My Solutions for "Mathematical Methods of Physics (Second Edition)" by J. Mathews, R. L. Walker

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0 Introduction

This document is an archive of my solutions to J. Mathews and R. L. Walker's "Mathematical Methods of Physics" textbook. None of the solutions have been verified by anyone other than myself, whom I do not consider a reliable source. Hence, it is strongly advised against to use this solution for any application where accuracy matters, especially for assignments and any academic work. However, I have not yet been able to find any other solutions. Please just use this manual as just a suggestion.

1 Chapter 1

1. We first acknowledge that $y = 0$ is a solution, so we seek solutions that are not identically zero.

Let $y = x \cdot v$.

$$y' = v + xv' \Rightarrow x^2(v + xv') + x^2v^2 = x \cdot xv \cdot (v + xv')$$

$$\Rightarrow v + xv' = xvv' \Rightarrow \left(1 - \frac{1}{v}\right) dv = \frac{1}{x} dx \Rightarrow v - \ln v = \ln x + C$$

$$\therefore \frac{y}{x} - \ln y = C$$

2.

$$\frac{y}{\sqrt{1+y^2}} dy = \frac{x}{\sqrt{1+x^2}} dx$$

$$\therefore \sqrt{1+y^2} = \sqrt{1+x^2} + C$$

3. Let $v := x + y$.

$$v' = 1 + y' \Rightarrow v' - 1 = \frac{a^2}{v^2} \Rightarrow \frac{v^2}{a^2+v^2}dv = dx$$

$$x + C = \int \left(1 - \frac{a^2}{a^2+v^2}\right) dv = v - a\tan^{-1}\left(\frac{v}{a}\right) = x + y - a\tan^{-1}\left(\frac{x+y}{a}\right)$$

$$\therefore y - a\tan^{-1}\left(\frac{x+y}{a}\right) = C$$

4. We first seek the complement solutions.

$$y_c' + y_c \cos x = 0 \Rightarrow \frac{dy_c}{y_c} + \cos x dx = 0 \Rightarrow y_c = Ce^{-\sin x}$$

For the particular solution, we first observe that $\frac{1}{2}\sin 2x = \sin x \cos x$. Thus, we shall try the ansatz $y_p = \sin x + A$.

$$\cos x + (\sin x + A)\cos x = \sin x \cos x \Rightarrow A = -1$$

$$\therefore y = Ce^{-\sin x} + \cos x - 1$$

5.

$$\left(1+x^2\right) y' = xy(y+1) \Rightarrow \frac{dy}{y(y+1)} = \frac{x}{1-x^2}dx$$

$$\Rightarrow \ln\left(\frac{y}{y+1}\right) = -\frac{1}{2}\ln\left(1-x^2\right) + C_1$$

$$\Rightarrow \frac{y+1}{y} = e^{-C_1}\sqrt{1-x^2} = C\sqrt{1-x^2}$$

$$\therefore y = \frac{1}{C\sqrt{1-x^2}-1}$$

6. We first take note that the equation is dimension-consistent with $[y] = [x^{-2}]$. Thus, we define $v := x^2 y$ or $y = \frac{v}{x^2}$.

$$y' = \frac{v'}{x^2} - \frac{2v}{x^3} \Rightarrow 2xv' - 4v = 1 + \sqrt{1+4v}$$

$$\Rightarrow \frac{v'}{1+4v+\sqrt{1+4v}} = \frac{1}{2x}$$

$$\Rightarrow \left(\frac{1}{\sqrt{1+4v}} - \frac{1}{\sqrt{1+4v}+1}\right) dv = \frac{dx}{2x}$$

Basic calculus yields $\int \frac{dv}{\sqrt{1+4v}+1} = \frac{1}{2}\left(\sqrt{1+4v} - \ln\left(1+\sqrt{1+4v}\right)\right) + C$. Hence,

$$\frac{1}{2}\sqrt{1+4v} - \frac{1}{2}\left(\sqrt{1+4v} - \ln\left(1+\sqrt{1+4v}\right)\right) = \frac{1}{2}\ln x + C_1$$

$$\Rightarrow 1 + \sqrt{1+4v} = e^{2C_1}x = Cx$$

$$\therefore \sqrt{1+4x^2y} = Cx - 1$$

7. Let $v' := y'$.

$$v' + v^2 + 1 = 0 \Rightarrow \frac{dv}{v^2+1} = -1 \Rightarrow v = \tan(C_1 - x)$$

$$\therefore y = \int dx v = \ln\left(\cos(C_1 - x)\right) + C_2$$

8.

$$y'y'' = e^y y' \Rightarrow \int dy' y' = \int dy e^y \Rightarrow \frac{1}{2}y'^2 = e^y + A$$

$$\Rightarrow \frac{dy}{\sqrt{e^y + A}} = \sqrt{2}dx$$

Basic calculus yields

$$\int \frac{dy}{\sqrt{e^y + A}} = \frac{1}{\sqrt{A}} \ln\left(\frac{\sqrt{e^y + A} - \sqrt{A}}{\sqrt{e^y + A} + \sqrt{A}}\right) + C_1.$$

$$\therefore \ln\left(\frac{\sqrt{e^y + A} - \sqrt{A}}{\sqrt{e^y + A} + \sqrt{A}}\right) = \sqrt{2A}x + C$$

9. Notice how $(x(1-x))' = 1 - 2x$.

$$\begin{aligned}0 &= x(1-x)y'' + 4y' + 2y \\ &= x(1-x)y'' + (x(1-x))'y' + (2x+3)y' + 2y \\ &= (x(1-x)y')' + ((2x+3)y)'\end{aligned}$$

$$\Rightarrow x(1-x)y' + (2x+3)y = A$$

Let us define the integrating factor λ:

$$\begin{aligned}\lambda &:= \exp\left(\int dx \frac{2x+3}{x(1-x)}\right) \\ &= \exp\left(\int dx \left(\frac{3}{x} - \frac{5}{x-1}\right)\right) \\ &= \exp\left(3\ln x - 5\ln(x-1)\right) \\ &= \frac{x^3}{(x-1)^5}\end{aligned}$$

$$\Rightarrow (\lambda y)' = \frac{A}{x(1-x)} \cdot \lambda = \frac{Ax^2}{(x-1)^6}$$

$$
\begin{aligned}
\Rightarrow \lambda y &= \int dx \frac{Ax^2}{(x-1)^6} \\
&= A \int dx \left(\frac{1}{(x-1)^4} + \frac{2}{(x-1)^5} + \frac{1}{(x-1)^6} \right) \\
&= -A \frac{10x^2 - 5x + 1}{30(x-1)^5} + C_2
\end{aligned}
$$

$$\therefore y = C_1 \frac{10x^2 - 5x + 1}{x^3} + C_2 \frac{(1-x)^5}{x^3}$$

10.

$$\frac{dy}{y^2} = \frac{1-x}{x^3} dx \Rightarrow -\frac{1}{y} = -\frac{1}{2x^2} + \frac{1}{x} + A$$

$$\therefore y = \frac{2x^2}{Cx^2 + 2x - 1}$$