

# MTH 230 HW 1

Esteban Morales

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**Exercise 1.** 1. Express each statement using P, Q and logical connectives.

- (a) P whenever Q.  $Q \Rightarrow P$
- (b) P is necessary for Q  $P \Rightarrow Q$
- (c) P is sufficient for Q.  $P \Rightarrow Q$
- (d) P only if Q.  $Q \Rightarrow P$
- (e) P is necessary and sufficient for Q  $P \Leftrightarrow Q$

**Exercise 2.** Prove Pierce's Law:

For any proposition P, Q, the proposition  $((P \Rightarrow Q) \Rightarrow P) \Rightarrow P$  is a tautology.

*Proof.*

$P$	$Q$	$P \Rightarrow Q$	$(P \Rightarrow Q) \Rightarrow P$	$((P \Rightarrow Q) \Rightarrow P) \Rightarrow P$
$T$	$T$	$T$	$T$	$T$
$T$	$F$	$F$	$T$	$T$
$F$	$T$	$T$	$F$	$T$
$F$	$F$	$T$	$F$	$T$

□

**Exercise 3.** Prove that  $((P \Rightarrow Q) \wedge (Q \Rightarrow R) \wedge P) \Rightarrow R$  is a tautology

*Proof.*

$P$	$Q$	$R$	$P \Rightarrow Q$	$Q \Rightarrow R$	$(P \Rightarrow Q) \wedge (Q \Rightarrow R)$	$((P \Rightarrow Q) \wedge (Q \Rightarrow R) \wedge P) \Rightarrow R$
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