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MTH 230 Fall '24

HW 1

1.) a. $Q \to P$

b.) $P \to Q$

c.) $P \to Q$

d.) $Q \to P$

e.) $P \leftrightarrow Q$

2.)

(a) P	(b) Q	(c) $P \to Q$	(d) $(P \to Q) \to P$	$(d) \to (a)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

Hence Pierce's law is a tautology.

3.)

(a) P	(b) Q	(c) R	(d) $P \to Q$	(e) $Q \to R$	(f) $(d) \land (e)$	(g) $(d) \land (e) \land (a)$	$(g) \to (c)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	F	T
F	T	F	T	F	F	F	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

Hence $((P \to Q) \land (Q \to R) \land P) \to R$ is a tautology

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3.) con'td

direct pf: $((P \to Q) \wedge (Q \to R) \wedge P) \to R$

We assume that $(P \to Q) \wedge (Q \to R) \wedge P$ holds a True truth value, from this we can deduce that P must contain a True Truth value from the conjunction. In order to make the first conjunct, $(P \to Q)$, Q must be True by assumption by similar logic, R cannot be false as it would make the second conjunct false so R is True. Thus, an implication where the precedent and antecedent a True is also True by def.

4.) $\forall_t \exists_p f(p)|^t \wedge \forall_p \exists_t f(p)|^t \wedge \forall_p \forall_t \neg f(p)|^t$

fool person 'p' at time 't'

5.)

(a.) $\forall x$, x a person, $\neg H(x) \to R(x)$

$H(x)$: person x studies History

$R(x)$: person x repeats History

(b.) $\exists x, \neg H(x) \wedge \neg R(x)$

using the equivalent expression of : $\alpha \to \beta \equiv \neg\alpha \vee \beta$

and demorgen's law $\neg(\neg\alpha \vee \beta) \equiv \alpha \wedge \neg\beta$

6.) $\lim_{x \to x_0} f(x) = L \in \mathbb{R} :\Longleftrightarrow \forall \varepsilon > 0, \exists \delta > 0$ s.t. $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$

$f(x)$ is cts. at x_0 iff

) $x_0 \in \operatorname{dom}(f(x))$

) $\lim_{x \to x_0} f(x) = f(x_0)$

(a.) Def. $\lim_{x \to 0} f(x) = f(0) :\Leftrightarrow \forall \varepsilon > 0, \exists \delta > 0$ s.t. $0 < |x - 0| < \delta \Rightarrow |f(x) - f(0)| < \varepsilon$

$\boxed{\text{negation}}$: $\exists \varepsilon > 0, \forall \delta > 0$ s.t. $(0 < |x - 0| < \delta) \wedge |f(x) - f(0)| \geq \varepsilon$

In order to show that f was not continues, I would use the negation and show that its value would be True thus making the original statement False, and hence our function would not be continues.

b.) $\lim_{x \to 0} x = 0$, by def, $\forall \varepsilon > 0, \exists \delta > 0\ \forall x$, $0 < |x - 0| < \delta \Rightarrow |x - 0| < \varepsilon$

pick $\varepsilon = 1/2$ arbitrarily, $0 < |x| < \delta \Rightarrow |x| < 1/2$, if $\delta = 1/2$ or less then our Statement is true. thus $\delta := \varepsilon$ and this works for all ε we choose