

**Practice final exam     MTH 224-O, Spring 2024**

**Question 1.** (a) Representatives from 20 countries, including Russia and Ukraine, are attending an event at the UN. Participants are randomly seated around a rectangular table with 20 seats. What is the probability that the Russian diplomat is seated next to the Ukrainian diplomat?

(b) We throw a fair six-sided die two times and get two different numbers. What is the probability that none of the two numbers is a 6?

(c) A bag contains 6 packs of pretzel and 9 packs of cookies. We randomly take 5 packs out one-by-one and without replacement. Find the probability that we select 2 cookies and then 3 pretzels.

(d) Given that it will not rain in exactly 6 out of the next 10 days, find the conditional probability that the next three days will be not rainy, rainy, and rainy, respectively. Assume that each day, independently of other days, is not rainy with probability  $p > 0$ .

**Question 2.** The number of earthquakes, each year, of magnitude 5 or larger in the US is modeled by  $N = 30 + Y$ , where  $Y \sim \text{Bin}(100, \frac{1}{2})$ . The magnitude of each such earthquake is modeled by  $M = 5 + T$ , where  $T \sim \text{Exp}(\ln 10)$  (an exponential random variable with parameter  $\ln 10$ ). You also assume that the magnitude of each earthquake is independent of  $N$  and of all other earthquakes; and that the number of earthquakes each year is independent of other years.

(a) Find the expected number of earthquakes of magnitude 6.5 or larger during the year 2023.

(b) Assume that the damage  $D$  (in \$B) caused by an earthquake of magnitude  $M$  is given by  $D = e^{M-5}Z$ , in which  $Z \sim \text{Bern}(0.1)$  is independent of  $M$ . Find  $E[D]$  and  $\text{Var}(D)$ .

(c) There were 80 earthquakes of magnitude 5 or larger during the year 2020. Assume that the amount of damage caused by the earthquakes are independent and that the model in part (b) applies to each earthquake. Estimate the probability that the total damage caused by the earthquakes during that year was at least \$20B.

**Question 3.** Given the bivariate sample:

X: 0.2, 0.7, 0.3, 0.0, 0.5, 1.5,

Y: 3.6, 2.7, 3.6, 3.9, 3.3, 1.1,

answer the following questions.

(a) Find the sample means and the sample variances of  $X$  and  $Y$ . Show your work.

(b) Find the regression line with  $X$  as the predictor and  $Y$  as the response variable. Show your work.

**Question 4.** Let  $X_1, \dots, X_N$  be independent and identically distributed  $N(\mu, 1)$  random variables, in which  $\mu$  is an unknown parameter.

(a) Find the maximum likelihood estimator (MLE) for  $\mu$ . Show all of your calculations.

(b) Find the mean squared error (MSE) for the MLE of part (a).