A Panel Unobserved Components Model *

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Abstract

This paper develops a Bayesian panel unobserved components model. We work with unrestricted PUC model and relying on the global-local shrinkage prior to deal with over-parameterization concerns. We extend the conventional domestic Phillips curve to a global Phillips curve. We take on the challenge of jointly estimating 34-country trend simultaneously. Our method is computationally efficient. Equation-by-equation estimation can be used. We demonstrate the merits of our model through a multi-country study involving 34 countries. We find evidence of "fragile inflation" EMEs. We also find our PUC model provide more precise estimates of trend. In a forecasting exercise, We find that our PUC model can beat all alternatives in most cases.

JEL classification: C11, C33, E31, F62
Keywords: unobserved components model, static interdependencies, dynamic interdependencies, Horseshoe prior

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1 Introduction

The concept of trend is central to macroeconomics. Multivariate unobserved components (UC) models have been shown to provide reasonable estimates of the trend. Much of the existing UC literature either focus on one country or impose independent assumption between economies in multi-country studies. But in the modern globalized economy, countries are linked together through trade and financial flows. Events in one country can spill over into others and the spillovers may influence the estimates of trend (see Canova and Ciccarelli, 2013; Taylor and Wieland, 2016). The UC model without interdependencies across countries is counter-intuitive. We use the general term interdependencies for dependencies that can take place both contemporaneously and with a lag.

Both interdependencies are allowed for in our proposed multi-country UC model: panel unobserved components model (PUC). In PUC, the dependent variables for all countries are modelled jointly in a single UC and, thus, the UC for each individual country is augmented with lagged dependent variables from other countries. The novelties of our approach are that i) we allow for interdependencies across countries, which may influence the estimates of trend; ii) we do not impose any zero restriction. This reduces the associated mis-specification risk and means that we work with unrestricted PUC model; and iii) we do so in a computationally efficient way. Equation-by-equation estimation can be used.

To explain the significance of the econometric contributions of this paper, we note a major need is to account for the panel structure in the data, and explicitly model interdependencies and commonalities across countries. We borrow the advantages from Panel Vector Autoregressions (PVARs) to fulfill this need. One advantage of PVARs (compared to the Global VARs) is that it is convenient to do forecasting and structural analysis. The PVARs literature includes two main approaches to consider the panel structure: imposing zero restrictions and relying on shrinkage priors (see Koop and Korobilis, 2019; Feldkircher et al., 2021; Bai et al., 2022). In this paper, we rely on shrinkage priors. We do not selectively model the dynamic links across countries while imposing zero restrictions on others (see Canova and Ciccarelli, 2009; Canova and Ciccarelli, 2013), because these zero restrictions, if wrongly chosen, potentially lead to mis-specification problems. It is clearly desirable to introduce restrictions in a data based fashion (see Feldkircher et al., 2021). One popular data-based approach is the stochastic search specification selection approach, developed by Koop and Korobilis (2016). Their approach produces posterior inclusion probabilities for every possible restrictions and these probabilities can be used to sort through restrictions in a data based fashion. Davidson et al. (2019) further extend the method of Koop and Korobilis (2016) to allow for a more detailed investigation of cross-country linkages. Another data-based approach is to rely on global-local shrinkage priors. Zero-restriction implies that the matrix is sparse. It is found that if a matrix is characterized by a relatively low number of non-zero elements, a possible solution is a global-local shrinkage prior (e.g., Polson and Scott, 2010; Kastner and Huber, 2020). Such advantage of global-local shrinkage prior shrinks strongly the parameter space but at the same time provides enough flexibility to allow for non-zero elements if necessary, thus imposing zero restriction for most elements but dropping the restriction if necessary. The global-local shrinkage prior has been applied to PVAR literature in Feldkircher et al. (2021).[1] In this paper, we follow their method, working with unrestricted PUC model and relying on the global-local shrinkage prior to deal with over-parameterization concerns.

[1] Leaving the model unrestricted will also make the computation cumbersome. They develop a method to speed up computation, which they call integrated rotated Gaussian approximation (IRGA). We have not applied this new method, because our model is not so huge, compared to their model.

Our paper also seeks to contribute to the empirical literature on the Phillips curve. Recent development in inflation dynamics has raised questions about whether the relationship between real economy and inflation has been altered and whether the Phillips curve is still valid. In the conventional Phillips curve, what links inflation and output together is their own country Phillips curve (see Stella and Stock, 2013; Chan et al., 2016; Kabundi et al., 2021 and Wu, 2021). One explanation of the muted response of domestic inflation is that inflation is determined globally. For instance, Forbes (2019) demonstrate empirically, using a Phillips curve with a set of global variables, that globalisation plays an increasingly more important role in explaining inflation dynamics. Kabundi et al. (2021) add global output to explain inflation dynamics. However, there are many linkages and inter-relationships between the economies. Modelling aggregate output for the world as a whole will miss many interesting country-specific patterns since different countries have differing trade exposure to different countries. These considerations justify why we want to extend the conventional Phillips curve to a global Phillips curve. The global Phillips curve is allowed through the panel structure in PUC model at a low cost. The equation-by-equation estimation can still be used.

A second empirical contribution lies in the estimates of trend. Precise estimates are important because they are thought to reflect the fundamental structure of the economy in the absence of shocks (see Zaman, 2021). Studies have guided important empirical features to get precise estimates. Canova (2011) point out that the model without dynamic interdependencies is suitable for estimates of trend[2] in a small open economy. But this paper includes 34 countries (23 advanced economies and 11 emerging market economies). The small open economy assumption is no longer satisfied and the spillovers may influence the estimates of trend (see Canova and Ciccarelli, 2013). Taylor and Wieland (2016) also emphasize that omitting variables can affect the reliability of the estimates of trend. Therefore, in this paper, we take on the challenge of jointly estimating 34-country trend simultaneously.

In our empirical work, we consider 23 advanced economies (AEs) and 11 emerging market economies (EMEs). We first provide evidence of interdependencies from impulse response analysis, our PUC model allows us to consider the effects of both variable-specific shocks and, more importantly, global shocks. We use the Generalized Impulse Response Functions (GIRFs), proposed in Koop et al. (1996). We find whatever the shock is, the GIRFs of EMEs inflation go back to zero more slowly than AEs, which implies that a shock has on average a longer-term effect on EMEs inflation. We call this "fragile inflation" EMEs.

We then proceed by comparing the performance of our PUC model to a range of alternatives including a multi-country UC model with factor stochastic volatility (FSV), and single country UC model. In terms of empirical results, we find our PUC model provide more precise estimates of trend. The precision of the estimates, as measured by the width of the 84% credible intervals. More specifically, we find that allowing for contemporaneous cross-country interdependencies can provide more precise estimates of trend, while omitting cross-country interdependencies that occur with a lag will overestimate trend output.

In the forecasting exercise, we divide our out-of-sample forecasting results into three parts: forecasting inflation, forecasting output and jointly forecasting inflation and output. We find that our PUC model

[2]Precisely, Canova (2011) use the term "steady state ". We use the term "trend ". There are subtle differences between them, but they can be interpreted as the same for the purpose of this paper.

can beat all alternatives in most cases. The only exception is to forecast 1-quarter-ahead inflation.

This paper is organized as follows. In Section 2, we start from the multi-country unobserved components model that allows for contemporaneous cross-country interdependencies, then introduce the panel unobserved components model with factor stochastic volatility, which allows for dependencies that can take place both contemporaneously and with a lag, both across variables within a country and across countries. After introducing the new model, we describe the global-local shrinkage priors on parameters. In Section 3, we describe the 34-country data and provide evidence of interdependencies. After showing the existence of interdependencies, we show the importance of interdependencies in the following two sections. In Section 4, we present the estimates of trend. We find that allowing for contemporaneous cross-country interdependencies can provide more precise estimates of trend, while omitting cross-country interdependencies that occur with a lag will overestimate trend output. In Section 5, we show that PUC-FSV model can provide higher in-sample fit and more accurate density forecasts compared to existing models in the literature. Finally, Section 6 concludes.

2 A Panel Unobserved Components model with Factor Stochastic Volatility

In this section, we develop the panel unobserved components model with factor stochastic volatility. We do not impose any zero restrictions. To deal with the over-parameterization concerns, we rely on a global-local shrinkage prior (the Horseshoe prior). The priors are described after introducing the PUC-FSV model.

2.1 PUC-FSV Model Specification

We begin with the multi-country unobserved component (UC-FSV) model developed in Wu (2021). They allow for contemporaneous interdependencies across countries through factor stochastic volatility structure. In particular, for country i, $i = 1, \dots, N$, $\pi_{i,t}$ is the inflation of country i at time t and $y_{i,t}$ is the output of country i, $\tau^{\pi}_{i,t}$ and $\tau^{y}_{i,t}$ are their trends. The UC-FSV model for N-country inflation and output is defined as:

$$
\begin{aligned}
\pi_t - \tau^{\pi}_t &= P(\pi_{t-1} - \tau^{\pi}_{t-1}) + A(y_t - \tau^{y}_t) + L_\pi f_t + u^{\pi}_t,\ f_t \sim \mathcal{N}(0,\ \Omega^{\pi}_t),\ u^{\pi}_t \sim \mathcal{N}(0,\ \Sigma^{\pi}_t)\\
y_t - \tau^{y}_t &= \Phi(y_{t-1} - \tau^{y}_{t-1}) + \Theta(y_{t-2} - \tau^{y}_{t-2}) + L_y g_t + u^{y}_t,\ g_t \sim \mathcal{N}(0,\ \Omega^{y}_t),\ u^{y}_t \sim \mathcal{N}(0,\ \Sigma^{y}_t)\\
\tau^{\pi}_{i,t} &= \tau^{\pi}_{i,t-1} + \epsilon^{\tau\pi}_{i,t},\quad \epsilon^{\tau\pi}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{\tau\pi}),\ i = 1, \dots, N\\
\tau^{y}_{i,t} &= \tau^{y}_{i,t-1} + \epsilon^{\tau y}_{i,t},\quad \epsilon^{\tau y}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{\tau y})\\
h_{j,t} &= h_{j,0} + \omega^h_j \tilde{h}_{j,t}\\
\tilde{h}_{j,t} &= \tilde{h}_{j,t-1} + \epsilon^h_{j,t},\quad \epsilon^h_{j,t} \sim \mathcal{N}(0,\ 1),\ j = 1, \dots, 2N + r_\pi + r_y
\end{aligned}
\tag{1}
$$

where $\pi_t = (\pi_{1,t}, \dots, \pi_{N,t})'$ is an $N \times 1$ vector, $\tau^{\pi}_t = (\tau^{\pi}_{1,t}, \dots, \tau^{\pi}_{N,t})'$ is an $N \times 1$ vector, $P = \text{diag}(\rho_1, \dots, \rho_N)$ is an $N \times N$ matrix, $A = \text{diag}(\alpha_1, \dots, \alpha_N)$ is an $N \times N$ matrix, $y_t = (y_{1,t}, \dots, y_{N,t})'$ is an $N \times 1$ vector, $\tau^{y}_t = (\tau^{y}_{1,t}, \dots, \tau^{y}_{N,t})'$ is an $N \times 1$ vector, L_π is an $N \times r_\pi$ matrix, f_t is a $r_\pi \times 1$ vector, u^{π}_t is an $N \times 1$ vector, $\Phi = \text{diag}(\phi_1, \dots, \phi_N)$ is an $N \times N$ matrix, $\Theta = \text{diag}(\theta_1, \dots, \theta_N)$ is an $N \times N$ matrix, L_y is an $N \times r_y$ matrix, g_t is a $r_y \times 1$ vector, u^{y}_t is an $N \times 1$ vector. $\Sigma^{\pi}_t = \text{diag}(e^{h_{1,t}}, \dots, e^{h_{N,t}})$, $\Sigma^{y}_t = \text{diag}(e^{h_{N+1,t}}, \dots, e^{h_{2N,t}})$, $\Omega^{\pi}_t = \text{diag}(e^{h_{2N+1,t}}, \dots, e^{h_{2N+r_\pi,t}})$, and

$\Omega_t^y = \text{diag}(e^{h_{2N+r_\pi+1,t}}, \ldots, e^{h_{2N+r_\pi+r_y,t}})$.

The assumption that the errors are driven by latent factors (f_t and g_t) allows for contemporaneous interdependencies across countries. However, Wu (2021) assume the coefficient matrices P, A, Φ and Θ are diagonal. It is with this assumption that we part with them. One would expect that country i variables depend on other countries' variables, either contemporaneously or with a lag. Therefore, we relax this diagonal assumption to allow for a more comprehensive investigation of dependencies. Specifically, we assume that the coefficient matrices P, A, Φ and Θ are full matrices:

$$\begin{aligned}\pi_{i,t} - \tau_{i,t}^\pi =& \rho_{i,1}(\pi_{1,t-1} - \tau_{1,t-1}^\pi) + \rho_{i,2}(\pi_{2,t-1} - \tau_{2,t-1}^\pi) + \cdots + \rho_{i,N}(\pi_{N,t-1} - \tau_{N,t-1}^\pi) \\ &+ \alpha_{i,1}(y_{1,t} - \tau_{1,t}^y) + \alpha_{i,2}(y_{2,t} - \tau_{2,t}^y) + \cdots + \alpha_{i,N}(y_{N,t} - \tau_{N,t}^y) \\ &+ L_{i,\pi} f_t + u_{i,t}^\pi\end{aligned} \tag{2}$$

$$\begin{aligned}y_{i,t} - \tau_{i,t}^y =& \phi_{i,1}(y_{1,t-1} - \tau_{1,t-1}^y) + \phi_{i,2}(y_{2,t-1} - \tau_{2,t-1}^y) + \cdots + \phi_{i,N}(y_{N,t-1} - \tau_{N,t-1}^y) \\ &+ \theta_{i,1}(y_{1,t-2} - \tau_{1,t-2}^y) + \theta_{i,2}(y_{2,t-2} - \tau_{2,t-2}^y) + \cdots + \theta_{i,N}(y_{N,t-2} - \tau_{N,t-2}^y) \\ &+ L_{i,y} g_t + u_{i,t}^y\end{aligned} \tag{3}$$

where $\rho_{i,j}$ for $i, j = 1, \ldots, N$ represents the affect of country j inflation gap on country i inflation. Similarly, $\alpha_{i,j}$ for $i, j = 1, \ldots, N$ represents the affect of country j output gap on country i inflation. $\phi_{i,j}$ and $\theta_{i,j}$ represent the affect of country j output gap on country i output. Equation (2)-(3) specify the model for country i.

Written in matrix, we can obtain the multi-country PUC-FSV model specification:

$$\begin{aligned}\pi_t - \tau_t^\pi &= P(\pi_{t-1} - \tau_{t-1}^\pi) + A(y_t - \tau_t^y) + L_\pi f_t + u_t^\pi,\ f_t \sim \mathcal{N}(0,\ \Omega_t^\pi),\ u_t^\pi \sim \mathcal{N}(0,\ \Sigma_t^\pi) \\ y_t - \tau_t^y &= \Phi(y_{t-1} - \tau_{t-1}^y) + \Theta(y_{t-2} - \tau_{t-2}^y) + L_y g_t + u_t^y,\ g_t \sim \mathcal{N}(0,\ \Omega_t^y),\ u_t^y \sim \mathcal{N}(0,\ \Sigma_t^y) \\ \tau_{i,t}^\pi &= \tau_{i,t-1}^\pi + \epsilon_{i,t}^{\tau\pi}, \quad \epsilon_{i,t}^{\tau\pi} \sim \mathcal{N}(0,\ \sigma_{\tau\pi}^2),\ i = 1, \ldots, N \\ \tau_{i,t}^y &= \tau_{i,t-1}^y + \epsilon_{i,t}^{\tau y}, \quad \epsilon_{i,t}^{\tau y} \sim \mathcal{N}(0,\ \sigma_{\tau y}^2) \\ h_{j,t} &= h_{j,0} + \omega_j^h \tilde{h}_{j,t} \\ \tilde{h}_{j,t} &= \tilde{h}_{j,t-1} + \epsilon_{j,t}^h, \quad \epsilon_{j,t}^h \sim \mathcal{N}(0,\ 1),\ j = 1, \ldots, 2N + r_\pi + r_y\end{aligned} \tag{4}$$

We refer to this specification as Panel Unobserved Components model with Factor Stochastic Volatility (PUC-FSV). Our PUC-FSV has three important features.

First, full matrices (P, Φ and Θ) allow for cross-country interdependencies that take place with a lag. It is this feature which allows for what are called dynamic interdependencies (DIs) (see, Canova and Ciccarelli, 2013 and Davidson et al., 2019). More specifically, a full matrix P allows that country i inflation depends on the first lag of other countries' inflation gap. For instance, if US inflation gap last quarter has an affect on UK this quarter, then we say there is an inflation DI from the US to UK. The magnitude of inflation DI is measured by the off-diagonal elements of matrix P. If the corresponding coefficient is (or close to) zero, then there is no inflation DI from the US to UK. Full matrices Φ and Θ allow that country i output depends on the first and second lag of other countries' output gap. For instance, if US output gap last two quarters have an affect on UK output this quarter, then we say there is an output DI from the US to UK. The magnitude of output DI is measured by the off-diagonal elements of matrices Φ and Θ. The magnitude measured by matrix Φ represents a faster transmission between two countries and we call this as output DI_{fast}. The magnitude measured by matrix Θ represents a slower transmission between two countries and we call this as output DI_{slow}. If the corresponding coefficients are (or close to) zero, then there is no output DI from the US to UK.

Second, full matrix A and factor stochastic volatility allow for contemporaneous dependencies. It is this feature which allows for what are called static interdependencies (SIs). More specifically, full matrix A allows that country i inflation depends on contemporaneous output gap, including own country output gaps and other countries' output gap. This is the second new feature of PUC-FSV. For instance, if US output gap this quarter has an affect on UK inflation this quarter, then we say there is a Phillips SI from the US to UK. The magnitude of Phillips SI is measured by the off-diagonal elements of matrix A. If the corresponding coefficient is (or close to) zero, then there is no Phillips SI from the US to UK. Factor stochastic volatility assumes that all countries' errors in the inflation gap equation are driven by common factors f_t and all countries' errors in the output gap equation are driven by g_t. And we call such SIs as inflation error SIs and output error SIs. The magnitude of two error SIs is measured by factor loading matrices L_π and L_y.

Third, we do not impose any zero restriction on the coefficient matrices, which means that we work with the unrestricted PUC-FSV. Leaving panel model unrestricted can lead to enormous parameters to estimate. And to deal with over-parameterization concerns, we rely on a global-local shrinkage prior. The Horseshoe prior is a global-local shrinkage prior and empirically successful. Next, we describe the Horseshoe prior and the priors for all other parameters.

2.2 Priors

We first introduce the prior for full matrices P, A, Φ and Θ. Then we introduce the prior for other parameters.

The prior for full matrices is the Horseshoe prior. The Horseshoe prior was proposed by Carvalho et al. (2010). It involves a global shrinkage parameter (τ) and a local shrinkage parameter (λ). Carvalho et al. (2010) have shown that the induced distributions over the global and local shrinkage parameters allow for optimal rates of shrinkage near zero, while having sufficiently thick tails (see Cross et al. (2020), 2020).

More specifically, we use the inverse-Gamma representation of Horseshoe prior for elements in the full matrix P, A, Φ and Θ. We assume that the global shrinkage parameter (τ) is specified to differ across types of parameters, that is, each full matrix has two global shrinkage parameters: one for own country coefficients and one for other country coefficients. For instance, suppose that $\rho_{i,m}$ is an element in P, if $i = m$, then $\rho_{i,i}$ is own country coefficient, and if $i \neq m$, then $\rho_{i,m}$ is other country coefficient, then the Horseshoe prior for own country coefficient $\rho_{i,i}$ is:

$$\begin{aligned}
&\rho_{i,i} \mid \lambda^{\rho}_{i,i}, \tau^{\rho,d} \sim \mathcal{N}(0,\ \lambda^{\rho}_{i,i}\tau^{\rho,d}),\ i = 1, \ldots, N \\
&\lambda^{\rho}_{i,i} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu^{\rho}_{i,i}}),\quad \tau^{\rho,d} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{\rho,d}}) \\
&\nu^{\rho}_{i,i} \sim \mathcal{IG}(\frac{1}{2},\ 1),\qquad \xi^{\rho,d} \sim \mathcal{IG}(\frac{1}{2},\ 1)
\end{aligned} \tag{5}$$

the Horseshoe prior for other country coefficient $\rho_{i,m}$ is:

$$\begin{aligned} &\rho_{i,m} \mid \lambda^{\rho}_{i,m}, \tau^{\rho,nd} \sim \mathcal{N}(0,\ \lambda^{\rho}_{i,m}\tau^{\rho,nd}),\ i=1,\dots,N,\ m=1,\dots,N,\ i\neq m \\ &\lambda^{\rho}_{i,m} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu^{\rho}_{i,m}}),\quad \tau^{\rho,nd} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{\rho,nd}}) \\ &\nu^{\rho}_{i,m} \sim \mathcal{IG}(\frac{1}{2},\ 1),\qquad \xi^{\rho,nd} \sim \mathcal{IG}(\frac{1}{2},\ 1) \end{aligned} \tag{6}$$

To ensure stationarity, we impose condition on P, A, Φ and Θ. Specifically, we first rewrite the first two equations in Equation (4) as a VAR(1):

$$\begin{pmatrix} \pi_t-\tau^{\pi}_t \\ y_{t+1}-\tau^{y}_{t+1} \\ y_t-\tau^{y}_t \\ y_{t-1}-\tau^{y}_{t-1} \end{pmatrix} = \begin{pmatrix} P & A & 0 & 0 \\ 0 & \Phi & \Theta & 0 \\ 0 & I_N & 0 & 0 \\ 0 & 0 & I_N & 0 \end{pmatrix} \begin{pmatrix} \pi_{t-1}-\tau^{\pi}_{t-1} \\ y_t-\tau^{y}_t \\ y_{t-1}-\tau^{y}_{t-1} \\ y_{t-2}-\tau^{y}_{t-2} \end{pmatrix} + \begin{pmatrix} L_{\pi}f_t+u^{\pi}_t \\ L_y g_{t+1}+u^{y}_{t+1} \\ 0 \\ 0 \end{pmatrix} \tag{7}$$

Then we obtain the VAR(1) representation. The stability condition requires that all the eigenvalues of coefficient matrix are smaller than one in modulus. And we use the command "eig" in Matlab to compute eigenvalues. The diagonal elements in A is the slope of Phillips curve, so we also constrain them to be positive.

The priors on other parameters are the same as that in Wu (2021). More specifically, we model the evolution of the log-volatility according to a random walk in non-centered parameterization and then use the Horseshoe prior to control time-variation. For each $j=1,\dots,2N+r_{\pi}+r_y$, the evolution of the log-volatility is modeled as:

$$\begin{aligned} h_{j,t} &= h_{j,0}+\omega^h_j\tilde{h}_{j,t} \\ \tilde{h}_{j,t} &= \tilde{h}_{j,t-1}++\epsilon^h_{j,t},\quad \epsilon^h_{j,t}\sim\mathcal{N}(0,\ 1) \end{aligned} \tag{8}$$

The non-centered parameterization decomposes a time-varying parameter $h_{j,t}$ into two parts: a time-invariant part $h_{j,0}$ and a time-varying part $\omega^h_j\tilde{h}_{j,t}$, which has a constant coefficient ω^h_j that controls the time-variation. For constant parameters ω^h_j and $h_{j,0}$, we use the Horseshoe prior:

$$\begin{aligned} &\omega^h_j \mid \lambda^{\omega^h}_j, \tau^{\omega^h} \sim \mathcal{N}(0,\ \lambda^{\omega^h}_j\tau^{\omega^h}),\ j=1,\dots,2N+r_{\pi}+r_y \\ &\lambda^{\omega^h}_j \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu^{\omega^h}_j}),\quad \tau^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{\omega^h}}) \\ &\nu^{\omega^h}_j \sim \mathcal{IG}(\frac{1}{2},\ 1),\qquad \xi^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ 1) \end{aligned} \tag{9}$$

$$\begin{aligned} &h_{j,0} \mid \lambda^{h_0}_j, \tau^{h_0} \sim \mathcal{N}(0,\ \lambda^{h_0}_j\tau^{h_0}),\ j=1,\dots,2N+r_{\pi}+r_y \\ &\lambda^{h_0}_j \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu^{h_0}_j}),\quad \tau^{h_0} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{h_0}}) \\ &\nu^{h_0}_j \sim \mathcal{IG}(\frac{1}{2},\ 1),\qquad \xi^{h_0} \sim \mathcal{IG}(\frac{1}{2},\ 1) \end{aligned} \tag{10}$$

The initial states are assumed to follow normal distribution with zero mean and variance ten, that is:

$$\tau^{\pi}_{i,1} \sim \mathcal{N}(0,\ 10),\ i = 1, \dots, N \tag{11}$$

$$\tau^{y}_{i,1} \sim \mathcal{N}(0,\ 10),\ i = 1, \dots, N \tag{12}$$

$$\tilde{h}_{j,1} \sim \mathcal{N}(0,\ 10),\ j = 1, \dots, 2N + r_\pi + r_y \tag{13}$$

The elements in factor loading matrices are assumed to follow a normal distribution with zero mean and variance ten, that is:

$$l_m \sim \mathcal{N}(0,\ 10),\ m = 1, \dots, n_{l,\pi} + n_{l,y} \tag{14}$$

where $n_{l,\pi}$ denotes the number of free elements in matrix L_π, and $n_{l,y}$ denotes the number of free elements in matrix L_y.

The error variances are assumed to follow inverse gamma distribution, that is:

$$\sigma^2_{\tau\pi} \sim \mathcal{IG}(10,\ 0.18),\ i = 1, \dots, N \tag{15}$$

$$\sigma^2_{\tau y} \sim \mathcal{IG}(10,\ 0.09),\ i = 1, \dots, N \tag{16}$$

We use the Markov Chain Monte Carlo (MCMC) algorithm to sample all parameters. More specifically, to sample $\tau^{\pi}_{i,t}$, the prior still follows a random walk process, but the likelihood will come from N equations and each equation is defined through Equation (2). To sample $\tau^{y}_{i,t}$, the prior still follows a random walk process, but the likelihood will come from two parts: the first part is N equations in Equation (2), the second part is N equations in Equation (3). It is standard to sample other parameters and we refer readers to Chan et al. (2016) and Chan (2021) for details.

3 Data and Evidence of Dependencies

In this section, we first introduce the data, then we provide evidence of interdependencies in two ways. The first way is through analyzing the matrices that measure dependencies. The second way is through impulse response analysis.

3.1 Data

The data are the quarterly consumer price index (CPI) and the quarterly real gross domestic product (GDP) for 34 countries, 23 advanced economies (AEs)[3] and 11 emerging market economies (EMEs)[4]. They span the period from 1995Q1 to 2018Q1. We transform the data to annualized growth rates as: $400\log(z_t/z_{t-1})$. And because the output gap equation follows an AR(2) process, our estimation start from 1995Q4. We assume that there is one common factor driving 34-country inflation, that is $r_\pi = 1$. We assume that there is one common factor driving 34-country output, that is $r_y = 1$. This assumption comes from the empirical results in Wu (2021). They find there is one global factor driving 34-country inflation and one global factor driving 34-country output. Posterior results are based on 100000 draws after a burn-in period of 20000.

[3]Australia, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Israel, Italy, Latvia, Lithuania, Netherlands, Portugal, Slovakia, South Korea, Spain, Sweden, Switzerland, UK, USA.

[4]Bolivia, Brazil, China, Hungary, Indonesia, Mexico, Philippines, Russia, South Africa, Thailand, Turkey.

3.2 Evidence of Interdependencies

In this section, we present evidence of interdependencies across countries in two different ways. The first way analyzes matrices that measure interdependencies, discussing estimates of coefficients, which countries support interdependencies and whether these interdependencies occur contemporaneously or with a lag. The estimates show that the Horseshoe prior can achieve sensible shrinkage. Events in both AEs and EMEs can spill over into AEs, contemporaneously and with a lag. By contrast, there are only several spillovers into EMEs. We find more evidence of static interdependencies (SI) than dynamic interdependencies (DI). The reason may be that we are using quarterly data.

Then we move to impulse response analysis, discussing how a shock affects the 34 countries analyzed. We do so by computing the generalised impulse response functions (GIRFs) for each shock. We find the GIRFs are much larger when a global shock hits the system than when a US shock hits the system. The GIRFs also show that whatever the shock is, the GIRFs of EMEs inflation go back to zero more slowly than AEs. We call this "fragile inflation"EMEs.

We want to emphasize that the above features are supported by data, and it is impossible to know all of them before estimating the model. They show the power of working with unrestricted model and selecting the appropriate restrictions in a data based manner.

3.2.1 Estimates of coefficients

For each of 34 countries, we have two blocks of interdependencies: one takes place contemporaneously (Static Interdependency, SI), the other takes place with a lag (Dynamic Interdependency, DI). Within each block, we have three matrices measuring the corresponding affect. Matrices A, L_π and L_y measure SIs, while matrice P, Θ, Φ measure DIs. We thus have six matrices summarizing the affects corresponding to the SIs (Figure 1 and Table 5) and the DIs (Figure 2-4). The table presents the estimates of factor loading matrices L_π and L_y. The figures plot the four full matrices A, P, Θ and Φ. The Y axis in each figure is the left hand in the equation (see Equation (4)). The X axis is the corresponding right hand in the equation. For instance, matrix P is the coefficient matrix for lag of inflation gap in the inflation gap equations. We plot the estimates of matrix P in Figure 2. Then, the Y axis in Figure 2 is the 34-country inflation gap this quarter ($\pi_t - \tau_t^\pi$), and the X axis in Figure 2 is the 34-country inflation gap last quarter ($\pi_{t-1} - \tau_{t-1}^\pi$).

The general pattern is that our modelling strategy induces a high degree of sparsity, since most elements in full matrices (A, P, Θ and Φ) are close to zero. More importantly, the induced shrinkage is sensible, as evidenced by that own country information are treated more important than other countries' information. For instance, the diagonal elements of P are larger than the off-diagonal elements (see Figure 2), which means that our model believes own country inflation lag is more important than other countries' inflation lag. Similar pattern is observed from Phillips SIs (matrix A in Figure 1), output DI$_{fast}$ (matrix Φ in Figure 3) and output DI$_{slow}$ (matrix Θ in Figure 4).

We observe evidence of interdependencies. First, we consider 23 AEs and 11 EMEs. We find events in both AEs and EMEs can spill over into AEs, contemporaneously and with a lag. By contrast, there are only several spillovers into EMEs. Second, we allow for both static interdependencies and dynamic interdependencies. We find more evidence of SIs than DIs. We observe considerable evidence of SIs from

Figure 1 and Table 5. They provide evidence that some countries' information is treated as important as own country information, and there is a common factor that drives all countries' inflation (output). However, most off-diagonal elements, in DI figures (Figure 2-4), are close to zero. Several non-zero off-diagonal elements seem to support that DIs occur within AEs and within EMEs. We describe the details of SIs and DIs below.

The first evidence of SIs comes from matrix A in Figure 1. Matrix A is the coefficient matrix of output gap in the inflation gap equations. It links all countries' inflation gap and output gap together and measures the Phillips SIs. The diagonal element is the slope of own country Phillips curve. Except the diagonal elements, we find some off-diagonal elements have a value that is quite comparable to the diagonal elements. This implies that our model deems these information (from other countries) as important as own country information. "Belgium" row to "Canada" row show the spillovers into AEs. We find events in both AEs and EMEs can spill over into AEs. However, in terms of spillovers into EMEs (see "South Africa" row to "Thailand" row), events in EMEs can spill over into EMEs. There is only one small degree of positive spillover from AEs into EMEs, from USA into China. Except this, we do not see other evident spillovers from AEs into EMEs. The second evidence of SIs comes from factor loading matrices L_π and L_y (see Table 5). We find the posterior means of factor loading matrices are quite large and 84% credible intervals do not include zero, supporting that 34-country inflation are driven by a common factor f_t and that 34-country output are driven by a common factor g_t.

With respect to DIs, we have three matrices (P, Θ and Φ) measuring DIs and we find the three matrices have different characteristics. Figure 2 plots the estimates of matrix P. Matrix P is associated with the lag of inflation gap and measures inflation DI. First, we find the diagonal cells for EMEs are darker green than the diagonal cells for AEs, which implies that EMEs exhibit a higher inflation gap persistence, while AEs exhibit a lower inflation gap persistence. In other words, inflation process in AEs is no longer adaptive (see, Cogley and Sargent, 2005; Stock and Watson, 2007 and Chan et al., 2016). While expectation formation in EMEs is more adaptive. Second, several non-diagonal elements are green, describing inflation DIs that occur with a lag. Most spillovers are positive (green cells) except that events in France have a small negative effect on inflation in Turkey (red cell).

Figure 3-4 plot the output DIs, which are measured by matrices Φ and Θ. The two matrices are the coefficients in output gap equation. Figure 3 the estimates of matrix Φ, which is associated with the first lag of output gap and measures output DI_{fast}. We notice that the output DI_{fast} occurs within AEs and within EMEs, while we do not observe evident output DI_{fast} between AEs and EMEs.

Figure 4 plots Θ, which is associated with the second lag of output gap and measures output DI_{slow}. Non-diagonal elements also describe cross-country spillovers that occur with a lag, but it has a different pattern from Φ. "Belgium" row to "Canada" row show the spillovers into AEs. We find events in both AEs and EMEs can spill over into AEs. However, in terms of spillovers into EMEs (see "South Africa" row to "Thailand" row), there is only one small degree of negative spillover into EMEs, from Switzerland into Thailand. Except this, we do not see other evident spillovers into EMEs.

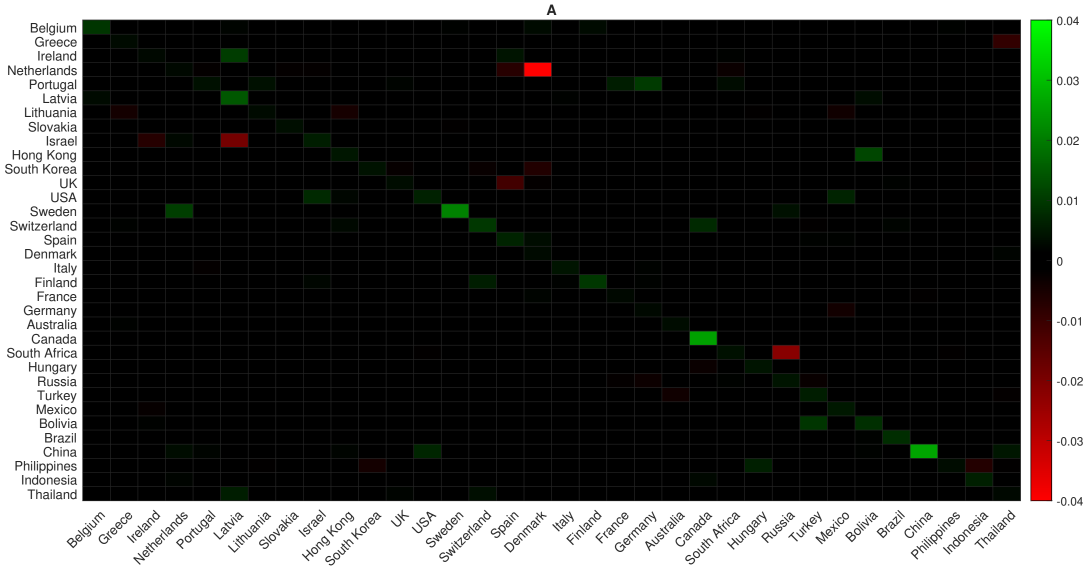

Figure 1: Heatmap of posterior means for the coefficients of output gap in inflation equations. The Y axis is the left hand in the equation. The X axis is the right hand in the equation. Then the Y axis denotes the 34-country inflation equations (this quarter), that is, $\pi_t - \tau_t^\pi$. The X axis denotes the 34-country output gap (this quarter), that is, $y_t - \tau_t^y$.

3.2.2 Impulse Response Analysis

The preceding discussion suggests the existence of interdependencies. An excellent aspect of panel model is that, after allowing for interdependencies, it can model the manner in which shocks are transmitted across countries (see Dees et al., 2007 and Canova and Ciccarelli, 2009). We will show that PUC-FSV can consider the effects of both variable-specific shocks and, more importantly, global shocks. With regard to global shocks, as pointed out by Dees et al. (2007), it is possible to view US shock as global shock in the case of a US equity market shock, but it might be less defensible for other types of shocks. Therefore, it might be desirable to consider the effects of global shocks which might not necessarily originate from a particular country, but could be common to the world economy as a whole. Examples of such shocks include major developments in technology. By using the factor stochastic volatility (FSV) specification, the proposed PUC-FSV directly allows us to consider the effects of global shocks.

Since PUC-FSV allows for dynamic dependencies, static dependencies and time variation in the parameters (e.g., stochastic volatility), we are interested in computing the responses of the endogenous variables to shocks in the variables/global factors and in describing their evolution over time. In this situation, we use the Generalized Impulse Response Functions (GIRFs), proposed in Koop et al. (1996).

With 68 endogenous variables (34 countries and 2 variables of each country) and time-varying parameters, there will be a different set of generalised impulse response functions (GIRFs) at each time in the sample period. However, for our study, we focus on the GIRFs for the end of sample period (2018Q1). we investigate the implications of four different shocks: (a) a 1-standard-deviation positive shock to US inflation; (b) a 1-standard-deviation positive shock to US output; (c) a 1-standard-deviation positive shock to global inflation; and (d) a 1-standard-deviation positive shock to global output.

More specifically, the computation of generalised impulse response function in Koop et al. (1996) is:

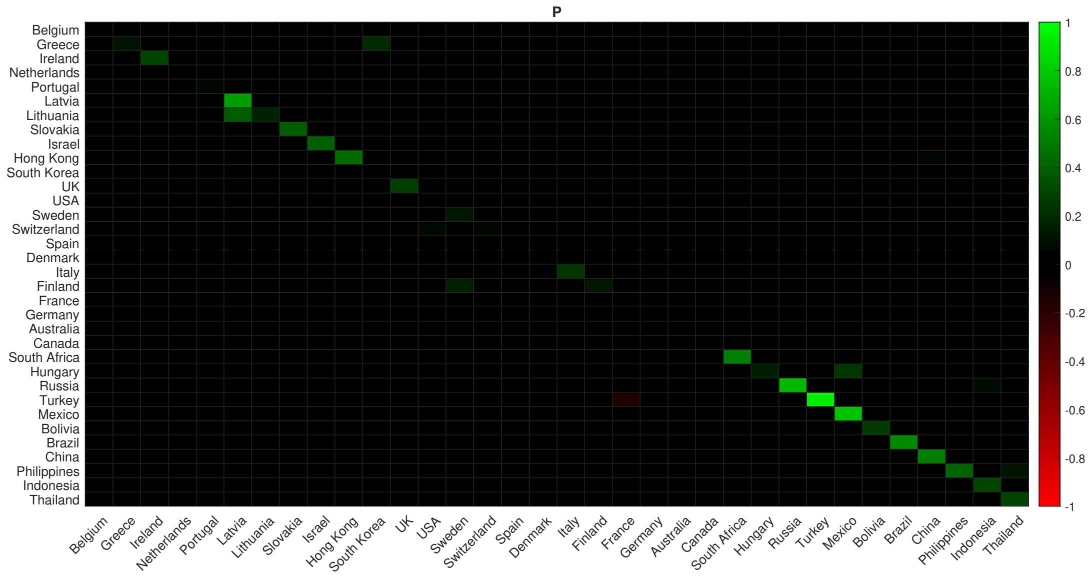

Figure 2: Heatmap of posterior means for the coefficients of inflation gap lag. The Y axis is the left hand in the equation. The X axis is the right hand in the equation. Then the Y axis denotes the 34-country inflation equations (this quarter), that is, $\pi_t - \tau_t^{\pi}$. The X axis denotes the 34-country inflation gap (last quarter), that is, $\pi_{t-1} - \tau_{t-1}^{\pi}$.

given the posterior draws, the GIRF is obtained from the difference between two alternative paths: in one a shock hits the system, and in the other this shock is absent:

$$GIRF_{t+k} = \mathbb{E}[\mathbf{z}_{t+k}|\mathbf{u}_t, \mathbf{I}_t] - \mathbb{E}[\mathbf{z}_{t+k}|\mathbf{I}_t] \tag{17}$$

where $\mathbf{z}_{t+k}$ is the forecast of the endogenous variables at the horizon k, $\mathbf{I}_t$ represent the current information set and $\mathbf{u}_t$ is the current structural disturbance terms. The computation of the generalised impulse response functions for a horizon k can be summarised in 4 steps:

Step1 : We first draw parameters from the posterior distributions within the Gibbs Sampler.

Step2 : We rewrite the errors in Equation (4). Suppose u_t is a vector and each element in u_t follows standard normal distribution $\mathcal{N}(0,\ 1)$, then:
$u_t^{\pi} = \Sigma_t^{\pi} u_t$ where $\Sigma_t^{\pi} = \text{diag}(e^{h_{1,t}}, \ldots, e^{h_{N,t}})$,
$u_t^{y} = \Sigma_t^{y} u_t$ where $\Sigma_t^{y} = \text{diag}(e^{h_{N+1,t}}, \ldots, e^{h_{N+N,t}})$,
$f_t = \Omega_t^{\pi} u_t$ where $\Omega_t^{\pi} = \text{diag}(e^{h_{2N+1,t}}, \ldots, e^{h_{2N+r_\pi,t}})$,
$g_t = \Omega_t^{y} u_t$ where $\Omega_t^{y} = \text{diag}(e^{h_{2N+r_\pi+1,t}}, \ldots, e^{h_{2N+r_\pi+r_y,t}})$.

Step3 : We draw u_t from the standard normal distribution, then generate two paths: one with the shock and the other without shock. For the latter case, we just compute the errors in *Step2* using u_t and then stochastically simulate a random path of length k using the coefficients drawn from *Step1*. For the former case, we set $u_{i,t}$ to the corresponding shock that we are interested in. For example, say we are interested in USA inflation shock (we consider 34 countries and the order for USA is 13), then the disturbance term will be $\tilde{u}_t = (u_{1,t}, u_{2,t}, \ldots, u_{13,t} + 1, \ldots, u_{N,t})'$. Thus, we compute the errors in *Step2* using $\tilde{u}_t$ and then stochastically simulate another random path of length k. If we are interested in global inflation shock (we consider one global factor), then the disturbance term will be $\tilde{u}_t = u_t + 1$.

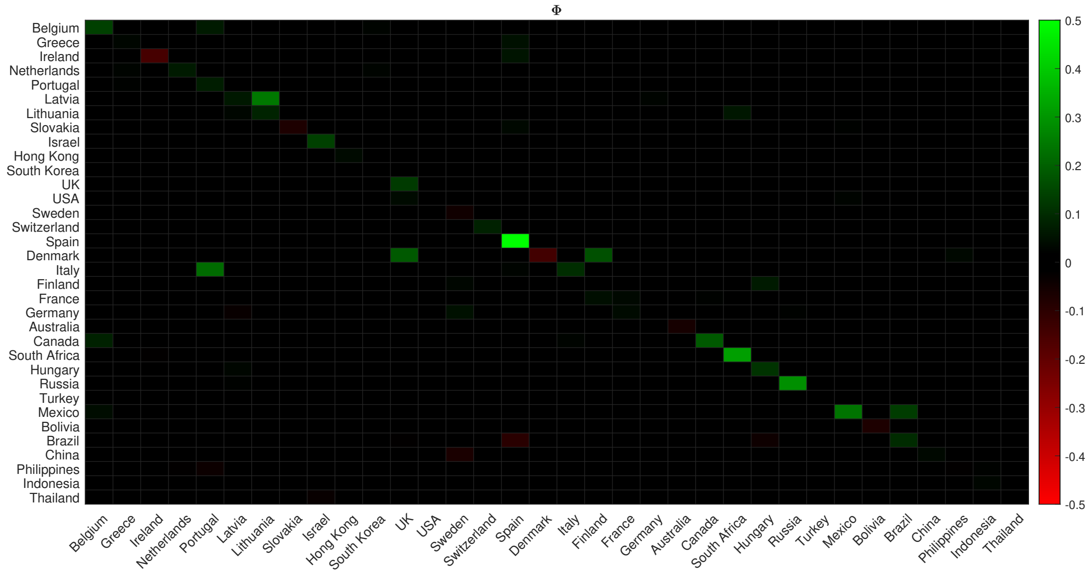

Figure 3: Heatmap of posterior means for the coefficients of output gap lag. The Y axis is the left hand in the equation. The X axis is the right hand in the equation. Then the Y axis denotes the 34 output equations (this quarter), that is, $y_t - \tau_t^y$. The X axis denotes the first lag of 34-country output gap (last quarter), that is, $y_{t-1} - \tau_{t-1}^y$.

Step4 :To compute the impulse response function, we take difference between the two paths.

We report the generalised impulse response functions in Figure 5-10. Figure 5 is about the US inflation shock, Figure 6-7 are about the US output shock, Figure 8 is about global inflation shock and Figures 9-10 are about global output shock. A quick visual inspection shows that the GIRFs go back to zero over the next 20 quarters and most GIRFs settle down quickly, possibly resulting from the stability condition we impose on the coefficient matrices (see Equation (7)).

We observe two notable differences. The first difference is between US shocks and global shocks. Figure 5-7 are about US shocks, while Figure 8-10 are about global shocks. The GIRFs are much larger when a global shock hits the system than when a US shock hits the system. We think this is because almost all countries load on the global factors and the loadings are quite large (see Appendix B). This provides evidence that it is important to allow for SIs (static interdependencies) as is done in this paper, where we assume that the covariance matrices are driven by latent factors. This possibly further confirms that it is not defensible to view US inflation shock as global shock (see Dees et al., 2007).

The second difference is the GIRFs of inflation between AEs and EMEs. Whatever the shock is, the GIRFs of EMEs inflation go back to zero more slowly than AEs, which implies that a shock has on average a longer-term effect on EMEs inflation. We call this “fragile inflation”EMEs. We also observe some other differences among different shocks and we describe them below.

US Inflation Shock:

Figure 5 report the GIRFs of 34-country inflation to a 1-standard-deviation increase in US inflation. We

Figure 4: Heatmap of posterior means for the coefficients of output gap lag. The Y axis is the left hand in the equation. The X axis is the right hand in the equation. Then the Y axis denotes the 34 output equations (this quarter), that is, $y_t - \tau_t^y$. The X axis denotes the second lag of 34-country output gap (the quarter before last), that is, $y_{t-2} - \tau_{t-2}^y$.

have in total 68 endogenous variables (34-country inflation and 34-country output), but We find that, from Equation (7), output only depends on the lag of output and does not depend on inflation. This means that US inflation shock will affect 34-country inflation, while does not affect 34-country output, so we only plot the GIRFs of 34-country inflation to a 1-standard-deviation increase in US inflation over the next 20 quarters. We report the posterior means and the 84% credible intervals.

US inflation increases by 0.4 on impact, and it quickly goes back to zero, which implies that US inflation shock has on average only a short-term effect on US inflation. For the cross-country spillovers, we observe some differences across the remaining 33 countries. First, US inflation shock has positive effects on 11 countries (Belgium, South Korea, Sweden, Switzerland, Italy, Finland, France, Australia, Mexico, Bolivia and China). In these 11 countries, the strongest affect occurs in Switzerland. The mean impact is 0.03. Second, US inflation shock has negative effects on 13 countries (Ireland, Netherlands, Latvia, Lithuania, Israel, Spain, Denmark, Germany, Canada, South Africa, Hungary, Turkey and Thailand) and the absolute effects are quite small (absolute values are smaller than 0.003). Finally, the GIRFs of the remaining 9 countries (Greece, Portugal, Slovakia, Hong Kong, UK, Russia, Brazil, Philippines and Indonesia) oscillate between positive and negative territories and then converge back to zero.

US Output Shock:

Figure 6-7 report the GIRFs of 34-country inflation and output to a 1-standard-deviation increase in USA output respectively. We report the posterior means and the 84% credible intervals.

Figure 6 reports the GIRFs of 34-country inflation to a 1-standard-deviation increase in US output. US inflation increases by 0.01 on impact, and it quickly goes back to zero, which implies that US out-

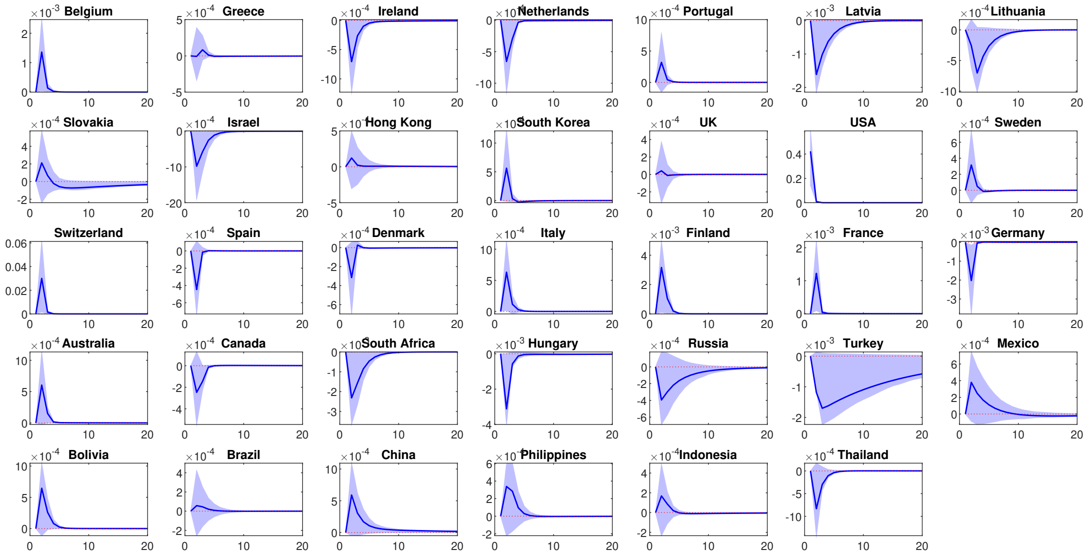

Figure 5: The mean generalised impulse responses of inflation to a 1-standard-deviation increase in US inflation rate over the next 20 quarters. The shaded areas indicate the 84% posterior credible intervals.

put shock has on average only a short-term effect on US inflation. For the cross-country spillovers, we find the largest impact on China inflation, which is close to 0.01 at the beginning and takes almost 10 quarters to go back to zero. The large and immediate impact provides evidence of interdependencies.

Figure 7 reports the GIRFs of 34-country output to a 1-standard-deviation increase in US output. US output increases by 1.7 on impact, and it quickly goes back to zero, which implies that US output shock has a large but short-term effect on US output. For the cross-country spillovers, we find US output shock has a positive effect on most AEs, while US output shock has a negative effect on most EMEs. This provides evidence of dependencies between AEs, and more importantly, between AEs and EMEs. Imposing the cross-sectional homogeneity restriction may lead to mis-specification.

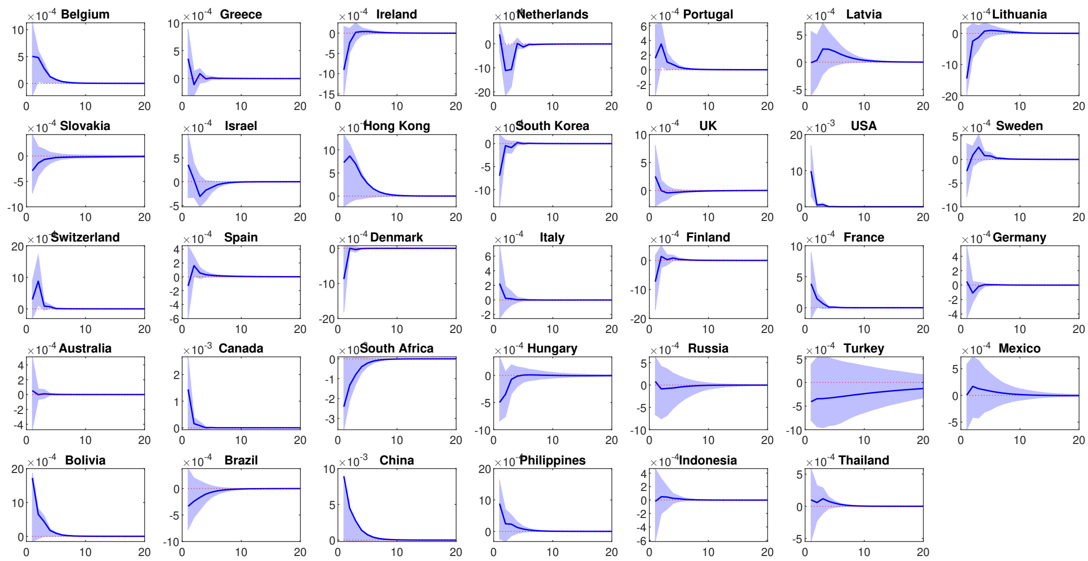

Figure 6: The mean generalised impulse responses of inflation to a 1-standard-deviation increase in US output over the next 20 quarters. The shaded areas indicate the 84% posterior credible intervals.

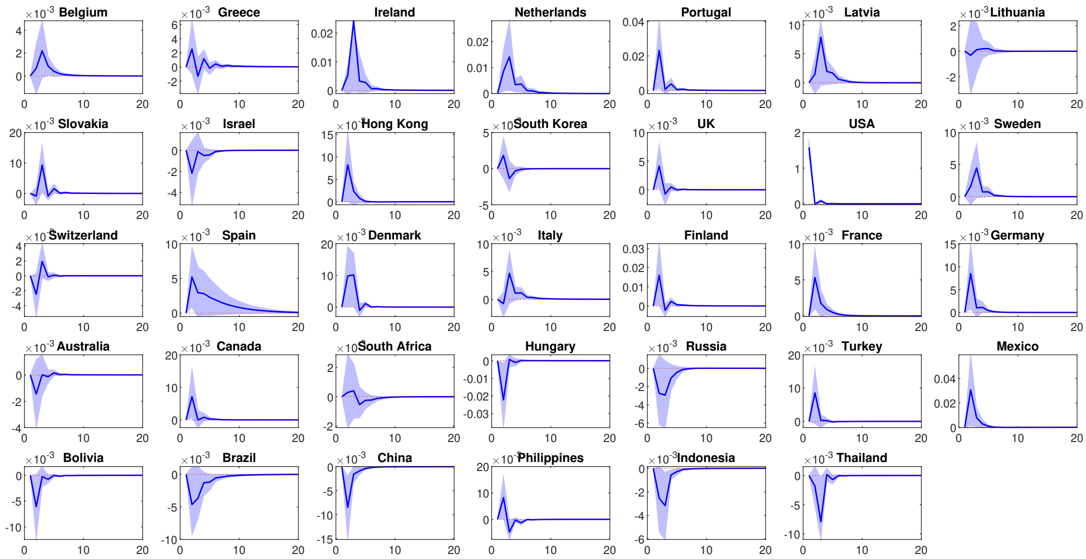

Figure 7: The mean generalised impulse responses of output to a 1-standard-deviation increase in US output over the next 20 quarters. The shaded areas indicate the 84% posterior credible intervals.

Global Inflation Shock:

Figure 8 reports the generalised impulse response functions of 34-country inflation to a 1-standard-deviation increase in global inflation. The impacts are unambiguously positive in all countries. The GIRFs go back to zero over the next 20 quarters, except one country, Turkey. The GIRFs in Turkey decrease slowly and arrives at 0.76 after 20 quarters.

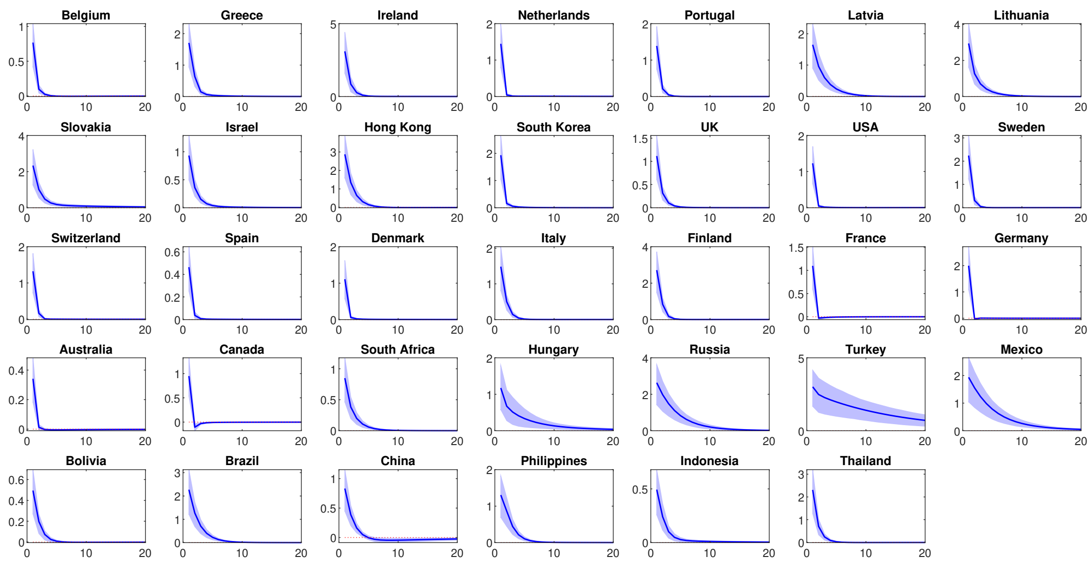

Figure 8: The mean generalised impulse responses of inflation to a 1-standard-deviation increase in global inflation over the next 20 quarters. The shaded areas indicate the 84% posterior credible intervals.

Global Output Shock:

Figures 9 and 10 report the generalised impulse response functions of inflation and output to a 1 standard deviation increase in global output. Similar to the impacts of global inflation shock on 34-country inflation, the impacts of global inflation shock on output are positive and quickly goes back to zero. By contrast, the impacts on inflation need more time to converge back to zero compared to the impacts on output.

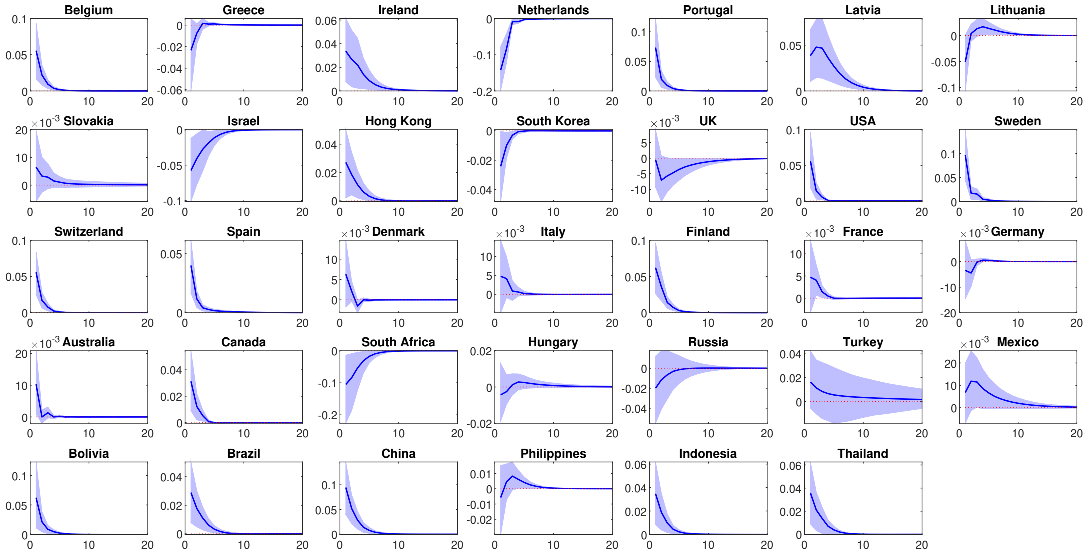

Figure 9: The mean generalised impulse responses of inflation to a 1-standard-deviation increase in global output over the next 20 quarters. The shaded areas indicate the 84% posterior credible intervals.

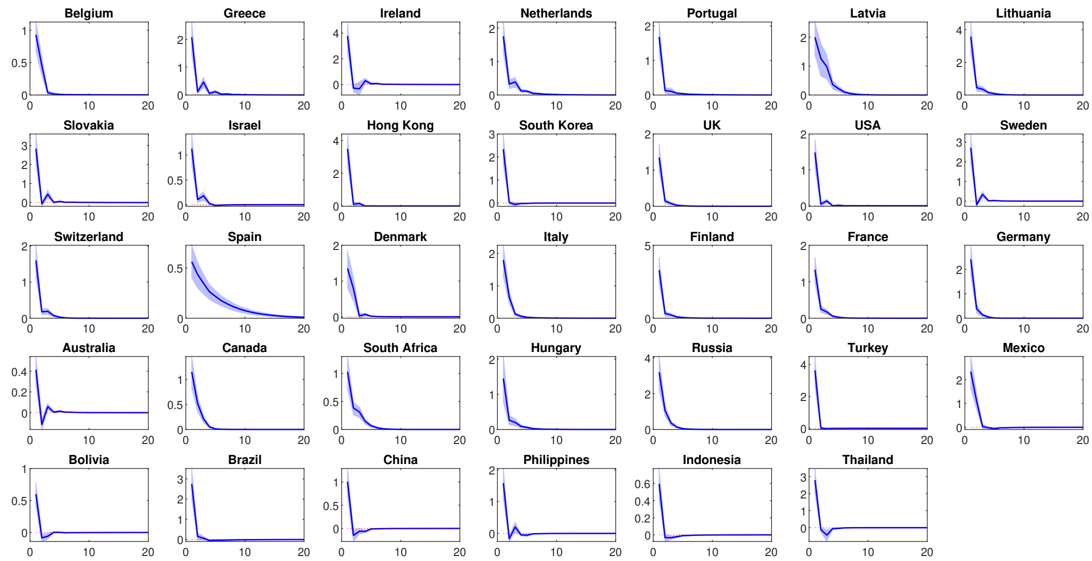

Figure 10: The mean generalised impulse responses of output to a 1-standard-deviation increase in global output over the next 20 quarters. The shaded areas indicate the 84% posterior credible intervals.

4 Estimates of Trends

Figure 11-12 plot the posterior estimates for trend inflation and trend output respectively, estimated using the full sample. We consider 34 countries: the first 23 countries are AEs (from Belgium to Canada) and the following 11 countries are EMEs (from South Africa to Thailand). We compare three models: (a) Bi-UC-SV (the model in Chan et al., 2016. The coefficients are constant, but it allows for stochastic volatility in inflation gap equation); (b) UC-FSV (the model in Wu, 2021. The number of common factors is set to one, that is, $r_\pi = 1$, $r_y = 1$); and (c) PUC-FSV model. The differences are: the Bi-UC-SV model does not allow for any interdependencies across countries, the UC-FSV model allows for cross-country interdependencies that occur contemporaneously, while the PUC-FSV model allows for cross-country interdependencies that occur both contemporaneously and with a lag. In both figures, the solid blue lines are the posterior means under PUC-FSV. The dotted blue lines are the 16% and 84% quantiles under PUC-FSV, the solid black lines are posterior means under UC-FSV, while the solid red lines are posterior means under Bi-UC-SV.

Figure 11 plots the posterior estimates for trend inflation τ^π. The title of each sub-figure is the country name, followed by the official inflation targets (point target or target bands). For example, the title of the first sub-figure is "Belgium (2) ", then the first sub-figure depicts the estimates of trend inflation for Belgium and the official inflation target set by Belgium central bank is 2%.

The broad contours reflected in the posterior mean from the three models are similar. However, we observe some interesting differences. First, we find that both UC-FSV and PUC-FSV can provide narrower width than Bi-UC-SV. The precision of the estimates, as measured by the width of the 84% credible intervals. This indicates that allowing for contemporaneous cross-country interdependencies will provide more precise estimates of trend inflation. We report the width of the 84% credible intervals in Appendix A.

Second, we find that allowing for cross-country interdependencies has more effects on AEs than on EMEs. This is in line with the analysis of cross-country spillovers (Through analyzing P, A, Φ and Θ, we find evident spillovers into AEs, while only several spillovers into EMEs. See Section 3.2.1). We take the US for example. The official inflation target is 2%. Trend inflation from the model without any cross-country interdependencies (that is the Bi-UC-SV model, plotted using solid red lines) increases to 2.5% in 2000s and has decreased to 1.8% in 2015. However, our PUC-FSV model, which allows for cross-country interdependencies (solid blue lines), chooses to estimate trend inflation as a slight increase to 2.3%in 2000s. After the financial crisis has hit, trend inflation decreases to 1.9% in 2010 and then it begins to recover (that is, increase). The effect of cross-country interdependencies that take place with a lag is analyzed by comparing the PUC-FSV (the solid blue line) to the UC-FSV (the solid black line). The effect is observed before 2002 and allowing for cross-country interdependencies that take place with a lag will generate a higher estimates of trend inflation. Trend inflation under PUC-FSV fluctuates in a range between 2.0% and 2.3%, while trend inflation under UC-FSV fluctuates in a range between 1.9% and 2.2%.

Figure 12 plots the posterior estimates for trend output τ^y. First, a similar pattern to trend inflation is observed. We find that both UC-FSV and PUC-FSV can provide narrower width than Bi-UC-SV. This indicates that allowing for contemporaneous cross-country interdependencies will provide more precise estimates of trend output. We report the width of the 84% credible intervals in Appendix A. Second, a general effect of cross-country interdependencies that take place with a lag is observed. Note that the UC-FSV model allows for cross-country interdependencies that occur contemporaneously, while the PUC-FSV model allows for cross-country interdependencies that occur both contemporaneously and with a lag. We find the estimates of trend output under UC-FSV (the solid black line) is higher than the estimates under PUC-FSV (the solid blue line). Although the two models can both provide precise estimates, we find the UC-FSV provides a lower model fit than the PUC-FSV (as shown in Table 1, 885.51 against 918.44). This suggests that PUC-FSV fits the data better and omitting cross-country interdependencies that occur with a lag will overestimate trend output.

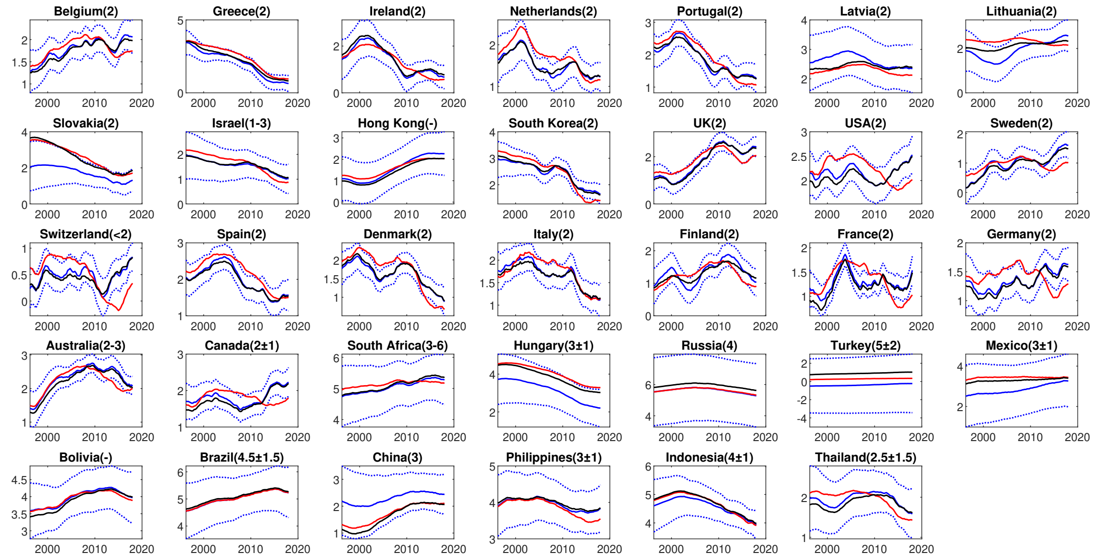

Figure 11: Posterior estimates for trend inflation τ^{π}. The title of each sub-figure is the country name, followed by the official inflation targets (point target or target bands). For Hong Kong and Bolivia, we do not find the official inflation targets, so we use "- ". The solid blue lines are the posterior means under PUC-FSV. The dotted blue lines are the 16% and 84% quantiles under PUC-FSV, the solid black lines are posterior means under UC-FSV, while the solid red lines are posterior means under Bi-UC-SV.

5 Model Comparison

We divide this section into two sub-sections. In both sub-sections, we compare our PUC-FSV to Bi-UC-SV (the model in Chan et al., 2016. The coefficients are constant, but it allows for stochastic volatility in inflation gap equation) and UC-FSV (the model in Wu, 2021. The number of common factors is set to one, that is, $r_{\pi} = 1$, $r_y = 1$). The first sub-section, section 5.1, reports the in-sample fit results. The second sub-section, section 5.2, reports the out-of-sample forecasting results.

5.1 In-sample fit

The gold standard is using marginal likelihood, however, in our settings where we allow for time-variation in volatility, the computation of marginal likelihood requires integrating out all the states, making it a nontrivial task. Therefore, we use an approximation to the marginal likelihood (e.g., Geweke, 2001; Cross et al., 2020). They propose that conditioning on the estimation period, the sums of one-step-ahead joint log predictive likelihoods of 34 countries can be viewed as an approximation to the marginal likelihood, therefore provides a direct measure of in-sample fit.

Before computing the the sums of one-step-ahead joint log predictive likelihoods, we need to define some basics. Let $\hat{y}^{(i,j)}_{t+k}$ denote, at time t, the k-step-ahead forecast of the j-th variable in the i-th country, and

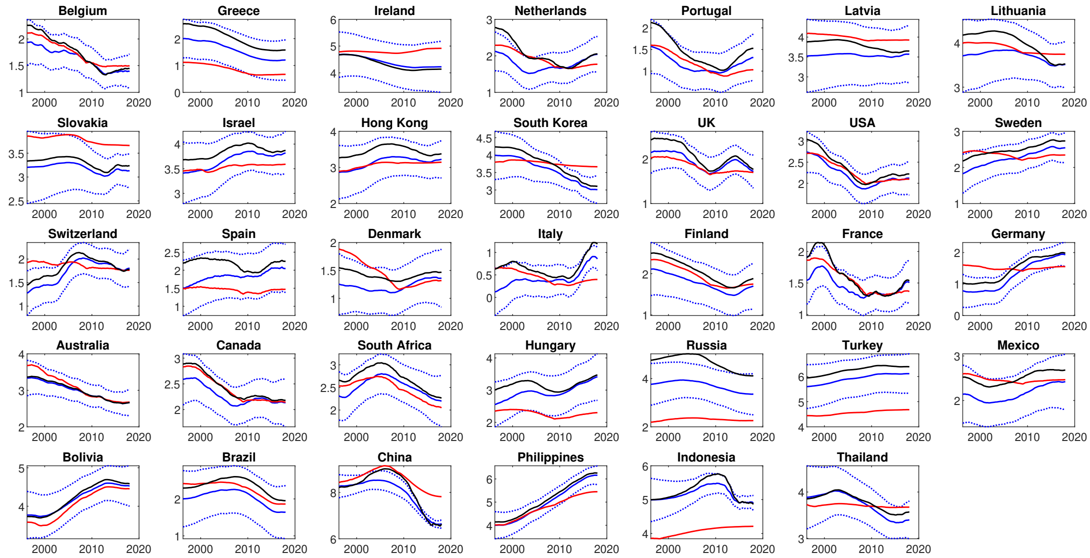

Figure 12: Posterior estimates for trend inflation τ^y. The solid blue lines are the posterior means under PUC-FSV. The dotted blue lines are the 16% and 84% quantiles under PUC-FSV, the solid black lines are posterior means under UC-FSV, while the solid red lines are posterior means under Bi-UC-SV.

$y_{t+k}^{(i,j)}$ denote the actual value. In our empirical work, $i = 1, \ldots, N$ with $n = 34$, $j = 1, 2$ where $j = 1$ denote inflation and $j = 2$ denote output. $\mathrm{Y}_{1:t}^{(i,j)}$ stores the data up to time t, so $\hat{y}_{t+k}^{(i,j)} = \mathrm{E}\ (y_{t+k}^{(i,j)} \mid \mathrm{Y}_{1:t}^{(i,j)})$. Then we compute the k-step-ahead log predictive likelihoods (LPL) of the j-th variable in the i-th country at time t:

$$\mathrm{LPL}_{t,i,j,k} = \log\, p(\hat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \mathrm{Y}_{1:t}^{(i,j)}),\ t = T_0, \ldots, T - k$$

Then the sums of one-step-ahead joint log predictive likelihoods is computed using:

$$\mathrm{LPL}_{\cdot,\cdot,\cdot,1} = \sum_{t=T_0}^{T-1} \sum_{i=1}^{n} \sum_{j=1}^{2} \log\, p(\hat{y}_{t+1}^{(i,j)} = y_{t+1}^{(i,j)} | \mathrm{Y}_{1:t}^{(i,j)})$$

Our estimation period starts from 1995Q4 (to 2018Q1), and the forecasting evaluation period starts from 2003Q1. We provide the sums of one-step-ahead joint log predictive likelihoods of 34 countries in Table 1.

In Table 1, results are presented relative to the forecast performance of the Bi-UC-SV: we take differences, so that a positive number indicates a model is forecasting better than Bi-UC-SV. (Please note that we only take the sum, and no average. That may be why the number seems so large. For example, the sums of LPL under PUC-FSV is 918.44. If we take average over time, then it is 15.06. If we take further average across country, then it is 0.44). The results show that the PUC-FSV provides the highest model fit.

Table 1: Sum of one-step-ahead log predictive likelihood

Model	against Bi-UC-SV
Bi-UC-SV	0
UC-FSV	885.51
PUC-FSV	918.44

5.2 Out-of-sample Forecasting

Having shown that PUC-FSV provides competitive in-sample fit, we now compare the out-of-sample forecast performance of the three models. We use the data from 1995Q4 to 2002Q4 as an initial estimation period, and use data through 2002Q4 to produce k-step-ahead forecast distributions. We consider forecast horizons of $k = 1, 2, 3, 4, 6$ quarters. So our forecast evaluation period begins in 2003Q1. We divide our out-of-sample forecasting results into three parts: forecasting inflation, forecasting output and jointly forecasting inflation and output. For each part, we discuss the results in three dimensions. The first dimension is aggregate forecasting performance over time and over countries (the aggregate LPL, by summing all countries and all time periods). The second dimension is about forecasting performance over time (we can study how the sums of LPL changes over time, by summing all countries at time t). After providing evidence that PUC-FSV can produce more accurate estimate in economic recession, we further study whether such good forecast performance is driven by particular countries, so the third dimension is about the forecasting performance at country level. All results are presented relative to the forecast under Bi-UC-SV: we take differences, so a positive number indicates a model is forecasting better than Bi-UC-SV.

5.2.1 Forecasting inflation

We first report the aggregate forecasting performance for inflation over time and over countries in Table 2. It is calculated by summing the LPL for the N countries over T_0 to $T - k$ (and recall that $j = 1$ denote inflation):

$$\text{LPL}_{\cdot,\cdot,1,k} = \sum_{t=T_0}^{t=T-k} \sum_{i=1}^{n} \log p(\hat{y}_{t+k}^{(i,1)} = y_{t+k}^{(i,1)} | \text{Y}_{1:t}^{(i,1)})$$

First, the positive values provide evidence that PUC-FSV forecasts inflation more accurately at all horizons. Second, except the case when $k = 1$, PUC-FSV provides the most accurate forecast of inflation at other (longer) horizons.

Table 2: Sum of k-step-ahead log predictive likelihood for 34-country inflation

Model	k=1	k=2	k=3	k=4	k=6
Bi-UC-SV	0	0	0	0	0
UC-FSV	**124.57**	210.18	216.28	223.02	286.31
PUC-FSV	114.75	**252.52**	**280.23**	**286.59**	**387.72**

The second dimension of discussion for inflation is sums of LPL over time (by summing all countries at time t), which can be calculated by:

$$\text{LPL}_{t,\cdot,1,k} = \sum_{i=1}^{n} \log p(\hat{y}_{t+k}^{(i,1)} = y_{t+k}^{(i,1)} | \text{Y}_{1:t}^{(i,1)})$$

We plot the results (against Bi-UC-SV) in Figure 13. To forecast inflation during periods of uncertainty (like 2008), we find overall good forecast performance for UC-FSV and PUC-FSV at all horizons. And PUC-FSV forecasts better than UC-FSV at long horizons ($k = 4$ and $k = 6$).
The third dimension of discussion for inflation is the forecasting result for individual countries. The

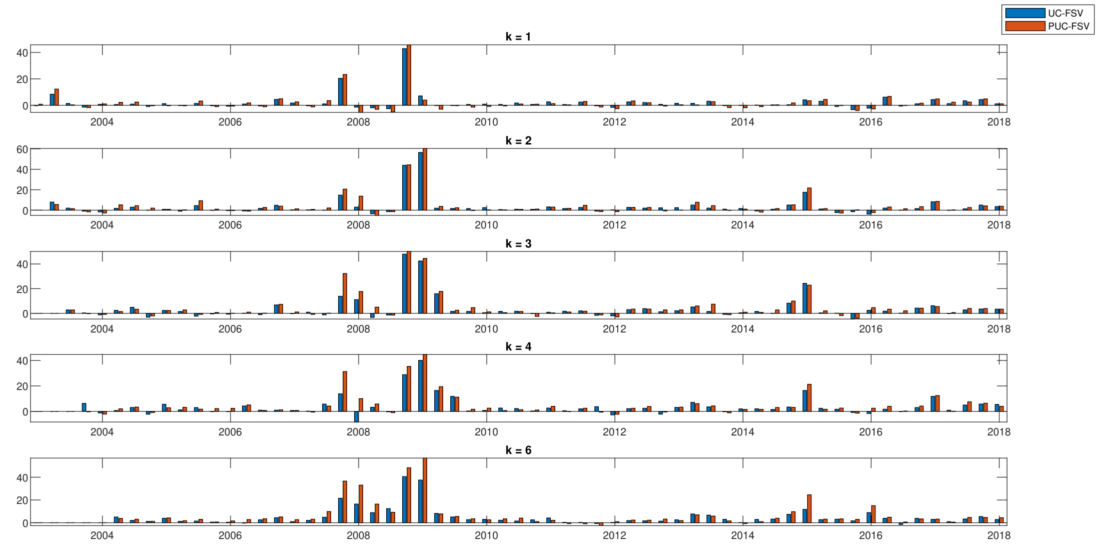

Figure 13: Sums of k-step ahead LPL of inflation for PUC-FSV and UC-FSV relative to Bi-UC-SV over time. The X axis is $t + k$ and represents when to forecast.

LPL of inflation for country i at time t, which can be calculated by:

$$\text{LPL}_{t,i,1,k} = \log p(\hat{y}^{(i,1)}_{t+k} = y^{(i,1)}_{t+k} | \text{Y}^{(i,1)}_{1:t})$$

We plot the results (against Bi-UC-SV) in Figure 14. Here the period of uncertainty that we plot is 2008Q4, so time to forecast is 2008Q4 ($t + k = 2008Q4$). If $k = 1$, then the time we make forecast is 2008Q3, and we find overall good forecast performance for most countries with more pronounced gains in advanced economies (The first 23 countries are AEs, and the following 11 countries are EMEs). A similar pattern is found if $k = 6$. The time we make forecast is 2007Q2, and we also find overall good forecast performance for most countries. In Figure 14, we only plot the shortest horizon $k = 1$ and the longest horizon $k = 6$, for middle horizons ($k = 2, 3, 4$), we find good forecasting result across most countries and did not find particular country which is important in driving good forecasting results. Overall, We find good forecast performance for most countries and such good forecast performance is not driven by particular countries.

5.2.2 Forecasting output

With regard to output, we report the sums of LPL of output over time and over countries in Table 3. It is calculated by summing the LPL for the N countries over T_0 to $T - k$ (and recall that $j = 2$ denote output):

$$\text{LPL}_{\cdot,\cdot,2,k} = \sum_{t=T_0}^{t=T-k} \sum_{i=1}^{n} \log p(\hat{y}^{(i,2)}_{t+k} = y^{(i,2)}_{t+k} | \text{Y}^{(i,2)}_{1:t})$$

The results show that PUC-FSV provides the most accurate forecast for output at all horizons.

Similar to the analysis of inflation, the second dimension of discussion for output is sums of LPL over

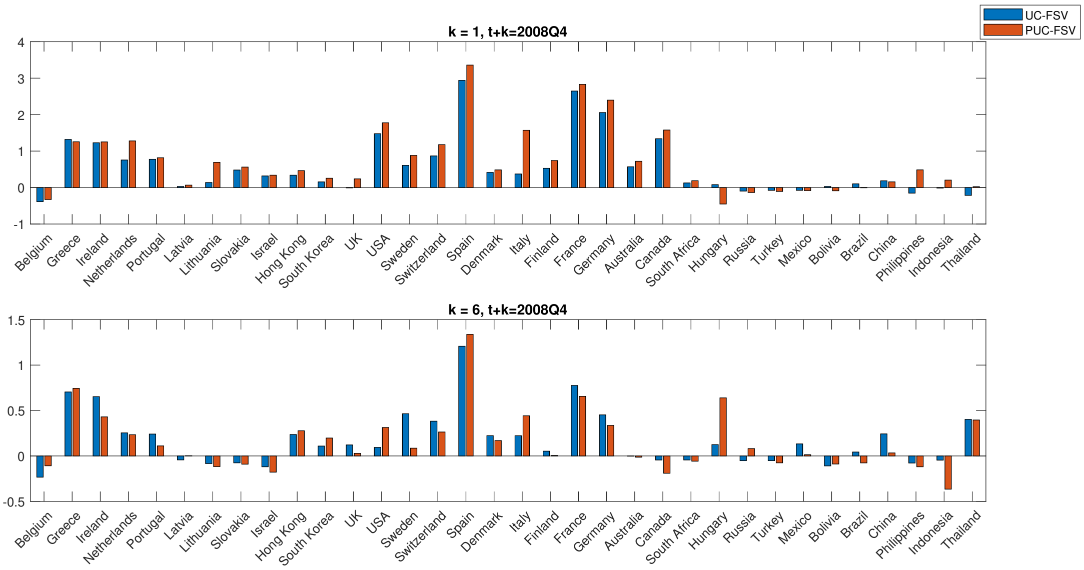

Figure 14: Sums of k-step ahead LPL of inflation for country i under PUC-FSV and UC-FSV relative to Bi-UC-SV.

Table 3: Sum of k-step-ahead log predictive likelihood for 34-country output

Model	k=1	k=2	k=3	k=4	k=6
Bi-UC-SV	0	0	0	0	0
UC-FSV	750.25	979.29	969.58	830.92	942.70
PUC-FSV	**791.86**	**1010.76**	**1010.26**	**1041.74**	**1112.64**

time (by summing all countries at time t), which can be calculated by:

$$\text{LPL}_{t,\cdot,2,k} = \sum_{i=1}^{n} \log p(\hat{y}^{(i,2)}_{t+k} = y^{(i,2)}_{t+k} | \text{Y}^{(i,2)}_{1:t})$$

We plot the results (against Bi-UC-SV) in Figure 15. To forecast output during periods of uncertainty (like 2008), we find overall good forecast performance for PUC-FSV and UC-FSV at all horizons.

To investigate whether the good forecast performance is driven by particular countries, we calculate the sums of LPL of output for country i at time t by:

$$\text{LPL}_{t,i,2,k} = \log p(\hat{y}^{(i,2)}_{t+k} = y^{(i,2)}_{t+k} | \text{Y}^{(i,2)}_{1:t})$$

We plot the results (against Bi-UC-SV) in Figure 16. We choose 2008Q4 to represent the period of uncertainty. For $k = 1$ and $k = 6$, we both find overall good forecast performance for PUC-FSV and UC-FSV for all countries. For several countries (like Spain, Italy and Germany), PUC-FSV forecasts better than UC-FSV.

5.2.3 Jointly Forecasting inflation and output

With regard to the joint predictive density for inflation and output, we first report the sums of joint LPL over time and over countries in Table 4. It is calculated by summing the LPL for the N countries

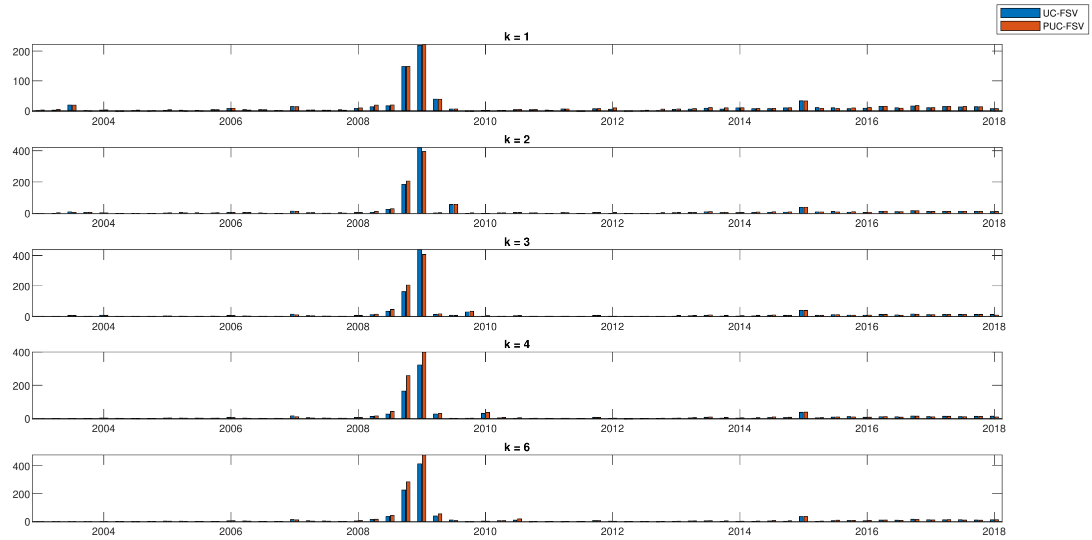

Figure 15: Sums of k-step ahead LPL of output for PUC-FSV and UC-FSV relative to Bi-UC-SV over time. The X axis is $t+k$ and represents when to forecast.

over T_0 to $T-k$ (and for all j, recall that $j=1$ denote inflation, $j=2$ denote output):

$$\text{LPL}_{\cdot,\cdot,\cdot,k} = \sum_{t=T_0}^{t=T-k} \sum_{i=1}^{n} \sum_{j=1}^{2} \log\, p(\hat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \text{Y}_{1:t}^{(i,j)})$$

The results show that PUC-FSV provides the most accurate joint forecast for inflation and output at all horizons. Next, we study the time-variation in forecast performance to see whether the benefits

Table 4: Sum of k-step-ahead joint log predictive likelihood for 34-country inflation and output

Model	k=1	k=2	k=3	k=4	k=6
Bi-UC-SV	0	0	0	0	0
UC-FSV	885.51	1111.32	1221.00	1044.97	1093.88
PUC-FSV	**918.44**	**1169.58**	**1291.92**	**1318.78**	**1253.67**

arise from the forecast during periods of uncertainty. So the second dimension of discussion for joint predictive density for inflation and output is sums of joint LPL over time (by summing all j and all countries at time t), which can be calculated by:

$$\text{LPL}_{t,\cdot,\cdot,k} = \sum_{i=1}^{n} \sum_{j=1}^{2} \log\, p(\hat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \text{Y}_{1:t}^{(i,j)})$$

We plot the results (against Bi-UC-SV) in Figure 17. A similar pattern to inflation and output was found. To jointly forecast inflation and output during periods of uncertainty (like 2008), we find overall good forecast performance under PUC-FSV and UC-FSV at all horizons.

Finally, we investigate whether the good forecast performance of periods of uncertainty is driven by particular countries, so the third dimension of discussion for joint predictive density for inflation and output is sums of joint LPL at the country level (by summing all j for country i), which can be calculated

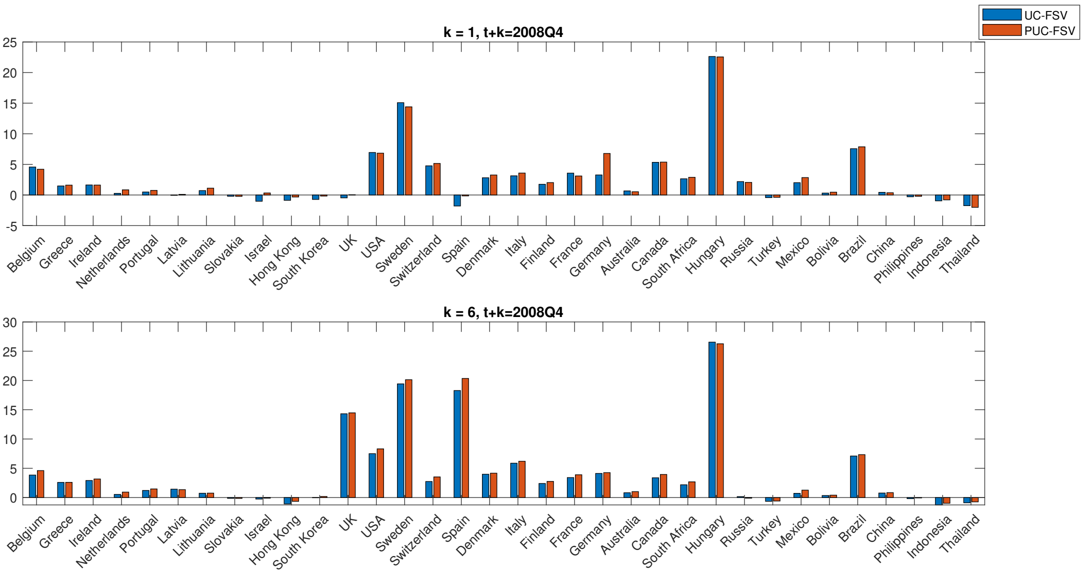

Figure 16: Sums of k-step ahead LPL of output in country i for PUC-FSV and UC-FSV relative to Bi-UC-SV.

by:

$$\text{LPL}_{t,i,\cdot,k} = \sum_{t=T_0}^{t=T-k} \sum_{j=1}^{2} \log p(\hat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \text{Y}_{1:t}^{(i,j)})$$

We plot the results (against Bi-UC-SV) in Figure 18. A similar pattern to output is found. (This is sensible since the gains in output are much larger than gains in inflation, see Figure 14 and Figure 16). We find overall good forecast performance for PUC-FSV for all countries.

6 Conclusions

In a globalized world, countries are linked together and cross-country interdependencies may influence the estimates of trend. However, such influence has not been considered in unobserved components models, which are popular to estimate the trend. In this paper, we develop a panel unobserved components model that allows for dependencies that can take place both contemporaneously and with a lag, both across variables within a country and across countries. We introduce three novel features. First, we allow for interdependencies across countries, which has been shown to influence the precision of estimates of trend. Second, we do not impose any zero restriction. This reduces the associated mis-specification risk. To deal with over-parameterization concerns, we rely on a global-local shrinkage prior. Third, we do so in a computationally efficient way. Equation-by-equation estimation can be used.

We demonstrate the merits of our model through a multi-country study involving 34 countries. The estimates of coefficients and generalised impulse response functions (GIRFs) provide evidence of interdependencies. The estimates of coefficients show that the Horseshoe prior can achieve sensible shrinkage. Events in both AEs and EMEs can spill over into AEs, contemporaneously and with a lag. By contrast, there are only several spillovers into EMEs. We find more evidence of static interdependencies than dynamic interdependencies. The GIRFs are much larger when a global shock hits the system than when

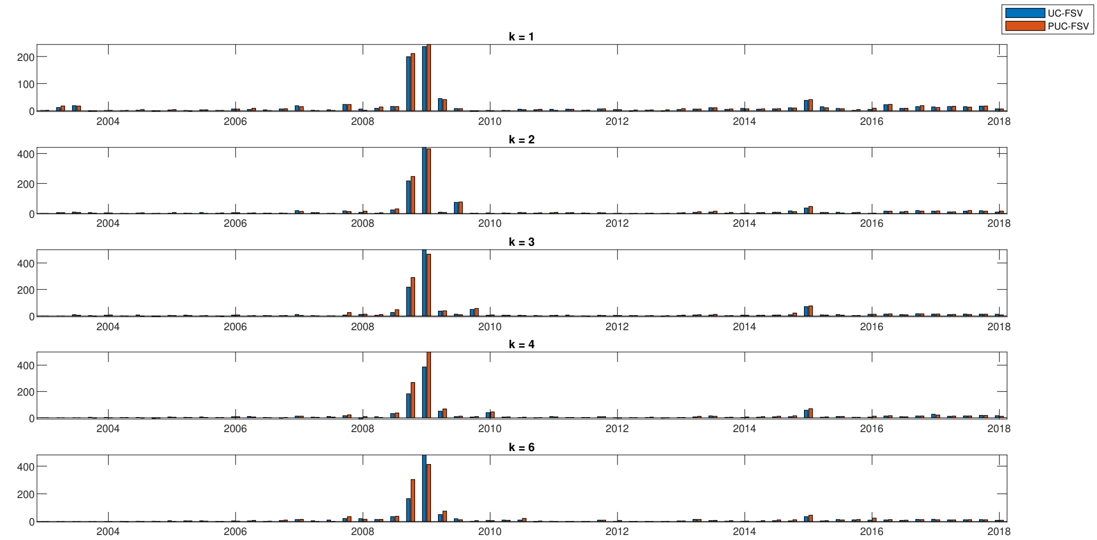

Figure 17: Sums of k-step ahead joint LPL for PUC-FSV and UC-FSV relative to Bi-UC-SV over time. The X axis is $t + k$ and represents when to forecast.

a US shock hits the system. The GIRFs also show that whatever the shock is, the GIRFs of EMEs inflation go back to zero more slowly than AEs. We call this "fragile inflation"EMEs. These features show the power of working with unrestricted model and selecting the appropriate restrictions in a data based manner.

We also present the importance of interdependencies. First, we find that allowing for contemporaneous cross-country interdependencies can provide more precise estimates of trend, while omitting cross-country interdependencies that occur with a lag will overestimate trend output. Second, our proposed model provides a superior in-sample fit and accurate density forecasts compared to existing models in the literature.

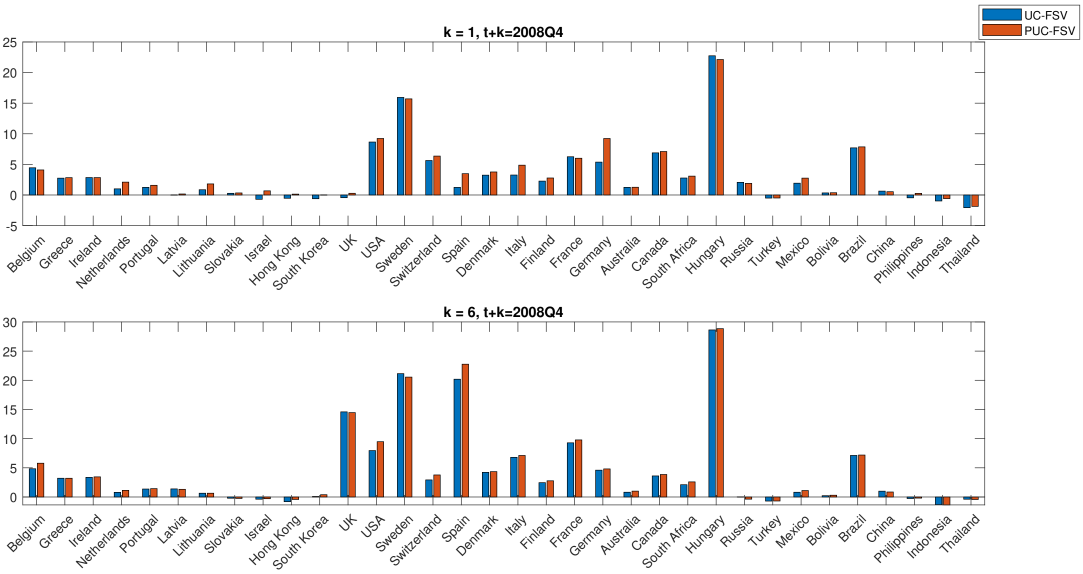

Figure 18: Sums of k-step ahead joint LPL in country i for PUC-FSV and UC-FSV relative to Bi-UC-SV.

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Appendices

A Width of Credible Intervals for Trends

In this appendix, we report the width of 84% credible intervals for trends under three models: Bi-UC-FSV, UC-FSV and PUC-FSV. Figure 19 reports the width of credible interval for trend inflation. Figure 20 reports the width of credible interval for trend output.

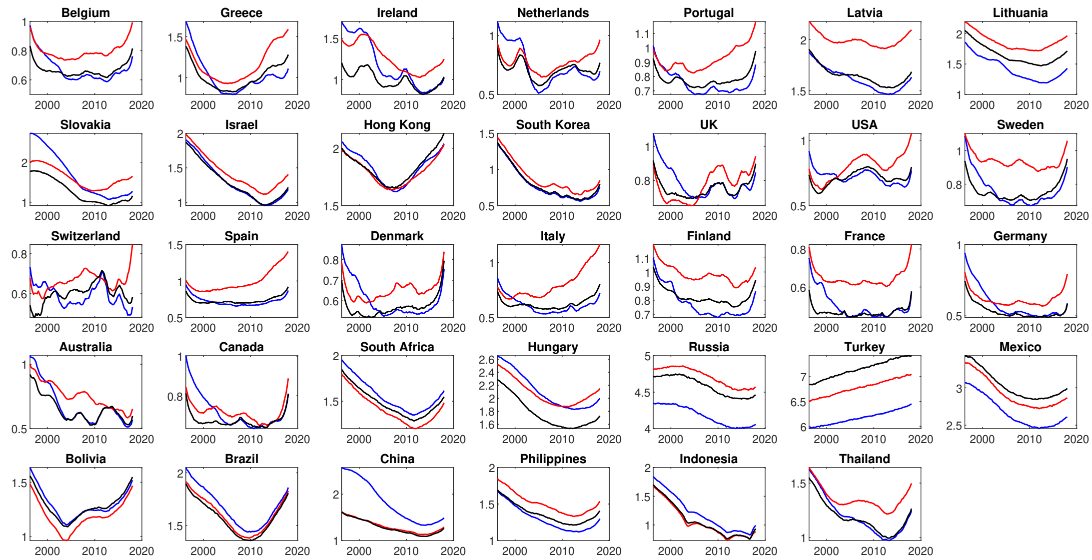

Figure 19: The width of 84% credible interval for trend inflation under three models: Bi-UC-SV, UC-FSV and PUC-FSV. The red lines are the width under Bi-UC-SV. The black lines are the width under UC-FSV. The blue lines are the width under PUC-FSV.

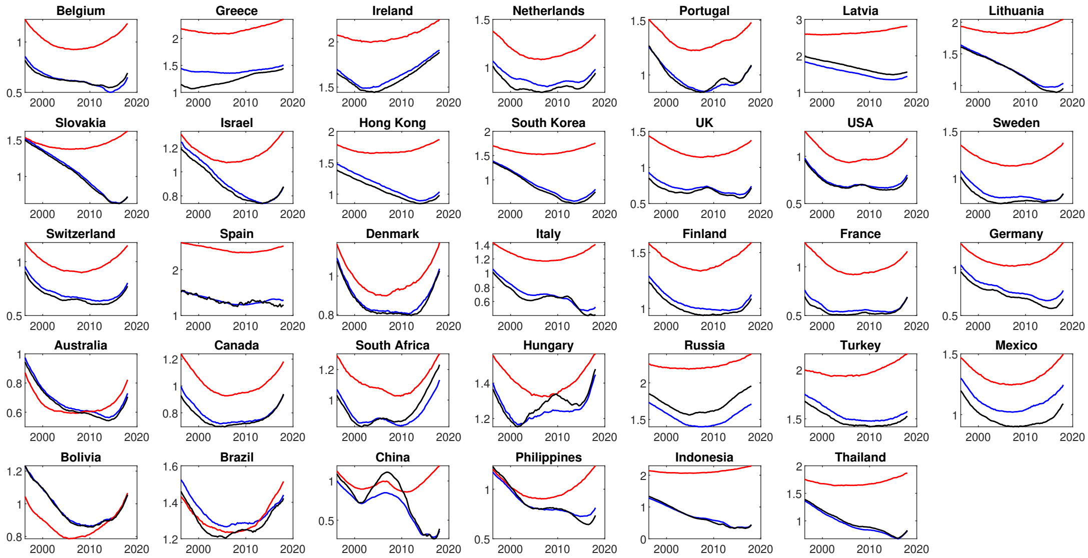

Figure 20: The width of 84% credible interval for trend output under three models: Bi-UC-SV, UC-FSV and PUC-FSV. The red lines are the width under Bi-UC-SV. The black lines are the width under UC-FSV. The blue lines are the width under PUC-FSV.

B Estimates of factor loading matrices

In this appendix, we report the posterior estimates of factor loading matrices under PUC-FSV. Basically, we have two factors:

(1): global inflation factor f_t, and its loading matrix is L_π, L_π is $n \times r_\pi$, in our empirical application, $n = 34$, $r_\pi = 1$. (2): global output factor g_t, and its loading matrix is L_y, L_y is $n \times r_y$, in our empirical application, $n = 34$, $r_y = 1$.

We report the posterior mean and the 84% quantiles of both L_π and L_y. And for identification, we assume the factor loading matrices are lower triangular matrices with ones on the main diagonal, so some elements in L_π and L_y are 1 or 0.

Table 5: Posterior Estimates of factor loading matrix L_π and L_y

	L_π			L_y		
country	mean	16%	84%	mean	16%	84%
Belgium	1	1	1	1	1	1
Greece	1.54	1.03	2.11	2.28	1.54	3.01
Ireland	1.42	0.96	1.93	3.94	2.62	5.27
Netherlands	1.21	0.82	1.66	1.91	1.50	2.32
Portugal	1.18	0.77	1.64	1.83	1.41	2.26
Latvia	1.37	0.85	1.92	2.18	1.16	3.21
Lithuania	1.31	0.77	1.85	3.91	2.70	5.13
Slovakia	1.58	1.07	2.17	3.11	2.16	4.08
Israel	1.38	0.87	1.91	1.23	0.86	1.60
Hong Kong	0.72	0.17	1.27	3.77	2.91	4.63
South Korea	1.08	0.71	1.48	2.54	1.87	3.21
UK	1.21	0.83	1.66	1.47	1.07	1.87
USA	1.96	1.34	2.67	1.62	1.23	2.01
Sweden	1.51	1.03	2.08	2.95	2.33	3.57
Switzerland	1.22	0.84	1.66	1.74	1.39	2.10
Spain	2.09	1.45	2.86	0.61	0.42	0.81
Denmark	1.17	0.80	1.60	1.49	0.85	2.11
Italy	0.92	0.63	1.26	1.96	1.56	2.36
Finland	1.04	0.69	1.43	3.59	2.82	4.35
France	1.44	1.00	1.96	1.45	1.17	1.73
Germany	1.39	0.96	1.91	2.64	2.13	3.16
Australia	1.45	1.00	1.98	0.45	0.15	0.75
Canada	1.72	1.16	2.36	1.26	0.90	1.62
South Africa	1.18	0.67	1.69	1.12	0.84	1.40
Hungary	2.27	1.42	3.16	1.54	0.67	2.37
Russia	0.27	-0.28	0.83	3.46	2.67	4.24
Turkey	2.16	1.21	3.15	3.94	2.87	5.02
Mexico	0.38	0.08	0.68	2.58	2.02	3.14
Bolivia	0.48	-0.05	1.02	0.65	0.18	1.12
Brazil	0.08	-0.32	0.47	3.02	2.22	3.81
China	0.49	0.14	0.83	1.11	0.69	1.52
Philippines	0.87	0.45	1.30	1.73	1.11	2.35
Indonesia	0.41	0.09	0.73	0.65	0.36	0.94
Thailand	1.71	1.11	2.34	3.05	2.40	3.71

C Estimates of Uncertainty

In this appendix, we report the posterior estimates of uncertainty. There are four uncertainties in PUC-FSV:

(1): Idiosyncratic Inflation Uncertainty: the idiosyncratic inflation uncertainty is the standard deviation of the shocks to the inflation gap, $\exp(h_t^\pi/2)$. We report this in Figure 21. The title of each sub-figure is the country name. Each sub-figure plots the posterior estimates under three competing models (PUC-FSV, UC-FSV and Bi-UC-SV). The blue lines are the posterior means, 16% and 84% quantiles under PUC-FSV, the red lines are posterior means under Bi-UC-SV, while the black lines are posterior means under UC-FSV.

(2): Idiosyncratic Output Uncertainty: the idiosyncratic output uncertainty is the standard deviation of the shocks to the output gap, $\exp(h_t^y/2)$. We report this in Figure 22. The title of each sub-figure is the country name. Each sub-figure plots the posterior estimates under two competing models (PUC-FSV and UC-FSV). The stochastic volatility is not allowed in Bi-UC-SV, so we only have the estimates under two models. The blue lines are the posterior means, 16% and 84% quantiles under PUC-FSV, while the black lines are posterior means under UC-FSV.

(3): Global Inflation Uncertainty: the global inflation uncertainty is the standard deviation of the shocks to the global inflation factor, $\exp(h_t^f/2)$. We report this in Figure 23. The figure plots the posterior estimates under two competing models (PUC-FSV and UC-FSV). There is no global factors in Bi-UC-SV model, so we only have the estimates under two models. The blue lines are the posterior means, 16% and 84% quantiles under PUC-FSV, while the black lines are posterior means under UC-FSV.

(4): Global Output Uncertainty: the global output uncertainty is the standard deviation of the shocks to the global output factor, $\exp(h_t^g/2)$. We report this in Figure 24. The figure plots plots the posterior estimates under two competing models (PUC-FSV and UC-FSV). The blue lines are the posterior means, 16% and 84% quantiles under PUC-FSV, while the black lines are posterior means under UC-FSV.

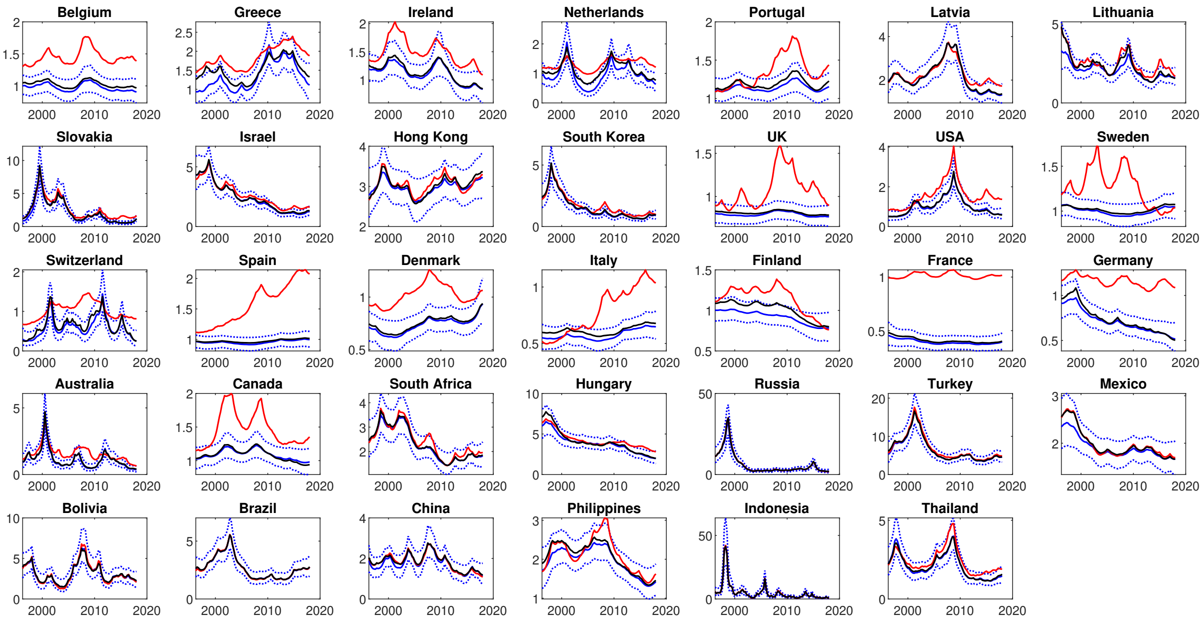

Figure 21: Posterior estimates for idiosyncratic inflation uncertainty $\exp(h_t^\pi/2)$. The blue lines are the posterior means, 16% and 84% quantiles under PUC-FSV, the red lines are posterior means under Bi-UC-SV, while the black lines are posterior means under UC-FSV.

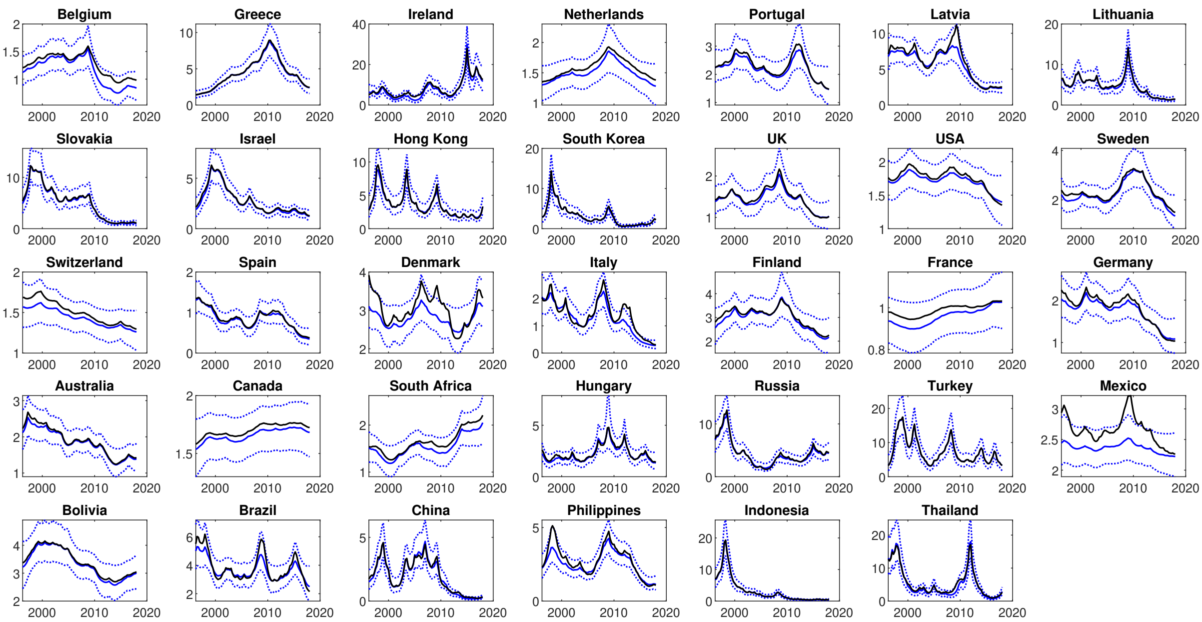

Figure 22: Posterior estimates for idiosyncratic inflation uncertainty $\exp(h_t^y/2)$. The blue lines are the posterior means, 16% and 84% quantiles under PUC-FSV, the red lines are posterior means under Bi-UC-SV, while the black lines are posterior means under UC-FSV.

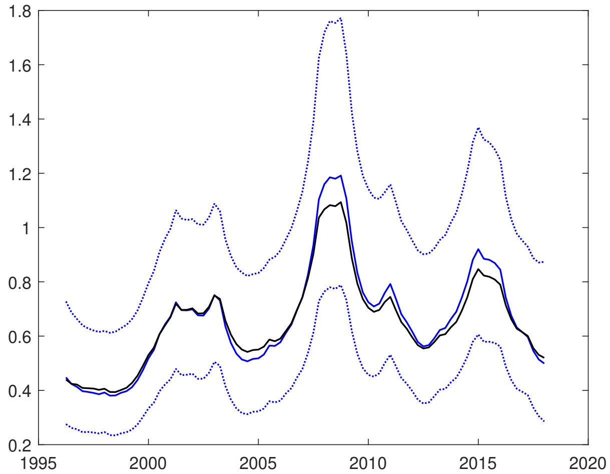

Figure 23: Posterior estimates for global inflation uncertainty $\exp(h_t^f/2)$. The blue lines are the posterior means, 16% and 84% quantiles under PUC-FSV, while the black lines are posterior means under UC-FSV.

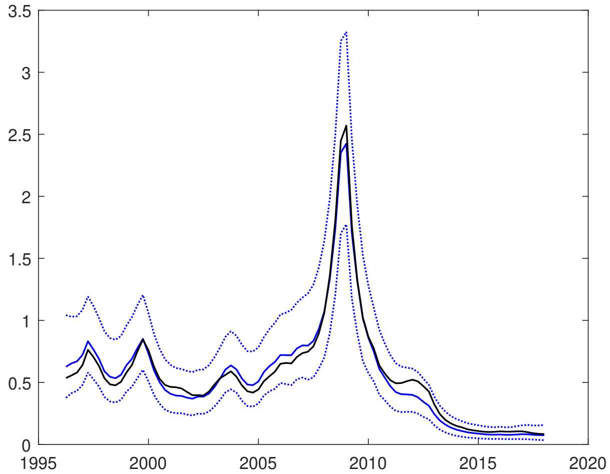

Figure 24: Posterior estimates for global output uncertainty $\exp(h_t^g/2)$. The blue lines are the posterior means, 16% and 84% quantiles under PUC-FSV, while the black lines are posterior means under UC-FSV.