Unchirped Laser Pulse Stretching

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Abstract

The Unchirped Laser Pulse Stretching method is pivotal in laser and optics research, enhancing control over laser pulse characteristics essential in material processing, medical technologies, and scientific research. This method extends the duration of short, intense laser pulses into longer, temporally extended pulses with reduced peak power, crucial for the laser implication application. This paper explores not only existing techniques but also introduces a novel approach to unchirped pulse stretching that offsets frequency variations. The new method discussed involves the superposition of positive and negative linearly chirped pulses, facilitated by metasurfaces. The output of the superposed chirped pulses is investigated about its time duration, frequency sweep, and phase profiles. In the end, two-metasurfaces system is introduced to achieve such superposition with several advantages.

Contents

1 Blackground and Introduction

The unchirped pulse stretching method is a pivotal technique in the field of laser and optics research. This method involves stretching a short, intense laser pulse into a longer, temporally extended pulse with lower peak power. It is one of the important part in the pulse amplification. By stretching the pulse duration, the peak power is reduced, making it safer and more manageable for the amplification process.
In industrial applications, unchirped pulse stretching is crucial in processes that require precise control over laser intensity and duration. For example, it is widely used in the field of materials processing, where lasers are used for cutting, welding, and surface treatment. The ability to stretch and subsequently compress laser pulses allows for the precise deposition of energy, which can enhance the quality and precision of the machining process. Additionally, this technique is instrumental in medical technology, particularly in laser surgery and phototherapy, where controlled pulse durations ensure minimal damage to surrounding tissues [1].
Moreover, unchirped pulse stretching has significant applications in scientific research, such as in spectroscopy and in the generation of attosecond pulses. These applications underscore its versatility and critical role in advancing laser technology across various sectors.
One important pulse amplification technique which uses the pulse stretching is the CPA (Chirped Pulse Amplification). The principle of CPA is shown in FIG. 1:

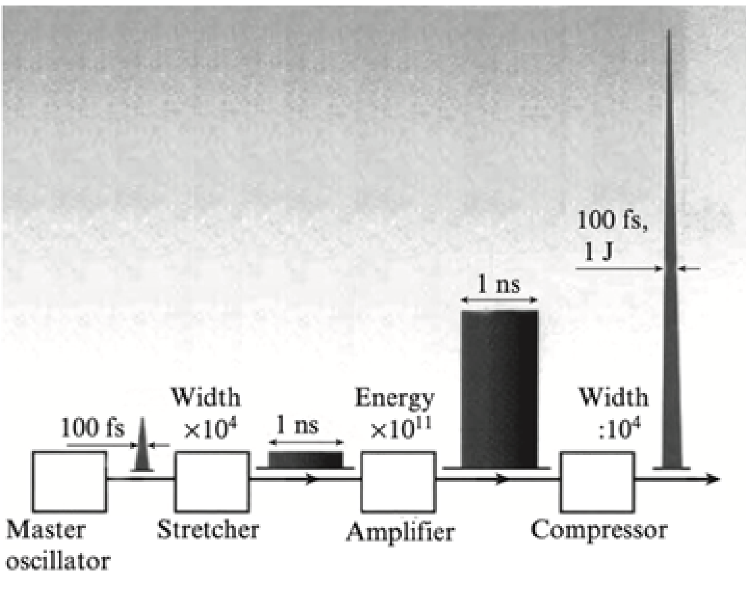


Figure 1: The Chirped Pulse Amplification Working Principle

The initial pulse is firstly stretched by the stretcher to its initial $\times 10^4$ fs width. And the peak power of pulse is considerably reduced with a wider time duration. After the amplifier which contains the amplification medium, it uses one inverse structure of the stretcher which is the compressor to compress the amplified pulse to its initial width and generate the ultra-

high-power laser with a short time. Through history, there are different kinds of optical systems to stretch and compress the pulse, mainly including optical fiber, double grating, four-prism sequence system. [2], [3], [4].
Due to this process, the very high-power laser will not damage the equipment since the power is first stretched and distributed. Thus, the pulse stretching is very important part in this process. In this report, one new stretching method is discussed. It is unchirped meanings it will not introduce the frequency variation of the pulse in the time domain.

1.1 Pulse Property

Assume the electric field can be expressed as:

$$E = A_0 \exp\left(j(\omega t - \beta z)\right)$$

where there is one important factor: the propagation constant of the electric field.The propagation constant depends on the frequency as the dispersion relation:

$$\beta = \beta(\omega)$$

In addition, the definition of phase velocity is found (or the velocity of each frequency):

$$v_{\mathrm{ph}} = \frac{\omega}{\beta}$$

And also the group velocity is defined (or the velocity of the envelope):

$$v_g = \frac{d\omega}{d\beta}$$

The group and phase velocity together define the propagation of the pulse. In fact, the following plot describes the relation between the group and phase velocity more precisely in FIG.2:

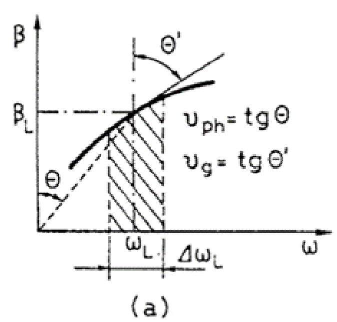


Figure 2: The relation between the group and phase velocity in the plot of β (propagation constant) and ω (frequency). The phase and group velocity describes different angle θ and θ'

Based on the group and phase velocity, the concept of dispersion and the arrival time can be described by the following:

- Group delay describes the time a pulse needs to arrive at its destination, propagated based on group velocity:
$$\tau_g = \frac{l}{v_g} = l \left(\frac{d\beta}{d\omega} \right)_{\omega_l}$$
- Group delay dispersion (GDD): With broader pulse bandwidth, the dispersion of the pulse cannot be approximated by a linear relation and needs to consider the dispersion due to the group velocity of different spectral regions, which is also called second order dispersion, GDD:
$$\Delta\tau_d \approx |\phi''(\omega_L)| \, \Delta\omega_L \approx l \left| \left(\frac{d^2\beta}{d\omega^2} \right)_{\omega_l} \right| \Delta\omega_L$$

Based on the GDD, it can easily define group velocity dispersion (GVD):

$$GVD = \left(\frac{d^2\omega}{d\beta^2} \right)_{\omega_l}$$

In one pulse passing through the normal-dispersion medium, it will have positive GDD meaning the normal dispersion due to different frequencies have different propagation constants and different phase terms. To be specific, for positive GDD (normal dispersion), if there are two pulses with different frequencies ω, the pulse with a higher frequency (or shorter wavelength) will propagate slower than the pulse with lower frequency in the normal-dispersion medium. After propagating a certain distance or time, the pulse will be chirped, meaning the higher frequency will delay in the tail of the entire pulse and the lower frequency will lead in the head of the entire pulse. This separation makes the entire pulse's frequency varied within the pulse, which can be measured in the time domain. That is the chirped pulse. The chirped pulse is unwanted in many experiment. To eliminate this chirping, one method is that by using certain and desirable optical path difference to introduce another GDD, which is negative (or the opposed sign of existing GDD) (such as prism, grating). These two GDD will cancel with each other to generate one unchirped pulse in the normal-dispersion medium. To be specific, for instance, to compensate the fast speed of lower-frequency part in bandwidth, artificially we make lower-frequency part propagates longer OPL than the high-frequency part. Thus, using different optical structure (for example for the prism, it uses designed OPL for certain frequency to compensate or compress the initial pulse dispersion, or for double grating since different frequency will reflect in different angles, by certain design of frequency and OPL, double grating can achieve using OPL to compensate the change of propagation constant), it can achieve optical stretcher (same sign to existing dispersion) or compressor (opposed sign to existing dispersion) [2], [5].

1.2 Chirped Pulse and SPM

As described above, the chirped pulse is the frequency variation in the time domain within the pulse. In fact, there are two kinds of chirping: up-chirped pulse (frequency increases linearly) or down-chirped pulse (frequency decreases linearly). One way to also create the chirping is the self-phase-modulation (SPM), which the material has the Kerr effect, which is the refractive index of the medium depends on the intensity by:

$$n = n_0 + n_2 I$$

where n_0 is the linear refractive index, n_2 is the nonlinear index coefficient, and I is the intensity of the light.
For the material with the Kerr effect, substituting the Kerr effect equation into the phase term of the field, we find the equation of angular frequency:

$$\omega = \omega_L - \left(\frac{\omega_L n_2 z}{c}\right) \frac{\partial I}{\partial t}$$

It can find the frequency depends on the intensity by the following plot 3:

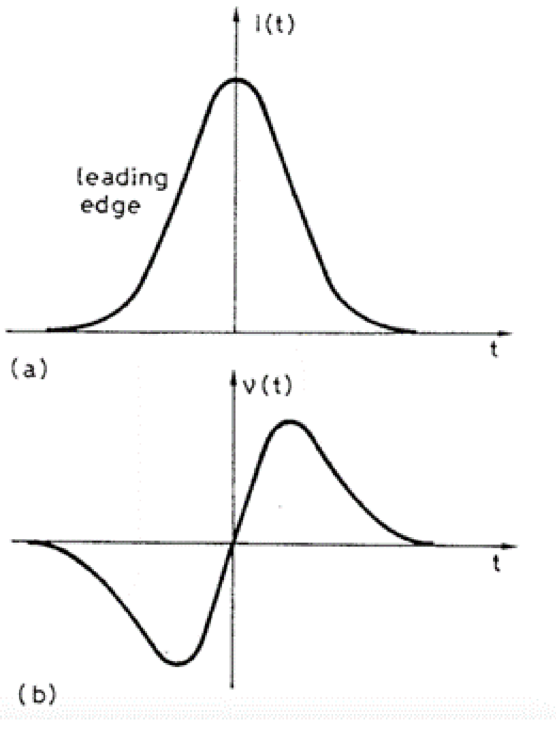


Figure 3: Intensity and frequency plot in time domain for the medium exiting Kerr Effect

It can find for the medium exiting the Kerr effect the pulse can also be chriped, which is also called self-phase-modulation (SPM).

1.3 SPM and GDD

Both SPM and GDD have the effect on the pulse chirping. GDD is due to the dispersion relation of the material:

$$\beta = \beta(\omega).$$

For light of different frequencies, it will experience different propagation factors, and then it will have different group velocities. However, for SPM, it is due to the intensity profile of the pulse. Due to the Kerr effect, where the refractive index is proportional to the square of the electric field or intensity:

$$n = n_0 + n_2 I$$

different frequencies within the pulse, having different intensities, will experience different refractive indices, and therefore, different speeds. For the optical fiber experiment [5], the output of both SPM and GDD (existing at the same time) are shown in FIG.4:

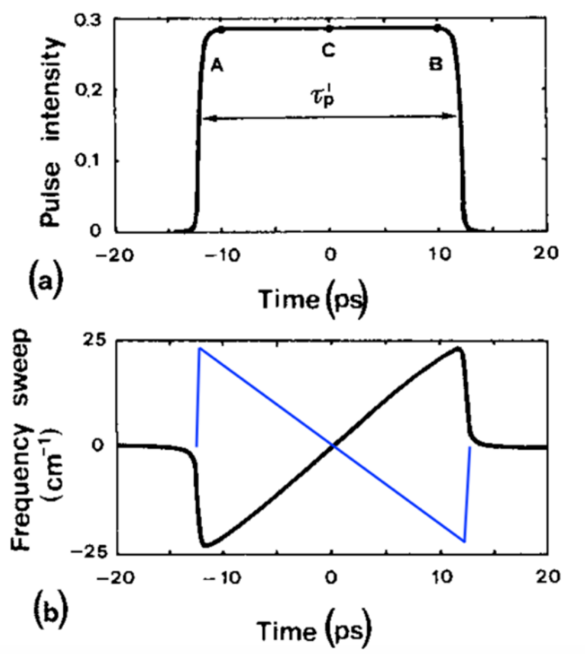


Figure 4: Output of pulse after fiber with both GDD and SPM: Intesnity and frequency sweep with time

It can find the output of the fiber with both GDD and SPM is linearly chirped (frequency of one pulse increases with the time). In fact, certain optical system with negative chirping can be created and added to the system (blue line). It can find the blue line added with the initial black line can achieve unchirping. In addition, the spectrum of the output of fiber system is shown in FIG.5:

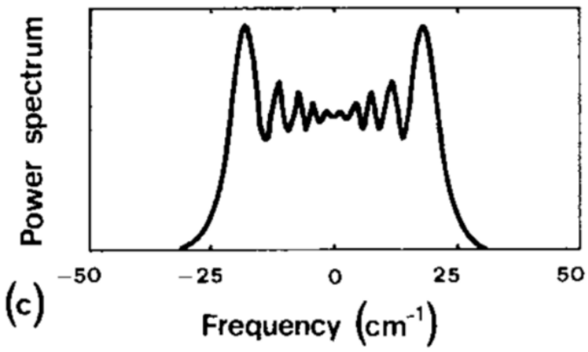


Figure 5: Output pulse spectrum

It can clearly find that SPM actually introduces broader spectrum (introduce more frequency) in the spectrum of the pulse. That is something only GDD cannot do. Through Fourier Transform, broden spectrum means narrower time duration. But there is the limit which is called Transform limit (or Fourier limit, Fourier transform limit)

1.4 Transform limit

Transform limit is the lower limit for the pulse duration for a given pulse spectrum. For instance, for one spectrum, through the Fourier transform from the frequency domain (Spectrum) to the time domain (Time Duration), there is a minimum time duration that this pulse can achieve. Quantitatively, the time-bandwidth product (TBP) describes the transform limit of the pulse by [6]:

$$TBP = \Delta\tau \cdot \Delta\nu$$

where $\Delta\tau$ is the time duration of the pulse and $\Delta\nu$ is the spectral width of the pulse. So, even if SPM can introduce more frequency to the spectrum of light. The time duration of the resulted pulse is still limited by the transform limit. Vise Versa for the narrower spectrum. Two different pulse profile's TBP are shown:[6]:

- For Gaussian profiles

$$TBP_{\text{Gaussian}} = 0.441,$$

- For sech^2 profiles

$$TBP_{\text{sech}^2} = 0.315.$$

The frequency and time domain relation are shown in FIG.6 and FIG.7:

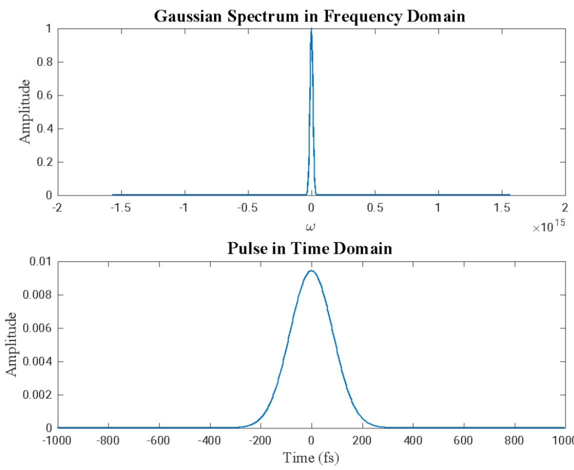


Figure 6: Gaussian time and spectrum

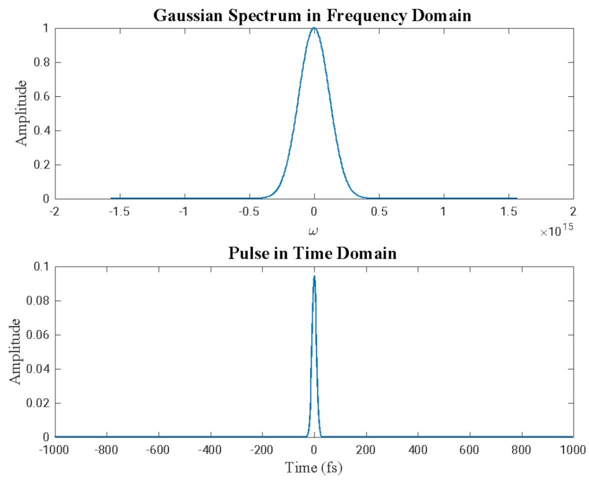


Figure 7: Gaussian time and spectrum

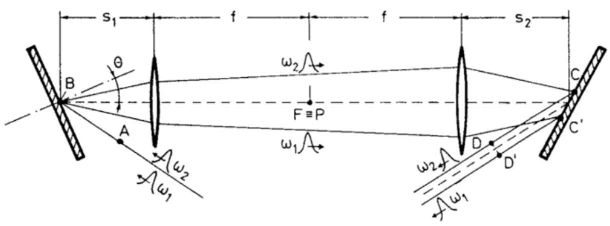
The reason why transform limit is important for chirping pulse is that when the pulse is at the transform limit, where $TBP_{\min} = \Delta\tau \cdot \Delta\nu$, the pulse has a constant instantaneous frequency (constant spectral phase) and is unchirped. Once the pulse exhibits an increasing or varying instantaneous frequency, which indicates it is chirped, $\Delta\tau \cdot \Delta\nu > TBP_{\min}$.

2 Existing Unchirped Pulse Stretching Method

Until now, there are several methods which can be the potential unchirped pulse stretching method choice.

2.1 Grating or prism or fiber to compensate the existing GDD

After passing through the medium, the pulse will be chriped. Then, one grating or prism or fiber can be used to add the opposed-sign GDD to each wavelength so that the existing GDD can be balanced. The structure is shown in FIG.8:



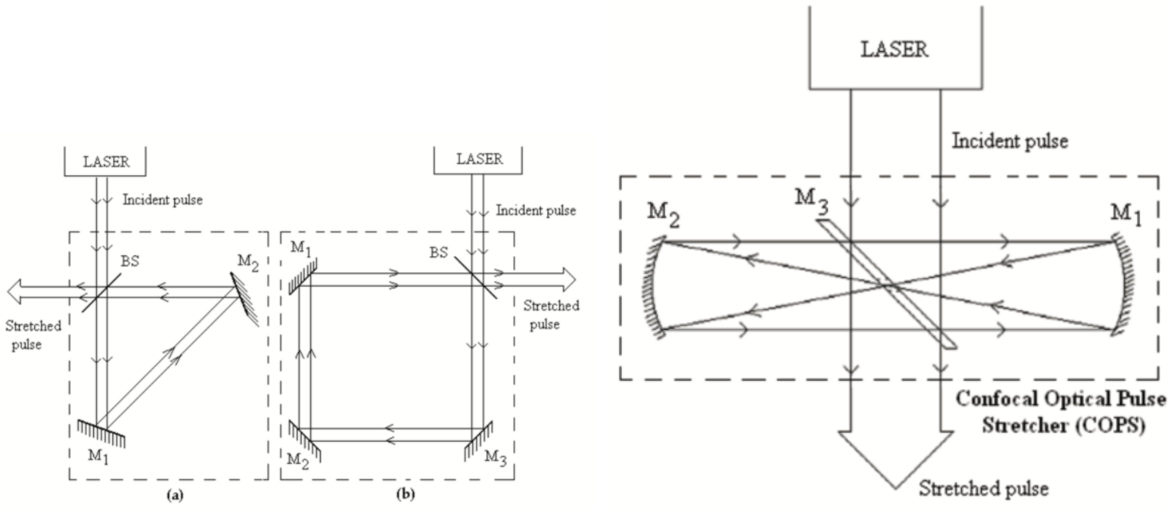
$$\text{GDD} = \frac{d^2\phi}{d\omega^2} = \frac{4\pi^2 c}{\omega^3 d^2 \cos^2\theta}\,(2f - s_1 - s_2)$$

Figure 8: Grating GDD Compensation Method with the provided GDD below [5]

Two gratings can be used to correct the delay between two ω_1 and ω_2 pulses. This method has certain problems: very hard to construct in the lab. The structure needs to have a precise construction which provides suitable GDD to compensate the existing GDD.

2.2 Optical ring cavity

The cavity can be used to split the amplitude of the incident laser pulse and introduce optical delays by having larger OPL, and recombine the temporally delayed portions of the pulse to provide a temporally stretched laser pulse. In detail, when one pulse incidences the BS, part of it will reflect while remaining will transmit. Through each round-trip in the cavity, part of the remaining transmit pulse will get out after introduce nL/c delay where n is the round-trip number, L is the total length of cavity. Thus, the cavity divides an initially amplitude laser pulse into many smaller amplitude pulses and introduce different nL/c delay to them and recombine at BS to have stretching pulse. It can also use the Fabry–Perot interferometer with the same principle. Since using the cavity and BS does not affect the spectral property of the pulse, no chirping will be introduced during this process. Two kinds of cavities are shown in FIG. 9a and FIG.9b:



(a) Cavity formed by Beam slipter

(b) Confocal Optical Pulse Stretcher (COPS)

Figure 9: Cavity Unchirped Pulse Stretching[7]

Stretching the pulse using cavity indeed does not influence the spectral property of pulse. But its stretching value is limited and the output stretched pulse profile has high intensity (first comes-out) and generally decreases.

Specifically, for the COPS system, the part (amplitude) of input laser light is transmitted while other is reflected into the confocal cavity. The confocal cavity will apply a time delay for the reflected part and after one round it will continue to transmit part of the light (remaining in the cavity) and reflect other light (exit the cavity). Through this cavity, it will continue applying a certain delay to the remaining amplitude in the cavity which stretches the pulse in the time and spatial domain. The output of the cavity stretching method using FIG.9 is shown in FIG.10:

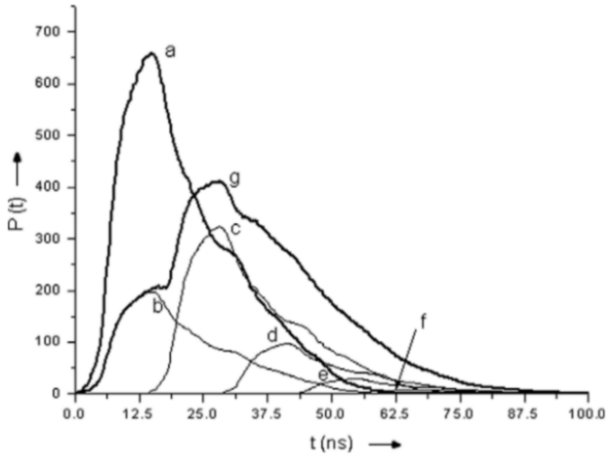


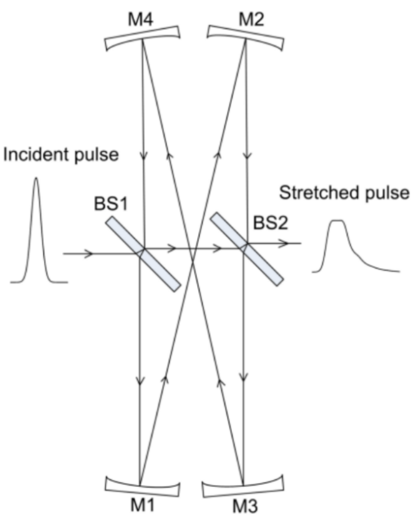
Figure 10: The COPS input (a) and output(g) pulse, (b, c, d, e, f) are the 1st to 5th term in the P(t) equation and (g) is the superposition of (b, c, d, e, f) [7]

And the result of stretching using COPS is shown in Table.1:

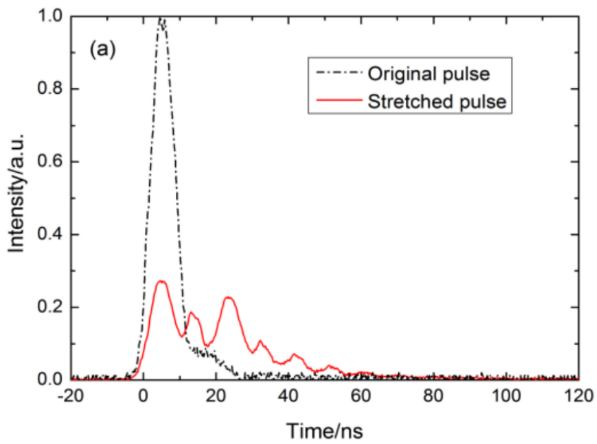
Characteristic	**Before COPS**	**After COPS**
Pulse $1/e^2$ width	40 ns	55 ns
Peak power	53.57 kW	38.96 kW (average power does not change)
Peak power location	15.25 ns	28.5 ns

Table 1: Result of Pulse Characteristics Before and After COPS [7]

In addition, COPS can also use two cavities shown in FIG.11a and result shown in FIG.11b:



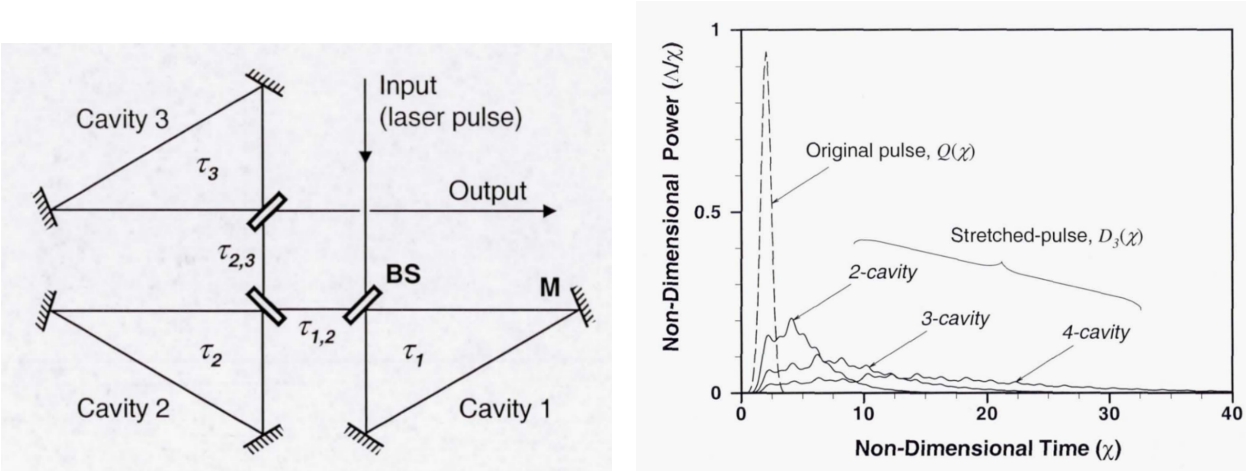
(a) Two mirrors two cavity structures



(b) Output temporal profile of two mirror COPS

Figure 11: The structure and output temporal profile of two mirror COPS [7]: the output is indeed not in good shape

Additionally, the cavity number can continuously increase, which is Multiple Optical Ring-Cavities. The structure and result of different number of cavities are shown in FIG.12a and FIG.12b [8]:



(a) Multiple Optical Ring-Cavities Structure (b) Multiple Optical Ring-Cavities temporal output

Figure 12: Structure and output of the Multiple Optical Ring-Cavities Structure and Output

2.3 Compensation of nonlinear phase shifts with third-order dispersion TOD

The gain of amplifier can be designed to have certain non linear phase shift and introduce TOD within the stretcher to compensate the SPM effect [9]. The dechirped process occurs when TOD compensates the nonlinear phase shift, shown in FIG.13:

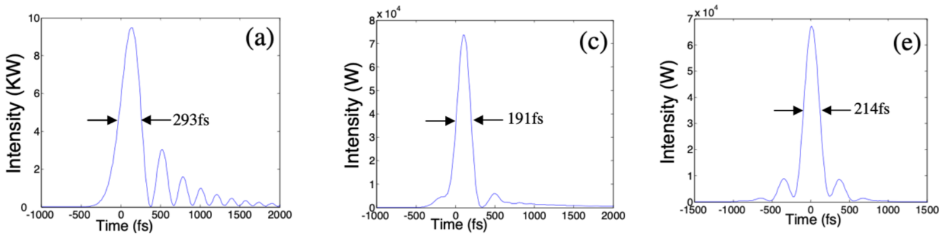


Figure 13: (a) fiber stretcher, $\Phi^{NL} = 0.4\pi$. (c) fiber stretcher, $\Phi^{NL} = 1.9\pi$. (e) grating stretcher, $\Phi^{NL} = 1.9\pi$

3 Positive and Negative Chirped Pulse Superposition

Based on the existing method to achieve unchirped pulse stretching, there is one hypothesis which can be another method to generate the unchirped pulse. Two chirped pulses are generated, one is positively linearly chirped and the other is negatively linearly chirped. Each of them has the same chirping rate. When a certain technique is used to overlap these two pulses in the time domain, the two chirped pulses will cancel each other's chirping and generate one unchirped pulse. To test this, two Gaussian profile chirped pulses are used. Consider a chirped pulse, where the pulse shape G_t and its corresponding electric field E_t are defined as follows:

$$G_t = A_0 \cdot \exp\left(-\left(\frac{t}{\Delta t}\right)^2\right)$$

$$E_t = G_t \cdot \exp\left(i\left(\omega_{\text{center}} \cdot t - \phi_t + \phi_m\right)\right)$$

where:

- A_0 is the unit amplitude
- Δt represents the pulse duration
- $\omega_{\text{center}} = 2\pi \cdot f_{center}$ represents the carrier frequency.
- $\phi_t = -\beta_t \cdot t^2$ is the chirped term.
- ϕ_m can actually be added to achieve the misalignment problem of two pulses. For perfectly alignment, $\phi_m = 0$

By selecting β_t with both negative and positive values of the same magnitude, one can generate both negatively and positively chirped pulses. To obtain the instantaneous frequency with respect to time, the formula is given by:

$$\omega_{\text{ins}}(t) = \omega_{center} - \frac{d\phi}{dt}$$

where $\frac{d\phi}{dt}$ represents the rate of change of the phase with respect to time. The plots of these two pulses are shown in FIG.14

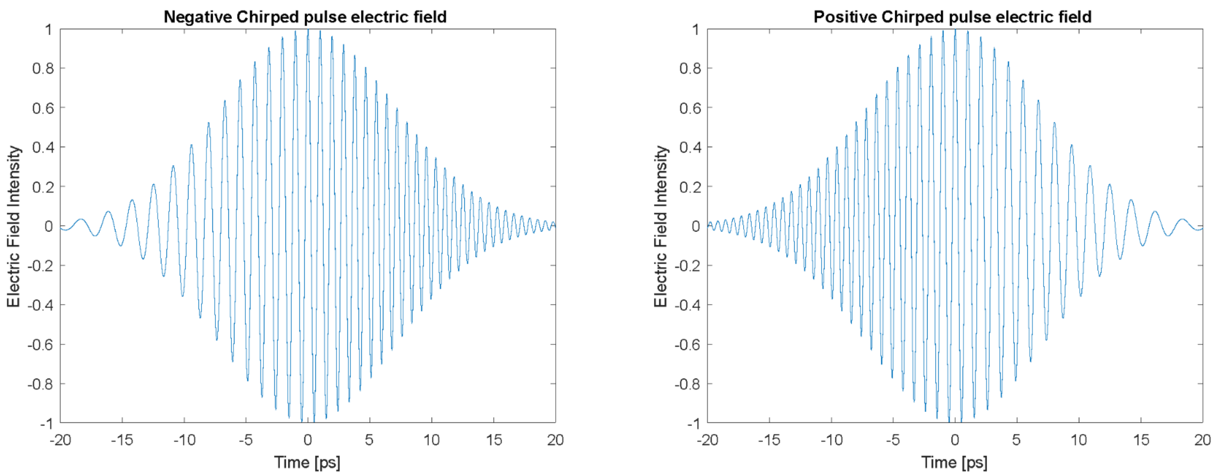


Figure 14: Two Chirped Pulses Intensity with time, where chirping rate $\beta_t = \pm 0.1$

Assume there is no misalignment problem. Then, two chirped pulses with the same but opposed chirping rate, named E_1 and E_2 are superposed together. To show that superposing positive and negative chirping pulses can indeed stretch the pulse in the time domain. One reference pulse is chosen E_r. The reference pulse is unchirped. The spectrum of $E_1 + E_2$ and E_r are adjusted to be the same (same FWHM) to ensure that $E_1 + E_2$ and E_r have the same spectrum so that it can compare the time duration of $E_1 + E_2$ and E_r under the same width of spectrum. The spectrum of $E_1 + E_2$ and E_r are shown in FIG. 15:

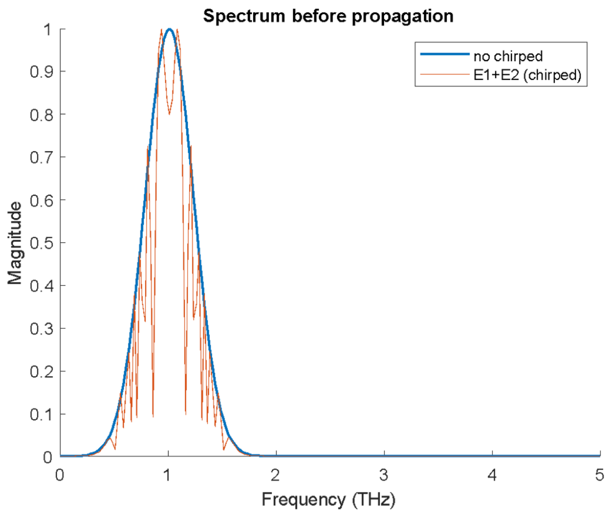


Figure 15: Spectrum of $E_1 + E_2$ and E_r (no chirped)

Notices that although $E_1 + E_2$ and E_r have the same FWHM. In $E_1 + E_2$, some wavelengths on the spectrum have lower intensity value. The reason for this is when superposing E_1 and E_2 there are interference that some wavelengths are destructively interfered out. In that case, the spectrum is not a smooth curve but with peak and trough. Then, FFT of the spectrum of two pulses to find the time domain profile of these two pulses as shown in FIG.16:

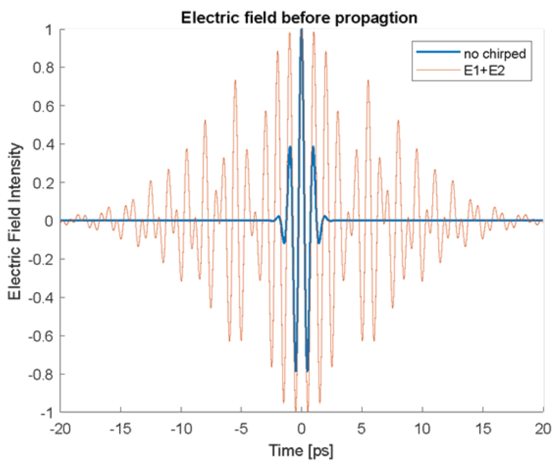


Figure 16: Time Profile of $E_1 + E_2$ and E_r under the same spectrum width

It can find that with same spectrum (no new wavelength), the positive-negative-superposed pulse (red or $E_1 + E_2$) has larger pulse time duration in time domain than the reference pulse (blue to E_r). It means positive-negative-superposing makes **the time domain pulse stretching**. To further proof that positive-negative-superposing can make the time domain pulse unchirped stretching, $\omega_{\text{ins}}(t)$ of E_1, E_2, and $E_1 + E_2$ are found in FIG.17:

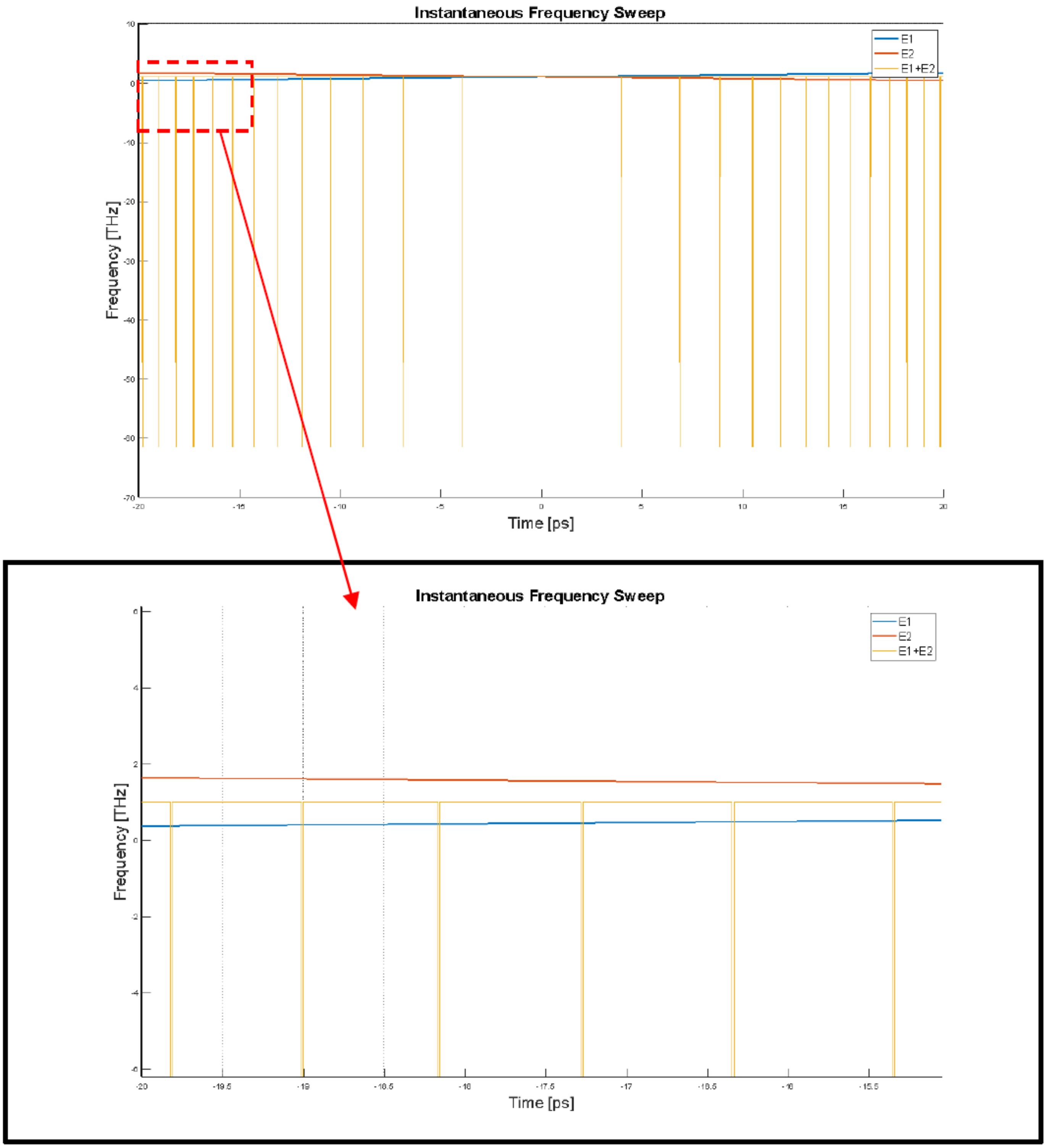


Figure 17: Frequency Sweep Result of E_1, E_2, and $E_1 + E_2$

Based on the plot, it can find that indeed the $E_1 + E_2$ superposed pattern in FIG.16 is unchirped (yellow line is flat). The wired drop or toughs of yellow line is due to the phase problems, which the unwrap and wrap phase of $E_1 + E_2$ superposed pattern in FIG.16 is shown in FIG.18 and FIG.19:

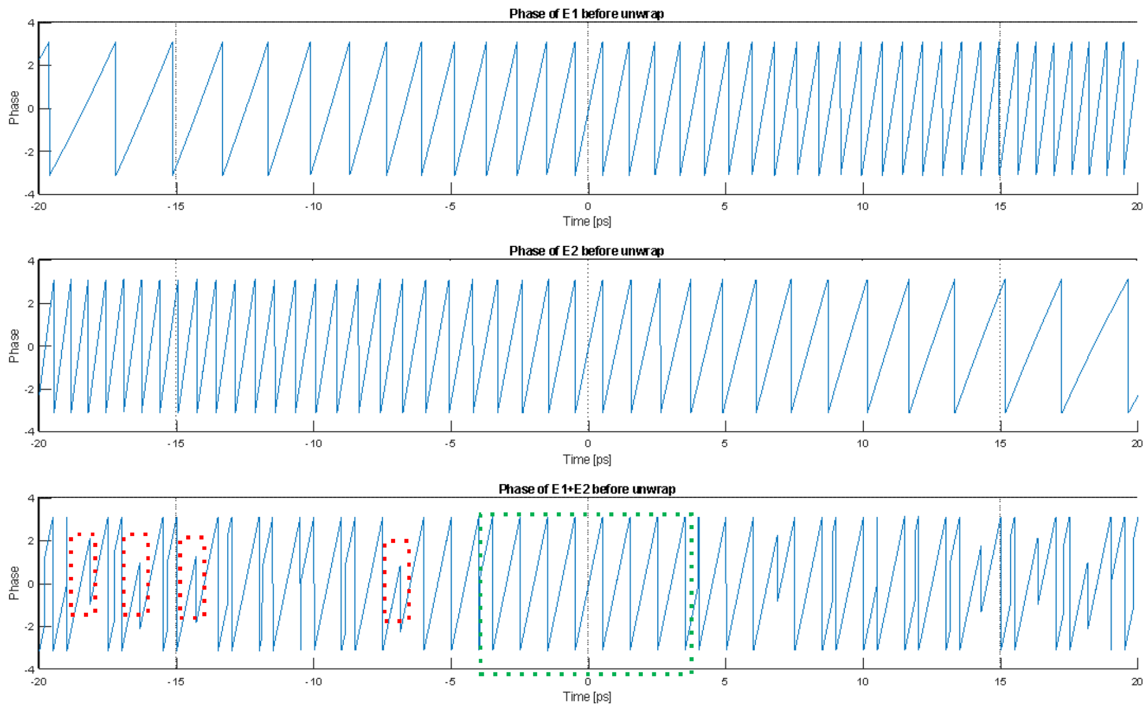


Figure 18: E1 E2 and E1+E2 phase term before unwrap (red boxes are the sudden phase discontinuous and green box is the safe region)

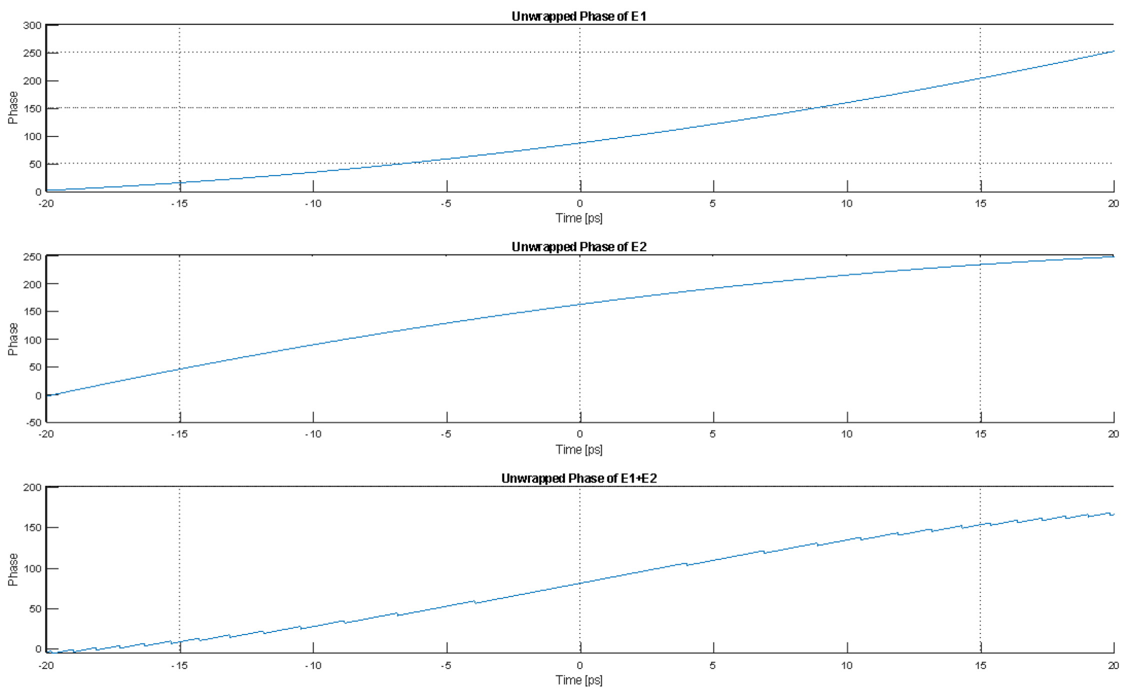


Figure 19: E1 E2 and E1+E2 phase term after unwrap (E1+E2 has zigzag shapes)

Compared the unwrap and wrap phase term of $E_1 + E_2$, it can find at the red box region, phase is not continuous, made the wrap phase term of $E_1 + E_2$ does not continue at certain points. This sudden phase change will cause the infinity frequency detection which can explain the unusual trough.

- **In general, superposing positive and negative linearly-chirped pulse can achieve unchirped pulse stretching in time domain. However, it will introduce some sudden frequency changes.**

Then, the misalignment problem is considered with E1 and E2 have 0.7pi phase shift. The spectrum and electric fields of reference pulse and misaligned E1+E2 are shown in FIG.20:

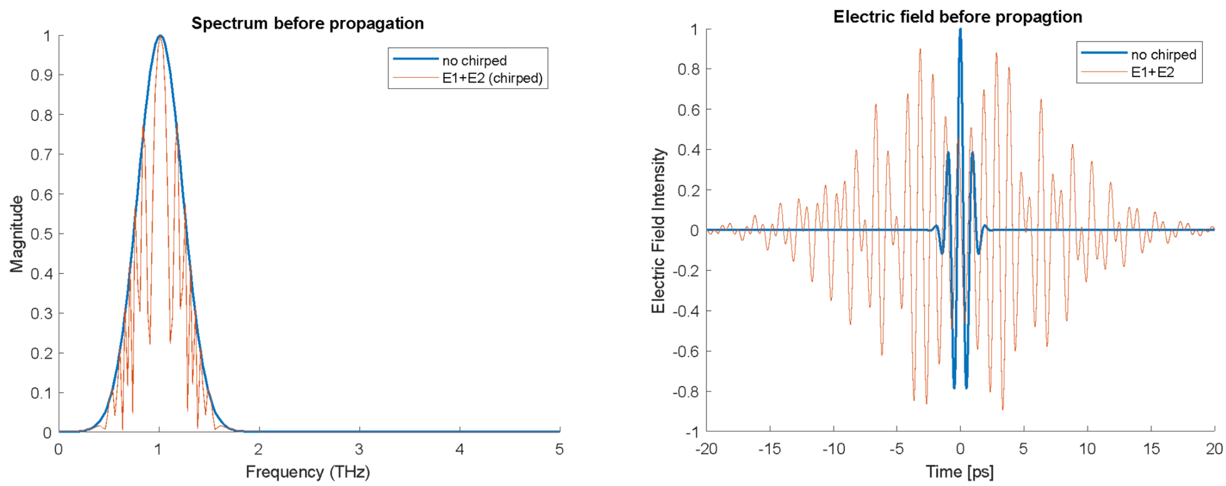


Figure 20: The spectrum and electric fields of reference pulse and misaligned E1+E2

It can find with misalignment, superposing E1 and E2 still can have pulse stretching in the time domain under the same spectrum. The chirping testing of misaligned E1+E2 is shown in FIG.21:

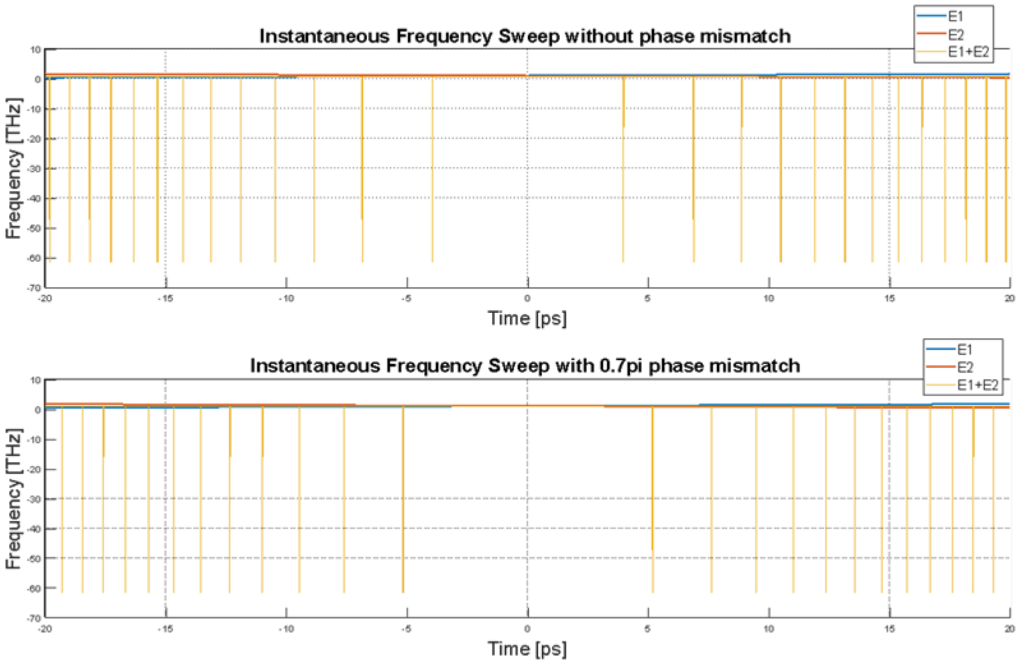


Figure 21: Frequency Sweep Result of E_1, E_2, and $E_1 + E_2$ under 0.7pi phase shift

Compared with before, it can state:

- **In general, with phase mismatch, superposing positive and negative linearly-chirped pulse can still achieve unchirped pulse stretching in time domain. However, the sudden frequency changes positions will be different.**

4 Method

Based on the investigation above, superposing the positive and negative linearly chirped pulses (with same but opposed sign chriped rate) can indeed achieve unchirped pulse stretching in time domain. This hypothesis can be achieved by using metasurface, as shown in FIG.22:

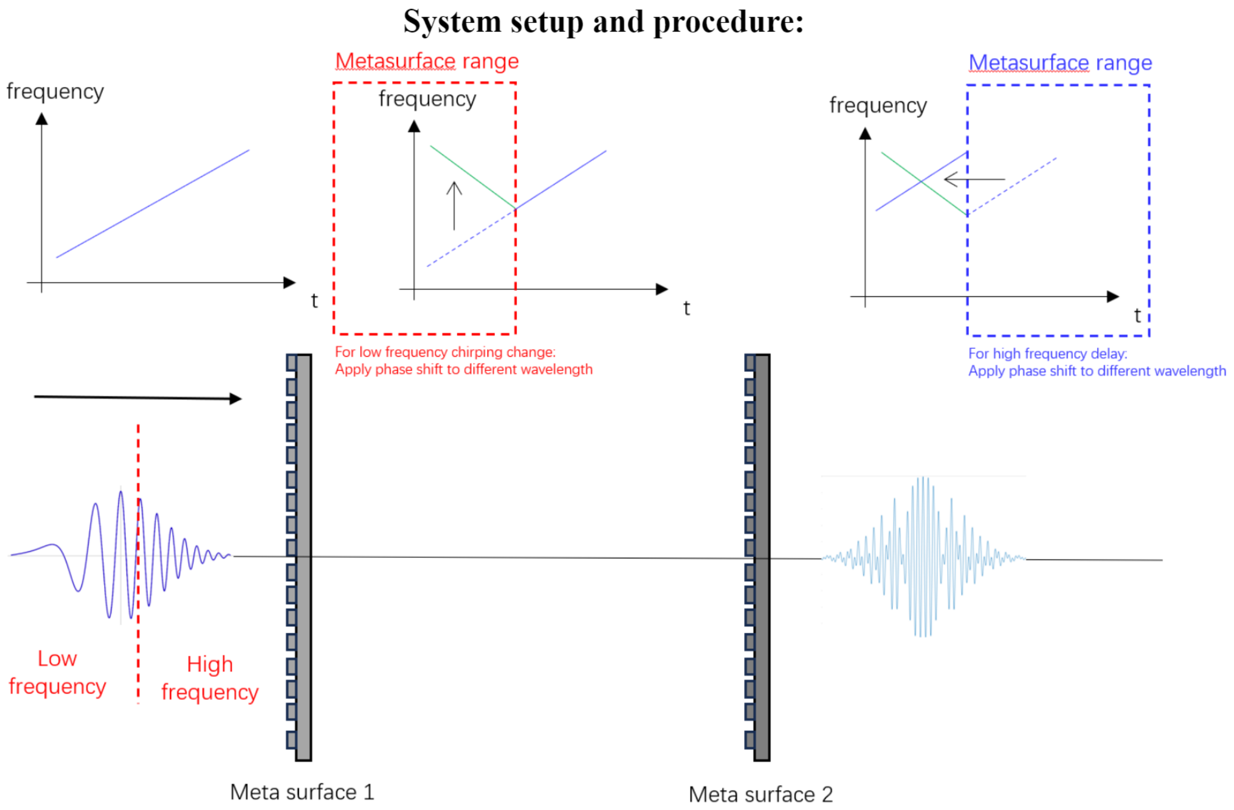


Figure 22: Two Metasurface structures to achieve superposing the positive and negative linearly chirped pulses

Instead of two pulses, this system just relays on one chirped pulse and use metasurface to separate this pulse to create the positive and negative linearly chirped pulse.

- Incoming Pulse: one positive or negative linearly-chirped pulse incidents from left.
- The pulse interacts with the first metasurface (Meta surface 1). The Metasurface 1 has a certain effective working frequency range (for higher frequency metasurface 1 has no effect, so the high frequency part will directly transmit). For lower frequency part, metasurface 1 will apply certain phase shifts to different wavelengths, which can make the initial positive chirping to negative chirping. And metasurface 1 needs to be included nonlinear crystal which can double the frequency of lower frequency part. Thus, we can split the initial chirped pulse to two pulses with opposed chirping.
- Then, the split positive and negative chirped pulses still have certain time delay. Metasurface 2 is used to compensate such time delay to make two split positive and negative chirped pulse coincidences in the time domain, by applied certain phase shift to certain wavelength.

To achieve this, the metasurface needs to have certain requirements:

- High transmission for both high and low frequency part

- Have certain effective working frequency range and the boundary is in the middle of initial positive or negative chirped pulse spectrum
- can apply enough phase shift depend on the wavelength for switching chirping and pulse delay.

5 Discussion

In this report, it mainly discussed the unchirped pulse stretching in the time domain. By superposing the positive and negative linearly chirped pulses (with same but opposed sign chirped rate), it can achieve pulse stretching unchirply in the time domain. The stretching effect is compared with the reference unchirped pulse with the same spectrum width. The output of this superposed pulses is investigated about its time duration, frequency sweep, and phase profile. The unusual drops or the toughs are discussed and explained. In the end, two metasurfaces (one with non-linear liquid crystal) are used to (1) slipt the incoming chirped pulse into two (2) apply certain phase shifts to switch the chirping and double the frequency (3) apply time delay to superpose these two portions coming from the incident chirped pulse. The advantages of superposing the positive and negative linearly chirped pulses to stretch pulse is that it has relatively low requirement for the alignment, have minor effect on the output stretched pulse profile, and enable to achieve relatively high stretching value.

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