

LAB REPORT – 1

Signal Processing

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AIM: Fourier Series Analysis

1.1 Finding Fourier Series Coefficients

For a periodic signal $x(t)$, the Fourier Series Coefficients are determined by the formula :

$$a_k = \frac{1}{T} \int_{\langle T \rangle} x(t) e^{-jk\omega_0 t} dt, k \in \mathbb{Z}$$

Function for fouriercoeff:

```
function F = fourierCoeff(t, xt, T, t1, t2, N)
F = zeros(2*N+1, 1);

k = 1;
while k <= 2*N+1
    F(k) = 1/T * int(xt * exp(-1j * (k - N - 1) * 2 * pi * t / T), t, t1, t2);
    k = k + 1;
end
end
```

a. Finding Fourier Series Coefficients of:

$$x(t) = 2\cos(2\pi t) + \cos(6\pi t), T = 1 \text{ and } N = 5$$

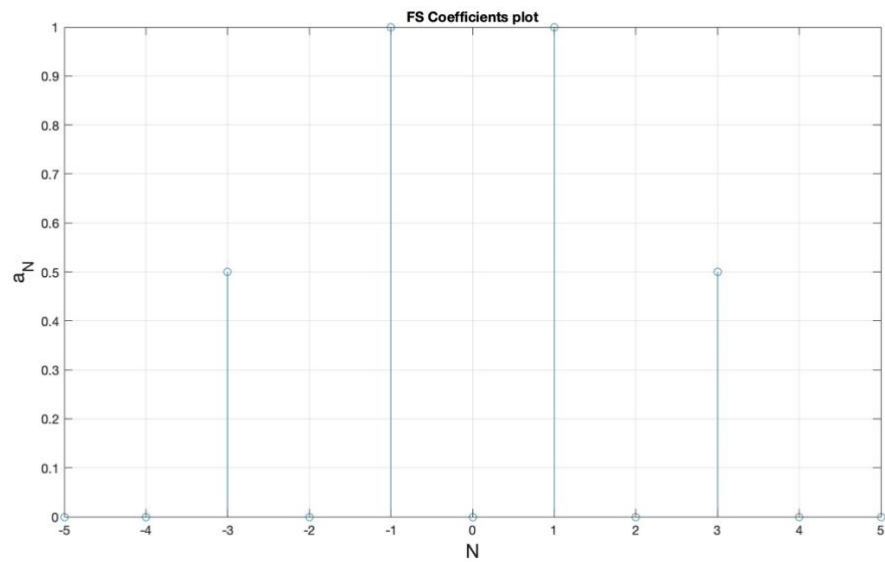
Code:

```
% part a
clc, clearvars, close all

syms t;
T = 1; xt = 2*cos(2*pi*t) + cos(6*pi*t); N = 5;
t1 = -(T/2); t2 = T/2;
F = fourierCoeff(t,xt,T,t1,t2,N);

FS_idx = -1*N:N;
figure; stem(FS_idx,F);
grid on;
xlabel('N','FontSize',15); ylabel('a_{N}','FontSize',15); title('FS Coefficients plot')
```

Plot:



Calculations:

$$\begin{aligned}
 x(t) &= 2\cos(2\pi t) + \cos(6\pi t) ; \quad T=1. \\
 a_k &= \frac{1}{T} \int_{\langle T \rangle} x(t) e^{-jk\omega_0 t} dt ; \quad \omega_0 = \frac{2\pi}{T} = 2\pi \text{ rad/sec} \\
 \text{we express } x(t) &\underline{as} \quad x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \\
 \Rightarrow x(t) &= 2 \left[\frac{e^{j2\pi t} + e^{-j2\pi t}}{2} \right] + \left[\frac{e^{j6\pi t} + e^{-j6\pi t}}{2} \right] \\
 &= \frac{e^{-j3\omega_0 t}}{2} + e^{-j\omega_0 t} + e^{j\omega_0 t} + \frac{e^{j3\omega_0 t}}{2} \\
 &= 0.5 e^{-j3\omega_0 t} + e^{-j\omega_0 t} + e^{j\omega_0 t} + 0.5 e^{j3\omega_0 t} \\
 \Rightarrow a_3 &= 0.5 = a_{-3} \quad \& \quad a_1 = 1 = a_{-1}
 \end{aligned}$$

b. Finding Fourier series coefficients of:

$$x(t) = \begin{cases} 1, & -T_1 \leq t \leq T_1 \\ 0, & T_1 < |t| < T/2 \end{cases}$$

$$N = 10, T=1, T = 4T_1$$

Code:

```

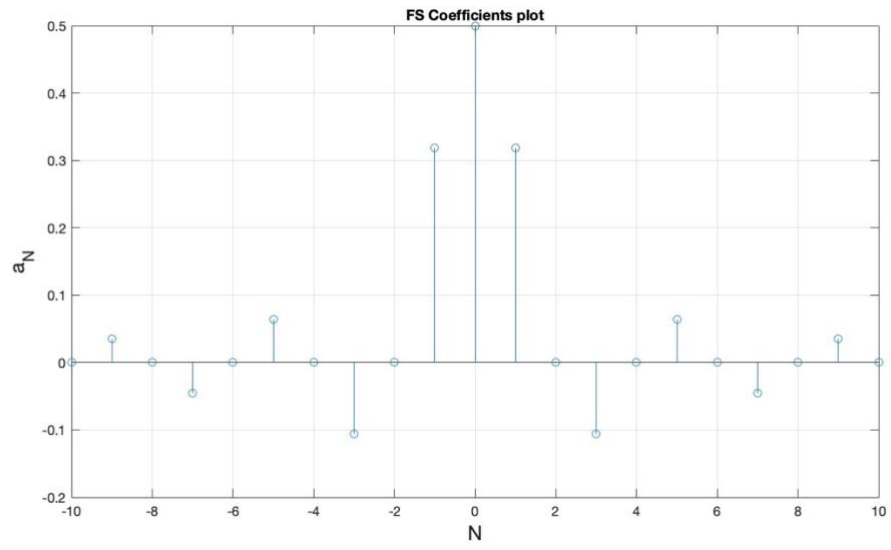
% part b
clc, clearvars, close all

syms t;
T = 1; t1 = -(T/2); t2 = T/2; N = 10;
T1 = 0.25;
xt = piecewise(t >= -(T1) & t <= T1, 1, T1 < abs(t) & abs(t) < T/2, 0);
F = fourierCoeff(t, xt, T, t1, t2, N);

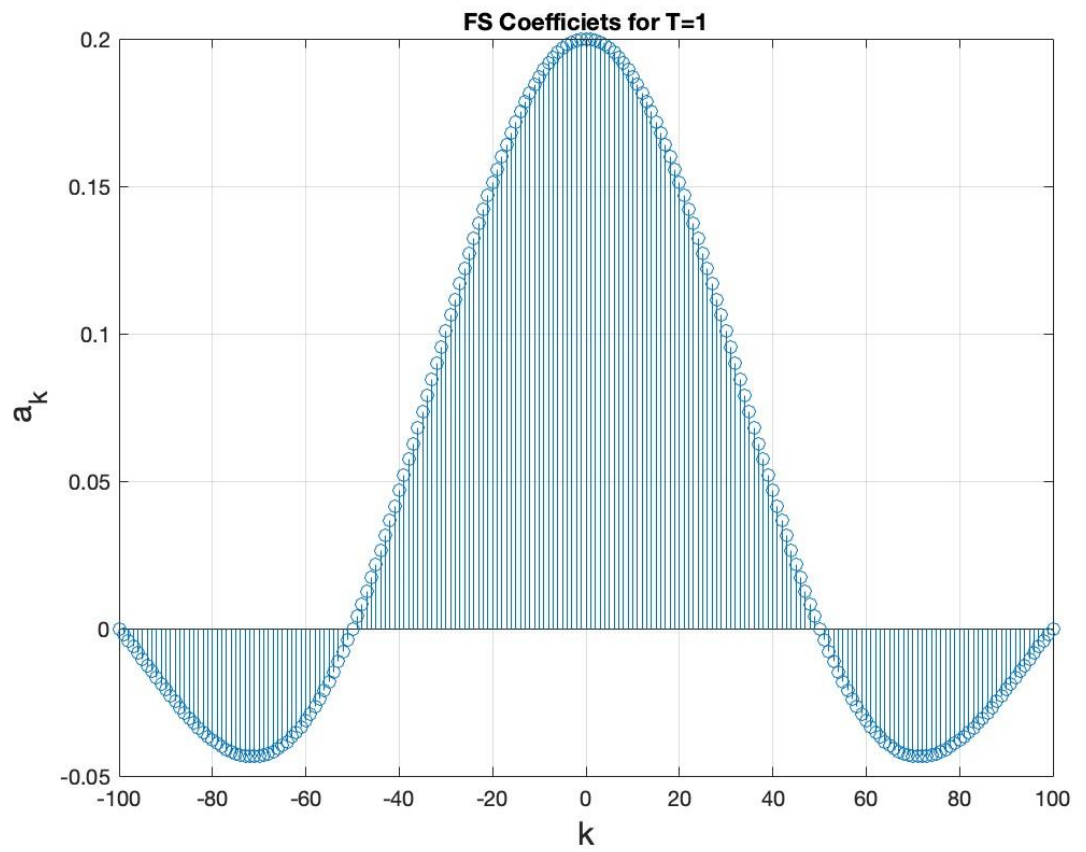
FS_idx = -N:N;
figure; stem(FS_idx, F);
grid on;
xlabel('N', 'FontSize', 15); ylabel('a_{N}', 'FontSize', 15); title('FS Coefficients plot')

```

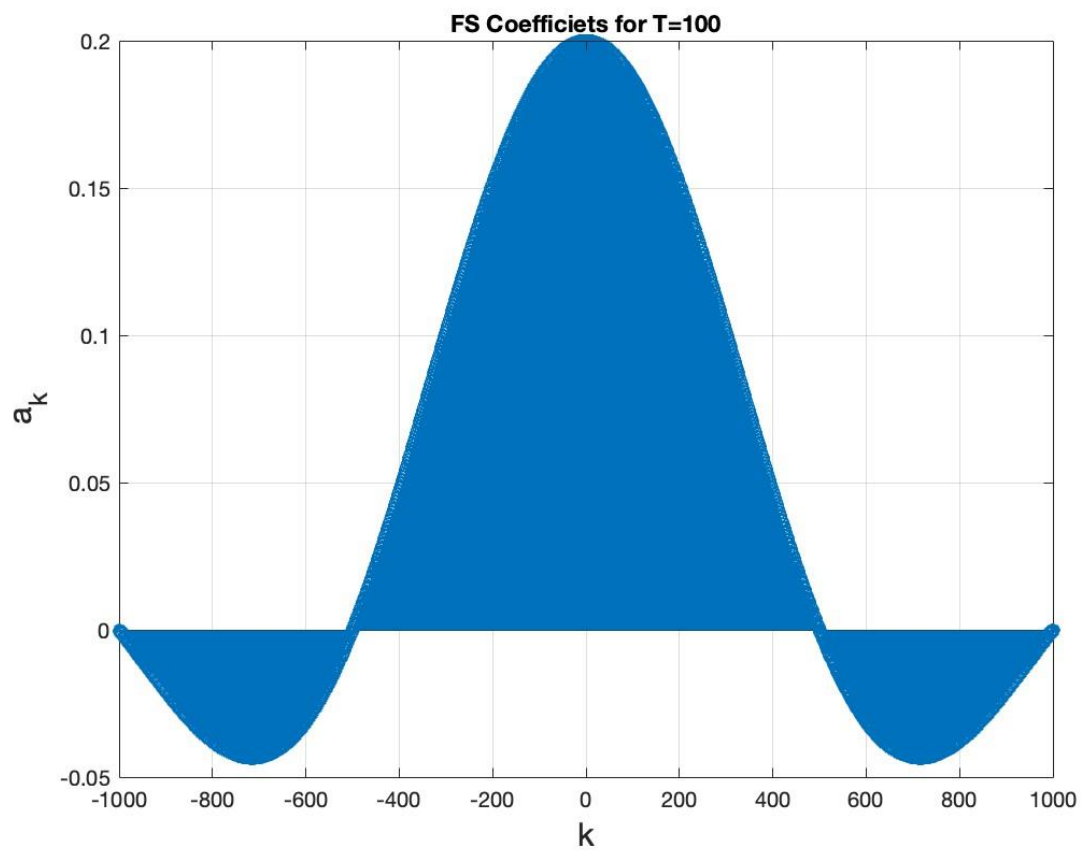
Plot:



$T = 10$



$T = 100$



Calculations:

$$(c) \quad x(t) = \begin{cases} 1, & -T_1 \leq t \leq T_1 \\ 0, & T_1 < |t| < T/2 \end{cases}$$

$$T_1 = 1/4, \quad T = 1$$

$$a_k = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jk\omega_0 t} dt.$$

$$T = 1 \quad \omega_0 = 2\pi/T = 2\pi \text{ rad/sec.}$$

$$a_k = \frac{1}{T} \int_{-1/2}^{1/2} x(t) e^{-jk\omega_0 t} dt$$

$$= \int_{-1/2}^{1/4} x(t) e^{-jk\omega_0 t} dt + \int_{-1/4}^{1/4} x(t) e^{-jk\omega_0 t} dt + \int_{1/4}^{1/2} x(t) e^{-jk\omega_0 t} dt.$$

$$= \int_{-1/2}^{1/4} e^{-jk\omega_0 t} dt.$$

$$= \left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0} \right]_{-1/2}^{1/4}$$

$$= \frac{1}{-jk\omega_0} \left[e^{-jk\omega_0/4} - e^{jk\omega_0/4} \right]$$

$$= \frac{1}{k\pi} \sin\left(\frac{k\pi}{2}\right)$$

$$a_k = \frac{1}{k\pi} \sin\left(\frac{k\pi}{2}\right) \quad \text{for } k \in \mathbb{Z}, \quad k \neq 0.$$

$$\text{for } k=0, \quad a_0 = \frac{1}{T} \int_{-T/2}^{T/2} x(t) dt = \frac{1}{T} \times (\text{Area under one time period})$$

$$a_0 = 1 \times 1 \times \frac{1}{2} = 0.5$$

$$a_0 = 0.5$$

$$a_k = \frac{1}{k\pi} \sin\left(\frac{k\pi}{2}\right)$$


1.2 FS reconstruction and finite FS aprox. Error

The partial fourier sum for a signal $x(t)$ with fourier series coeff.

a_k is given by: $\hat{x}(t) = \sum_{k=-N}^N a_k e^{jk\omega_0 t}$

As $N \rightarrow \infty$, we construct the original signal.

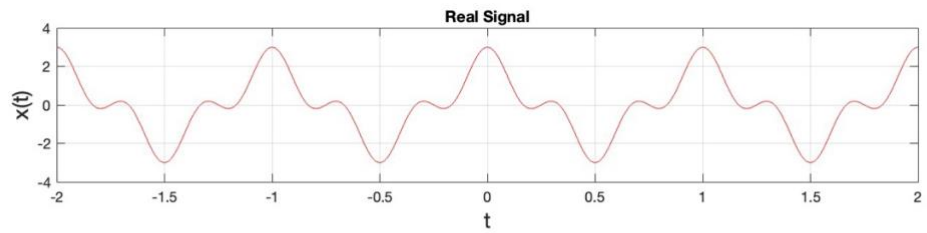
Parital fourier sum function:



```
function y = partialfouriersum(A,T,time_grid)
y = zeros(size(time_grid)); N = (length(A)-1)/2;
for k = -N:N
    y = y+A(k+N+1)*exp(j*k*(2*pi*time_grid)/T);
end
end
```

a.

Plot:



b.

Code:

```

clc, clearvars, clear all
syms t;
time_grid = - 0.5:0.01:0.5;
xt = 2*cos(2*pi*t)+cos(6*pi*t); T = 1; N = 5; t1 = -T/2; t2 = T/2;
F = fourierCoeff(t,xt,T,t1,t2,N);
y = partialfouriersum(F,T,time_grid);
time = -2:0.01:2; nums = double(subs(xt,t,time));

subplot(3,1,1); plot(time,nums, "Color",'r'); grid on;
xlabel('t','FontSize',15); ylabel('x(t)','FontSize',15); title('Real Signal');

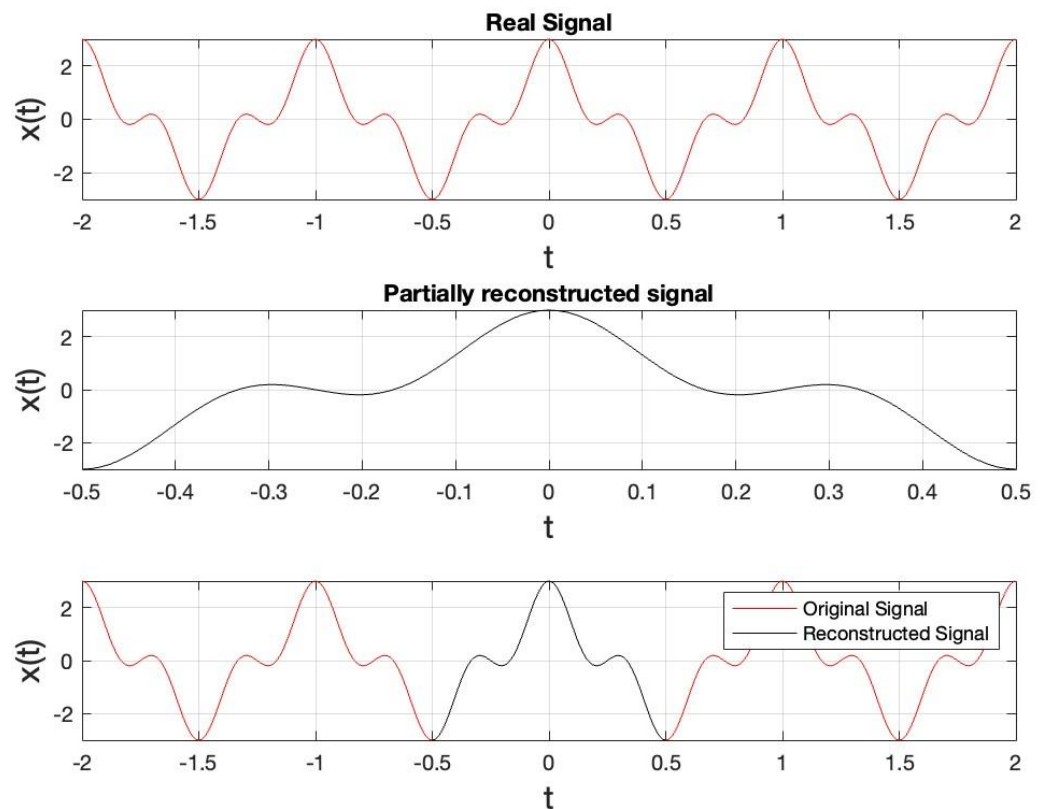
% hold on;

subplot(3,1,2); plot(time_grid,y, "Color", 'black'); grid on;
xlabel('t','FontSize',15); ylabel('x(t)','FontSize',15); title('Partially reconstructed signal');

% hold on;

subplot(3,1,3);
orig = plot(time, nums, "Color",'r'); grid on; hold on;
recons = plot(time_grid, y, "Color",'black'); hold on;
xlabel('t','FontSize',15); ylabel('x(t)','FontSize',15); legend('Original Signal', 'Reconstructed Signal');
```

Plot:



c. Code for finding error:

Code:

```

● ● ●

clc, clearvars, clear all
syms t;
T=1; N=5;
xt=2*cos(2*pi*t)+cos(6*pi*t); A=fourierCoeff(t,xt,T,-T/2,T/2,N);
time_grid = -0.5:0.01:0.5;
xtrecon = partialfouriersum(A,T,time_grid); xtnew=2*cos(2*pi*time_grid)+cos(6*pi*time_grid); mae=0;
rmssum=0;
for k = 1:length(time_grid)
    xtarr=2*cos(2*pi*time_grid(k))+cos(6*pi*time_grid(k)); er = abs(xtarr-xtrecon(k));
    rmssum=rmssum+er*er;
    if(er>mae)
        mae=er;
    end
end
rms=sqrt(rmssum/length(time_grid));
disp('Max. abs error = '); disp(mae);
disp('sqrt error = '); disp(rms);

```

Answer:

Command Window

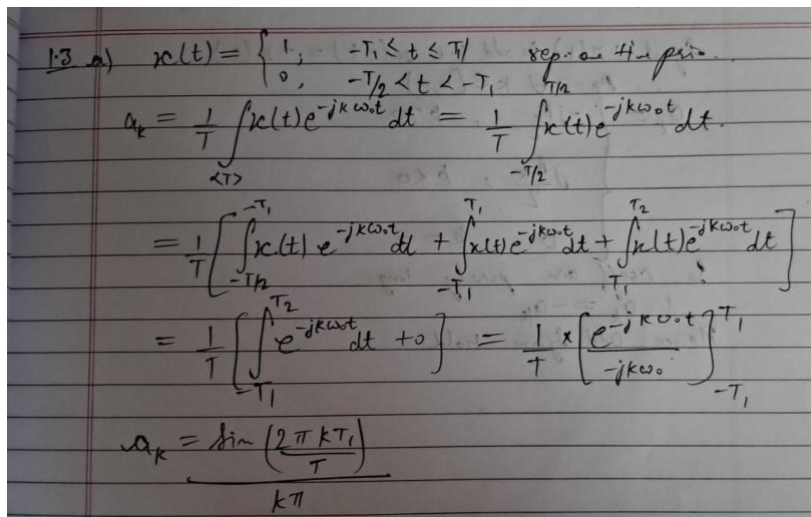
Max. abs error =
8.6221e-16

sqrt error =
2.9071e-16

f_x >>

1.3 Gibbs Phenomenon revisit square wave

a. Calculations:



$$1.3 \ a) \ x(t) = \begin{cases} 1, & -T/2 \leq t \leq T/2 \\ 0, & \text{elsewhere} \end{cases}$$

$$a_k = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jk\omega_0 t} dt = \frac{1}{T} \int_{-T/2}^{T/2} e^{-jk\omega_0 t} dt$$

$$= \frac{1}{T} \left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0} \right]_{-T/2}^{T/2} = \frac{1}{T} \left(\frac{e^{-jk\omega_0 T/2} - e^{jk\omega_0 T/2}}{-jk\omega_0} \right)$$

$$= \frac{1}{T} \left(\frac{e^{-jk\pi} - e^{jk\pi}}{-jk\pi} \right) = \frac{1}{T} \left(\frac{e^{-jk\pi} - e^{jk\pi}}{-jk\pi} \right)$$

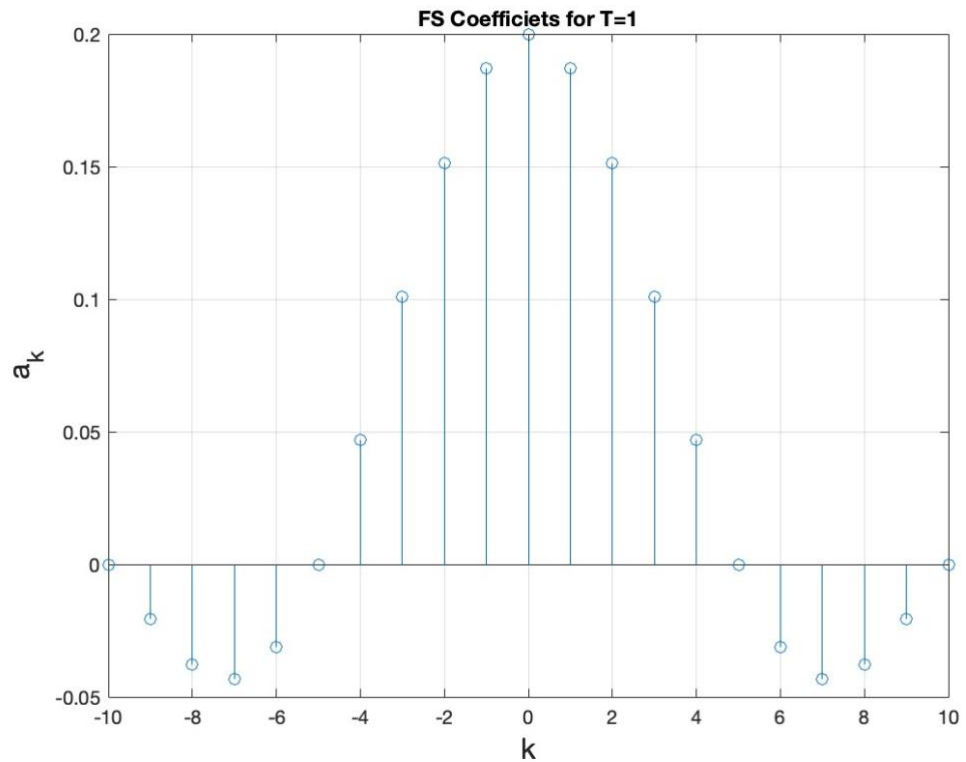
$$a_k = \frac{\sin(k\pi)}{k\pi}$$

b. Code to find coefficients:

```
syms t;
T = 1; N = 10*T; t1 = -T/2; t2 = T/2; T1 = 0.1;
xt = piecewise((-T/2 < t) & (t < -T1), 0, ((-T1 <= t) & (t <= T1)), 1, (T1 < t) & (t < T/2), 0);
FS_idx = -N:N;
F = fourierCoeff(t,xt,T,t1,t2,N); y = T*F;

figure(1); stem(FS_idx,y); grid on;
xlabel('k', FontSize = 15); ylabel('a_k', FontSize=15); title('FS Coefficients for T=1');
```

Plot:



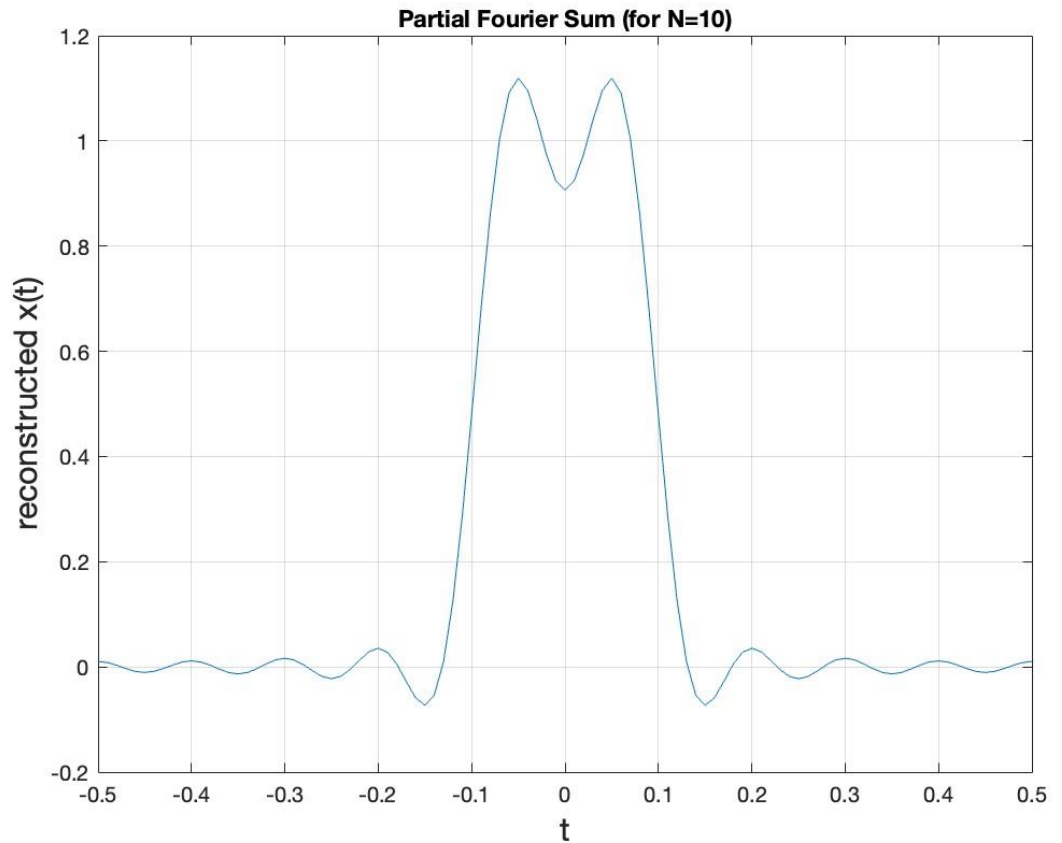
c. Code:

```

syms t;
T = 1; N = 10; t1 = -T/2; t2 = T/2; T1 = 0.1; time_grid = -0.5:0.01:0.5;
xt = piecewise((-T/2<t)&(t<=-T1),0,((-T1<=t)&(t<=T1)),1,((T1<=t)&(t<T/2)),0);
F = fourierCoeff(t,xt,T,t1,t2,N);
xReconstructed = partialfouriersum(F,T,time_grid);
plot(time_grid,xReconstructed); grid on; xlabel('t', FontSize = 15);
ylabel('reconstructed x(t)', FontSize = 15);
title('Partial Fourier Sum (for N=10)');

```

Plot:



We observe that as N increases, the error decreases, and the reconstructed signal starts resembling the original signal more and more.

1.4 Fourier Series more examples and symmet.

Prop.

PART A :

Code:

```

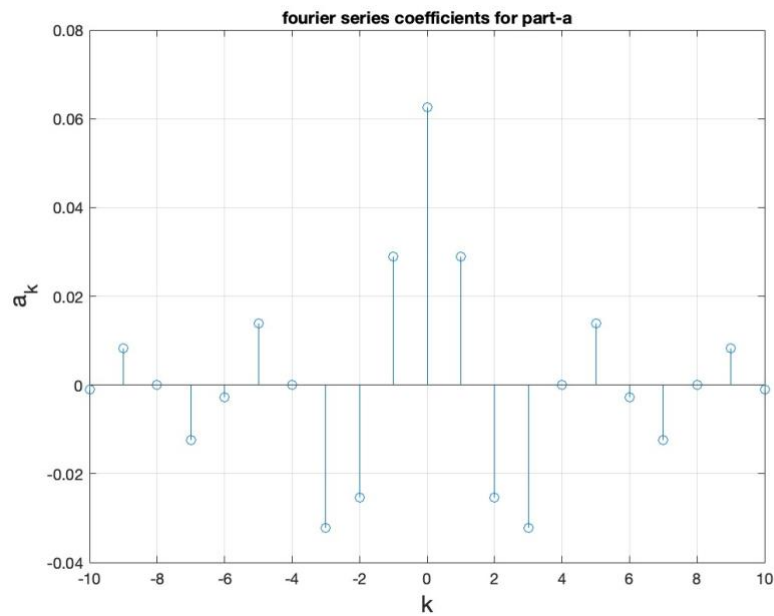
clc, clearvars, clear all
syms t;
T = 1; t1 = -T/2; t2 = T/2; N = 10;

% part-a
xt = piecewise(t>-1/4 & t<0, -t, t>0 & t<1/4, t, 1/4 < abs(t) & abs(t)<T/2,0);
F = fourierCoeff(t, xt, T, t1, t2, N); index = -N:N; figure(1); stem(index, F); grid on;
xlabel('k', 'FontSize',15); ylabel('a_k', 'FontSize',15); title('fourier series coefficients for part-a');

% part-b
xt_ = piecewise(t>-1/4 & t<1/4, t, t<-1/4 & t>t1, 0, t>1/4 & t<t2, 0);
F_ = fourierCoeff(t,xt_,T,t1,t2,N); figure(2); stem(index, -1j*F_); grid on;
xlabel('k', 'FontSize',15); ylabel('Im(a_k)', 'FontSize',15); title('Fourier Series Coeff of part-b');

```

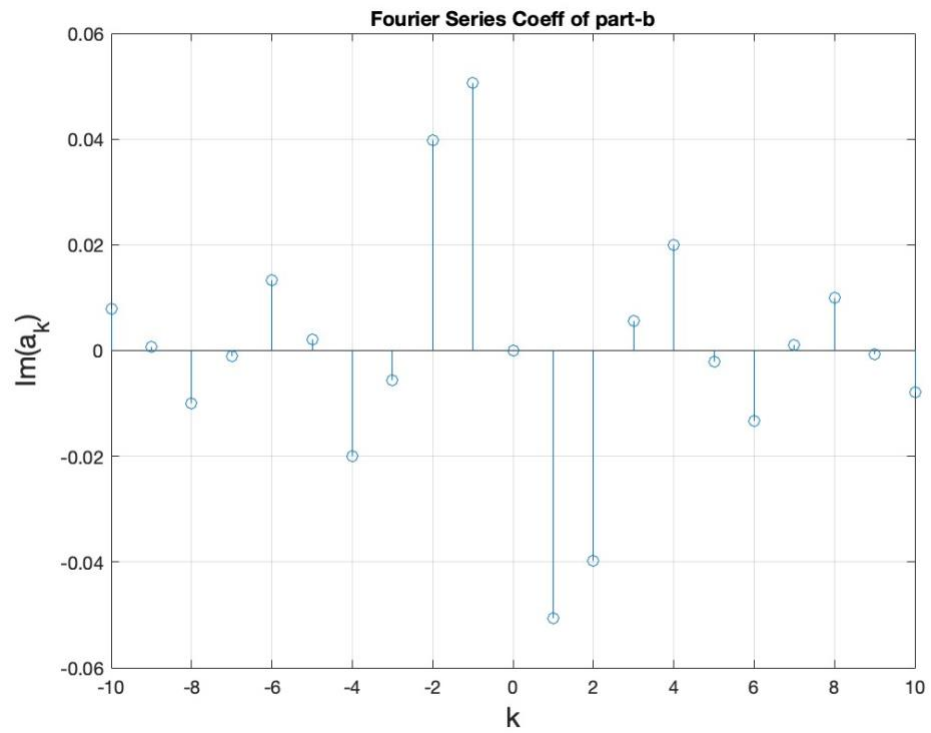
Plot:



We observe a Even Symmetry here.

PART B :

Plot:



We observe a Odd Symmetry in this.

PART C :

Calculations:

1.4 c) 1.4(a) is an even signal.
 $\rightarrow x(t) = \sum_{k=-\infty}^{\infty} a_k e^{-jk\omega_0 t} = A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} B_k \sin(k\omega_0 t)$

$\therefore x(t)$ is even, $B_k = 0 \forall k \in [1, \infty)$

$$a_k = \begin{cases} A_0 & ; k=0 \\ \frac{A_k - jB_k}{2} & ; k > 0 \\ \frac{A_k + jB_k}{2} & ; k < 0 \end{cases} = \begin{cases} A_0 & , k=0 \\ \frac{A_k}{2} & , k > 0 \\ \frac{A_{-k}}{2} & ; k < 0 \end{cases}$$

$$a_k = \begin{cases} A_0 & , k=0 \\ \frac{A_{|k|}}{2} & , k \neq 0 \end{cases}$$

for $k \neq 0$, $a_k = a_{-k}$. So the graph a_k vs k is symm. w.r.t y axis.

\rightarrow Coeff. are real

Hence, Even Symm. holds.

Signal 1.7(b) is odd i.e. $x(-t) = -x(t)$

$\therefore A_k = 0 \forall k \in [0, \infty)$

$$a_k = \begin{cases} -j \frac{B_k}{2} & , k > 0 \\ j \frac{B_{-k}}{2} & , k < 0 \\ 0 & k=0 \end{cases}$$

So, coeff are purely imag.

$$a_k = -a_{-k}$$

Hence, Odd Symm. holds.