Regularização

Estrutura

- Problemas Inversos
  - Introdução
- Sistemas lineares
  - Determinante $\neq 0$
  - Determinante $= 0$
  - Determinante ≈ 0
- Problemas lineares
- Problemas não-lineares
- Regularização

Problemas Inversos

(Introdução)

$$\bar{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}_{M \times 1}$$

parâmetros

$$\bar{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}_{N \times 1}$$

dados observados

$$\bar{g}(\bar{p}) = \begin{bmatrix} g_1(\bar{p}) \\ \vdots \\ g_N(\bar{p}) \end{bmatrix}_{N \times 1}$$

dados preditos

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dados
preditos

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

norma L2
(função escalar)

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$$\bar{\nabla}\phi(\bar{p}^*) = \bar{0}_{M\times 1}$$

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Problema linear

Problema não-linear

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norma L2
(função escalar)

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M\times 1}$$

Problema linear

$$\bar{g}(\bar{p}) = \bar{\bar{B}}\bar{p} + \bar{b}$$

$$\bar{p}^* = \left(\bar{\bar{B}}^T \bar{\bar{B}}\right)^{-1} \bar{\bar{B}}^T [\bar{d} - \bar{b}]$$

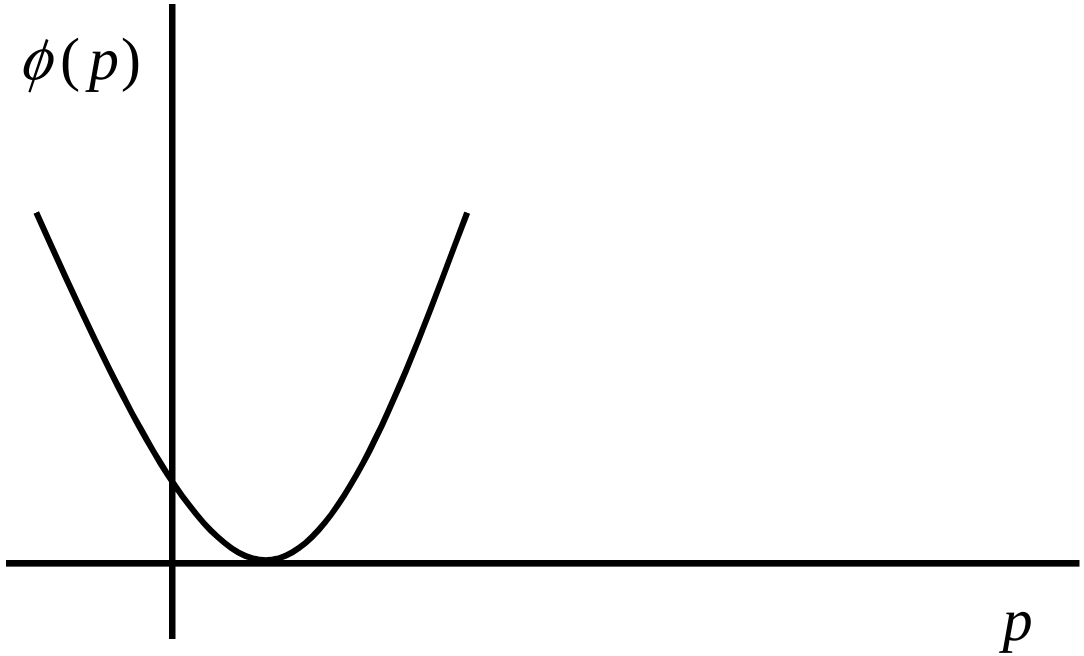
Problema não-linear

$$\bar{g}(\bar{p}) \neq \bar{\bar{B}}\bar{p} + \bar{b}$$

$$\Delta\bar{p} = \left(\bar{\bar{G}}(\bar{p}_0)^T \bar{\bar{G}}(\bar{p}_0)\right)^{-1} \bar{\bar{G}}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Problemas Inversos
(Introdução)

Exemplo
linear 1D

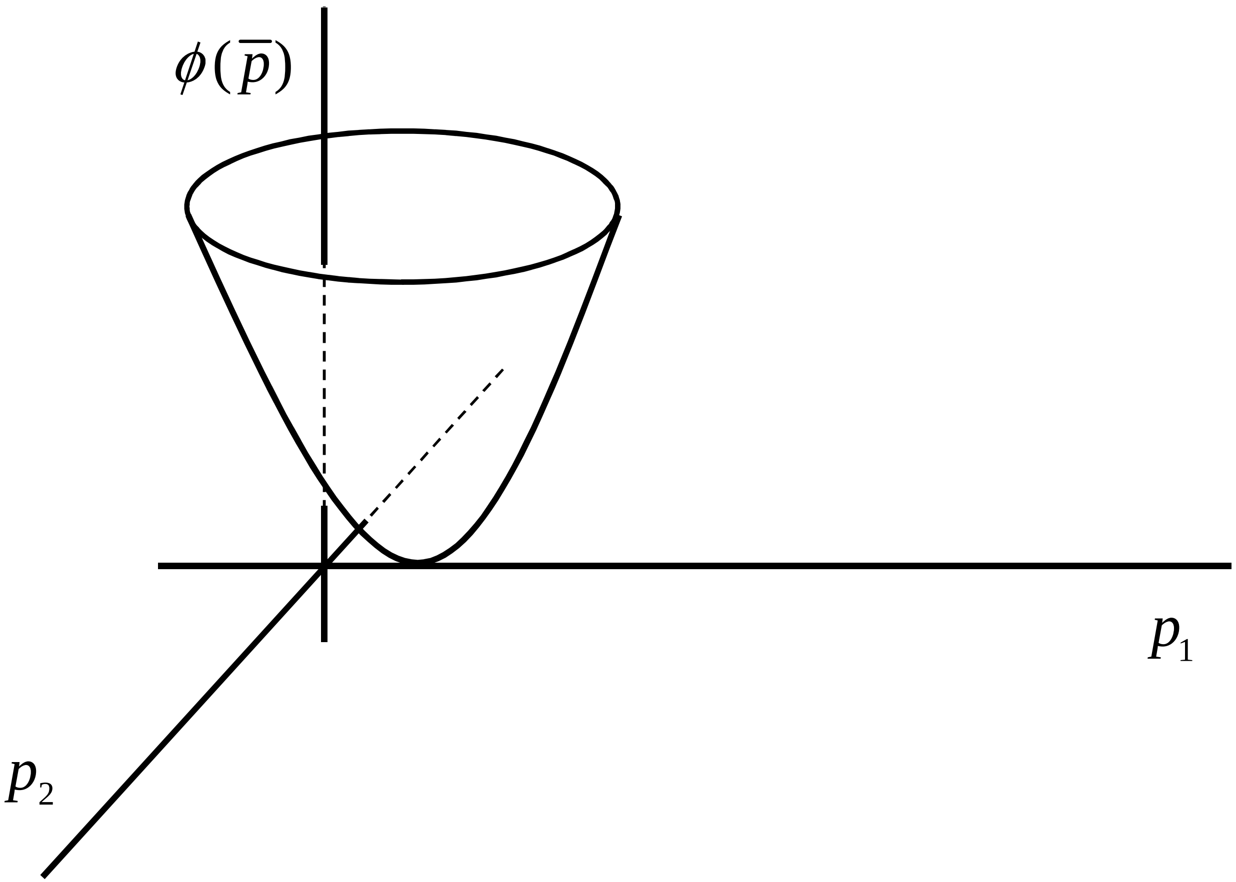


$$\phi(p) = [\bar{d} - \bar{g}(p)]^T [\bar{d} - \bar{g}(p)]$$

Problemas Inversos

(Introdução)

Exemplo
linear 2D

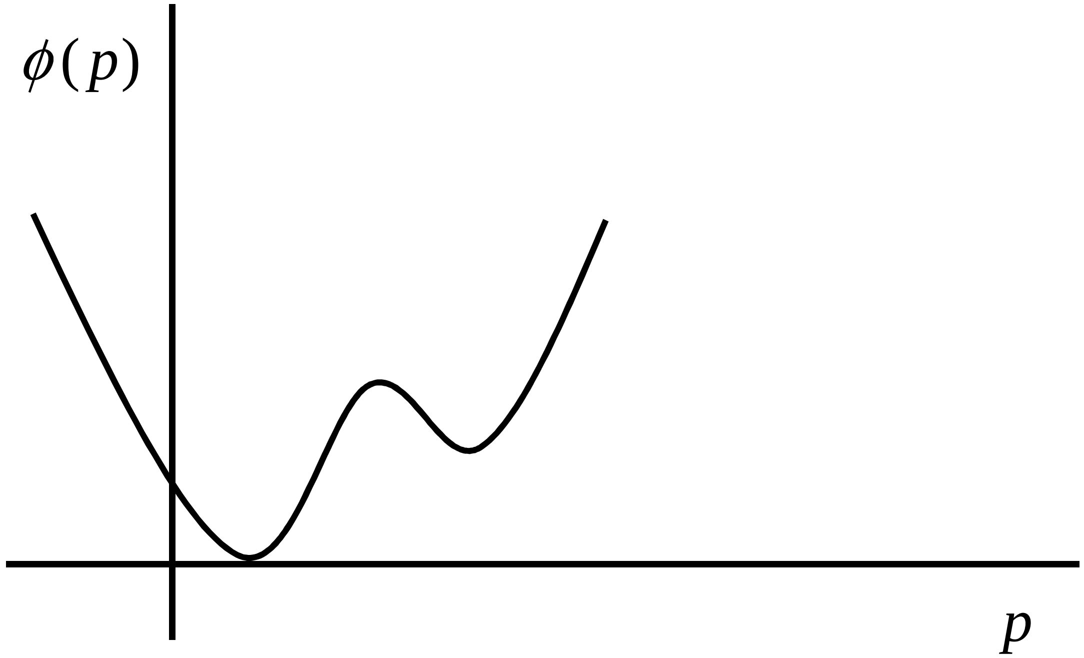


$$\phi(\overline{p}) = [\overline{d} - \overline{g}(\overline{p})]^T [\overline{d} - \overline{g}(\overline{p})]$$

Problemas Inversos

(Introdução)

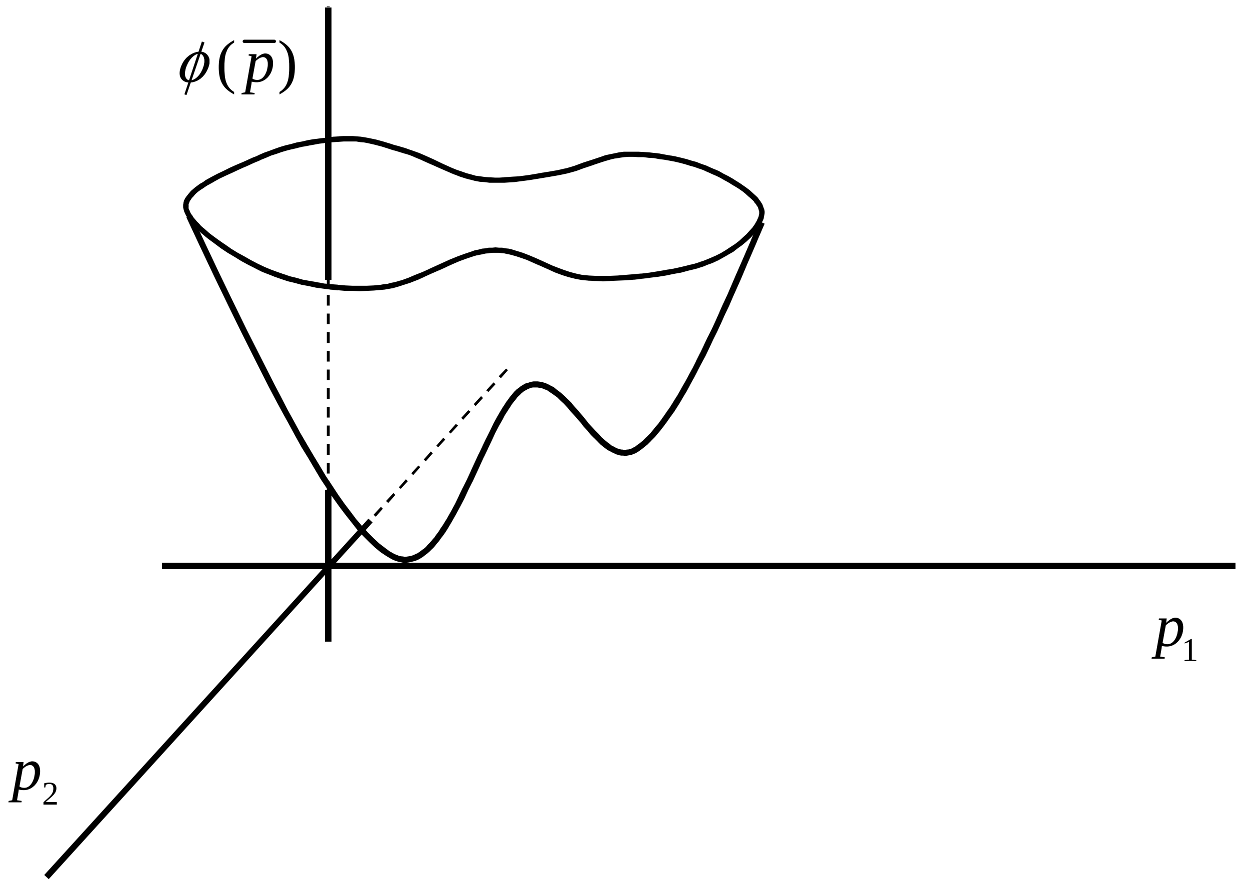
Exemplo
não-linear 1D



$$\phi(p) = [\bar{d} - \bar{g}(p)]^T [\bar{d} - \bar{g}(p)]$$

Problemas Inversos
(Introdução)

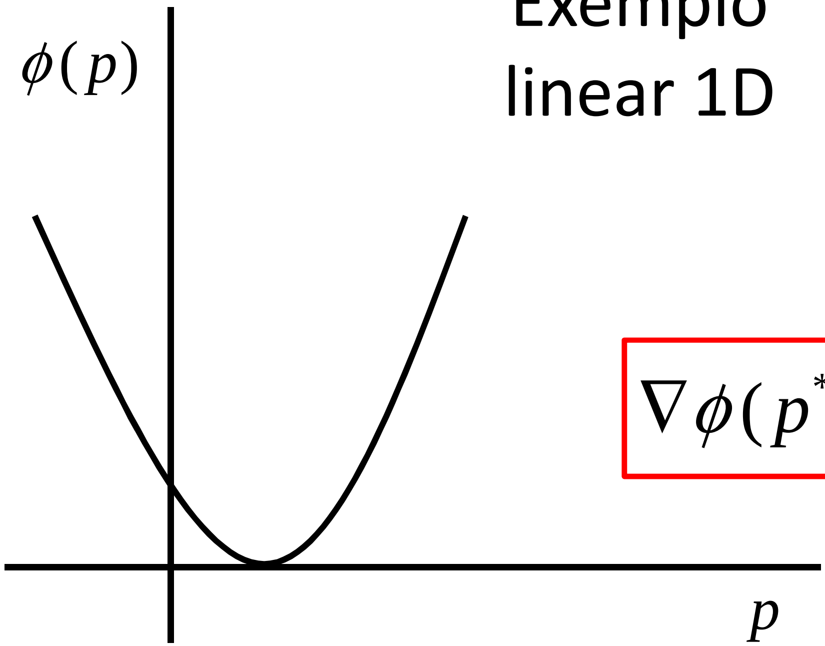
Exemplo
não-linear 2D



$$\phi(\overline{p}) = [\overline{d} - \overline{g}(\overline{p})]^T [\overline{d} - \overline{g}(\overline{p})]$$

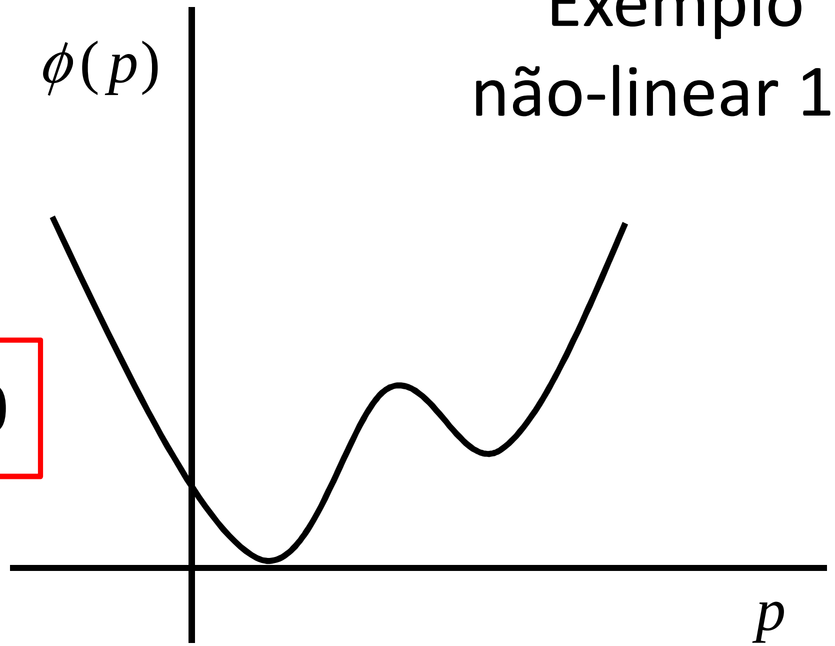
Problemas Inversos
(Introdução)

Exemplo
linear 1D



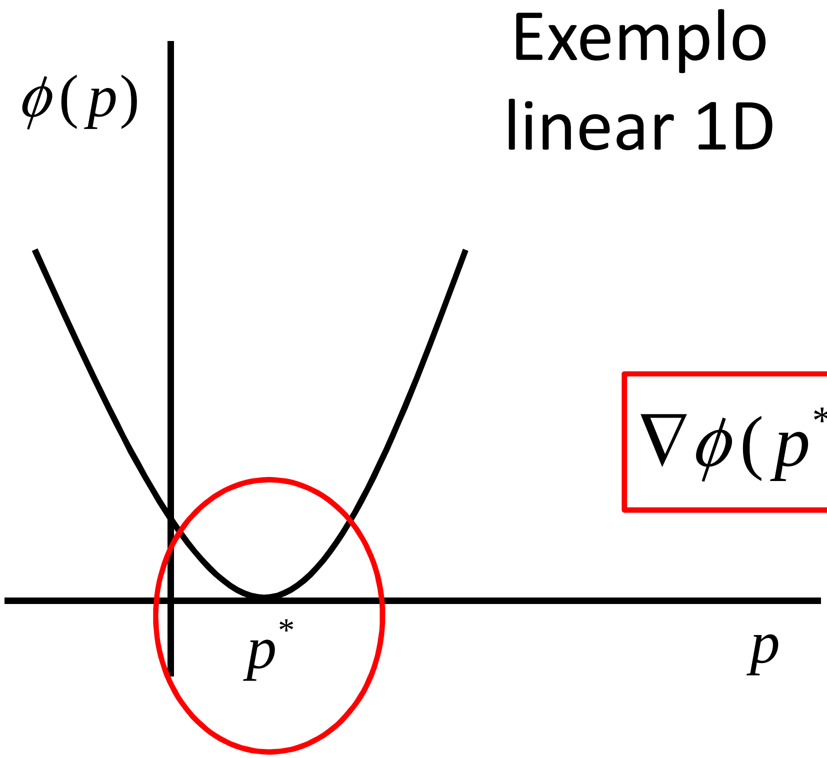
$$\boxed{\nabla \phi(p^*) = 0}$$

Exemplo
não-linear 1D

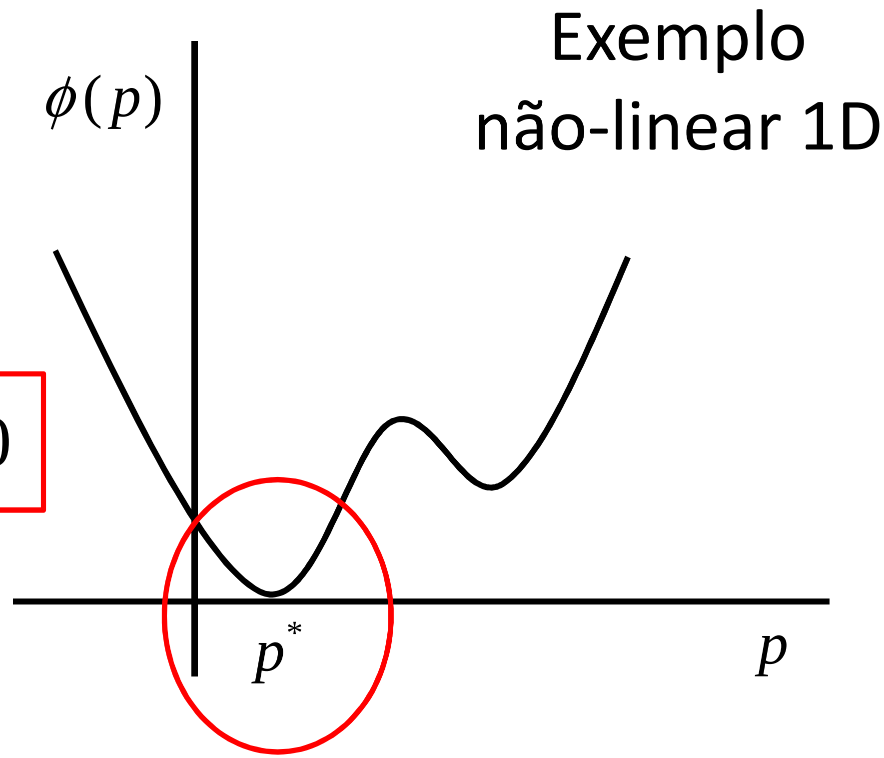


Problemas Inversos

(Introdução)



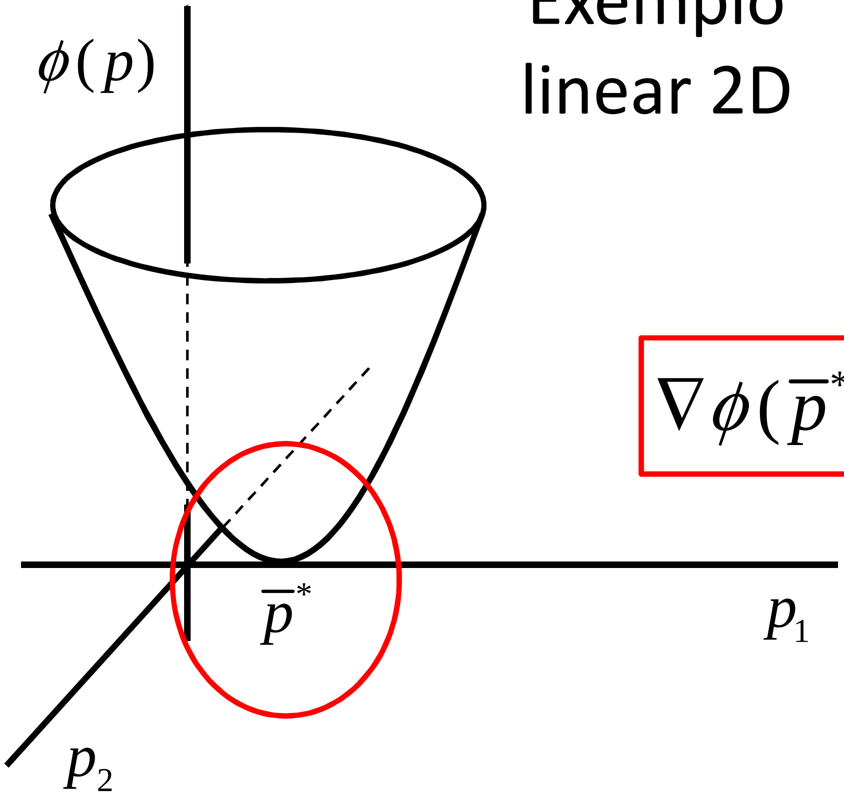
$$\nabla\phi(p^{*}) = 0$$



Problemas Inversos

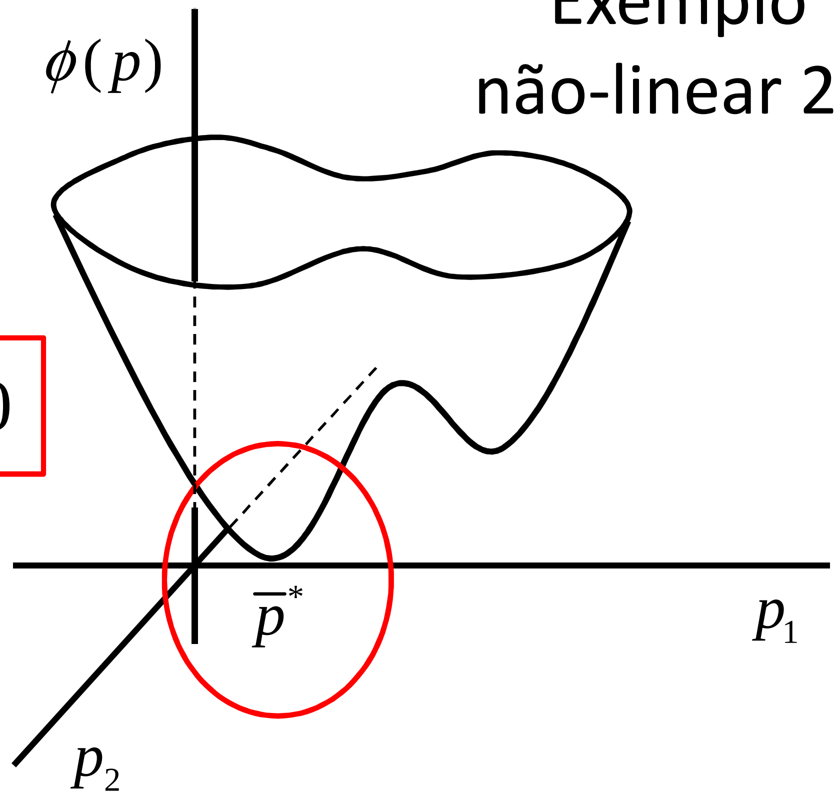
(Introdução)

Exemplo linear 2D



$$\boxed{\nabla \phi(\overline{p}^*) = 0}$$

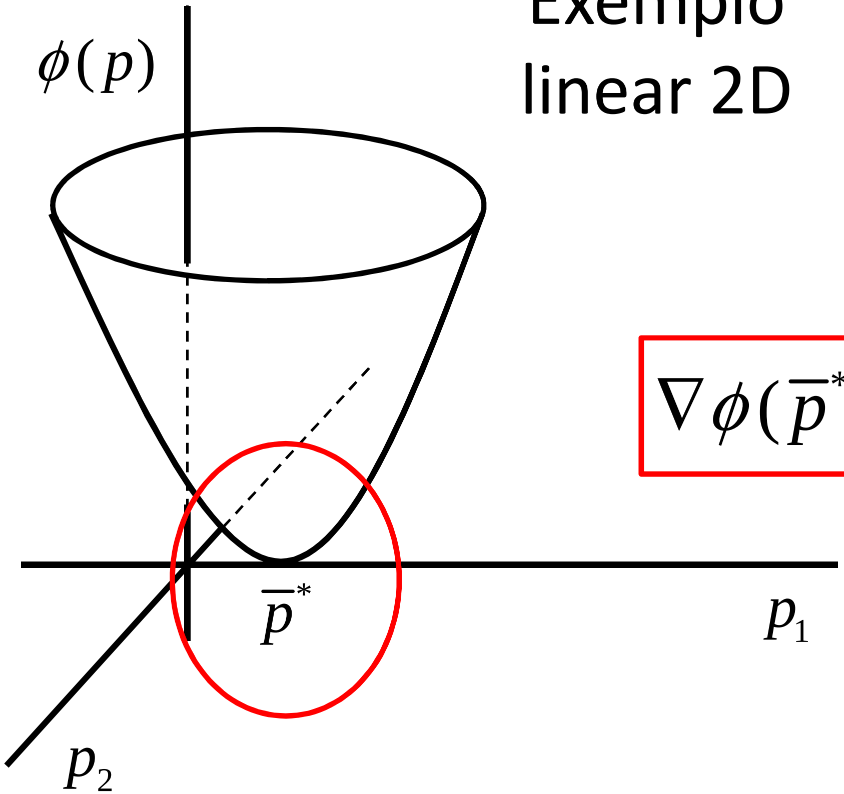
Exemplo não-linear 2D



Problemas Inversos

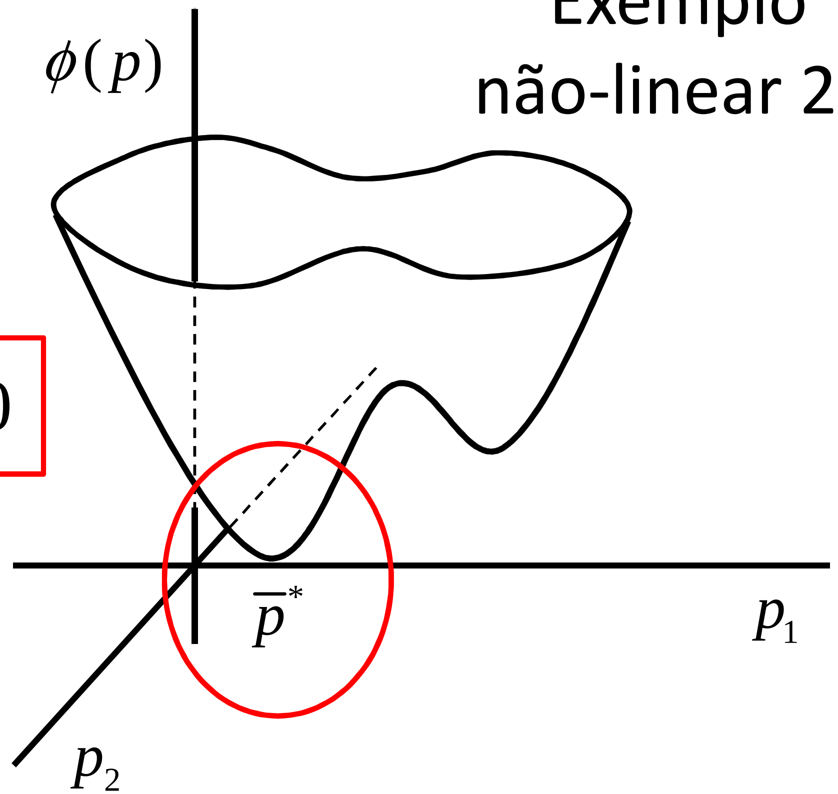
(Introdução)

Exemplo linear 2D



$$\boxed{\nabla\phi(\bar{p}^*) = 0}$$

Exemplo não-linear 2D



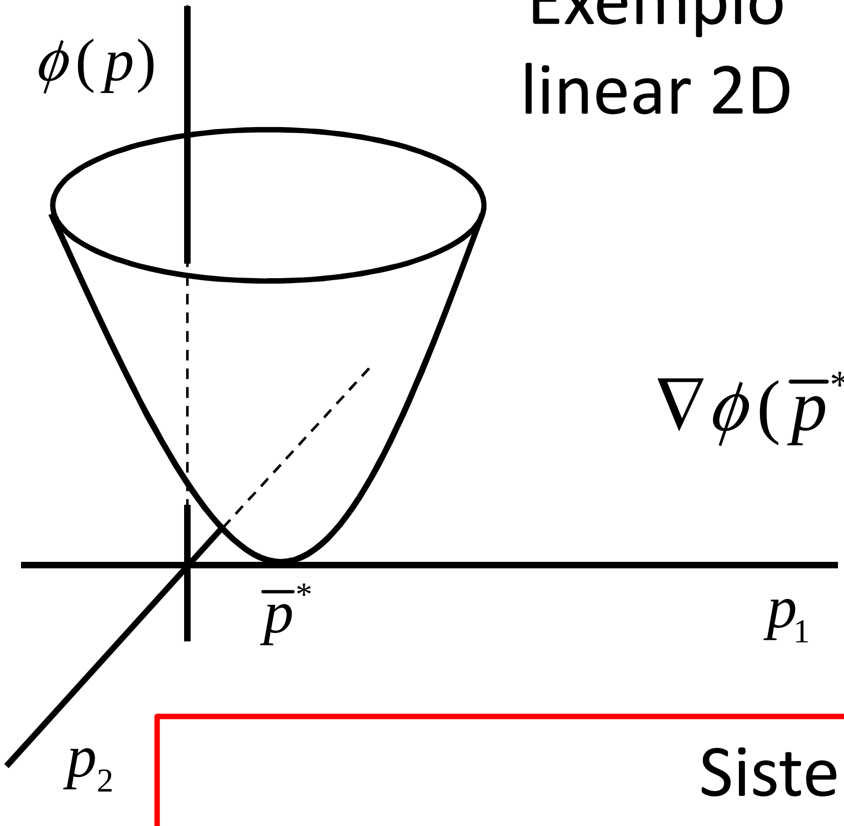
$$\bar{p}^* = \left(\bar{\bar{B}}^T \bar{\bar{B}}\right)^{-1} \bar{\bar{B}}^T [\bar{d} - \bar{b}]$$

$$\Delta\bar{p} = \left(\bar{\bar{G}}(\bar{p}_0)^T \bar{\bar{G}}(\bar{p}_0)\right)^{-1} \bar{\bar{G}}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Problemas Inversos

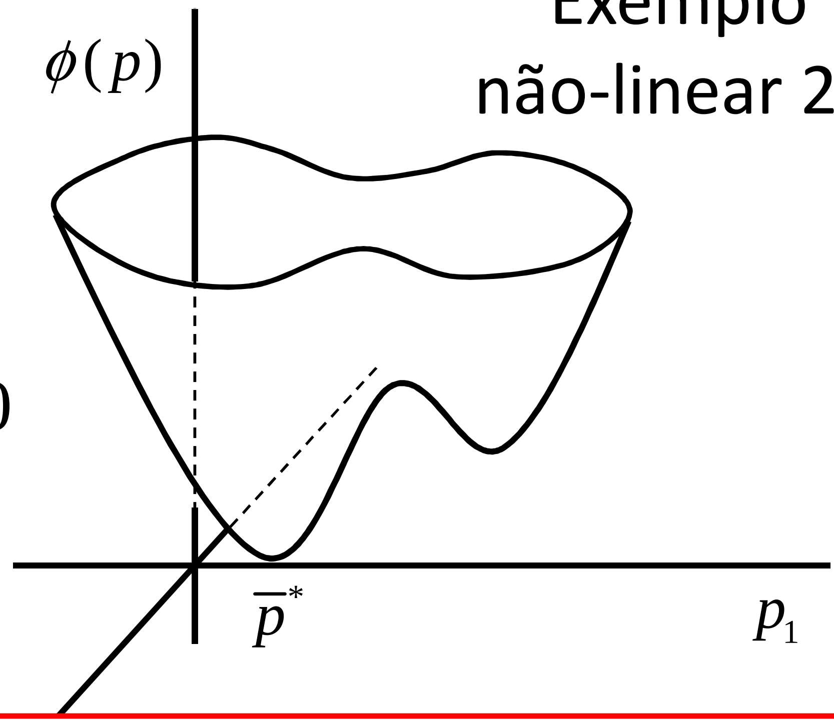
(Introdução)

Exemplo
linear 2D



$$\nabla\phi(\overline{p}^*) = 0$$

Exemplo
não-linear 2D



Sistemas lineares

$$\overline{p}^* = \left(\overline{\overline{B}}^T \overline{\overline{B}}\right)^{-1} \overline{\overline{B}}^T [\overline{d} - \overline{b}]$$

$$\Delta\overline{p} = \left(\overline{\overline{G}}(\overline{p}_0)^T \overline{\overline{G}}(\overline{p}_0)\right)^{-1} \overline{\overline{G}}(\overline{p}_0)^T [\overline{d} - \overline{g}(\overline{p}_0)]$$

Sistemas lineares

$$\bar{b} = \left(\bar{\bar{\mathrm{B}}}^T \bar{\bar{\mathrm{B}}} \right)^{-1} \bar{\bar{\mathrm{B}}}^T \bar{r}$$

Sistemas lineares

$$\bar{b} = \left(\bar{\bar{\mathrm{B}}}^{T} \bar{\bar{\mathrm{B}}} \right)^{-1} \bar{\bar{\mathrm{B}}}^{T} \bar{r}$$

Sistemas lineares

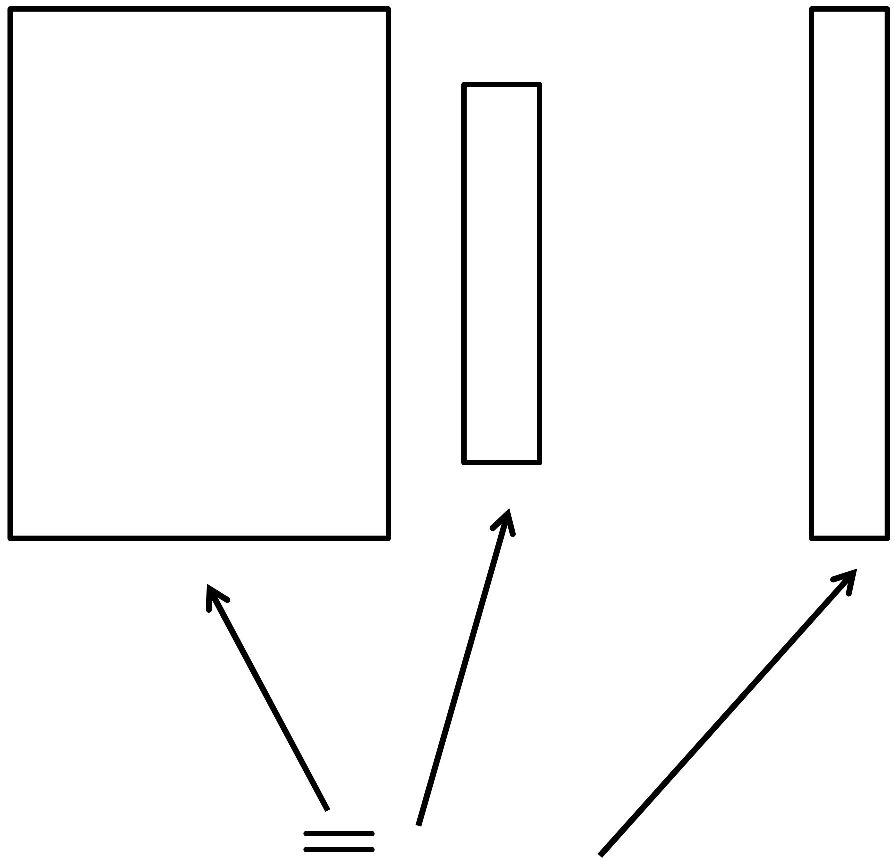
$$\left(\overline{\overline{\mathrm{B}}}^{T}\,\overline{\overline{\mathrm{B}}}\right)\bar{b}=\overline{\overline{\mathrm{B}}}^{T}\,\bar{r}$$

Sistemas lineares

$$\underbrace{\left(\overline{\overline{\mathrm{B}}}^{T}\,\overline{\overline{\mathrm{B}}}\right)}\overline{b} = \underbrace{\overline{\overline{\mathrm{B}}}^{T}\,\overline{r}}$$

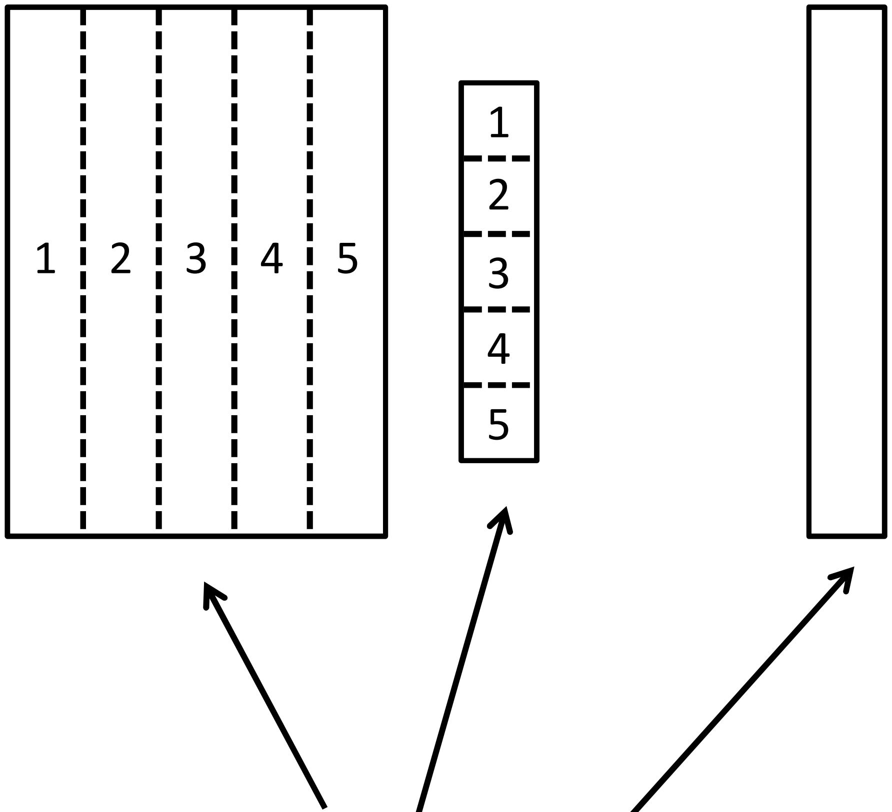
$$\overline{\overline{\mathrm{A}}}\,\overline{b} = \overline{w}$$

Sistemas lineares



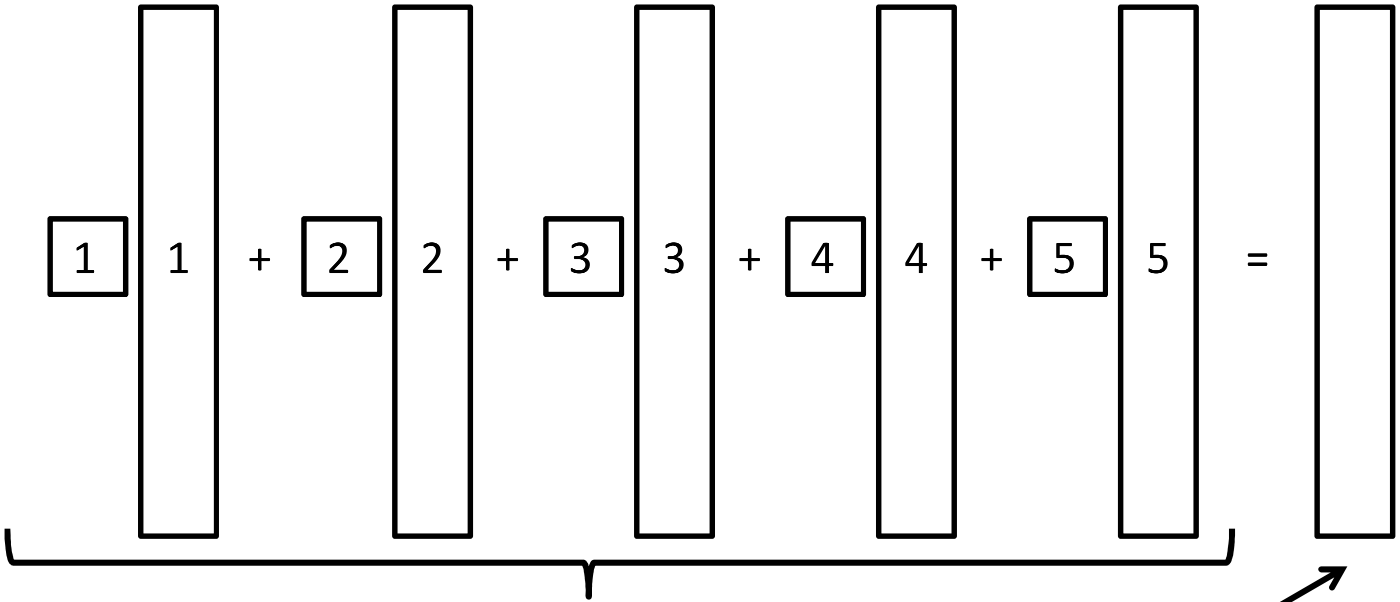
$$\overline{\overline{\mathrm{A}}}\,\overline{b} = \overline{w}$$

Sistemas lineares



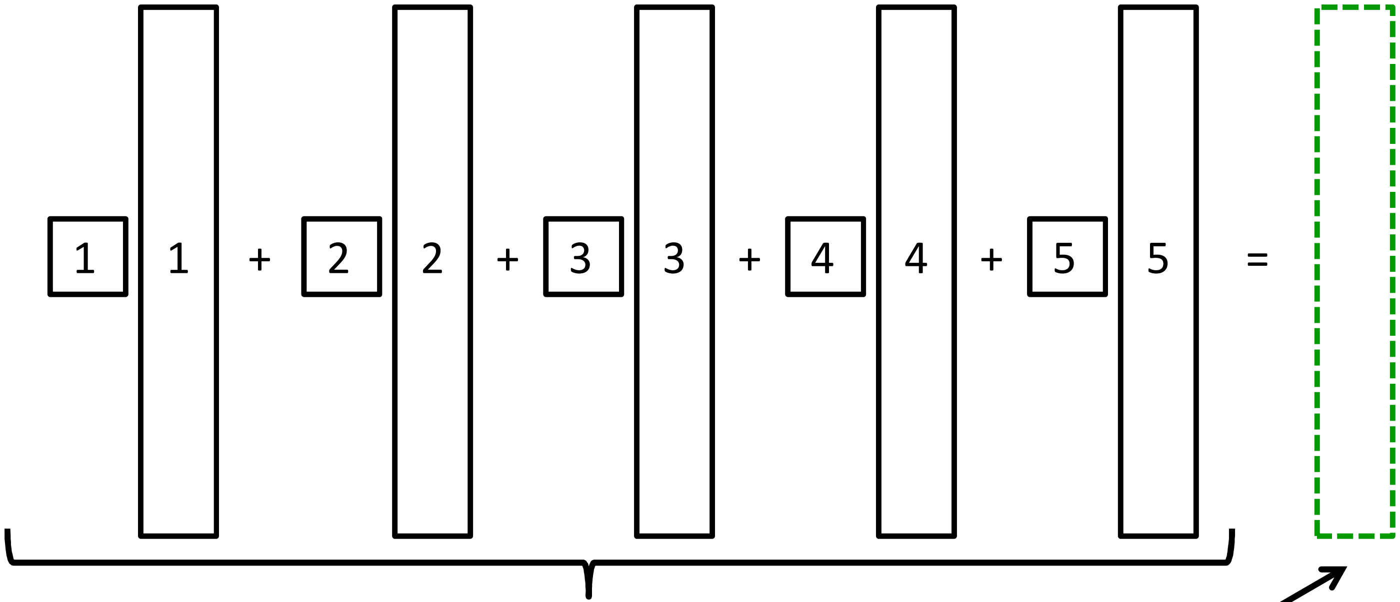
$$\overline{\overline{\mathrm{A}}}\,\overline{b} = \overline{w}$$

Sistemas lineares



$$\boxed{\overline{\overline{A}}\,\overline{b}} = \overline{w}$$

Sistemas lineares

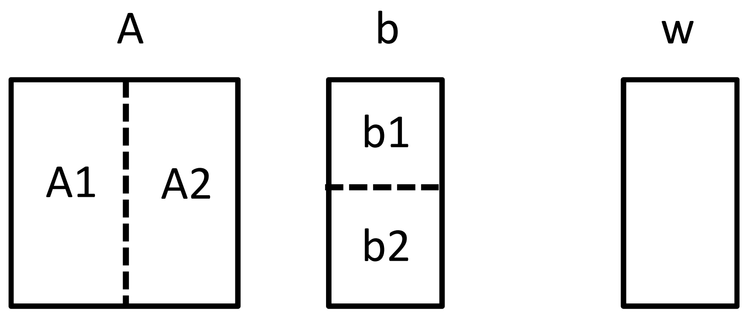


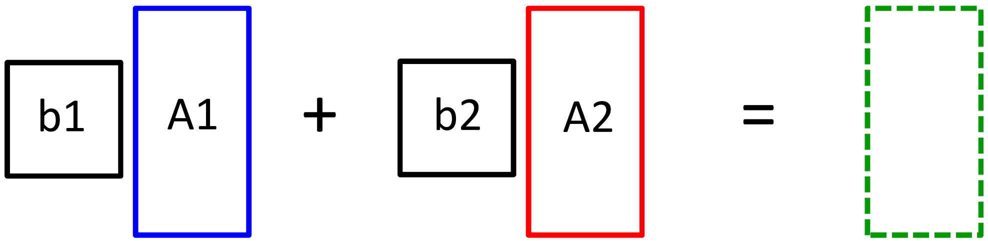
O vetor do lado direito é uma *combinação linear* dos vetores do lado esquerdo

$$\boxed{\overline{\overline{A}}\,\overline{b}} = \overline{w}$$

Sistemas lineares

Exemplo 2D

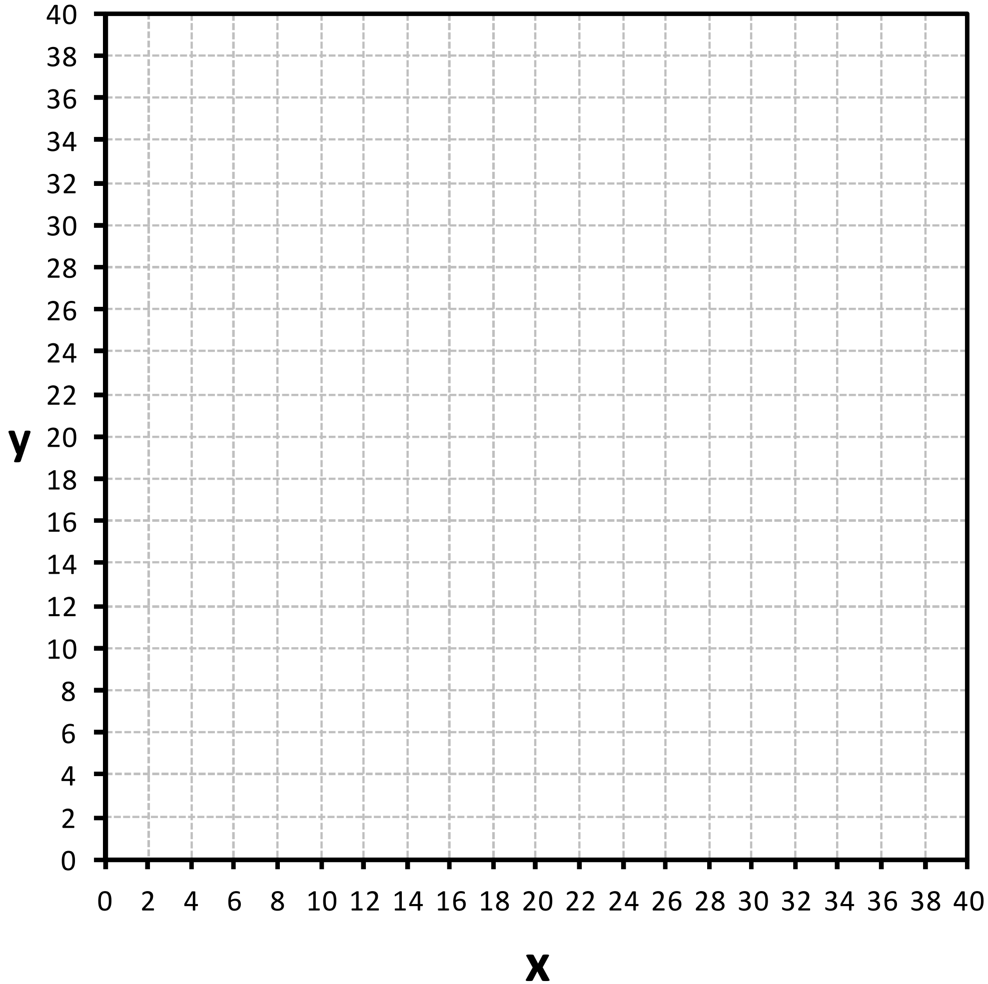




Sistemas lineares

Exemplo 2D

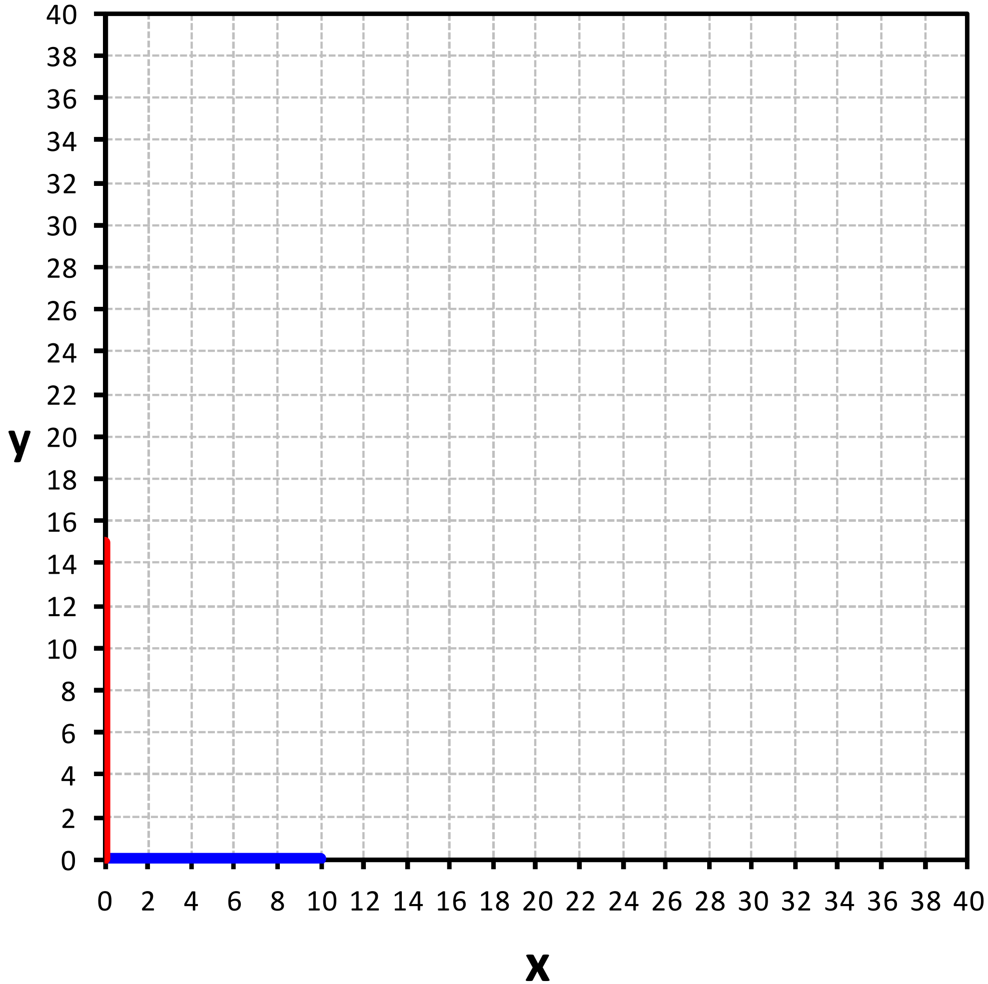
b1 A1 + b2 A2 =



Sistemas lineares

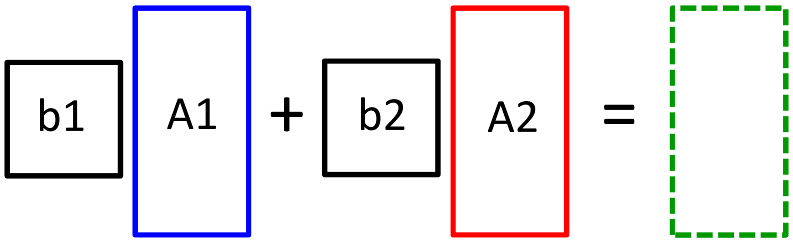
Exemplo 2D

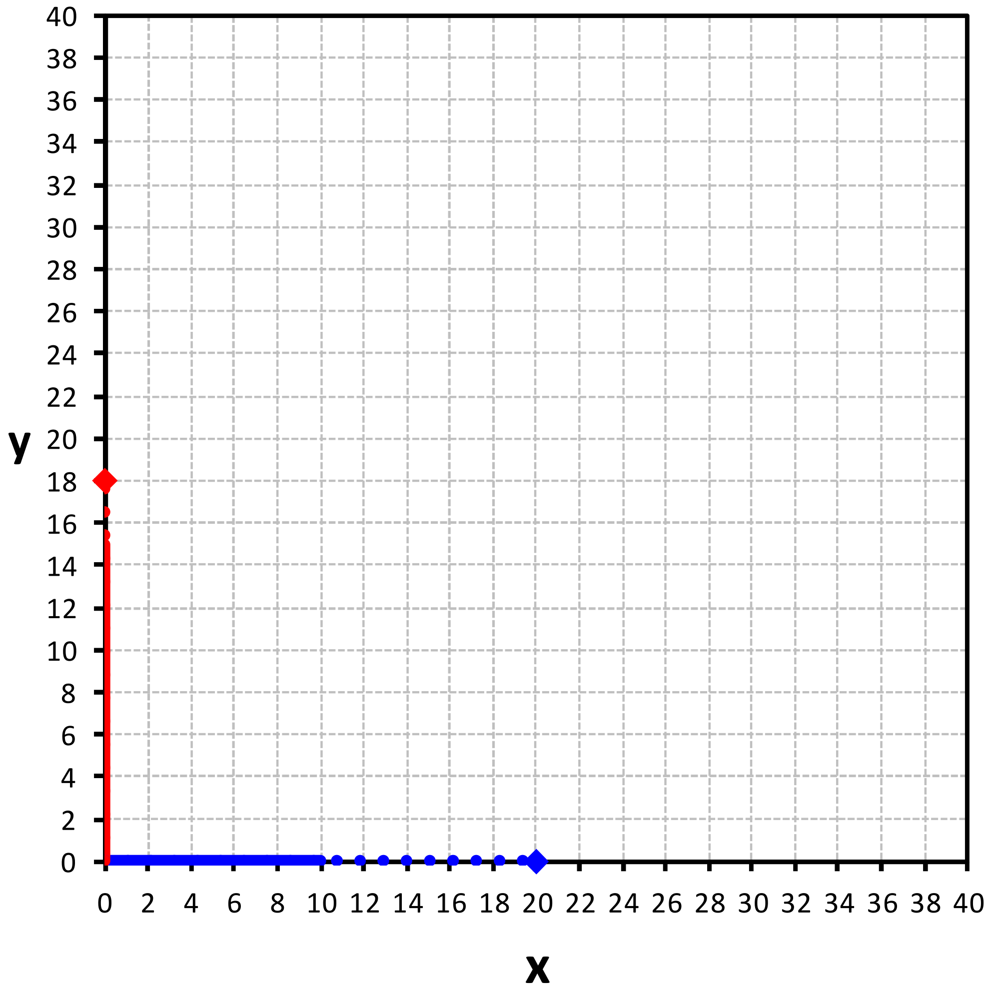
b1 A1 + b2 A2 =



Sistemas lineares

Exemplo 2D





Sistemas lineares

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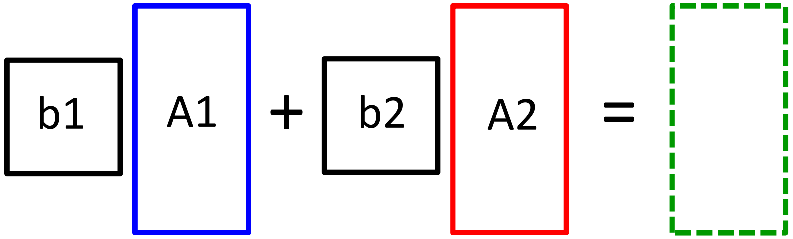

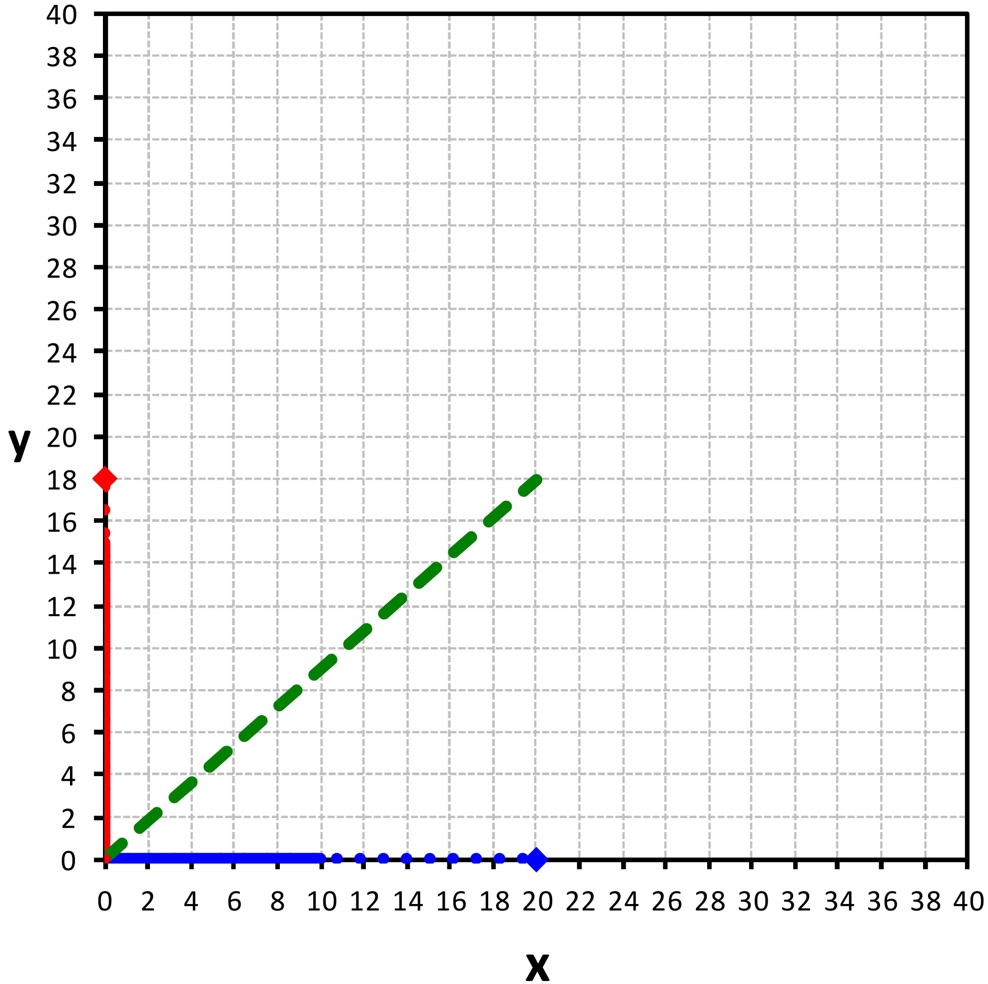

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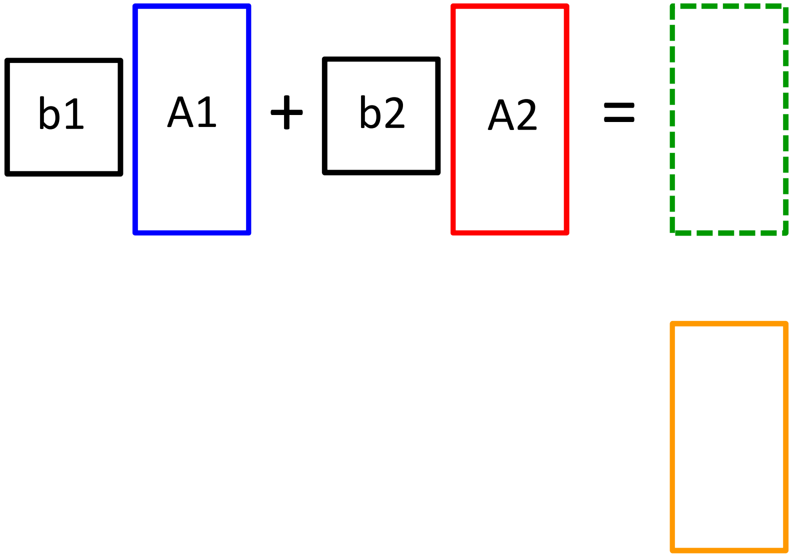

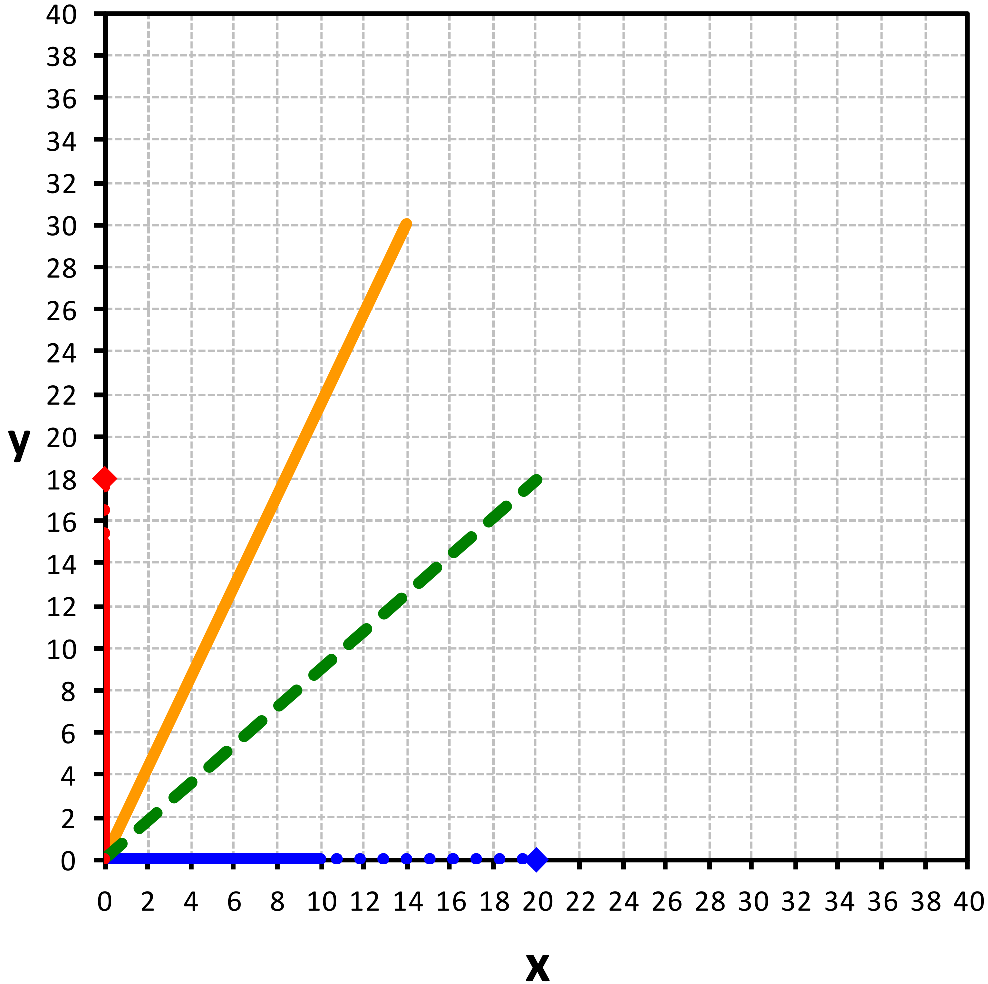

Sistemas lineares

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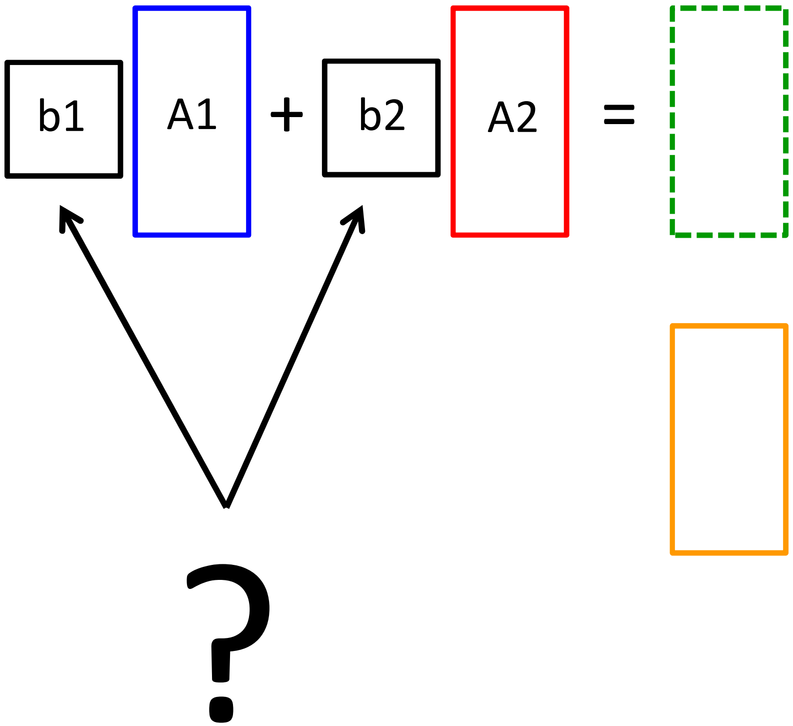

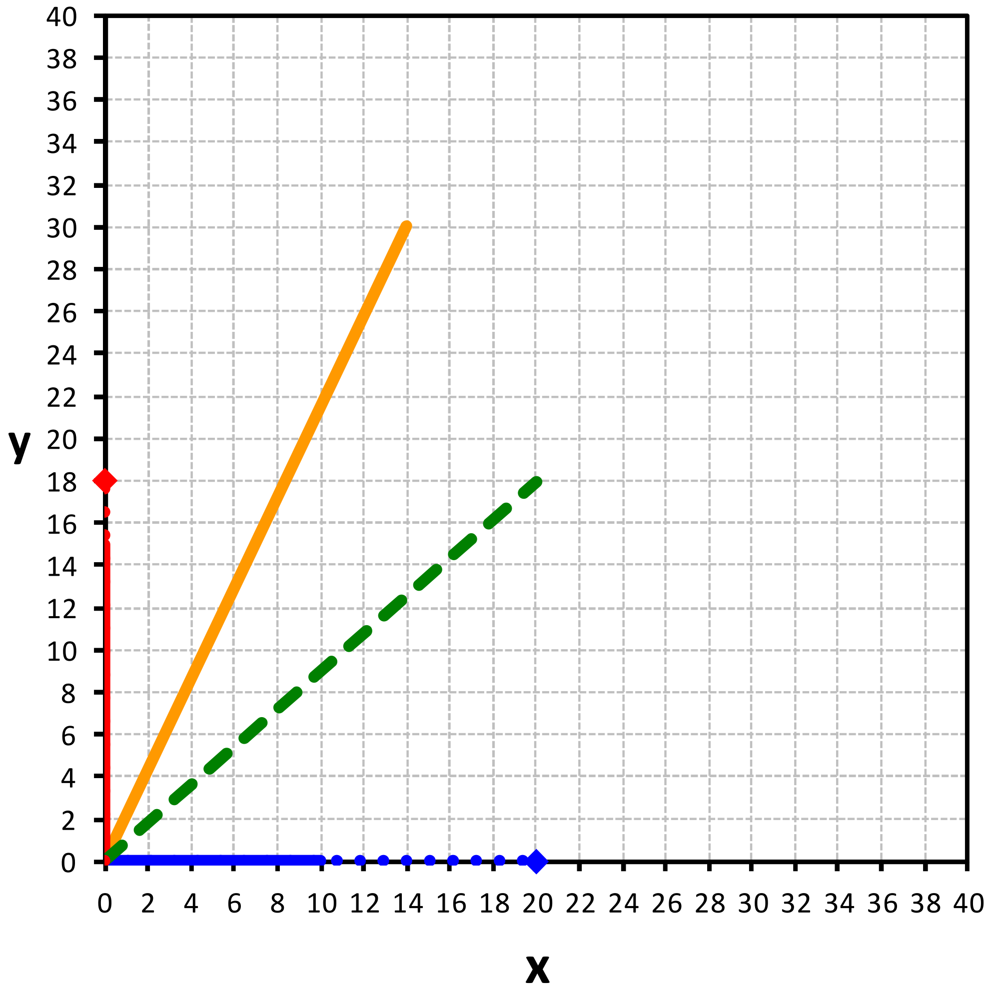

Sistemas lineares

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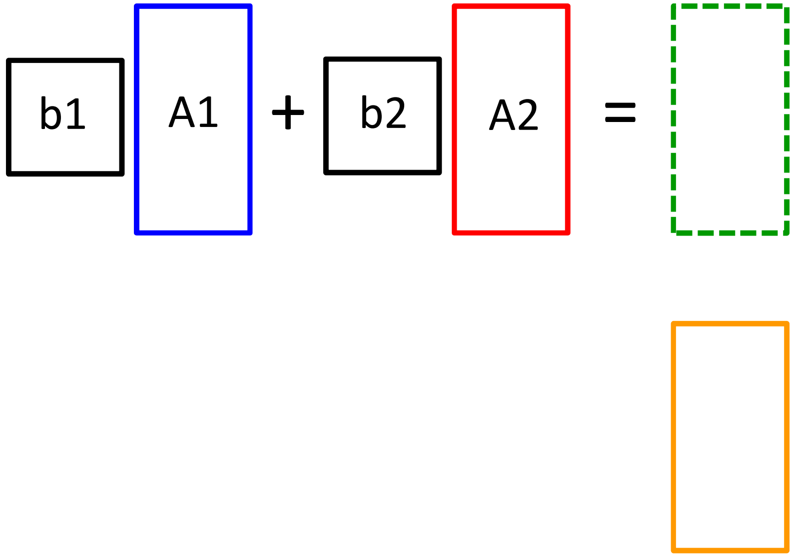

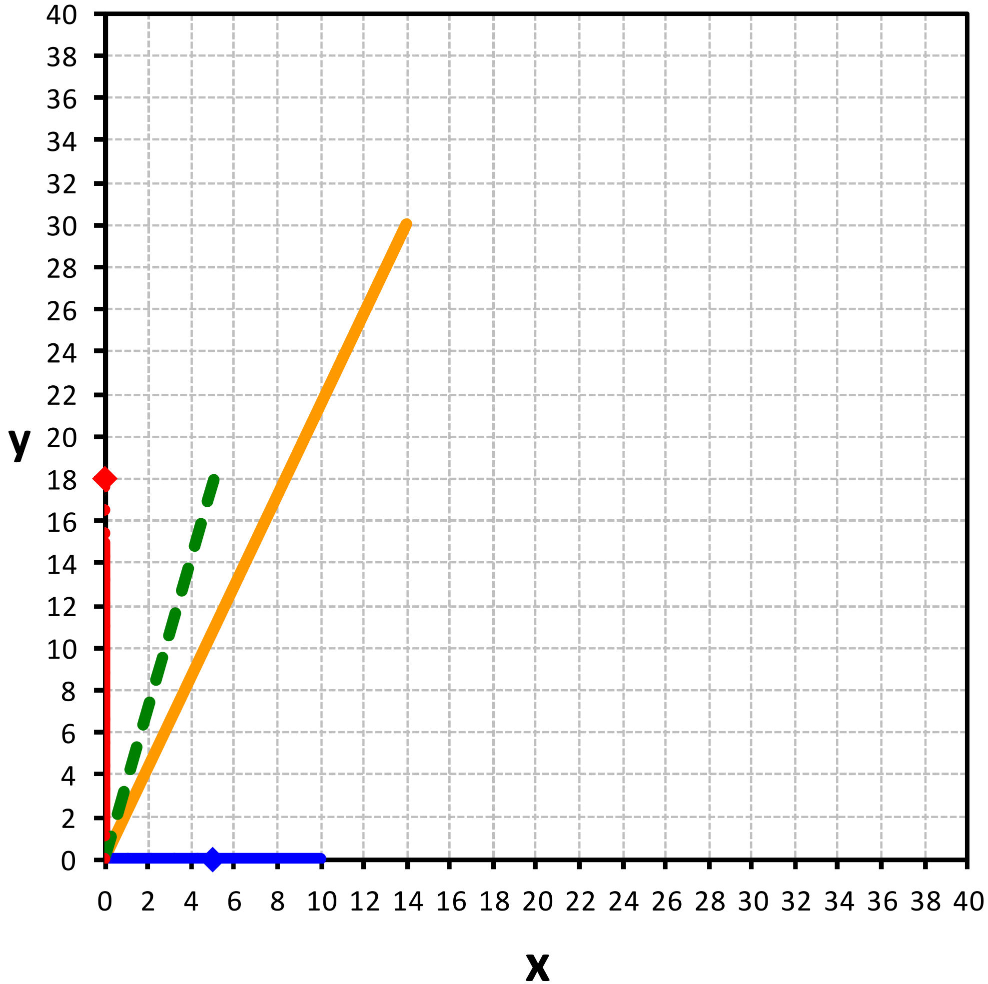

Sistemas lineares

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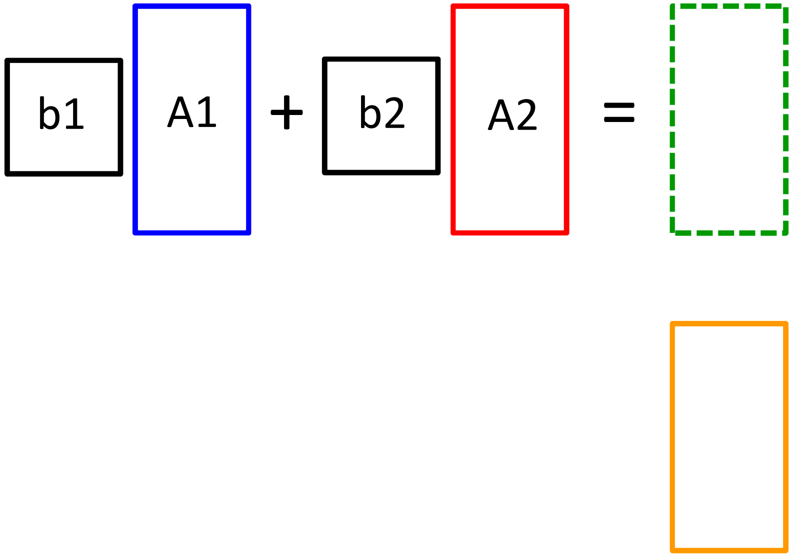

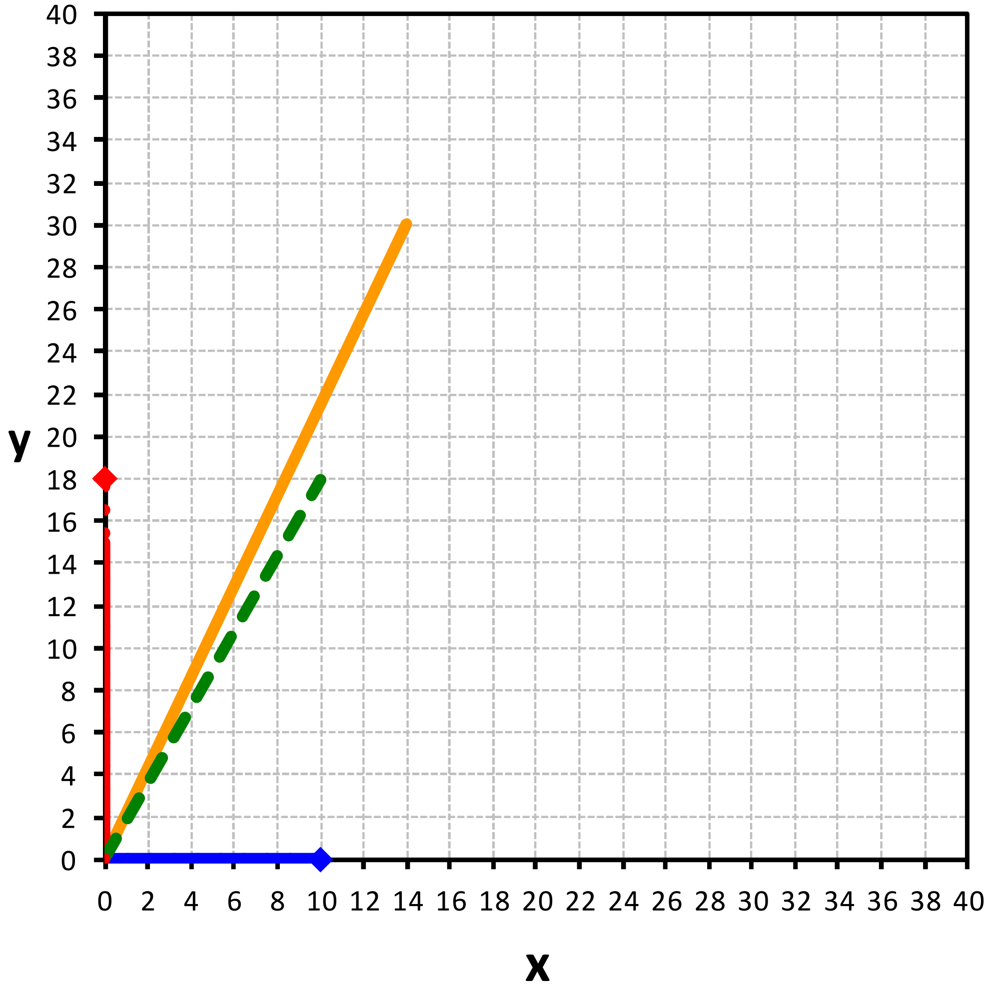

Sistemas lineares

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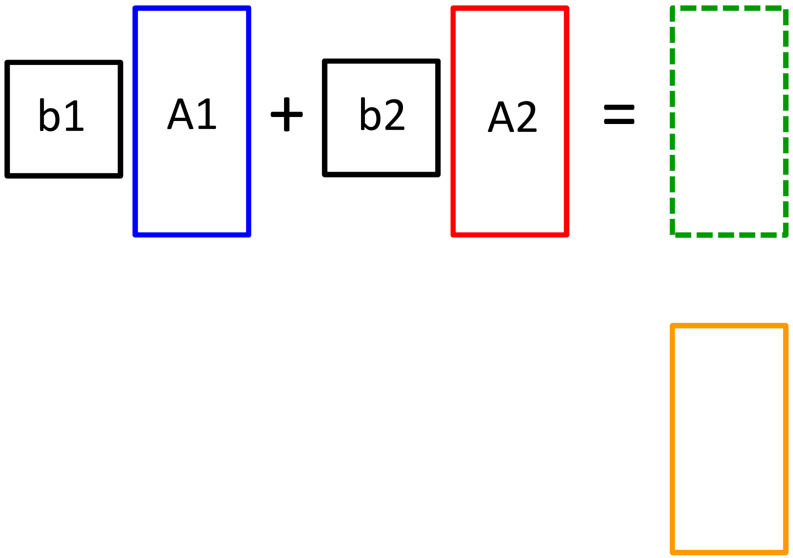

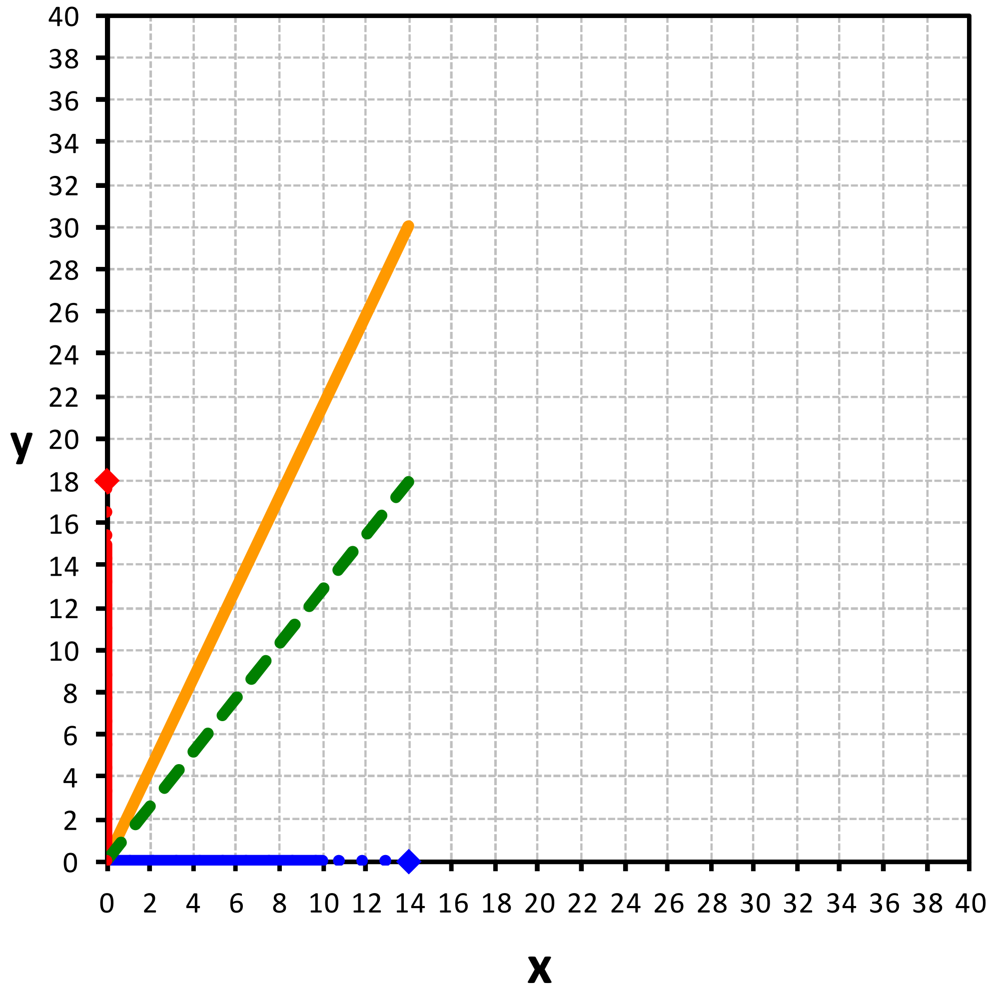

Sistemas lineares

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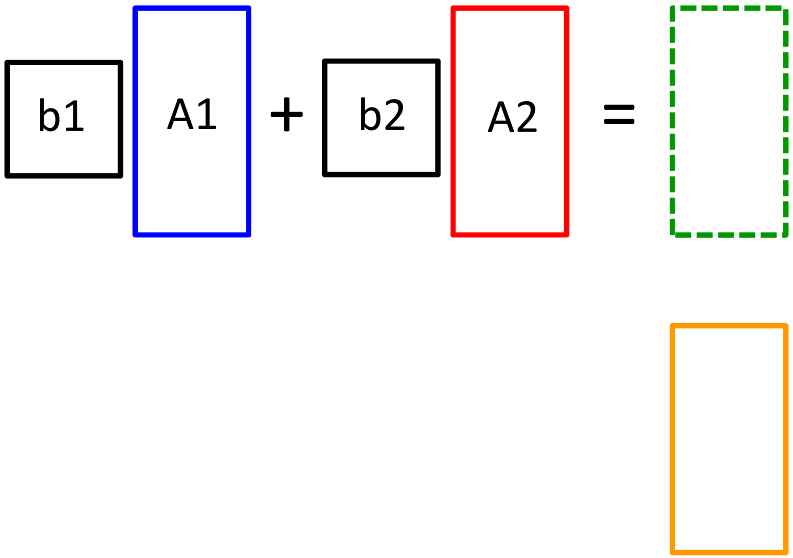

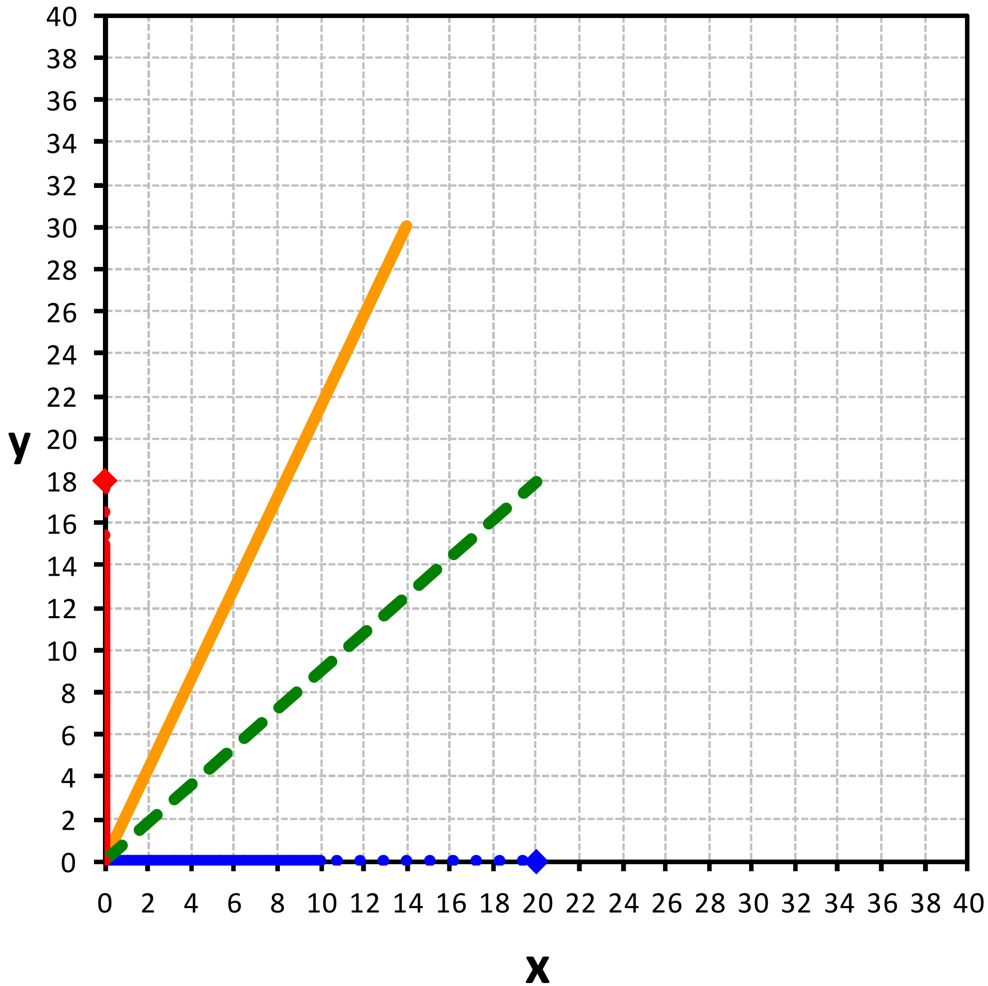

Sistemas lineares

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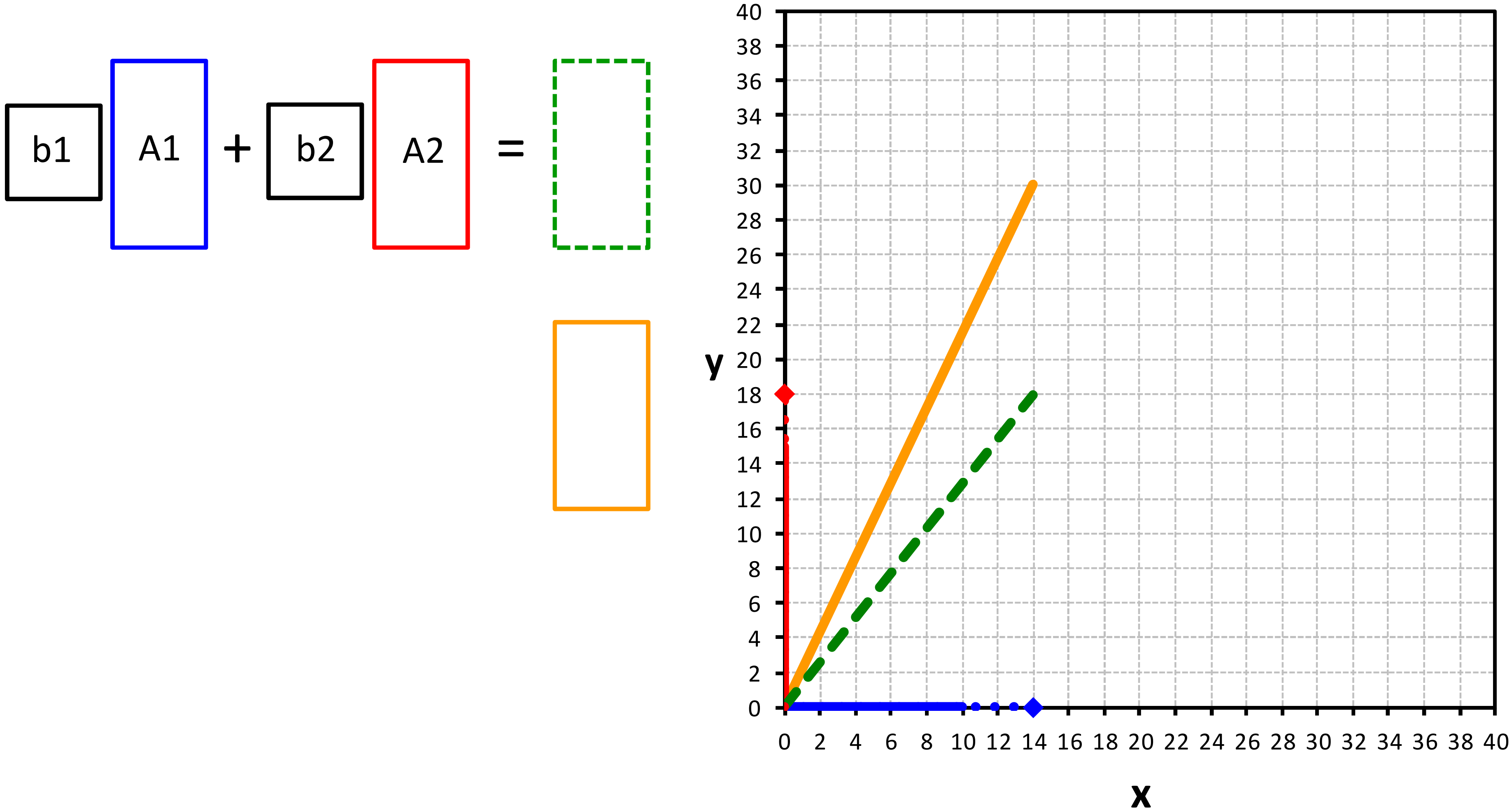

Sistemas lineares

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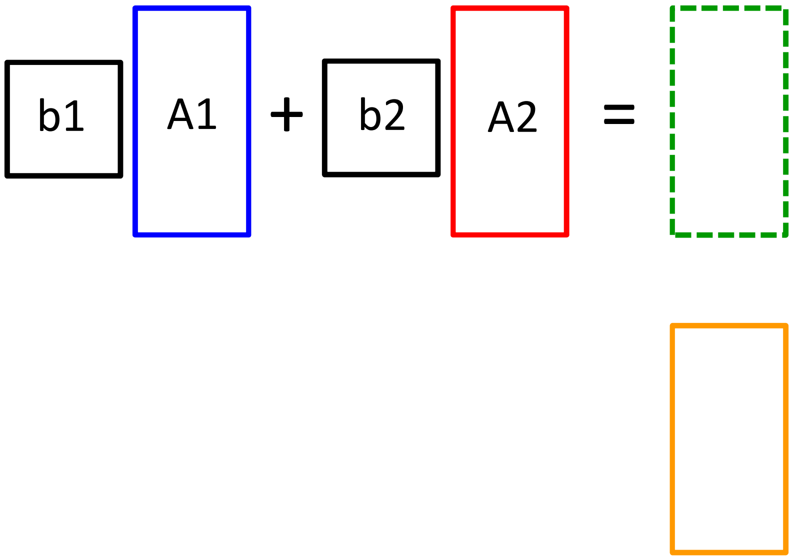
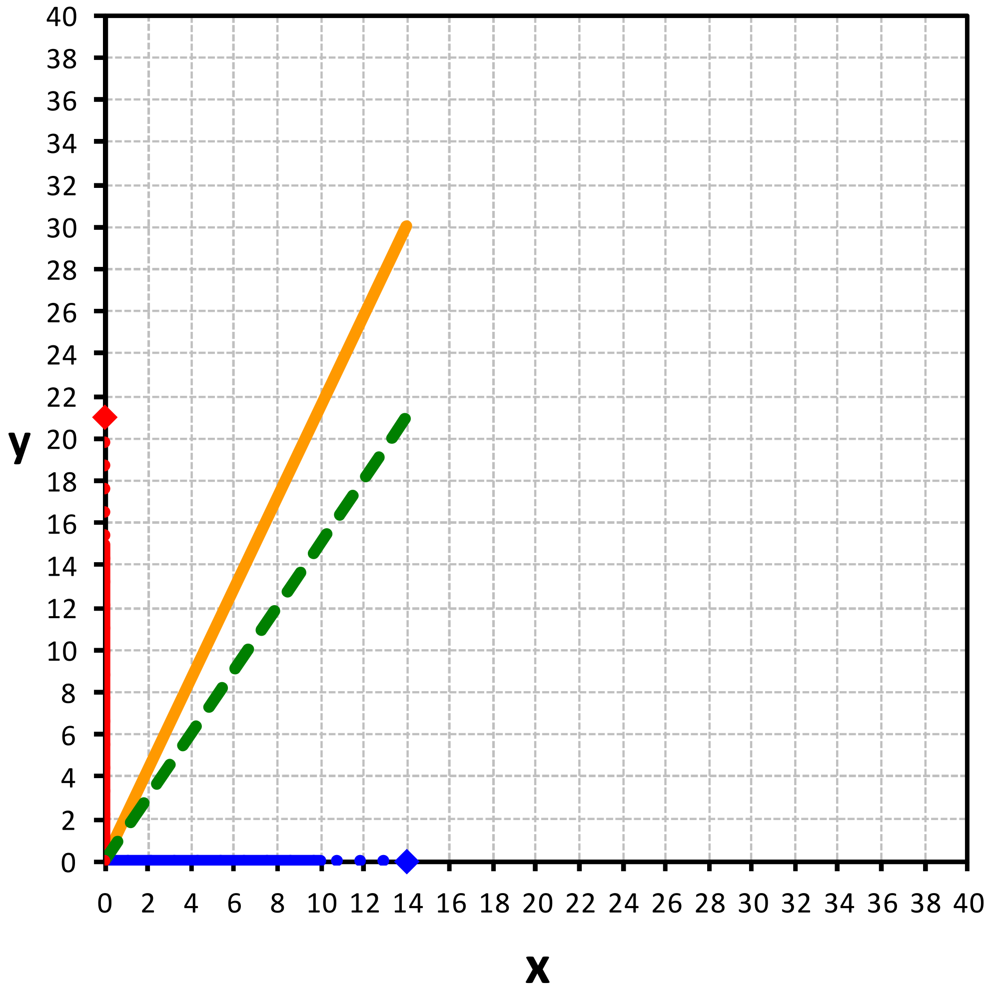
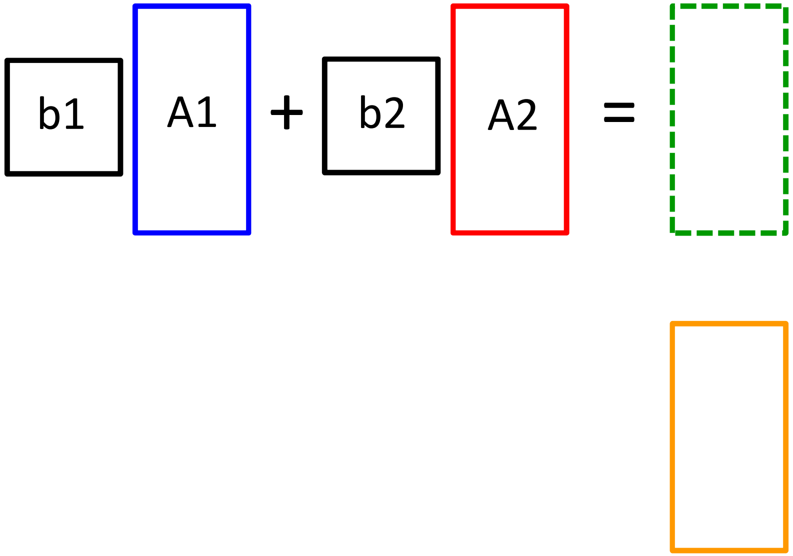
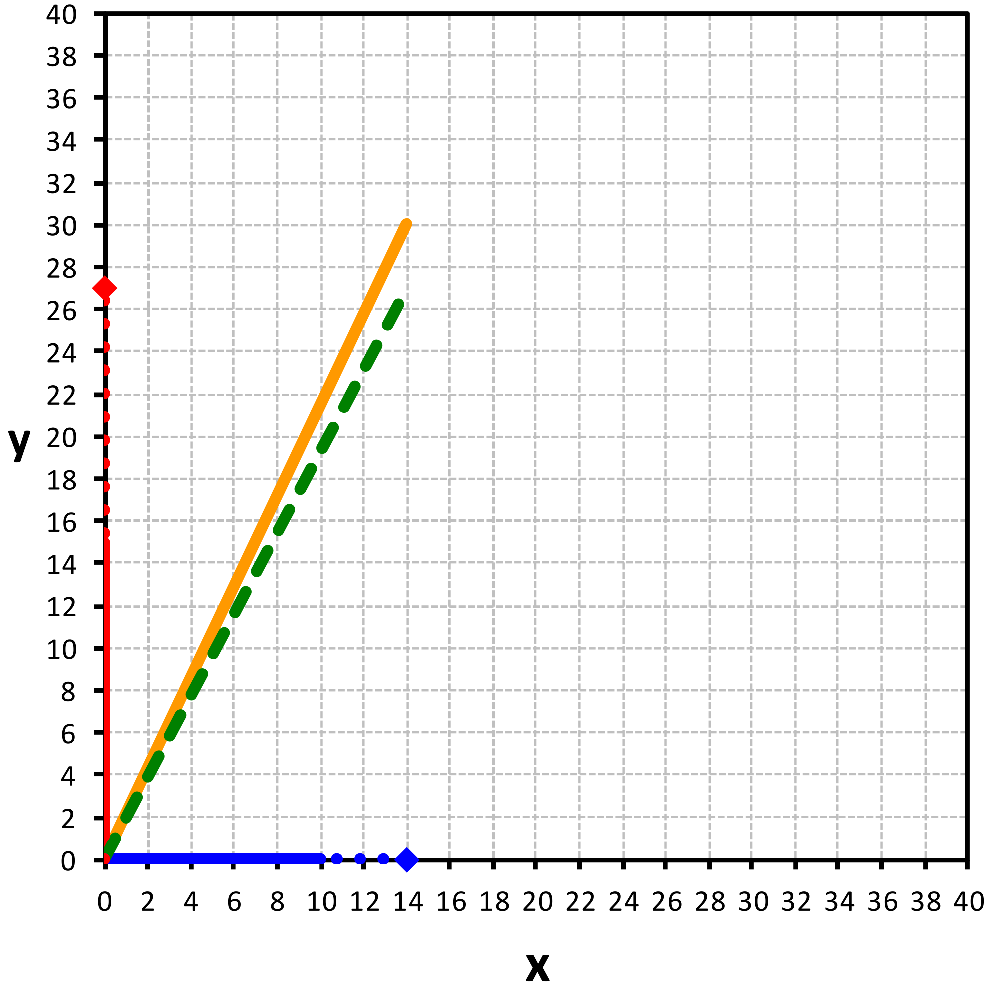




Sistemas lineares

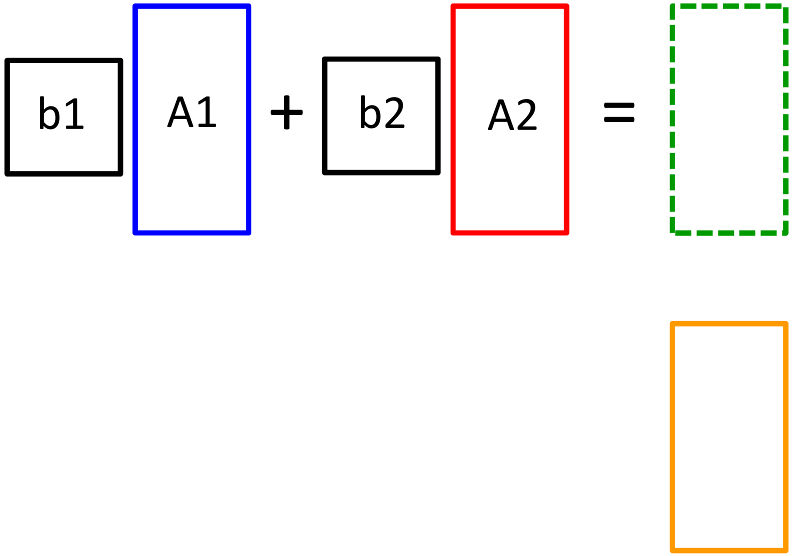
Exemplo 2D

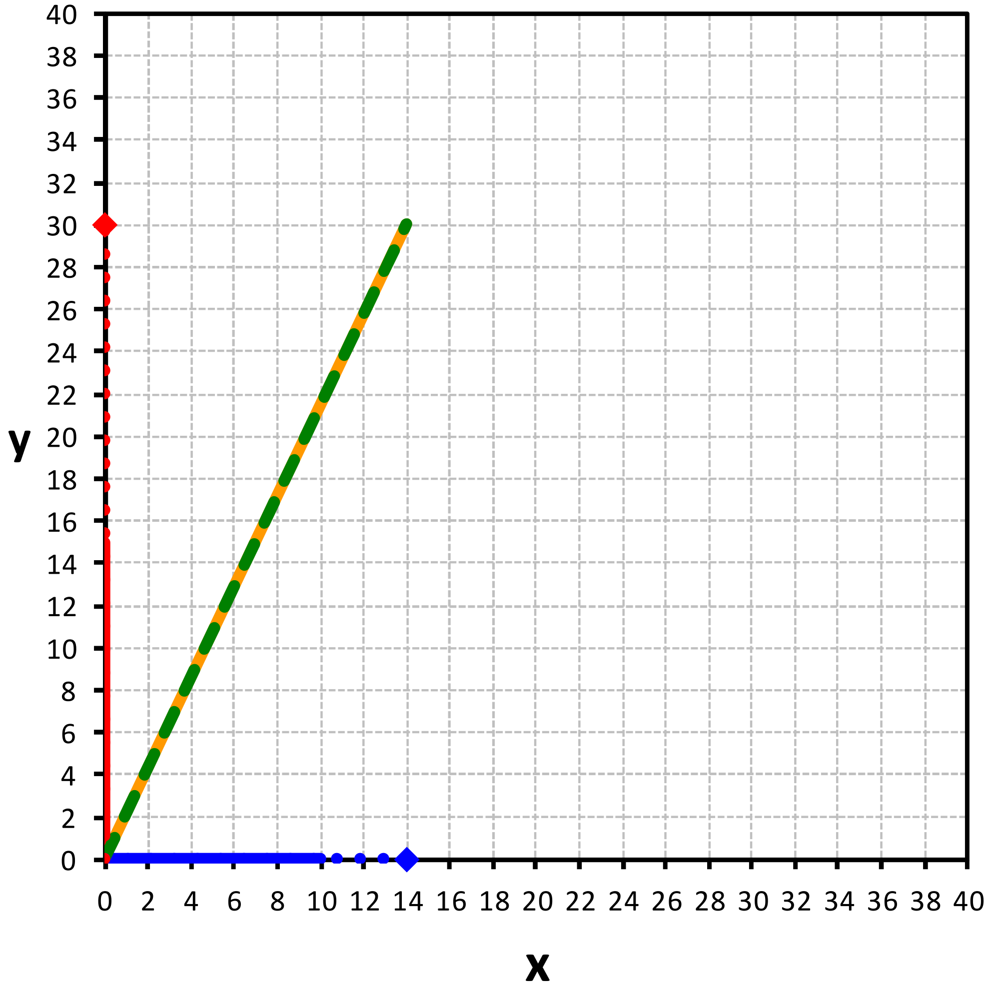




Sistemas lineares

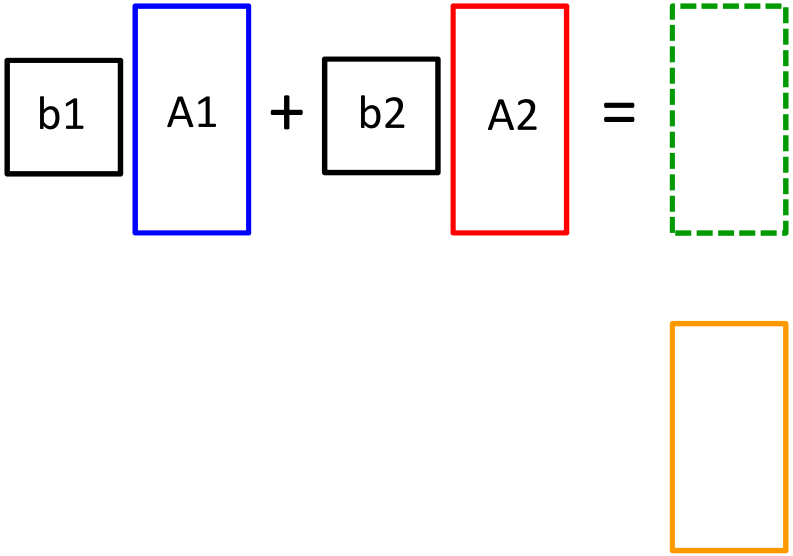
Exemplo 2D

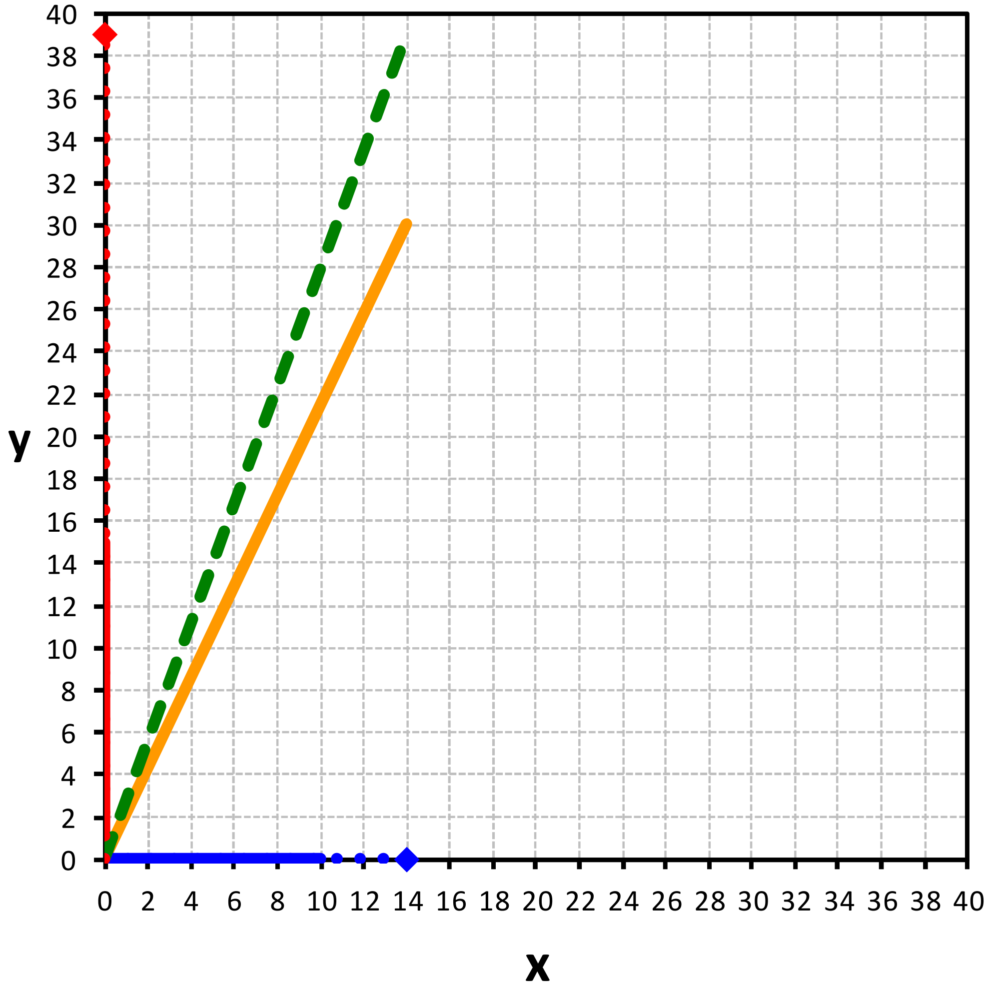




Sistemas lineares

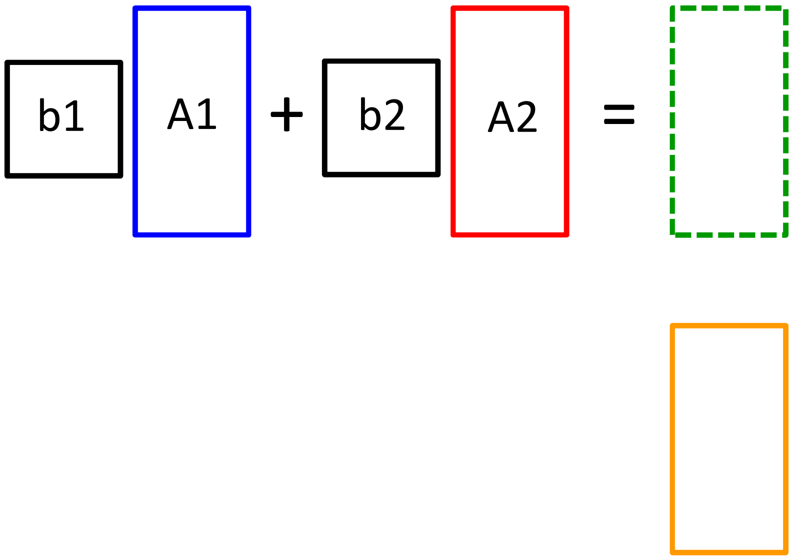
Exemplo 2D

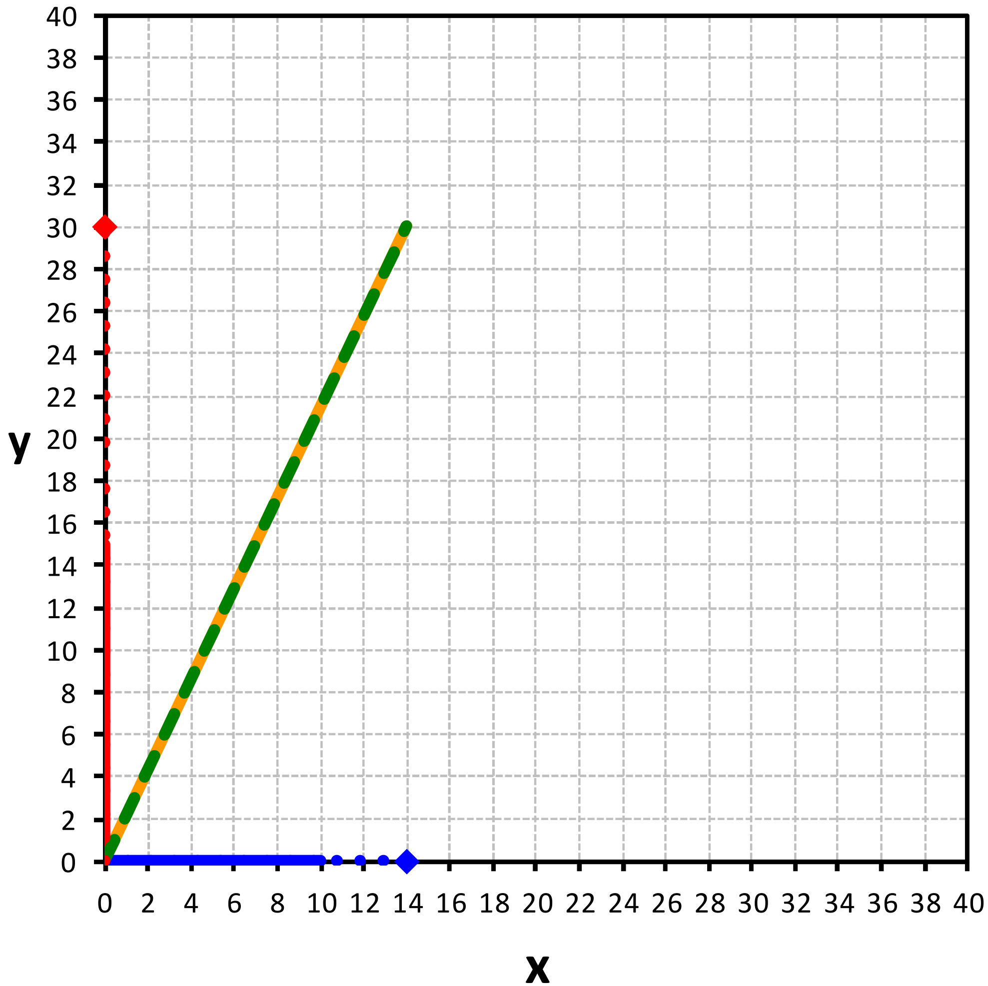




Sistemas lineares

Exemplo 2D

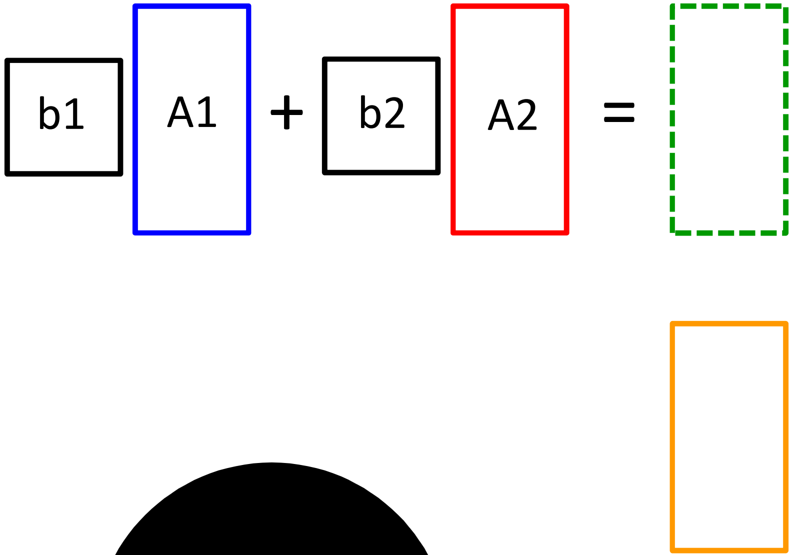


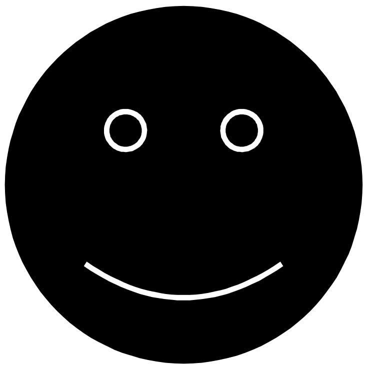


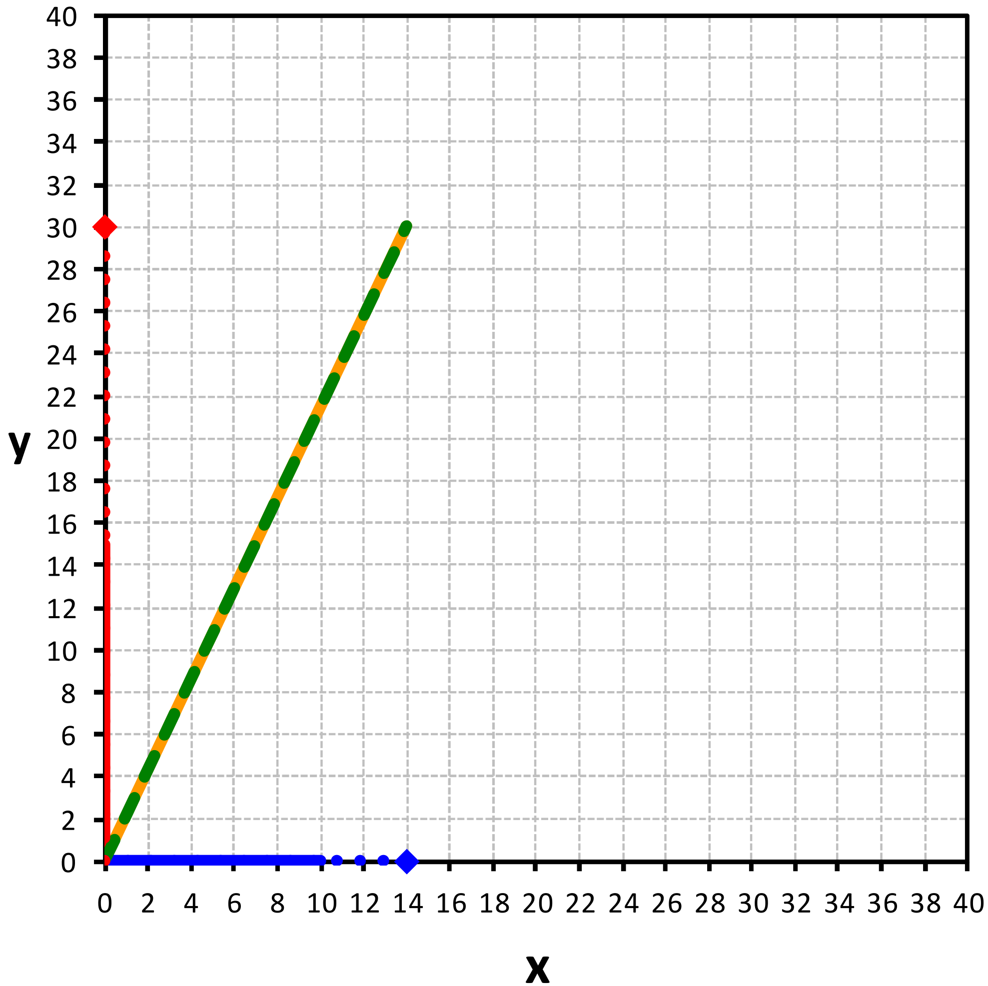
Neste caso, os vetores A1 e A2 são *linearmente independentes* e os coeficientes b1 e b2 são únicos

Sistemas lineares

Exemplo 2D

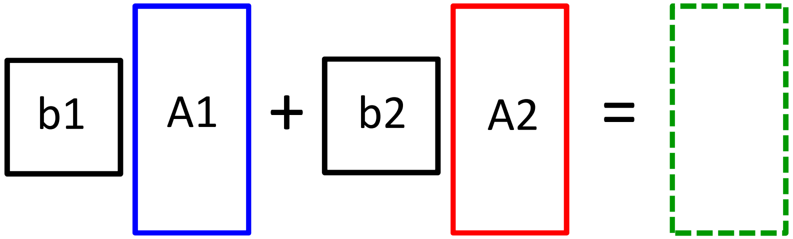


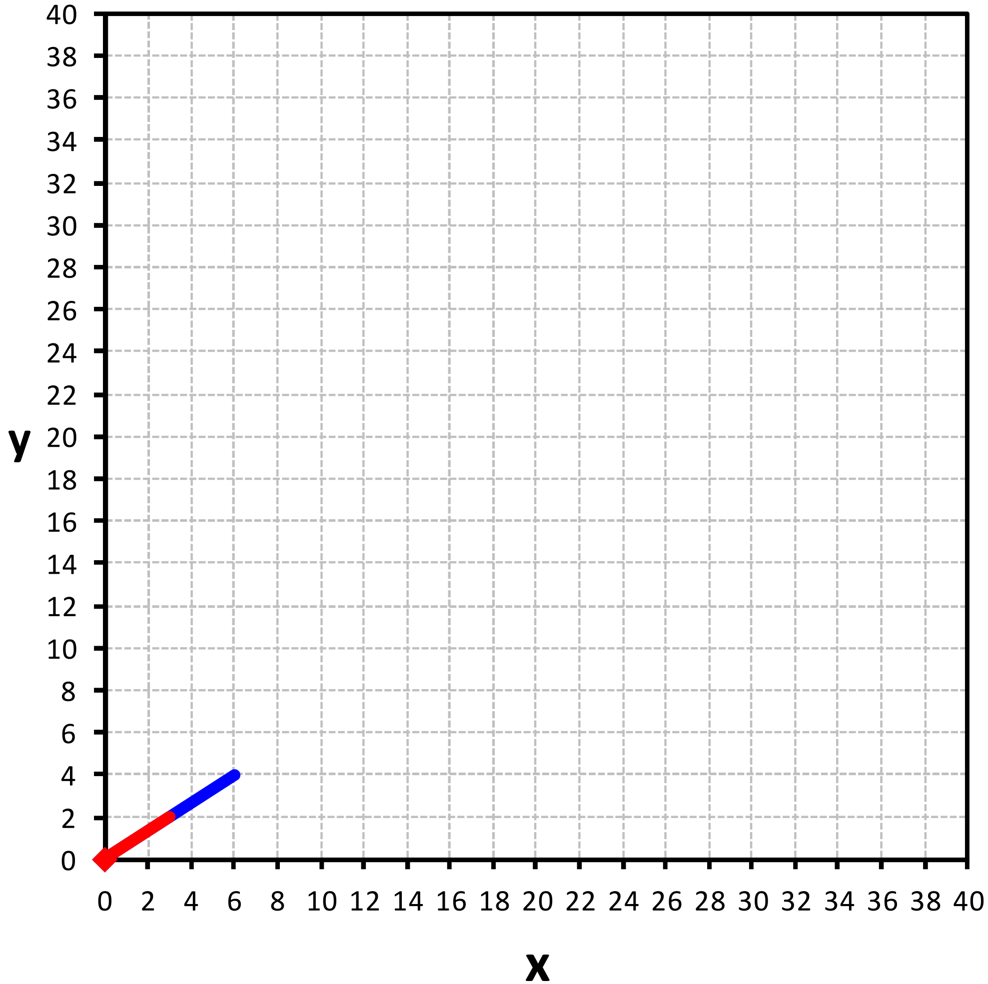




Sistemas lineares

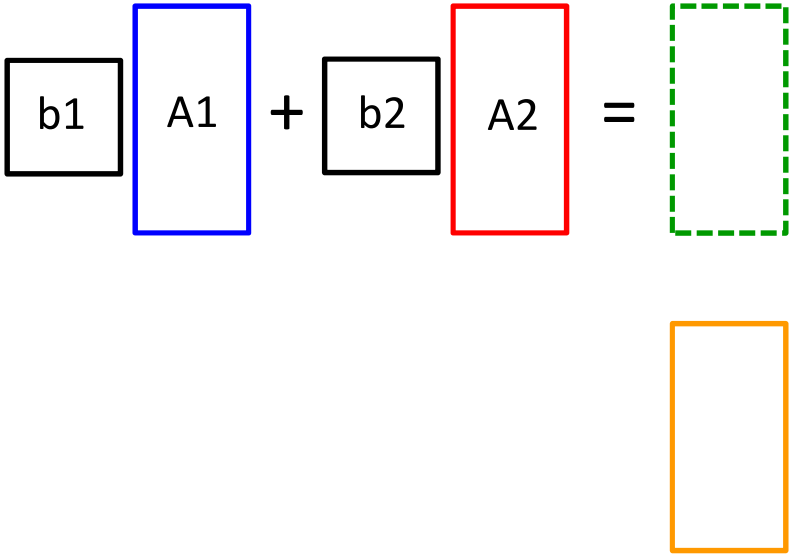
Exemplo 2D

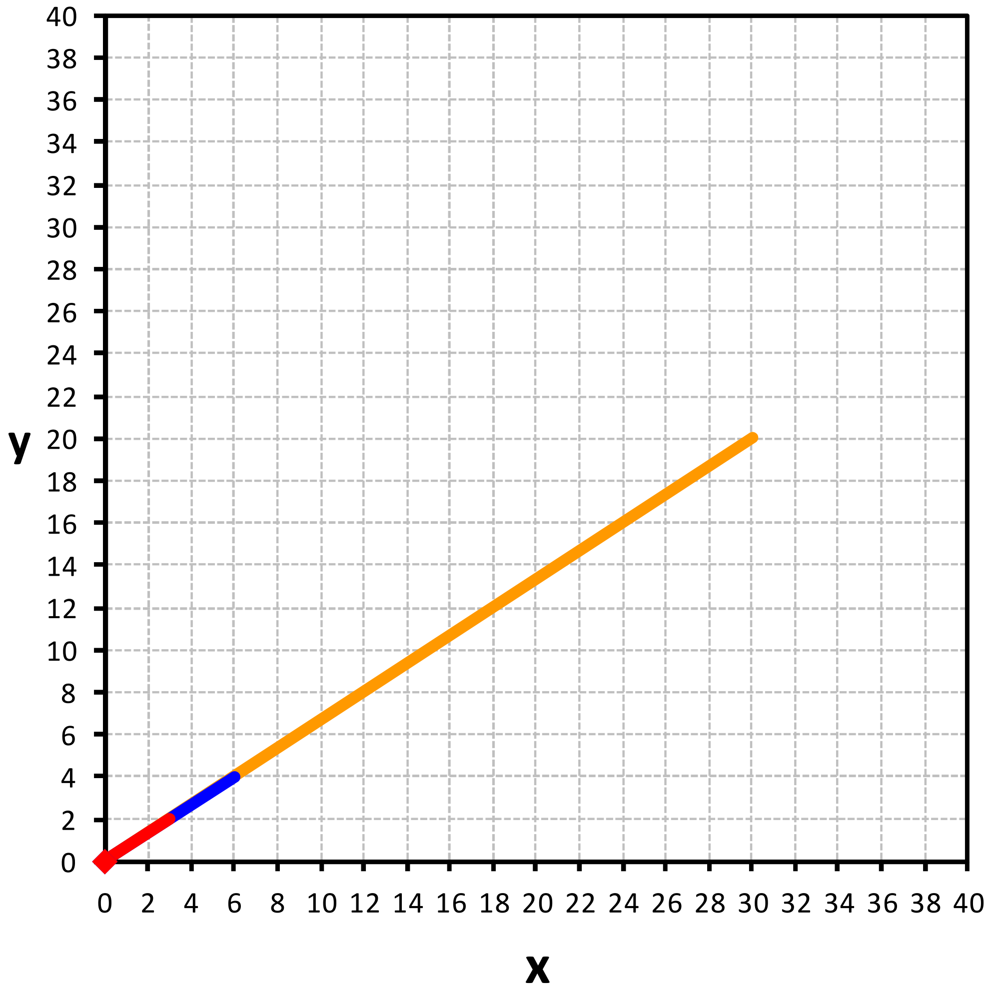




Sistemas lineares

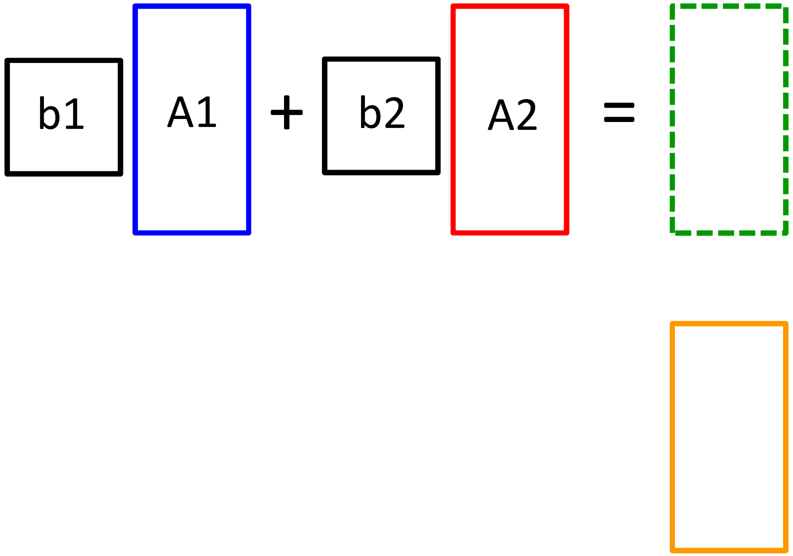
Exemplo 2D

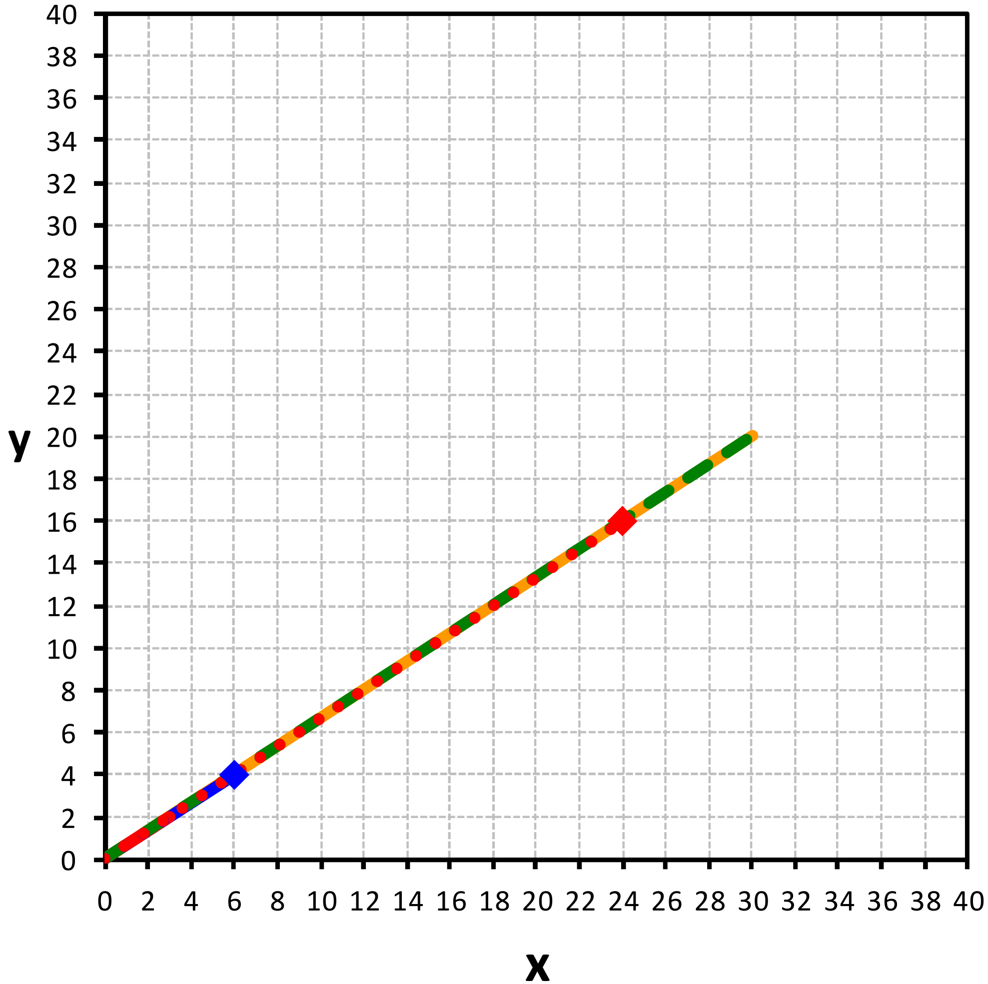




Sistemas lineares

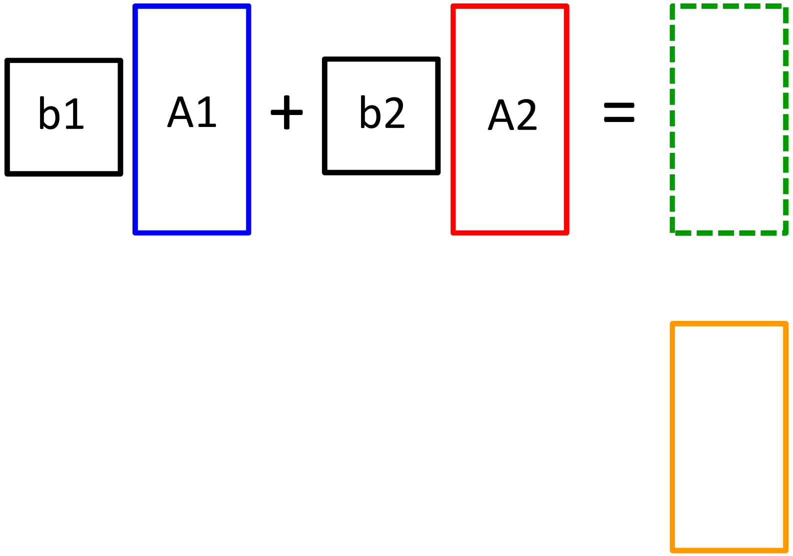
Exemplo 2D

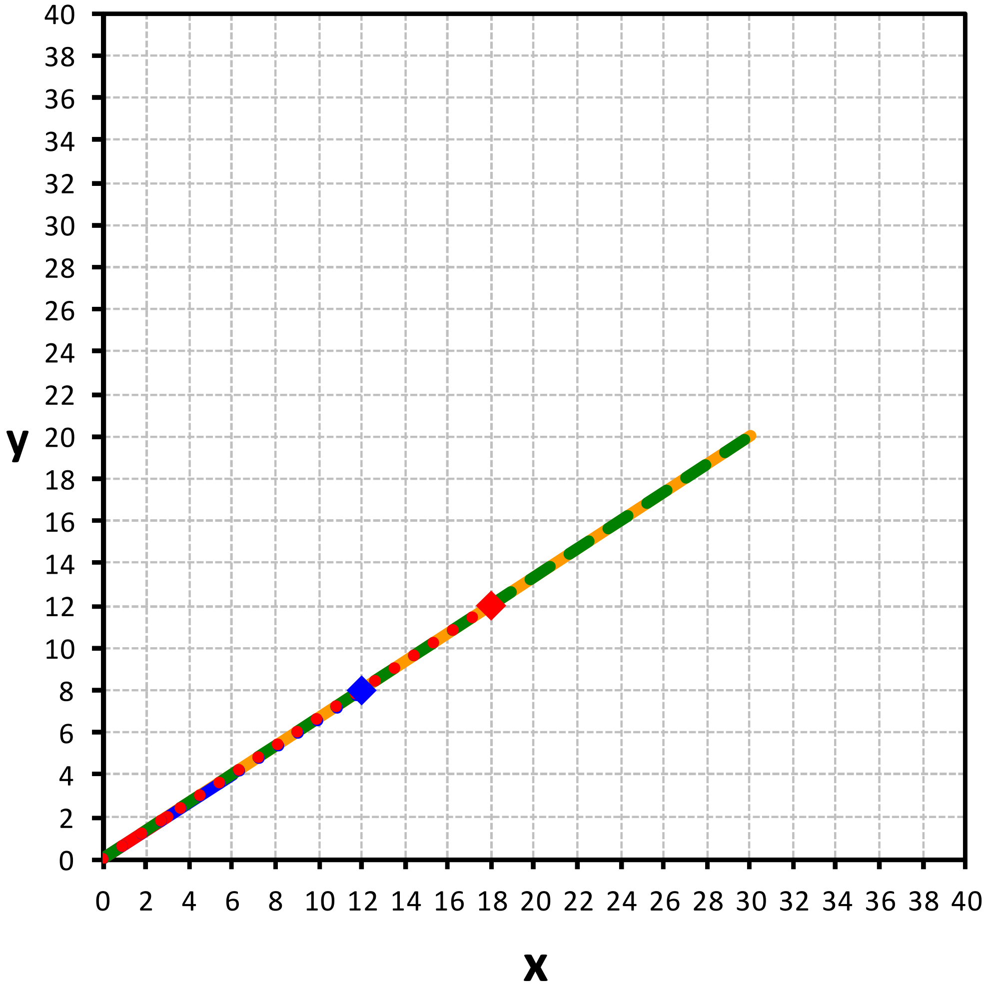




Sistemas lineares

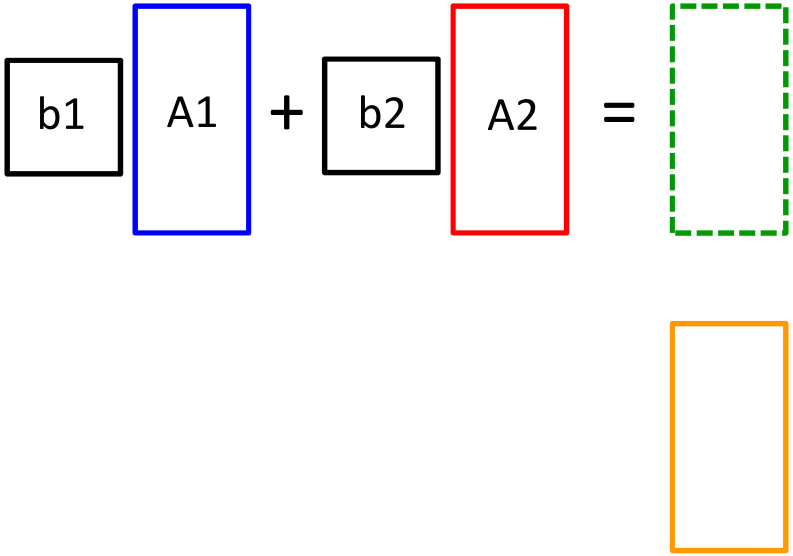
Exemplo 2D

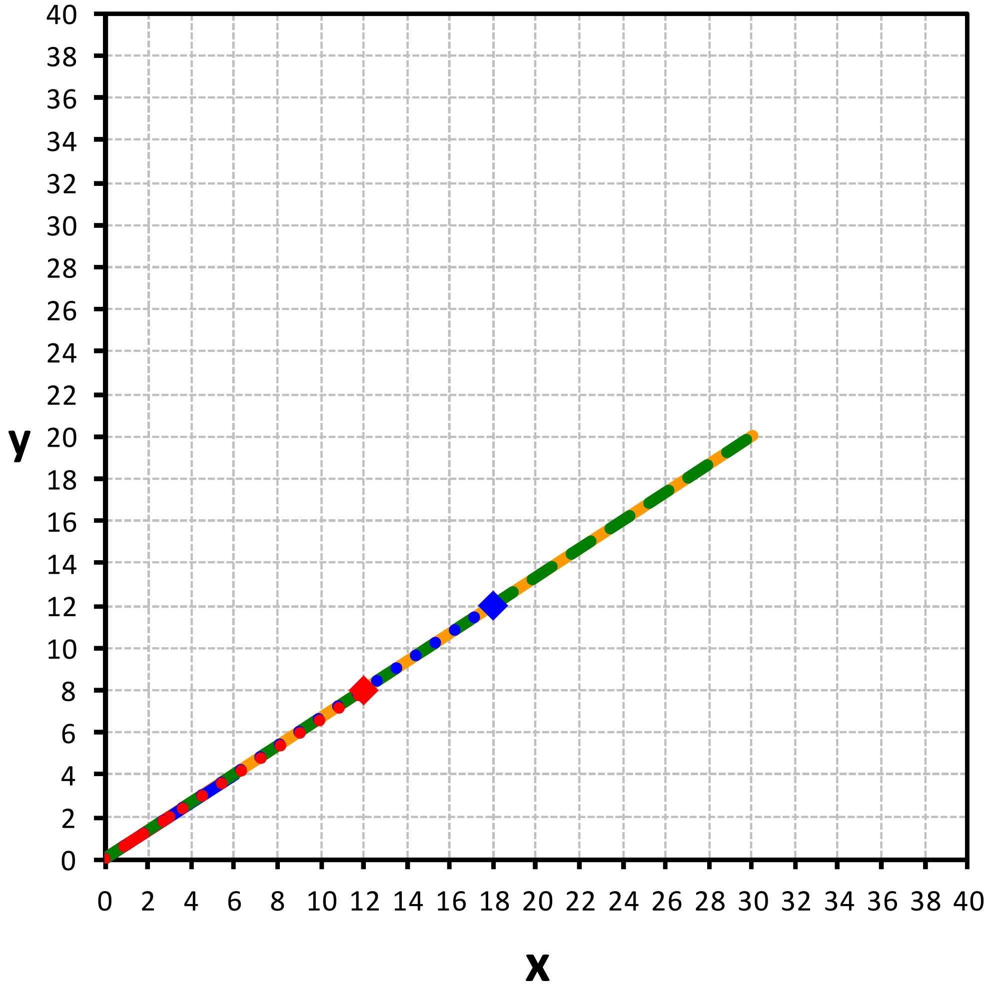




Sistemas lineares

Exemplo 2D

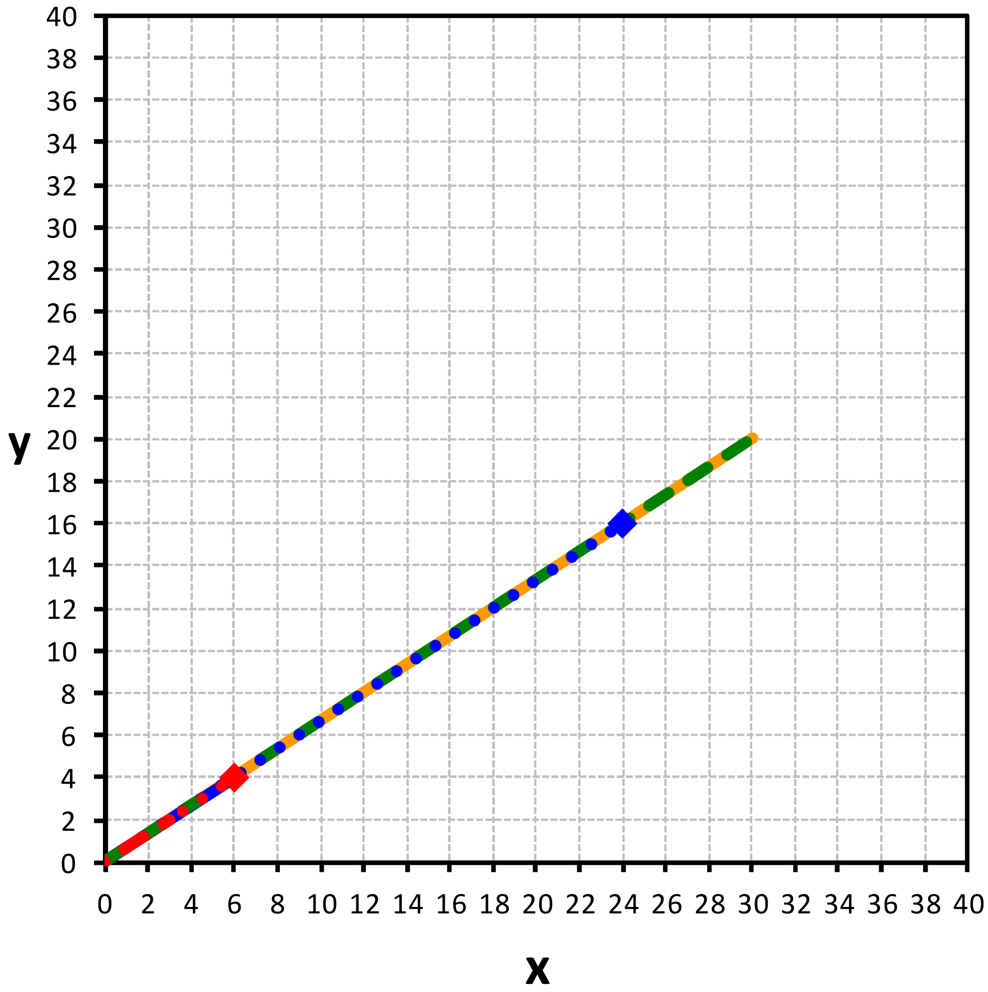




Sistemas lineares

Exemplo 2D

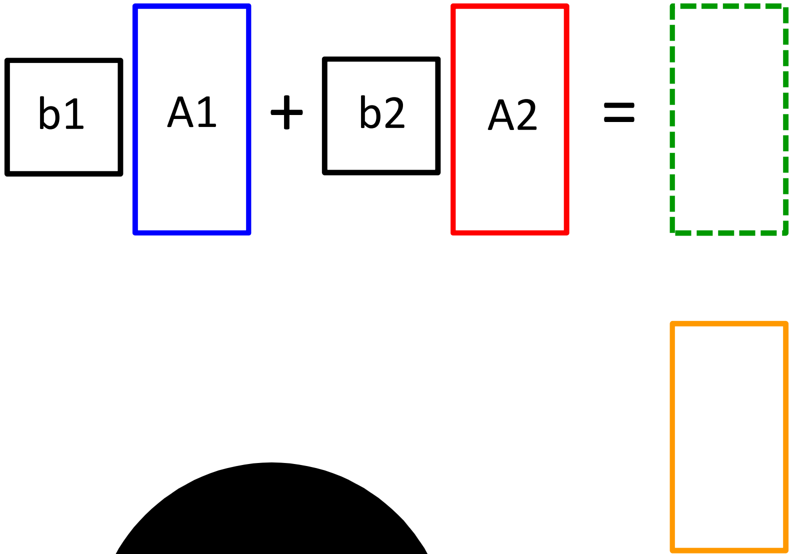
b1 A1 + b2 A2 =

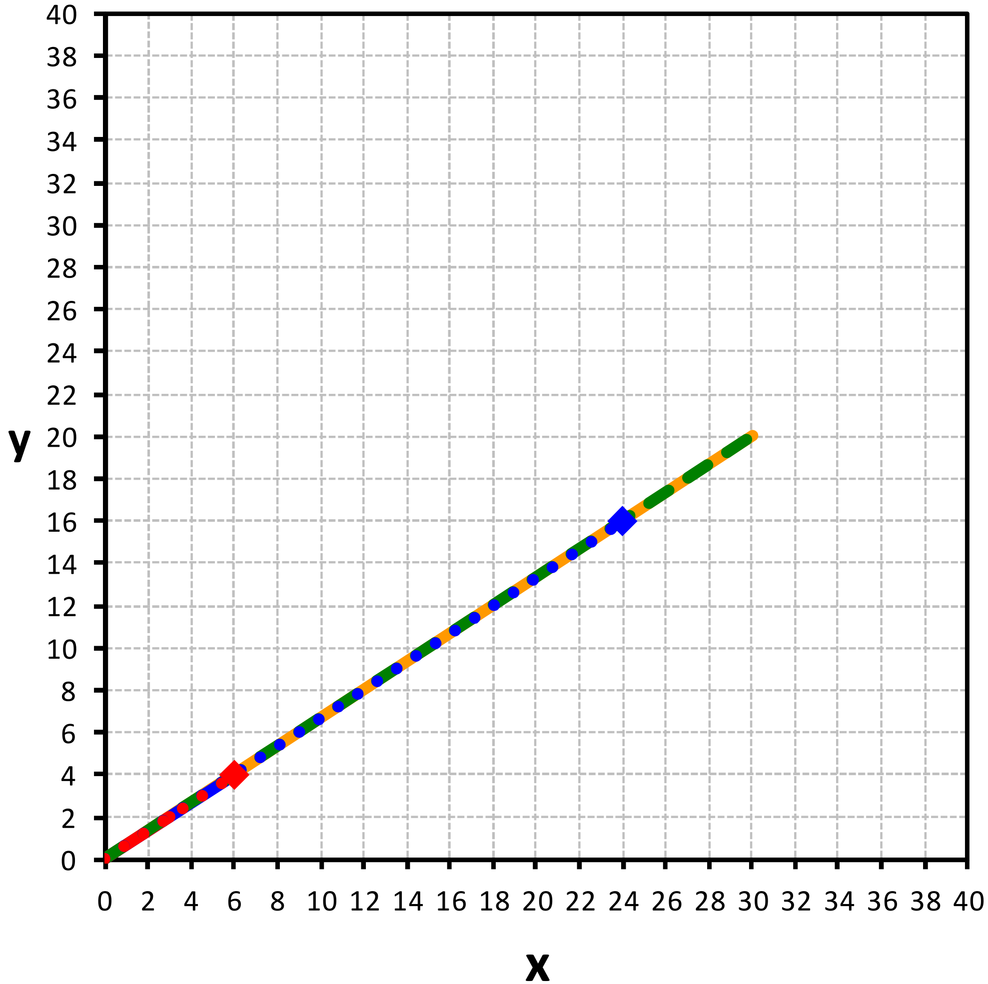


Neste caso, os vetores A1 e A2 são *linearmente dependentes* e existem infinitos pares b1 e b2 que produzem o mesmo resultado

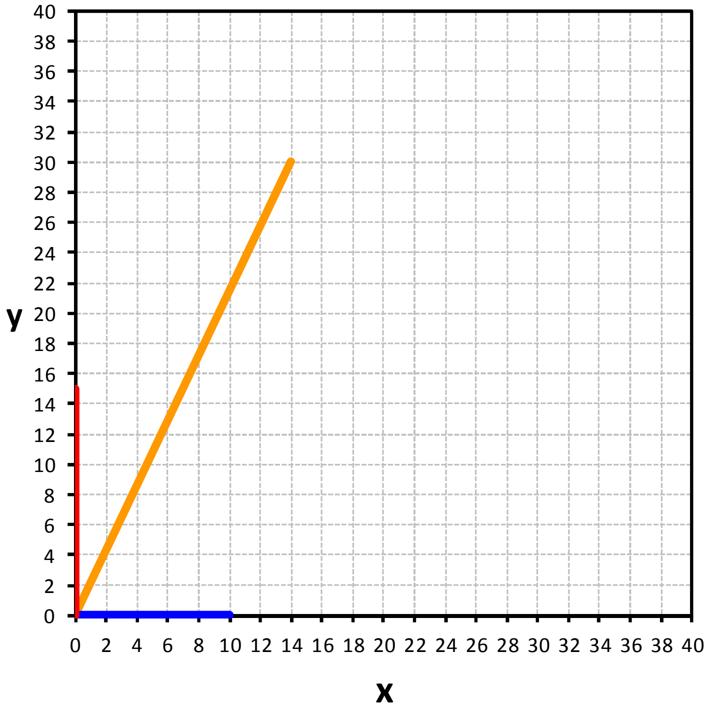
Sistemas lineares

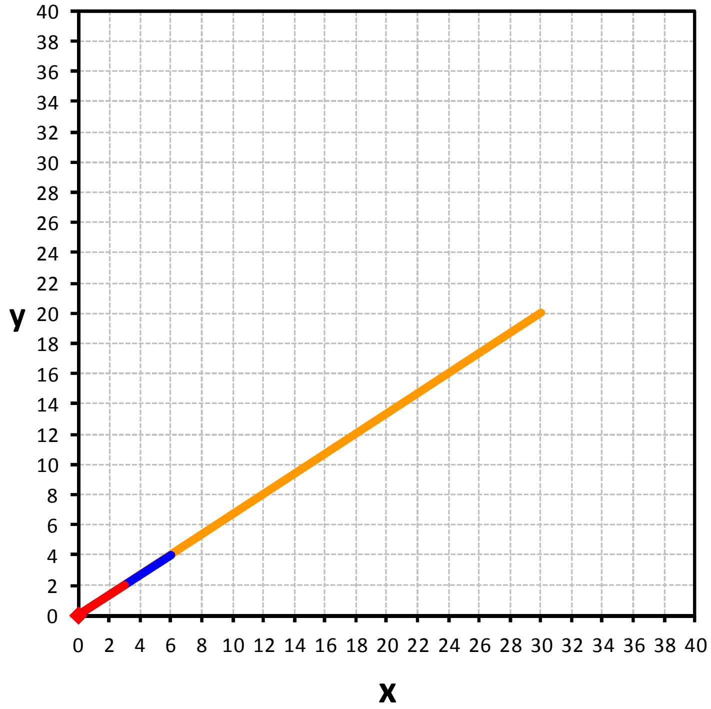
Exemplo 2D

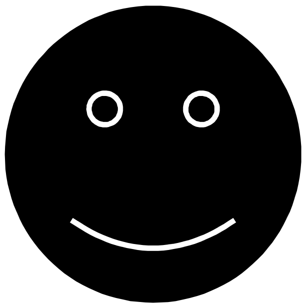


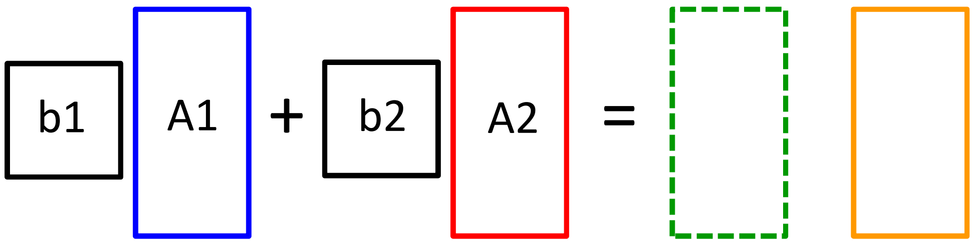


Sistemas lineares

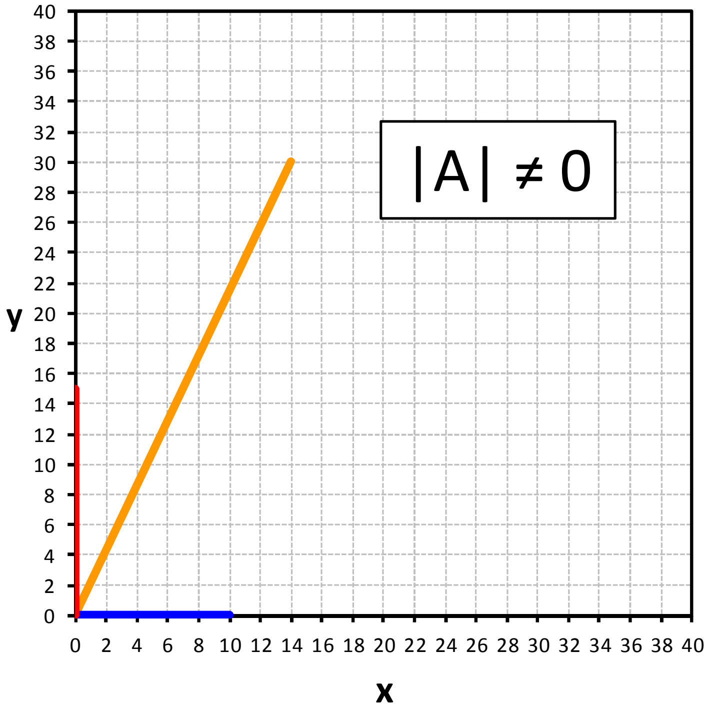


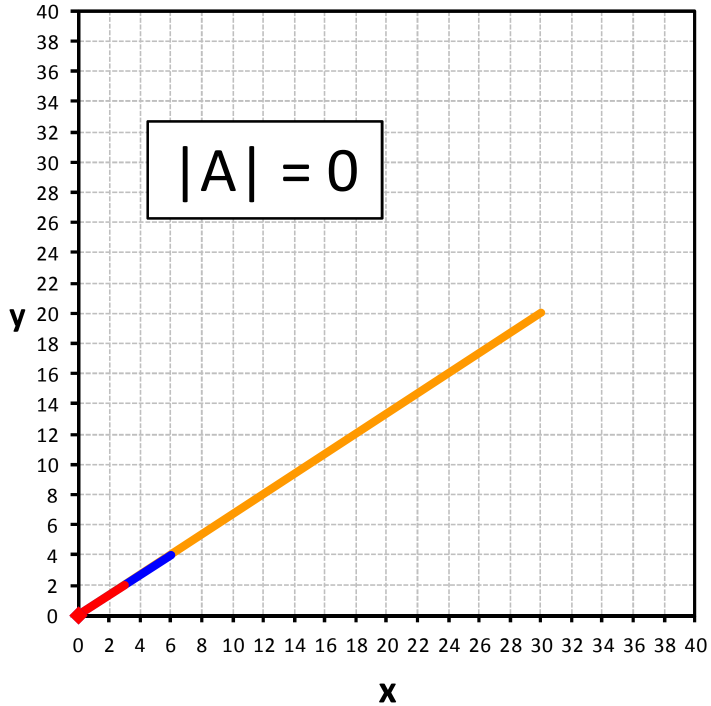


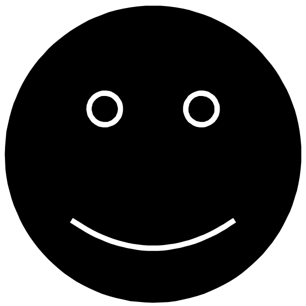


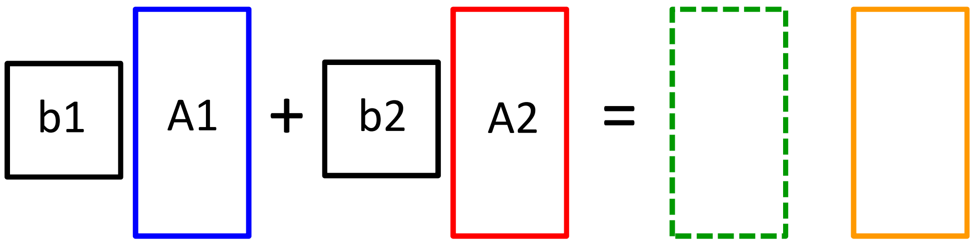


Sistemas lineares



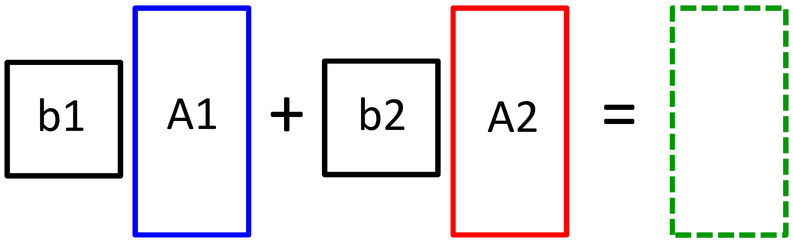


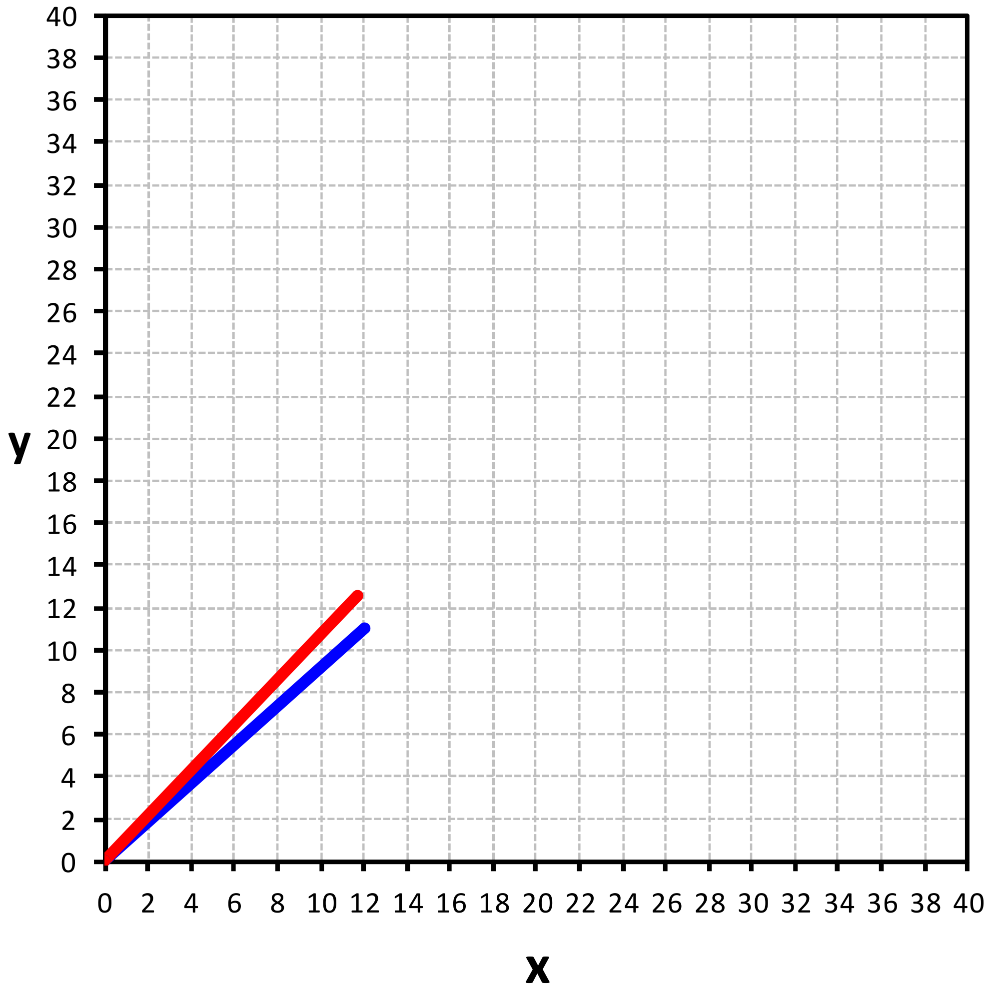




Sistemas lineares

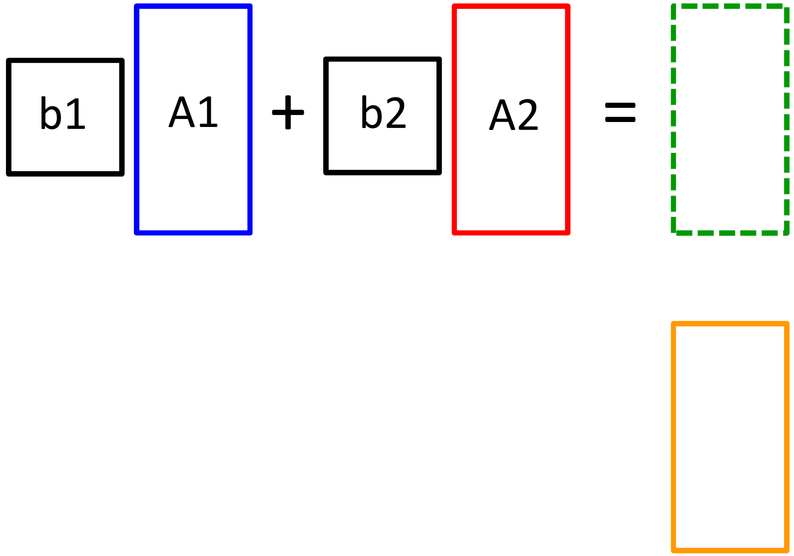
Exemplo 2D

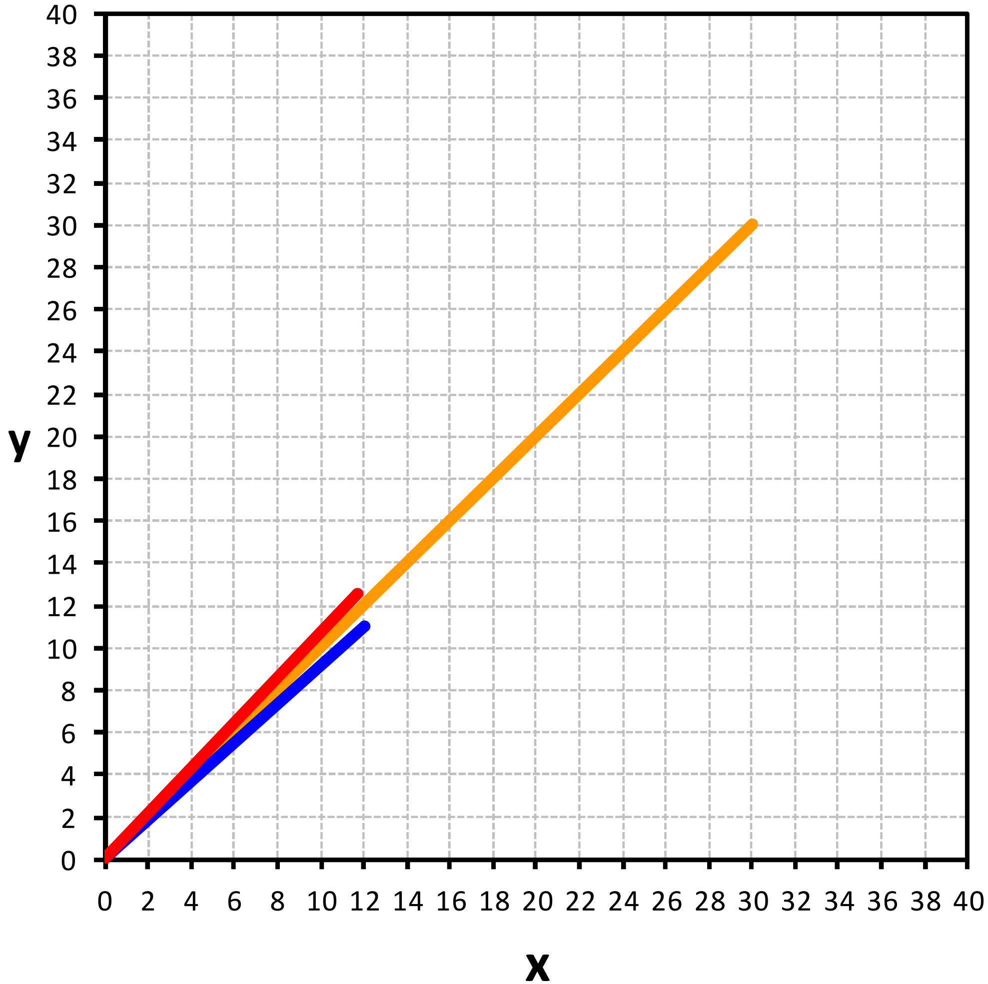




Sistemas lineares

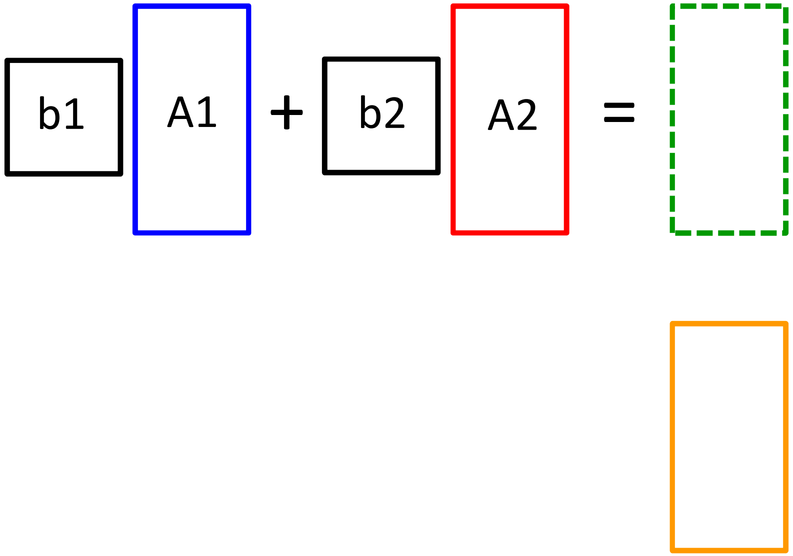
Exemplo 2D

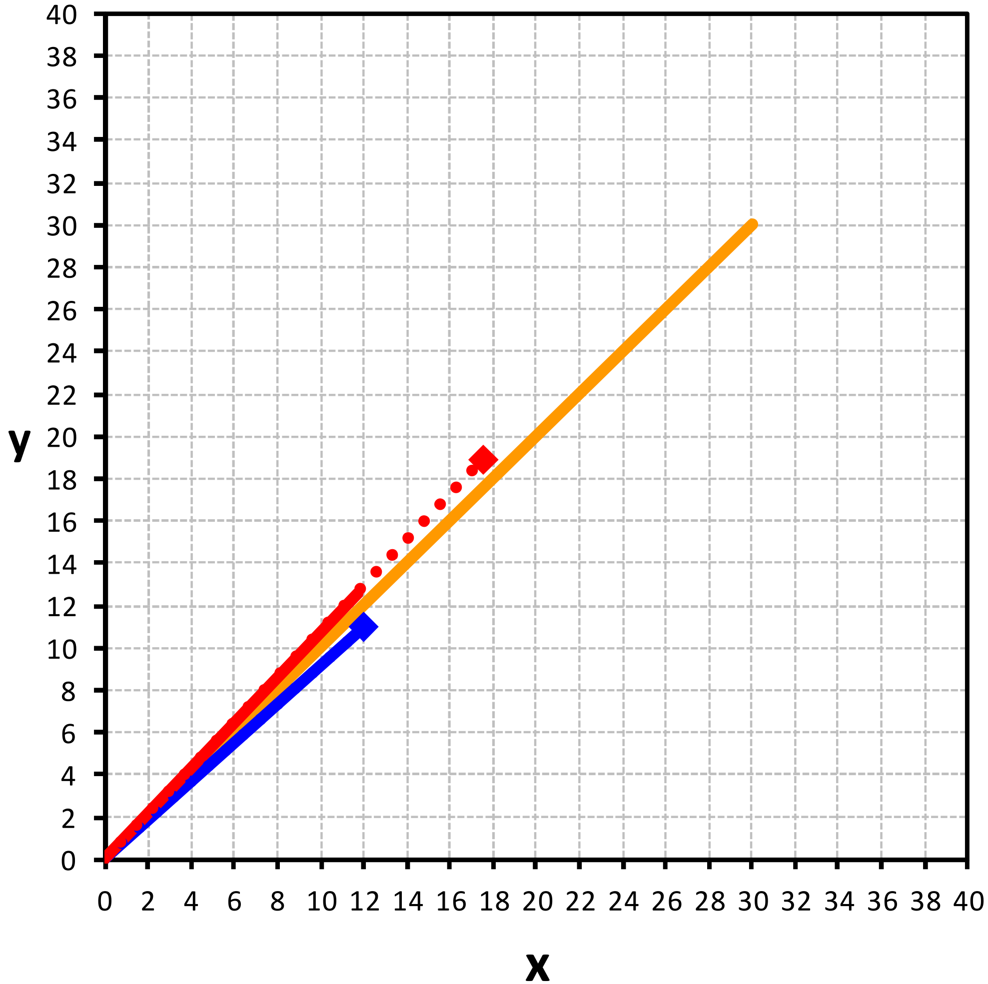




Sistemas lineares

Exemplo 2D





Sistemas lineares

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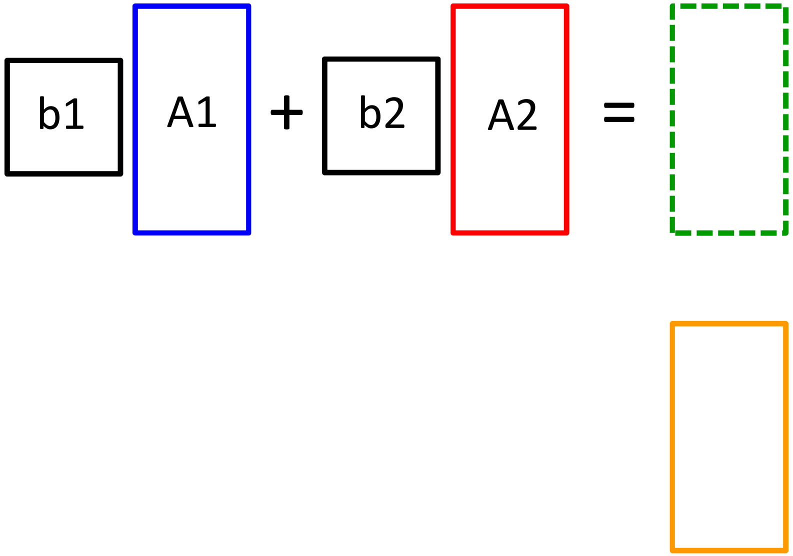

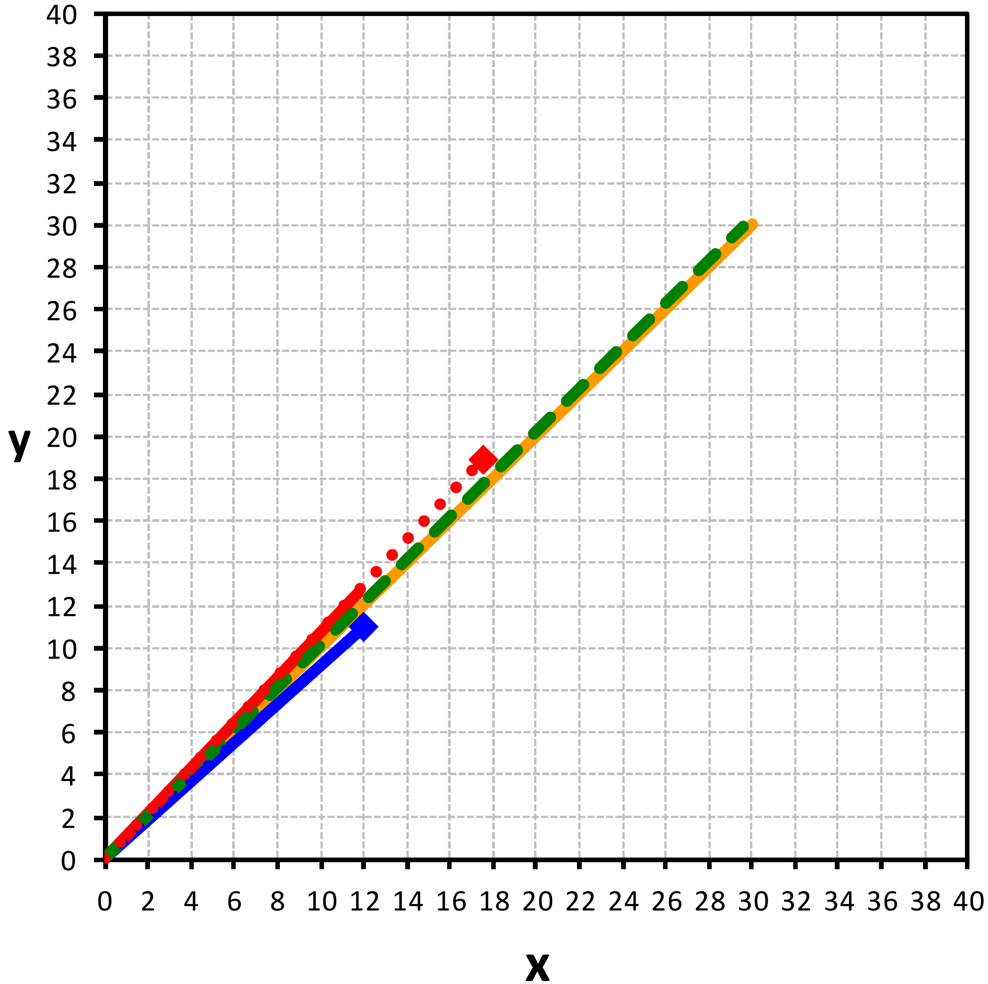

Sistemas lineares

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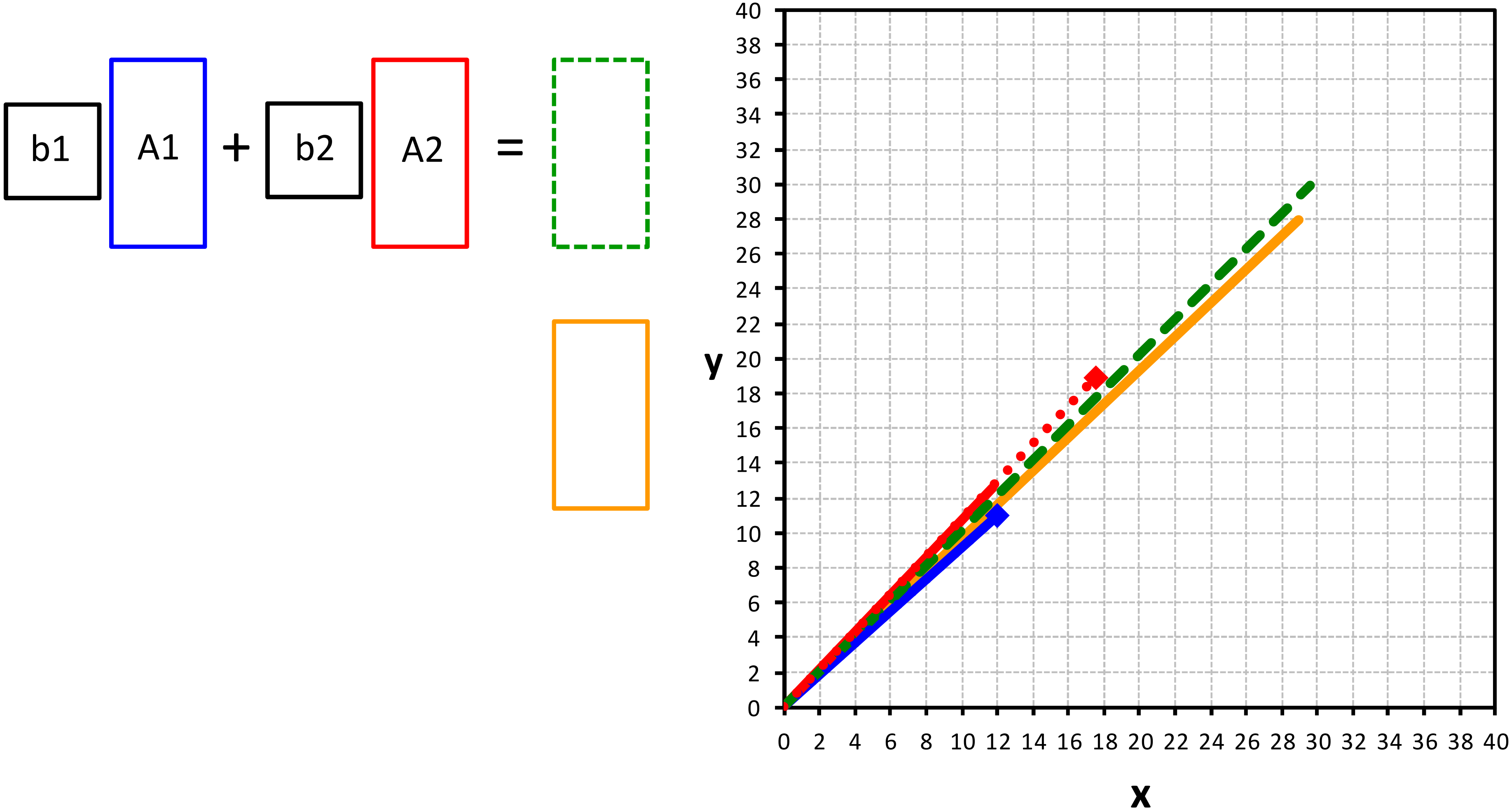

Sistemas lineares

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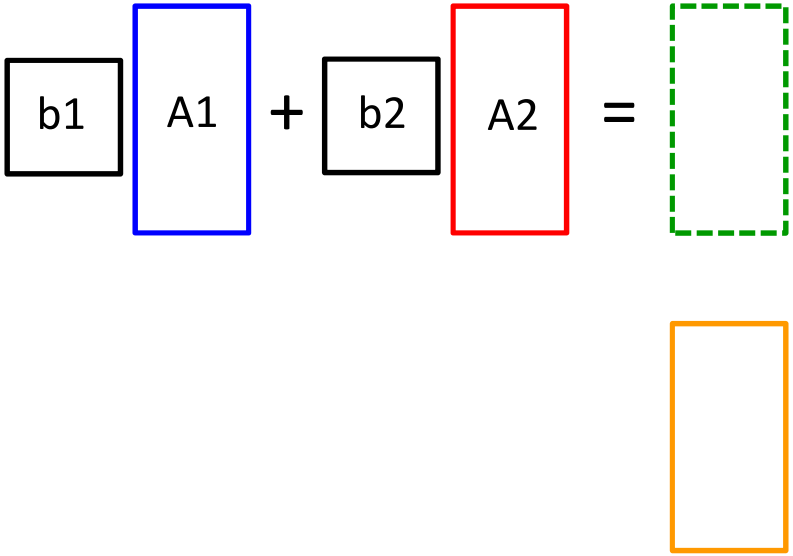
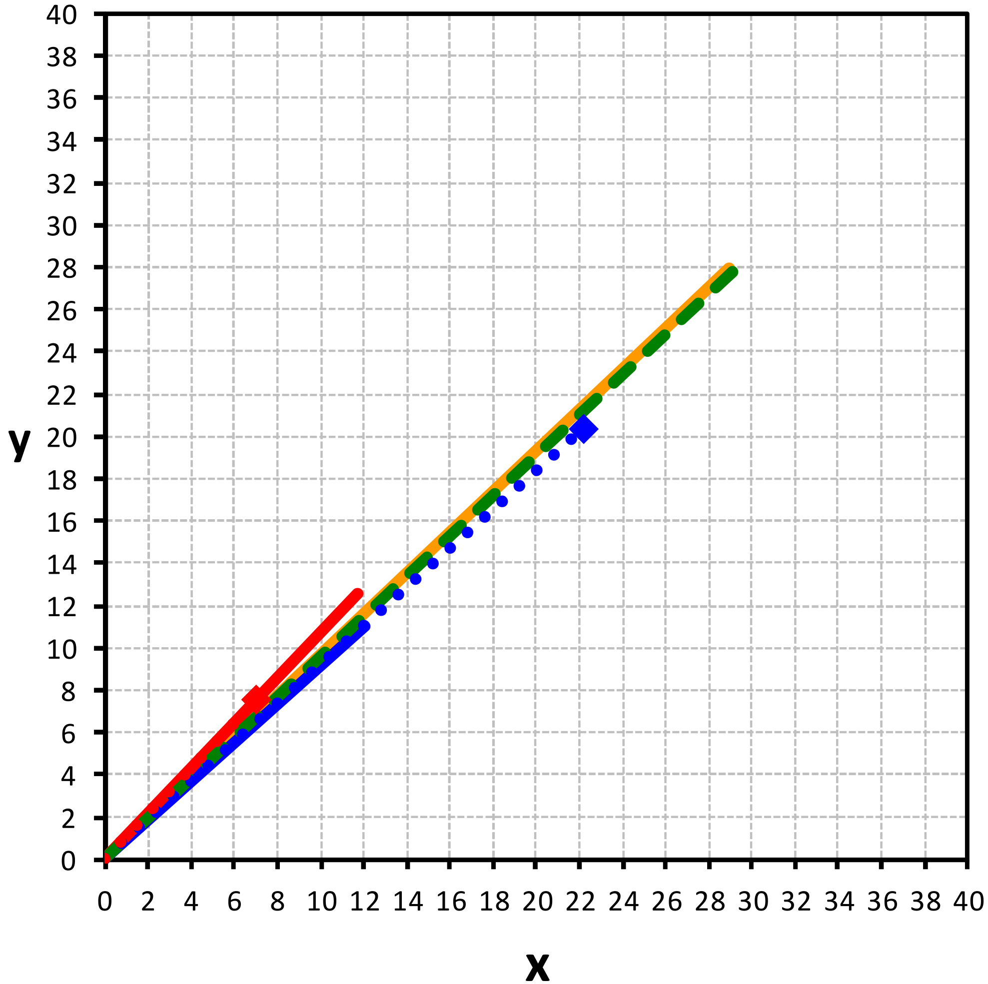
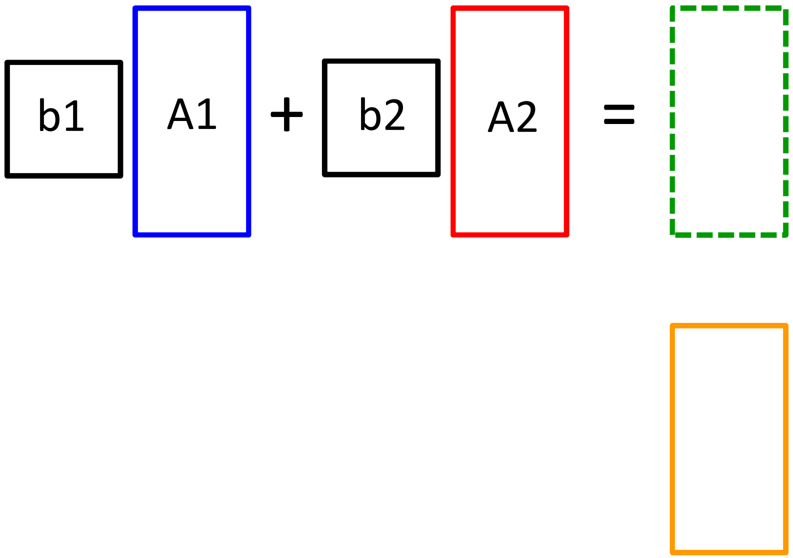
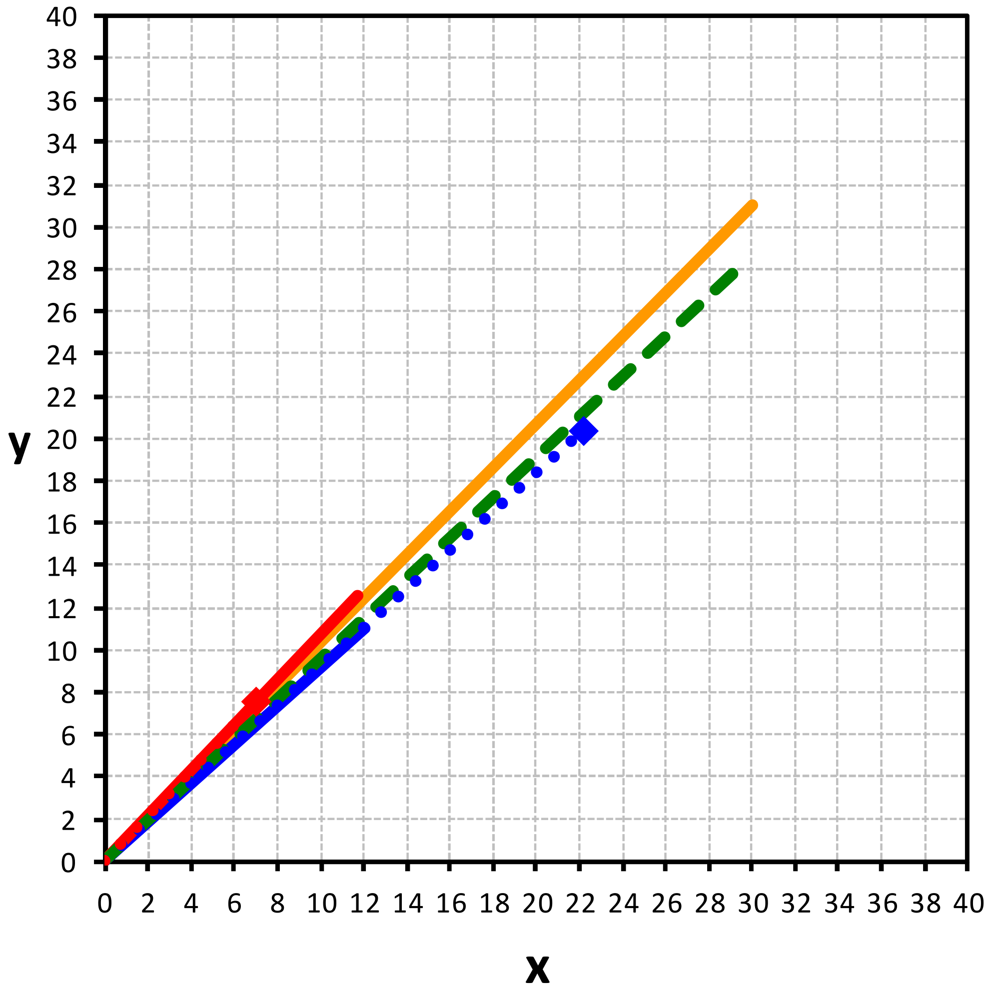




Sistemas lineares

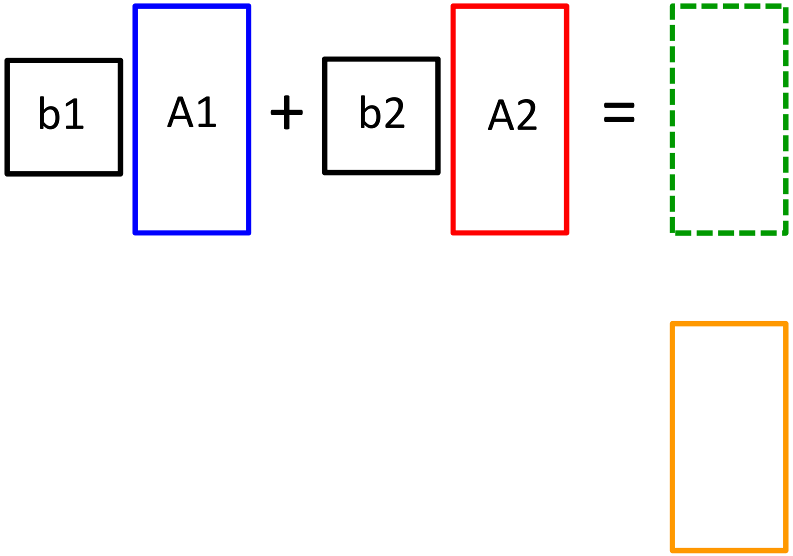
Exemplo 2D

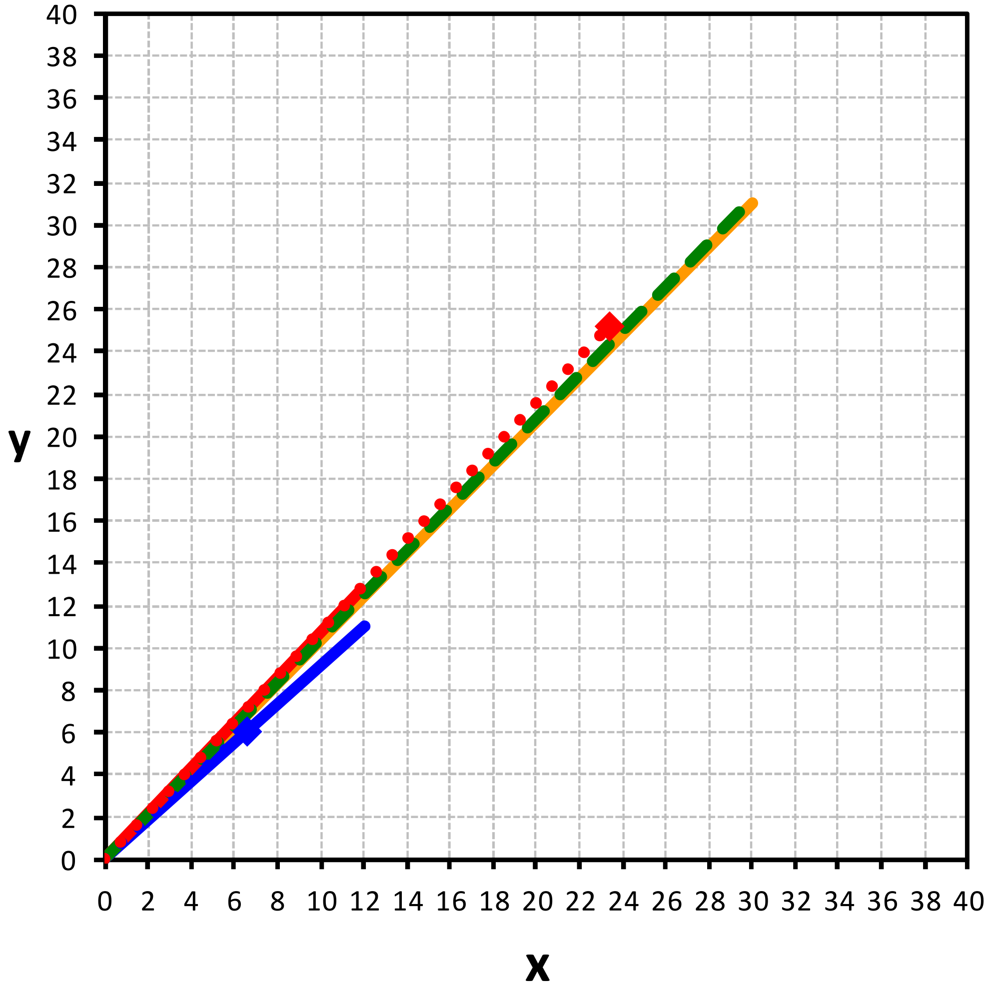




Sistemas lineares

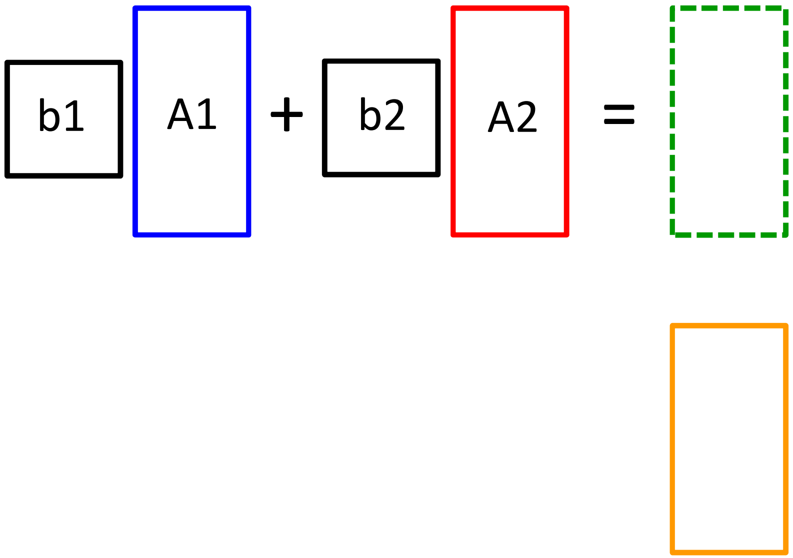
Exemplo 2D

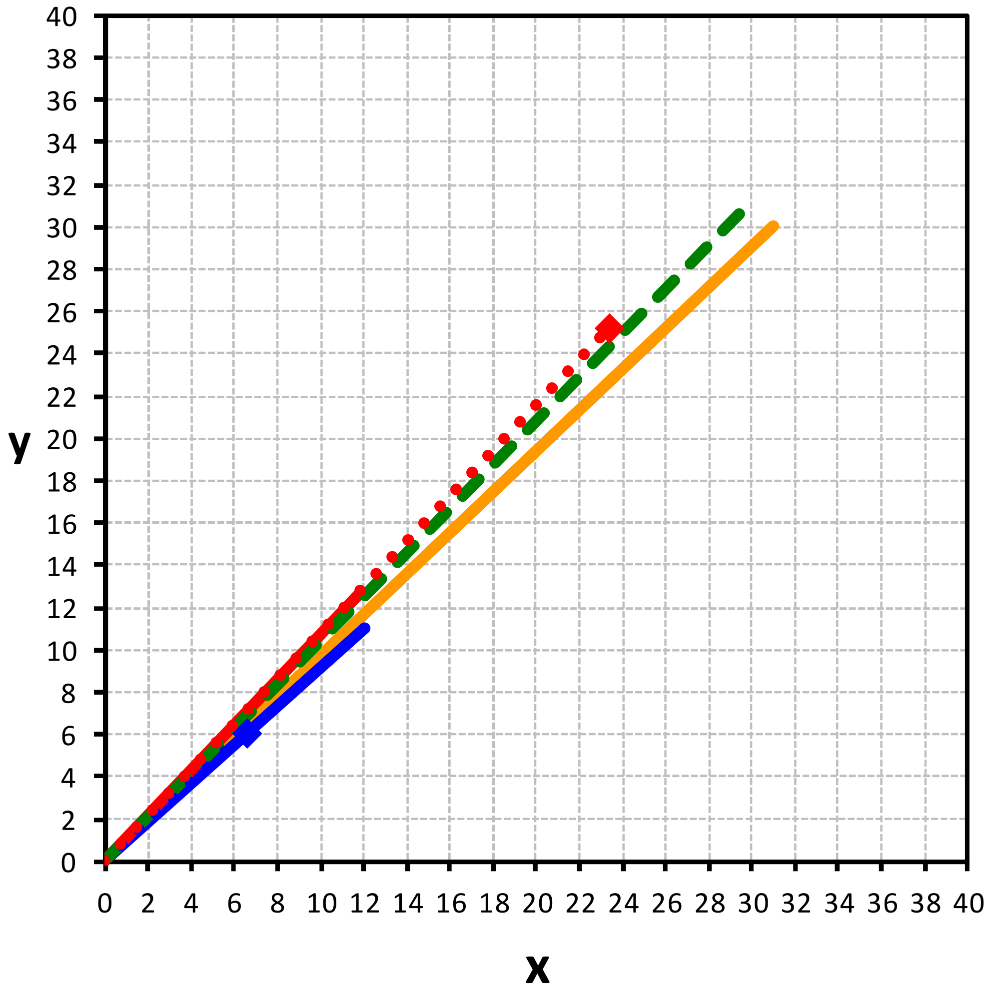




Sistemas lineares

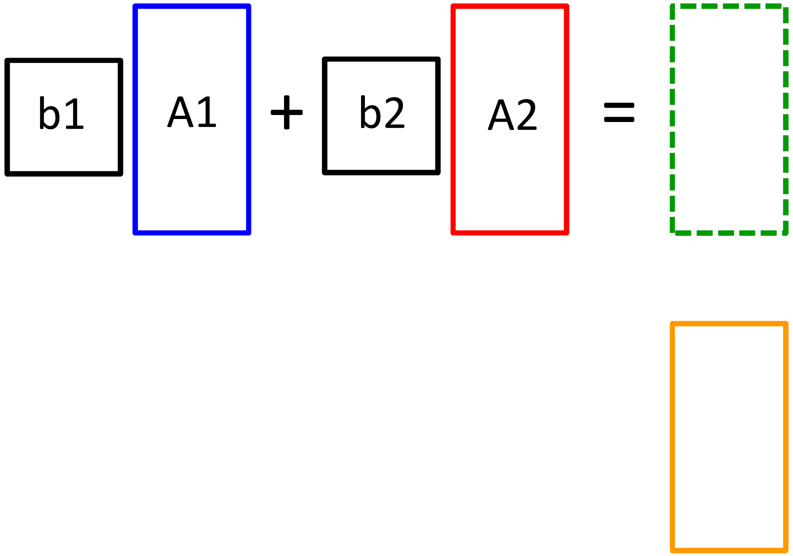
Exemplo 2D

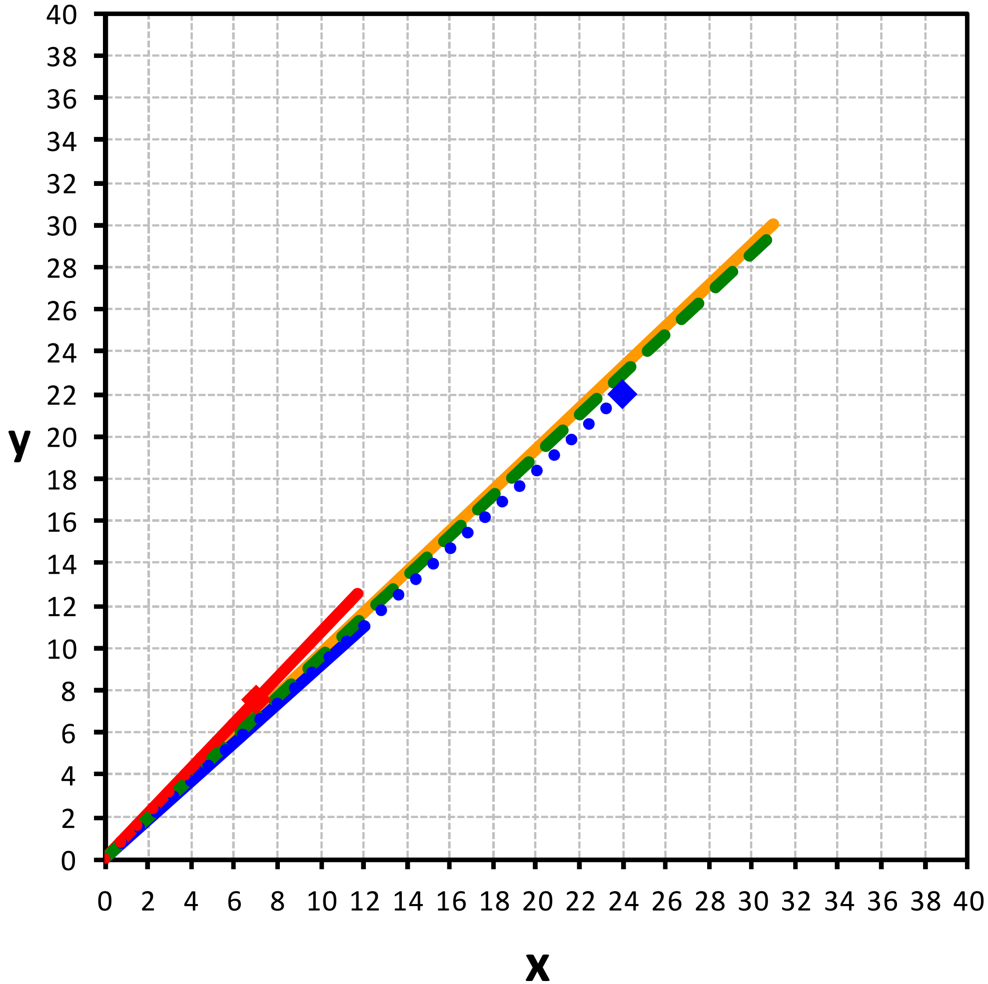




Sistemas lineares

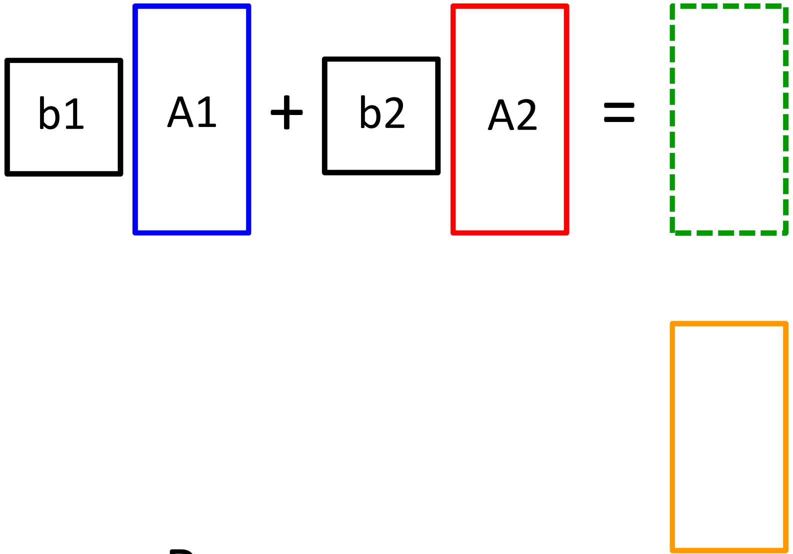
Exemplo 2D

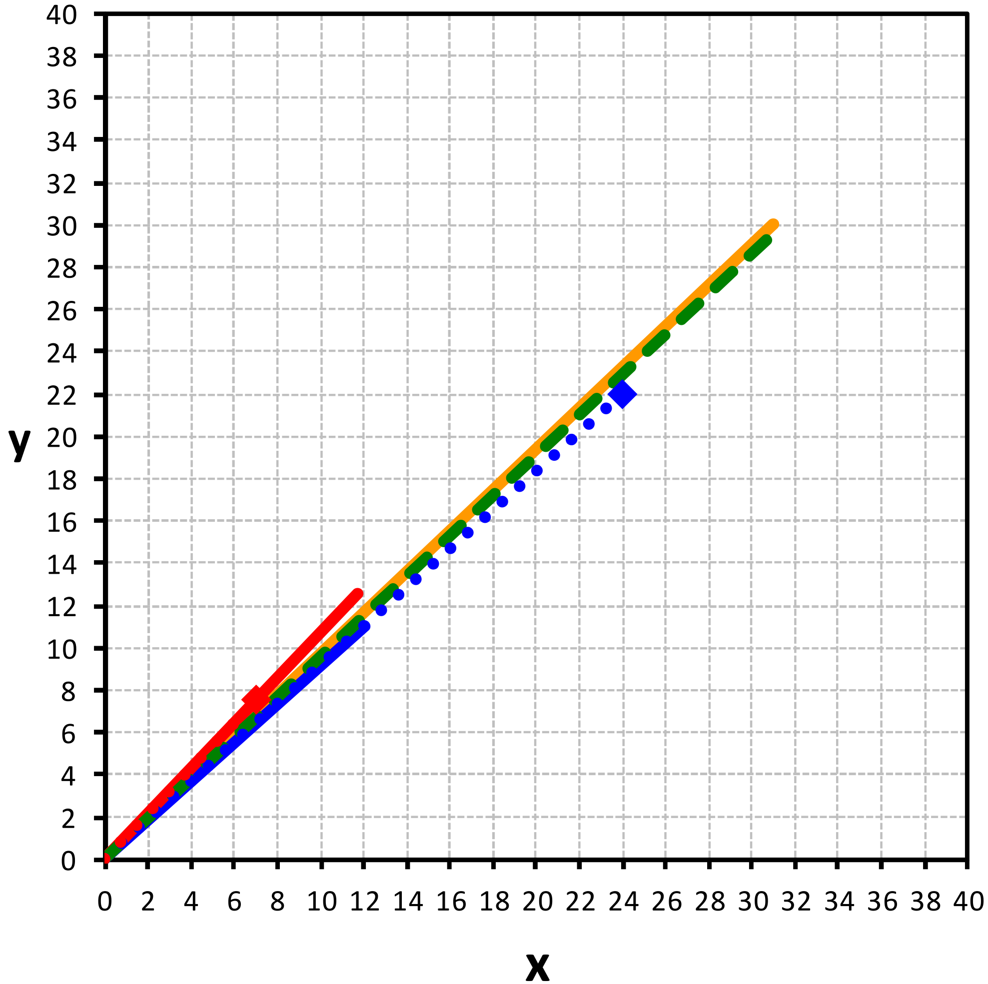




Sistemas lineares

Exemplo 2D

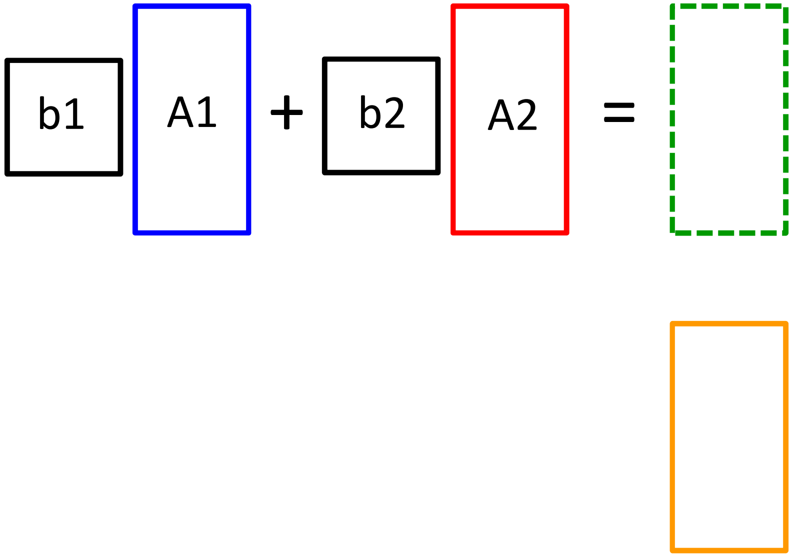


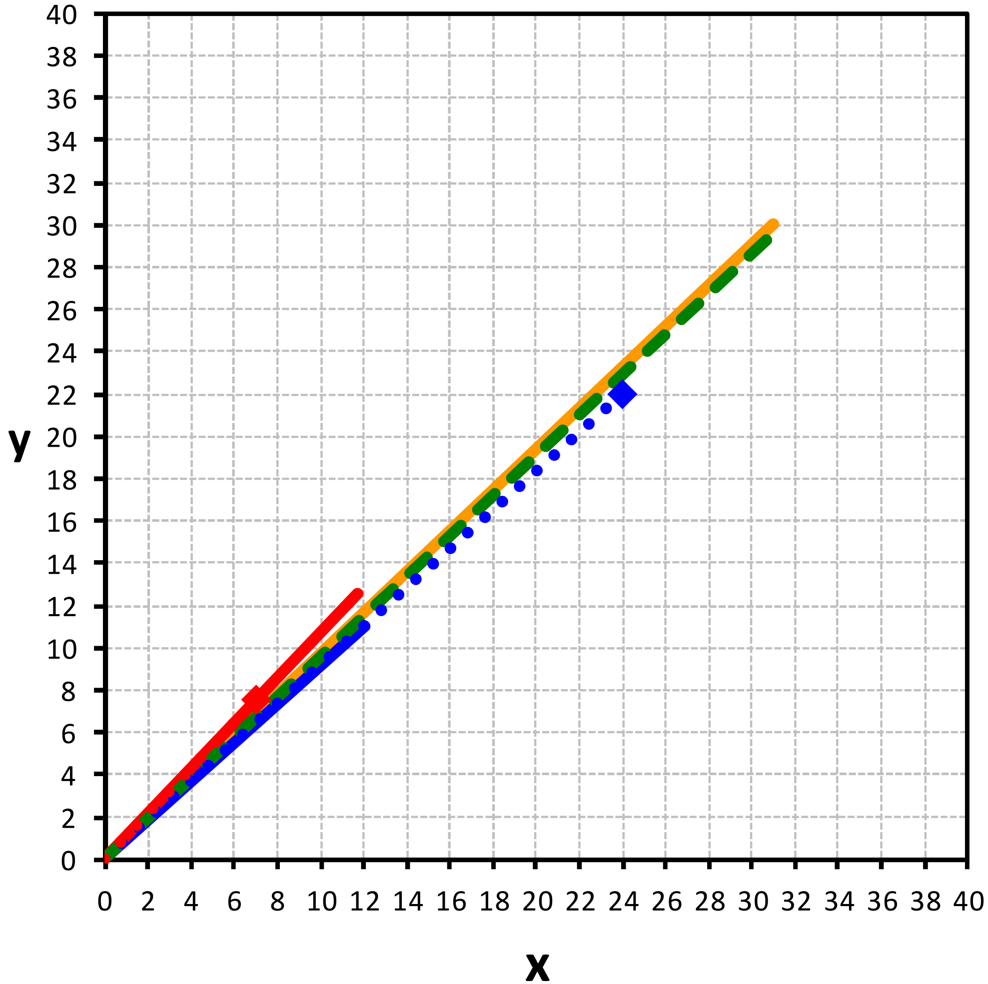


Pequenas perturbações do lado direito causam grandes perturbações nos coeficientes b1 e b2

Sistemas lineares

Exemplo 2D

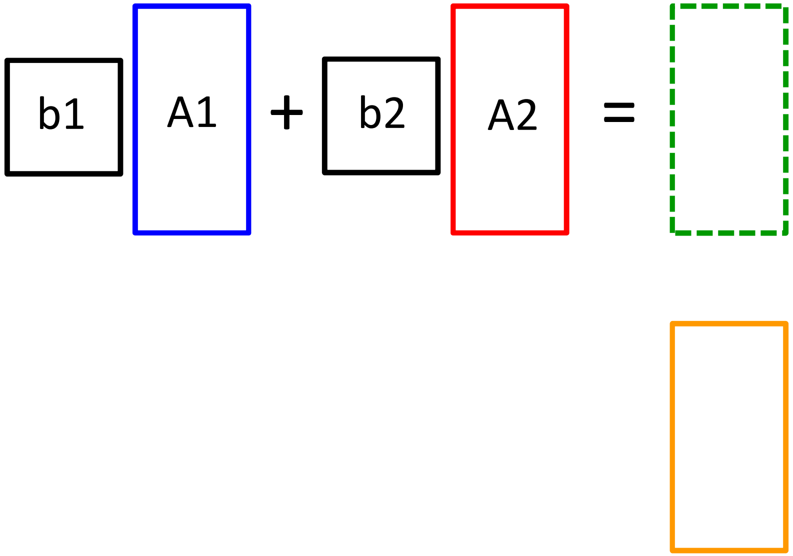


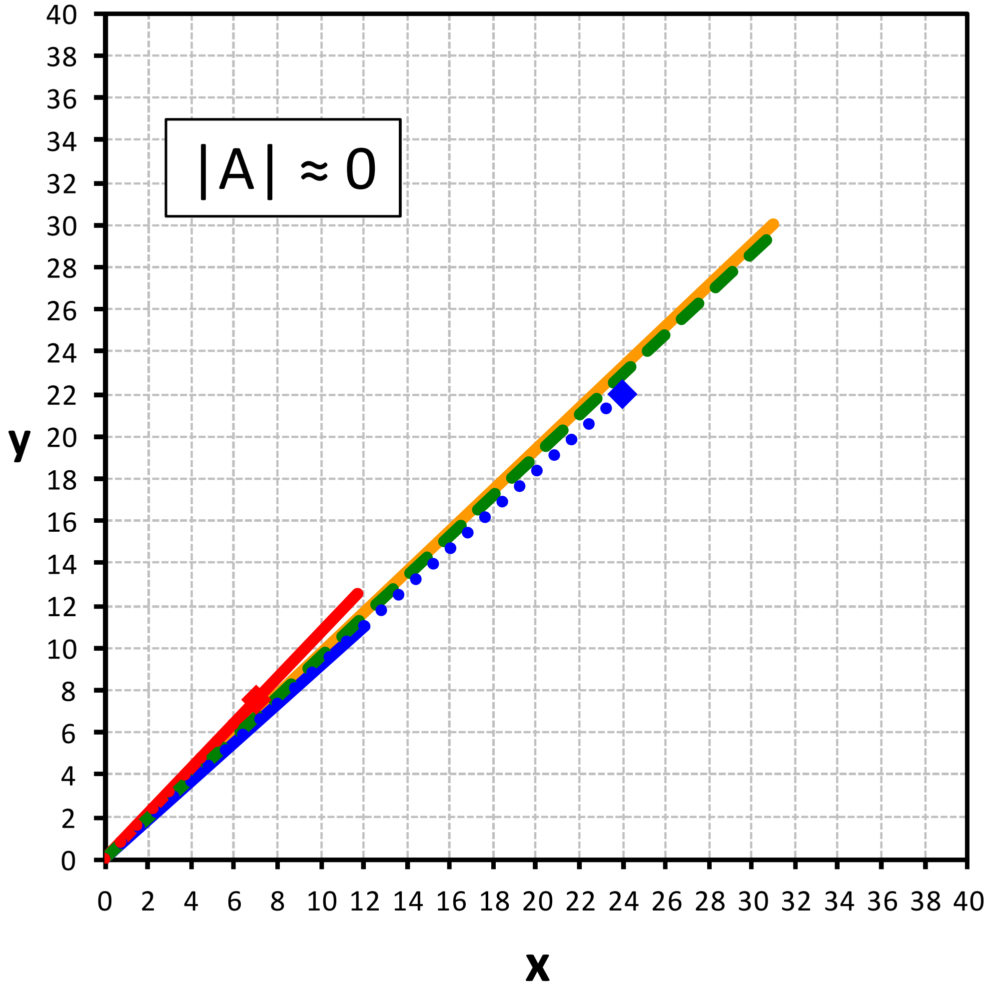


Nesse caso, diz-se que o sistema linear é *instável*

Sistemas lineares

Exemplo 2D

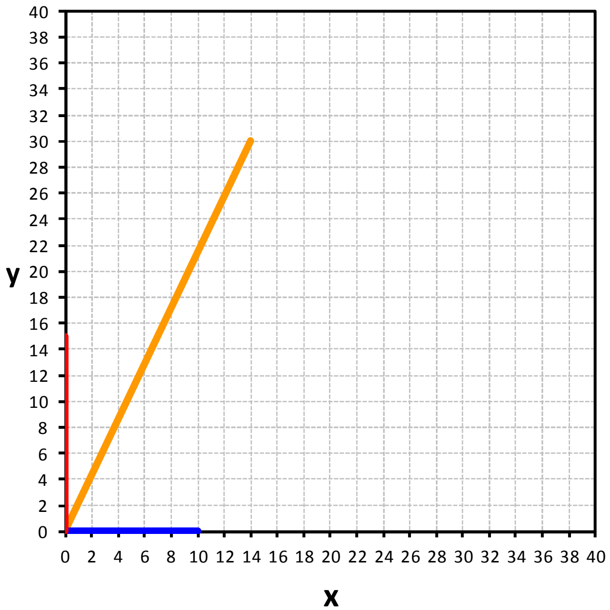




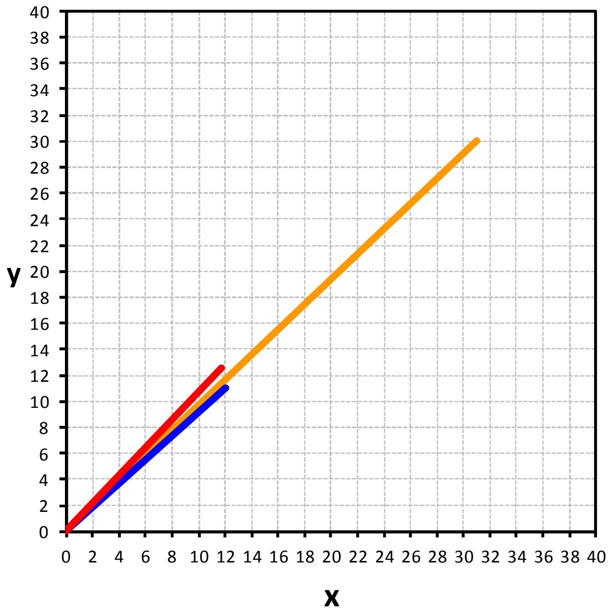
Nesse caso, diz-se que o sistema linear é *instável*

Sistemas lineares

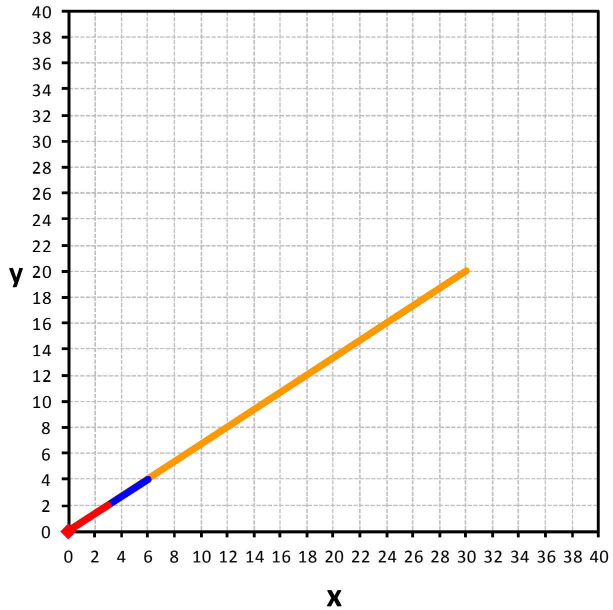
$|A| \neq 0$

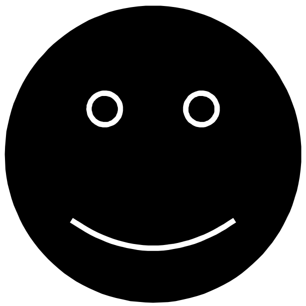


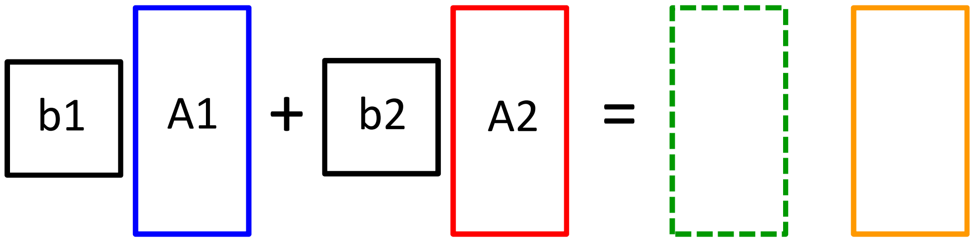
$|A| \approx 0$



$|A| = 0$







Problemas lineares

$$\bar{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}_{M\times1}$$

parâmetros

$$\bar{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}_{N\times1} \qquad \bar{g}(\bar{p}) = \begin{bmatrix} g_1(\bar{p}) \\ \vdots \\ g_N(\bar{p}) \end{bmatrix}_{N\times1}$$

dados observados — dados preditos

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

norma L2
(função escalar)

$$\bar{\nabla}\phi(\bar{p}^*) = \bar{0}_{M\times1}$$

Problema linear

$$\bar{g}(\bar{p}) = \bar{\bar{B}}\bar{p} + \bar{b}$$

$$\bar{p}^* = \left(\bar{\bar{B}}^T \bar{\bar{B}}\right)^{-1} \bar{\bar{B}}^T [\bar{d} - \bar{b}]$$

Problema não-linear

$$\bar{g}(\bar{p}) \neq \bar{\bar{B}}\bar{p} + \bar{b}$$

$$\Delta\bar{p} = \left(\bar{\bar{G}}(\bar{p}_0)^T \bar{\bar{G}}(\bar{p}_0)\right)^{-1} \bar{\bar{G}}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Problemas lineares

$$\phi(\overline{p}) = [\overline{d} - \overline{g}(\overline{p})]^T [\overline{d} - \overline{g}(\overline{p})]$$

$$\overline{g}(\overline{p}) = \overline{\overline{B}}\,\overline{p} + \overline{b}$$

$$\overline{\nabla}\phi(\overline{p}^*) = \overline{0}_{M\times 1}$$

$$\overline{p}^* = \left(\overline{\overline{B}}^T \overline{\overline{B}}\right)^{-1} \overline{\overline{B}}^T [\overline{d} - \overline{b}]$$

Problemas lineares

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Problemas lineares

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Sistema linear

Problemas lineares

$$\phi(\overline{p}) = [\overline{d} - \overline{g}(\overline{p})]^T [\overline{d} - \overline{g}(\overline{p})]$$

$$\overline{g}(\overline{p}) = \overline{\overline{B}}\,\overline{p} + \overline{b}$$

$$\overline{\nabla}\phi(\overline{p}^*) = \overline{0}_{M\times 1}$$

$$\left(\overline{\overline{B}}^T \overline{\overline{B}}\right)\overline{p}^* = \overline{\overline{B}}^T [\overline{d} - \overline{b}]$$

Sistema linear

Caso 1)

$$\det\left(\overline{\overline{B}}^T \overline{\overline{B}}\right) \neq 0$$

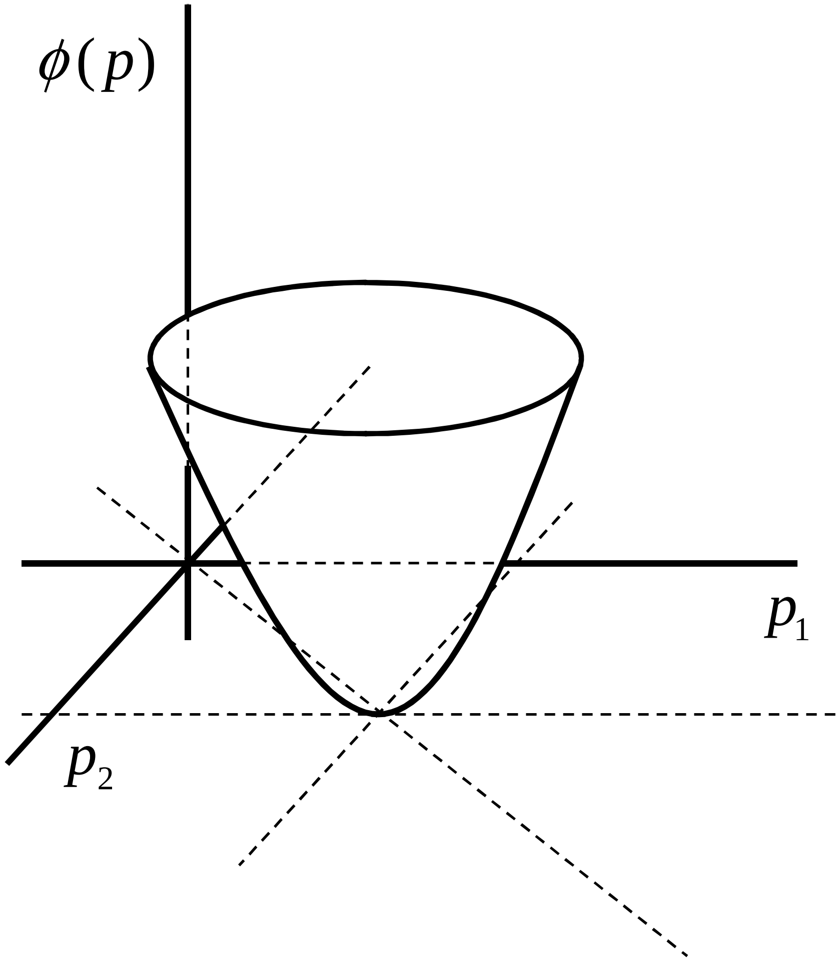
Caso 2)

$$\det\left(\overline{\overline{B}}^T \overline{\overline{B}}\right) = 0$$

Caso 3)

$$\det\left(\overline{\overline{B}}^T \overline{\overline{B}}\right) \approx 0$$

Problemas lineares



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Caso 2)

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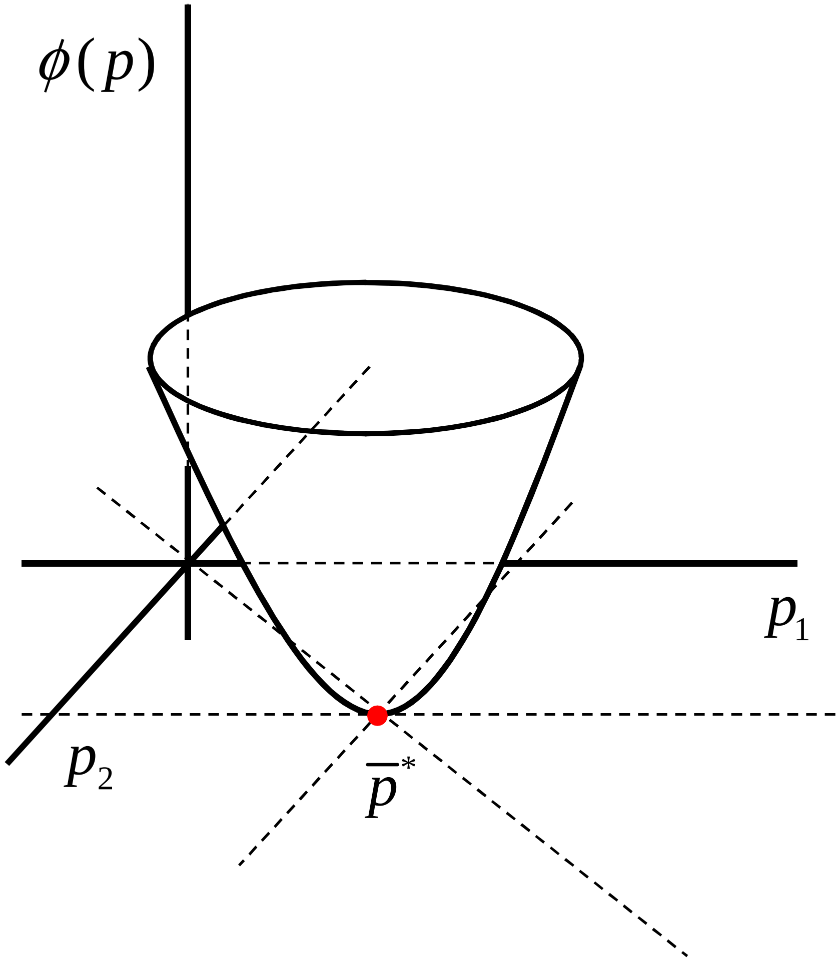
$$\det\left(\overline{\overline{B}}^T\,\overline{\overline{B}}\right)\approx 0$$

Problemas lineares



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Caso 3)

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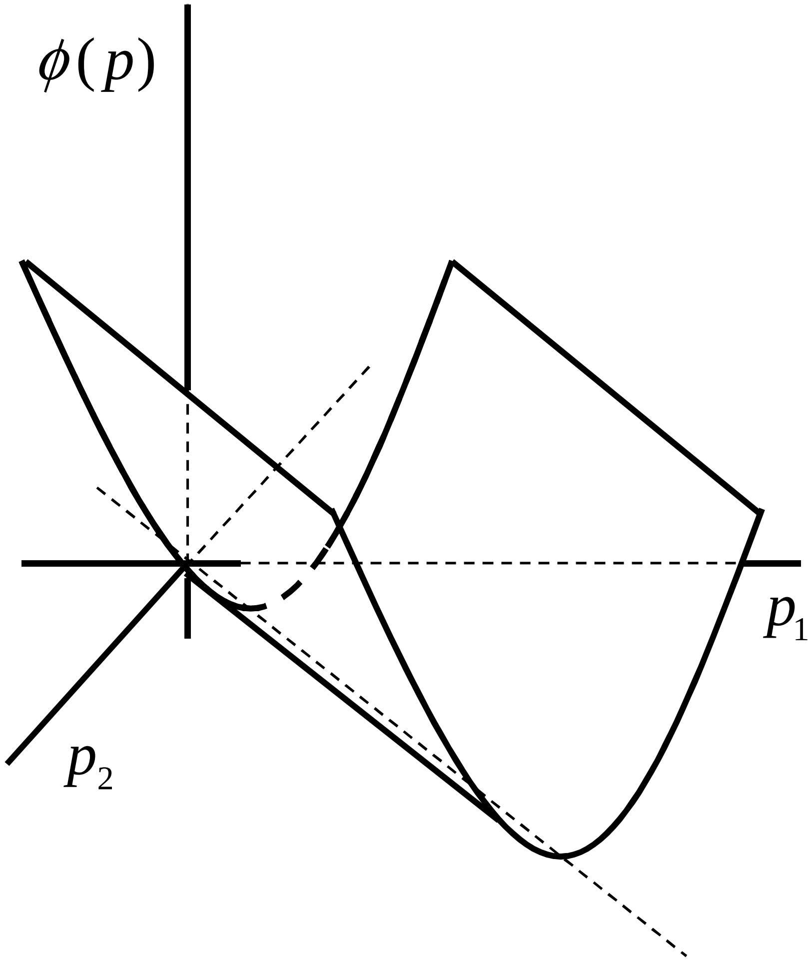
Problemas lineares



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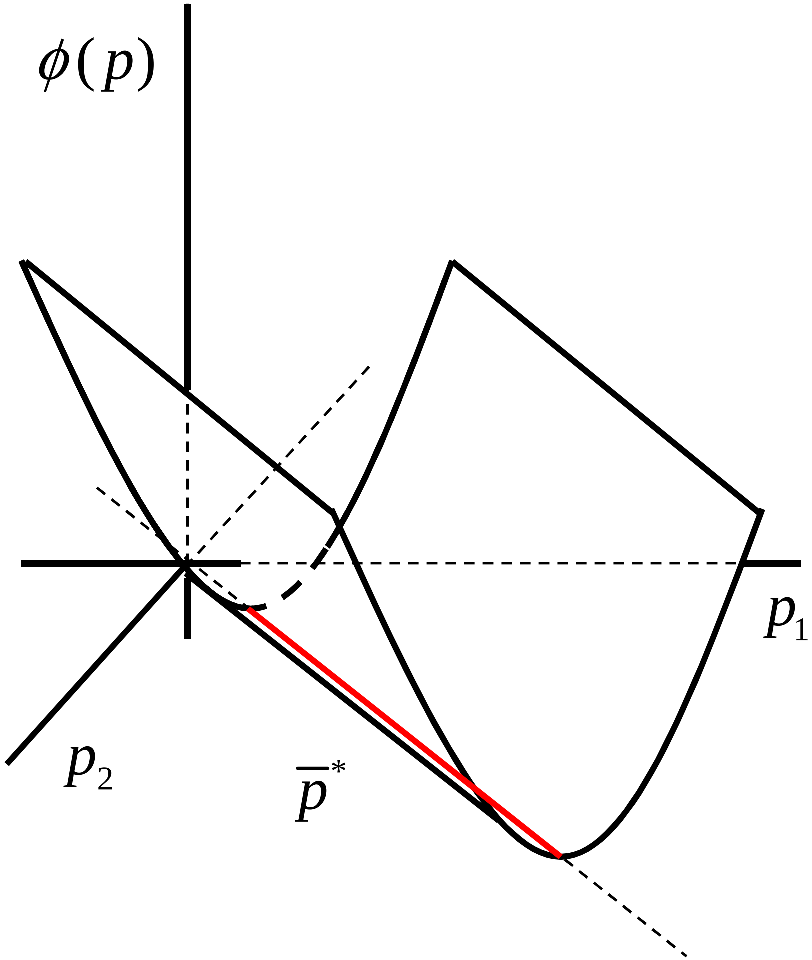
Problemas lineares



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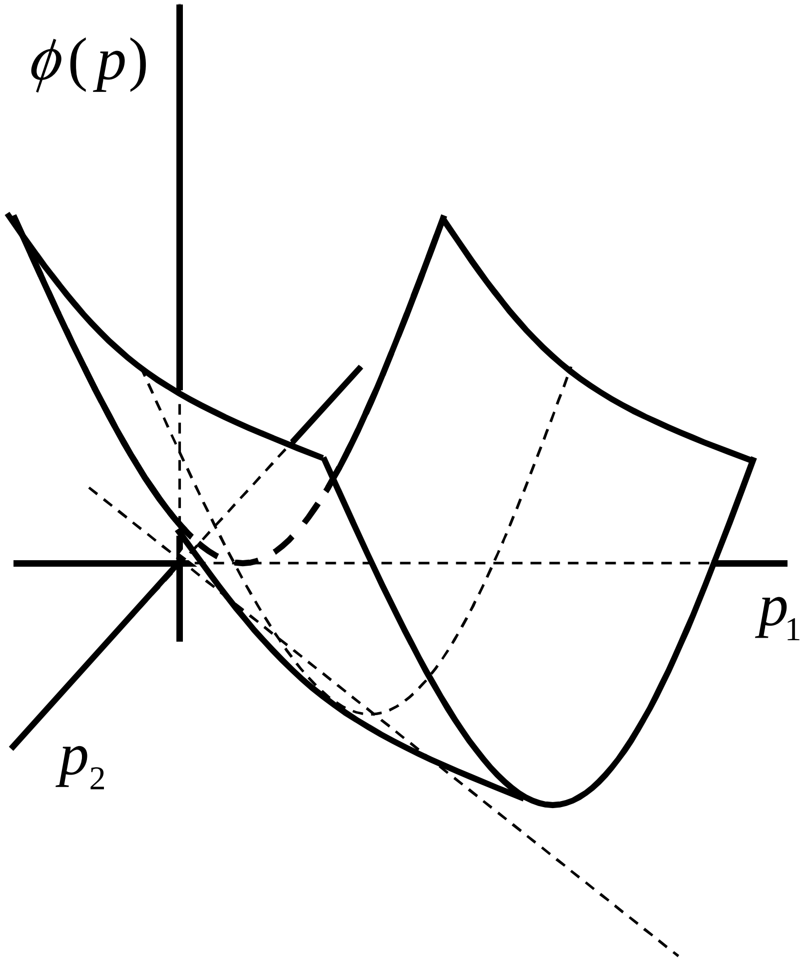
Problemas lineares



Caso 1)

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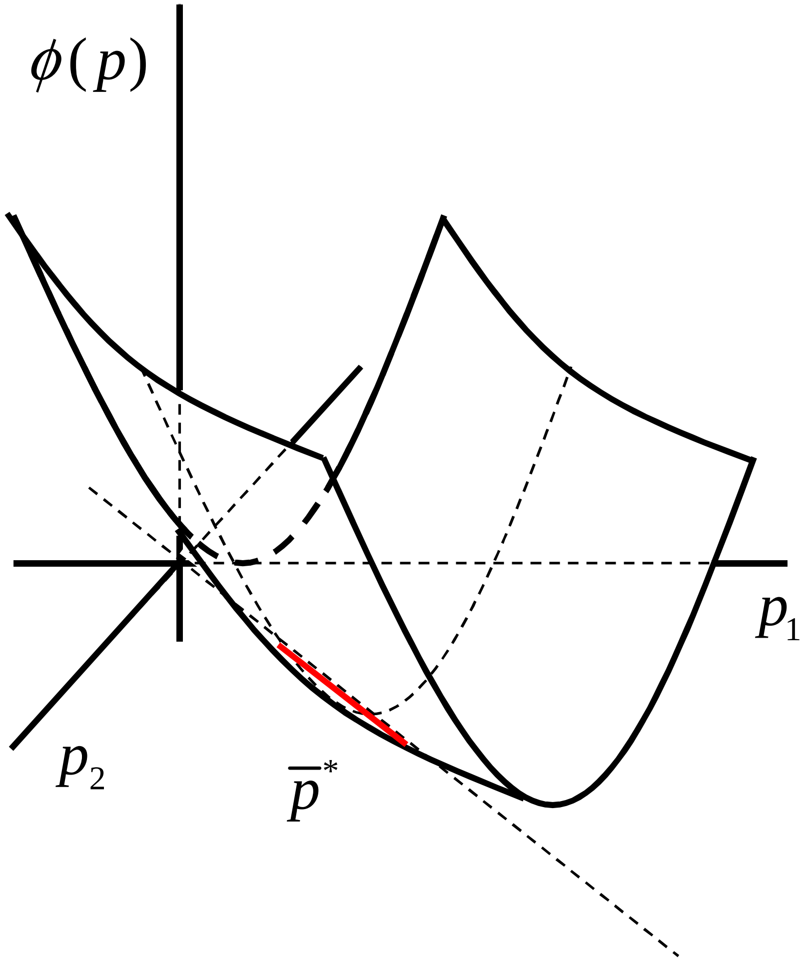
Caso 2)

$$\det\left(\overline{\overline{B}}^T \overline{\overline{B}}\right) = 0$$

Caso 3)

$$\det\left(\overline{\overline{B}}^T \overline{\overline{B}}\right) \approx 0$$

Problemas lineares



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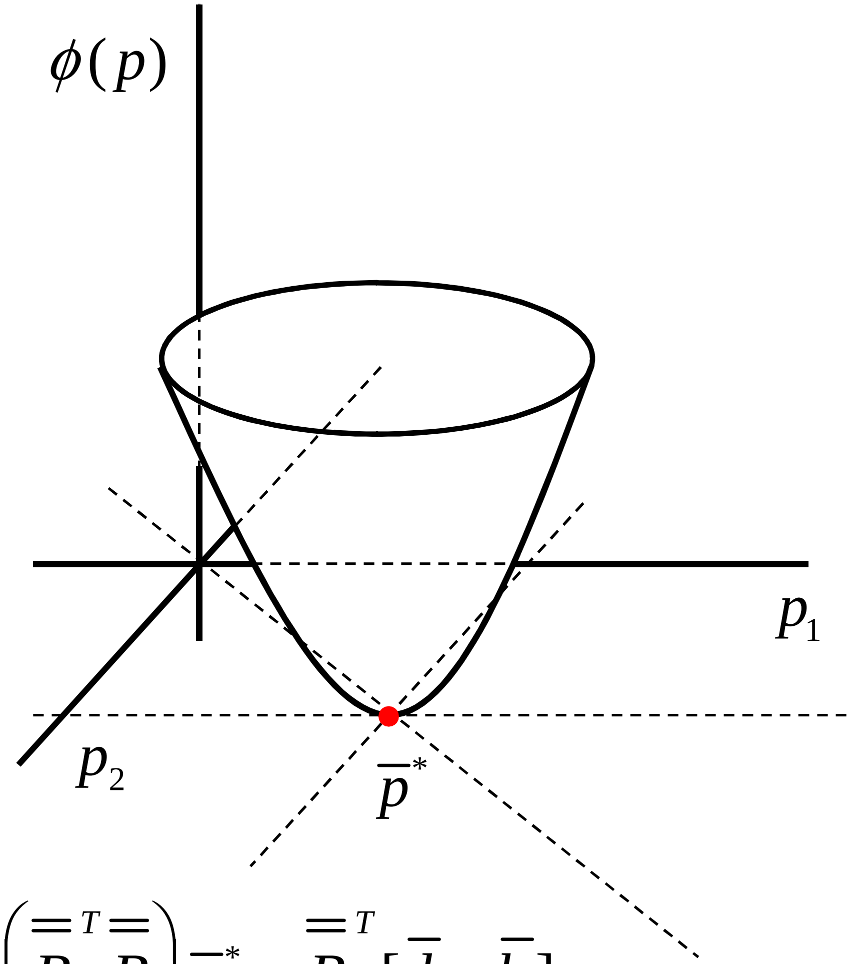
Problemas lineares



$$\left(\overline{\overline{B}}^T \overline{\overline{B}}\right)\overline{p}^* = \overline{\overline{B}}^T [\overline{d} - \overline{b}]$$

Solução única

Caso 1)

$$\det\left(\overline{\overline{B}}^T \overline{\overline{B}}\right) \neq 0$$

Caso 2)

$$\det\left(\overline{\overline{B}}^T \overline{\overline{B}}\right) = 0$$

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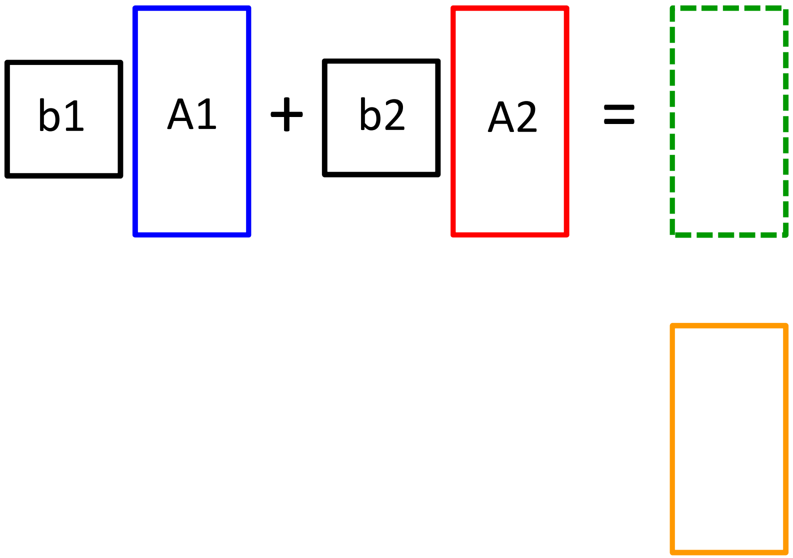
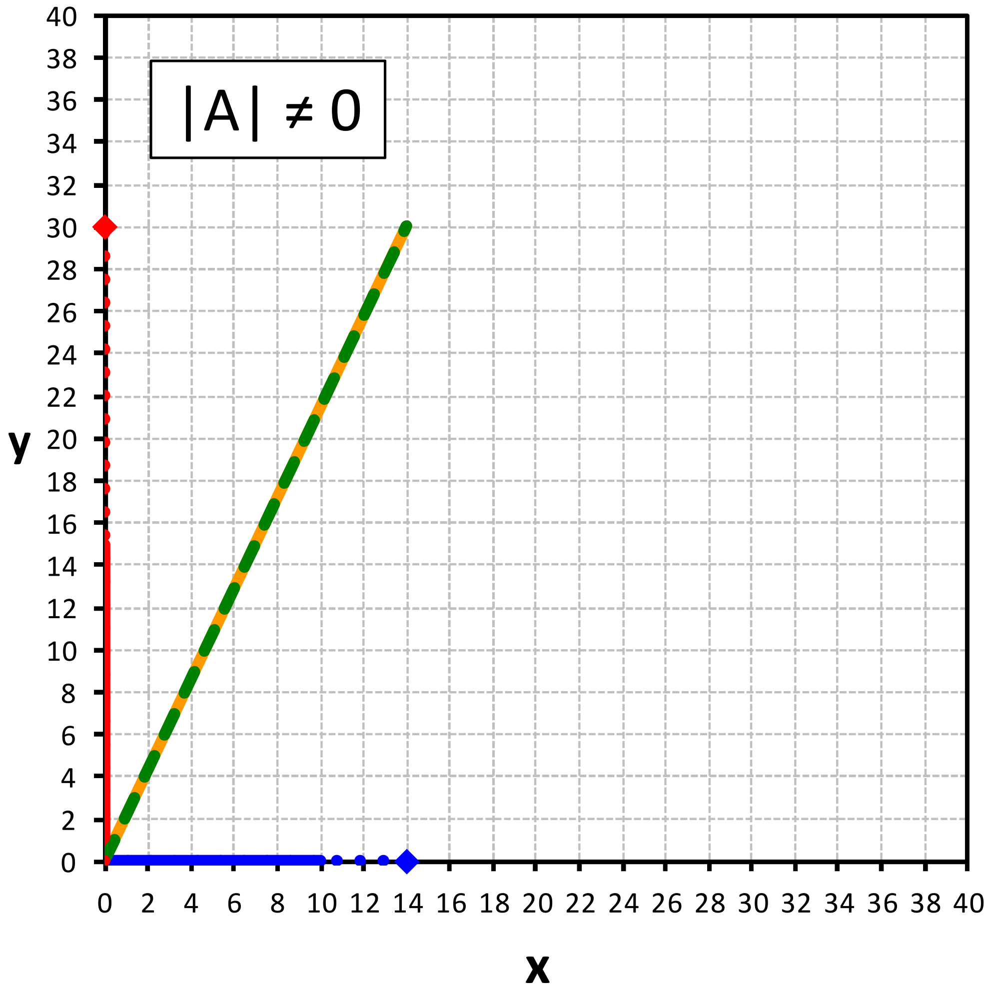
Problemas lineares

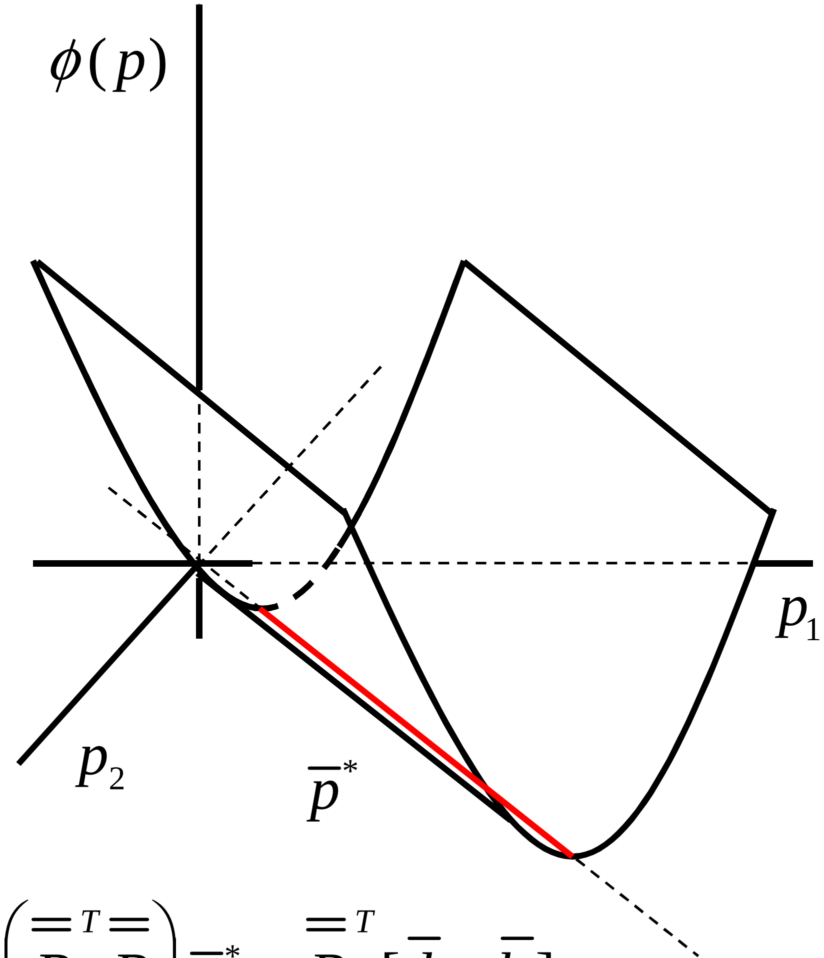
Exemplo 2D





Neste caso, os vetores A1 e A2 são *linearmente independentes* e os coeficientes b1 e b2 são únicos

Problemas lineares



$$\left(\overline{\overline{B}}^T \overline{\overline{B}}\right)\overline{p}^* = \overline{\overline{B}}^T [\overline{d} - \overline{b}]$$

Infinitas soluções

Caso 1)

$$\det\left(\overline{\overline{B}}^T \overline{\overline{B}}\right) \neq 0$$

Caso 2)

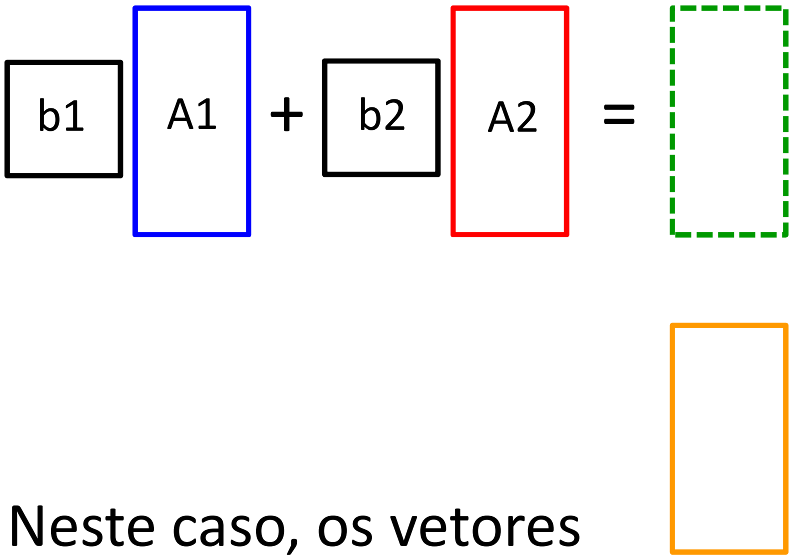
$$\det\left(\overline{\overline{B}}^T \overline{\overline{B}}\right) = 0$$

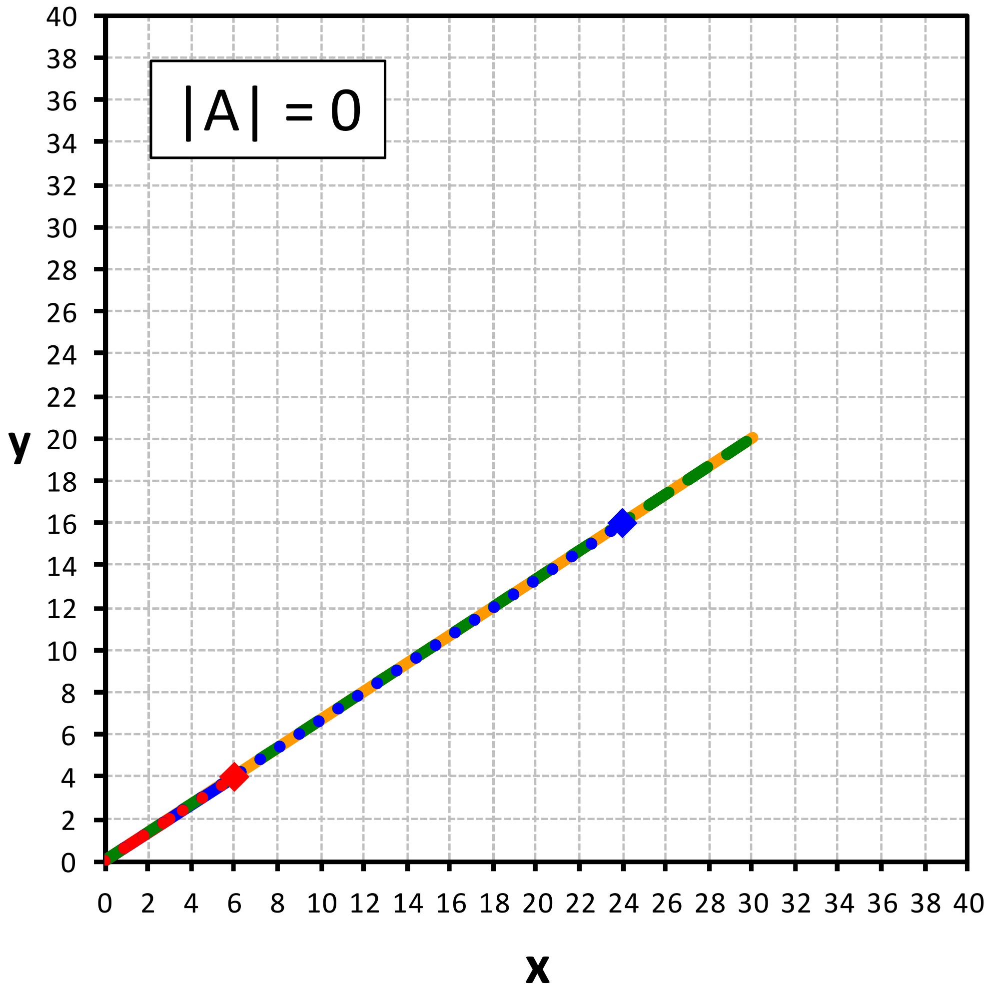
Caso 3)

$$\det\left(\overline{\overline{B}}^T \overline{\overline{B}}\right) \approx 0$$

Problemas lineares

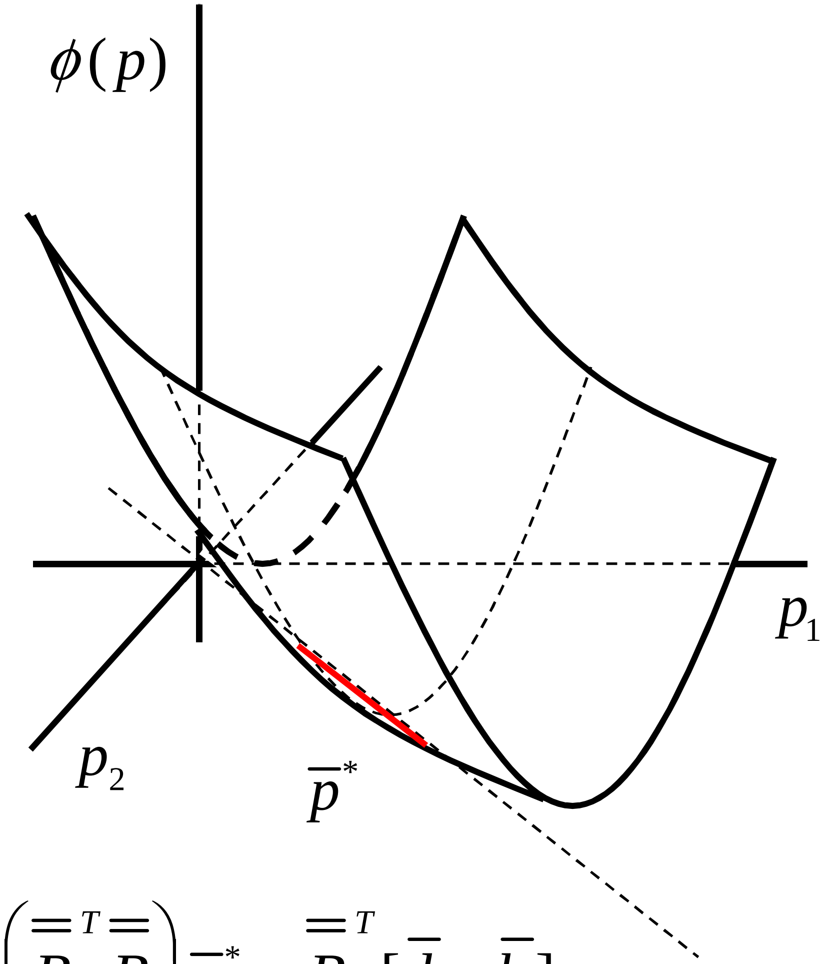
Exemplo 2D





Neste caso, os vetores A1 e A2 são *linearmente dependentes* e existem infinitos pares b1 e b2 que produzem o mesmo resultado

Problemas lineares



$$\left(\overline{\overline{B}}^T \overline{\overline{B}}\right)\overline{p}^* = \overline{\overline{B}}^T [\overline{d} - \overline{b}]$$

Sistema instável

Caso 1)

$$\det\left(\overline{\overline{B}}^T \overline{\overline{B}}\right) \neq 0$$

Caso 2)

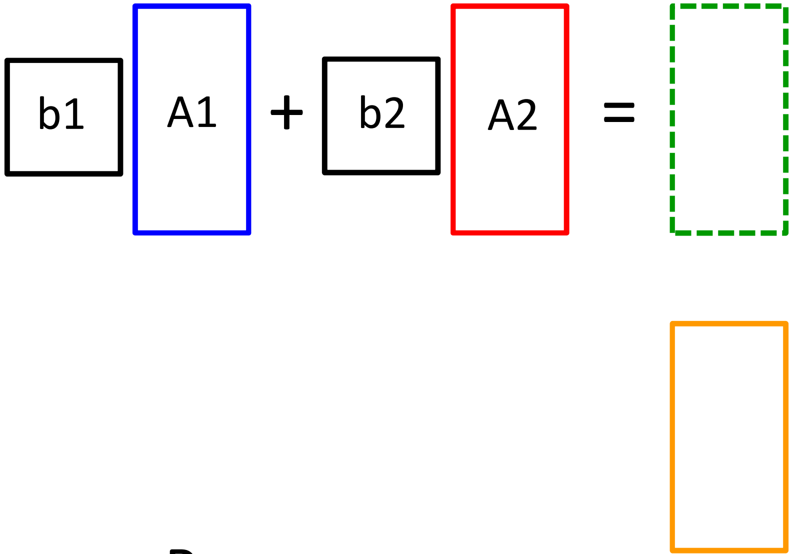
$$\det\left(\overline{\overline{B}}^T \overline{\overline{B}}\right) = 0$$

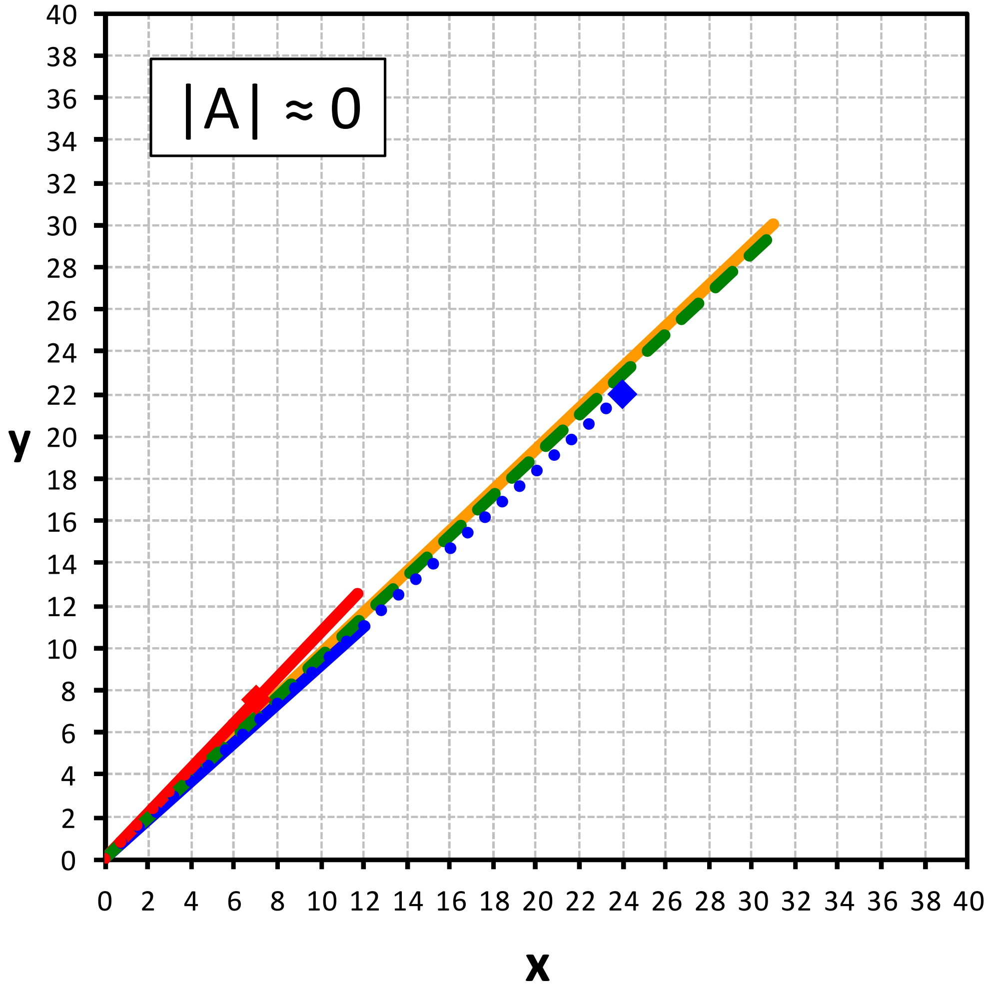
Caso 3)

$$\det\left(\overline{\overline{B}}^T \overline{\overline{B}}\right) \approx 0$$

Problemas lineares

Exemplo 2D





Pequenas perturbações do lado direito causam grandes perturbações nos coeficientes b1 e b2

Problemas não-lineares

$$\bar{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}_{M\times 1}$$

parâmetros

$$\bar{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}_{N\times 1}$$

dados
observados

$$\bar{g}(\bar{p}) = \begin{bmatrix} g_1(\bar{p}) \\ \vdots \\ g_N(\bar{p}) \end{bmatrix}_{N\times 1}$$

dados
preditos

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

norma L2
(função escalar)

$$\bar{\nabla}\phi(\bar{p}^*) = \bar{0}_{M\times 1}$$

Problema linear

$$\bar{g}(\bar{p}) = \bar{\bar{B}}\bar{p} + \bar{b}$$

$$\bar{p}^* = \left(\bar{\bar{B}}^T \bar{\bar{B}}\right)^{-1} \bar{\bar{B}}^T [\bar{d} - \bar{b}]$$

Problema não-linear

$$\bar{g}(\bar{p}) \neq \bar{\bar{B}}\bar{p} + \bar{b}$$

$$\Delta\bar{p} = \left(\bar{\bar{G}}(\bar{p}_0)^T \bar{\bar{G}}(\bar{p}_0)\right)^{-1} \bar{\bar{G}}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Problemas não-lineares

$$\phi(\overline{p}) = [\overline{d} - \overline{g}(\overline{p})]^T [\overline{d} - \overline{g}(\overline{p})]$$

$$\overline{g}(\overline{p}) \neq \overline{\overline{B}}\,\overline{p} + \overline{b}$$

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$$\overline{p} = \overline{p}_0 + \Delta\overline{p}$$

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Sistema linear

Problemas não-lineares

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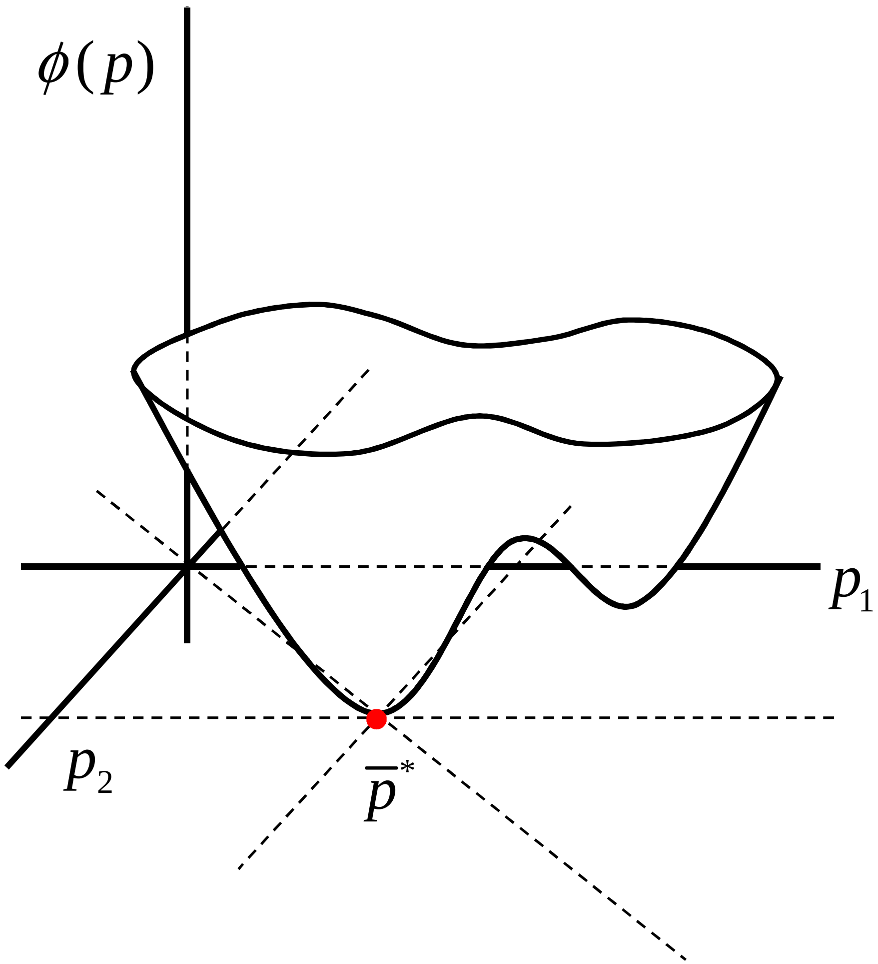
Sistema linear



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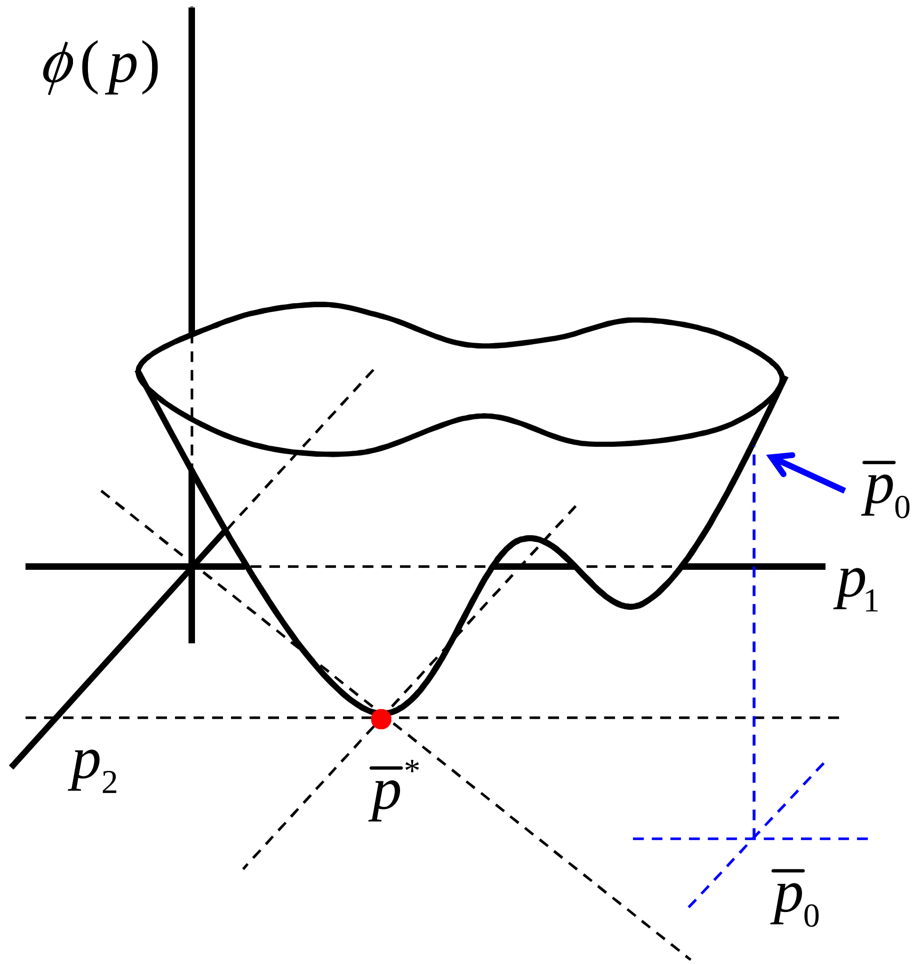

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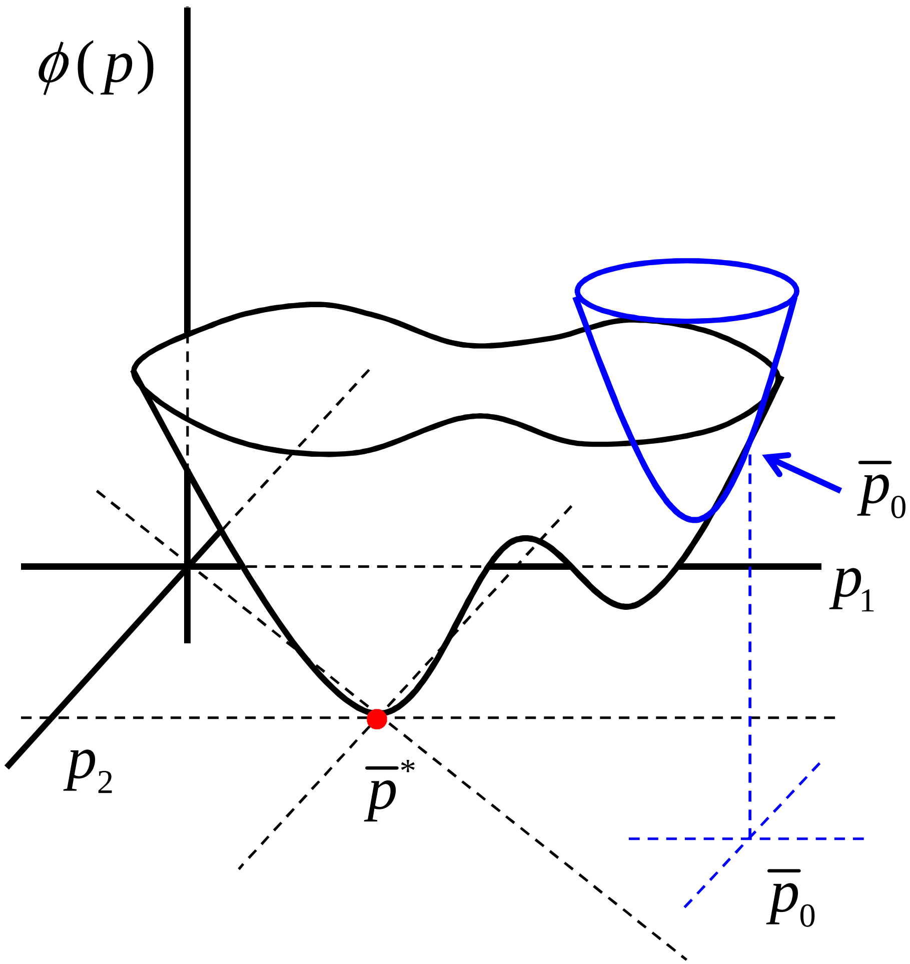

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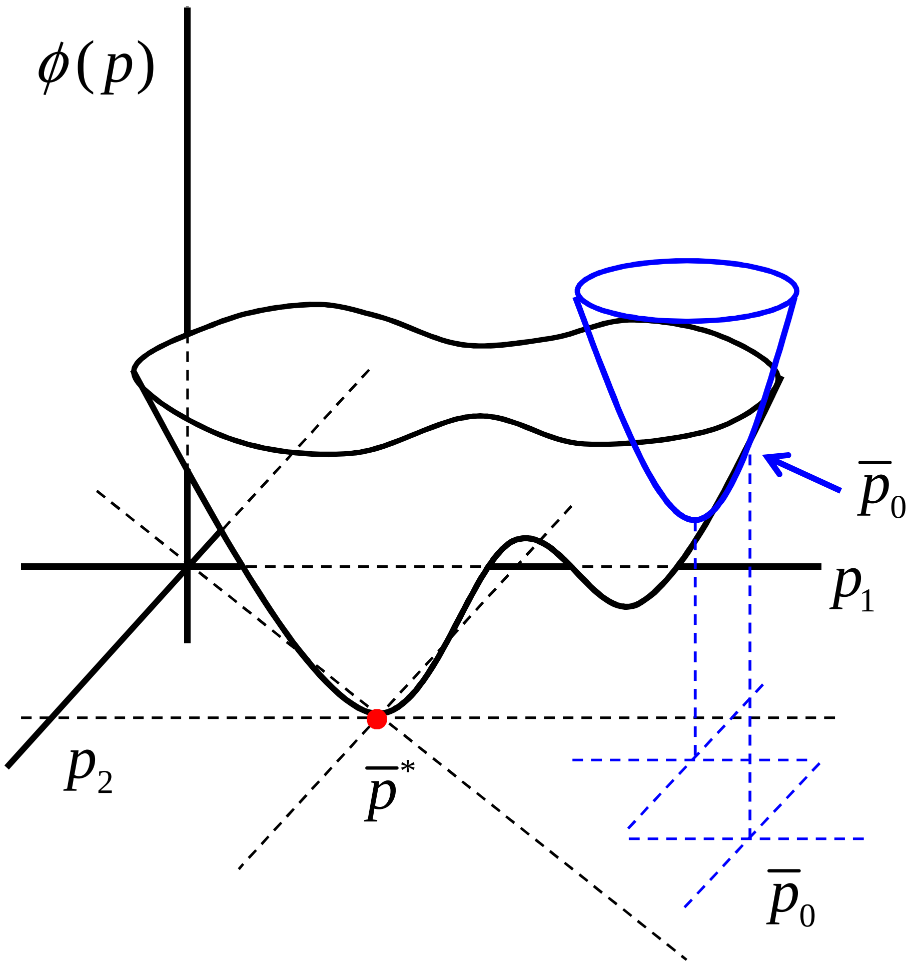

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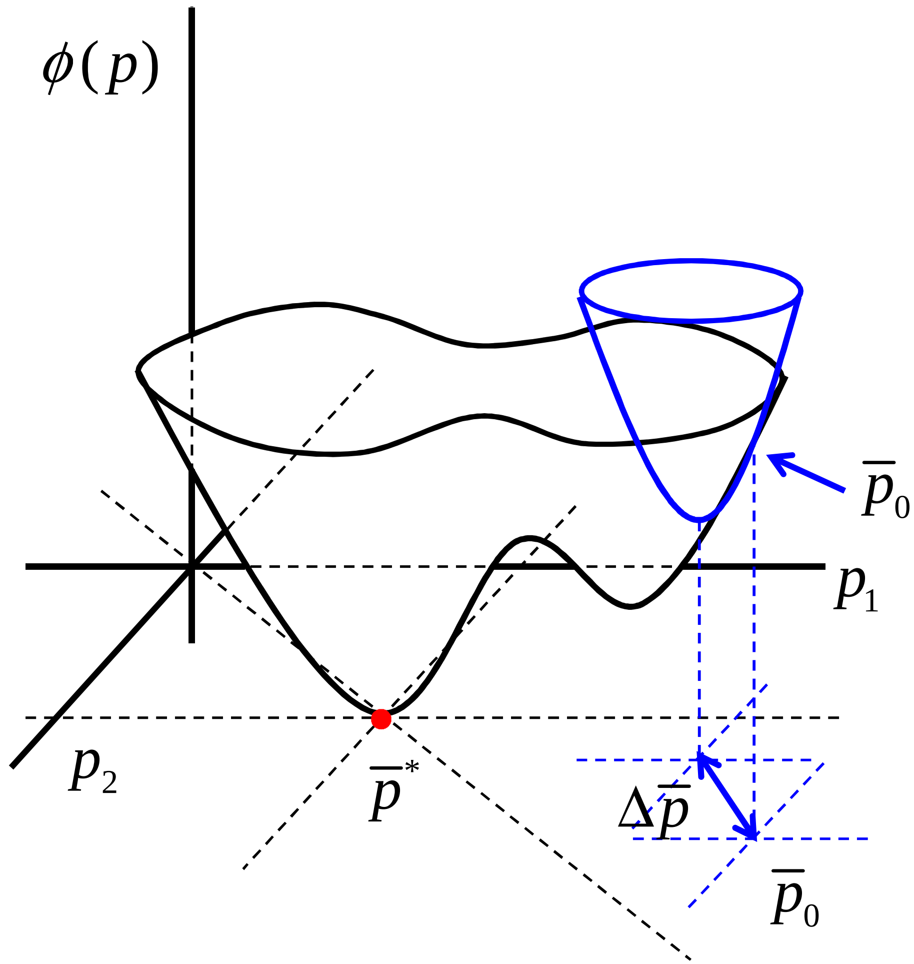

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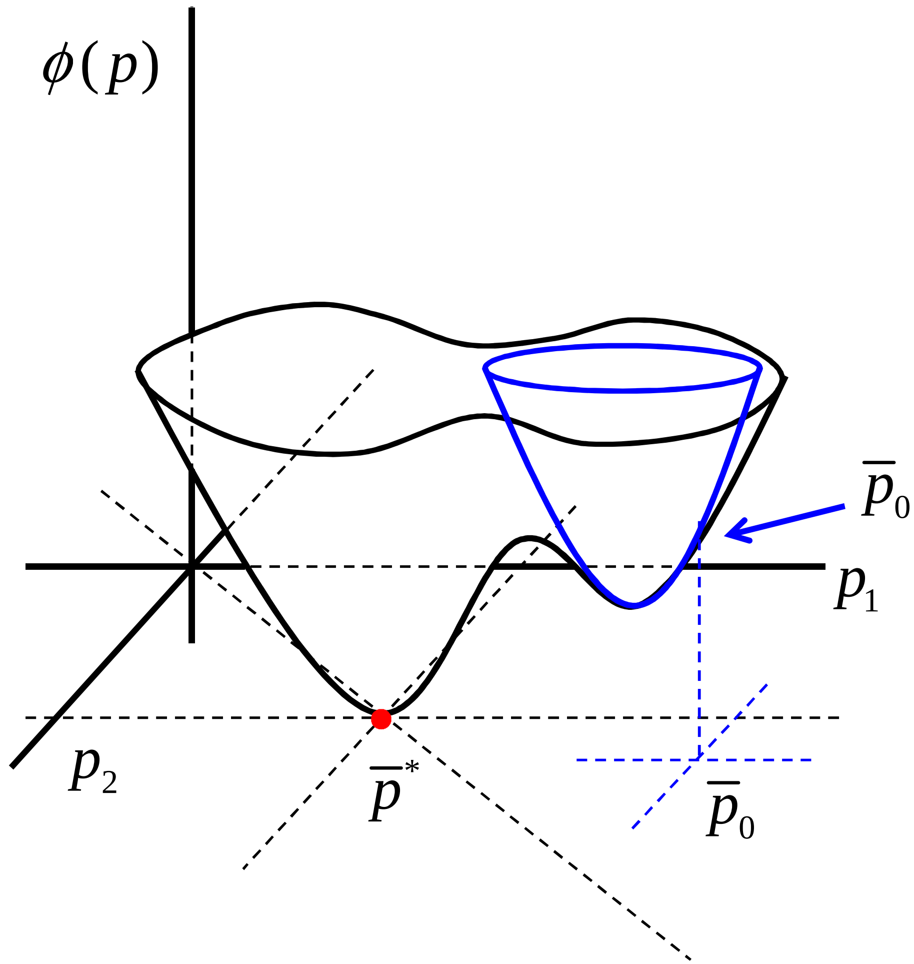

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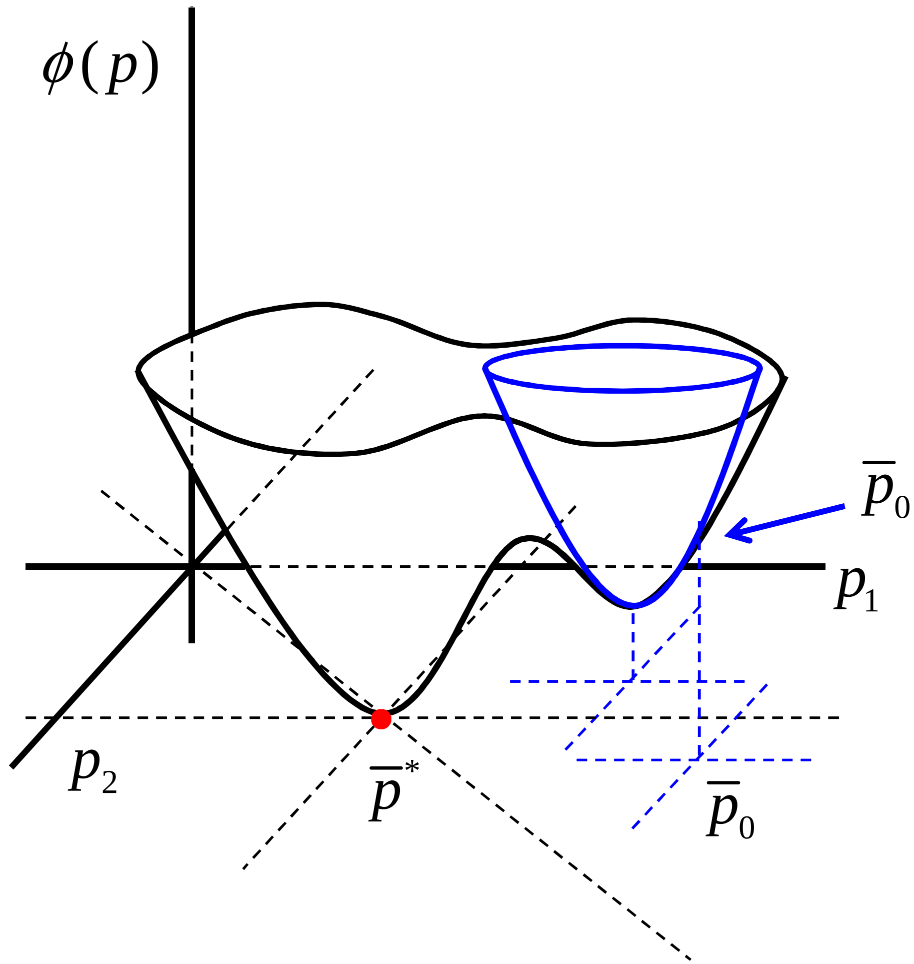

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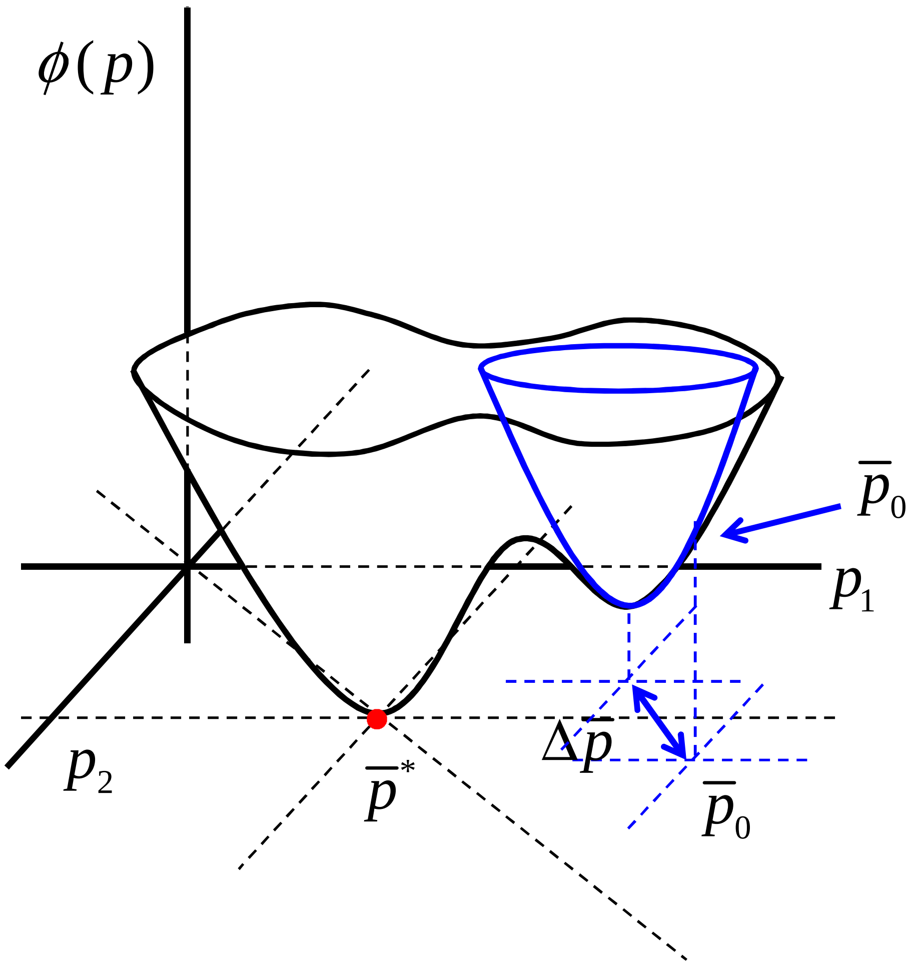
Sistema linear



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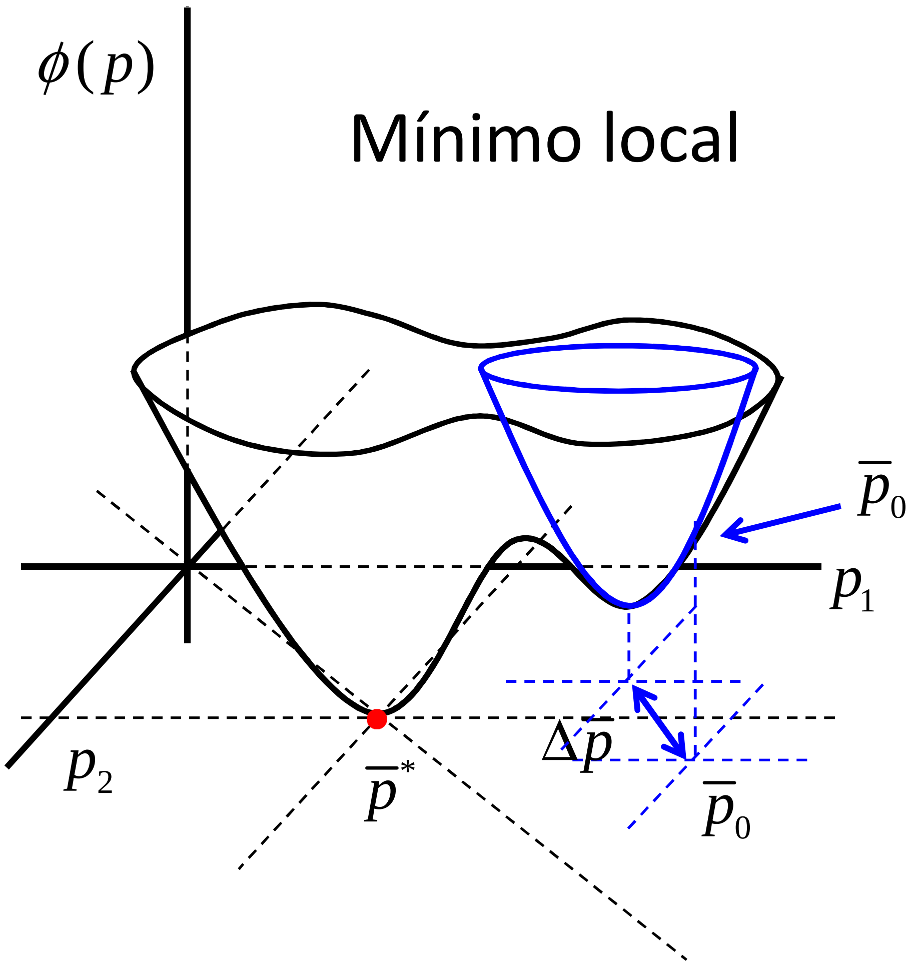
Sistema linear



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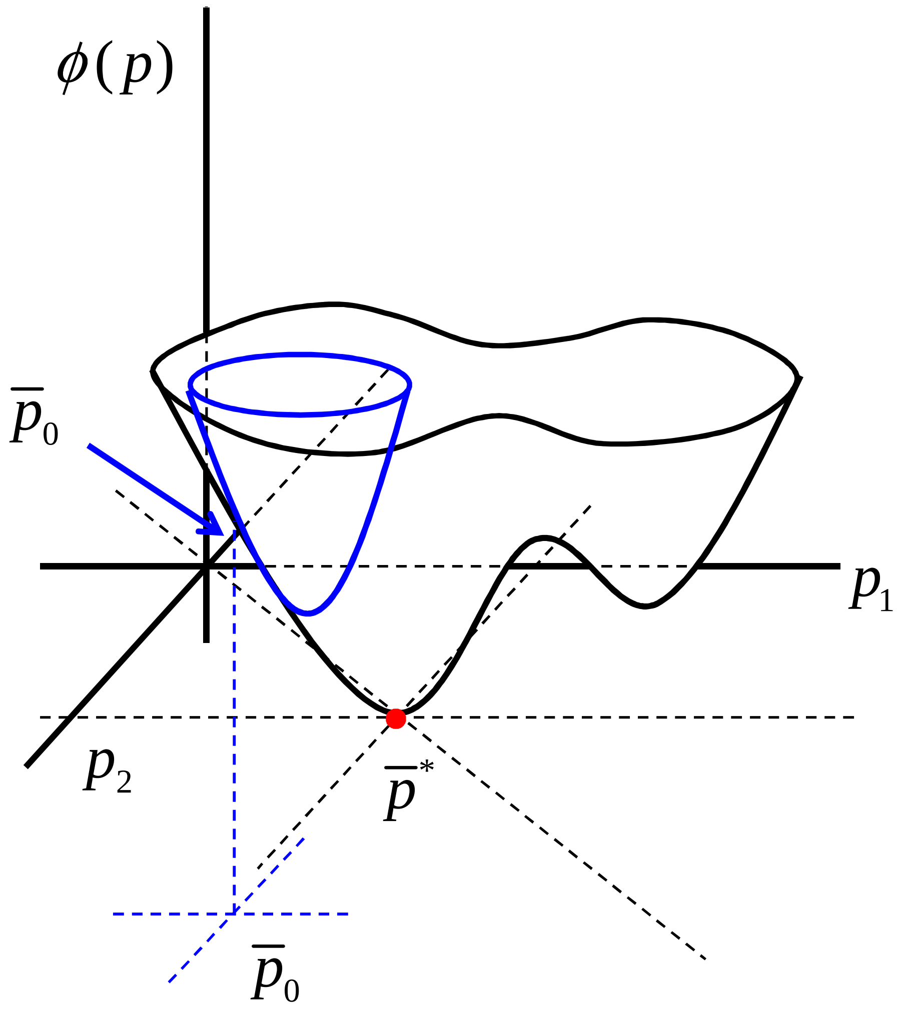
Sistema linear



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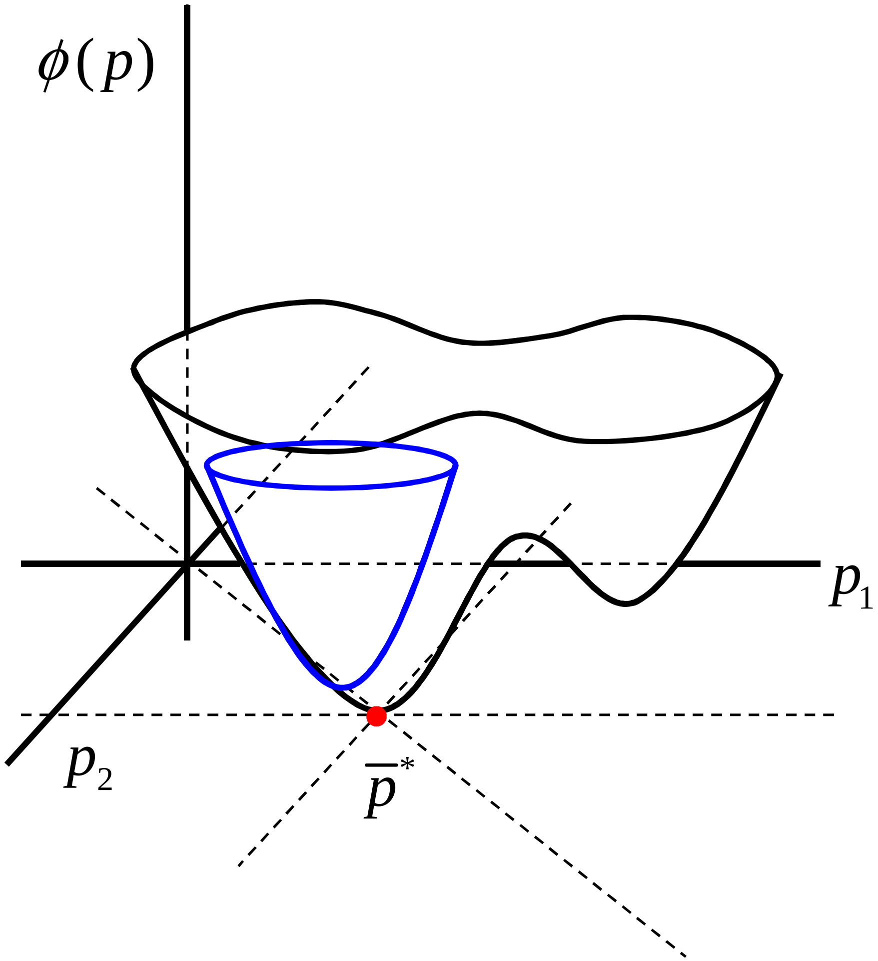
Sistema linear



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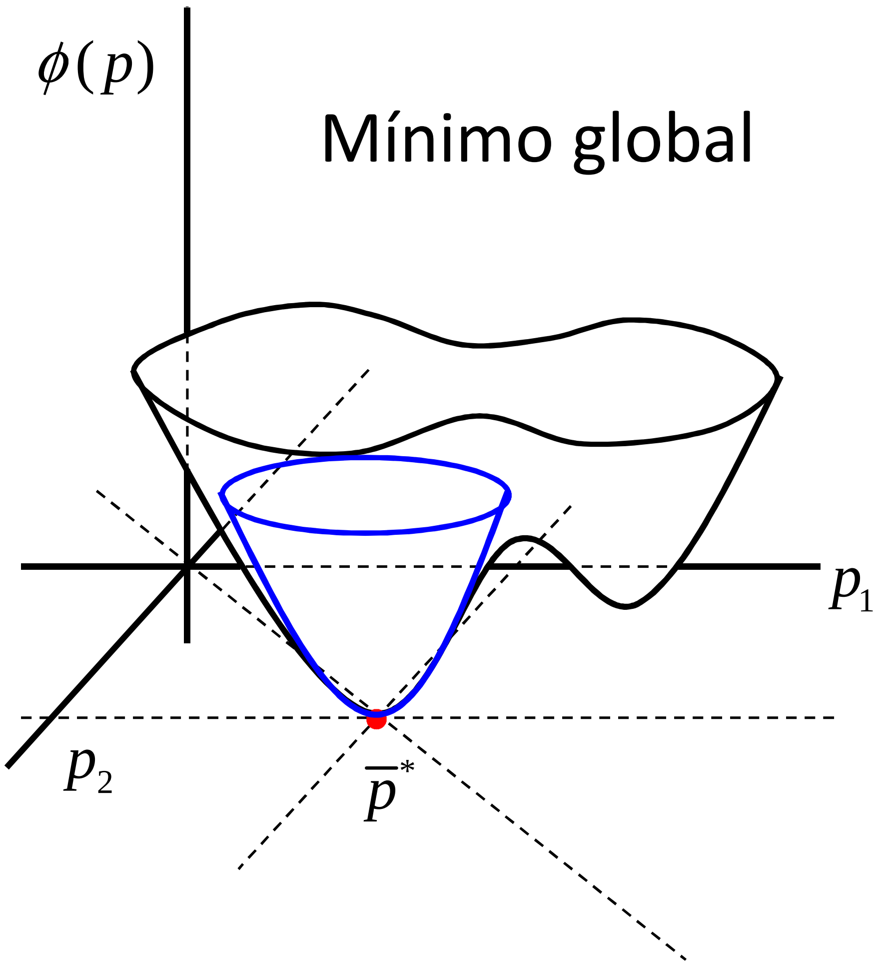
Sistema linear



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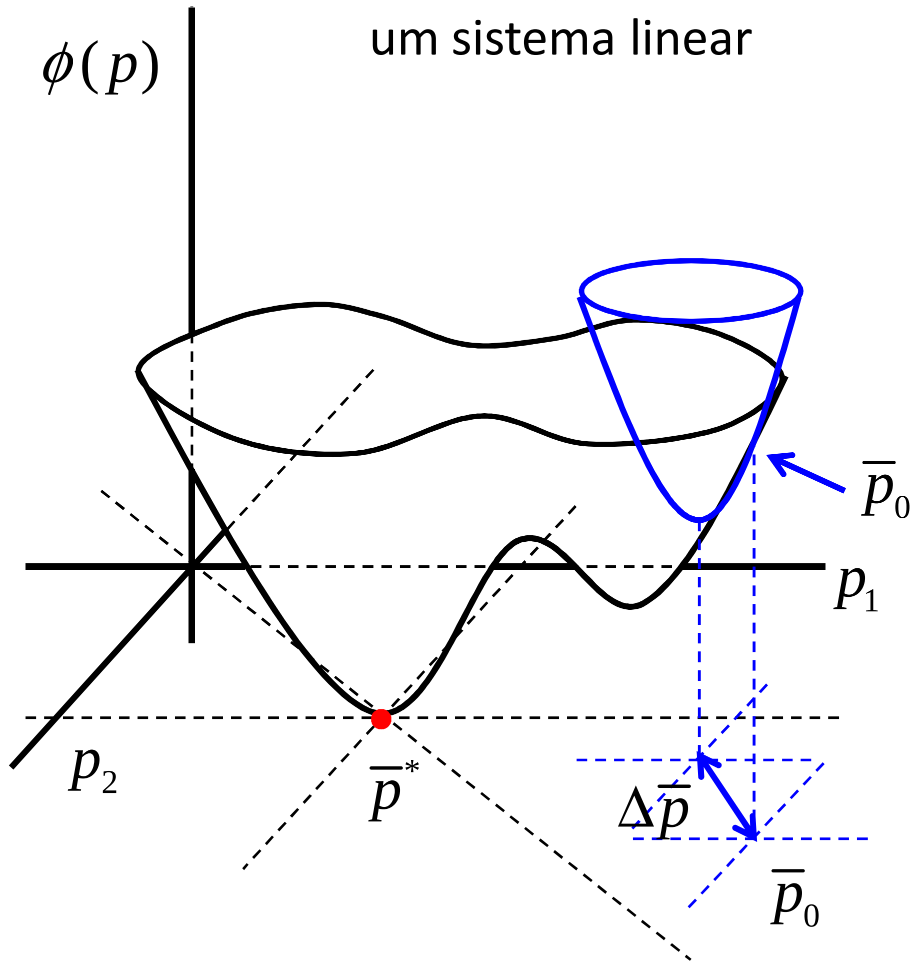
Sistema linear

Em cada iteração é resolvido um sistema linear



Problemas não-lineares

$$\phi(\overline{p}) = [\overline{d} - \overline{g}(\overline{p})]^T[\overline{d} - \overline{g}(\overline{p})]$$

$$\overline{g}(\overline{p}) \neq \overline{\overline{B}}\,\overline{p} + \overline{b}$$

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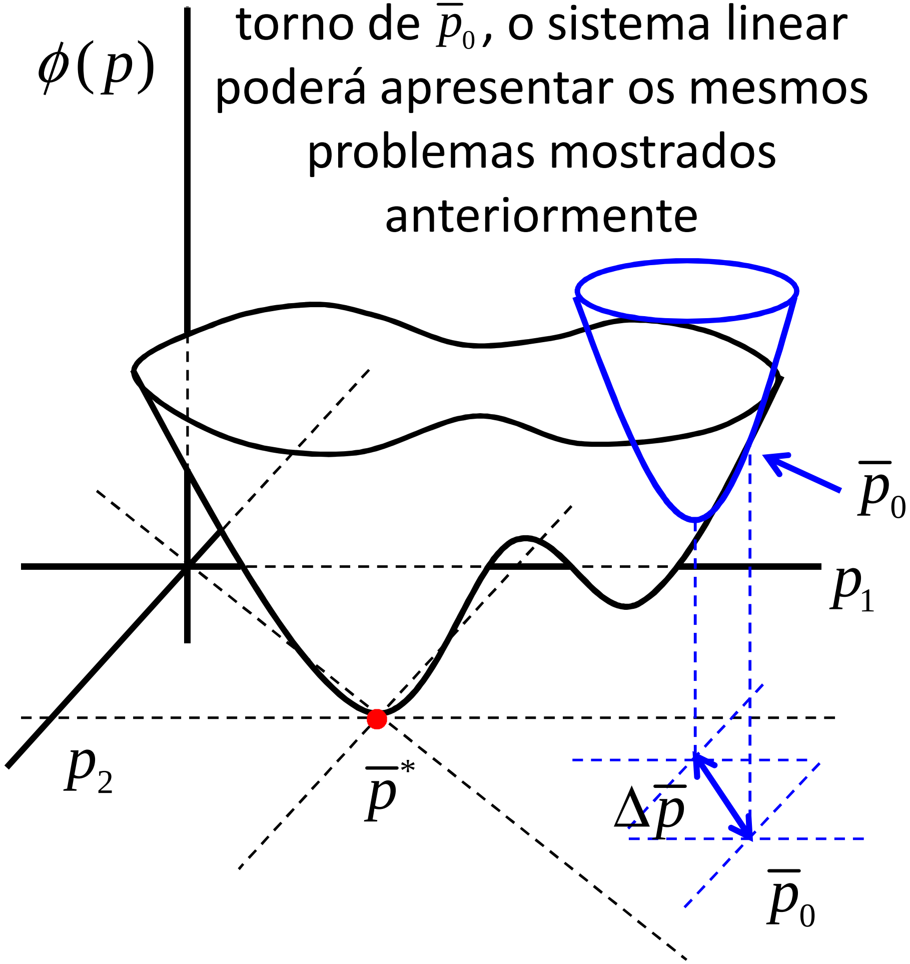
Sistema linear

Dependendo da função $\phi(\overline{p}_0)$ em torno de $\overline{p}_0$, o sistema linear poderá apresentar os mesmos problemas mostrados anteriormente



Regularização

Problema linear

$$\phi(\overline{p}) = [\overline{d} - \overline{g}(\overline{p})]^T [\overline{d} - \overline{g}(\overline{p})]$$

Problema não-linear

$$\overline{g}(\overline{p}) = \overline{\overline{B}}\,\overline{p} + \overline{b}$$

$$\overline{g}(\overline{p}) \neq \overline{\overline{B}}\,\overline{p} + \overline{b}$$

$$\overline{\nabla}\phi(\overline{p}^*) = \overline{0}_{M\times 1}$$

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$$\overline{p} = \overline{p}_0 + \Delta\overline{p}$$

$$\left(\overline{\overline{B}}^T \overline{\overline{B}}\right)\overline{p}^* = \overline{\overline{B}}^T [\overline{d} - \overline{b}]$$

Sistema linear

$$\left(\overline{\overline{G}}(\overline{p}_0)^T \overline{\overline{G}}(\overline{p}_0)\right)\Delta\overline{p} = \overline{\overline{G}}(\overline{p}_0)^T [\overline{d} - \overline{g}(\overline{p}_0)]$$

Sistema linear

Regularização

Problema linear

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

Problema não-linear

$$\bar{g}(\bar{p}) = \bar{\bar{B}}\bar{p} + \bar{b}$$

$$\bar{g}(\bar{p}) \neq \bar{\bar{B}}\bar{p} + \bar{b}$$

$$\boxed{\det \approx 0}$$

$$\bar{\nabla}\phi(\bar{p}^*) = \bar{0}_{M\times 1}$$

$$\boxed{\det = 0}$$

$$\bar{\nabla}\phi(\bar{p}^*) = \bar{0}_{M\times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta\bar{p}$$

$$\boxed{\left(\bar{\bar{B}}^T \bar{\bar{B}}\right)} \bar{p}^* = \bar{\bar{B}}^T [\bar{d} - \bar{b}]$$

Sistema linear

$$\boxed{\left(\bar{\bar{G}}(\bar{p}_0)^T \bar{\bar{G}}(\bar{p}_0)\right)} \Delta\bar{p} = \bar{\bar{G}}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear

Regularização

A regularização é um procedimento que objetiva aumentar o determinante das matrizes envolvidas na solução dos problemas inversos lineares e não-lineares

Regularização

Problema linear

$$\left(\overline{\overline{B}}^T \overline{\overline{B}}\right)\overline{p}^* = \overline{\overline{B}}^T [\overline{d} - \overline{b}]$$

Problema não-linear

$$\left(\overline{\overline{G}}(\overline{p}_0)^T \overline{\overline{G}}(\overline{p}_0)\right)\Delta\overline{p} = \overline{\overline{G}}(\overline{p}_0)^T [\overline{d} - \overline{g}(\overline{p}_0)]$$

Estas equações levam ao vetor de parâmetros que ajustam os dados, ou seja, estimam um vetor de parâmetros que produz os dados preditos mais próximos possíveis aos dados observados

Regularização

Problema linear

$$\left(\overline{\overline{B}}^T \overline{\overline{B}}\right)\overline{p}^* = \overline{\overline{B}}^T [\overline{d} - \overline{b}]$$

Problema não-linear

$$\left(\overline{\overline{G}}(\overline{p}_0)^T \overline{\overline{G}}(\overline{p}_0)\right)\Delta\overline{p} = \overline{\overline{G}}(\overline{p}_0)^T [\overline{d} - \overline{g}(\overline{p}_0)]$$

E se quiséssemos que o vetor de parâmetros ajustasse um vetor $\overline{h}$ (diferente dos dados observados) dado pela relação linear abaixo?

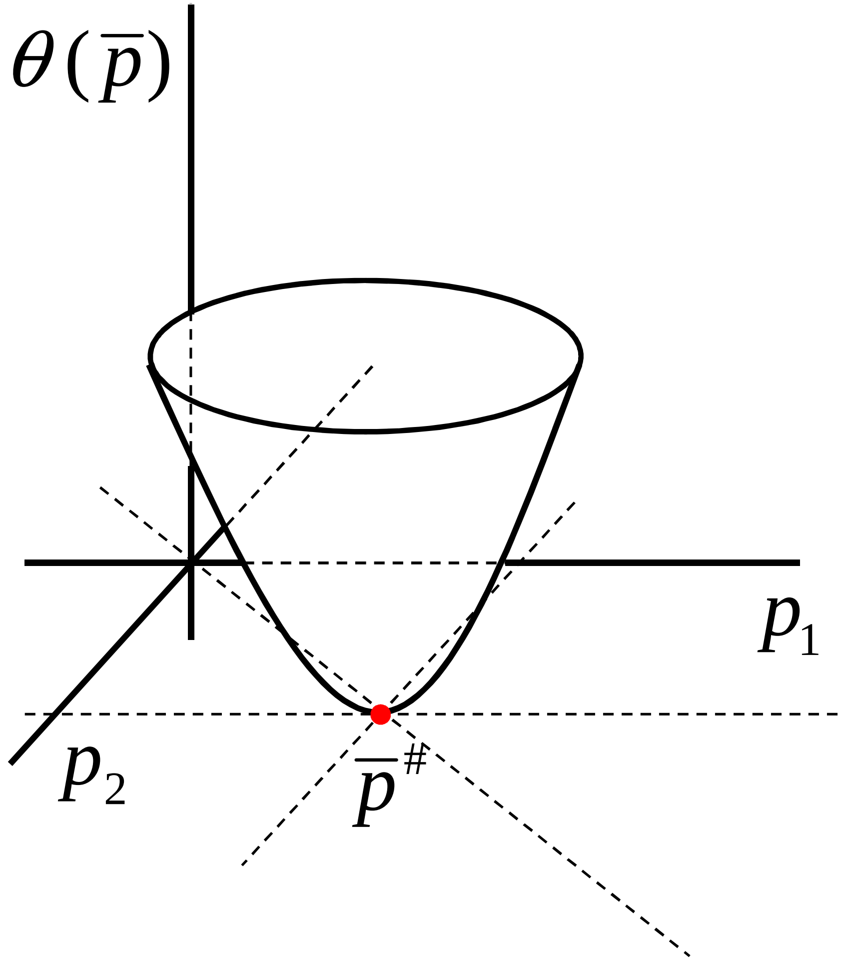
$$\overline{\overline{H}}\,\overline{p} = \overline{h}$$

Regularização

$$\theta(\overline{p}) = [\overline{h} - \overline{p}]^T [\overline{h} - \overline{p}]$$

$$\overline{\nabla}\theta(\overline{p}^{\#}) = \overline{0}_{M\times 1}$$

$$\left(\overline{\overline{H}}^T \overline{\overline{H}}\right)\overline{p}^{\#} = \overline{\overline{H}}^T \overline{h}$$



Regularização

Problema linear

$$\left(\overline{\overline{B}}^{T}\,\overline{\overline{B}}\right)\overline{p}^{*}=\overline{\overline{B}}^{T}[\overline{d}-\overline{b}]$$

Problema não-linear

$$\left(\overline{\overline{G}}(\overline{p}_0)^{T}\,\overline{\overline{G}}(\overline{p}_0)\right)\Delta\overline{p}=\overline{\overline{G}}(\overline{p}_0)^{T}[\overline{d}-\overline{g}(\overline{p}_0)]$$

E se agora quiséssemos que o vetor de parâmetros ajustasse o vetor $\overline{h}$ e também o vetor de dados observados ao mesmo tempo?

$$\overline{\overline{H}}\,\overline{p}=\overline{h}$$

Regularização

$$\overline{\overline{B}}\,\overline{p}+\overline{b}\approx\overline{d}$$

$$\overline{\overline{H}}\,\overline{p}\approx\overline{h}$$

Regularização

$$\overline{\overline{B}}\,\overline{p}+\overline{b}\approx\overline{d}$$

$$\overline{\overline{H}}\,\overline{p}\approx\overline{h}$$

$$\underbrace{\begin{bmatrix}\overline{\overline{B}}\\ \overline{\overline{H}}\end{bmatrix}}_{\overline{\overline{\mathrm{A}}}}\overline{p}\approx\underbrace{\begin{bmatrix}\overline{d}-\overline{b}\\ \overline{h}\end{bmatrix}}_{\overline{w}}$$

Regularização

$$\overline{\overline{B}}\,\overline{p} + \overline{b} \approx \overline{d}$$

$$\overline{\overline{H}}\,\overline{p} \approx \overline{h}$$

$$\underbrace{\begin{bmatrix} \overline{\overline{B}} \\ \overline{\overline{H}} \end{bmatrix}}_{\overline{\overline{\mathrm{A}}}} \overline{p} \approx \underbrace{\begin{bmatrix} \overline{d} - \overline{b} \\ \overline{h} \end{bmatrix}}_{\overline{w}}$$

$$\Omega\left(\overline{p}\right) = \left[\overline{w} - \overline{\overline{\mathrm{A}}}\,\overline{p}\right]^{T} \left[\overline{w} - \overline{\overline{\mathrm{A}}}\,\overline{p}\right]$$

Regularização

$$\overline{\overline{B}}\,\overline{p} + \overline{b} \approx \overline{d}$$

$$\overline{\overline{H}}\,\overline{p} \approx \overline{h}$$

$$\underbrace{\begin{bmatrix} \overline{\overline{B}} \\ \overline{\overline{H}} \end{bmatrix}}_{\overline{\overline{\mathrm{A}}}} \overline{p} \approx \underbrace{\begin{bmatrix} \overline{d} - \overline{b} \\ \overline{h} \end{bmatrix}}_{\overline{w}}$$

$$\Omega(\overline{p}) = \phi(\overline{p}) + \theta(\overline{p})$$

$$\Omega(\overline{p}) = \left[\overline{w} - \overline{\overline{\mathrm{A}}}\,\overline{p}\right]^T \left[\overline{w} - \overline{\overline{\mathrm{A}}}\,\overline{p}\right]$$

$$\phi(\overline{p}) = \left[\overline{d} - \overline{g}(\overline{p})\right]^T \left[\overline{d} - \overline{g}(\overline{p})\right]$$

$$\theta(\overline{p}) = \left[\overline{h} - \overline{\overline{H}}\,\overline{p}\right]^T \left[\overline{h} - \overline{\overline{H}}\,\overline{p}\right]$$

Regularização

$$\overline{p} = \overline{p}_0 + \Delta\overline{p}$$

$$\left[\overline{\overline{\nabla}}\phi(\overline{p}_0) + \mu\overline{\overline{\nabla}}\theta(\overline{p}_0)\right]\Delta\overline{p} = -\left[\overline{\nabla}\phi(\overline{p}_0) + \mu\overline{\nabla}\theta(\overline{p}_0)\right]$$

Regularização

$$\overline{p} = \overline{p}_0 + \Delta \overline{p}$$

$$\underbrace{\left[\overline{\overline{\nabla}} \phi(\overline{p}_0) + \mu \overline{\overline{\nabla}} \theta(\overline{p}_0)\right]}_{\substack{\text{Espera-se que} \\ \text{essa matriz} \\ \text{resultante tenha} \\ \det \neq 0}} \Delta \overline{p} = -\left[\overline{\nabla} \phi(\overline{p}_0) + \mu \overline{\nabla} \theta(\overline{p}_0)\right]$$