MTH 230 HW 2

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1.) $A = \{1, 2, 3, 4\}$ $B = \{2, 5, 8\}$

a.) $A \cap B = \{2\}$

b.) $A \cup B = \{1, 2, 3, 4, 5, 8\}$

c.) $A \times B = \{(1,2), (1,5), (1,8), (2,2), (2,5), (2,8), (3,2), (3,5), (3,8), (4,2), (4,5), (4,8)\}$

d. $\mathcal{P}(A) = \{\{1\}, \{2\}, \{3\}, \{4\}, \emptyset, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}, \{1,2,4\}\}$ Has 16 elements since $|\mathcal{P}(A)| = 2^{|A|}$

2.) Show $A \cup B = B \iff A \subseteq B$

$\boxed{\Rightarrow}$

Assume $A \cup B = B$:

$A \cup B := \{x \mid x \in A \vee x \in B\} = B = \{x \mid x \in B\}$

$\Rightarrow \{x \mid x \in A \vee x \in B\} = \{x \mid x \in B\}$ by Assumption

for this to be true if $x \in A \Rightarrow x \in B$, since they are equal.

$\Rightarrow \{x \mid x \in A \Rightarrow x \in B \vee x \in B\}$

$\Rightarrow$ $A \subseteq B$ since the inclusion of A adds no new information or elements to the set $A \cup B$.

$\boxed{\Leftarrow}$ Assume $A \subseteq B$:

$A \subseteq B := \{x \mid x \in A \Rightarrow x \in B\}$, by assumption

$A \cup B := \{x \mid x \in A \vee x \in B\}$, by def.

$\iff \{x \mid (x \in A \Rightarrow x \in B) \vee x \in B\}$, by assumption

$\Rightarrow \{x \mid x \in B \vee x \in B\}$ by def of subset

$\iff \{x \mid x \in B\} = B \Rightarrow A \cup B = B$.

3.) Show

$$\overset{X}{A \cap (B \cup C)} = \overset{Y}{(A \cap B) \cup (A \cap C)}$$

to show equality it is enough to show that

$X \subseteq Y$ and $Y \subseteq X$

$\boxed{X \subseteq Y}$ $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

let $\alpha \in A \cap (B \cup C)$ such that $\alpha \in A$ and $\alpha \in B \cup C$

so either α is in B or C, or both,

If $\alpha \in B$ then $\alpha \in (A \cap B) \cup (A \cap C)$

If $\alpha \in C$ then $\alpha \in (A \cap C) \cup (A \cap B)$

thus for all possibilities, $\alpha \in A \cap (B \cup C) \implies \alpha \in (A \cap B) \cup (A \cap C)$

therefore $X \subseteq Y$

$\boxed{Y \subseteq X}$

let $b \in (A \cap B) \cup (A \cap C)$

so $b \in A \cap B$ or $b \in A \cap C$ or both. Irregardless

we can see that $b \in A$

If $b \in B \implies b \in A \cap (B \cup C)$

If $b \in C \implies b \in A \cap (C \cup B)$

in both cases we can see that if $b \in (A \cap B) \cup (A \cap C) \implies b \in A \cap (B \cup C)$ $\square$

4.) yes $A \in 2^A$ $(A \in \mathcal{P}(A))$

and yes $A \subseteq 2^A$ $(A \subseteq \mathcal{P}(A)$

Ex. $A = \{4, 5, 6\}$ $2^A = \mathcal{P}(A) = \{\{4\}, \{5\}, \{6\}, \{4,5\}, \{5,6\}, \{4,6\}, \{4,5,6\}, \emptyset\}$

As you can see $\{4,5,6\} \in 2^A$

and $\{4,5,6\} \subseteq 2^A$

5.) for all P, $\left(\forall x \in S,\ P(x) \Rightarrow \exists x \in S,\ P(x)\right)$

by our assumption, every single x in S makes our predicate, $P(x)$ true so of course there exists at least 1 x such that $P(x)$ is true. This holds for when $S \neq \emptyset$

If $S = \emptyset$, then our antecedent of the implication is false, which makes our implication vaccuously true.

6.) If $n, m \in \mathbb{Z}$ and $m \cdot n = 2k+1$, $k \in \mathbb{Z}$

$\Rightarrow n = 2l + 1,\ m = 2v + 1,\ l, v \in \mathbb{Z}$

Case 1 n, m are both even

$n = 2l,\ m = 2v \Rightarrow \underbrace{n \cdot m = 4vl = 2(q)}_{n \cdot m \text{ even}}$, which is even

Case 2 if n odd, m even (w.l.o.g.)

$n = 2l+1,\ m = 2v \Rightarrow n \cdot m = 4lv + 2v = 2\underbrace{(2lv + v)}_{p \in \mathbb{Z}}$ $\Rightarrow n \cdot m$ even

Case 3 If $n = 2l+1$ $m = 2v+1$

$\Rightarrow n \cdot m = 4lv + 2l + 2v + 1 = 2\underbrace{(2lv + l + v)}_{r \in \mathbb{Z}} + 1$

$= 2r + 1$, which is odd

thus, it must be the case that n and m are both odd