# MTH 310 Fall HW 1

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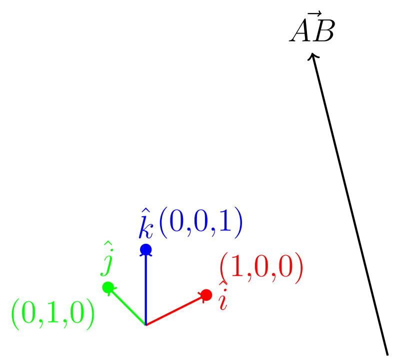
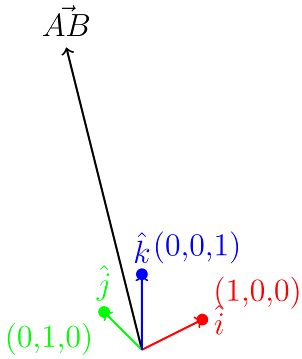
August 28, 2024

**Section 10.2.** Excercise 8.
Find a vector $\vec{a}$ with representation given by the directed line segment $\vec{AB}$. Draw $\vec{AB}$ and the equivealent representation starting at the origin.




*Solution.*




Explanation: We observe that $\|\vec{A}\|$ is smaller than $\|\vec{B}\|$ i.e. (4,0,-2) is closer to the origin than (4,2,1) and thus we select B as our head and A as our tail.

From this, we must reposition A to the origin while maintaining the magnitude and direction of the vector. We can do this by summing each entry in A by it's additive inverse, and repeating the same for B.

This yields our equivalent representation of the vector starting at the origin of $\mathbb{R}^3$ This new vector has representation $\langle 0, 2, 3 \rangle$

□

**Section 10.2.** Excercise 18
Find a vector that has the same direction as <-2,4,2>but has length 6

*Solution.* We begin be recalling the definition of distance in the Euclidean plane.

**Recall:** $Dist(a,b) := \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2}$, provided that a = $(x_0, y_0)$ and b = $(x_1, y_1)$

Let us calculate the distance(length) of the given vector
$Dist(a,0) = \sqrt{(-2)^2 + (4)^2) + (2)^2)} = \sqrt{24}$ We can notice that this is $\frac{6}{\sqrt{24}}$times the desired length, as so we can multiply the entire vector by a scalar multiple of $\frac{6}{\sqrt{24}}$ in order to get the length of 6, with the same direction.
Our Result is $\langle \frac{-12}{\sqrt{24}}, \frac{24}{\sqrt{24}}, \frac{12}{\sqrt{24}} \rangle$

□

**Section 10.3.** Excercise 16
Find the angle between the vectors. (First find an exact expression and then approximate to the
nearest degree)

$\mathbf{a} = \langle 4, 0, 2 \rangle$, $\mathbf{b} = \langle 2, -1, 0, \rangle$

*Solution.* We will first recall the theorem proved in class, namely that a $\cdot$ b $= ||a||||b||cos(\theta)$ where $\theta$ is the angle between the two vectors. we know that a $\cdot b = 4*2+0*-1+2*0 = 8$
$||a|| = \sqrt{4^2 + 0^2 + 2^2} = \sqrt{20}$
$||b|| = \sqrt{2^2 + -1^2 + 0^2} = \sqrt{5}$
$=> \sqrt{20} * \sqrt{5} = \sqrt{100} = 10$

Thus, we know that $8 = cos(\theta) * 10 => cos(\theta) = \frac{8}{10}$ Reducing this and utilizing inverse functions gives that $\theta = \arccos(\frac{8}{10})$, so $\theta$ is $\sim$0.64350...

□

**Section 10.3.** Excercise 24
Find two unit vectors that make an angle of 60°with $\mathbf{v}$ =$\langle 3, 4\rangle$
For our two unit vectors, they must have a $cos(\theta) = \frac{1}{2}$ since the angle between them is 60°and cos(60) = $\frac{1}{2}$.

If we take the dot product with our supposed unit vector $\vec{k}$ we get that $v \cdot k = 3 * a + 4 * b$ where a,b are the coordinates of a unit vector k
We will first recall the theorem proved in class, namely that a · b = $||a||||b||cos(\theta)$ where $\theta$ is the angle between the two vectors.
Using this, $3 * a + 4 * b = ||k||||v|| * cos(\theta)$ Substituting, we get $3 * a + 4 * b = 1 * ||v|| * \frac{1}{2}$
Using the Euclidean defintion of distance $||v|| = \sqrt{(3^2 + 4^2)} = \sqrt{25} = |5|$
$=> 3 * a + 4 * b = 1 * 5 * \frac{1}{2}$

$=> 3 * a + 4 * b = 2.5$ At this point, we can arbitrarily select two values a and b which satisfy the requirement of being a unit vector(having their distance from the origin be 1) we can make a system of equations where $a^2+b^2 = 1$ and $3a+4b = 2.5$ Hence: $3a = (2.5-4b) =>$ $a = (2.5 - 4b)/3$ Substituting, $(\frac{2.5-4b}{3})^2 + b^2 = 1$ Simplifying, $(\frac{6.25-20b+16b^2}{9}) + b^2 = 1, =>$ $(6.25 - 20b + 16b^2 + 9b^2 = 9) => 25b^2 - 20b - 2.75 = 0$ We can solve this quadratic trivially and get that $b = \frac{2 \pm 15\sqrt{3}}{50} => b = 2/5 \pm \frac{3\sqrt{3|}}{10} => b = 0.919615 \vee b = -0.119615$ Plugging this result back into our other equation in our system gives : when b = 0.919615, $a = (2.5 - 4 * 0.919615)/3 => a = (2.5 - 3.6784)/3 => a = -0.3928$

when b = -0.119615, $a = (2.5 - 4 * -0.119615)/3 => a = (2.5 + 0.4784)/3 => a = 0.9928$

The two unit vectors that have an angle of 60°with the vector $\langle 3, 4\rangle$are:

$\vec{k_0}$ =$\langle -0.3928, 0.919615\rangle$ and $\vec{k_1}$ =$\langle 0.9928, -0.119615\rangle$