MATH 210 EXAM 1

The test is due at the beginning of class next Tuesday. You must turn in the exam in person. You may use your text and notes, but you may not discuss any aspect of the exam with anyone else, other than the instructor, until you have turned in the exam on Tuesday. You may not use a calculator. If you do the problems correctly your answers will not involve very complicated arithmetic.

Note that the eigenvalues of a matrix A are the numbers λ for which the equation A X $= \lambda$ X has a solution X which is not the zero vector.

1. Let A be the 3 x 3 matrix $\begin{pmatrix} 1 & 9 & 8 \\ 2 & 15 & 4 \\ 3 & 21 & 18 \end{pmatrix}$. Find det(A) and A^{-1}. Find the LU decomposition of A. Solve the system A$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 63 \\ 69 \\ 159 \end{pmatrix}$ four different ways: by row operations, by using the inverse, by Cramer's rule, and by using the LU decomposition.

2. Suppose A = $\begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}$ and B = $\begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix}$. Find a formula for A^n and for $A\,B$. Suppose C is an n x n upper triangular matrix, with 0's on the main diagonal. What is C^{n+1}?

3. Suppose the linear transformation on R^2 corresponds to rotation by 60 degrees, counterclockwise. Find the matrix A for this transformation. What is the matrix for A^3?

4. Let L be the linear transformation on R^2 whose matrix is given by A= $\begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix}$. Find the eigenvalues of A and corresponding eigenvectors. (You should get 2 and 3 as the eigenvectors.) Write the matrix for L in terms of these eigenvectors and use this to find a general formula for A^n .

5. Suppose the linear transformation is given by the matrix A = $\begin{pmatrix} 1/4 & 2/3 \\ 3/4 & 1/3 \end{pmatrix}$. Find the eigenvalues of A. (You should get 1 and -5/12). Find the eigenvector E = $\begin{pmatrix} x \\ y \end{pmatrix}$ corresponding to the eigenvalue 1 with x +y = 1700. Show that if $\begin{pmatrix} a \\ b \end{pmatrix}$ is any vector

with a + b = 1700 then in the limit, $A^n\begin{pmatrix} a \\ b \end{pmatrix}$ tends to E . In other words, the limiting distribution is that of E, no matter where you start, as long as the sum of the two entries is 1700.

6. (i) Suppose A is the matrix in problem 1. What is the volume of the parallelepiped whose sides are given by the rows of A? If S is any solid in R^3 whose volume is 1, what is the volume of the solid L(S), where L is the linear transformation given by A?

(ii) Find the point in the plane x + y + z =3 in R^3 that is nearest to the origin (0,0,0).

7. Consider the system given below, of 5 equations in only three variables. How do we know that the system is going to have solutions only for certain values of A, B, C, D and E ? (Regardless of the coefficients of x, y and z we will only have solutions for certain values of A, B, C, D and E.) For the system below, find the relations among A,...,E which are necessary and sufficient for the existence of a solution to the system. For values of A,..,E which admit a solution, how many solutions are there?

$$\begin{aligned} x + y + z &= A \\ 2x - y + 5z &= B \\ 4x + 2y - z &= C \\ 9x + 4y + 7z &= D \\ 14x + 7y + 7z &= E. \end{aligned}$$