LAB REPORT – 1
Signal Processing

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AIM: Fourier Series Analysis

1.1 Finding Fourier Series Coefficients

For a periodic signal x(t), the Fourier Series Coefficients are determined by the formula :

$$\boldsymbol{a_k = \frac{1}{T}\int_{<T>} x(t)e^{-jk\omega_0 t}\,dt \quad , k \in \mathbb{Z}}$$

Function for fouriercoeff:

```
function F = fourierCoeff(t, xt, T, t1, t2, N)
F = zeros(2*N+1, 1);

k = 1;
while k <= 2*N+1
    F(k) = 1/T * int(xt * exp(-1j * (k - N - 1) * 2 * pi * t / T), t, t1, t2);
    k = k + 1;
end
end
```

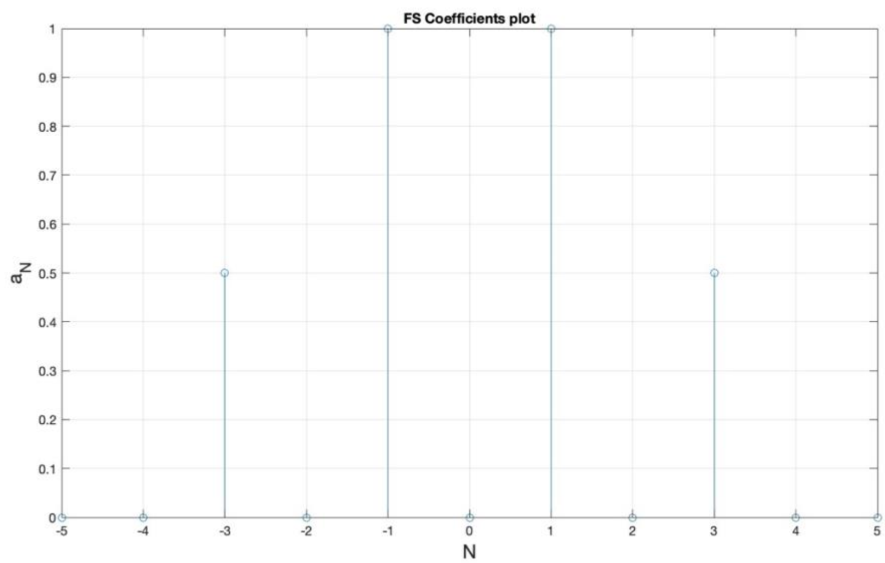
a. Finding Fourier Series Coefficients of:

$$x(t) = 2Cos(2\pi t) + Cos(6\pi t) \qquad , T = 1 \; and \; N = 5$$

Code:

```
% part a
clc, clearvars, close all

syms t;
T = 1; xt = 2*cos(2*pi*t) + cos(6*pi*t); N = 5;
t1 = -(T/2); t2 = T/2;
F = fourierCoeff(t,xt,T,t1,t2,N);

FS_idx = -1*N:N;
figure; stem(FS_idx,F);
grid on;
xlabel('N','FontSize',15); ylabel('a_{N}','FontSize',15); title('FS Coefficients plot')
```

Plot:



Calculations:

$$x(t) = 2\cos(2\pi t) + \cos(6\pi t) \quad ; \quad T = 1.$$

$$a_k = \frac{1}{T}\int_{\langle T\rangle} x(t)\, e^{-jk\omega_0 t}\, dt \quad ; \quad \omega_0 = \frac{2\pi}{T} = 2\pi \text{ rad/sec}$$

we express $x(t)$ as $\quad x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

$$\Rightarrow x(t) = 2\left[\frac{e^{j2\pi t} + e^{-j2\pi t}}{2}\right] + \left[\frac{e^{j6\pi t} + e^{-j6\pi t}}{2}\right]$$

$$= \frac{e^{-j3\omega_0 t}}{2} + e^{-j\omega_0 t} + e^{j\omega_0 t} + \frac{e^{j3\omega_0 t}}{2}$$

$$= .5e^{-j3\omega_0 t} + e^{-j\omega_0 t} + e^{j\omega_0 t} + 0.5e^{3j\omega_0 t}$$

$$\Rightarrow a_3 = .5 = a_{-3} \quad \& \quad a_1 = 1 = a_{-1}$$

b. Finding Fourier series coefficients of:

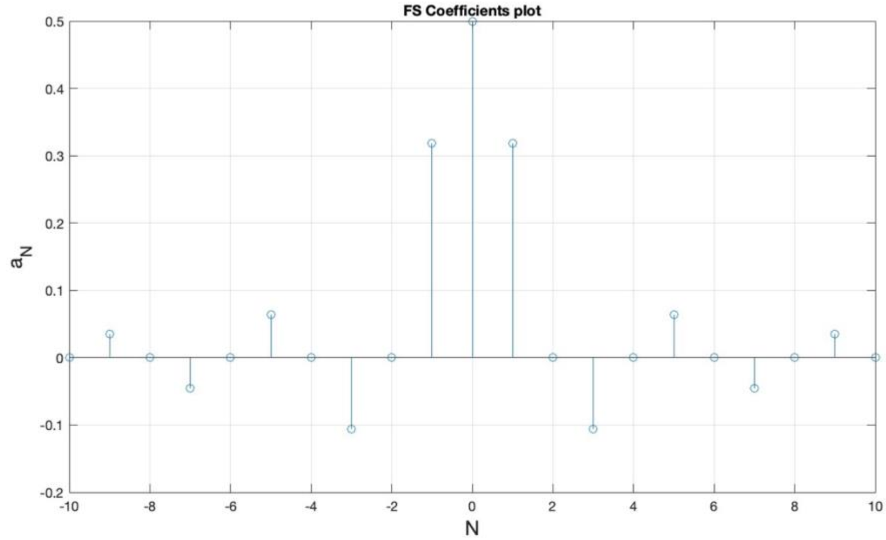
$$x(t) = \begin{cases} 1, & -T_1 \leq t \leq T_1 \\ 0, & T_1 < |t| < T/2 \end{cases}$$

N = 10, T=1, T =4T_1

Code:

```
% part b
clc, clearvars, close all

syms t;
T = 1; t1 = -(T/2); t2 = T/2; N = 10;
T1 = 0.25;
xt = piecewise(t>=-(T1) & t<=T1, 1, T1<abs(t) & abs(t)<T/2 ,0);
F = fourierCoeff(t,xt,T,t1,t2,N);

FS_idx = -N:N;
figure; stem(FS_idx,F);
grid on;
xlabel('N','FontSize',15); ylabel('a_{N}','FontSize',15); title('FS Coefficients plot')
```

Plot:



Calculations:

(6) $x(t) = \begin{cases} 1, & -T_1 \leq t \leq T_1 \\ 0, & T_1 < |t| < T/2 \end{cases}$

$T_1 = 1/4, \quad T = 1$

$$a_k = \frac{1}{T}\int_{<T>} x(t)\, e^{-jk\omega_0 t}\, dt.$$

$T = 1 \quad \omega_0 = 2\pi/T = 2\pi$ rad/sec.

$$a_k = \frac{1}{T}\int_{-1/2}^{1/2} x(t) e^{-jk\omega_0 t}$$

$$= \int_{-1/2}^{1/4} \cancel{x(t) e^{-jk\omega_0 t}}^{0}\, dt + \int_{-1/4}^{1/4} x(t) e^{-jk\omega_0 t}\, dt + \int_{1/4}^{1/2} \cancel{x(t) e^{-jk\omega_0 t}}^{0}\, dt.$$

$$= \int_{-1/4}^{1/4} e^{-jk\omega_0 t}\, dt.$$

$$= \left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right]_{-1/4}^{1/4}$$

$$= \frac{1}{-jk\omega_0}\left[e^{-jk\omega_0/4} - e^{jk\omega_0/4}\right]$$

$$= \frac{1}{k\pi}\sin\left(\frac{k\pi}{2}\right)$$

$a_k = \frac{1}{k\pi}\sin\left(\frac{k\pi}{2}\right)$ for $k \in \mathbb{Z}$, $k \neq 0$.

for $k=0$, $a_0 = \frac{1}{T}\int_{<T>} x(t)\, dt = \frac{1}{T} \times$ (Area under one time period)

$a_0 = 1 \times 1 \times \frac{1}{2} = .5 \quad \boxed{a_0 = 0.5} \quad \boxed{a_k = \frac{1}{k\pi}\sin\left(\frac{k\pi}{2}\right)}$

1.2 FS reconstruction and finite FS aprox. Error

The partial fourier sum for a signal x(t) with fourier series coeff.

a_k is given by: $\hat{x}(t) = \sum_{k=-N}^{N} a_k \, e^{jk\omega_0 t}$

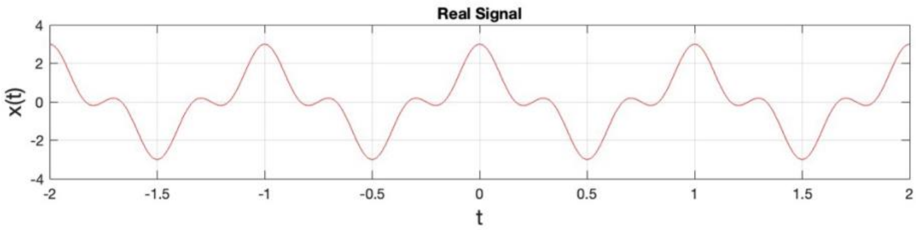
As $N \rightarrow \infty$, we construct the original signal.

Parital fourier sum function:

```
function y = partialfouriersum(A,T,time_grid)
y = zeros(size(time_grid)); N = (length(A)-1)/2;
for k = -N:N
    y = y+A(k+N+1)*exp(j*k*(2*pi*time_grid)/T);
end
end
```

a.

Plot:

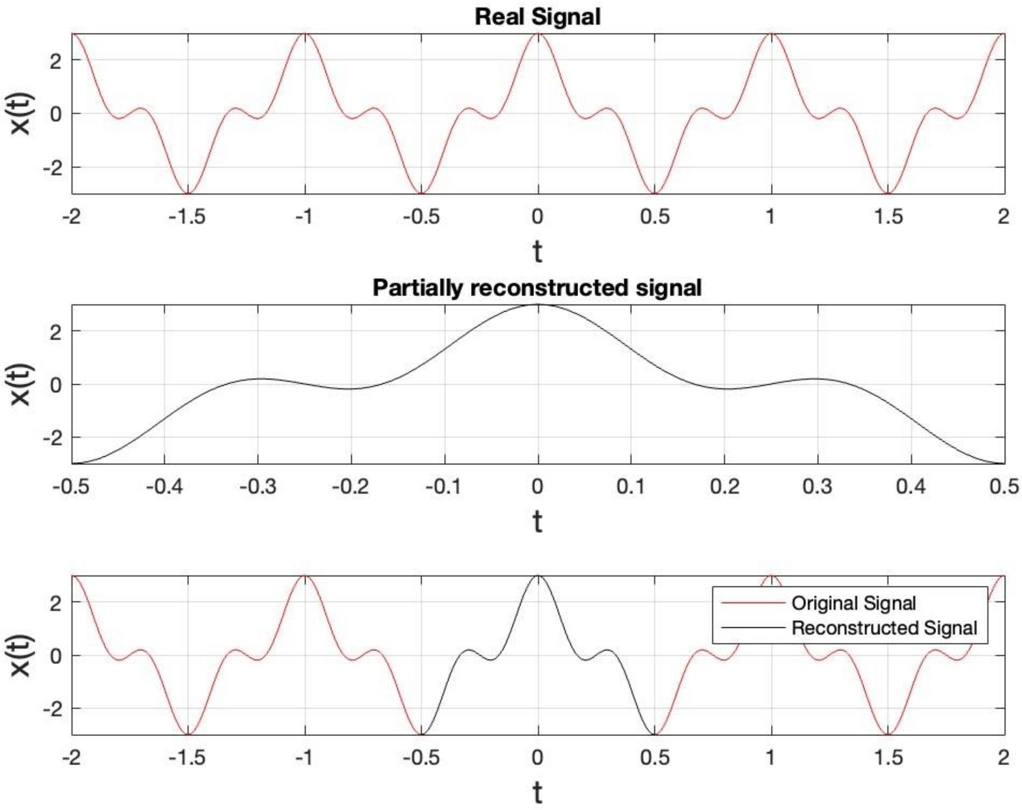


b.

Code:

```
clc, clearvars, clear all
syms t;
time_grid = - 0.5:0.01:0.5;
xt = 2*cos(2*pi*t)+cos(6*pi*t); T = 1; N = 5; t1 = -T/2; t2 = T/2;
F = fourierCoeff(t,xt,T,t1,t2,N);
y = partialfouriersum(F,T,time_grid);
time = -2:0.01:2; nums = double(subs(xt,t,time));

subplot(3,1,1); plot(time,nums, "Color",'r'); grid on;
xlabel('t','FontSize',15); ylabel('x(t)','FontSize',15); title('Real Signal');

% hold on;

subplot(3,1,2); plot(time_grid,y, "Color", 'black'); grid on;
xlabel('t','FontSize',15); ylabel('x(t)','FontSize',15); title('Partially reconstructed signal');

% hold on;

subplot(3,1,3);
orig = plot(time, nums, "Color",'r'); grid on; hold on;
recons = plot(time_grid, y, "Color",'black'); hold on;
xlabel('t','FontSize',15); ylabel('x(t)','FontSize',15); legend('Original Signal', 'Reconstructed Signal');
```

Plot:



c. Code for finding error:

Code:

```
clc, clearvars, clear all
syms t;
T=1; N=5;
xt=2*cos(2*pi*t)+cos(6*pi*t); A=fourierCoeff(t,xt,T,-T/2,T/2,N);
time_grid = -0.5:0.01:0.5;
xtrecon = partialfouriersum(A,T,time_grid); xtnew=2*cos(2*pi*time_grid)+cos(6*pi*time_grid); mae=0;
rmssum=0;
for k = 1:length(time_grid)
xtarr=2*cos(2*pi*time_grid(k))+cos(6*pi*time_grid(k)); er = abs(xtarr-xtrecon(k));
rmssum=rmssum+er*er;
if(er>mae)
mae=er;
end
end
rms=sqrt(rmssum/length(time_grid));
disp('Max. abs error = '); disp(mae);
disp('sqrt error = '); disp(rms);
```

Answer:

```
Command Window
Max. abs error =
   8.6221e-16

sqrt error =
   2.9071e-16

fx >>
```

1.3 Gibbs Phenomenon revisit square wave

a. Calculations:

1.3 a) $x(t) = \begin{cases} 1; & -T_1 \leq t \leq T_1 \\ 0, & -T/2 < t < -T_1 \end{cases}$ rep. over the period.

$$a_k = \frac{1}{T}\int_{\langle T \rangle} x(t)e^{-jk\omega_0 t}\,dt = \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-jk\omega_0 t}\,dt.$$

$$= \frac{1}{T}\left[\int_{-T/2}^{-T_1} x(t)\,e^{-jk\omega_0 t}\,dt + \int_{-T_1}^{T_1} x(t)e^{-jk\omega_0 t}\,dt + \int_{T_1}^{T_2} x(t)e^{-jk\omega_0 t}\,dt\right]$$

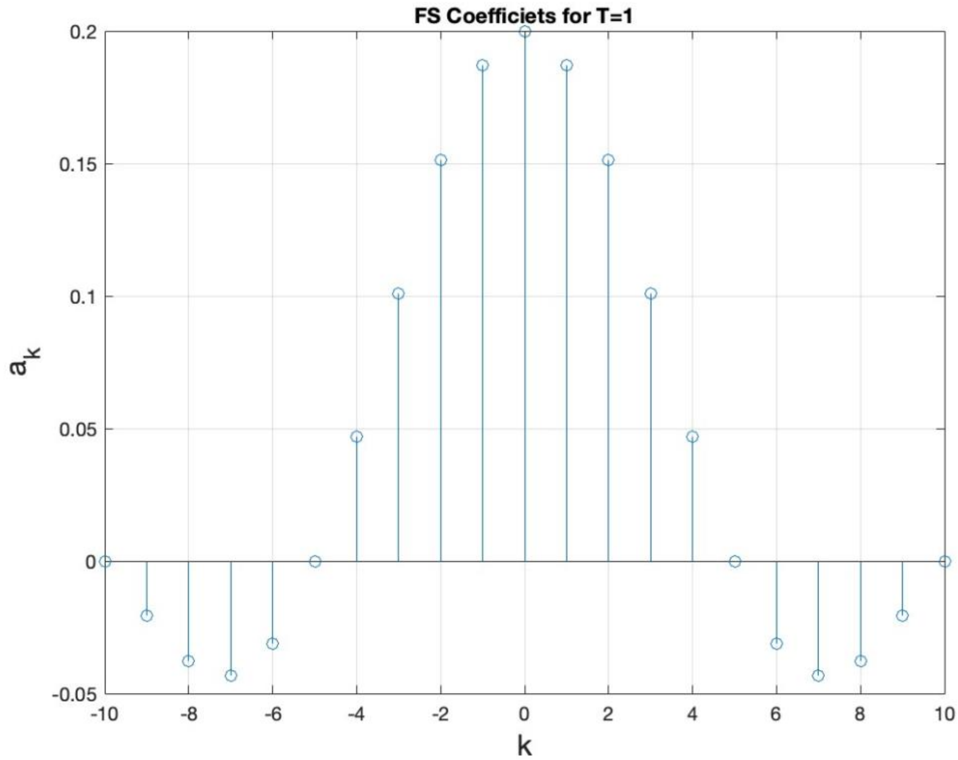
$$= \frac{1}{T}\left[\int_{-T_1}^{T_2} e^{-jk\omega_0 t}\,dt + 0\right] = \frac{1}{T} \times \left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right]_{-T_1}^{T_1}$$

$$a_k = \frac{\sin\left(\frac{2\pi k T_1}{T}\right)}{k\pi}$$

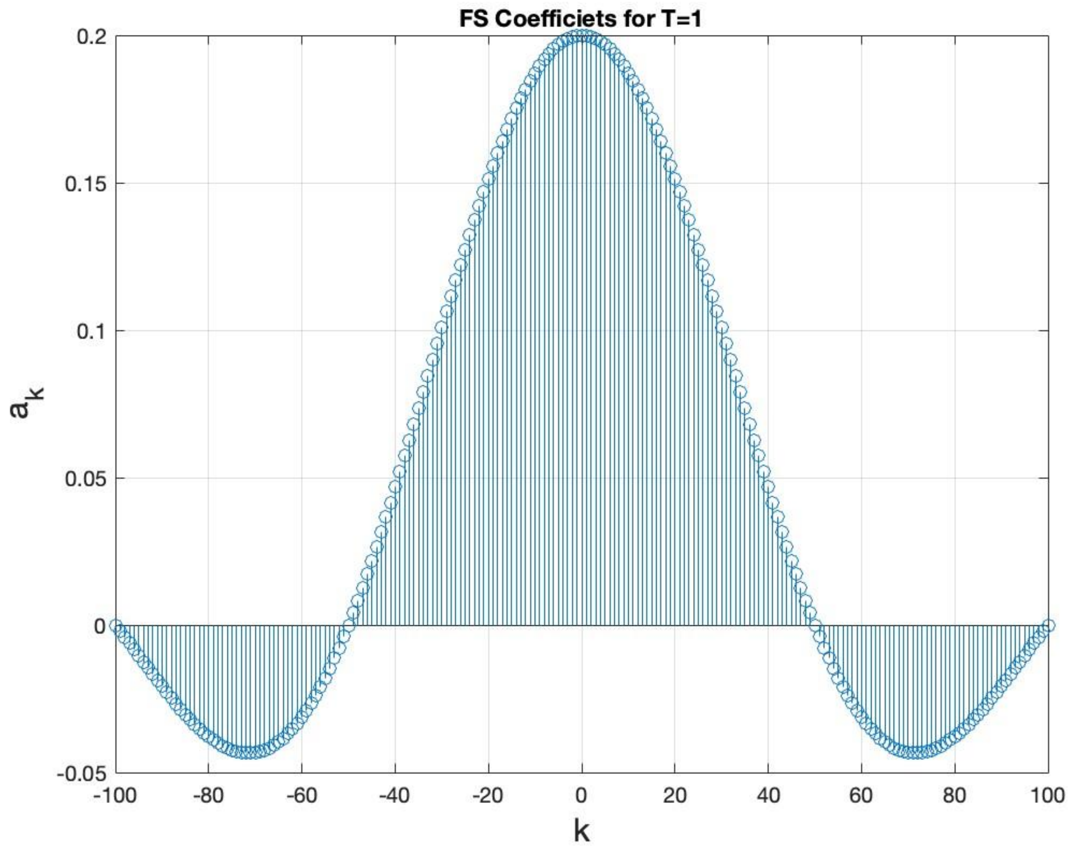
b. Code to find coefficients:

```
syms t;
T = 1; N = 10*T; t1 = -T/2; t2 = T/2; T1 = 0.1;
xt = piecewise(((-T/2<t)&(t<-T1)),0,((-T1<=t)&(t<=T1)),1,((T1<t)&(t<T/2)),0);
FS_idx = -N:N;
F = fourierCoeff(t,xt,T,t1,t2,N); y = T*F;

figure(1); stem(FS_idx,y); grid on;
xlabel('k', FontSize = 15); ylabel('a_k', FontSize=15); title('FS Coefficiets for T=1');
```

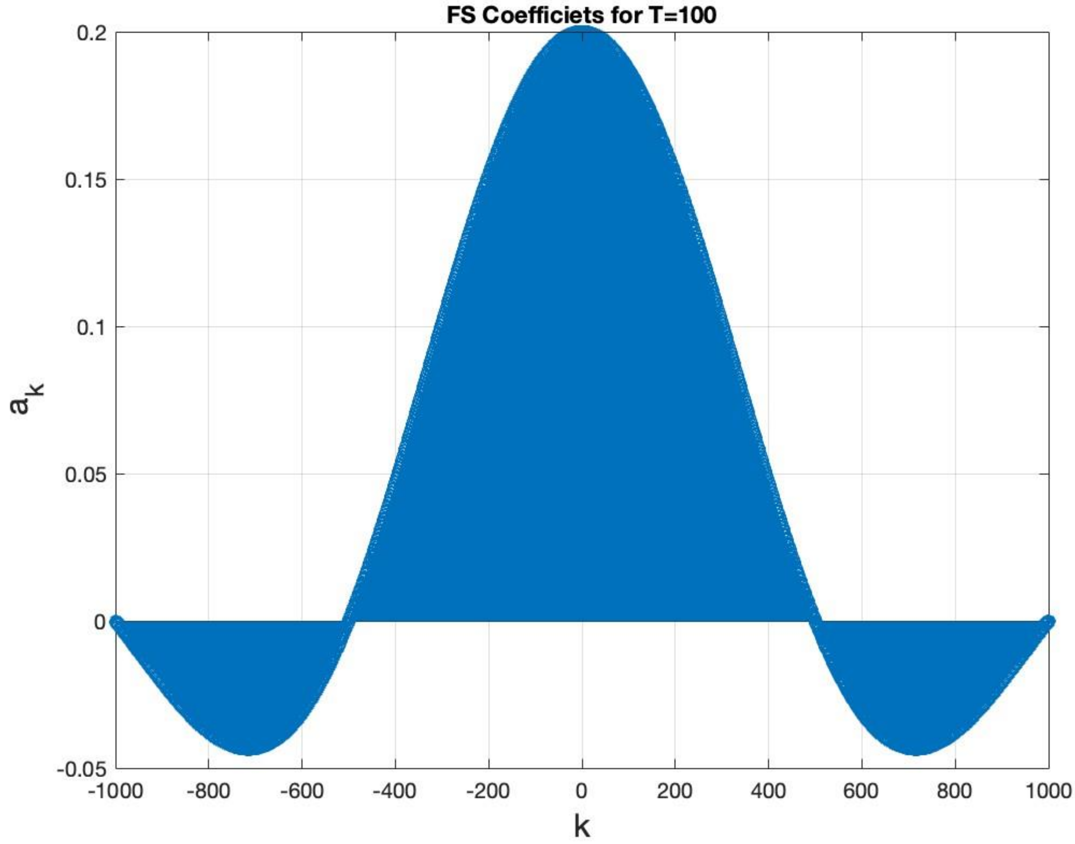
Plot:


FS Coefficiets for T=1
0.2
0.15
0.1
0.05
0
-0.05
-10
-8
-6
-4
-2
0
2
4
6
8
10
a_k
k

T=10


FS Coefficiets for T=1
0.2
0.15
0.1
0.05
0
-0.05
-100
-80
-60
-40
-20
0
20
40
60
80
100
a_k
k

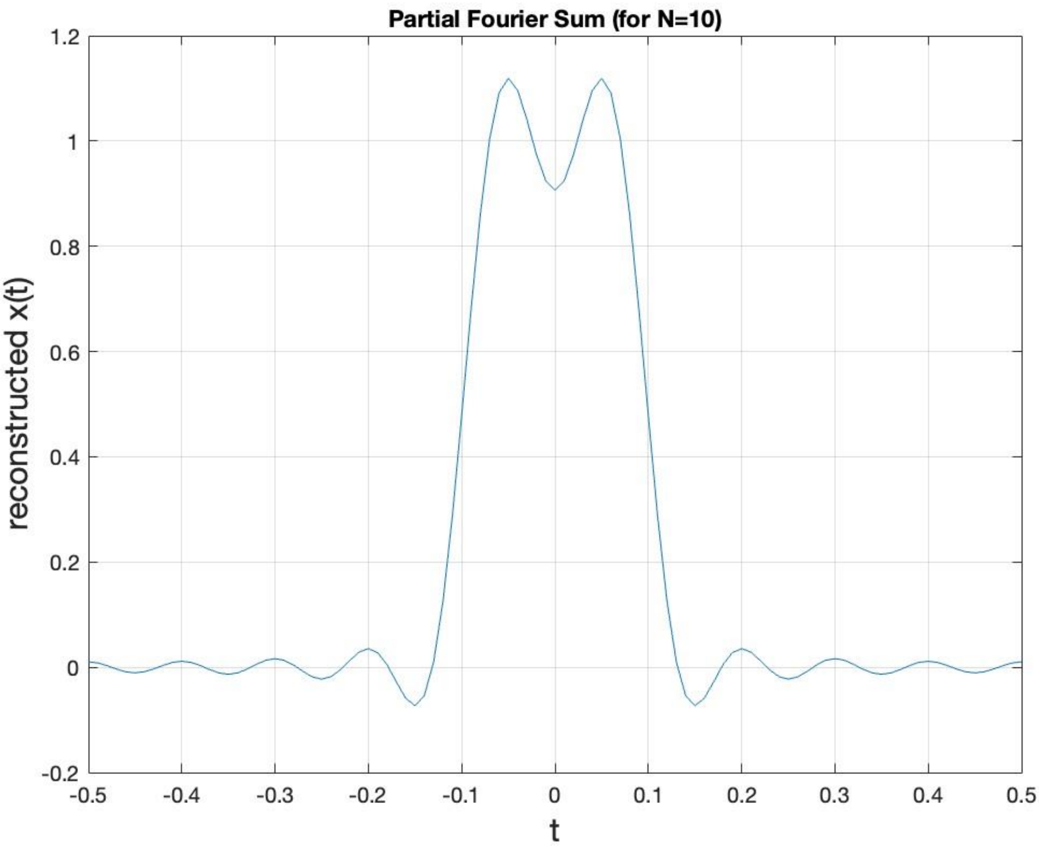
T = 100



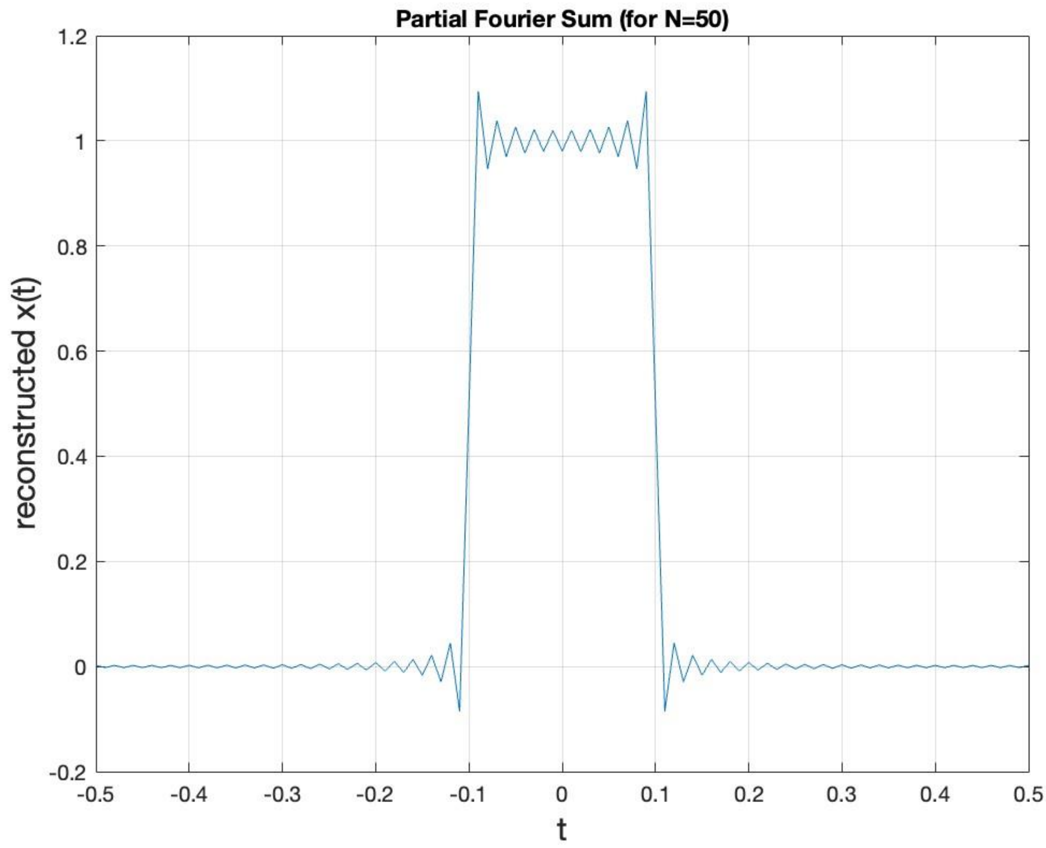
c. Code:

```
syms t;
T = 1; N = 10; t1 = -T/2; t2 = T/2; T1 = 0.1; time_grid = -0.5:0.01:0.5;
xt = piecewise(((-T/2<t)&(t<=-T1)),0,((-T1<=t)&(t<=T1)),1,((T1<=t)&(t<T/2)),0);
F = fourierCoeff(t,xt,T,t1,t2,N);
xReconstructed = partialfouriersum(F,T,time_grid);
plot(time_grid,xReconstructed); grid on; xlabel('t', FontSize = 15);
ylabel('reconstructed x(t)', FontSize = 15);
title('Partial Fourier Sum (for N=10)');
```

Plot:


Partial Fourier Sum (for N=10)
reconstructed x(t)
t
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5

N=50



We observe that as N increases, the error decreases, and the reconstructed signal starts resembling the original signal more and more.

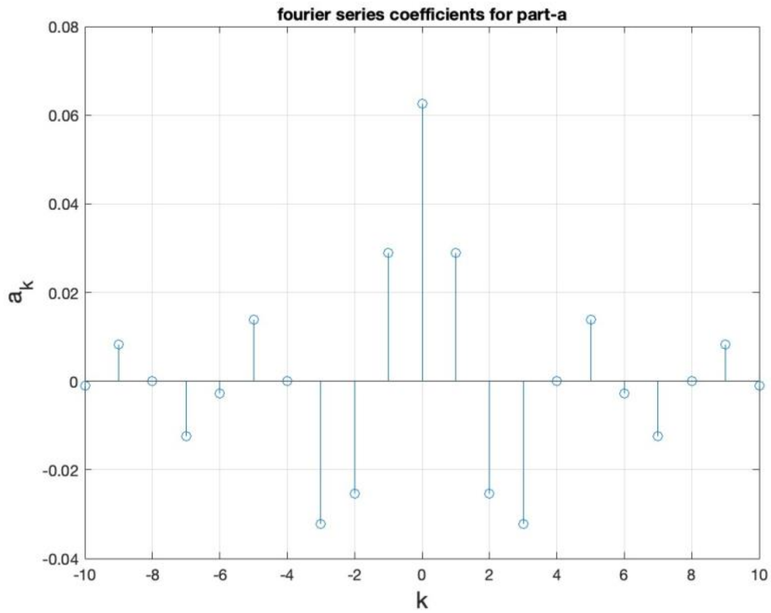
1.4 Fourier Series more examples and symmet. Prop.

PART A :

Code:

```
clc, clearvars, clear all
syms t;
T = 1; t1 = -T/2; t2 = T/2; N = 10;

% part-a
xt = piecewise(t>-1/4 & t<0, -t, t>0 & t<1/4, t, 1/4 < abs(t) & abs(t)<T/2,0);
F = fourierCoeff(t, xt, T, t1, t2, N); index = -N:N; figure(1); stem(index, F); grid on;
xlabel('k', 'FontSize',15); ylabel('a_k', 'FontSize',15); title('fourier series coefficients for part-a');


% part-b
xt_ = piecewise(t>-1/4 & t<1/4, t, t<-1/4 & t>t1, 0, t>1/4 & t<t2, 0);
F_ = fourierCoeff(t,xt_,T,t1,t2,N); figure(2); stem(index, -1j*F_); grid on;
xlabel('k', 'FontSize',15); ylabel('Im(a_k)', 'FontSize',15); title('Fourier Series Coeff of part-b');
```

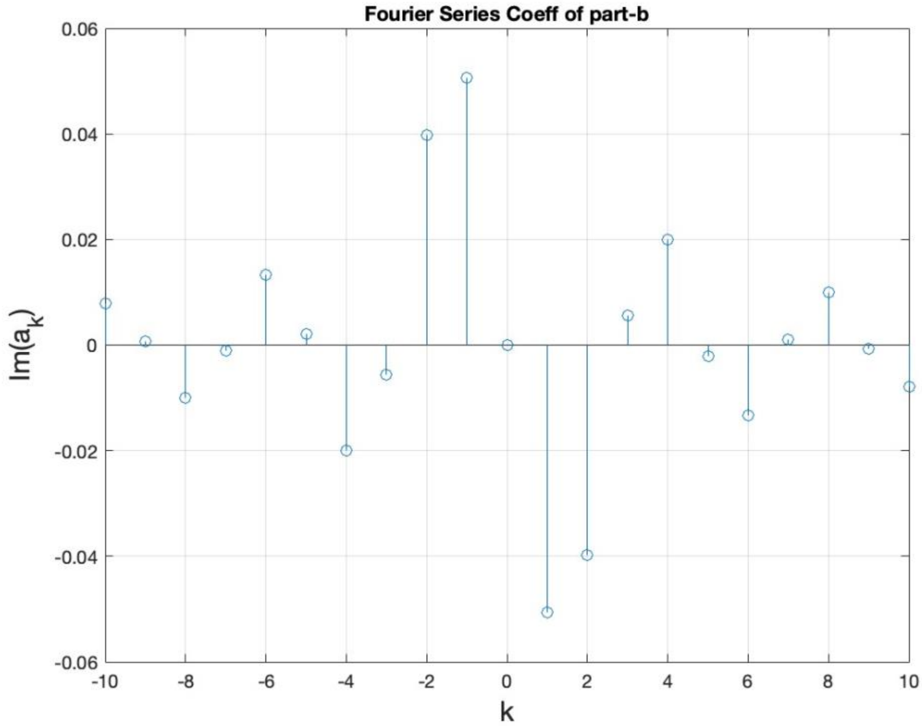
Plot:



We observe a Even Symmetry here.

PART B :

Plot:



We observe a Odd Symmetry in this.

PART C :

Calculations:

1.4 c) 1.4 (a) is an Even Signal.

$$\rightarrow x(t) = \sum_{k=-\infty}^{\infty} a_k e^{-jk\omega_0 t} = A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} B_k \sin(k\omega_0 t)$$

$\because x(t)$ is even, $B_k = 0 \ \forall k \in [1, \infty)$

$$a_k = \begin{cases} A_0 & ; \ k=0 \\ \frac{A_k - jB_k}{2} & ; \ k>0 \\ \frac{A_k + jB_k}{2} & ; \ k<0 \end{cases} = \begin{cases} A_0 & , k=0 \\ \frac{A_k}{2} & , A>0 \\ \frac{A_{-k}}{2} & ; A<0 \end{cases}$$

$$a_k = \begin{cases} A_0 & , k=0 \\ \frac{A_{|k|}}{2} & , k \neq 0 \end{cases}$$

for $k \neq 0$, $a_k = a_{-k}$. So the graph a_k v/s k is symm. w.r.t y axis.

$\rightarrow$ Coeff. are real

Hence, Even Symm. holds.

Signal 1.4 (b) is odd i.e $x(-t) = -x(t)$

$\therefore A_k = 0 \ \forall k \in [0, \infty)$

$$a_k = \begin{cases} -j\frac{B_k}{2} & , k>0 \\ j\frac{B_{-k}}{2} & , k<0 \\ 0 & k=0 \end{cases}$$

So, coeff are purely img.

& $a_k = -a_{-k}$

Hence, Odd Symm. hold.