Some Class
Random Examples

Esteban Morales

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Chapter 1

1.1 Random Examples

Limit of Sequence in Let $\{s_n\}$ be a sequence in . We say

$$\lim_{n\to\infty} s_n = s$$

where $s \in$ if $\forall$ real numbers > 0 $\exists$ natural number N such that for $n > N$

$$s- < s_n < s + i.e. |s - s_n| <$$

Is the set $x-$axis$\setminus\{Origin\}$ a closed set We have to take its complement and check whether that set is a open set i.e. if it is a union of open balls We will do topology in Normed Linear Space (Mainly ^{n} and occasionally ^{n})using the language of Metric Space TopologyTopology is cool Open Set and Close Set

Open Set:
- ϕ
- $\bigcup\limits_{x\in X} B_r(x)$ (Any $r > 0$ will do)
- $B_r(x)$ is open

Closed Set:
- $X,\ \phi$
- $\overline{B_r(x)}$

$x-$axis $\cup$ $y-$axis

If $x \in$ open set V then $\exists$ $\delta > 0$ such that $B_\delta(x) \subset V$ By openness of V, $x \in B_r(u) \subset V$

[red] (0,0) circle [x radius=3.5cm, y radius=2cm] ; (3,1.6) node[red]V; [blue] (1,0) circle (1.45cm) ; [blue] (1,0) circle (1pt) node[anchor=north]u; (2.9,0.4) node[blue]$B_r(u)$; [green!40!black] (1.7,0) circle (0.5cm) node [yshift=0.7cm]$B_\delta(x)$; [green!40!black] (1.7,0) circle (1pt) node[anchor=west]x;

Given $x \in B_r(u) \subset V$, we want $\delta > 0$ such that $x \in B_\delta(x) \subset B_r(u) \subset V$. Let $d = d(u, x)$. Choose δ such that $d + \delta < r$ (e.g. $\delta < \frac{r-d}{2}$)

If $y \in B_\delta(x)$ we will be done by showing that $d(u, y) < r$ but

$$d(u, y) \leq d(u, x) + d(x, y) < d + \delta < r$$

By the result of the proof, we can then show... Suppose $\vec{v_1}, \ldots, \vec{v_n} \in [n]$ is subspace of ^{n}. $1 + 1 = 2$.

1.2 Random

Normed Linear Space and Norm $\|\cdot\|$Let V be a vector space over (or). A norm on V is function $\|\cdot\|$ $V \to_{\geq 0}$ satisfying

[label=**0**] $\|x\| = 0 \iff x = 0 \ \forall\ x \in V$ $\|\lambda x\| = |\lambda|\|x\| \ \forall\ \lambda \in$**(or),** $x \in V$ $\|x+y\| \leq \|x\| + \|y\| \ \forall\ x, y \in V$ **(Triangle Inequality/Subadditivity)**

And V is called a normed linear space.

• Same definition works with V a vector space over (again $\|\cdot\| \to_{\geq 0}$) where **??** becomes $\|\lambda x\| = |\lambda|\|x\| \ \forall\ \lambda \in$, $x \in V$, where for $\lambda = a + ib$, $|\lambda| = \sqrt{a^2+b^2}$

$p-$Norm$V = {}^m$, $p \in_{\geq 0}$. Define for $x = (x_1, x_2, \cdots, x_m) \in^m$

$$\|x\|_p = \Big(|x_1|^p + |x_2|^p + \cdots + |x_m|^p\Big)^{\frac{1}{p}}$$

(In school $p = 2$) **Special Case** $p = 1$: $\|x\|_1 = |x_1| + |x_2| + \cdots + |x_m|$ is clearly a norm by usual triangle inequality.

Special Case $p \to \infty$ (^{m} **with** $\|\cdot\|_\infty$): $\|x\|_\infty = \max\{|x_1|, |x_2|, \cdots, |x_m|\}$ For $m = 1$ these $p-$norms are nothing but $|x|$. Now exercise Prove that triangle inequality is true if $p \geq 1$ for $p-$norms. (What goes wrong for $p < 1$?) **For Property ?? for norm-2**

When field is :

We have to show $\sum_i (x_i + y_i)^2 \leq \left(\sqrt{\sum_i x_i^2} + \sqrt{\sum_i y_i^2}\right)^2$
$\sum_i (x_i^2 + 2x_iy_i + y_i^2) \leq \sum_i x_i^2 + 2\sqrt{\left[\sum_i x_i^2\right]\left[\sum_i y_i^2\right]} + \sum_i y_i^2$
$\left[\sum_i x_iy_i\right]^2 \leq \left[\sum_i x_i^2\right]\left[\sum_i y_i^2\right]$So in other words prove $\langle x, y\rangle^2 \leq \langle x, x\rangle\langle y, y\rangle$ where

$$\langle x, y\rangle = \sum_i x_iy_i$$

- $\|x\|^2 = \langle x, x\rangle$
- $\langle x, y\rangle = \langle y, x\rangle$
- $\langle \cdot, \cdot\rangle$ is $-$linear in each slot i.e. $\langle rx + x', y\rangle = r\langle x, y\rangle + \langle x', y\rangle and similarly for second slot$Here in $\langle x, y\rangle$ x is in first slot and y is in second slot.

Now the statement is just the Cauchy-Schwartz Inequality. For proof

$$\langle x, y\rangle^2 \leq \langle x, x\rangle\langle y, y\rangle$$

expand everything of $\langle x - \lambda y, x - \lambda y\rangle$ which is going to give a quadratic equation in variable λ $\langle x - \lambda y, x - \lambda y\rangle = \langle x, x - \lambda y\rangle - \lambda\langle y, x - \lambda y\rangle$
$= \langle x, x\rangle - \lambda\langle x, y\rangle - \lambda\langle y, x\rangle + \lambda^2\langle y, y\rangle$
$= \langle x, x\rangle - 2\lambda\langle x, y\rangle + \lambda^2\langle y, y\rangle$Now unless $x = \lambda y$ we have $\langle x - \lambda y, x - \lambda y\rangle > 0$ Hence the quadratic equation has no root therefore the discriminant is greater than zero.

When field is :

Modify the definition by

$$\langle x, y \rangle = \sum_i \overline{x_i} y_i$$

Then we still have $\langle x, x \rangle \geq 0$

1.3 Algorithms

[H] This is some input This is some output This is a comment some code here

$x \leftarrow 0$ $y \leftarrow 0$ $x > 5$ x is greater than 5 *This is also a comment x is less than or equal to 5 y in 0..5 $y \leftarrow y + 1$ y in 0..5 $y \leftarrow y - 1$ $x > 5$ $x \leftarrow x - 1$ Return something here what