Simulation of solid fuel engine

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1 Calculations regarding engine thrust

1.1 Thrust

We calculate engine thrust by usage of following equation, which as input takes P_{ch}, A_t and P_e.

$$F = A_t P_0 \sqrt{\frac{2k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left[1-\left(\frac{P_e}{P_0}\right)^{\frac{k-1}{k}}\right]} + A_e(P_e - P_a) \tag{1}$$

1.2 Exit pressure

Unfortunately, we don't have P_e so we need to calculate it numerically from following equation for expansion ratio.

$$E = \frac{A_t}{A_e} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}\left(\frac{P_e}{P_0}\right)^{\frac{1}{k}}\sqrt{\left(\frac{k+1}{k-1}\right)\left[1-\left(\frac{P_e}{P_0}\right)^{\frac{k-1}{k}}\right]} \tag{2}$$

Where E is:

$$E = \frac{A_t}{A_e} = \frac{1}{\text{Expansion ratio}}$$

Numerical calculations will be performed by taking the minimum of ΔE in order to obtain the corresponding P_e for which, upon substitution into the formula, we will achieve a value closest to the E calculated from the nozzle parameters.

$$0 \approx \Delta E = \left|E - \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}\left(\frac{P_e}{P_0}\right)^{\frac{1}{k}}\sqrt{\left(\frac{k+1}{k-1}\right)\left[1-\left(\frac{P_e}{P_0}\right)^{\frac{k-1}{k}}\right]}\right| \tag{3}$$

2 Calulations regarding engine chamber parameters

Calculations related to the combustion chamber focus on a key equation for solid fuel rocket engines, namely the equation for pressure in the chamber.

$$P_{ch}(A_b) = K_n^{\frac{1}{1-n}}(c^* \rho_p a)^{\frac{1}{1-n}}, \qquad K_n = \frac{A_b}{A_t} \tag{4}$$

Therefore, we treat P_{ch} as a function of A_b. To conduct simulations, we need the function $A_b(x)$, where x is the burn distance. Since this simulator considers only the BATES geometry, below is the formula for precisely this geometry.

$$A_b(x) = 2\pi N((R+x)(L-2x) + \left(\frac{D}{2}\right)^2 - (R+x)^2) \tag{5}$$

Where $R = d_p/2$ and D is external grain diameter, L is lenght of one segment and N is number of segments.

When it comes to computation of x, we solve it numerically. That means using rectangle method of solving integrals. In this case it's a simple one.

$$x(t) = \int_0^{t_c} r dt = \sum r\Delta t \tag{6}$$

Where regresion rate r is a function of $P_c h$, therefore for each $t_c + = \Delta t$ we will calculate new value of regression, which we will use for corresponding addition to burn distance.

$$r(P_{ch}) = a(P_{ch})^n$$

As a closing I want to also mention that we use exactly same method to calculate total impuls, that is simple numerical integration.