2D gravity inversion with isostatic constraint applied to passive rifted margins

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Running head: **2D gravity inversion for passive rifted margins**

ABSTRACT

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INTRODUCTION

Several methods have been proposed for using gravity data to estimate the relief of an interface separating two layers.

These methods usually presume that the two layers have constant and known density values. Additionally, they also presume that the interface oscillates around a known reference depth.

For different combinations of density values and reference depths, it is possible to find different interfaces producing the same gravity data.

To deal with this inherent ambiguity (Roy, 1962; Skeels, 1947) and obtain meaningful solutions, the interpreter must use a priori information obtained from seismic data and/or boreholes, for example, in order to constrain the range of possible models.

One of the first automated methods for estimating the interface separating two homogeneous layers was presented by Bott (1960), in space domain.

Since then, several other approaches have been developed in space domain (Tanner, 1967; Dyrelius and Vogel, 1972; Pedersen, 1977; Richardson and MacInnes, 1989; Barbosa et al., 1997, 1999b,a; Chakravarthi and Sundararajan, 2007; Silva et al., 2010; Camacho et al., 2011; Lima et al., 2011; Martins et al., 2011; Barnes and Barraud, 2012; Silva et al., 2014; Santos et al., 2015; Silva and Santos, 2017) and also in the Fourier domain (Oldenburg, 1974; Granser, 1987; Reamer and Ferguson, 1989; Guspí, 1993; Braitenberg et al., 1997; Braitenberg and Zadro, 1999).

Methods in Fourier domain use Parker's formula (Parker, 1973).

Faltam os papers do pilkington, o paper do salem e os papers de satelite regional (Leo e Sampietro) e os globais (pegar no trabalho do sampietro e do reguzzoni)

Dentre estes trabalhos, tem que dividir quais impem equilbrio isosttico e como eles fazem isso.

METHODOLOGY

Forward problem

Let $\mathbf{d}^o$ be the observed data vector, whose i-th element d_i^o, $i = 1, \ldots, N$, represent the observed gravity disturbance at the point (x_i, y_i, z_i), on a profile located over a rifted passive margin. The coordinates are referred to a topocentric Cartesian system, with z axis pointing down, y-axis along the profile and x-axis perpendicular to the profile. We assume that the observed gravity disturbance is produced by an anomalous mass distribution defined as the difference between the actual mass distribution in the subsurface, which is schematically represented in Figure **??**, and a reference mass distribution (Figure **??**). In doing it, we implicitly assume that Figure **??** represents the outer layers of a global mass distribution producing the normal gravity field.

The anomalous mass distribution producing the observed data is approximated by an interpretation model (Figure **??**) formed by N adjacent columns. For convenience, we presume that the observed data are regularly spaced, so that there is one observation at the centre of the top of each column forming the interpretation model.The i-th column is formed by four vertically adjacent layers, , which in turn are composed of vertically adjacent prisms having infinite length along the x-axis. The first and shallowest layer represents the water layer, is formed by a single prism, has thickness t_i^w and a constant density contrast $\Delta\rho^w = \rho^w - \rho^r$, where ρ^w and ρ^r represents, respectively, the densities of water and the reference mass distribution (Figure **??**) at the same point. The third layer represents the crust, it is also formed by a single prism, has thickness t_i^c and density contrast $\Delta\rho_i^c = \rho^c - \rho^r$, with ρ^c being the crust density. For simplicity, we presume that the crust density ρ_i^c may be equal to ρ^{cc}, for $y_i \leq y_{COT}$, which represents continental crust, or equal to ρ^{oc}, for $y_i > y_{COT}$, which represents oceanic crust. The crust density depends on the position of the i-th column with respect to y_{COT}, which defines an abrupt Crust-Ocean Transition (COT). Consequently, the crust may have two possible density contrasts: $\Delta\rho_i^c = \rho^{cc} - \rho r$ or $\Delta\rho_i^c = \rho^{oc} - \rho^r$. The top of this layer defines the basement relief and its bottom the relief of the Moho. The fourth layer represents the mantle, it is divided into two parts, each one formed by a single prism having the same density ρ^m and, consequently, the same density contrast $\Delta\rho^m = \rho^m - \rho^r$. The shallowest portion of this layer has thickness t_i^m. Its top and bottom define, respectively, the depths of Moho and the planar isostatic compensation layer S_0. The deepest portion of the fourth layer has thickness ΔS_0, top at the surface S_0 and bottom at the planar surface $S_0 + \Delta S_0$, which defines the Moho in the reference mass distribution model (Figure **??**). Finally, the second layer forming the t-th column of the interpretation model is defined by the interpreter, according to the geological environment to be studied and the a priori information availability. As a general rule, this layer can be defined by a set of Q vertically adjacent prisms, each one with thickness t_i^q, density ρ^q and density contrast $\Delta\rho^q = \rho^q - \rho^r$, $q = 1, \dots Q$.

Given the density contrasts, the COT position y_{COT}, the isostatic compensation surface S_0, the thickness of the water layer and of the $Q-1$ prisms forming the shallowest portion of the second layer, it is possible to describe the interpretation model in terms of an $M \times 1$

parameter vector $\mathbf{p}$, $M = 2N + 1$, defined as follows:

$$\mathbf{p} = \begin{bmatrix} \mathbf{t}^Q \\ \mathbf{t}^m \\ \Delta S_0 \end{bmatrix} , \tag{1}$$

where $\mathbf{t}^Q$ and $\mathbf{t}^m$ are $N \times 1$ vectors whose i-th elements t_i^Q and t_i^m represent, respectively, the thickness of the prism forming the deepest portion of the second layer and the thickness of the prism forming the shallowest portion of the fourth layer of the interpretation model. In this case, the gravity disturbance produced by the interpretation model (the predicted gravity disturbance) at the position (x_i, y_i, z_i) can be written as the sum of the vertical component of the gravitational attraction exerted by the L prisms forming the interpretation model as follows:

$$d_i(\mathbf{p}) = k_g \, G \sum_{j=1}^{L} f_{ij}(\mathbf{p}) \, , \tag{2}$$

where $f_{ij}(\mathbf{p})$ represents an integral over the volume of the j-th prism. Here, these volume integrals are computed with the expressions proposed by Nagy et al. (2000), by using the open-source Python package *Fatiando a Terra* (Uieda et al., 2013).

Inverse problem

Let $\mathbf{d}(\mathbf{p})$ be the predicted data vector, whose i-th element $d_i(\mathbf{p})$ is defined by Equation 2. Estimating the particular parameter vector $\mathbf{p} = \hat{\mathbf{p}}$ producing a predicted data vector $\mathbf{d}(\mathbf{p})$ as close as possible to the observed data vector $\mathbf{d}^o$ can be formulated as the problem of minimizing the goal function

$$\Gamma(\mathbf{p}) = \Phi(\mathbf{p}) + \mu \sum_{k=0}^{3} \alpha_k \Psi_k(\mathbf{p}) \, , \tag{3}$$

subject to all elements of $\hat{\mathbf{p}}$ be positive. In Equation 3, μ represents the regularizing parameter, $\Phi(\mathbf{p})$ represents the misfit function given by

$$\Phi(\mathbf{p}) = \frac{1}{N}\|\mathbf{d}^o - \mathbf{d}(\mathbf{p})\|_2^2 \, , \tag{4}$$

where $\|\cdot\|_2^2$ represents the squared Euclidean norm, α_k represent the weights assigned to the regularizing functions $\Psi_k(\mathbf{p})$, with define the constraints on the parameters to be estimated, $k = 0, 1, 2, 3$.

Airy constraint

Consider that the interpretation model is in isostatic equilibrium according to the Airy model (Turcotte and Schubert, 2002; Hofmann-Wellenhof and Moritz, 2005; Lowrie, 2007). In this case, the pressure (or lithostatic stress) exerted by the model is constant on the isostatic compensation surface S_0. The pressure per unit area exerted by the i-th column of the model on S_0, divided by gravity, is given by:

$$t_i^w \rho^w + t_i^1 \rho_i^1 + \cdots + t_i^Q \rho_i^Q + t_i^c \rho_i^c + t_i^m \rho^m = \sigma_0 \, , \tag{5}$$

where σ_0 is an arbitrary positive constant. Rearranging terms in Equation 5 and using the relation

$$S_0 = t_i^w + t_i^1 + \cdots + t_i^Q + t_i^c + t_i^m \, , \tag{6}$$

it is possible to show that:

$$(\rho_i^Q - \rho_i^c)\, t_i^Q + (\rho^m - \rho_i^c)\, t_i^m + (\rho^w - \rho_i^c)\, t_i^w + (\rho_i^1 - \rho_i^c)\, t_i^1 + \cdots + (\rho_i^{Q-1} - \rho_i^c)\, t_i^{Q-1} + \rho_i^c\, S_0 = \sigma_0 . \tag{7}$$

In order to describe the pressure exerted by all columns forming the interpretation model on the surface S_0, Equation 7 can be written, in matrix notation, as follows:

$$\mathbf{M}^Q \mathbf{t}^Q + \mathbf{M}^m \mathbf{t}^m + \mathbf{M}^w \mathbf{t}^w + \mathbf{M}^1 \mathbf{t}^1 + \cdots + \mathbf{M}^{Q-1} \mathbf{t}^{Q-1} + \boldsymbol{\rho}^c S_0 = \sigma_0 \mathbf{1} \, , \tag{8}$$

where $\mathbf{1}$ is an $N \times 1$ vector with all elements equal to one, $\mathbf{t}^{\alpha}$ are $N \times 1$ vectors with i-th element defined by the thickness t_i^{α} of a prism forming the i-th column, $\alpha = w, 1, \ldots, Q-1, Q, m$, and $\mathbf{M}^Q$, $\mathbf{M}^m$, $\mathbf{M}^w$, $\mathbf{M}^1$, $\ldots$, $\mathbf{M}^{Q-1}$ are $N \times N$ diagonal matrices with elements ii of main diagonal are given by density contrasts $(\rho_i^Q - \rho_i^c)$, $(\rho^m - \rho_i^c)$, $(\rho^w - \rho_i^c)$, $(\rho_i^1 - \rho_i^c)$ and $\ldots$, $(\rho_i^{Q-1} - \rho_i^c)$, respectively, and $\boldsymbol{\rho}^c$ is an $N \times 1$ vector containing the densities of the prisms representing the crust. By applying the first-order Tikhonov regularization (Aster et al., 2005) to the constant vector $\sigma_0 \mathbf{1}$, we obtain the following expression:

$$\mathbf{R}\left(\mathbf{Cp} + \mathbf{Dt}\right) = \mathbf{0}\,, \tag{9}$$

where $\mathbf{0}$ is a vector with null elements and the remaining terms are given by:

$$\mathbf{C} = \begin{bmatrix} \mathbf{M}^Q & \mathbf{M}^m & \mathbf{0} \end{bmatrix}_{N \times M}\,, \tag{10}$$

$$\mathbf{D} = \begin{bmatrix} \mathbf{M}^w & \mathbf{M}^1 & \cdots & \mathbf{M}^{Q-1} & \boldsymbol{\rho}^c \end{bmatrix}_{N \times (QN+1)}\,, \tag{11}$$

$$\mathbf{t} = \begin{bmatrix} \mathbf{t}^w \\ \mathbf{t}^1 \\ \vdots \\ \mathbf{t}^{Q-1} \\ S_0 \end{bmatrix}_{(QN+1) \times 1}\,, \tag{12}$$

$\mathbf{p}$ is the parameter vector (Equation 1) and $\mathbf{R}$ is an $(N-1) \times N$ matrix, whose element ij is defined as follows:

$$[\mathbf{R}]_{ij} = \begin{cases} 1 & , \quad j = i \\ -1 & , \quad j = i+1 \\ 0 & , \quad \text{otherwise} \end{cases}\,. \tag{13}$$

Finally, from Equation 9, it is possible to define the regularizing function $\Psi_0(\mathbf{p})$ (Equation 3):

$$\Psi_0(\mathbf{p}) = \| \mathbf{R} \left(\mathbf{C}\mathbf{p} + \mathbf{D}\mathbf{t} \right) \|_2^2 \, . \tag{14}$$

We call this function as *Airy constraint*. Notice that minimizing this function imposes smoothness on the pressure exerted by the interpretation model on the isostatic compensation surface S_0.

Smoothness constraint

This constraint imposes smoothness on the adjacent thickness of the prisms forming the deepest portion of the second layer and the shallowest part of the fourth layer of the interpretation model by applying the first-order Tikhonov regularization (Aster et al., 2005) to the vectors $\mathbf{t}^Q$ and $\mathbf{t}^m$ (Equation 1). Mathematically, this constraint is represented by the regularizing function $\Psi_1(\mathbf{p})$ (Equation 3):

$$\Psi_1(\mathbf{p}) = \| \mathbf{S}\mathbf{p} \|_2^2 \, , \tag{15}$$

where $\mathbf{S}$ is an $(N-1) \times M$ matrix given by:

$$\mathbf{S} = \begin{bmatrix} \mathbf{R} & \mathbf{R} & \mathbf{0} \end{bmatrix} \, , \tag{16}$$

where $\mathbf{R}$ is defined by Equation 13 and $\mathbf{0}$ is a vector with all elements equal to zero.

Equality constraint

Equality constraint on basement depths

Let $\mathbf{a}$ be a vector whose k-th element a_k, $k = 1, \ldots, A$, is the known basement depth at the horizontal coordinate y_k^A of the profile. These known basement depth values are used to define the regularizing function $\Psi_2(\mathbf{p})$ (Equation 3):

$$\Psi_2(\mathbf{p}) = \|\mathbf{A}\mathbf{p} - \mathbf{a}\|_2^2 \,, \tag{17}$$

where $\mathbf{A}$ is an $A \times M$ matrix whose k-th line has one element equal to one and all the remaining elements equal to zero. The location of the single non-null element in the k-th line of $\mathbf{A}$ depends on the coordinate y_k^A of the known basement depth a_k. Let us consider, for example, an interpretation model formed by $N = 10$ columns. Consider also that the basement depth at the coordinates $y_1^A = y_4$ and $y_2^A = y_9$ of the profile are equal to 25 and 35.7 km, respectively. In this case, $A = 2$, $\mathbf{a}$ is a 2×1 vector with elements $a_1 = 25$ and $a_2 = 35.7$ and $\mathbf{A}$ is a $2 \times M$ matrix ($M = 2N + 1 = 21$). The element 4 of the first line and the element 9 of the second line of $\mathbf{A}$ are equal to 1 and all its remaining elements are equal to zero.

Equality constraint on Moho depths

Let $\mathbf{b}$ be a vector whose k-th element b_k, $k = 1, \ldots, B$, is the difference between the isostatic compensation depth S_0 and the known Moho depth at the horizontal coordinate y_k^B of the profile. These differences, which must be positive, are used to define the regularizing function $\Psi_3(\mathbf{p})$ (Equation 3):

$$\Psi_3(\mathbf{p}) = \|\mathbf{B}\mathbf{p} - \mathbf{b}\|_2^2 \,, \tag{18}$$

where $\mathbf{B}$ is a $B \times M$ matrix whose k-th line has one element equal to one and all the remaining elements equal to zero. This matrix is defined in the same way as matrix $\mathbf{A}$ (Equation 17).

CONCLUSIONS

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