MATH 210 EXAM

The exam must be returned in person at the beginning of class next Thursday. You must not consult anyone about the exam- if you have questions please contact me. THERE WILL BE NO CLASS THIS COMING TUESDAY.

1. We have five "data points" : (1,3), (2,5), (3,6), (4,7) and (5,9). We want to get the line of best fit: y = mx + b. The answer depends on what we mean by "best fit".

(i) Find the best "least squares" approximation, that is, the line that minimizes the total squared error. For this line find the total squared error and the correlation coefficient. YOU MUST DO THIS BY HAND.

(ii) Suppose by "best fit" we mean the line that gives the smallest individual error, that is, for x = 1, 2, 3, 4 or 5 we look at the difference between the given value and the value given by our line, and the "error" is the maximum of these differences. For instance, if y = 2x is our line then the errors are 1, 1, 0, 1 and 1, so the maximum error is 1. Find a line that gives a smaller "maximum error" than the line in part (i). (You don't have to find the best line, just one that is better.)

2. Let A be the following 3 x 3 matrix:

$$\begin{pmatrix} 5 & -2 & 1 \\ -2 & 5 & -1 \\ 1 & -1 & 8 \end{pmatrix}.$$

Verify that this matrix is symmetric. Find the eigenvalues and the corresponding eigenvectors for this matrix. Verify that eigen-

vectors corresponding to the different eigenvalues are orthogonal. YOU MUST DO THIS BY HAND. (If you do this correctly the answers you get will not be too complicated.) What is the diagonal form of this matrix? Use this to find a formula for A^n ,

3. Do problems 6.3-13, 6.3-15 and 6.4-9 from the book. Make sure to explain your work clearly.

4. Let $\sum$ be the vector space of all continuous functions on the interval [0,1]. (We add and multiply by scalars as usual.) This is a vector space, of infinite dimension.

i. Show that the three polynomials 1, x, and x^2 are linearly independent.

ii. Show that the functions 1, Sin(x) and Cos(x) are linearly independent.

iii. Show that the functions 1, Sin^2(x) and Cos^2(x) are NOT linearly independent.