Permanent and Transitory shocks: Implications from A Panel Unobserved Components Model *

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Abstract

This paper develops a Bayesian panel unobserved components model. The conventional domestic Phillips curve is extended to a multi-country Phillips curve, where the inflation gap in one country is determined by both the output gap in own country and the output gaps in other countries. The model can be used to study the permanent trend shocks and transitory gap shocks. We use this model to study the 34-country dynamics of various shocks. We find evidence of "fragile inflation"in emerging market economies. We also find a shock to US trend output will generate a permanent influence on the output in some countries.

Keywords: unobserved components model, permanent trend shock, transitory gap shock

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1 Introduction

The concept of trend is central to macroeconomics. Multivariate unobserved components (UC) models have been shown to provide reasonable estimates of the trend. Much of the existing UC literature either focus on one country or impose independent assumption between economies in multi-country studies. But in the modern globalized economy, countries are linked together through trade and financial flows. Events in one country can spill over into others and the spillovers may influence the estimates of trend (see Canova and Ciccarelli, 2013; Taylor and Wieland, 2016). The UC model without interdependencies across countries is counter-intuitive.

This paper develops a panel unobserved components (PUC) model that allows for interdependencies that can take place both contemporaneously and with a lag. The conventional bivariate UC model for domestic inflation and domestic output is extended to a version that domestic inflation is determined by both domestic output and the output in other countries (we call this foreign output). The novelties of our approach are that i) the model can be used to study effects of the permanent trend shocks and transitory gap shocks; ii) we do not impose any zero restriction. This reduces the associated mis-specification risk and means that we work with unrestricted PUC model.

To explain the significance of the econometric contributions of this paper, we note a major difference between Vector Autoregressions (VARs) and UC models is that the UC model decomposes a variable into two parts: trend and gap. For instance, the bivariate UC model has equations for trend inflation and trend output, and has equations for inflation gap and output gap (see Stella and Stock, 2013; Chan et al., 2016). The two sets of equations allows us to distinguish the permanent trend shocks and transitory gap shocks. But this feature is not easily allowed in VARs.[1] In this paper, we extend the bivariate UC model to a multi-country version, where domestic inflation is determined by both domestic output and foreign outputs.

Another econometric contribution of this paper is that we note a major need is to account for the panel structure in the data, and explicitly model interdependencies and commonalities across countries. We borrow the advantages from Panel Vector Autoregressions

[1] In recent years a literature using VARs with common trends has developed (Johannsen and Mertens (2021)). It has something in common with the model in this paper. But the permanent/transitory shocks have not been studied in UC models.

(PVARs) to fulfill this need. One advantage of PVARs (compared to the Global VARs) is that it is convenient to do forecasting and structural analysis. The PVARs literature includes two main approaches to consider the panel structure: imposing zero restrictions and relying on shrinkage priors (see Koop and Korobilis, 2019; Feldkircher et al., 2021; Bai et al., 2022). In this paper, we rely on shrinkage priors. We do not selectively model the dynamic links across countries while imposing zero restrictions on others (see Canova and Ciccarelli, 2009; Canova and Ciccarelli, 2013), because these zero restrictions, if wrongly chosen, potentially lead to mis-specification problems. It is clearly desirable to introduce restrictions in a data based fashion (see Feldkircher et al., 2021). One popular data-based approach is the stochastic search specification selection approach, developed by Koop and Korobilis (2016). Their approach produces posterior inclusion probabilities for every possible restrictions and these probabilities can be used to sort through restrictions in a data based fashion. Davidson et al. (2019) further extend the method of Koop and Korobilis (2016) to allow for a more detailed investigation of cross-country linkages. Another data-based approach is to rely on global-local shrinkage priors. Zero-restriction implies that the matrix is sparse. It is found that if a matrix is characterized by a relatively low number of non-zero elements, a possible solution is a global-local shrinkage prior (e.g., Polson and Scott, 2010; Kastner and Huber, 2020). Such advantage of global-local shrinkage prior shrinks strongly the parameter space but at the same time provides enough flexibility to allow for non-zero elements if necessary, thus imposing zero restriction for most elements but dropping the restriction if necessary. The global-local shrinkage prior has been applied to PVAR literature in Feldkircher et al. (2021).[2] In this paper, we follow their method, working with unrestricted PUC model and relying on the global-local shrinkage prior to deal with over-parameterization concerns.

Our paper also seeks to contribute to the empirical literature on estimates of trend. Precise estimates are important because they are thought to reflect the fundamental structure of the economy in the absence of shocks (see Zaman, 2021). Studies have guided important empirical features to get precise estimates. Canova (2011) point out that the model without dynamic interdependencies is suitable for estimates of trend[3] in a small open economy. But this paper includes 34 countries (23 advanced economies and 11

[2]Leaving the model unrestricted will also make the computation cumbersome. They develop a method to speed up computation, which they call integrated rotated Gaussian approximation (IRGA). We have not applied this new method, because our model is not so huge, compared to their model.

[3]Precisely, Canova (2011) use the term "steady state ". We use the term "trend ". There are subtle differences between them, but they can be interpreted as the same for the purpose of this paper.

emerging market economies). The small open economy assumption is no longer satisfied and the spillovers may influence the estimates of trend (see Canova and Ciccarelli, 2013). Taylor and Wieland (2016) also emphasize that omitting variables can affect the reliability of the estimates of trend. Therefore, in this paper, we take on the challenge of jointly estimating 34-country trends simultaneously.

A second empirical contribution lies in the Phillips curve. Recent development in inflation dynamics has raised questions about whether the relationship between real economy and inflation has been altered and whether the Phillips curve is still valid. In the conventional Phillips curve, what links inflation and output together is their own country Phillips curve. One explanation of the muted response of domestic inflation is that inflation is determined globally. For instance, Forbes (2019) demonstrate empirically, using a Phillips curve with a set of global variables, that globalisation plays an increasingly more important role in explaining inflation dynamics. However, there are many linkages and inter-relationships between the economies. Modelling aggregate output for the world as a whole will miss many interesting country-specific patterns since different countries have differing trade exposure to different countries. These considerations justify why we want to extend the conventional Phillips curve to a multi-country Phillips curve. The multi-country Phillips curve is allowed through the panel structure in PUC model at a low cost. The equation-by-equation estimation can still be used.

In our empirical work, we consider 23 advanced economies (AEs) and 11 emerging market economies (EMEs). We first provide evidence that the PUC model generates more precise estimates of trend. The precision of the estimates are measured by the width of the 84% credible intervals. In the forecasting exercise, we find that the PUC model can beat all alternatives in most cases. The only exception is to forecast 1-quarter-ahead inflation.

We then proceed by the Generalized Impulse Response Functions (GIRFs), proposed in Koop et al. (1996). We compare the permanent trend shocks and transitory gap shocks. On the responses of inflation, we find whatever the shock is, the GIRFs of EMEs inflation go back to zero more slowly than AEs, which implies that a shock has on average a longer-term effect on EMEs inflation. We call this “fragile inflation” EMEs. On the responses of output, we find a shock to US trend output will generate a permanent influence on the output in some countries.

This paper is organized as follows. In Section 2, we start from the multi-country unobserved components model with factor stochastic volatility (FSV), then introduce the panel unobserved components model with FSV, which allows for interdependencies that can take place both contemporaneously and with a lag. After introducing the PUC model, we describe the global-local shrinkage priors on parameters. In Section 3, we describe the 34-country data and provide evidence of more precise estimates of trend. This is followed by a forecasting exercise. In Section 4, we present the responses to various shocks. Finally, Section 5 concludes.

2 A Panel Unobserved Components model with Factor Stochastic Volatility

In this section, we develop the panel unobserved components model with factor stochastic volatility. We do not impose any zero restrictions. To deal with the over-parameterization concerns, we rely on a global-local shrinkage prior (the Horseshoe prior). The priors are described after introducing the PUC-FSV model.

2.1 PUC-FSV Model Specification

We begin with the multi-country unobserved component (UC-FSV) model developed in Wu (2022). They allow for contemporaneous interdependencies across countries through factor stochastic volatility structure. In particular, for country i, $i = 1, \ldots, N$, $\pi_{i,t}$ is the inflation of country i at time t and $y_{i,t}$ is the output growth of country i, $\tau^{\pi}_{i,t}$ and $\tau^{y}_{i,t}$ are their trends. The UC-FSV model for N-country inflation and output is defined as:

$$
\begin{aligned}
\pi_t - \tau_t^\pi &= P(\pi_{t-1} - \tau_{t-1}^\pi) + A(y_t - \tau_t^y) + L_\pi f_t + u_t^\pi,\ f_t \sim \mathcal{N}(0,\ \Omega_t^\pi),\ u_t^\pi \sim \mathcal{N}(0,\ \Sigma_t^\pi) \\
y_t - \tau_t^y &= \Phi(y_{t-1} - \tau_{t-1}^y) + \Theta(y_{t-2} - \tau_{t-2}^y) + L_y g_t + u_t^y,\ g_t \sim \mathcal{N}(0,\ \Omega_t^y),\ u_t^y \sim \mathcal{N}(0,\ \Sigma_t^y) \\
&\tau_{i,t}^\pi = \tau_{i,t-1}^\pi + \varepsilon_{i,t}^{\tau\pi},\quad \varepsilon_{i,t}^{\tau\pi} \sim \mathcal{N}(0,\ \sigma_{\tau\pi}^2),\ i = 1, \ldots, N \\
&\tau_{i,t}^y = \tau_{i,t-1}^y + \varepsilon_{i,t}^{\tau y},\quad \varepsilon_{i,t}^{\tau y} \sim \mathcal{N}(0,\ \sigma_{\tau y}^2) \\
&h_{j,t} = h_{j,0} + \omega_j^h \widetilde{h}_{j,t} \\
&\widetilde{h}_{j,t} = \widetilde{h}_{j,t-1} + \varepsilon_{j,t}^h,\quad \varepsilon_{j,t}^h \sim \mathcal{N}(0,\ 1),\ j = 1, \ldots, 2N + r_\pi + r_y
\end{aligned}
\tag{1}
$$

where $\pi_t = (\pi_{1,t}, \ldots, \pi_{N,t})'$ is an $N \times 1$ vector, $\tau_t^\pi = (\tau_{1,t}^\pi, \ldots, \tau_{N,t}^\pi)'$ is an $N \times 1$ vector, $P = \text{diag}(\rho_1, \ldots, \rho_N)$ is an $N \times N$ matrix, $A = \text{diag}(\alpha_1, \ldots, \alpha_N)$ is an $N \times N$ matrix, $y_t = (y_{1,t}, \ldots, y_{N,t})'$ is an $N \times 1$ vector, $\tau_t^y = (\tau_{1,t}^y, \ldots, \tau_{N,t}^y)'$ is an $N \times 1$ vector, L_π is an $N \times r_\pi$ matrix, f_t is a $r_\pi \times 1$ vector, u_t^π is an $N \times 1$ vector, $\Phi = \text{diag}(\phi_1, \ldots, \phi_N)$ is an $N \times N$ matrix, $\Theta = \text{diag}(\theta_1, \ldots, \theta_N)$ is an $N \times N$ matrix, L_y is an $N \times r_y$ matrix, g_t is a $r_y \times 1$ vector, u_t^y is an $N \times 1$ vector. $\Sigma_t^\pi = \text{diag}(e^{h_{1,t}}, \ldots, e^{h_{N,t}})$, $\Sigma_t^y = \text{diag}(e^{h_{N+1,t}}, \ldots, e^{h_{2N,t}})$, $\Omega_t^\pi = \text{diag}(e^{h_{2N+1,t}}, \ldots, e^{h_{2N+r_\pi,t}})$, and
$\Omega_t^y = \text{diag}(e^{h_{2N+r_\pi+1,t}}, \ldots, e^{h_{2N+r_\pi+r_y,t}})$.

The assumption that the errors are driven by latent factors (f_t and g_t) allows for contemporaneous interdependencies across countries. However, Wu (2022) assume the coefficient matrices P, A, Φ and Θ are diagonal. It is with this assumption that we part with them. One would expect that country i variables depend on other countries' variables, either contemporaneously or with a lag. Therefore, we relax this diagonal assumption to allow for a more comprehensive investigation of dependencies. Specifically, we assume that the coefficient matrices P, A, Φ and Θ are full matrices:

$$
\begin{aligned}
\pi_{i,t} - \tau_{i,t}^\pi = &\rho_{i,1}(\pi_{1,t-1} - \tau_{1,t-1}^\pi) + \rho_{i,2}(\pi_{2,t-1} - \tau_{2,t-1}^\pi) + \cdots + \rho_{i,N}(\pi_{N,t-1} - \tau_{N,t-1}^\pi) \\
&+ \alpha_{i,1}(y_{1,t} - \tau_{1,t}^y) + \alpha_{i,2}(y_{2,t} - \tau_{2,t}^y) + \cdots + \alpha_{i,N}(y_{N,t} - \tau_{N,t}^y) \\
&+ L_{i,\pi} f_t + u_{i,t}^\pi
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
y_{i,t} - \tau_{i,t}^y = &\phi_{i,1}(y_{1,t-1} - \tau_{1,t-1}^y) + \phi_{i,2}(y_{2,t-1} - \tau_{2,t-1}^y) + \cdots + \phi_{i,N}(y_{N,t-1} - \tau_{N,t-1}^y) \\
&+ \theta_{i,1}(y_{1,t-2} - \tau_{1,t-2}^y) + \theta_{i,2}(y_{2,t-2} - \tau_{2,t-2}^y) + \cdots + \theta_{i,N}(y_{N,t-2} - \tau_{N,t-2}^y) \\
&+ L_{i,y} g_t + u_{i,t}^y
\end{aligned}
\tag{3}
$$

where $\rho_{i,j}$ for $i,j = 1,\ldots,N$ represents the affect of country j inflation gap on country i inflation. Similarly, $\alpha_{i,j}$ for $i,j = 1,\ldots,N$ represents the affect of country j output gap on country i inflation. $\phi_{i,j}$ and $\theta_{i,j}$ represent the affect of country j output gap on country i output. Equation (2)-(3) specify the model for country i.

Written in matrix, we can obtain the multi-country PUC-FSV model specification:

$$\begin{aligned}
\pi_t - \tau_t^{\pi} &= P(\pi_{t-1} - \tau_{t-1}^{\pi}) + A(y_t - \tau_t^{y}) + L_{\pi} f_t + u_t^{\pi},\ f_t \sim \mathcal{N}(0,\ \Omega_t^{\pi}),\ u_t^{\pi} \sim \mathcal{N}(0,\ \Sigma_t^{\pi}) \\
y_t - \tau_t^{y} &= \Phi(y_{t-1} - \tau_{t-1}^{y}) + \Theta(y_{t-2} - \tau_{t-2}^{y}) + L_y g_t + u_t^{y},\ g_t \sim \mathcal{N}(0,\ \Omega_t^{y}),\ u_t^{y} \sim \mathcal{N}(0,\ \Sigma_t^{y}) \\
\tau_{i,t}^{\pi} &= \tau_{i,t-1}^{\pi} + \varepsilon_{i,t}^{\tau\pi},\quad \varepsilon_{i,t}^{\tau\pi} \sim \mathcal{N}(0,\ \sigma_{\tau\pi}^2),\ i = 1,\ldots,N \\
\tau_{i,t}^{y} &= \tau_{i,t-1}^{y} + \varepsilon_{i,t}^{\tau y},\quad \varepsilon_{i,t}^{\tau y} \sim \mathcal{N}(0,\ \sigma_{\tau y}^2) \qquad (4) \\
h_{j,t} &= h_{j,0} + \omega_j^h \widetilde{h}_{j,t} \\
\widetilde{h}_{j,t} &= \widetilde{h}_{j,t-1} + \varepsilon_{j,t}^{h},\quad \varepsilon_{j,t}^{h} \sim \mathcal{N}(0,\ 1),\ j = 1,\ldots,2N + r_{\pi} + r_y
\end{aligned}$$

We refer to this specification as Panel Unobserved Components model with Factor Stochastic Volatility (PUC-FSV). Our PUC-FSV has two important features.

Firstly, we have two sets of equations. The first set is for the gaps (the first two lines in Equation (4)) and it allows us to study the effects of transitory gap shocks. The second set is for the trends (the third and fourth lines in Equation (4)) and it allows us to study the effects of permanent trend shocks.

Secondly, we do not impose any zero restriction on the coefficient matrices, which means that we work with the unrestricted PUC-FSV. Leaving panel model unrestricted can lead to enormous parameters to estimate. And to deal with over-parameterization concerns, we rely on a global-local shrinkage prior. The Horseshoe prior is a global-local shrinkage prior and empirically successful. Next, we describe the Horseshoe prior and the priors for all other parameters.

2.2 Priors

We first introduce the prior for full matrices P, A, Φ and Θ. Then we introduce the prior for other parameters.

The prior for full matrices is the Horseshoe prior. The Horseshoe prior was proposed by

Carvalho et al. (2010). It involves a global shrinkage parameter (τ) and a local shrinkage parameter (λ). Carvalho et al. (2010) have shown that the induced distributions over the global and local shrinkage parameters allow for optimal rates of shrinkage near zero, while having sufficiently thick tails (see Cross et al. (2020), 2020).

ore specifically, we use the inverse-Gamma representation of Horseshoe prior for elements in the full matrix P, A, Φ and Θ. We assume that the global shrinkage parameter (τ) is specified to differ across types of parameters, that is, each full matrix has two global shrinkage parameters: one for own country coefficients and one for other country coefficients. For instance, suppose that $\rho_{i,m}$ is an element in P, if $i = m$, then $\rho_{i,i}$ is own country coefficient, and if $i \neq m$, then $\rho_{i,m}$ is other country coefficient, then the Horseshoe prior for own country coefficient $\rho_{i,i}$ is:

$$\begin{aligned}
&\rho_{i,i} \mid \lambda^{\rho}_{i,i}, \tau^{\rho,d} \sim \mathcal{N}(0,\ \lambda^{\rho}_{i,i}\tau^{\rho,d}),\ i = 1, \ldots, N \\
&\lambda^{\rho}_{i,i} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu^{\rho}_{i,i}}), \quad \tau^{\rho,d} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{\rho,d}}) \\
&\nu^{\rho}_{i,i} \sim \mathcal{IG}(\frac{1}{2},\ 1), \qquad \xi^{\rho,d} \sim \mathcal{IG}(\frac{1}{2},\ 1)
\end{aligned} \tag{5}$$

the Horseshoe prior for other country coefficient $\rho_{i,m}$ is:

$$\begin{aligned}
&\rho_{i,m} \mid \lambda^{\rho}_{i,m}, \tau^{\rho,nd} \sim \mathcal{N}(0,\ \lambda^{\rho}_{i,m}\tau^{\rho,nd}),\ i = 1, \ldots, N,\ m = 1, \ldots, N,\ i \neq m \\
&\lambda^{\rho}_{i,m} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu^{\rho}_{i,m}}), \quad \tau^{\rho,nd} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{\rho,nd}}) \\
&\nu^{\rho}_{i,m} \sim \mathcal{IG}(\frac{1}{2},\ 1), \qquad \xi^{\rho,nd} \sim \mathcal{IG}(\frac{1}{2},\ 1)
\end{aligned} \tag{6}$$

To ensure stationarity, we impose condition on P, A, Φ and Θ. Specifically, we first rewrite the first two equations in Equation (4) as a VAR(1):

$$\begin{pmatrix} \pi_t - \tau^{\pi}_t \\ y_{t+1} - \tau^{y}_{t+1} \\ y_t - \tau^{y}_t \\ y_{t-1} - \tau^{y}_{t-1} \end{pmatrix} = \begin{pmatrix} P & A & 0 & 0 \\ 0 & \Phi & \Theta & 0 \\ 0 & I_N & 0 & 0 \\ 0 & 0 & I_N & 0 \end{pmatrix} \begin{pmatrix} \pi_{t-1} - \tau^{\pi}_{t-1} \\ y_t - \tau^{y}_t \\ y_{t-1} - \tau^{y}_{t-1} \\ y_{t-2} - \tau^{y}_{t-2} \end{pmatrix} + \begin{pmatrix} L_\pi f_t + u^{\pi}_t \\ L_y g_{t+1} + u^{y}_{t+1} \\ 0 \\ 0 \end{pmatrix} \tag{7}$$

Then we obtain the VAR(1) representation. The stability condition requires that all the eigenvalues of coefficient matrix are smaller than one in modulus. And we use the command "eig" in Matlab to compute eigenvalues. The diagonal elements in A is the slope of Phillips curve, so we also constrain them to be positive.

The priors on other parameters are the same as that in Wu (2022). More specifically, we model the evolution of the log-volatility according to a random walk in non-centered parameterization and then use the Horseshoe prior to control time-variation. For each $j = 1, \ldots, 2N + r_\pi + r_y$, the evolution of the log-volatility is modeled as:

$$
\begin{aligned}
h_{j,t} &= h_{j,0} + \omega_j^h \widetilde{h}_{j,t} \\
\widetilde{h}_{j,t} &= \widetilde{h}_{j,t-1} + + \varepsilon_{j,t}^h, \quad \varepsilon_{j,t}^h \sim \mathcal{N}(0,\ 1)
\end{aligned} \tag{8}
$$

The non-centered parameterization decomposes a time-varying parameter $h_{j,t}$ into two parts: a time-invariant part $h_{j,0}$ and a time-varying part $\omega_j^h \widetilde{h}_{j,t}$, which has a constant coefficient ω_j^h that controls the time-variation. For constant parameters ω_j^h and $h_{j,0}$, we use the Horseshoe prior:

$$
\begin{aligned}
&\omega_j^h \mid \lambda_j^{\omega^h}, \tau^{\omega^h} \sim \mathcal{N}(0,\ \lambda_j^{\omega^h} \tau^{\omega^h}),\ j = 1, \ldots, 2N + r_\pi + r_y \\
&\lambda_j^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu_j^{\omega^h}}), \quad \tau^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{\omega^h}}) \\
&\nu_j^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ 1), \qquad \xi^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ 1)
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
&h_{j,0} \mid \lambda_j^{h_0}, \tau^{h_0} \sim \mathcal{N}(0,\ \lambda_j^{h_0} \tau^{h_0}),\ j = 1, \ldots, 2N + r_\pi + r_y \\
&\lambda_j^{h_0} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu_j^{h_0}}), \quad \tau^{h_0} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{h_0}}) \\
&\nu_j^{h_0} \sim \mathcal{IG}(\frac{1}{2},\ 1), \qquad \xi^{h_0} \sim \mathcal{IG}(\frac{1}{2},\ 1)
\end{aligned} \tag{10}
$$

The initial states are assumed to follow normal distribution with zero mean and variance ten, that is:

$$\tau_{i,1}^\pi \sim \mathcal{N}(0,\ 10),\ i = 1, \ldots, N \tag{11}$$

$$\tau_{i,1}^y \sim \mathcal{N}(0,\ 10),\ i = 1, \ldots, N \tag{12}$$

$$\widetilde{h}_{j,1} \sim \mathcal{N}(0,\ 10),\ j = 1, \ldots, 2N + r_\pi + r_y \tag{13}$$

The elements in factor loading matrices are assumed to follow a normal distribution with zero mean and variance ten, that is:

$$l_m \sim \mathcal{N}(0,\ 10),\ m = 1, \ldots, n_{l,\pi} + n_{l,y} \tag{14}$$

where $n_{l,\pi}$ denotes the number of free elements in matrix L_π, and $n_{l,y}$ denotes the number of free elements in matrix L_y.

The error variances are assumed to follow inverse gamma distribution, that is:

$$\sigma^2_{\tau\pi} \sim \mathcal{IG}(10,\ 0.18),\ i = 1, \ldots, N \tag{15}$$
$$\sigma^2_{\tau y} \sim \mathcal{IG}(10,\ 0.09),\ i = 1, \ldots, N \tag{16}$$

We use the Markov Chain Monte Carlo (MCMC) algorithm to sample all parameters. More specifically, to sample $\tau^{\pi}_{i,t}$, the prior still follows a random walk process, but the likelihood will come from N equations and each equation is defined through Equation (2). To sample $\tau^{y}_{i,t}$, the prior still follows a random walk process, but the likelihood will come from two parts: the first part is N equations in Equation (2), the second part is N equations in Equation (3). It is standard to sample other parameters and we refer readers to Chan et al. (2016) and Chan (2021) for details.

3 Data, Estimates and Forecasting

In this section, we first introduce the data, then we provide evidence of more precise estimates of trend. This is followed by a forecasting exercise to show that the PUC model generates competitive forecasting results.

3.1 Data

The data are the quarterly consumer price index (CPI) and the quarterly real gross domestic product (GDP) for 34 countries, 23 advanced economies (AEs)[4] and 11 emerging

[4] Australia, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Israel, Italy, Latvia, Lithuania, Netherlands, Portugal, Slovakia, South Korea, Spain, Sweden, Switzerland, UK, USA.

market economies (EMEs)[5]. They span the period from 1995Q1 to 2018Q1. We transform the data to annualized growth rates as: $400\log(z_t/z_{t-1})$. And because the output gap equation follows an AR(2) process, our estimation start from 1995Q4. We assume that there is one common factor driving 34-country inflation, that is $r_\pi = 1$. We assume that there is one common factor driving 34-country output, that is $r_y = 1$. This assumption comes from the empirical results in Wu (2022). They find there is one global factor driving 34-country inflation and one global factor driving 34-country output. Posterior results are based on 100000 draws after a burn-in period of 20000.

3.2 Estimates of Trend

We consider 34 countries: the first 23 countries are AEs (from Belgium to Canada) and the following 11 countries are EMEs (from South Africa to Thailand). We compare three models: (a) Bi-UC-SV (the model in Chan et al., 2016. The coefficients are constant, but it allows for stochastic volatility in inflation gap equation); (b) UC-FSV (the model in Wu, 2022. The number of common factors is set to one, that is, $r_\pi = 1$, $r_y = 1$); and (c) PUC-FSV model. The differences are: the Bi-UC-SV model does not allow for any interdependencies across countries, the UC-FSV model allows for cross-country interdependencies that occur contemporaneously, while the PUC-FSV model allows for cross-country interdependencies that occur both contemporaneously and with a lag.

The precision of the estimates are measured by the width of the 84% credible intervals. We report the width of the 84% credible intervals in Figures 1-2. Figure 1 is for the trend inflation and Figure 2 is for the trend output. The broad contours reflected in both figures is that the PUC model generates more precise estimates of trend. And it is the Bi-UC-SV that always generate least estimates of trend. This emphasizes the importance of allowing for interdependencies.

[5]Bolivia, Brazil, China, Hungary, Indonesia, Mexico, Philippines, Russia, South Africa, Thailand, Turkey.

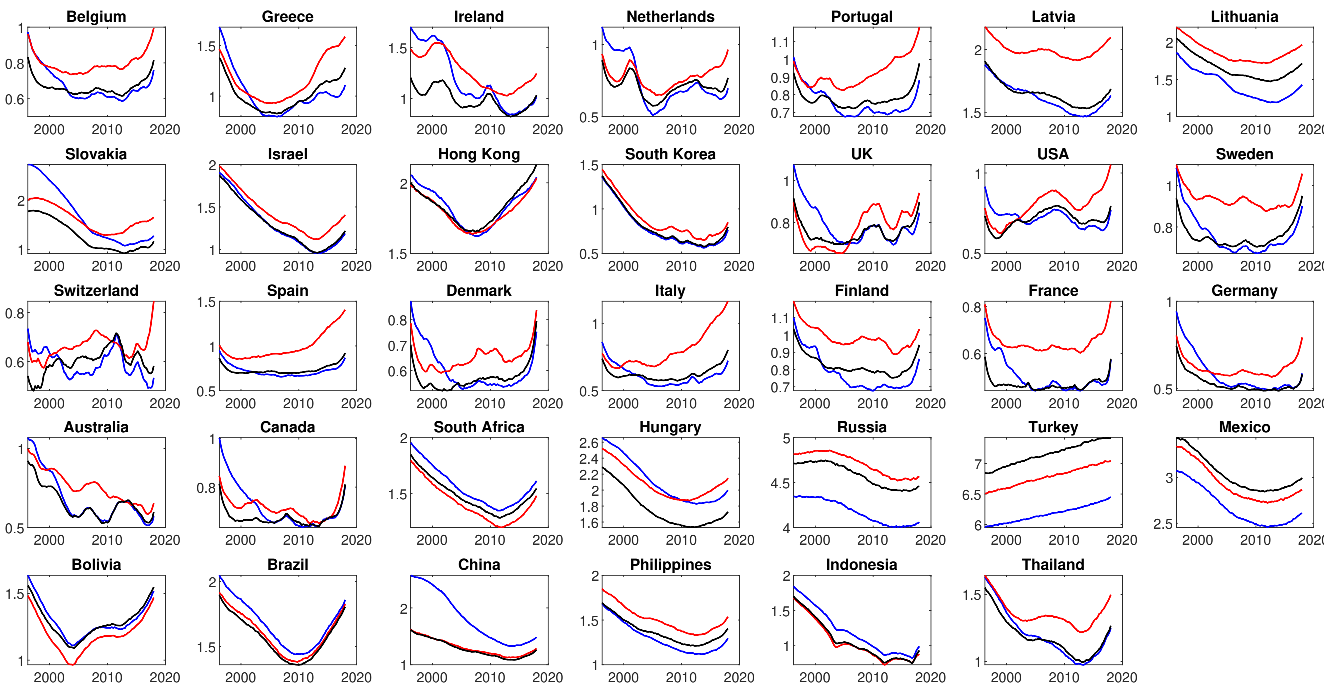

Figure 1: The width of 84% credible interval for trend inflation under three models: Bi-UC-SV, UC-FSV and PUC-FSV. The red lines are the width under Bi-UC-SV. The black lines are the width under UC-FSV. The blue lines are the width under PUC-FSV.

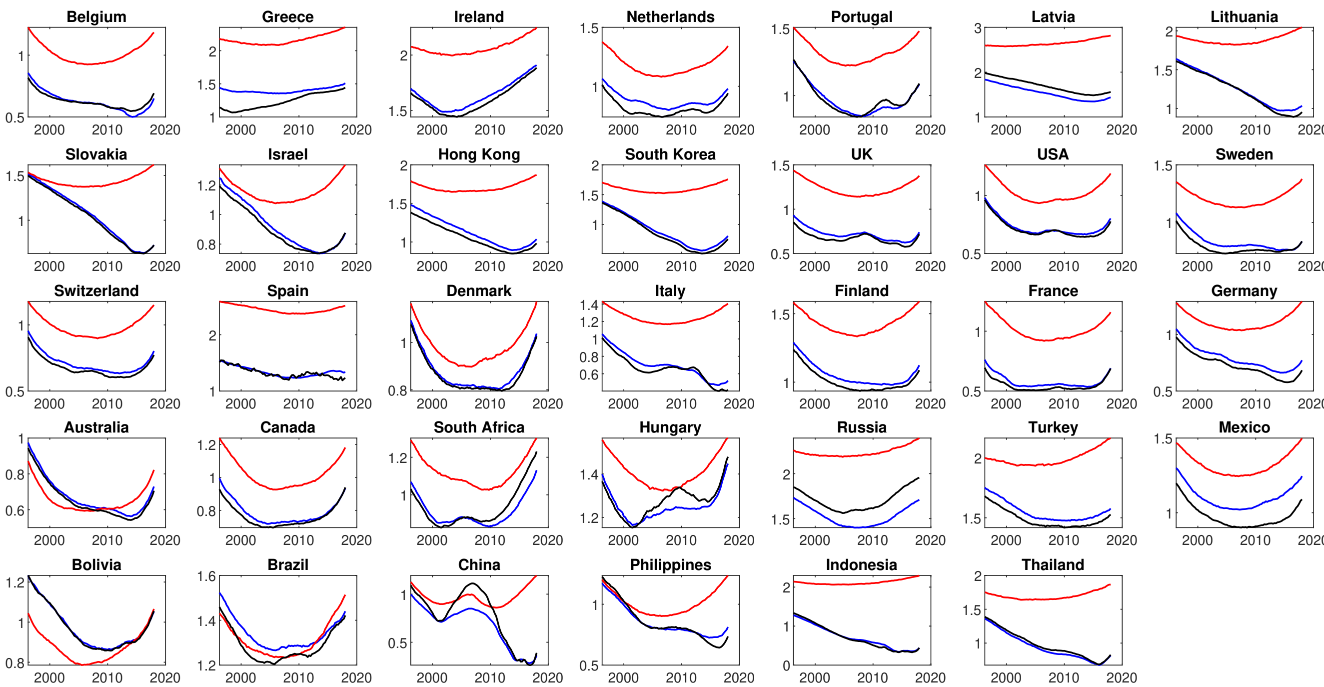

Figure 2: The width of 84% credible interval for trend output under three models: Bi-UC-SV, UC-FSV and PUC-FSV. The red lines are the width under Bi-UC-SV. The black lines are the width under UC-FSV. The blue lines are the width under PUC-FSV.

3.3 Forecasting Performance

Before reporting the out-of-sample forecasting results, we first report the in-sample fit results. The gold standard is using marginal likelihood, however, in our settings where we allow for time-variation in volatility, the computation of marginal likelihood requires integrating out all the states, making it a nontrivial task. Therefore, we use an approximation to the marginal likelihood (e.g., Geweke, 2001; Cross et al., 2020). They propose that conditioning on the estimation period, the sums of one-step-ahead joint log predictive likelihoods of 34 countries can be viewed as an approximation to the marginal likelihood, therefore provides a direct measure of in-sample fit.

Our estimation period starts from 1995Q4 (to 2018Q1), and the forecasting evaluation period starts from 2003Q1. We provide the sums of one-step-ahead joint log predictive likelihoods of 34 countries in Table 1. Results are presented relative to the forecast performance of the Bi-UC-SV: we take differences, so that a positive number indicates a model is forecasting better than Bi-UC-SV. (Please note that we only take the sum, and no average. That may be why the number seems so large. For example, the sums of LPL under PUC-FSV is 918.44. If we take average over time, then it is 15.06. If we take further average across country, then it is 0.44). The results show that the PUC-FSV provides the highest model fit.

Table 1: Sum of one-step-ahead log predictive likelihood

Model	against Bi-UC-SV
Bi-UC-SV	0
UC-FSV	885.51
PUC-FSV	918.44

we now compare the out-of-sample forecast performance of the three models. We use the data from 1995Q4 to 2002Q4 as an initial estimation period, and use data through 2002Q4 to produce k-step-ahead forecast distributions. We consider forecast horizons of $k = 1, 2, 3, 4, 6$ quarters. So our forecast evaluation period begins in 2003Q1. We divide our out-of-sample forecasting results into three parts: forecasting inflation, forecasting output and jointly forecasting inflation and output. We report the aggregate forecasting performance over time and across countries (the aggregate LPL, by summing all countries and all time periods). In Appendix A, we discuss the results in another two dimensions.

The first dimension is about forecasting performance over time (we can study how the sums of LPL changes over time, by summing all countries at time t). After providing evidence that PUC-FSV can produce more accurate estimate in economic recession, we further study whether such good forecast performance is driven by particular countries, so the second dimension is about the forecasting performance at country level. All results are presented relative to the forecast under Bi-UC-SV: we take differences, so a positive number indicates a model is forecasting better than Bi-UC-SV.

The results in Table 2 show that PUC-FSV provides the most accurate density forecast for inflation, output, and joint forecast at all horizons. The only exception is one-quarter-ahead forecast of inflation, but the loss is quite small (PUC-FSV is 114.75, while the UC-FSV is 124.57).

Table 2: Sum of k-step-ahead log predictive likelihood

Forecasting Inflation					
Model	k=1	k=2	k=3	k=4	k=6
Bi-UC-SV	0	0	0	0	0
UC-FSV	**124.57**	210.18	216.28	223.02	286.31
PUC-FSV	114.75	**252.52**	**280.23**	**286.59**	**387.72**
Forecasting Output					
Model	k=1	k=2	k=3	k=4	k=6
Bi-UC-SV	0	0	0	0	0
UC-FSV	750.25	979.29	969.58	830.92	942.70
PUC-FSV	**791.86**	**1010.76**	**1010.26**	**1041.74**	**1112.64**
Joint Forecasting					
Model	k=1	k=2	k=3	k=4	k=6
Bi-UC-SV	0	0	0	0	0
UC-FSV	885.51	1111.32	1221.00	1044.97	1093.88
PUC-FSV	**918.44**	**1169.58**	**1291.92**	**1318.78**	**1253.67**

4 Impulse Response Analysis

A good aspect of panel model is that, after allowing for interdependencies, it can model the manner in which shocks are transmitted across countries (see Dees et al., 2007 and Canova and Ciccarelli, 2009). An excellent aspect of PUC model is that it allows us to study the permanent trend shocks and transitory gap shocks. We are interested in computing the responses of the endogenous variables to shocks and in describing their evolution over time. In this situation, we use the Generalized Impulse Response Functions (GIRFs), proposed in Koop et al. (1996).

With 68 endogenous variables (34 countries and 2 variables of each country) and time-varying parameters, there will be a different set of generalised impulse response functions (GIRFs) at each time in the sample period. However, for our study, we focus on the GIRFs for the end of sample period (2018Q1). we investigate the implications of four different shocks: (a) a 0.01 positive shock to US trend inflation; (b) a 0.01 positive shock to US inflation gap; (c) a 0.01 positive shock to US trend output; and (d) a 0.01 positive shock to US output gap.

More specifically, the computation of generalised impulse response function in Koop et al. (1996) is: given the posterior draws, the GIRF is obtained from the difference between two alternative paths: in one a shock hits the system, and in the other this shock is absent:

$$GIRF_{t+k} = \mathbb{E}[\mathbf{z}_{t+k}|\mathbf{u}_t, \mathbf{I}_t] - \mathbb{E}[\mathbf{z}_{t+k}|\mathbf{I}_t] \tag{17}$$

where $\mathbf{z}_{t+k}$ is the forecast of the endogenous variables at the horizon k, $\mathbf{I}_t$ represent the current information set and $\mathbf{u}_t$ is the current structural disturbance terms.

From the effects of various shocks, we observe two notable differences. The first difference is the effects on inflation between AEs and EMEs. Whatever the shock is, the GIRFs of EMEs inflation go back to zero more slowly than AEs, which implies that a shock has on average a longer-term effect on EMEs inflation. We call this “fragile inflation”in EMEs. One instance is reported in Figure 3. This is the inflation responses to a 0.01 shock to US inflation gap.

The second difference is the effects of shock to US trend output and shock to US output

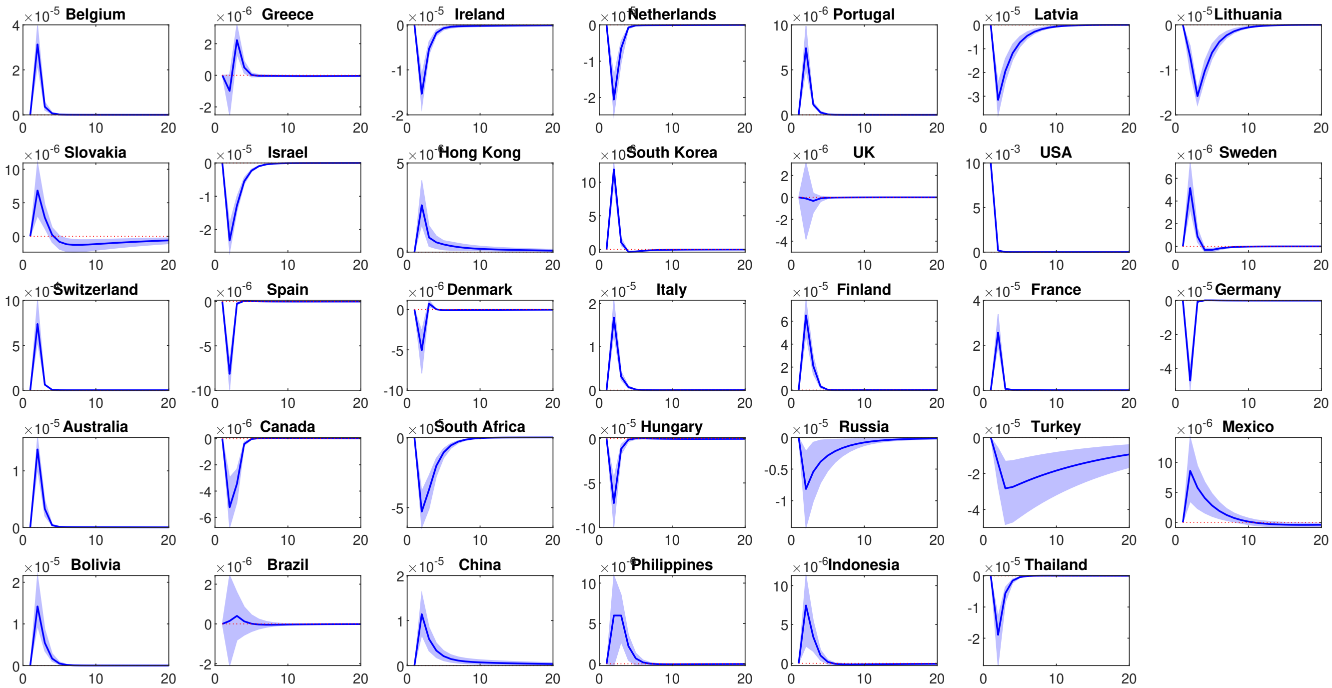

Figure 3: Inflation responses to a 0.01 shock to US inflation gap: Posterior means and 84% credible intervals.

gap. The responses to shock to output gap will be zero after some periods, but the responses to shock to trend output will not be zero for most countries. As shown in Figure 4 (red lines), the output in AEs settles down at a higher value, while the output first increases but then settles down at a lower value. Overall, we find a shock to US trend output will influence the output in other countries and the influence seems to be permanent in some countries.

5 Conclusions

In a globalized world, countries are linked together and cross-country interdependencies may influence the estimates of trend. However, such influence has not been considered in unobserved components models, which are popular to estimate the trend. In this paper, we develop a panel unobserved components model that allows for dependencies that can take place both contemporaneously and with a lag. The model can be used to study effects of the permanent trend shocks and transitory gap shocks. The impulse response

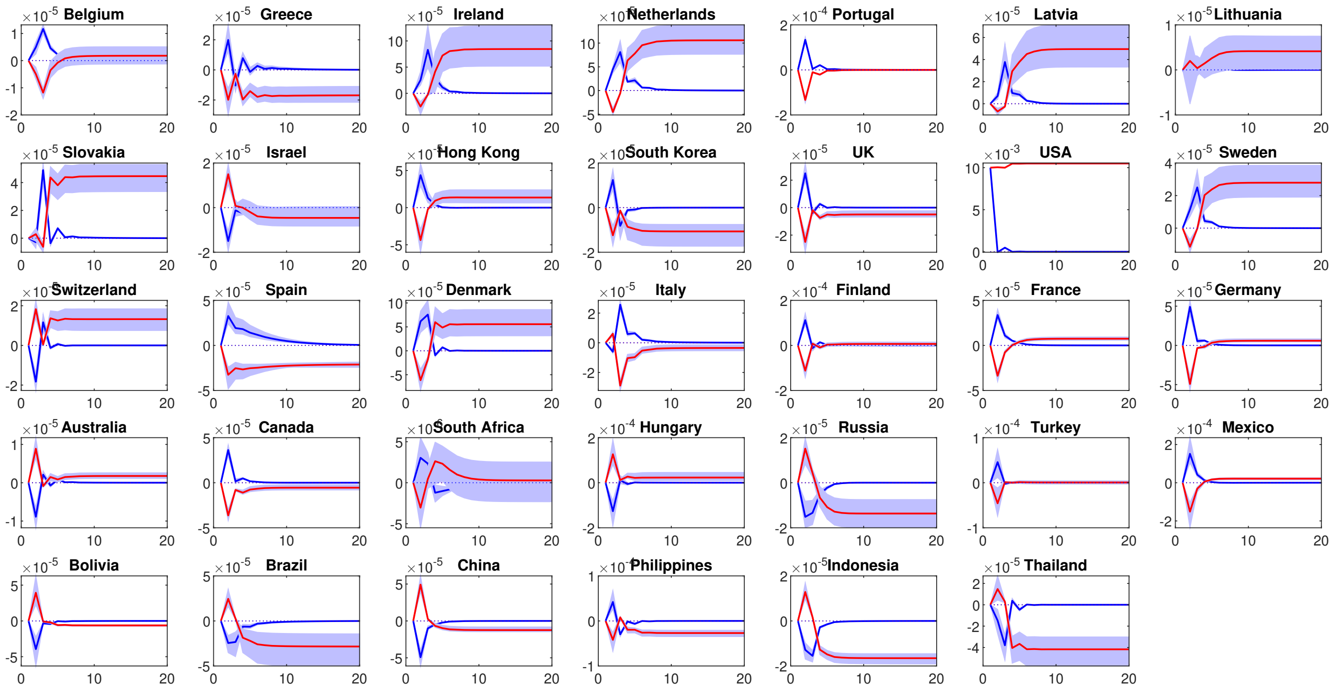

Figure 4: Output responses: Blue lines are the output responses to a 0.01 shock to US output gap (Posterior means and 84% credible intervals). Red lines are the output responses to a 0.01 shock to US trend output.

analysis provide evidence of "fragile inflation"in EMEs. It also shows that a shock to US trend output will generate a permanent influence on the output in some countries. We also find evidence that the model will generate more precise estimates of trend.

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Appendices

A More Forecasting Results

This section reports the forecasting results in another two dimensions. The first dimension is the time-variation feature, and the second dimension is the country feature.

A.1 Forecasting inflation

The first dimension of discussion for inflation is sums of LPL over time (by summing all countries at time t). We plot the results (against Bi-UC-SV) in Figure 5. To forecast inflation during periods of uncertainty (like 2008), we find overall good forecast performance for UC-FSV and PUC-FSV at all horizons. And PUC-FSV forecasts better than UC-FSV at long horizons ($k = 4$ and $k = 6$).

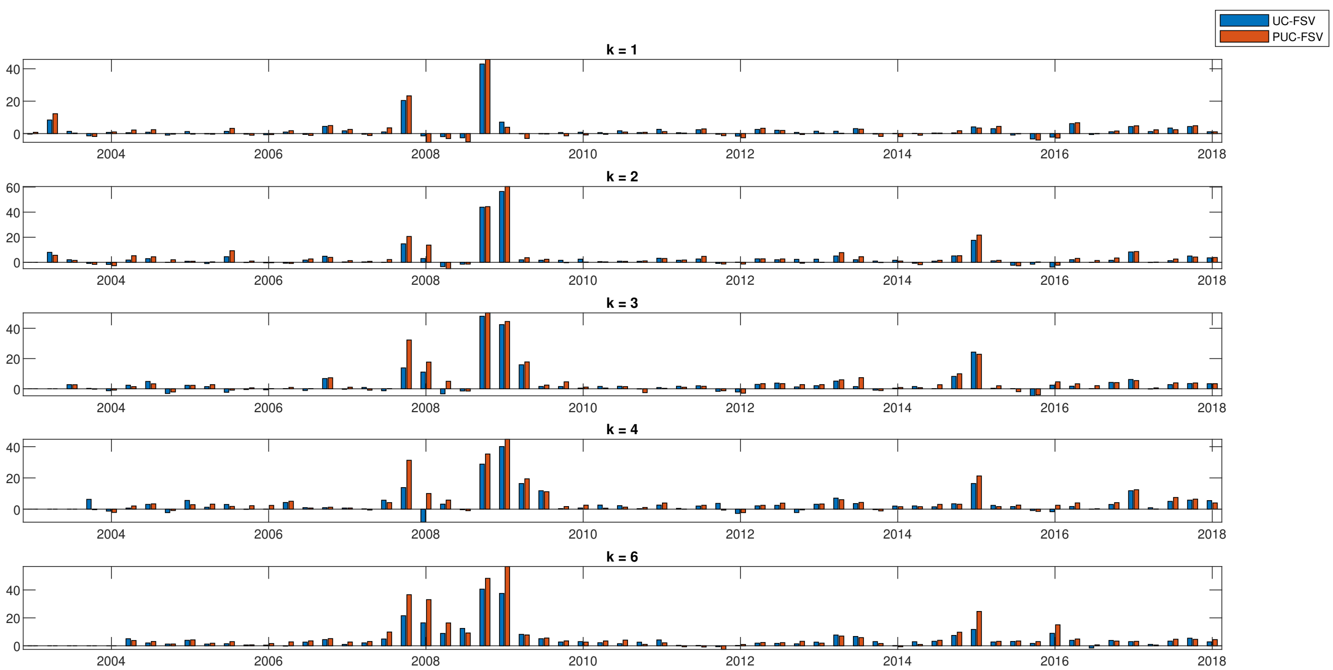

Figure 5: Sums of k-step ahead LPL of inflation for PUC-FSV and UC-FSV relative to Bi-UC-SV over time. The X axis is $t + k$ and represents when to forecast.

The scond dimension of discussion for inflation is the forecasting result for individual countries. We plot the results (against Bi-UC-SV) in Figure 6. Here the period of uncertainty that we plot is 2008Q4, so time to forecast is 2008Q4 ($t + k = 2008Q4$). If $k = 1$, then the time we make forecast is 2008Q3, and we find overall good forecast performance for most countries with more pronounced gains in advanced economies (The first 23 countries are AEs, and the following 11 countries are EMEs). A similar pattern is found if $k = 6$. The time we make forecast is 2007Q2, and we also find overall good forecast performance for most countries. In Figure 6, we only plot the shortest horizon $k = 1$ and the longest horizon $k = 6$, for middle horizons ($k = 2, 3, 4$), we find good forecasting result across most countries and did not find particular country which is important in driving good forecasting results. Overall, We find good forecast performance for most countries and such good forecast performance is not driven by particular countries.

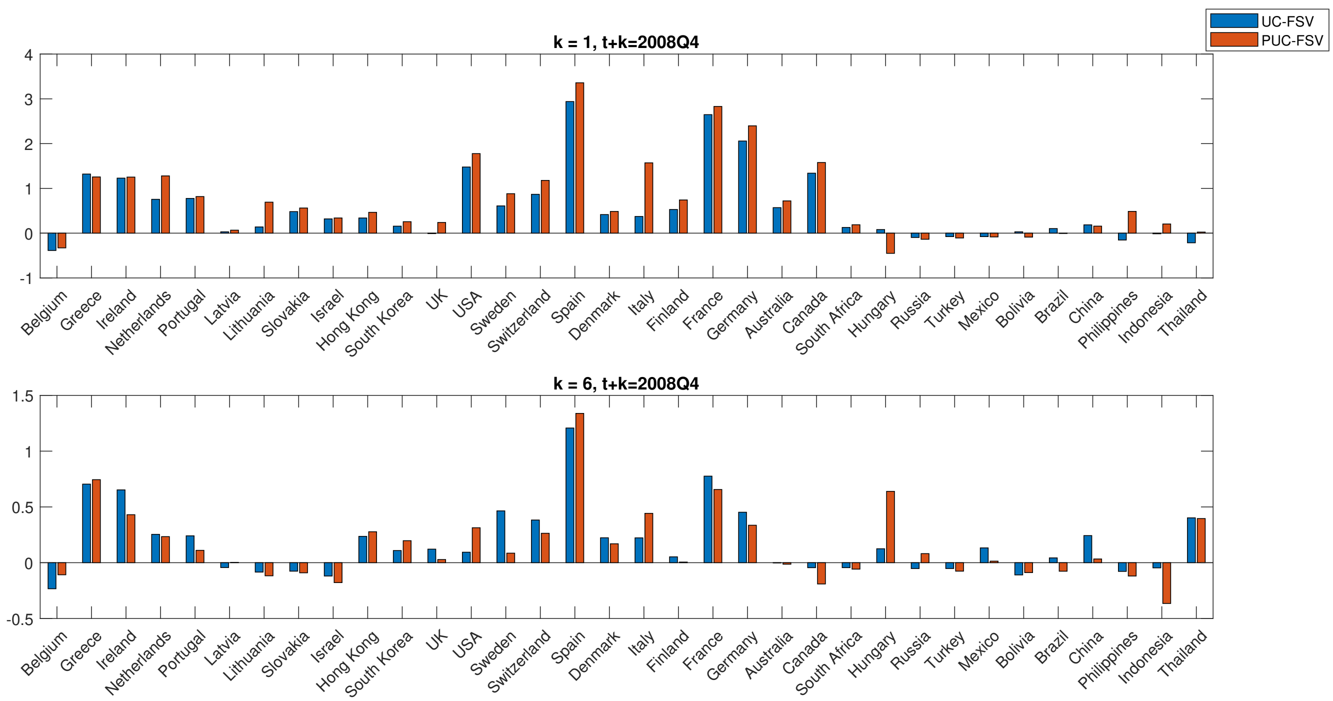

Figure 6: Sums of k-step ahead LPL of inflation for country i under PUC-FSV and UC-FSV relative to Bi-UC-SV.

A.2 Forecasting output

Similar to the analysis of inflation, the second dimension of discussion for output is sums of LPL over time (by summing all countries at time t). We plot the results (against Bi-UC-SV) in Figure 7. To forecast output during periods of uncertainty (like 2008), we find overall good forecast performance for PUC-FSV and UC-FSV at all horizons.

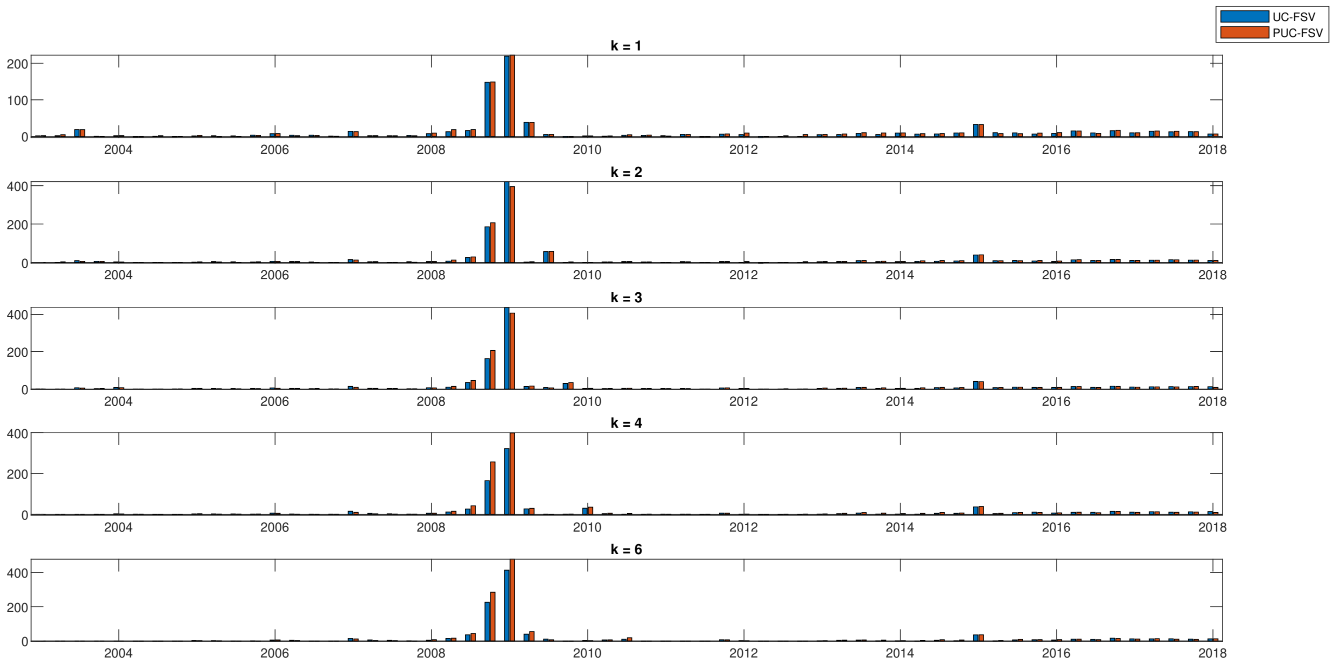

Figure 7: Sums of k-step ahead LPL of output for PUC-FSV and UC-FSV relative to Bi-UC-SV over time. The X axis is $t + k$ and represents when to forecast.

To investigate whether the good forecast performance is driven by particular countries, we plot the results (against Bi-UC-SV) in Figure 8. We choose 2008Q4 to represent the period of uncertainty. For $k = 1$ and $k = 6$, we both find overall good forecast performance for PUC-FSV and UC-FSV for all countries. For several countries (like Spain, Italy and Germany), PUC-FSV forecasts better than UC-FSV.

A.3 Jointly Forecasting inflation and output

we study the time-variation in forecast performance to see whether the benefits arise from the forecast during periods of uncertainty. So the second dimension of discussion for joint

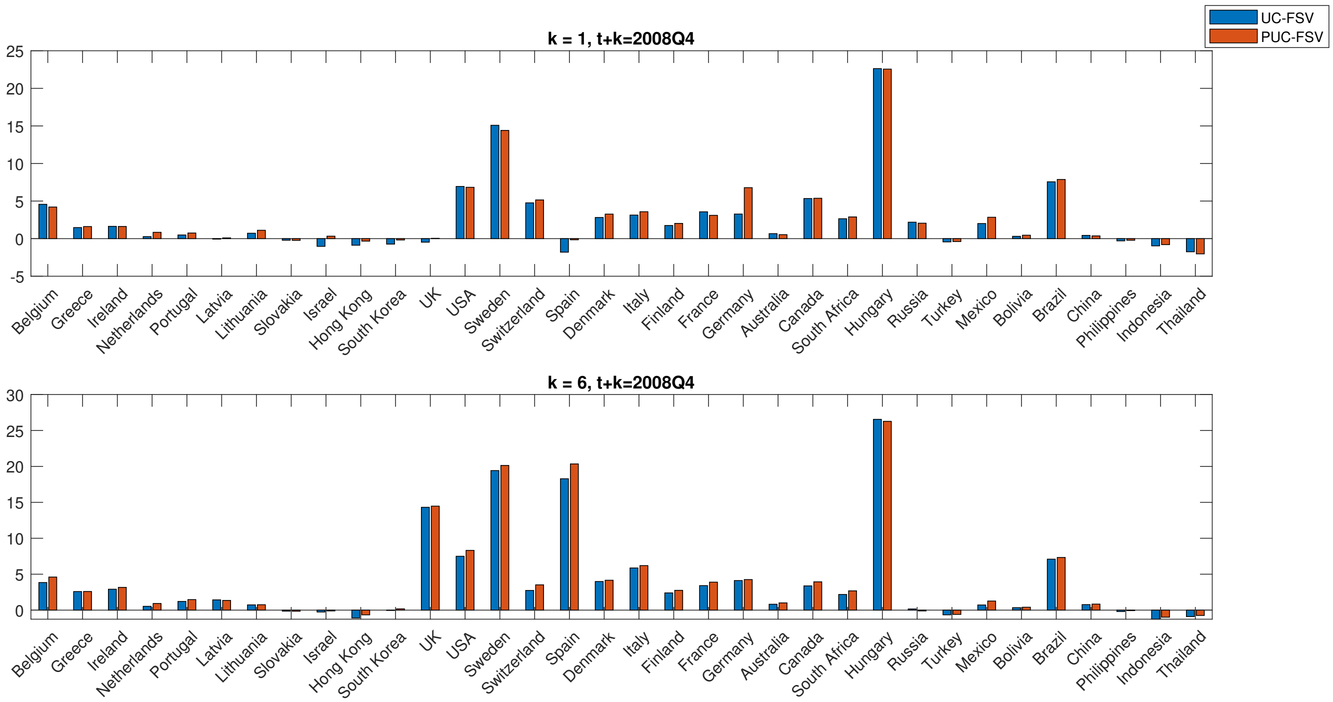

Figure 8: Sums of k-step ahead LPL of output in country i for PUC-FSV and UC-FSV relative to Bi-UC-SV.

predictive density for inflation and output is sums of joint LPL over time. We plot the results (against Bi-UC-SV) in Figure 9. A similar pattern to inflation and output was found. To jointly forecast inflation and output during periods of uncertainty (like 2008), we find overall good forecast performance under PUC-FSV and UC-FSV at all horizons.

Finally, we investigate whether the good forecast performance of periods of uncertainty is driven by particular countries, so the third dimension of discussion for joint predictive density for inflation and output is sums of joint LPL at the country level. We plot the results (against Bi-UC-SV) in Figure 10. A similar pattern to output is found. (This is sensible since the gains in output are much larger than gains in inflation, see Figure 6 and Figure 8). We find overall good forecast performance for PUC-FSV for all countries.

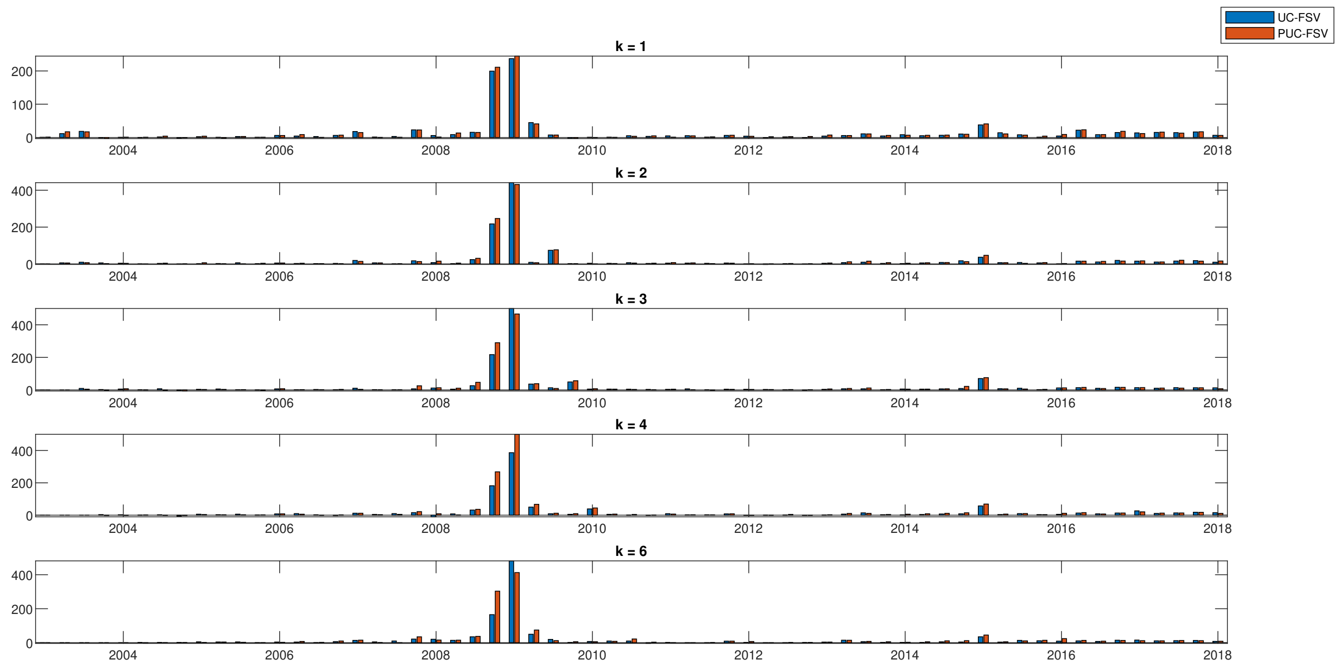

Figure 9: Sums of k-step ahead joint LPL for PUC-FSV and UC-FSV relative to Bi-UC-SV over time. The X axis is $t + k$ and represents when to forecast.

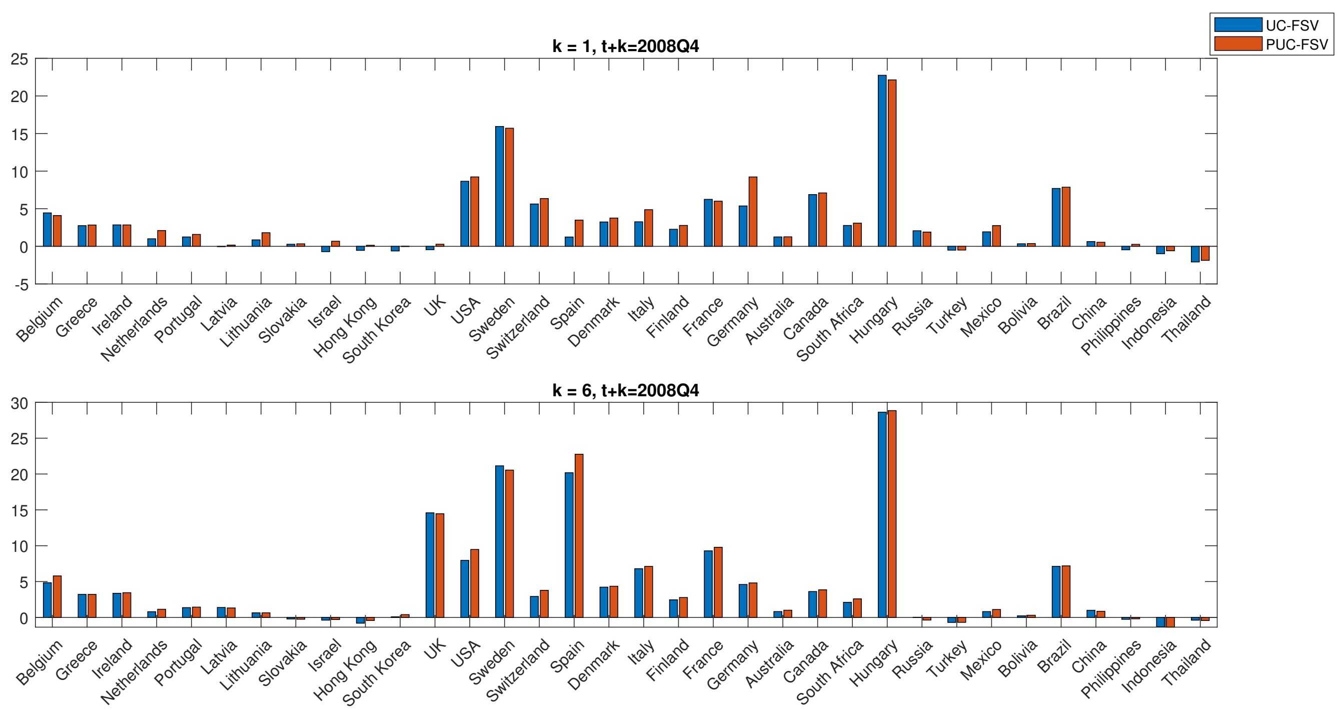

Figure 10: Sums of k-step ahead joint LPL in country i for PUC-FSV and UC-FSV relative to Bi-UC-SV.