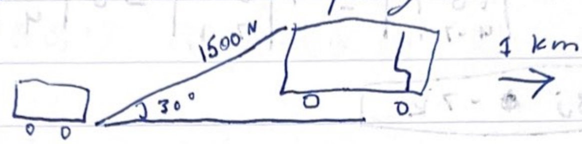
HW 2 MTH 310 Fall 2024 Esteban Morales

section 10.3

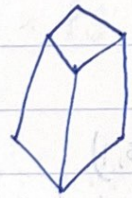
38.) A tow truck drags a stalled car along a road. the chain makes an angle of 30° the tension in the chain is 1500 N how much work is done by pulling the car 1 km.



$W = F \cdot D = |F|\,|D| \cdot \cos(30°) = 1500 \cdot 1000 \cdot \cos(30)$

$= 750000\sqrt{3}$

44.) Find the angle between a diagonal of a Cube and a diagonal of one of its faces.



We can use the theorem about dot products to calculate the $\cos(\theta)$

Recall: $a \cdot b = \|a\|\,\|b\| \cdot \cos(\theta)$

$\Rightarrow \langle 1,1,0\rangle \cdot \langle 1,1,1\rangle = \sqrt{2} \cdot \sqrt{3} \cdot \cos(\theta)$

$\Rightarrow 2 = \sqrt{6} \cdot \cos(\theta) \Rightarrow \cos(\theta) = \frac{2}{\sqrt{6}}$

$\Rightarrow \theta = \arccos\left(\frac{2}{\sqrt{6}}\right)$

Section 10.4

18.) $a = \langle 1,0,1\rangle$ $b = \langle 2,1,-1\rangle$ and $c = \langle 0,1,3\rangle$

Show that $a \times (b \times c) \neq (a \times b) \times c$

18.) cont'd $b \times c = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 0 & 1 & 3 \end{vmatrix} = i \begin{vmatrix} 1 & -1 \\ 1 & 3 \end{vmatrix} - j \begin{vmatrix} 2 & -1 \\ 0 & 3 \end{vmatrix} + k \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix}$

$= 4i - 7j + k = \langle 4, -7, 1 \rangle$

$a \times (b \times c) = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 4 & -7 & 1 \end{vmatrix} = i \begin{vmatrix} 0 & 1 \\ -7 & 1 \end{vmatrix} - j \begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix} + k \begin{vmatrix} 1 & 0 \\ 4 & -7 \end{vmatrix}$

$= \boxed{7i - 3j - 7k}$

$a \times b = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 2 & 1 & -1 \end{vmatrix} = i \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} - j \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} + k \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix}$

$= -i - 3j - k = \langle -1, -3, -1 \rangle$

$(a \times b) \times c = \begin{vmatrix} i & j & k \\ -1 & -3 & -1 \\ 0 & 1 & 3 \end{vmatrix} = i \begin{vmatrix} -3 & -1 \\ 1 & 3 \end{vmatrix} - j \begin{vmatrix} -1 & -1 \\ 0 & 3 \end{vmatrix} + k \begin{vmatrix} -1 & -3 \\ 0 & 1 \end{vmatrix}$

$= 8i + 2j - k$

30.) $P(0,0,-3)$ $Q(4,2,0)$ $R(3,3,1)$

Find a non-zero vector orthogonal to the plane through the points P, Q, R and (b) find the Area of the triangle

Take the vector $\vec{PQ} = \langle 4, 2, 3 \rangle$ and $\vec{PR} = \langle 3, 3, 4 \rangle$

$\vec{PQ} \times \vec{PR}$ generates a vector perpendicular to both and thus, to the plane

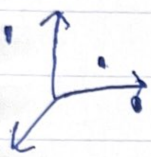
$\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 4 & 2 & 3 \\ 3 & 3 & 4 \end{vmatrix} = i \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} - j \begin{vmatrix} 4 & 3 \\ 3 & 4 \end{vmatrix} + k \begin{vmatrix} 4 & 2 \\ 3 & 3 \end{vmatrix}$

$= -i - 7j + 6k = \boxed{\langle -1, -7, 6 \rangle}$

the Area of the triangle formed in the plane bx PQR is $\frac{1}{2} \cdot \|\vec{PQ} \times \vec{PR}\| = \frac{1}{2} \cdot \sqrt{-1^2 + -7^2 + 6^2} = \boxed{\frac{\sqrt{86}}{2}}$

36.) Find the Volume of the parallelpiped with adjacent edges PQ, PR and PS.

$P = (3,0,1) \quad Q = (-1,2,5) \quad R = (5,1,-1) \quad S = (0,4,2)$



$\vec{PQ} = (-4,2,4), \quad \vec{PR} = (2,1-2)$

$\vec{PS} = (-3,4,1)$

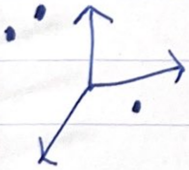
The volume of the parallelipiped is the magnitude of their scalar triple product

$V = \| \vec{PQ} \cdot (\vec{PR} \times \vec{PS}) \|$, where PQ is the height and PR×PS is Base

$\vec{PR} \times \vec{PS} = \begin{vmatrix} i & j & k \\ 2 & 1 & -2 \\ -3 & 4 & 1 \end{vmatrix} = i\begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} - j\begin{vmatrix} 2 & -2 \\ -3 & 1 \end{vmatrix} + k\begin{vmatrix} 2 & 1 \\ -3 & 4 \end{vmatrix}$

$= 9i + 4j + 11k = \langle 9,4,11 \rangle$

$\vec{PQ} \cdot \langle 9,4,11 \rangle = \cancel{18}\ -36+8+44 = \cancel{8}\ \boxed{16.}$



$(\vec{PQ} \times \vec{PS}) \cdot \vec{PR} =$

$\vec{PQ} \times \vec{PS} = \begin{vmatrix} i & j & k \\ -4 & 2 & 4 \\ -3 & 4 & 1 \end{vmatrix} = i\begin{vmatrix} 2 & 4 \\ 4 & 1 \end{vmatrix} - j\begin{vmatrix} -4 & 4 \\ -3 & 1 \end{vmatrix} + k\begin{vmatrix} -4 & 2 \\ -3 & 4 \end{vmatrix}$

$-6i - 8j - 10k = \langle \cancel{-14}, -8, -10 \rangle$

$\langle -6,-8-10 \rangle \cdot \vec{PR} =$

$= -28 + -8 + 20 \ \boxed{=16}$

38.) $\vec{AB} = \langle 2,-4,4 \rangle$ $\vec{AC} = \langle 4,-1-2 \rangle$

$AD = \langle 2, 3, -6 \rangle$

$\vec{AB} \cdot (\vec{AC} \times \vec{AD}) =$

$$\vec{AC} \times \vec{AD} = \begin{vmatrix} i & j & k \\ 4 & -1 & -2 \\ 2 & 3 & -6 \end{vmatrix} = i \begin{vmatrix} -1 & -2 \\ 3 & -6 \end{vmatrix} - j \begin{vmatrix} 4 & -2 \\ 2 & -6 \end{vmatrix} + k \begin{vmatrix} 4 & -1 \\ 2 & 3 \end{vmatrix}$$

$= 12i + 20j + 10k = \langle 12, 20, 10 \rangle$

$AB \cdot \langle 12, 20, 10 \rangle = 24 - 80 + 56 = 0$

Since the volume of the paralelipiped is 0, the vectors must be co-planar.