Spring 2024 MTH 162
Exam 1 Review

Exam 1 will cover Chapter 5 (except sections 5.5 and 5.7), section 6.1

- Section 5.1:
  - Find an expression for the inverse function of a one–to–one function.
  - Use the following theorem; If f is a one–to–one continuous, differentiable function and $f'(f^{-1}(a) \neq 0$, then $(f^{-1})'(a) = \dfrac{1}{f'(f^{-1}(a))}$
- Sections 5.2, 5.3, 5.4.
  - Find the derivative of a function involving logarithmic/exponential functions.
  - Evaluate the limit of a function involving logarithmic/exponential functions.
  - Evaluate the integral of a function involving logarithmic/exponential functions.
  - Find critical point(s), maximum/minimum, intervals of increase/decrease of a function, point(s) of inflection, concavity.
- Section 5.6.
  - Domain, range of $y = \sin^{-1} x$, $y = \cos^{-1} x$, $y = \tan^{-1} x$
  - Simplify an expression containing an inverse trigonometric function.
  - Derivatives, integrals of an inverse trigonometric function.
- Section 5.8.
  - Limit of an indeterminate forms such as indeterminate quotients, products, difference and powers.
- Section 6.1
  - Integration by parts

- Sample problems.

1. Let $y = -\ln x$. Domain : $(0, \infty)$

   (a) Show that y is one–to–one. $y' = -\frac{1}{x} < 0$, when $x > 0$

   (b) Find an expression for the inverse function of y.

2. Let $f(x) = 2x - 3 + \sin\left(\frac{\pi}{2}x\right)$. Evaluate $(f^{-1})'(0)$.

3. Solve the equation. $\ln(x+1) + \ln(x-1) = 1$

4. Differentiate $y = x^{x^3}$ $\rightarrow \ln y = \ln x^{x^3} = x^3 \ln x \Rightarrow \frac{1}{y}\frac{dy}{dx} = 3x^2 \ln x + x^2$

5. Let $f(x) = e^{2x - x^2}$. Find the critical number(s) of f, and the interval(s) of increase/decrease of f.

6. Find the exact value of $\csc(2\sin^{-1} 2/x)$

7. At what point(s) does the graph of $f(x) = x + \ln(x^2 + 1)$ have a horizontal tangent?

8. Let $f(x) = 2\sin^{-1}(x - 3)$. State the domain and range of f. Find the critical number(s) of f.

   $-1 \le x - 3 \le 1$

   $2 \le x \le 4$ Domain

   Range: $[-\pi, \pi]$

   $-\frac{2\pi}{2} \le 2\sin^{-1}(x-3) \le \frac{2\pi}{2}$

9. On which interval(s) is the function $y = \dfrac{e^x + e^{-x}}{2}$ increasing?

10. Evaluate the limit.

    (a) $\displaystyle\lim_{x\to\infty} \frac{1 + 2^x}{1 - 2^x}$

    (b) $\displaystyle\lim_{x\to 0} (1 - 2x)^{1/x}$

    $\lim_{x\to 0} \ln(1-2x)^{1/x} = \lim_{x\to 0} \frac{\ln(1-2x)}{x} \; \frac{\to 0}{\to 0}$

    $= \lim_{x\to 0} \frac{\frac{-2}{1-2x}}{1} = \lim_{x\to 0} \frac{-2}{1-2x} = -2$

    $\lim_{x\to 0} (1-2x)^{1/x} = e^{-2}$

11. Evaluate the integral.

    (a) $\displaystyle\int \frac{x}{\sqrt{1 - x^4}}\, dx$

    (b) $\displaystyle\int \left(\frac{1 - x}{x}\right)^2 dx = \int \frac{1 - 2x + x^2}{x^2} dx = \int \left(\frac{1}{x^2} - \frac{2}{x} + 1\right) dx$

    (c) $\displaystyle\int \tan x \ln(\cos x)\, dx$

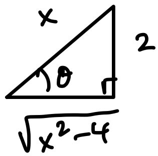
    (d) $\displaystyle\int_2^4 \frac{1 + x - x^2}{x^2}\, dx$

> Chapter 5 additional practice problems from the textbook.
> Section 5.1: 21– 27 odd, 33, 37
> Section 5.2: 13– 29 odd, 37, 41, 51.
> Section 5.3: 5, 9, 15, 17–21 odd , 27–33 odd, 37, 61–67 odd
> Section 5.4: 25–37 odd, 41–45 odd
> Section 5.6: 1–5 odd, 9, 21–27 odd, 39–47 odd
> Section 5.8: 11–37 odd

⑥ $\csc\left(2\underbrace{\sin^{-1}\left(\frac{2}{x}\right)}_{\theta}\right) = \csc(2\theta) = \frac{1}{\sin(2\theta)} = \frac{1}{2\sin\theta\cos\theta}$

$= \frac{1}{2}\csc\theta\sec\theta$

$\sin\theta = \frac{2}{x}$



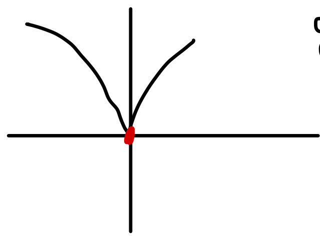
$= \frac{1}{2}\,\frac{x}{2}\cdot\frac{x}{\sqrt{x^2-4}}$

⑦ $f(x) = x + \ln(x^2+1)$

$f'(x) = 1 + \frac{2x}{x^2+1} = \frac{x^2+1+2x}{x^2} = \frac{(x+1)^2}{x^2} = 0$

when $x=-1$

$f(-1) = -1 + \ln(2)$ $\quad (-1, -1+\ln 2)$



$y = x^{2/3}$

$y' = \frac{2}{3}x^{-1/3}$

$= \frac{2}{3x^{1/3}}$

– Section 6.1.

Evaluate the integral.

1. $\int x^2 \cos(3x)\ dx$
2. $\int (4x^3 - 2x^2 + 5) \sin x\ dx$ (Tabulation Method)
3. $\int x \tan^2 x\ dx$ (Hint: Use the identity $1 + \tan^2 x = \sec^2 x$.)

   $\int x(\sec^2 x - 1)dx = \int x\sec^2 x\, dx - \int x\, dx$

   $u = x \quad dv = \sec^2 x \qquad \frac{x^2}{2}$

   $du = dx \quad v = \tan$

   $x \tan x -$
4. $\int \cos(\sqrt{x})\ dx$
5. $\int_0^1 x \ln(1 + x)\ dx$

Chapter 6 additional practice problems from the textbook.

Section 6.1: 1–29 odd

(5) $u = \ln(1+x) \qquad dv = x\,dx$

$du = \frac{1}{1+x} \qquad v = \frac{1}{2}x^2$

$$\int_0^1 x\ln(1+x)\,dx = \frac{1}{2}x^2\ln(1+x)\Big|_0^1 - \frac{1}{2}\int_0^1 \frac{x^2}{1+x}\,dx$$

$u = 1+x \qquad du = dx$

$x = (u-1)$

$$= \frac{1}{2}\ln 2 - \frac{1}{2}\Big($$

$$\int_1^2 \frac{(u-1)^2}{u}\,du = \int_1^2 \frac{u^2 - 2u + 1}{u}\,du$$

$$= \int_1^2 \left(u - 2 + \frac{1}{u}\right)du$$

$$= \frac{u^2}{2} - 2u + \ln|u|\Big|_1^2$$

$$= 2 - 4 + \ln 2 - \left(\frac{1}{2} - 2\right)$$

- Answers.

  Sample Problems.

  1. (a) $y' = -\dfrac{1}{x} < 0$ for $x > 0$

     (b) $y = e^{-x}$
  2. $(f^{-1})'(0) = \dfrac{1}{2}$
  3. $x = \sqrt{e+1}$
  4. $y' = (3x^2 \ln x + x^2)x^{x^3}$
  5. $x = 1,$ f is decreasing on $(1, \infty)$. f is increasing on $(-\infty, 1)$
  6. $\dfrac{x^2}{4\sqrt{x^2-4}}$
  7. $(-1, -1 + \ln 2)$
  8. Domain $[2, 4]$. Range $[-\pi, \pi]$. Critical numbers $x = 2,$ $x = 4$
  9. $[0, \infty)$
  10. -1
  11. e^{-2}
  12. $\dfrac{1}{2}\sin^{-1}(x^2) + C$
  13. $x - \dfrac{1}{x} - 2\ln|x| + C$
  14. $-\dfrac{1}{2}(\ln(\cos x))^2 + C$
  15. $\ln 2 - \dfrac{7}{4}$

- Section 6.1

  1. $\dfrac{1}{27}((9x^2 - 2)\sin(3x) + 6x\cos(3x)) + C$
  2. $4(3x^2 - x - 6)\sin x + (-4x^3 + 2x^2 + 24x - 9)\cos x + C$
  3. $\tan x + \ln|\cos x| - \dfrac{1}{2}x^2 + C$
  4. $2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$
  5. $\dfrac{1}{4}$