MTH 224O - Spring 2024

Practice Problems for Exam 1

Problem 1: Answer each question below.

(a) We toss a fair coin 6 times. Find the probability of getting exactly 3 heads.

(b) We draw 4 cards randomly from a standard deck of playing cards. Calculate the probability that the suits are different?

(c) We randomly place 8 castles (also known as rooks) on a chessboard. What is the probability that none of the castles capture another.

If you haven't played chess before: A chess board is an eight-by-eight square, consisting of 64 smaller squares arranged in 8 rows and 8 columns. Each chess piece can occupy only one of the 64 smaller squares. Two castles capture each other if they are on the same row or the same column.

Problem 2: We roll a die, then flip a coin. If we get heads, we stop. Otherwise, we roll the die again and we flip the coin again. If it is heads, we stop. And so forth. Let N be the number of times that we role the die. Let S be the total sum of the outcomes of rolling the die. Find the probability of the following events:

(a) $N = i$ for $i \geq 1$.

(b) $N = 2$ given $S = 4$.

(c) N is even.

(d) $S = 4$ given N is even.

Problem 3: (a) 10 boys and 10 girls are randomly lined up. Find the expected number of boys who stand next to a girl.

(b) Now assume that 10 boys and 10 girls are randomly seated at a round table. Find the expected number of boys who stand next to a girl.