MTH 224O - Spring 2024

Practice Problems for Exam 2

Problem 1: Let X be the lifetime (in years) of a certain type of light bulb. We know that the pdf of X is given by

$$f_X(x) = \begin{cases} \frac{c}{x^2}; & x > 10, \\ 0; & x \leq 10. \end{cases}$$

(a) Determine the constant c.

(b) What is $\mathbb{E}(X)$, the expected lifetime of a light bulb? How about its variance $\mathrm{Var}(X)$?

(c) You have just installed 6 new light bulbs. What is the probability that at least 3 of them will be working after 15 years? What assumptions are you making?

Problem 2: Jane rolls a die repeatedly, and counts how many times the result is 1. She stops the first time the result is a 6. Let N be the number of rolls until she stops, and X be the number of 1's that appear before she stops.

(a) Find $p_{N,X}(n, x)$, that is, the joint probability mass function (pmf) of N and X.

(b) Find $P(X = N - 1)$.

Problem 3: Phileas has read The Daily Telegraph every morning for the last 40 years. He has found that, on average, there is one misprint in every 10 pages of the newspaper. When answering the following questions, be sure to explain what probability distribution you are using and why.

(a) If he thoroughly reads page 3 of today's newspaper, what is the probability that he will find at least one misprint?

(b) Assume that, while looking over page 5 (but not having read it thoroughly), he finds a mistake. What is the probability that there are exactly 2 more mistakes on page 5?

(c) He finds out that two editors, "A" and "B", are responsible for editing the newspaper. With equal probability, one of the editors will be in charge of editing tomorrow's newspaper. The average number of misprints per page is different for the two editors: 3 per page for "A", and 4.2 per page for "B." With this information, what is the probability that page 8 of tomorrow's newspaper will have no misprint?