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HW 4

MTH 310 Fall 24'

Section 10.9

10.) $r(t) = < t^2 i, 2t\,j, \ln(t)\,k >$

Velocity $V(t) = 2t\,i + 2j + \frac{1}{t}k$

Acceleration $a(t) = 2i + 0j - \frac{1}{t^2}k$

Speed $s(t) = \| v(t) \| = \sqrt{2t^2 + 2^2 + (\frac{1}{t})^2}$

$= \sqrt{4t^2 + 4 + \frac{1}{t^2}}$

12.) Find the velocity and position vectors of a particle that has the given acceleration and the given initial velocity and position.

$a(t) = 2i + 6t\,j + 12t^2 k \quad V(0) = i \quad r(0) = j - k$

$\int_0^x a(t)\,dt = V(x) - V(0)$

$\int a(t)\,dt = 2t\,i + \frac{6t^2}{2}j + 4t^3 k + c_1 + c_2 + c_3$

$V(t) = (2t + c_1)i + (3t^2 + c_2)j + (4t^3 + c_3)k$

$V(0) = i \Rightarrow (2(0) + c_1)i + (3\cdot 0 + c_2)j + (4\cdot 0 + c_3)k$

$\Rightarrow c_1 = 1, \quad c_2 = 0, \quad c_3 = 0$

12. cont'd

$V(t) = (2t+1)i + (3t^2)j + (4t^3)k$

$r(0) = j - k$

$\int v(t) = r(t) = (t^2 + t + c_1)i + (t^3 + c_2)j + (t^4 + c_3)k$

since $r(0) = j - k \Rightarrow (0^2 + 0 + c_1)i + (0^3 + c_2)j + (0^4 + c_3)k$

$= c_1 = 0 \quad c_2 = 1 \quad c_3 = -1$

so $r(t) = (t^2 + t + c + 0)i + (t^3 + 1)i + (t^4 - 1)k$

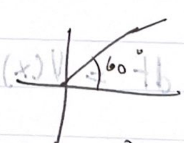
16.) What force is required so that particle of mass m has the position function $r(t) = t^3 i + t^2 j + t^3 k$

$F = m \cdot a(t) \Rightarrow r''(t) = (3t^2 i + 2tj + 3t^2 k)'$

$\Rightarrow 6ti + tj + 6tk$

$F = m((6t)i + (t)j + (6t)k)$

19.) $\|V(0)\| = \dfrac{200\ m}{s}$



$X(t) = V_0 \cos(\alpha)_t t + V_0 \sin(\alpha)t = \dfrac{t^2}{2} g$

$x(t) = 200 \cdot \frac{1}{2} t + \left(\dfrac{200\sqrt{3}}{2}\right) t - \dfrac{t^2}{2} g$

(a) $\dfrac{200^2 \frac{\sqrt{3}}{2}}{g} = 3{,}531.194\ m$

(b) $\text{Height} = \dfrac{200^2 \cdot \sin^2(60^\circ)}{2g} = \dfrac{40{,}000 \cdot 0.75}{19.62}$

$= 1529.051 \text{ m}$

(c) $T = \dfrac{2V_0}{g} = \dfrac{2}{9.81} \cdot 173.21 = 35.29 \text{ s}$

22.) A Gun is fired with elevation 30° ($\pi/6$) what is the muzzle speed if the maximum height of the shell is 500 m.

$r(t) = V_0 \,) \cdot \frac{\sqrt{3}}{2} t \;,\; V_0 \cdot \frac{1}{2} t - \frac{t}{2} g t^2$

What is V_0 if $\quad r_y(t) = V_0 \cdot \frac{1}{2} t - \frac{t^2}{2} g \leq 500$

$V_y(t) = r_y'(t) = \frac{V_0}{2} - \cancel{2} t g =$

$\frac{V_0}{2} - tg = 0 \iff \frac{V_0}{2} = \frac{t \cdot g}{g} \Rightarrow t = \frac{V_0 \cdot g}{2g}$

back substituting

$V_0 \cdot \frac{1}{2} \frac{V_0}{2g} g - \frac{1}{2} g \frac{V_0^2 \cdot g}{2^2 g^2} = 500$

$V_0^2 \left(\frac{1}{2} \cdot \frac{1}{2} \frac{g}{g} - \frac{1}{2} \frac{g}{2^2 g^3} \right) = 500 \Rightarrow$

$V_0^2 \left(\frac{1}{39.24} \; \frac{1}{7.629} \right) = 500 \Rightarrow V_0^2 = 500 \cdot 0.0127$

$\Rightarrow V^2 = 175$

$\boxed{V_0 = 198.09}$

23. Range $= \dfrac{V_0^{\,2} \cdot \sin(2\alpha)}{g}$

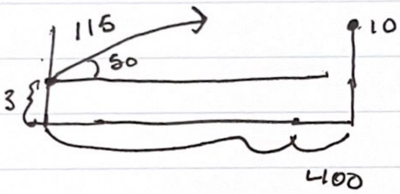
$800 = \dfrac{150^2 \cdot \sin(2\alpha)}{9.81} \Rightarrow \sin(2\alpha) = \dfrac{800 \cdot 9.81}{150^2}$

$2\theta = \arcsin\left(\dfrac{800 \cdot 9.81}{120^2}\right) \Rightarrow \theta = \dfrac{\pi\, 0.336}{2} \Rightarrow \theta \approx \cancel{30}\, 10.7^\circ$

$\sin(\pi - x) = \sin(x) \Rightarrow 2\theta_2 = \pi - 2\theta_1$

$\Rightarrow \theta_2 = \dfrac{\pi}{2} - \theta_1 \Rightarrow \dfrac{\pi}{2} - 10.7 = \theta_2 \Rightarrow \theta_2 \approx 79.3$

24.)



$r(t) = 115\cos(50)\,t + 115\sin(50)\,t - \dfrac{t^2}{2} g$

$t = \dfrac{400}{v_{0,x}} =$ $\Rightarrow 73.920\,t + 88.09\,t - t^2 \cdot 4.9$

let $400 = 115\cos(50)\,t \Rightarrow 400 = 73.920\,t$

$\Rightarrow t = 5.411$

$r(5.44) = \cancel{73.920 \cdot 5.44} + \cancel{88.09 \cdot 5.44 - (5.44)^2 \cdot 4.9}$

$\Rightarrow 3 + V_{0y} \cdot \dfrac{400}{V_{0x}} - 4.5g \cdot \left(\dfrac{400}{v_{0x}}\right)^2$

$\Rightarrow 3 + 88.1 \cdot 5.41 - 0.5 \cdot 32.2 \cdot 5.41^2$

$\Rightarrow 41.44$ ft.

Yes It does clear the fence