SPRING 24 MTH 162 F31
Final Exam Review

- **Chapter 5: Sections 5.1, 5.2, 5.3, 5.4, 5.6, 5.8.**

1. Let $y = -\ln x$.
   (a) Show that y is one–to–one.
   (b) Find an expression for the inverse function of y.
2. Let $f(x) = 2x - 3 + \sin\left(\frac{\pi}{2}x\right)$. Evaluate $(f^{-1})'(0)$.
3. Solve the equation. $\ln(x+1) + \ln(x-1) = 1$
4. Differentiate $y = x^{x^3}$
5. Let $f(x) = e^{2x-x^2}$. Find the critical number(s) of f, and the interval(s) of increase/decrease of f.
6. Find the exact value of $\csc(2\sin^{-1} 2/x)$
7. At what point(s) does the graph of $f(x) = x + \ln(x^2+1)$ have a horizontal tangent?
8. Let $f(x) = 2\sin^{-1}(x-3)$. State the domain and range of f.
   Find the critical number(s) of f.
9. On which interval(s) is the function $y = \dfrac{e^x + e^{-x}}{2}$ increasing?
10. Evaluate the limit.
    (a) $\lim\limits_{x\to\infty} \dfrac{1+2^x}{1-2^x}$
    (b) $\lim\limits_{x\to 0}(1-2x)^{1/x}$
11. Evaluate the integral.
    (a) $\displaystyle\int \frac{x}{\sqrt{1-x^4}}\ dx$
    (b) $\displaystyle\int \left(\frac{1-x}{x}\right)^2\ dx$
    (c) $\displaystyle\int \tan x \ln(\cos x)\ dx$
    (d) $\displaystyle\int_2^4 \frac{1+x-x^2}{x^2}\ dx$

Chapter 5 additional practice problems from the textbook.
Section 5.1: 21– 27 odd, 33, 37
Section 5.2: 13– 29 odd, 37, 41, 51.
Section 5.3: 5, 9, 15, 17–21 odd , 27–33 odd, 37, 61–67 odd
Section 5.4: 25–37 odd, 41–45 odd
Section 5.6: 1–5 odd, 9, 21–27 odd, 39–47 odd
Section 5.8: 11–37 odd

- **Chapter 6. Sections 6.1, 6.2, 6.3, 6.6.**

Evaluate the integral.

1. $\int x^2 \cos(3x)\ dx$
2. $\int (4x^3 - 2x^2 + 5) \sin x\ dx$ (Tabulation Method)
3. $\int x \tan^2 x\ dx$ (Hint: Use the identity $1 + \tan^2 x = \sec^2 x$.)
4. $\int \cos(\sqrt{x})\ dx$
5. $\int_0^1 x \ln(1 + x)\ dx$
6. $\int \tan^{-1}\left(\frac{1}{x}\right)\ dx$
7. $\int \tan^4 x\ dx$
8. $\int \cos^2 x \tan^3 x\ dx$
9. $\int \sqrt{1 - 4x^2}\ dx$
10. $\int \frac{\sqrt{x^2 - 4}}{x}\, dx$
11. $\int \frac{x^2 - x + 6}{x^3 + 3x}\ dx$
12. $\int \frac{x^2 + 2x - 1}{x^3 - x}\ dx$
13. $\int \frac{2}{x - 3\sqrt{x + 10}}\ dx$

Chapter 6 Additional practice problems from the textbook.
Section 6.1: 1–29 odd
Section 6.2: 1–59 odd
Section 6.3: 7–31 odd, 35–39 odd

- **Chapter 8. Sections 8.2 through 8.7**

1. Determine whether the series is convergent or divergent. Justify your answer.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{1+4^n}{1+3^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n}+(-\pi)^n}{5^{n-1}}$

   (e) $\displaystyle\sum_{n=1}^{\infty} n^{-1/3}$

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$

2. Find the sum of

$$\sum_{n=1}^{\infty} 2^n 3^{1-n}$$

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3. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\arctan n}{n^2}$

   (b) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$

4. Find the radius, and the interval of convergence for

$$\sum_{n=1}^{\infty} \frac{3^n (x-2)^n}{n+1}$$

5. Write the power series representation for $f(x) = \dfrac{x^2}{(3x+1)^2}$

6. Let $f(x) = \dfrac{1}{3-2x}$. Find the first four non zeros for the power series representation of $f'(x)$.

7. Write the power series representation for $f(x) = \ln(3+x)$

8. Find the Taylor Series for $f(x) = \sin x$ centered at $a = \dfrac{\pi}{2}$.

9. Find the McLaurin series for $f(x) = x^2 \ln(1+x^3)$

10. Find the sum of the series $\displaystyle\sum_{n=2}^{\infty} \frac{3^n}{5^n n!}$

Chapter 8 additional practice problems from the textbook.
Section 8.2. 9–27 odd.
Section 8.3. 11-29 odd
Section 8.4. 5, 7, 13, 15 19–35 odd
Section 8.5. 3–21 odd
Section 8.6. 5–9 odd, 13, 15, 17, 21, 23, 25
Section 8.7: 5–17 odd, 21, 31, 43, 51, 59–63 odd

- **Sections 9.1, 9.2**

1. Consider the following equations $x = 1 - t^2$, $y = t - 2$, $-2 \leq t \leq 2$. Eliminate the parameter t and sketch the graph of the curve. Indicate with an arrow the direction in which the curved is traced as t increases.
2. Find the point(s) on the graph of the curve $x = t^3 - 3t$, $y = t^2 - 3$ where the tangent line is horizontal or vertical.
3. Find the length of the curve $x = e^t + e^{-t}$, $y = 5 - 2t$, $0 \leq t \leq 2$.

Chapter 9 additional practice problems from the textbook.
Section 9.1 1–17 odd.
Section 9.2 1–15, 37, 39 odd

ANSWERS

Chapter 5.

1. (a) $y' = -\dfrac{1}{x} < 0$ for $x > 0$
   (b) $y = e^{-x}$
2. $(f^{-1})'(0) = \dfrac{1}{2}$
3. $x = \sqrt{e+1}$
4. $y' = (3x^2 \ln x + x^2)x^{x^3}$
5. $x = 1$, f is decreasing on $(1, \infty)$. f is increasing on $(-\infty, 1)$
6. $\dfrac{x^2}{4\sqrt{x^2-4}}$
7. $(-1, -1 + \ln 2)$
8. Domain $[2, 4]$. Range $[-\pi, \pi]$. Critical numbers $x = 2,\ x = 4$
9. $[0, \infty)$
10. (a) -1
    (b) e^{-2}
11. (a) $\dfrac{1}{2}\sin^{-1}(x^2) + C$
    (b) $x - \dfrac{1}{x} - 2\ln|x| + C$
    (c) $-\dfrac{1}{2}(\ln(\cos x))^2 + C$
    (d) $\ln 2 - \dfrac{7}{4}$

Chapter 6.

1. $\dfrac{1}{27}((9x^2-2)\sin(3x) + 6x\cos(3x)) + C$
2. $4(3x^2 - x - 6)\sin x + (-4x^3 + 2x^2 + 24x - 9)\cos x + C$
3. $x\tan x + \ln|\cos x| - \dfrac{1}{2}x^2 + C$
4. $2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$
5. $\dfrac{1}{4}$
6. $\dfrac{1}{2}\ln(x^2+1) + x\tan^{-1}\left(\dfrac{1}{x}\right) + C$

7. $x + \frac{1}{3}\tan x(\sec^2 x - 4) + C$
8. $\frac{1}{2}(\cos^2 x + 2\ln|\cos x|) + C$
9. $\frac{1}{4}\left(2x\sqrt{1-4x^2} + \sin^{-1}(2x)\right) + C$
10. $\sqrt{x^2-4} - 2\tan^{-1}\left(\sqrt{x^2-4}2\right) + C$
11. $-\frac{1}{2}\ln(x^2+3) + 2\ln x - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$
12. $\ln|1-x| + \ln|x| - \ln|x+1| + C$
13. $\frac{4}{7}\left(5\ln\left(5-\sqrt{x+10}\right) + 2\ln\left(\sqrt{x+10}+2\right)\right) + C$

Chapter 8

1. (a) Divergent. (Use the divergence test.)
   (b) Convergent (Geometric series with $r = \frac{2}{3}$
   (c) Divergent. (Use the divergence test.)
   (d) Convergent (sum of 3 geometric series with $r = \frac{4}{5}$ and $r = \frac{-\pi}{5}$.
   (e) Divergent (p–series with $p < 1$.)
   (f) Convergent. (Comparison test or limit comparison test with $b_n = \frac{1}{n^2}$.
2. 6
3. (a) Absolutely convergent (Comparison test with $b_n = \frac{1}{n^2}$.)
   (b) Conditionally convergent.
   (c) Absolutely convergent (Ratio Test)
4. $R = \frac{1}{3}$, $I = [5/3, 7/3)$
5. $\sum_{n=1}^{\infty}(-3)^{n-1}nx^{n+1}$
6. $\frac{2}{9} + \frac{8}{27}x + \frac{8}{27}x^2 + \frac{16}{243}x^3$
7. $\ln 3 + \sum_{n=1}^{\infty}\frac{(-1)^{n-1}x^n}{n3^n}$
8. $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}\left(x - \frac{\pi}{2}\right)^{2n}$

9. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^{3n+2}}{n}$

10. $e^{3/5} - \dfrac{8}{5}$

Chapter 9

1. $x = -y^2 - 4y - 3$ for $-4 \leq y \leq 0$.

2. Horizontal tangent $(0, -3)$, vertical tangent $(-2, -2)$, $(2, -2)$

3. $e^2 - e^{-2}$