Notes 230

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Section Strong induction$\Rightarrow$ standard inducion.

Proof. Suppose strong induction holds, then for any predicate P on $\mathbb{N}$.
If P(n) holds and $\forall k P(0) \wedge P(1) \wedge ...P(k) \Rightarrow P(k+1)$
Then $\forall n \geq 0, P(n) holds$
Suppose P(0) holds, and $\forall k \geq 0 P(K) \Rightarrow P(K+1)$
If P(K) is true, then P(K + 1) is true. Assume P(0) and P(1) and ... P(K) holds true. So, so does P(K + 1). Hence, strong induction lets us calculate $\forall n \geq 0, P(N)$ □

Section Standard induction implies well ordering principle. *Proof.* Let P(n) be a predication on $\mathbb{N}$
If $\exists i \leq n \in S \subseteq \mathbb{N}$
Then S has a a least element. If S is non empty, then there exists an n in S so P(N) $\Rightarrow$ well ordering principles holds for S.
Now we will prove by induction that $\forall n \geq 0 P(N)$ is true.
Base Case: P(O) states if $0 \in S$, S has a least element, namely 0
Inductive Step: Assume P(K) If $\exists j \leq k \in S$ then $\exists l \in S$ such that $m \in S \Rightarrow l \leq m$ □

Section Well Ordering Principle implies Strong Induction. *Proof.* Assume Well ordering principle P(0)
Assume by contradiction $\exists n$ such that P(n) is false.
Let S be the set of all the Positive integers,n, such that P(n) is false. Then S is nonempty, let l be the elast element. $1 \neq 0$ because P(0) is true. As l is the least element of S, P(0), P(1) ... are all true.
Hence $P(0) \wedge P(1) \wedge ...P(l-1)$ is true, a contradiction! □

Section standard induction implies strong induction. *Proof.* Given a predicate P(n) let Q(n) be if $0 \leq i \leq n$ then P(i) is true
Suppose t □