"It's Math Time!"

# Math Union

**University of Miami**

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## *Set Theory and Mathematical Logic!*

The **Math Union** is here for students to get together and discuss topics in the exciting world of mathematics!

We are holding student presentations, every **Monday at 6:30 in Ungar 402** until the semester comes to a close.

## The Continuum Hypothesis and the Characterizations of Infinity.

**By ESTEBAN MORALES**

*"Set theory has a dual role in mathematics. In pure mathematics, it is the place where questions about the infinite are studied. Although this is a fascinating study of permanent interest, it does not account for the importance of set theory in applied areas. There the importance stems from the fact that set theory provides an incredibly versatile toolbox for building mathematical models of various phenomena."*

- Jon Barwise and Lawrence Moss

This Monday, we'll discuss the puzzling world of Set Theory and Mathematical Logic. More specifically, we'll delve into different characterizations and propositions relating to infinity and demonstrate the existence of these peculiar objects called ordinals.

**Def.** $x$ is an ordinal if and only if $x$ is a transitive set and $(x, \in)$ is a well ordering.

Before we dive in headfirst, we must first examine, and eventually refine, the area in which we discuss and reason. We will begin in Ancient Greece and travel through the 19th and 20th centuries to try and encapsulate the elusive representations of infinity. It is on this journey that we will uncover why mathematicians axiomatized set theory and realize just how powerful the ZFC axioms are.

Next, we will look at how our set of natural numbers was constructed, the size of the naturals when compared to the reals, and the Continuum Hypothesis (CH):

$$2^{\aleph_0} = \aleph_1$$

In 1940, Kurt Gödel showed that, within ZFC, the Continuum Hypothesis cannot be disproven. It was a monumental result, even if it did not put the question of CH to bed. In the 1960s Paul Cohen was able to deduce that CH cannot be proven within ZFC. Thus, if the statement cannot be proven or disproven within a theory, it is considered independent of it.

If time permits, we will explore Cohen's method of notions of forcing and generic filters on sets.

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