Exam 3 Review

Test 3 will cover Sections 8.1 through 8.7.

- Section 8.1: Sequences
  - Determine whether a sequence is convergent or divergent by evaluating $\lim_{n\to\infty} a_n$.
  - Convergence of a recursive sequence, The Monotonic Bounded Sequence Theorem.
- Section 8.2–8.4. Infinite Series.
  - Convergence of a geometric series, p–series.
  - The Divergence Test.
  - The Integral Test
  - Comparison and Limit Comparison Tests
  - The Alternating Series Test
  - Absolute convergence, conditional Convergence
  - The Ratio and the Root Tests
- Section 8.5 Power Series
  - Radius and Interval of Convergence for a Power Series
- Section 8.6 Power Series representation for a function.
  - Use $\dfrac{1}{1-x} = \displaystyle\sum_{n=0}^{\infty} x^n$ for $|x| < 1$ as well as differentiation, integration to find the power series representation for a function.
- Section 8.7: Taylor and McLaurin Series.
  - Find the Taylor and/or McLaurin series of a function.
  - Derivative, integral of a Taylor and/or McLaurin series.

- Sample Problems

Chapter 8: Sections 8.1 through 8.7

1. Determine whether the sequence is convergent or divergent. Justify your answer.
   (a) $b_n = \dfrac{(3n-2)!}{(3n+1)!}$ (Convergent)
   (b) $a_n = \dfrac{5n^3+4n^2}{2n^3+1}$ (Convergent)
2. Let a_n be the following recursive sequence: $a_0 = \sqrt{2}$, $a_{n+1} = \sqrt{3a_n}$. It is known that a_n is increasing and bounded above by 4.
   (a) Show that a_n is convergent.
   (b) Where does a_n converge to? **Answer:** $L = 3$
3. Determine whether the series is convergent or divergent. Justify your answer.
   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{1+4^n}{1+3^n}$ (Divergent)
   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^n}$ (Convergent)
   (c) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$ (Divergent)
   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n}+(-\pi)^n}{5^{n-1}}$ (Convergent)
   (e) $\displaystyle\sum_{n=1}^{\infty} n^{-1/3}$ (Divergent)
   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$ (Convergent)
4. Find the sum of
$$\sum_{n=1}^{\infty} 2^n 3^{1-n}$$
   . **Answer:** 6
5. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.
   (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\arctan n}{n^2}$ (Absolutely Convergent)
   (b) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln n}$ (Conditionally Convergent)
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$ (Absolutely Convergent)

6. Find the radius, and the interval of convergence for

$$\sum_{n=1}^{\infty} \frac{3^n (x-2)^n}{n+1}$$

Answer: $R = \frac{1}{3}$, $I = [5/3, 7/3)$

7. Write the power series representation for $f(x) = \frac{x^2}{(3x+1)^2}$

Answer: $\sum_{n=1}^{\infty} (-3)^{n-1} n x^{n+1}$

8. Let $f(x) = \frac{1}{3-2x}$. Find the first four non zeros for the power series representation of $f'(x)$.

Answer: $\frac{2}{9} + \frac{8}{27}x + \frac{8}{27}x^2 + \frac{16}{243}x^3$

9. Write the power series representation for $f(x) = \ln(3+x)$

Answer: $\ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n 3^n}$

10. Find the Taylor Series for $f(x) = \sin x$ centered at $a = \frac{\pi}{2}$.

Answer: $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \left(x - \frac{\pi}{2}\right)^{2n}$

11. Find the McLaurin series for $f(x) = x^2 \ln(1 + x^3)$

Answer: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{3n+2}}{n}$

12. Find the sum of the series $\sum_{n=2}^{\infty} \frac{3^n}{5^n n!}$

Answer: $e^{3/5} - \frac{8}{5}$

Chapter 8 additional practice problems from the textbook.
Section 8.2. 9–27 odd.
Section 8.3. 11-29 odd
Section 8.4. 5, 7, 13, 15 19–35 odd
Section 8.5. 3–21 odd
Section 8.6. 5–9 odd, 13, 15, 17, 21, 23, 25
Section 8.7: 5–17 odd, 21, 31, 43, 51, 59–63 odd