Spring 2024 MTH 162
Exam 1 Review

Exam 1 will cover Chapter 5 (except sections 5.5 and 5.7), section 6.1

- Section 5.1:
  - Find an expression for the inverse function of a one–to–one function.
  - Use the following theorem; If f is a one–to–one continuous, differentiable function and $f'(f^{-1}(a) \neq 0$, then $(f^{-1})'(a) = \dfrac{1}{f'(f^{-1}(a))}$
- Sections 5.2, 5.3, 5.4.
  - Find the derivative of a function involving logarithmic/exponential functions.
  - Evaluate the limit of a function involving logarithmic/exponential functions.
  - Evaluate the integral of a function involving logarithmic/exponential functions.
  - Find critical point(s), maximum/minimum, intervals of increase/decrease of a function, point(s) of inflection, concavity.
- Section 5.6.
  - Domain, range of $y = \sin^{-1} x$, $y = \cos^{-1} x$, $y = \tan^{-1} x$
  - Simplify an expression containing an inverse trigonometric function.
  - Derivatives, integrals of an inverse trigonometric function.
- Section 5.8.
  - Limit of an indeterminate forms such as indeterminate quotients, products, difference and powers.
- Section 6.1
  - Integration by parts

- Sample problems.

1. Let $y = -\ln x$.
   (a) Show that y is one–to–one.
   (b) Find an expression for the inverse function of y.
2. Let $f(x) = 2x - 3 + \sin\left(\frac{\pi}{2}x\right)$. Evaluate $(f^{-1})'(0)$.
3. Solve the equation. $\ln(x+1) + \ln(x-1) = 1$
4. Differentiate $y = x^{x^3}$
5. Let $f(x) = e^{2x-x^2}$. Find the critical number(s) of f, and the interval(s) of increase/decrease of f.
6. Find the exact value of $\csc(2\sin^{-1} 2/x)$
7. At what point(s) does the graph of $f(x) = x + \ln(x^2+1)$ have a horizontal tangent?
8. Let $f(x) = 2\sin^{-1}(x-3)$. State the domain and range of f.
   Find the critical number(s) of f.
9. On which interval(s) is the function $y = \dfrac{e^x + e^{-x}}{2}$ increasing?
10. Evaluate the limit.
    (a) $\lim\limits_{x\to\infty} \dfrac{1+2^x}{1-2^x}$
    (b) $\lim\limits_{x\to 0}(1-2x)^{1/x}$
11. Evaluate the integral.
    (a) $\displaystyle\int \frac{x}{\sqrt{1-x^4}}\ dx$
    (b) $\displaystyle\int \left(\frac{1-x}{x}\right)^2\ dx$
    (c) $\displaystyle\int \tan x \ln(\cos x)\ dx$
    (d) $\displaystyle\int_2^4 \frac{1+x-x^2}{x^2}\ dx$

Chapter 5 additional practice problems from the textbook.
Section 5.1: 21– 27 odd, 33, 37
Section 5.2: 13– 29 odd, 37, 41, 51.
Section 5.3: 5, 9, 15, 17–21 odd , 27–33 odd, 37, 61–67 odd
Section 5.4: 25–37 odd, 41–45 odd
Section 5.6: 1–5 odd, 9, 21–27 odd, 39–47 odd
Section 5.8: 11–37 odd

– Section 6.1.

Evaluate the integral.

1. $\int x^2 \cos(3x)\ dx$
2. $\int (4x^3 - 2x^2 + 5) \sin x\ dx$ (Tabulation Method)
3. $\int x \tan^2 x\ dx$ (Hint: Use the identity $1 + \tan^2 x = \sec^2 x$.)
4. $\int \cos(\sqrt{x})\ dx$
5. $\int_0^1 x \ln(1 + x)\ dx$

Chapter 6 additional practice problems from the textbook.

Section 6.1: 1–29 odd

- Answers.

  Sample Problems.

  1. (a) $y' = -\dfrac{1}{x} < 0$ for $x > 0$

     (b) $y = e^{-x}$
  2. $(f^{-1})'(0) = \dfrac{1}{2}$
  3. $x = \sqrt{e+1}$
  4. $y' = (3x^2 \ln x + x^2)x^{x^3}$
  5. $x = 1$, f is decreasing on $(1, \infty)$. f is increasing on $(-\infty, 1)$
  6. $\dfrac{x^2}{4\sqrt{x^2-4}}$
  7. $(-1, -1 + \ln 2)$
  8. Domain $[2, 4]$. Range $[-\pi, \pi]$. Critical numbers $x = 2,\ x = 4$
  9. $[0, \infty)$
  10. -1
  11. e^{-2}
  12. $\dfrac{1}{2}\sin^{-1}(x^2) + C$
  13. $x - \dfrac{1}{x} - 2\ln|x| + C$
  14. $-\dfrac{1}{2}(\ln(\cos x))^2 + C$
  15. $\ln 2 - \dfrac{7}{4}$

- Section 6.1

  1. $\dfrac{1}{27}((9x^2 - 2)\sin(3x) + 6x\cos(3x)) + C$
  2. $4(3x^2 - x - 6)\sin x + (-4x^3 + 2x^2 + 24x - 9)\cos x + C$
  3. $x\tan x + \ln|\cos x| - \dfrac{1}{2}x^2 + C$
  4. $2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$
  5. $\dfrac{1}{4}$