MTH 230 Midterm Esteban Morales

1.)

P	Q	R	$P\to Q$	α: $(P\to Q)\to R$	$P\to R$	β: $(P\to Q)\to(P\to R)$	$\alpha\to\beta$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	T	T
T	F	F	F	T	F	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	F	T	T	T

thus $((P\to Q)\to R)\to (P\to Q)\to(P\to R)$ is a tautology

2.)

(a) $\neg\left(\exists x\in S,\ \forall y\in T,\ P(x,y)\right)$

$\iff \forall x\in S,\ \exists y\in T,\ \neg P(x,y)$

(b) Assume: $\exists x\in S,\ \forall y\in T,\ P(x,y)$. fix $x^*\in S$

by assumption $\forall y\in T,\ P(x^*,y)\equiv T$

$\Rightarrow \forall y\in T,\ \exists x\in S,\ P(x,y)\equiv T$ ▫

(c) let $S:=\{x \mid x$ is a ~~student~~ alumni at V.M.$\}$ $P(x,y):=$ diploma y belongs to ~~student~~ alumni x

$T:=\{y \mid y$ is a diploma from V.M.$\}$

Thus, $\exists x\in S,\ \forall y\in T$ diploma y belongs to ~~student~~ alumni x is FALSE (every Diploma does not belong to 1 student)

but $\forall y\in T\ \exists x\in S$ diploma y belongs to alumnus x is True (every alumnus has a diploma by def)

3) Pf: $A \cap B = B \iff B \subseteq A$

$A \cap B := \{x \mid x \in A \wedge x \in B\} = \{x \mid x \in B\} \iff \{x \mid x \in B \Rightarrow x \in A\}$

thus we prove
the first direction

$\underline{\Longrightarrow}$ Assume $\{x \mid x \in A \wedge x \in B\} = \{x \mid x \in B\}$
$\qquad A \cap B \quad = \quad B$

$\Rightarrow$ fix $x_0 \in B$. then $x_0 \in A$ since $A \cap B = B$
for any x_0 in B, we can deduce that it is in A

$\Rightarrow \{x \mid x \in B \Rightarrow x \in A\} \Rightarrow B \subseteq A$.

$\underline{\Longleftarrow}$ Assume that $B \subseteq A$

$\Leftarrow \{x \mid x \in B \Rightarrow x \in A\} \Leftarrow B = A \vee B \subset A \Leftarrow$

(i) If $B = A \Rightarrow B \cap A = B$.

(ii) If $B \subset A \Rightarrow B \cap A = B$.

$\{x \mid (x \in B \Rightarrow x \in A) \wedge B \neq A\}$
$\Rightarrow \{x \mid x \in B \wedge x \in A\}$ eqval

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4.) $\forall n \geq 0, \sum_{i=0}^{n} 2^i = 2^{n+1} - 1$

base Case: for $n=0$, $\sum_{i=0}^{0} 2^i = 2^{0+1} - 1 \iff 2^0 = 2^1 - 1 \iff 1 = 1$ ✓

(IH) Assume that for $k \in \mathbb{N}$, $\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$

We need to show that $\sum_{i=0}^{k+1} 2^i = 2^{(k+1)+1} - 1$

$\sum_{i=0}^{k} 2^i = 1 + 2 + 4 + \ldots 2^k \iff \sum_{i=0}^{k} 2^i + 2^{k+1} = 2^{k+1} - 1 + 2^{k+1}$

$\Rightarrow \sum_{i=0}^{k+1} 2^i = 2^{k+1} + 2^{k+1} - 1 = 2(2^{k+1}) - 1 = 2^{k+2} - 1 = 2^{(k+1)+1} - 1.$ ✓

thus, by the Principle of Mathematical Induction

$\forall n \geq 0, \sum_{i=0}^{n} 2^i = 2^{n+1} - 1.$

5.)

5.) $13x + 12 \equiv 0 \mod (23)$

$\Leftrightarrow \exists k \in \mathbb{Z} ;\ 23k = 13x + 12 - 0$

$\Rightarrow 23k - 13x = 12$

$23 = (1)13 + 10$

$13 = (1)10 + 3$

$10 = (3)3 + 1$

$3 = (1)3 + 0$

$1 = 10 - 3(3)$

$1 = 10 - 3(13 - (1)10) = 10 - 3(13) + 3(10) = 4(10) - 3(13)$

$1 = -3(13) + 4(23 - 13) = -7(13) + 4(23)$

$\Rightarrow \quad 12 = 12(4(23) - 7(13))$

$\Rightarrow \quad k = 48,\ x = 84$ is 1 sol.

In general, we can say that the Solutions to this linear diophantine equation are

$k = 48 + 13q, \quad x = 84 + 23q, \quad q \in \mathbb{Z}$

6.) $41s + 17t = 3$

$41 = 2(17) + 7$
$17 = 2(7) + 3$
$7 = 2(3) + 1$
$3 = 3(1) + 0$

$1 = 7 - 2(3)$

$1 = -2(17 - 2(7)) + 7 = +5(7) - 2(17)$

$1 = 5(41 - 2(17)) - 2(17) = -12(17) + 5(41)$

$\Rightarrow 3 = 3\left(-12(17) + 5(41)\right)$

$\Rightarrow t = -36$, $s = 15$ is a sol.

and more generally we can solve it as

$S = 15 + 17l$, $t = -36 - 41l$