METODE PERAMALAN DERET WAKTU

Pertemuan ke-1

Yenni Angraini, M.Si dan Akbar Rizki, M.Si

OUTLINE

1. Pokok Bahasan
2. Pendahuluan

OUTLINE

1. Pokok Bahasan

2. Pendahuluan

1. Pokok Bahasan

Pokok Bahasan

- Pengertian, ruang lingkup, dan karakteristik data deret waktu serta overview berbagai metode peramalan
- Metode pemulusan rataan bergerak sederhana dan rataan bergerak ganda
- Metode pemulusan eksponensial sederhana dan eksponensial ganda
- Metode pemulusan Winter aditif dan multiplikatif
- Model regresi untuk data deret waktu
- Model regresi dengan peubah lag

UJIAN TENGAH SEMESTER (UTS)

- Konsep Dasar Pemodelan Data Deret Waktu
- Model Deret Waktu Stasioner
- Model Deret Waktu Tidak Stasioner
- Identifikasian Model ARIMA
- Pendugaan Parameter Model, Diagnostik dan Peramalan
- Presentasi Tugas (PENILAIAN UAS)

1. Pokok Bahasan

Pustaka yang digunakan

1. Montgomery, D.C., et.al. 2008. Forecasting Time Series Analysis 2nd. John Wiley
2. Cryer, J.D. and Chan, K.S. 2008. Time Series Analysis with Application in R. Springer
3. Abraham, B and Ledolter, J. 2005. Statistical Methods for Forecasting, John Wiley
4. Hyndman, R.J and Athanasopoulos, G. 2013. Forecasting: principles and practice

OUTLINE

1. Pokok Bahasan

2. Pendahuluan

2. Pendahuluan

Why we need study time series analysis?

Manusia hidup dalam ruang dan waktu sehingga tidak bisa dipisahkan dengan data deret waktu.

DEFINISI DATA DERET WAKTU

“ Data yang diamati berdasarkan urutan waktu dengan rentang yang sama (jam, hari, minggu, bulan, tahun, dsb)”

Misalnya: data harian covid-19 di Indonesia, data nilai tukar rupiah harian, dsb.

2. Pendahuluan

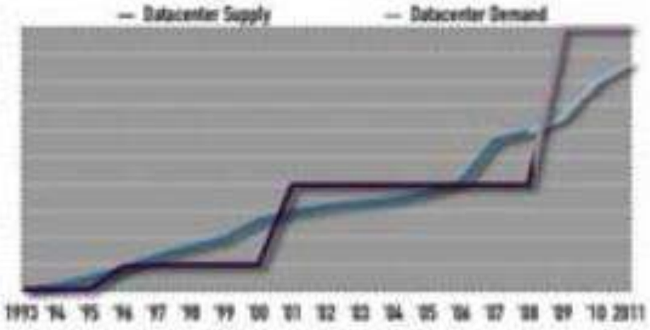
BIDANG KESEHATAN

2. Pendahuluan

BIDANG EKONOMI

Data keuangan

Data Stok Barang

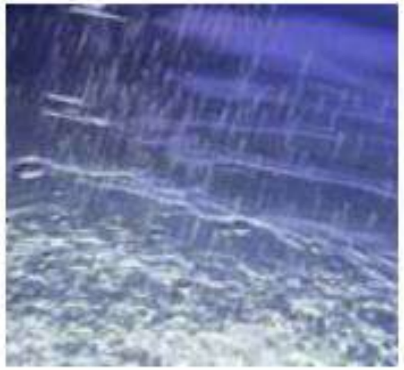


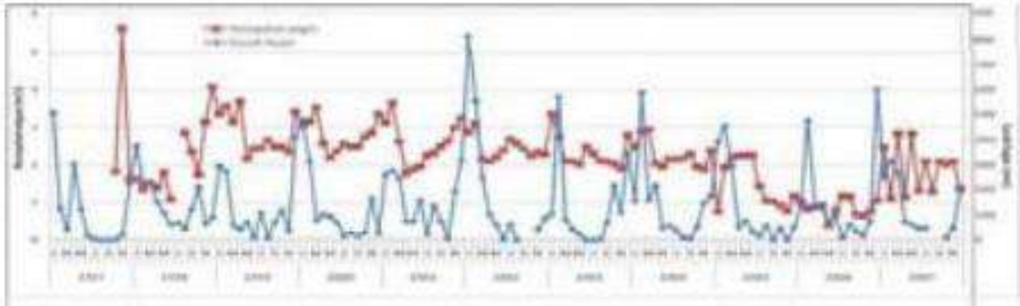
Data supply demand

Data daya tukar nilai uang

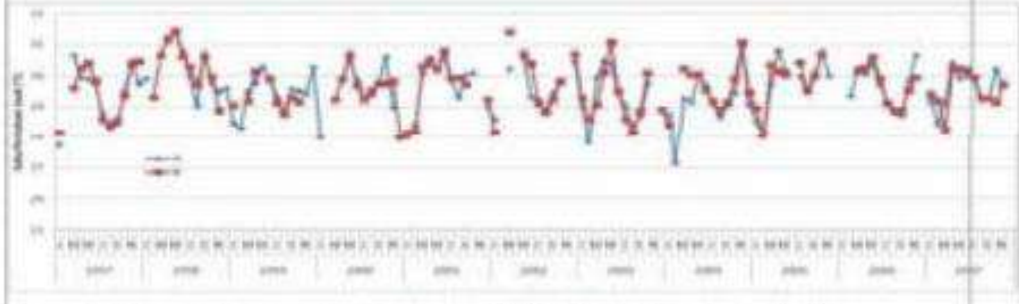
2. Pendahuluan

BIDANG KLIMATOLOGI





Gambar 3. Curah hujan dan kecepatan angin di Teluk Jakarta berdasarkan data stasiun BMG Tanjung Priok



Gambar 4. Variasi temporal suhu permukaan laut dari sensor AVHRR di Teluk Jakarta wilayah A dan B

2. Pendahuluan

Kapan data didekati dengan metode deret waktu?

Kalau diduga kuat bahwa keragaman dalam data ada **faktor waktu yang dominan** (faktor-faktor lain yang mempengaruhi, juga dipengaruhi waktu)

DATA DERET WAKTU

Data deret waktu secara teoritis ditulis sebagai:

$$x_t = b_1 z_1(t) + b_2 z_2(t) \ldots + b_k z_k(t) + \varepsilon_t$$

dimana

b_k = Parameter ke - k

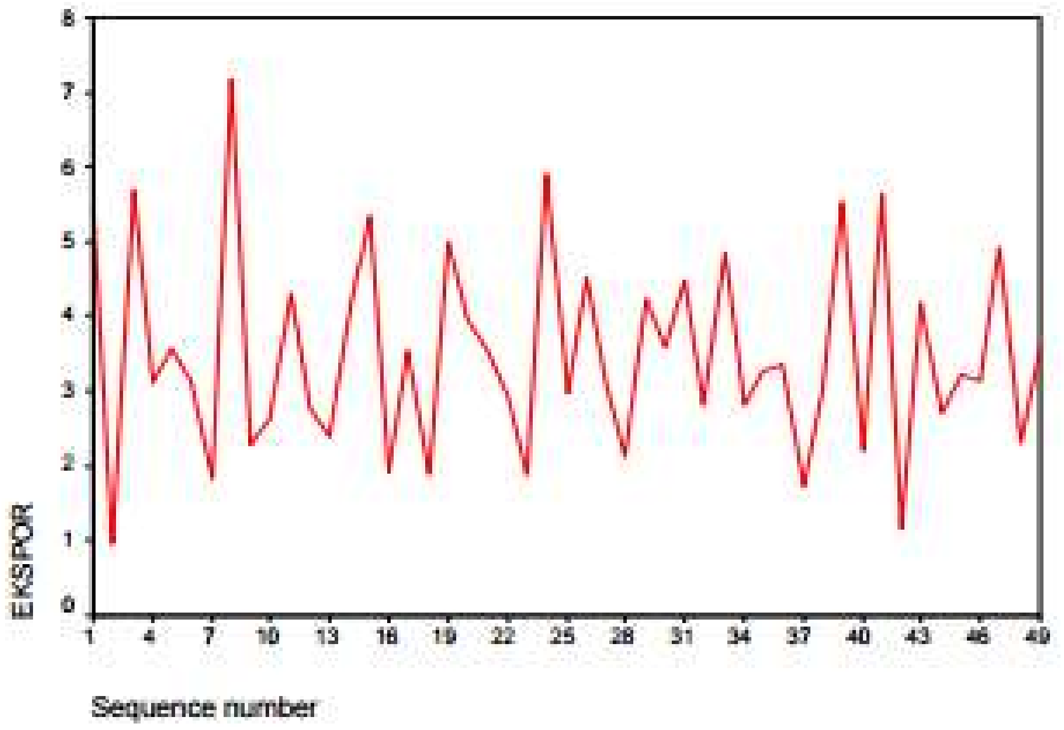
$z_k(t)$ = Fungsi Matematik ke - k pada t

ε_k = Komponen Acak ke - k

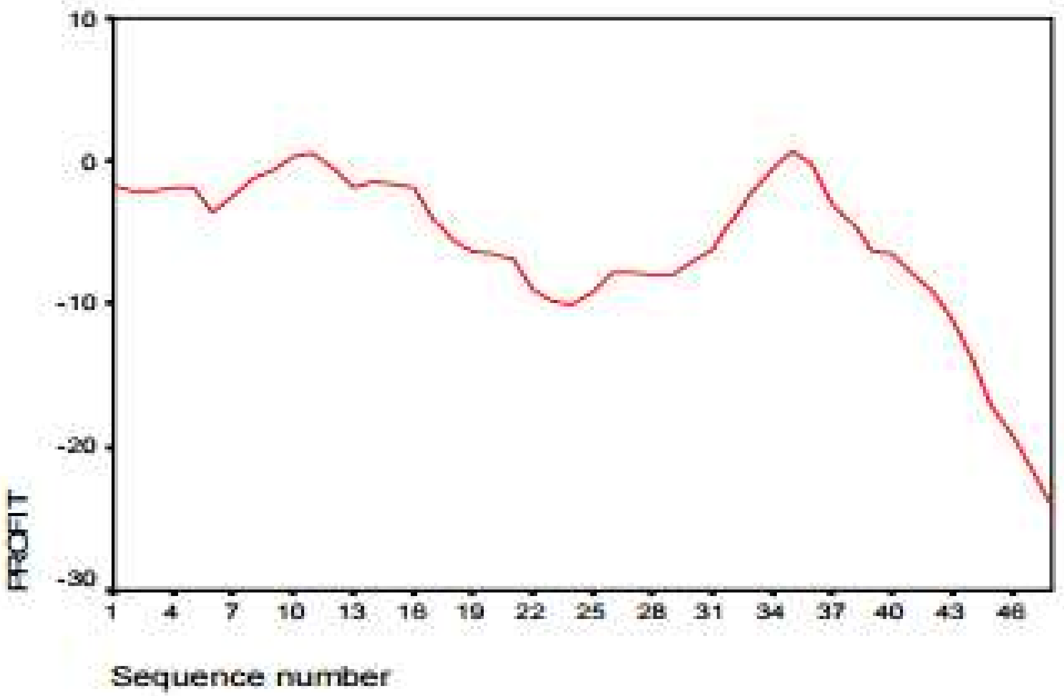
2. Pendahuluan

KARAKTERISTIK DATA DERET WAKTU

- Secara garis besar, data DW dibedakan menjadi dua, yaitu stasioner dan tidak stasioner
- Dikatakan stasioner apabila data DW memiliki nilai tengah (rataan) dan ragam (fluktuasi) yang konstan dari waktu ke waktu



Contoh data DW Statsioner



Contoh data DW tidak Statsioner

2. Pendahuluan

POLA DATA DERET WAKTU

Secara garis besar pola data time series adalah:

- Pola Data Horizontal

  →Terjadi bila data berfluktuasi di sekitar rata-rata yang konstan.

  Contoh: Data penjualan yang konstan

- Pola Data Musiman

  →Terjadi bilamana suatu deret dipengaruhi oleh faktor musiman (misalnya kuartal tahun tertentu, bulanan, atau hari-hari pada minggu tertentu)

  Contoh: Data produksi tanaman

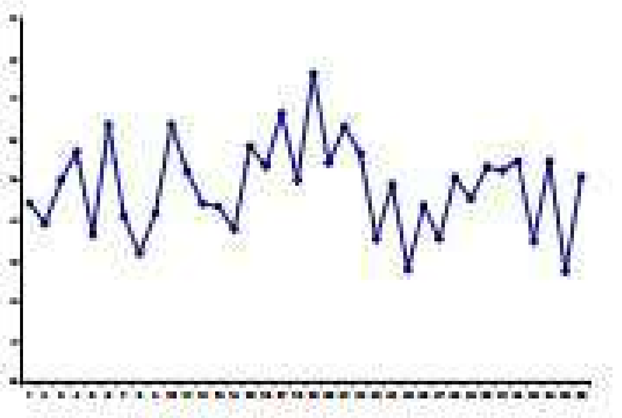
2. Pendahuluan

POLA DATA DERET WAKTU

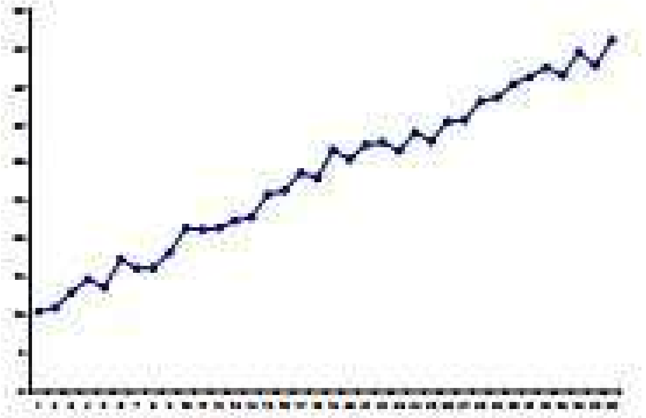
- Pola Data Siklis
  →Terjadi bila data dipengaruhi oleh fluktuasi ekonomi jangka panjang seperti yang berhubungan dengan siklus bisnis.
  Contoh: Penjualan mobil
- Pola Data Trend
  →Terjadi bilamana kenaikan atau penurunan sekuler jangka panjang dalam data
  Contoh: GNP
- Pola Gabungan antara beberapa pola yang telah disebutkan diatas

2. Pendahuluan

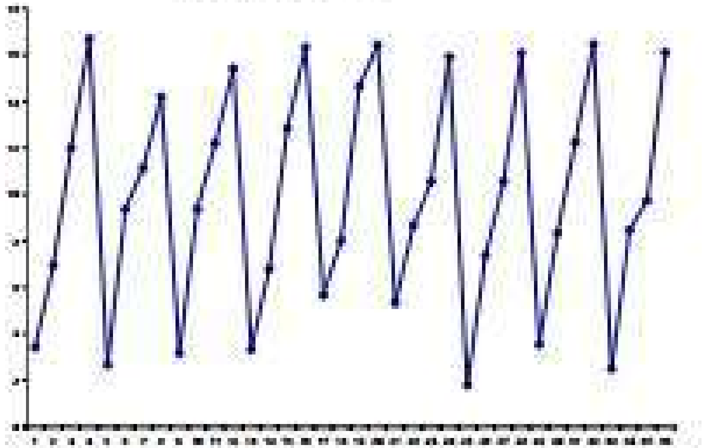
POLA DATA DERET WAKTU



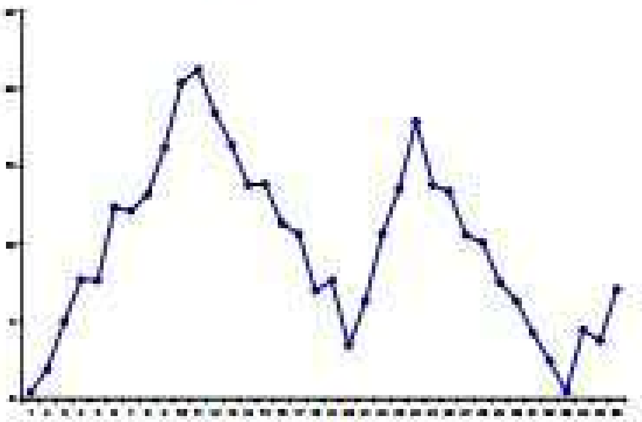
Konstan



Trend



Seasonal

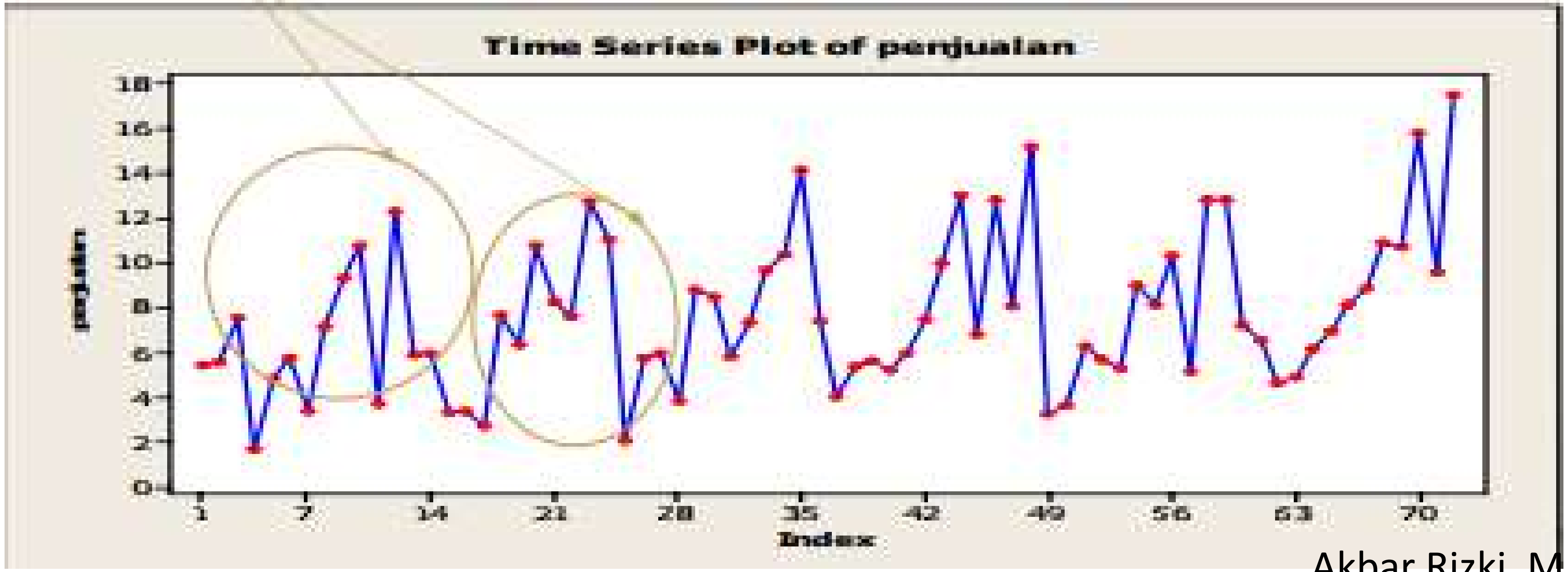


Cyclic

2. Pendahuluan

PLOT DATA DERET WAKTU

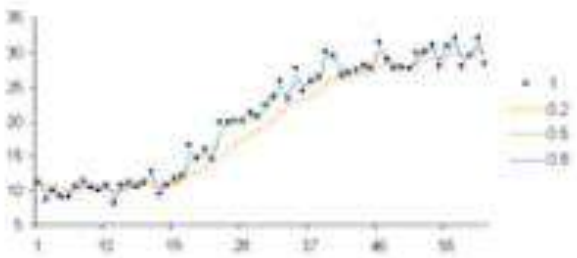
Time Series plot sangat penting untuk melihat pola data deret waktu yang akan kita analisa lebih lanjut.
Dibawah ini adalah contoh data deret waktu penjualan yang memiliki **pola musiman**.



2. Pendahuluan

RUANG LINGKUP ANALISIS DATA DERET WAKTU

Pemulusan



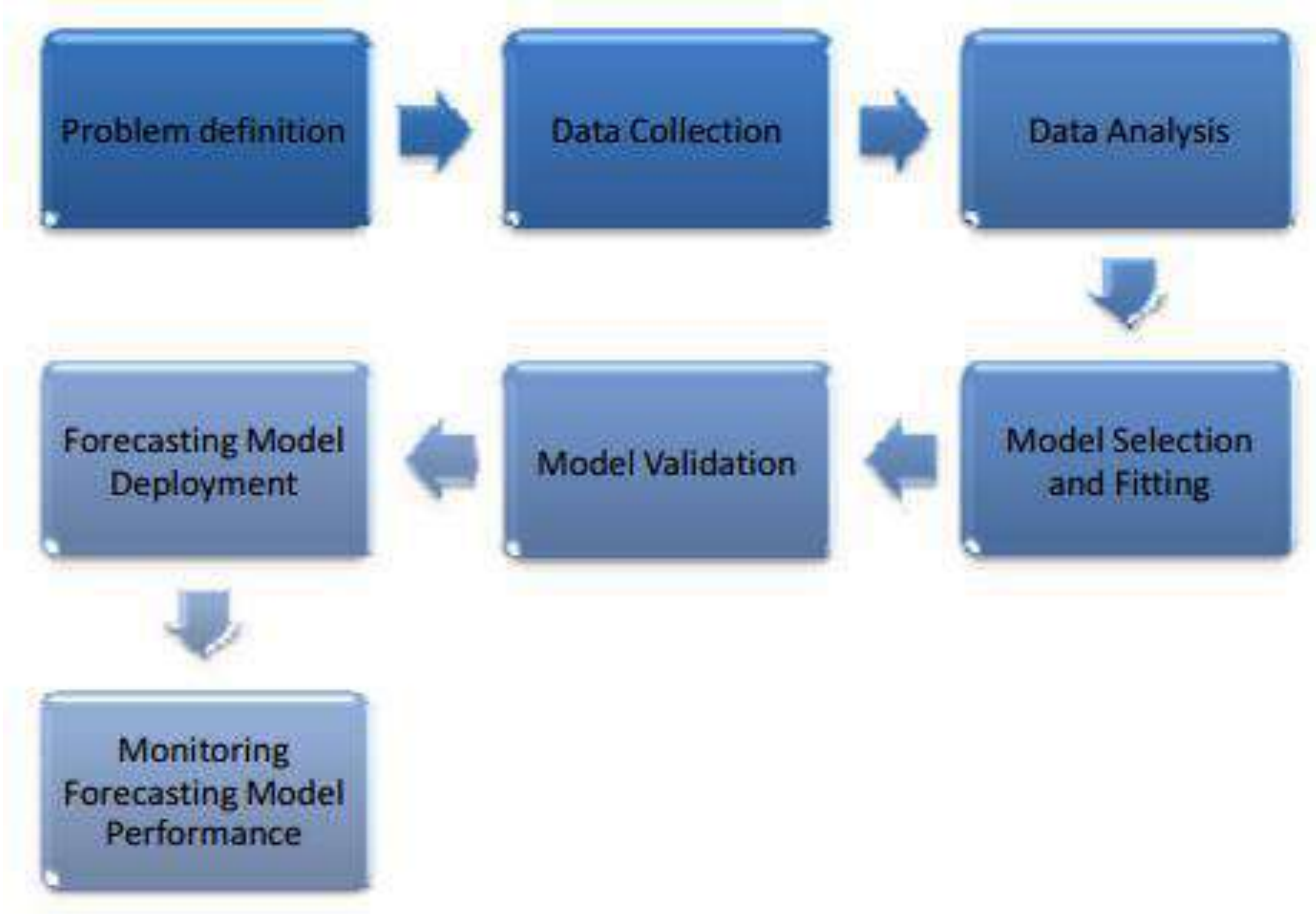
Peramalan

$$X_t = c + \varepsilon_t + \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}$$

Pemodelan

2. Pendahuluan

PROSES PERAMALAN



2. Pendahuluan

METODE DALAM ANALISIS DERET WAKTU

- **ARIMA** (Autoregressive Integrated Moving Average) pada dasarnya menggunakan fungsi deret waktu, metode ini memerlukan pendekatan model identifikasi serta penaksiran awal dari paramaternya.

  Sebagai contoh: peramalan nilai tukar mata uang asing, pergerakan nilai IHSG.
- **Regresi** menggunakan dummy variabel dalam formulasi formulasi matematisnya. Sebagai contoh: kemampuan dalam meramal sales suatu produk berdasarkan harganya.
- **Bayesian** merupakan metode yang menggunakan state space berdasarkan model dinamis linear (dynamical (dynamical linear model). Sebagai contoh: menentukan diagnosa suatu penyakit berdasarkan data-data gejala (hipertensi atau sakit jantung), mengenali warna berdasarkan fitur indeks warna RGB, mendeteksi warna kulit (skin detection) berdasarkan fitur warna chrominant.
- **Metode smoothing** dipakai untuk mengurangi mengurangi ketidakteraturan data yang bersifat musiman dengan cara membuat keseimbangan rata-rata dari data masa lampau.

2. Pendahuluan

METODE PERAMALAN KUANTITATIF

- **Metode Pemulusan (Smoothing)**
  - ✓Rata-rata bergerak tunggal (single moving average) – utk data stasioner
  - ✓Rata-rata bergerak ganda (double moving average) – utk data berpola trend
  - ✓Pemulusan exponensial tunggal (single exponential smoothing) – utk data stasioner
  - ✓Pemulusan Pemulusan exponensial exponensial ganda (double exponential exponential smoothing smoothing) – utk data berpola trend
  - ✓Pemulusan Metode Winter – utk data yang ada faktor musiman
- **Metode Pemodelan Box Jenkins (ARIMA)**

MATODE PERAMALAN KUALITATIF

“qualitative forecasting techniques relied on human judgments and intuition more than manipulation of past historical data,” atau metode yang hanya didasarkan kepada penilaian dan intuisi, bukan kepada pengolahan data historis.

2. Pendahuluan

ACCURACY MEASURES

Beberapa ukuran yang dapat dipakai untuk penilaian seberapa baik metode mengepas data:

- Mean Absolute Deviation (MAD)

$$MAD = \frac{1}{n}\sum_{t=1}^{n}|X_t - \hat{X}_t|$$

- Mean Squared Deviation (MSD)

$$MSD = \frac{1}{n}\sum_{t=1}^{n}(X_t - \hat{X}_t)^2$$

- Mean Absolute Percentage Error (MAPE)

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{X_t - \hat{X}_t}{X_t}\right| \times 100\%$$

- AIC (Akaike information criterion)
- BIC (Bayesian information criterion)

IPB University
Bogor Indonesia

TERIMAKASIH

METODE PEMULUSAN RATAAN BERGERAK SEDERHANA (RBS) & RATAAN BERGERAK GANDA (RBG)

Topik ke-2

Akbar Rizki, M.Si

OUTLINE

1. Sekilas Tentang Pemulusan
2. Rataan Bergerak Sederhana
3. Rataan Bergerak Ganda
4. Ilustrasi dengan R

OUTLINE

1. Sekilas Tentang Pemulusan
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1. SEKILAS TENTANG PEMULUSAN

- Prinsip dasar: pengenalan pola data dengan menghaluskan variasi lokal.
- Prinsip penghalusan umumnya berupa rata-rata.
- Beberapa metode penghalusan hanya cocok untuk pola data tertentu.

Metode yang akan dibahas:

- Single Moving Average
- Double Moving Average
- Single Exponential Smoothing
- Double Exponential Smoothing
- Metode Winter untuk musiman aditif
- Metode Winter untuk musiman multiplikatif

1. SEKILAS TENTANG PEMULUSAN

PERBEDAAN ANTARA NILAI RAMALAN (FORECAST VALUE) DAN NILAI DUGAAN (FITTED VALUE)

Generally, we will need to distinguish between a **forecast** or **predicted value** of y_t that was made at some previous time period, say, $t - \tau$, and a **fitted value** of y_t that has resulted from estimating the parameters in a time series model to historical data. Note that τ is the forecast lead time. The forecast made at time period $t - \tau$ is denoted by $\hat{y}_t(t - \tau)$. There is a lot of interest in the **lead – 1** forecast, which is the forecast of the observation in period t, y_t, made one period prior, $\hat{y}_t(t - 1)$. We will denote the fitted value of y_t by $\hat{y}_t$.

forecast value

fitted value

1. SEKILAS TENTANG PEMULUSAN

PERBEDAAN ANTARA FORECAST ERROR DAN RESIDUAL

We will also be interested in analyzing **forecast errors**. The forecast error that results from a forecast of y_t that was made at time period $t-\tau$ is the **lead** $-\tau$ **forecast error**

$$e_t(\tau) = y_t - \hat{y}_t(t-\tau)$$

For example, the lead $-$ 1 forecast error is

$$\boxed{e_t(1) = y_t - \hat{y}_t(t-1)}$$

OUTLINE

1. Sekilas Tentang Pemulusan

2. Rataan Bergerak Sederhana

3. Rataan Bergerak Ganda

4. Ilustrasi dengan R

2. RATAAN BERGERAK SEDERHANA (RBS)

- Ide: data pada suatu periode dipengaruhi oleh data beberapa periode sebelumnya
- Cocok untuk pola data konstan/stasioner
- Prinsip dasar:
  - Data *smoothing* pada periode ke-t merupakan rata-rata dari m buah data dari data periode ke-t hingga ke-(t-m+1) ➔ $$S_t = \frac{1}{m}\sum_{i=t-m+1}^{t} X_i$$
  - Data *smoothing* pada periode ke-t berperan sebagai nilai *forecasting* pada periode ke-t+1

$$F_t = S_{t-1} \text{ dan } F_{n,h} = S_n$$

- $Var(S_t) < Var\ (X_t)$

2. RATAAN BERGERAK SEDERHANA (RBS)

Bulan (t)	Data (X_t)
1	5
2	7
3	6
4	4
5	5
6	6
7	8
8	7
9	8
10	7

CONTOH:

Berikut data profit bulanan (dalam milyar) suatu perusahaan di bidang ekspor impor selama 10 bulan terakhir.

a. Tentukan data termuluskan melalui teknik rataan bergerak sederhana dengan rentang m=3. kemudian buat time series plot nya bersama dengan data asal

b. Tentukan ramalan besarnya profit pada setiap satu waktu ke depan. Berapa ramalan profit pada bulan ke-11 dan ke-12

2. RATAAN BERGERAK SEDERHANA (RBS)

a. Tentukan data termuluskan melalui teknik rataan bergerak sederhana dengan rentang m=3. kemudian buat time series plot nya bersama dengan data asal.

Bulan (*t*)	Data (X_t)	*Smoothing* (S_t)
1	5	-
2	7	-
3	6	6
4	4	5.6
5	5	5
6	6	5
7	8	6.3
8	7	7
9	8	7.6
10	7	7.3

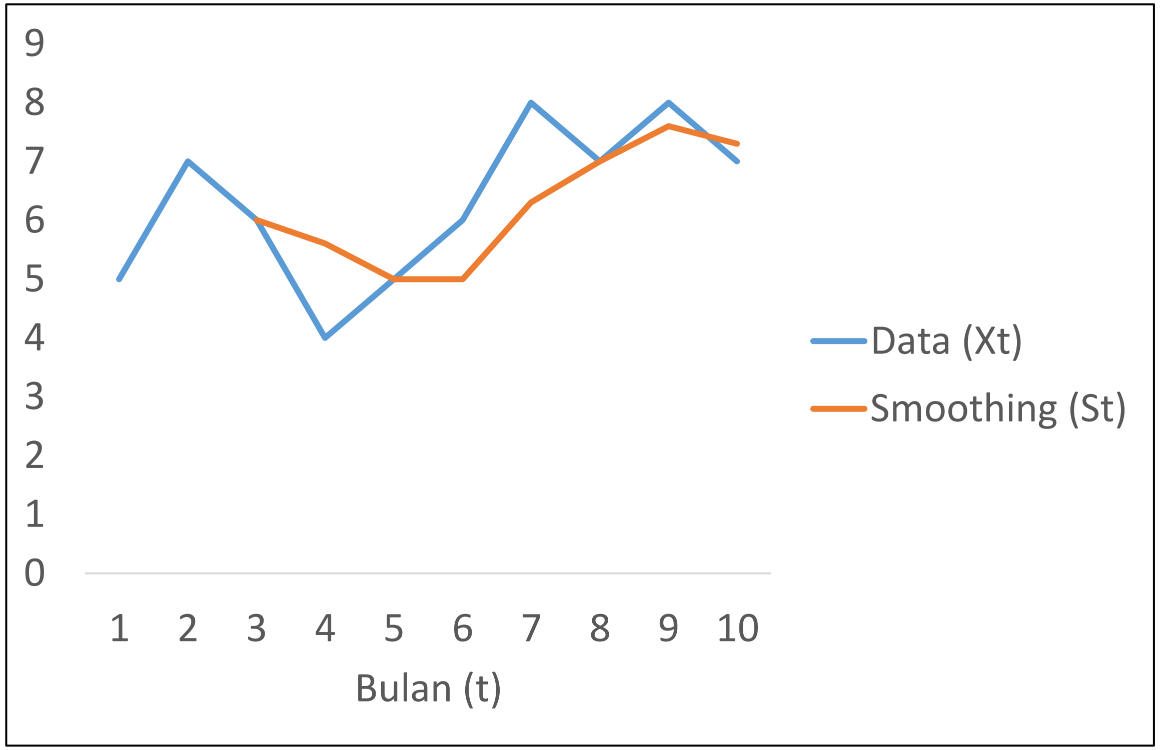
$$S_t = \frac{1}{m} \sum_{i=t-m+1}^{t} X_i$$

$$S_3 = \frac{1}{3}(X_1 + X_2 + X_3) = \frac{1}{3}(5 + 7 + 6) = 6$$

$$S_4 = \frac{1}{3}(X_2 + X_3 + X_4) = \frac{1}{3}(7 + 6 + 4) = 5.6$$

2. RATAAN BERGERAK SEDERHANA (RBS)

a. Tentukan data termuluskan melalui teknik rataan bergerak sederhana dengan rentang m=3. kemudian buat time series plot nya bersama dengan data asal.



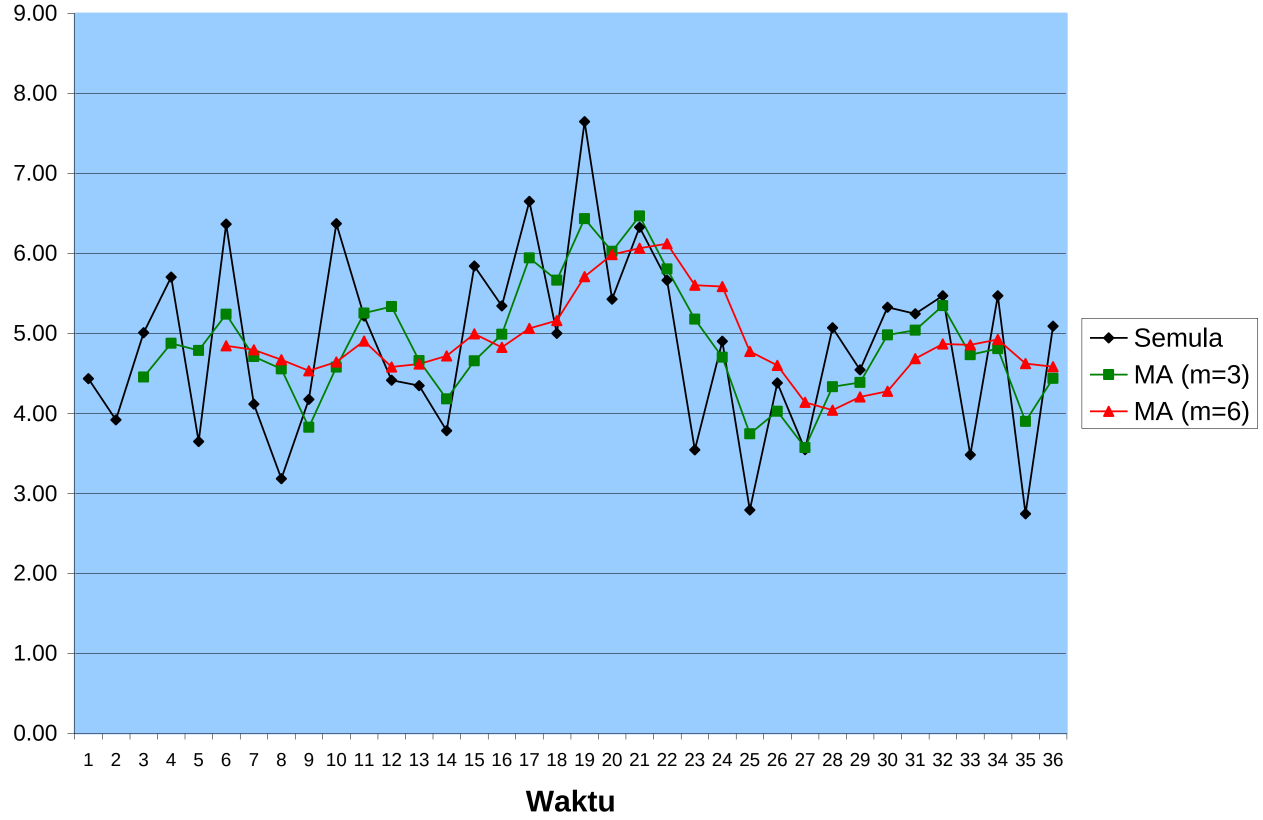
2. RATAAN BERGERAK SEDERHANA (RBS)

Periode (t)	Data (X_t)	*Smoothing* (S_t)	*Forecasting* (F_t)
1	5	-	-
2	7	-	-
3	6	6	-
4	4	5.6	6
5	5	5	5.6
6	6	5	5
7	8	6.3	5
8	7	7	6.3
9	8	7.6	7
10	7	7.3	7.6
11			7.3
12			7.3

b. Tentukan ramalan besarnya profit pada setiap satu waktu ke depan. Berapa ramalan profit pada bulan ke-11 dan ke-12

2. RATAAN BERGERAK SEDERHANA (RBS)

PENGARUH PEMILIHAN NILAI m



MA dengan *m* yang lebih besar menghasilkan pola data yang lebih halus.

OUTLINE

1. Sekilas Tentang Pemulusan
2. Rataan Bergerak Sederhana
3. Rataan Bergerak Ganda
4. Ilustrasi dengan R

3. RATAAN BERGERAK GANDA (RBG)

- Mirip dengan *single moving average*
- Cocok untuk data yang berpola tren
- Proses penghalusan dengan rata-rata dilakukan dua kali
  - Tahap I:

$$S_{1,t} = \frac{1}{m} \sum_{i=t-m+1}^{t} X_i$$

  - Tahap II:

$$S_{2,t} = \frac{1}{m} \sum_{i=t-m+1}^{t} S_{1,i}$$

3. RATAAN BERGERAK GANDA (RBG)

- Forecasting dilakukan dengan formula

$$F_{2,t,t+h} = A_t + B_t(h)$$

dengan

$$A_t = 2S_{1,t} - S_{2,t}$$

$$B_t = \frac{2}{m-1}\left(S_{1,t} - S_{2,t}\right)$$

3. RATAAN BERGERAK GANDA (RBG)

CONTOH:

Berikut data omset bulanan (dalam milyar) suatu perusahaan selama 9 bulan terakhir.

a. Tentukan data termuluskan melalui teknik rataan bergerak berganda dengan rentang m=3. kemudian buat time series plot nya bersama dengan data asal

b. Tentukan ramalan besarnya omset pada setiap satu waktu ke depan. Berapa ramalan omset pada bulan ke-10, ke-11, dan ke-12

t	X_t
1	12.50
2	11.80
3	12.85
4	13.95
5	13.30
6	13.95
7	15.00
8	16.20
9	16.10

3. RATAAN BERGERAK GANDA (RBG)

a. Tentukan data termuluskan melalui teknik rataan bergerak berganda dengan rentang m=3. kemudian buat time series plot nya bersama dengan data asal

t	X_t	$S_{1,t}$	$S_{2,t}$
1	12.50		
2	11.80		
3	12.85	12.38	
4	13.95	12.87	
5	13.30	13.37	12.87
6	13.95	13.73	13.32
7	15.00	14.08	13.73
8	16.20	15.05	14.29
9	16.10	15.77	14.97

$$S_{1,t} = \frac{1}{m}\sum_{i=t-m+1}^{t} X_i$$

$$S_{1,3} = \frac{1}{3}(X_1 + X_2 + X_3) = \frac{1}{3}(12.5 + 11.8 + 12.85) = 12.38$$

$$S_{1,4} = \frac{1}{3}(X_2 + X_3 + X_4) = \frac{1}{3}(11.8 + 12.85 + 13.95) = 12.87$$

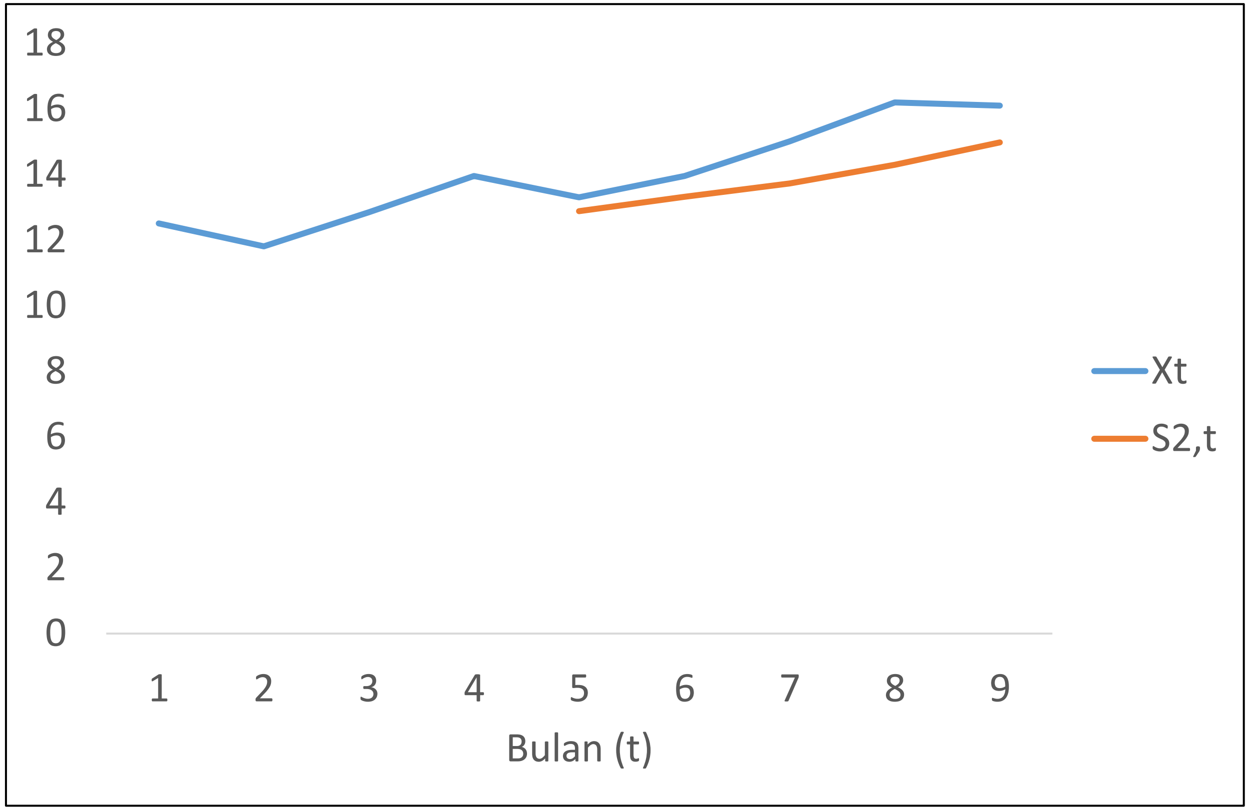
$$S_{2,t} = \frac{1}{m}\sum_{i=t-m+1}^{t} S_{1,i}$$

$$S_{2,5} = \frac{1}{3}\left(S_{1,3} + S_{1,4} + S_{1,5}\right) = \frac{1}{3}(12.38 + 12.87 + 13.37)$$
$$= 12.87$$

$$S_{2,6} = \frac{1}{3}\left(S_{1,4} + S_{1,5} + S_{1,6}\right) = \frac{1}{3}(12.87 + 13{,}37 + 13.73)$$
$$= 13.32$$

3. RATAAN BERGERAK GANDA (RBG)

a. Tentukan data termuluskan melalui teknik rataan bergerak berganda dengan rentang m=3. kemudian buat time series plot nya bersama dengan data asal



3. RATAAN BERGERAK GANDA (RBG)

t	X_t	$S_{1,t}$	$S_{2,t}$	A_t	B_t	$F_{2,t}$
1	12.50					
2	11.80					
3	12.85	12.38				
4	13.95	12.87				
5	13.30	13.37	12.87	13.87	0.50	
6	13.95	13.73	13.32	14.14	0.41	14.37
7	15.00	14.08	13.73	14.43	0.35	14.55
8	16.20	15.05	14.29	15.81	0.76	14.78
9	16.10	15.77	14.97	16.57	0.80	16.57
10						17.37
11						18.17
12						18.97

$$A_t = 2S_{1,t} - S_{2,t}$$

$A_5 = 2S_{1,5} - S_{2,5} = 2(13.37) - 12.87$
=13.87

$$B_t = \frac{2}{m-1}\left(S_{1,t} - S_{2,t}\right)$$

$B_5 = \frac{2}{3-1}\left(S_{1,5} - S_{2,5}\right)$
$= \frac{2}{2}(13.37 - 12.87)$=0.5

$$F_{2,t,t+h} = A_t + B_t(h)$$

$F_{2,9,12} = A_9 + B_9(3) = 16.57 + 0.8(3)$
=18.97

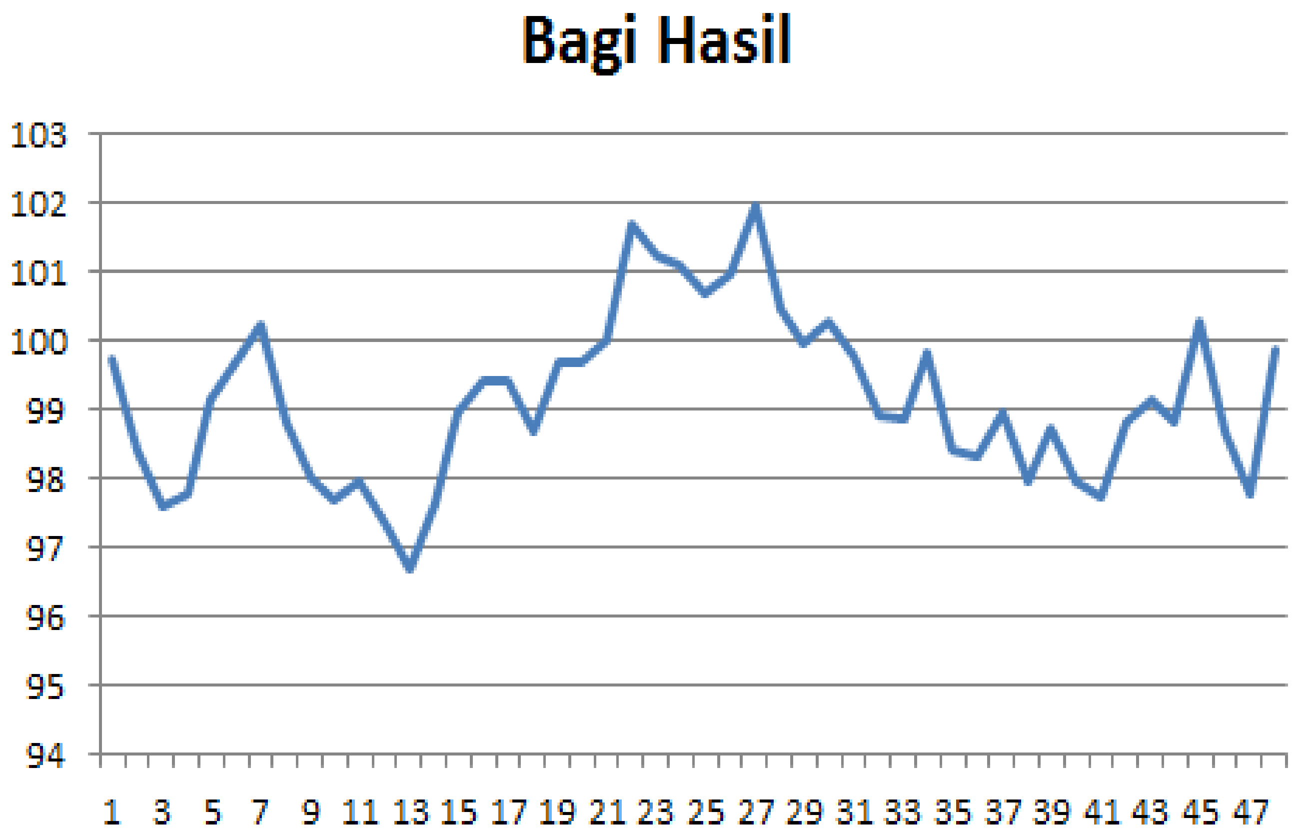
OUTLINE

1. Sekilas Tentang Pemulusan
2. Rataan Bergerak Sederhana
3. Rataan Bergerak Ganda
4. Ilustrasi dengan R

4. ILUSTRASI DENGAN R

Sebagai ilustrasi, tersedia data bagi hasil suatu bank syariah per bulan (**File excel Moving Average.csv**). Data ini dicatat setiap tanggal 1 di masing-masing bulan. Periodenya dari Januari 1989 hingga Desember 1992, sehingga terdapat 48 pengamat

Data Contoh: SMA



4. ILUSTRASI DENGAN R

SMA dengan R

Sintaks R

```
library("forecast")
library("TTR")
library("graphics")
Data1<-read.csv("D:/campus/work/Bahan Mandiri/Moving Average.csv",
header=TRUE)
#membentuk objek time  series
Data1.ts<-ts(Data1)
#melakukan Single Moving  Average dengan n=3
Data1.sma<-SMA(Data1.ts,  n=3)
ramal.sma<-c(NA,Data1.sma)
Data<-cbind (bagihasil_aktual=c (Data1.ts,NA),pemulusan=c
(Data1.sma,NA),ramal.sma)
```

Output R

	bagihasil_aktual	pemulusan	ramal.sma
[1,]	99.72244	NA	NA
[2,]	98.38826	NA	NA
[3,]	97.57348	98.56139	NA
[4,]	97.75673	97.90616	98.56139
[5,]	99.12783	98.15268	97.90616
[6,]	99.65564	98.84673	98.15268
[7,]	100.21011	99.66453	98.84673
[8,]	98.79006	99.55194	99.66453
[9,]	97.99188	98.99735	99.55194
[10,]	97.68087	98.15427	98.99735
[11,]	97.93829	97.87034	98.15427
[12,]	97.37835	97.66583	97.87034
[13,]	96.68437	97.33367	97.66583
[14,]	97.62021	97.22764	97.33367
[15,]	98.92994	97.74484	97.22764
.			
.			
[44,]	98.81002	98.91963	98.56444
[45,]	100.26684	99.40174	98.91963
[46,]	98.68675	99.25453	99.40174
[47,]	97.75455	98.90271	99.25453
[48,]	99.84867	98.76332	98.90271
[49,]	NA	NA	98.76332

ty

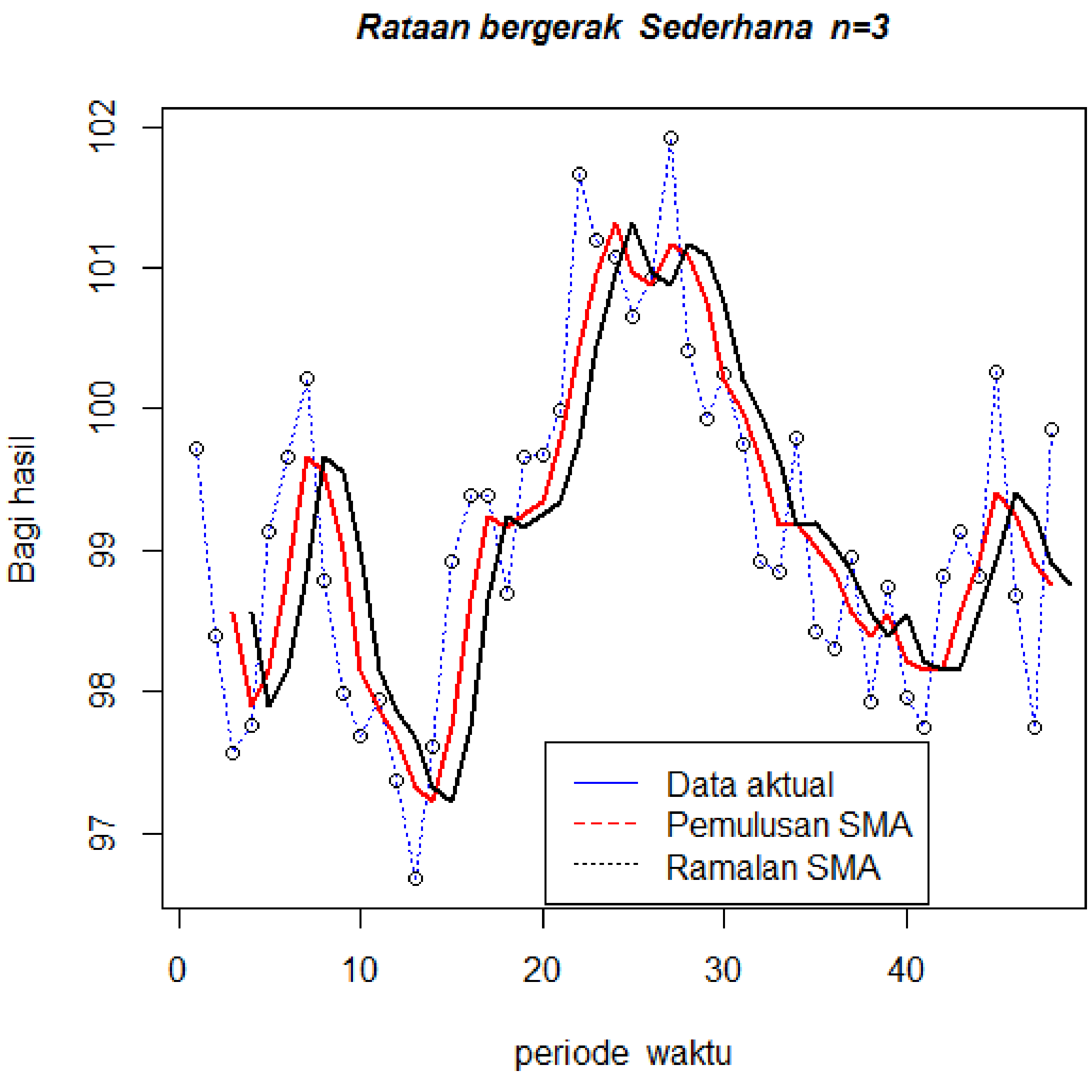
4. ILUSTRASI DENGAN R

SMA dengan R

Sintaks R

```
#membuat plot
ts.plot (Data1.ts,xlab="periode  waktu",ylab="Bagi hasil",
col="blue",lty=3)
points(Data1.ts)
lines (Data1.sma,col="red",lwd=2)
lines (ramal.sma,col="black",lwd= 2)
title("Rataan bergerak  Sederhana  n=3",cex.main=1,font.main=4
,col.main="black")
legend(locator(1),legend=c ("Data aktual","Pemulusan
SMA","Ramalan SMA"),lty=1:3,col=c ("blue","red","black"))
```

Output R



4. ILUSTRASI DENGAN R

Mencari nilai keakuratan

Mean Absolute Percentage Error (MAPE):

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{x_t - \hat{x}_t}{x_t}\right| x100\%$$

Sintaks R

```
#menghitung nilai keakuratan
error<-Data1.ts-ramal.sma[1:length(Data1.ts)]
MAPE<-
mean(abs((error[4:length(Data1.ts)]/ramal.sma[4:length(Data1.ts)])*100))
```

Output R

```
> MAPE
[1] 0.82229
```

4. ILUSTRASI DENGAN R

DMA dengan R

Sintaks R

```
bagihasil.dma<-SMA(Data1.sma,n=3)
At<-2*Data1.sma-bagihasil.dma
Bt<-Data1.sma-bagihasil.dma
pemulusan.dma<-At+Bt
ramal.dma<-c(NA,pemulusan.dma)
Data.dma<-
cbind(bagihasil_aktual=c(Data1.ts,NA),pemulusan
.dma=c(pemulusan.dma,NA),ramal.dma)
```

Output R

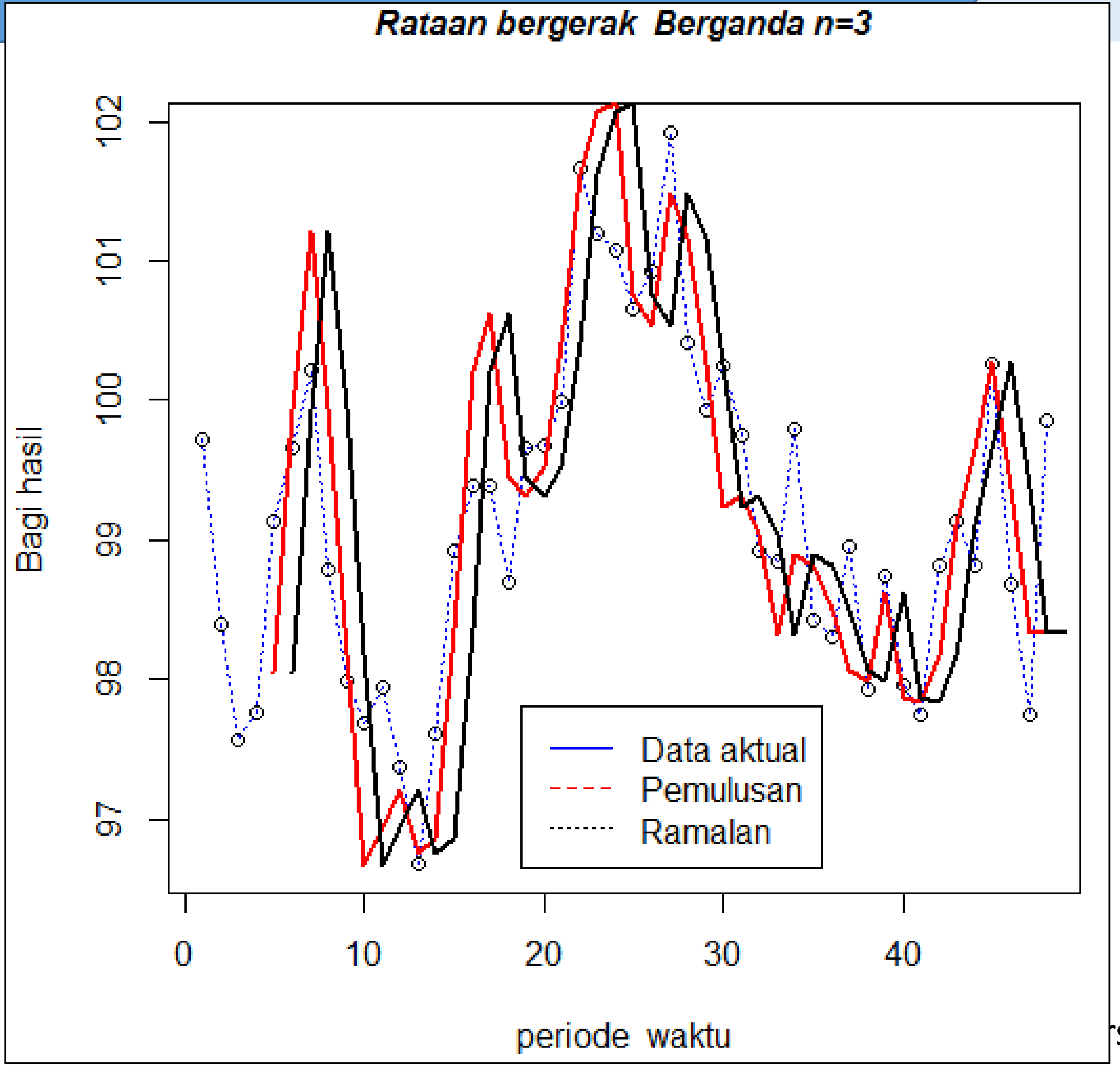
	bagihasil_aktual	pemulusan.dma	ramal.dma
[1,]	99.72244	NA	NA
[2,]	98.38826	NA	NA
[3,]	97.57348	NA	NA
[4,]	97.75673	NA	NA
[5,]	99.12783	98.04455	NA
[6,]	99.65564	99.93649	98.04455
[7,]	100.21011	101.21762	99.93649
[8,]	98.79006	99.94702	101.21762
[9,]	97.99188	98.18284	99.94702
.			
.			
.			
[47,]	97.75455	98.33548	99.37967
[48,]	99.84867	98.34292	98.33548
[49,]	NA	NA	98.34292

4. ILUSTRASI DENGAN R

DMA dengan R

Sintaks R

```
#membuat plot
ts.plot (Data1.ts,xlab="periode  waktu",ylab="Bagi
hasil",  col="blue",lty=3)
points(Data1.ts)
lines (pemulusan.dma,col="red",lwd=2)
lines (ramal.dma,col="black",lwd= 2)
title("Rataan bergerak  Berganda
n=3",cex.main=1,font.main=4 ,col.main="black")
legend(locator(1),legend=c ("Data
aktual","Pemulusan","Ramalan"),lty=1:3,col=c
("blue","red","black"))
```



sity

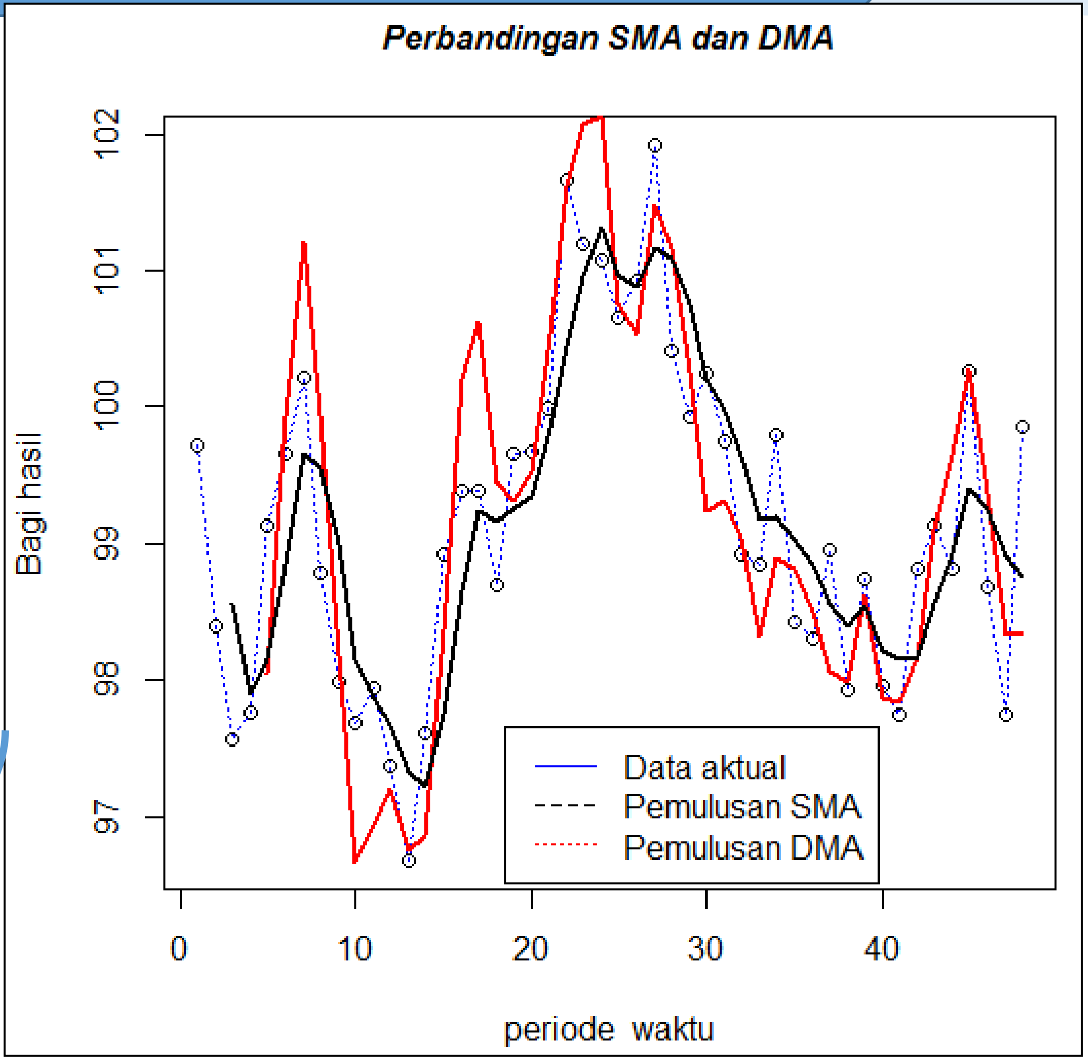
4. ILUSTRASI DENGAN R

Perbandingan SMA dan DMA

Sintaks R

```
#perbandingan SMA dan DMA
ts.plot (Data1.ts,xlab="periode  waktu",ylab="Bagi
hasil",  col="blue",lty=3)
points(Data1.ts)
lines (pemulusan.dma,col="red",lwd=2)
lines (Data1.sma,col="black",lwd= 2)
title("Perbandingan SMA dan
DMA",cex.main=1,font.main=4 ,col.main="black")
legend(locator(1),legend=c ("Data
aktual","Pemulusan SMA","Pemulusan
DMA"),lty=1:3,col=c ("blue","black","red"))
```

Output R



MAPE :
0.82229
0.8887221

4. ILUSTRASI DENGAN R

TUGAS PRAKTIKUM 1

Gunakan data (the sales of mature pharmaceutical product) di dalam buku Montgomery (Appendix B, Table B.2, halaman 587)

a. Tentukan data termuluskan melalui teknik rataan bergerak sederhana dengan rentang m=4. hitung ramalan untuk 5 waktu ke depan.
b. Tentukan data termuluskan melalui teknik rataan bergerak sederhana dengan rentang m=6. hitung ramalan untuk 5 waktu ke depan.
c. Buat time series plotnya masing-masing dengan data asal.
d. Tentukan nilai SSE, MSE, dan MAPE masing-masing untuk (a) dan (b). Apa kesimpulan Anda.

Catatan: Kerjakan terlebih dahulu poin (a) s.d (d) di atas menggunakan Excel. Kemudian bandingkan hasilnya dengan keluaran dari program R.

4. ILUSTRASI DENGAN R

TUGAS PRAKTIKUM 2

Gunakan data (the sales of mature pharmaceutical product) di dalam buku Montgomery (Appendix B, Table B.2, halaman 587)

a. Tentukan data termuluskan melalui teknik rataan bergerak berganda dengan rentang m=4. hitung ramalan untuk 5 waktu ke depan.
b. Tentukan data termuluskan melalui teknik rataan berganda sederhana dengan rentang m=6. hitung ramalan untuk 5 waktu ke depan.
c. Buat time series plotnya masing-masing dengan data asal.
d. Tentukan nilai SSE, MSE, dan MAPE masing-masing untuk (a) dan (b) serta bandingkan pula dengan hasil pada tugas praktikum 1. Apa kesimpulan Anda.

Catatan: Kerjakan terlebih dahulu poin (a) s.d (d) di atas menggunakan Excel. Kemudian bandingkan hasilnya dengan keluaran dari program R.

4. ILUSTRASI DENGAN R

TUGAS PRAKTIKUM

TABLE B.2 Pharmaceutical Product Sales

Week	Sales (In Thousands)	Week	Sales (In Thousands)	Week	Sales (In Thousands)	Week	Sales (In Thousands)
1	10618.1	31	10334.5	61	10538.2	91	10375.4
2	10537.9	32	10480.1	62	10286.2	92	10123.4
3	10209.3	33	10387.6	63	10171.3	93	10462.7
4	10553.0	34	10202.6	64	10393.1	94	10205.5
5	9934.9	35	10219.3	65	10162.3	95	10522.7
6	10534.5	36	10382.7	66	10164.5	96	10253.2
7	10196.5	37	10820.5	67	10327.0	97	10428.7
8	10511.8	38	10358.7	68	10365.1	98	10615.8
9	10089.6	39	10494.6	69	10755.9	99	10417.3
10	10371.2	40	10497.6	70	10463.6	100	10445.4
11	10239.4	41	10431.5	71	10080.5	101	10690.6
12	10472.4	42	10447.8	72	10479.6	102	10271.8
13	10827.2	43	10684.4	73	9980.9	103	10524.8
14	10640.8	44	10176.5	74	10039.2	104	9815.0
15	10517.8	45	10616.0	75	10246.1	105	10398.5
16	10154.2	46	10627.7	76	10368.0	106	10553.1
17	9969.2	47	10684.0	77	10446.3	107	10655.8
18	10260.4	48	10246.7	78	10535.3	108	10199.1
19	10737.0	49	10265.0	79	10786.9	109	10416.6
20	10430.0	50	10090.4	80	9975.8	110	10391.3
21	10689.0	51	9881.1	81	10160.9	111	10210.1
22	10430.4	52	10449.7	82	10422.1	112	10352.5
23	10002.4	53	10276.3	83	10757.2	113	10423.8
24	10135.7	54	10175.2	84	10463.8	114	10519.3
25	10096.2	55	10212.5	85	10307.0	115	10596.7
26	10288.7	56	10395.5	86	10134.7	116	10650.0
27	10289.1	57	10545.9	87	10207.7	117	10741.6
28	10589.9	58	10635.7	88	10488.0	118	10246.0
29	10551.9	59	10265.2	89	10262.3	119	10354.4
30	10208.3	60	10551.6	90	10785.9	120	10155.4

4. ILUSTRASI DENGAN R

TUGAS PRAKTIKUM 3

Gunakan data profit sebuah perusahaan berikut ini:

a. Tentukan data termuluskan melalui teknik rataan bergerak sederhana dengan rentang m=4. hitung ramalan untuk 5 waktu ke depan.
b. Tentukan data termuluskan melalui teknik rataan bergerak sederhana dengan rentang m=6. hitung ramalan untuk 5 waktu ke depan.
c. Buat time series plotnya masing-masing dengan data asal.
d. Tentukan nilai SSE, MSE, dan MAPE masing-masing untuk (a) dan (b). Apa kesimpulan Anda.

Catatan: Kerjakan terlebih dahulu poin (a) s.d (d) di atas menggunakan Excel. Kemudian bandingkan hasilnya dengan keluaran dari program R.

periode	profit	periode	profit	periode	profit
1	140385.5	11	151378	21	205837.7
2	134759.9	12	135571	22	215129.8
3	129560.6	13	141933.1	23	219035.5
4	133791.5	14	135256.7	24	227126.9
5	144560.6	15	168587.7	25	228542.1
6	147848.8	16	165804.5	26	229196.3
7	150515.9	17	173212.5	27	238975.8
8	142275.6	18	193385.4	28	243293
9	150644.6	19	196898.5	29	239453.6
10	148968.9	20	199971.6	30	238108.9

4. ILUSTRASI DENGAN R

TUGAS PRAKTIKUM 4

Gunakan data profit sebuah perusahaan berikut ini:

a. Tentukan data termuluskan melalui teknik rataan bergerak ganda dengan rentang m=4. hitung ramalan untuk 5 waktu ke depan.
b. Tentukan data termuluskan melalui teknik rataan bergerak ganda dengan rentang m=6. hitung ramalan untuk 5 waktu ke depan.
c. Buat time series plotnya masing-masing dengan data asal.
d. Tentukan nilai SSE, MSE, dan MAPE masing-masing untuk (a) dan (b) serta bandingkan pula dengan hasil pada tugas praktikum 3. Apa kesimpulan Anda.

Catatan: Kerjakan terlebih dahulu poin (a) s.d (d) di atas menggunakan Excel. Kemudian bandingkan hasilnya dengan keluaran dari program R.

periode	profit	periode	profit	periode	profit
1	140385.5	11	151378	21	205837.7
2	134759.9	12	135571	22	215129.8
3	129560.6	13	141933.1	23	219035.5
4	133791.5	14	135256.7	24	227126.9
5	144560.6	15	168587.7	25	228542.1
6	147848.8	16	165804.5	26	229196.3
7	150515.9	17	173212.5	27	238975.8
8	142275.6	18	193385.4	28	243293
9	150644.6	19	196898.5	29	239453.6
10	148968.9	20	199971.6	30	238108.9

IPB University
Bogor Indonesia

TERIMAKASIH

METODE PEMULUSAN EKSPONENSIAL TUNGGAL DAN EXPONENSIAL GANDA

Pertemuan ke-3
Akbar Rizki, M.Si

OUTLINE

1. Single Exponential Smoothing (SES)

2. Double Exponential Smoothing (DES

3. Tugas Praktikum

OUTLINE

1. Single Exponential Smoothing (SES)

2. Double Exponential Smoothing (DES

3. Tugas Praktikum

1. SINGLE EXPONENTIAL SMOOTHING

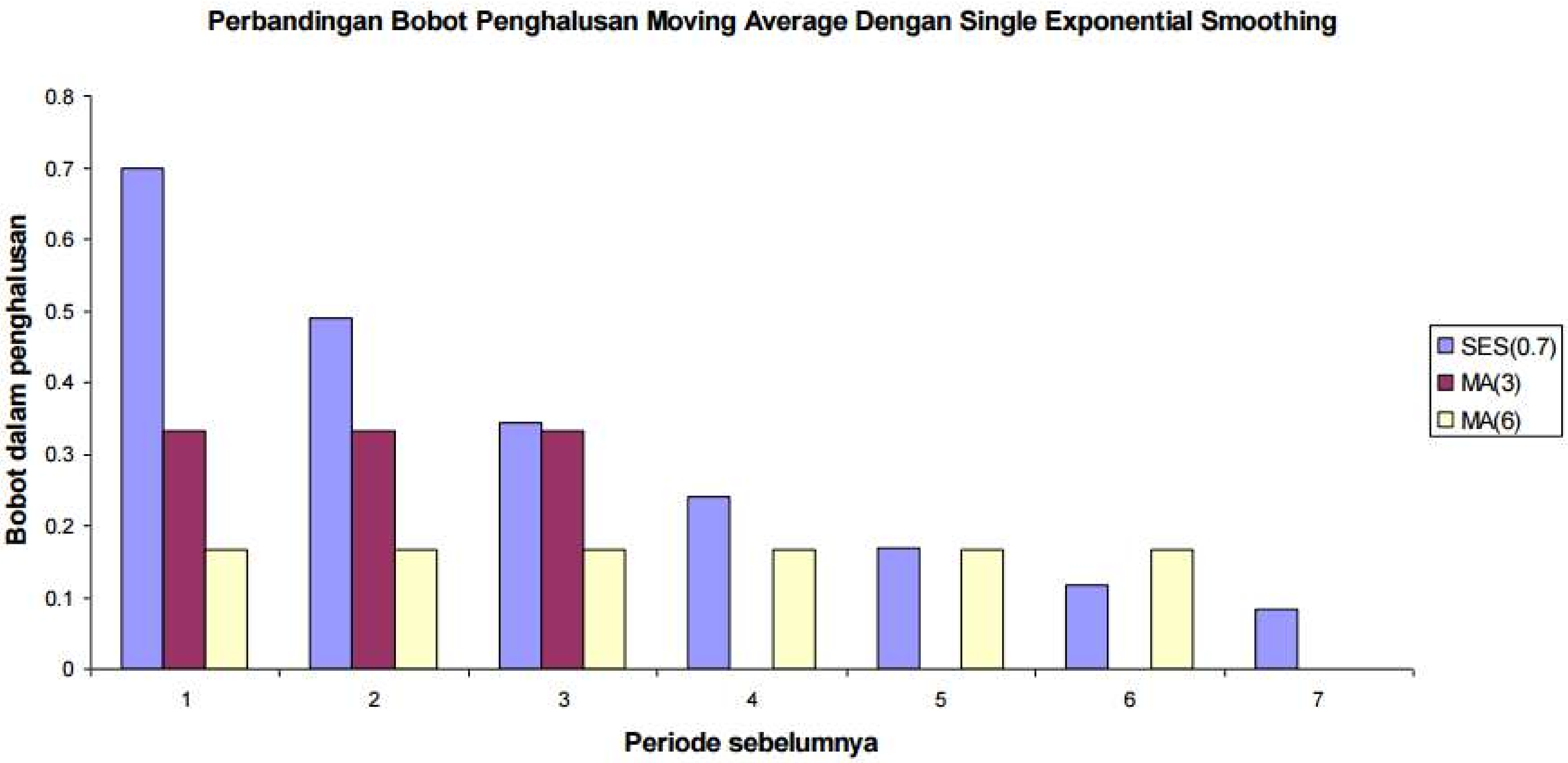
EXPONENTIAL SMOOTHING

- Metode Exponential Smoothing adalah metode pemulusan dengan melakukan pembobotan menurun secara eksponensial
- Nilai yang lebih baru diberi bobot yang lebih besar dari nilai terdahulu.
- Terdapat satu atau lebih parameter pemulusan yang ditentukan secara eksplisit, dan hasil pemilihan parameter tersebut akan menentukan bobot yang akan diberikan pada nilai pengamatan.
- Ada dua macam model, yaitu model tunggal dan ganda

1. SINGLE EXPONENTIAL SMOOTHING

- Metode Moving Average mengakomodir pengaruh data beberapa periode sebelumnya melalui pemberian bobot yang sama dalam proses merata-rata.
- Hal ini berarti bobot pengaruh sekian periode data tersebut dianggap sama.
- Dalam kenyataannya, bobot pengaruh data yang lebih baru mestinya lebih besar.
- Adanya perbedaan bobot pengaruh ini diakomodir metode SES dengan menetapkan bobot secara eksponensial.

1. SINGLE EXPONENTIAL SMOOTHING



1. SINGLE EXPONENTIAL SMOOTHING

- Nilai smoothing pada periode ke-t: $\tilde{y}_T = \lambda y_T + (1 - \lambda)\tilde{y}_{T-1}$
- Nilai λ merupakan parameter pemulusan dengan nilai 0 < λ< 1.
- Inisialisasi smoothing pada periode ke-0 (montgomerry hlm:241)

1. Set $\tilde{y}_0 = y_1$. If the changes in the process are expected to occur early and fast, this choice for the starting value for $\tilde{y}_T$ is reasonable.
2. Take the average of the available data or a subset of the available data, $\bar{y}$, and set $\tilde{y}_0 = \bar{y}$. If the process is at least at the beginning locally constant, this starting value may be preferred.

- Nilai smoothing pada periode ke-t bertindak sebagai nilai forecast pada periode ke- $(\mathrm{T} + \tau)$

$$\hat{y}_{T+\tau}(T) = \tilde{y}_T$$

1. SINGLE EXPONENTIAL SMOOTHING

- Pada data file excel terdapat data pada sheet ses yang merupakan data index saham Dow Jones dari juni-99 sampai juni-06

1. Lakukan pemulusan terhadap data tersebut dengan Single Exponential Smoothing untuk nilai $\lambda = 0.2\ dan\ 0.4$, dengan formula:

$$\tilde{y}_T = \lambda y_T + (1 - \lambda)\tilde{y}_{T-1}$$

2. Hitung nilai peramalan satu periode ke depan untuk harga saham Dow Jones dengan formula sebagai berikut: $\hat{y}_{T+\tau}(T) = \tilde{y}_T$

3. hitunglah: $e_T(1) = y_{T+1} - \hat{y}_{T+1}(T)$

1. SINGLE EXPONENTIAL SMOOTHING

4. Hitunglah nilai MAD, MAPE, dan MSE dari setiap hasil peramalan yang dilakukan. Bandingkan, bagaimana hasil yang Anda peroleh?
5. Hitunglah SSE

$$SS_E(\lambda) = \sum_{t=1}^{T} e_{t-1}^2(1)$$

Nilai λ berapa yang memberikan nilai SSE paling kecil?

OUTLINE

1. Single Exponential Smoothing (SES)
2. Double Exponential Smoothing (DES)
3. Tugas Praktikum

2. DOUBLE EXPONENTIAL SMOOTHING

- Digunakan untuk data yang memiliki pola tren
- Semacam SES, hanya saja dilakukan dua kali
  - ✓Pertama untuk tahapan ‘level’
  - ✓Kedua untuk tahapan ‘tren

2. DOUBLE EXPONENTIAL SMOOTHING

- Pada data file excel terdapat data pada sheet des yang merupakan data consumer price index (CPI)

1. Lakukan pemulusan terhadap data tersebut dengan Double Exponential Smoothing untuk nilai $\lambda = 0.3\ dan\ \gamma = 0.3$, dengan formula:

$$\tilde{y}_T = \lambda y_T + (1-\lambda)\tilde{y}_{T-1}$$

$$\tilde{y}_T^{(2)} = \lambda \tilde{y}_T^{(1)} + (1-\lambda)\,\tilde{y}_{T-1}^{(2)}$$

Dengan initial value dari

$$\tilde{y}_0^{(1)} = \hat{\beta}_{0,0} - \frac{1-\lambda}{\lambda}\hat{\beta}_{1,0}$$

$$\tilde{y}_0^{(2)} = \hat{\beta}_{0,0} - 2\left(\frac{1-\lambda}{\lambda}\right)\hat{\beta}_{1,0}$$

2. DOUBLE EXPONENTIAL SMOOTHING

2. Hitung nilai peramalan satu periode ke depan untuk CPI dengan formula sebagai berikut:

$$\hat{y}_{T+\tau}(\tau) = \left(2\tilde{y}_T^{(1)} - \tilde{y}_T^{(2)}\right) + \tau\frac{\lambda}{1-\lambda}\left(\tilde{y}_T^{(1)} - \tilde{y}_T^{(2)}\right)$$
$$= \left(2 + \frac{\lambda}{1-\lambda}\tau\right)\tilde{y}_T^{(1)} - \left(1 + \frac{\lambda}{1-\lambda}\tau\right)\tilde{y}_T^{(2)}.$$

3. hitunglah:

$$e_T(1) = y_{T+1} - \hat{y}_{T+1}(T)$$

2. DOUBLE EXPONENTIAL SMOOTHING

4. Hitunglah nilai MAD, MAPE, dan MSE dari setiap hasil peramalan yang dilakukan. Bandingkan, bagaimana hasil yang Anda peroleh?
5. Hitunglah SSE

$$SS_E(\lambda) = \sum_{t=1}^{T} e_{t-1}^2(1)$$

Nilai λ berapa yang memberikan nilai SSE paling kecil?

OUTLINE

1. Single Exponential Smoothing (SES)
2. Double Exponential Smoothing (DES
3. Tugas Praktikum

3. TUGAS PRAKTIKUM

Soal diambil dari Buku Montgomerry hlm: 315

Nomor 1.
Lakukan dengan excel dan R

4.12 Table B.2 contains data on pharmaceutical product sales.

a. Make a time series plot of the data.

b. Use simple exponential smoothing with $\lambda = 0.1$ to smooth this data. How well does this smoothing procedure work?

c. Make one-step-ahead forecasts of the last 10 observations. Determine the forecast errors.

Nomor 2.

a. Lakukan Single Exponential Smoothing dan Double Exponential Smoothing pada berbagai nilai λ!
b. Lakukan evaluasi pada metode pemulusan manakah dan pada λ berapa yang menghasilkan peramalan terbaik!

Lakukan dengan R.

TERIMAKASIH

METODE PEMULUSAN WINTER ADITIF & WINTER MULTIPLIKATIF

Pertemuan ke-4
Akbar Rizki, M.Si

OUTLINE

1. Pemulusan untuk Data Musiman
2. Pemulusan Winter Aditif
3. Pemulusan Winter Multiplikatif

OUTLINE

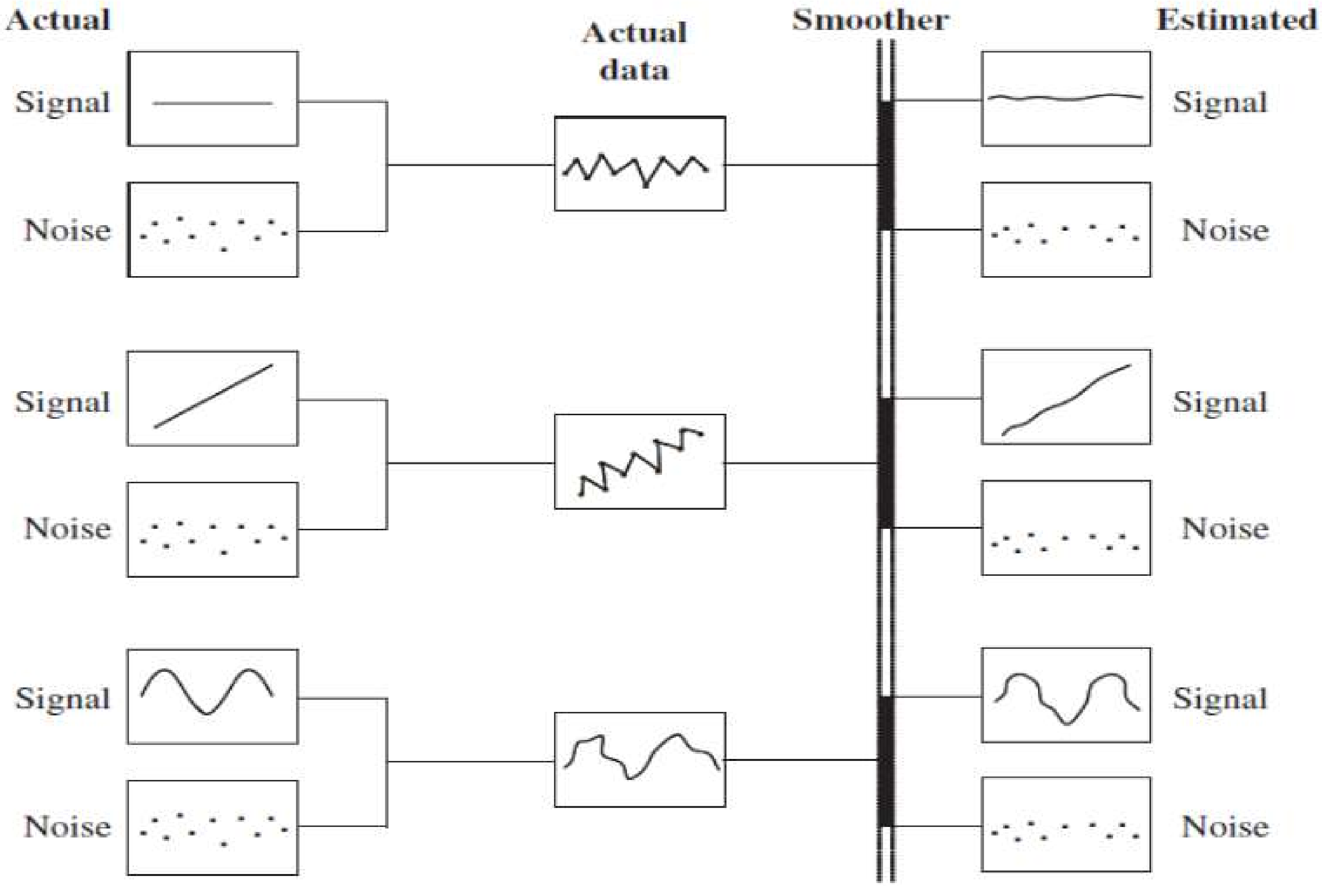
1. Pemulusan untuk Data Musiman

2. Pemulusan Winter Aditif

3. Pemulusan Winter Multiplikatif

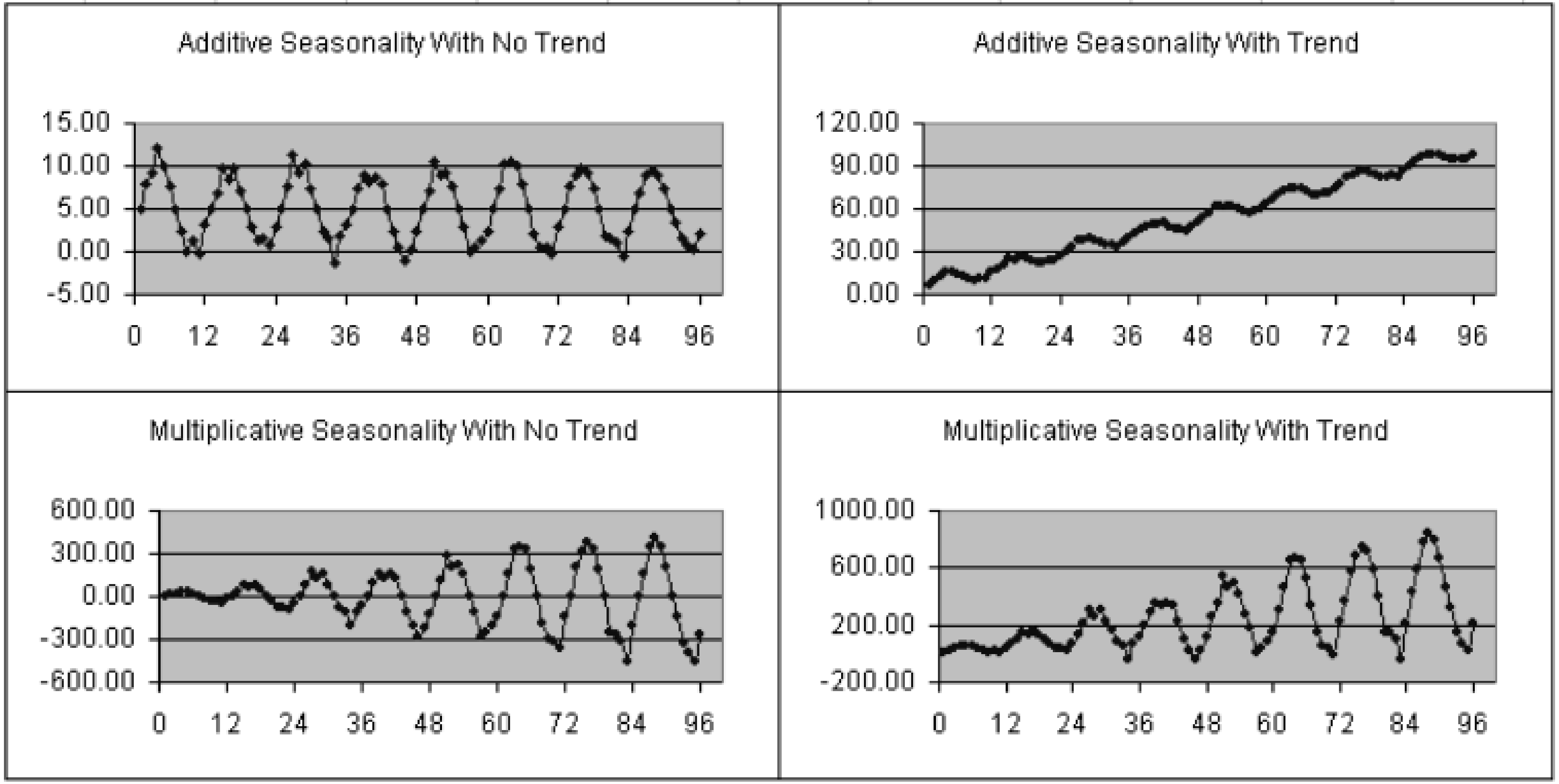
1. PEMULUSAN UNTUK DATA MUSIMAN

REVIEW PEMULUSAN



1. PEMULUSAN UNTUK DATA MUSIMAN

DATA MUSIMAN



1. PEMULUSAN UNTUK DATA MUSIMAN

ILUSTRASI

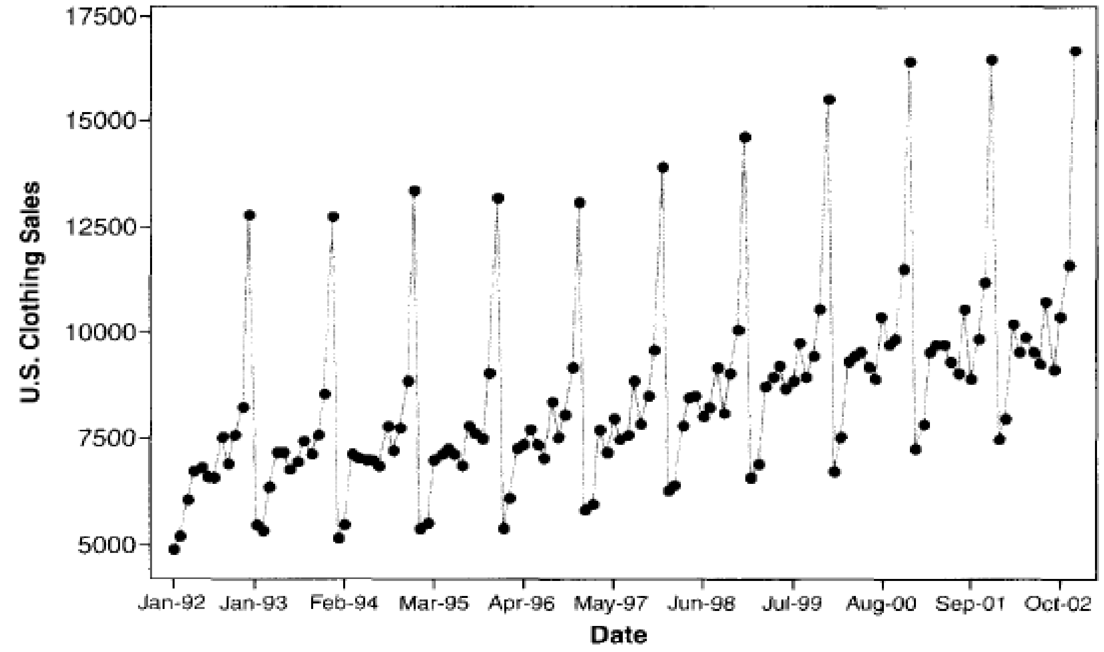


FIGURE 4.26 Time series plot of U.S. clothing sales from January 1992 to December 2003.

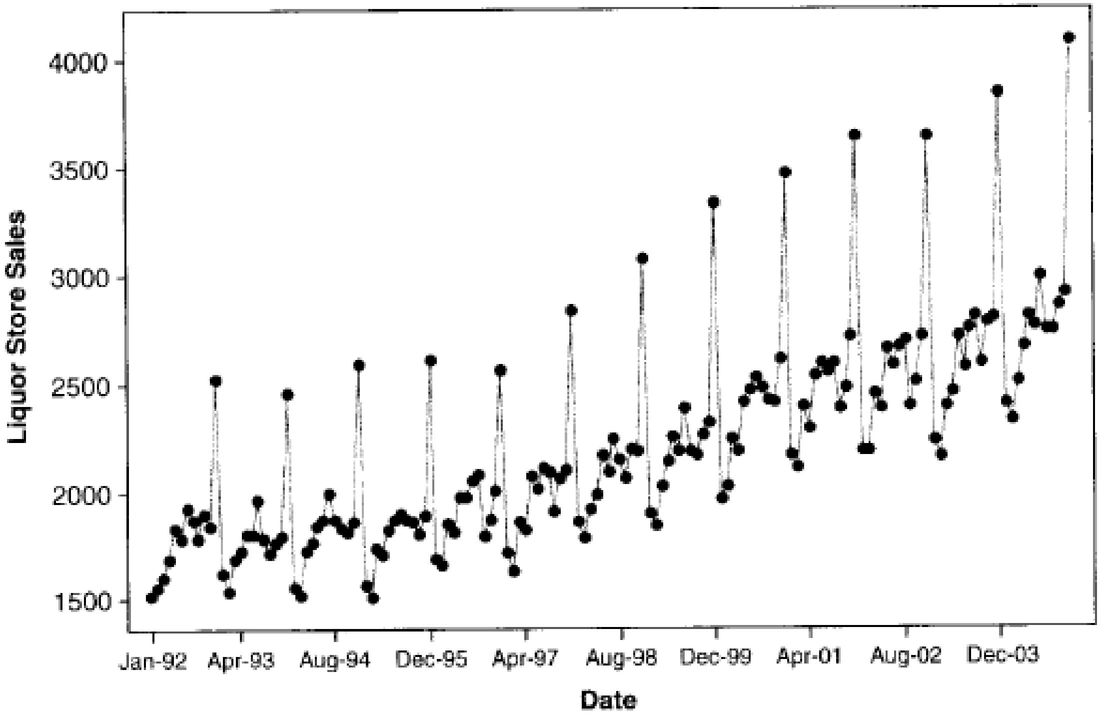


FIGURE 4.29 Time series plot of liquor store sales data from January 1992 to December 2004.

1. PEMULUSAN UNTUK DATA MUSIMAN

EXPONENTIAL SMOOTHING FOR SEASONAL DATA

- Originally introduced by Holt (1957)and Winters (1960)
- Generally known as Winters'method
- Basic idea: seasonal adjustment -> linear trend model
- Two types of adjustments are suggested:
    1. Additive
    2. Multiplicative

OUTLINE

1. Pemulusan untuk Data Musiman

2. Pemulusan Winter Aditif

3. Pemulusan Winter Multiplikatif

2. PEMULUSAN WINTER ADITIF

Additive Model

$$y_t = \boxed{L_t} + \boxed{S_t} + \varepsilon_t$$

level or linear trend component
can in turn be represented by $\beta_0 + \beta_1 t$

the seasonal adjustment

$S_t = S_{t+m} = S_{t+2m} = \ldots$ for $t = 1, \ldots, m - 1$

length of the season (period) of the cycles

$$\sum_{t=1}^{m} S_t = 0.$$

2. PEMULUSAN WINTER ADITIF

Double Exponential Vs Additive Holt-Winter's Method

Double Exponential: $\hat{y}_{t+h}(t) = \underbrace{L_t}_{\text{Level}} + \underbrace{T_t}_{\text{Trend}} h$

Holt-Winter: $\hat{y}_{t+h}(t) = \underbrace{L_t}_{\text{Level}} + \underbrace{B_t}_{\text{Trend}} h + \underbrace{S_{t+h-m}}_{\text{Seasonal}}$

Holt Winter≈Triple Exponential Smoothing

2. PEMULUSAN WINTER ADITIF

Holt-Winters Additive Formulation

- Suppose the time series is denoted by $y_1, \dots, y_n$ with m seasonal period.

Estimate of the level:

$$l_t = \alpha(y_t - s_{t-m}) + (1-\alpha)(l_{t-1} + b_{t-1})$$

Estimate of the trend:

$$b_t = \gamma(l_t - l_{t-1}) + (1-\gamma)b_{t-1}$$

Estimate of the seasonal factor:

$$s_t = \delta(y_t - l_t) + (1-\delta)s_{t-m}$$

- Let $\hat{y}_{t+h}(t)$ be the h-step forecast made using data to time t

$$\hat{y}_{t+h}(t) = l_t + b_t h + s_{t+h-m}$$

2. PEMULUSAN WINTER ADITIF

The Procedure

Step 1: Initialize the value of l_t , b_t, and s_t

Step 2: Update the estimate of l_t

Step 3: Update the estimate of b_t

Step 4: Update the estimate of s_t

Step 5: Conduct the h-step-ahead forecast

2. PEMULUSAN WINTER ADITIF

Initializing the Holt-Winters method

Montgomery (2015):

use the least squares estimates of the following model:

$$y_t = \underbrace{\beta_0}_{l_0} + \underbrace{\beta_1}_{b_0} t + \sum_{i=1}^{s-1} \gamma_i (I_{t,i} - I_{t,s}) + \varepsilon_t,$$

where

$$I_{t,i} = \begin{cases} 1, & t = i, i+s, i+2s, \ldots \\ 0, & \text{otherwise} \end{cases}.$$

$\hat{s}_{j-s} = \hat{y}_j$ for $1 \le j \le m-1$, and $\hat{s}_0 = -\sum_{j=1}^{m-1} \hat{y}_j$

2. PEMULUSAN WINTER ADITIF

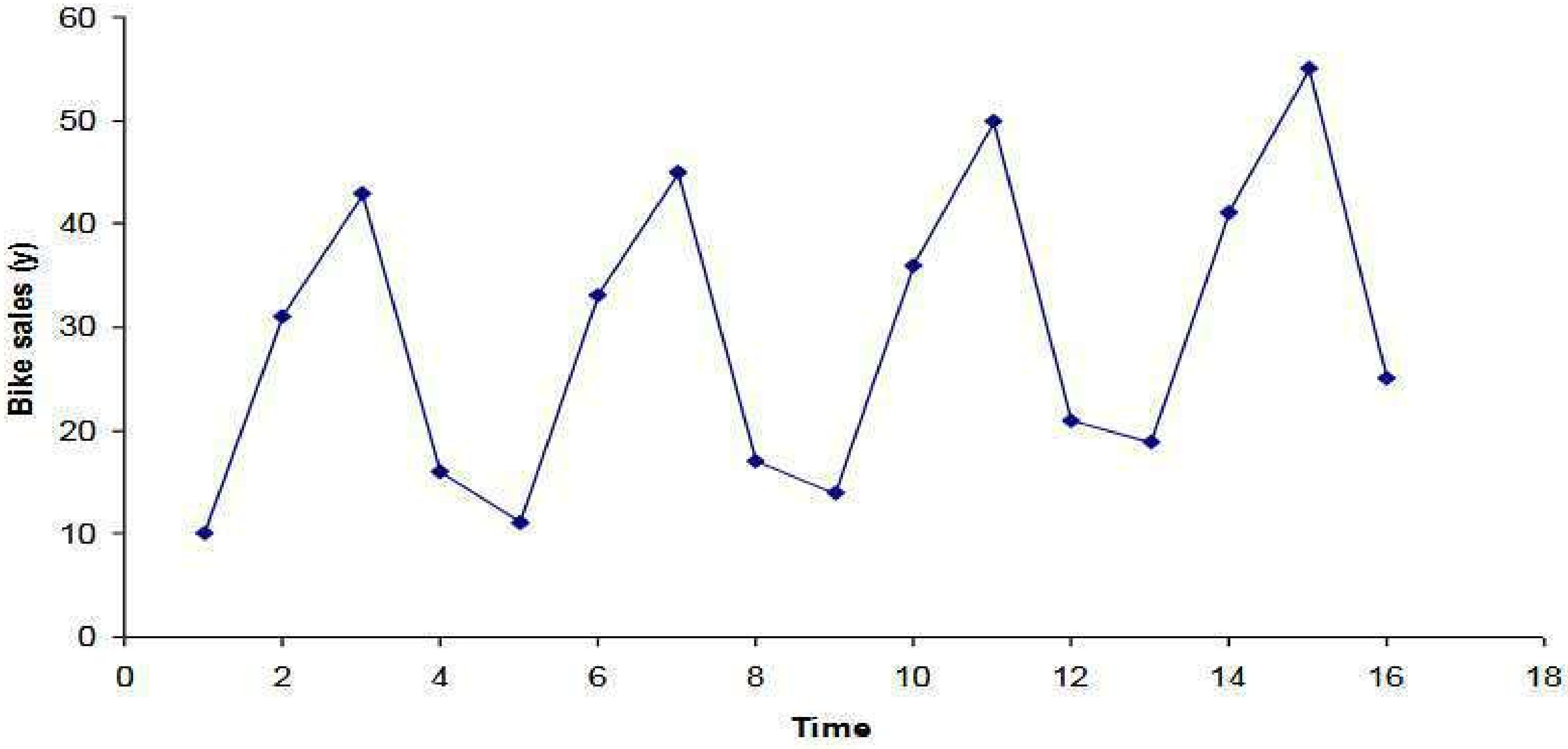
Procedures of Additive Holt-Winters Method

Consider the Mountain Bike example,

Quarterly sales of the TRK-50 Mountain Bike				
	Year			
Quarter	**1**	**2**	**3**	**4**
1	10	11	14	19
2	31	33	36	41
3	43	45	50	55
4	16	17	21	25

2. PEMULUSAN WINTER ADITIF

Procedures of Additive Holt-Winters Method



Observations:

- Linear upward trend over the 4-year period
- Magnitude of seasonal span is almost constant as the level of the time series increases

→ *Additive Holt-Winters method can be applied to forecast future sales*

2. PEMULUSAN WINTER ADITIF

Procedures of Additive Holt-Winters Method

Step 1: Obtain initial values for the level ℓ_0, the growth rate b_0, and the seasonal factors sn_{-3}, sn_{-2}, sn_{-1}, and sn_0, by fitting a least squares trend line to <u>at least four or five years</u> of the historical data.

- y-intercept = ℓ_0; slope = b_0

Example

- Fit a least squares trend line to all 16 observations
- Trend line

$$\hat{y}_t = 20.85 + 0.980882t$$

$$\ell_0 = 20.85;\ b_0 = 0.9809$$

SUMMARY OUTPUT	
Regression Statistics	
Multiple R	0.320508842
R Square	0.102725918
Adjusted R Square	0.038634912
Standard Error	14.28614022
Observations	16
ANOVA	
	df
Regression	1
Residual	14
Total	15
	Coefficients
Intercept	20.85
Time	0.980882353

2. PEMULUSAN WINTER ADITIF

Procedures of Additive Holt-Winters Method

Step 2: Find the initial seasonal factors

1. Compute $\hat{y}_t$ for each time period that is used in finding the least squares regression equation. In this example, t = 1, 2, ..., 16.

$$\hat{y}_1 = 20.85 + 0.980882(1) = 21.8309$$
$$\hat{y}_2 = 20.85 + 0.980882(2) = 22.8118$$
$$\ldots\ldots$$
$$\hat{y}_{16} = 20.85 + 0.980882(16) = 36.5441$$

Step 2: Find the initial seasonal factors

2. Detrend the data by computing $S_t = y_t - \hat{y}_t$ for each observation used in the least squares fit. In this example, t = 1, 2, ..., 16.

$$S_1 = y_1 - \hat{y}_1 = 10 - 21.8309 = -11.8309$$
$$S_2 = y_2 - \hat{y}_2 = 31 - 22.8112 = 8.1882$$
$$\ldots\ldots$$
$$S_{16} = y_{16} - \hat{y}_{16} = 25 - 36.5441 = -11.5441$$

Step 2: Find the initial seasonal factors

3. Compute the average seasonal values for each of the L seasons. The L averages are found by computing the average of the detrended values for the corresponding season. For example, for quarter 1,

$$\bar{S}_{[1]} = \frac{S_1 + S_5 + S_9 + S_{13}}{4}$$
$$= \frac{(-11.8309) + (-14.7544) + (-15.6779) + (-14.6015)}{4} = -14.2162$$

Step 2: Find the initial seasonal factors

4. Compute the average of the L seasonal factors. The average should be 0.

2. PEMULUSAN WINTER ADITIF

Procedures of Additive Holt-Winters Method

Step 3: Calculate a point forecast of y_1 from time 0 using the initial values

$$\hat{y}_{T+p}(T) = \lambda_T + pb_T + sn_{T+p-L} \qquad (T = 0, p = 1)$$

$$\hat{y}_1(0) = \lambda_0 + b_0 + sn_{1-4} = \lambda_0 + b_0 + sn_{-3}$$
$$= 20.85 + 0.9809 + (-14.2162) = 7.6147$$

Step 4: Update the estimates ℓ_T, b_T, and sn_T by using some predetermined values of smoothing constants.

Example: let $\alpha = 0.2$, $\gamma = 0.1$, and $\delta = 0.1$

$$\lambda_1 = \alpha(y_1 - sn_{1-4}) + (1-\alpha)(\lambda_0 + b_0)$$
$$= 0.2(10 - (-14.2162)) + 0.8(20.85 + 0.9808) = 22.3079$$

$$b_1 = \gamma(\lambda_1 - \lambda_0) + (1-\gamma)b_0$$
$$= 0.1(22.3079 - 20.85) + 0.9(0.9809) = 1.0286$$

$$sn_1 = \delta(y_1 - \lambda_1) + (1-\delta)sn_{1-4}$$
$$= 0.1(10 - 22.3079) + 0.9(-14.2162) = -14.0254$$

$$\hat{y}_2(1) = \lambda_1 + b_1 + sn_{2-4} = \lambda_1 + b_1 + sn_{-2}$$
$$= 22.3079 + 1.0286 + 6.5529 = 29.8895$$

2. PEMULUSAN WINTER ADITIF

Procedures of Additive Holt-Winters Method

1	n	alpha	gamma	delta	SSE	MSE	s	
2	16	0.2000	0.1000	0.1000	25.2166	1.9397	1.3927	
3								
4								
5						Forecast		Squared
6				Growth	Seasonal	Made Last	Forecast	Forecast
7	Time	y	Level	Rate	Factor	Period	Error	Error
8	-3				-14.2162			
9	-2				6.5529			
10	-1				18.5721			
11	0		20.85	0.9809	-10.9088			
12	1	10	22.30794	1.0286	-14.0254	7.6147	2.3853	5.6896
13	2	31	23.55864	1.0508	6.6418	29.8895	1.1105	1.2333
14	3	43	24.57314	1.0472	18.5575	43.1815	-0.1815	0.0329
15	4	16	25.87801	1.0729	-10.8057	14.7115	1.2885	1.6603
16	5	11	26.56583	1.0344	-14.1794	12.9256	-1.9256	3.7079
17	6	33	27.35185	1.0096	6.5424	34.2420	-1.2420	1.5427
18	7	45	27.97764	0.9712	18.4040	46.9190	-1.9190	3.6825
19	8	17	28.72023	0.9483	-10.8972	18.1431	-1.1431	1.3067
20	9	14	29.37074	0.9186	-14.2985	15.4892	-1.4892	2.2176
21	10	36	30.12295	0.9019	6.4759	36.8317	-0.8317	0.6918
22	11	50	31.1391	0.9133	18.4497	49.4289	0.5711	0.3262
23	12	21	32.0214	0.9102	-10.9096	21.1553	-0.1553	0.0241
24	13	19	33.00502	0.9176	-14.2692	18.6331	0.3669	0.1346
25	14	41	34.04291	0.9296	6.5240	40.3985	0.6015	0.3618
26	15	55	35.28807	0.9612	18.5759	53.4222	1.5778	2.4894
27	16	25	36.18131	0.9544	-10.9368	25.3396	-0.3396	0.1153

2. PEMULUSAN WINTER ADITIF

Procedures of Additive Holt-Winters Method

p-step-ahead forecast made at time T

$$\hat{y}_{T+p}(T) = \lambda_T + pb_T + sn_{T+p-L} \qquad (p = 1, 2, 3, \ldots)$$

Example

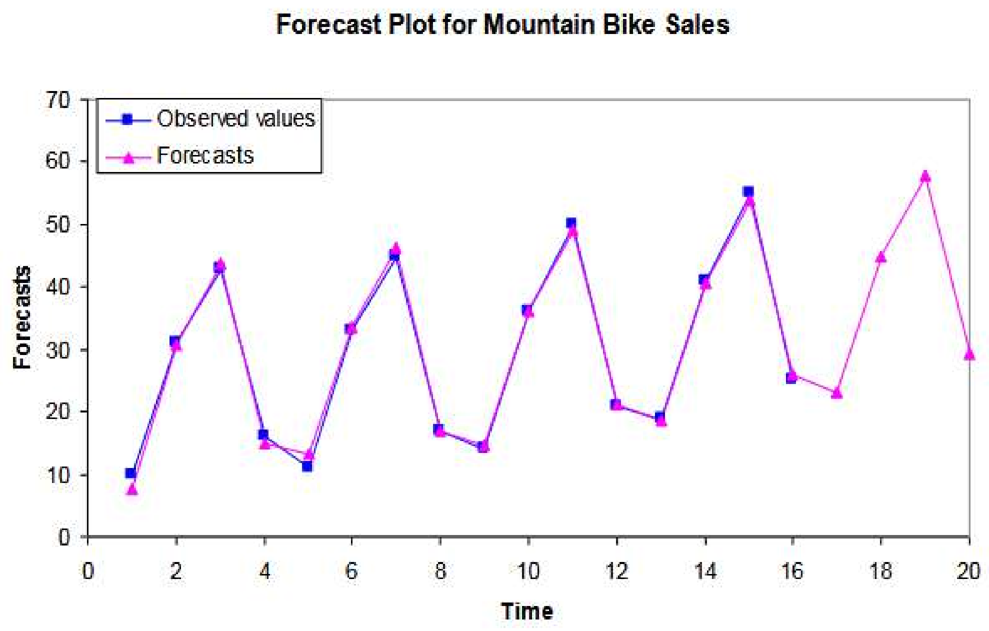
$$\hat{y}_{17}(16) = \lambda_{16} + b_{16} + sn_{17-4} = 36.3426 + 0.9809 - 14.2162 = 23.1073$$

$$\hat{y}_{18}(16) = \lambda_{16} + 2b_{16} + sn_{18-4} = 36.3426 + 2(0.9809) + 6.5529 = 44.8573$$

$$\hat{y}_{19}(16) = \lambda_{16} + 3b_{16} + sn_{19-4} = 36.3426 + 3(0.9809) + 18.5721 = 57.8573$$

$$\hat{y}_{20}(16) = \lambda_{16} + 4b_{16} + sn_{20-4} = 36.3426 + 4(0.9809) - 10.9088 = 29.3573$$

Example



OUTLINE

1. Pemulusan untuk Data Musiman

2. Pemulusan Winter Aditif

3. Pemulusan Winter Multiplikatif

3. PEMULUSAN WINTER MULTIPLIKATIF

Multiplicative ModeL

$$Y_t = \boxed{(L_t)}\boxed{(S_t)} + \varepsilon_t$$

level or linear trend component
can in turn be represented by $\beta_0 + \beta_1 t$

the seasonal adjustment

$S_t = S_{t+m} = S_{t+2m} = \dots$ for $t = 1, \dots, m - 1$

length of the season (period) of the cycles

$$\sum_{t=1}^{m} S_t = 0.$$

3. PEMULUSAN WINTER MULTIPLIKATIF

Holt-Winters Multiplicative Formulation

- Suppose the time series is denoted by $y_1, \ldots, y_n$ with m seasonal period.

Estimate of the level:

$$L_t = \alpha\left(\frac{Y_t}{S_{t-m}}\right) + (1-\alpha)(L_{t-1} + B_{t-1})$$

Estimate of the trend:

$$B_t = \gamma(L_t - L_{t-1}) + (1-\gamma)B_{t-1}$$

Estimate of the seasonal factor:

$$S_t = \delta\left(\frac{Y_t}{L_t}\right) + (1-\delta)S_{t-m}$$

- Let $\hat{Y}_{t+h}(t)$ be the h-step forecast made using data to time t

$$\hat{Y}_{t+h}(t) = (L_t + B_t h)S_{t+h-m}$$

3. PEMULUSAN WINTER MULTIPLIKATIF

The Procedure

Step 1: Initialize the value of L_t , B_t, and S_t

Step 2: Update the estimate of L_t

Step 3: Update the estimate of B_t

Step 4: Update the estimate of S_t

Step 5: Conduct the h-step-ahead forecast

3. PEMULUSAN WINTER MULTIPLIKATIF

Initializing the Holt-Winters method

Montgomery (2015):

Suppose a dataset consist of k seasons.

- $\hat{L}_0 = \frac{\bar{y}_k - \bar{y}_1}{(k-1)m}$ where $\bar{y}_i = \frac{1}{m}\sum_{t=(i-1)m+1}^{im} y_t$
- $\hat{B}_0 = \bar{y}_1 - \frac{m}{2}\hat{L}_0$
- $\hat{S}_{j-m} = m\left(\frac{\hat{S}_j^*}{\sum_{i=1}^{m}\hat{S}_j^*}\right)$, for $1 \le j \le s$, where $\hat{S}_j^* = \frac{1}{k}\sum_{t=1}^{k}\frac{y_{(t-1)m+j}}{\bar{y}_t - \left(\frac{s+1}{2-j}\right)\hat{\beta}_0}$

3. PEMULUSAN WINTER MULTIPLIKATIF

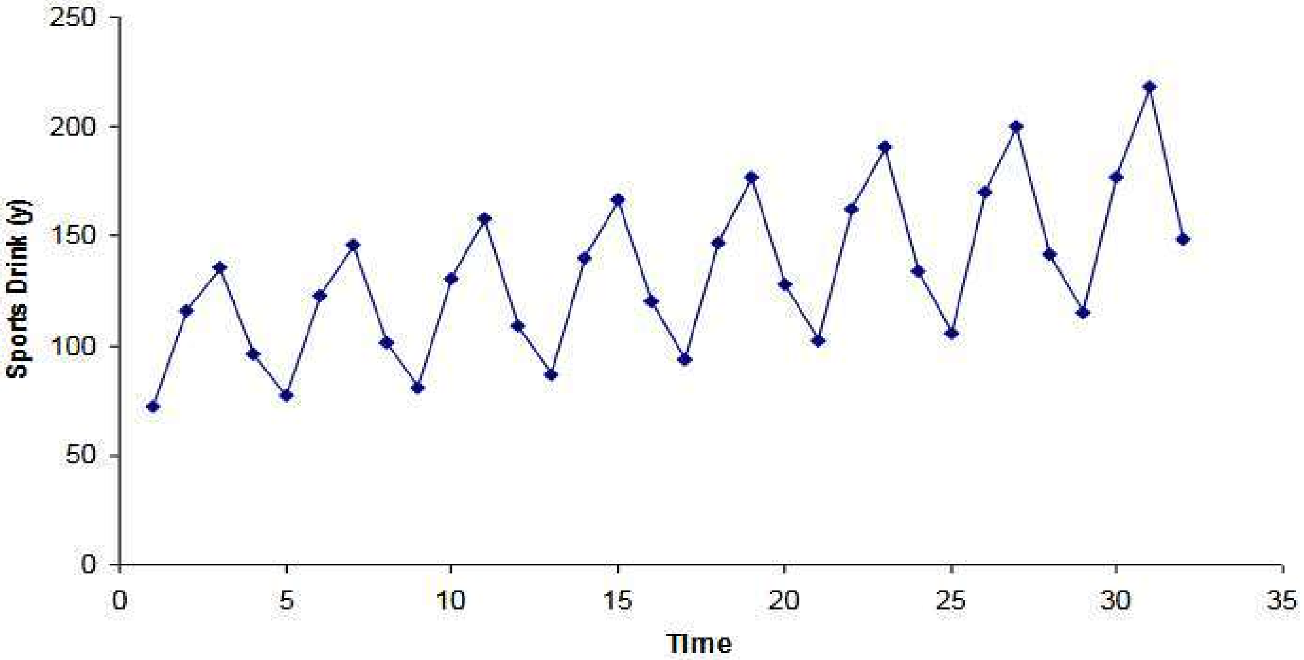
Procedures of Multiplicative Holt-Winters Method

Use the Sports Drink example as an illustration

Quarterly sales of Tiger Sports Drink								
	Year							
Quarter	1	2	3	4	5	6	7	8
1	72	77	81	87	94	102	106	115
2	116	123	131	140	147	162	170	177
3	136	146	158	167	177	191	200	218
4	96	101	109	120	128	134	142	149

3. PEMULUSAN WINTER MULTIPLIKATIF

Procedures of Multiplicative Holt-Winters Method



Observations:

- Linear upward trend over the 8-year period
- Magnitude of the seasonal span increases as the level of the time series increases

⇒ Multiplicative Holt-Winters method can be applied to forecast future sales

3. PEMULUSAN WINTER MULTIPLIKATIF

Procedures of Multiplicative Holt-Winters Method

Step 1: Obtain initial values for the level ℓ_0, the growth rate b_0, and the seasonal factors s_{-3}, s_{-2}, s_{-1}, and s_0, by fitting a least squares trend line to at least four or five years of the historical data.

- y-intercept = ℓ_0; slope = b_0

Example

- Fit a least squares trend line to the first 16 observations
- Trend line

$$\hat{y}_t = 95.2500 + 2.4706t$$

- $\ell_0 = 95.2500$; $b_0 = 2.4706$

SUMMARY OUTPUT	
Regression Statistics	
Multiple R	0.403809754
R Square	0.163062318
Adjusted R Square	0.103281055
Standard Error	27.58325823
Observations	16
ANOVA	
	df
Regression	1
Residual	14
Total	15
	Coefficients
Intercept	95.25
X Variable 1	2.470588235

3. PEMULUSAN WINTER MULTIPLIKATIF

Procedures of Multiplicative Holt-Winters Method

Step 2: Find the initial seasonal factors

1. Compute $\hat{y}_t$ for the in-sample observations used for fitting the regression. In this example, t = 1, 2, ..., 16.

$$\hat{y}_1 = 95.2500 + 2.4706(1) = 97.7206$$
$$\hat{y}_2 = 95.2500 + 2.4706(2) = 100.1912$$
$$\ldots\ldots$$
$$\hat{y}_{16} = 95.2500 + 2.4706(16) = 134.7794$$

Step 2: Find the initial seasonal factors

2. Detrend the data by computing $S_{0,t} = y_t / \hat{y}_t$ for each time period that is used in finding the least squares regression equation. In this example, t = 1, 2, ..., 16.

$$S_{0,1} = y_1 / \hat{y}_1 = 72 / 97.7206 = 0.7368$$
$$S_{0,2} = y_2 / \hat{y}_2 = 116 / 100.1912 = 1.1578$$
$$\ldots\ldots$$
$$S_{0,16} = y_{16} / \hat{y}_{16} = 120 / 134.7794 = 0.8903$$

Step 2: Find the initial seasonal factors

3. Compute the average seasonal values for each of the k seasons. The k averages are found by computing the average of the detrended values for the corresponding season. For example, for quarter 1,

$$\bar{S}_{[1]} = \frac{S_1 + S_5 + S_9 + S_{13}}{4}$$
$$= \frac{0.7368 + 0.7156 + 0.6894 + 0.6831}{4} = 0.7062$$

3. PEMULUSAN WINTER MULTIPLIKATIF

Procedures of Multiplicative Holt-Winters Method

Step 2: Find the initial seasonal factors

4. Multiply the average seasonal values by the normalizing constant

$$CF = \frac{m}{\sum_{i=1}^{m} \bar{S}_{[i]}}$$

such that the average of the seasonal factors is 1. The initial seasonal factors are

$$S_{i-m} = \bar{S}_{[i]}(CF) \qquad (i = 1, 2, \ldots, L)$$

Step 2: Find the initial seasonal factors

4. Multiply the average seasonal values by the normalizing constant such that the average of the seasonal factors is 1.
   - Example

     CF = 4/3.9999 = 1.0000

$$S_{-3} = S_{1-4} = \bar{S}_{[1]}(CF) = 0.7062(1) = 0.7062$$

$$S_{-2} = S_{2-4} = \bar{S}_{[2]}(CF) = 1.1114(1) = 1.1114$$

$$S_{-1} = S_{3-4} = \bar{S}_{[3]}(CF) = 1.2937(1) = 1.2937$$

$$S_{0} = S_{4-4} = \bar{S}_{[1]}(CF) = 0.8886(1) = 0.8886$$

3. PEMULUSAN WINTER MULTIPLIKATIF

Procedures of Multiplicative Holt-Winters Method

Step 3: Calculate a point forecast of y_1 from time 0 using the initial values

$$\hat{y}_{t+h}(t) = (L_t + B_t h)S_{t+h-m} \qquad (t = 1, h = 0)$$

$$\hat{y}_1(0) = (L_0 + B_0)S_{1-4} = (L_0 + B_0)S_{-3}$$

$$= (95.25 + 2.4706)0.7062$$

$$= 69.0103$$

3. PEMULUSAN WINTER MULTIPLIKATIF

Procedures of Multiplicative Holt-Winters Method

Step 4: Update the estimates ℓ_T, b_T, and sn_T by using some predetermined values of smoothing constants.

Example: let α = 0.2, γ = 0.1, and $\delta = 0.1$

$$\begin{aligned} l_1 &= \alpha(y_1 / s_{1-4}) + (1-\alpha)(l_0 + b_0) \\ &= 0.2(72/0.7062) + 0.8(95.2500 + 2.4706) = 98.5673 \end{aligned}$$

$$\begin{aligned} b_1 &= \gamma(l_1 - l_0) + (1-\gamma)b_0 \\ &= 0.1(98.5673 - 95.2500) + 0.9(2.4706) = 2.5553 \end{aligned}$$

$$\begin{aligned} sn_1 &= \delta(y_1 / l_1) + (1-\delta)s_{1-4} \\ &= 0.1(72/98.5673) + 0.9(0.7062) = 0.7086 \end{aligned}$$

$$\begin{aligned} \hat{y}_2(1) &= (l_1 + b_1)s_{2-4} \\ &= (98.5673 + 2.5553)(1.1114) = 112.3876 \end{aligned}$$

3. PEMULUSAN WINTER MULTIPLIKATIF

Procedures of Multiplicative Holt-Winters Method

$$\begin{aligned}\lambda_2 &= \alpha(y_2/s_{2-4}) + (1-\alpha)(\lambda_1 + b_1)\\ &= 0.2(116/1.1114) + 0.8(98.5673 + 2.5553)\\ &= 101.7727\end{aligned}$$

$$\begin{aligned}b_2 &= \gamma(\lambda_2 - \lambda_1) + (1-\gamma)b_1\\ &= 0.1(101.7727 - 98.5673) + 0.9(2.5553)\\ &= 2.62031\end{aligned}$$

$$\begin{aligned}sn_2 &= \delta(y_2/\lambda_2) + (1-\delta)s_{2-4}\\ &= 0.1(116/101.7727) + 0.9(1.1114)\\ &= 1.114239\end{aligned}$$

$$\begin{aligned}\hat{y}_3(2) &= (\lambda_2 + b_2)s_{3-4}\\ &= (101.7727 + 2.62031)(1.2937)\\ &= 135.053\end{aligned}$$

$$\begin{aligned}\lambda_4 &= \alpha(y_4/s_{4-4}) + (1-\alpha)(\lambda_3 + b_3)\\ &= 0.2(96/0.8886) + 0.8(104.5393 + 2.6349)\\ &= 107.3464\end{aligned}$$

$$\begin{aligned}b_4 &= \gamma(\lambda_4 - \lambda_3) + (1-\gamma)b_3\\ &= 0.1(107.3464 - 104.5393) + 0.9(2.6349)\\ &= 2.65212\end{aligned}$$

$$\begin{aligned}sn_4 &= \delta(y_4/\lambda_4) + (1-\delta)s_{4-4}\\ &= 0.1(96/107.3464) + 0.9(0.8886)\\ &= 0.889170\end{aligned}$$

$$\begin{aligned}\hat{y}_5(4) &= (\lambda_4 + b_4)s_{5-4}\\ &= (107.3464 + 2.65212)(0.7086)\\ &= 77.945\end{aligned}$$

3. PEMULUSAN WINTER MULTIPLIKATIF

Procedures of Multiplicative Holt-Winters Method

1	n	alpha	gamma	delta	SSE	MSE	s	
2	32	0.2	0.1	0.1	177.3223	6.1146	2.4728	
3								
4								
5						Forecast		Squared
6				Growth	Seasonal	Made Last	Forecast	Forecast
7	Time	y	Level	Rate	Factor	Period	Error	Error
8	-3				0.7062			
9	-2				1.1114			
10	-1				1.2937			
11	0		95.25	2.4706	0.8886			
12	1	72	98.56729	2.5553	0.7086	69.0103	2.9897	8.9384
13	2	116	101.7726	2.6203	1.1142	112.3876	3.6124	13.0494
14	3	136	104.5393	2.6349	1.2944	135.0531	0.9469	0.8967
15	4	96	107.3464	2.6521	0.8892	95.2350	0.7650	0.5853
16	5	77	109.731	2.6254	0.7079	77.9478	-0.9478	0.8984
17	6	123	111.9629	2.5860	1.1127	125.1919	-2.1919	4.8043
18	7	146	114.1974	2.5509	1.2928	148.2750	-2.2750	5.1755
19	8	101	116.1165	2.4877	0.8872	103.8091	-2.8091	7.8911
20	9	81	117.7668	2.4040	0.7059	83.9641	-2.9641	8.7858
21	10	131	119.6835	2.3552	1.1109	133.7108	-2.7108	7.3482
22	11	158	122.0734	2.3587	1.2930	157.7754	0.2246	0.0504
23	12	109	124.1164	2.3271	0.8863	110.4005	-1.4005	1.9615
24	13	87	125.8035	2.2631	0.7045	89.2593	-2.2593	5.1044
25	14	140	127.6589	2.2224	1.1094	142.2642	-2.2642	5.1268
26	15	167	129.7369	2.2079	1.2924	167.9337	-0.9337	0.8718
	....					......		
38	27	200	156.1396	2.1752	1.2903	202.0396	-2.0396	4.1601
39	28	142	158.5505	2.1988	0.8908	140.9508	1.0492	1.1008
40	29	115	161.2803	2.2519	0.7047	113.1314	1.8686	3.4918
41	30	177	162.8178	2.1804	1.1046	180.9529	-3.9529	15.6252
42	31	218	165.7889	2.2595	1.2928	212.8988	5.1012	26.0220
43	32	149	167.8899	2.2437	0.8905	149.7057	-0.7057	0.4981

3. PEMULUSAN WINTER MULTIPLIKATIF

Procedures of Multiplicative Holt-Winters Method

p-step-ahead forecast made at time *T*

$$\hat{y}_{t+h}(t) = (L_t + B_t h)S_{t+h-m} \qquad (h = 1, 2, 3, \ldots.)$$

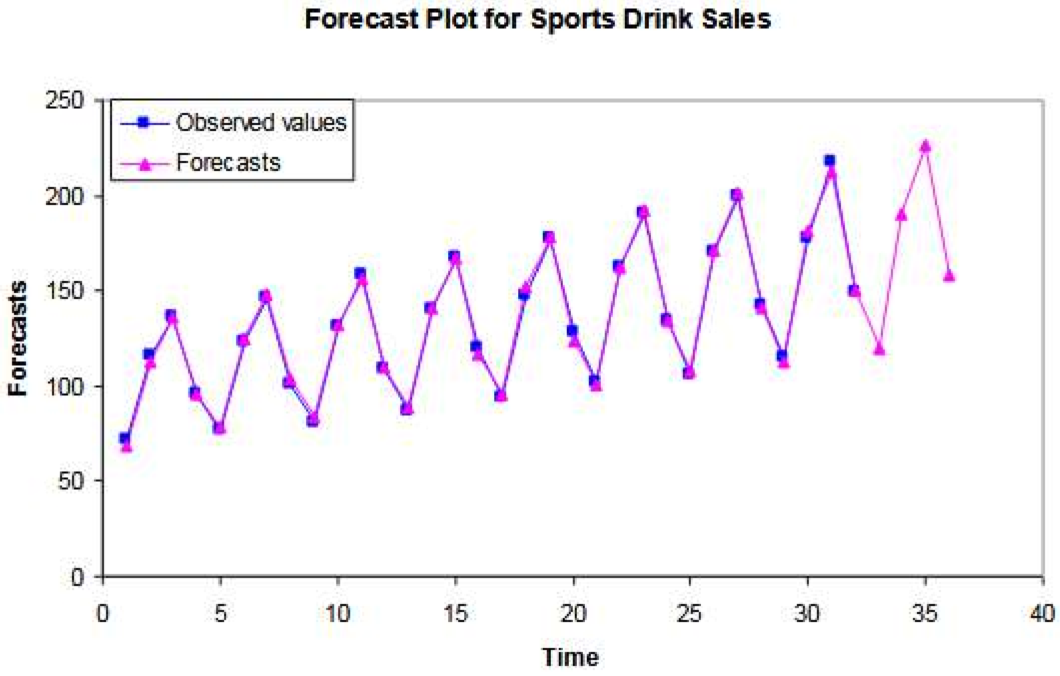
Example

$$\hat{y}_{33}(32) = (l_{32} + b_{32})\, s_{33-4} = (168.1213 + 2.3028)(0.7044) = 120.0467$$

$$\hat{y}_{34}(32) = (l_{32} + 2b_{32})\, s_{34-4} = [168.1213 + 2(2.3028)](1.1038) = 190.6560$$

$$\hat{y}_{35}(32) = (l_{32} + 3b_{32})\, s_{35-4} = [(168.1213 + 3(2.3028)](1.2934) = 226.3834$$

$$\hat{y}_{36}(32) = (l_{32} + 4b_{32})\, s_{36-4} = [(168.1213 + 4(2.3028)](0.8908) = 157.9678$$



3. PEMULUSAN WINTER MULTIPLIKATIF

Holt-Winters Additive Vs Multiplicative Formulation

Suppose the time series is denoted by $y_1, \dots, y_n$ with m seasonal period.

	Additive	Multiplicative
Est. of level	$L_t = \alpha(Y_t - S_{t-m}) + (1-\alpha)(L_{t-1} + B_{t-1})$	$L_t = \alpha\left(\frac{Y_t}{S_{t-m}}\right) + (1-\alpha)(L_{t-1} + B_{t-1})$
Est. of trend	$B_t = \gamma(L_t - L_{t-1}) + (1-\gamma)B_{t-1}$	$B_t = \gamma(L_t - L_{t-1}) + (1-\gamma)B_{t-1}$
Est. of seasonal	$S_t = \delta(Y_t - L_t) + (1-\delta)S_{t-m}$	$S_t = \delta\left(\frac{Y_t}{L_t}\right) + (1-\delta)S_{t-m}$
Forecast	$\hat{Y}_{t+h}(t) = L_t + B_t h + S_{t+h-m}$	$\hat{Y}_{t+h}(t) = (L_t + B_t h)S_{t+h-m}$

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Bogor Indonesia

TERIMAKASIH

Model Regresi untuk Data Deret Waktu (1)

Review model regresi

Model Regresi Linier Sederhana

(yang hubungannya linier $\rightarrow$ ordo x=1)

- Linier dalam parameter
- Sederhana = banyaknya peubah bebas/penjelas hanya satu
- Hubungan antara X dan Y dinyatakan dalam fungsi linier/ordo 1
- Perubahan Y diasumsikan karena adanya perubahan X
- Model populasi regresi linier sederhana yang hubungannya linier (selanjutnya cukup sebut "regresi linier sederhana") :

$$\boxed{Y = \beta_0 + \beta_1 x + \varepsilon}$$

Dengan :

β_0 dan β_1 adalah parameter regresi

ε adalah sisaan/galat (peubah acak)

Y adalah peubah tak bebas (peubah acak)

X adalah peubah bebas yang nilainya diketahui dan presisinya sangat tinggi (bukan peubah acak)

Regresi Linier Berganda

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$$
$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots. + \beta_k x_k + \varepsilon$$

Dengan :

β_0 , β_1 ... β_k	adalah parameter regresi
ε	adalah sisaan/galat (peubah acak)
Y	adalah peubah tak bebas (peubah acak)
X_1 ,..., X_k	adalah peubah bebas yang nilainya diketahui dan presisinya sangat tinggi (bukan peubah acak)

Asumsi Model Regresi Linier

- Bentuk hubungannya linear (Y merupakan fungsi linier dari X, plus sisaan yang acak)
- Sisaan ε_i adalah peubah acak yang bebas thdp nilai x
- Sisaan merupakan peubah acak yang menyebar Normal dengan rataan 0 dan memiliki ragam konstan, σ^2

  (sifat ragam yang konstan/homogen ini disebut homoscedasticity)

$$\boxed{E[\varepsilon_i] = 0 \quad \text{dan} \quad E[\varepsilon_i{}^2] = \sigma^2 \qquad \text{untuk } (i = 1, \ldots, n)}$$

- Sisaan ε_i, tidak berkorelasi satu dengan yang lainnya, sehingga atau

$$\boxed{E[\varepsilon_i \varepsilon_j] = 0 \quad , \quad i \neq j} \qquad \boxed{\text{cov}[\varepsilon_i, \varepsilon_j] = 0 \ , i \neq j}$$

- Tidak terjadi multikolinearitas antar peubah bebas (asumsi tambahan untuk Regresi Linear Berganda)

Permasalahan kebebasan data dalam model regresi serta konsekuensinya

- Sisaan ε_i, berkorelasi satu dengan yang lainnya
- Akibatnya :
  - Hasil OLS tetap tidak berbias, namun ragamnya bukan lagi ragam yang paling minimum
  - Jika sisaan berkorelasi diri maka

$$\hat{\sigma}^2 = s_e^2 = KT_{sisaan} = \frac{\text{JKS}}{n-2} = \frac{\sum_{i=1}^{n} e_i^2}{n-2}$$

Menjadi underestimate

$$s_{b_1}^2 = \frac{s_e^2}{\sum (x_i - \bar{x})^2} = \frac{s_e^2}{(n-1)s_x^2}$$

Mengakibatkan Sb1 menjadi kecil

  - Selang kepercayaan dan uji hipotesis yang berbasis uji t dan uji F → SUDAH TIDAK TEPAT

Model regresi untuk data deret waktu

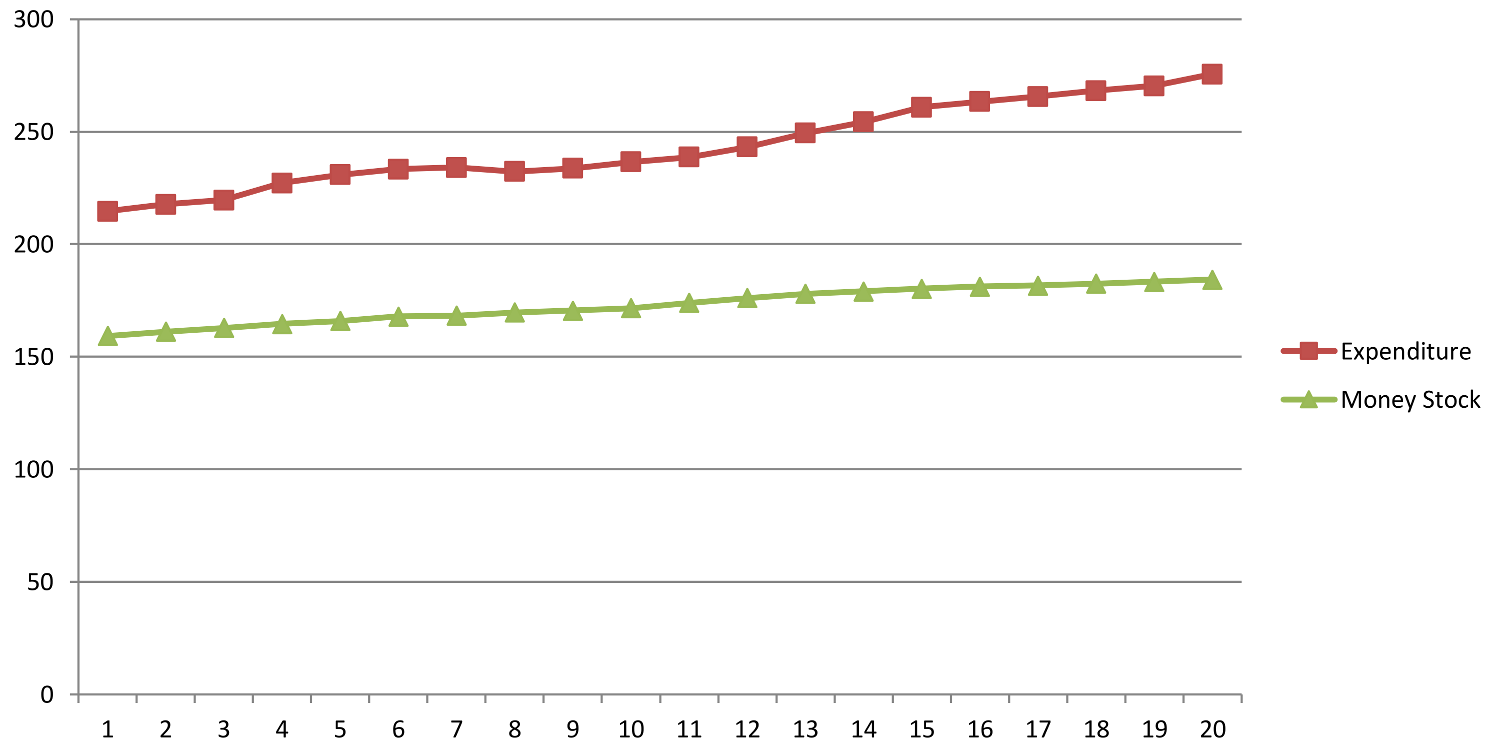
$$y_t = \beta_0 + \beta_1 t + \varepsilon, \quad t = 1, 2, \ldots, T$$

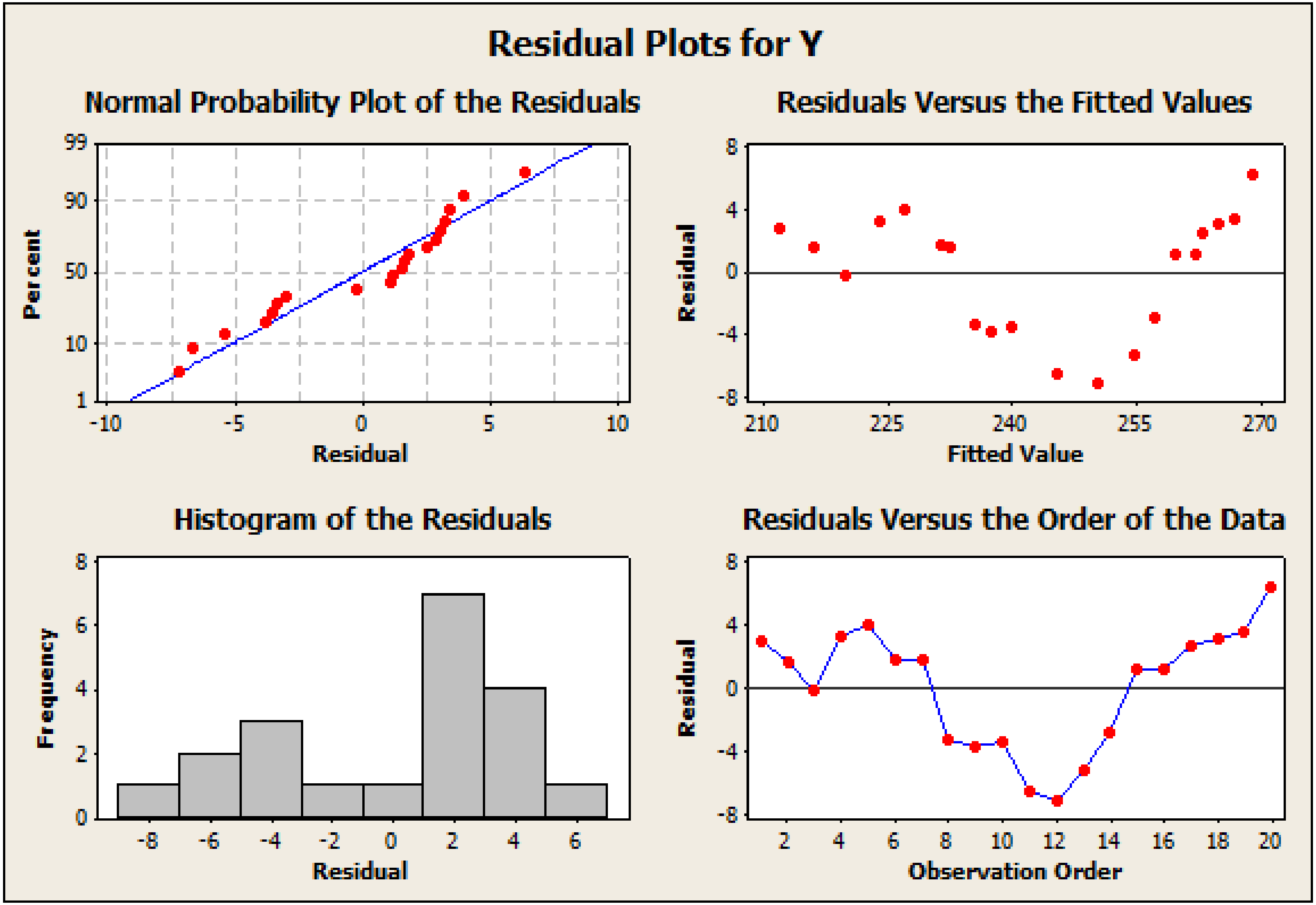
$$y_t = \beta_0 + \beta_1 \sin \frac{2\pi}{d} t + \beta_2 \cos \frac{2\pi}{d} t + \varepsilon$$

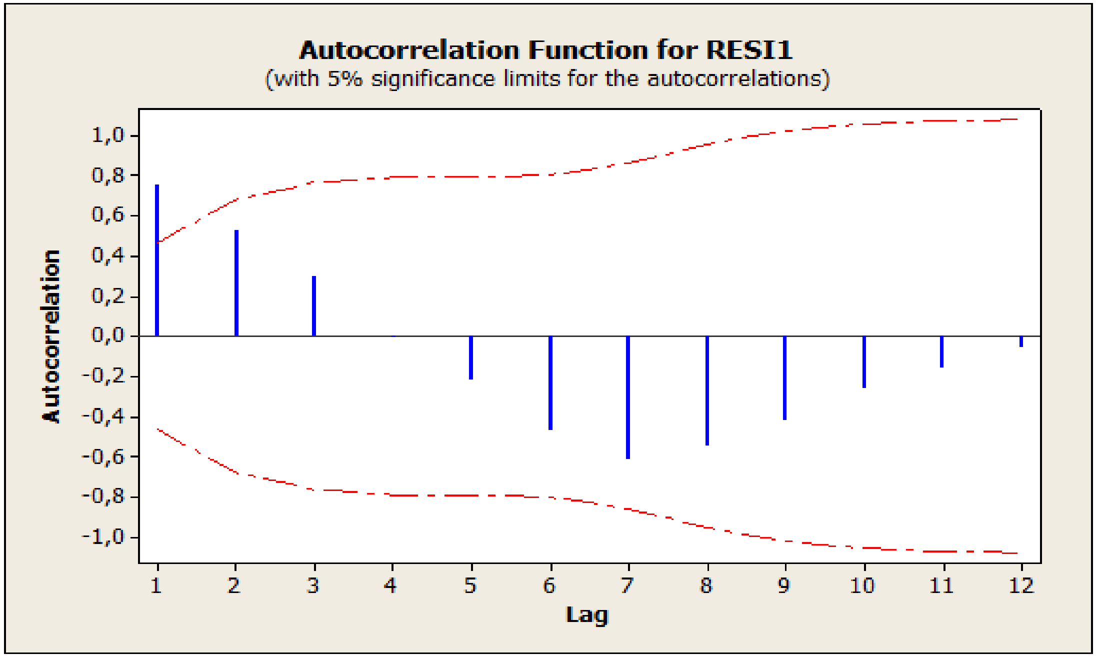
$$y_t = \beta_0 + \beta_1 x_1(t) + \cdots + \beta_k x_k(t) + \varepsilon_t, \quad t = 1, 2, \ldots, T$$

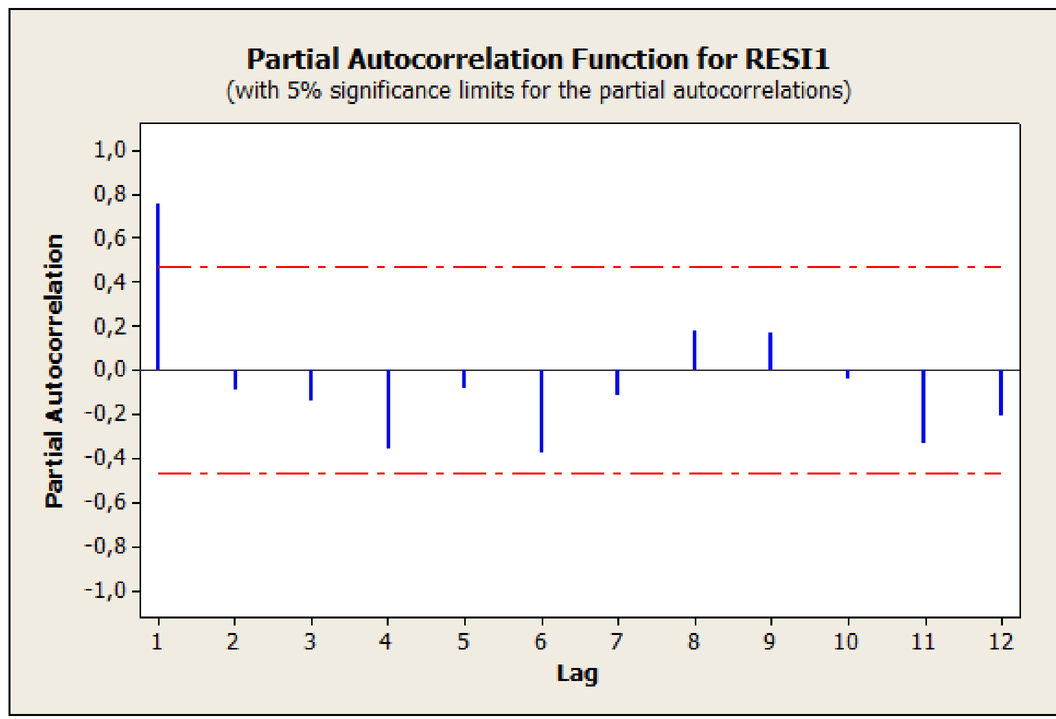
Model regresi untuk data deret waktu

The data gives quarterly data from 1952 to 1956 on consumer expenditure ***(Y)*** and the stock of money *(X),* both measured in billions of current dollars for the United States




Residual Plots for Y
Normal Probability Plot of the Residuals
Percent
99
90
50
10
1
-10
-5
0
5
10
Residual
Residuals Versus the Fitted Values
Residual
8
4
0
-4
-8
210
225
240
255
270
Fitted Value
Histogram of the Residuals
Frequency
8
6
4
2
0
-8
-6
-4
-2
0
2
4
6
Residual
Residuals Versus the Order of the Data
Residual
8
4
0
-4
-8
2
4
6
8
10
12
14
16
18
20
Observation Order


Autocorrelation Function for RESI1
(with 5% significance limits for the autocorrelations)
Autocorrelation
1,0
0,8
0,6
0,4
0,2
0,0
-0,2
-0,4
-0,6
-0,8
-1,0
1
2
3
4
5
6
7
8
9
10
11
12
Lag


Partial Autocorrelation Function for RESI1
(with 5% significance limits for the partial autocorrelations)
Partial Autocorrelation
1,0
0,8
0,6
0,4
0,2
0,0
-0,2
-0,4
-0,6
-0,8
-1,0
1
2
3
4
5
6
7
8
9
10
11
12
Lag

Terima Kasih

MODEL REGRESI UNTUK DATA DERET WAKTU (2)

Pertemuan ke-6

Akbar Rizki, M.Si

OUTLINE

1. Reviu
2. Uji Kebebasan Antar Sisaan
3. Penanganan Autokorelasi Diri

OUTLINE

1. Reviu

2. Uji Kebebasan Antar Sisaan

3. Penanganan Autokorelasi Diri

1. REVIU

- Salah satu asumsi regresi linear klasik:

$$cov(e_i, e_j) = 0$$

dengan e_i menunjukkan galat pengamatan ke- i dan e_j menunjukkan galat pengamatan ke-j.

- Sebab Umum Terjadinya Autokorelasi pada Galat:
  a. Terdapat peubah yang tidak disertakan dalam model
  b. Mispesifikasi model
  c. Measurement error

1. REVIU

Konsekuensi Pelanggaran Asumsi Kebebasan Sisaan

- Jika asumsi tidak terpenuhi:

a. Penduga masih bersifat tak bias dan konsisten
b. Jika ukuran contoh besar, masih bisa diasumsikan normal
c. Namun, penduga menjadi tidak efisien (bukan penduga tak bias terbaik (BLUE)).
d. Penduga galat baku menjadi tidak reliable, sehingga hasil uji-T dan F dapat menjadi tidak valid.

1. REVIU

Deteksi Autokorelasi

- Deteksi Autokorelasi:
  a. Pendekatan grafik (residual plot, ACF plot)
  b. Uji Durbin-Watson
  c. Uji Breusch-Godfrey (BG)
  d. Run-test, etc

1. REVIU

Deteksi Autokorelasi dengan Grafik

Plot sisaan vs order → Bila plot tidak membentuk pola tertentu, maka asumsi kebebasan terpenuhi

ACF dan PACF Sisaan → Bila ACF dan PACF tidak ada yang signifikan, maka sisaan saling bebas

OUTLINE

1. Reviu

2. Uji Kebebasan Antar Sisaan

3. Penanganan Autokorelasi Diri

2. UJI KEBEBASAN ANTAR SISAAN

UJI DURBIN WATSON

- Hipotesis:
  - ✓ $H0$: ϕ = 0 lawan $H1$: ϕ > 0 (terdapat autokorelasi positif)
  - ✓ $H0$: ϕ = 0 lawan $H1$: ϕ < 0 (terdapat autokorelasi negatif)
  - ✓ $H0$: ϕ = 0 lawan $H1$: ϕ ≠ 0 (tidak terdapat autokorelasi)
- Statistik uji durbin-Watson (d) didefinisikan sbb:

$$d = \frac{\sum_{t=1}^{t=n}(e_t - e_{t-1})^2}{\sum_{t=1}^{t=n} e_t^2}$$

2. UJI KEBEBASAN ANTAR SISAAN

UJI DURBIN WATSON

- Asumsi pada Uji Durbin-Watson:

1. Terdapat intersep pada model regresi
2. Seluruh peubah penjelas bersifat tetap (fixed) pada penarikan contoh berulang
3. Galat mengikuti skema Autoregressive (AR) ordo ke-1 :

$$u_t = \rho u_{t-1} + v_t$$

   dengan ρ adalah koefisien autokorelasi yang bernilai -1 s.d 1
4. Galat menyebar normal
5. Lag dari peubah respon tidak disertakan sebagai peubah penjelas dalam model

2. UJI KEBEBASAN ANTAR SISAAN

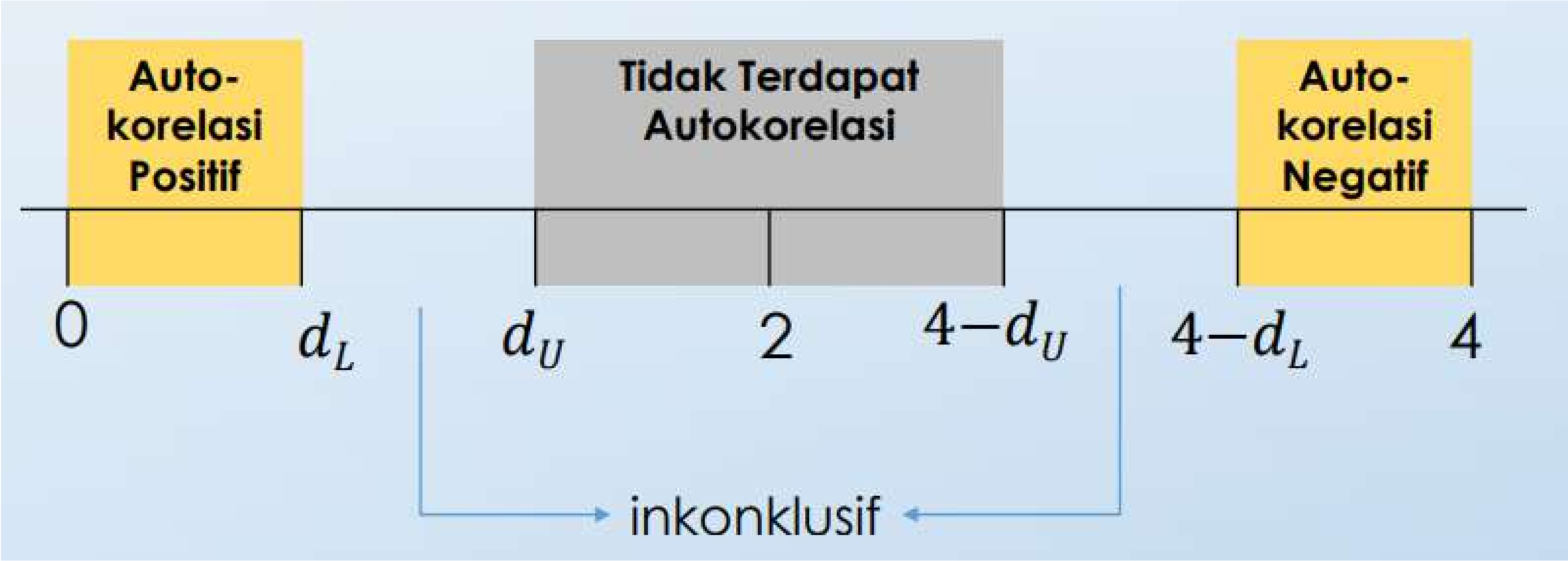
UJI DURBIN WATSON

- Menggunakan dua titik kritis, yaitu batas bawah dL dan batas atas dU
- Nilai d selalu terletak di antara 0 dan 4
- Gambaran tentang statistik Durbin-Watson:
  - Jika d mendekati nol→ semakin besar kemungkinan adanya autokorelasi positif
  - Jika d mendekati 4→semakin besar kemungkinan adanya autokorelasi negatif.
  - Jika d mendekati 2→belum cukup bukti adanya autokorelasi negatif atau positif

2. UJI KEBEBASAN ANTAR SISAAN

UJI DURBIN WATSON

- Kriteria Penarikan Kesimpulan:



2. UJI KEBEBASAN ANTAR SISAAN

UJI DURBIN WATSON

- Ilustrasi

Berikut adalah data deret waktu selama 24 periode:

Periode	Y	X	Periode	Y	X
1	32	38	13	69	74
2	49	40	14	64	132
3	50	44	15	60	52
4	39	62	16	51	32
5	38	50	17	47	56
6	55	106	18	46	14
7	57	50	19	40	18
8	50	52	20	49	36
9	58	132	21	72	42
10	81	138	22	60	18
11	81	100	23	54	42
12	67	96	24	40	10

2. UJI KEBEBASAN ANTAR SISAAN

UJI DURBIN WATSON

- Ilustrasi

```
> summary(model)

Call:
lm(formula = y ~ x, data = contoh)

Residuals:
     Min       1Q   Median       3Q      Max
-17.9921  -6.0457  -0.9104   5.4266  21.1712

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 42.04340    4.08925  10.281 7.27e-10 ***
x            0.20918    0.05808   3.602  0.00159 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 10.6 on 22 degrees of freedom
Multiple R-squared:  0.3709,	Adjusted R-squared:  0.3423
F-statistic: 12.97 on 1 and 22 DF,  p-value: 0.001585
```

Periode	Sisaan	Periode	Sisaan
1	-18.0	13	11.5
2	-1.4	14	-5.7
3	-1.2	15	7.1
4	-16.0	16	2.3
5	-14.5	17	-6.8
6	-9.2	18	1.0
7	4.5	19	-5.8
8	-2.9	20	-0.6
9	-11.7	21	21.2
10	10.1	22	14.2
11	18.0	23	3.2
12	4.9	24	-4.1

2. UJI KEBEBASAN ANTAR SISAAN

UJI DURBIN WATSON

- Ilustrasi

$$d = \frac{\sum_{t=2}^{t=n}(e_t - e_{t-1})^2}{\sum_{t=1}^{t=n} e_t^2}$$

$$= \frac{(e_2 - e_1)^2 + (e_3 - e_2)^2 + \cdots + (e_{24} - e_{23})^2}{e_1^2 + e_2^2 + \cdots + e_{24}^2}$$

$$= \frac{(-1.4 - (-18))^2 + (-1.2 - (-1.4))^2 + \cdots + (-4.1 - 3.2)^2}{(-18)^2 + (-1.4)^2 + \cdots + (-4.1)^2}$$

$$= 1.208767$$

2. UJI KEBEBASAN ANTAR SISAAN

UJI DURBIN WATSON

- Ilustrasi

$$d = \frac{\sum_{t=2}^{t=n}(e_t - e_{t-1})^2}{\sum_{t=1}^{t=n} e_t^2}$$

$$= \frac{(e_2 - e_1)^2 + (e_3 - e_2)^2 + \cdots + (e_{24} - e_{23})^2}{e_1^2 + e_2^2 + \cdots + e_{24}^2}$$

$$= \frac{(-1.4 - (-18))^2 + (-1.2 - (-1.4))^2 + \cdots + (-4.1 - 3.2)^2}{(-18)^2 + (-1.4)^2 + \cdots + (-4.1)^2}$$

$$= 1.208767$$

2. UJI KEBEBASAN ANTAR SISAAN

UJI DURBIN WATSON

Durbin-Watson Statistic: 5 Per Cent Significance Points of dL and dU

	k'=1		k'=2		k'=3		k'=4		k'=5		k'=6		k'=7		k'=8	
n	dL	dU	dL	dU	dL	dU	dL	dU	dL	dU	dL	dU	dL	dU	dL	dU
6	0.610	1.400	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
7	0.700	1.356	0.467	1.896	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
8	0.763	1.332	0.559	1.777	0.367	2.287	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
9	0.824	1.320	0.629	1.699	0.455	2.128	0.296	2.588	-----	-----	-----	-----	-----	-----	-----	-----
10	0.879	1.320	0.697	1.641	0.525	2.016	0.376	2.414	0.243	2.822	-----	-----	-----	-----	-----	-----
11	0.927	1.324	0.758	1.604	0.595	1.928	0.444	2.283	0.315	2.645	0.203	3.004	-----	-----	-----	-----
12	0.971	1.331	0.812	1.579	0.658	1.864	0.512	2.177	0.380	2.506	0.268	2.832	0.171	3.149	-----	-----
13	1.010	1.340	0.861	1.562	0.715	1.816	0.574	2.094	0.444	2.390	0.328	2.692	0.230	2.985	0.147	3.266
14	1.045	1.350	0.905	1.551	0.767	1.779	0.632	2.030	0.505	2.296	0.389	2.572	0.286	2.848	0.200	3.111
15	1.077	1.361	0.946	1.543	0.814	1.750	0.685	1.977	0.562	2.220	0.447	2.471	0.343	2.727	0.251	2.979
16	1.106	1.371	0.982	1.539	0.857	1.728	0.734	1.935	0.615	2.157	0.502	2.388	0.398	2.624	0.304	2.860
17	1.133	1.381	1.015	1.536	0.897	1.710	0.779	1.900	0.664	2.104	0.554	2.318	0.451	2.537	0.356	2.757
18	1.158	1.391	1.046	1.535	0.933	1.696	0.820	1.872	0.710	2.060	0.603	2.258	0.502	2.461	0.407	2.668
19	1.180	1.401	1.074	1.536	0.967	1.685	0.859	1.848	0.752	2.023	0.649	2.206	0.549	2.396	0.456	2.589
20	1.201	1.411	1.100	1.537	0.998	1.676	0.894	1.828	0.792	1.991	0.691	2.162	0.595	2.339	0.502	2.521
21	1.221	1.420	1.125	1.538	1.026	1.669	0.927	1.812	0.829	1.964	0.731	2.124	0.637	2.290	0.546	2.461
22	1.239	1.429	1.147	1.541	1.053	1.664	0.958	1.797	0.863	1.940	0.769	2.090	0.677	2.246	0.588	2.407
23	1.257	1.437	1.168	1.543	1.078	1.660	0.986	1.785	0.895	1.920	0.804	2.061	0.715	2.208	0.628	2.360
24	1.273	1.446	1.188	1.546	1.101	1.656	1.013	1.775	0.925	1.902	0.837	2.035	0.750	2.174	0.666	2.318
25	1.288	1.454	1.206	1.550	1.123	1.654	1.038	1.767	0.953	1.886	0.868	2.013	0.784	2.144	0.702	2.280
26	1.302	1.461	1.224	1.553	1.143	1.652	1.062	1.759	0.979	1.873	0.897	1.992	0.816	2.117	0.735	2.246
27	1.316	1.469	1.240	1.556	1.162	1.651	1.084	1.753	1.004	1.861	0.925	1.974	0.845	2.093	0.767	2.216

d_L=1.273 dan d_U=1.446

- Ilustrasi

2. UJI KEBEBASAN ANTAR SISAAN

UJI DURBIN WATSON

- Ilustrasi

Kesimpulan: cukup bukti utk mengatakan bahwa terdapat autokorelasi positif pada taraf 5%.

```
> library(lmtest)
> dwtest(model)

        Durbin-Watson test

data:  model
DW = 1.2088, p-value = 0.01364
alternative hypothesis: true
autocorrelation is greater than 0
```

OUTLINE

1. Reviu

2. Uji Kebebasan Antar Sisaan

3. Penanganan Autokorelasi Diri

3. PENANGANAN AUTOKORELASI DIRI

- Cochrane-Orcutt
- Hildreth-Lu
- Regresi dengan Peubah Lag

3. PENANGANAN AUTOKORELASI DIRI

Pendahuluan

Perhatikan model berikut:

$$y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + \ldots + \beta_k x_{kt} + u_t \qquad (1)$$

dengan

$$u_t = \rho u_{t-1} + \varepsilon_t$$

Jika model tdb di-lag-kan dan dikalikan dengan ρ

$$\rho y_{t-1} = \beta_1 \rho + \beta_2 \rho x_{2t-1} + \beta_3 \rho x_{3t-1} + \ldots + \beta_k \rho x_{kt-1} + \rho u_{t-1} \qquad (2)$$

Model pada persamaan (1) dikurangi dengan (2) akan menjadi:

$$y_t - \rho y_{t-1} = \beta_1 (1-\rho) + \beta_2 \left(x_{2t} - \rho x_{2t-1}\right) + \ldots + \beta_k \left(x_{kt} - \rho x_{kt-1}\right) + \left(u_t - \rho u_{t-1}\right) \qquad (3)$$

3. PENANGANAN AUTOKORELASI DIRI

Pendahuluan

Pers. (3) dapat ditulis sbb.

$$y_t^* = \beta_0^* + \beta_1 x_{t1}^* + \beta_2 x_{t2}^* + \cdots + \beta_k x_{tk}^* + u_t$$

dengan

$$y_t^* = y_t - \rho y_{t-1}, \quad \beta_0^* = \beta_0(1-\rho),$$

$$x_{ti}^* = x_{ti} - \rho x_{t-1,i}, \; \varepsilon_t = \rho \varepsilon_{t-1} + u_t$$

Notes

for $t = 2, 3, \ldots, T$ and $i = 1, \ldots, k$. Note that the error term satisfies all the properties needed for applying the OLS procedure. If ρ were known, we could apply OLS to the transformed y^* and x^* and obtain estimates that are BLUE. However, ρ is unknown and has to be estimated from the sample.

3. PENANGANAN AUTOKORELASI DIRI

Cochrane-Orcutt

Pendekatan dilakukan secara iterative agar mendapatkan penduga ρ yg lebih baik.

Tahapan prosedur:

1. Meregresikan Y terhadap X untuk memperoleh galat e_t
2. Menduga koefisien korelasi serial ordo ke-1 ($\hat{\rho}$) dengan meregresikan e_t terhadap e_{t-1}

$$e_t = \rho e_{t-1} + u_t$$

3. Melakukan transformasi terhadap X dan Y:

$$y_t^* = y_t - \hat{\rho} y_{t-1}, \quad x_{t1}^* = x_{t1} - \hat{\rho} x_{t-1,1}$$

3. PENANGANAN AUTOKORELASI DIRI

Cochrane-Orcutt

4. Meregresikan Y^* terhadap X^* sehingga diperoleh penduga koefisien β_0^*, β_1^*, dst…

5. Menghitung $\hat{\beta}_0 = \frac{\widehat{\beta}_0^*}{1-\rho}$, substitusikan $\hat{\beta}_0$ dan β_1^*, β_2^*, dst… pada persamaan regresi pada tahap (1) sehingga dapat dihitung gugus data galat e_t yg baru.

6. Ulangi tahap (2) s.d tahap (5) hingga nilai $\hat{\rho}$ dianggap konvergen

3. PENANGANAN AUTOKORELASI DIRI

Cochrane-Orcutt

- Ilustrasi

Berikut adalah data deret waktu selama 24 periode:

Periode	Y	X
1	32	38
2	49	40
3	50	44
4	39	62
5	38	50
6	55	106
7	57	50
8	50	52
9	58	132
10	81	138
11	81	100
12	67	96

Periode	Y	X
13	69	74
14	64	132
15	60	52
16	51	32
17	47	56
18	46	14
19	40	18
20	49	36
21	72	42
22	60	18
23	54	42
24	40	10

3. PENANGANAN AUTOKORELASI DIRI

Cochrane-Orcutt

- Ilustrasi

```
> model<-lm(y~x)
> library(lmtest)
> dwtest(model)

        Durbin-Watson test

data:  model
DW = 1.2088, p-value = 0.01364
alternative hypothesis: true autocorrelation is
greater than 0
```

```
> library(orcutt)
> cochrane.orcutt(model)

Cochrane-orcutt estimation for first order
autocorrelation

Call:
lm(formula = y ~ x)

 number of interaction: 13
 rho 0.441367

Durbin-Watson statistic
(original):    1.20877 , p-value: 1.364e-02
(transformed): 1.66348 , p-value: 1.992e-01

 coefficients:
(Intercept)           x
  47.908320    0.132056
```

3. PENANGANAN AUTOKORELASI DIRI

Cochrane-Orcutt

- Ilustrasi

```
> rho<-cochrane.orcutt(model)$rho
> y.transformed<-y[-1]-(y[-24]*rho)
> x.transformed<-x[-1]-(x[-24]*rho)
> model.t<-lm(y.transformed~x.transformed)
```

before

```
> summary(model)

Call:
lm(formula = y ~ x)

Residuals:
     Min       1Q   Median       3Q      Max 
-17.9921  -6.0457  -0.9104   5.4266  21.1712 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 42.04340    4.08925  10.281 7.27e-10 ***
x            0.20918    0.05808   3.602  0.00159 ** 
---
```

after

```
> summary(model.t)

Call:
lm(formula = y.transformed ~ x.transformed)

Residuals:
    Min      1Q  Median      3Q     Max 
-15.454  -6.105  -1.869   5.291  20.162 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)   26.76315    2.74122   9.763 2.94e-09 ***
x.transformed  0.13206    0.05898   2.239   0.0361 *  
```

3. PENANGANAN AUTOKORELASI DIRI

Cochrane-Orcutt

- Ilustrasi

```
> dwtest(model.t)

        Durbin-Watson test

data:  model.t
DW = 1.6635, p-value = 0.1992
alternative hypothesis: true autocorrelation is greater than 0
```

- Penduga koefisien regresi setelah tranformasi ke persamaan awal:
  - $b_0 = \frac{b_0^*}{1-r} = \frac{26.76315}{1-0.441367} = 47.90829$
  - $b_1 = b_1^* = 0.13206$

  → $\hat{y}_t = \mathbf{47.908 + 0.132}\ x_t$

3. PENANGANAN AUTOKORELASI DIRI

Hildreth-Lu

STEP 1: Choose a value of ρ (say ρ_1). Using this value, transform the variables and estimate the transformed regression by OLS.

STEP 2: From these estimates, derive $\hat{u}_t$ from Equation (10.1) and the error sum of squares associated with it. Call it $\mathrm{SSR}_{\hat{u}}(\rho_1)$. Nest choose a different ρ (ρ_2) and repeat Steps 1 and 2.

STEP 3: By varying ρ from -1 to $+1$ in some systematic way (say, at steps of length 0.05 or 0.01), we can get a series of values of $\mathrm{SSR}_{\hat{u}}(\rho)$. Choose that ρ for which $\mathrm{SSR}_{\hat{u}}$is a minimum. This is the final ρ that globally minimizes the error sum of squares of the transformed model. Equation (10.1) is then estimated with the final ρ as the optimum solution.

3. PENANGANAN AUTOKORELASI DIRI

Hildreth-Lu

- Ilustrasi

```
# Hildreth-Lu (does not require iterations)
rho = c(seq(0.1,0.8,by=0.1),seq(0.90,0.99,by=0.01))

hildreth.lu <- function(rho, model){
  x <- model.matrix(model)[, -1]
  y <- model.response(model.frame(model))
  n <- length(y)
  t <- 2:n
  y <- y[t] - rho * y[t-1]
  x <- x[t] - rho * x[t-1]

  return(lm(y ~ x))
}
```

3. PENANGANAN AUTOKORELASI DIRI

Hildreth-Lu

- Ilustrasi

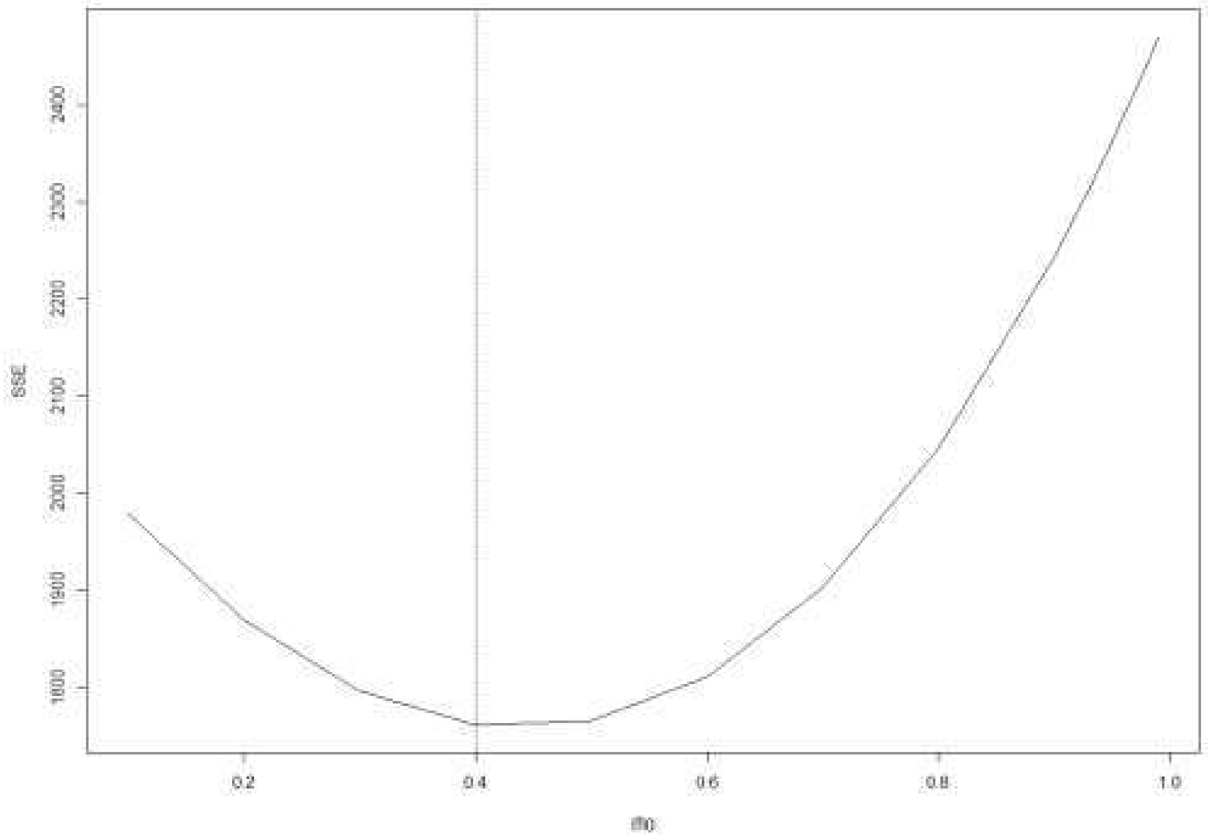
```
> fit <- lm(y ~ x)
> tab <- data.frame('rho' = rho,
+                   'SSE' = sapply(rho, function(r) {deviance(hildreth.lu(r, fit))}))
> round(tab, 4)
    rho      SSE
1  0.10 1979.103
2  0.20 1869.163
3  0.30 1796.786
4  0.40 1761.665
5  0.50 1765.243
6  0.60 1810.768
7  0.70 1902.698
8  0.80 2045.622
9  0.90 2243.228
10 0.91 2266.095
11 0.92 2289.534
12 0.93 2313.546
13 0.94 2338.132
14 0.95 2363.293
15 0.96 2389.029
16 0.97 2415.341
17 0.98 2442.231
18 0.99 2469.697
```

3. PENANGANAN AUTOKORELASI DIRI

Hildreth-Lu

- Ilustrasi

```
plot(SSE ~ rho, tab, type = 'l')
abline(v = tab[tab$SSE == min(tab$SSE), 'rho'], lty = 3)
```

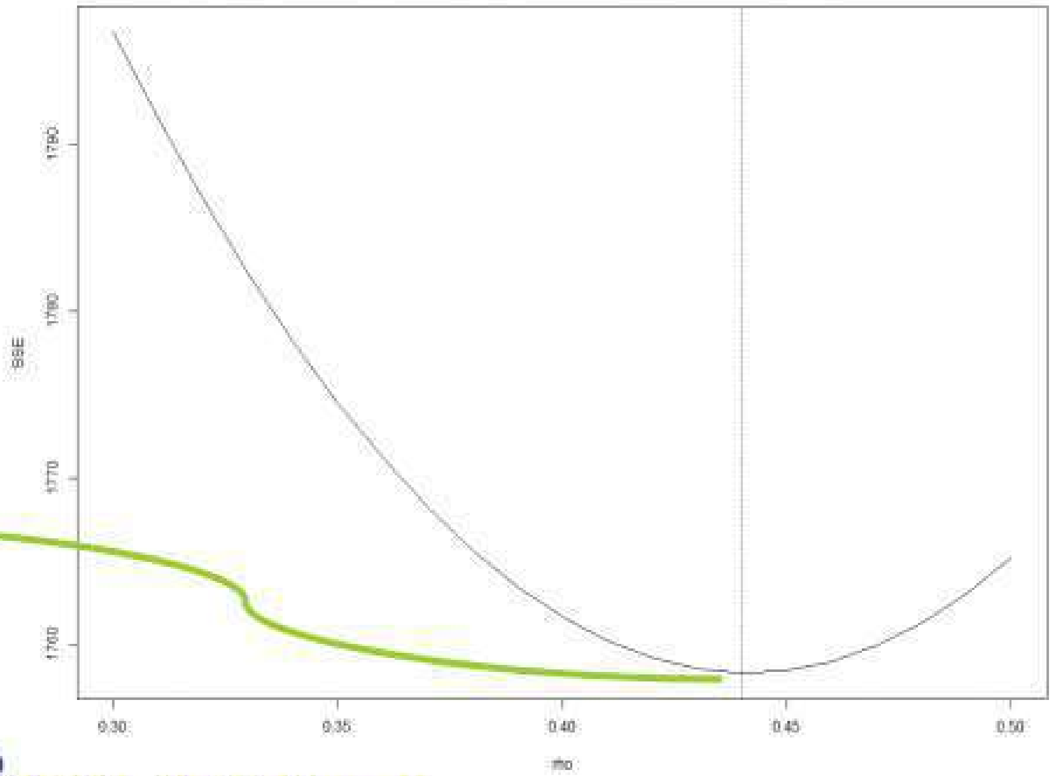


3. PENANGANAN AUTOKORELASI DIRI

Hildreth-Lu

- Ilustrasi

```
> rho = c(seq(0.3,0.5,by=0.01))
>
> tab <- data.frame('rho' = rho,
+                   'SSE' = sapply(rho, function(r) {deviance(hildreth.lu(r, fit))}))
> round(tab, 4)
    rho      SSE
1  0.30 1796.786
2  0.31 1791.588
3  0.32 1786.762
4  0.33 1782.309
5  0.34 1778.229
6  0.35 1774.523
7  0.36 1771.193
8  0.37 1768.241
9  0.38 1765.667
10 0.39 1763.475
11 0.40 1761.665
12 0.41 1760.241
13 0.42 1759.205
14 0.43 1758.560
15 0.44 1758.307
16 0.45 1758.452
17 0.46 1758.996
18 0.47 1759.944
19 0.48 1761.298
20 0.49 1763.063
21 0.50 1765.243
> plot(SSE ~ rho, tab, type = 'l')
> abline(v = tab[tab$SSE == min(tab$SSE), 'rho'], lty = 3)
```



3. PENANGANAN AUTOKORELASI DIRI

Hildreth-Lu

- Ilustrasi

```
> fit <- hildreth.lu(0.44, fit)
> summary(fit)

Call:
lm(formula = y ~ x)

Residuals:
    Min      1Q  Median      3Q     Max 
-15.460  -6.101  -1.872   5.278  20.160 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 26.82008    2.74456   9.772  2.9e-09 ***
x            0.13228    0.05897   2.243   0.0358 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 9.15 on 21 degrees of freedom
Multiple R-squared:  0.1933,    Adjusted R-squared:  0.1549 
F-statistic: 5.031 on 1 and 21 DF,  p-value: 0.03581

> 
> dwtest(fit)

        Durbin-Watson test

data:  fit
DW = 1.6625, p-value = 0.1984
alternative hypothesis: true autocorrelation is greater than 0
```

3. PENANGANAN AUTOKORELASI DIRI

Hildreth-Lu

- Ilustrasi

```
> cat("Y = ", coef(fit)[1] / (1 - 0.44), " + ",
coef(fit)[2], "X", sep = "")

Y = 47.893 + 0.1322756X
```

Persamaan regresi setelah di transformasi ke persamaan awal

IPB University
Bogor Indonesia

TERIMAKASIH

4. TUGAS PRAKTIKUM

NOMOR 1

- Berikut ini adalah data pangsa pasar produk pasta gigi selama 20 periode:

Periode	Pangsa Pasar	Harga
1	3.63	0.97
2	4.20	0.95
3	3.33	0.99
4	4.54	0.91
5	2.89	0.98
6	4.87	0.90
7	4.90	0.89
8	5.29	0.86
9	6.18	0.85
10	7.20	0.82

Periode	Pangsa Pasar	Harga
11	7.25	0.79
12	6.09	0.83
13	6.80	0.81
14	8.65	0.77
15	8.43	0.76
16	8.29	0.80
17	7.18	0.83
18	7.90	0.79
19	8.45	0.76
20	8.23	0.78

- Lakukan pemodelan regresi antara pangsa pasar (Y) terhadap harga (X).
- Periksalah apakah terdapat autokorelasi pada sisaan model tersebut dengan pendekatan:
  a) Grafik
  b) Uji Durbin-Watson

4. TUGAS PRAKTIKUM

NOMOR 2

Periode	X	Y	Periode	X	Y
1	0	6.3	**10**	3	3.4
2	0	6.2	**11**	3	3.5
3	0	6.4	**12**	3	3.6
4	1	5.3	**13**	4	2.6
5	1	5.4	**14**	4	2.5
6	1	5.5	**15**	4	2.4
7	2	4.5	**16**	5	1.3
8	2	4.4	**17**	5	1.4
9	2	4.4	**18**	5	1.5

- Periksalah apakah terdapat korelasi serial pada sisaan model regresi $y_t = b_0 + b_1 x_t + e_t$?
- Jika ada, lakukan penanganan dengan metode Cochrane-Orcutt.

- Lakukan pula penanganan dengan metode Hildreth-Lu.

4. TUGAS PRAKTIKUM

NOMOR 3

Tahun	Penjualan	Biaya Iklan
1975	11.7	9.4
1976	12.0	9.6
1977	12.3	10
1978	12.8	10.4
1979	13.1	10.8
1980	13.6	10.9
1981	13.9	11.7
1982	14.4	12.2
1983	14.7	12.5
1984	15.3	12.9
1985	15.5	13.0
1986	15.8	13.2
1987	16.1	13.8
1988	16.6	14.2
1989	16.9	14.6
1990	16.7	14.4
1991	16.9	15.0
1992	17.4	15.4
1993	17.6	15.7
1994	17.9	15.9

Tahun	Penjualan	Biaya Iklan
1995	18.0	15.9
1996	17.9	16.0
1997	18.0	16.3
1998	18.2	16.2
1999	18.2	16.8
2000	18.3	17.3
2001	18.6	17.6
2002	19.2	18.1
2003	19.3	18.3
2004	19.5	18.5
2005	19.2	18.7
2006	19.3	18.9
2007	19.5	19.2
2008	20.0	20.0
2009	20.0	20.0
2010	19.9	20.3
2011	19.8	20.4
2012	19.9	21.0
2013	20.2	21.5
2014	21.0	22.1

- Periksalah apakah terdapat korelasi serial pada sisaan model regresi $y_t = b_0 + b_1 x_t + e_t$?
- Jika ada, lakukan penanganan dengan metode Cochrane-Orcutt.
- Lakukan pula penanganan dengan metode Hildreth-Lu.

MODEL REGRESI DENGAN PEUBAH LAG

Pertemuan ke-7
Akbar Rizki, M.Si

OUTLINE

1. Konsep Dasar Regresi dengan Peubah Lag

2. Autoregressive Distributed Lag (ARDL) models

3. Koyck Lag

MODEL REGRESI

- Static Regression

$$Y_t = \beta_0 + \beta_1 X_{t,1} + \cdots + \beta_k X_{t,k} + u_t$$

- Regression with distributed lag

$$Y_t = \beta_0 + \beta_1 X_{t,1} + \beta_2 X_{t-1,1} + \beta_3 X_{t-2,1} + \beta_4 X_{t,2} + u_t$$

- Dynamic regression/ autoregressive model

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \beta_3 X_{t,1} + \beta_4 X_{t,2} + u_t$$

MODEL AUTOREGRESSIVE

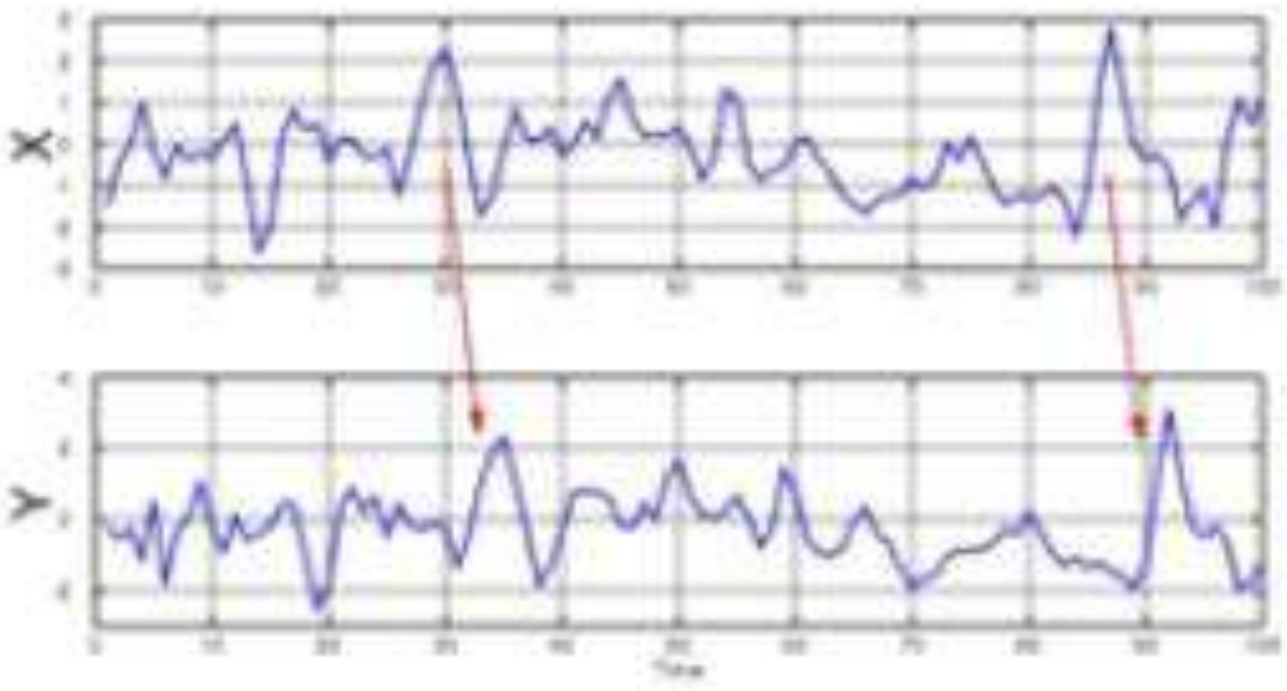
Apabila peubah dependen dipengaruhi oleh peubah independen pada waktu sekarang, serta dipengaruhii juga oleh peubah dependen itu sendiri pada satu waktu yang lalu maka model tersebut disebut autoregressive (Gujarati, 2004)

$$Y_t = \alpha + \beta X_t + \gamma Y_{t-1} + u_t$$

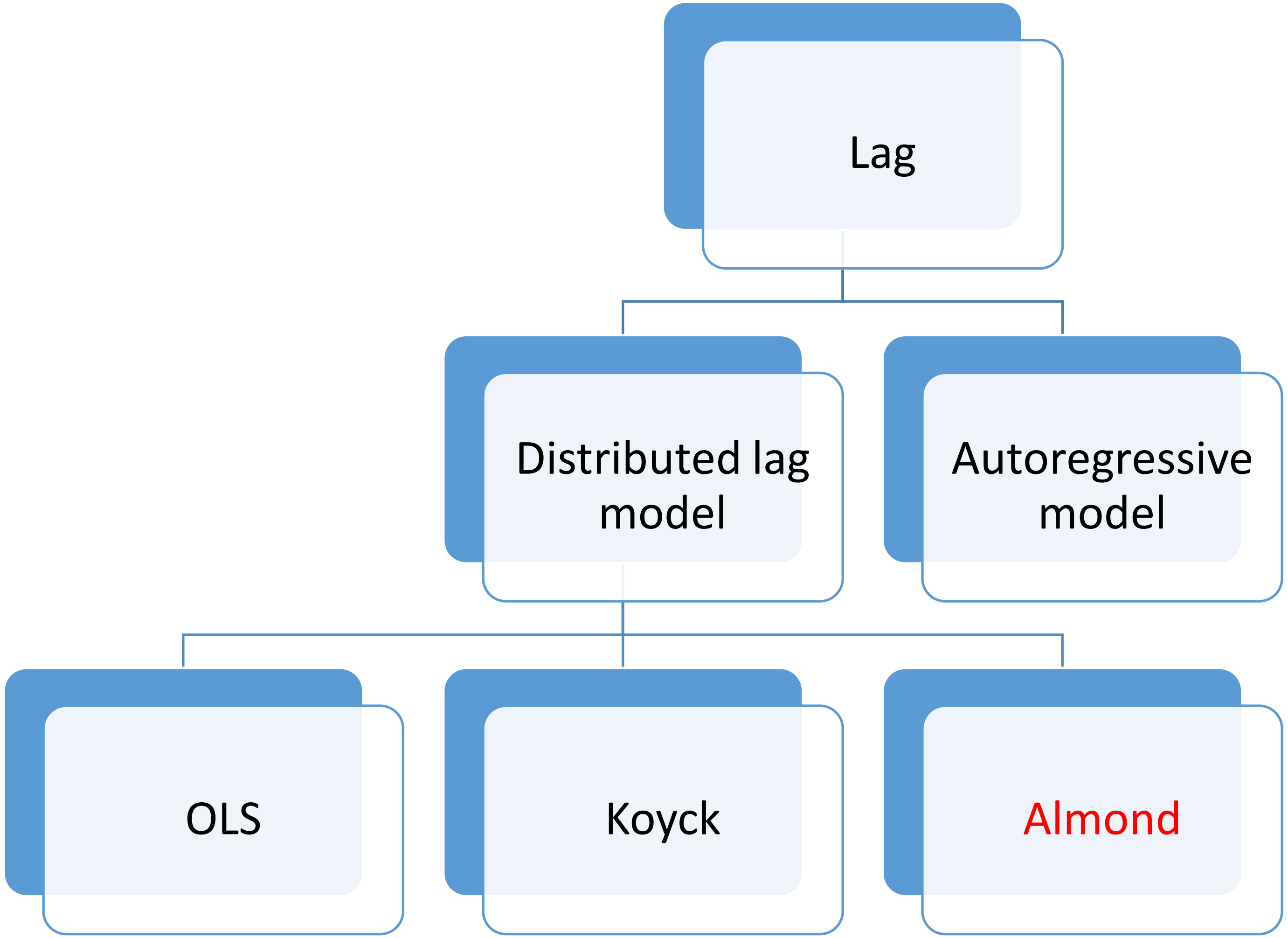
LAG

Waktu yang diperlukan bagi peubah bebas X dalam mempengaruhi peubah tak bebas Y disebut lag.

Contoh: Butuh waktu untuk membangun jalan raya, efek dari investasi public ini pada pertumbuhan GNP akan muncul dengan lag dan efek ini mungkin akan bertahan selama beberapa tahun.



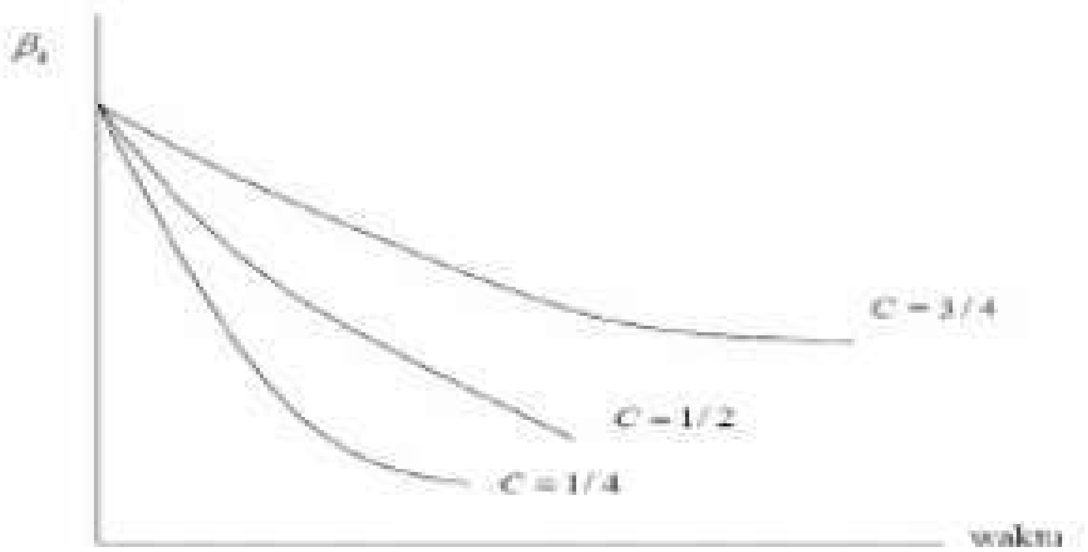
MIND MAP


Lag
Distributed lag model
Autoregressive model
OLS
Koyck
Almond

PENDEKATAN KOYCK UNTUK DITRIBUTED LAG MODEL

- Metode Koyck didasarkan asumsi bahwa semakin jauh jarak lag peubah independen dari periode sekarang maka semakin kecil pengaruh peubah lag terhadap peubah dependen
- Koyck mengusulkan suatu metode untuk menduga model dinamis distributed lag dengan mengasumsikan bahwa semua koefisien β mempunyai tanda sama.
- Model Koyck merupakan jenis paling umum dari model *infinite distributed lag* dan juga dikenal sebagai *geometric lag*

MODEL KOYCK



Koyck menganggap bahwa koefisien menurun secara geometris sebagai berikut:

$$\beta_k = \beta_0 C^k, k = 0, 1, \ldots$$

Dengan:

C: rata-rata tingkat penurunan dari distribusi lag dengan nilai $0 < C < 1$

$1 - C$: kecepatan peyesuaian

MODEL KOYCK

$\hat{\beta}_0 = \beta_0$

$\hat{\beta}_1 = \beta_0 C$

$\hat{\beta}_2 = \beta_0 C^2$

$\hat{\beta}_k = \beta_0 C^k$

$Y_t = \alpha + \beta_0 X_t + \beta_0 C X_{t-1} + \beta_0 C^2 X_{t-2} + \ldots + \varepsilon_t$ (1.1)

$Y_{t-1} = \alpha + \beta_0 X_{t-1} + \beta_0 C X_{t-2} + \beta_0 C^2 X_{t-3} + \ldots + \varepsilon_{t-1}$ (1.2)

$CY_{t-1} = \alpha C + \beta_0 C X_{t-1} + \beta_0 C^2 X_{t-2} + \beta_0 C^3 X_{t-3} + \ldots + \varepsilon_{t-1}$ (1.3)

Jika persamaan (1.1) - (1.3) maka didapat

$Y_t - CY_{t-1} = \alpha(1-C) + \beta_0 X_t + (\varepsilon_t - C\varepsilon_{t-1})$

$Y_t = \alpha(1-C) + \beta_0 X_t + CY_{t-1} + V_t$ (1.4)

Model (1.4) merupakan **model Koyck.**

MODEL KOYCK

Ilustrasi

Penelitian dilakukan untuk mengetahui hubungan antara pembelian perlengkapan dan hasil penjualan suatu perusahaan selama 20 tahun.

Berdasarkan data pembelian perlengkapan dan hasil penjualan dalam tabel akan ditunjukkan persamaan dinamis distribusi lag dengan menggunakan metode Koyck

t	Yt	Xt
1	52.9	30.3
2	53.8	30.9
3	54.9	30.9
4	58.2	33.4
5	60	35.1
6	63.4	37.3
7	68.2	41
8	78	44.9
9	84.7	46.5
10	90.6	50.3
11	98.2	53.5
12	101.7	52.8
13	102.7	55.9
14	108.3	63
15	124.7	73
16	157.9	84.8
17	158.2	86.6
18	170.2	98.9
19	180	110.8
20	198	124.7

MODEL KOYCK

t	Yt	Y(t-1)	Xt
1	52.9		30.3
2	53.8	52.9	30.9
3	54.9	53.8	30.9
4	58.2	54.9	33.4
5	60	58.2	35.1
6	63.4	60	37.3
7	68.2	63.4	41
8	78	68.2	44.9
9	84.7	78	46.5
10	90.6	84.7	50.3
11	98.2	90.6	53.5
12	101.7	98.2	52.8
13	102.7	101.7	55.9
14	108.3	102.7	63
15	124.7	108.3	73
16	157.9	124.7	84.8
17	158.2	157.9	86.6
18	170.2	158.2	98.9
19	180	170.2	110.8
20	198	180	124.7

Regresikan Yt dengan Y(t-1) dan Xt, sehingga diperoleh persamaan regresi sebagai berikut:

$Y_t = 2.727 + 0.941\ X_t + 0.468\ Y_{t-1}$

Model dugaan dapat dituliskandalam bentuk persamaan dinamis distribusi *lag* dugaan dengan cara sebagai berikut.Berdasarkan persamaan di atas diketahui :

$\hat{C} = 0.468$

$\hat{\alpha}(1 - \hat{C}) = 2.727$ -> $\hat{\alpha} = 5.1275$

$\hat{\beta}_0 = \beta_0 = 0.941$

$\hat{\beta}_1 = \beta_0\, C = 0.4403$

$\hat{\beta}_2 = \beta_0 C^2 = 0.206$

Jadi model lag dugaannya adalah

$$\hat{Y} = 5.1275 + 0.941\ X_{t-1} + \ 0.4403\ X_{t-1} + \ 0.206\ X_{t-2} + \cdots$$

Bisa diamati bahwa pengaruh dari *lag Y* menurun secara geometris dilihat dari persamaan $Y_t = 2.727 + 0.941\ X_t + 0.468\ Y_{t-1}$. Diketahui bahwa nilai koefisien dari Y_{t-1} bernilai positif yaitu sebesar 0.468. Nilai 0.4682 berarti bahwa apabila penjualan naik sebesar 1% makapengeluaran perlengkapan akan naik sebesar 0.468%.

MODEL KOYCK

```
install.packages("dLagM")
library("dLagM")
data<-read.delim("clipboard")
model.koyck=koyckDlm(x=data$Xt, y=data$Yt)
```

To deal with infinite DLMs, we can use the Koyck transformation. When we apply Koyck transformation, we get the following:

$$Y_t - \phi Y_{t-1} = \alpha(1-\phi) + \beta X_t + (\epsilon_t - \phi\epsilon_{t-1}).$$

When we solve this equation for Y_t, we obtain Koyck DLM as follows:

$$Y_t = \delta_1 + \delta_2 Y_{t-1} + \delta_3 X_t + \nu_t,$$

where $\delta_1 = \alpha(1-\phi), \delta_2 = \phi, \delta_3 = \beta$ and the random error after the transformation is $\nu_t = (\epsilon_t - \phi\epsilon_{t-1})$ (Judge and Griffiths, 2000).

```
> model.koyck
$model

Call:
"Y ~ (Intercept) + Y.1 + X.t"

Coefficients:
(Intercept)          Y.1          X.t
     2.0804       0.5726       0.7826


$geometric.coefficients
                            alpha      beta       phi
Geometric coefficients:  4.867444 0.7826327 0.5725805
```

AUTOREGRESSIVE DISTRIBUTED LAG

- Ilustrasi
  - Different specifications of consumption function dari buku Econometric Analysis by William H. Greene (2003)
  - **Data:** Quarterly US macroeconomic data tahun1950(1) – 2000(4) yang disediakan oleh USMacroG, a "ts" time series. Terdiri dari disposable income dpi dan consumption (billion USD).

AUTOREGRESSIVE DISTRIBUTED LAG

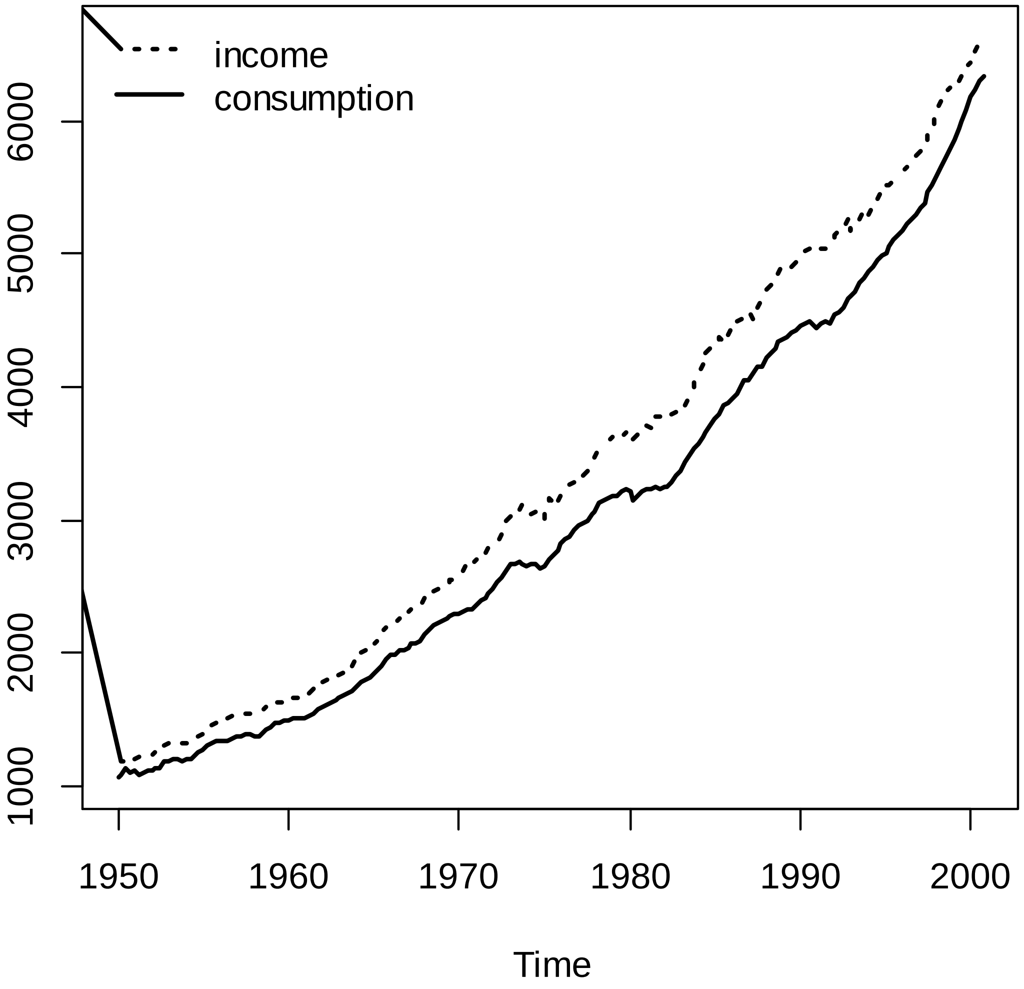
Models: Greene (2003) considers

$$\begin{aligned}
\texttt{consumption}_i &= \beta_1 + \beta_2\,\texttt{dpi}_i + \beta_3\,\texttt{dpi}_{i-1} + \varepsilon_i \\
\texttt{consumption}_i &= \beta_1 + \beta_2\,\texttt{dpi}_i + \beta_3\,\texttt{consumption}_{i-1} + \varepsilon_i.
\end{aligned}$$

Interpretation:

- Distributed lag model: consumption responds to changes in income only over two periods.
- Autoregressive distributed lag: effects of income changes persist.

AUTOREGRESSIVE DISTRIBUTED LAG




```
data("USMacroG",package="AER")

plot(USMacroG[,c("dpi","consumption")],lty=c(3,1),lwd=2,plot
.type="single",ylab="")

legend("topleft",
legend=c("income","consumption"),lwd=2,lty=c(3,1),bty="n")
```

AUTOREGRESSIVE DISTRIBUTED LAG

```
library("dynlm")
cons_lm1<-dynlm(consumption~dpi+L(dpi),data=USMacroG)
cons_lm2<-dynlm(consumption~dpi+L(consumption),data=USMacroG)
summary(cons_lm1)
summary(cons_lm2)
deviance(cons_lm1)
deviance(cons_lm2)
```

```
> summary(cons_lm1)

Time series regression with "ts" data:
Start = 1950(2), End = 2000(4)

Call:
dynlm(formula = consumption ~ dpi + L(dpi), data = USMacroG)

Residuals:
    Min      1Q  Median      3Q     Max
-190.02  -56.68    1.58   49.91  323.94

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -81.07959   14.50814  -5.589 7.43e-08 ***
dpi           0.89117    0.20625   4.321 2.45e-05 ***
L(dpi)        0.03091    0.20754   0.149    0.882
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 87.58 on 200 degrees of freedom
Multiple R-squared:  0.9964,	Adjusted R-squared:  0.9964
F-statistic: 2.785e+04 on 2 and 200 DF,  p-value: < 2.2e-16
```

```
> summary(cons_lm2)

Time series regression with "ts" data:
Start = 1950(2), End = 2000(4)

Call:
dynlm(formula = consumption ~ dpi + L(consumption), data = USMacroG)

Residuals:
     Min       1Q   Median       3Q      Max
-101.303   -9.674    1.141   12.691   45.322

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)     0.535216   3.845170   0.139    0.889
dpi            -0.004064   0.016626  -0.244    0.807
L(consumption)  1.013111   0.018161  55.785   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 21.52 on 200 degrees of freedom
Multiple R-squared:  0.9998,	Adjusted R-squared:  0.9998
F-statistic: 4.627e+05 on 2 and 200 DF,  p-value: < 2.2e-16
> deviance(cons_lm1)
[1] 1534001
> deviance(cons_lm2)
[1] 92644.15
```

$$\mathtt{consumption}_i = \beta_1 + \beta_2\,\mathtt{dpi}_i + \beta_3\,\mathtt{dpi}_{i-1} + \varepsilon_i$$

$$\mathtt{consumption}_i = \beta_1 + \beta_2\,\mathtt{dpi}_i + \beta_3\,\mathtt{consumption}_{i-1} + \varepsilon_i$$

The Fundamental Concepts in the theory of time series

YENNI ANGRAINI

Outline

- Time Series and Stochastic Processes
- Means, Variances, and Covariances
- Stationarity

Time Series and Stochastic Processes

- **A stochastic process** is a collection or ensemble of random variables indexed by a variable t, usually representing time.
- A *stochastic process* means that one has a system for which there are observations at certain times, and that the outcome, that is, the observed value at each time is a random variable.
- The sequence of random variables $\{Y_t : t = 0, \pm 1, \pm 2, \pm 3, \ldots\}$ is called a **stochastic process**
- Another simple **stochastic process** is the random walk, $Y_t = Y_{t-1} + e_t$

Means

For a stochastic process $\{Y_t: t = 0, \pm 1, \pm 2, \pm 3, \ldots\}$, the **mean function** is defined by

$$\mu_t = E(Y_t) \qquad \text{for } t = 0, \pm 1, \pm 2, \ldots \tag{2.2.1}$$

That is, μ_t is just the expected value of the process at time t. In general, μ_t can be different at each time point t.

Properties of Variance

$$Var(X) \geq 0$$

$$Var(a + bX) = b^2 Var(X)$$

If X and Y are independent, then

$$Var(X + Y) = Var(X) + Var(Y)$$

In general, it may be shown that

$$Var(X) = E(X^2) - [E(X)]^2$$

Properties of Covariance

$$Cov(a + bX, c + dY) = bdCov(X, Y)$$

$$Var(X + Y) = Var(X) + Var(Y) + 2Cov(X, Y)$$

$$Cov(X + Y, Z) = Cov(X, Z) + Cov(Y, Z)$$

$$Cov(X, X) = Var(X)$$

$$Cov(X, Y) = Cov(Y, X)$$

If X and Y are independent,

$$Cov(X, Y) = 0$$

Properties of Correlation

The **correlation coefficient** of X and Y, denoted by $Corr(X, Y)$ or ρ, is defined as

$$\rho = Corr(X, Y) = \frac{Cov(X, Y)}{\sqrt{Var(X)Var(Y)}}$$

$$-1 \le Corr(X, Y) \le 1$$

$$Corr(a + bX, c + dY) = sign(bd)Corr(X, Y)$$

$$\text{where } sign(bd) = \begin{cases} 1 \text{ if } bd > 0 \\ 0 \text{ if } bd = 0 \\ -1 \text{ if } bd < 0 \end{cases}$$

Variances, and Covariances

The **autocovariance function**, $\gamma_{t,s}$, is defined as

$$\gamma_{t,s} = Cov(Y_t, Y_s) \qquad \text{for } t, s = 0, \pm 1, \pm 2, \ldots \tag{2.2.2}$$

where $Cov(Y_t, Y_s) = E[(Y_t - \mu_t)(Y_s - \mu_s)] = E(Y_t Y_s) - \mu_t \mu_s$.

The **autocorrelation function**, $\rho_{t,s}$, is given by

$$\rho_{t,s} = Corr(Y_t, Y_s) \qquad \text{for } t, s = 0, \pm 1, \pm 2, \ldots \tag{2.2.3}$$

where

$$Corr(Y_t, Y_s) = \frac{Cov(Y_t, Y_s)}{\sqrt{Var(Y_t)Var(Y_s)}} = \frac{\gamma_{t,s}}{\sqrt{\gamma_{t,t}\gamma_{s,s}}} \tag{2.2.4}$$

The following important properties of Variances and Covariances

$$\left.\begin{array}{ll} \gamma_{t,t} = Var(Y_t) & \rho_{t,t} = 1 \\ \gamma_{t,s} = \gamma_{s,t} & \rho_{t,s} = \rho_{s,t} \\ |\gamma_{t,s}| \le \sqrt{\gamma_{t,t}\gamma_{s,s}} & |\rho_{t,s}| \le 1 \end{array}\right\}$$

- ❑ Values of $\rho_{t,s}$ near ±1 indicate strong (linear) dependence,
- ❑ Values of $\rho_{t,s}$ near zero indicate weak (linear) dependence.
- ❑ If $\rho_{t,s} = 0$, we say that Y_t and Y_s are *uncorrelated*.

The Random Walk

Let $e_1, e_2, \ldots$ be a sequence of independent, identically distributed random variables each with zero mean and variance σ_e^2.

The observed time series, $\{Yt : t = 1, 2, \ldots\}$, is constructed as follows:

$$\left.\begin{aligned} Y_1 &= e_1 \\ Y_2 &= e_1 + e_2 \\ &\vdots \\ Y_t &= e_1 + e_2 + \cdots + e_t \end{aligned}\right\} \Rightarrow \quad Y_t = Y_{t-1} + e_t$$

Mean of Random Walk

$$\mu_t = E(Y_t) = E(e_1 + e_2 + \cdots + e_t) = E(e_1) + E(e_2) + \cdots + E(e_t)$$
$$= 0 + 0 + \cdots + 0$$

$$\mu_t = 0 \quad \text{for all } t$$

Variance of Random Walk

$$\begin{aligned} Var(Y_t) &= Var(e_1 + e_2 + \cdots + e_t) = Var(e_1) + Var(e_2) + \cdots + Var(e_t) \\ &= \sigma_e^2 + \sigma_e^2 + \cdots + \sigma_e^2 \end{aligned}$$

$$Var(Y_t) = t\sigma_e^2$$

Notice that the process variance increases linearly with time.

Covariance of Random Walk

To investigate the covariance function, suppose that $1 \le t \le s$. Then we have

$$\gamma_{t,s} = Cov(Y_t, Y_s) = Cov(e_1 + e_2 + \cdots + e_t, e_1 + e_2 + \cdots + e_t + e_{t+1} + \cdots + e_s)$$

$$\gamma_{t,s} = \sum_{i=1}^{s} \sum_{j=1}^{t} Cov(e_i, e_j)$$

However, these covariances are zero unless $i = j$, in which case they equal $Var(e_i) = \sigma_e^2$. There are exactly t of these so that $\gamma_{t,s} = t\sigma_e^2$.

Autocovariance and Autocorrelation function for Random Walk

Since $\gamma_{t,s} = \gamma_{s,t}$, this specifies the autocovariance function for all time points t and s and we can write

$$\boxed{\gamma_{t,s} = t\sigma_e^2 \qquad \text{for } 1 \le t \le s}$$

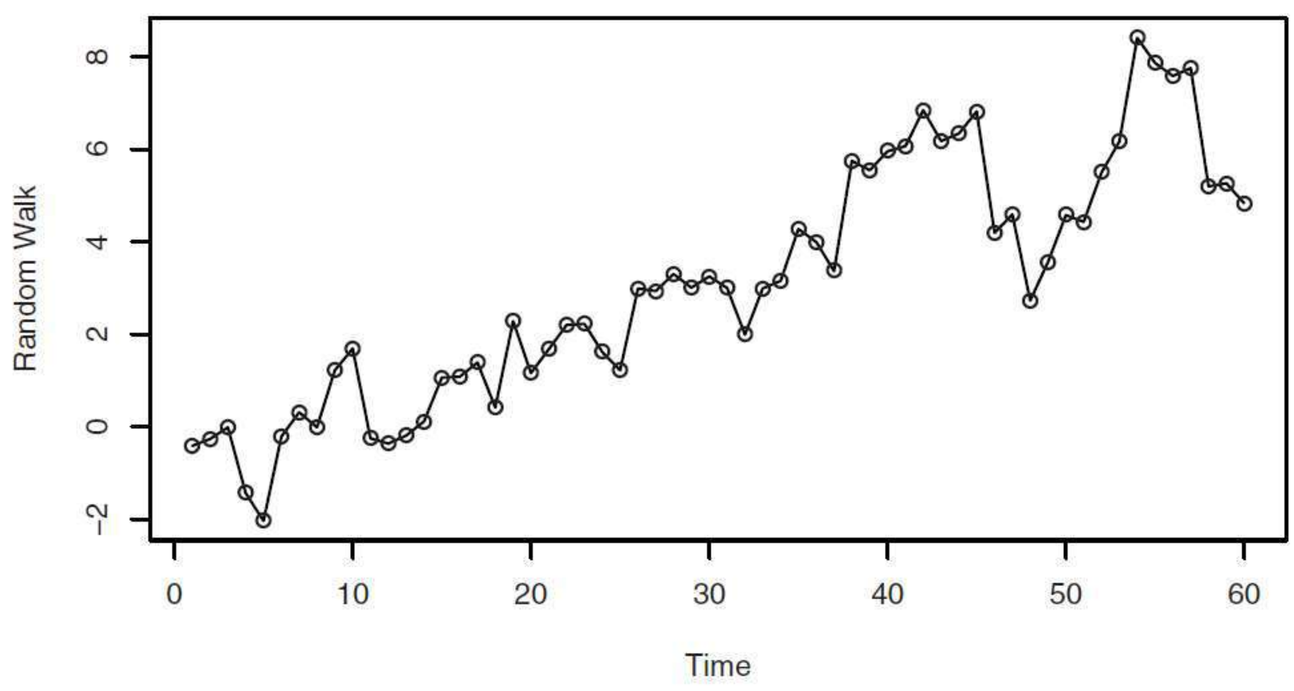
The autocorrelation function for the random walk is now easily obtained as

$$\rho_{t,s} = \frac{\gamma_{t,s}}{\sqrt{\gamma_{t,t}\gamma_{s,s}}} = \sqrt{\frac{t}{s}} \qquad \text{for } 1 \le t \le s$$

The Random Walk

The simple random walk process provides a good model (at least to a first approximation) for phenomena as diverse as the movement of common stock price, and the position of small particles suspended in a fluid—so-called Brownian motion.

Time Series Plot of a Random Walk



A Moving Average

Suppose that $\{Y_t\}$ is constructed as

$$Y_t = \frac{e_t + e_{t-1}}{2}$$

the e's are assumed to be independent and identically distributed with zero mean and variance σ_e^2

Mean of A Moving Average

$$Y_t = \frac{e_t + e_{t-1}}{2}$$

$$\begin{aligned}\mu_t &= E(Y_t) = E\left\{\frac{e_t + e_{t-1}}{2}\right\} = \frac{E(e_t) + E(e_{t-1})}{2} \\ &= 0\end{aligned}$$

Variance of A Moving Average

$$Y_t = \frac{e_t + e_{t-1}}{2}$$

$$Var(Y_t) = Var\left\{\frac{e_t + e_{t-1}}{2}\right\} = \frac{Var(e_t) + Var(e_{t-1})}{4}$$

$$= 0.5\sigma_e^2$$

Covariance of A Moving Average

$$Y_t = \frac{e_t + e_{t-1}}{2}$$

$$\begin{aligned}
Cov(Y_t, Y_{t-1}) &= Cov\left\{\frac{e_t + e_{t-1}}{2}, \frac{e_{t-1} + e_{t-2}}{2}\right\} \\
&= \frac{Cov(e_t, e_{t-1}) + Cov(e_t, e_{t-2}) + Cov(e_{t-1}, e_{t-1})}{4} \\
&\qquad + \frac{Cov(e_{t-1}, e_{t-2})}{4} \\
&= \frac{Cov(e_{t-1}, e_{t-1})}{4} \qquad \text{(as all the other covariances are zero)} \\
&= 0.25\sigma_e^2
\end{aligned}$$

$$\gamma_{t,t-1} = 0.25\sigma_e^2 \qquad \text{for all } t$$

Covariance of A Moving Average

$$Y_t = \frac{e_t + e_{t-1}}{2}$$

Furthermore,

$$Cov(Y_t, Y_{t-2}) = Cov\left\{\frac{e_t + e_{t-1}}{2}, \frac{e_{t-2} + e_{t-3}}{2}\right\}$$

$$= 0 \qquad \text{since the } e\text{'s are independent.}$$

Similarly, $Cov(Y_t, Y_{t-k}) = 0$ for $k > 1$, so we may write

$$\gamma_{t,s} = \begin{cases} 0.5\sigma_e^2 & \text{for } |t-s| = 0 \\ 0.25\sigma_e^2 & \text{for } |t-s| = 1 \\ 0 & \text{for } |t-s| > 1 \end{cases}$$

Autocorrelation function for A Moving Average

$$\rho_{t,s} = \begin{cases} 1 & \text{for } |t-s| = 0 \\ 0.5 & \text{for } |t-s| = 1 \\ 0 & \text{for } |t-s| > 1 \end{cases}$$

Stationarity

- To make statistical inferences about the structure of a stochastic process on the basis of an observed record of that process, we must usually make some simplifying (and presumably reasonable) assumptions about that structure.
- The most important such assumption is that of **stationarity**.
- A process $\{Y_t\}$ is said to be **strictly stationary** if the joint distribution of $Y_{t_1}, Y_{t_2}, \dots, Y_{t_n}$ is the same as the joint distribution of $Y_{t_1-k}, Y_{t_2-k}, \dots, Y_{t_n-k}$ for all choices of time points $t_1, t_2, \dots, t_n$ and all choices of time lag k.
- When $n = 1$, the (univariate) distribution of Y_t is the same as that of Y_{t-k} for all t and k
- It then follows that $E(Y_t) = E(Y_{t-k})$ for all t and k so that the mean function is constant for all time
- $Var(Y_t) = Var(Y_{t-k})$ or all t and k so that the variance is also constant over time.

Stationarity

Setting $n = 2$ in the stationarity definition we see that the bivariate distribution of Y_t and Y_s must be the same as that of Y_{t-k} and Y_{s-k} from which it follows that $Cov(Y_t, Y_s) = Cov(Y_{t-k}, Y_{s-k})$ for all t, s, and k. Putting $k = s$ and then $k = t$, we obtain

$$\begin{aligned}\gamma_{t,s} &= Cov(Y_{t-s}, Y_0)\\ &= Cov(Y_0, Y_{s-t})\\ &= Cov(Y_0, Y_{|t-s|})\\ &= \gamma_{0,|t-s|}\end{aligned}$$

Stationarity

That is, the covariance between Y_t and Y_s depends on time only through the time difference $|t-s|$ and not otherwise on the actual times t and s. Thus, for a stationary process, we can simplify our notation and write

$$\gamma_k = Cov(Y_t, Y_{t-k}) \qquad \text{and} \qquad \rho_k = Corr(Y_t, Y_{t-k})$$

Note also that

$$\rho_k = \frac{\gamma_k}{\gamma_0}$$

$$\left.\begin{array}{ll} \gamma_0 = Var(Y_t) & \rho_0 = 1 \\ \gamma_k = \gamma_{-k} & \rho_k = \rho_{-k} \\ |\gamma_k| \le \gamma_0 & |\rho_k| \le 1 \end{array}\right\}$$

Important example of as stationarity process – White Noise

- **White noise** process is defined as a sequence of independent, identically distributed random variables $\{e_t\}$
- Many useful processes can be constructed from **white noise**
- The fact that $\{e_t\}$ is strictly stationary

$\mu_t = E(e_t)$ is constant and

$$\gamma_k = \begin{cases} Var(e_t) & \text{for } k = 0 \\ 0 & \text{for } k \neq 0 \end{cases} \qquad \rho_k = \begin{cases} 1 & \text{for } k = 0 \\ 0 & \text{for } k \neq 0 \end{cases}$$

$$\boxed{Y_t = \frac{e_t + e_{t-1}}{2}}$$

is another example of a stationary process constructed from white noise.

Many Thanks

Stationary Time Series

YENNI ANGRAINI

General Linear Processes

A *general linear process*, $\{Y_t\}$, is one that can be represented as a weighted linear combination of present and past white noise terms as

$$Y_t = e_t + \psi_1 e_{t-1} + \psi_2 e_{t-2} + \cdots$$

$$\sum_{i=1}^{\infty} \psi_i^2 < \infty$$

We should also note that since $\{e_t\}$ is unobservable, there is no loss in the generality of $\sum_{i=1}^{\infty} \psi_1^2 < \infty$ if we assume that the coefficient on e_t is 1; effectively, $\psi_0 = 1$.

General Linear Processes

An important nontrivial example to which we will return often is the case where the ψ's form an exponentially decaying sequence

$$\psi_j = \phi^j$$

where ϕ is a number strictly between -1 and $+1$. Then

$$Y_t = e_t + \phi e_{t-1} + \phi^2 e_{t-2} + \cdots$$

For this example,

$$E(Y_t) = E(e_t + \phi e_{t-1} + \phi^2 e_{t-2} + \cdots) = 0$$

- $\{e_t\}$ represent an unobserved white noise series, that is, a sequence of identically distributed, zero-mean, independent random variables.
- The assumption of independence could be replaced by the weaker assumption that the $\{e_t\}$ are uncorrelated random variables

General Linear Processes -example

$$\begin{aligned} Var(Y_t) &= Var(e_t + \phi e_{t-1} + \phi^2 e_{t-2} + \cdots) \\ &= Var(e_t) + \phi^2 Var(e_{t-1}) + \phi^4 Var(e_{t-2}) + \cdots \\ &= \sigma_e^2(1 + \phi^2 + \phi^4 + \cdots) \\ &= \frac{\sigma_e^2}{1-\phi^2} \text{ (by summing a geometric series)} \end{aligned}$$

General Linear Processes -example

$$\begin{aligned}
Cov(Y_t, Y_{t-1}) &= Cov(e_t + \phi e_{t-1} + \phi^2 e_{t-2} + \cdots, e_{t-1} + \phi e_{t-2} + \phi^2 e_{t-3} + \cdots) \\
&= Cov(\phi e_{t-1}, e_{t-1}) + Cov(\phi^2 e_{t-2}, \phi e_{t-2}) + \cdots \\
&= \phi\sigma_e^2 + \phi^3\sigma_e^2 + \phi^5\sigma_e^2 + \cdots \\
&= \phi\sigma_e^2(1 + \phi^2 + \phi^4 + \cdots) \\
&= \frac{\phi\sigma_e^2}{1-\phi^2} \quad \text{(again summing a geometric series)}
\end{aligned}$$

$$Cov(Y_t, Y_{t-k}) = \frac{\phi^k\sigma_e^2}{1-\phi^2}$$

$$Corr(Y_t, Y_{t-1}) = \left[\frac{\phi\sigma_e^2}{1-\phi^2}\right] \Big/ \left[\frac{\sigma_e^2}{1-\phi^2}\right] = \phi$$

$$Corr(Y_t, Y_{t-k}) = \phi^k$$

Moving Average Processes

$$Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}$$

A **moving average of order *q*** and abbreviate the name to MA(q)

The First-Order Moving Average Processes

MA(1)

$$Y_t = e_t - \theta_1 e_{t-1}$$

$e_t \sim iid(0, \sigma_e^2)$

$$E(Y_t) = E(e_t - \theta_1 e_{t-1}) = 0$$

$$Var(Y_t) = Var(e_t - \theta_1 e_{t-1}) = \sigma_e^2(1 + \theta^2)$$

$$\begin{aligned} Cov(Y_t, Y_{t-1}) &= Cov(e_t - \theta e_{t-1}, e_{t-1} - \theta e_{t-2}) \\ &= Cov(-\theta e_{t-1}, e_{t-1}) = -\theta\sigma_e^2 \end{aligned}$$

$$\begin{aligned} Cov(Y_t, Y_{t-2}) &= Cov(e_t - \theta e_{t-1}, e_{t-2} - \theta e_{t-3}) \\ &= 0 \end{aligned}$$

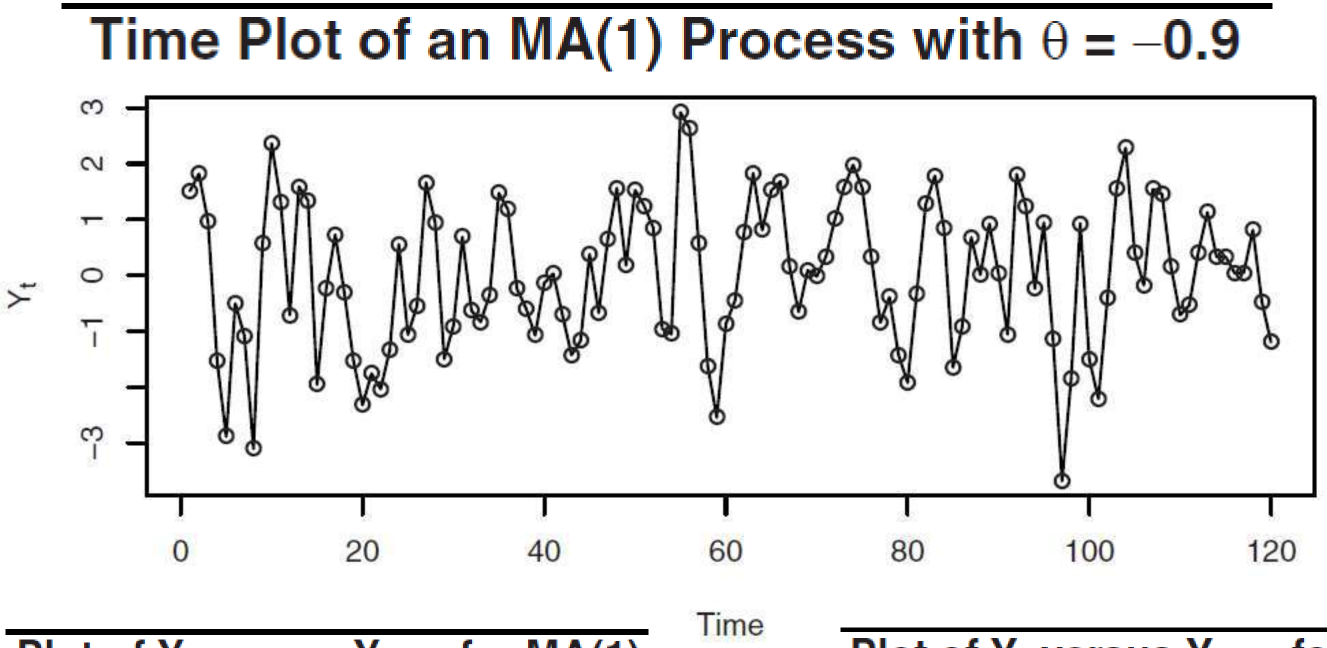
$$Cov(Y_t, Y_{t-k}) = 0 \text{ whenever } k \geq 2$$

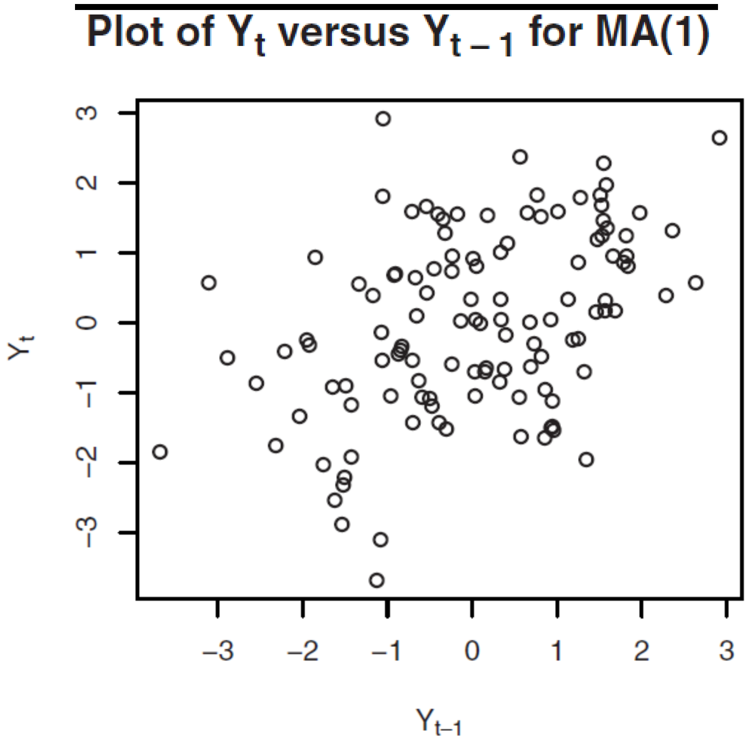
The First-Order Moving Average Processes

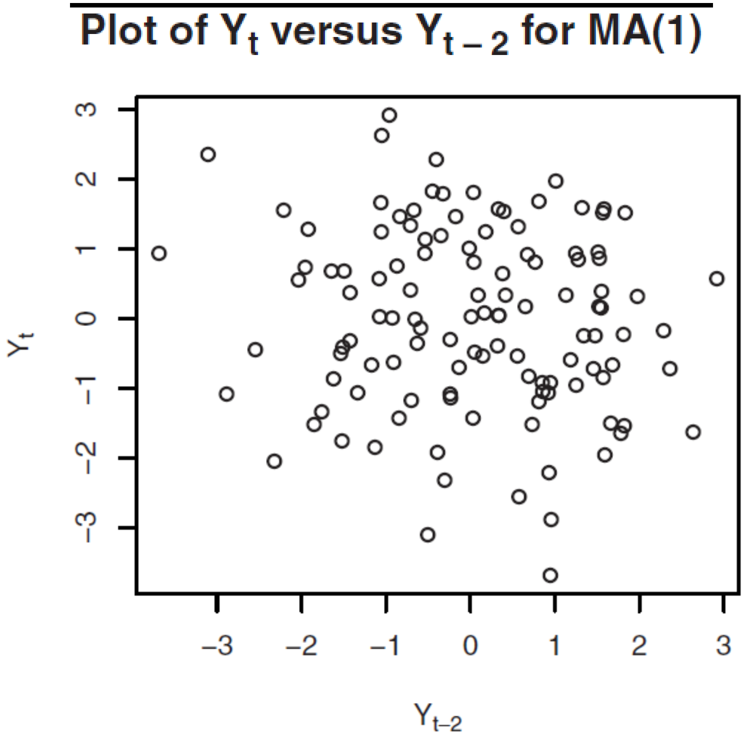
In summary, for an MA(1) model $Y_t = e_t - \theta e_{t-1}$,

$$\left.\begin{aligned} E(Y_t) &= 0 \\ \gamma_0 &= Var(Y_t) = \sigma_e^2(1+\theta^2) \\ \gamma_1 &= -\theta\sigma_e^2 \\ \rho_1 &= (-\theta)/(1+\theta^2) \\ \gamma_k &= \rho_k = 0 \quad \text{for } k \geq 2 \end{aligned}\right\}$$

MA(1)







$$Y_t = e_t + 0.9e_{t-1}$$

$$\gamma_1 = -\theta\sigma_e^2$$

$$\rho_1 = (-\theta)/(1+\theta^2)$$

$$\gamma_k = \rho_k = 0 \quad \text{for } k \geq 2$$

$$\gamma_1 = 0.9\sigma_e^2$$

$$\rho_1 = \frac{0.9}{(1+(-0.9)^2)}$$

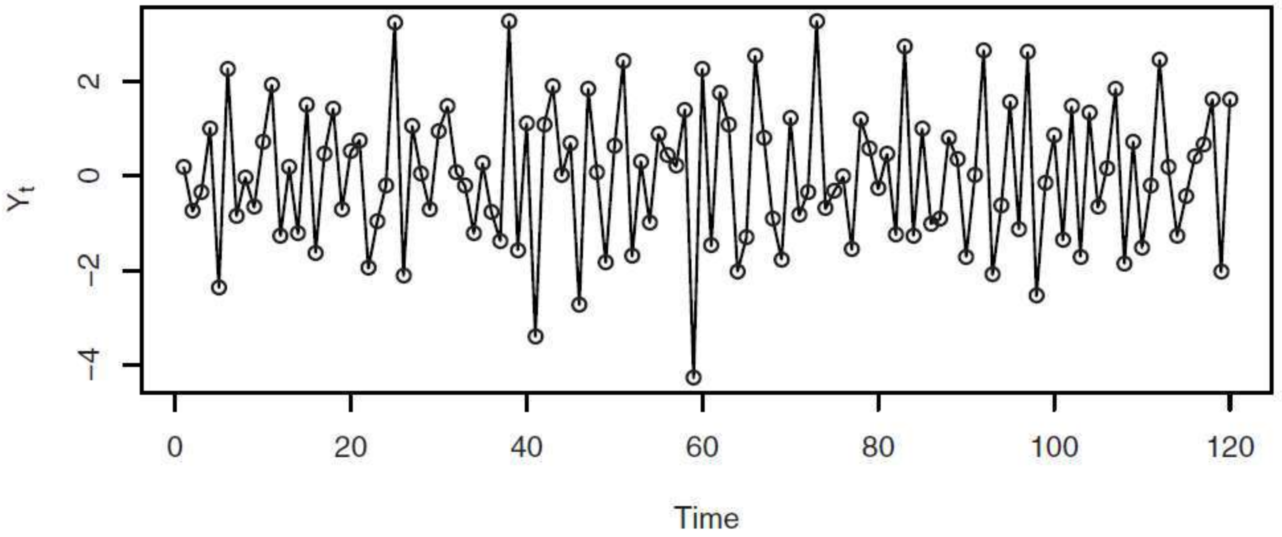
$$\rho_2 = 0$$

MA(1)

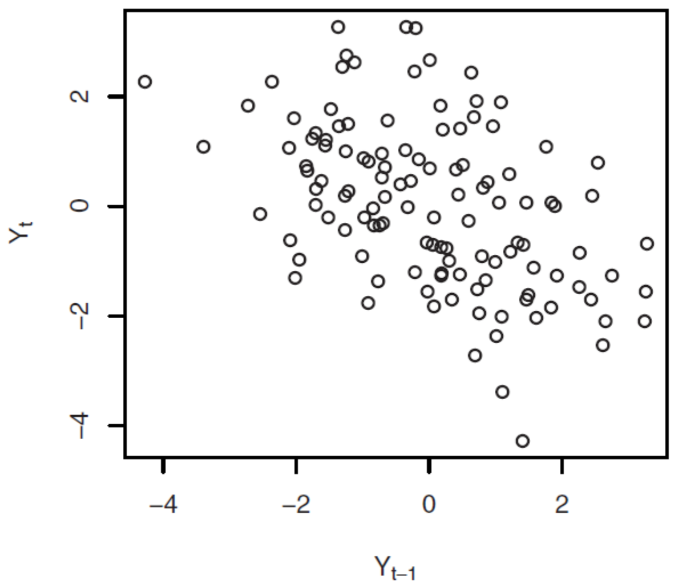
$$Y_t = e_t - 0.9e_{t-1}$$

$$\gamma_1 = -\theta\sigma_e^2$$
$$\rho_1 = (-\theta)/(1+\theta^2)$$
$$\gamma_k = \rho_k = 0 \quad \text{for } k \geq 2$$

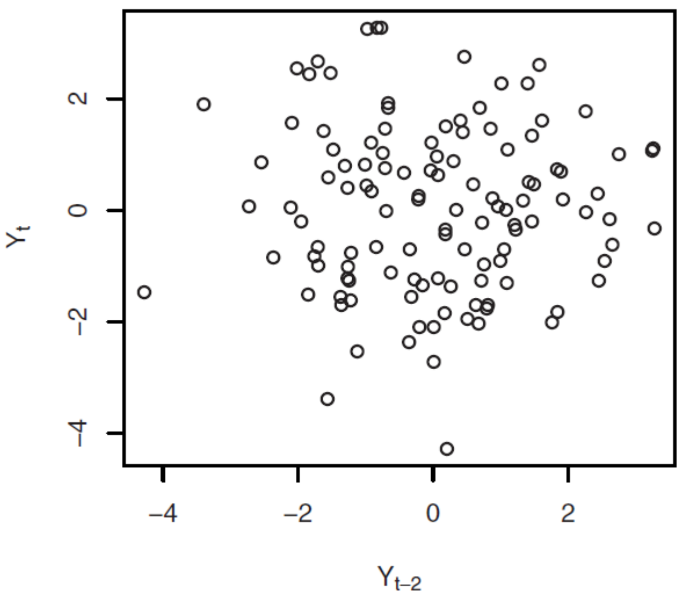
Time Plot of an MA(1) Process with θ = +0.9



Plot of Y_t versus Y_{t-1} for MA(1)



Plot of Y_t versus Y_{t-2} for MA(1)



$$\gamma_1 = -0.9\sigma_e^2$$
$$\rho_1 = \frac{-0.9}{(1+0.9^2)}$$
$$\rho_2 = 0$$

The Second-Order Moving Average Process

$$Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}$$

$$\gamma_0 = Var(Y_t) = Var(e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}) = (1 + \theta_1^2 + \theta_2^2)\sigma_e^2$$

$$\begin{aligned}
\gamma_1 &= Cov(Y_t, Y_{t-1}) = Cov(e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}, e_{t-1} - \theta_1 e_{t-2} - \theta_2 e_{t-3}) \\
&= Cov(-\theta_1 e_{t-1}, e_{t-1}) + Cov(-\theta_1 e_{t-2}, -\theta_2 e_{t-2}) \\
&= [-\theta_1 + (-\theta_1)(-\theta_2)]\sigma_e^2 \\
&= (-\theta_1 + \theta_1\theta_2)\sigma_e^2
\end{aligned}$$

$$\begin{aligned}
\gamma_2 &= Cov(Y_t, Y_{t-2}) = Cov(e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}, e_{t-2} - \theta_1 e_{t-3} - \theta_2 e_{t-4}) \\
&= Cov(-\theta_2 e_{t-2}, e_{t-2}) \\
&= -\theta_2\sigma_e^2
\end{aligned}$$

The Second-Order Moving Average Process

$$Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}$$

$$\rho_1 = \frac{-\theta_1 + \theta_1\theta_2}{1 + \theta_1^2 + \theta_2^2}$$

$$\rho_2 = \frac{-\theta_2}{1 + \theta_1^2 + \theta_2^2}$$

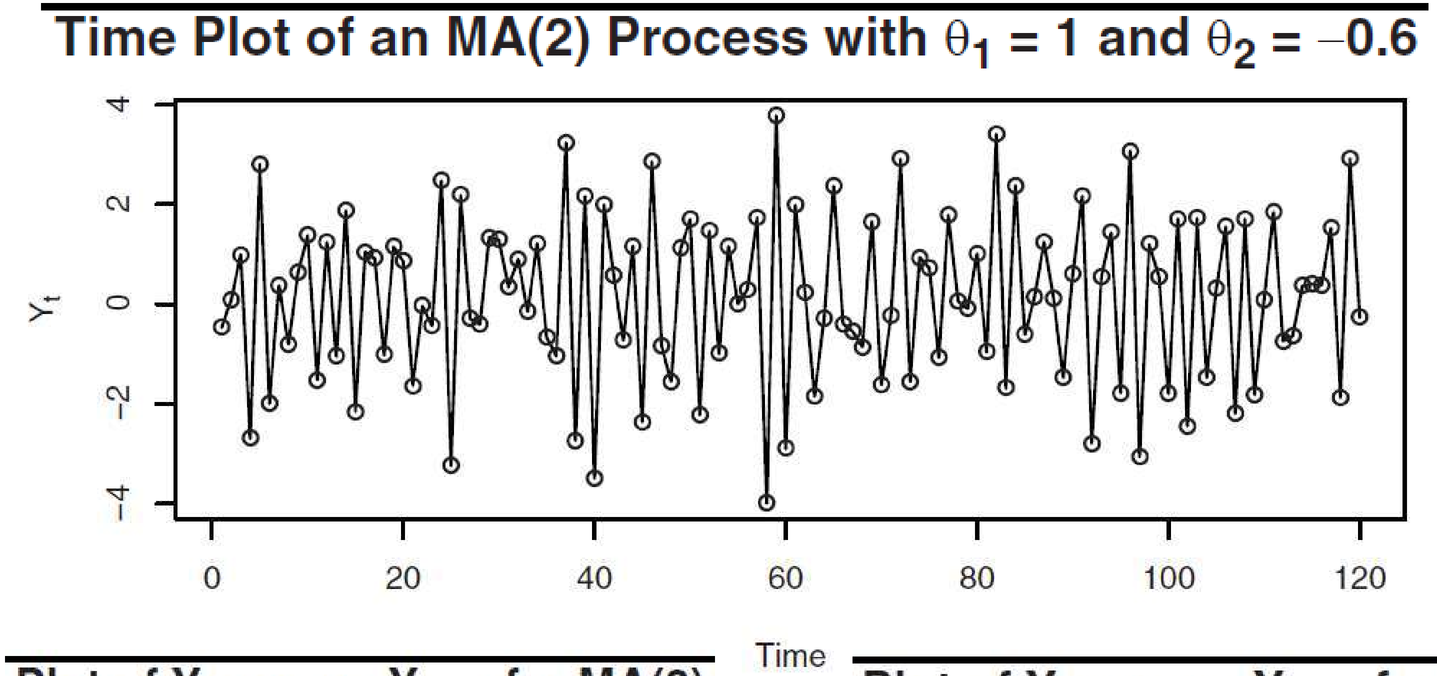
$$\rho_k = 0 \quad \text{for } k = 3, 4, \ldots$$

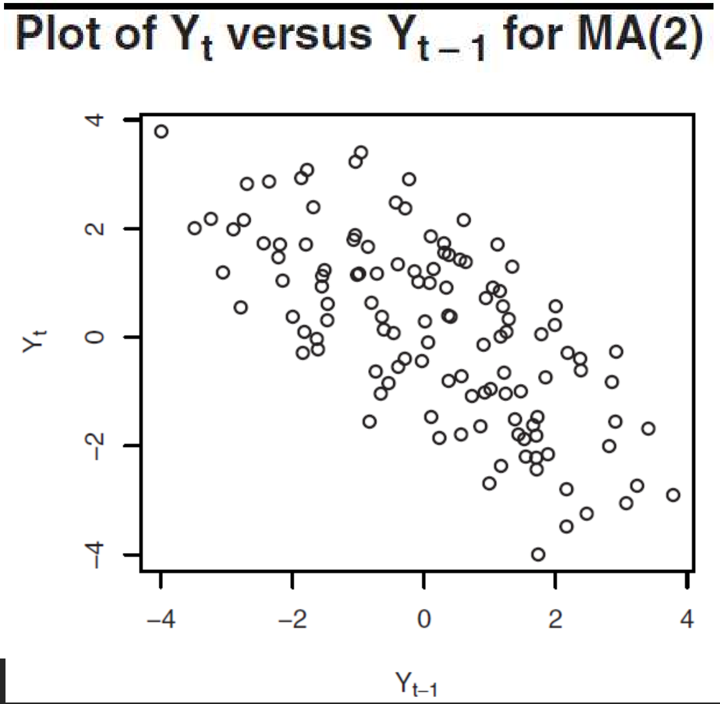
MA(2)

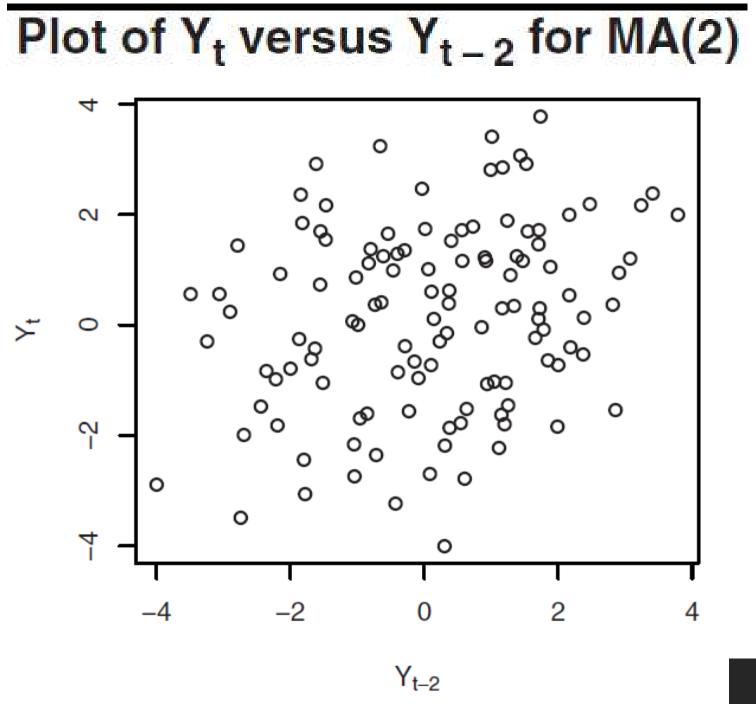
$$Y_t = e_t - e_{t-1} + 0.6e_{t-2}$$

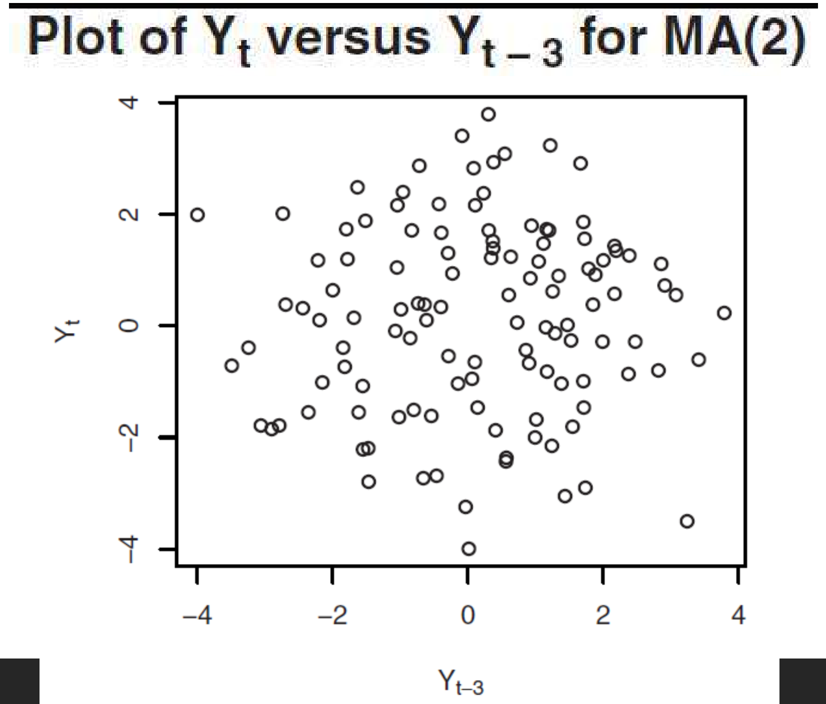
$$\rho_1 = \frac{-1 + (1)(-0.6)}{1 + (1)^2 + (-0.6)^2} = \frac{-1.6}{2.36} = -0.678$$

$$\rho_2 = \frac{0.6}{2.36} = 0.254$$









The General MA(q) Process

For the general MA(q) process $Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}$, similar calculations show that

$$\gamma_0 = (1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2)\sigma_e^2$$

and

$$\rho_k = \begin{cases} \dfrac{-\theta_k + \theta_1\theta_{k+1} + \theta_2\theta_{k+2} + \cdots + \theta_{q-k}\theta_q}{1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2} & \text{for } k = 1, 2, \ldots, q \\ 0 & \text{for } k > q \end{cases}$$

Thanks

Stationary Time Series

YENNI ANGRAINI

Autoregressive Processes

Autoregressive processes are as their name suggests—regressions on themselves. Specifically, a *p*th-order **autoregressive process** $\{Y_t\}$ satisfies the equation

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + e_t$$

The current value of the series Y_t is a linear combination of the p most recent past values of itself plus an "innovation" term e_t that incorporates everything new in the series at time t that is not explained by the past values. Thus, for every t, we assume that e_t is independent of $Y_{t-1}, Y_{t-2}, Y_{t-3}, \ldots$. Yule (1926) carried out the original work on autoregressive processes

The First-Order Autoregressive Processes

AR(1) $Y_t = \phi Y_{t-1} + e_t$

$e_t \sim iid(0, \sigma_e^2)$

we are assuming that Y_t has zero mean.

$E(Y_t) = 0$

we also assume that e_t is independent of $Y_{t-1}, Y_{t-2}, Y_{t-3}, \dots$.

$E(e_t, Y_{t-1}) = 0, E(e_t, Y_{t-2}) = 0$, ,,,dst

$Var(Y_t) = Var(\phi Y_{t-1} + e_t)$

$$\gamma_0 = \phi^2 \gamma_0 + \sigma_e^2$$

Solving for γ_0 yields

$$\gamma_0 = \frac{\sigma_e^2}{1 - \phi^2}$$

$\phi^2 < 1$ or that $|\phi| < 1$.

The First-Order Autoregressive Processes

AR(1) $Y_t = \phi Y_{t-1} + e_t$

$$e_t \sim iid(0, \sigma_e^2)$$

we are assuming that Y_t has zero mean.

$$E(Y_t) = 0$$

we also assume that e_t is independent of $Y_{t-1}, Y_{t-2}, Y_{t-3}, \ldots$.

$$E(e_t, Y_{t-1}) = 0, E(e_t, Y_{t-2}) = 0, \text{,,,dst}$$

$$E(e_t Y_{t-k}) = E(e_t)E(Y_{t-k}) = 0$$

$$E(Y_{t-k}Y_t) = \phi E(Y_{t-k}Y_{t-1}) + E(e_t Y_{t-k})$$

or

$$\gamma_k = \phi\gamma_{k-1} + E(e_t Y_{t-k})$$

$$\gamma_k = \phi\gamma_{k-1} \qquad \text{for } k = 1, 2, 3, \ldots$$

Setting $k = 1$, we get $\gamma_1 = \phi\gamma_0 = \phi\sigma_e^2/(1-\phi^2)$. With $k = 2$, we obtain $\gamma_2 = \phi^2\sigma_e^2/(1-\phi^2)$. Now it is easy to see that in general

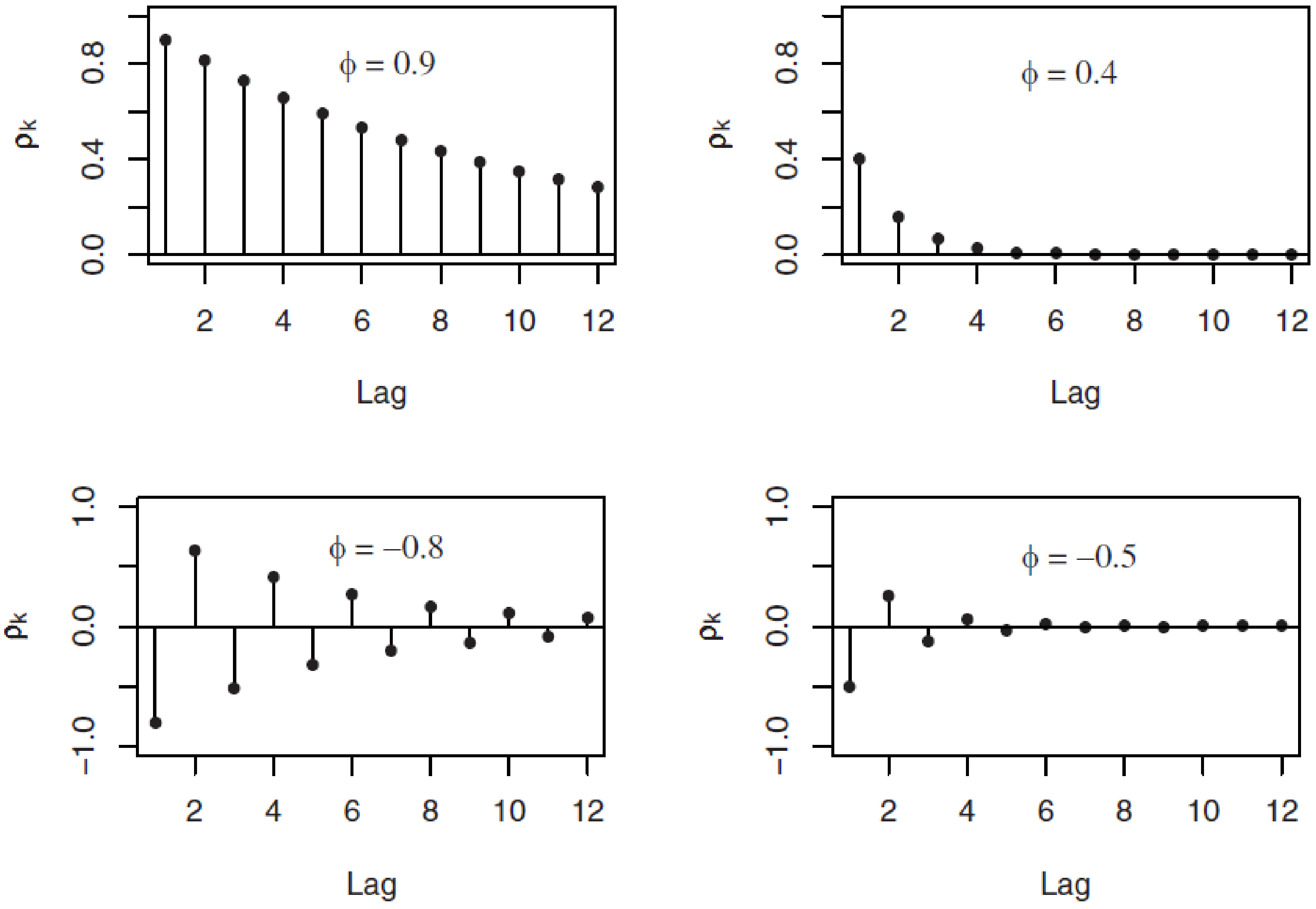
$$\gamma_k = \phi^k \frac{\sigma_e^2}{1-\phi^2}$$

and thus

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \phi^k \qquad \text{for } k = 1, 2, 3, \ldots$$

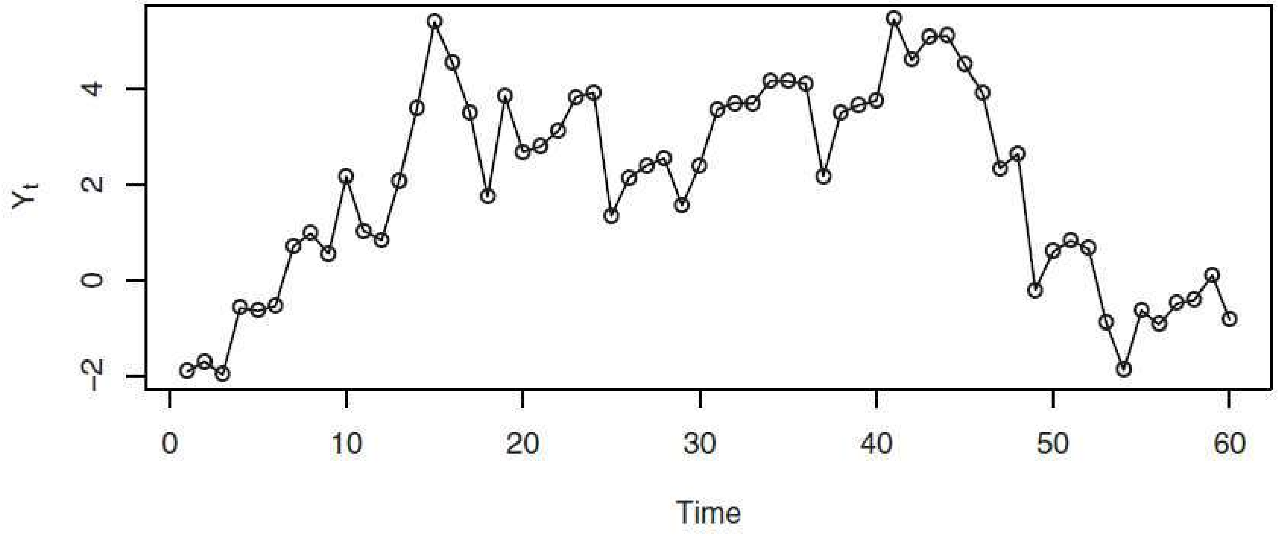
The First-Order Autoregressive Processes

- Since $|\phi| < 1$, the magnitude of the autocorrelation function decreases exponentially as the number of lags, k, increases.
- If $0 < \phi < 1$, all correlations are positive;
- If $-1 < \phi < 0$, the lag 1 autocorrelation is negative ($\rho_1 = \phi$) and the signs of successive autocorrelations alternate from positive to negative, with their magnitudes decreasing exponentially.



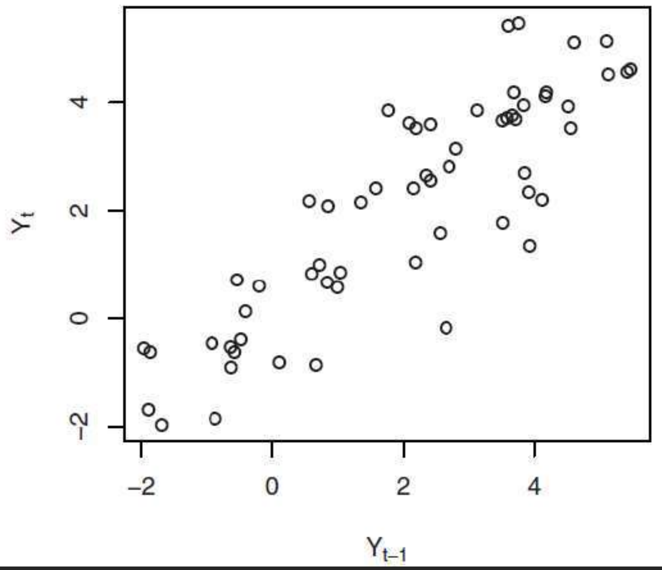
AR(1)

Time Plot of an AR(1) Series with $\phi = 0.9$

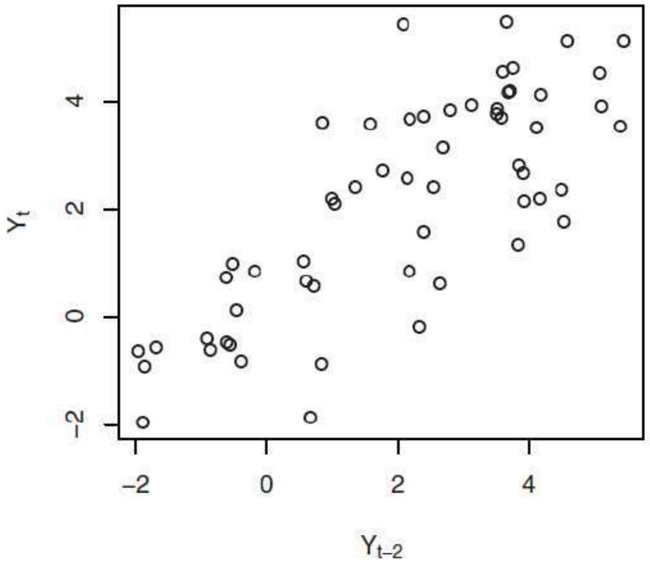


$$Y_t = \phi Y_{t-1} + e_t$$

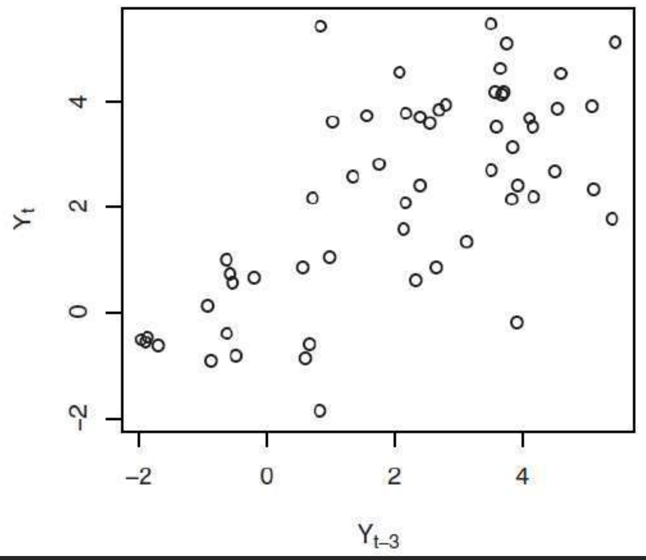
Plot of Y_t versus Y_{t-1} for AR(1)



Plot of Y_t versus Y_{t-2} for AR(1)



Plot of Y_t versus Y_{t-3} for AR(1)



The General Linear Process Version of the AR(1) Model

The General Linear Process Version of the AR(1) Model can be written as follows :

$$Y_t = e_t + \phi e_{t-1} + \phi^2 e_{t-2} + \cdots + \phi^{k-1} e_{t-k+1} + \phi^k Y_{t-k}$$

Assuming $|\phi| < 1$ and letting k increase without bound, it seems reasonable (this is almost a rigorous proof) that we should obtain the infinite series representation

$$Y_t = e_t + \phi e_{t-1} + \phi^2 e_{t-2} + \phi^3 e_{t-3} + \cdots$$

The Second-Order Autoregressive Processes

AR(2) $$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t$$

$$e_t \sim iid(0, \sigma_e^2)$$

we also assume that e_t is independent of $Y_{t-1}, Y_{t-2}, Y_{t-3}, \ldots$

$E(e_t, Y_{t-1}) = 0, E(e_t, Y_{t-2}) = 0$, ,,,dst

How to get the variance for AR(2) model ?

$$\gamma_k = \phi_1 \gamma_{k-1} + \phi_2 \gamma_{k-2} \qquad \text{for } k = 1, 2, 3, \ldots$$

or, dividing through by γ_0,

$$\rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} \qquad \text{for } k = 1, 2, 3, \ldots$$

These Equations are usually called
the **Yule-Walker equations**

Setting $k = 1$ and using $\rho_0 = 1$ and $\rho_{-1} = \rho_1$, we get and so

$$\rho_1 = \frac{\phi_1}{1 - \phi_2}$$

Using the now known values for ρ_1 (and ρ_0) and ρ_k, for k = 2 we can obtain

$$\rho_2 = \phi_1 \rho_1 + \phi_2 \rho_0$$

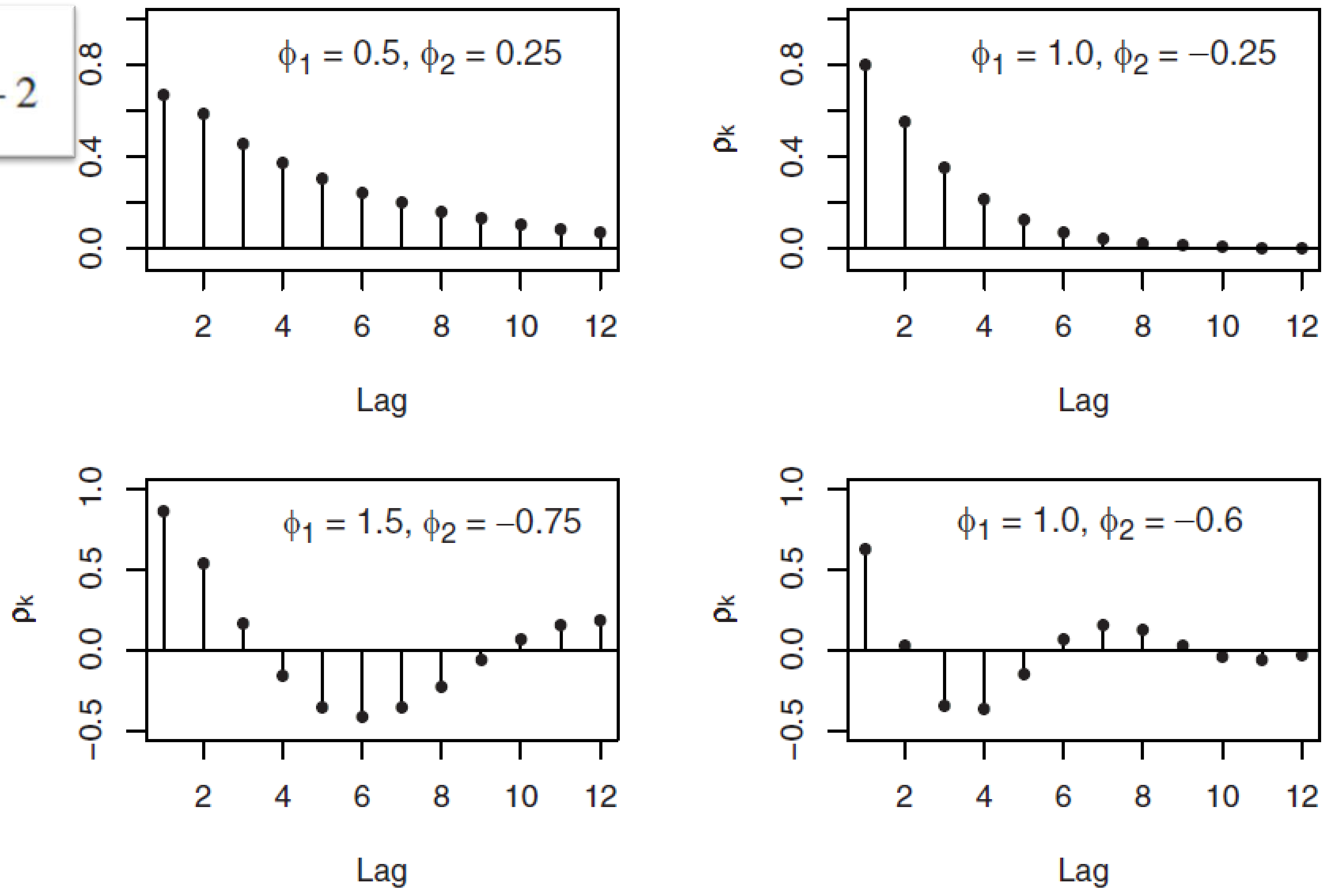
$$= \frac{\phi_2(1 - \phi_2) + \phi_1^2}{1 - \phi_2}$$

Autocorrelation Functions for Several AR(2) Models

$$\rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2}$$

$$\rho_1 = \frac{\phi_1}{1 - \phi_2}$$

$$\rho_0 = 1$$



The Mixed Autoregressive Moving Average Model

If we assume that the series is partly autoregressive and partly moving average, we obtain a quite general time series model. In general, if

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}$$

we say that $\{Y_t\}$ is a mixed autoregressive moving average process of orders p and q, respectively; we abbreviate the name to ARMA(p,q)

The ARMA(1,1) Model

The defining equation can be written

$$Y_t = \phi Y_{t-1} + e_t - \theta e_{t-1}$$

To derive Yule-Walker type equations, we first note that

$$\begin{aligned} E(e_t Y_t) &= E[e_t(\phi Y_{t-1} + e_t - \theta e_{t-1})] \\ &= \sigma_e^2 \end{aligned}$$

$$\begin{aligned} E(e_{t-1} Y_t) &= E[e_{t-1}(\phi Y_{t-1} + e_t - \theta e_{t-1})] \\ &= \phi\sigma_e^2 - \theta\sigma_e^2 \\ &= (\phi - \theta)\sigma_e^2 \end{aligned}$$

$$\left.\begin{aligned} \gamma_0 &= \phi\gamma_1 + [1 - \theta(\phi - \theta)]\sigma_e^2 \\ \gamma_1 &= \phi\gamma_0 - \theta\sigma_e^2 \\ \gamma_k &= \phi\gamma_{k-1} \quad \text{for } k \geq 2 \end{aligned}\right\}$$

Solving the first two equations yields

$$\gamma_0 = \frac{(1 - 2\phi\theta + \theta^2)}{1 - \phi^2}\sigma_e^2$$

and solving the simple recursion gives

$$\rho_k = \frac{(1 - \theta\phi)(\phi - \theta)}{1 - 2\theta\phi + \theta^2}\phi^{k-1} \quad \text{for } k \geq 1$$

Stationarity of AR(1)

- It can be shown that, subject to the restriction that e_t be independent of $Y_{t-1}, Y_{t-2}, Y_{t-3}, \ldots$ and that $\sigma_e^2 > 0$,
- The solution of the AR(1) defining recursion $Y_t = \phi Y_{t-1} + e_t$ will be stationary if and only $|\phi| < 1$.
- The requirement $|\phi| < 1$ is usually called the stationary condition for the AR(1) process.

Stationarity of AR(2)

The AR(2) process is called the stationary if and only if three conditions are satisfied:

$$\phi_1 + \phi_2 < 1, \qquad \phi_2 - \phi_1 < 1, \qquad \text{and} \qquad |\phi_2| < 1$$

As with the AR(1) model, we call these the stationarity conditions for the AR(2) model.

Invertibility

An MA(1) model: $Y_t = e_t - \theta e_{t-1}$

First rewriting this as $e_t = Y_t + \theta e_{t-1}$ and then replacing t by $t-1$ and substituting for e_{t-1} above, we get

$$\begin{aligned} e_t &= Y_t + \theta(Y_{t-1} + \theta e_{t-2}) \\ &= Y_t + \theta Y_{t-1} + \theta^2 e_{t-2} \end{aligned}$$

If $|\theta| < 1$, we may continue this substitution "infinitely" into the past and obtain the expression

$$e_t = Y_t + \theta Y_{t-1} + \theta^2 Y_{t-2} + \cdots$$

or

$$Y_t = (-\theta Y_{t-1} - \theta^2 Y_{t-2} - \theta^3 Y_{t-3} - \cdots) + e_t$$

- If $|\theta| < 1$, we see that the MA(1) model can be inverted into an infinite-order autoregressive model.
- We say that the MA(1) model is invertible if and only if $|\theta| < 1$.

Thanks

Model for Non-Stationary Time Series

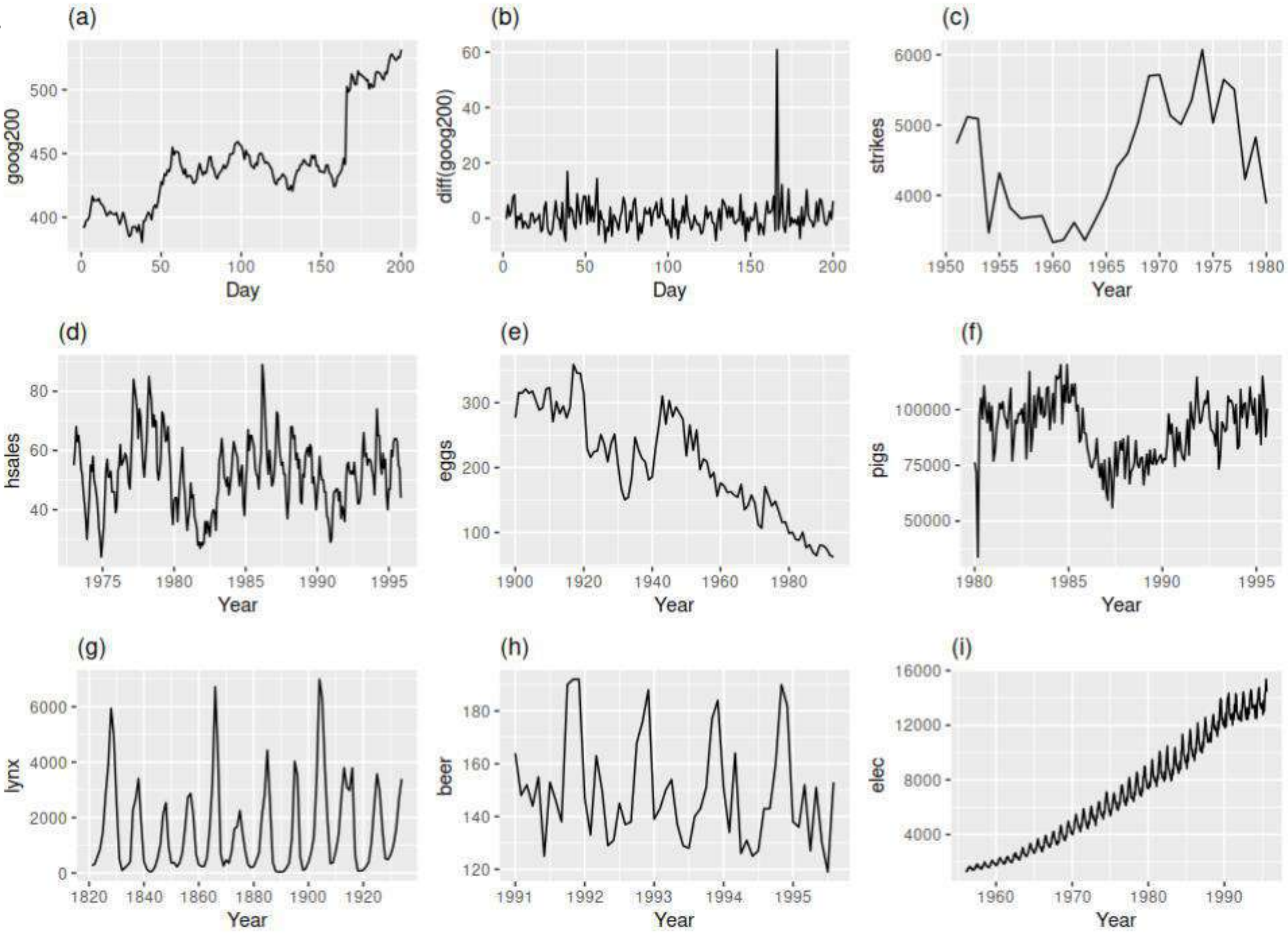
YENNI ANGRAINI

Outline

- Differencing
- ARIMA Model

Which of these series are stationary?

(a) Google stock price for 200 consecutive days;
(b) Daily change in the Google stock price for 200 consecutive days;
(c) Annual number of strikes in the US;
(d) Monthly sales of new one-family houses sold in the US;
(e) Annual price of a dozen eggs in the US (constant dollars);
(f) Monthly total of pigs slaughtered in Victoria, Australia;
(g) Annual total of lynx trapped in the McKenzie River district of north-west Canada;
(h) Monthly Australian beer production;
(i) Monthly Australian electricity production



We normally restrict autoregressive models to stationary data, in which case some constraints on the values of the parameters are required.

- For an AR(1) model: $-1 < \phi_1 < 1$.
- For an AR(2) model: $-1 < \phi_2 < 1, \phi_1 + \phi_2 < 1, \phi_2 - \phi_1 < 1$.

The invertibility constraints for other models are similar to the stationarity constraints.

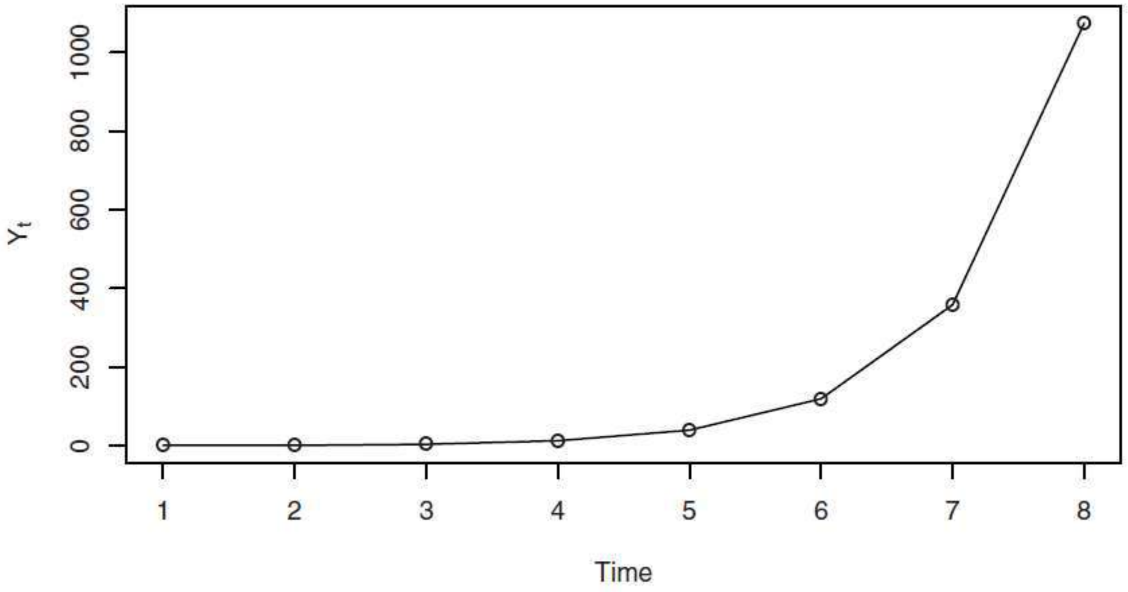
- For an MA(1) model: $-1 < \theta_1 < 1$.
- For an MA(2) model: $-1 < \theta_2 < 1, \ \ \theta_2 + \theta_1 > -1, \ \ \theta_1 - \theta_2 < 1$.

$$Y_t = 3Y_{t-1} + e_t \qquad e_t \sim iid(0, \sigma_e^2)$$

Iterating into the past as we have done before yields

$$Y_t = e_t + 3e_{t-1} + 3^2 e_{t-2} + \cdots + 3^{t-1}e_1 + 3^t Y_0 \qquad Y_0 = 0$$

An Explosive "AR(1)" Series



Simulation of the Explosive "AR(1) Model" $Y_t = 3Y_{t-1} + e_t$

t	1	2	3	4	5	6	7	8
e_t	0.63	−1.25	1.80	1.51	1.56	0.62	0.64	−0.98
Y_t	0.63	0.64	3.72	12.67	39.57	119.33	358.63	1074.91

$$Var(Y_t) = \frac{1}{8}(9^t - 1)\sigma_e^2$$

and

$$Cov(Y_t, Y_{t-k}) = \frac{3^k}{8}(9^{t-k} - 1)\sigma_e^2$$

respectively. Notice that we have

$$Corr(Y_t, Y_{t-k}) = 3^k\left(\frac{9^{t-k} - 1}{9^t - 1}\right) \approx 1 \qquad \text{for large } t \text{ and moderate } k$$

Differencing

$AR(1): Y_t = \phi Y_{t-1} + e_t \qquad e_t \sim iid(0, \sigma_e^2)$

if $|\phi| \geq 1$, the AR(1) is non stationary model

Suppose $\phi = 1$

$$Y_t = Y_{t-1} + e_t$$
$$Y_t - Y_{t-1} = e_t$$
$$\nabla^1 Y_t = e_t$$

$$E(\nabla^1 Y_t) = E(e_t) = 0$$
$$Var(\nabla^1 Y_t) = Var(e_t) = \sigma_e^2$$

$\Rightarrow$ Stationary model

Differencing

Backshift (B) :

$$B(Y_t) = Y_{t-1}$$
$$B^2(Y_t) = Y_{t-2}$$
$$B^k(Y_t) = Y_{t-k}$$

Backward (∇) :

$$\nabla = 1 - B$$
$$\nabla^2 = (1 - B)^2 = (1 - 2B - B^2)$$

$$\nabla Y_t = (1 - B)Y_t = Y_t \ -Y_{t-1}$$
$$\nabla^2 Y_t = (1 - B)^2 Y_t = (1 - 2B - B^2)Y_t = Y_t - 2Y_{t-1} + Y_{t-2}$$

ARIMA Models

A time series $\{Y_t\}$ is said to follow an **integrated autoregressive moving average** model if the dth difference $W_t = \nabla^d Y_t$ is a stationary ARMA process. If $\{W_t\}$ follows an ARMA(p,q) model, we say that $\{Y_t\}$ is an ARIMA(p,d,q) process. Fortunately, for practical purposes, we can usually take $d = 1$ or at most 2.

Consider then an ARIMA$(p,1,q)$ process. With $W_t = Y_t - Y_{t-1}$, we have

$$W_t = \phi_1 W_{t-1} + \phi_2 W_{t-2} + \cdots + \phi_p W_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}$$

or, in terms of the observed series,

$$\begin{aligned} Y_t - Y_{t-1} &= \phi_1(Y_{t-1} - Y_{t-2}) + \phi_2(Y_{t-2} - Y_{t-3}) + \cdots + \phi_p(Y_{t-p} - Y_{t-p-1}) \\ &\quad + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q} \end{aligned}$$

ARIMA Models

$$Y_t - Y_{t-1} = \phi_1(Y_{t-1} - Y_{t-2}) + \phi_2(Y_{t-2} - Y_{t-3}) + \cdots + \phi_p(Y_{t-p} - Y_{t-p-1})$$
$$+ e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}$$

which we may rewrite as

$$Y_t = (1 + \phi_1)Y_{t-1} + (\phi_2 - \phi_1)Y_{t-2} + (\phi_3 - \phi_2)Y_{t-3} + \cdots$$
$$+ (\phi_p - \phi_{p-1})Y_{t-p} - \phi_p Y_{t-p-1} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}$$

We call this the **difference equation form** of the model.
Notice that it appears to be an ARMA(p + 1,q) process

The IMA(1,1) Model

$d = 1, q = 1$

$Y_t = Y_{t-1} + e_t - \theta e_{t-1}$ $\longrightarrow$ Y_t is Non-stationary process

$Y_t - Y_{t-1} = e_t - \theta e_{t-1}$

$W_t = e_t - \theta e_{t-1}$ $\longrightarrow$ W_t is stationary process

The IMA(2,2) Model

$d = 2, q = 2$

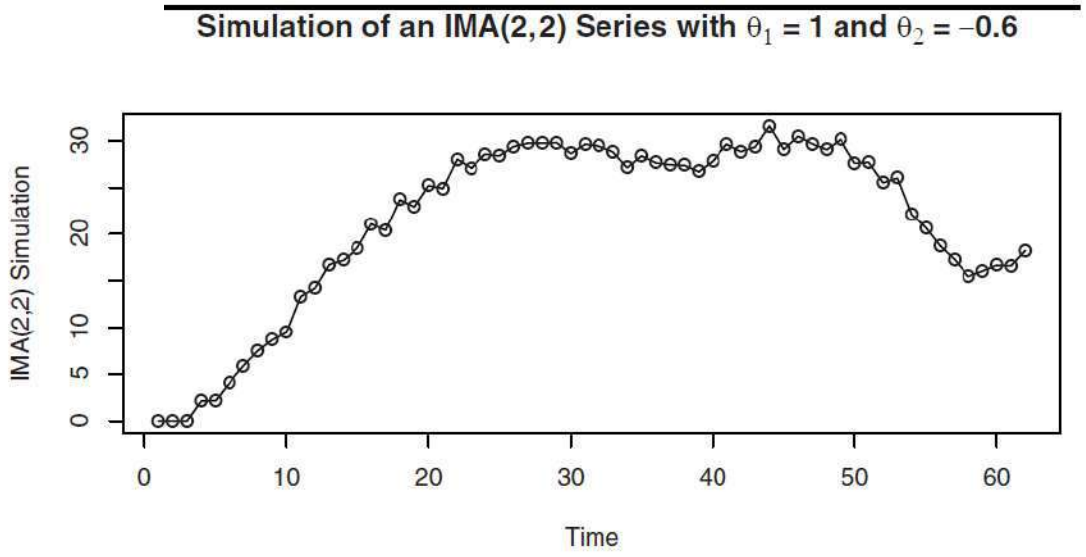
$Y_t = 2Y_{t-1} - Y_{t-2} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}$ $\Rightarrow$ Y_t is Non-stationary process

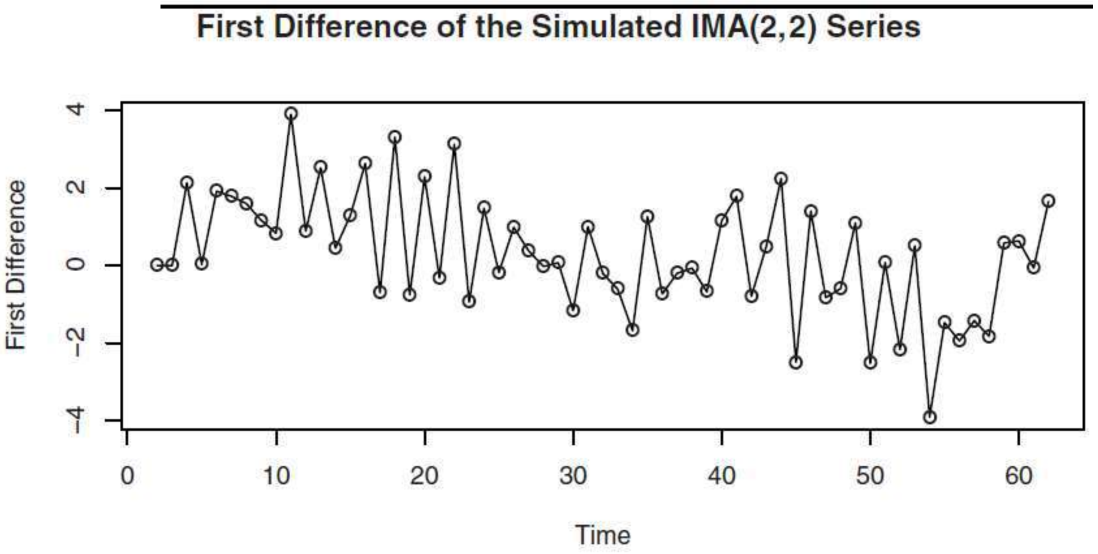
$Y_t - 2Y_{t-1} + Y_{t-2} = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}$

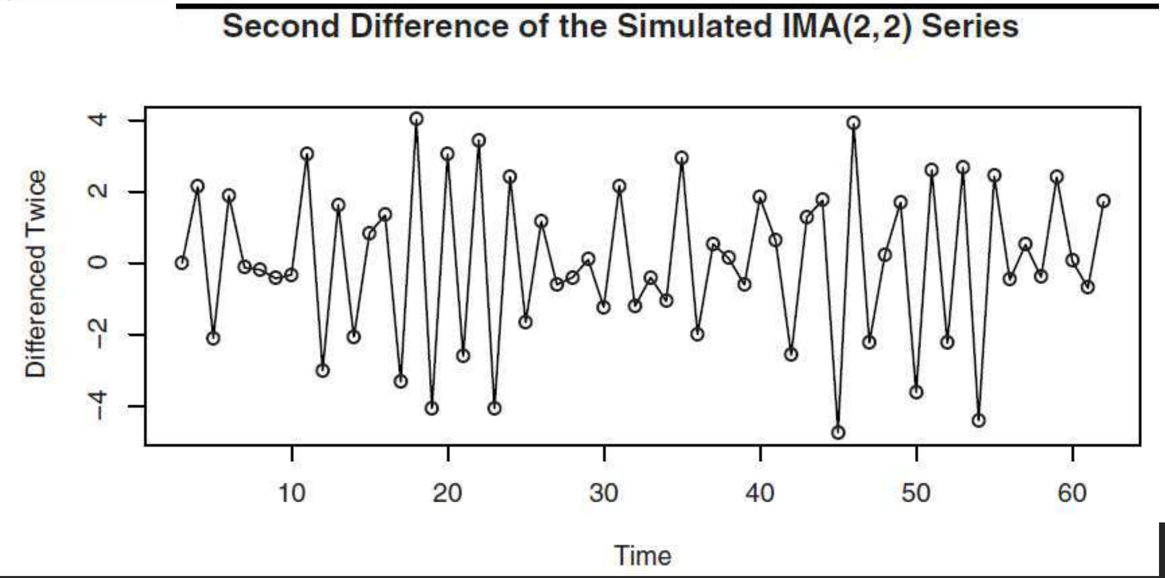
$\nabla^2 Y_t = e_t - \theta e_{t-1} - \theta_2 e_{t-2}$

$W_t = e_t - \theta e_{t-1} - \theta_2 e_{t-2}$ $\Rightarrow$ W_t is stationary process

The IMA(2,2) Model







The ARI(1,1) Model

$p = 1, d = 1$

$\nabla Y_t = \phi \nabla Y_{t-1} + e_t$ → ∇Y_t is stationary process

$Y_t - Y_{t-1} = \phi(Y_{t-1} - Y_{t-2}) + e_t$

$Y_t = (1 + \phi)Y_{t-1} - \phi Y_{t-2} + e_t$ → Y_t is Non-stationary process

ARIMA Models

ARIMA(0, 0, 1) : $Y_t = e_t - \theta e_{t-1}$

ARIMA (1,1,0) : $\nabla Y_t = \phi \nabla Y_{t-1} + e_t$

ARIMA(0, 1, 1) : $\nabla Y_t = e_t - \theta e_{t-1}$

ARIMA (0,2,2) : $\nabla^2 Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}$

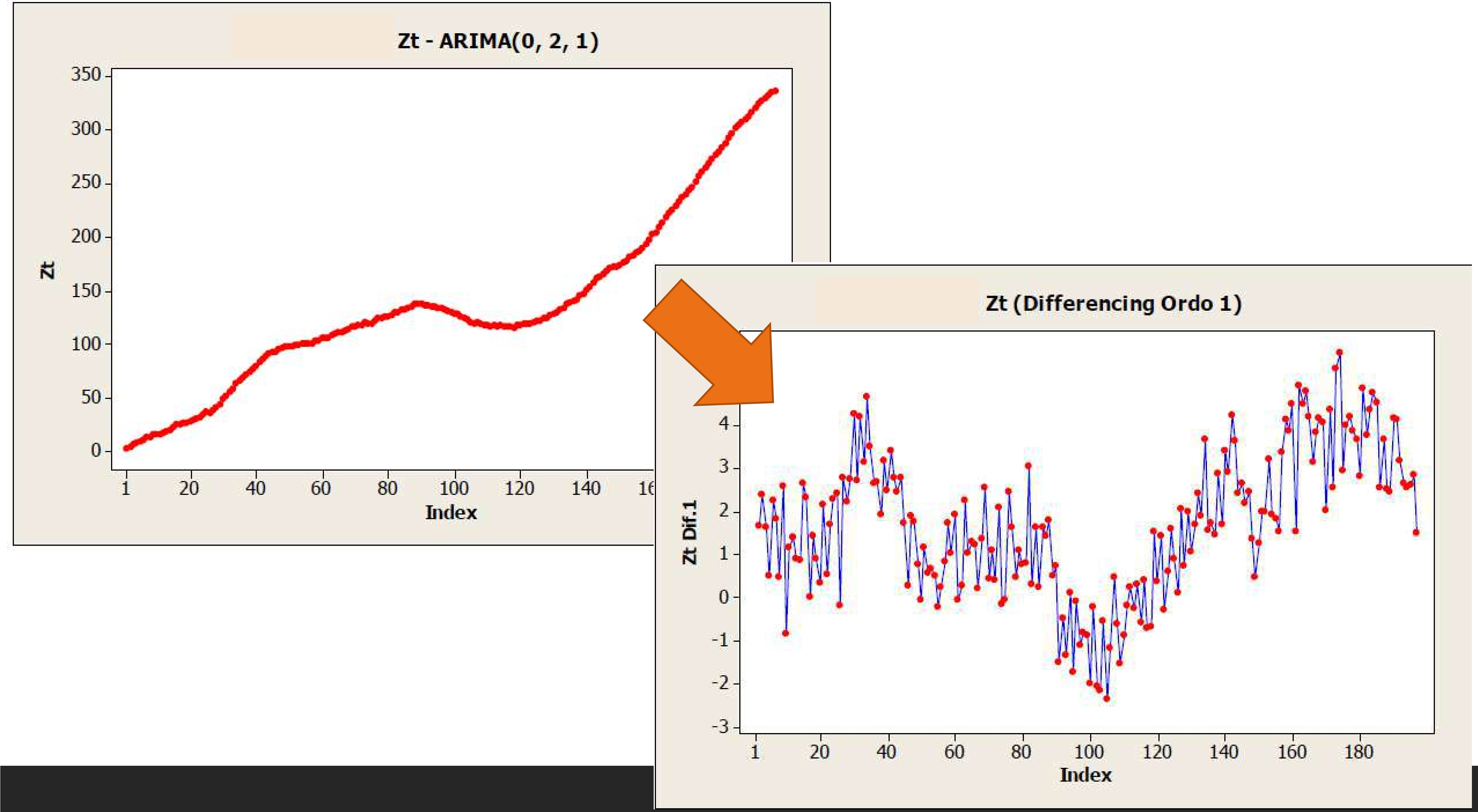
ARIMA(1, 0, 0) : $Y_t = \phi Y_{t-1} + e_t$

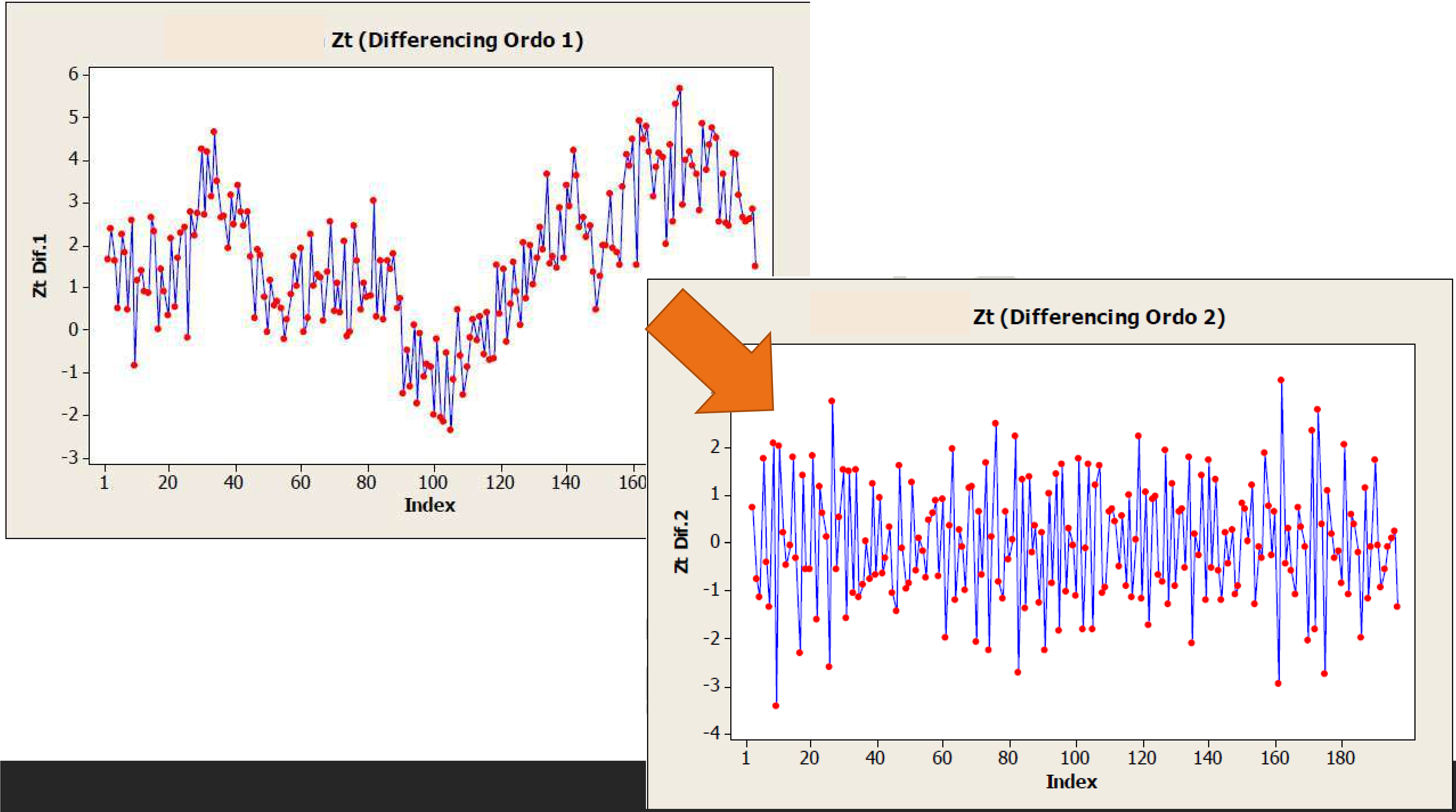
ARIMA (1,1,1) : $\nabla Y_t = \phi \nabla Y_{t-1} + e_t - \theta_1 e_{t-1}$

Exercise

Consider the process $Y_t = Y_{t-1} + e_t + \frac{1}{4}e_{t-1}$

Is the process invertible?
Is the process stationary?


Zt - ARIMA(0, 2, 1)
Zt
Index
Zt (Differencing Ordo 1)
Zt Dif.1
Index

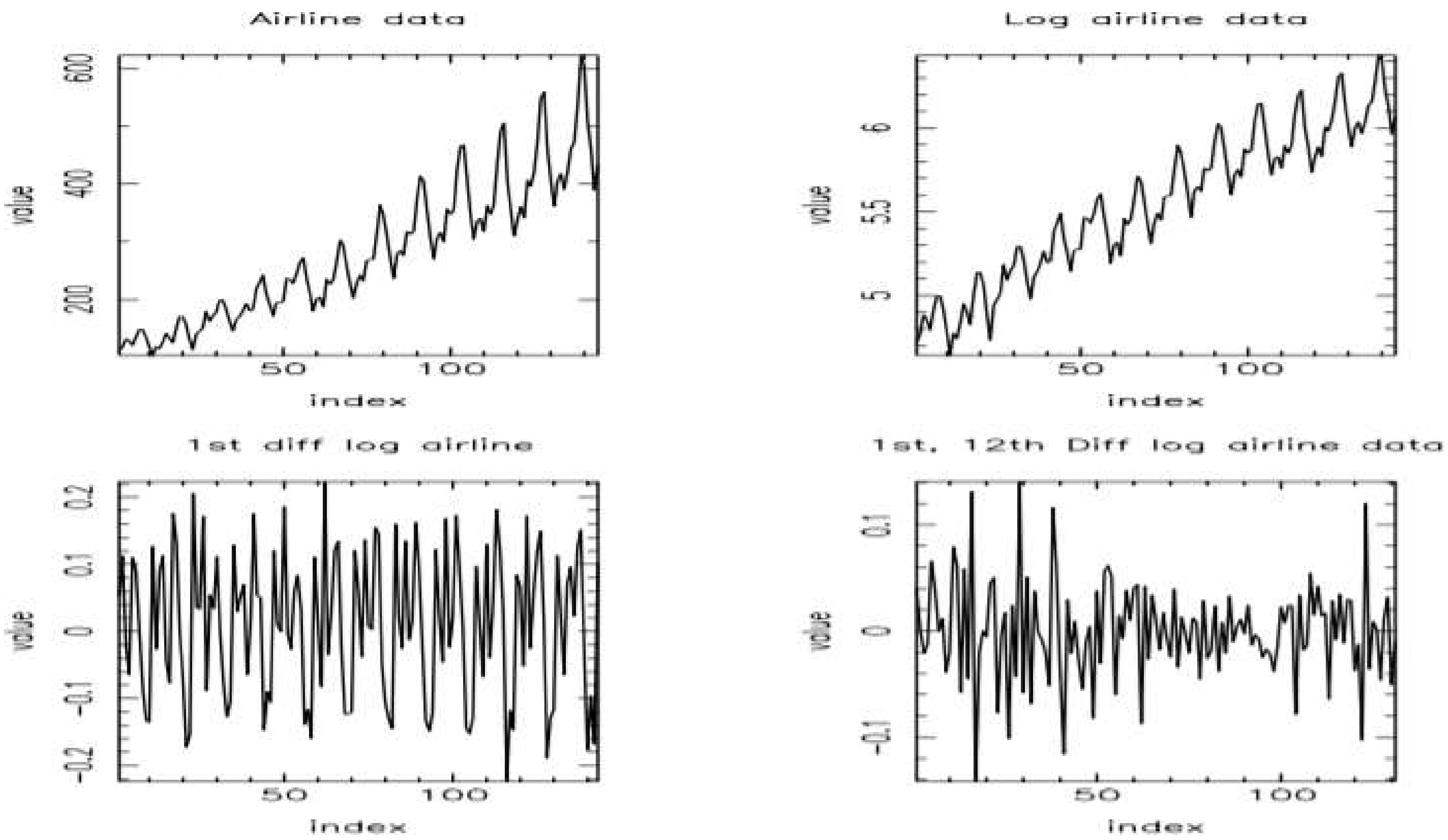

Zt (Differencing Ordo 1)
Zt Dif.1
Index
Zt (Differencing Ordo 2)
Zt Dif.2
Index

Other Transformations

- We have seen how differencing can be a useful transformation for achieving stationarity.
- However, the other transformation is also a useful method for achieving stationarity.

Common Box-Cox Transformations

Lambda	Suitable Transformation
-2	$Y^{-2} = 1/Y^2$
-1	$Y^{-1} = 1/Y^1$
-0.5	$Y^{-0.5} = 1/(Sqrt(Y))$
0	log(Y)
0.5	$Y^{0.5} = Sqrt(Y)$
1	$Y^1 = Y$
2	Y^2


Airline data
Log airline data
1st diff log airline
1st, 12th Diff log airline data
value
index
600
400
200
50
100
6
5.5
5
0.2
0.1
0
-0.1
-0.2

The next meetings

- Model Specification
- Parameter estimation
- Model Diagnostic
- Forecasting

Thanks

MODEL SPECIFICATION

YENNI ANGRAINI

Outline

- Characteristic of ACF : AR(p), MA(q) and ARMA(p,q)
- Characteristic of PACF : AR(p), MA(q) and ARMA(p,q)
- Illustrations

The subjects of the next three meeting, respectively, are:

1. how to choose appropriate values for p, d, and q for a given series;
2. how to estimate the parameters of a specific ARIMA(p,d,q) model;
3. how to check on the appropriateness of the fitted model and improve it if needed.

The Autocorrelation Functions

The **autocovariance function**, $\gamma_{t,s}$, is defined as

$$\gamma_{t,s} = Cov(Y_t, Y_s) \qquad \text{for } t, s = 0, \pm 1, \pm 2, \ldots$$

where $Cov(Y_t, Y_s) = E[(Y_t - \mu_t)(Y_s - \mu_s)] = E(Y_t Y_s) - \mu_t \mu_s$.

$Cor(x,y) = \dfrac{Cov(x,y)}{\sqrt{Var(x)\cdot Var(y)}}$

The **autocorrelation function**, $\rho_{t,s}$, is given by

$$\rho_{t,s} = Corr(Y_t, Y_s) \qquad \text{for } t, s = 0, \pm 1, \pm 2, \ldots$$

lag $k = 1$

where

$Corr(y_t, y_{t-1}) = Corr(y_{t-1}, y_{t-2}) = Corr(y_{t+1}, y_t)$

$$Corr(Y_t, Y_s) = \frac{Cov(Y_t, Y_s)}{\sqrt{Var(Y_t)Var(Y_s)}} = \frac{\gamma_{t,s}}{\sqrt{\gamma_{t,t}\gamma_{s,s}}}$$

lag $k = 2$

t	y_t	y_{t-1}	y_{t-2}
1	y_1	$\cancel{y_0}$	y_{-1}
2	y_2	y_1	y_0
3	y_3	y_2	y_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$
T	y_T	y_{T-1}	y_{T-2}

$Corr(y_t, y_{t-2}) = r_2$

$Corr(y_{t-1}, y_{t-3}) =$

we can simplify our notation and write

$$\gamma_k = Cov(Y_t, Y_{t-k}) \quad \text{and} \quad \rho_k = Corr(Y_t, Y_{t-k})$$

Note also that

$$\rho_k = \frac{Cov(y_t, y_{t-k})}{\sqrt{Var(y_t)\,Var(y_{t-k})}} = \frac{\gamma_k}{\sqrt{\gamma_0\,\gamma_0}} = \frac{\gamma_k}{\gamma_0}$$

$$\underbrace{Var(y_t, y_t)}_{\gamma_0} \qquad \underbrace{Var(y_{t-k}, y_{t-k})}_{\gamma_0}$$

$$\rho_0 = \frac{\gamma_0}{\gamma_0} = 1$$

$$\rho_k = \frac{\gamma_k}{\gamma_0}$$

$$\gamma_0 = Var(Y_t) \qquad \boxed{\rho_0 = 1}$$

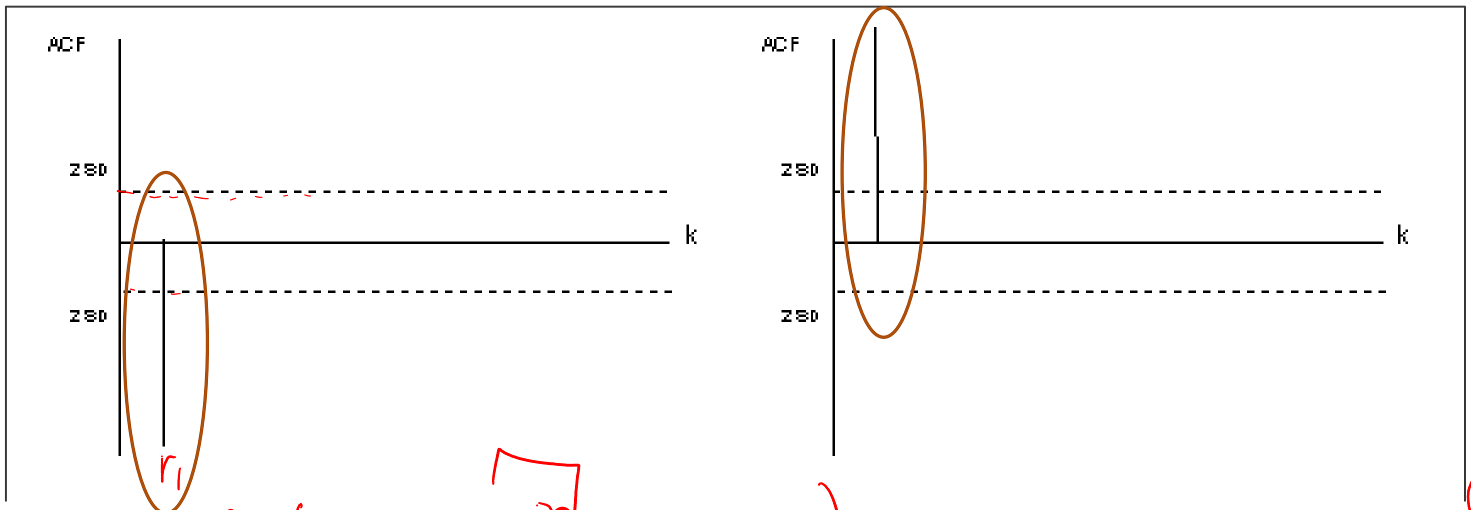
$$\gamma_k = \gamma_{-k} \qquad \rho_k = \rho_{-k}$$

$$|\gamma_k| \le \gamma_0 \qquad |\rho_k| \le 1$$

$$-1 < \rho_k < 1$$

If a process is strictly stationary and has finite variance, then the covariance function must depend only on the time lag.

ACF for MA (1)



$$Y_t = e_t - \theta e_{t-1}$$

$$E(Y_t) = 0, Var(Y_t) = \gamma_0 = \sigma_e^2(1+\theta^2)$$

$$Cov(Y_t, Y_{t-1}) = \gamma_1 = -\theta\sigma_e^2$$

$$Cov(Y_t, Y_{t-2}) = 0,\ Cov(Y_t, Y_{t-3}) = 0, \ldots Cov(Y_t, Y_{t-k}) = 0\ for\ k = 2,3, \ldots$$

$$\boxed{\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{-\theta}{(1+\theta^2)}}\ \rho_2 = 0, \ldots, \rho_k = 0\ for\ k = 2,3, \ldots$$

$Cov(y_t, y_{t-1}) = Cov(e_t - \theta_1 e_{t-1}, e_{t-1} - \theta_1 e_{t-2})$

$= -\theta_1 Var(e_{t-1})$

$= -\theta_1 \sigma_e^2$

$Cov(y_t, y_{t-2}) = Cov(e_t - \theta_1 e_{t-1}, e_{t-2} - \theta_1 e_{t-3})$

$= 0$

$e_t \sim (0, \sigma_e^2)$

$E(y_t) = E(e_t - \theta_1 e_{t-1})$

$= E(e_t) - \theta_1 E(e_{t-1})$

$= 0 - \theta_1(0) = 0$

$Var(y_t) = Var(e_t - \theta_1 e_{t-1})$

$= Var(e_t) + \theta_1^2 Var(e_{t-1})$

$= \sigma_e^2 + \theta_1^2\sigma_e^2$

$= \sigma_e^2(1+\theta_1^2)$

$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{-\theta\cancel{\sigma_e^2}}{\cancel{\sigma_e^2}(1+\theta^2)}$

ACF for MA (2)

$Cov(y_t, y_{t-1}) = Cov(e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2},\ e_{t-1} - \theta_1 e_{t-2} - \theta_2 e_{t-3})$

$= -\theta_1 Var(e_{t-1}) + \theta_1\theta_2 Var(e_{t-2})$

$= (-\theta_1 + \theta_1\theta_2)\sigma_e^2$

$$\boxed{Y_t = e_t - \theta e_{t-1} - \theta_2 e_{t-2}}$$

$e_t \overset{iid}{\sim} (0, \sigma_e^2)$

$Cov(y_t, y_{t-2}) = Cov(e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2},\ e_{t-2} - \theta_1 e_{t-3} - \theta_2 e_{t-4})$

$= -\theta_2 \sigma_e^2$

$$E(Y_t) = 0,$$

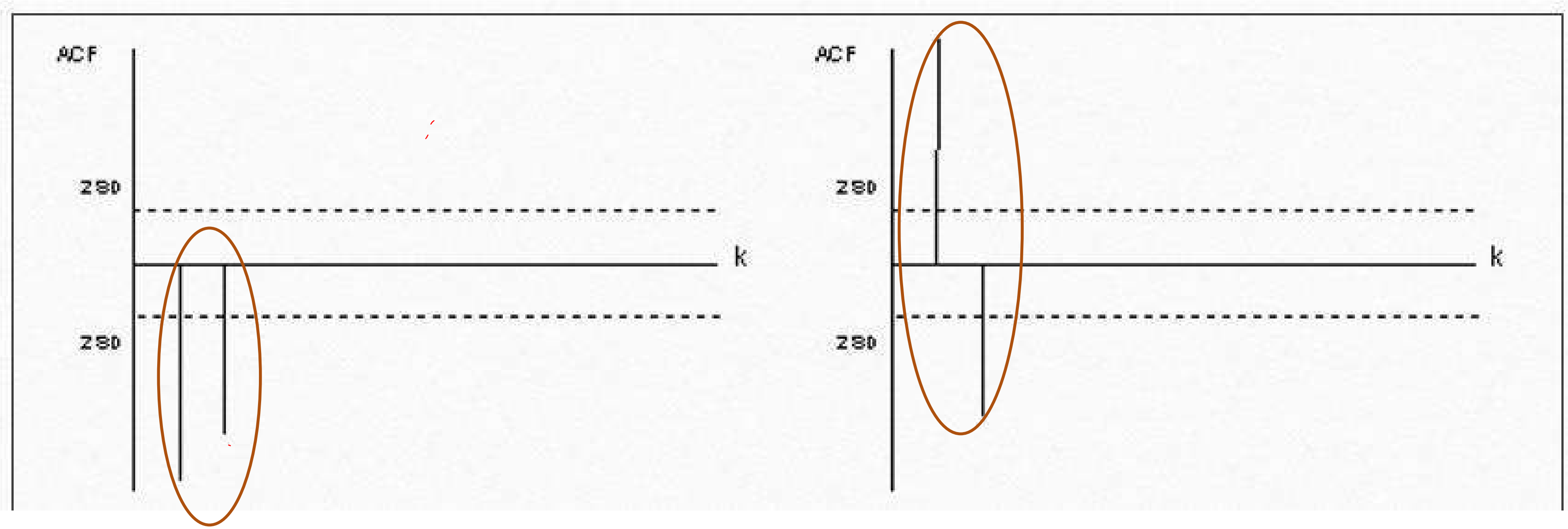
$$Var(Y_t) = \gamma_0 = (1 + \theta_1^2 + \theta_2^2)\sigma_e^2$$

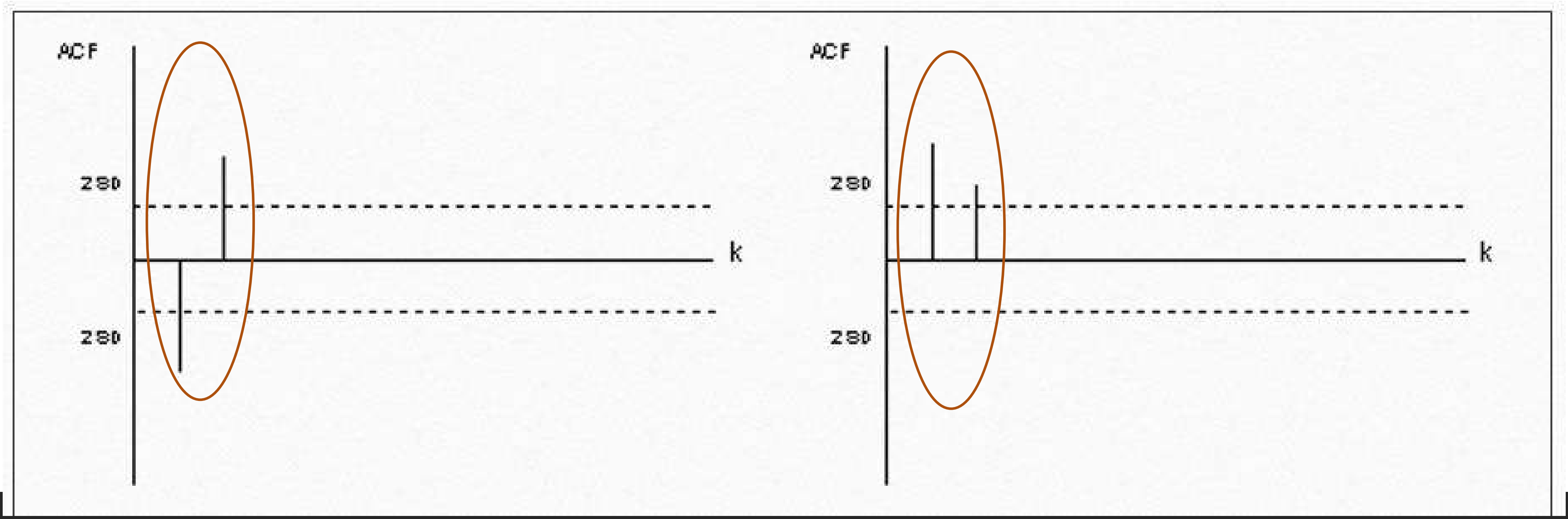
$$Cov(Y_t, Y_{t-1}) = \gamma_1 = (-\theta_1 + \theta_1\theta_2)\sigma_e^2$$

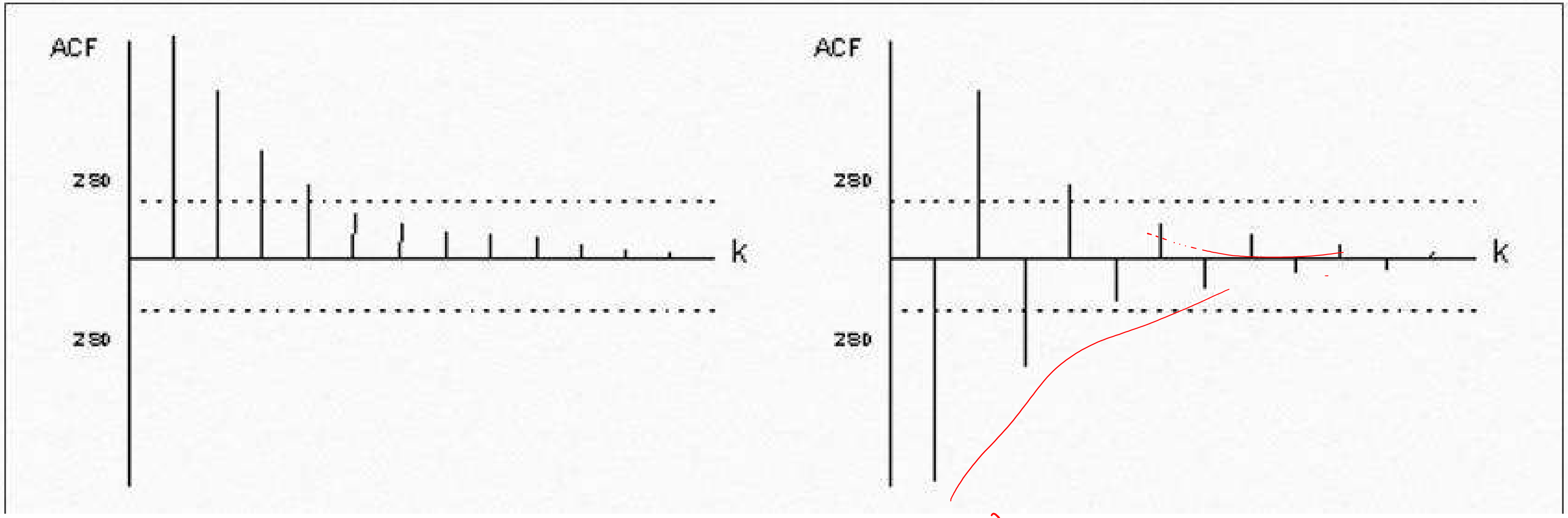
$$Cov(Y_t, Y_{t-2}) = \gamma_2 = -\theta_2\sigma_e^2,\ Cov(Y_t, Y_{t-3}) = \gamma_3 = 0$$

$$\rho_1 = \frac{-\theta_1 + \theta_1\theta_2}{1 + \theta_1^2 + \theta_2^2},\ \rho_2 = \frac{-\theta_2}{1 + \theta_1^2 + \theta_2^2},\ \rho_k = 0 \text{ for } k = 3,4,\ldots$$

ACF for MA (2)







ACF for AR (1)

$\mathrm{Cov}(y_t, y_{t-1}) = \mathrm{Cov}(\phi_1 y_{t-1} + e_t,\ \phi_1 y_{t-2} + e_t)$

$\gamma_1 = \phi_1 \gamma_0 \qquad -1 < \phi < 1$

ϕ_1 neg

$$Y_t = \phi_1 Y_{t-1} + e_t \qquad e_t \overset{iid}{\sim} (0, \sigma_e^2)$$

$$\boxed{E(Y_t) = 0},$$

$$Var(Y_t) = \gamma_0 = \frac{\sigma_e^2}{1-\phi^2}$$

$$\gamma_k = \phi \gamma_{k-1} = \boxed{\phi^k \frac{\sigma_e^2}{1-\phi^2}} \quad for\ k = 1,2,3, \ldots$$

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \phi^k \quad for\ k = 1,2,3, \ldots$$

$$\rho_k = \frac{\phi^k \cancel{\frac{\sigma_e^2}{1-\phi^2}}}{\cancel{\frac{\sigma_e^2}{1-\phi_1^2}}} = \phi_1^k$$

$Var(y_t) = \phi_1^2 Var(y_{t-1}) + Var(e_t)$

$\gamma_0 = \phi_1^2 \gamma_0 + \sigma_e^2$

$\gamma_0 - \phi_1^2 \gamma_0 = \sigma_e^2$

$(1 - \phi_1^2)\gamma_0 = \sigma_e^2$

$\gamma_0 = \dfrac{\sigma_e^2}{1-\phi_1^2}$

$\rho_1 = \phi_1 \rho_0 + \phi_2 \rho_1 = \phi_1 + \phi_2 \rho_1$

$\rho_2 = \phi_1 \rho_1 + \phi_2 \rho_0$

ACF for AR (2)

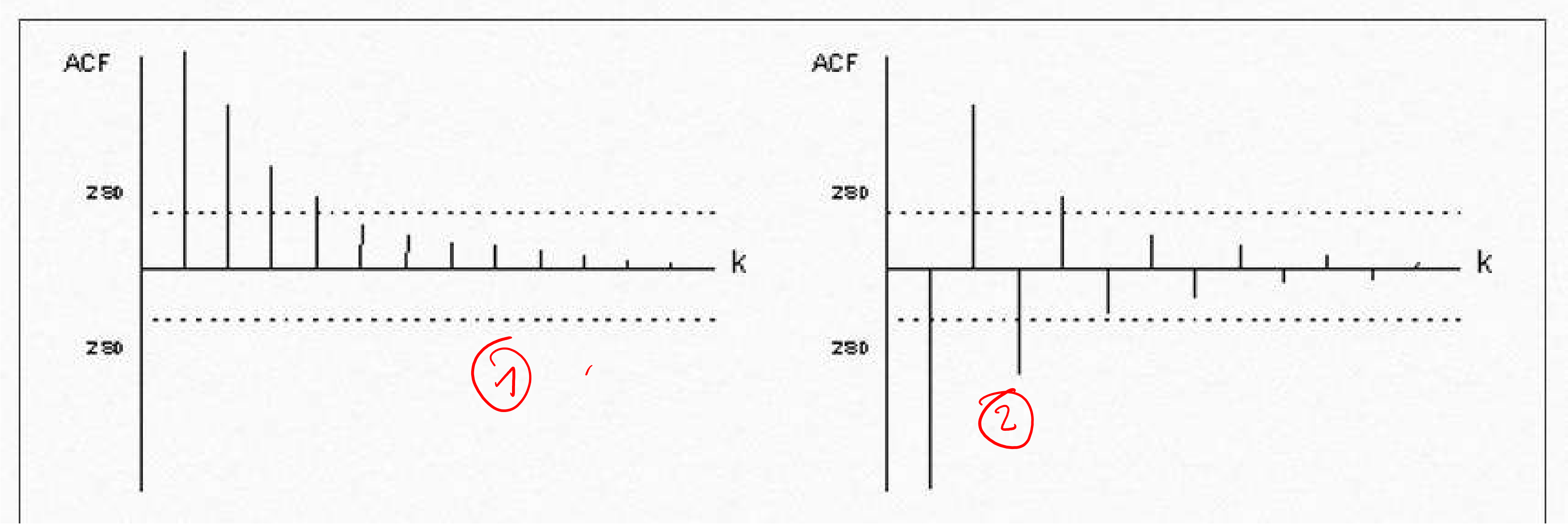
$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t$$

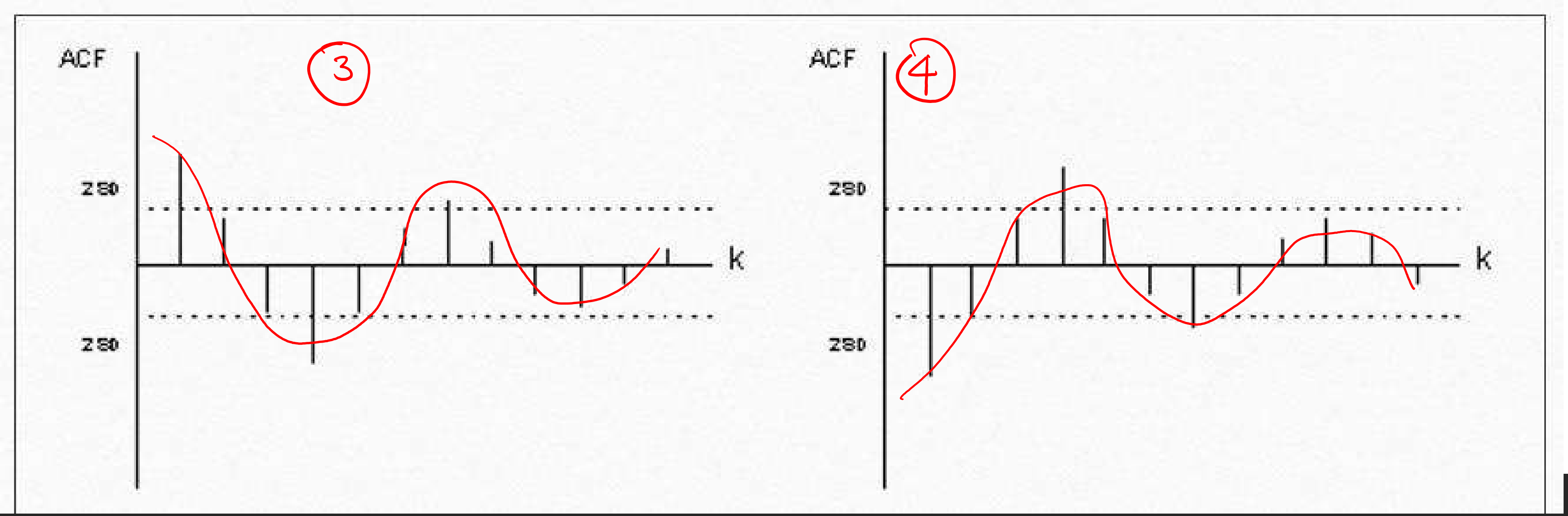
$$E(Y_t) = 0,$$
$$Var(Y_t) = \gamma_0 = \phi_1 \gamma_1 + \phi_2 \gamma_2 + \sigma_e^2$$

$$Cov(Y_t, Y_{t-k}) = \gamma_k = \phi_1 \gamma_{k-1} + \phi_2 \gamma_{k-2} \; for \; k = 1,2,3, \ldots$$

$$\boxed{\rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2}} \; for \; k = 1,2,3, \ldots$$

ACF for AR (2)





The Partial Autocorrelation Functions

- Since for MA(q) models the autocorrelation function is zero for lags beyond q, the sample autocorrelation is a good indicator of the order of the process.
- However, the autocorrelations of an AR(p) model do not become zero after a certain number of lags—they die off rather than cut off. **So a different function is needed to help determine the order of autoregressive models**.
- A function may be defined as the correlation between Y_t and Y_{t-k} *after removing the effect of the intervening variables* $Y_{t-1}, Y_{t-2}, Y_{t-3}, \ldots, Y_{t-k+1}$
- This coefficient is called **the partial autocorrelation at lag *k*** and will be denoted by ϕ_{kk}.

The Partial Autocorrelation Functions

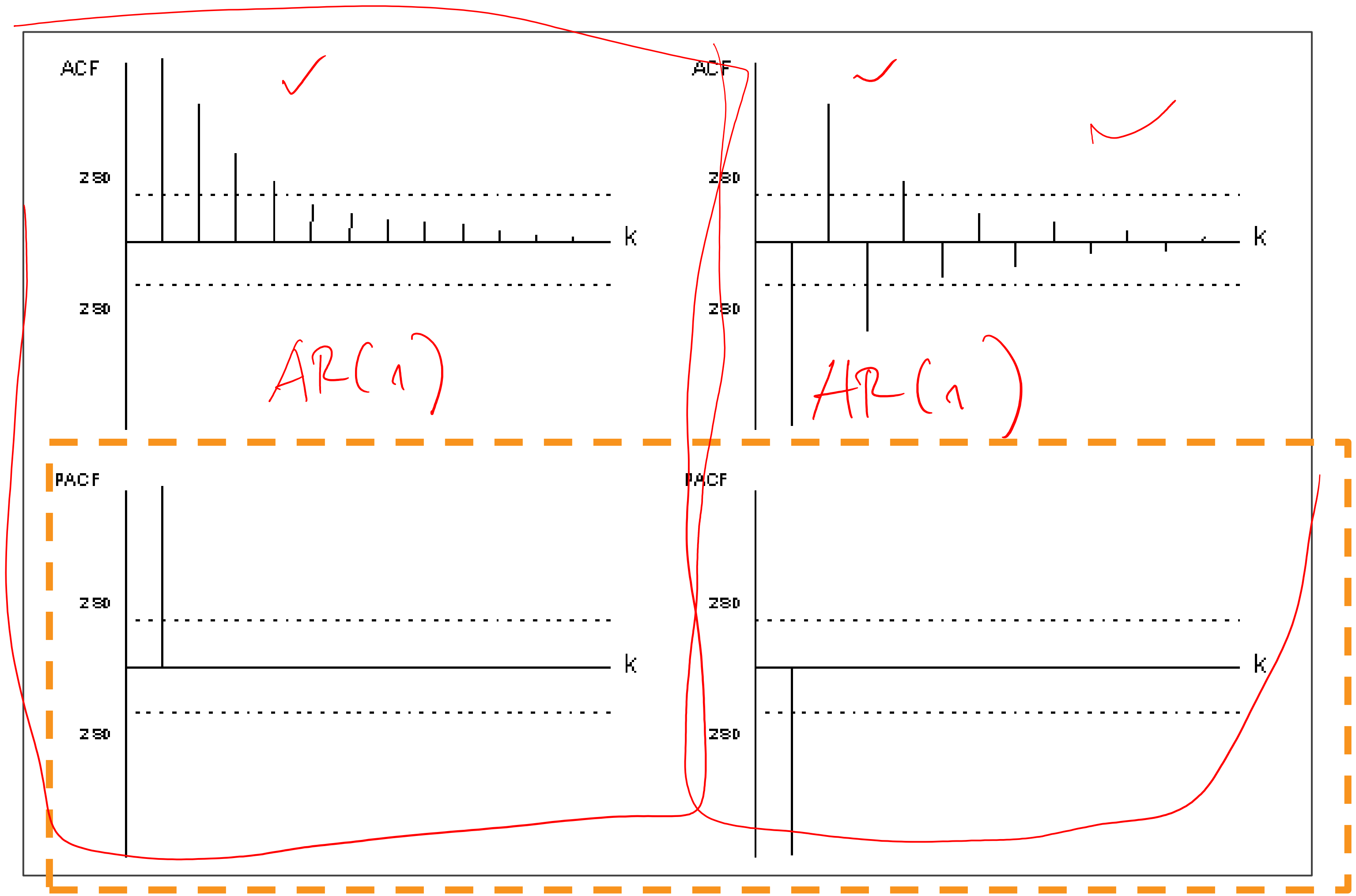
A general method for finding the partial autocorrelation function for any stationary process with autocorrelation function ρ_k is as follows (see Anderson 1971, pp. 187–188, for example). For a given lag k, it can be shown that the ϕ_{kk} satisfy the Yule-Walker equations

$$\rho_j = \phi_{k1}\rho_{j-1} + \phi_{k2}\rho_{j-2} + \phi_{k3}\rho_{j-3} + \cdots + \phi_{kk}\rho_{j-k} \quad \text{for } j = 1, 2, \ldots, k$$

More explicitly, we can write these k linear equations as

$$\left.\begin{aligned}
\phi_{k1} + \rho_1\phi_{k2} + \rho_2\phi_{k3} + \cdots + \rho_{k-1}\phi_{kk} &= \rho_1 \\
\rho_1\phi_{k1} + \phi_{k2} + \rho_1\phi_{k3} + \cdots + \rho_{k-2}\phi_{kk} &= \rho_2 \\
&\vdots \\
\rho_{k-1}\phi_{k1} + \rho_{k-2}\phi_{k2} + \rho_{k-3}\phi_{k3} + \cdots + \phi_{kk} &= \rho_k
\end{aligned}\right\}$$

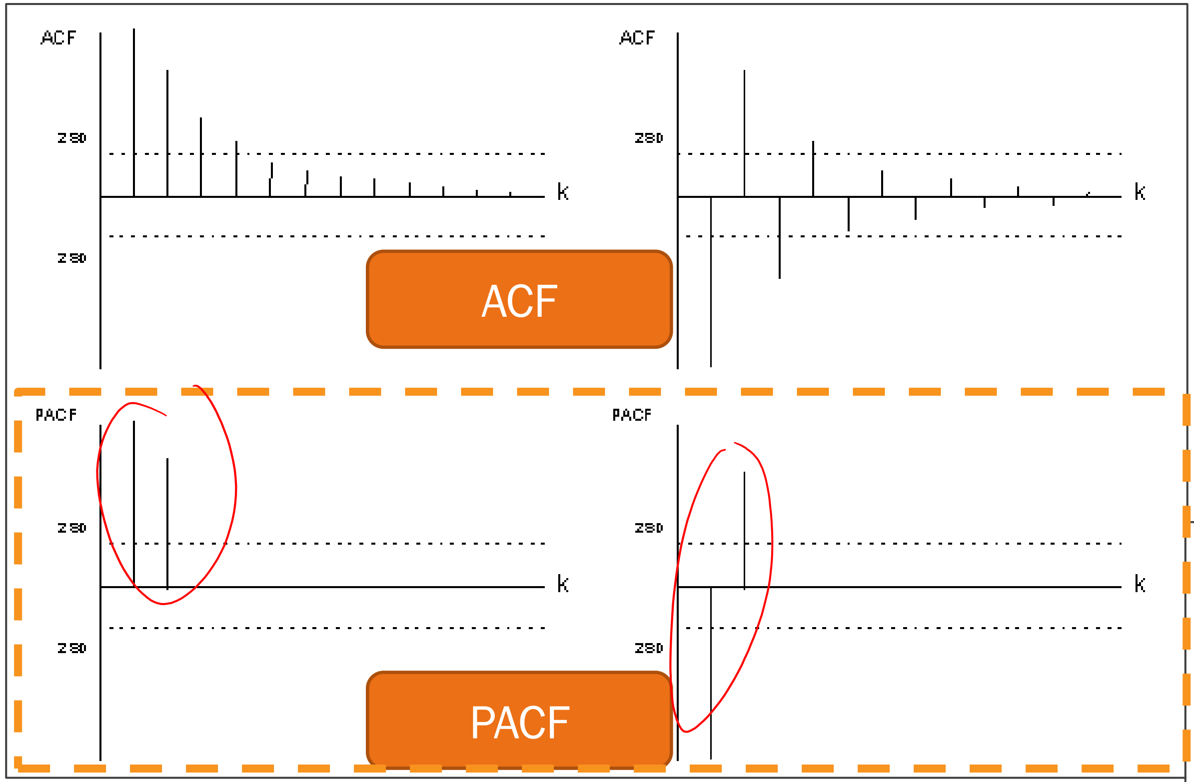
PACF for AR (1)

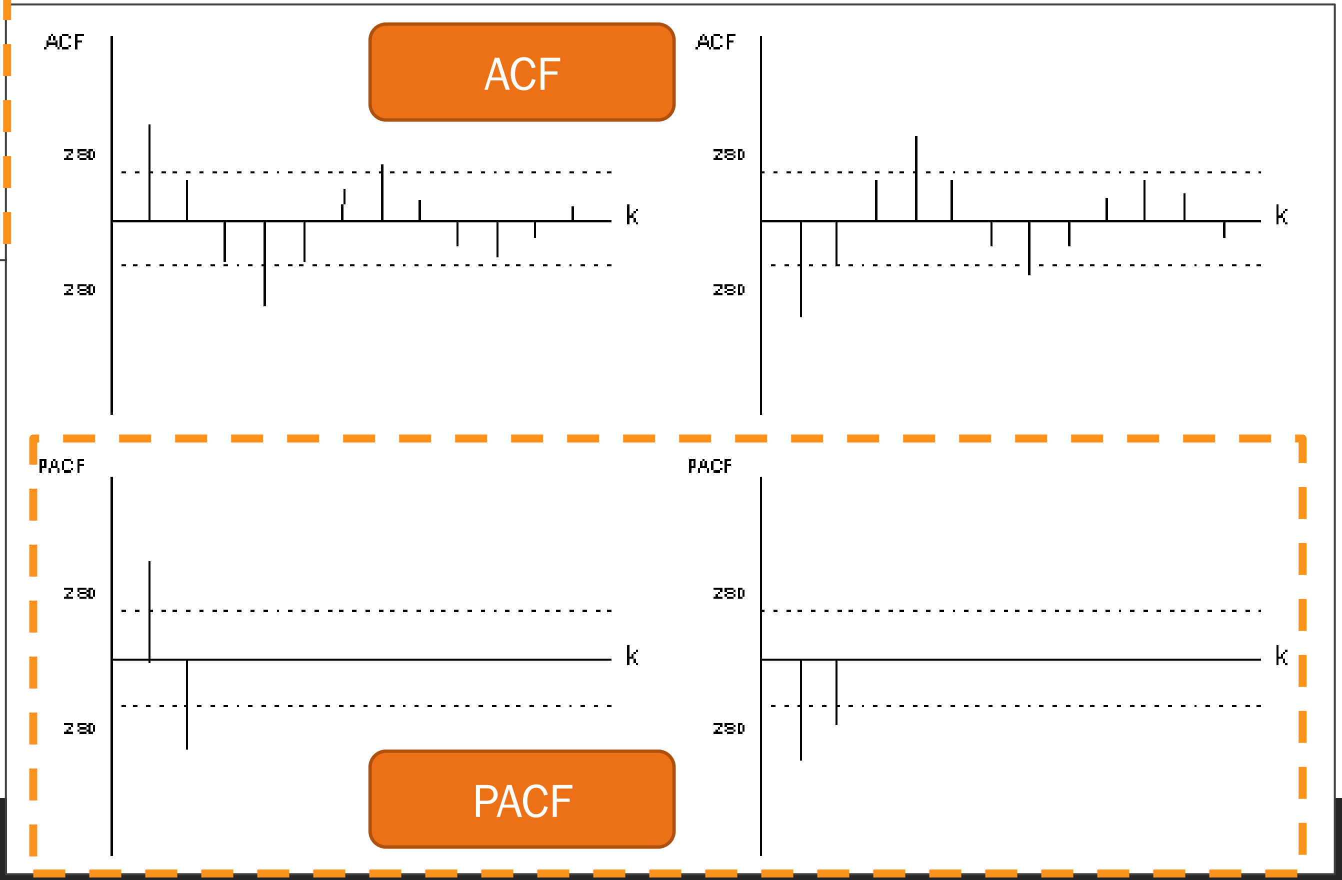


ACF

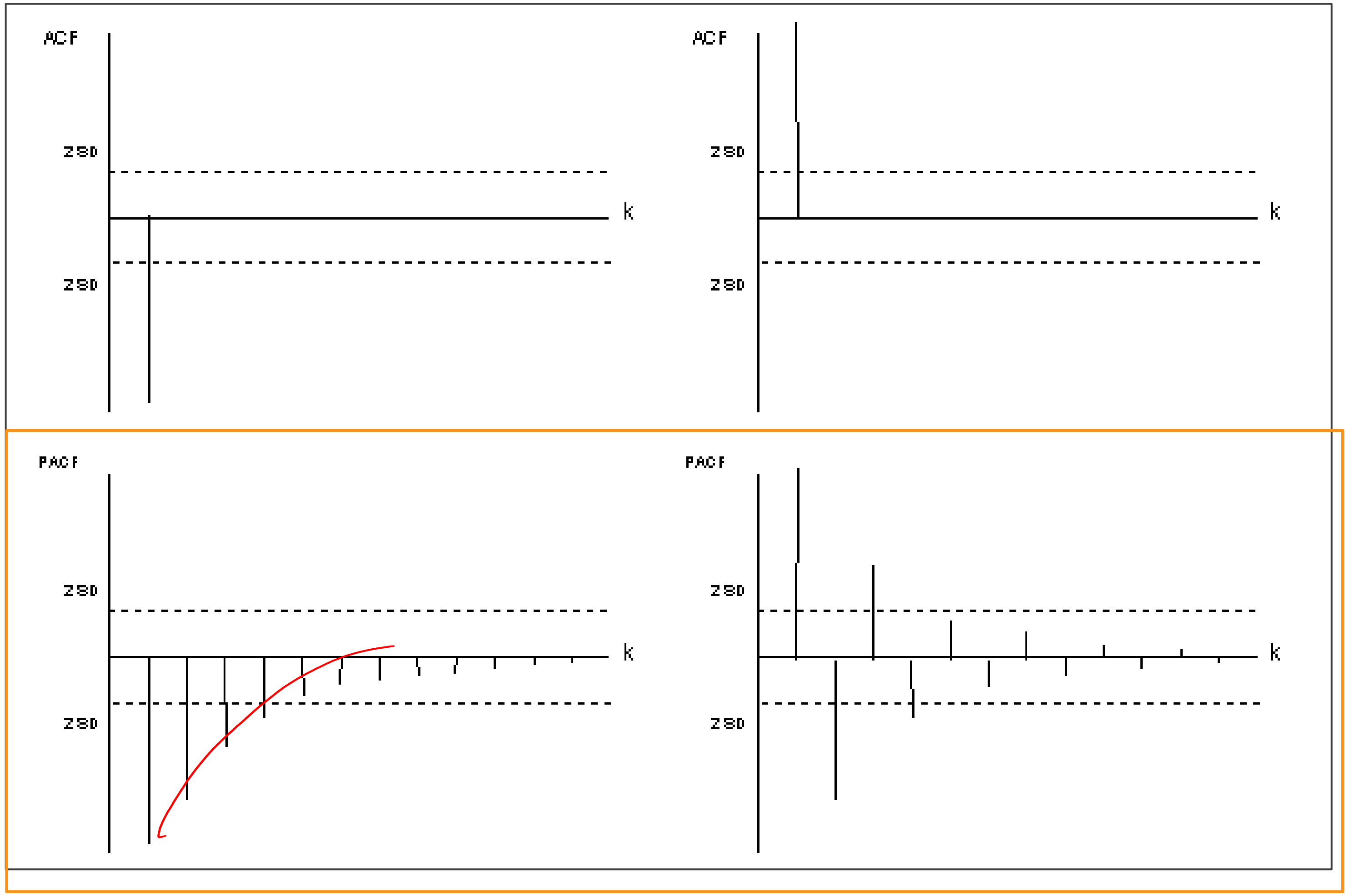
PACF

PACF
for AR (2)





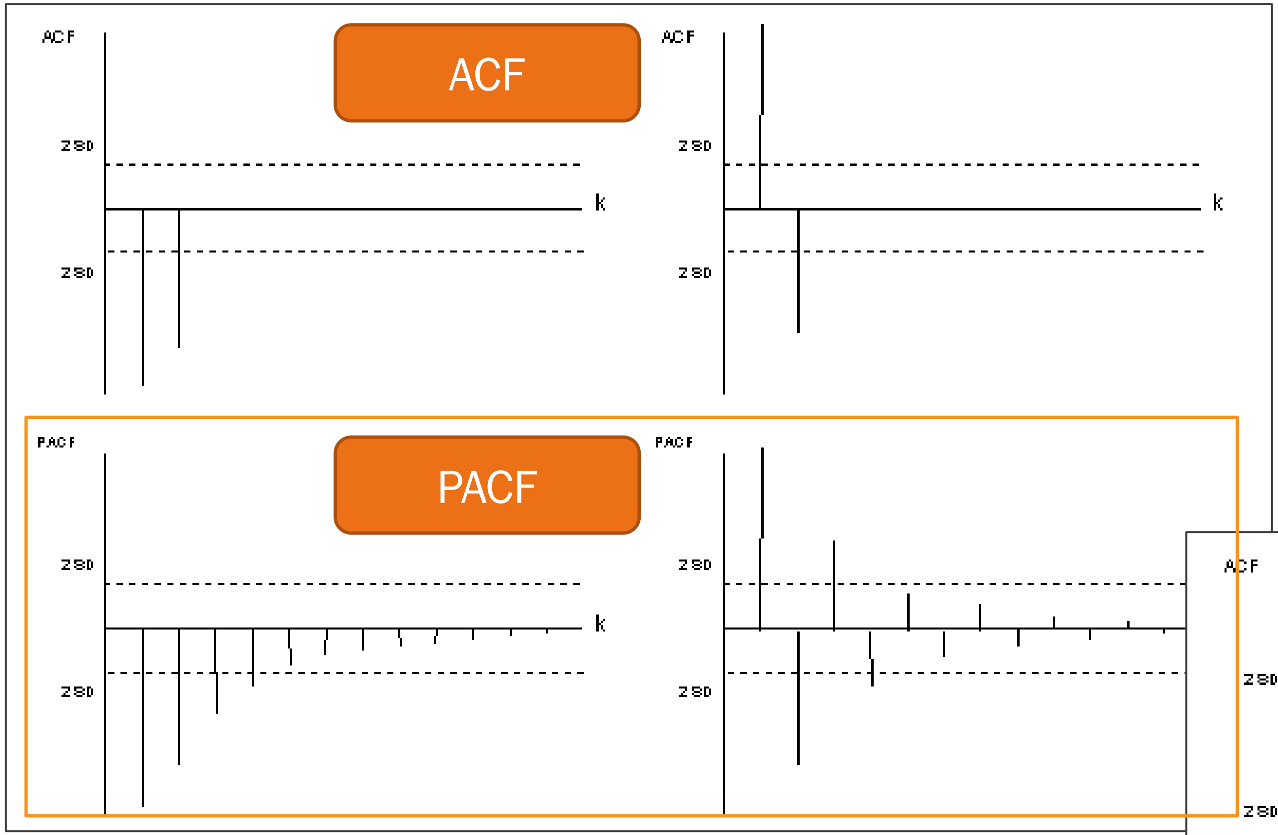
PACF for MA (1)

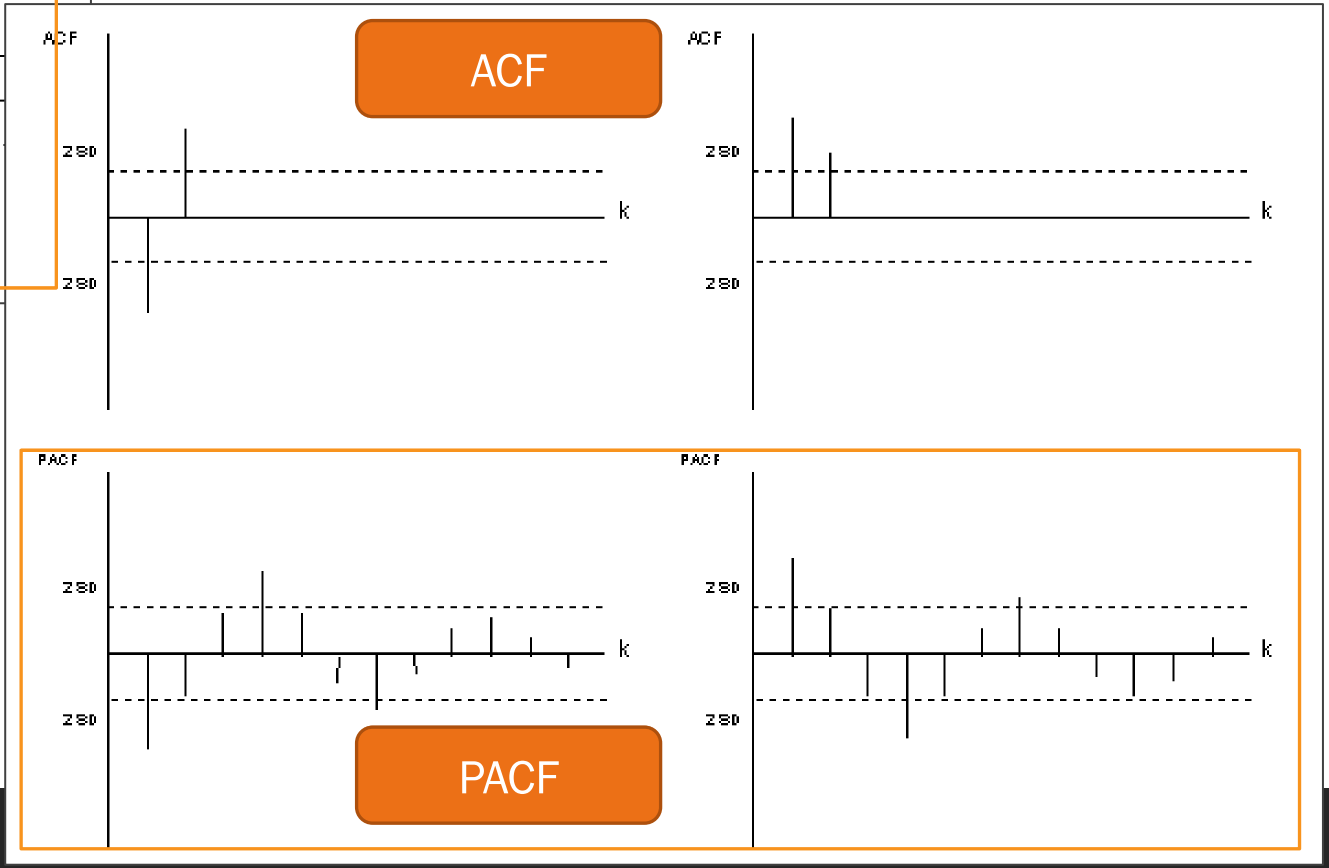


ACF

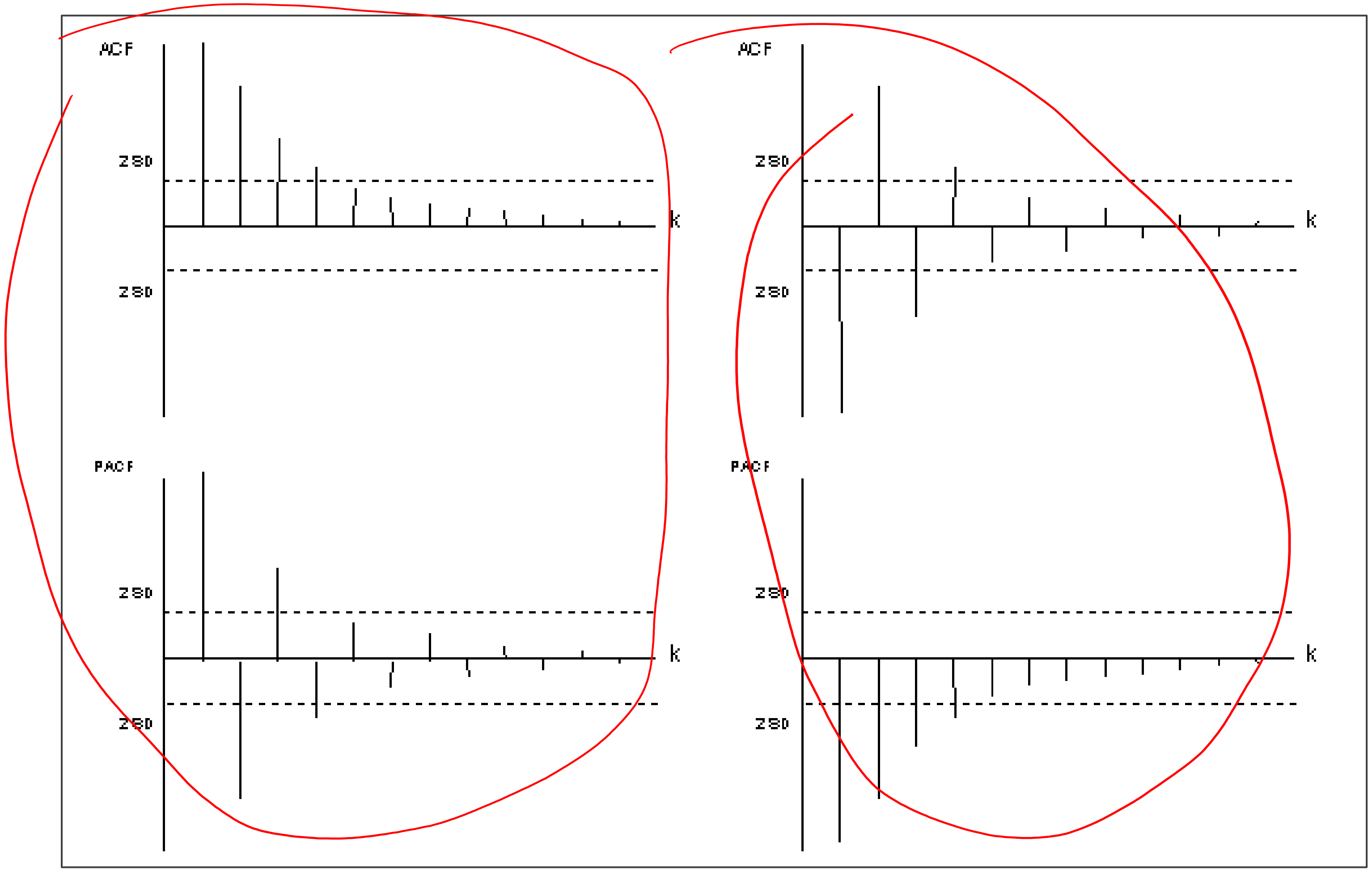
PACF

PACF for MA (2)


ACF
PACF
ACF
2SD
2SD
k
PACF
2SD
2SD
k


ACF
PACF
ACF
2SD
2SD
k
PACF
2SD
2SD
k

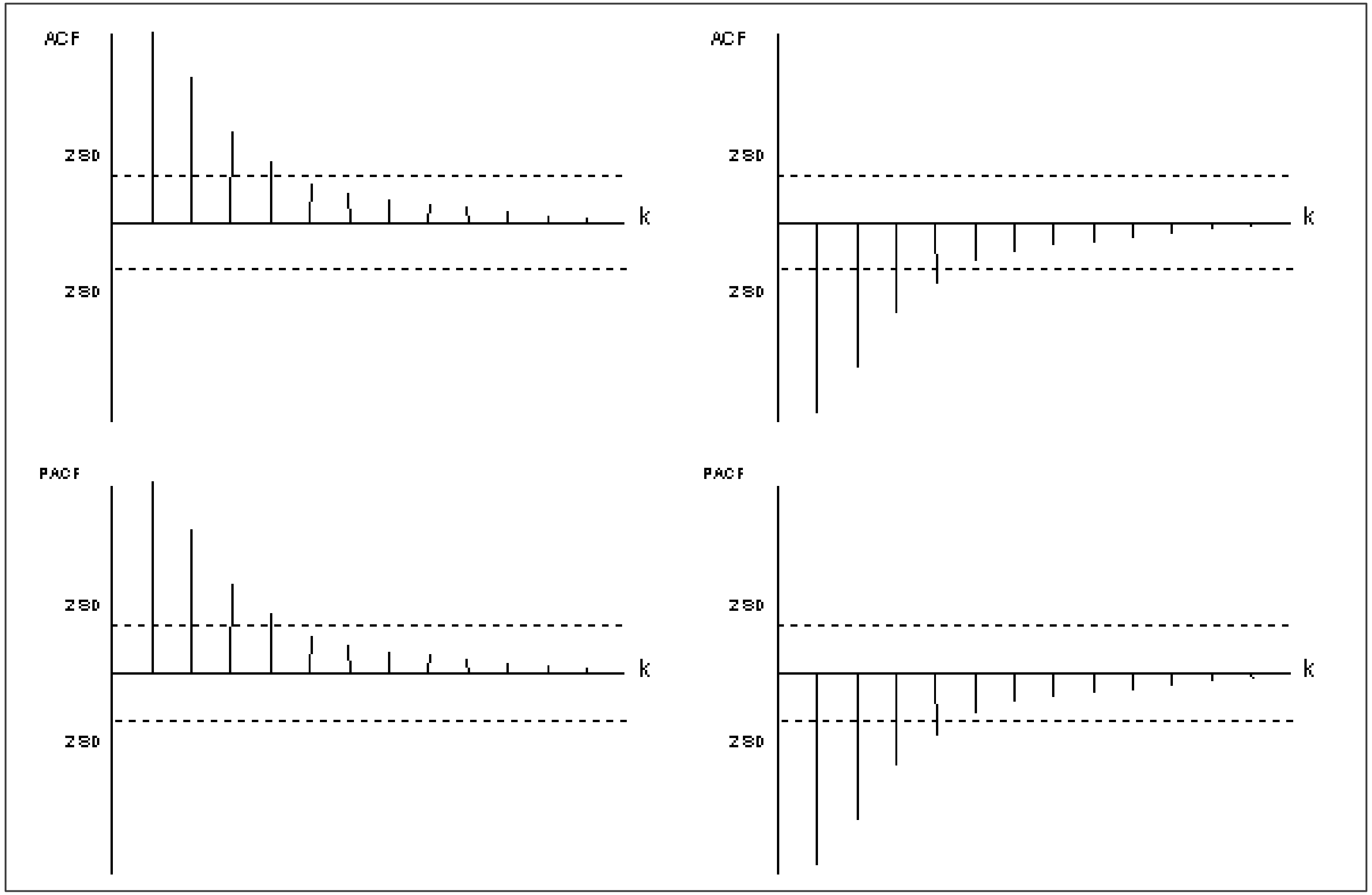
ACF and PACF for ARMA(1,1)



ACF

PACF

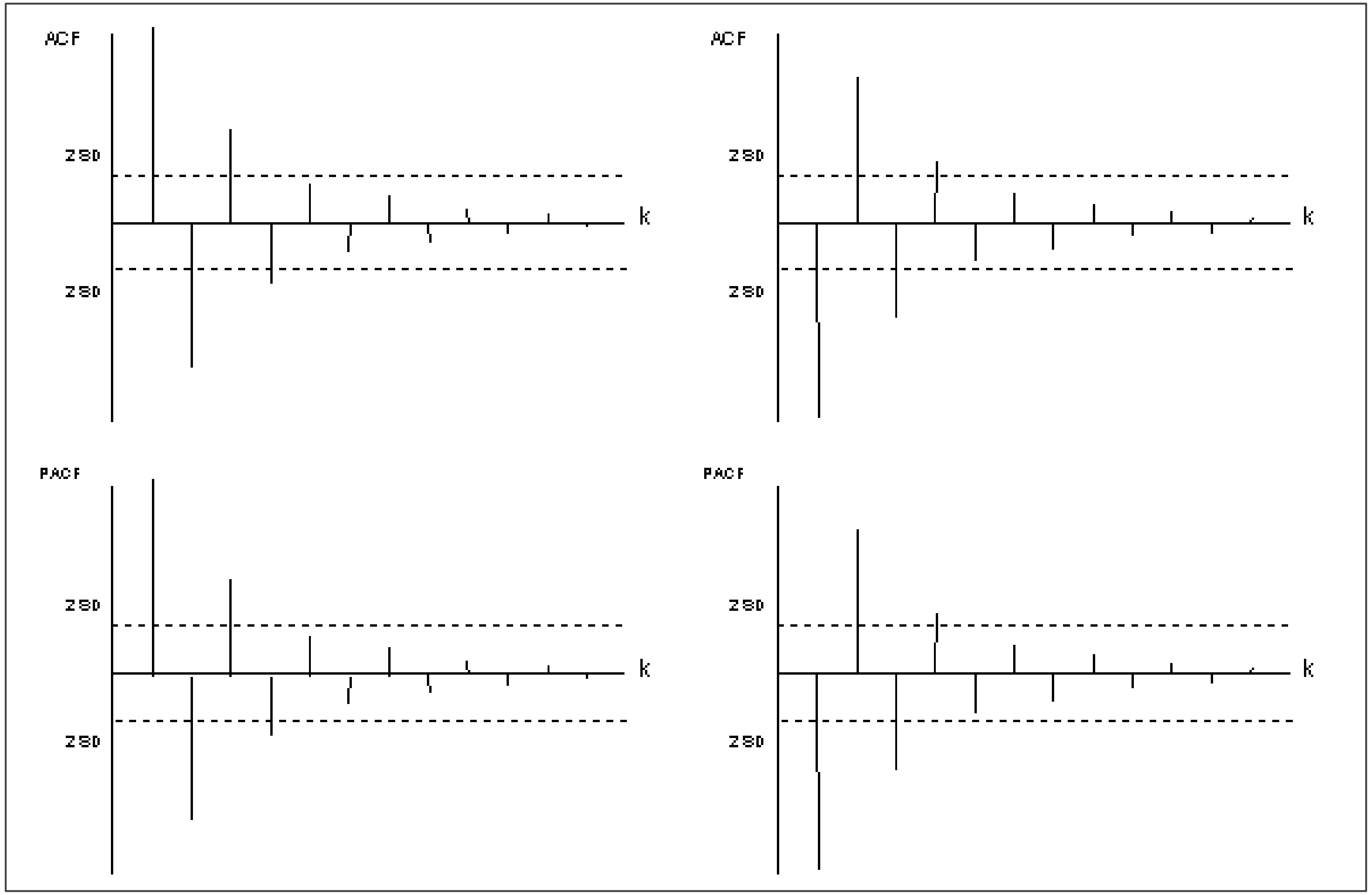
ACF and PACF for ARMA(1,1)



ACF

PACF

ACF and PACF for ARMA(1,1)



ACF

PACF

General Behavior of the ACF and PACF for ARMA Models

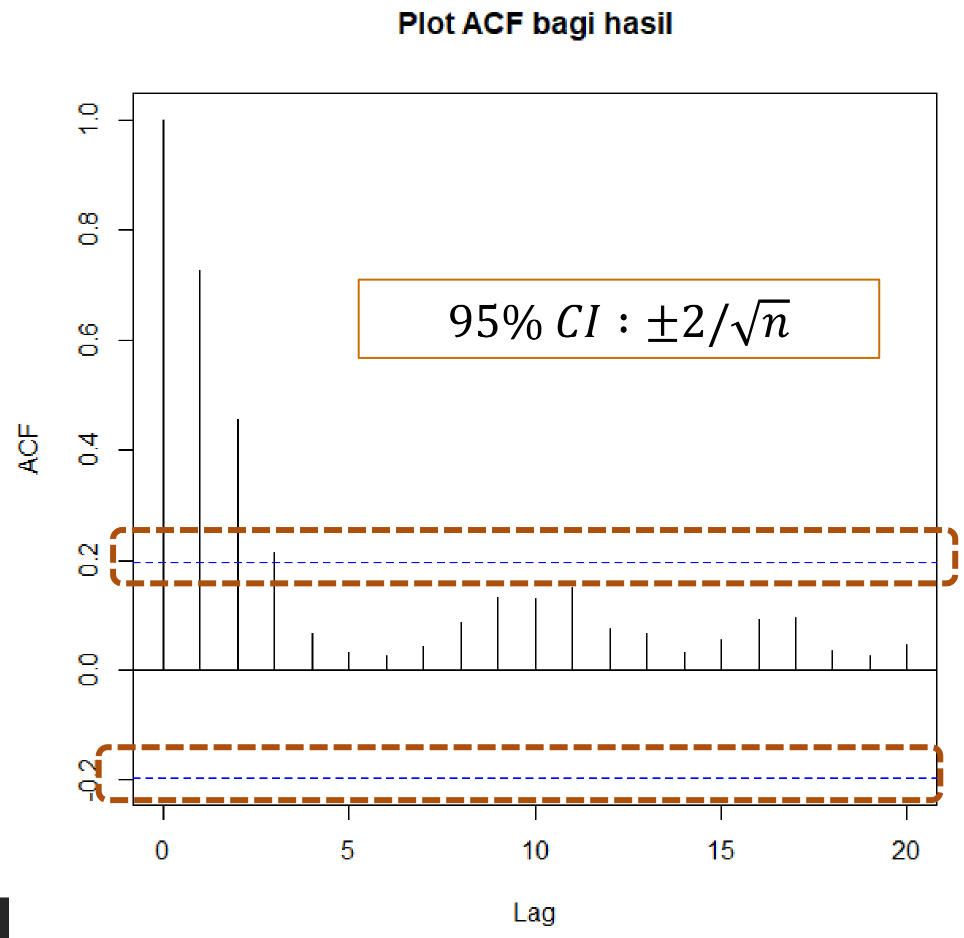
	AR(p)	MA(q)	ARMA(p,q), $p>0$, and $q>0$
ACF	Tails off	Cuts off after lag q	Tails off
PACF	Cuts off after lag p	Tails off	Tails off

The **sample autocorrelation function**, r_k, at lag k as

$$r_k = \frac{\sum_{t=k+1}^{n} (Y_t - \bar{Y})(Y_{t-k} - \bar{Y})}{\sum_{t=1}^{n} (Y_t - \bar{Y})^2} \quad \text{for } k = 1, 2, \ldots$$

$$Var(r_k) \approx \frac{1}{n}$$

$$S(r_k) = 1/\sqrt{n}$$



The **sample partial autocorrelation function**, $\hat{\phi}_{kk}$

$$\phi_{kk} = \frac{\rho_k - \sum_{j=1}^{k-1} \phi_{k-1,j}\,\rho_{k-j}}{1 - \sum_{j=1}^{k-1} \phi_{k-1,j}\,\rho_j}$$

where

$$\phi_{k,j} = \phi_{k-1,j} - \phi_{kk}\phi_{k-1,k-j} \quad \text{for } j = 1, 2, \ldots, k-1$$

For example, using $\phi_{11} = \rho_1$ to get started, we have

$$\phi_{22} = \frac{\rho_2 - \phi_{11}\rho_1}{1 - \phi_{11}\rho_1} = \frac{\rho_2 - \rho_1^2}{1 - \rho_1^2}$$

(as before) with $\phi_{21} = \phi_{11} - \phi_{22}\phi_{11}$, which is needed for the next step. Then

$$\phi_{33} = \frac{\rho_3 - \phi_{21}\rho_2 - \phi_{22}\rho_1}{1 - \phi_{21}\rho_1 - \phi_{22}\rho_2}$$

$$Var(\hat{\phi}_{kk}) = 1/n$$

$$S(\hat{\phi}_{kk}) = 1/\sqrt{n}$$

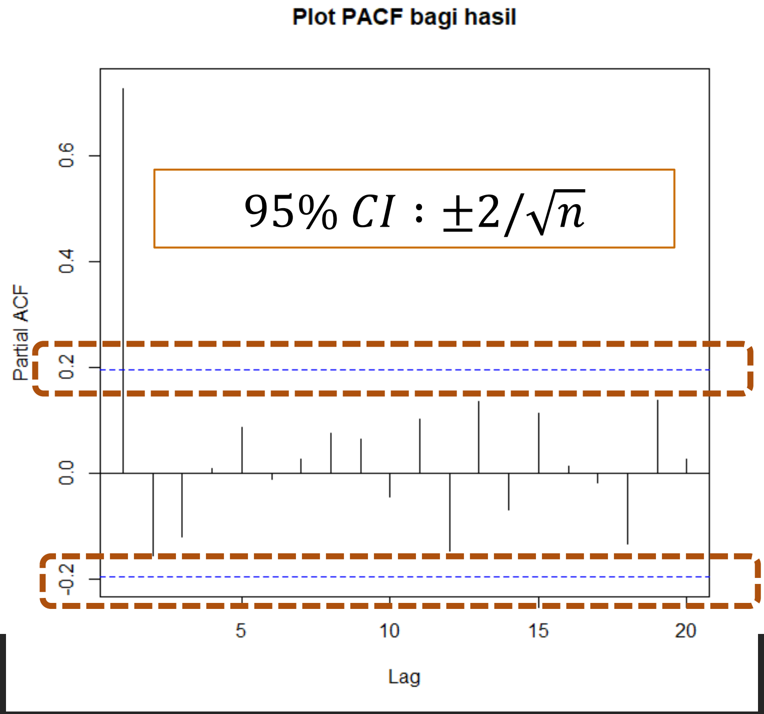
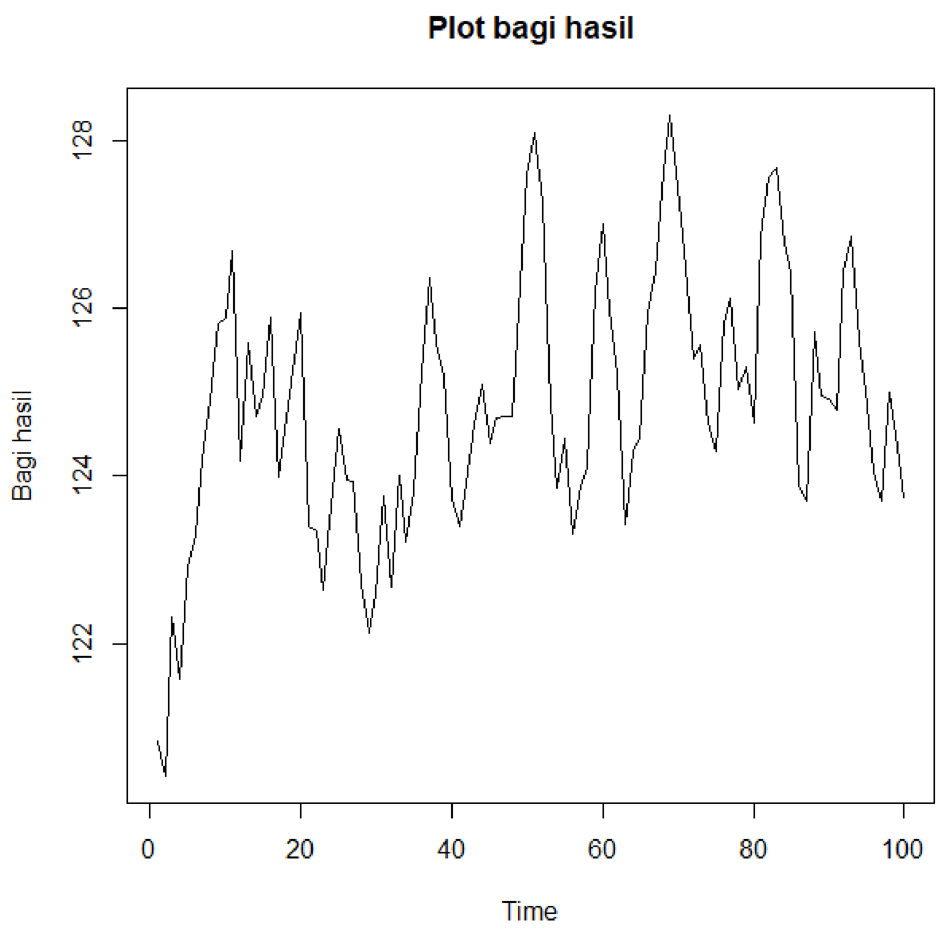
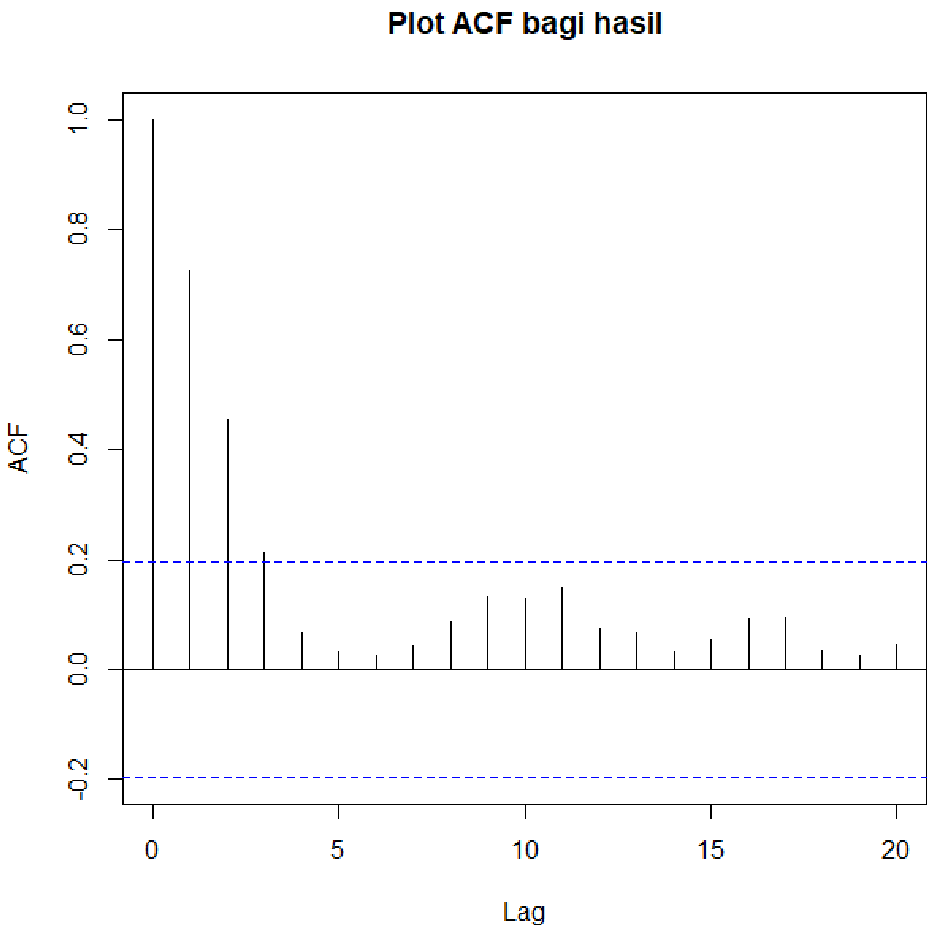
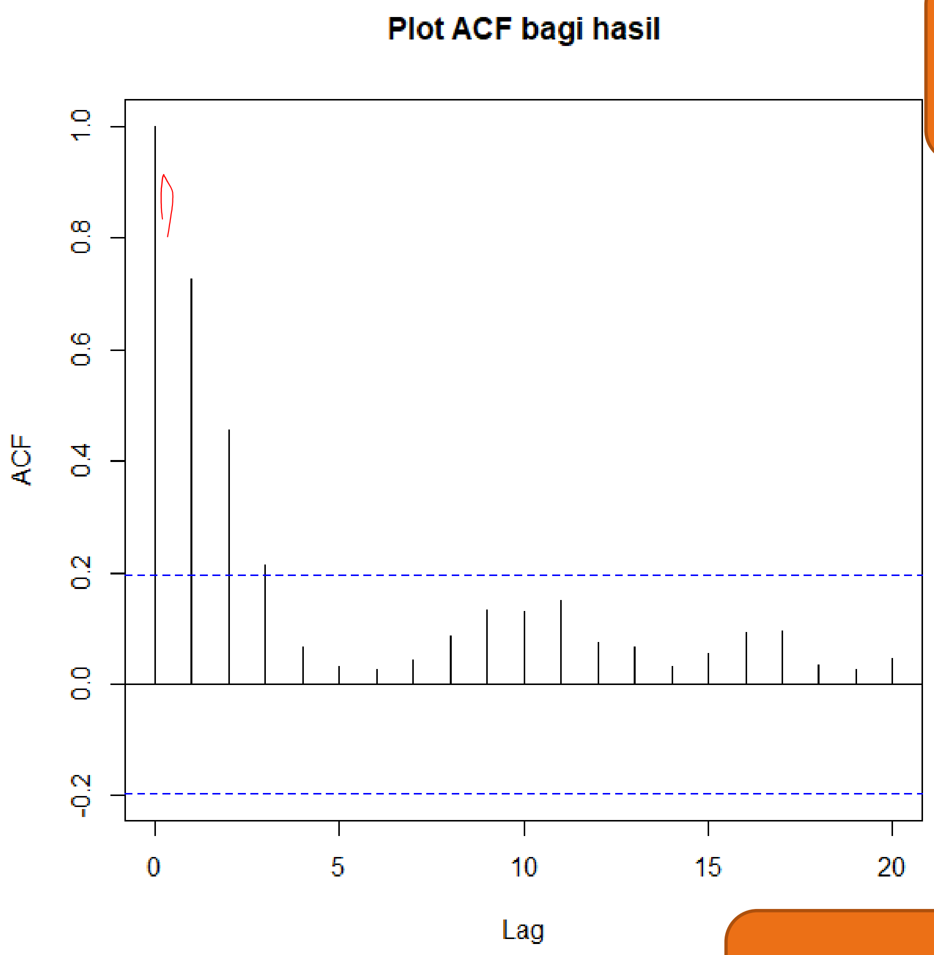


Illustration 1

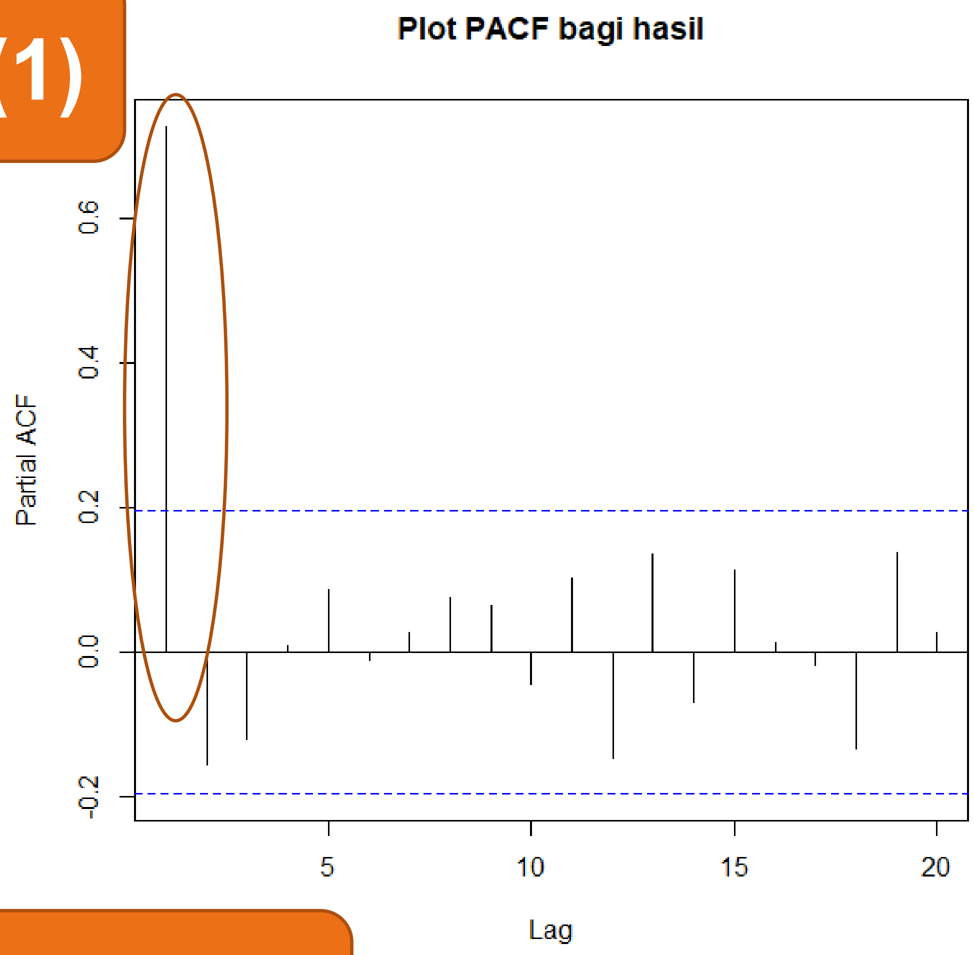
```
> Bagi.hasil
Time Series:
Start = 1
End = 100
Frequency = 1
  [1] 120.8399 120.4160 122.3064 121.5694 122.8942 123.3013 124.1932 124.8930
  [9] 125.8147 125.8902 126.6755 124.1835 125.5775 124.7087 124.9495 125.8825
 [17] 123.9903 124.6912 125.2508 125.9463 123.3904 123.3333 122.6446 123.6364
 [25] 124.5462 123.9536 123.9251 122.7099 122.1322 122.5747 123.7589 122.6723
 [33] 124.0033 123.2013 123.8163 125.2421 126.3553 125.5886 125.1816 123.7251
 [41] 123.4024 123.9143 124.6083 125.0931 124.3820 124.6854 124.7064 124.7050
 [49] 126.2886 127.5864 128.0930 127.2421 124.8627 123.8595 124.4472 123.3074
 [57] 123.8689 124.1007 126.1777 126.9949 125.9848 125.1947 123.4171 124.2836
 [65] 124.4681 125.9491 126.4359 127.6295 128.2976 127.4389 126.4682 125.3810
 [73] 125.5677 124.6353 124.2816 125.8262 126.1171 125.0234 125.2856 124.6220
 [81] 126.8500 127.5602 127.6669 126.7697 126.4150 123.8775 123.6915 125.7046
 [89] 124.9605 124.9237 124.7858 126.4416 126.8498 125.6157 124.8311 124.0431
 [97] 123.6912 124.9995 124.3380 123.7311
```


Plot bagi hasil
Bagi hasil
122
124
126
128
0
20
40
60
80
100
Time


Plot ACF bagi hasil
ACF
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20
Lag



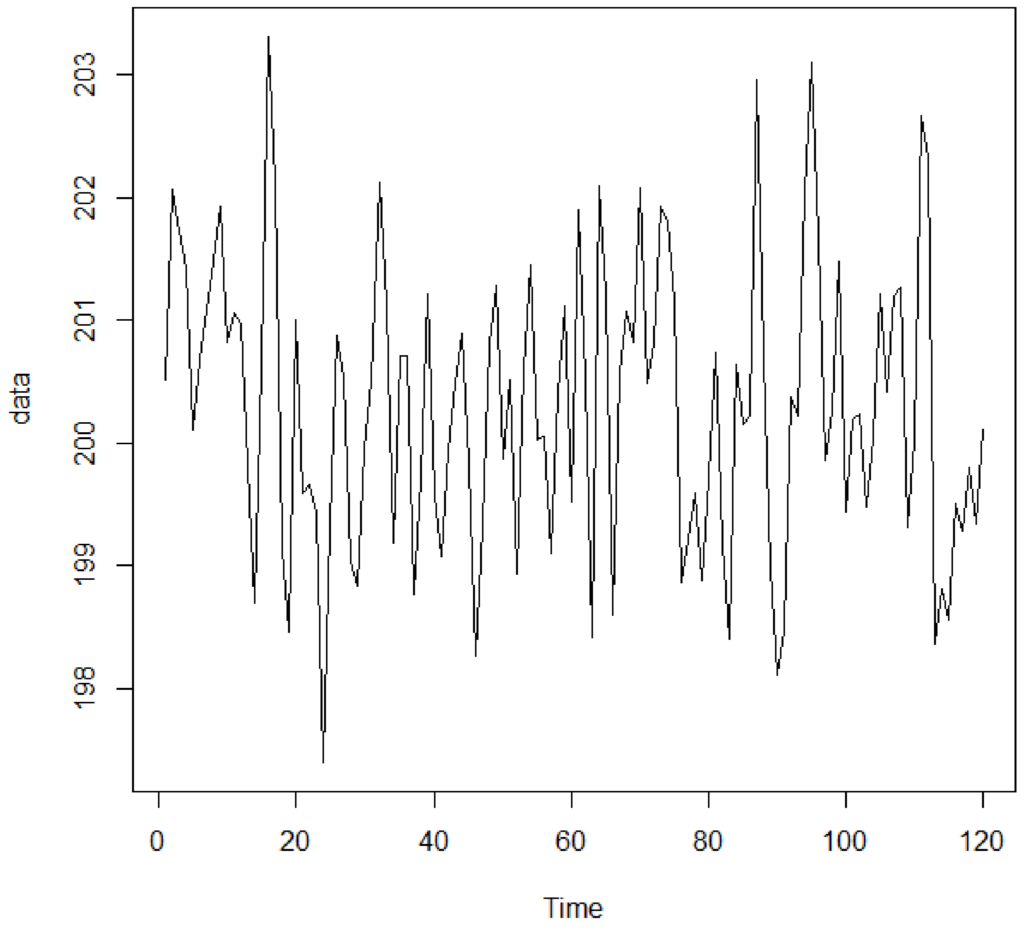
AR(1)



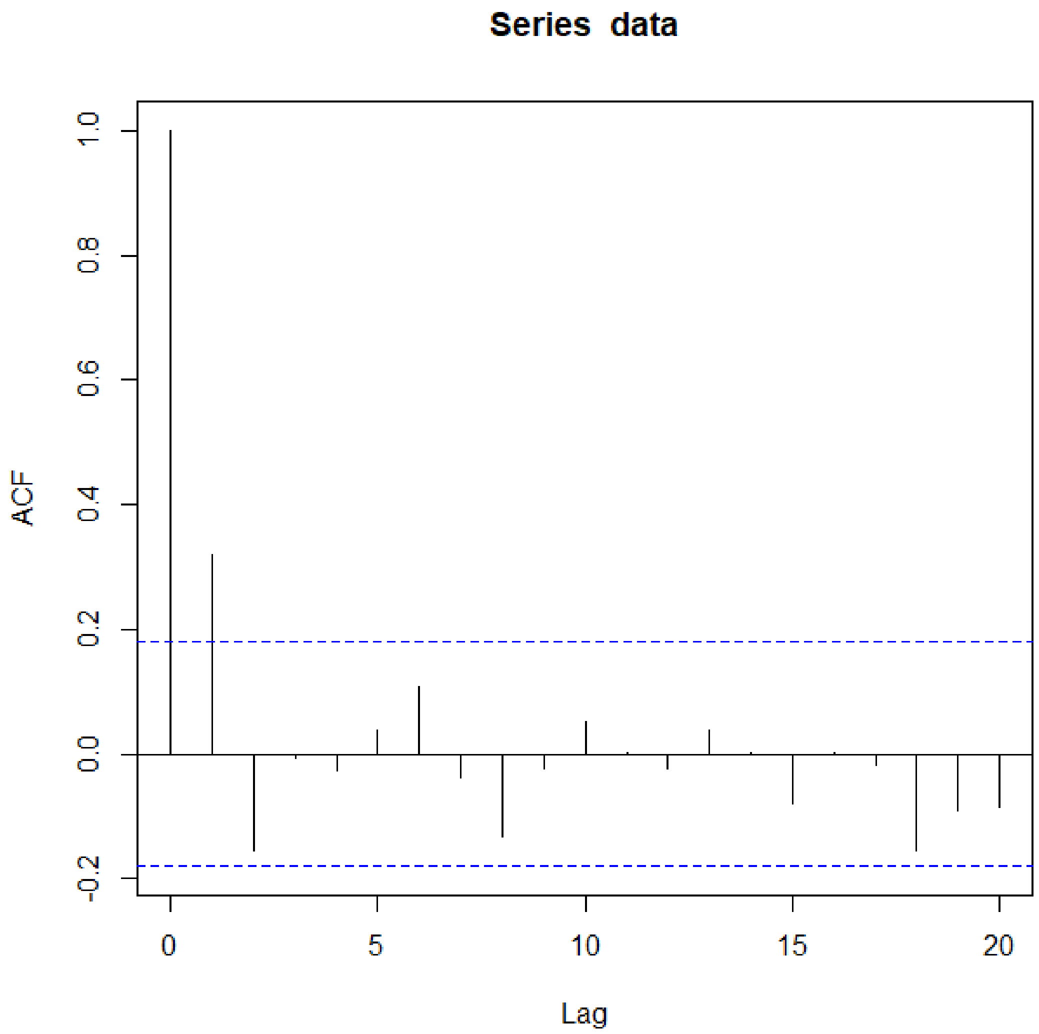
→ **ACF tails off**
→ **PACF cuts off after lag 1**

Illustration 2

```
Time Series:
Start = 1
End = 120
Frequency = 1
  [1] 200.5173 202.0572 201.7297 201.4259 200.1057 200.6729 201.0916 201.4741 201.9279 200.8178 201.0537 200.9672 199.8111 198.6983 200.6401 203.3064 202.1264 199.1470 198.4510 201.0010
 [21] 199.5935 199.6556 199.4319 197.3983 199.3218 200.8780 200.5457 199.0316 198.8325 199.9403 200.4977 202.1172 201.1243 199.1779 200.7120 200.7010 198.7608 199.7088 201.2135 199.5722
 [41] 199.0691 199.9331 200.4803 200.8937 199.8262 198.2573 199.3214 200.7999 201.2875 199.8618 200.5080 198.9363 200.5931 201.4473 200.0211 200.0435 199.0973 200.4031 201.1087 199.5232
 [61] 201.9022 200.5117 198.4120 202.0972 201.2107 198.5957 200.5630 201.0777 200.8211 202.0754 200.4868 200.7986 201.9214 201.7936 201.1510 198.8624 199.2071 199.5827 198.8699 199.7079
 [81] 200.7313 199.1806 198.3952 200.6342 200.1485 200.2179 202.9627 200.7069 198.9806 198.1063 198.4641 200.3711 200.2231 202.0403 203.1007 201.6661 199.8561 200.2652 201.4809 199.4389
[101] 200.1917 200.2379 199.4787 200.0520 201.2102 200.4082 201.1976 201.2709 199.3109 200.0213 202.6683 202.3267 198.3642 198.8088 198.5504 199.5064 199.2750 199.7942 199.3422 200.1082
```

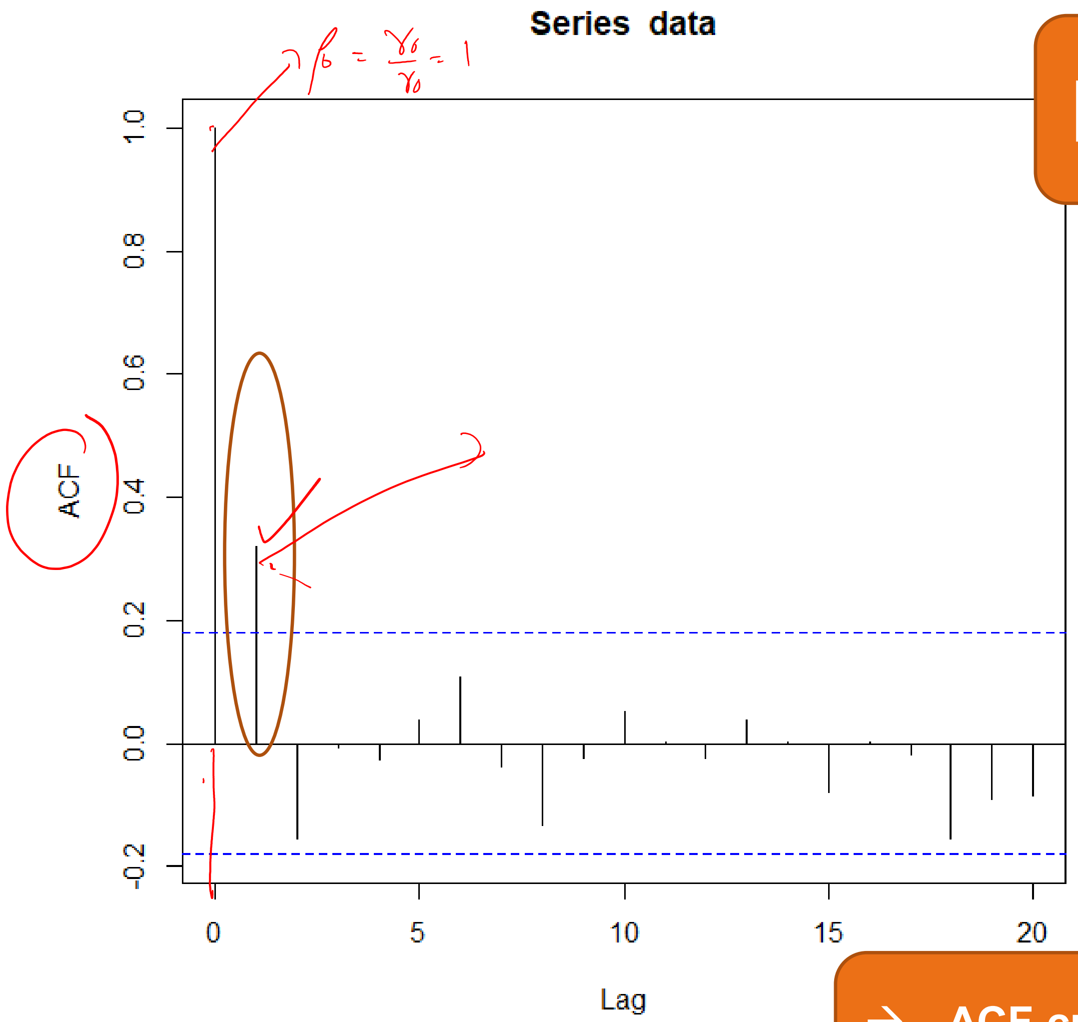


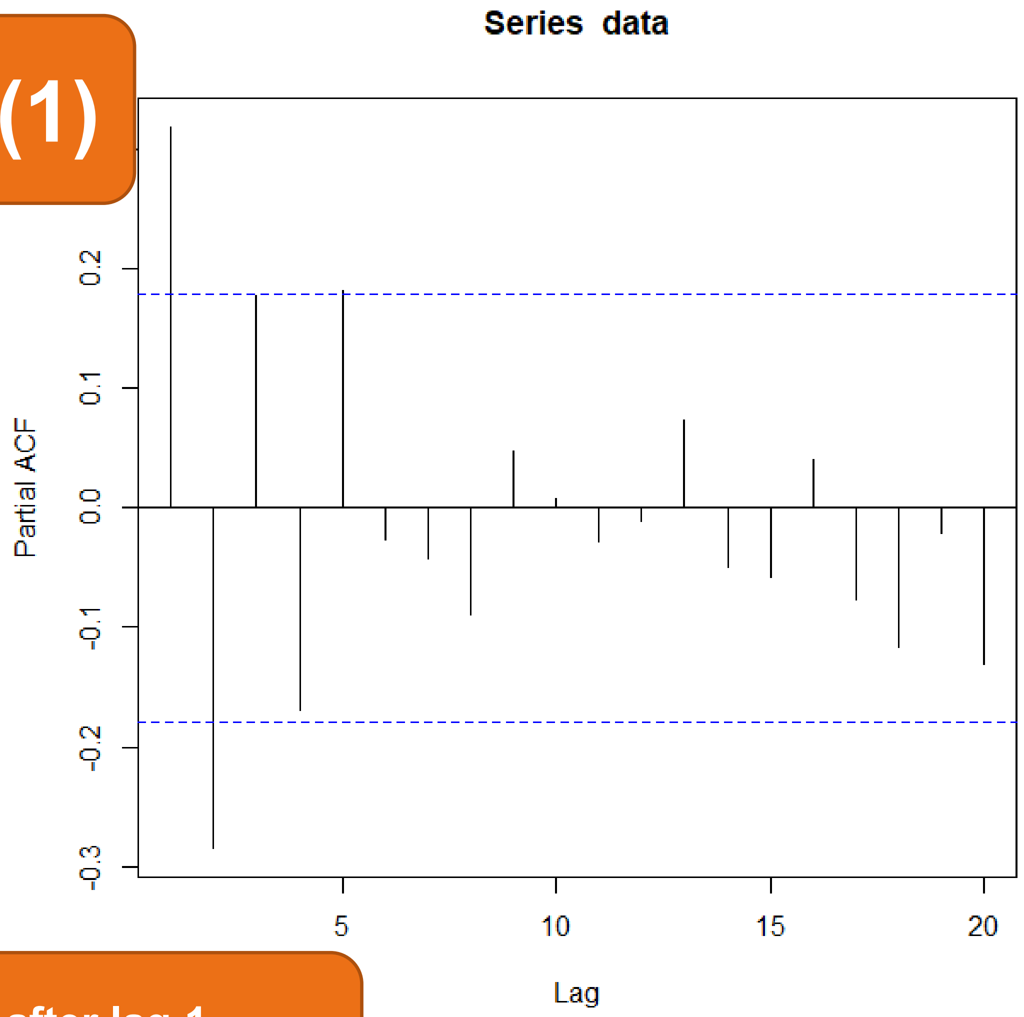
data
Time
198
199
200
201
202
203
0
20
40
60
80
100
120



Series data
ACF
Lag
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20

MA(1)

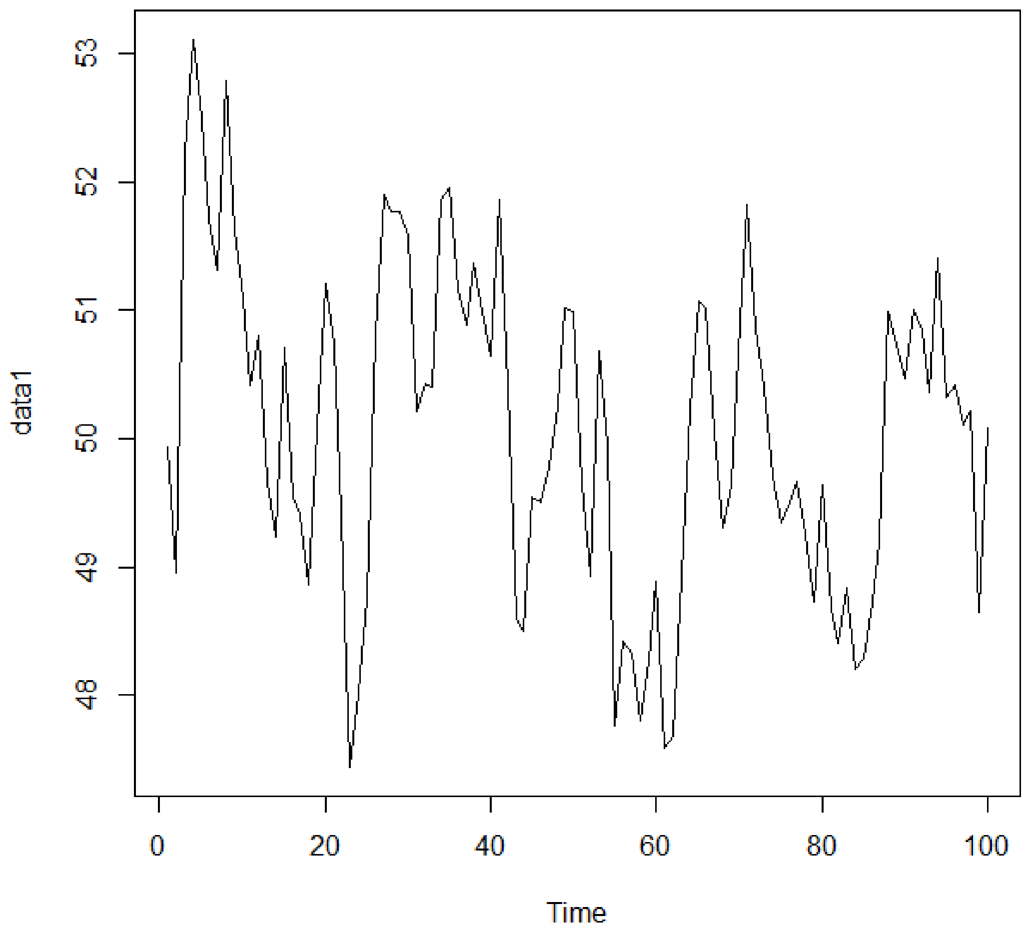




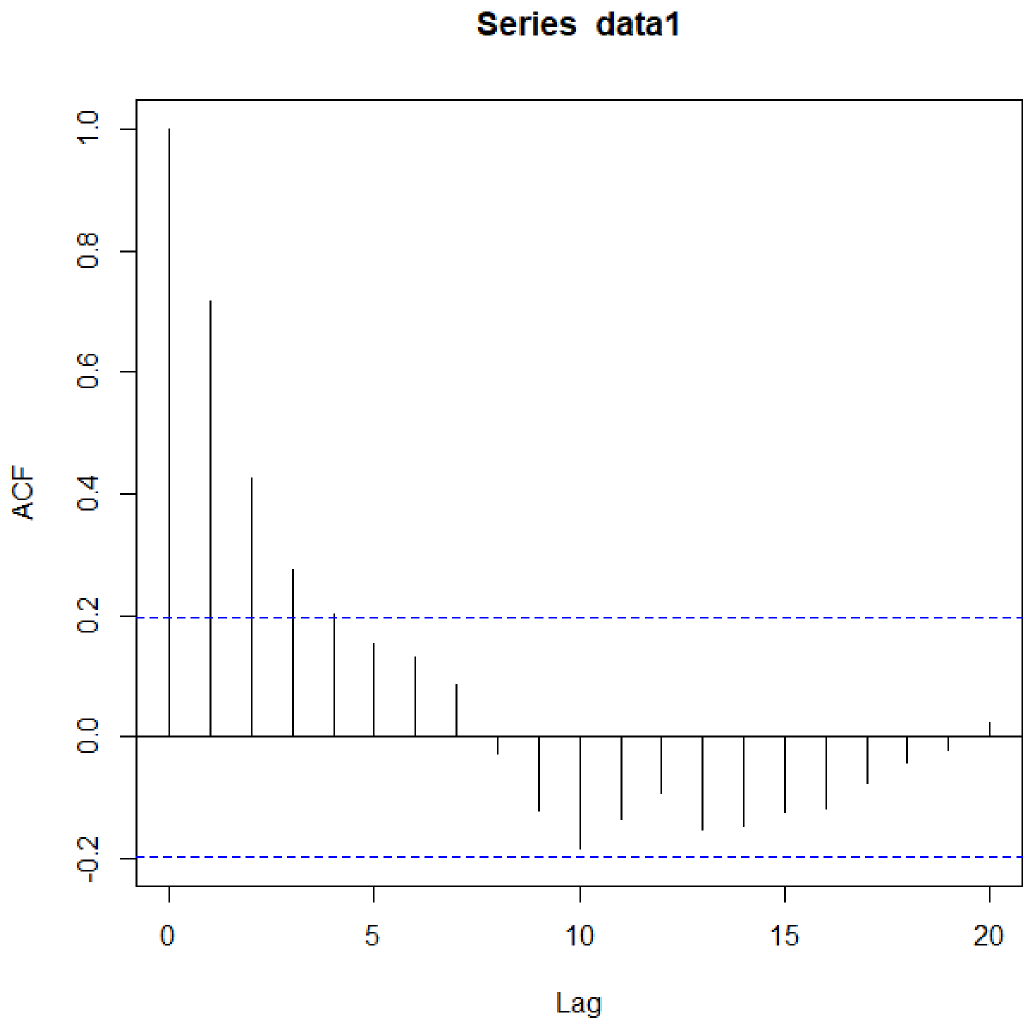
- → ACF cuts off after lag 1
- → PACF tails off

Illustration 3

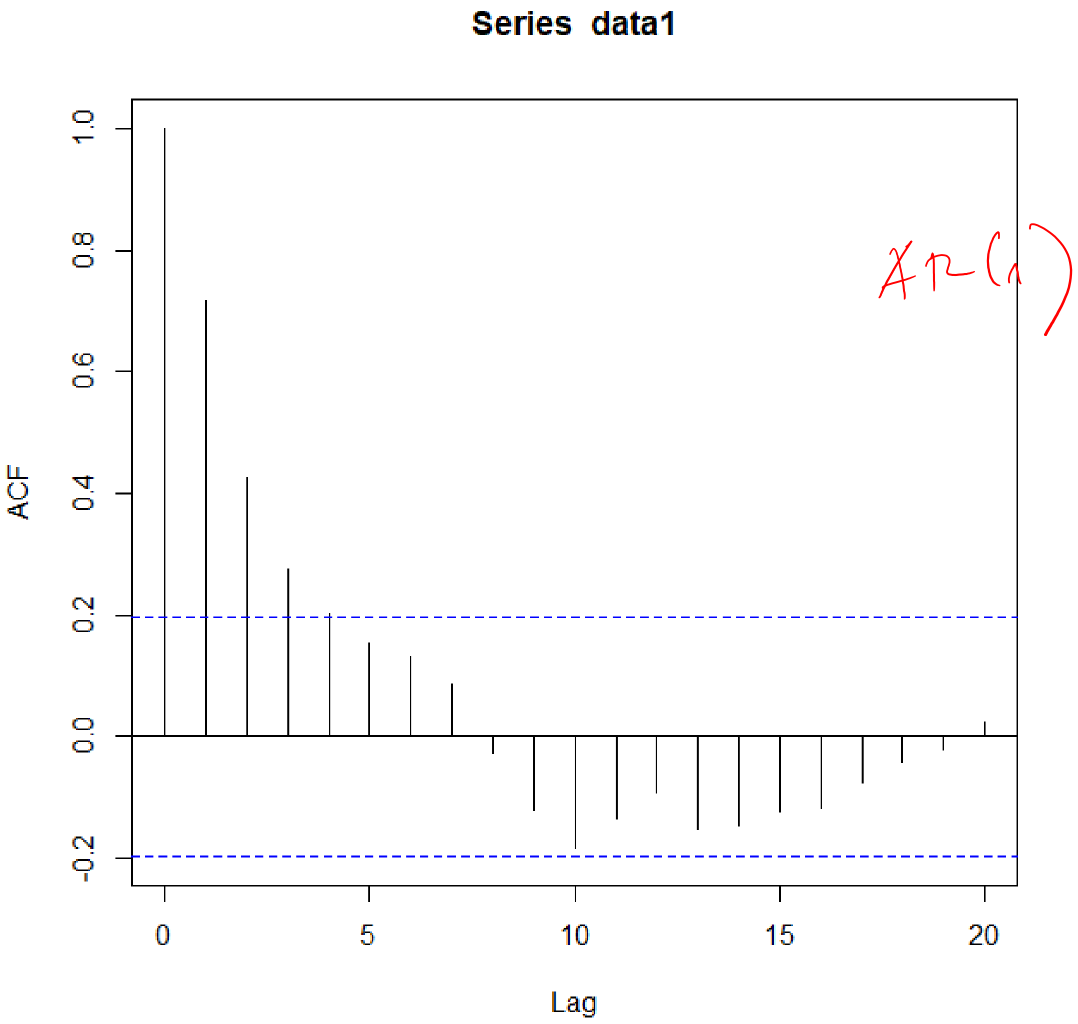
```
Time Series:
Start = 1
End = 100
Frequency = 1
  [1] 49.92942 48.95245 52.22844 53.10579 52.49634 51.72033 51.31350 52.78676
  [9] 51.61496 51.09523 50.41426 50.80596 49.66480 49.24230 50.71685 49.54151
 [17] 49.41205 48.86938 50.08358 51.20780 50.73118 49.40406 47.44811 48.02236
 [25] 48.75842 50.80060 51.89719 51.77418 51.76429 51.58101 50.20945 50.42741
 [33] 50.40793 51.86786 51.95139 51.16796 50.88727 51.36737 50.99544 50.64541
 [41] 51.86159 50.40773 48.60134 48.50495 49.54614 49.50129 49.78693 50.25483
 [49] 51.01725 50.98829 49.64737 48.93415 50.68030 49.94146 47.76121 48.42248
 [57] 48.32285 47.80088 48.27708 48.88886 47.59616 47.68154 48.92910 50.23003
 [65] 51.07214 51.01725 50.05644 49.30724 49.63453 50.91134 51.82234 50.86288
 [73] 50.40121 49.71097 49.34301 49.50765 49.67234 49.26819 48.73062 49.64235
 [81] 48.67032 48.40517 48.84276 48.21299 48.29200 48.72733 49.20141 50.99337
 [89] 50.73869 50.47096 51.00631 50.84143 50.35898 51.40961 50.32619 50.41951
 [97] 50.10471 50.21408 48.64214 50.07742
```

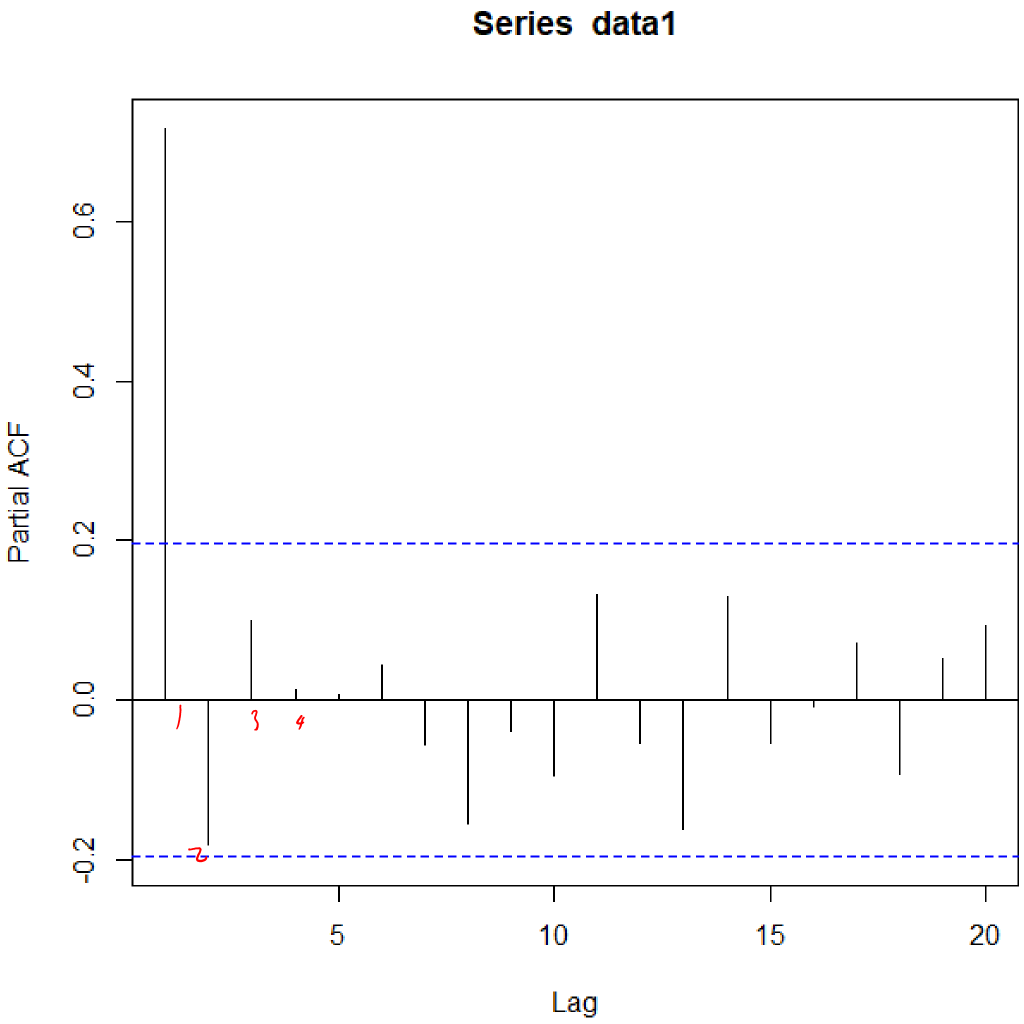


data1
53
52
51
50
49
48
0
20
40
60
80
100
Time



Series data1
ACF
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
0
5
10
15
20
Lag


Series data1
ACF
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
0
5
10
15
20
Lag
AR(1)


Series data1
Partial ACF
0.6
0.4
0.2
0.0
-0.2
5
10
15
20
Lag
1
2
3
4

General Behavior of the ACF and PACF for ARMA Models

	AR(p)	MA(q)	ARMA(p,q), $p>0$, and $q>0$
ACF	Tails off	Cuts off after lag q	Tails off
PACF	Cuts off after lag p	Tails off	Tails off

The Extended Autocorrelation Function

EACF'

EACF (hyun)

ARMA(1,1)

$q=1$

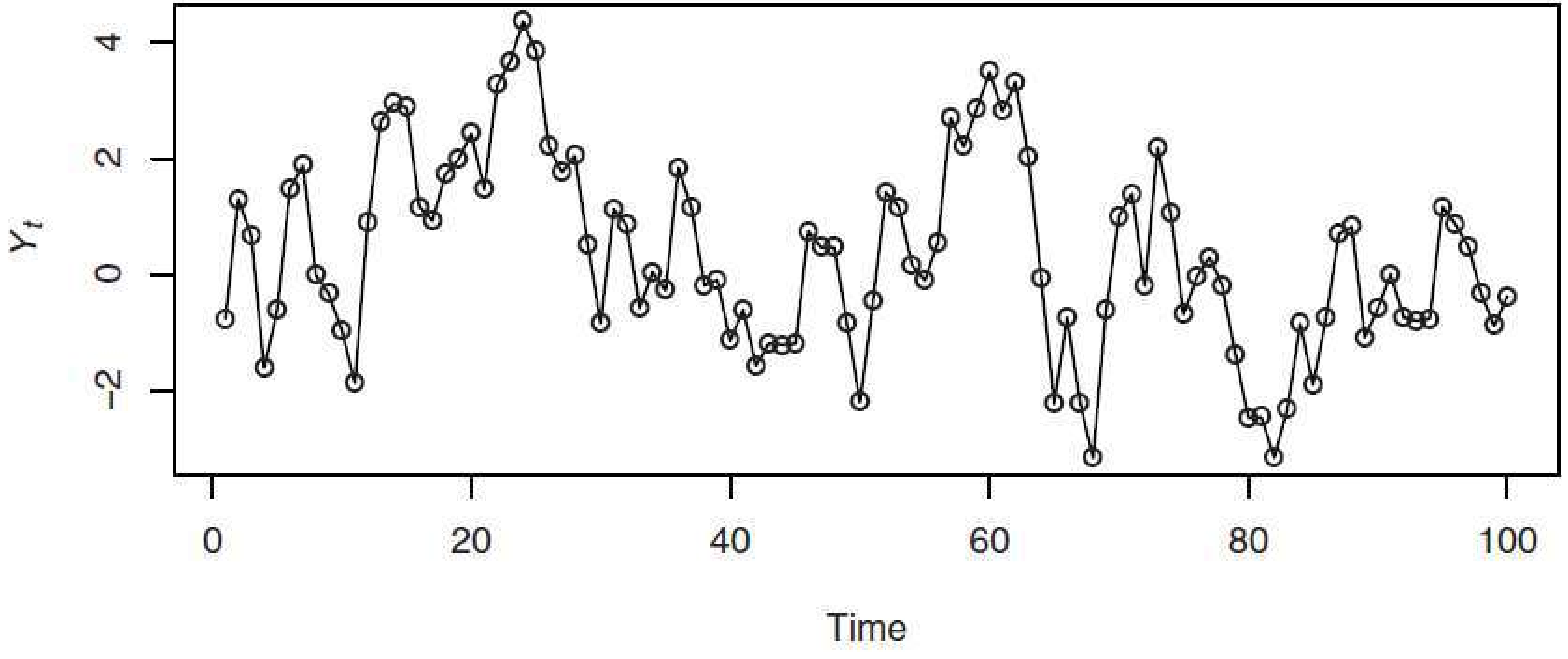
$p=1$

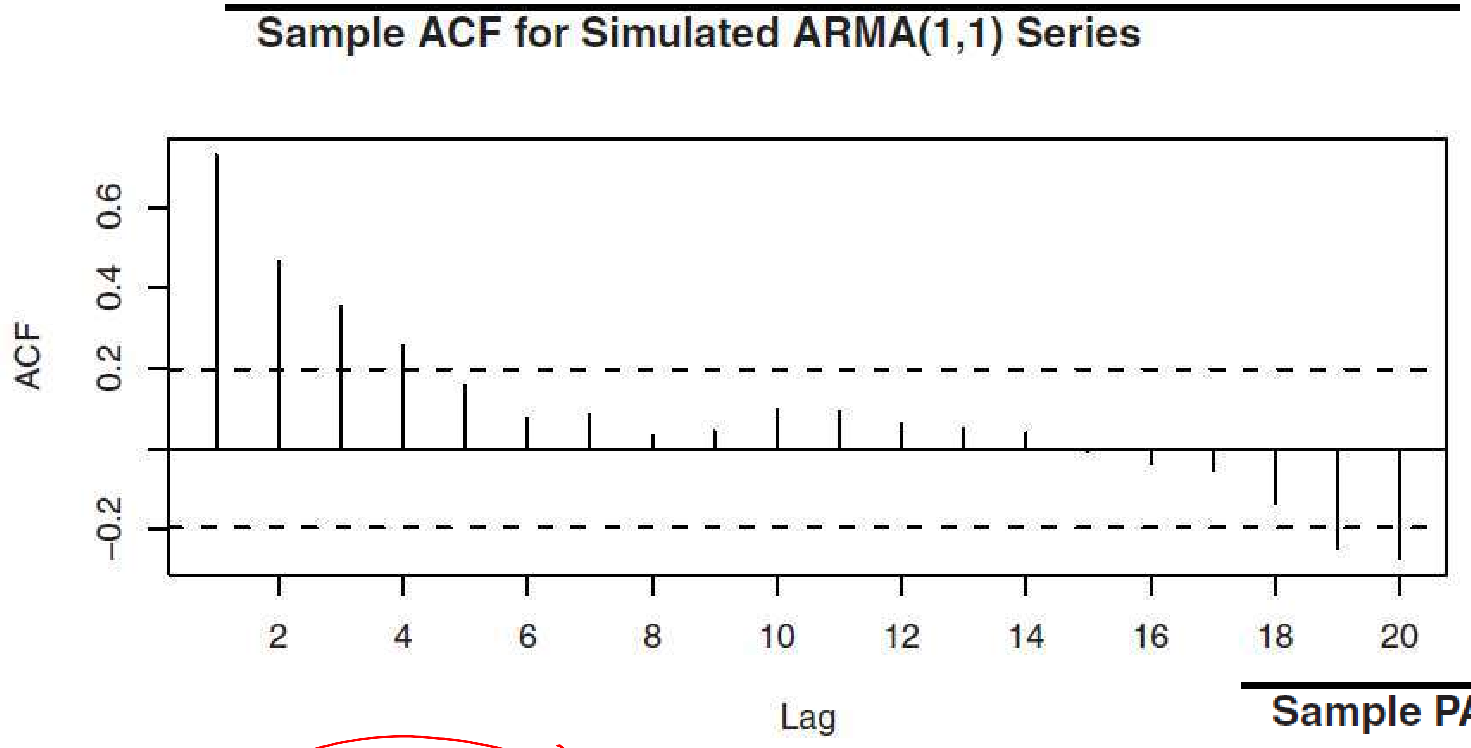
Theoretical Extended ACF (EACF) for an ARMA(1,1) Model

AR/MA	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	x	x	x	x	x	x	x	x	x	x	x	x	x	x
1	x	0*	0	0	0	0	0	0	0	0	0	0	0	0
2	x	x	0	0	0	0	0	0	0	0	0	0	0	0
3	x	x	x	0	0	0	0	0	0	0	0	0	0	0
4	x	x	x	x	0	0	0	0	0	0	0	0	0	0
5	x	x	x	x	x	0	0	0	0	0	0	0	0	0
6	x	x	x	x	x	x	0	0	0	0	0	0	0	0
7	x	x	x	x	x	x	x	0	0	0	0	0	0	0

Specification of ARMA model

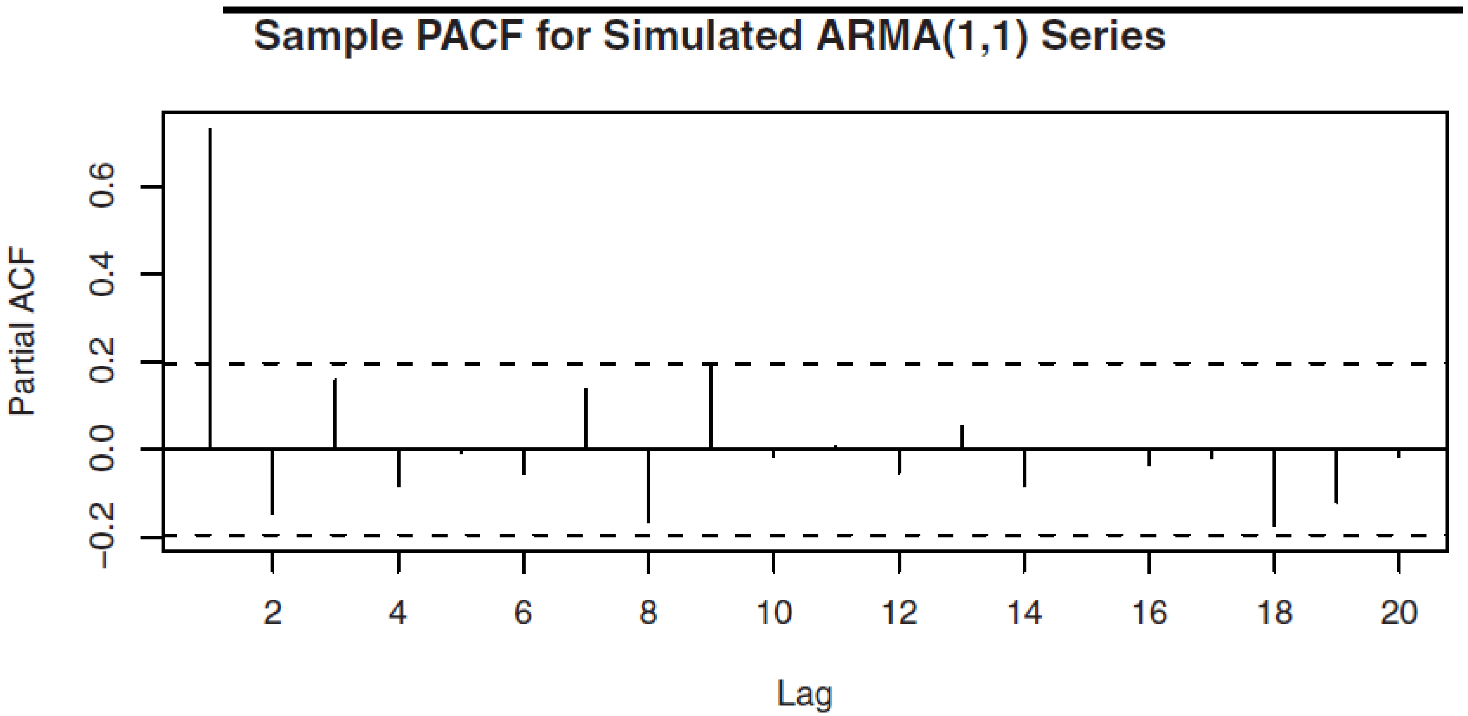
Simulated ARMA(1,1) Series with $\phi = 0.6$ and $\theta = -0.3$.





AR(1)

- → **ACF tails off**
- → **PACF cuts off after lag 1**



AR(1)
ARMA(1,1)
ARMA(2,1)

Sample EACF for Simulated ARMA(1,1) Series

q=1

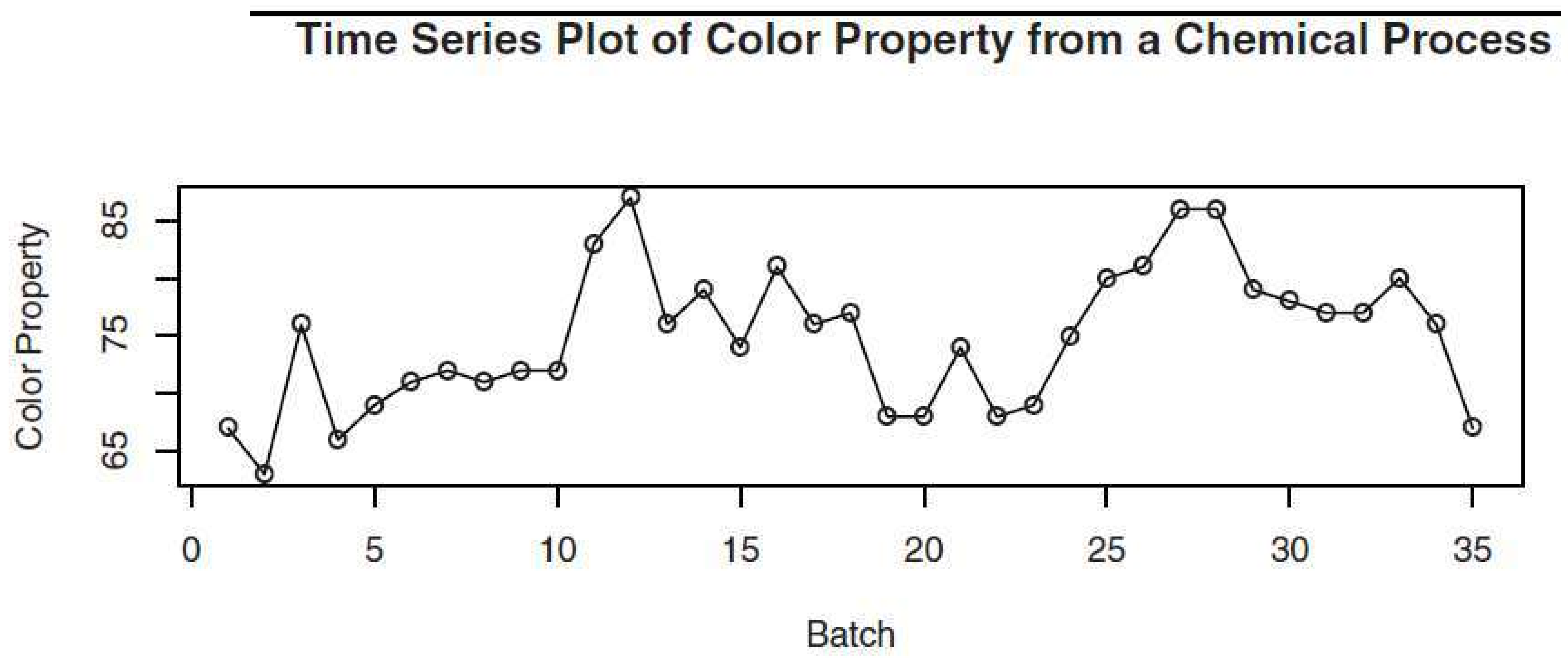
AR / MA	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	x	x	x	x	o	o	o	o	o	o	o	o	o	o
p=1 1	x	o	o	o	o	o	o	o	o	o	o	o	o	o
p=2 2	x	o	o	o	o	o	o	o	o	o	o	o	o	o
3	x	x	o	o	o	o	o	o	o	o	o	o	o	o
4	x	o	x	o	o	o	o	o	o	o	o	o	o	o
5	x	o	o	o	o	o	o	o	o	o	o	o	o	o
6	x	o	o	o	x	o	o	o	o	o	o	o	o	o
7	x	o	o	o	x	o	o	o	o	o	o	o	o	o

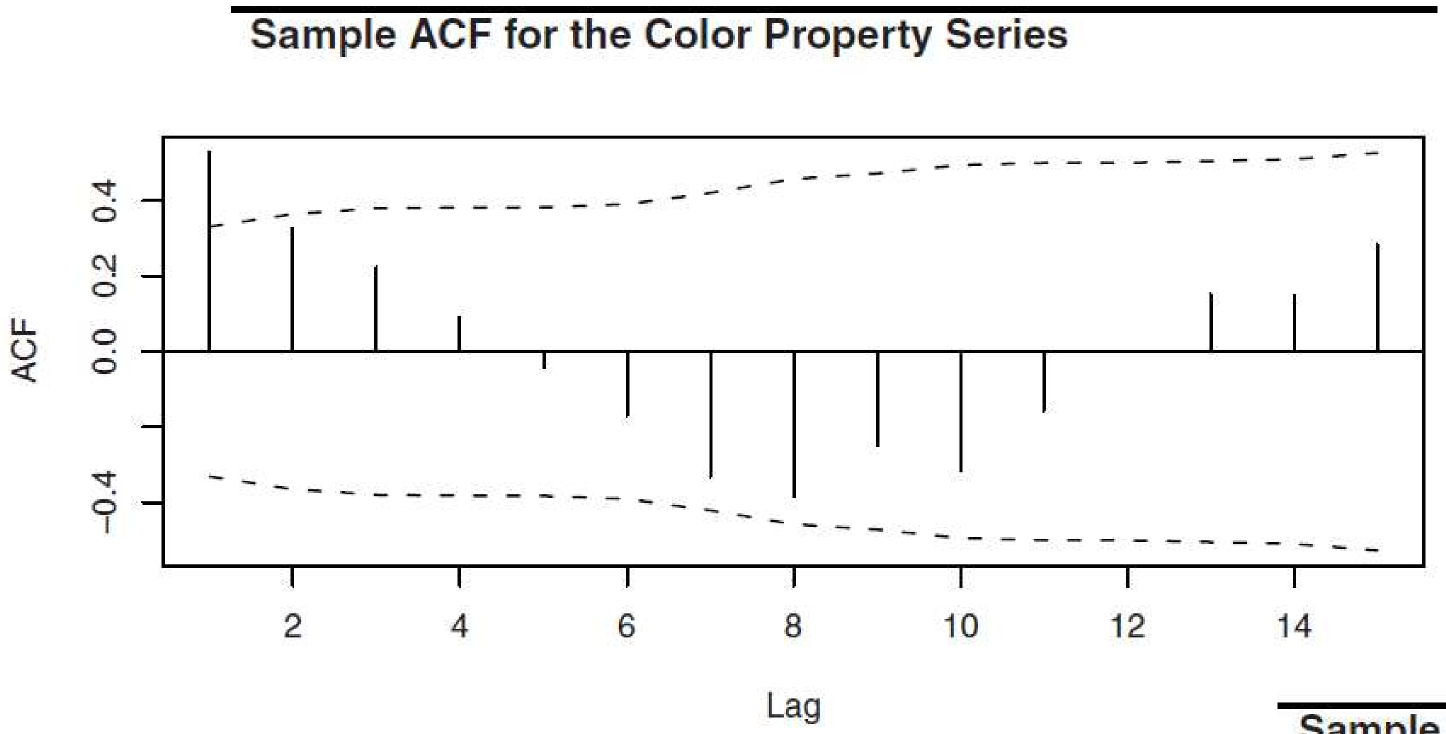
$p = 1, q = 1$ ✓

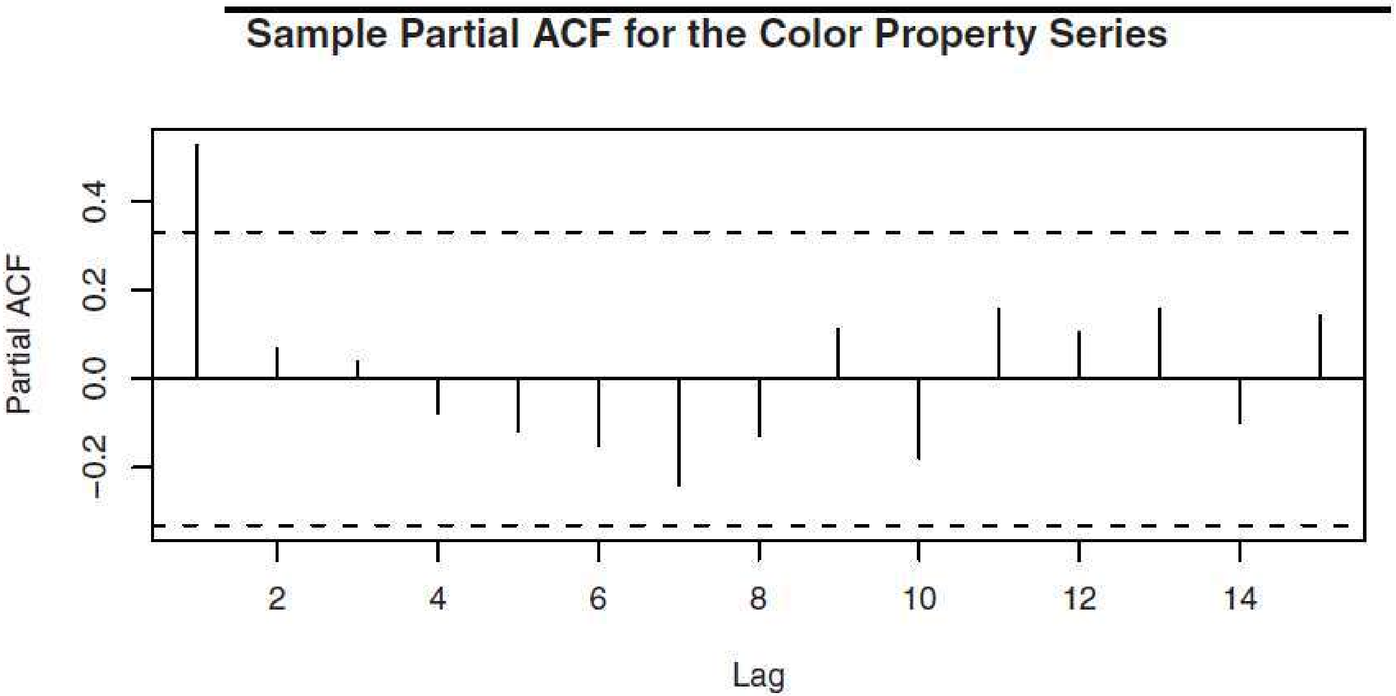
$p = 2, q = 1$ ✓

Specification for Some actual time series data

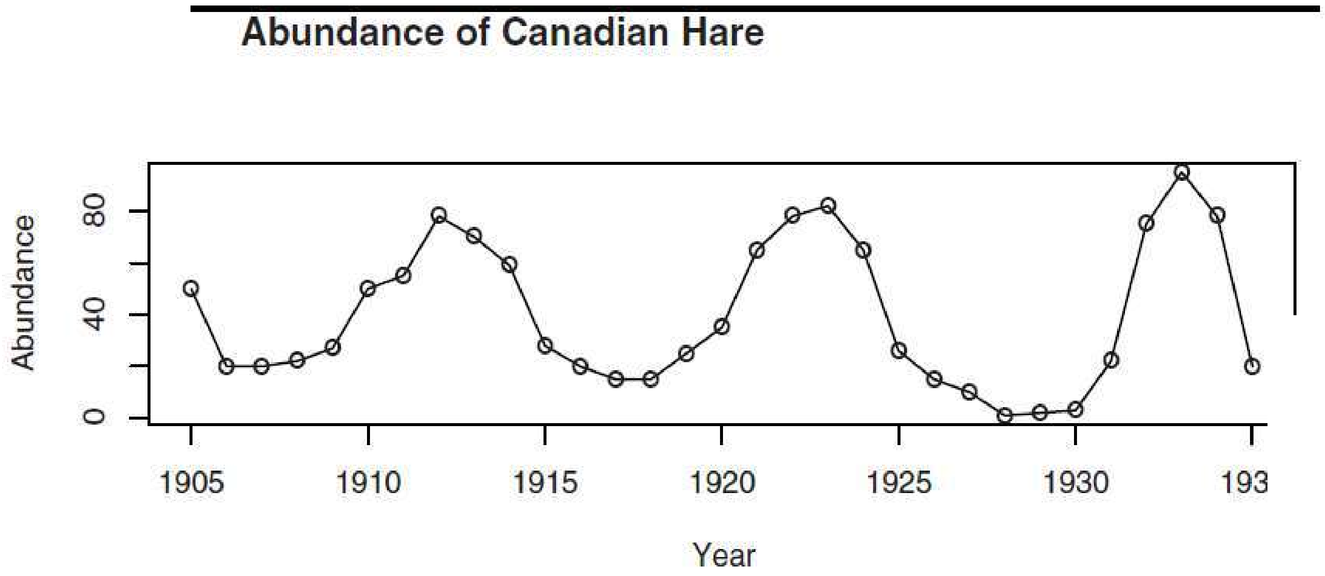
The Chemical Process Color Property Series

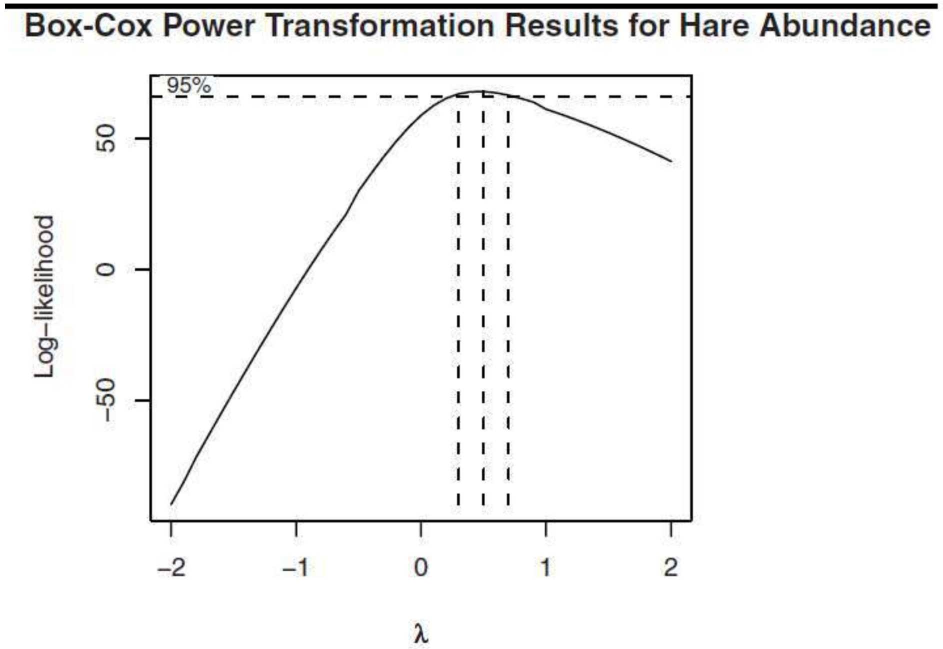



Sample ACF for the Color Property Series
ACF
0.4
0.2
0.0
−0.4
2
4
6
8
10
12
14
Lag


Sample Partial ACF for the Color Property Series
Partial ACF
0.4
0.2
0.0
−0.2
2
4
6
8
10
12
14
Lag

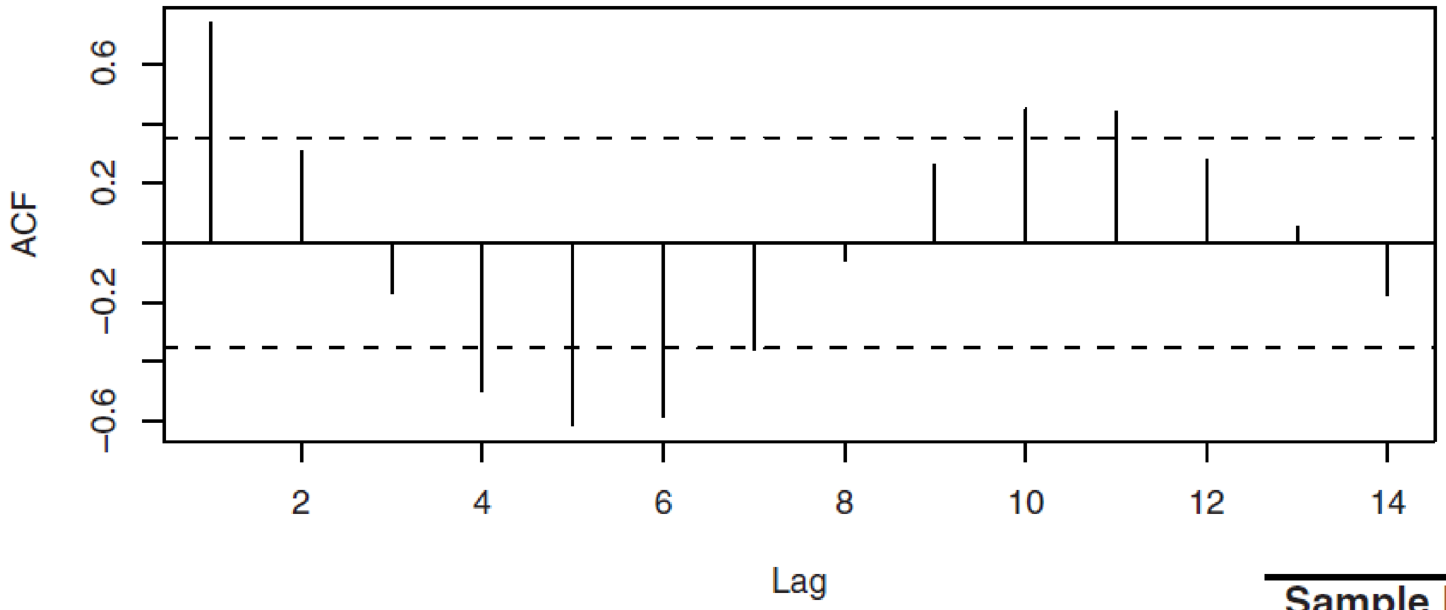
The Annual Abundance of Canadian Hare Series



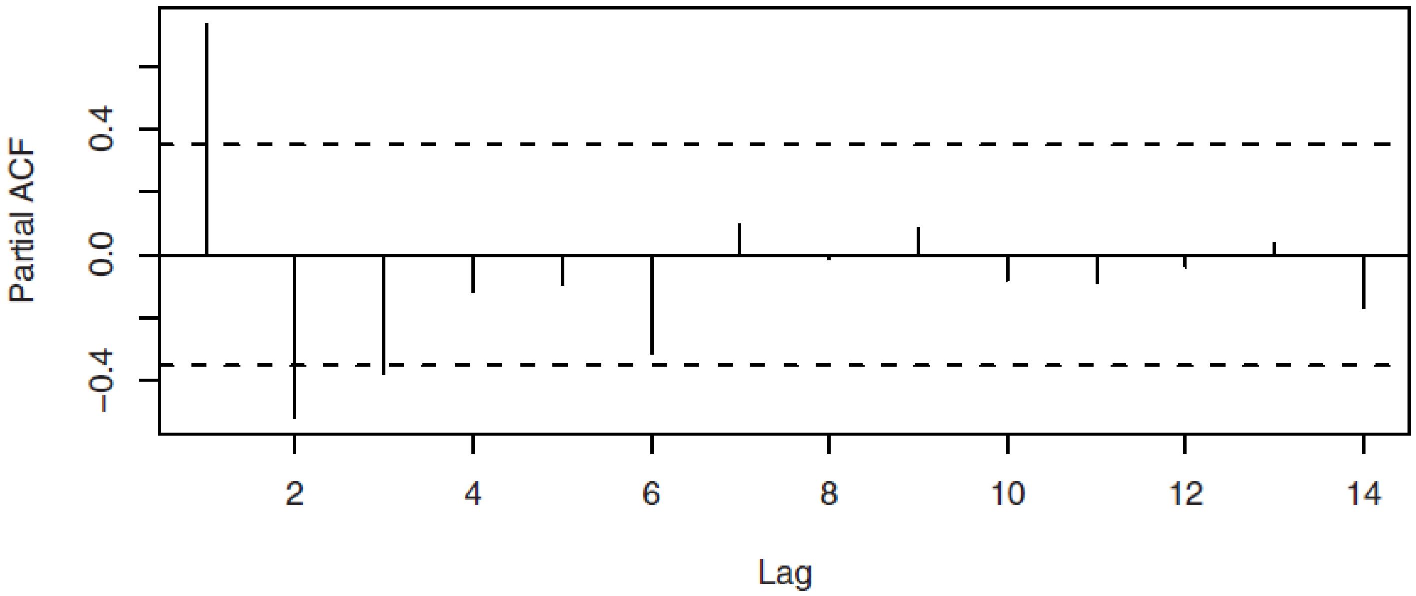


λ	Transformed Data
-2	y^{-2}
-1	y^{-1}
-0.5	$1/\sqrt{y}$
0	ln(y)
0.5	$\sqrt{y}$
1	y
2	y^2

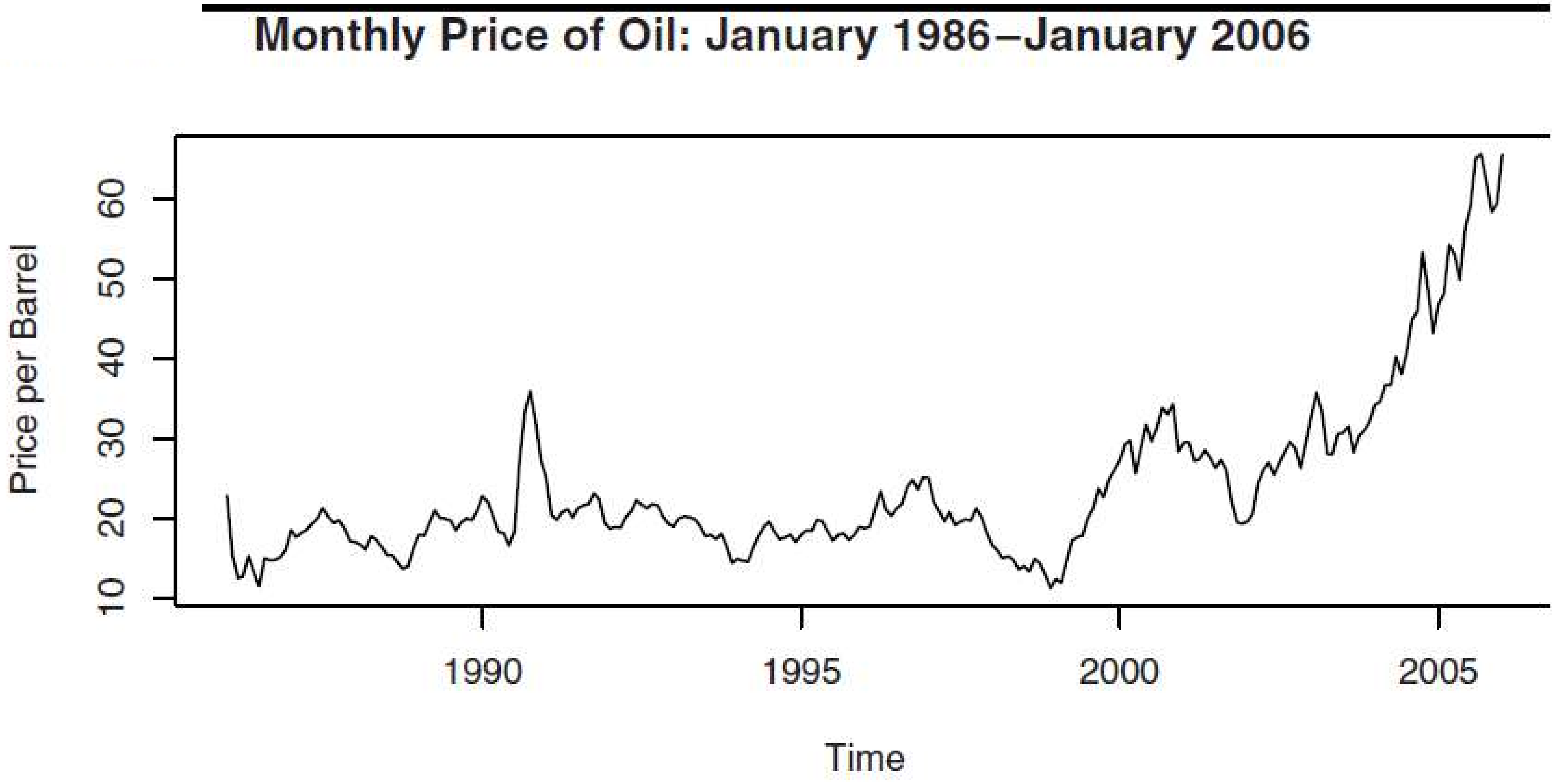
Sample ACF for Square Root of Hare Abundance



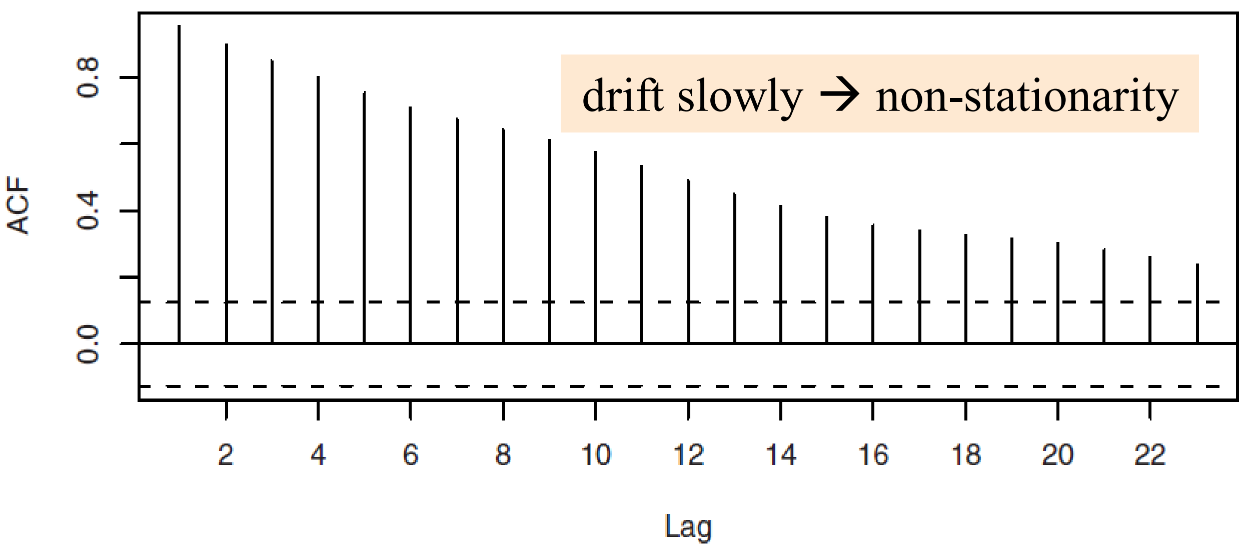
Sample Partial ACF for Square Root of Hare Abundance



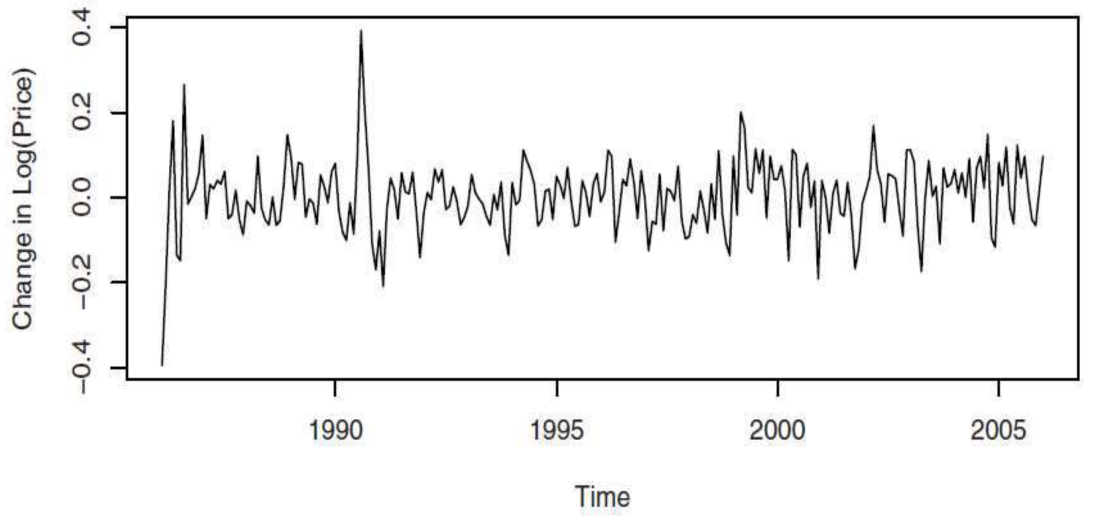
Monthly Price of Oil: January 1986-January 2006



Sample ACF for the Oil Price Time Series

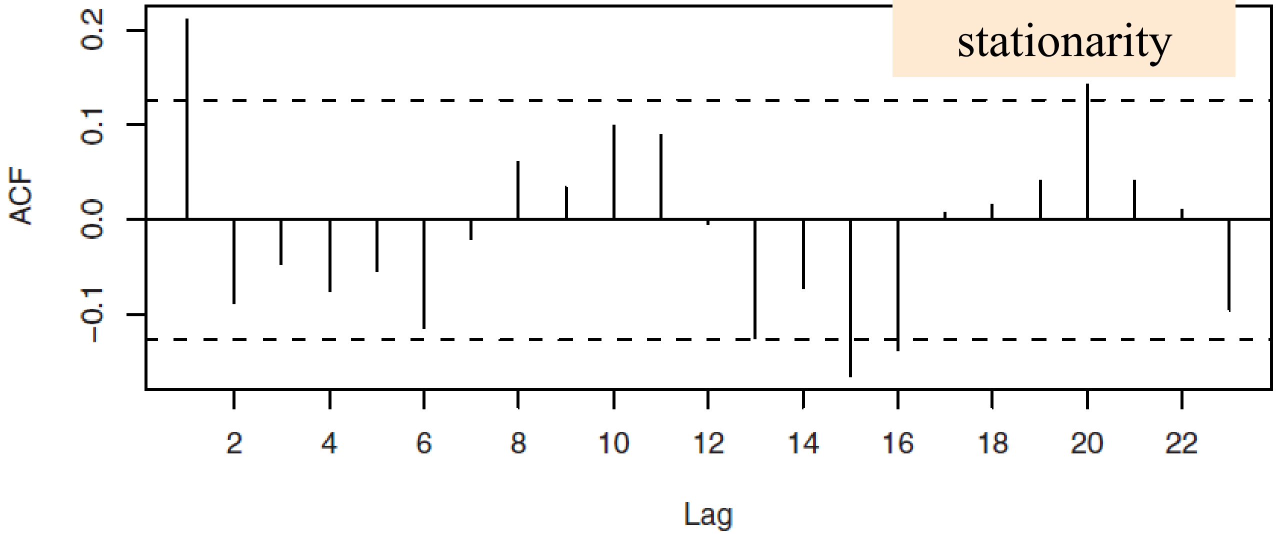


The Difference Series of the Logs of the Oil Price Time



Monthly Price of Oil → Box-cox transformation using log → differencing ($d = 1$)

Sample ACF for the Difference of the Log Oil Price Series



Extended ACF for Difference of Logarithms of Oil Price Series

AR / MA	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	x	o	o	o	o	o	o	o	o	o	o	o	o	o
1	x	x	o	o	o	o	o	o	o	o	x	o	o	o
2	o	x	o	o	o	o	o	o	o	o	o	o	o	o
3	o	x	o	o	o	o	o	o	o	o	o	o	o	o
4	o	x	x	o	o	o	o	o	o	o	o	o	o	o
5	o	x	o	x	o	o	o	o	o	o	o	o	o	o
6	o	x	o	x	o	o	o	o	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	o
7	x	x	o	x	o	o	o	o	o	o	o	o	o	o

$p = 0, q = 1$

Exercises

6.23 Simulate an AR(1) time series with $\phi = 0.6$, with

(a) $n = 24$, and estimate $\rho_1 = \phi = 0.6$ with r_1;
(b) $n = 60$, and estimate $\rho_1 = \phi = 0.6$ with r_1;
(c) $n = 120$, and estimate $\rho_1 = \phi = 0.6$ with r_1.
(d) For each of the series in parts (a), (b), and (c), compare the estimated values with the theoretical value. Use Equation (6.1.5) on page 111, to quantify the comparisons. In general, describe how the precision of the estimate varies with the sample size.

6.24 Simulate an MA(1) time series with $\theta = 0.7$, with

(a) $n = 24$, and estimate ρ_1 with r_1;
(b) $n = 60$, and estimate ρ_1 with r_1;
(c) $n = 120$, and estimate ρ_1 with r_1.
(d) For each of the series in parts (a), (b), and (c), compare the estimated values of ρ_1 with the theoretical value. Use Exhibit 6.2 on page 112, to quantify the comparisons. In general, describe how the precision of the estimate varies with the sample size.

Pemodelan Box-Jenkins

1. Cek stasioner data — plot TS / Plot ACF / Uji ADF → pembedaan

↓ Ragam → transformasi Box-cox

Stasioner dlm rataan & ragam.

2. Identifikasi model (p, d, q) → ACF, PACF & EACF

→ Model tentatif

3. Pendugaan Parameter

→ 1. Apakah semua parameter dr setiap model nyata

2. AIC & BIC → yg nilainya paling kecil

→ Satu model dg AIC & BIC terkecil? AR(1)

4. Diagnostik Model → asumsi sisaan (AHN) AR(1) ✓

5. Overfitting → AR(2) $\boxed{\text{AIC \& BIC ? < asumsi sisaan ?}}$ AIC & BIC AR(1) ✓

c. Validasi Model

6. Forecasting.

$H_0: \sigma_1^2 = \sigma_2^2$

80% 20%

trans

test

AR(2)

e?

MAPE

MSE

MAD

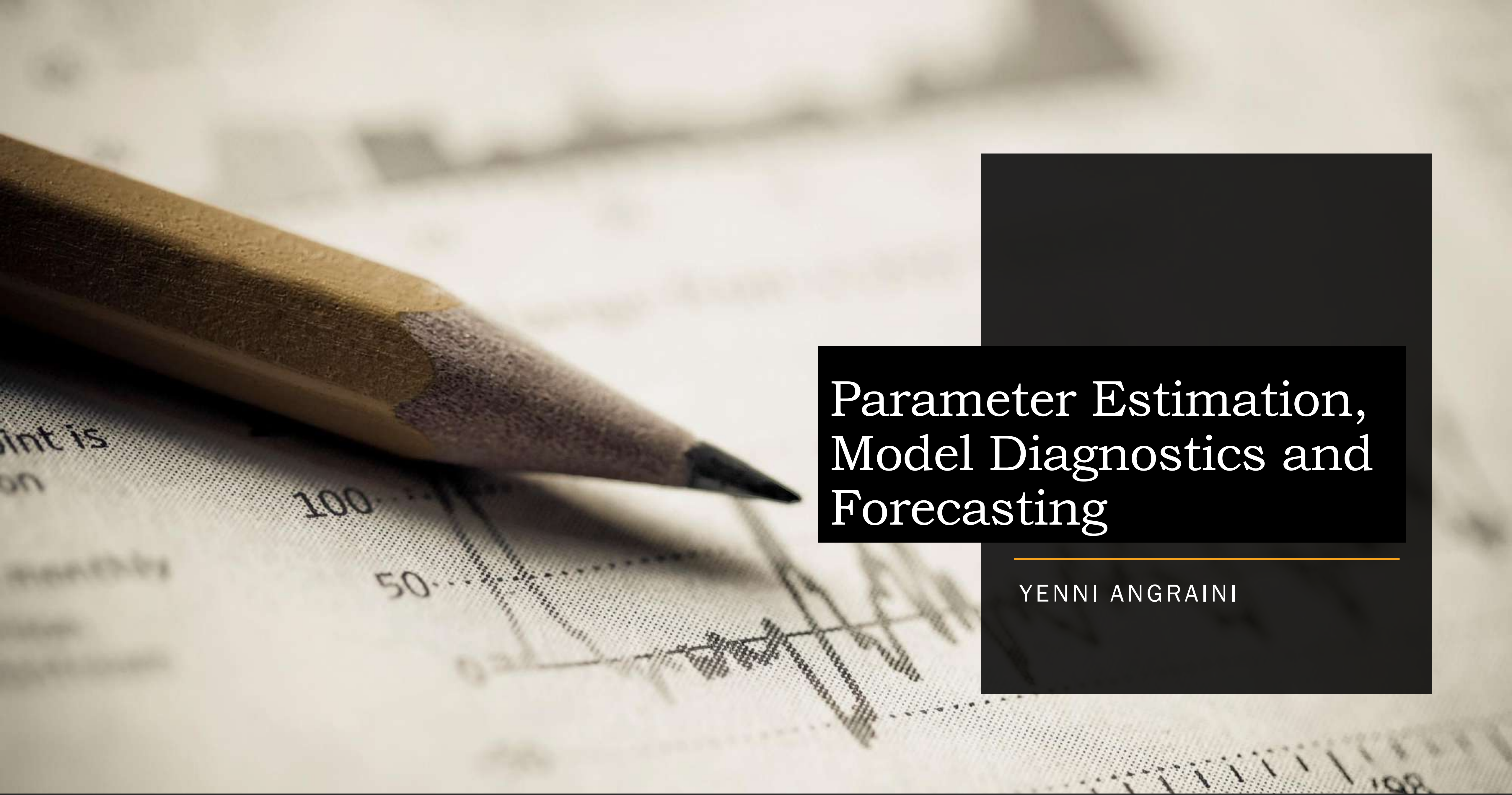
ARMA(1,1)

↓

ARMA(2,1)

ARMA(1,2)


THANK
YOU



Parameter Estimation, Model Diagnostics and Forecasting

YENNI ANGRAINI

Outline

- Parameter estimation method for AR, MA and ARMA :
  - The method of moments,
  - Least Square Estimation and
  - Maximum Likelihood
- Model Diagnostics : Residual Analysis and Overfitting
- Forecasting
- Illustration

Parameter estimation : The method of moments

- The method of moments is frequently one of the easiest
- The method consists of equating sample moments to corresponding theoretical moments and solving the resulting equations to obtain estimates of any unknown parameters

Parameter estimation :
The method of moments for **Autoregressive Models**

- Consider first the AR(1) case.
- For this process, we have the simple relationship $\rho_1 = \phi$.
- In the method of moments, ρ_1 is equated to r_1, the lag 1 sample autocorrelation.
- Thus we can estimate ϕ by $\hat{\phi} = r_1$

Parameter estimation :
The method of moments for **Autoregressive Models**

Now consider the AR(2) case.

The relationships between the parameters ϕ_1 and ϕ_2 and various moments are given by the Yule-Walker equations

$$\rho_1 = \phi_1 + \rho_1\phi_2 \; and \; \rho_2 = \rho_1\phi_1 + \phi_2 \; \rightarrow r_1 = \phi_1 + r_1\phi_2 \; and \; r_2 = r_1\phi_1 + \phi_2$$

which are then solved to obtain $\hat{\phi}_1 = \frac{r_1(1-r_2)}{1-r_1^2} \; and \; \hat{\phi}_2 = \frac{r_2-r_1^2}{1-r_1^2}$

Parameter estimation : The method of moments for **Autoregressive Models**

The general AR(p) case proceeds similarly. Replace ρ_k by r_k throughout the Yule-Walker equations to obtain

$$\left.\begin{array}{rrrcrcl}
\phi_1 + & r_1\phi_2 + & r_2\phi_3 & + \cdots + & r_{p-1}\phi_p & = & r_1 \\
r_1\phi_1 + & \phi_2 + & r_1\phi_3 & + \cdots + & r_{p-2}\phi_p & = & r_2 \\
 & & & & & \vdots & \\
r_{p-1}\phi_1 + & r_{p-2}\phi_2 + & r_{p-3}\phi_3 & + \cdots + & \phi_p & = & r_p
\end{array}\right\}$$

Parameter estimation : The method of moments for **Moving Average Models**

The method of moments is not nearly as convenient when applied to moving average models.

Consider the simple MA(1) case $\rho_1 = -\frac{\theta}{1+\theta^2}$, equating ρ_1 and r_1, we are led to solve a quadratic equation in θ.

$$\hat{\theta} = \frac{-1 + \sqrt{1 - 4r_1^2}}{2r_1}$$

For higher-order MA models, the method of moments quickly gets complicated

Parameter estimation :
The method of moments for **ARMA Models**

The ARMA(1,1) case

$$\rho_k = \frac{(1-\theta\phi)(\phi-\theta)}{1-2\theta\phi+\theta^2}\phi^{k-1} \quad \text{for } k \geq 1$$

Noting that $\rho_2/\rho_1 = \phi$, we can first estimate ϕ as

$$\hat{\phi} = \frac{r_2}{r_1}$$

Having done so, we can then use

$$r_1 = \frac{(1-\theta\hat{\phi})(\hat{\phi}-\theta)}{1-2\theta\hat{\phi}+\theta^2}$$

Note again that a quadratic equation must be solved.

Parameter estimation :
The method of moments for **the Noise Variance**

For the AR(p) models

$$\hat{\sigma}_e^2 = (1 - \hat{\phi}_1 r_1 - \hat{\phi}_2 r_2 - \cdots - \hat{\phi}_p r_p) s^2$$

$$s^2 = \frac{1}{n-1} \sum_{t=1}^{n} (Y_t - \bar{Y})^2$$

In particular, for an AR(1) process,

$$\hat{\sigma}_e^2 = (1 - r_1^2) s^2$$

since $\hat{\phi} = r_1$

Parameter estimation :
The method of moments for **the Noise Variance**

For the MA(q) case

$$\hat{\sigma}_e^2 = \frac{s^2}{1 + \hat{\theta}_1^2 + \hat{\theta}_2^2 + \cdots + \hat{\theta}_q^2}$$

$$s^2 = \frac{1}{n-1}\sum_{t=1}^{n}(Y_t - \bar{Y})^2$$

For the ARMA(1,1) process

$$\hat{\sigma}_e^2 = \frac{1 - \hat{\phi}^2}{1 - 2\hat{\phi}\hat{\theta} + \hat{\theta}^2}s^2$$

Numerical example for method of moments parameter estimation

	True Parameters			Method-of-Moments Estimates			
Model	θ	ϕ_1	ϕ_2	θ	ϕ_1	ϕ_2	n
MA(1)	–0.9			–0.554			120
MA(1)	0.9			0.719			120
MA(1)	–0.9			NA†			60
MA(1)	0.5			–0.314			60
AR(1)		0.9			0.831		60
AR(1)		0.4			0.470		60
AR(2)		1.5	–0.75		1.472	–0.767	120

† No method-of-moments estimate exists since $r_1 = 0.544$ for this simulation.

Parameter estimation :
Least Squares Estimation for **Autoregressive Models**

Consider the first-order case where

$$Y_t - \mu = \phi(Y_{t-1} - \mu) + e_t$$

AR(1)

We can view this as a regression model with predictor variable Y_{t-1} and response variable Y_t. Least squares estimation then proceeds by minimizing the sum of squares of the differences

$$(Y_t - \mu) - \phi(Y_{t-1} - \mu)$$

Since only $Y_1, Y_2, \ldots, Y_n$ are observed, we can only sum from $t = 2$ to $t = n$. Let

$$S_c(\phi, \mu) = \sum_{t=2}^{n} [(Y_t - \mu) - \phi(Y_{t-1} - \mu)]^2$$

Conditional sum-of squares function

Parameter estimation : Least Squares Estimation for **Autoregressive Models**

Consider the equation $\partial S_c / \partial \mu = 0$. We have

$$\frac{\partial S_c}{\partial \mu} = \sum_{t=2}^{n} 2[(Y_t - \mu) - \phi(Y_{t-1} - \mu)](-1 + \phi) = 0$$

or, simplifying and solving for μ,

$$\boxed{\mu = \frac{1}{(n-1)(1-\phi)}\left[\sum_{t=2}^{n} Y_t - \phi \sum_{t=2}^{n} Y_{t-1}\right]}$$

Parameter estimation : Least Squares Estimation for **Autoregressive Models**

Now, for large n,

$$\frac{1}{n-1}\sum_{t=2}^{n} Y_t \approx \frac{1}{n-1}\sum_{t=2}^{n} Y_{t-1} \approx \bar{Y}$$

$$\hat{\mu} \approx \frac{1}{1-\phi}(\bar{Y} - \phi\bar{Y}) = \bar{Y}$$

We sometimes say, except for end effects, $\hat{\mu} = \bar{Y}$.

Parameter estimation : Least Squares Estimation for **Autoregressive Models**

Consider now the minimization of $S_c(\phi, \bar{Y})$ with respect to ϕ. We have

$$\frac{\partial S_c(\phi, \bar{Y})}{\partial \phi} = \sum_{t=2}^{n} 2[(Y_t - \bar{Y}) - \phi(Y_{t-1} - \bar{Y})](Y_{t-1} - \bar{Y})$$

Setting this equal to zero and solving for ϕ yields

$$\boxed{\hat{\phi} = \frac{\sum_{t=2}^{n}(Y_t - \bar{Y})(Y_{t-1} - \bar{Y})}{\sum_{t=2}^{n}(Y_{t-1} - \bar{Y})^2}}$$

Parameter estimation : Least Squares Estimation for **Moving Average Models**

Consider now the least-squares estimation of θ in the MA(1) model:

$$Y_t = e_t - \theta e_{t-1}$$

$$Y_t = -\theta Y_{t-1} - \theta^2 Y_{t-2} - \theta^3 Y_{t-3} - \cdots + e_t$$

an autoregressive model but of infinite order. Thus least squares can be meaningfully carried out by choosing a value of θ that minimizes

Parameter estimation : Least Squares Estimation for **Moving Average Models**

$$S_c(\theta) = \sum(e_t)^2 = \sum[Y_t + \theta Y_{t-1} + \theta^2 Y_{t-2} + \theta^3 Y_{t-3} + \cdots]^2$$

where, implicitly, $e_t = e_t(\theta)$ is a function of the observed series and the unknown parameter θ.

- It is clear from this equation that the least squares problem is *nonlinear* in the parameters.
- We will not be able to minimize $Sc(\theta)$ by taking a derivative with respect to θ, setting it to zero, and solving→ numerical optimization

Parameter estimation : Maximum Likelihood

- For any set of observations, Y_1, Y_2,…, Y_n, time series or not, the likelihood function L is defined to be the joint probability density of obtaining the data actually observed
- It is considered as a function of the unknown parameters in the model with the observed data held fixed
- For ARIMA models, L will be a function of the ϕ's, θ's, μ, and σ_e^2 given the observations Y_1, Y_2,…, Y_n.
- The maximum likelihood estimators are then defined as those values of the parameters for which the data actually observed are *most likely*, that is, the values that maximize the likelihood function.

Parameter estimation : Maximum Likelihood AR(1)

The likelihood function for an AR(1) model is given by

$$L(\phi,\mu,\sigma_e^2) = (2\pi\sigma_e^2)^{-\frac{n}{2}}(1-\phi^2)^{1/2} exp\left[-\frac{1}{2\sigma_e^2}S(\phi,\mu)\right]$$

Where

$$S(\phi,\mu) = \sum_{t=2}^{n}[(Y_t-\mu)-\phi(Y_{t-1}-\mu)]^2 + (1-\phi^2)(Y_1-\mu)$$

Unconditional sum-of squares function

The log-likelihood function for an AR(1) model is given by

$$l(\phi,\mu,\sigma_e^2) = -\frac{n}{2}\log(2\pi) - -\frac{n}{2}\log(\sigma_e^2) + \frac{1}{2}\log(1-\phi^2) - \frac{1}{2\sigma_e^2}S(\phi,\mu)$$

Properties of the Estimates

For large n, the estimators are approximately unbiased and normally distributed. The variances and correlations are as follows:

$$\text{AR(1)}: \ Var(\hat{\phi}) \approx \frac{1-\phi^2}{n}$$

$$\text{AR(2)}: \begin{cases} Var(\hat{\phi}_1) \approx Var(\hat{\phi}_2) \approx \dfrac{1-\phi_2^2}{n} \\ Corr(\hat{\phi}_1, \hat{\phi}_2) \approx -\dfrac{\phi_1}{1-\phi_2} = -\rho_1 \end{cases}$$

$$\text{MA(1)}: \ Var(\hat{\theta}) \approx \frac{1-\theta^2}{n}$$

$$\text{MA(2)}: \begin{cases} Var(\hat{\theta}_1) \approx Var(\hat{\theta}_2) \approx \dfrac{1-\theta_2^2}{n} \\ Corr(\hat{\theta}_1, \hat{\theta}_2) \approx -\dfrac{\theta_1}{1-\theta_2} \end{cases}$$

$$\text{ARMA(1,1)}: \begin{cases} Var(\hat{\phi}) \approx \left[\dfrac{1-\phi^2}{n}\right]\left[\dfrac{1-\phi\theta}{\phi-\theta}\right]^2 \\ Var(\hat{\theta}) \approx \left[\dfrac{1-\theta^2}{n}\right]\left[\dfrac{1-\phi\theta}{\phi-\theta}\right]^2 \\ Corr(\hat{\phi}, \hat{\theta}) \approx \dfrac{\sqrt{(1-\phi^2)(1-\theta^2)}}{1-\phi\theta} \end{cases}$$

Illustrations of Parameter Estimation

Parameter Estimation for Simulated AR(1) Models

Parameter ϕ	Method-of-Moments Estimate	Conditional SS Estimate	Unconditional SS Estimate	Maximum Likelihood Estimate	n
0.9	0.831	0.857	0.911	0.892	60
0.4	0.470	0.473	0.473	0.465	60

$$\sqrt{\hat{Var}(\hat{\phi})} \approx \sqrt{\frac{1-\hat{\phi}^2}{n}} = \sqrt{\frac{1-(0.831)^2}{60}} \approx 0.07$$

$$\sqrt{\hat{Var}(\hat{\phi})} = \sqrt{\frac{1-(0.470)^2}{60}} \approx 0.11$$

Illustrations of Parameter Estimation

Parameter Estimation for a Simulated AR(2) Model

Parameters	Method-of-Moments Estimates	Conditional SS Estimates	Unconditional SS Estimates	Maximum Likelihood Estimate	n
$\phi_1 = 1.5$	1.472	1.5137	1.5183	1.5061	120
$\phi_2 = -0.75$	–0.767	–0.8050	–0.8093	–0.7965	120

$$\sqrt{\widehat{Var}(\hat{\phi}_1)} \approx \sqrt{\widehat{Var}(\hat{\phi}_2)} \approx \sqrt{\frac{1-\phi_2^2}{n}} = \sqrt{\frac{1-(0.75)^2}{120}} \approx 0.06$$

Illustrations of Parameter Estimation

Parameter Estimation for a Simulated ARMA(1,1) Model

Parameters	Method-of-Moments Estimates	Conditional SS Estimates	Unconditional SS Estimates	Maximum Likelihood Estimate	n
$\phi = 0.6$	0.637	0.5586	0.5691	0.5647	100
$\theta = -0.3$	–0.2066	–0.3669	–0.3618	–0.3557	100

Illustrations of Parameter Estimation

Parameter Estimation for the Color Property Series

Parameter	Method-of-Moments Estimate	Conditional SS Estimate	Unconditional SS Estimate	Maximum Likelihood Estimate	n
ϕ	0.5282	0.5549	0.5890	0.5703	35

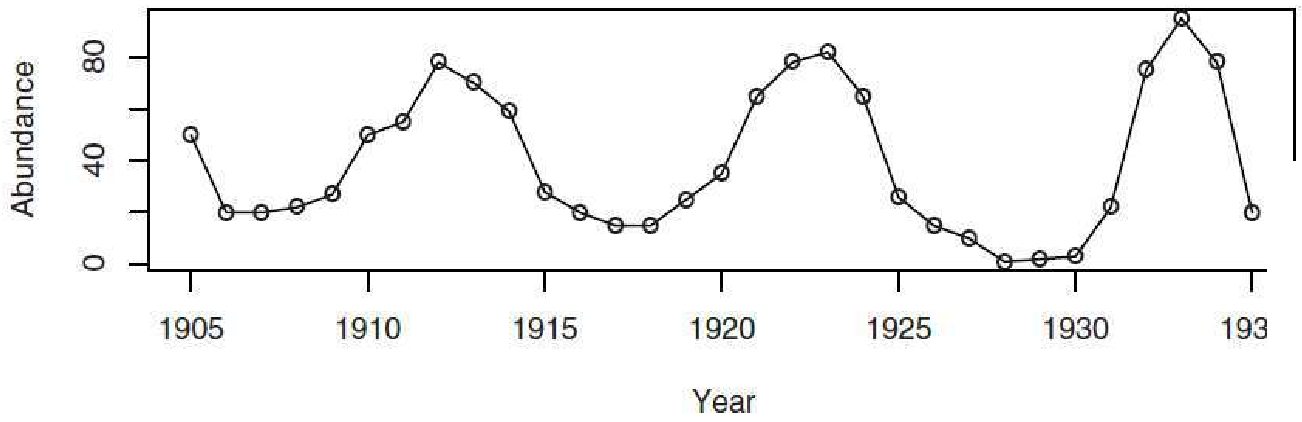
the standard error of the estimates is about

$$\sqrt{\hat{Var}(\hat{\phi})} \approx \sqrt{\frac{1-(0.57)^2}{35}} \approx 0.14$$

Illustrations of Parameter Estimation

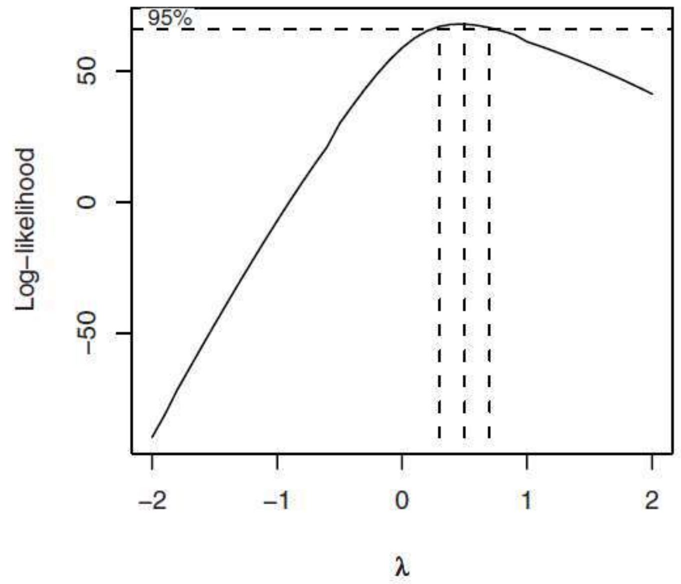
The Annual Abundance of Canadian Hare Series

Abundance of Canadian Hare

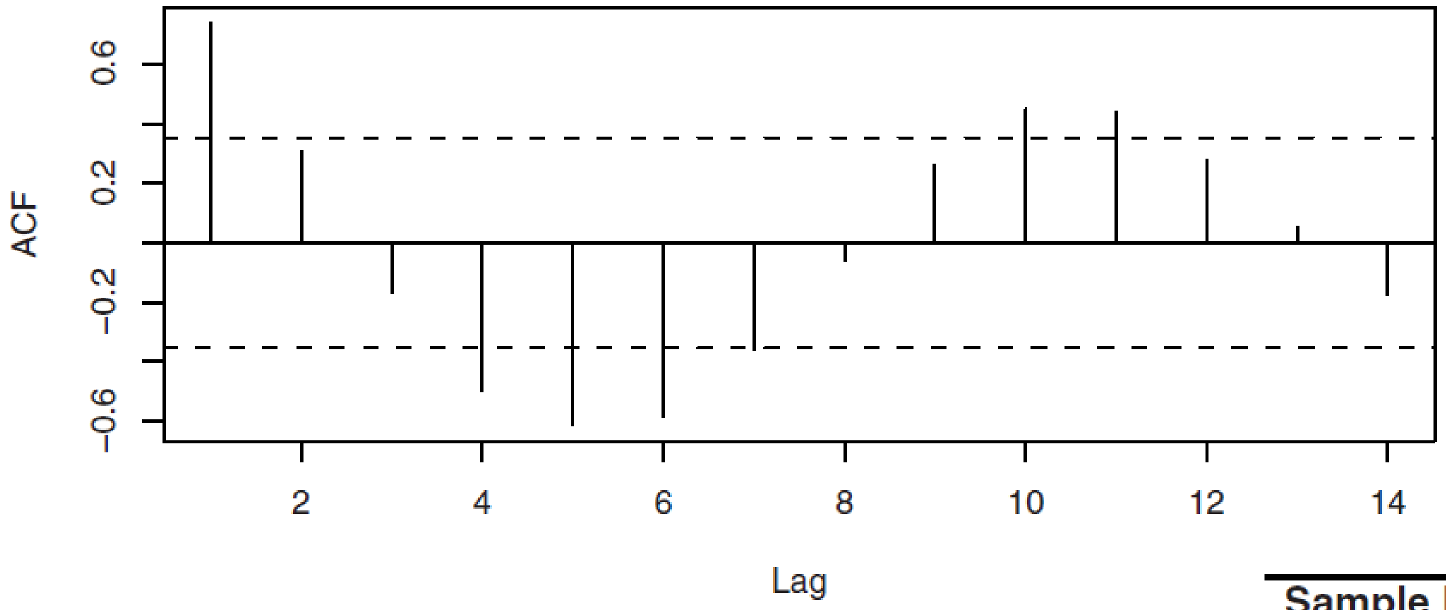


λ	Transformed Data
-2	y^{-2}
-1	y^{-1}
-0.5	$1/\sqrt{y}$
0	$\ln(y)$
0.5	$\sqrt{y}$
1	y
2	y^2

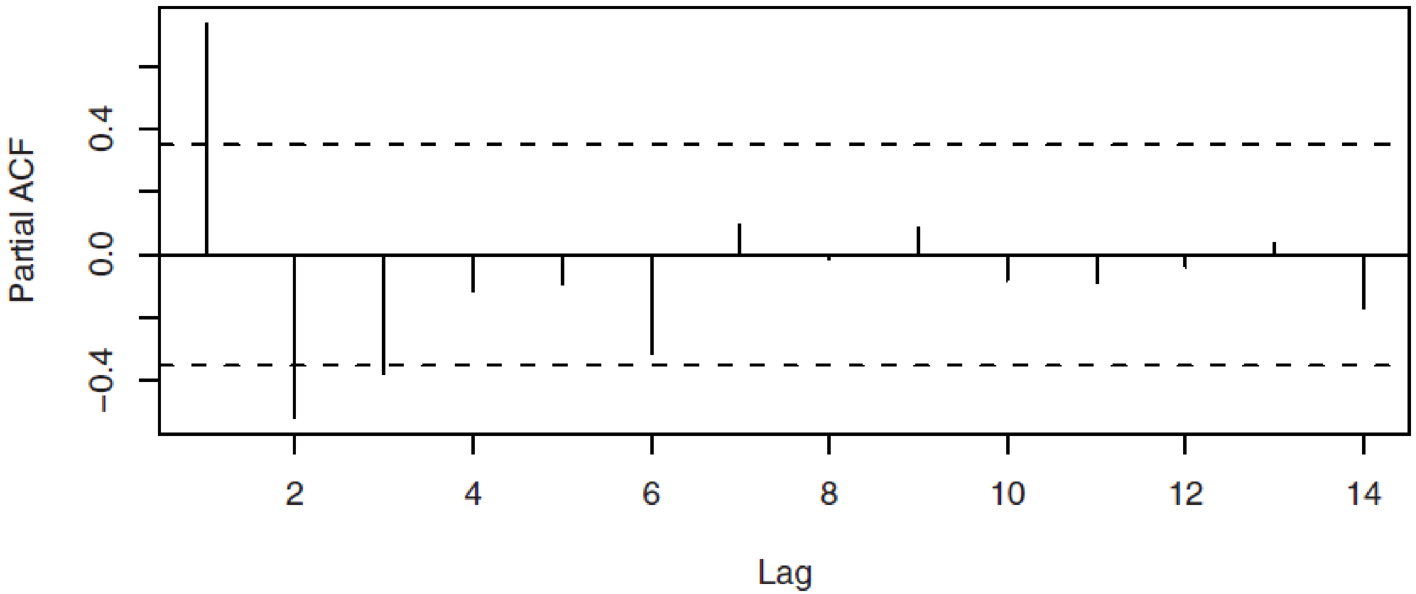
Box-Cox Power Transformation Results for Hare Abundance



Sample ACF for Square Root of Hare Abundance



Sample Partial ACF for Square Root of Hare Abundance



→ **ACF tails off**
→ **PACF cuts off after lag 3**

Maximum Likelihood Estimates from R Software: Hare Series

Coefficients:	ar1	ar2	ar3	Intercept[†]
	1.0519	–0.2292	–0.3931	5.6923
s.e.	0.1877	0.2942	0.1915	0.3371

`sigma^2 estimated as 1.066: log-likelihood = -46.54, AIC = 101.08`

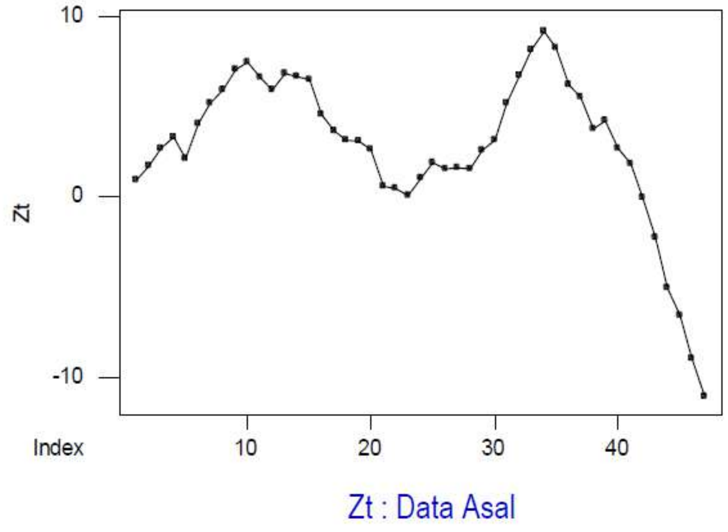
[†] The intercept here is the estimate of the process mean μ—not of θ_0.

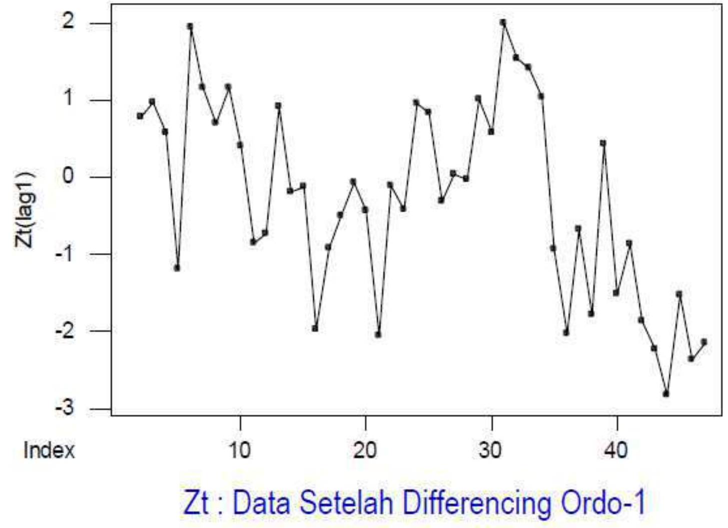
$$t_{ar1} = \frac{1.0519}{0.1877} = 5.604$$

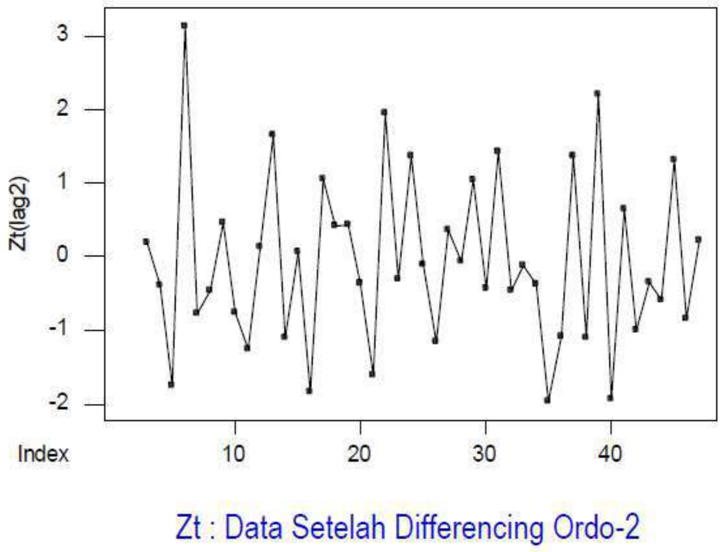
$$\boxed{t_{ar2} = -\frac{0.2292}{0.2941} = -0.779}$$

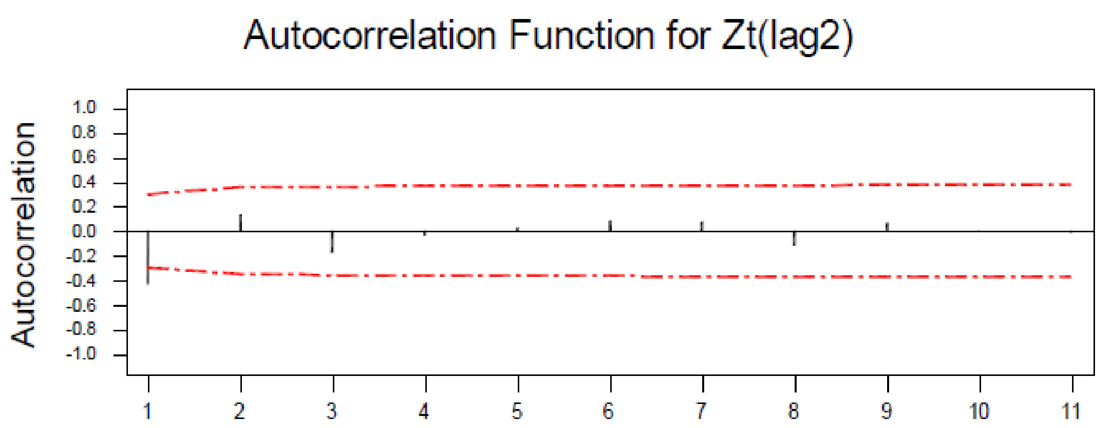
$$t_{ar3} = -\frac{0.3931}{0.1915} = -2.053$$

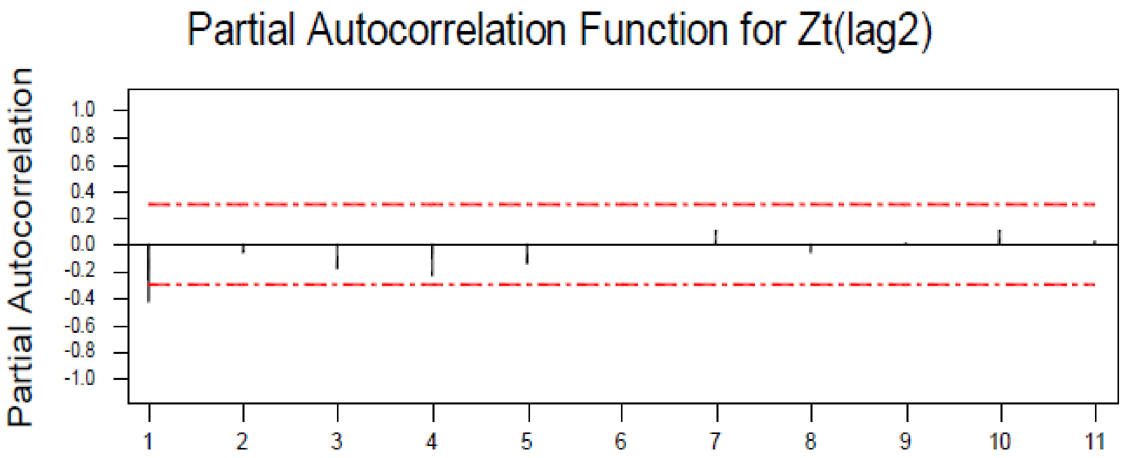
Illustrations of Parameter Estimation











ARIMA(0,2,1) & ARIMA(1,2,0)

ARIMA(0, 2, 1)

Final Estimates of Parameters

Type	Coef	SE Coef	T	P
MA 1	-0.4393	0.1371	-3.20	0.003
Constant	-0.0995	0.1581	-0.63	0.533

ARIMA(1, 2, 0)

Final Estimates of Parameters

Type	Coef	SE Coef	T	P
AR 1	0.5958	0.1225	4.86	0.000
Constant	-0.06673	0.06299	-1.06	0.295


THANK YOU
MERCI
OBRIGADO
ASANTE
WELALIN
GRAZIE
CHOKRANE
RAIBH MAITH AGAT
MULTUMESC
SPASIBO
NIRRINGRAZZJAK
MOCHCHAKKERAM
MATUR NUWUN
THANK YOU VERY MUCH
SAAGHA XALDA HWA
SUWUN
СПАСИБО
ZIKOMO
どうも
БАЯРЛАЛАА
MOCHCHAKKERAM
EIO
MAAKE
VINAKA
МНОГО ВИ БЛАГОДАРЯ
QUYANAQ
KHUBLEI
MATONDO

Parameter Estimation, Model Diagnostics and Forecasting

YENNI ANGRAINI

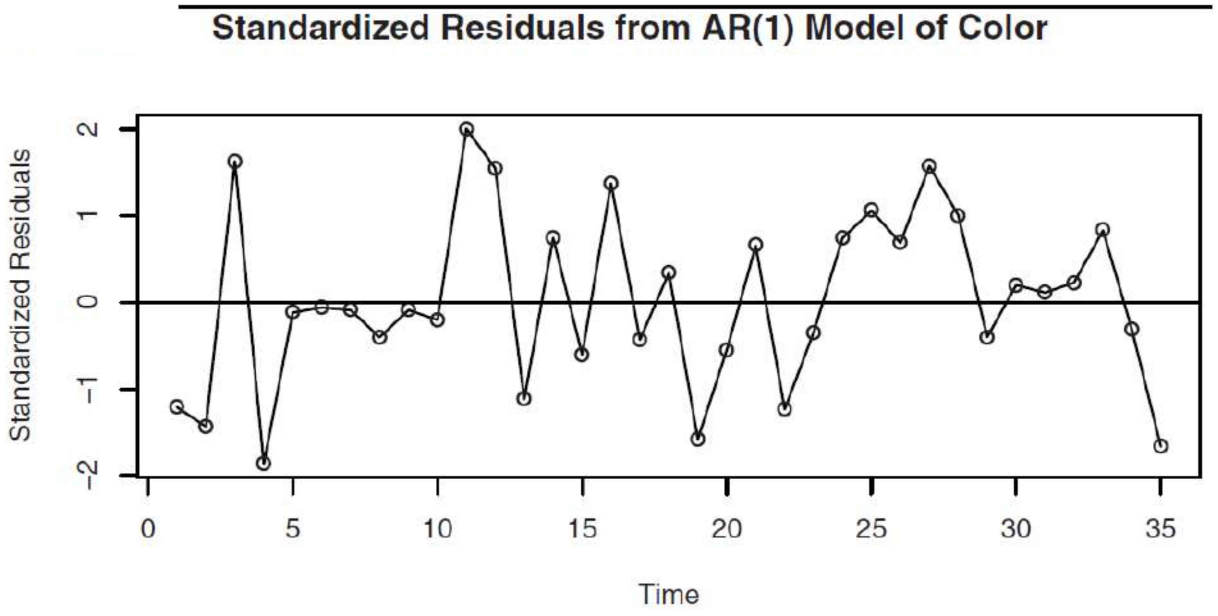
Outline

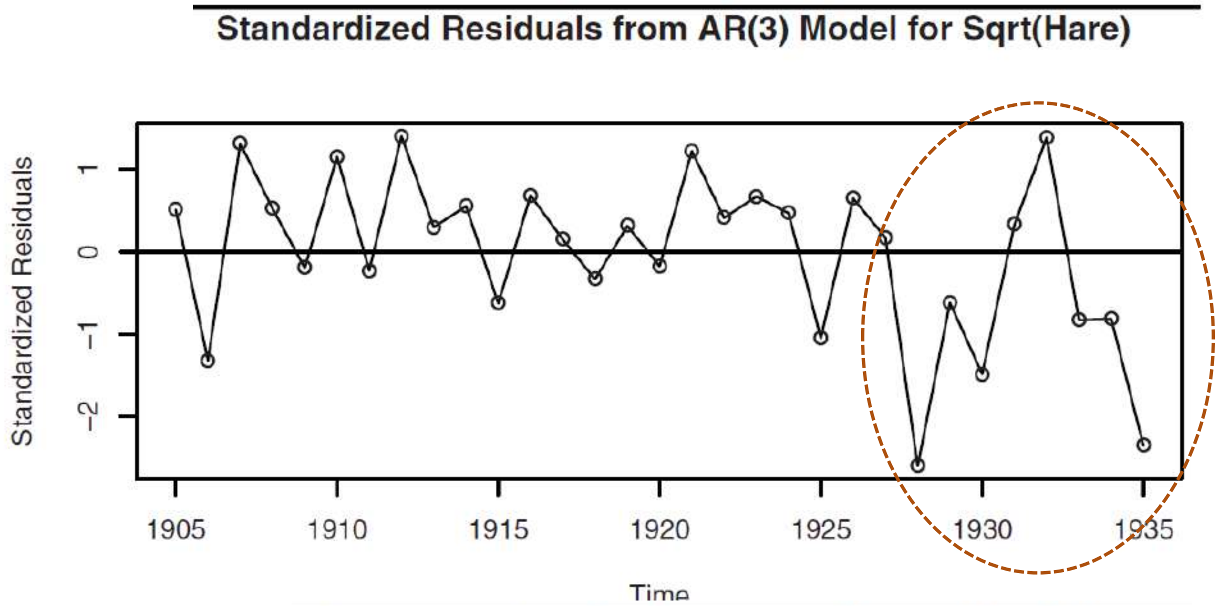
- Residual Analysis
  - Plot of the Residuals
  - Normality of Residuals
  - Autocorrelation of the Residuals
  - The Ljung-Box test
- Overfitting
- Forecasting

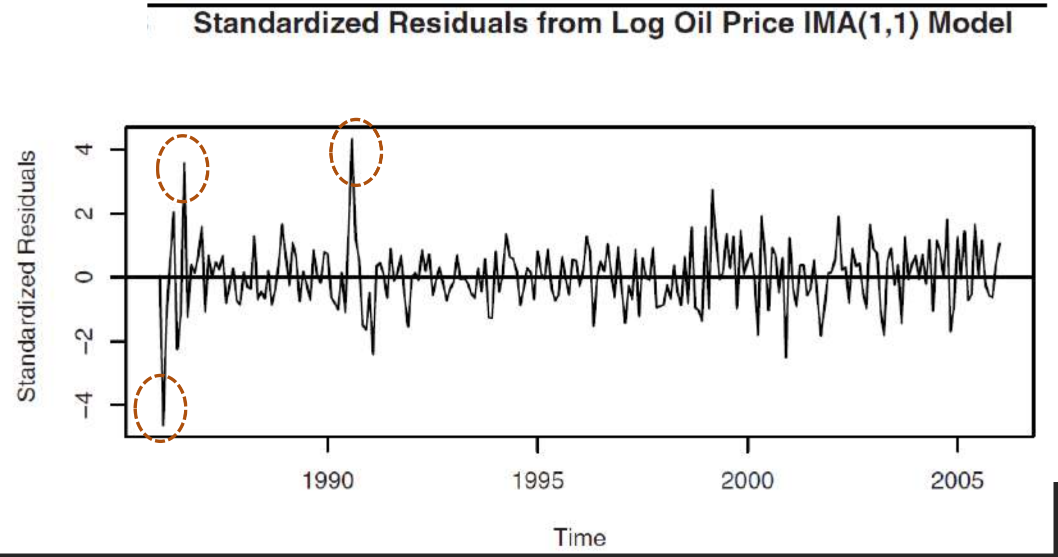
Model Diagnostics: Residual Analysis,

- Residual = actual – predicted
- If the model is correctly specified and the parameter estimates are reasonably close to the true values, then the residuals should have nearly the properties of white noise
- They should behave roughly like independent, identically distributed normal variables with zero means and common standard deviations

Model Diagnostics:
Residual Analysis : Plots of the Residuals



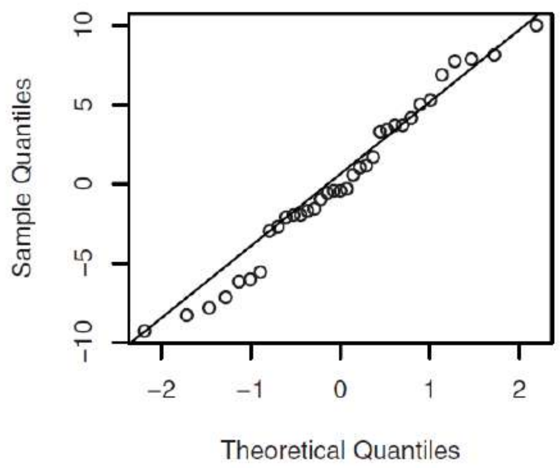




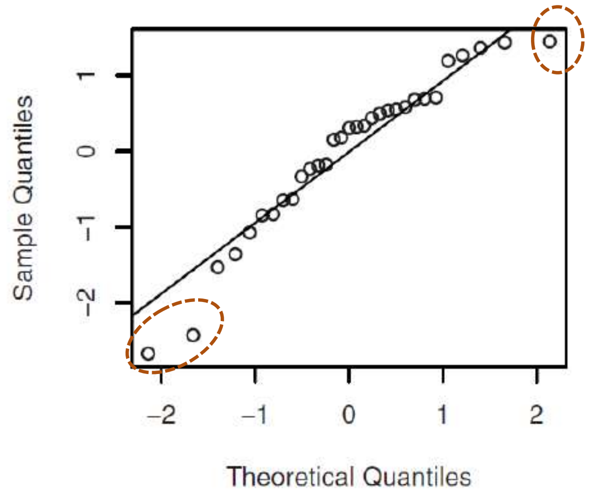
If the model is adequate, we expect the plot to suggest a rectangular scatter around a zero horizontal level with no trends whatsoever.

Model Diagnostics:
Residual Analysis : Normality of the Residuals

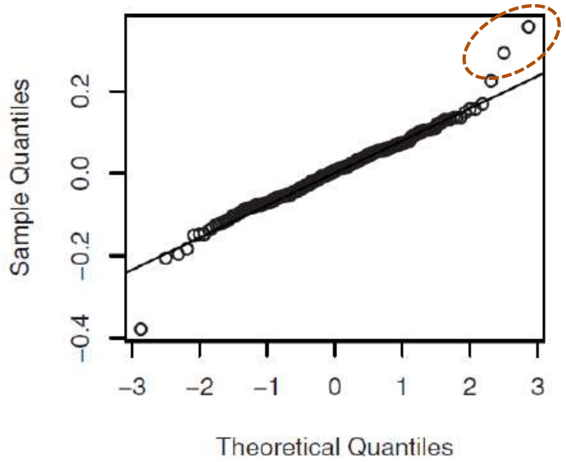
Quantile-Quantile Plot: Residuals from AR(1) Color Model



Quantile-Quantile Plot: Residuals from AR(3) for Hare



Quantile-Quantile Plot: Residuals from IMA(1,1) Model for Oil



Model Diagnostics:
Residual Analysis : Autocorrelation of the Residuals

As an example of these results, consider a correctly specified and efficiently estimated AR(1) model. It can be shown that, for large *n*,

$$Var(\hat{r}_1) \approx \frac{\phi^2}{n}$$

$$Var(\hat{r}_k) \approx \frac{1-(1-\phi^2)\phi^{2k-2}}{n} \quad \text{for } k > 1$$

$$Corr(\hat{r}_1, \hat{r}_k) \approx -sign(\phi)\frac{(1-\phi^2)\phi^{k-2}}{1-(1-\phi^2)\phi^{2k-2}} \quad \text{for } k > 1$$

$$sign(\phi) = \begin{cases} 1 & \text{if } \phi > 0 \\ 0 & \text{if } \phi = 0 \\ -1 & \text{if } \phi < 0 \end{cases}$$

Model Diagnostics:
Residual Analysis : Autocorrelation of the Residuals

AR(2)

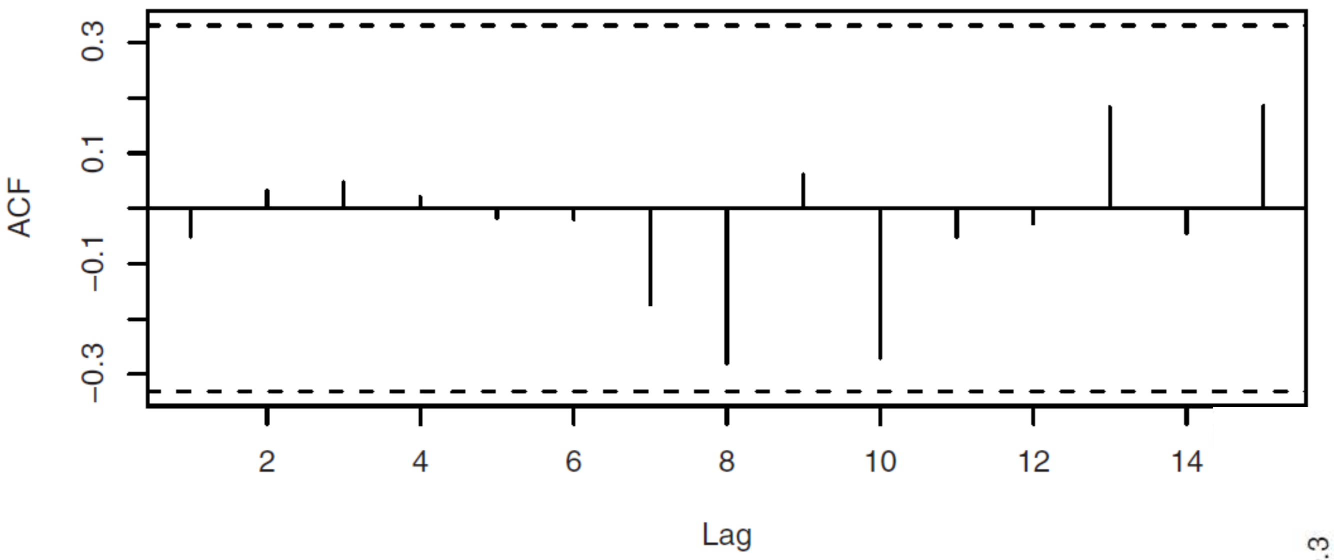
$$Var(\hat{r}_1) \approx \frac{\phi_2^2}{n}$$

$$Var(\hat{r}_2) \approx \frac{\phi_2^2 + \phi_1^2(1 + \phi_2)^2}{n}$$

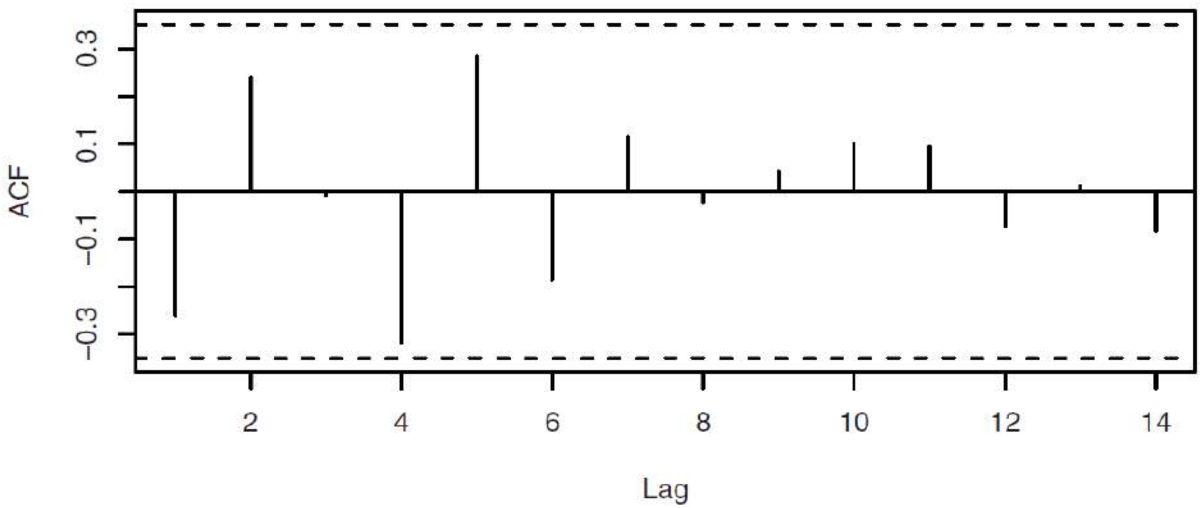
$$Var(\hat{r}_k) \approx \frac{1}{n} \quad \text{for } k \geq 3$$

Model Diagnostics:
Residual Analysis : Autocorrelation of the Residuals

Sample ACF of Residuals from AR(1) Model for Color



Sample ACF of Residuals from AR(2) Model for Hare



Model Diagnostics:
Residual Analysis : The Ljung-Box Test

H0 : the error terms are uncorrelated
H1 : the error terms are correlated

$$Q = n(\hat{r}_1^2 + \hat{r}_2^2 + \cdots + \hat{r}_K^2)$$

Box-Pierce Test

$$Q_* = n(n+2)\left(\frac{\hat{r}_1^2}{n-1} + \frac{\hat{r}_2^2}{n-2} + \cdots + \frac{\hat{r}_K^2}{n-K}\right)$$

Ljung-Box Test

Q *or* Q_* has an approximate chi-square distribution with $K - p - q$

Model Diagnostics:
Residual Analysis : The Ljung-Box Test

Residual Autocorrelation Values from AR(1) Model for Color

Lag k	1	2	3	4	5	6
Residual ACF	–0.051	0.032	0.047	0.021	–0.017	–0.019

```
> acf(residuals(m1.color),plot=F)$acf
> signif(acf(residuals(m1.color),plot=F)$acf[1:6],2)
> # display the first 6 acf values to 2 significant digits
```

H0 : the error terms are uncorrelated
H1 : the error terms are correlated

The Ljung-Box test statistic with $K = 6$ is equal to

$$Q_* = 35(35+2)\left(\frac{(-0.051)^2}{35-1} + \frac{(0.032)^2}{35-2} + \frac{(0.047)^2}{35-3} + \frac{(0.021)^2}{35-4} + \frac{(-0.017)^2}{35-5} + \frac{(-0.019)^2}{35-6}\right) \approx 0.28$$

This is referred to a chi-square distribution with $6 - 1 = 5$ degrees of freedom.
The *p*-value= 0.998

Model Diagnostics: Overfitting

After specifying and fitting what we believe to be an adequate model, we fit a slightly more general model; that is, a model “close by” that contains the original model as a special case.

For example, if an AR(2) model seems appropriate, we might over fit with an AR(3) model. The original AR(2) model would be confirmed if:

- The estimate of the additional parameter, ϕ_3, is not significantly different from zero, and
- The estimates for the parameters in common, ϕ_1 and ϕ_2, do not change significantly from their original estimates.

Model Diagnostics: Overfitting

AR(1) Model Results for the Color Property Series

Coefficients:†	**ar1**	**Intercept**‡
	0.5705	74.3293
s.e.	0.1435	1.9151

sigma^2 estimated as 24.83: log-likelihood = -106.07, AIC = 216.15

AR(2) Model Results for the Color Property Series

Coefficients:	**ar1**	**ar2**	**Intercept**
	0.5173	0.1005	74.1551
s.e.	0.1717	0.1815	2.1463

sigma^2 estimated as 24.6: log-likelihood = -105.92, AIC = 217.84

Model Diagnostics: Overfitting

AR(1) Model Results for the Color Property Series

Coefficients:†	ar1	Intercept‡
	0.5705	74.3293
s.e.	0.1435	1.9151

sigma^2 estimated as 24.83: log-likelihood = -106.07, AIC = 216.15

Overfit of an ARMA(1,1) Model for the Color Series

Coefficients:	ar1	ma1	Intercept
	0.6721	–0.1467	74.1730
s.e.	0.2147	0.2742	2.1357

sigma^2 estimated as 24.63: log-likelihood = -105.94, AIC = 219.88

Model Diagnostics: Overfitting

The implications for fitting and overfitting models are as follows:

- Specify the original model carefully. If a simple model seems at all promising, check it out before trying a more complicated model.
- When overfitting, do not increase the orders of both the AR and MA parts of the model simultaneously.
- Extend the model in directions suggested by the analysis of the residuals. For example, if after fitting an MA(1) model, substantial correlation remains at lag 2 in the residuals, try an MA(2), not an ARMA(1,1).

Model Diagnostics: Overfitting

AR(1) Model Results for the Color Property Series

Coefficients:†	ar1	Intercept‡
	0.5705	74.3293
s.e.	0.1435	1.9151

sigma^2 estimated as 24.83: log-likelihood = -106.07, AIC = 216.15

Overfitted ARMA(2,1) Model for the Color Property Series

Coefficients:	ar1	ar2	ma1	Intercept
	0.2189	0.2735	0.3036	74.1653
s.e.	2.0056	1.1376	2.0650	2.1121

sigma^2 estimated as 24.58: log-likelihood = –105.91, AIC = 219.82

Forecasting

We shall first illustrate many of the ideas with the simple AR(1) process with a nonzero mean that satisfies

$$Y_t - \mu = \phi(Y_{t-1} - \mu) + e_t \tag{9.3.1}$$

Consider the problem of forecasting one time unit into the future. Replacing t by $t + 1$ in Equation (9.3.1), we have

$$Y_{t+1} - \mu = \phi(Y_t - \mu) + e_{t+1} \tag{9.3.2}$$

Given $Y_1, Y_2, \ldots, Y_{t-1}, Y_t$, we take the conditional expectations of both sides of Equation (9.3.2) and obtain

$$\hat{Y}_t(1) - \mu = \phi[E(Y_t|Y_1, Y_2, \ldots, Y_t) - \mu] + E(e_{t+1}|Y_1, Y_2, \ldots, Y_t) \tag{9.3.3}$$

Forecasting

$$\hat{Y}_t(1) - \mu = \phi[E(Y_t|Y_1, Y_2, \ldots, Y_t) - \mu] + E(e_{t+1}|Y_1, Y_2, \ldots, Y_t) \tag{9.3.3}$$

Now, from the properties of conditional expectation, we have

$$E(Y_t|Y_1, Y_2, \ldots, Y_t) = Y_t \tag{9.3.4}$$

Also, since e_{t+1} is independent of $Y_1, Y_2, \ldots, Y_{t-1}, Y_t$, we obtain

$$E(e_{t+1}|Y_1, Y_2, \ldots, Y_t) = E(e_{t+1}) = 0 \tag{9.3.5}$$

Thus, Equation (9.3.3) can be written as

$$\hat{Y}_t(1) = \mu + \phi(Y_t - \mu) \tag{9.3.6}$$

Forecasting

$$Y_{t+l} - \mu = \phi(Y_{t+l-1} - \mu) + e_{t+l}$$

$l = 2$

$$Y_{t+2} - \mu = \phi(Y_{t+1} - \mu) + e_{t+2}$$

$$\hat{Y}_t(2) - \mu = \phi\left(\hat{Y}_t(1) - \mu\right) + 0$$
$$\hat{Y}_t(2) = \phi\left(\hat{Y}_t(1) - \mu\right) + \mu$$

$$\hat{Y}_t(3) = \phi\left(\hat{Y}_t(2) - \mu\right) + \mu$$
$$\hat{Y}_t(3) = \phi\left(\phi\left(\hat{Y}_t(1) - \mu\right) + \mu \ - \mu\right) + \mu$$
$$\hat{Y}_t(3) = \phi^2\left(\hat{Y}_t(1) - \mu\right) + \mu$$
$$\hat{Y}_t(3) = \phi^2(\phi(Y_t - \mu) + \mu - \mu) + \mu$$

$$\hat{Y}_t(3) = \phi^3(Y_t - \mu) + \mu$$

Forecasting

$$Y_t - \mu = \phi(Y_{t-1} - \mu) + e_t \tag{9.3.1}$$

Now consider a general lead time ℓ. Replacing t by $t + \ell$ in Equation (9.3.1) and taking the conditional expectations of both sides produces

$$\hat{Y}_t(\ell) = \mu + \phi[\hat{Y}_t(\ell - 1) - \mu] \quad \text{for } \ell \geq 1 \tag{9.3.7}$$

since $E(Y_{t+\ell-1}|Y_1, Y_2, \ldots, Y_t) = \hat{Y}_t(\ell - 1)$ and, for $\ell \geq 1$, $e_{t+\ell}$ is independent of Y_1, $Y_2, \ldots, Y_{t-1}, Y_t$.

$$\begin{aligned}
\hat{Y}_t(\ell) &= \phi[\hat{Y}_t(\ell - 1) - \mu] + \mu \\
&= \phi\{\phi[\hat{Y}_t(\ell - 2) - \mu]\} + \mu \\
&\vdots \\
&= \phi^{\ell-1}[\hat{Y}_t(1) - \mu] + \mu
\end{aligned}$$

or

$$\boxed{\hat{Y}_t(\ell) = \mu + \phi^{\ell}(Y_t - \mu)} \tag{9.3.8}$$

Forecasting—Example

Maximum Likelihood Estimation of an AR(1) Model for Color

Coefficients:	ar1	intercept[†]
	0.5705	74.3293
s.e.	0.1435	1.9151

sigma^2 estimated as 24.8: log-likelihood = –106.07, AIC = 216.15

[†]Remember that the intercept here is the estimate of the process mean μ—not θ_0.

The last observed value of the color property is 67, so we would forecast one time period ahead as[†]

$$\begin{aligned}\hat{Y}_t(1) &= 74.3293 + (0.5705)(67 - 74.3293)\\ &= 74.3293 - 4.181366\\ &= 70.14793\end{aligned}$$

$\hat{Y}_t(2)$?
$\hat{Y}_t(6)$?
$\hat{Y}_t(8)$?
$\hat{Y}_t(10)$?

Forecasting

MA(1)

$$Y_t = \mu + e_t - \theta e_{t-1}$$

$$Y_{t+1} = \mu + e_{t+1} - \theta e_t$$

$$\hat{Y}_t(1) = E[\mu + e_{t+1} - \theta e_t | \mathrm{Y_t, Y_{t-1}, \ldots, Y_1}]$$

$$\hat{Y}_t(1) = \mu + 0 - \theta e_t$$

$$\hat{Y}_t(1) = \mu - \theta e_t$$

$$Y_{t+2} = \mu + e_{t+2} - \theta e_{t+1}$$

$$\hat{Y}_t(2) = E[\mu + e_{t+2} - \theta e_{t+1} | \mathrm{Y_t, Y_{t-1}, \ldots, Y_1}]$$

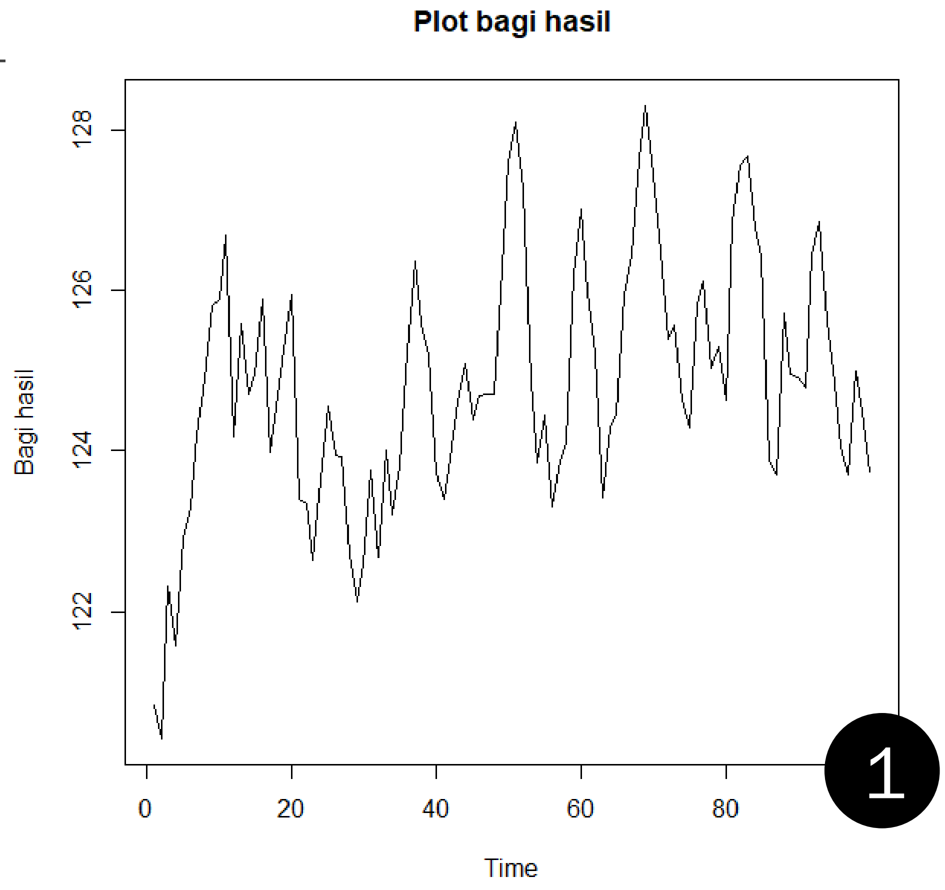
$$\hat{Y}_t(2) = \mu + 0 - 0$$

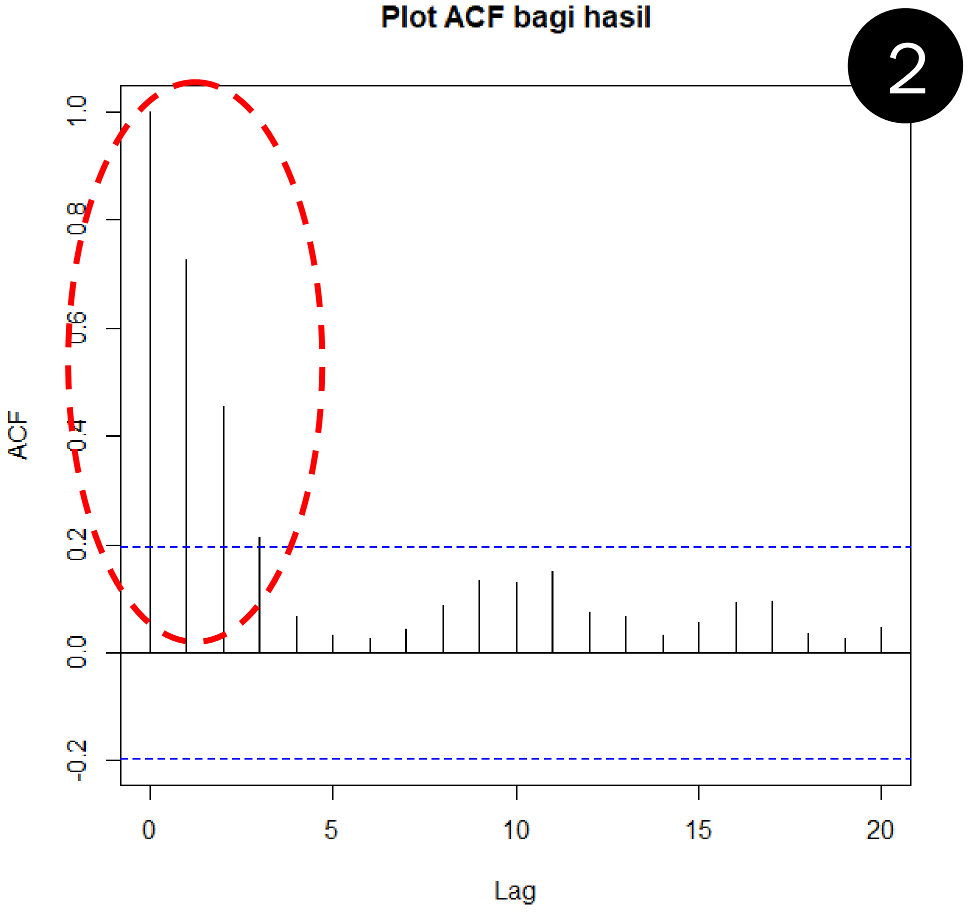
$$\hat{Y}_t(2) = \mu$$

Illustration

```
> Bagi.hasil
Time Series:
Start = 1
End = 100
Frequency = 1
  [1] 120.8399 120.4160 122.3064 121.5694 122.8942 123.3013 124.1932 124.8930
  [9] 125.8147 125.8902 126.6755 124.1835 125.5775 124.7087 124.9495 125.8825
 [17] 123.9903 124.6912 125.2508 125.9463 123.3904 123.3333 122.6446 123.6364
 [25] 124.5462 123.9536 123.9251 122.7099 122.1322 122.5747 123.7589 122.6723
 [33] 124.0033 123.2013 123.8163 125.2421 126.3553 125.5886 125.1816 123.7251
 [41] 123.4024 123.9143 124.6083 125.0931 124.3820 124.6854 124.7064 124.7050
 [49] 126.2886 127.5864 128.0930 127.2421 124.8627 123.8595 124.4472 123.3074
 [57] 123.8689 124.1007 126.1777 126.9949 125.9848 125.1947 123.4171 124.2836
 [65] 124.4681 125.9491 126.4359 127.6295 128.2976 127.4389 126.4682 125.3810
 [73] 125.5677 124.6353 124.2816 125.8262 126.1171 125.0234 125.2856 124.6220
 [81] 126.8500 127.5602 127.6669 126.7697 126.4150 123.8775 123.6915 125.7046
 [89] 124.9605 124.9237 124.7858 126.4416 126.8498 125.6157 124.8311 124.0431
 [97] 123.6912 124.9995 124.3380 123.7311
```

Step 1. Stationary





Time Series Plot and ACF

Step 1. Stationary

Uji *Augmented Dickey-Fuller (ADF)*

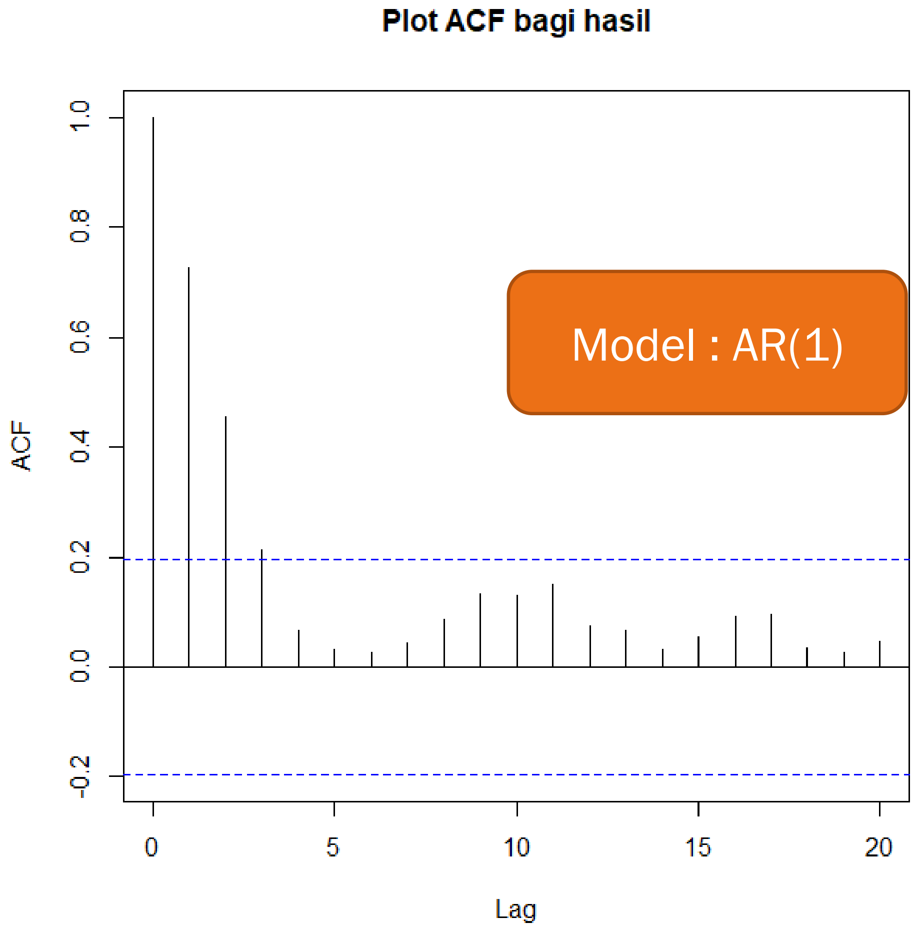
```
> #Melalui Uji Augmented Dickey-Fuller (ADF)
> adf.test(Bagi.hasil, alternative="stationary")

        Augmented Dickey-Fuller Test

data:  Bagi.hasil
Dickey-Fuller = -3.7862, Lag order = 4, p-value = 0.02236
alternative hypothesis: stationary
```

H_0: the data needs to be differenced to make it stationary

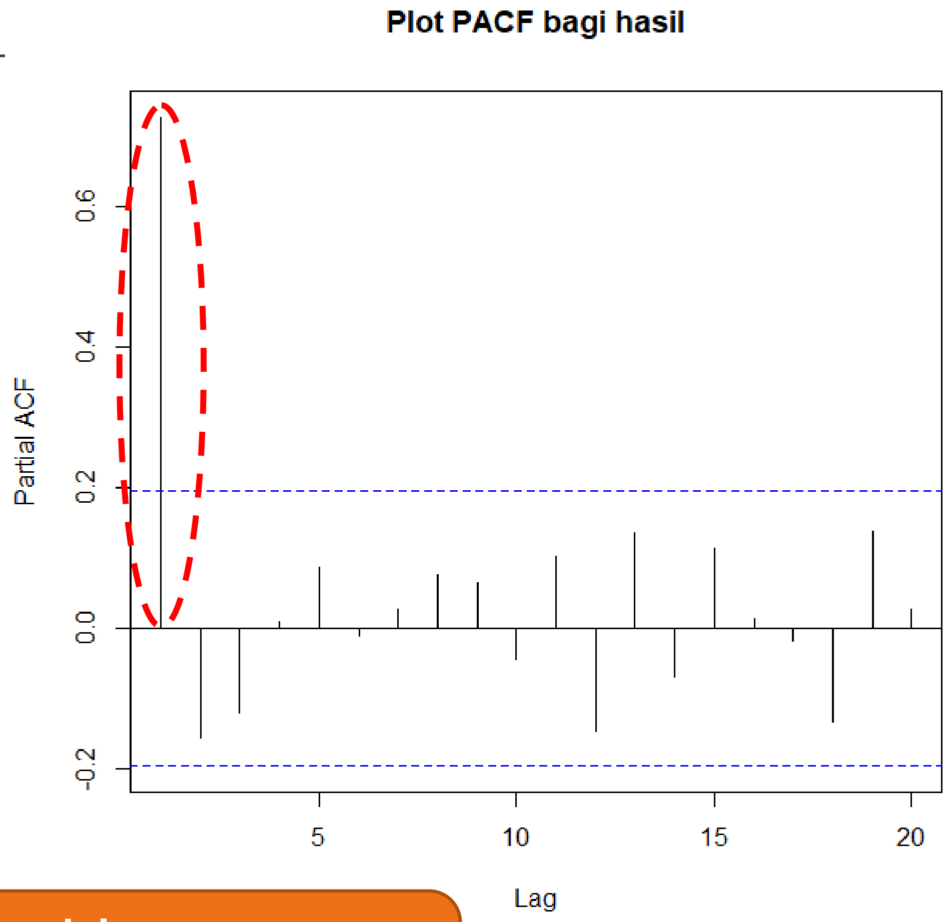
H_1: the data is stationary and doesn't need to be differenced

If $p - value < \alpha$, reject the null hypothesis, and it means that the data is stationary

Step 2. Model Specification



Model : AR(1)



- → **ACF menurun eksponensial**
- → **PACF signifikan pada lag ke-1 dan cut off pada lag ke-2**

Step 3. Parameter Estimation

```
> #Fit Model
> fit<-Arima(Bagi.hasil, order=c(1,0,0))
> summary(fit)
Series: Bagi.hasil
ARIMA(1,0,0) with non-zero mean

Coefficients:
         ar1   intercept
      0.7758    124.6629
s.e.  0.0665      0.4363

sigma^2 estimated as 1.025:  log likelihood=-142.58
AIC=291.15   AICc=291.4   BIC=298.97
```

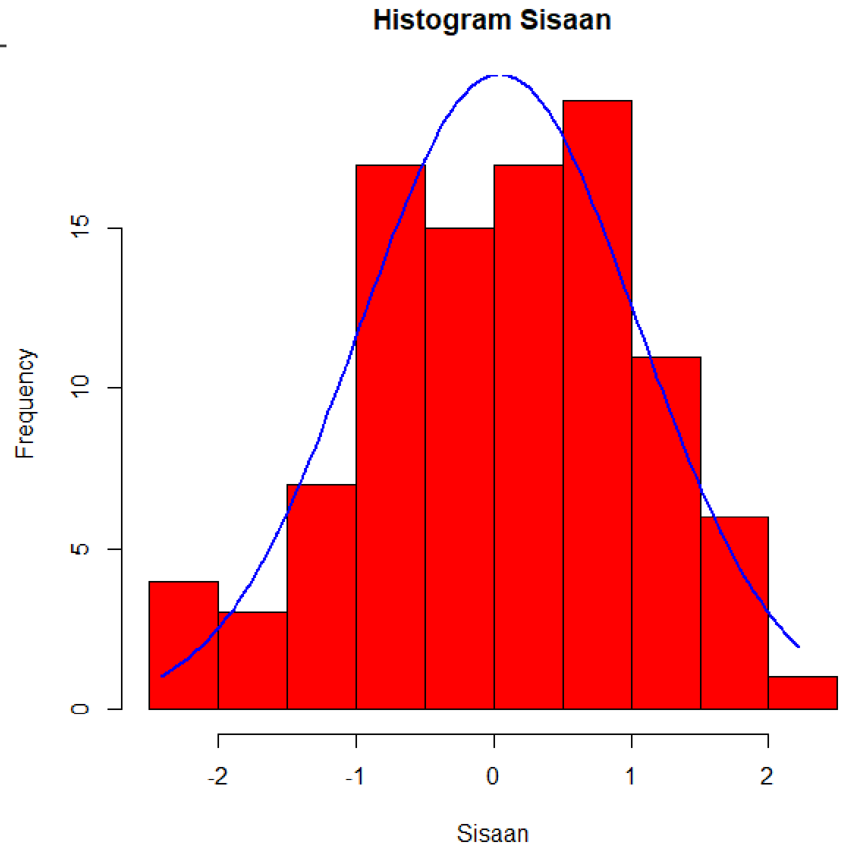
Step 3. Parameter Estimation

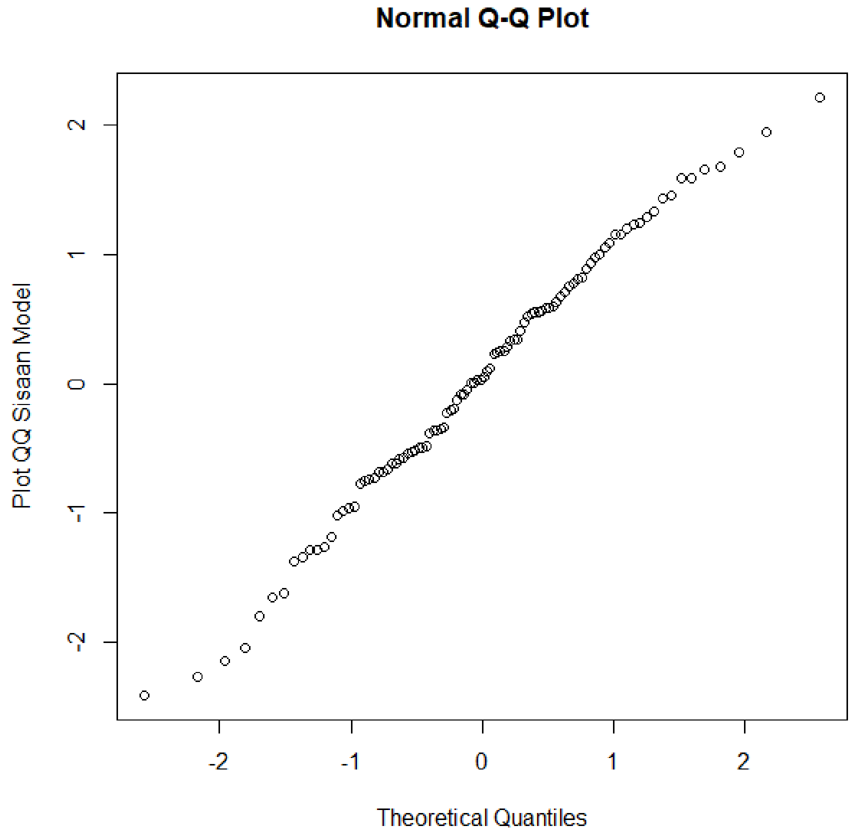
```
> #t hitung untuk ar1
> thit.ar1<-fit$coef[1]/0.0665
> thit.ar1
     ar1
11.66651
> p.value.ar1 = 2*pt(-abs(thit.ar1), df=length(Bagi.hasil)-1)
> p.value.ar1
         ar1
2.660125e-20
>
> #t hitung untuk intersep
> thit.int<-fit$coef[2]/0.4363
> thit.int
intercept
 285.7276
> p.value.int = 2*pt(-abs(thit.int), df=length(Bagi.hasil)-1)
> p.value.int
    intercept
3.325919e-146
```

$78y_{t-1} + e_t$

Step 4. Model Diagnostics

Normality of the Residuals

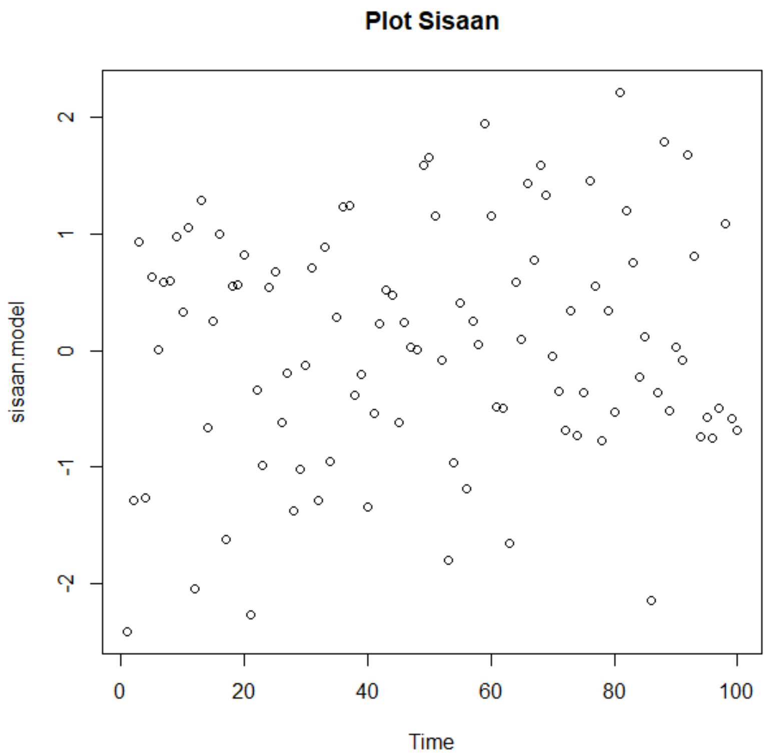


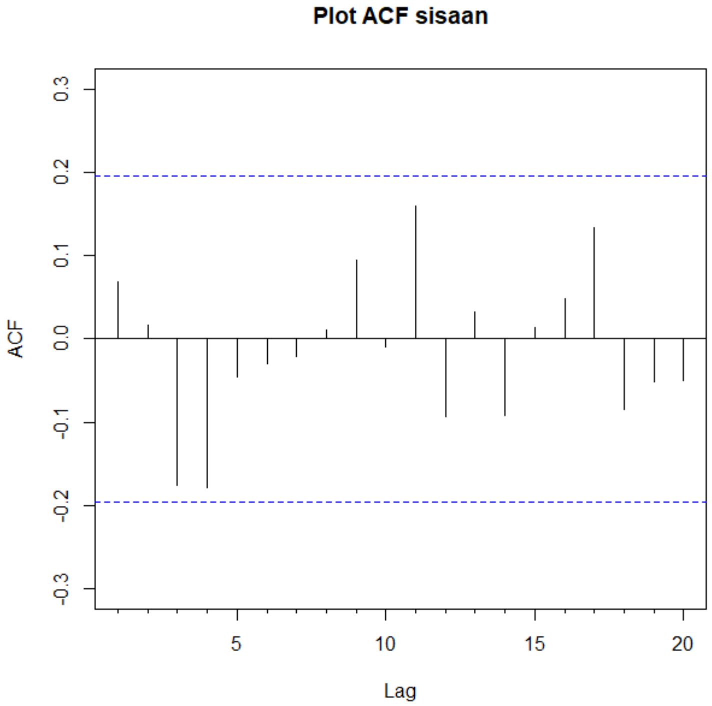


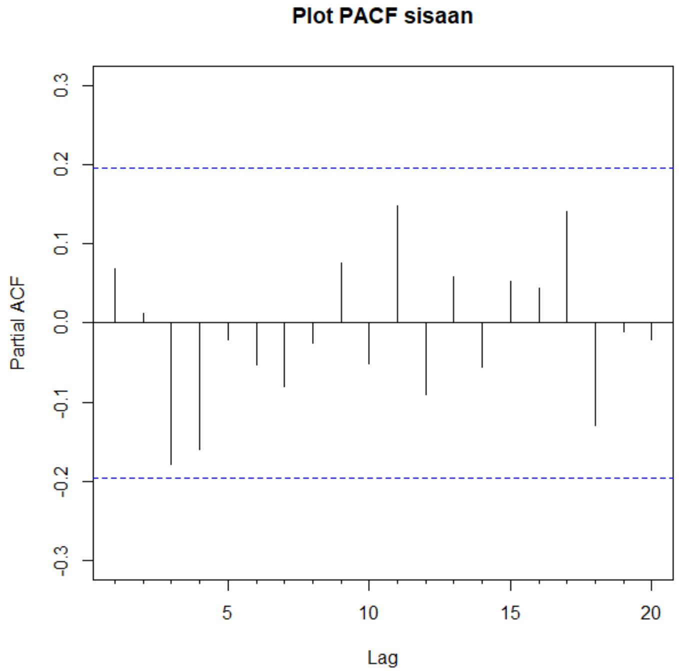
Step 4. Model Diagnostics

Residuals Plot

ACF and PACF of Residuals







Step 4. Model Diagnostics

Uji L-jung Box

```
> #L-Jung Box test
> test<-Box.test(sisaan.model, lag=24, type="Ljung")
> test

        Box-Ljung test

data:  sisaan.model
X-squared = 23.376, df = 24, p-value = 0.4977
```

H0 : the error terms are uncorrelated
H1 : the error terms are correlated

Reject $H0$ if $p - value < \alpha$

Step 5. Overfitting

The fitting model is AR(1), we might over fit with an AR(2)

```
> #Overfitting
> fit.ar2<-Arima(Bagi.hasil, order=c(2,0,0))
> summary(fit.ar2)
Series: Bagi.hasil
ARIMA(2,0,0) with non-zero mean

Coefficients:
         ar1      ar2  intercept
      0.8724  -0.1358   124.7157
s.e.  0.0987   0.1032     0.3725

sigma^2 estimated as 1.018:  log likelihood=-141.72
AIC=291.44   AICc=291.86   BIC=301.86
```

Parameter Estimation for AR(2)

$$t_{ar1} = \frac{0.8724}{0.0987} = 8.839$$

$$t_{ar2} = -\frac{0.1358}{0.1032} = -1.316$$

$$t_{int} = -\frac{124.7157}{0.3725} = -334.807$$

Step 5. Overfitting

```
> #Fit Model
> fit<-Arima(Bagi.hasil, order=c(1,0,0))
> summary(fit)
Series: Bagi.hasil
ARIMA(1,0,0) with non-zero mean

Coefficients:
         ar1  intercept
      0.7758   124.6629
s.e.  0.0665     0.4363

sigma^2 estimated as 1.025:  log likelihood=-142.58
AIC=291.15   AICc=291.4   BIC=298.97
```

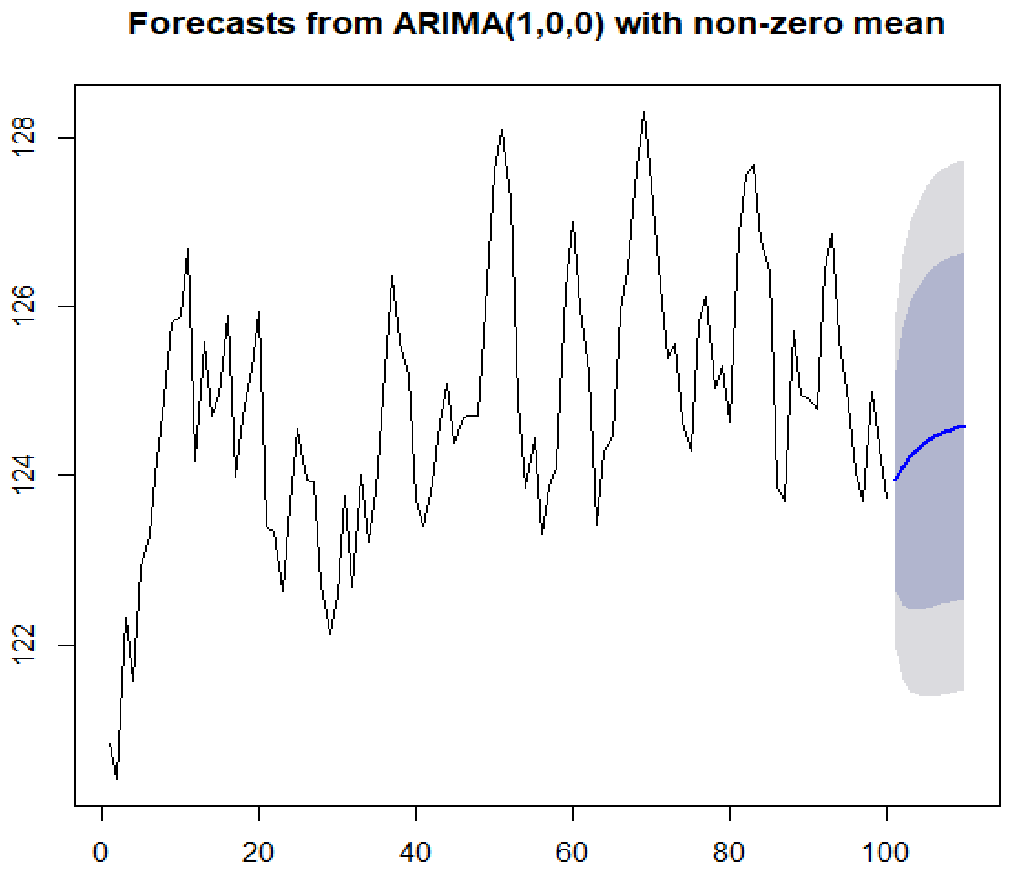
The Best Model

```
> #Overfitting
> fit.ar2<-Arima(Bagi.hasil, order=c(2,0,0))
> summary(fit.ar2)
Series: Bagi.hasil
ARIMA(2,0,0) with non-zero mean

Coefficients:
         ar1      ar2  intercept
      0.8724  -0.1358   124.7157
s.e.  0.0987   0.1032     0.3725

sigma^2 estimated as 1.018:  log likelihood=-141.72
AIC=291.44   AICc=291.86   BIC=301.86
```

Step 6. Forecasting

- Based on parameter estimation and *overfitting*, the best model is AR(1)
- The Model can be written as : $y_t = 124.66 + 0.78y_{t-1} + e_t$

```
> #Forecast
> Forecast.ar1<-forecast(fit)
> Forecast.ar1
    Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
101       123.9400 122.6425 125.2374 121.9557 125.9242
102       124.1021 122.4599 125.7442 121.5906 126.6135
103       124.2278 122.4094 126.0461 121.4469 127.0087
104       124.3253 122.4087 126.2420 121.3941 127.2566
105       124.4010 122.4276 126.3745 121.3829 127.4191
106       124.4597 122.4529 126.4666 121.3905 127.5289
107       124.5053 122.4786 126.5320 121.4057 127.6048
108       124.5406 122.5021 126.5792 121.4230 127.6583
109       124.5680 122.5224 126.6137 121.4395 127.6966
110       124.5893 122.5394 126.6392 121.4543 127.7244
>
> plot(Forecast.ar1)
>
```



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Tree-rings	35
Micro-Economic	22
Sales	37
Production	48
Labour market	24
Macro-Economic	56
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Search results: 564

Monthly milk production: pounds per cow. Jan 62 – Dec 75

Monthly time series (Jan 1962–Dec 1975). It was last modified on 1 Feb 2014 at 19:52. Categorized as Agriculture.

Annual barley yields per acre in England & Wales 1884 – 1939

Yearly time series (1884–1939). It was last modified on 1 Feb 2014 at 19:52. Categorized as Agriculture.

Monthly total number of pigs slaughtered in Victoria. Jan 1980 – August 1995

Monthly time series (Jan 1980–Aug 1995). It was last modified on 1 Feb 2014 at 19:52. Categorized as Agriculture.

Annual sheep population (1000s) in England & Wales 1867 – 1939

Yearly time series (1867–1939). It was last modified on 1 Feb 2014 at 19:52. Categorized as Agriculture. It is tagged population, demographics.


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