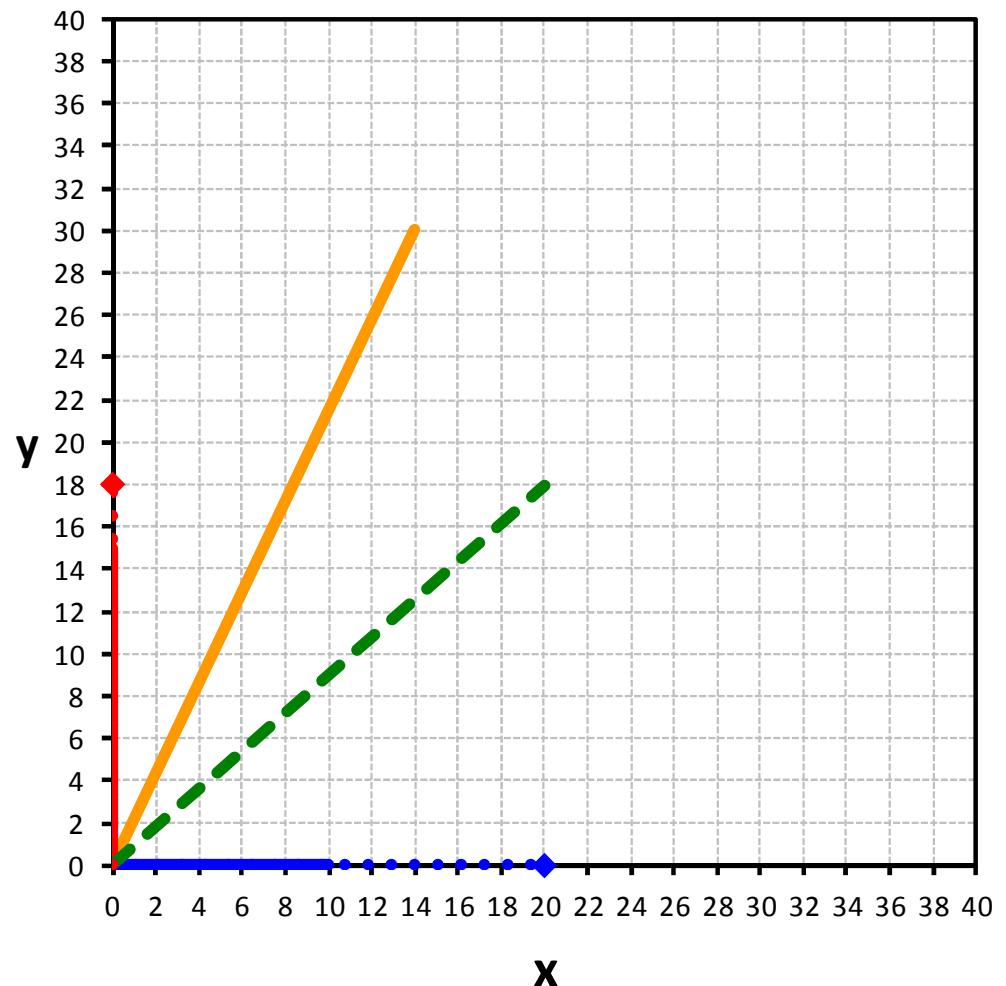
$$\boxed{\quad}$$



Sistemas lineares

Exemplo 2D

$$\begin{matrix} b_1 \\ A_1 \end{matrix} + \begin{matrix} b_2 \\ A_2 \end{matrix} = \begin{matrix} ? \\ \text{orange box} \end{matrix}$$

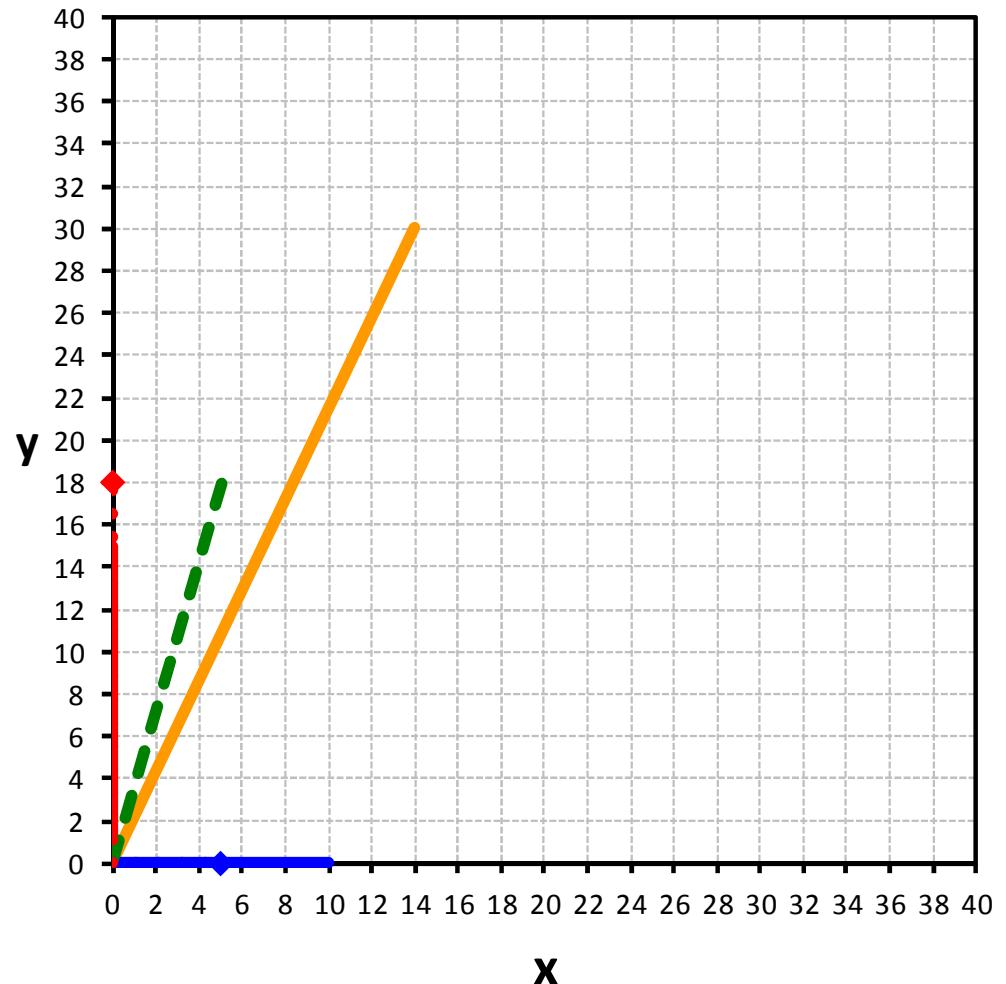


Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$

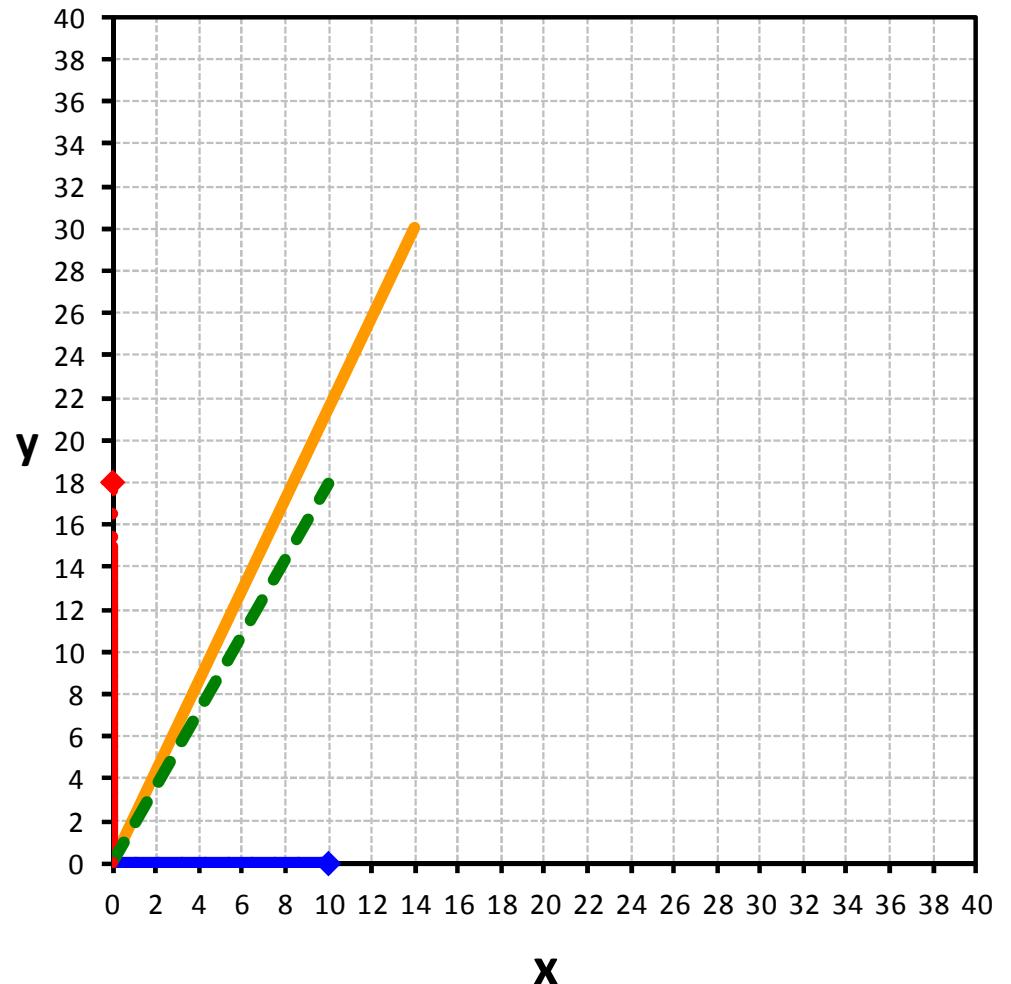


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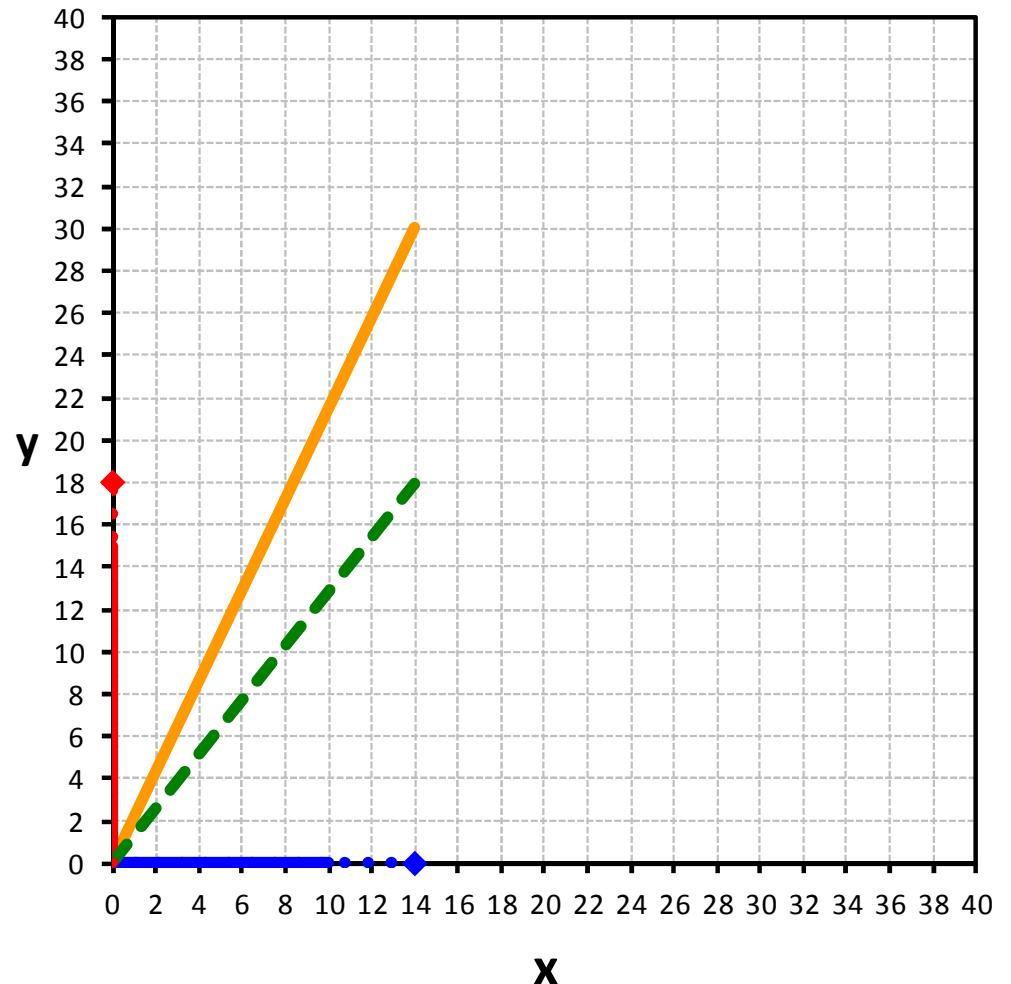


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$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

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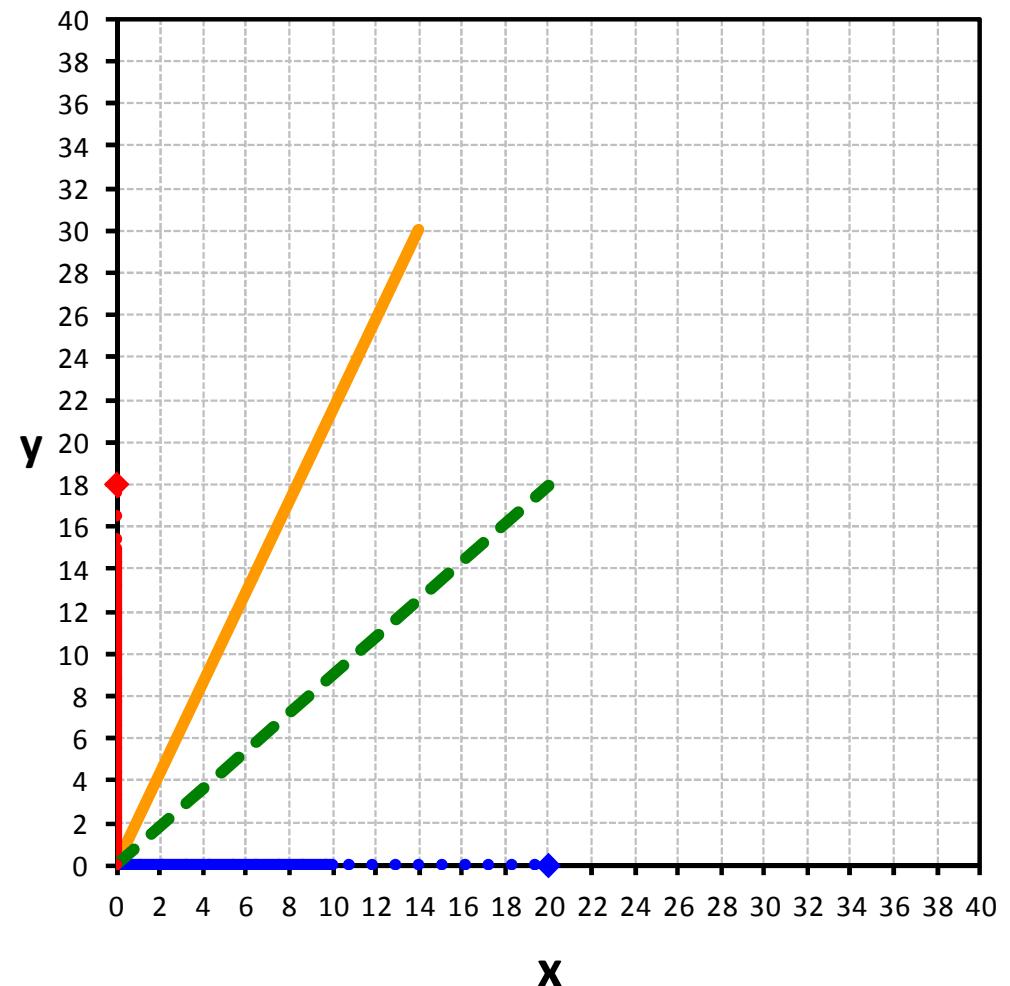


Sistemas lineares

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$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

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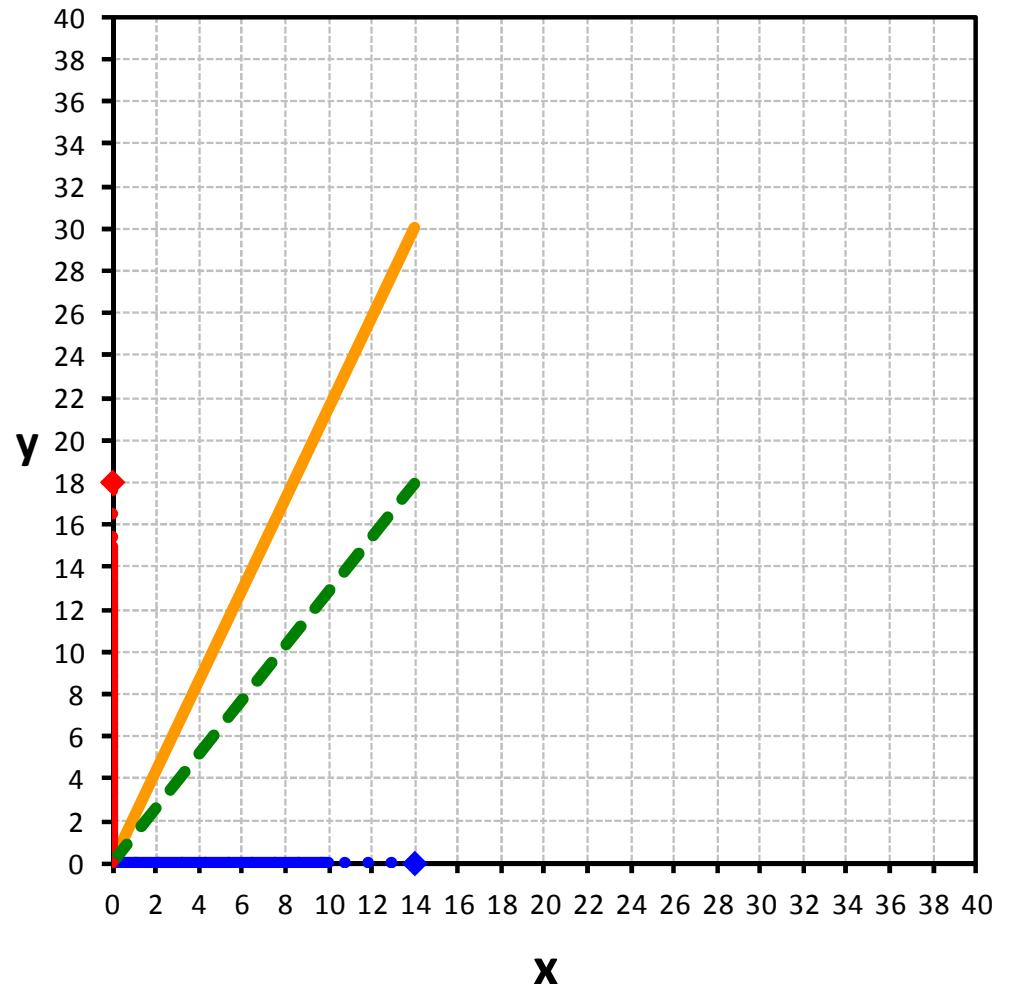


Sistemas lineares

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$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

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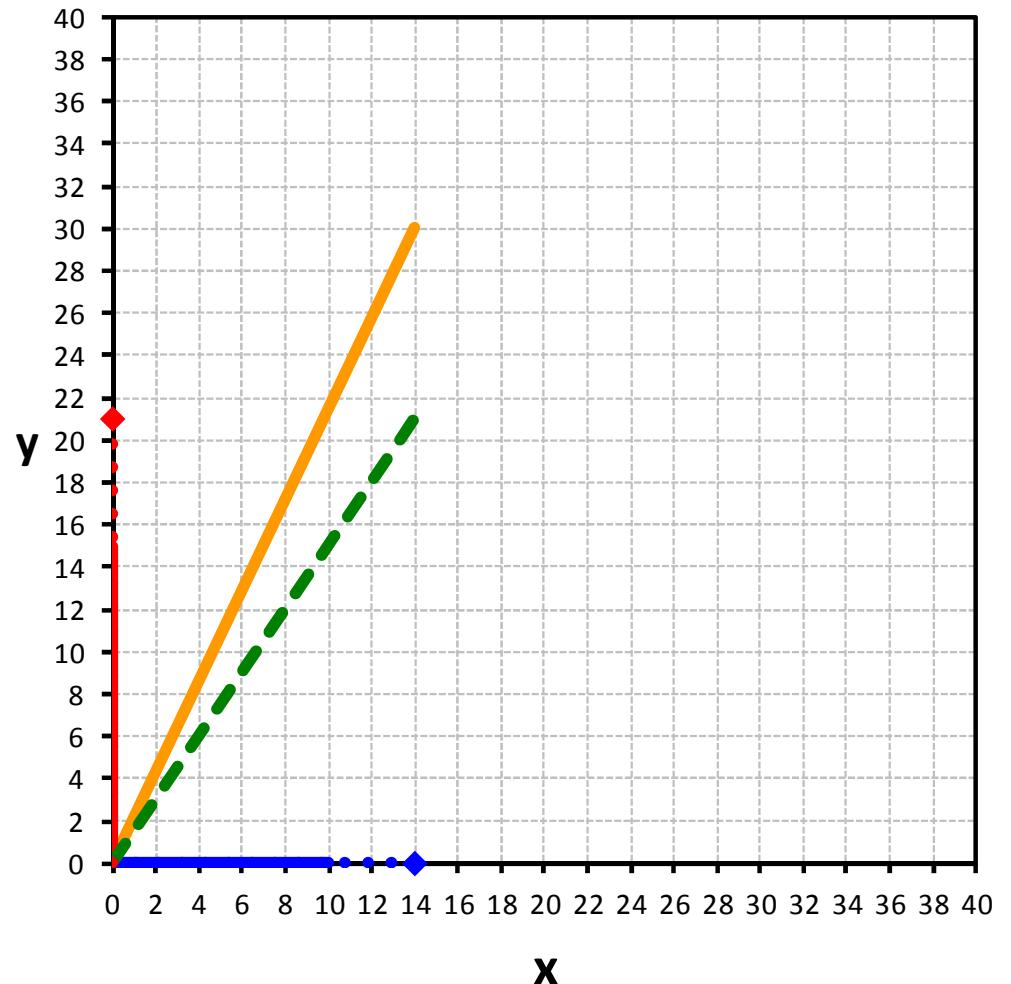


Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$

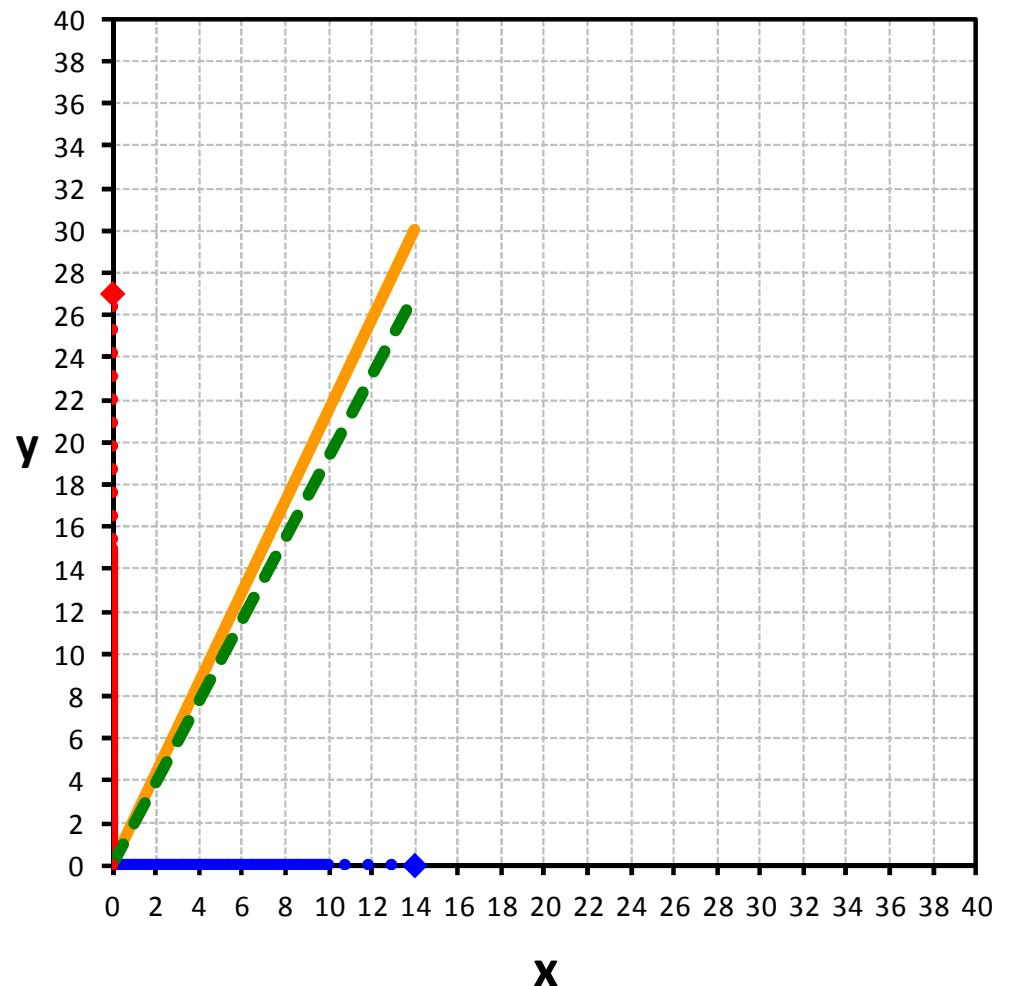


Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$

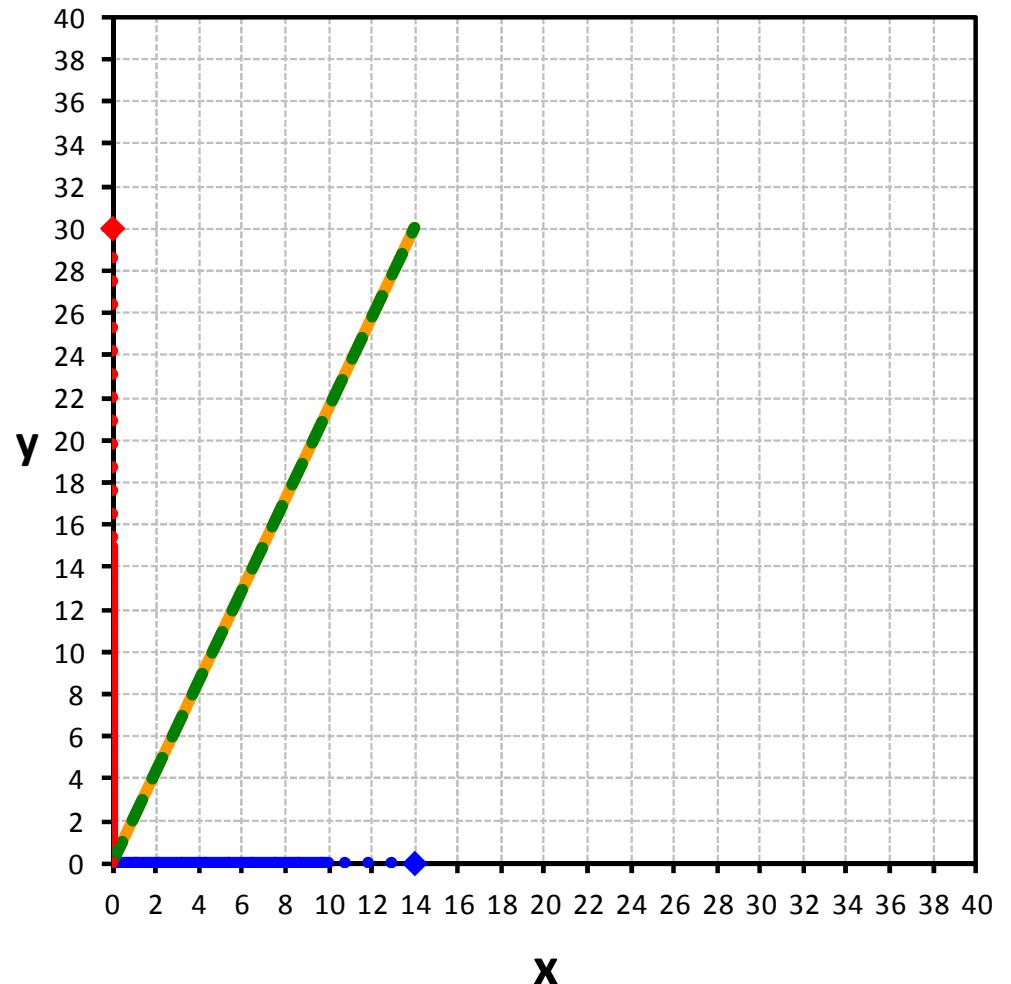


Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$

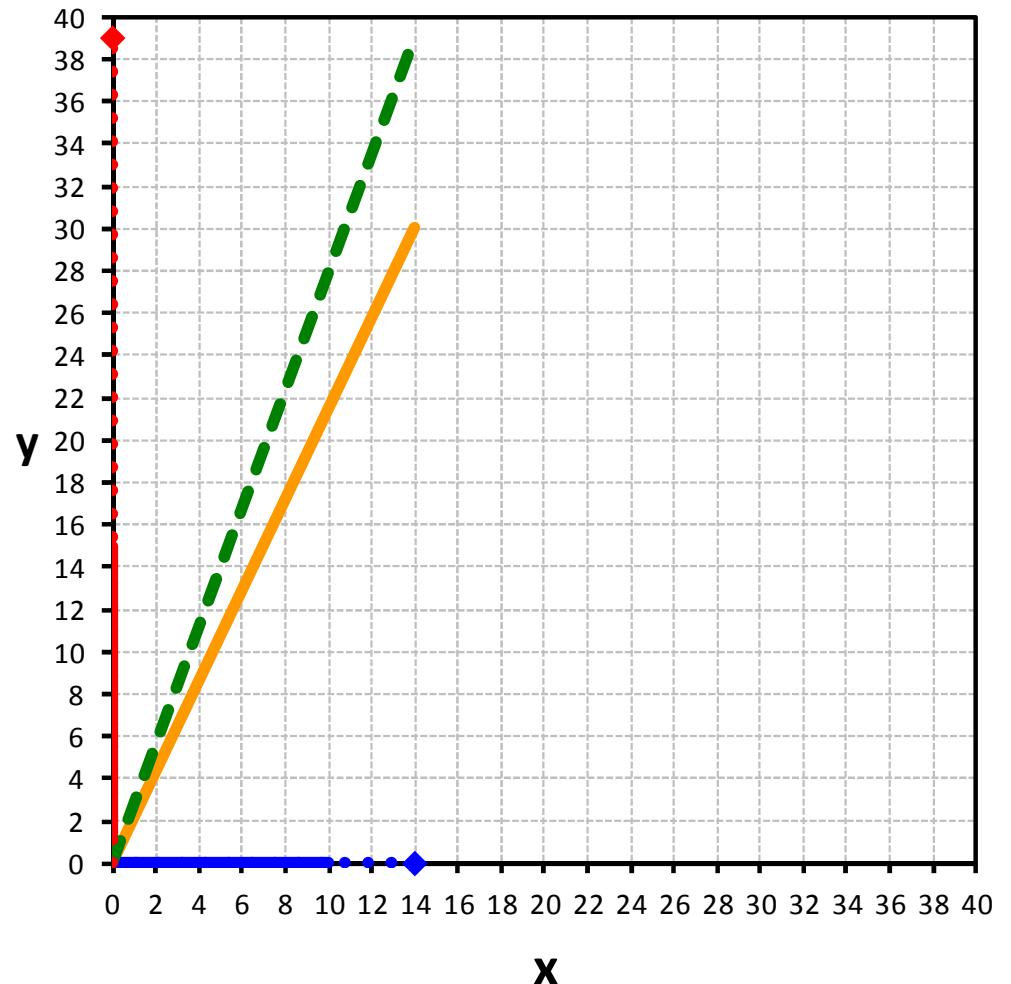


Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$



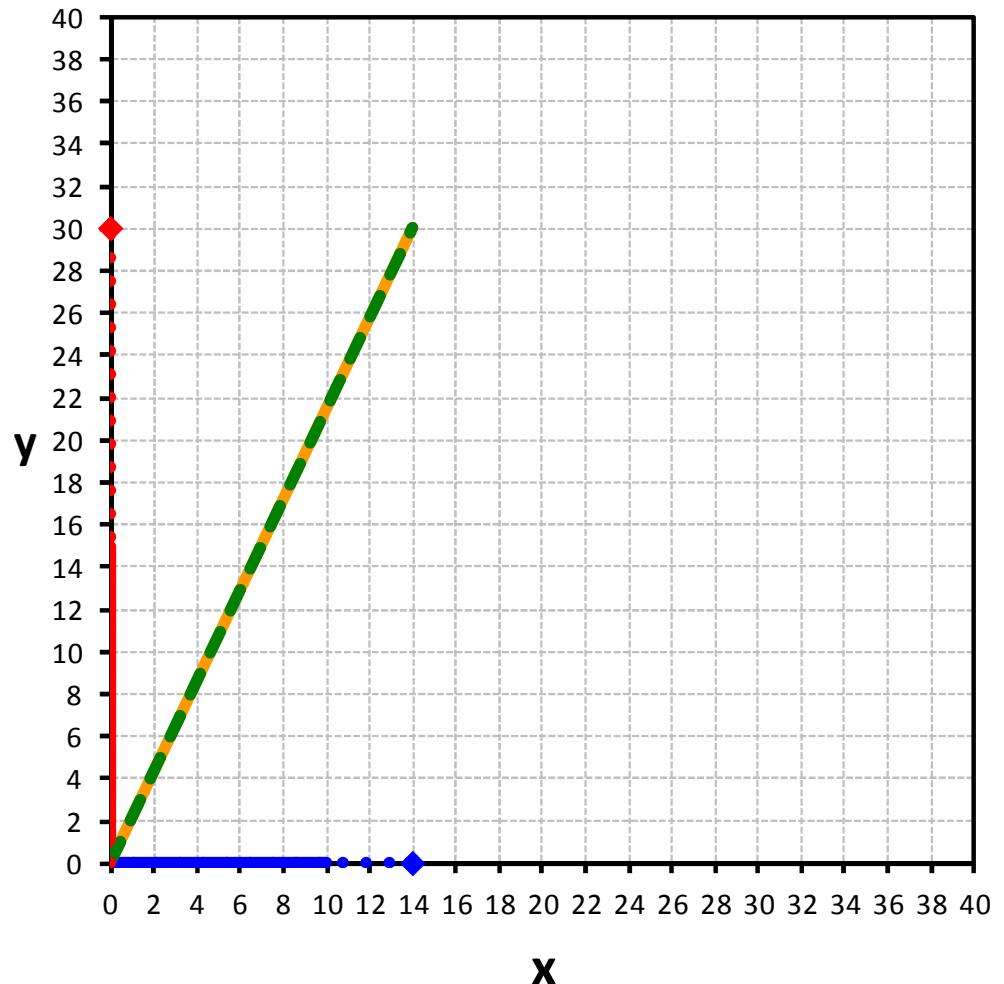
Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$

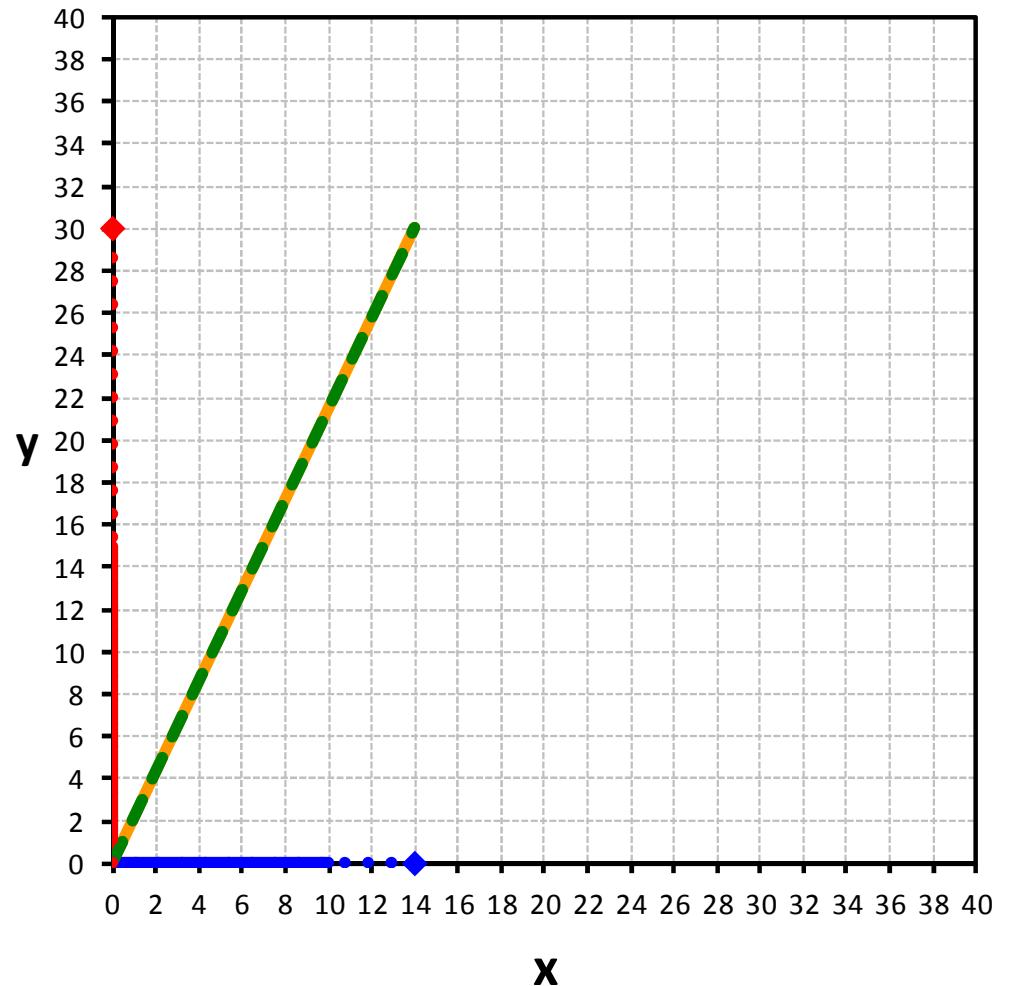
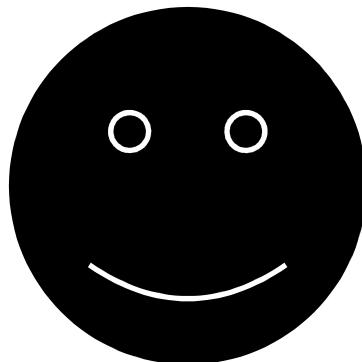
Neste caso, os vetores A_1 e A_2 são *linearmente independentes* e os coeficientes b_1 e b_2 são únicos



Sistemas lineares

Exemplo 2D

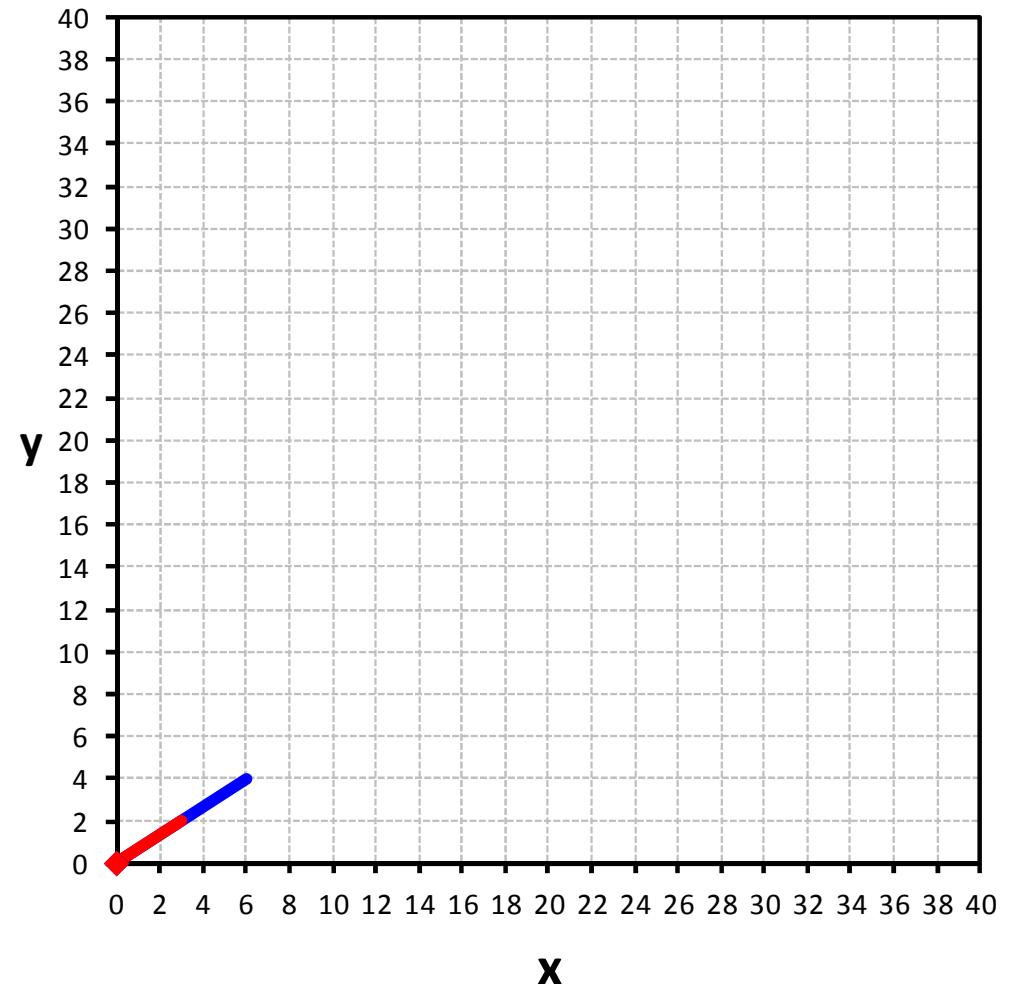
$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$



Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{}$$

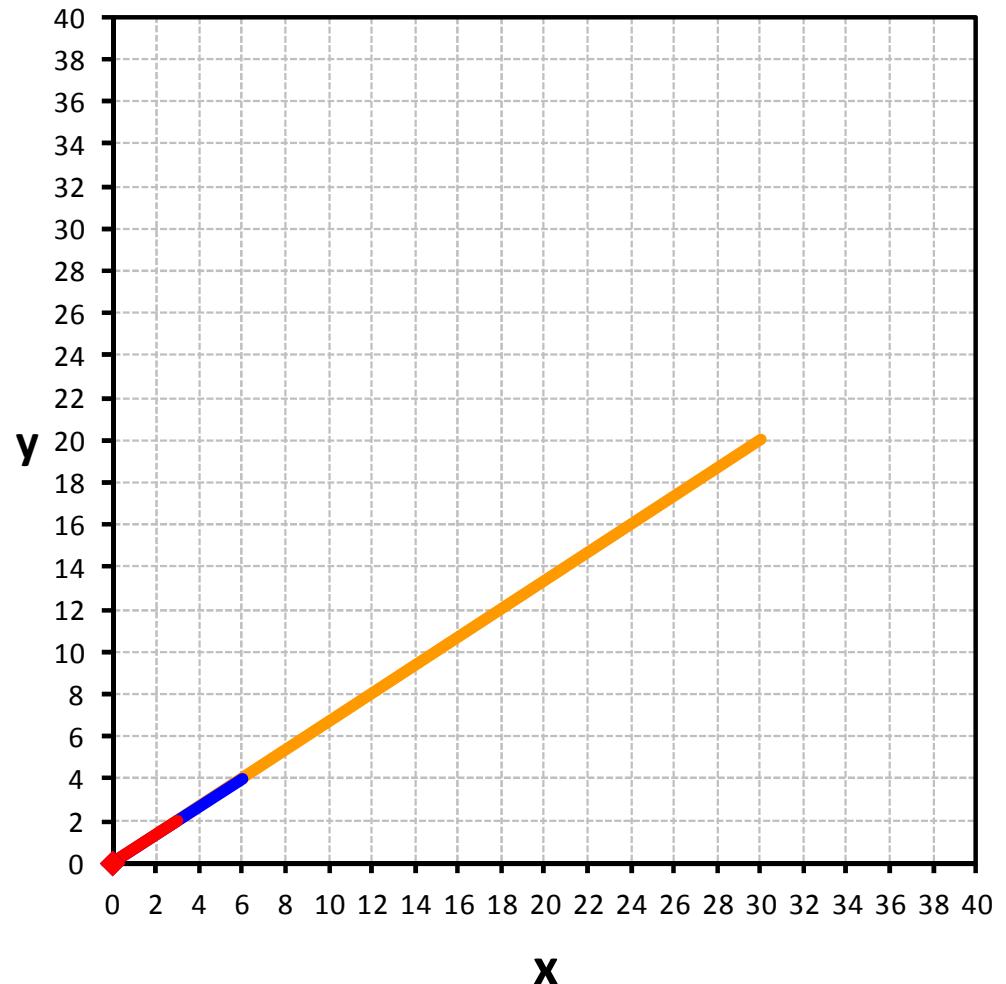


Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$

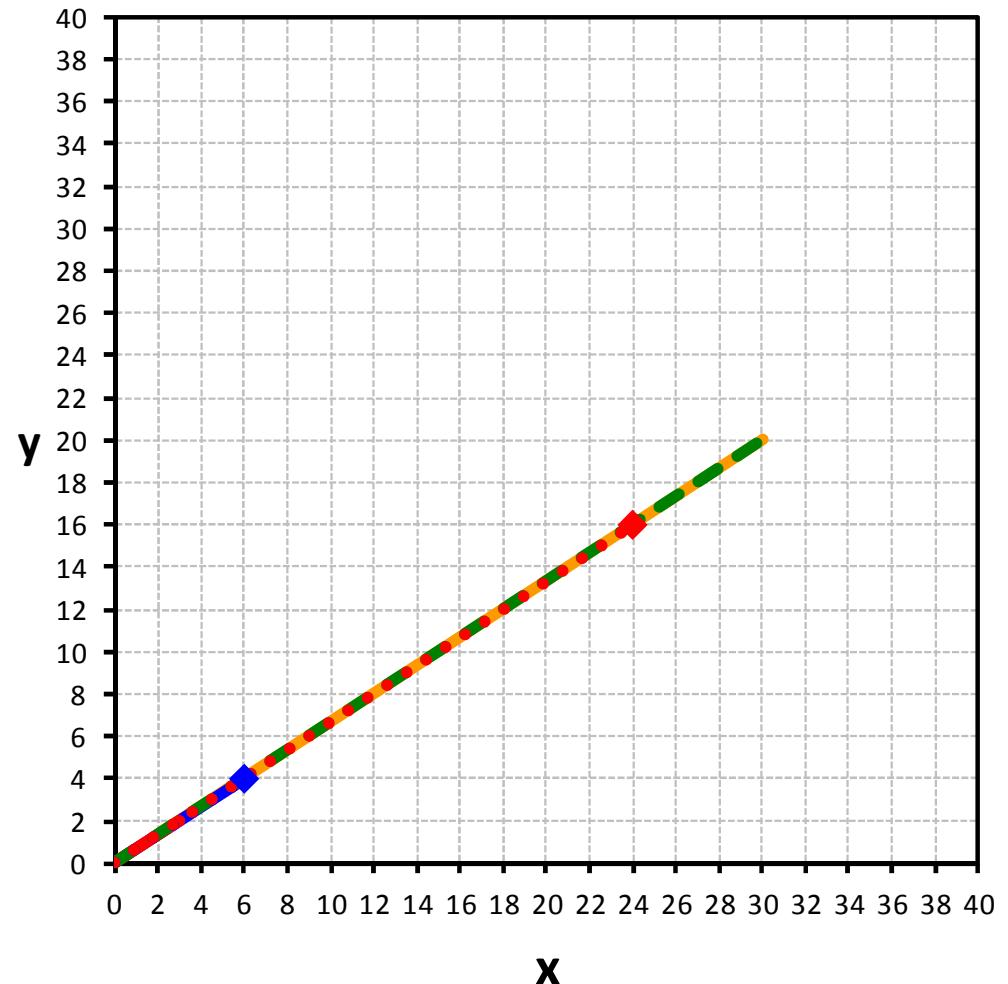


Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$

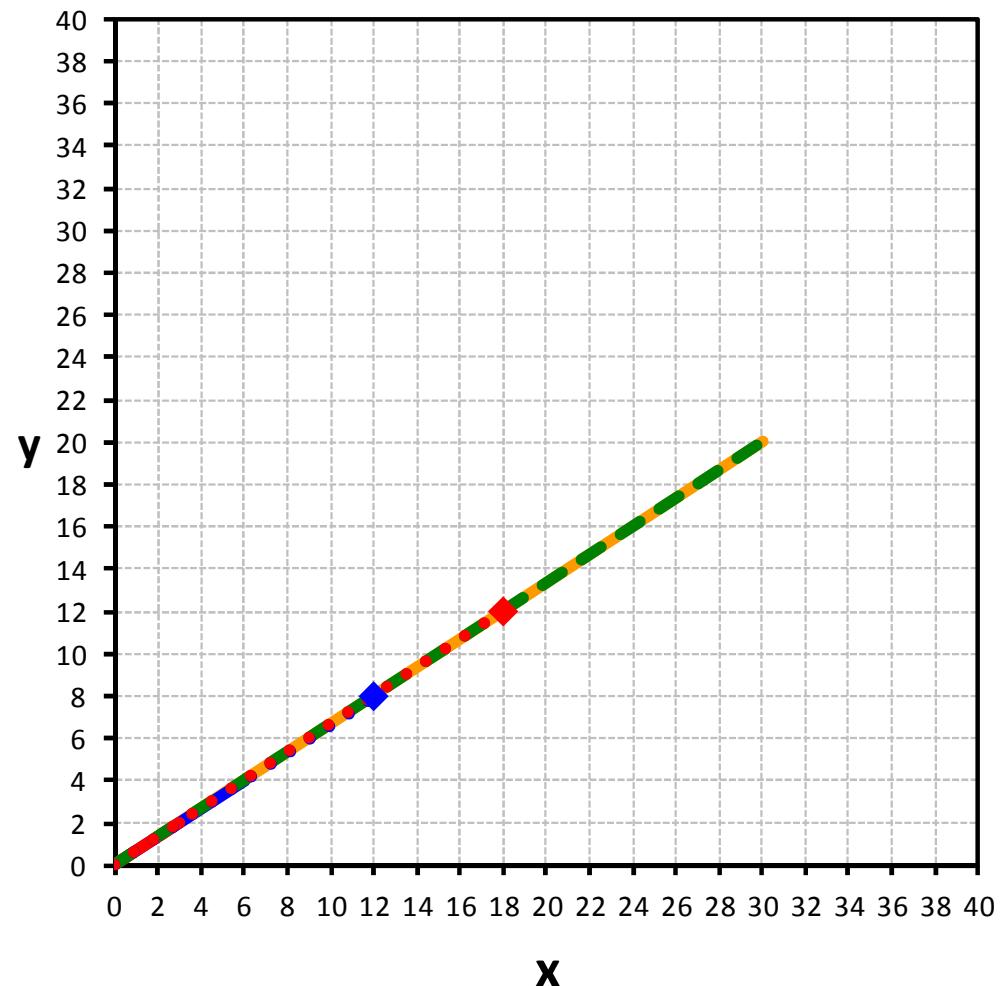


Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$

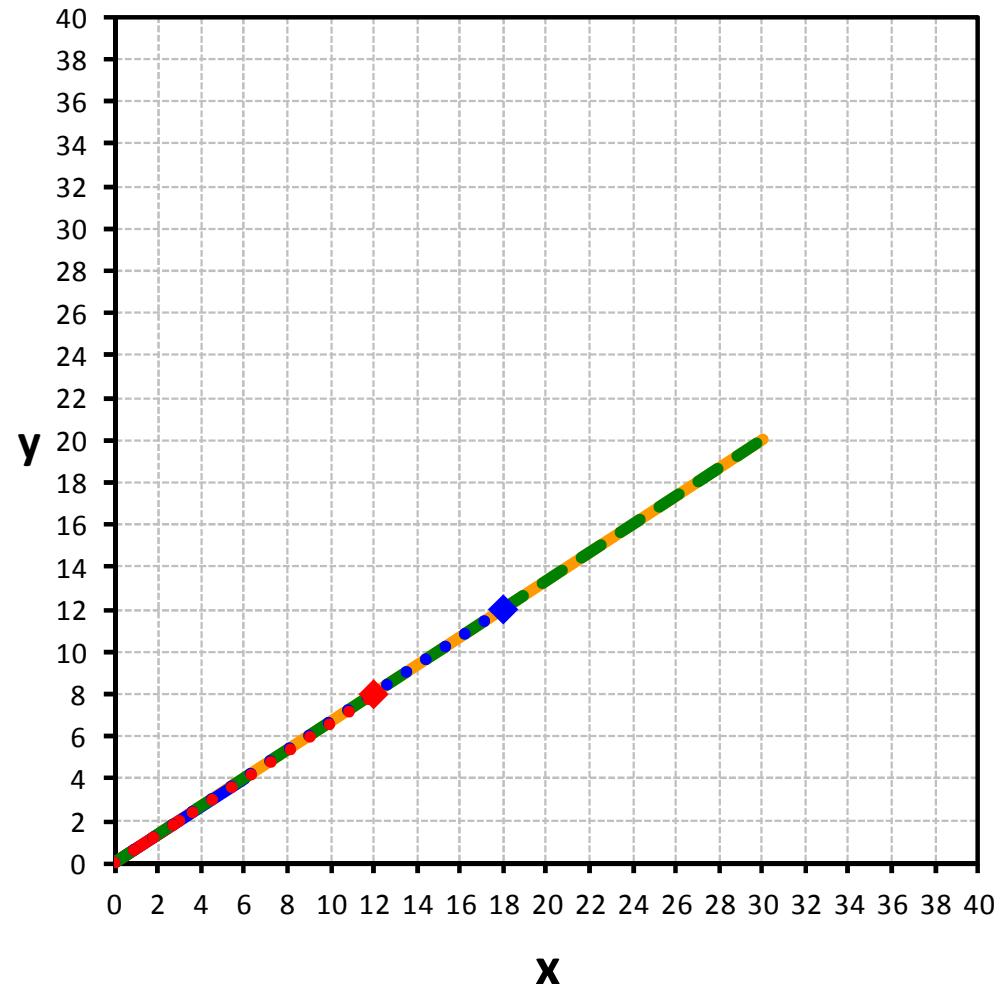


Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$

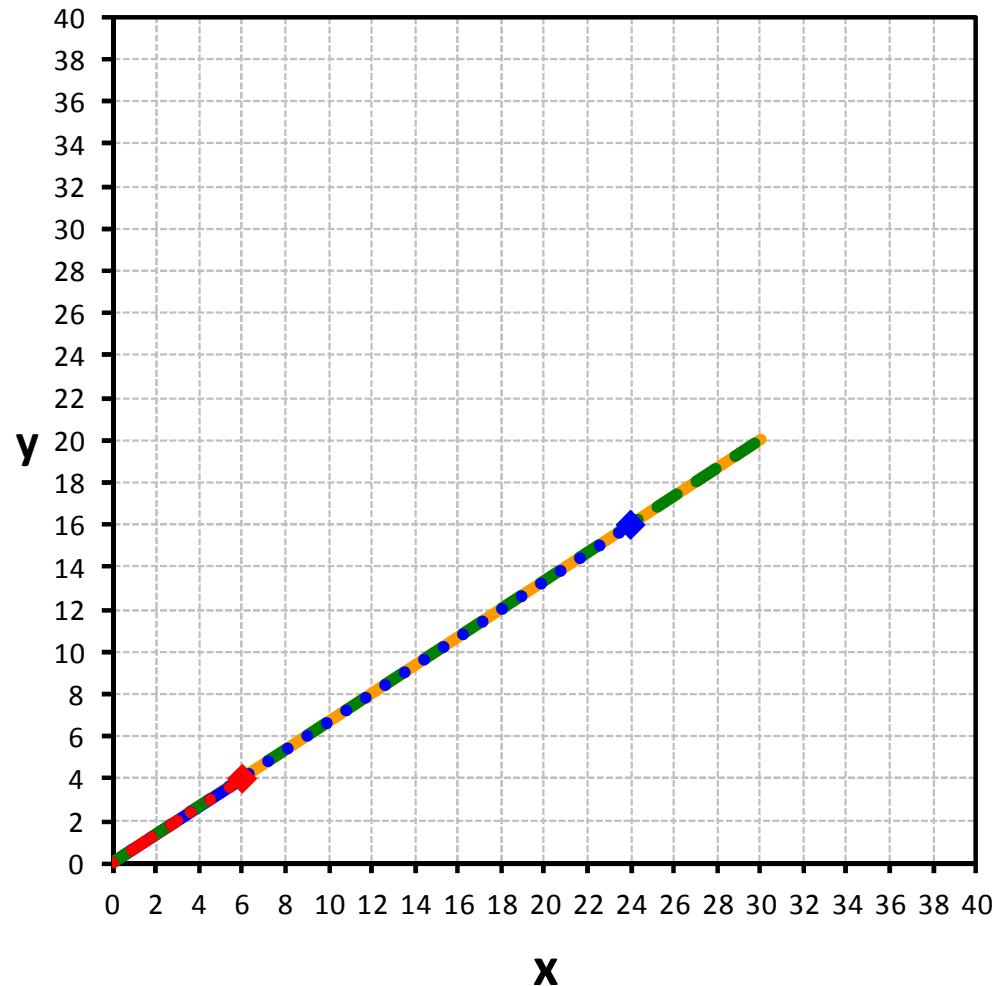


Sistemas lineares

Exemplo 2D

$$\begin{array}{|c|} \hline b_1 \\ \hline \end{array} + \begin{array}{|c|} \hline A_1 \\ \hline \end{array} + \begin{array}{|c|} \hline b_2 \\ \hline \end{array} + \begin{array}{|c|} \hline A_2 \\ \hline \end{array} = \begin{array}{|c|} \hline \quad \\ \hline \end{array}$$

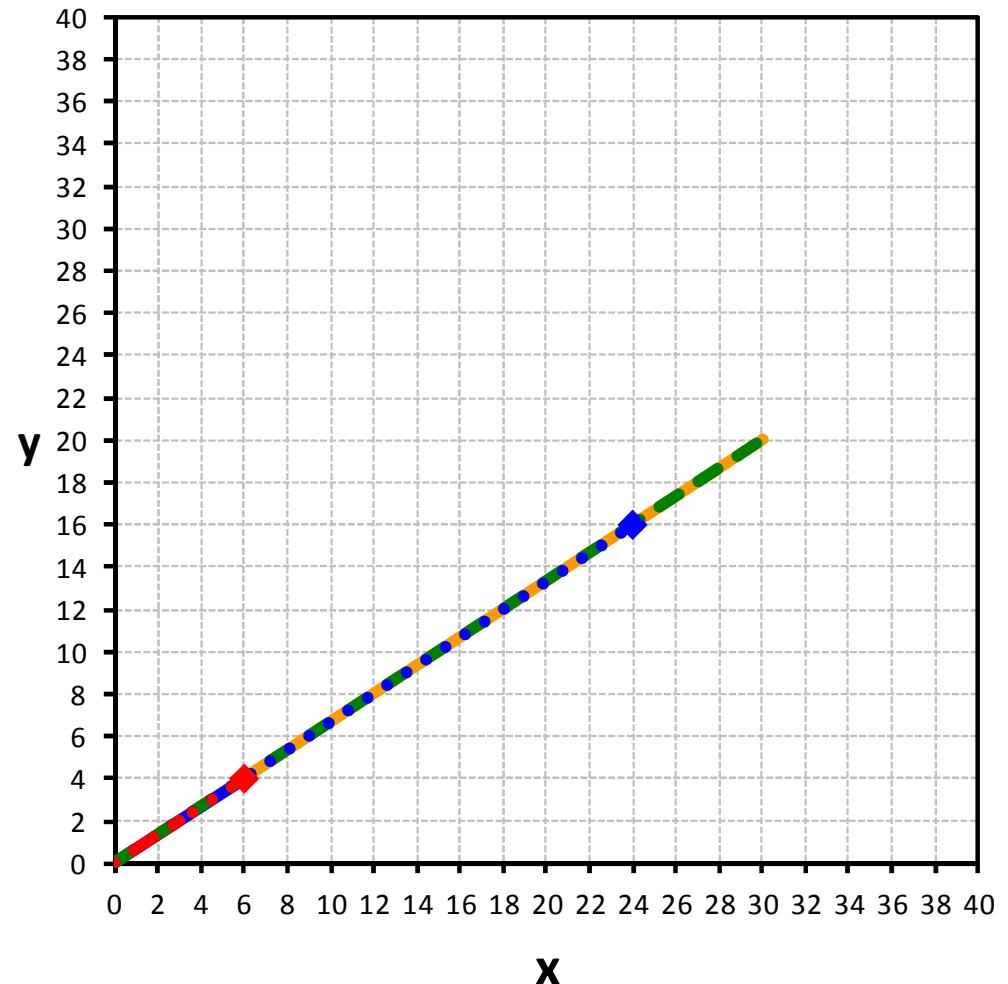
Neste caso, os vetores **A1** e **A2** são *linearmente dependentes* e existem infinitos pares b_1 e b_2 que produzem o mesmo resultado



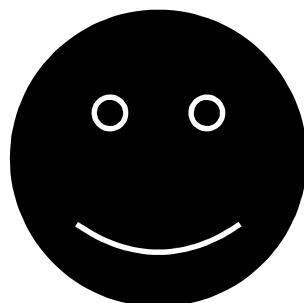
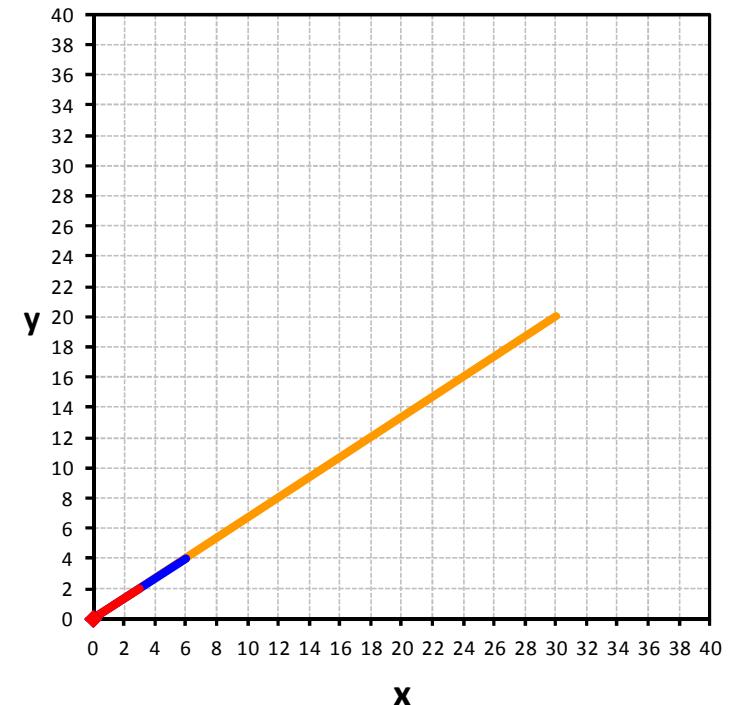
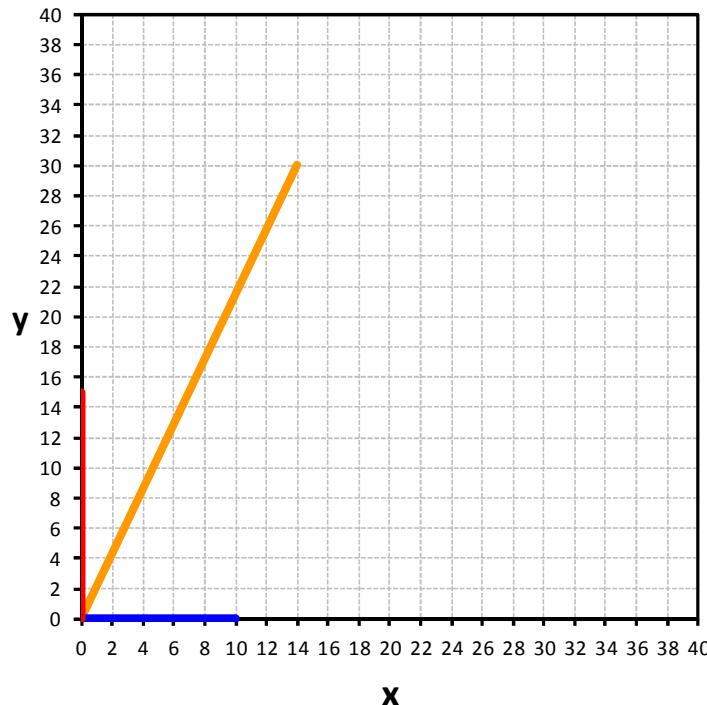
Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$



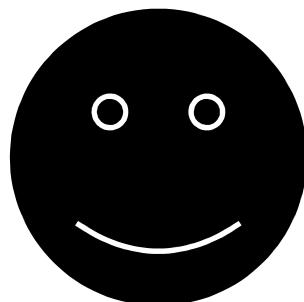
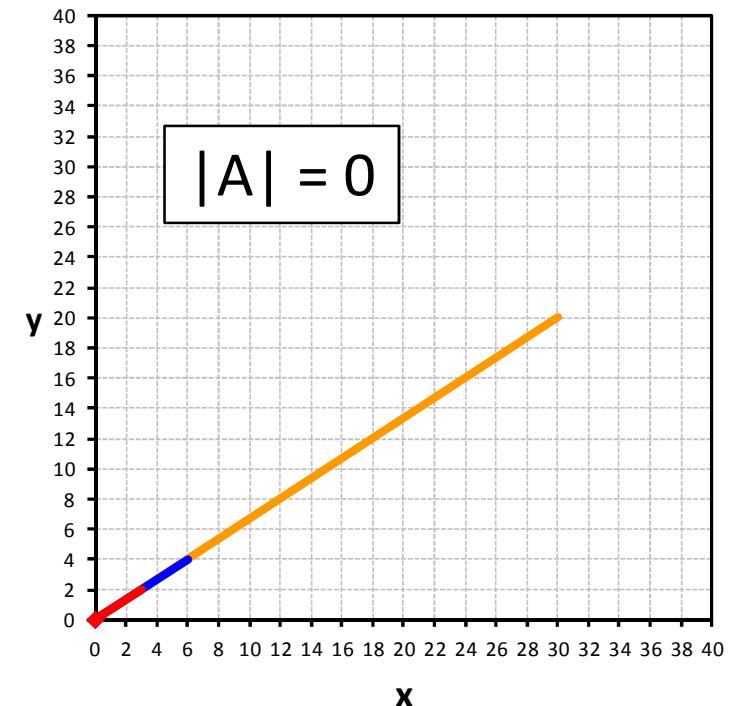
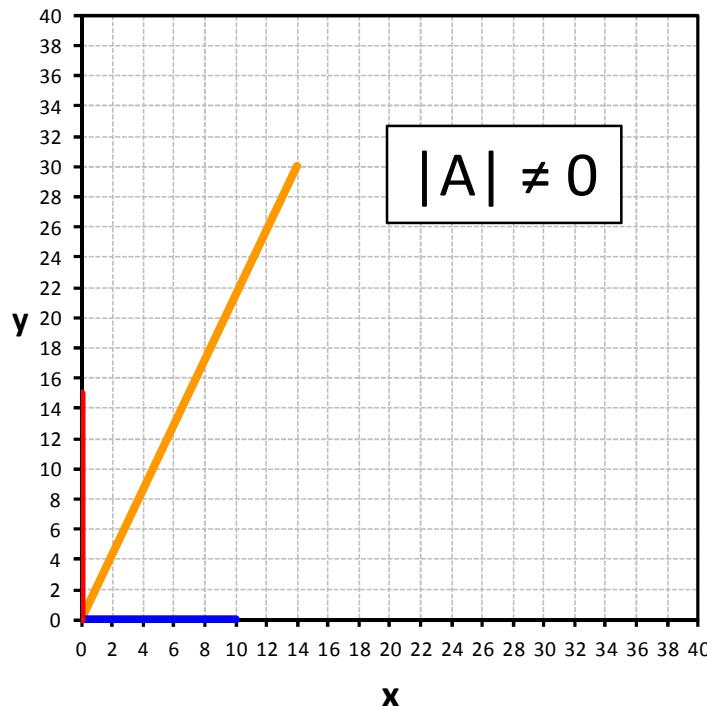
Sistemas lineares



$$b_1 \begin{array}{|c|} \hline \text{A1} \\ \hline \end{array} + b_2 \begin{array}{|c|} \hline \text{A2} \\ \hline \end{array} = \begin{array}{|c|} \hline \text{ } \\ \hline \end{array} \quad \begin{array}{|c|} \hline \text{ } \\ \hline \end{array}$$



Sistemas lineares



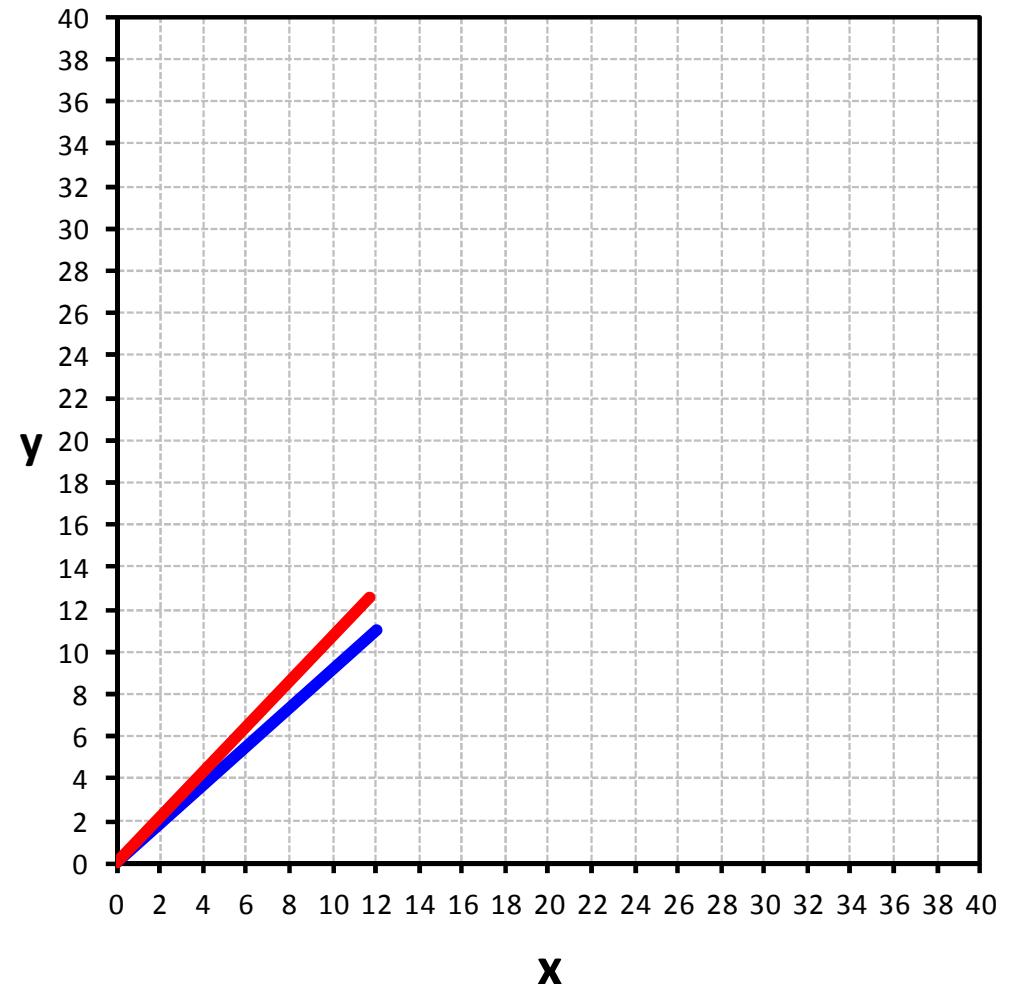
$$b_1 \quad A_1 + b_2 \quad A_2 = \boxed{} \quad \boxed{}$$



Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{}$$

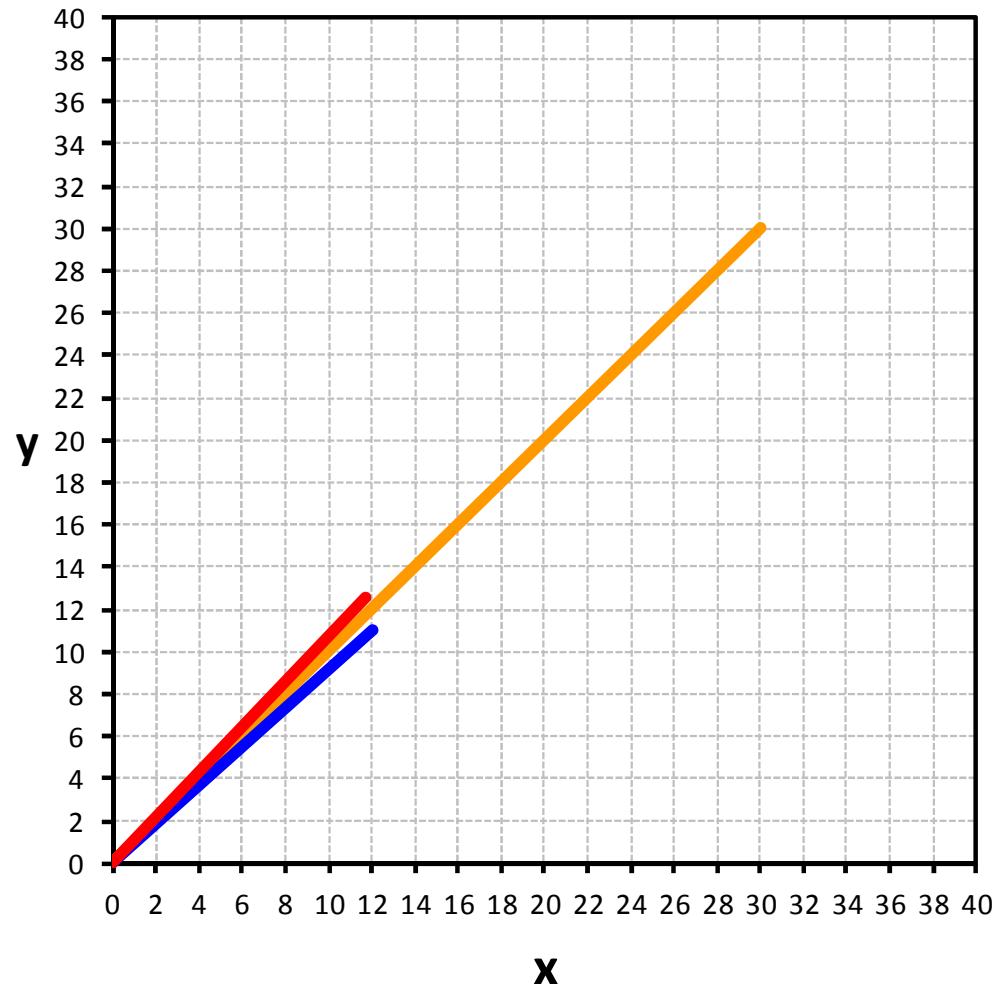


Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$

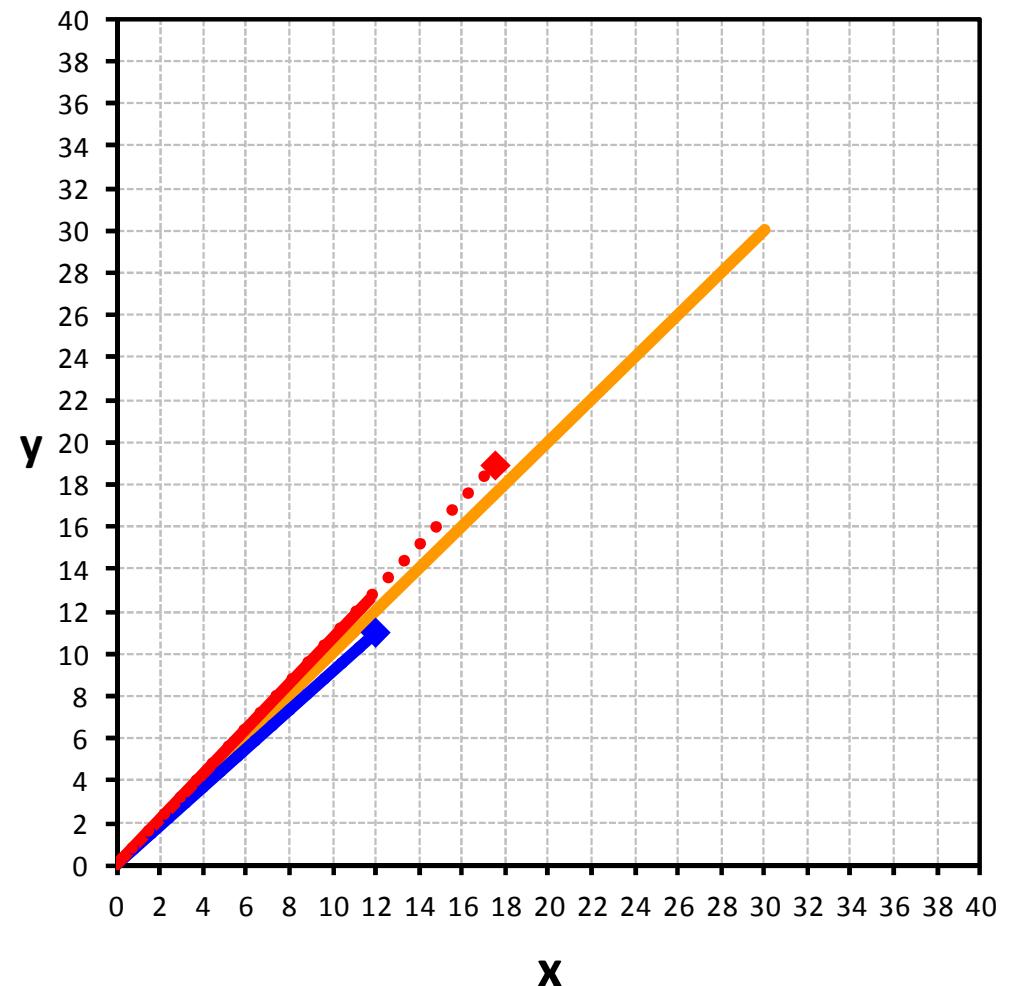


Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$

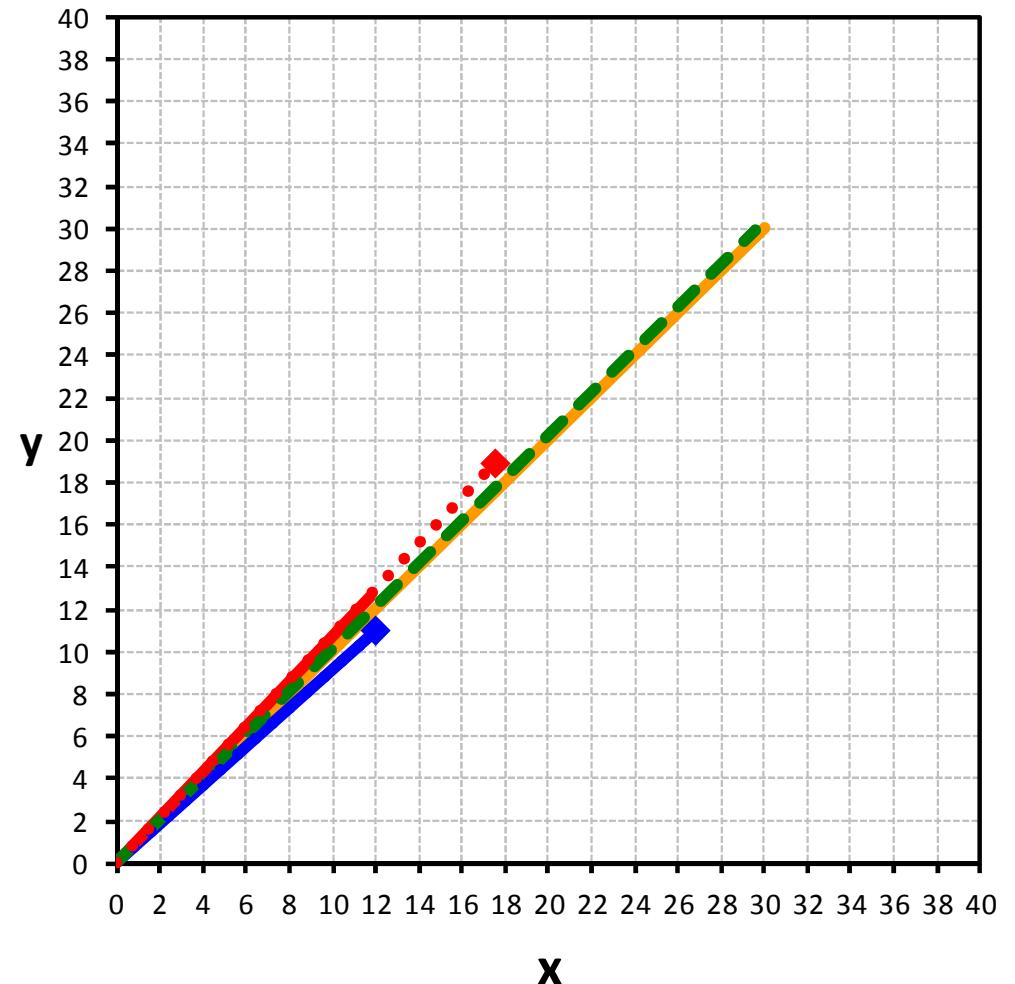


Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$

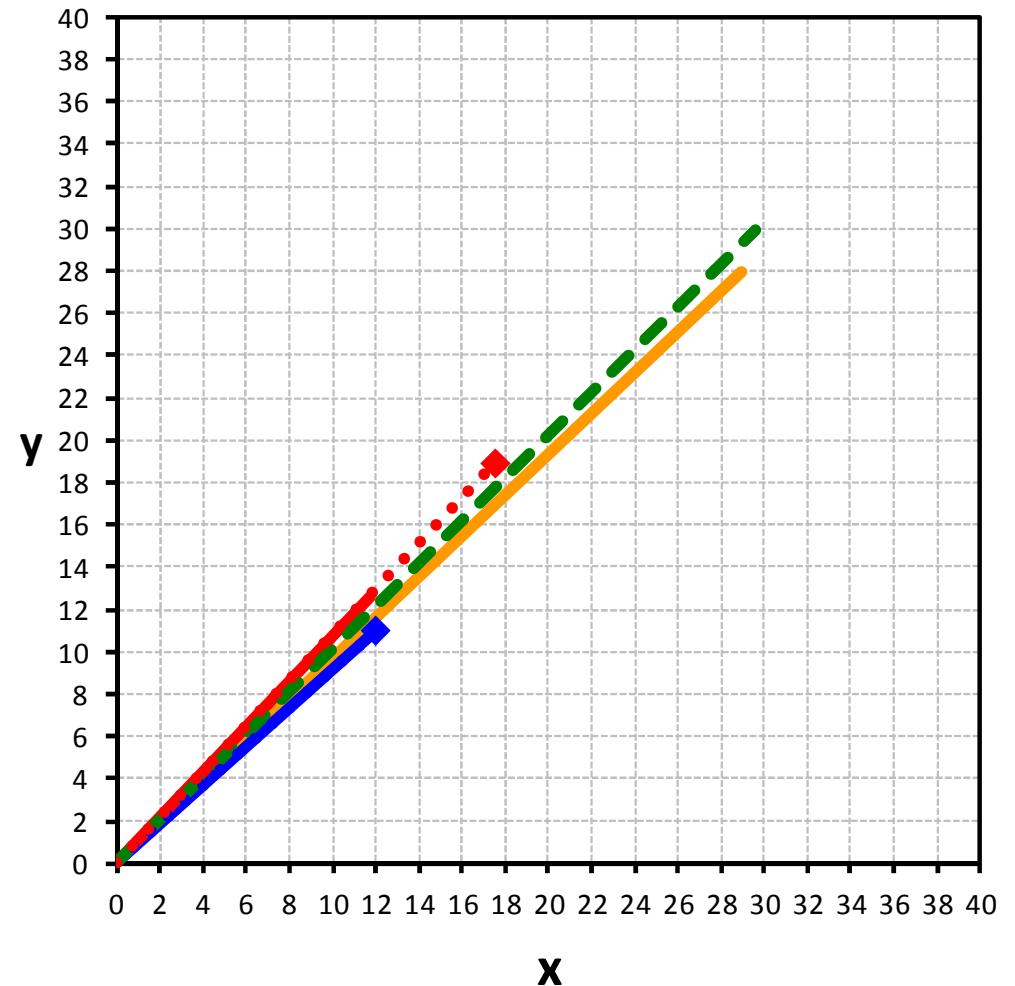


Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$

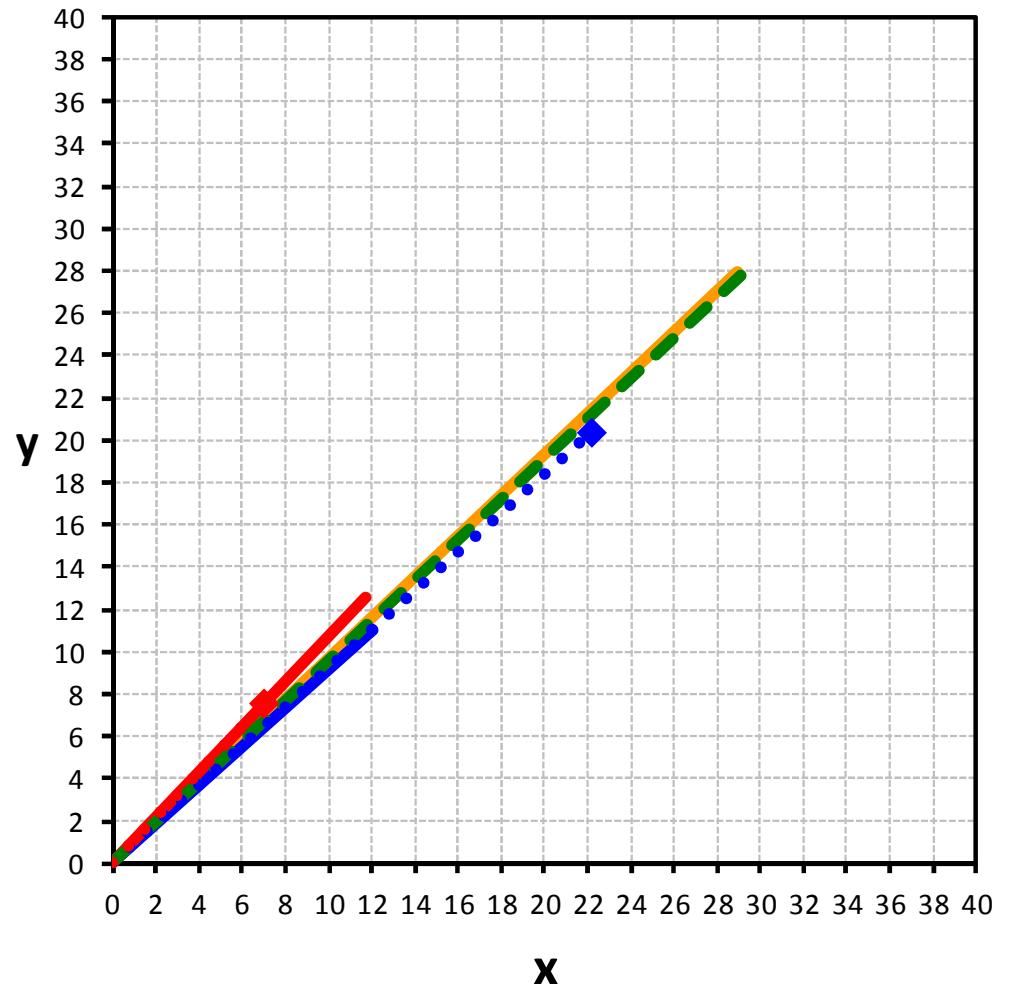


Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$

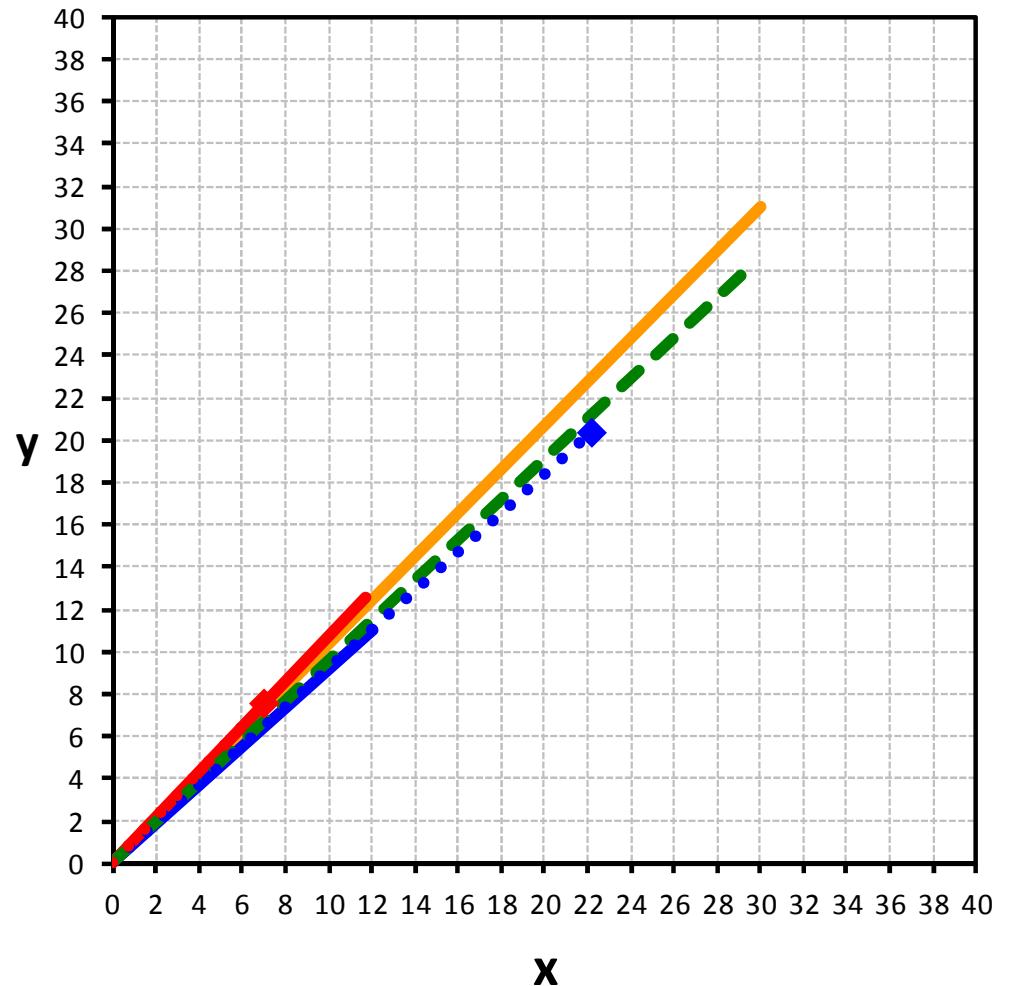


Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$

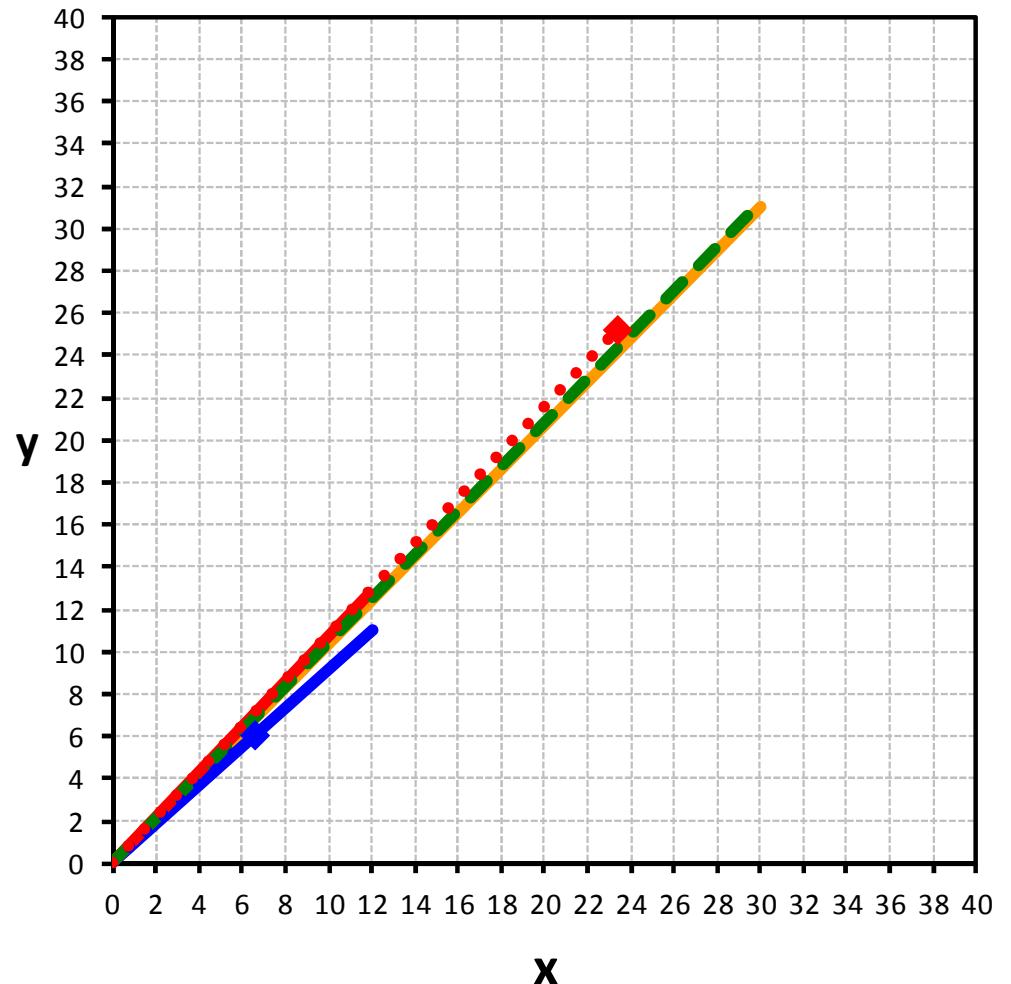


Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$

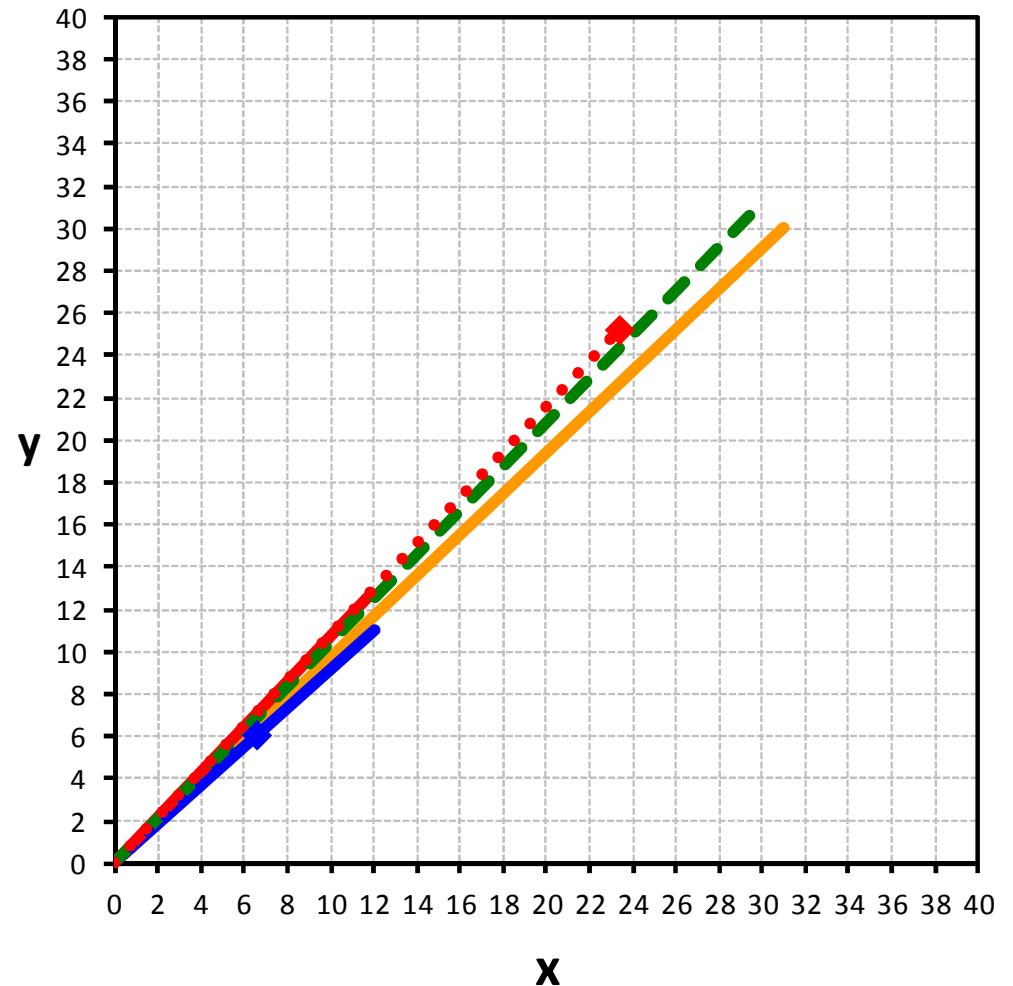


Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$

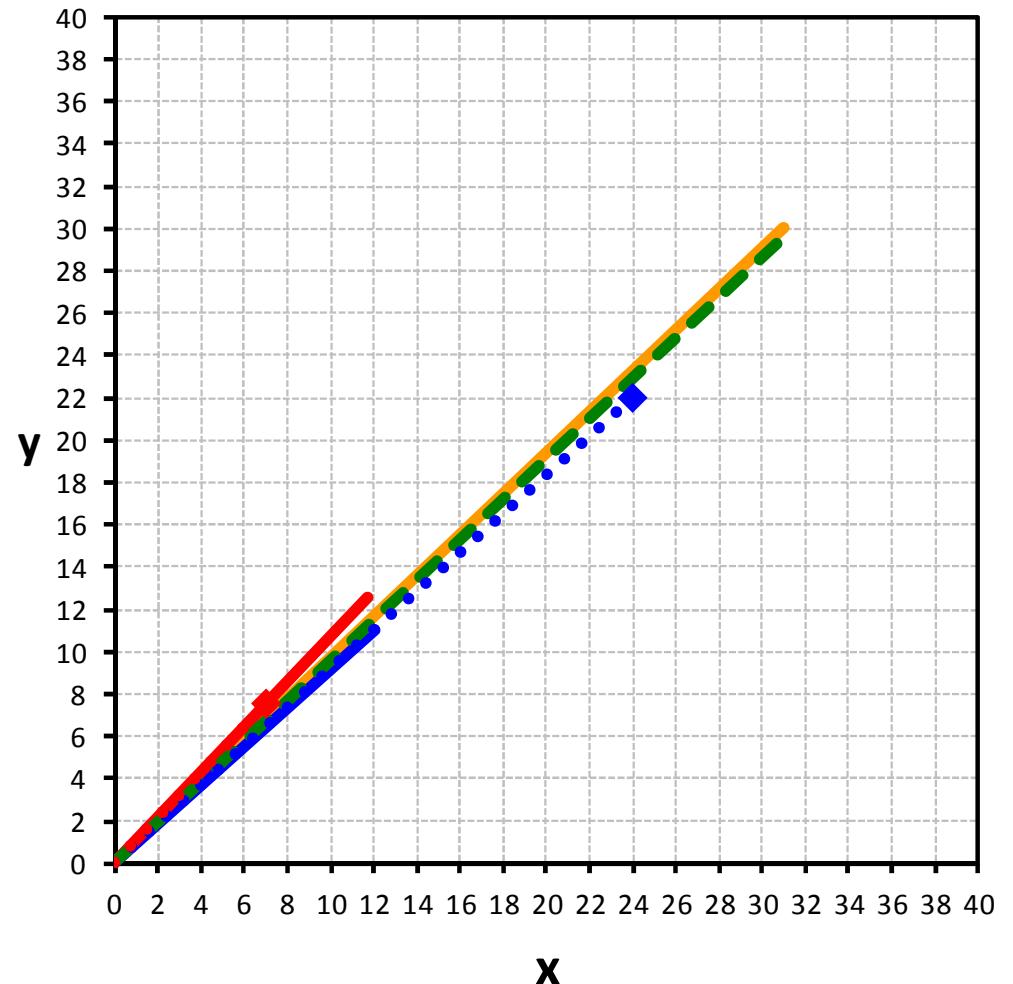


Sistemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$

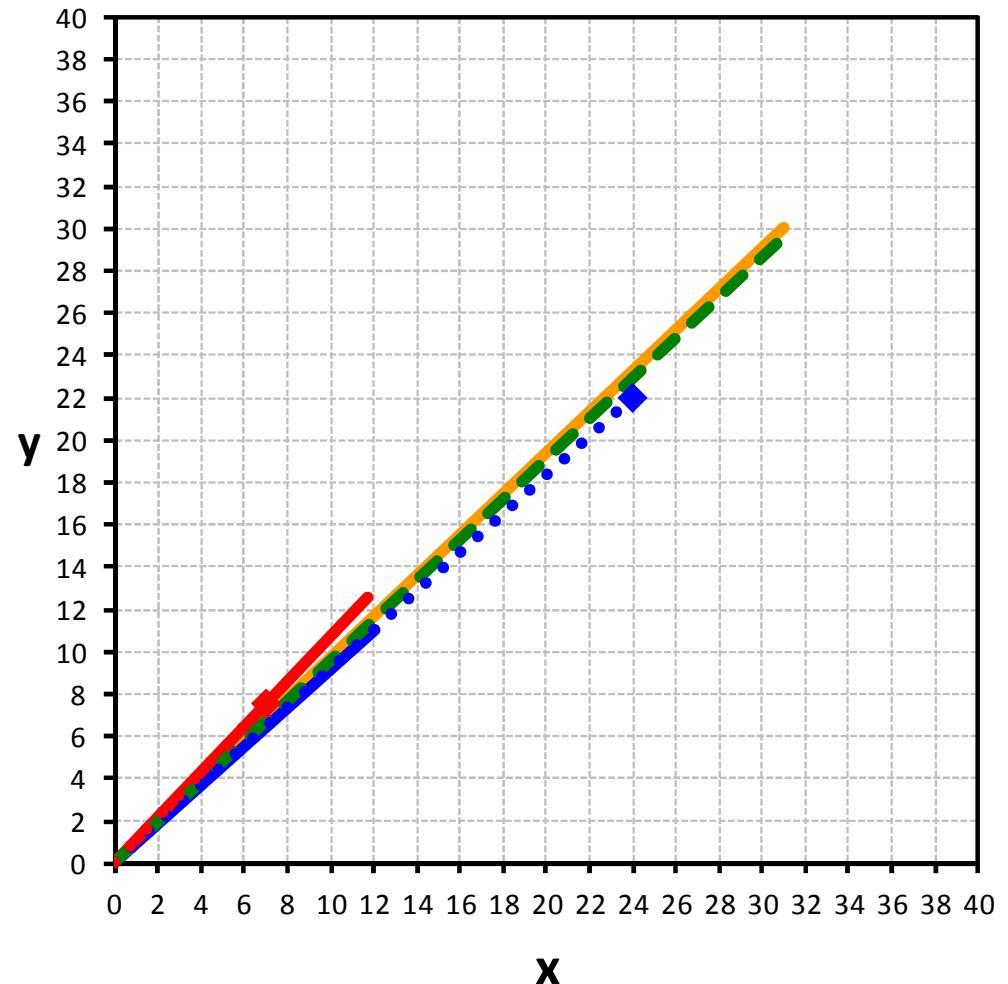


Sistemas lineares

Exemplo 2D

$$\boxed{b_1} \quad \boxed{A_1} + \boxed{b_2} \quad \boxed{A_2} = \boxed{\quad} = \boxed{\quad}$$

Pequenas
perturbações do lado
direito causam grandes
perturbações nos
coeficientes b_1 e b_2

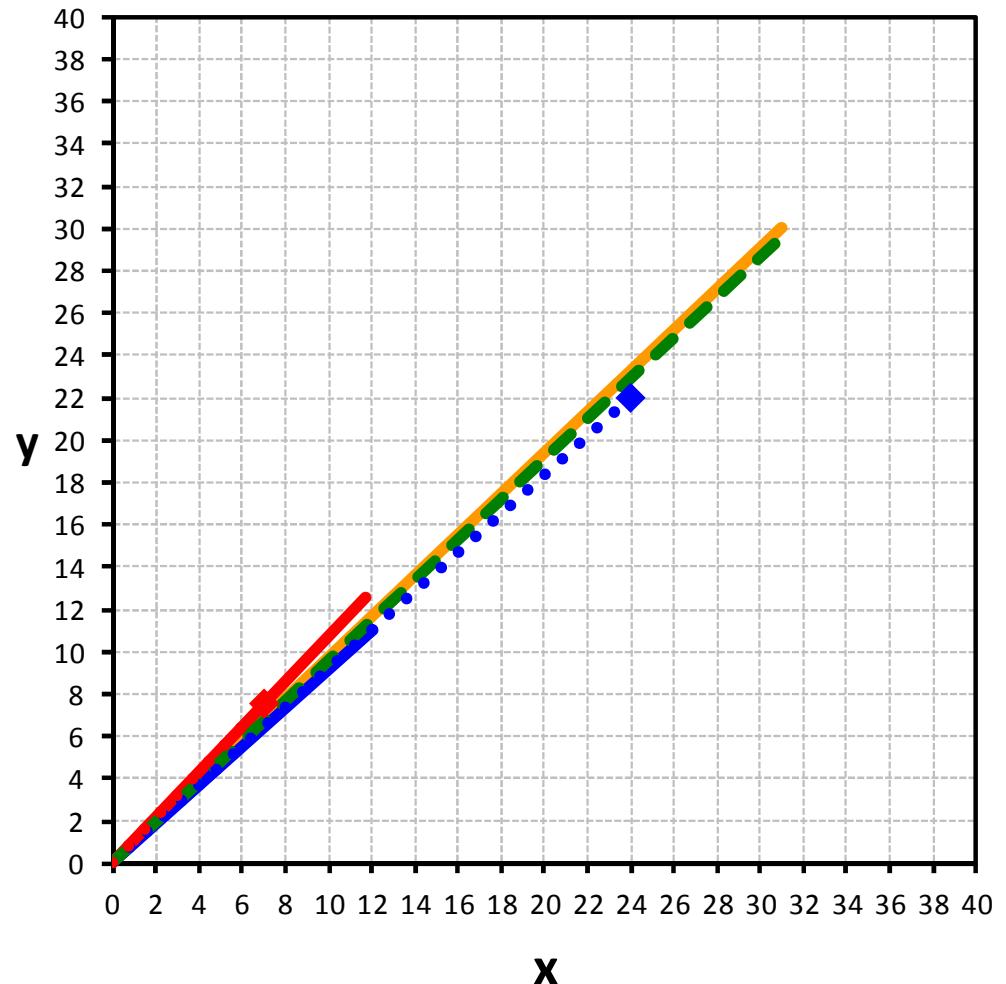


Sistemas lineares

Exemplo 2D

$$\boxed{b_1} \quad \boxed{A_1} + \boxed{b_2} \quad \boxed{A_2} = \boxed{\quad} = \boxed{\quad}$$

Nesse caso, diz-se que
o sistema linear é
instável

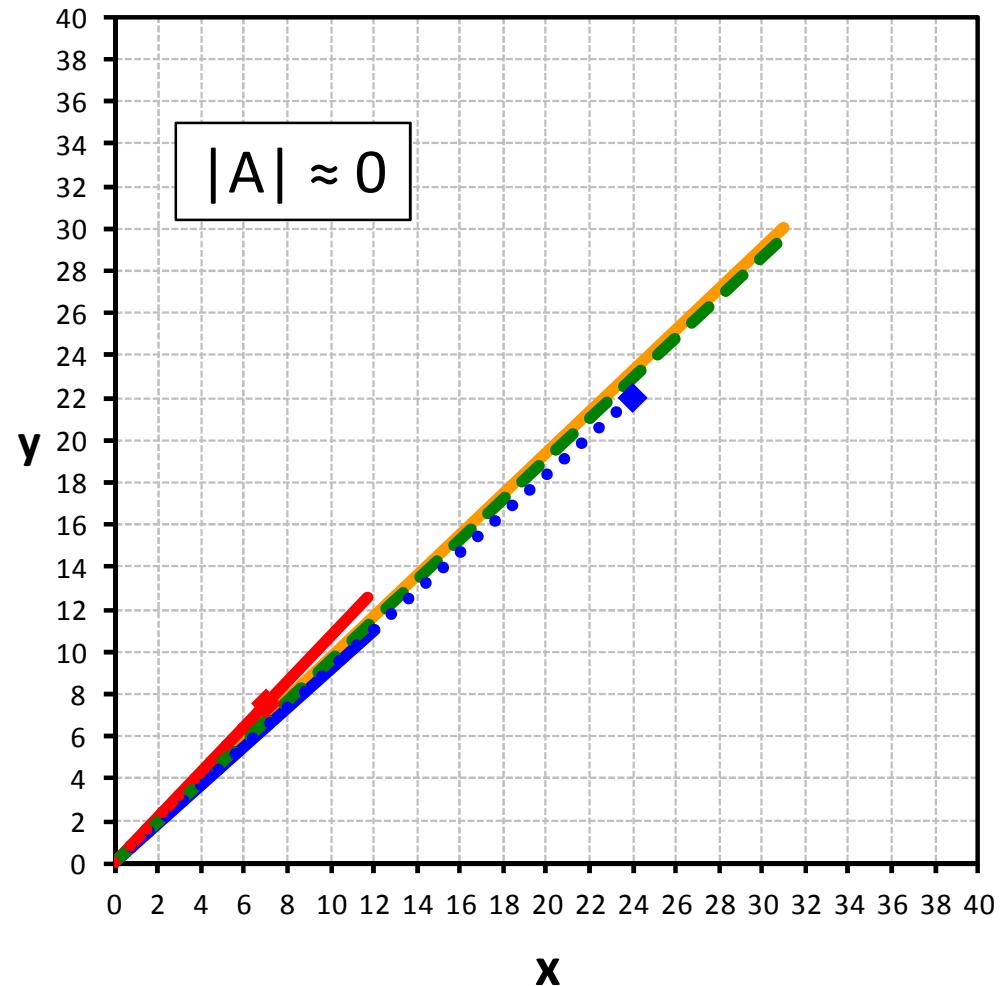


Sistemas lineares

Exemplo 2D

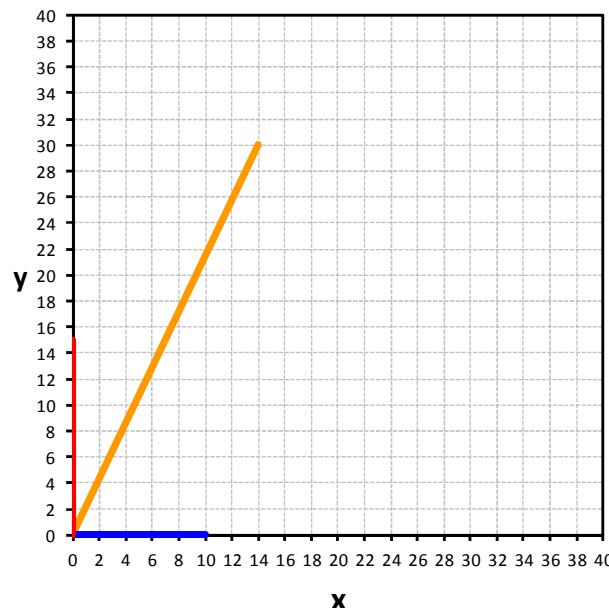
$$b1 + A1 + b2 + A2 = \boxed{}$$

Nesse caso, diz-se que
o sistema linear é
instável

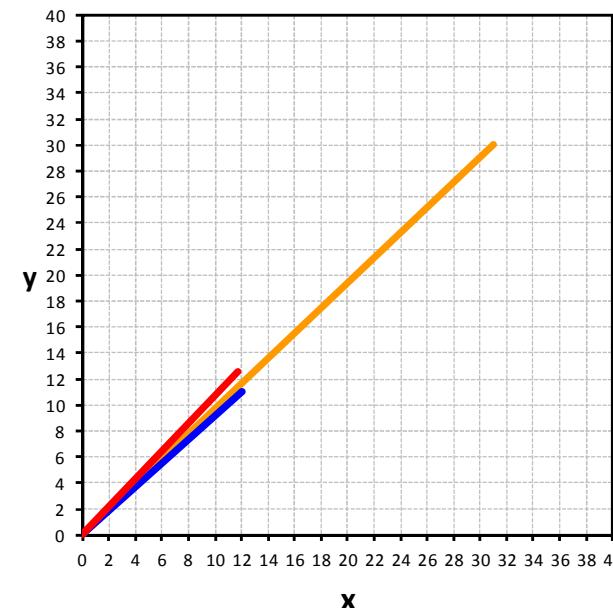


Sistemas lineares

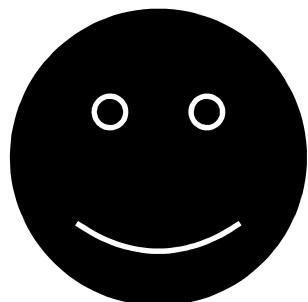
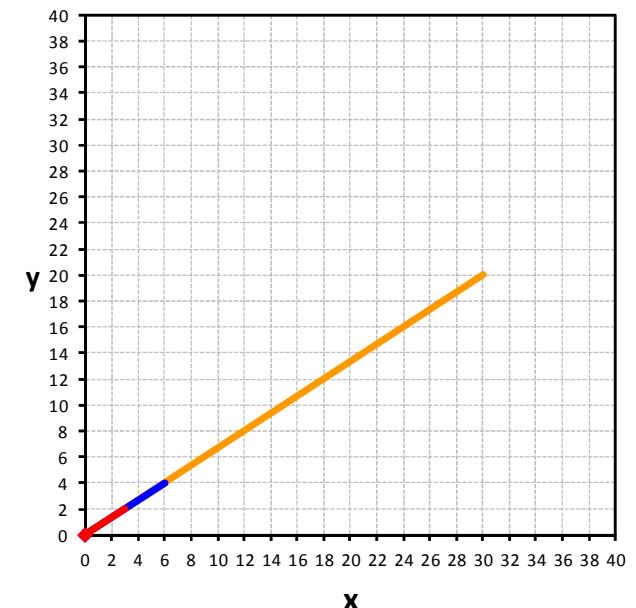
$$|A| \neq 0$$



$$|A| \approx 0$$



$$|A| = 0$$



$$\boxed{b_1} \quad \boxed{A_1} + \boxed{b_2} \quad \boxed{A_2} = \boxed{} \quad \boxed{}$$



Problemas lineares

$$\bar{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}_{M \times 1}$$

parâmetros

$$\bar{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}_{N \times 1}$$

dados
observados

$$\bar{g}(\bar{p}) = \begin{bmatrix} g_1(\bar{p}) \\ \vdots \\ g_N(\bar{p}) \end{bmatrix}_{N \times 1}$$

dados
preditos

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

norma L2
(função escalar)

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

Problema linear

$$\bar{g}(\bar{p}) = \bar{B}\bar{p} + \bar{b}$$

$$\bar{p}^* = \left(\bar{B}^T \bar{B} \right)^{-1} \bar{B}^T [\bar{d} - \bar{b}]$$

Problema não-linear

$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\Delta\bar{p} = \left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right)^{-1} \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Problemas lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

$$\bar{g}(\bar{p}) = \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p}^* = \begin{pmatrix} \bar{B} & \bar{B} \end{pmatrix}^{-1} \bar{B}^T [\bar{d} - \bar{b}]$$

Problemas lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

$$\bar{g}(\bar{p}) = \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p}^* = \begin{pmatrix} \bar{B} & \bar{B} \end{pmatrix}^{-1} \bar{B}^T [\bar{d} - \bar{b}]$$

Problemas lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

$$\bar{g}(\bar{p}) = \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\begin{pmatrix} \bar{B}^T \bar{B} \\ \bar{B} \end{pmatrix} \bar{p}^* = \bar{B}^T [\bar{d} - \bar{b}]$$

Sistema linear

Problemas lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

Caso 1)

$$\det \begin{pmatrix} \bar{B}^T & \bar{B} \end{pmatrix} \neq 0$$

$$\bar{g}(\bar{p}) = \bar{B}\bar{p} + \bar{b}$$

Caso 2)

$$\det \begin{pmatrix} \bar{B}^T & \bar{B} \end{pmatrix} = 0$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

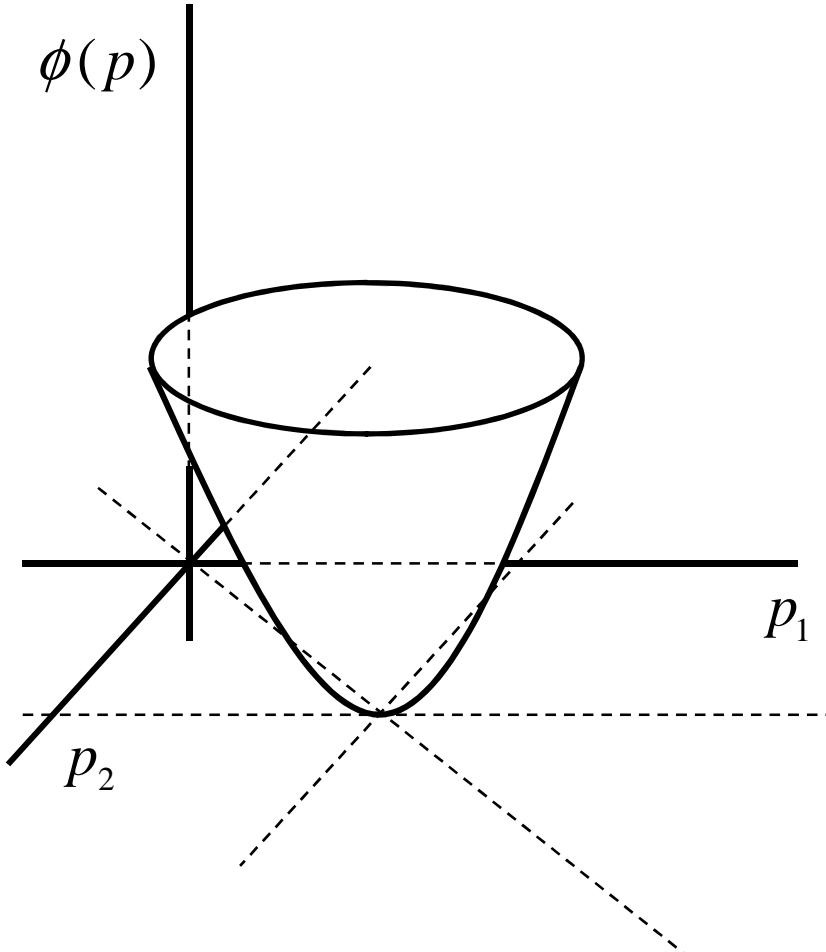
Caso 3)

$$\det \begin{pmatrix} \bar{B}^T & \bar{B} \end{pmatrix} \approx 0$$

$$\begin{pmatrix} \bar{B}^T & \bar{B} \end{pmatrix} \bar{p}^* = \bar{B}^T [\bar{d} - \bar{b}]$$

Sistema linear

Problemas lineares



Caso 1)

$$\det \begin{pmatrix} =^T= \\ B \quad B \end{pmatrix} \neq 0$$

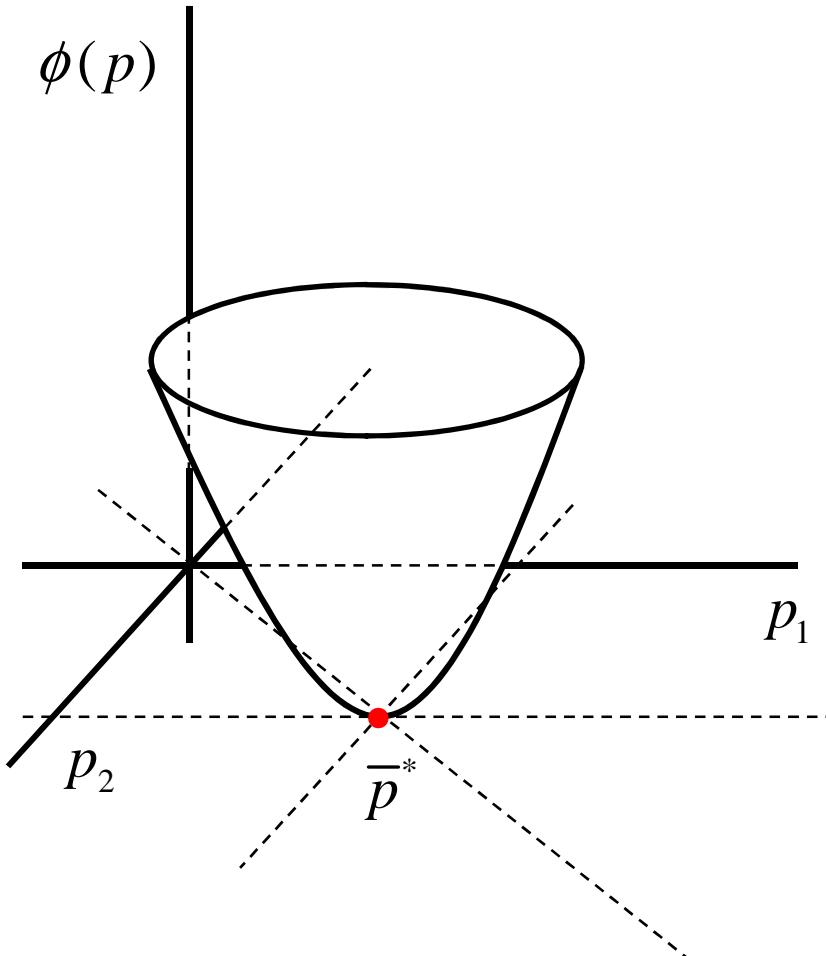
Caso 2)

$$\det \begin{pmatrix} =^T= \\ B \quad B \end{pmatrix} = 0$$

Caso 3)

$$\det \begin{pmatrix} =^T= \\ B \quad B \end{pmatrix} \approx 0$$

Problemas lineares



Caso 1)

$$\det \begin{pmatrix} =^T= \\ B \quad B \end{pmatrix} \neq 0$$

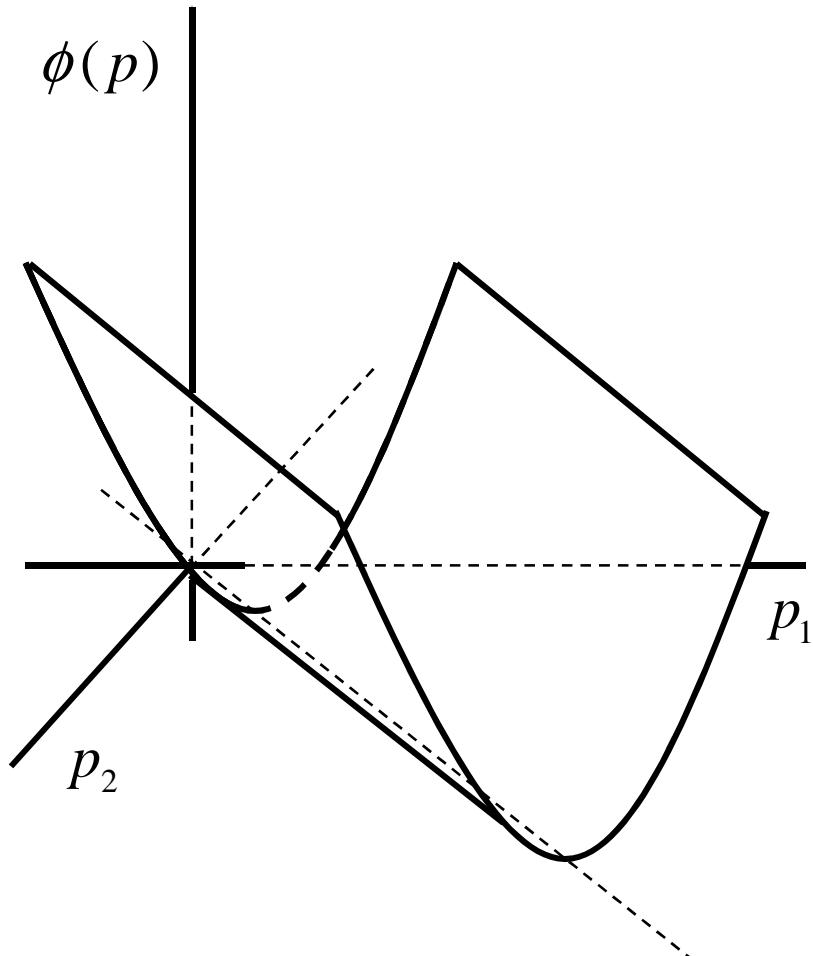
Caso 2)

$$\det \begin{pmatrix} =^T= \\ B \quad B \end{pmatrix} = 0$$

Caso 3)

$$\det \begin{pmatrix} =^T= \\ B \quad B \end{pmatrix} \approx 0$$

Problemas lineares



Caso 1)

$$\det \begin{pmatrix} =^T= \\ B \quad B \end{pmatrix} \neq 0$$

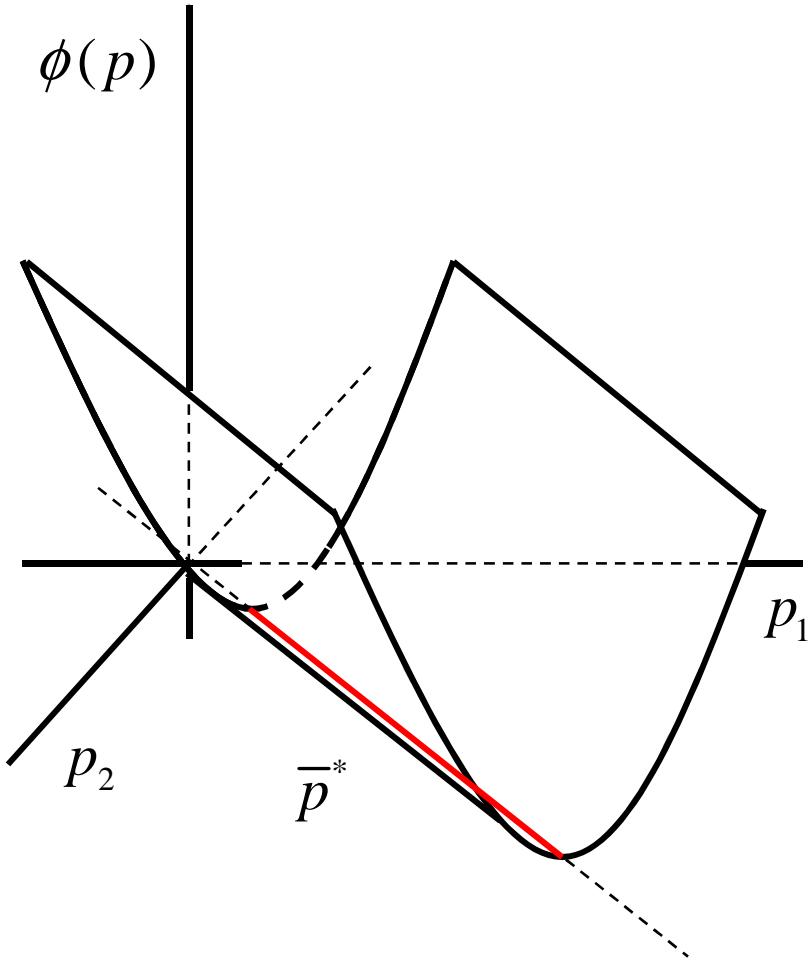
Caso 2)

$$\det \begin{pmatrix} =^T= \\ B \quad B \end{pmatrix} = 0$$

Caso 3)

$$\det \begin{pmatrix} =^T= \\ B \quad B \end{pmatrix} \approx 0$$

Problemas lineares



Caso 1)

$$\det \begin{pmatrix} = & T & = \\ B & B \end{pmatrix} \neq 0$$

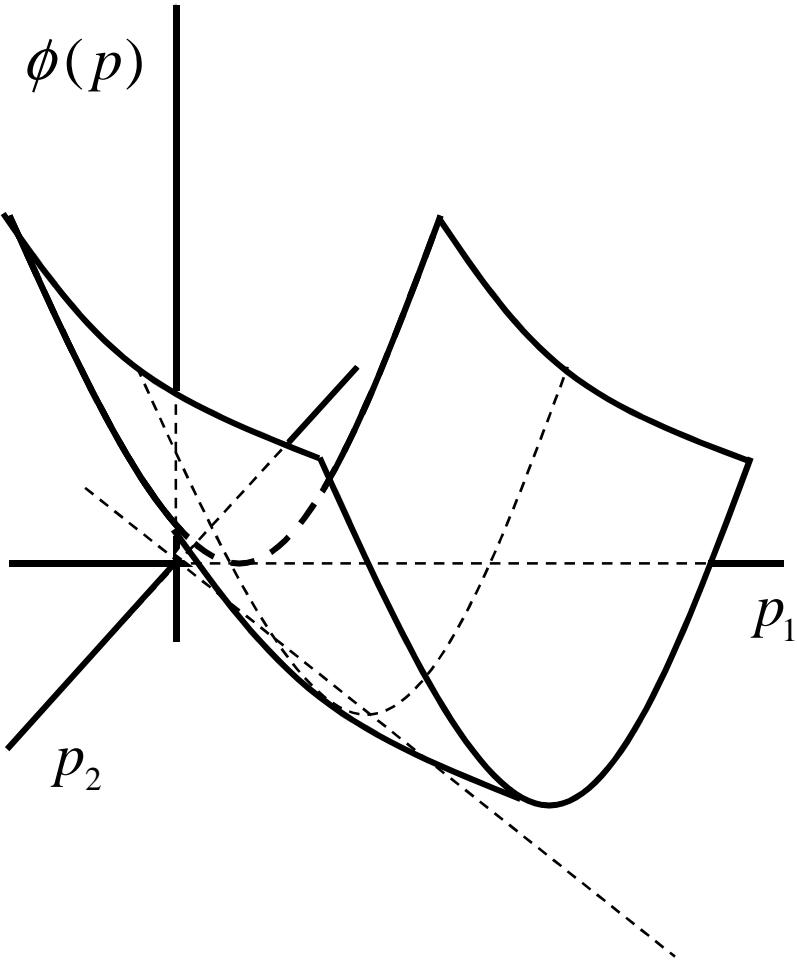
Caso 2)

$$\det \begin{pmatrix} = & T & = \\ B & B \end{pmatrix} = 0$$

Caso 3)

$$\det \begin{pmatrix} = & T & = \\ B & B \end{pmatrix} \approx 0$$

Problemas lineares



Caso 1)

$$\det \begin{pmatrix} =^T= \\ B^T B \end{pmatrix} \neq 0$$

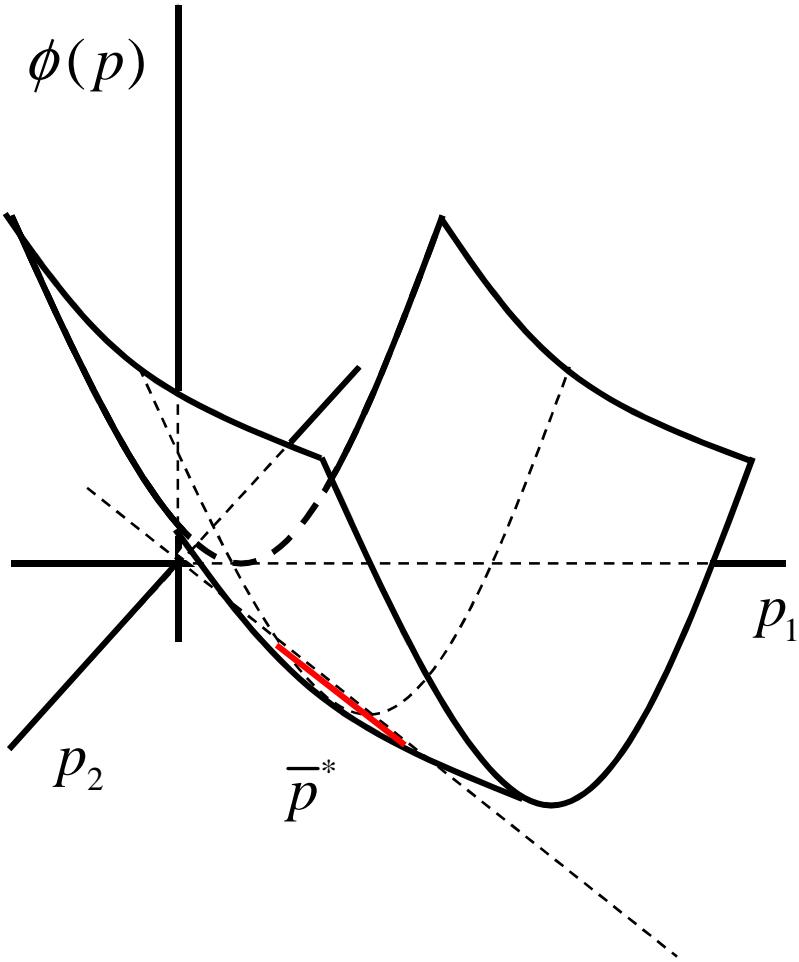
Caso 2)

$$\det \begin{pmatrix} =^T= \\ B^T B \end{pmatrix} = 0$$

Caso 3)

$$\det \begin{pmatrix} =^T= \\ B^T B \end{pmatrix} \approx 0$$

Problemas lineares



Caso 1)

$$\det \begin{pmatrix} =^T= \\ B \quad B \end{pmatrix} \neq 0$$

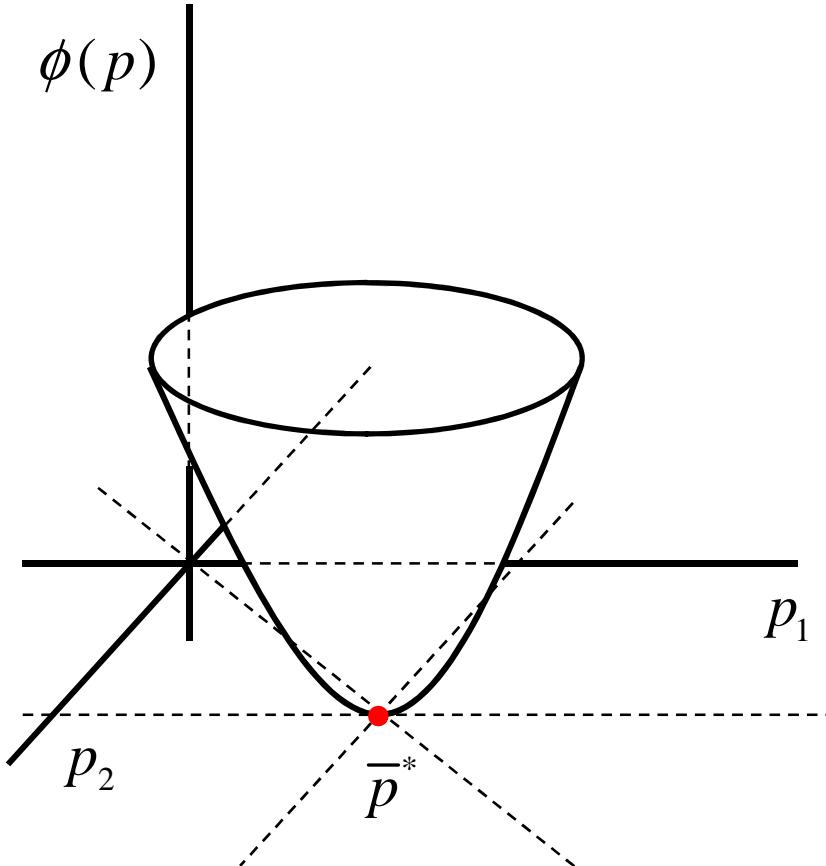
Caso 2)

$$\det \begin{pmatrix} =^T= \\ B \quad B \end{pmatrix} = 0$$

Caso 3)

$$\det \begin{pmatrix} =^T= \\ B \quad B \end{pmatrix} \approx 0$$

Problemas lineares



$$\begin{pmatrix} \mathbf{B}^T \\ \mathbf{B} \end{pmatrix} \bar{\mathbf{p}}^* = \mathbf{B}^T [\bar{\mathbf{d}} - \bar{\mathbf{b}}]$$

Solução única

Caso 1)

$$\det \begin{pmatrix} \mathbf{B}^T \\ \mathbf{B} \end{pmatrix} \neq 0$$

Caso 2)

$$\det \begin{pmatrix} \mathbf{B}^T \\ \mathbf{B} \end{pmatrix} = 0$$

Caso 3)

$$\det \begin{pmatrix} \mathbf{B}^T \\ \mathbf{B} \end{pmatrix} \approx 0$$

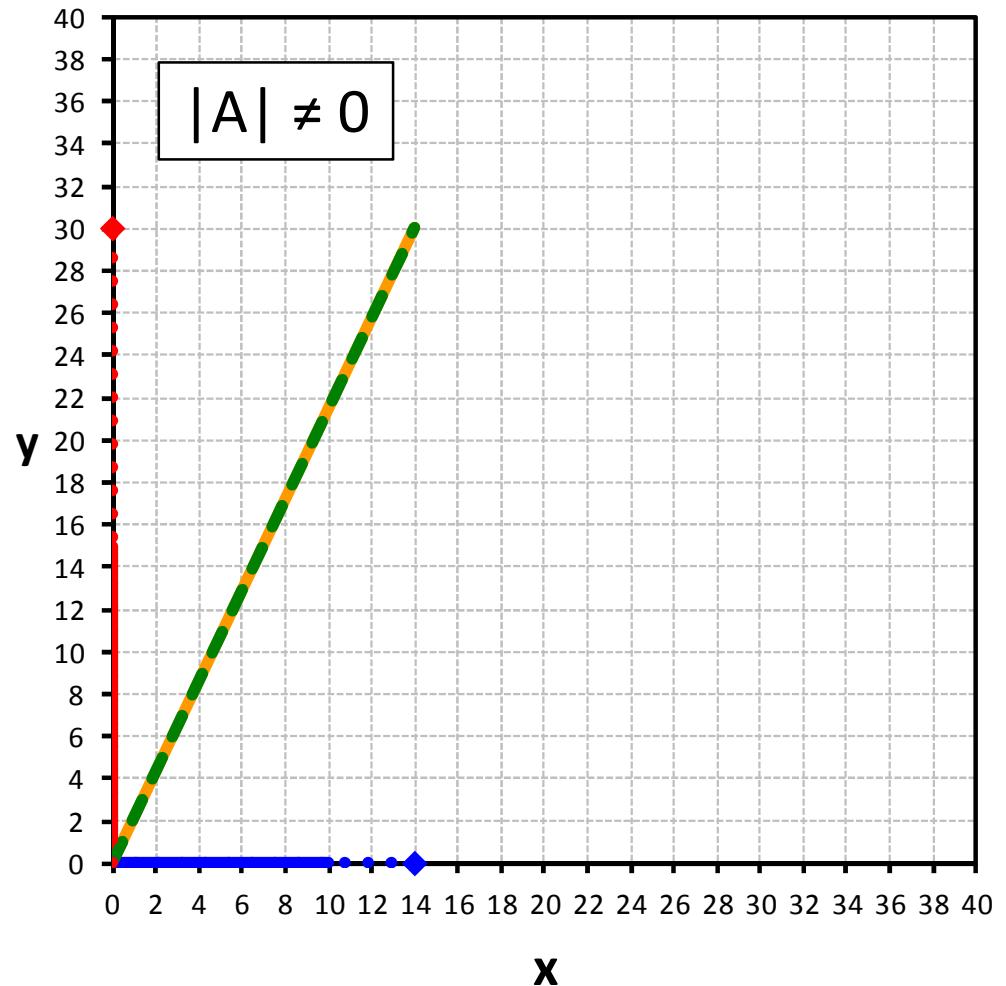
Problemas lineares

Exemplo 2D

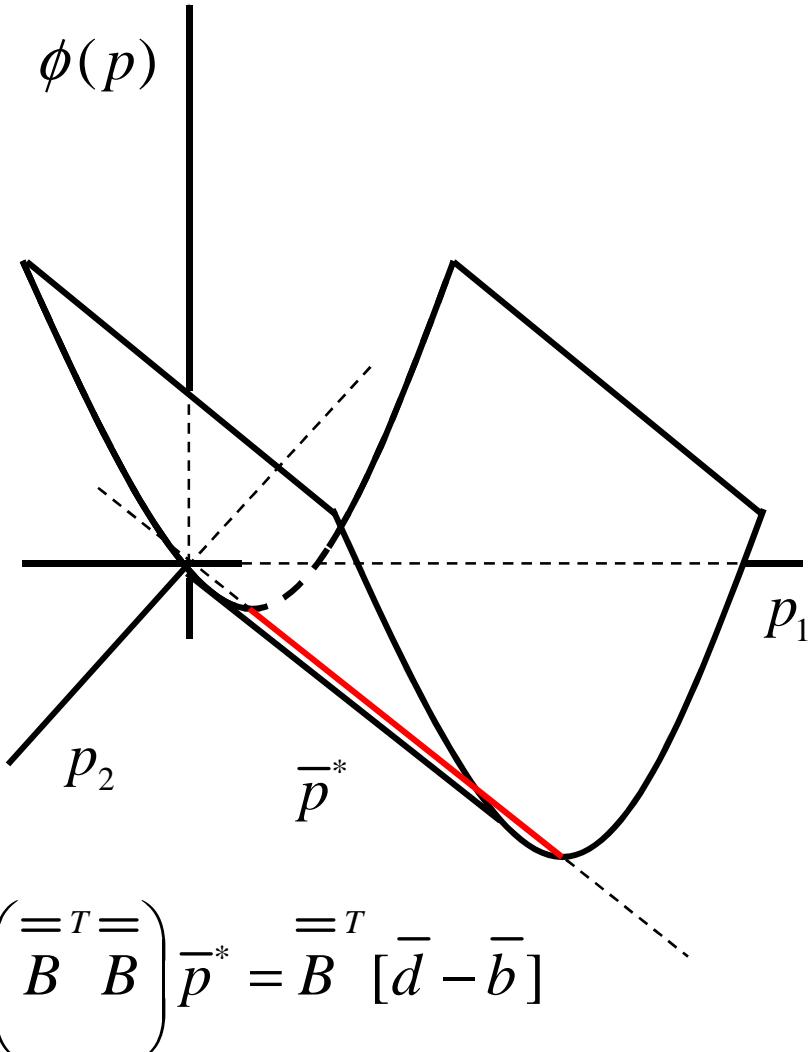
$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

$$\boxed{\quad}$$

Neste caso, os vetores A_1 e A_2 são *linearmente independentes* e os coeficientes b_1 e b_2 são únicos



Problemas lineares



Infinitas soluções

Caso 1)

$$\det \begin{pmatrix} =^T= \\ B \quad B \end{pmatrix} \neq 0$$

Caso 2)

$$\det \begin{pmatrix} =^T= \\ B \quad B \end{pmatrix} = 0$$

Caso 3)

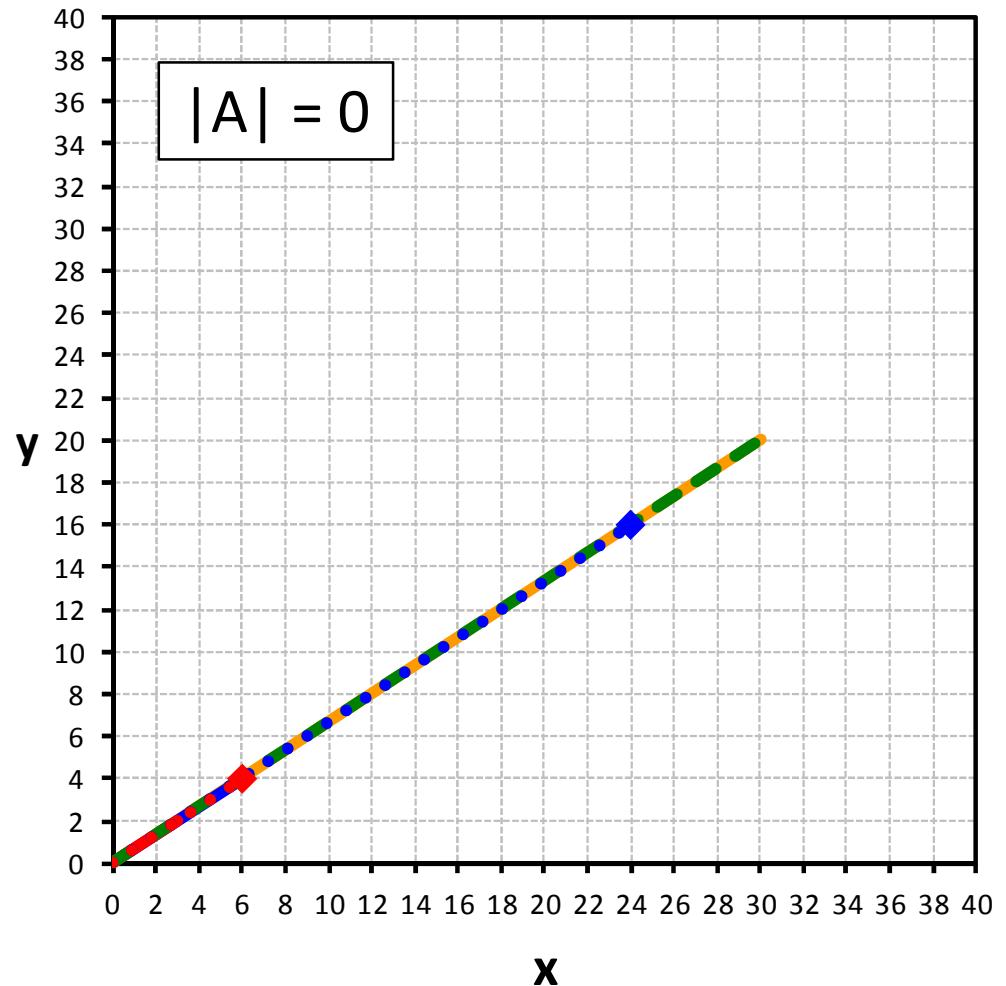
$$\det \begin{pmatrix} =^T= \\ B \quad B \end{pmatrix} \approx 0$$

Problemas lineares

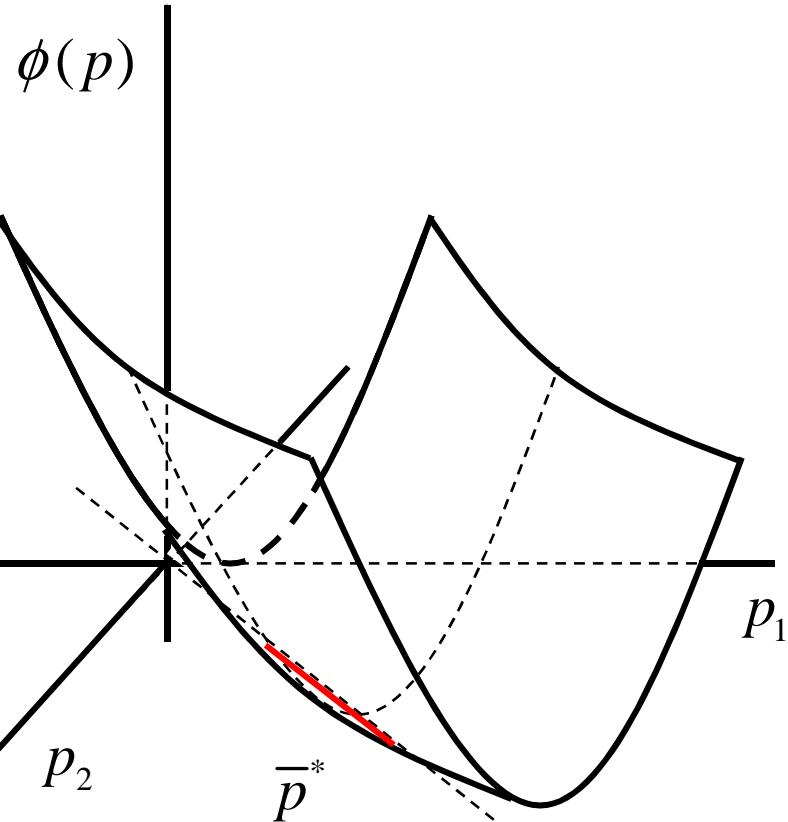
Exemplo 2D

$$\begin{array}{|c|} \hline b_1 \\ \hline \end{array} + \begin{array}{|c|} \hline A_1 \\ \hline \end{array} + \begin{array}{|c|} \hline b_2 \\ \hline \end{array} + \begin{array}{|c|} \hline A_2 \\ \hline \end{array} = \begin{array}{|c|} \hline \quad \\ \hline \end{array}$$

Neste caso, os vetores A_1 e A_2 são *linearmente dependentes* e existem infinitos pares b_1 e b_2 que produzem o mesmo resultado



Problemas lineares



$$\begin{pmatrix} =^T= \\ B \quad B \end{pmatrix} \bar{p}^* = \begin{pmatrix} =^T= \\ B \end{pmatrix} [\bar{d} - \bar{b}]$$

Sistema instável

Caso 1)

$$\det \begin{pmatrix} =^T= \\ B \quad B \end{pmatrix} \neq 0$$

Caso 2)

$$\det \begin{pmatrix} =^T= \\ B \quad B \end{pmatrix} = 0$$

Caso 3)

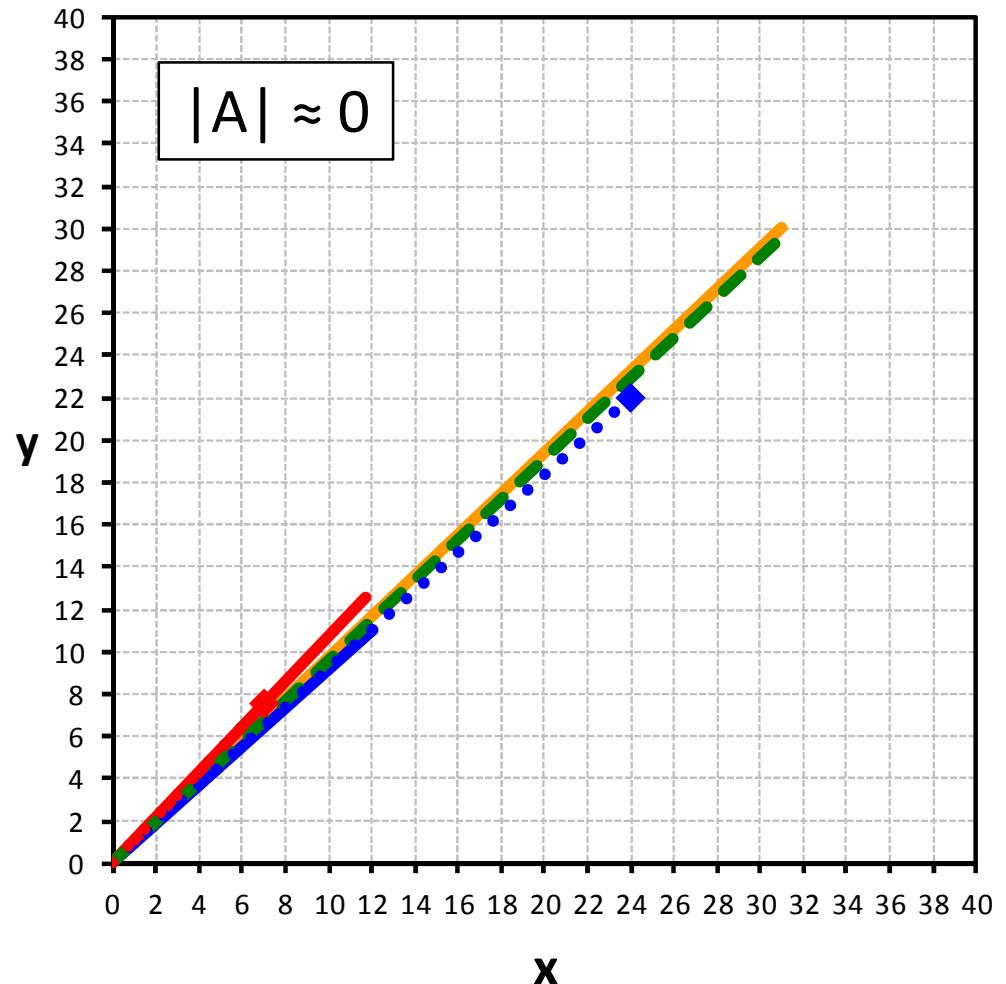
$$\det \begin{pmatrix} =^T= \\ B \quad B \end{pmatrix} \approx 0$$

Problemas lineares

Exemplo 2D

$$b_1 \boxed{A_1} + b_2 \boxed{A_2} = \boxed{\quad}$$

Pequenas
perturbações do lado
direito causam grandes
perturbações nos
coeficientes b_1 e b_2



Problemas não-lineares

$$\bar{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}_{M \times 1}$$

parâmetros

$$\bar{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}_{N \times 1}$$

dados
observados

$$\bar{g}(\bar{p}) = \begin{bmatrix} g_1(\bar{p}) \\ \vdots \\ g_N(\bar{p}) \end{bmatrix}_{N \times 1}$$

dados
preditos

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

norma L2
(função escalar)

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

Problema linear

$$\bar{g}(\bar{p}) = \bar{B}\bar{p} + \bar{b}$$

$$\bar{p}^* = \left(\bar{B}^T \bar{B} \right)^{-1} \bar{B}^T [\bar{d} - \bar{b}]$$

Problema não-linear

$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\Delta \bar{p} = \left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right)^{-1} \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\Delta \bar{p} = \left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right)^{-1} \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\Delta \bar{p} = \left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right)^{-1} \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$



Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{\bar{G}}(\bar{p}_0)^T \bar{\bar{G}}(\bar{p}_0) \right) \Delta \bar{p} = \bar{\bar{G}}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear

Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

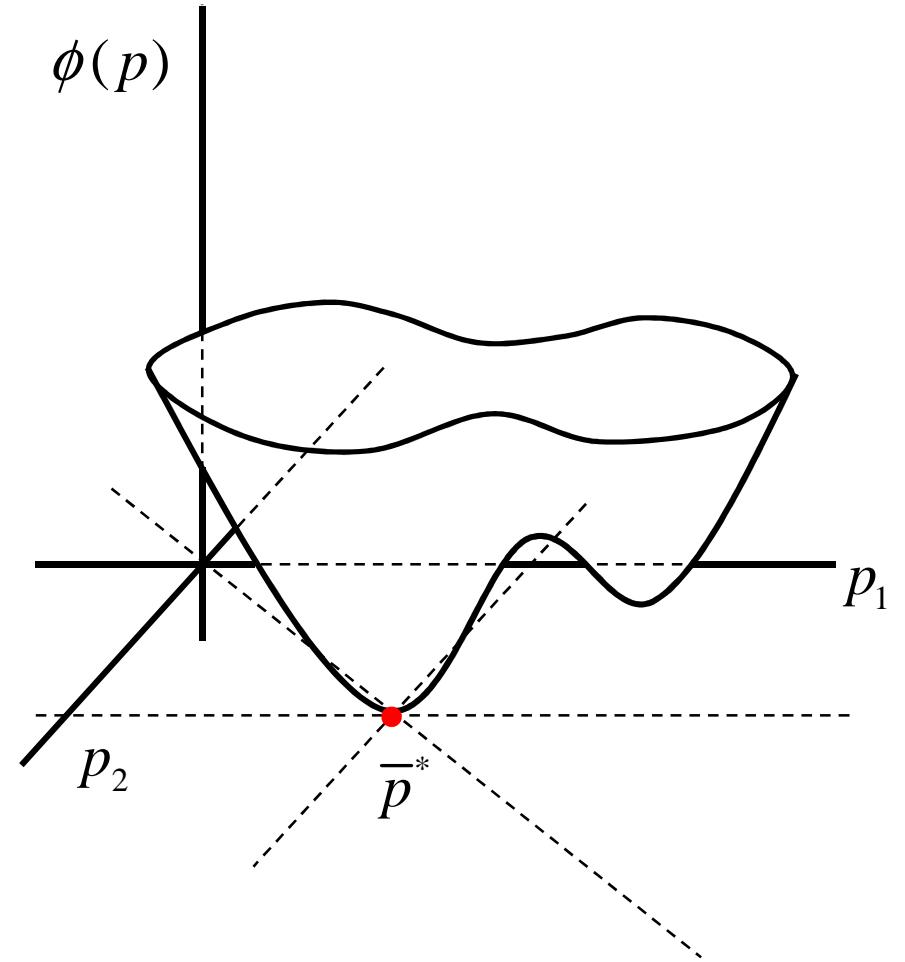
$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear



Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

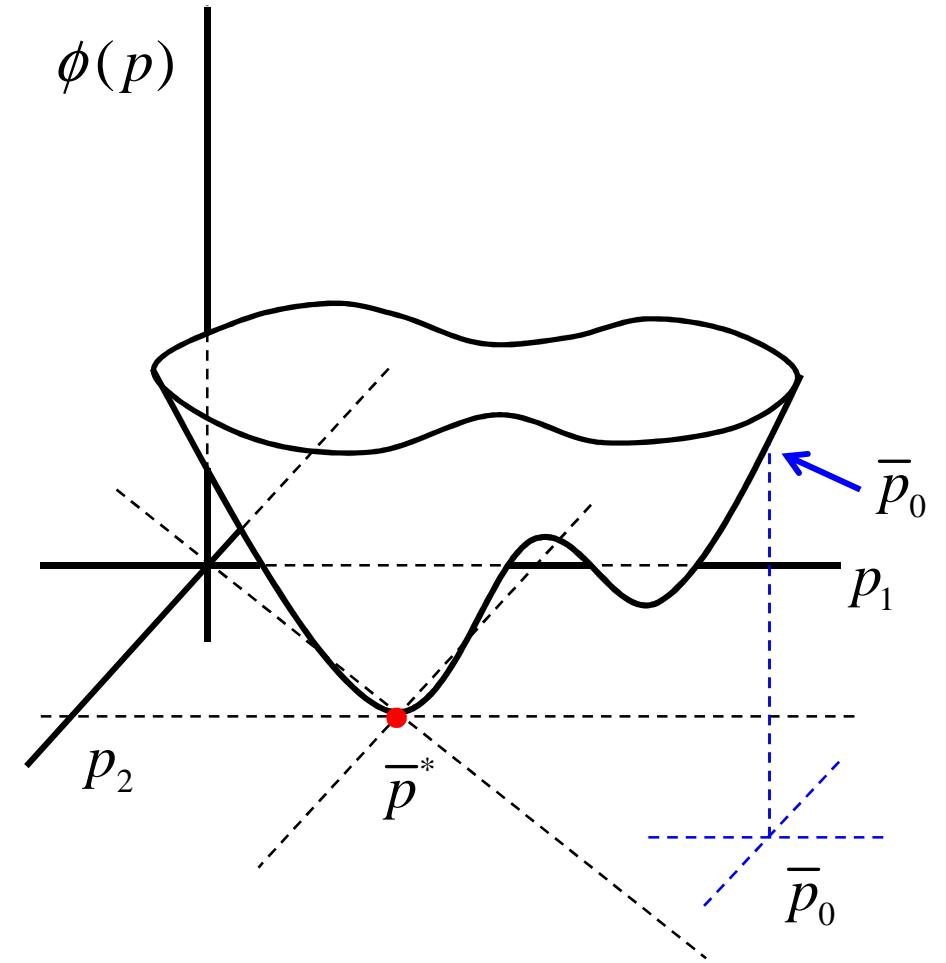
$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear



Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

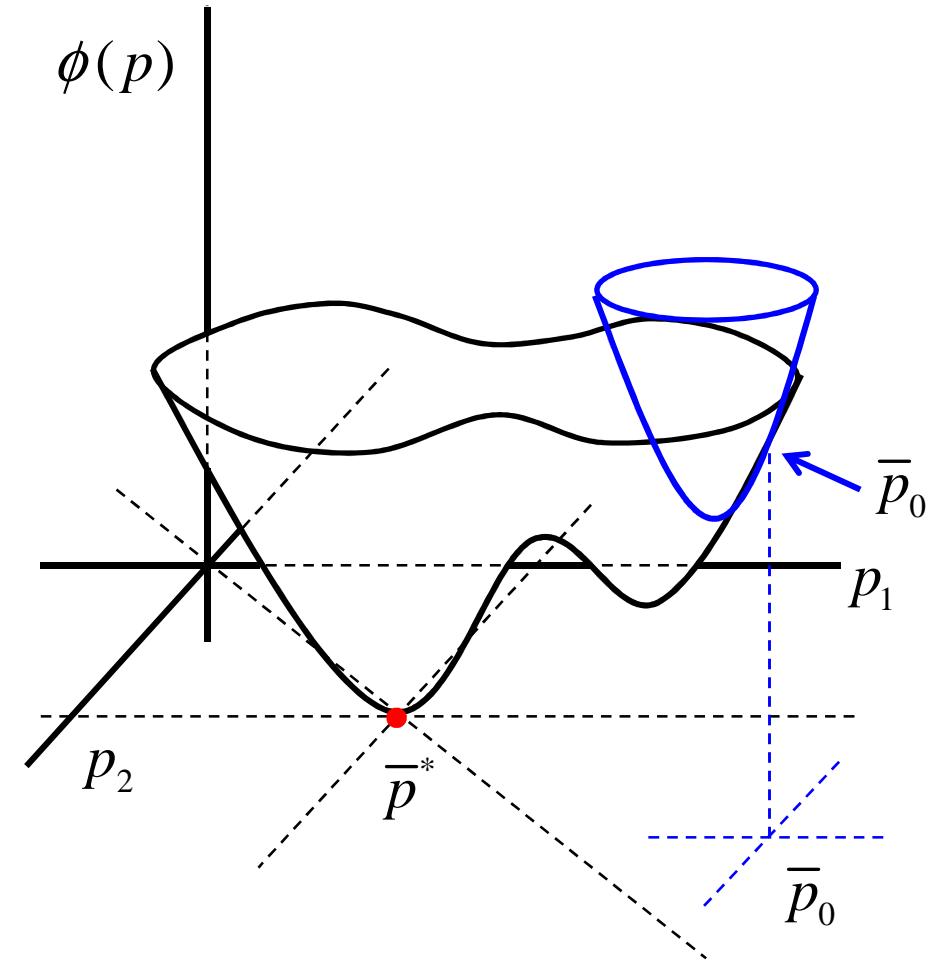
$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear



Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

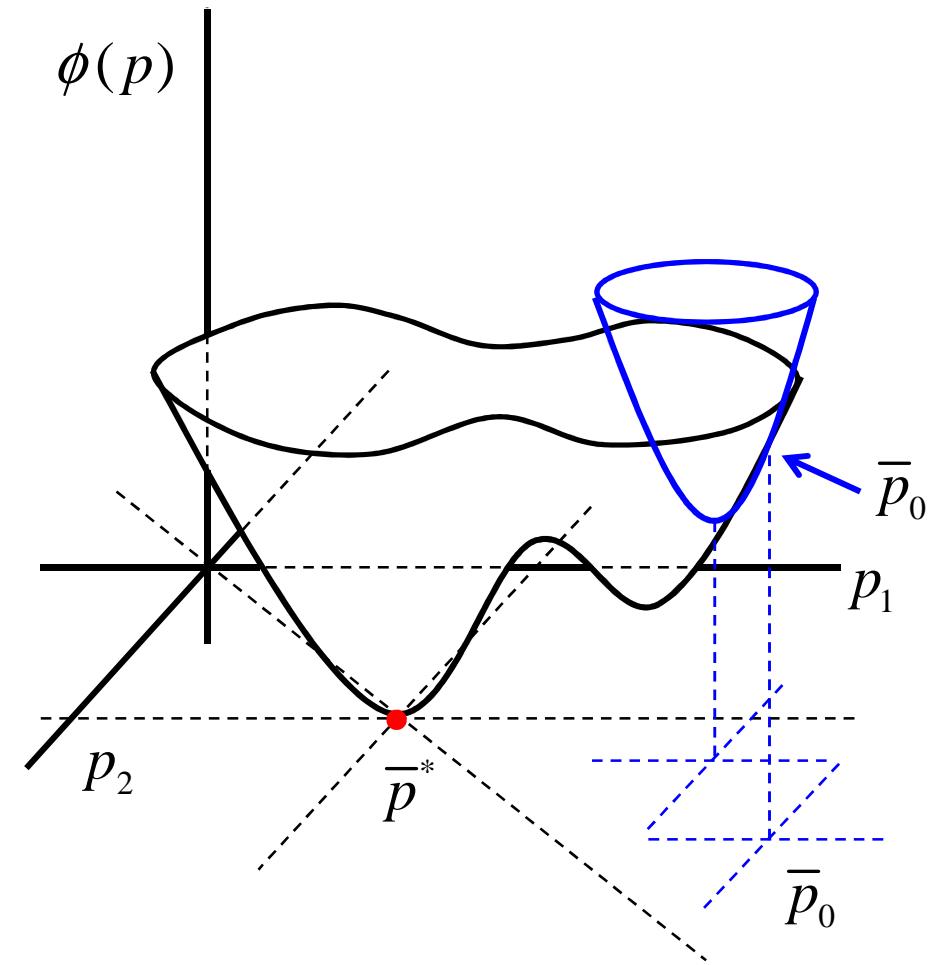
$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear



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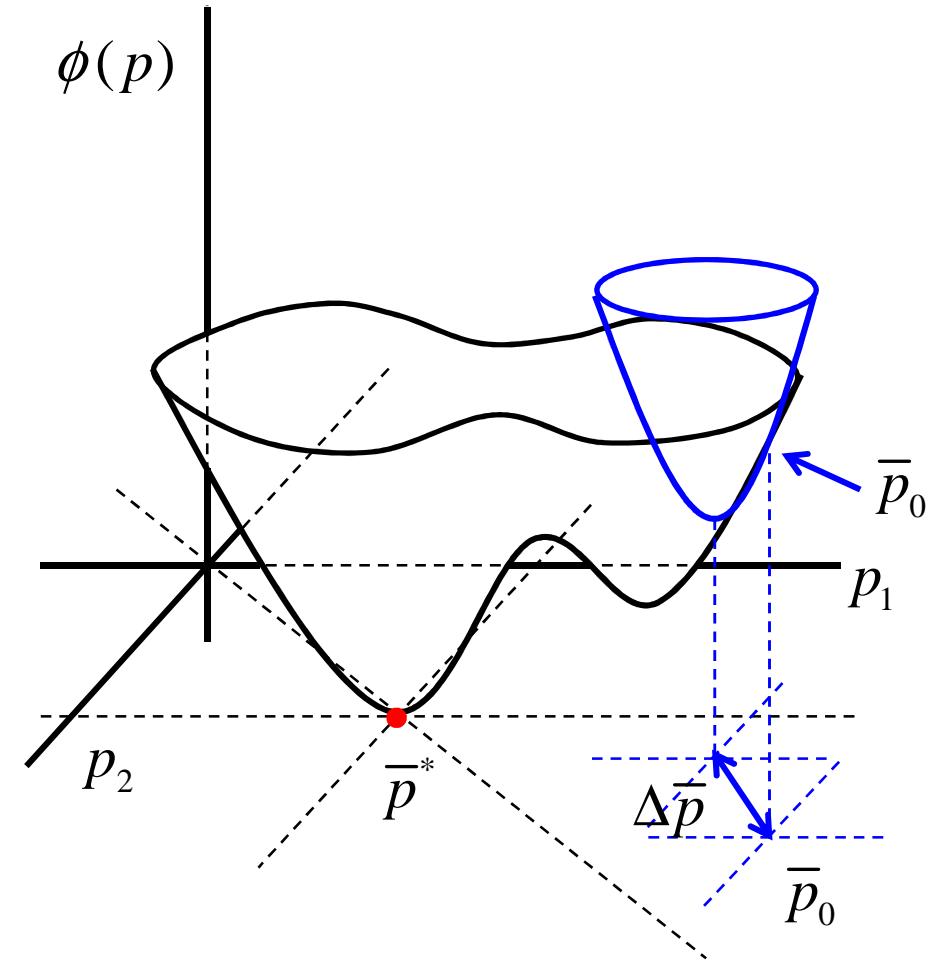
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Sistema linear



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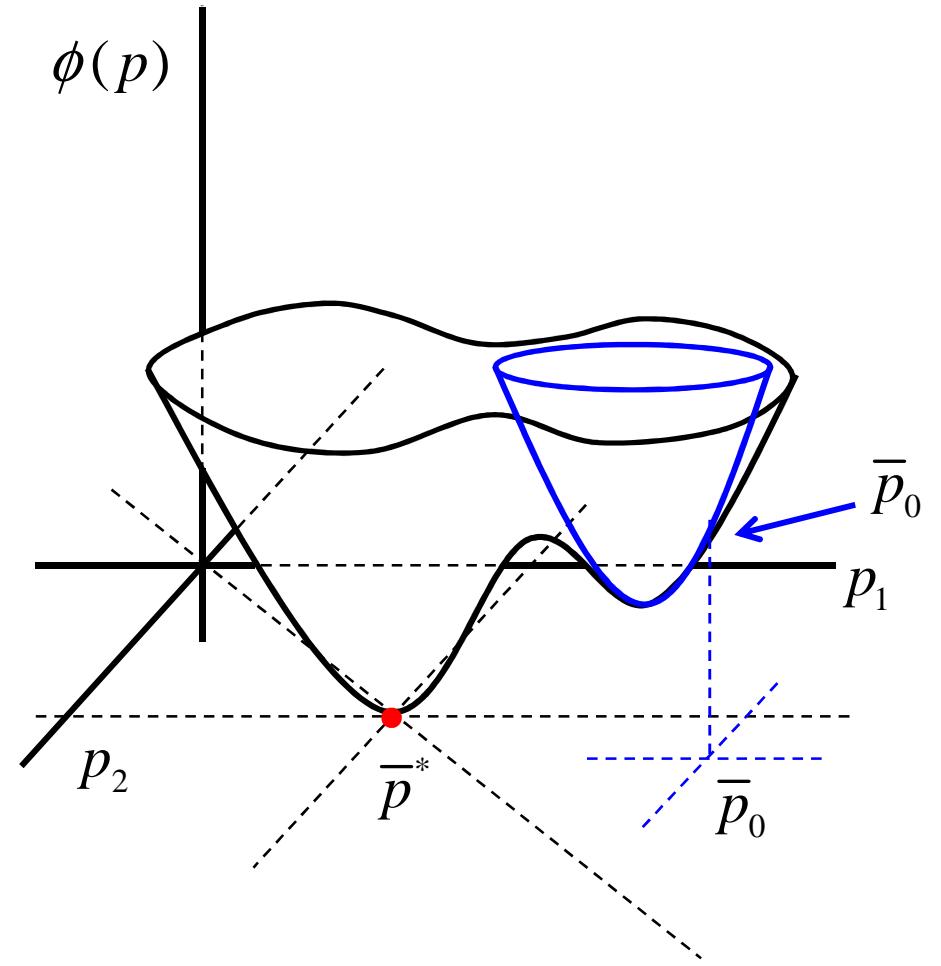
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Sistema linear



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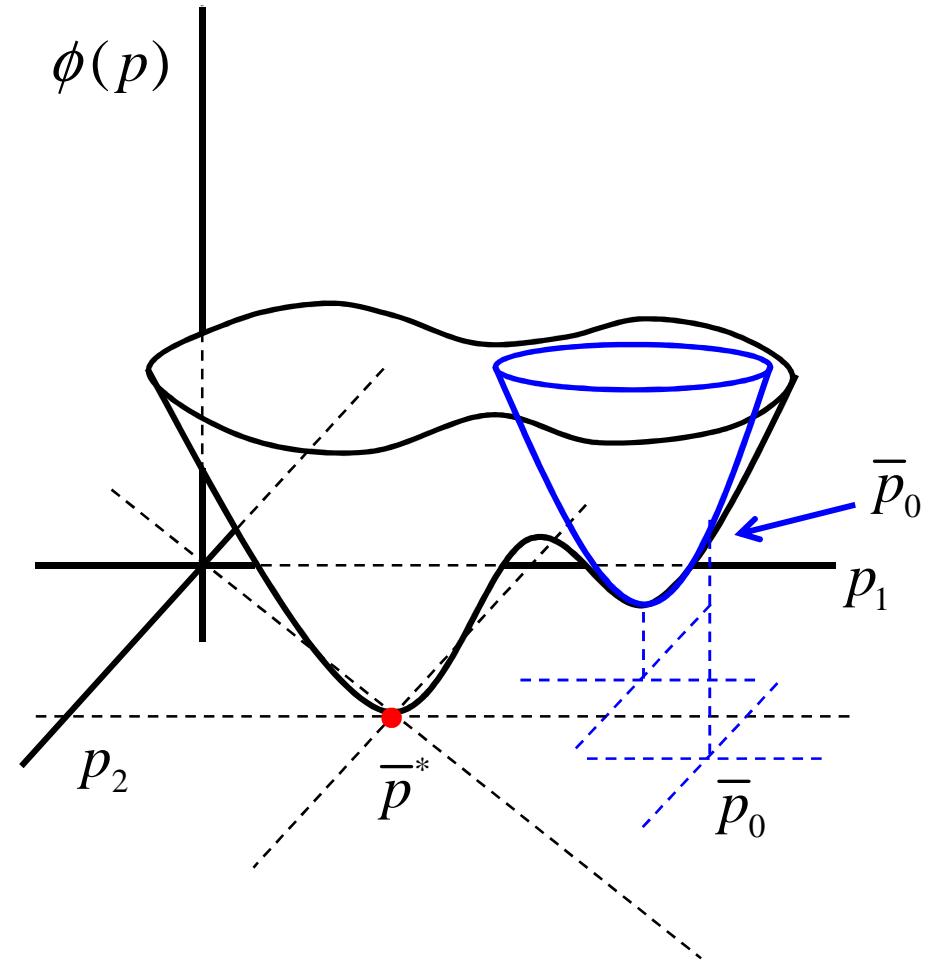
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Sistema linear



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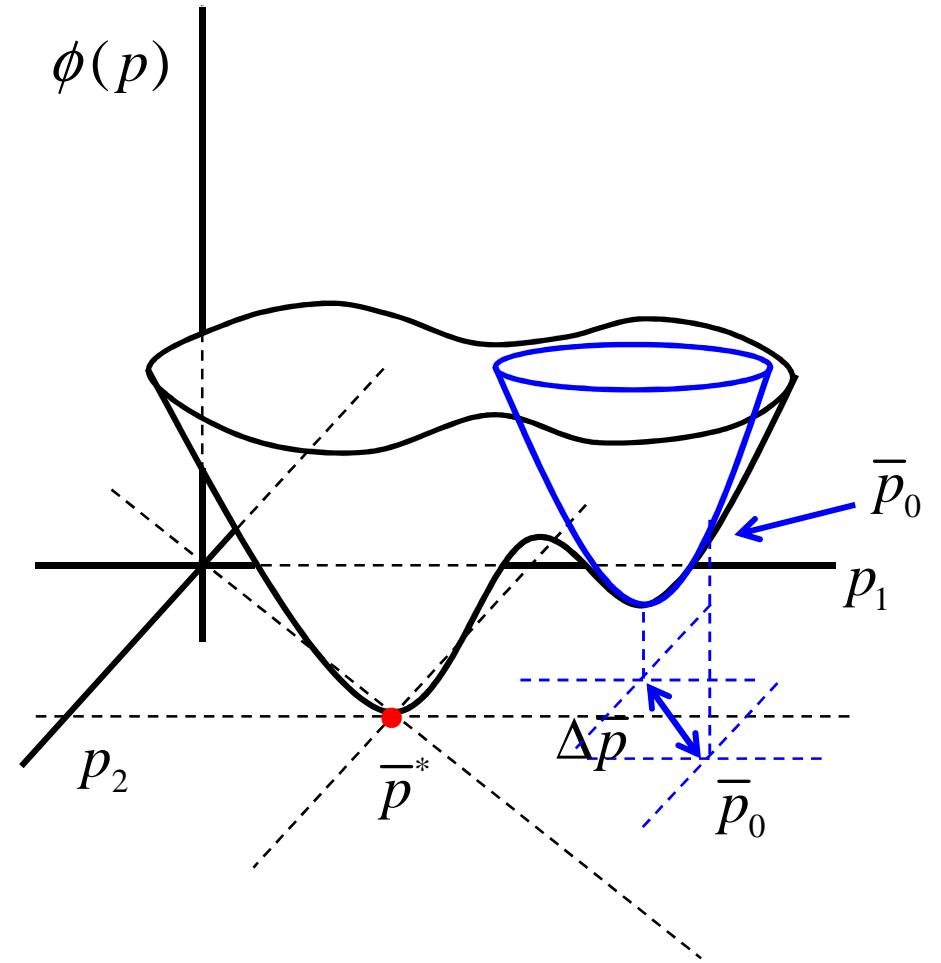
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Sistema linear



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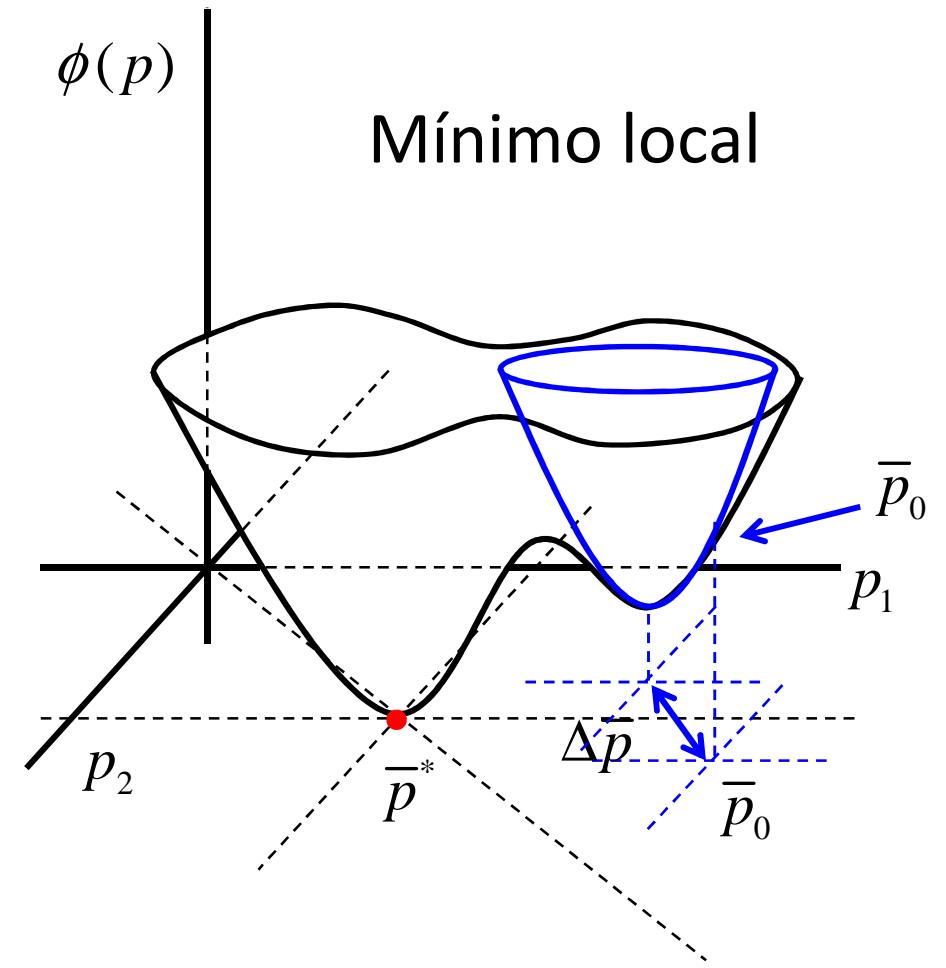
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Sistema linear



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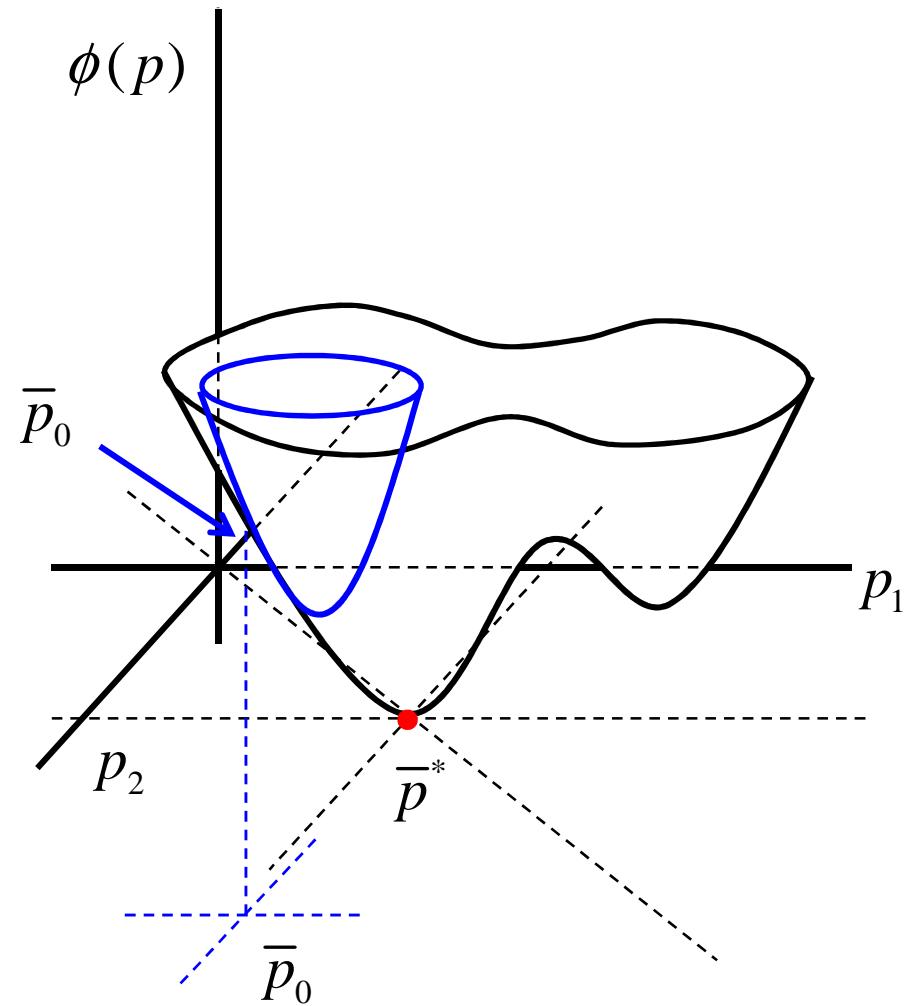
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Sistema linear



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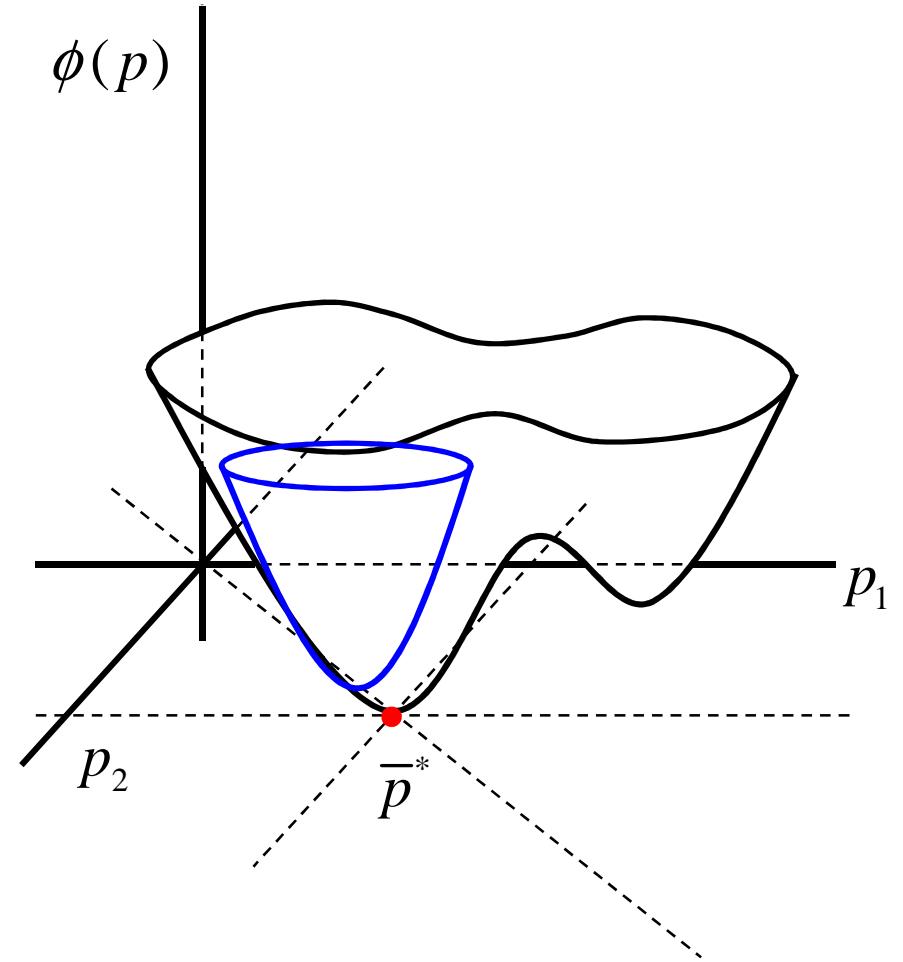
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Sistema linear



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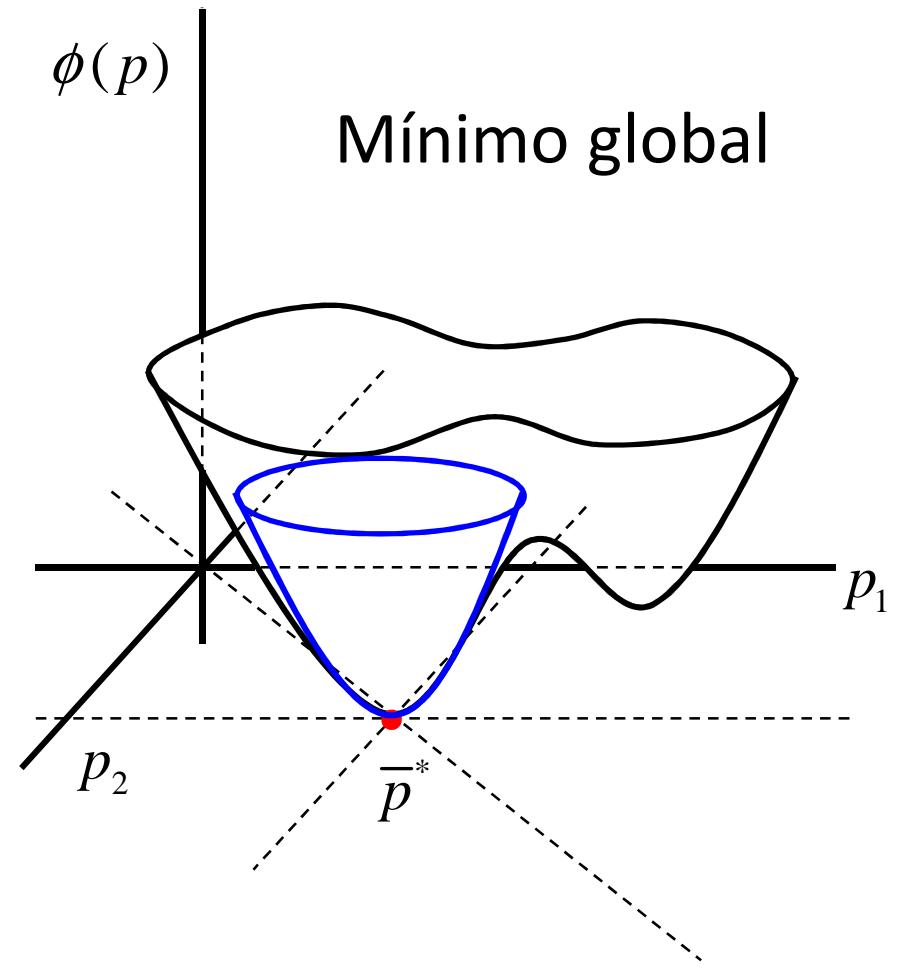
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Sistema linear



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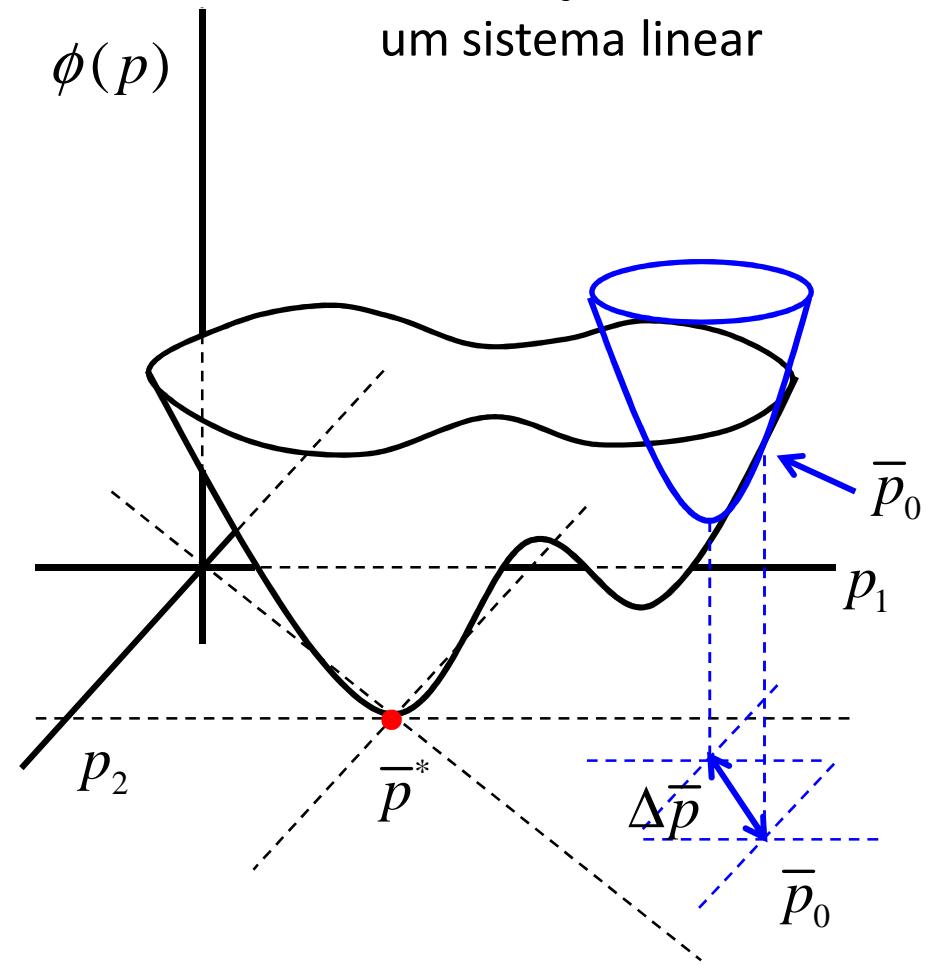
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Sistema linear

Em cada iteração é resolvido
um sistema linear



Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

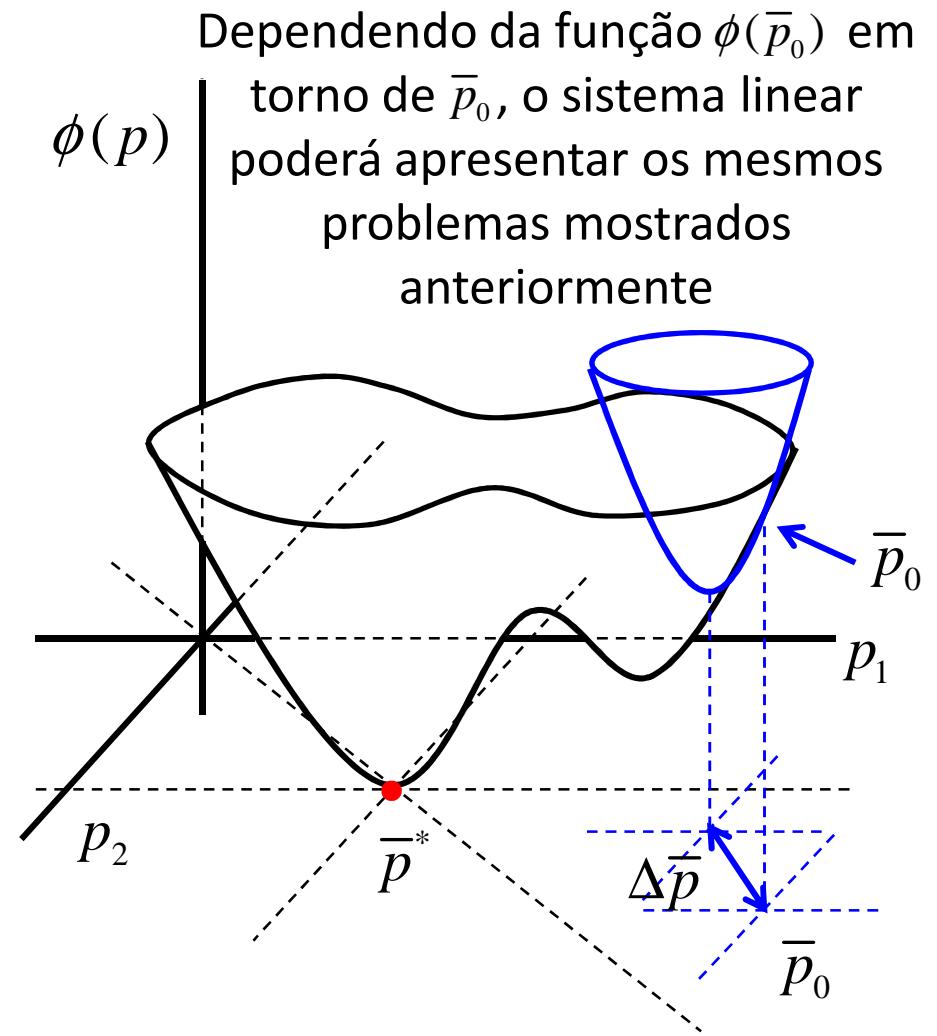
$$\bar{g}(\bar{p}) \neq B\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear



Regularização

Problema
linear

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

Problema
não-linear

$$\bar{g}(\bar{p}) = \bar{B}\bar{p} + \bar{b}$$

$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\begin{pmatrix} \bar{B}^T & \bar{B} \\ \bar{B} & \bar{B} \end{pmatrix} \bar{p}^* = \bar{B}^T [\bar{d} - \bar{b}]$$

Sistema linear

$$\begin{pmatrix} \bar{G}(\bar{p}_0)^T & \bar{G}(\bar{p}_0) \end{pmatrix} \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear

Regularização

Problema
linear

$$\bar{g}(\bar{p}) = \bar{B}\bar{p} + \bar{b}$$

$$\det \approx 0$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\det = 0$$

$$\begin{pmatrix} \bar{B}^T \\ \bar{B} \end{pmatrix} \bar{p}^* = \bar{B}^T [\bar{d} - \bar{b}]$$

Sistema linear

$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

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Sistema linear

Regularização

A regularização é um procedimento que objetiva aumentar o determinante das matrizes envolvidas na solução dos problemas inversos lineares e não-lineares

Regularização

Problema linear

$$\begin{pmatrix} \mathbf{B}^T \mathbf{B} \\ \mathbf{B} \end{pmatrix} \bar{\mathbf{p}}^* = \mathbf{B}^T [\bar{\mathbf{d}} - \bar{\mathbf{b}}]$$

Problema não-linear

$$\begin{pmatrix} \bar{\mathbf{G}}(\bar{\mathbf{p}}_0)^T \bar{\mathbf{G}}(\bar{\mathbf{p}}_0) \\ \bar{\mathbf{G}}(\bar{\mathbf{p}}_0) \end{pmatrix} \Delta \bar{\mathbf{p}} = \bar{\mathbf{G}}(\bar{\mathbf{p}}_0)^T [\bar{\mathbf{d}} - \bar{g}(\bar{\mathbf{p}}_0)]$$

Estas equações levam ao vetor de parâmetros que ajustam os dados, ou seja, estimam um vetor de parâmetros que produz os dados preditos mais próximos possíveis aos dados observados

Regularização

Problema linear

$$\begin{pmatrix} \mathbb{B}^T \mathbb{B} \\ \mathbb{B} \end{pmatrix} \bar{p}^* = \mathbb{B}^T [\bar{d} - \bar{b}]$$

Problema não-linear

$$\begin{pmatrix} \mathbb{\bar{G}}(\bar{p}_0)^T \mathbb{\bar{G}}(\bar{p}_0) \\ \mathbb{\bar{G}}(\bar{p}_0) \end{pmatrix} \Delta \bar{p} = \mathbb{\bar{G}}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

E se quiséssemos que o vetor de parâmetros ajustasse um vetor \bar{h} (diferente dos dados observados) dado pela relação linear abaixo?

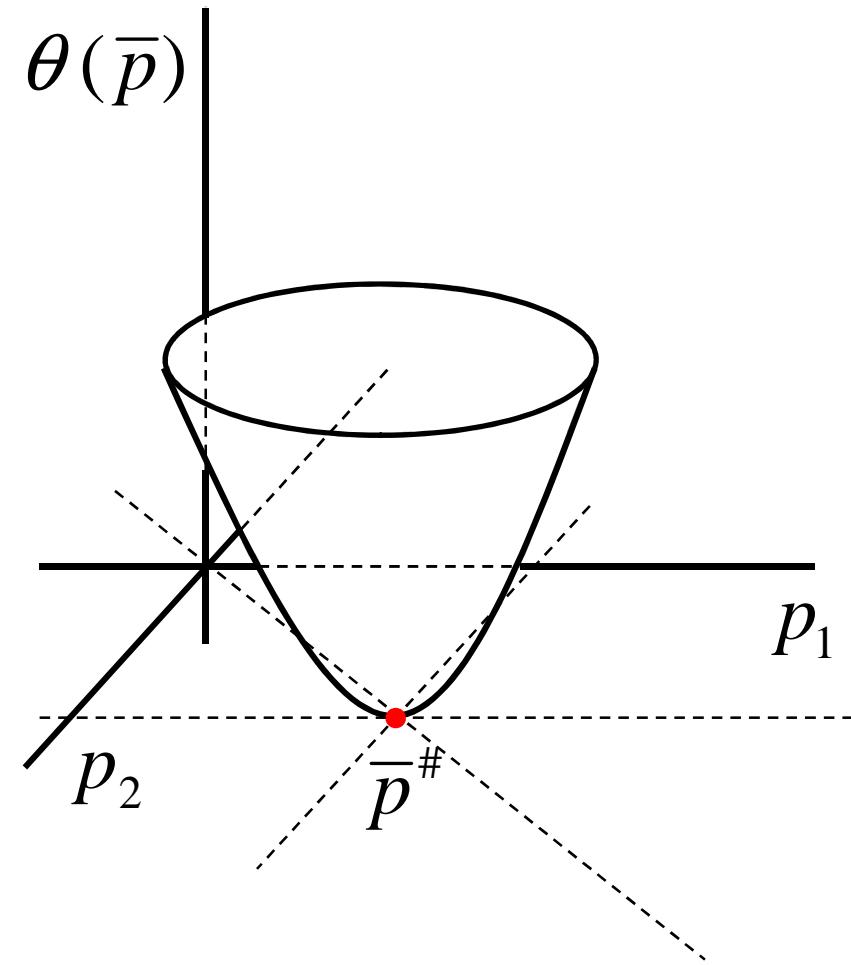
$$\mathbb{\bar{H}} \bar{p} = \bar{h}$$

Regularização

$$\theta(\bar{p}) = [\bar{h} - \bar{p}]^T [\bar{h} - \bar{p}]$$

$$\bar{\nabla} \theta(\bar{p}^\#) = \bar{0}_{M \times 1}$$

$$\begin{pmatrix} \bar{H}^T & \bar{H} \end{pmatrix} \bar{p}^\# = \bar{H}^T \bar{h}$$



Regularização

Problema linear

$$\begin{pmatrix} \mathbf{B}^T \mathbf{B} \\ \mathbf{B} \end{pmatrix} \bar{\mathbf{p}}^* = \mathbf{B}^T [\bar{\mathbf{d}} - \bar{\mathbf{b}}]$$

Problema não-linear

$$\begin{pmatrix} \bar{\mathbf{G}}(\bar{\mathbf{p}}_0)^T \bar{\mathbf{G}}(\bar{\mathbf{p}}_0) \\ \Delta \bar{\mathbf{p}} \end{pmatrix} \Delta \bar{\mathbf{p}} = \bar{\mathbf{G}}(\bar{\mathbf{p}}_0)^T [\bar{\mathbf{d}} - \bar{g}(\bar{\mathbf{p}}_0)]$$

E se agora quiséssemos que o vetor de parâmetros ajustasse o vetor \bar{h} e também o vetor de dados observados ao mesmo tempo?

$$\bar{\mathbf{H}} \bar{\mathbf{p}} = \bar{h}$$

Regularização

$$\overline{\overline{B}} \bar{p} + \bar{b} \approx \bar{d}$$

$$\overline{\overline{H}} \bar{p} \approx \bar{h}$$

Regularização

$$\bar{\bar{B}}\bar{p} + \bar{b} \approx \bar{d}$$

$$\bar{\bar{H}}\bar{p} \approx \bar{h}$$

$$\underbrace{\begin{bmatrix} \bar{\bar{B}} \\ \bar{\bar{H}} \end{bmatrix}}_{\bar{A}} \bar{p} \approx \underbrace{\begin{bmatrix} \bar{d} - \bar{b} \\ \bar{h} \end{bmatrix}}_{\bar{w}}$$

Regularização

$$\bar{\bar{B}}\bar{p} + \bar{b} \approx \bar{d}$$

$$\bar{\bar{H}}\bar{p} \approx \bar{h}$$

$$\underbrace{\begin{bmatrix} \bar{\bar{B}} \\ \bar{\bar{H}} \end{bmatrix}}_{\bar{A}} \bar{p} \approx \underbrace{\begin{bmatrix} \bar{d} - \bar{b} \\ \bar{h} \end{bmatrix}}_{\bar{w}}$$

$$\Omega(\bar{p}) = [\bar{w} - \bar{\bar{A}}\bar{p}]^T [\bar{w} - \bar{\bar{A}}\bar{p}]$$

Regularização

$$\overline{\overline{B}}\overline{p} + \overline{b} \approx \overline{d}$$

$$\overline{\overline{H}}\overline{p} \approx \overline{h}$$

$$\underbrace{\begin{bmatrix} \overline{\overline{B}} \\ \overline{\overline{H}} \end{bmatrix}}_{\overline{\mathbf{A}}} \overline{p} \approx \underbrace{\begin{bmatrix} \overline{d} - \overline{b} \\ \overline{h} \end{bmatrix}}_{\overline{w}}$$

$$\Omega(\overline{p}) = \phi(\overline{p}) + \theta(\overline{p})$$

$$\Omega(\overline{p}) = \left[\overline{w} - \overline{\overline{\mathbf{A}}} \overline{p} \right]^T \left[\overline{w} - \overline{\overline{\mathbf{A}}} \overline{p} \right]$$

$$\phi(\overline{p}) = \left[\overline{d} - \overline{g}(\overline{p}) \right]^T \left[\overline{d} - \overline{g}(\overline{p}) \right]$$

$$\theta(\overline{p}) = \left[\overline{h} - \overline{\overline{H}} \overline{p} \right]^T \left[\overline{h} - \overline{\overline{H}} \overline{p} \right]$$

Regularização

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left[\bar{\nabla} \phi(\bar{p}_0) + \mu \bar{\nabla} \theta(\bar{p}_0) \right] \Delta \bar{p} = - \left[\bar{\nabla} \phi(\bar{p}_0) + \mu \bar{\nabla} \theta(\bar{p}_0) \right]$$

Regularização

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\underbrace{\left[\bar{\nabla} \phi(\bar{p}_0) + \mu \bar{\nabla} \theta(\bar{p}_0) \right]}_{\text{Espera-se que essa matriz resultante tenha } \det \neq 0} \Delta \bar{p} = - \left[\bar{\nabla} \phi(\bar{p}_0) + \mu \bar{\nabla} \theta(\bar{p}_0) \right]$$

Espera-se que
essa matriz
resultante tenha
 $\det \neq 0$