Exam 2 Review

Exam 2 will cover Sections 6.1 through 6.3, 6.6.

- **Sections 6.1 through 6.3.**

1. Integration by parts.

   Examples: Evaluate the integral.

   (a) $\int x^2 \cos(3x)\ dx$

   (b) $\int \tan^{-1}\left(\frac{1}{x}\right)\ dx$

2. Trigonometric Integrals. Evaluate

   (a) $\int \tan^4 x\ dx$

   (b) $\int \cos^2 x \tan^3 x\ dx$

3. Trigonometric Substitution. Evaluate

   (a) $\int \sqrt{1-4x^2}\ dx$

   (b) $\int \frac{\sqrt{x^2-4}}{x}\, dx$

4. Partial fractions.

   (a) Write out the form of the partial fraction decomposition for the function. Do not determine the coefficients.
   $\frac{x^2-1}{x^3+x^2+x}$

   (b) Evaluate the integral.

   i. $\int \frac{x^2-x+6}{x^3+3x}\ dx$

   ii. $\int \frac{x^2+2x-1}{x^3-x}\ dx$

5. Rationalizing substitution.

   Examples: Evaluate the integral.

   $\int \frac{2}{x-3\sqrt{x+10}}\ dx$

- **Section 6.6. Improper integrals.**

Use the definition of an improper integral to determine whether the integral is convergent or divergent. If it is convergent, find its value.

Use a comparison test to determine if the integral is convergent or divergent.

Sample Problems

Chapter 6: Section 6.6

1. Use the definition of an improper integral to determine whether the integral is convergent or divergent. If it is convergent, find its value.
   $\int_0^3 \frac{1}{x^2 - 4x}\ dx$
2. Use a comparison test to determine if the integral is convergent or divergent.
   $\int_4^\infty \frac{1}{\sqrt[5]{x-2}}\ dx$

Chapter 6 additional practice problems from the textbook.
Section 6.1 Section 6.2 Section 6.3 Section 6.6 11–25 odd, 41–45 odd.
Review the problems from the worksheet

Answers.

- Sections 6.1–6.3.

1. (a) $\frac{1}{27}((9x^2 - 2)\sin(3x) + 6x\cos(3x)) + C$

   (b) $\frac{1}{2}\ln(x^2 + 1) + x\tan^{-1}\left(\frac{1}{x}\right) + C$

2. (a) $x + \frac{1}{3}\tan x(\sec^2 x - 4) + C$

   (b) $\frac{1}{2}(\cos^2 x + 2\ln|\cos x|) + C$

3. (a) $\frac{1}{4}\left(2x\sqrt{1 - 4x^2} + \sin^{-1}(2x)\right) + C$

   (b) $\sqrt{x^2 - 4} - 2\tan^{-1}\left(\frac{\sqrt{x^2 - 4}}{2}\right) + C$ or $\sqrt{x^2 - 4} - 2\sec^{-1}\left(\frac{x}{2}\right)$

4. (a) $\frac{A}{x} + \frac{Bx + C}{x^2 + x + 1}$

   (b) (i) $-\frac{1}{2}\ln(x^2 + 3) + 2\ln x - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$

   (ii) $\ln|1 - x| + \ln|x| - \ln|x + 1| + C$

5. $\frac{4}{7}\left(5\ln\left(5-\sqrt{x+10}\right)+2\ln\left(\sqrt{x+10}+2\right)\right)+C$

- Section 6.6

1. Divergent

2. Divergent (For $x \geq 4, \frac{1}{\sqrt[5]{x-2}} > \frac{1}{\sqrt[5]{x}}$. Since $\int_4^\infty \frac{1}{\sqrt[5]{x}}\ dx$ is divergent, $\int_4^\infty \frac{1}{\sqrt[5]{x-2}}\ dx$ is also divergent.)