MTH 810

HW 3

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Sec. 10.5

8.) the line through (2,1,0) and $\perp$ $i+j$ and $j+k$

$v_1 = i+j$, $v_2 = j+k$ => $v_1 \times v_2 = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$

$= i\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} - j\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + k\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$

$= i - j + k = \langle 1, -1, 1 \rangle$

$\begin{cases} x = 2 + t \\ y = 1 - t \\ z = 0 + t \end{cases}$ Parametric eq.

$t = x - 2 = 1 - y = z$ => $z = f(x,y) = \frac{x-2}{1} = \frac{1-y}{1}$

symmetric equations.

22.) the plane through (2,0,1) and $\perp$ $x = 3t$, $y = 2-t$, $z = 3+4t$

our direction vector $\langle 3, -1, 4 \rangle$

Equation of a plane

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

=> $3(x-2) + -1(y-0) + 4(z-1) = 0$

=> $3x - 6 - y + 4z - 4 = 0$

$3x - y + 4z = 10$

30.

$5x + 4y - 2z = 0$

$\vec{PQ} = \langle -1, 5, -4 \rangle$

$\vec{n}_1 = \langle 5, 4, -2 \rangle$

Consider $\vec{a} = \vec{PQ} \times \vec{n}_1 = \begin{vmatrix} i & j & k \\ -1 & 5 & -4 \\ 5 & 4 & -2 \end{vmatrix} = i \begin{vmatrix} -9 & -5 \\ 5 & 42 \end{vmatrix}$

$- j \begin{vmatrix} -1 & -4 \\ 5 & -2 \end{vmatrix} + k \begin{vmatrix} -1 & 5 \\ 5 & 4 \end{vmatrix} = \cancel{-} i = 6i - j22 - 29k$

$= \langle 6, -22, -29 \rangle$

so to fit this plane to our ~~orginal~~ original points we use

$a(x - x_0) + b(y - x_0) + c(z - z_0) = 0$

$\Rightarrow 6(x+1) - 22(y-3) - 29(z-1) = 0$

$\Rightarrow 6x + 6 - 22x + 66 - 29z + 29 = 0$

$101 + 6x - 22x - 29z = 0$

38.) Find if planes are parallel, perpendicular or neither

$x + 2y + 2z = 1 \qquad 2x - y + 2z = 1$

$\vec{n}_1 = \langle 1, 2, 2 \rangle \qquad \vec{n}_2 = \langle 2, -1, 2 \rangle$

$\vec{n}_1 \cdot \vec{n}_2 = 2 - 2 + 4 = 4$, not perpendicular (not $= 0$)

not parralel (not $= 1$)

~~parrallel~~ $\cancel{\alpha \vec{n}_1 = \vec{n}_2 \Rightarrow \langle \alpha, 2\alpha, 2\alpha \rangle = \langle 2, -1, 2 \rangle}$

$\cos(\theta) = \dfrac{\vec{n}_1 \cdot \vec{n}_2}{\|\vec{n}_1\| \cdot \|\vec{n}_2\|} = \dfrac{4}{3 \cdot 3} = \dfrac{4}{9}$

$\Rightarrow \theta = \arccos(4/9)$

Sec 10.7

14.) Find vector and parametric equations for the line segment that joins $\overline{PQ}$

$P(-1, 2, -2)$, $Q(-3, 5, 1)$

- the vector equation can be expressed as

$r(t) = P + t(Q-P)$

$\Rightarrow r(t) = \langle -1, 2, -2 \rangle + t\langle -2, 3, 3 \rangle \quad t \in \mathbb{R}$

- Parametric equation

$x(t) = -1 \cdot -2t$

$y(t) = 2 + 3t$

$z(t) = -2 + 3t$

Find a vector fn. that represents the curve of intersection

28.) The cylinder $x^2 + y^2 = 4$ and the surface $z = xy$

let ~~...~~ $x^2 + y^2 = 4$

Circle with radius 2, center $(0,0)$

$x = 2\cos(t)$, $y = 2\sin(t)$, $r = 2$

$z = x \cdot y = 2\cos(t) \cdot 2\sin(t) = 4\sin(t)\cos(t)$

$= 2\sin(2t) \Rightarrow z = 2\sin(2t)$

$r(t) = \langle 2\cos(t), 2\sin(t), 2\sin(2t) \rangle$

40.)

rct

=>

r'c

50

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eq

40.) find the derivative of the vector function

$r(t) = \langle \tan(t), \sec(t), t^{-2} \rangle$

$\Rightarrow$ by Linearity of derivatives

$r'(t) = \langle \sec^2(t), \sec(t)\cdot\tan(t), -2t^{-3} \rangle$

50.) Find the parametric equations for the tangent line to the Curve with the given parameteric equations at the point

$$\begin{cases} x(t) = e^t \\ y(t) = te^t \\ z(t) = te^{t^2} \end{cases}$$ at the point $(1,0,0)$

$r(t) = \langle 1,0,0 \rangle \iff t=0$

$r(t) = \langle e^t, te^t, te^{t^2} \rangle$

$r'(t) = \langle e^t, e^t + te^t, e^{t^2} + 2e^{x^2}\cdot t^2 \rangle$ Direction vector

$r'(0) = \langle 1, 1+0\cdot 1, 1+0 \rangle = \langle 1, 1, 1 \rangle$

$$tl(t) = \begin{cases} x(t) = 1+t \\ y(t) = t \\ z(t) = t \end{cases}$$ This are the equations for the tangent line at $\langle 1,0,0 \rangle$