MTH 310 HW 6

section 11.4

(4.) $f(x,y) = xe^{x \cdot y}$

$f_x = 1e^{x\cdot y} + xe^{x\cdot y}\cdot y = e^{x\cdot y}(1+xy)$

$f_y = x^2 e^{x\cdot y}$, $f_x(1,1) = 2e$ $f_y(1,1) = e$

$Z-1 = 2e(x-1) + e(y-1) \Rightarrow z = 2e(x-1) + e(y-1) + 1$

(6.) $f(x,y) = \ln(x-2y)$ $(3,1,0)$

$f_x = \frac{1}{x-2y} \cdot 1-0$ $f_y = \frac{1}{x-2y} \cdot 0-2$

$f_x(3,1) = \frac{1}{3-\cancel{4}2} = +1$ $f_y(3,1) = \frac{1}{3-4} \cdot -2 = -2$

$Z - 0 = 1(x-3) + -2(y-1)$

$\Rightarrow Z = x - 2 - 2y$

(16.) $f(0,0) = \sqrt{0+\cos^2(0)} = 1$

$f_x(x,y) = \frac{1}{2\sqrt{y+\cos^2(x)}} \cdot 2\cos(x) \cdot -\sin(x) = \frac{-\cos(x)\sin(x)}{\sqrt{y+\cos^2(x)}}$

$f_x(0,0) = \frac{\sin(0)\cdot\cos(0)}{\sqrt{0+\cos^2(0)}} = \frac{0}{1}$

$$f_y(x,y) = \frac{1}{2\sqrt{y+\cos^2(x)}} \cdot 1+0$$

$$f_y(0,0) = \frac{1}{2\sqrt{1}} = \frac{1}{2}$$

$$\Rightarrow \quad f(x,y) \approx 1 + 0(x) + \frac{1}{2}(y)$$

11.5

2.)

$$z = \cos(x+4y) \qquad x = 5t^4 \qquad y = \frac{1}{t}$$

$$z = \cos(5t^4 + \frac{1}{t})$$

$$\Rightarrow \frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$\Rightarrow \frac{\partial z}{\partial x} = -\sin(x+4y) \qquad \frac{dx}{dt} = 20t^3$$

$$\frac{\partial z}{\partial y} = -4\sin(x+4y) \qquad \frac{dy}{dt} = -t^{-2}$$

$$\Rightarrow \frac{dz}{dt} = -20t^3 \sin(x+4y) + \frac{4}{t^2} \sin(x+4y)$$

8.) $z = \tan\left(\frac{u}{v}\right)$ $u = 2s + 3t$ $v = 3s - 2t$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial s} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial t} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial t}$$

$$\frac{\partial z}{\partial u} = \sec^2\left(\frac{u}{v}\right) \cdot \frac{1}{v} \qquad \frac{\partial z}{\partial v} = \sec^2\left(\frac{u}{v}\right) - \frac{u}{v^2}$$

$$\frac{\partial u}{\partial s} = 2 \qquad \frac{\partial u}{\partial t} = 3$$

$$\frac{\partial v}{\partial s} = 3 \qquad \frac{\partial v}{\partial t} = 2$$

$$\Rightarrow \frac{\partial z}{\partial s} = 2\sec^2\left(\frac{u}{v}\right) \cdot \frac{1}{v} + 3\sec^2\left(\frac{u}{v}\right) \cdot \left(-\frac{u}{v^2}\right)$$

$$\frac{\partial z}{\partial t} = \sec^2\left(\frac{u}{v}\right) \cdot \frac{1}{v} \cdot 3 + \sec^2\left(\frac{u}{v}\right) \left(-\frac{u}{v^2}\right) \cdot (-2)$$

18.) $T = \frac{v}{2u + v}$ $u = pq\sqrt{r}$ $v = p\sqrt{q}\, r$

$$\frac{\partial T}{\partial p} = \frac{\partial T}{\partial u} \cdot \frac{\partial u}{\partial p} + \frac{\partial T}{\partial v} \cdot \frac{\partial v}{\partial p} =$$

11.6

8.) $f(x,y) = \frac{x}{x^2+y^2}$ $(1,2)$ $V = (3,5)$

normalize $V \Rightarrow$ $V_{unit} = \frac{1}{\|V\|} \cdot V$

$V_{unit} = \langle \frac{1}{\sqrt{34}} \cdot 3, \frac{1}{\sqrt{34}} 5 \rangle$

$f_x(x,y) = \frac{x^2+y^2 - x(2x)}{(x^2+y^2)^2} = \frac{x^2+y^2-2x^2}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$

$f_y(x,y) = \frac{0(x^2+y^2) - x(2y)}{(x^2+y^2)^2} = \frac{-2xy}{(x^2+y^2)^2}$

$\Rightarrow \nabla f(x,y) = \frac{y^2-x^2}{(x^2+y^2)^2}, \frac{-2xy}{(x^2+y^2)^2}$

$\Rightarrow \nabla f(1,2) = \frac{2^2-1^2}{(1^2+2^2)^2}, \frac{(-2)(2)}{(1^2+2^2)^2} = \langle \frac{3}{25}, \frac{-4}{25} \rangle$

$\Rightarrow D_v f(1,2) = \langle \frac{3}{25}, \frac{-4}{25} \rangle \cdot \langle \frac{3}{\sqrt{34}}, \frac{5}{\sqrt{34}} \rangle$

$\Rightarrow \frac{3}{25} \cdot \frac{3}{\sqrt{34}} + \frac{-4}{25} \cdot \frac{5}{\sqrt{36}} = \frac{-11}{25\sqrt{34}}$