Sparse Factor Stochastic Volatility for A Multi-country Unobserved Components Model *

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Abstract

This paper develops a multi-country unobserved components model that allows for cross-country linkages and models all economies jointly. We extend the existing literature in two ways. First, cross-country linkages are realized through the factor stochastic volatility specification. Second, our framework allows us to coherently estimate the factors and the number of factors in one step, rather than rely on a two-step approach common in the factor literature. We demonstrate the merits of our model through a multi-country study of inflation and output involving 34 economies. We provide evidence of significant commonality in output volatility, with one common factor driving strong comovement across economies. The results show that global uncertainty will flatten the Phillips Curve. The estimates of trends are in line with previous studies and, for certain economies, the estimates indicate that they are influenced by both domestic factors and global factors. We find that our proposed model provides a superior in-sample fit and accurate density forecasts compared to existing models in the literature, especially if the aim is to forecast periods of uncertainty.

JEL classification: F44, C11, C55
Kerwords: unobserved components models, cross-country linkages, factor stochastic volatility, global uncertainty, sparsification

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1 Introduction

Since the seminal work by Stock and Watson (2007), the unobserved components (hereafter UC) model with stochastic volatility (hereafter SV) is commonly used for modeling latent state vectors that can be interpreted as long-run equilibrium levels and has enjoyed great popularity. However, the existing literature is limited to country-specific stochastic volatility, and imposes an independent assumption across economies. No UC model has allowed for global uncertainty.

The last 30 years have witnessed a dramatic increase in globalisation. The globalisation has prompted the emergence of a recent literature which classifies uncertainty as a global phenomenon and assesses its effects on the global economy and on individual economies.

Extending prior work on the UC model, in this paper we develop a multi-country unobserved components model that allows for cross-country linkages and models all economies jointly. To that end, we start from the UC model for individual economies, then allow for cross-country linkages by introducing the factor stochastic volatility (hereafter FSV) specification. The FSV specification provides a measurement of global uncertainty and supports us to analyse the effects of global uncertainty on individual economies. The economies considered in this paper include 23 Advanced Economies (hereafter AEs) and 11 Emerging Market Economies (hereafter EMEs). We include two variables in each economy (quarterly CPI inflation and output).

Our results suggest that the factors, which we label as the Global Inflation Factor and Global Output Factor, drive strong co-movement across economies. The spikes in the volatility associated with these global factors coincide with major economic events.

Our results show that global uncertainty will flatten the Phillips Curve. This is in line with Forbes (2019). They include comprehensive controls for globalisation because globalisation is often cited as causing the flattening of the Phillips Curve.

Our results also include evaluation of forecast performance. We find that the model with global uncertainty can generate more accurate density forecasts compared to the model without global uncertainty, especially if the aim is to forecast periods of uncertainty. And such good forecast performance is for most economies and not driven by particular economies.

This paper is organized as follows. Section 2 reviews the related empirical literature and explains our contributions. In Section 3, we first discuss the UC model for individual economies, then introduce our new model, which models all economies jointly and allows for cross-country linkages through FSV. The details of our new model also includes an elaborated account of the sparsification. We use the sparsification to coherently estimate the factors and the number of factors in one step. We also use the sparsification to remove stochastic volatility in a data-based manner. Section 4 illustrates our modeling approach by fitting our model to 34 economies. We divide them into four parts. In the first part, we present estimates of global uncertainty. The second part is the evidence of using the sparsification. The third part is Bayesian model comparison, where we show that allowing for cross-country linkages will improve in-sample fit. After justifying the importance of cross-country linkages, we present, in the fourth part, estimates of country-specific parameters with the effects of global uncertainty on them. Section 5 is an out-of-sample forecasting exercise. We provide evidence that our proposed model can improve the inflation forecasting, output forecasting, and jointly forecasting. Finally, Section 6 concludes.

2 Relationship to prior work

To make clear our contributions, we first briefly summarize the most closely related studies of the unobserved components model and global uncertainty. We then detail key differences in our analysis compared to the most closely related studies. In broad terms, our work extends the literature by a combination of allowing for cross-country linkages, the use of sparsification, and considering a large number of economies.

The unobserved components model A large body of research has emerged on extending the UC model. One strand of extensions has focused on introducing more indicators into the conditional mean (e.g., Stella and Stock, 2013; Chan et al., 2018; Zaman, 2021 and Kabundi et al., 2021). Another strand of extensions of the UC model has focused on adding bounds on parameters (e.g., Chan et al., 2013; Chan et al., 2016; Zaman, 2021 and Kabundi et al., 2021).

There has been a lot of recent research devoted to introducing suitable indicators into the UC model. These indicators are guided by either economic theory or empirical research. For example, inspired by the Phillips curve, Stella and Stock (2013) extend the univariate UC model in Stock and Watson (2007) to bivariate UC model, and assume that it is inflation gap and unemployment gap[1] that drive the Phillips curve. Based on public commentary that central bankers pay considerable attention to measures of long-run inflation expectations, Chan et al. (2018) develop a bivariate model by introducing survey-based long-run forecasts of inflation into the UC model. To directly address critiques of omitted variable and omitted equation bias pointed out by Taylor and Wieland (2016), Zaman (2021) further extends the bivariate UC model of Stella and Stock (2013) to a large-scale UC model. In particular, they jointly estimate trends of several macroeconomic variables (they call them “stars”) and build up a rich structure for each star. The observed flattening of Phillips curve has generated various explanations of this conundrum and one camp

[1]Inflation gap is deviation of inflation from its trend, and similar interpretation of unemployment gap, deviation of unemployment rate from its trend.

highlights the role played by global factors. Therefore, Kabundi et al. (2021) introduce global factors (global output and oil price) into the bivariate UC model of Stella and Stock (2013) to estimate trends of inflation and output. In this paper, we follow Stella and Stock (2013) to incorporate Phillips curve into UC model. One may question the existence of Phillips curve, but McLeay and Tenreyro (2020) emphasize that the Phillips curve exists and policymakers are completely aware of its existence. Stock and Watson (2008) also raised the point that the Phillips curve is useful for conditional forecasting. So we expect that the Phillips curve still exists, even though we are observing that it has flattened (e.g., Ball and Mazumder, 2011; Hall et al., 2013 and Blanchard et al., 2015).

To constrain parameters to lie in reasonable intervals, we follow Chan et al. (2013). They first develop a method to constrain parameters to lie in intervals. This can avoid parameters move into undesirable regions. They find the model yields more sensible measures of trends than popular alternatives. Since their seminal work, there have been many applications that have employed the use of adding bounds on parameters (e.g., Chan et al., 2016; Zaman, 2021 and Kabundi et al., 2021).

As to the relationship of our paper to prior studies on the UC model, while our paper shares the two strands of extensions of UC model (introducing more indicators and constraining parameters to lie in reasonable intervals), we believe our paper provides further extensions. In this paper, we propose an approach to allow for cross-country linkages in the multi-country UC model. However, in the prior studies, the errors in the UC model are assumed to be independent with each other. This independent assumption is even used in multi-country studies (e.g., Chan et al., 2018 and Kabundi et al., 2021). A priori, one would expect that there exist linkages in the errors. And the linkages are more evident in multi-country studies due to globalisation, omitting these linkages may affect the estimate of latent states.

Global uncertainty Several ways to estimate global uncertainty have been proposed in the literature. Mumtaz and Theodoridis (2017) use a factor-augmented vector autoregression (hereafter VAR) model with common stochastic volatility and country-specific stochastic volatility. Pfarrhofer (2019) use a global vector autoregressive specification with FSV in the errors to estimate the impact of global uncertainty on six economies. Cuaresma et al. (2019) use a large-scale Bayesian VAR with FSV to investigate the macroeconomic consequences of international uncertainty shocks in G7 countries. Carriero et al. (2020) measure international macroeconomic uncertainty by featuring the error volatility with a factor structure containing time-varying global components and idiosyncratic components.

As to the relationship of our paper to prior studies on measuring global uncertainty, our paper is closely related to the FSV specification used in Pfarrhofer (2019) and Cuaresma et al. (2019). The contribution of this paper is that we use the sparsification to decide on the number of factors, whereas the prior studies either subjectively choose the number or rely on principal component-based analysis. The sparsification method, proposed by Chakraborty et al. (2020), obviates the need to specify a prior on the rank (in this paper, the rank is equivalent to the number of factors), and shrinks the regression matrix towards a low-rank structure. This sparsification method allows us to coherently estimate the factors and the number of factors in one step.

Our FSV specification shares with Mumtaz and Theodoridis (2017) the feature of allowing for both common and country-specific stochastic volatility. One difference is that we use the sparsification to remove stochastic volatility in a data-based manner.

As regards the relationship of our paper to Carriero et al. (2020), there are mainly two differences. The first difference is about the method. Carriero et al. (2020) rely on principal component-based analysis to decide on the number of factors, and assume that all variables are driven by same factors. Different from them, we use the sparsification and allow for the case that there may exist different global factors driving different variables across economies. Put another way, our contribution is to allow that inflation and output across economies are driven by different global factors. Our empirical work has provided evidence of this. We find the estimated global factors have different spikes. The second difference is the focus of empirical work. Carriero et al. (2020) focus on the effects of global uncertainty on the levels of the data, while we focus on the typical issue about the effects of global uncertainty on the Phillips curve.

One other difference between our paper and a number of others in the multi-country studies is that we study 34 economies, including both advanced economies and emerging market economies, whereas others focus on large economies, small advanced economies, or emerging market economies. As examples, Carriero et al. (2020) focus on large economies, Cross et al. (2018) focus on small advanced economies, Mumtaz and Theodoridis (2017) focus on eleven OECD countries, and Carrière-Swallow and Céspedes (2013) focus on emerging market economies.

3 Sparse Factor Stochastic Volatility for A Multi-country UC Model

This section begins by detailing the unobserved components model for individual economies, then introduces the factor stochastic volatilitye-henceforth referred to as a multi-country UC-FSV model-to allow for cross-country linkages. We then describe the sparsification. Finally, we summarize the model and complement priors.

3.1 A Multi-country UC-FSV Model Specification

We begin with the UC model for output and inflation, developed in Stella and Stock (2013) and Chan et al. (2016). In particular, we start from constant coefficient UC model for output, $y_{i,t}$, and inflation, $\pi_{i,t}$ of the form:

$$\pi_{i,t} - \tau^{\pi}_{i,t} = \rho_i(\pi_{i,t-1} - \tau^{\pi}_{i,t-1}) + \alpha_i(y_{i,t} - \tau^{y}_{i,t}) + \varepsilon^{\pi}_{i,t} \tag{1}$$

$$y_{i,t} - \tau^{y}_{i,t} = \varphi_{i,1}(y_{i,t-1} - \tau^{y}_{i,t-1}) + \varphi_{i,2}(y_{i,t-2} - \tau^{y}_{i,t-2}) + \varepsilon^{y}_{i,t} \tag{2}$$

$$\tau^{\pi}_{i,t} = \tau^{\pi}_{i,t-1} + \varepsilon^{\tau\pi}_{i,t}, \quad \varepsilon^{\tau\pi}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{\tau\pi}) \tag{3}$$

$$\tau^{y}_{i,t} = \tau^{y}_{i,t-1} + \varepsilon^{\tau y}_{i,t}, \quad \varepsilon^{\tau y}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{\tau y}) \tag{4}$$

where i denotes economy i, $i = 1, \ldots, N$. At time t, $\pi_{i,t}$ is the inflation of economy i and $y_{i,t}$ is the output of economy i, $\tau^{\pi}_{i,t}$ and $\tau^{y}_{i,t}$ are their trends. These trends are unobserved latent states which can be interpreted as long-run equilibrium level of inflation and output, also known as trend inflation and trend output.

This model is inspired by the Phillips curve and incorporates the properties that it is inflation gap and unemployment gap that drive the Phillips curve. These features are in common with the model of Stella and Stock (2013), Chan et al. (2013) and Chan et al. (2016).

Thus, the first equation embodies a Phillips curve, but we are assuming constant coefficients in the inflation gap equation. Many papers have emphasized that the Phillips curve has flattened post 2007 (see, Simon et al., 2013) and proposed to allow for time-variation in the coefficients to capture this behavior (see, Zaman, 2021). It seems to be more sensible to start from UC model with time-varying coefficients. However, using the data in our empirical work (observed from 1995Q1 to 2018Q1), we have considered a model where ρ_i and α_i vary over time, but find the Bayes Factor supports constant coefficients (see Appendix A). Accordingly, the main model we focus on does not have time-variation in the coefficients in inflation gap equation.

To ensure stationarity, we bound ρ_i and α_i to be positive and less than one, that is $0 < \rho_i < 1$ and $0 < \alpha_i < 1$, which also ensures that the Phillips curve has a positive slope. We also impose stationary condition on the output gap equation and assume $\varphi_{i,1} + \varphi_{i,2} < 1$, $\varphi_{i,2} - \varphi_{i,1} < 1$ and $|\varphi_{i,2}| < 1$. Chan et al. (2016) and Zaman (2021) also bound the coefficients and emphasize the importance of bounding.

The second equation implies AR(2) behavior for the output. The AR(2) assumption is empirically sensible and commonly-used. Note that we are assuming constant coefficients in the output gap equation. In existing UC literature, the assumption of constant coefficients has also been used in Chan et al. (2016), Zaman (2021) and Kabundi et al. (2021). In the broader output literature, Koop et al. (2020) and Carriero et al. (2020) also assume constant coefficients.

Thus far, we have specified a UC model for a single economy. In particular, it is a bivariate UC model used in Kabundi et al. (2021) and Chan et al. (2016), and incorporates the features from empirical findings (constant coefficients). However, conventional literature would next assume that the errors are independent across economies. It is with this assumption that we part with the existing literature.

As discussed earlier, the independent assumption across economies is not plausible when there is significant commonality across economies. To capture such commonality in uncertainty, we assume that, for all economies, their errors in the inflation gap equation are driven by several common factors and their errors in the output gap equation are also driven by common factors. This can be done through the factor stochastic volatility (FSV) specification.

To facilitate the FSV specification, at time t, we store the errors in inflation gap equation for all economies in an N-dimensional vector ε_t^π, that is, $\varepsilon_t^\pi = (\varepsilon_{1,t}^\pi, \ldots, \varepsilon_{N,t}^\pi)'$. Similarly, we store the errors in output gap equation for all economies in an N-dimensional vector ε_t^y, that is, $\varepsilon_t^y = (\varepsilon_{1,t}^y, \ldots, \varepsilon_{N,t}^y)'$. Through factor stochastic volatility specification, ε_t^π can be decomposed as:

$$\varepsilon_t^\pi = L_\pi f_t + u_t^\pi \tag{5}$$

$$\begin{pmatrix} u_t^\pi \\ f_t \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0_N \\ 0_{r_\pi} \end{pmatrix} , \begin{pmatrix} \Sigma_t^\pi & 0_{r_\pi} \\ 0_N & \Omega_t^\pi \end{pmatrix} \right) \tag{6}$$

and ε_t^y can be decomposed as:

$$\varepsilon_t^y = L_y g_t + u_t^y \tag{7}$$

$$\begin{pmatrix} u_t^y \\ g_t \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0_N \\ 0_{r_y} \end{pmatrix} , \begin{pmatrix} \Sigma_t^y & 0_{r_y} \\ 0_N & \Omega_t^y \end{pmatrix} \right) \tag{8}$$

where $f_t = (f_{1,t}, \ldots, f_{r_\pi,t})'$ is a r_π-dimensional vector of latent factors and L_π is the associated $N \times r_\pi$ loading matrix. Similarly, $g_t = (g_{1,t}, \ldots, g_{r_y,t})'$ is a r_y-dimensional vector of latent factors and L_y is the associated $N \times r_y$ loading matrix. This, without further assumption, can lead to an identification issue. Following Chan (2021), we assume that the factor loading matrices L_π and L_y are both a lower triangular matrix with ones on the main diagonal and $r_\pi \leqslant (N-1)/2$, $r_y \leqslant (N-1)/2$. Let $n_{l,\pi}$ denote the number of free elements in L_π, then $n_{l,\pi} = N \times r_\pi - \frac{(1+r_\pi)r_\pi}{2}$. Let $n_{l,y}$ denote the number of free elements in L_y, then $n_{l,y} = N \times r_y - \frac{(1+r_y)r_y}{2}$.

We assume that inflation gap equations across economies and output gap equations across economies are driven by different factors f_t and g_t. Based on preliminary empirical work that the errors in inflation gap equation exhibit stochastic volatility, we assume that the disturbances u_t^π exhibit stochastic volatility. This is why the error variance of u_t^π is Σ_t^π. While the previous literature would assume the errors in output gap equation remain homoscedastic, that is, u_t^y are homoscedastic, we assume the disturbances u_t^y exhibit stochastic volatility. This is why the error variance of u_t^y is Σ_t^y. Such specification will capture time variation in a economy's output volatility unique to that economy, and is used in Carriero et al. (2020). In the theoretical part of Cesa-Bianchi et al. (2020), they also assume that country-specific business-cycle components have a conditionally heteroscedastic variance-covariance matrix. If the error is homoscedastic, our specification of the log-volatility can (nearly) remove SV (see below).

For the latent factors f_t and g_t, we assume that they exhibit stochastic volatility. This is why the error variance of f_t is Ω_t^π, and the error variance of g_t is Ω_t^y.

Note that the time-varying variance matrix $\Sigma_t^\pi = \text{diag}(e^{h_{1,t}}, \ldots, e^{h_{N,t}})$,
$\Sigma_t^y = \text{diag}(e^{h_{N+1,t}}, \ldots, e^{h_{2N,t}})$, $\Omega_t^\pi = \text{diag}(e^{h_{2N+1,t}}, \ldots, e^{h_{2N+r_\pi,t}})$, and
$\Omega_t^y = \text{diag}(e^{h_{2N+r_\pi+1,t}}, \ldots, e^{h_{2N+r_\pi+r_y,t}})$.

We use $\exp(h_{j,t}/2), j = 1, \ldots, N$ to measure the **idiosyncratic inflation uncertainty**. For simplicity, we also use $\exp(h_t^\pi/2)$. We use $\exp(h_{j,t}/2), j = N+1, \ldots, 2N$ (also $\exp(h_t^y/2)$) to measure the **idiosyncratic output uncertainty**. We use $\exp(h_{j,t}/2), j = 2N+1, \ldots, 2N+r_\pi$ (also $\exp(h_t^f/2)$) to measure the **global inflation uncertainty**. And we use $\exp(h_{j,t}/2), j = 2N+r_\pi+1, \ldots, 2N+r_\pi+r_y$ (also $\exp(h_t^g/2)$) to measure the **global output uncertainty**. We summarize the definitions and descriptions of uncertainty in Table 1.

Table 1: Definitions and descriptions of uncertainty $\exp(h_{j,t}/2)$

Definitions	descriptions of uncertainty $\exp(h_{j,t}/2)$
idiosyncratic inflation uncertainty	$\exp(h_t^\pi/2)$, the standard deviation of u_t^π
idiosyncratic output uncertainty	$\exp(h_t^y/2)$, the standard deviation of u_t^y
global inflation uncertainty	$\exp(h_t^f/2)$, the standard deviation of f_t
global output uncertainty	$\exp(h_t^g/2)$, the standard deviation of g_t
global inflation factor	f_t
global output factor	g_t

3.2 Sparsification

One of our contributions is the use of the sparsification. We use the sparsification to coherently estimate the factors and the number of factors in one step. We also use the sparsification to remove stochastic volatility in a data-based manner. In this sub-section, we first talk about deciding on the number of factors, then removing stochastic volatility.

To decide on the number of factors This is done through the prior on the factor loading matrix. To avoid specifying a prior on the number of factors, Chakraborty et al. (2020) work within a parameter-expanded framework to consider a potentially full-rank decomposition assign shrinkage priors to shrink out the redundant columns. Then they post-process the posterior draws to decide on the rank of the matrix (in this paper, the rank of the matrix is also the number of factors). In this paper, we follow their method. More specifically, we set the number of factors to a large number, assign shrinkage priors on the factor loading matrix, post-process the posterior draws to decide on the number of factors.

The first step is to set the number of factors to a large number. The factor loading matrix in the inflation gap equation is the $N \times r_\pi$ matrix L_π, and the factor loading matrix in the output gap equation is the $N \times r_y$ matrix L_y. r_π and r_π are the number of factors. They are set to a large number (in this paper, we set $r_\pi = 5$, and $r_y = 2$. $r_y = 2$ is inspired by the prior studies which find that there is one factor driving global GDP).

The second step is to assign shrinkage priors on the factor loading matrix to shrink out the redundant columns. Let $L_{\pi,j}$ denote the j-th column of factor loading matrix L_π, then the prior on $L_{\pi,j}$ is:

$$
\begin{aligned}
& L_{\pi,j} \mid \lambda_j^{L_\pi}, \tau^{L_\pi} \sim \mathcal{N}(0,\ \lambda_j^{L_\pi}\tau^{L_\pi}),\ j = 1, \ldots, r_\pi \\
& \lambda_j^{L_\pi} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu_j^{L_\pi}}),\quad \tau^{L_\pi} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{L_\pi}}) \\
& \nu_j^{L_\pi} \sim \mathcal{IG}(\frac{1}{2},\ 1),\qquad \xi^{L_\pi} \sim \mathcal{IG}(\frac{1}{2},\ 1)
\end{aligned}
\tag{9}
$$

Let $L_{y,j}$ denote the j-th column of factor loading matrix L_y, then the prior on $L_{y,j}$ is:

$$
\begin{aligned}
& L_{y,j} \mid \lambda_j^{L_y}, \tau^{L_y} \sim \mathcal{N}(0,\ \lambda_j^{L_y}\tau^{L_y}),\ j = 1, \ldots, r_y \\
& \lambda_j^{L_y} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu_j^{L_y}}),\quad \tau^{L_y} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{L_y}}) \\
& \nu_j^{L_y} \sim \mathcal{IG}(\frac{1}{2},\ 1),\qquad \xi^{L_y} \sim \mathcal{IG}(\frac{1}{2},\ 1)
\end{aligned}
\tag{10}
$$

The third step is to post-process the posterior draws to decide on the number of factors. We threshold the singular values of the factor loading matrix, and estimate the rank as the number of nonzero thresholded singular values. We refer our readers to Chakraborty et al. (2020) for more details.

To remove stochastic volatility To allow the data to decide whether there is time-variation in their log-volatility, we model the evolution of the log-volatility according to a random walk in non-centered parameterization and then use global-local shrinkage prior (Horseshoe prior) to control time-variation. More specifically, for each $j = 1, \ldots, 2N + r_\pi + r_y$, the evolution of the log-volatility is modeled as:

$$\begin{aligned} h_{j,t} &= h_{j,0} + \omega_j^h \widetilde{h}_{j,t} \\ \widetilde{h}_{j,t} &= \widetilde{h}_{j,t-1} + +\varepsilon_{j,t}^h, \quad \varepsilon_{j,t}^h \sim \mathcal{N}(0,\ 1) \end{aligned} \tag{11}$$

The non-centered parameterization decomposes a time-varying parameter $h_{j,t}$ into two parts: a time-invariant part $h_{j,0}$ and a time-varying part $\omega_j^h \widetilde{h}_{j,t}$, which has a constant coefficient ω_j^h that controls the time-variation. Then we expect that some elements in ω_j^h may be (close to) zero, which means the error is homoscedastic, but at the same time several elements in ω_j^h may be different from zero, which means the error is heteroscedastic. This case is exactly the advantage of global-local shrinkage prior. Many papers have documented that global-local shrinkage prior can cope with the case where a matrix is characterized by a small number of non-zero elements (e.g., Polson and Scott, 2010; Kastner and Huber, 2020). we use the empirically successful global-local shrinkage prior, Horseshoe prior, and consider the inverse-Gamma representation of Horseshoe prior as in Cross et al. (2020):

$$\begin{aligned} &\omega_j^h \mid \lambda_j^{\omega^h}, \tau^{\omega^h} \sim \mathcal{N}(0,\ \lambda_j^{\omega^h} \tau^{\omega^h}),\ j = 1, \ldots, 2N + r_\pi + r_y \\ &\lambda_j^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu_j^{\omega^h}}), \quad \tau^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{\omega^h}}) \\ &\nu_j^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ 1), \qquad \xi^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ 1) \end{aligned} \tag{12}$$

If the error (or factor) really is homoscedastic, the Horseshoe prior will shrink ω_j^h to (nearly) zero and automatically remove (or nearly so) the SV from the error (or factor). τ^{ω^h} is the global shrinkage parameter that pushes all elements (ω_j^h) towards zero. We have assumed that all ω_j^h are forced to zero through a single global shrinkage parameter τ^{ω^h} for different factors, different economies and different equations within a given economy. This is a restricted version of the Horseshoe prior in Feldkircher et al. (2021). They specify the global shrinkage parameter to differ across economies and equations within a given economy. However, we notice that such a flexible prior is for the coefficients in panel VARs, and our Horseshoe prior is for the time-varying part of log-volatility. They all represent the uncertainty, so we expect that they have a single global shrinkage parameter. Our specification in Equation (10) does allow for the differences across factors, economies and equations within a given economy and this is realized through the local shrinkage parameter $\lambda_j^{\omega^h}$.

For the time-invariant part of log-volatility, $h_{j,0}$, we consider the inverse-Gamma representation of Horseshoe prior:

$$
\begin{aligned}
&h_{j,0} \mid \lambda_j^{h_0}, \tau^{h_0} \sim \mathcal{N}(0,\ \lambda_j^{h_0}\tau^{h_0}),\ j = 1, \ldots, 2N + r_\pi + r_y \\
&\lambda_j^{h_0} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu_j^{h_0}}), \quad \tau^{h_0} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{h_0}}) \\
&\nu_j^{h_0} \sim \mathcal{IG}(\frac{1}{2},\ 1), \qquad \xi^{h_0} \sim \mathcal{IG}(\frac{1}{2},\ 1)
\end{aligned} \tag{13}
$$

For ω_j^h, we also consider the inverse-Gamma representation of Horseshoe prior:

$$
\begin{aligned}
&\omega_j^h \mid \lambda_j^{\omega^h}, \tau^{\omega^h} \sim \mathcal{N}(0,\ \lambda_j^{\omega^h}\tau^{\omega^h}),\ j = 1, \ldots, 2N + r_\pi + r_y \\
&\lambda_j^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\nu_j^{\omega^h}}), \quad \tau^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ \frac{1}{\xi^{\omega^h}}) \\
&\nu_j^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ 1), \qquad \xi^{\omega^h} \sim \mathcal{IG}(\frac{1}{2},\ 1)
\end{aligned} \tag{14}
$$

The initial states are assumed to follow normal distribution with zero mean and variance ten, that is:

$$
\widetilde{h}_{j,1} \sim \mathcal{N}(0,\ 10),\ j = 1, \ldots, 2N + r_\pi + r_y \tag{15}
$$

3.3 Summarizing the model and priors

To summarize the model including all economies:

$$
\begin{aligned}
\pi_t - \tau_t^\pi &= P(\pi_{t-1} - \tau_{t-1}^\pi) + A(y_t - \tau_t^y) + L_\pi f_t + u_t^\pi,\ f_t \sim \mathcal{N}(0,\ \Omega_t^\pi),\ u_t^\pi \sim \mathcal{N}(0,\ \Sigma_t^\pi) \\
y_t - \tau_t^y &= \Phi_1(y_{t-1} - \tau_{t-1}^y) + \Phi_2(y_{t-2} - \tau_{t-2}^y) + L_y g_t + u_t^y,\ g_t \sim \mathcal{N}(0,\ \Omega_t^y),\ u_t^y \sim \mathcal{N}(0,\ \Sigma_t^y) \\
\tau_{i,t}^\pi &= \tau_{i,t-1}^\pi + \varepsilon_{i,t}^{\tau\pi}, \quad \varepsilon_{i,t}^{\tau\pi} \sim \mathcal{N}(0,\ \sigma_{\tau\pi}^2),\ i = 1, \ldots, N \\
\tau_{i,t}^y &= \tau_{i,t-1}^y + \varepsilon_{i,t}^{\tau y}, \quad \varepsilon_{i,t}^{\tau y} \sim \mathcal{N}(0,\ \sigma_{\tau y}^2) \\
h_{j,t} &= h_{j,0} + \omega_j^h \widetilde{h}_{j,t} \\
\widetilde{h}_{j,t} &= \widetilde{h}_{j,t-1} + \varepsilon_{j,t}^h, \quad \varepsilon_{j,t}^h \sim \mathcal{N}(0,\ 1),\ j = 1, \ldots, 2N + r_\pi + r_y
\end{aligned} \tag{16}
$$

where $\pi_t = (\pi_{1,t}, \ldots, \pi_{N,t})'$ is an $N \times 1$ vector, $\tau_t^\pi = (\tau_{1,t}^\pi, \ldots, \tau_{N,t}^\pi)'$ is an $N \times 1$ vector, $P = \text{diag}(\rho_1, \ldots, \rho_N)$ is an $N \times N$ matrix, $A = \text{diag}(\alpha_1, \ldots, \alpha_N)$ is an $N \times N$ matrix, $y_t = (y_{1,t}, \ldots, y_{N,t})'$ is an $N \times 1$ vector, $\tau_t^y = (\tau_{1,t}^y, \ldots, \tau_{N,t}^y)'$ is an $N \times 1$ vector, L_π is an $N \times r_\pi$ matrix, f_t is a $r_\pi \times 1$ vector, u_t^π is an $N \times 1$ vector, $\Phi_1 = \text{diag}(\phi_{1,1}, \ldots, \phi_{N,1})$ is an $N \times N$ matrix, $\Phi_2 = \text{diag}(\phi_{1,2}, \ldots, \phi_{N,2})$ is an $N \times N$ matrix, L_y is an $N \times r_y$ matrix, g_t is a $r_y \times 1$ vector, u_t^y is an $N \times 1$ vector. Σ_t^π, Σ_t^y, Ω_t^π and Ω_t^π are defined previously.

We will use the multi-country UC-FSV as an acronym for this model defined through equation (16). Many models can be written as restricted version of the multi-country UC-FSV model and can help to investigate some aspects of our specification. These models, along with their acronyms, are as follows:

1) UC-FSV-$r_y = 0$: this is the restricted version of the UC-FSV where there is no common factors in the output gap equation, that is, $r_y = 0$. And the error in output gap equation is allowed to exhibit stochastic volatility.

2) UC-FSV-$r_y, r_\pi = 0$: this is the restricted version of UC-FSV where there is no common factors in inflation and output gap equations, that is, $r_\pi = 0$, $r_y = 0$. The error in inflation and output gap equation is allowed to exhibit stochastic volatility.

3) UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$: this is the restricted version of UC-FSV where there is no common factors in inflation and output gap equations, that is, $r_\pi = 0$, $r_y = 0$, and the disturbances u_t^y are homoscedastic, while the disturbances u_t^π exhibit stochastic volatility. This is the model that is used in Stella and Stock (2013), and Chan et al. (2016).[2]

We summarize the definitions and descriptions of uncertainty in Table 2:

Table 2: Models which are restricted version of UC-FSV and the corresponding restrictions

Models	corresponding restrictions
UC-FSV-$r_y = 0$	$r_y = 0$, no common factors in the output gap equation
UC-FSV-$r_y, r_\pi = 0$	$r_y = 0$, $r_\pi = 0$, no common factors in inflation and output gap equations
UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$	$r_y = 0$, $r_\pi = 0$, $\omega_j^h = 0, j = N+1, \dots, 2N$, similar to Stella and Stock (2013), and Chan et al. (2016).

The constant coefficients are assume to follow normal distribution with zero mean and variance ten, that is:

$$\rho_i \sim \mathcal{N}(0,\ 10),\ i = 1, \dots, N \tag{17}$$
$$\alpha_i \sim \mathcal{N}(0,\ 10),\ i = 1, \dots, N \tag{18}$$
$$\varphi_{i,j} \sim \mathcal{N}(0,\ 10),\ i = 1, \dots, N,\ j = 1, 2 \tag{19}$$

To ensure stationarity, we bound ρ_i and α_i to be positive and less than one, that is $0 < \rho_i < 1$ and $0 < \alpha_i < 1$. We also impose the stationary condition on the output gap equation and assume $\varphi_{i,1} + \varphi_{i,2} < 1$, $\varphi_{i,2} - \varphi_{i,1} < 1$ and $|\varphi_{i,2}| < 1$.

[2]The coefficients in this paper are restricted to be constant.

The initial states are assumed to follow normal distribution with zero mean and variance ten, that is:

$$\tau^{\pi}_{i,1} \sim \mathcal{N}(0,\ 10),\ i = 1, \ldots, N \tag{20}$$
$$\tau^{y}_{i,1} \sim \mathcal{N}(0,\ 10),\ i = 1, \ldots, N \tag{21}$$

The error variances are assumed to follow inverse gamma distribution, that is:

$$\sigma^2_{\tau\pi} \sim \mathcal{IG}(10,\ 0.18),\ i = 1, \ldots, N \tag{22}$$
$$\sigma^2_{\tau y} \sim \mathcal{IG}(10,\ 0.09),\ i = 1, \ldots, N \tag{23}$$

4 Full-sample results

4.1 Data

The data are the quarterly consumer price index (CPI) and the quarterly real gross domestic product (GDP) for 34 economies, 23 advanced economies (AEs)[3] and 11 emerging market economies (EMEs)[4]. They span the period from 1995Q1 to 2018Q1. We transform the data to annualized growth rates as: 400log(z_t/z_{t-1}). And because the output gap equation follows an AR(2) process, our estimation start from 1995Q4. Posterior results are based on 100000 draws after a burn-in period of 20000.

4.2 Overview of empirical results

We divide our Full-sample results into four sub-sections. The first sub-section, section 4.3, is the multi-country UC-FSV estimate of global inflation uncertainty $\exp(h^f_t/2)$ and global output uncertainty $\exp(h^g_t/2)$. We set $r_\pi = 5$ and $r_y = 2$, that is, we include five factors in the inflation gap equation and two factors in the output gap equation.

The second sub-section, section 4.4, is about the evidence of using sparsification. We use the sparsification to decide on the number of factors and remove stochastic volatility in a data-based manner. We first report the result of deciding on the number of factors, then removing stochastic volatility.

[3] Australia, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Israel, Italy, Latvia, Lithuania, Netherlands, Portugal, Slovakia, South Korea, Spain, Sweden, Switzerland, UK, USA.

[4] Bolivia, Brazil, China, Hungary, Indonesia, Mexico, Philippines, Russia, South Africa, Thailand, Turkey.

The second sub-section, section 4.5, is about Bayesian model comparison. We compare the multi-country UC-FSV to alternative models (UC-FSV-$r_y = 0$, UC-FSV-$r_y, r_\pi = 0$, UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) described in Table 2.

After justifying that our multi-country UC-FSV model provides higher model fit, we show, in the third sub-section (section 4.6), that persistence and the slope of the Phillips curve will decrease under the multi-country UC-FSV, compared with the commonly-used UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$[5]. And the estimates of trend inflation, trend output, idiosyncratic inflation uncertainty and idiosyncratic output uncertainty are found to be sensible.

4.3 The multi-country UC-FSV Estimates of global uncertainty

Although the multi-country UC-FSV estimates of global uncertainty reflect contemporaneous effect of global factors on (the volatility of) macroeconomic data, the effect is also directly related to the loadings on the global factors. These loadings are reported in Appendix B. Table 10 is the loadings on global inflation factor. We report the posterior mean of the five factors' loadings (recall that we set $r_\pi = 5$), but only the 16% and 84% quantiles of first factor's loadings for brevity. Most of the economies have sizable loadings on the first global inflation factor, and the quantiles (except Russia and Brazil) do not include zero. Table 11 is the loadings on global output factor. We report the posterior mean and quantiles of the two factors' loadings (recall that we set $r_y = 2$). The quantiles of the first global output factor for all economies do not include zero. This provides strong evidence of significant commonality of output in the 34 economies. Carriero et al. (2020) obtain similar result in their case of the 19-country GDP dataset.

Figure 1 displays the posterior estimates of global uncertainty obtained from the multi-country UC-FSV using the full sample. The left panel, Figure 1a, is the estimate of global inflation uncertainty, and the right panel, Figure 1b, is the estimate of global output uncertainty. In both figures, the solid lines represent the posterior means of the first uncertainty, while the dotted lines are the associated 16% and 84% quantiles. The dashed lines represent the posterior means of the remaining uncertainties. For example, with regard to global inflation uncertainty, we set $r_\pi = 5$, so we obtain the posterior estimates of the five global inflation uncertainties from MCMC, including their posterior means and quantiles. Then, in Figure 1a, we plot the posteriror means and quantiles of the first global inflation uncertainty (see solid lines and dotted lines), but for brevity, we only plot the posterior means of the remaining uncertainties (the second, third, fourth and fifth uncertainty) using dashed lines.

As indicated in Figure 1a, we only observe evident and meaningful time-variation in

[5]This is the model that is used in Stella and Stock (2013), and Chan et al. (2016). The coefficients are restricted to be constant in this paper.

the first global inflation uncertainty. The estimated global inflation uncertainty show significant increases around some of the political and economic events that Bloom (2009) highlights as periods of uncertainty, including 9/11, the Enron scandal, the second Gulf war, and the global financial crisis period. These spikes in the volatility associated with the global factor are documented in Kastner and Huber (2020) using US macroeconomic data. Since our data comes from 34 economies, the consistency between the estimate in Kastner and Huber (2020) and our study indicates that global macroeconomic uncertainty is closely related to uncertainty in the US, which might not seem surprising given the tie of the international economy to the US economy. One spike that is not documented in Kastner and Huber (2020) is that volatility increases from 2013 onward. This may indicate that such increase is driven by economies other than US. In addition, at the end of our sample (2018Q1), the global inflation uncertainty still exists and continues to influence all economies under consideration. This is supported by a related study, Forbes (2019). They add commodity price volatility to explain inflation and find that commodity price volatility plays a large role for CPI inflation.

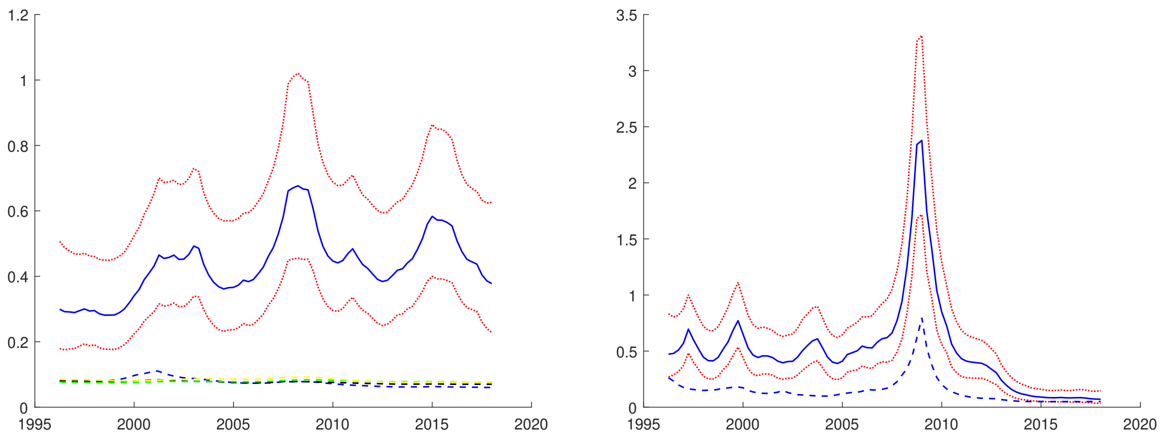
However, we find a different story with regard to the time-variation in the global output uncertainty from Figure 1b. First, the two global output uncertainties both increase during the GFC of 2008, but except this, we do not observe other meaningful time-variation form the second global output uncertainty.[6] Before the Global Financial Crisis (GFC) of 2008, there exists global output uncertainty but it does not show much time-variation. Then during the GFC, such uncertainty increases substantially. In the aftermath of the GFC, it decreases sharply and importantly, in the 2015, the global inflation uncertainty reaches a very low level.[7] These features are documented in Carriero et al. (2020) in their 19-country GDP data set.

4.4 Evidence of using sparsification

To decide on the number of factors To estimate the number, we post-process the posterior draws as: threshold the singular values of the factor loading matrix, and estimate the rank as the number of nonzero thresholded singular values. One choice of the threshold is proposed in Chakraborty et al. (2020). Using their choice, we get the result in Table 3. We have inflation equation and output equation. The second column is the result in the inflation equation, and the third column is the result in the the output equation. The first column reports the singular values of the factor loading matrix. The number of factors is determined as: the number of singular values larger than the threshold. In the output equation, we find that there is one singular value (119.82) that is larger than the threshold (= 108.46). This means that there is one global factor in the output equation. This finding is consistent to prior studies.

[6]This is the first reason of including only two factors in the output gap equation.

[7]This is the second reason of including only two factors in the output gap equation



(a) Global inflation uncertainty

(b) Global output uncertainty

Figure 1: Posterior estimates for global inflation uncertainty $\exp(h_t^f/2)$ and global output uncertainty $\exp(h_t^g/2)$ under the multi-country UC-FSV. The solid lines represent the posterior means of the first global uncertainty, while the dotted lines are the associated 16% and 84% percentiles. The dashed lines represent the posterior means of the remaining uncertainties.

In the inflation equation, we find that there is no singular value that is larger than the threshold (= 118.47). This means that there is no global factor in the inflation equation. Even if in the full-sample result we do not find strong evidence of global factors in the inflation equation, in the out-of-sample forecasting exercise we find the FSV specification does improve the forecast of inflation (although the improvement of forecasting inflation is smaller than the improvement of forecasting output. See Section 5).

Table 3: Posterior number of factors

Singular values (Descending)	The inflation equation threshold = 118.47	The output equation threshold = 108.46
First	49.08	119.82
Second	23.33	45.85
Third	5.17	20.62

To remove stochastic volatility We assess whether the Horseshoe prior can successfully shrink strongly the parameter space (ω^h) but at the same time provides enough flexibility to allow for non-zero elements if necessary.

We first test for time-variation in the volatility of inflation and output, then plot the estimated time-varying standard deviation ($\exp(h_{j,t}/2)$) to see whether it coincides with the test result. Of course, to test for time-variation in the volatility, a gold standard is using marginal likelihood (Bayes Factor is the ratio of two marginal likelihoods). However, in our settings where we allow for time-variation in volatility, the computation of marginal likelihood requires integrating out all the states, making it a nontrivial task. Therefore, we follow the method developed in Chan (2018). More specifically, since we notice that the model without SV is a restricted version of the model with SV, the Bayes Factor can be calculated using the Savage-Dickey density ratio, thus avoiding the computation of marginal likelihood. The Bayes Factor in favor of the unrestricted model (model with SV) can be obtained using the Savage-Dickey density ratio as

$$\mathrm{BF}_{h_j} = \frac{p(\omega_j^h = 0)}{p(\omega_j^h = 0 |\ y)}$$

So if BF_{h_j} is larger than 1, then the Bayes Factor is in favor of the unrestricted model. In this part, the unrestricted model is time-varying h_j. For simplicity, we compare the log Bayes Factor. So a positive log Bayes Factor supports a time-varying h_j.

On the computation of posterior density ($p(\omega_j^h = 0 |\ y)$), we can obtain the posterior distribution given output from MCMC algorithm, then it is direct to compute the posterior density. On the computation of prior density ($p(\omega_j^h = 0)$), since we use the Horseshoe prior on ω_j^h, $p(\omega_j^h = 0)$ does not have a convenient analytical form. But, given the hyperparameters $(\lambda_j^{\omega^h}, \tau^{\omega^h}, \nu_j^{\omega^h}, \xi^{\omega^h})$ in Horseshoe prior, $p(\omega_j^h = 0 |\ \lambda_j^{\omega^h}, \tau^{\omega^h}, \nu_j^{\omega^h}, \xi^{\omega^h})$ is Normal. Thus, if we have output from a prior simulator, we can approximate $p(\omega_j^h = 0)$ by

$$\widehat{p}(\omega_j^h = 0) = \frac{1}{S} \sum_{s=1}^{S} p(\omega_j^h = 0 |\ \lambda_j^{\omega^{h,s}}, \tau^{\omega^{h,s}}, \nu_j^{\omega^{h,s}}, \xi^{\omega^{h,s}})$$

This approximation applies for any prior which has a hierarchical form. The estimated log Bayes Factor is reported in Appendix D. For time-variation in the volatility of inflation in 34 economies, 12 economies are in favor of time-variation in the volatility (their log Bayes Factor are positive). For time-variation in the volatility of output in 34 economies, 14 economies are in favor of time-variation in the volatility.

To see whether the estimated time-varying standard deviation coincides with the estimated log Bayes Factor, we report the two in Figure 2 and Figure 3. Figure 2 depicts the estimated time-varying standard deviation for inflation. The title of each sub-figure is the economy name, followed by the estimated log Bayes Factor. For example, the title of the first sub-figure is "Belgium (-6.16) ", then the first sub-figure depicts the estimated time-varying standard deviation for Belgium inflation and the the estimated log Bayes Factor is -6.16, which is negative and implies that the log Bayes Factor does not supports time-variation in the volatility of Belgium inflation. Figure 3 depicts the estimated time-varying standard deviation for output. The title is named in the same way as inflation. The first sub-figure depicts the estimated time-varying standard deviation for Belgium output and the the estimated log Bayes Factor is -5.17, which is negative and implies that the log Bayes Factor does not support time-variation in the volatility of Belgium output.

We find that the estimates of both the log Bayes Factor and the time-varying standard deviation are sensible and coincide with past research. For USA, the log Bayes Factor supports time-varying volatility of inflation, while does not support time-varying volatility of output. This is consistent with what we observe from the estimated time-varying standard deviation for inflation and output. We observe marked spike in Figure 2, while it remains quite flat in Figure 3. Zaman (2021), Kabundi et al. (2021), among many others, assume that the error in the output gap equation remains homoscedastic. The consistency among the log Bayes Factor, the time-varying standard deviation and past literature implies that the Horseshoe prior can successfully remove unimportant small SV and at the same time provides enough flexibility to allow for SV if necessary.

In addition, we find that, for several economies, the log Bayes Factor supports time-varying volatility of output and we also observe marked spikes from the time-varying standard deviation. This result points towards a big advantage of our proposed model, which allows for SV in output gap equation. While past research assume the error in output gap equation is homoscedastic, such assumption displays a tendency to be over-restricted in multi-country study and ignores patterns observed under the model allowing for SV in output gap equation. Omitting the SV can severely affect the reliability of the estimates of the trend output.

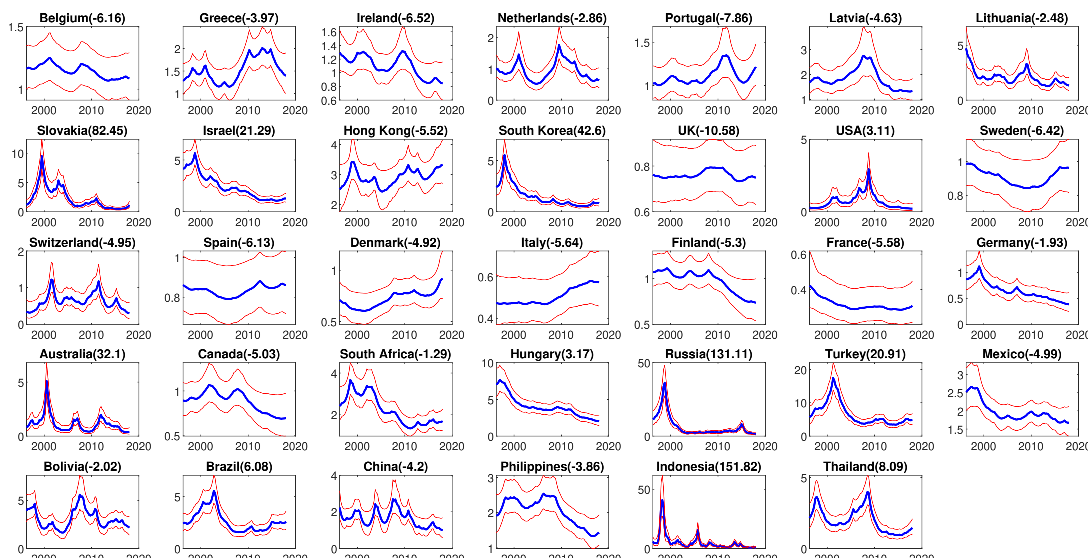


Figure 2: The estimated log Bayes Factor and time-varying standard deviation in inflation gap equation. The solid blue lines are the means, 16% and 84% quantiles of time-varying standard deviation.

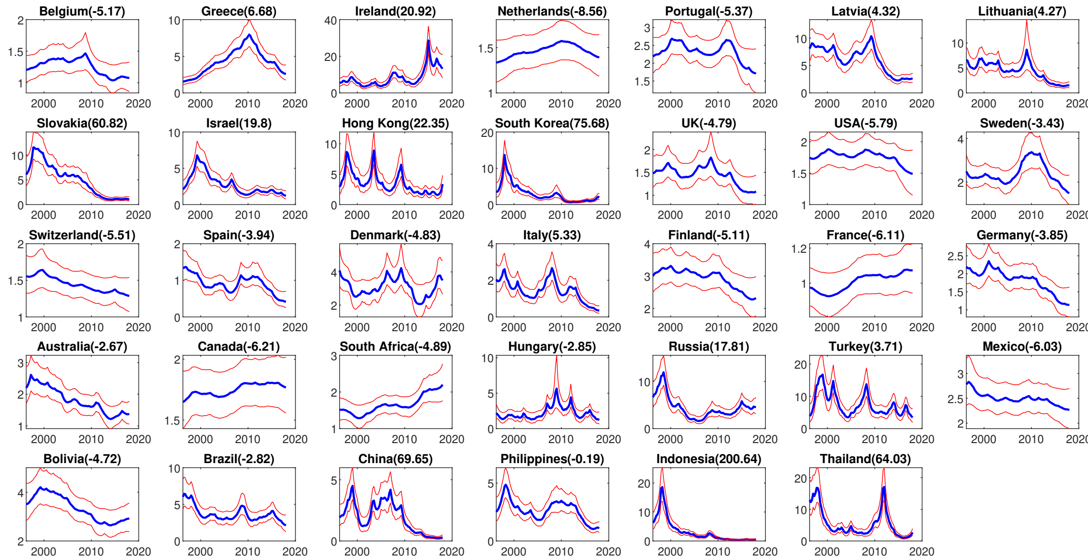


Figure 3: The estimated log Bayes Factor and time-varying standard deviation in output gap equation. The solid blue lines are the means, 16% and 84% quantiles of time-varying standard deviation.

4.5 Bayesian Model Comparison

As discussed previously, the computation of marginal likelihood can be a challenge when there are a large number of states. Therefore, we use an approximation to the marginal likelihood (e.g., Geweke, 2001; Cross et al., 2020). They propose that conditioning on the estimation period, the sums of one-step-ahead joint log predictive likelihoods of 34 economies can be viewed as an approximation to the marginal likelihood, therefore provides a direct measure of in-sample fit. We compare four competing models: the multi-country UC-FSV, UC-FSV-$r_y = 0$, UC-FSV-$r_y, r_\pi = 0$ and UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

Before computing the the sums of one-step-ahead joint log predictive likelihoods, we need to define some basics. Let $\widehat{y}_{t+k}^{(i,j)}$ denote, at time t, the k-step-ahead forecast of the j-th variable in the i-th economy, and $y_{t+k}^{(i,j)}$ denote the actual value. In our empirical work, $i = 1, \ldots, N$ with $n = 34$, $j = 1, 2$ where $j = 1$ denote inflation and $j = 2$ denote output. $\mathrm{Y}_{1:t}^{(i,j)}$ stores the data up to time t, so $\widehat{y}_{t+k}^{(i,j)} = \mathrm{E}\ (y_{t+k}^{(i,j)} \mid \mathrm{Y}_{1:t}^{(i,j)})$. Then we compute the k-step-ahead log predictive likelihoods (LPL) of the j-th variable in the i-th economy at time t:

$$\mathrm{LPL}_{t,i,j,k} = \log\, p(\widehat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \mathrm{Y}_{1:t}^{(i,j)}),\ t = T_0, \ldots, T-k$$

Then the sums of one-step-ahead joint log predictive likelihoods is computed using:

$$\text{LPL}_{\cdot,\cdot,\cdot,1} = \sum_{t=T_0}^{T-1} \sum_{i=1}^{n} \sum_{j=1}^{2} \log p(\widehat{y}_{t+1}^{(i,j)} = y_{t+1}^{(i,j)} | \text{Y}_{1:t}^{(i,j)})$$

Our estimation period starts from 1995Q4 (to 2018Q1), and the forecasting evaluation period starts from 2003Q1. We provide the sums of one-step-ahead joint log predictive likelihoods of 34 economies in Table 4.

In Table 4, results are presented relative to the forecast performance of the UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$: we take differences, so that a positive number indicates a model is forecasting better than the UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$. (Please note that we only take the sum, and no average. That may be why the number seems so large. For example, the sums of LPL under UC-FSV is 895.02. If we take average over time, then it is 14.67. If we take further average across economies, then it is 0.43). The results show that the multi-country UC-FSV provides the best fit compared to all other models. In addition, since we find UC-FSV-$r_y, r_\pi = 0$ provides higher model fit than UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$, we view this as another evidence in support of allowing for idiosyncratic stochastic volatility in output gap equation.

Table 4: Sum of one-step-ahead log predictive likelihood

Model	against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$
UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$	0
UC-FSV-$r_y, r_\pi = 0$	520.37
UC-FSV-$r_y = 0$	658.57
UC-FSV	883.34

4.6 Estimates under the multi-country UC-FSV

In the preceding sub-section, we provide evidence of successful shrinkage and flexibility realized by our NCP-HS-SV specification, and find that our multi-country UC-FSV provides higher model fit, justifying the importance of international macroeconomic uncertainty. In this subsection, we compare the estimates of parameters produced from the multi-country UC-FSV and UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$. Specifically:

1) Decrease of persistence and Flattening of the Phillips Curve: different from Kabundi et al. (2021), which focus on the time-variation of coefficients, we focus on the effects of international macroeconomic uncertainty on the constant coefficients. This can be done through comparing the relative change of constant coefficients under UC-FSV against

UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$. We find that considering contemporaneous cross-country linkages will decrease the persistence and flatten the Phillips Curve.

2) Estimates of country-specific latent states: we present the posterior estimates of country-specific latent states: Trend inflation τ^π, Trend output τ^y, Idiosyncratic inflation uncertainty $\exp(h_t^\pi/2)$ (In fact, $\exp(h_t^\pi/2)$ is the standard deviation, so what we compare is the standard deviation). Idiosyncratic output uncertainty $\exp(h_t^y/2)$.

Decrease of persistence and Flattening of the Phillips Curve We provide the posterior estimates of the coefficients in the Appendix C. Before describing the detailed characteristics of each constant coefficient (ρ, α, φ_1, φ_2), we first summarize the relative change of constant coefficients under the multi-country UC-FSV against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$, to assess the effects of contemporaneous cross-country linkages on them.

In Table 5, the number is number of economies. For example, the "decrease" row "ρ" column is 24, then out of 34 economies, there are 24 economies whose ρ is smaller under the multi-country UC-FSV than the ρ under UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$. ρ is the inflation gap persistence, α is the slope of the Phillips Curve. We find, for most economies, considering global inflation uncertainty will decrease the inflation gap persistence and the slope of the Phillips Curve. Also, we find output gap persistence φ_1 decreases, so allowing for idiosyncratic stochastic volatility in output and global output uncertainty will decrease the output gap persistence.

Table 5: Relative change under the multi-country UC-FSV against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$

	ρ	α	φ_1	φ_2	$\varphi_1 + \varphi_2$
decrease	24	25	29	9	29
no change	3	8	0	0	0
increase	7	1	5	25	5

Inflation gap persistence Table 12 reports the inflation gap persistence. A noticeable difference between AEs and EMEs is observed. Most AEs exhibit a smaller gap persistence which is attributed to a better anchoring of inflation expectations, suggesting that agents in AEs have become more forward looking than agents in EMEs. In other word, inflation process in AEs is no longer adaptive (see Cogley and Sargent, 2005, Stock and Watson, 2007 and Chan et al., 2016). While, expectation formation in EMEs is more adaptive. Such difference between AEs and EMEs is also documented in Kabundi et al. (2021).

The new finding (compared with Kabundi et al., 2021) is that considering global inflation uncertainty will decrease inflation gap persistence in both AEs and (most) EMEs. This seems to indicate that, by taking into account global inflation uncertainty, the central bank is more able to correct (or less willing to tolerate) the deviations of inflation from its target.

Output gap persistence Table 14 reports the output gap persistence. A similar pattern to inflation gap persistence is found for output gap persistence. We find considering global output uncertainty will decrease the AR(1) coefficient φ_1. Although the AR(2) coefficient φ_2 increases in most economies (25 out of 34 economies), the sum of the AR(1) and AR(2) coefficient decreases in most economies (29 out of 34 economies).

Phillips curve The coefficient controlling the slope of the Phillips curve, α, has decreased in most economies (25 out of 34 economies) after taking into account global uncertainty. This provide further evidence that global uncertainty will flatten the Phillips Curve. As done in Forbes (2019), they include comprehensive controls for globalization because globalization is often cited as causing the flattening of the Phillips Curve.

Then, we report the posterior estimates of four country-specific latent states: trend inflation τ_t^π, trend output τ_t^y, idiosyncratic inflation uncertainty $\exp(h_t^\pi)$ and idiosyncratic output uncertainty $\exp(h_{j,t}^y)$. For each country, we compare their country-specific latent states under four competing models: the multi-country UC-FSV, UC-FSV-$r_y = 0$, UC-FSV-$r_y, r_\pi = 0$ and UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

Estimates of Trend inflation In Figure 4, we report the posterior estimates of trend inflation under the four competing models. The title of each sub-figure is the economy name, followed by the official inflation targets (point target or target bands). For example, the title of the first sub-figure is "Belgium (2) ", then the first sub-figure depicts the estimate of trend inflation for Belgium and the official inflation target set by Belgium central bank is 2%. Each sub-figure plots the posterior estimates (mean, 16% and 84% quantiles) of trend inflation under the multi-country UC-FSV along with the posterior mean of trend inflation under three competing models. The solid blue lines are the means, 16% and 84% quantiles under UC-FSV, the dotted red lines are posterior means under UC-FSV-$r_y = 0$, the dashed black lines are posterior means under UC-FSV-$r_y, r_\pi = 0$, while the dashed green lines are posterior means under UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

The solid blue lines and the dotted red lines represent the estimate under the models considering global inflation uncertainty in inflation gap equation, while the dashed black lines and the dashed green lines represent the estimate under the models without global inflation uncertainty in inflation gap equation. The first 23 economies are AEs (from Belgium to Canada), followed by 11 EMEs. A pattern which emerges from the results is that considering global inflation uncertainty tend to influence the estimated trend inflation more in AEs than in EMEs. The posterior means under the four competing models are almost coincident in EMEs. However, global inflation uncertainty does generate some differences in AEs, such as Netherlands, USA, Switzerland, Denmark, Italy, France, Germany, Canada. And we observe that such differences indicate that trend inflation is driven by both domestic factors and global factors. For example, many papers without global inflation uncertainty document that trend inflation for USA has been below 2% since 2012, and this is also observed under our competing models UC-FSV-$r_y, r_\pi = 0$ and UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$, in contrast, the mean estimate under model with global inflation uncertainty decreases to a higher level in 2010. Then it begins to increase, rather than decreasing until 2015.

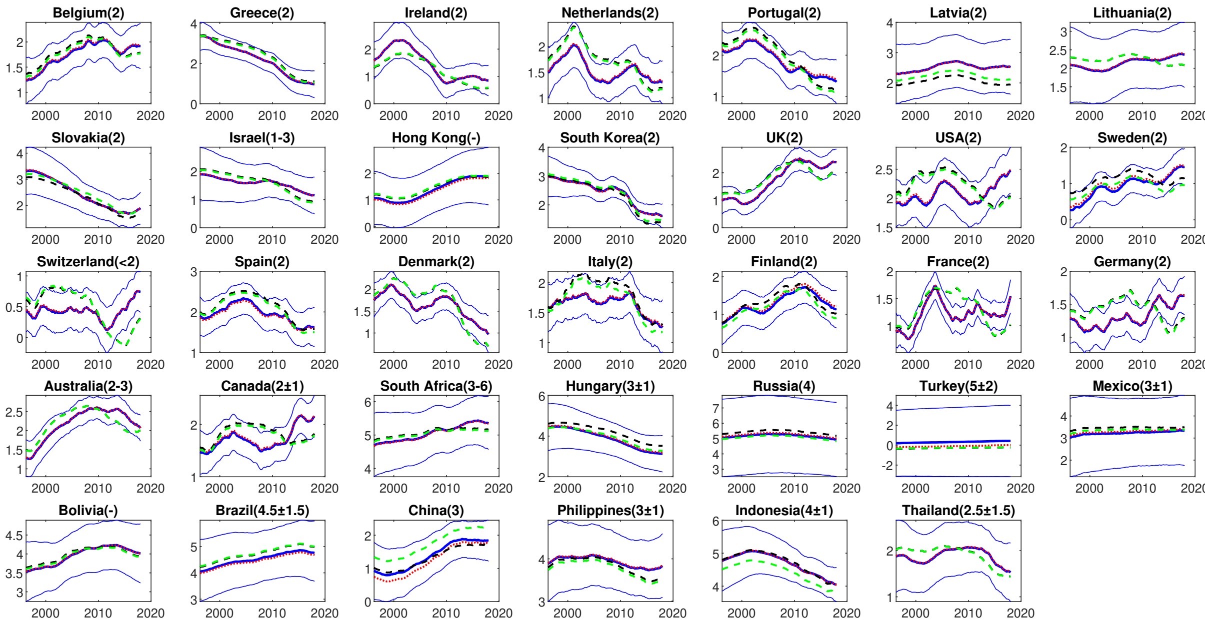


Figure 4: Posterior estimates for trend inflation τ^π. The title of each sub-figure is the economy name, followed by the official inflation targets (point target or target bands). For Hong Kong and Bolivia, we do not find the official inflation targets, so we use "-". The solid blue lines are the means, 16% and 84% quantiles under the multi-country UC-FSV, the dotted red lines are posterior means under the multi-country UC-FSV-$r_y = 0$, the dashed black lines are posterior means under UC-FSV-$r_y, r_\pi = 0$, while the dashed green lines are posterior means under UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

Estimates of Trend output In Figure 5, we report the posterior estimates of trend output under the four competing models. The title of each sub-figure is the economy name. Each sub-figure plots the posterior estimates (mean, 16% and 84% quantiles) of trend output under the multi-country UC-FSV along with the posterior mean of trend output under three competing models. The meaning of each line is the same as that in Figure 4. The solid blue lines are the means, 16% and 84% quantiles under the multi-country UC-FSV, the dotted red lines are posterior means under UC-FSV-$r_y = 0$, the dashed black lines are posterior means under UC-FSV-$r_y, r_\pi = 0$, while the dashed green lines are posterior means under UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

We have two modifications in the output gap equation: allowing for idiosyncratic output uncertainty (UC-FSV-$r_y, r_\pi = 0$ and UC-FSV-$r_y = 0$) and considering global output uncertainty (the multi-country UC-FSV).

We first analyze the effect of idiosyncratic output uncertainty on the estimate of trend output. This is done through comparing the estimates under UC-FSV-$r_y, r_\pi = 0$ (dashed black lines) with the estimate under UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$ (dashed greed lines), where the error in output gap equation remains homoscedastic. We find allowing for idiosyncratic output uncertainty will provide higher estimate of trend output in many economies (roughly 20 out of 34 economies).

Then, we analyze the effect of global output uncertainty on the estimate of trend output. This is done through comparing the estimates under the multi-country UC-FSV (solid blue lines) with the estimate under UC-FSV-$r_y = 0$ (dotted red lines). Similar to the finding in the effect of global inflation uncertainty, we find that considering global output uncertainty tend to influence the estimated trend inflation more in AEs than in EMEs.

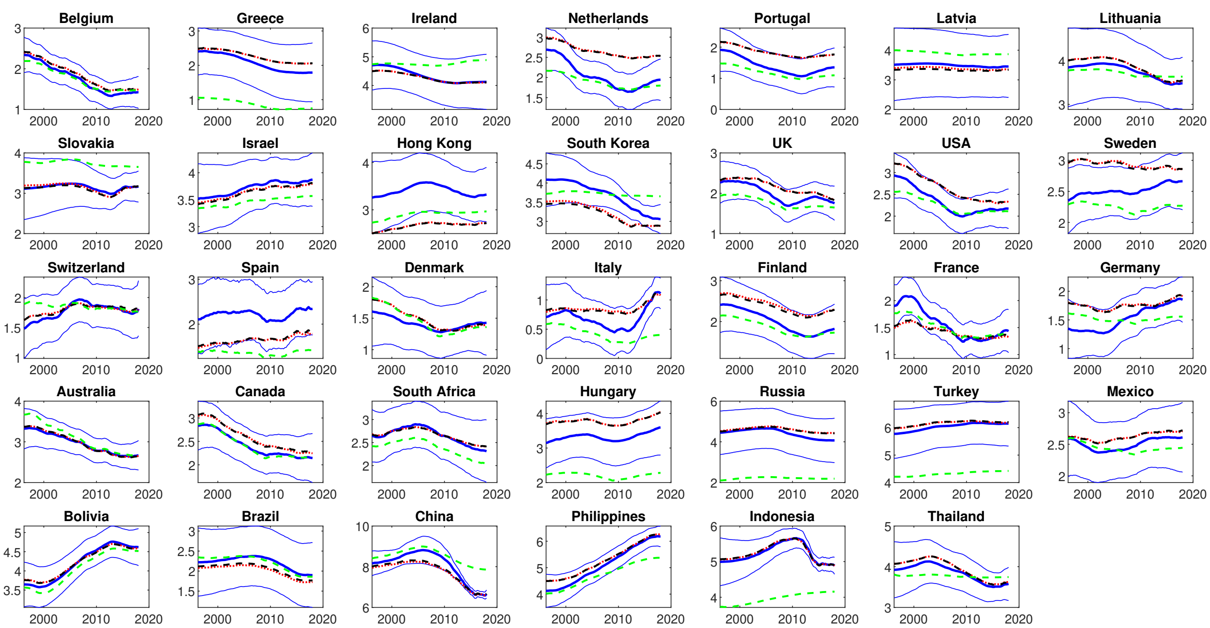


Figure 5: Posterior estimates for trend output τ^y. The solid blue lines are the means, 16% and 84% quantiles under the multi-country UC-FSV, the dotted red lines are posterior means under UC-FSV-$r_y = 0$, the dashed black lines are posterior means under UC-FSV-$r_y, r_\pi = 0$, while the dashed green lines are posterior means under UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

Estimates of idiosyncratic inflation uncertainty In Figure 6, we report the idiosyncratic inflation uncertainty estimates (i.e., the standard deviation of the shocks to the inflation gap, $\exp(h_t^\pi/2)$) under the four competing models. The title of each sub-figure is the economy name. Each sub-figure plots the posterior estimates (mean, 16% and 84% quantiles) under the multi-country UC-FSV along with the posterior mean under three competing models. The meaning of each line is the same as that in Figure 4. The solid blue lines are the means, 16% and 84% quantiles under the multi-country UC-FSV, the dotted red lines are posterior means under UC-FSV-$r_y = 0$, the dashed black lines are posterior means under UC-FSV-$r_y, r_\pi = 0$, while the dashed green lines are posterior means under UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

The solid blue lines and the dotted red lines represent the estimate under the models allowing for cross-country linkages in inflation gap equation, while the dashed black lines and the dashed green lines represent the estimate under the models without cross-country linkages in inflation gap equation. A quick visual inspection shows that allowing for cross-country linkages reduces the spike of idiosyncratic inflation uncertainty. This can be regarded as an evidence supporting that there exist factors driving strong co-movement of inflation across economies. In addition, the idiosyncratic inflation uncertainty in several economies becomes quite flat after allowing for cross-country linkages, suggesting that the uncertainty in their inflation gap equation is driven by global inflation uncertainty, rather than idiosyncratic inflation uncertainty.

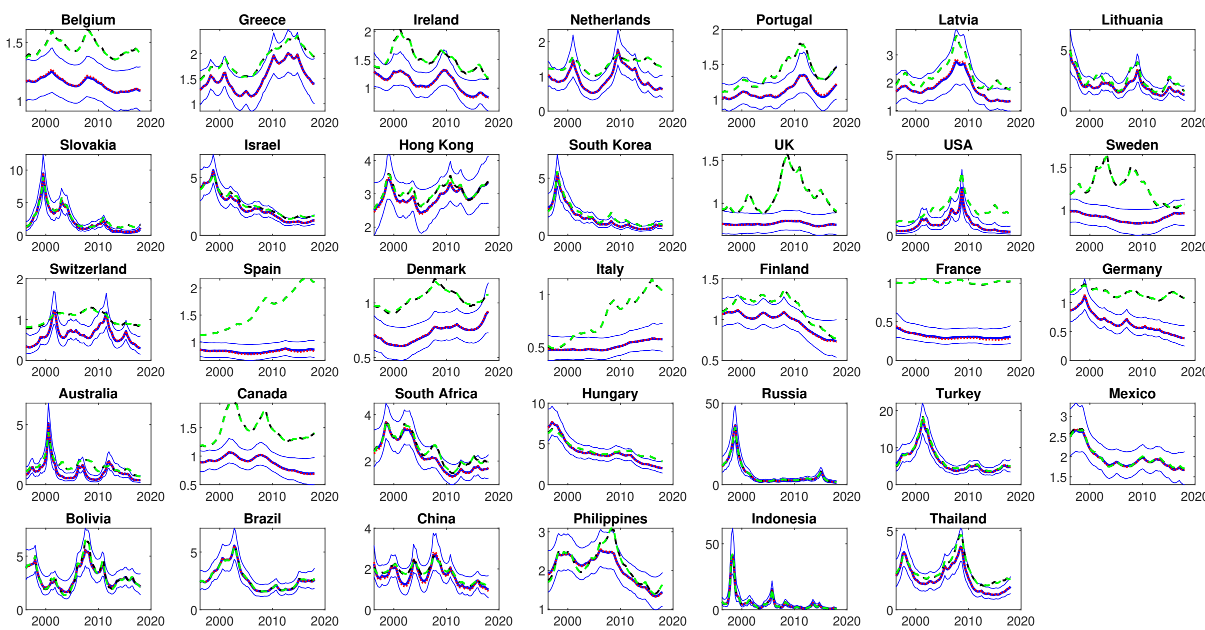


Figure 6: Posterior estimates for idiosyncratic inflation uncertainty $\exp(h_t^\pi/2)$. The solid blue lines are the means, 16% and 84% quantiles under the multi-country UC-FSV, the dotted red lines are posterior means under UC-FSV-$r_y = 0$, the dashed black lines are posterior means under UC-FSV-$r_y, r_\pi = 0$, while the dashed green lines are posterior means under UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

Estimates of idiosyncratic output uncertainty In Figure 7, we report the idiosyncratic output uncertainty estimates (i.e., the standard deviation of the shocks to the output gap, $\exp(h_t^y/2)$) under the four competing models. The title and the meaning of each line is the same as that in Figure 6.

The solid blue lines and the dotted red lines represent the estimate under the models allowing for cross-country linkages in output gap equation, while the dashed black lines and the dashed green lines represent the estimate under the models without cross-country linkages in output gap equation. The pattern found for idiosyncratic inflation uncertainty can also be found for idiosyncratic output uncertainty. We again observe that allowing for cross-country linkages reduces the spike of idiosyncratic output uncertainty. This indicates that there exist factors driving strong co-movement of output across economies. The idiosyncratic output uncertainty in many economies becomes quite flat after allowing for cross-country linkages, suggesting that the uncertainty in their output gap equation is driven by global output uncertainty, rather than idiosyncratic output uncertainty. This number of idiosyncratic output uncertainty becoming flat is higher than the number of idiosyncratic inflation uncertainty becoming flat, which provides evidence for papers assuming that the error in output gap equation is homoscedastic, but at the same time supports the need to allow for cross-country linkages.

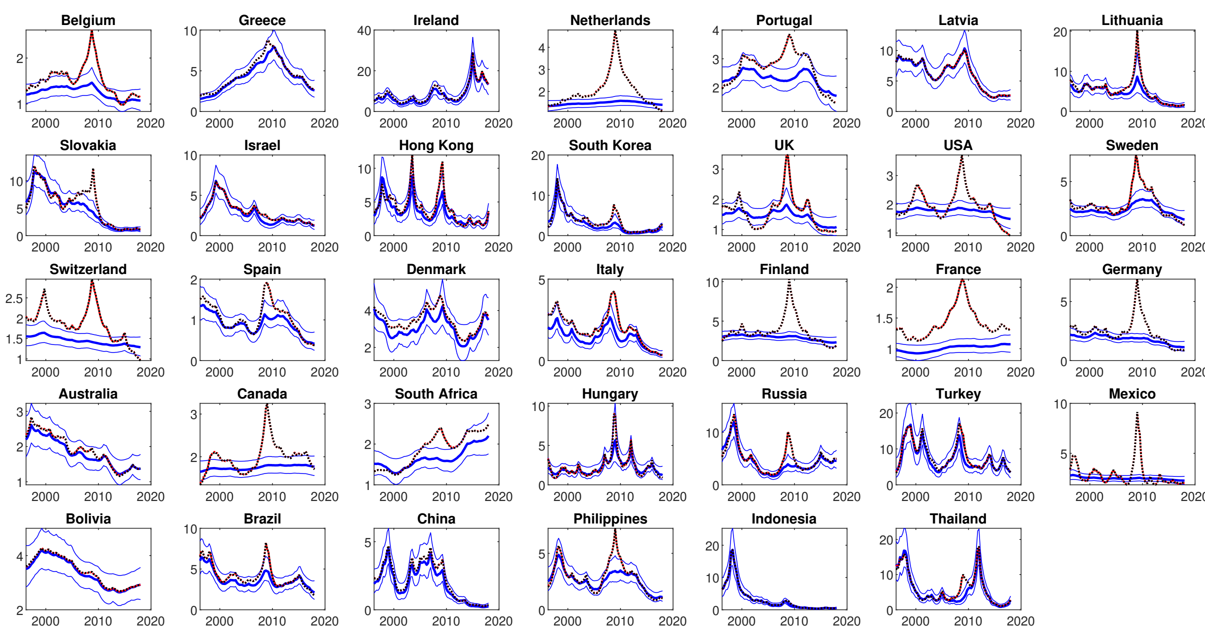


Figure 7: Posterior estimates for idiosyncratic output uncertainty $\exp(h_t^y/2)$. The solid blue lines are the means, 16% and 84% quantiles under the multi-country UC-FSV, the dotted red lines are posterior means under UC-FSV-$r_y = 0$, the dashed black lines are posterior means under UC-FSV-$r_y, r_\pi = 0$.

5 Out-of-sample Forecasting Results

Since our modifications are about uncertainty, we focus on the density forecast. We use the data from 1995Q4 to 2002Q4 as an initial estimation period, and use data through 2002Q4 to produce k-step-ahead forecast distributions. We consider forecast horizons of $k = 1, 4, 8, 12, 16$ quarters. So our forecast evaluation period begins in 2003Q1. We divide our out-of-sample forecasting results into three parts: forecasting inflation, forecasting output and jointly forecasting inflation and output. For each part, we discuss the results in three dimensions. The first dimension is aggregate forecasting performance over time and over economies (the aggregate LPL, by summing all economies and all time periods). Since we observe international macroeconomic uncertainty, it is natural to expect that considering such uncertainty will provide more accurate estimate in economic recession. Thus, the second dimension is about forecasting performance over time (we can study how the sums of LPL changes over time, by summing all economies at time t). After providing evidence that our multi-country UC-FSV can produce more accurate estimate in economic recession, we further study whether such good forecast performance is driven by particular economies, so the third dimension is about the forecasting performance at economy level. All results are presented relative to the forecast under UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$: we take differences, so a positive number indicates a model is forecasting better than the UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

5.1 Forecasting inflation

We first report the aggregate forecasting performance for inflation over time and over economies in Table 6. It is calculated by summing the LPL for the N economies over T_0 to $T - k$ (and recall that $j = 1$ denote inflation):

$$\text{LPL}_{\cdot,\cdot,1,k} = \sum_{t=T_0}^{t=T-k} \sum_{i=1}^{n} \log p(\widehat{y}_{t+k}^{(i,1)} = y_{t+k}^{(i,1)} | \text{Y}_{1:t}^{(i,1)})$$

The results show that the model with cross-country linkages in inflation (UC-FSV-$r_y = 0$ and UC-FSV) provides more accurate forecast for inflation than the model without cross-country linkages (UC-FSV-$r_y, r_\pi = 0$ and UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) at all horizons.
The forecasting result of inflation in Table 6 suggests the benefits of allowing for cross-country linkages, which is done through considering the global inflation uncertainty in our paper. It is natural to expect that the good forecasting result may largely arise from periods of uncertainty. To investigate this point, we calculate the sums of LPL over time. A common method is, as done in Feldkircher et al. (2021), to sum the LPL for the N

Table 6: Sum of k-step-ahead log predictive likelihood for 34-country inflation

Model	k=1	k=4	k=8	k=12	k=16
UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$	0	0	0	0	0
UC-FSV-$r_y, r_\pi = 0$	-4.27	71.83	127.82	138.85	185.26
UC-FSV-$r_y = 0$	98.92	**265.09**	286.02	350.53	333.11
UC-FSV	**101.63**	257.39	**294.76**	**379.19**	**356.89**

economies at time t:

$$\mathrm{LPL}_{t,\cdot,1,k} = \sum_{i=1}^{n} \log\, p(\widehat{y}_{t+k}^{(i,1)} = y_{t+k}^{(i,1)} | \mathrm{Y}_{1:t}^{(i,1)})$$

For example, suppose we are at the time point of 2007Q4, then $k = 1$ means we are forecasting the data in 2008Q1, and $k = 4$ means we are forecasting the data in 2008Q4. So this method helps to answer at time t, which model can provide the most accurate forecast in the future.

However, recall that global inflation uncertainty shows significant increases around 2008 and 2015 (see Figure 1a, and because our forecast starts from 2003Q1, so we omit the increase in 2001). Such global inflation uncertainty drives strong co-movement across economies. So a more interesting study is to investigate whether this global inflation uncertainty can improve the forecast performance during periods of uncertainty. For example, suppose that we want to know which model can provide the most accurate forecast of 2008Q1? Different forecast horizons will provide the forecast made at different time t. If $k = 1$, then this means the forecast is made at 2007Q4 (one-step-ago). If $k = 4$, then this means the forecast is made at 2007Q1 (four-step-ago). Overall, the difference is the X axis. Suppose that we are at time t, in Feldkircher et al. (2021), the X axis is t and represents when we make the forecast, but in our paper, the X axis is $t+k$ and represents when to forecast. That is how we produce Figure 8. About the starting time, since we make the first forecast at 2002Q4, if $k = 1$, the time to forecast (at 2002Q4) is 2003Q1, so in Figure 8, the X axis (time to forecast) starts from 2003Q1 when $k = 1$. If $k = 4$, the time to forecast (at 2002Q4) is 2003Q4, so in Figure 8, the X axis (time to forecast) starts from 2003Q4 when $k = 4$. Similarly, if $k = 16$, the time to forecast (at 2002Q4) is 2006Q4, so in Figure 8, the X axis (time to forecast) starts from 2006Q4 when $k = 16$.

We plot the results (against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Figure 8 (for brevity, we only plot the results of UC-FSV). To forecast inflation during periods of uncertainty (like 2008), we find overall good forecast performance for UC-FSV at all horizons, particularly at long horizons. This indicates the importance of taking into account cross-country linkages for improving forecasts of inflation, especially to forecast periods of uncertainty. To forecast more stable periods, it does not harm to take into account cross-country linkages.

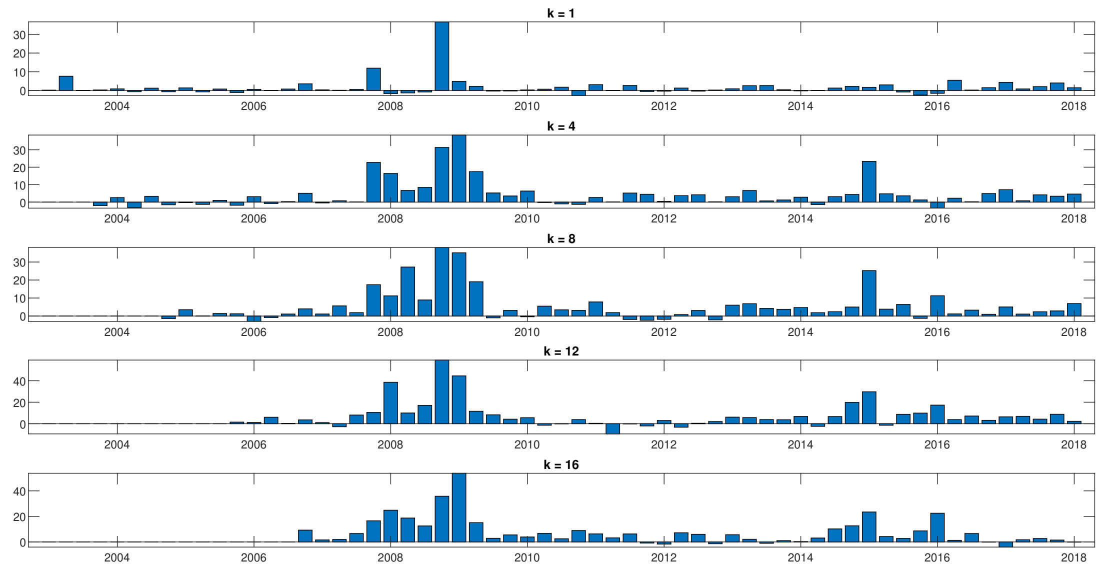


Figure 8: Sums of k-step ahead LPL of inflation for UC-FSV relative to UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$ over time. The X axis is $t+k$ and represents when to forecast.

The sums of LPL over time in Figure 8 is for the 34 economies. Someone may question whether the good forecasting result is driven by particular economies? To investigate this point, we present the forecasting result for individual economies. The LPL of inflation for economy i at time t, which can be calculated by:

$$\mathrm{LPL}_{t,i,1,k} = \log\, p(\widehat{y}_{t+k}^{(i,1)} = y_{t+k}^{(i,1)} | \mathrm{Y}_{1:t}^{(i,1)})$$

We plot the results (against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Figure 9. Here the period of uncertainty that we plot is 2008Q4, so time to forecast is 2008Q4 ($t+k = 2008Q4$). If $k = 1$, then the time we make forecast is 2008Q3, and we find overall good forecast performance for most economies with more pronounced gains in advanced economies (The first 23 economies are AEs, and the following 11 economies are EMEs). A similar pattern is found if $k = 16$. The time we make forecast is 2004Q4, and we also find overall good forecast performance for most economies. We also find significant gains in Spain and USA. The gain is not so significant if $k = 1$ as the gain if $k = 16$. In Figure 9, we only plot the shortest horizon $k = 1$ and the longest horizon $k = 16$, for middle horizons ($k = 4, 8, 12$), we find good forecasting result across most economies and did not find particular economy which is important in driving good forecasting results. Overall, We find good forecast performance for UC-FSV for most economies and such good forecast performance is not driven by particular economies.

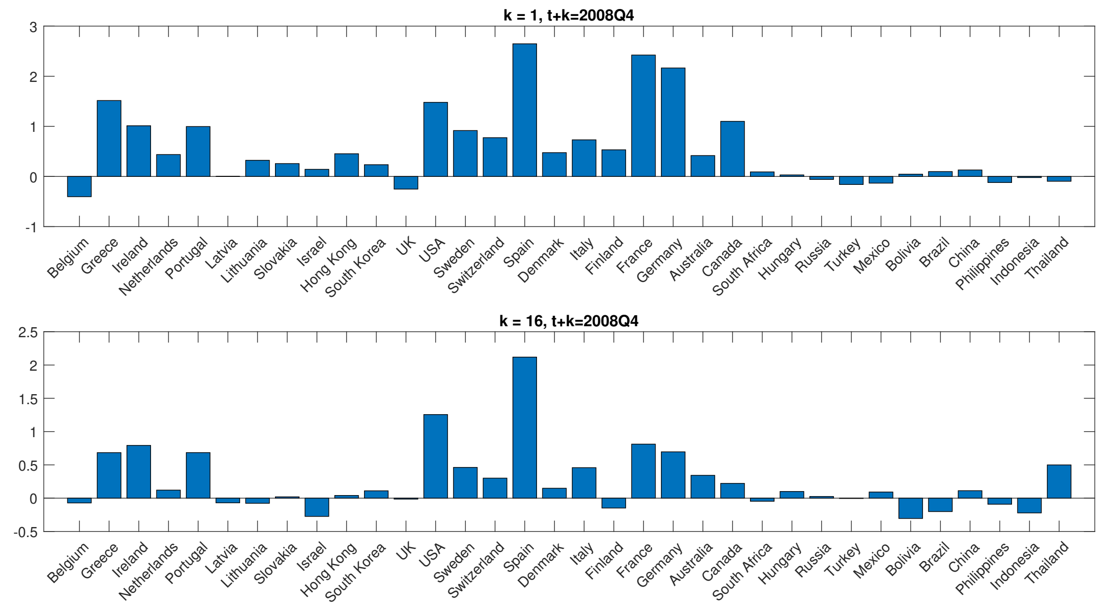


Figure 9: Sums of k-step ahead LPL of inflation for economy i under UC-FSV relative to UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

5.2 Forecasting output

With regard to output, we report the sums of LPL of output over time and over economies in Table 7. It is calculated by summing the LPL for the N economies over T_0 to $T - k$ (and recall that $j = 2$ denote output):

$$\text{LPL}_{\cdot,\cdot,2,k} = \sum_{t=T_0}^{t=T-k} \sum_{i=1}^{n} \log\, p(\widehat{y}_{t+k}^{(i,2)} = y_{t+k}^{(i,2)} | \mathrm{Y}_{1:t}^{(i,2)})$$

The results show that the model, which allows for both idiosyncratic stochastic volatility in output and cross-country linkages in output (UC-FSV), provides the most accurate forecast for output at all horizons.

Table 7: Sum of k-step-ahead log predictive likelihood for 34-economy output

Model	k=1	k=4	k=8	k=12	k=16
UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$	0	0	0	0	0
UC-FSV-$r_y, r_\pi = 0$	577.02	694.98	811.01	797.25	684.98
UC-FSV-$r_y = 0$	566.81	668.04	852.04	772.90	680.99
UC-FSV	**762.93**	**1194.99**	**1211.17**	**1208.10**	**1052.36**

Similar to the analysis of inflation, the second dimension of discussion for output is sums of LPL over time (by summing all economies at time t), which can be calculated by:

$$\text{LPL}_{t,\cdot,2,k} = \sum_{i=1}^{n} \log\, p(\widehat{y}_{t+k}^{(i,2)} = y_{t+k}^{(i,2)} | \text{Y}_{1:t}^{(i,2)})$$

We plot the results (against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Figure 10. To forecast output during periods of uncertainty (like 2008), we find overall good forecast performance for UC-FSV at all horizons. This indicates the importance of taking into account cross-country linkages for improving forecasts of output, especially to forecast periods of uncertainty. To forecast more stable periods, it does not harm to take into account cross-country linkages.

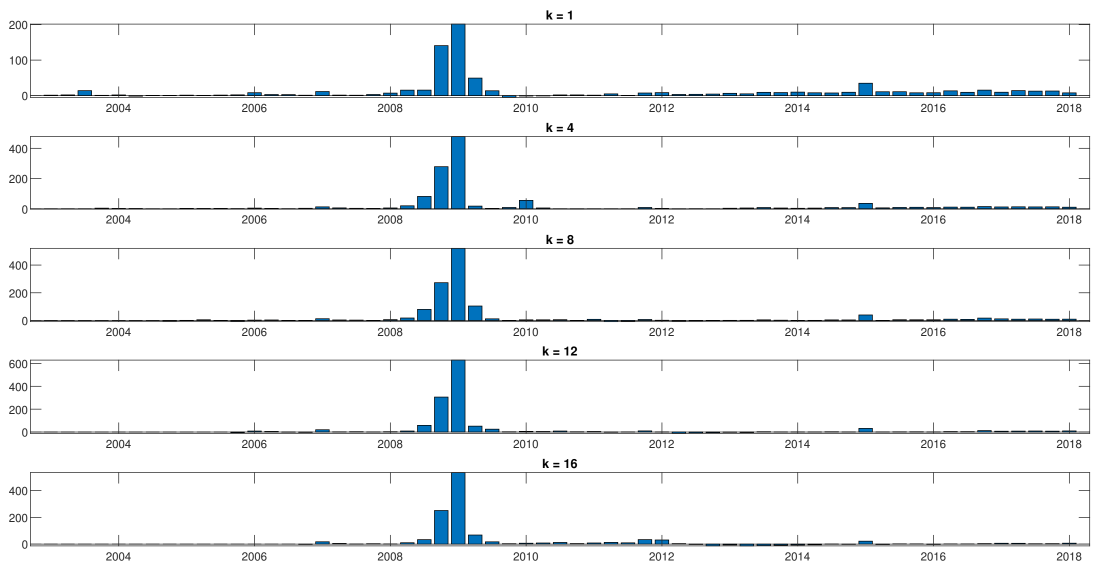


Figure 10: Sums of k-step ahead LPL of output for UC-FSV relative to UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$ over time. The X axis is $t + k$ and represents when to forecast.

To investigate whether the good forecast performance is driven by particular economies, we calculate the sums of LPL of output for economy i at time t by:

$$\text{LPL}_{t,i,2,k} = \log\, p(\widehat{y}_{t+k}^{(i,2)} = y_{t+k}^{(i,2)} | \text{Y}_{1:t}^{(i,2)})$$

We plot the results (against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Figure 11. We choose 2008Q4 to represent the period of uncertainty. For $k = 1$ and $k = 16$, we both find overall good forecast performance for UC-FSV for all economies. The highest gain is found for Hungary, followed by Sweden. However, different from the conclusion in the case of forecasting inflation that more pronounced gains are found in AEs, we find significant gains in both AEs and EMEs. This implies that allowing for idiosyncratic stochastic volatility in output and cross-country linkages in output is important for both AEs and EMEs.

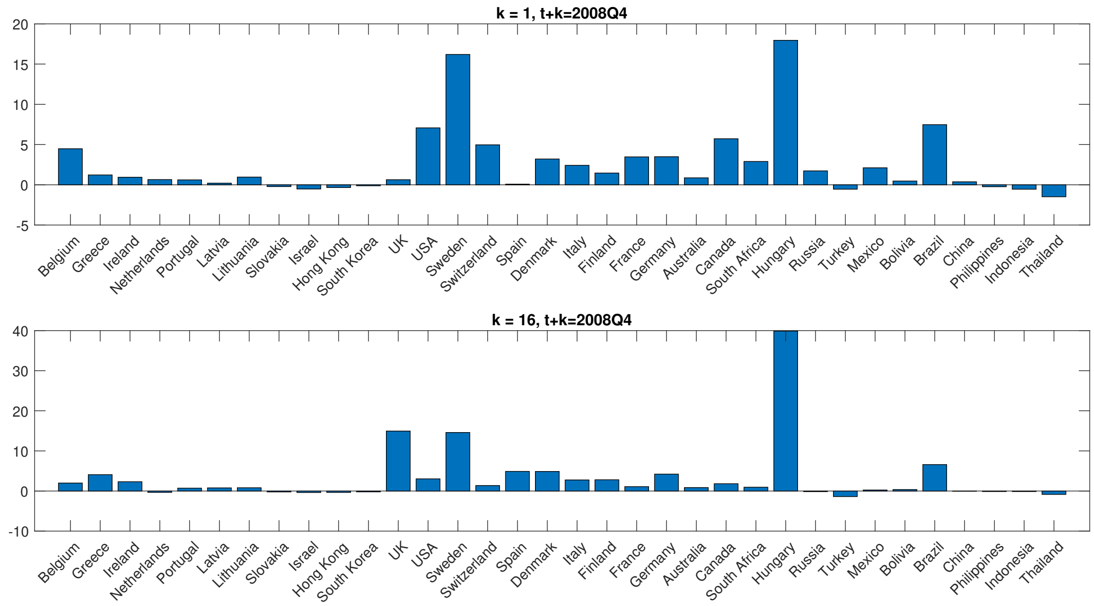


Figure 11: Sums of k-step ahead LPL of output in economy i for UC-FSV relative to UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

5.3 Jointly Forecasting inflation and output

With regard to the joint predictive density for inflation and output, we first report the sums of joint LPL over time and over economies in Table 8. It is calculated by summing the LPL for the N economies over T_0 to $T-k$ (and for all j, recall that $j = 1$ denote inflation, $j = 2$ denote output):

$$\text{LPL}_{\cdot,\cdot,\cdot,k} = \sum_{t=T_0}^{t=T-k} \sum_{i=1}^{n} \sum_{j=1}^{2} \log\, p(\widehat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \text{Y}_{1:t}^{(i,j)})$$

The results show that the model, which allows for idiosyncratic stochastic volatility in output and cross-country linkages in both inflation and output (UC-FSV), provides the most accurate joint forecast for inflation and output at all horizons. Next, we study the time-variation in forecast performance to see whether the benefits arise from the forecast during periods of uncertainty. So the second dimension of discussion for joint predictive density for inflation and output is sums of joint LPL over time (by summing all j and all

Table 8: Sum of k-step-ahead joint log predictive likelihood for 34-economy inflation and output

Model	k=1	k=4	k=8	k=12	k=16
UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$	0	0	0	0	0
UC-FSV-$r_y, r_\pi = 0$	520.37	679.42	751.62	794.16	615.81
UC-FSV-$r_y = 0$	658.57	898.62	1084.28	1084.13	1148.35
UC-FSV	**883.34**	**1513.05**	**1545.20**	**1824.70**	**1672.17**

economies at time t), which can be calculated by:

$$\text{LPL}_{t,\cdot,\cdot,k} = \sum_{i=1}^{n} \sum_{j=1}^{2} \log\, p(\widehat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \mathrm{Y}_{1:t}^{(i,j)})$$

We plot the results (against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Figure 12. A similar pattern to inflation and output was found. To jointly forecast inflation and output during periods of uncertainty (like 2008), we find overall good forecast performance under UC-FSV at all horizons. This indicates the importance of taking into account cross-country linkages (in inflation and output) for improving forecasts of inflation and output, especially during periods of uncertainty.

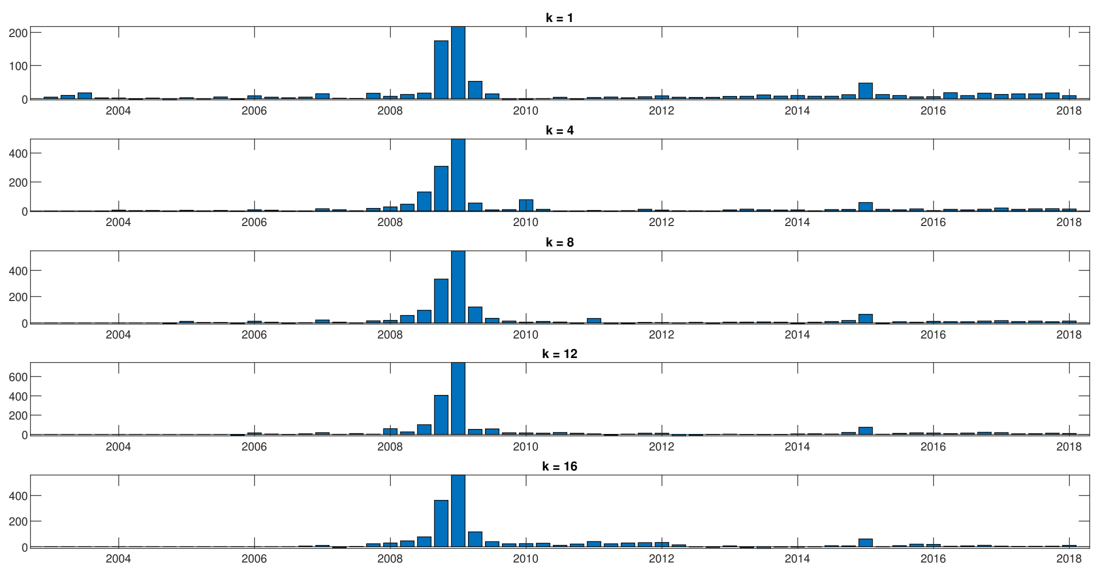


Figure 12: Sums of k-step ahead joint LPL for UC-FSV relative to UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$ over time. The X axis is $t + k$ and represents when to forecast.

Finally, we investigate whether the good forecast performance of periods of uncertainty is driven by particular economies, so the third dimension of discussion for joint predictive density for inflation and output is sums of joint LPL at the economy level (by summing all j for economy i), which can be calculated by:

$$\text{LPL}_{t,i,\cdot,k} = \sum_{t=T_0}^{t=T-k} \sum_{j=1}^{2} \log\, p(\widehat{y}_{t+k}^{(i,j)} = y_{t+k}^{(i,j)} | \text{Y}_{1:t}^{(i,j)})$$

We plot the results (against UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$) in Figure 13. A similar pattern to output is found. (This is sensible since the gains in output are much larger than gains in inflation, see Figure 9 and Figure 11). We find overall good forecast performance for UC-FSV for all economies.

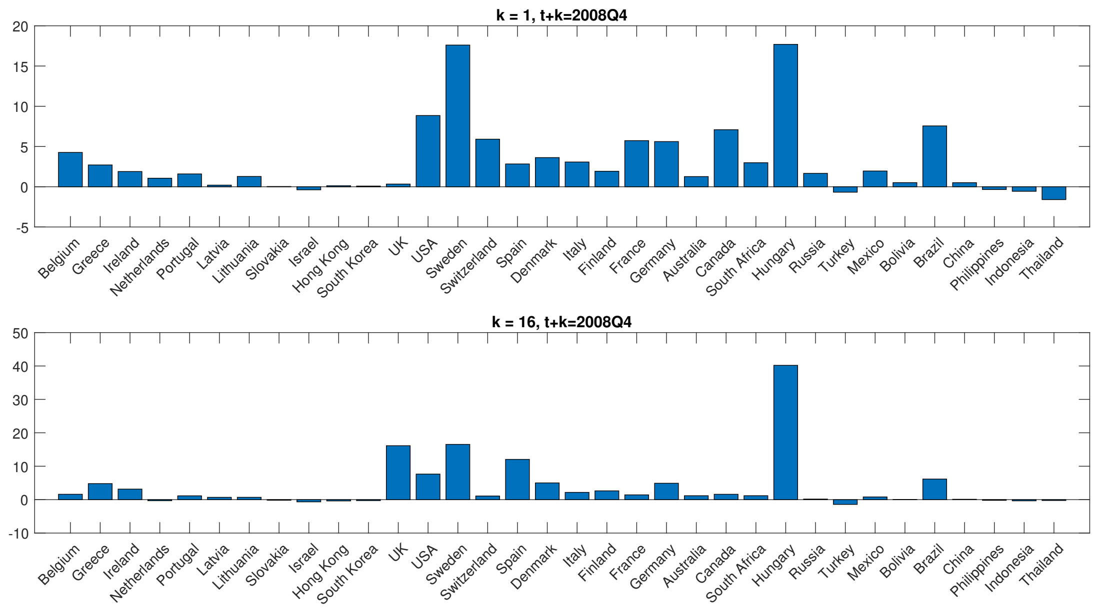


Figure 13: Sums of k-step ahead joint LPL in country i for UC-FSV relative to UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$.

6 Conclusion

This paper develops a multi-country unobserved components model that allows for cross-country linkages and models all economies jointly. The model includes 23 Advanced Economies and 11 Emerging Market Economies, with two variables (output and inflation) within each economy and their relationship being inspired by the Phillips curve. An important feature of our model is that we model all economies jointly and allow for cross-country linkages, through the factor stochastic volatility specification. Factor stochastic volatility specification enables us to study the commonality in international macroeconomic uncertainty (global uncertainty). Past research has highlighted importance of allowing for stochastic volatility. Accordingly, we allow for stochastic volatility in all equations and factors. Another important feature of our model is the use of the sparsification. We use the sparsification to coherently estimate the factors and the number of factors in one step. We also use the sparsification to remove stochastic volatility in a data-based manner. More specifically, we use the Horseshoe prior to achieve sparsification. The Horseshoe prior is a global-local shrinkage prior, which is found to strongly shrink the parameter but at the same time provide flexibility if necessary. Since the prior density of Horseshoe prior does not have an analytical form, which makes it difficult to compute the Bayes Factor for nested models, we propose to use prior simulation to solve this problem. This method applies for any prior which has a hierarchical form. Another advantage of our model is computational. Recent research has devoted to speeding up computation and and one prominent progress is performing equation-by-equation estimation. The factor stochastic volatility specification also enables us to estimate this high dimensional model equation-by-equation.

In an empirical application we first present evidence of global uncertainty and it coincides with major economic events. Then, we show the evidence of the use of the sparsification. The Bayesian model comparison results provide strong support to our proposed model which takes into account global uncertainty. After justifying that our model is sensible and supported, we present an expansive set of posterior estimates that we hope would be helpful. The posterior estimates of coefficients show that global uncertainty will decrease the persistence and flatten the Phillips curve. The posterior estimates of trends are found to be sensible and indicate that they are driven by both domestic factors and global factors. A by-product is that our model allows us to tell the uncertainty in a equation is driven by common international components, or components operating at an economy level, or both. Finally, we provide a detailed forecasting exercise to evaluate the merits of our model. We find our model can provide more accurate forecasts, especially if the aim is to forecast periods of uncertainty. And such good forecast performance is for most economies and not driven by particular economies.

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Appendices

A Testing for Time-Variation in Coefficients

In this appendix, we illustrate the method to test for time-variation in coefficients and report the estimated Bayes Factor, which support the constant coefficients models.

What we did is to allow the coefficients in UC-SV to be time-varying in the noncentered parameterization as follows:

$$\pi_{i,t} - \tau^{\pi}_{i,t} = (\rho_{i,0} + \omega^{\rho}_{i}\widetilde{\rho}_{i,t})(\pi_{i,t-1} - \tau^{\pi}_{i,t-1}) + (\alpha_{i,0} + \omega^{\alpha}_{i}\widetilde{\alpha}_{i,t})(y_{i,t} - \tau^{y}_{i,t}) + \varepsilon^{\pi}_{i,t}, \quad \varepsilon^{\pi}_{i,t} \sim \mathcal{N}(0,\ e^{h_{i,t}}) \tag{24}$$

$$y_{i,t} - \tau^{y}_{i,t} = \varphi_{i,1}(y_{i,t-1} - \tau^{y}_{i,t-1}) + \varphi_{i,2}(y_{i,t-2} - \tau^{y}_{i,t-2}) + \varepsilon^{y}_{i,t}, \quad \varepsilon^{y}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_y) \tag{25}$$

$$\tau^{\pi}_{i,t} = \tau^{\pi}_{i,t-1} + \varepsilon^{\tau\pi}_{i,t}, \quad \varepsilon^{\tau\pi}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{\tau\pi}) \tag{26}$$

$$\tau^{y}_{i,t} = \tau^{y}_{i,t-1} + \varepsilon^{\tau y}_{i,t}, \quad \varepsilon^{\tau y}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_{\tau y}) \tag{27}$$

$$h_{i,t} = h_{i,t-1} + \varepsilon^{h}_{i,t}, \quad \varepsilon^{h}_{i,t} \sim \mathcal{N}(0,\ \sigma^2_h) \tag{28}$$

$$\widetilde{\rho}_{i,t} = \widetilde{\rho}_{i,t-1} + \varepsilon^{\rho}_{i,t}, \quad \varepsilon^{\rho}_{i,t} \sim \mathcal{N}(0,\ 1) \tag{29}$$

$$\widetilde{\alpha}_{i,t} = \widetilde{\alpha}_{i,t-1} + \varepsilon^{\alpha}_{i,t}, \quad \varepsilon^{\alpha}_{i,t} \sim \mathcal{N}(0,\ 1) \tag{30}$$

We assume a normal prior with zero mean and variance ten for $\rho_{i,0}$, ω^{ρ}_{i}, $\alpha_{i,0}$, ω^{α}_{i}. The prior for other parameters are kept the same as UC-FSV.

Next, we calculate the Bayes factor in favor of the unrestricted model against the restricted version where $\omega^{\rho}_{i} = 0$ as:

$$BF_{\rho_i} = \frac{p(\omega^{\rho}_{i} = 0)}{p(\omega^{\rho}_{i} = 0 |\ y)} \tag{31}$$

So if BF_{ρ_i} is larger than 1, then the Bayes Factor is in favor of the unrestricted model. In this part, the unrestricted model is time-varying ρ_i. For simplicity, we compare the log Bayes Factor. So a positive log Bayes Factor supports the time-varying coefficient ρ_i. We can calculate the log Bayes Factor for ω^{α}_{i} similarly.

Using the data in the empirical section, we report the log Bayes Factor in Table 9. We find most log Bayes Factor are negative (except for 3 cases: log BF_{ρ_i} for Latvia, Turkey and Mexico), so we think this result strongly supports constant coefficients models.

Table 9: The estimated log Bayes factors for ω_i^ρ and ω_i^α

Economies	log BF_{ρ_i}	log BF_{α_i}
Belgium	-2.83	-1.03
Greece	-2.42	-0.99
Ireland	-2.00	-0.86
Netherlands	-2.21	-2.23
Portugal	-2.61	-1.52
Latvia	0.08	-1.81
Lithuania	-2.20	-2.01
Slovakia	-2.64	-2.33
Israel	-3.30	-3.48
Hong Kong	-2.54	-3.65
South Korea	-1.71	-2.63
UK	-1.44	-3.37
USA	-2.86	-3.63
Sweden	-2.79	-2.28
Switzerland	-2.80	-2.15
Spain	-3.15	-2.54
Denmark	-3.40	-2.45
Italy	-2.30	-2.85
Finland	-2.95	-3.02
France	-2.72	-2.82
Germany	-3.07	-2.94
Australia	-3.13	-3.09
Canada	-3.02	-1.27
South Africa	-1.97	-3.45
Hungary	-3.00	-3.57
Russia	-1.41	-3.37
Turkey	1.08	-3.42
Mexico	2.99	-3.44
Bolivia	-1.02	-3.68
Brazil	-2.18	-3.10
China	-2.09	-2.87
Philippines	-2.99	-2.81
Indonesia	-2.91	-2.39
Thailand	-3.37	-1.41

B Estimates of factor loading matrices

In this appendix, we report the posterior estimates of factor loading matrices under UC-FSV. Basically, we have two classes of factors:

(1): global inflation factor f_t, and its loading matrix is L_π, L_π is $n \times r_\pi$, in our empirical application, $n = 34$, $r_\pi = 5$. Table 10 is the loadings of global inflation factor. We report the posterior mean of the five factors' loadings, but only the quantiles of first factor's loadings for brevity.

(2): global output factor g_t, and its loading matrix is L_y, L_y is $n \times r_y$, in our empirical application, $n = 34$, $r_y = 2$. Table 11 is the loadings of global output factor. We report the posterior mean and quantiles of the two factors' loadings.

And for identification, we assume the factor loading matrices are lower triangular matrices with ones on the main diagonal, so some elements in L_π and L_y are 1 or 0.

Table 10: Posterior Estimates of factor loading matrix L_π

	1st factor			2nd factor	3rd factor	4th factor	5th factor
Economy	mean	16%	84%	mean	mean	mean	mean
Belgium	1	1	1	0	0	0	0
Greece	2.29	1.52	3.01	1	0	0	0
Ireland	2.09	1.40	2.74	1.05	1	0	0
Netherlands	1.79	1.17	2.39	-0.97	-0.69	1	0
Portugal	1.57	0.98	2.15	-0.71	-0.17	0.51	1
Latvia	1.96	1.17	2.75	0.66	0.28	0.20	-0.71
Lithuania	2.26	1.41	3.08	1.00	0.35	0.19	-0.82
Slovakia	2.05	1.35	2.72	-0.09	-0.07	0.50	0.23
Israel	2.06	1.31	2.81	-0.29	0.02	-0.12	0.34
Hong Kong	0.93	0.15	1.74	0.60	0.34	-0.90	-0.73
South Korea	1.53	0.99	2.05	-0.18	-0.21	-0.24	-0.63
UK	1.83	1.24	2.38	0.09	-0.13	0.00	-0.22
USA	2.94	2.02	3.81	-0.73	-0.45	0.29	-0.34
Sweden	1.98	1.32	2.61	0.03	0.15	-0.44	-0.59
Switzerland	1.73	1.17	2.26	-0.16	0.07	-0.27	0.34
Spain	3.08	2.11	3.97	-0.03	-0.01	0.71	0.87
Denmark	1.69	1.14	2.21	0.04	0.03	0.21	-0.03
Italy	1.32	0.87	1.74	-0.42	-0.11	0.11	0.75
Finland	1.23	0.77	1.68	0.42	0.38	-0.18	-0.31
France	2.10	1.46	2.68	0.69	0.59	-0.25	0.26
Germany	2.09	1.43	2.68	0.36	0.15	0.46	-0.23
Australia	2.16	1.47	2.80	0.01	0.29	-0.21	0.44
Canada	2.44	1.65	3.19	-0.08	-0.01	-0.54	-0.45
South Africa	1.72	1.01	2.43	-0.79	-0.68	0.45	-0.27
Hungary	3.00	1.86	4.15	0.14	0.11	0.46	-0.05
Russia	0.26	-0.65	1.17	0.48	0.52	-0.57	-0.04
Turkey	3.04	1.79	4.31	0.15	0.13	-0.12	0.04
Mexico	0.49	0.02	0.97	0.41	0.22	0.07	0.18
Bolivia	0.82	-0.04	1.69	0.65	0.25	-0.52	-0.85
Brazil	-0.36	-1.02	0.30	0.17	0.15	-0.50	-0.43
China	0.38	-0.11	0.89	0.50	0.29	-0.62	-0.86
Philippines	1.33	0.68	1.98	-0.25	-0.14	-0.19	-0.12
Indonesia	0.61	0.05	1.18	-0.26	-0.04	-0.16	0.12
Thailand	2.52	1.67	3.33	0.23	-0.04	-0.16	-0.70

Table 11: Posterior Estimates of factor loading matrix L_y

	1st factor			2nd factor		
Economy	mean	16%	84%	mean	16%	84%
Belgium	1	1	1	0	0	0
Greece	2.11	1.35	2.86	1	1	1
Ireland	3.91	2.61	5.21	-2.28	-5.00	0.40
Netherlands	1.75	1.23	2.28	2.61	1.59	3.89
Portugal	1.60	1.14	2.06	1.04	-0.15	2.22
Latvia	1.39	0.26	2.52	0.46	-2.13	3.04
Lithuania	3.70	2.22	5.18	5.54	1.06	10.09
Slovakia	2.99	1.84	4.12	4.63	1.79	7.37
Israel	1.08	0.74	1.41	-1.02	-2.19	0.03
Hong Kong	3.57	2.63	4.51	-0.18	-2.42	1.98
South Korea	2.93	2.20	3.66	-2.03	-4.41	0.21
UK	1.34	0.94	1.74	-1.19	-2.81	0.24
USA	1.60	1.20	1.99	-0.68	-2.00	0.57
Sweden	2.91	2.24	3.57	-0.12	-2.33	1.99
Switzerland	1.74	1.37	2.10	-0.95	-2.30	0.28
Spain	0.70	0.48	0.93	0.12	-0.65	0.84
Denmark	2.12	1.46	2.78	0.40	-1.49	2.26
Italy	2.08	1.62	2.54	-0.23	-1.65	1.05
Finland	3.28	2.28	4.27	4.23	2.38	6.28
France	1.41	1.13	1.70	-0.12	-1.00	0.72
Germany	2.56	1.95	3.18	2.23	0.91	3.72
Australia	0.58	0.23	0.93	-1.62	-2.80	-0.62
Canada	1.34	0.98	1.70	0.81	-0.24	1.87
South Africa	1.04	0.73	1.34	0.60	-0.27	1.49
Hungary	1.31	0.45	2.24	-1.53	-4.37	1.17
Russia	3.36	2.48	4.24	-0.62	-3.09	1.76
Turkey	3.83	2.65	5.00	1.05	-1.50	3.55
Mexico	2.52	1.79	3.26	3.37	2.01	5.04
Bolivia	0.83	0.34	1.33	-0.48	-2.01	0.98
Brazil	3.02	2.20	3.83	-2.27	-4.93	0.22
China	0.95	0.51	1.39	-1.15	-2.63	0.23
Philippines	1.37	0.69	2.05	3.03	1.29	4.94
Indonesia	0.60	0.26	0.94	-0.08	-1.39	1.14
Thailand	2.87	2.12	3.63	0.74	-1.21	2.63

C Estimates of constant coefficients

In this appendix, we report the posterior estimates of constant coefficients: ρ, α, φ_1, φ_2.

Table 12: Posterior estimates of inflation persistence ρ

Economy	UC-FSV mean	UC-FSV 16%	UC-FSV 84%	UC-FSV-$r_y = 0$ posterior mean	UC-FSV-$r_y, r_\pi = 0$ posterior mean	UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$ posterior mean
Belgium	0.28	0.19	0.37	0.28	0.33	0.33
Greece	0.36	0.27	0.45	0.36	0.41	0.41
Ireland	0.46	0.35	0.57	0.46	0.59	0.59
Netherlands	0.29	0.18	0.39	0.29	0.28	0.28
Portugal	0.36	0.27	0.46	0.36	0.47	0.47
Latvia	0.65	0.59	0.71	0.65	0.70	0.70
Lithuania	0.62	0.54	0.70	0.62	0.66	0.66
Slovakia	0.54	0.45	0.62	0.54	0.63	0.63
Israel	0.47	0.38	0.56	0.47	0.53	0.53
Hong Kong	0.56	0.45	0.68	0.57	0.58	0.58
South Korea	0.22	0.12	0.31	0.22	0.31	0.31
UK	0.43	0.34	0.52	0.43	0.41	0.41
USA	0.22	0.14	0.29	0.22	0.28	0.28
Sweden	0.37	0.28	0.45	0.37	0.52	0.52
Switzerland	0.34	0.26	0.42	0.34	0.28	0.28
Spain	0.24	0.17	0.30	0.23	0.39	0.40
Denmark	0.23	0.13	0.32	0.23	0.25	0.25
Italy	0.49	0.41	0.57	0.49	0.59	0.59
Finland	0.49	0.40	0.57	0.49	0.56	0.56
France	0.15	0.09	0.21	0.15	0.26	0.26
Germany	0.07	0.02	0.13	0.08	0.11	0.11
Australia	0.16	0.08	0.24	0.16	0.20	0.20
Canada	0.10	0.03	0.16	0.10	0.11	0.11
South Africa	0.55	0.46	0.65	0.55	0.56	0.56
Hungary	0.40	0.30	0.50	0.40	0.49	0.50
Russia	0.80	0.71	0.89	0.80	0.79	0.79
Turkey	0.93	0.90	0.97	0.94	0.94	0.94
Mexico	0.81	0.75	0.88	0.81	0.80	0.81
Bolivia	0.33	0.23	0.44	0.33	0.32	0.33
Brazil	0.63	0.52	0.74	0.63	0.60	0.61
China	0.53	0.44	0.62	0.52	0.58	0.58
Philippines	0.57	0.47	0.67	0.57	0.61	0.61
Indonesia	0.36	0.26	0.47	0.36	0.35	0.35
Thailand	0.41	0.31	0.51	0.41	0.53	0.53

Table 13: Estimates of slope of Phillips Curve α

	UC-FSV			UC-FSV-$r_y = 0$	UC-FSV-$r_y, r_\pi = 0$	UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$
Economy	mean	16%	84%	posterior mean	posterior mean	posterior mean
Belgium	0.16	0.09	0.23	0.16	0.22	0.22
Greece	0.02	0.00	0.03	0.02	0.02	0.02
Ireland	0.01	0.00	0.02	0.01	0.01	0.01
Netherlands	0.01	0.00	0.03	0.01	0.03	0.03
Portugal	0.05	0.01	0.08	0.05	0.05	0.05
Latvia	0.07	0.04	0.10	0.07	0.10	0.10
Lithuania	0.02	0.00	0.04	0.02	0.04	0.04
Slovakia	0.03	0.01	0.06	0.03	0.08	0.08
Israel	0.07	0.02	0.12	0.07	0.09	0.09
Hong Kong	0.05	0.01	0.09	0.05	0.06	0.06
South Korea	0.03	0.01	0.06	0.04	0.08	0.08
UK	0.02	0.00	0.04	0.02	0.04	0.04
USA	0.05	0.01	0.08	0.05	0.05	0.06
Sweden	0.09	0.06	0.13	0.09	0.14	0.14
Switzerland	0.07	0.03	0.10	0.07	0.15	0.15
Spain	0.08	0.03	0.13	0.07	0.12	0.11
Denmark	0.03	0.01	0.05	0.03	0.05	0.05
Italy	0.04	0.02	0.06	0.04	0.09	0.09
Finland	0.08	0.05	0.10	0.08	0.11	0.11
France	0.02	0.00	0.04	0.02	0.11	0.11
Germany	0.02	0.00	0.03	0.02	0.05	0.05
Australia	0.02	0.00	0.04	0.02	0.03	0.03
Canada	0.10	0.05	0.16	0.10	0.20	0.20
South Africa	0.04	0.01	0.07	0.04	0.06	0.06
Hungary	0.05	0.01	0.10	0.05	0.08	0.08
Russia	0.04	0.01	0.07	0.04	0.04	0.05
Turkey	0.06	0.02	0.11	0.07	0.08	0.09
Mexico	0.04	0.01	0.08	0.04	0.05	0.05
Bolivia	0.09	0.02	0.15	0.09	0.08	0.08
Brazil	0.09	0.03	0.15	0.09	0.08	0.08
China	0.22	0.13	0.30	0.21	0.23	0.22
Philippines	0.02	0.00	0.04	0.02	0.03	0.03
Indonesia	0.15	0.03	0.26	0.14	0.12	0.15
Thailand	0.01	0.00	0.02	0.01	0.02	0.02

Table 14: Estimates of output persistence φ_1

	UC-FSV			UC-FSV-$r_y = 0$	UC-FSV-$r_y, r_\pi = 0$	UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$
Economy	mean	16%	84%	posterior mean	posterior mean	posterior mean
Belgium	0.37	0.27	0.47	0.58	0.58	0.62
Greece	0.09	-0.02	0.20	0.17	0.17	0.10
Ireland	-0.17	-0.29	-0.05	-0.03	-0.03	-0.21
Netherlands	0.14	0.06	0.22	0.31	0.31	0.27
Portugal	0.12	0.02	0.23	0.28	0.28	0.21
Latvia	0.23	0.11	0.34	0.25	0.25	0.17
Lithuania	0.19	0.09	0.28	0.22	0.22	0.06
Slovakia	-0.06	-0.15	0.03	-0.06	-0.06	-0.21
Israel	0.28	0.16	0.39	0.34	0.34	0.12
Hong Kong	0.10	-0.03	0.23	0.39	0.39	0.13
South Korea	-0.06	-0.18	0.06	0.23	0.24	0.27
UK	0.22	0.10	0.35	0.38	0.38	0.53
USA	0.04	-0.07	0.16	0.26	0.26	0.23
Sweden	-0.07	-0.17	0.04	0.07	0.07	0.19
Switzerland	0.21	0.11	0.31	0.46	0.46	0.39
Spain	0.58	0.47	0.71	0.72	0.72	0.73
Denmark	-0.13	-0.24	-0.01	-0.02	-0.02	-0.05
Italy	0.29	0.19	0.39	0.54	0.54	0.44
Finland	0.05	-0.03	0.13	0.16	0.16	0.14
France	0.15	0.06	0.24	0.46	0.45	0.43
Germany	0.06	-0.02	0.14	0.20	0.20	0.25
Australia	-0.08	-0.20	0.03	-0.09	-0.09	-0.20
Canada	0.33	0.23	0.43	0.43	0.42	0.42
South Africa	0.39	0.28	0.50	0.57	0.57	0.39
Hungary	0.20	0.09	0.32	0.24	0.24	0.39
Russia	0.38	0.27	0.48	0.52	0.52	0.43
Turkey	-0.04	-0.14	0.06	0.03	0.03	-0.02
Mexico	0.30	0.22	0.39	0.29	0.29	0.38
Bolivia	-0.18	-0.30	-0.06	-0.17	-0.17	-0.28
Brazil	0.26	0.13	0.38	0.39	0.39	0.13
China	0.08	-0.05	0.22	0.19	0.18	0.07
Philippines	-0.06	-0.16	0.05	-0.02	-0.02	0.00
Indonesia	0.11	-0.03	0.26	0.13	0.14	0.43
Thailand	-0.02	-0.13	0.09	0.10	0.10	-0.10

Table 15: Estimates of output persistence φ_2

Economy	UC-FSV mean	UC-FSV 16%	UC-FSV 84%	UC-FSV-$r_y = 0$ posterior mean	UC-FSV-$r_y, r_\pi = 0$ posterior mean	UC-FSV-$r_y, r_\pi = 0, \omega_y^h = 0$ posterior mean
Belgium	-0.03	-0.12	0.06	-0.07	-0.07	-0.28
Greece	0.35	0.24	0.45	0.39	0.38	0.35
Ireland	0.10	0.01	0.21	0.15	0.15	-0.05
Netherlands	0.30	0.22	0.37	0.36	0.36	0.11
Portugal	0.27	0.17	0.37	0.33	0.33	0.18
Latvia	0.31	0.21	0.42	0.33	0.33	0.29
Lithuania	0.17	0.08	0.26	0.12	0.12	0.07
Slovakia	0.23	0.15	0.31	0.18	0.18	0.02
Israel	0.04	-0.06	0.13	0.06	0.06	0.08
Hong Kong	0.17	0.08	0.26	0.23	0.23	0.12
South Korea	0.12	0.03	0.21	0.18	0.18	-0.10
UK	0.16	0.05	0.27	0.10	0.09	-0.05
USA	0.22	0.12	0.31	0.15	0.15	0.12
Sweden	0.19	0.10	0.28	0.16	0.16	0.05
Switzerland	0.17	0.09	0.26	0.05	0.05	-0.08
Spain	0.22	0.11	0.32	0.18	0.18	-0.01
Denmark	0.07	-0.04	0.17	0.12	0.12	0.04
Italy	0.08	-0.01	0.16	0.01	0.01	-0.04
Finland	0.19	0.11	0.27	0.24	0.24	0.04
France	0.24	0.16	0.32	0.21	0.21	0.03
Germany	0.16	0.09	0.23	0.09	0.09	-0.05
Australia	0.10	-0.01	0.20	0.10	0.10	-0.04
Canada	0.04	-0.05	0.14	-0.02	-0.02	-0.18
South Africa	0.18	0.07	0.28	0.05	0.05	0.02
Hungary	0.16	0.06	0.25	0.12	0.13	-0.06
Russia	0.05	-0.04	0.15	0.07	0.07	-0.11
Turkey	0.06	-0.03	0.14	0.11	0.11	0.06
Mexico	-0.06	-0.14	0.02	0.03	0.03	-0.23
Bolivia	-0.13	-0.24	-0.01	-0.15	-0.15	-0.24
Brazil	0.14	0.03	0.24	0.08	0.08	-0.05
China	0.05	-0.05	0.16	0.10	0.09	0.01
Philippines	0.09	-0.01	0.18	0.09	0.10	-0.06
Indonesia	-0.01	-0.12	0.10	-0.05	-0.05	-0.11
Thailand	-0.05	-0.14	0.04	0.00	-0.01	0.08

D Testing for Time-Variation in volatilities

In this appendix, we report the estimated log Bayes factors to test for time-variation in volatilities.

Table 16: The estimated log Bayes factors for ω_i^h

Economies	log BF_{h_i} for inflation	log BF_{h_i} for output
Belgium	-6.16	-5.17
Greece	-3.97	6.68
Ireland	-6.52	20.92
Netherlands	-2.86	-8.56
Portugal	-7.86	-5.37
Latvia	-4.63	4.32
Lithuania	-2.48	4.27
Slovakia	82.45	60.82
Israel	21.29	19.80
Hong Kong	-5.52	22.35
South Korea	42.60	75.68
UK	-10.58	-4.79
USA	3.11	-5.79
Sweden	-6.42	-3.43
Switzerland	-4.95	-5.51
Spain	-6.13	-3.94
Denmark	-4.92	-4.83
Italy	-5.64	5.33
Finland	-5.30	-5.11
France	-5.58	-6.11
Germany	-1.93	-3.85
Australia	32.10	-2.67
Canada	-5.03	-6.21
South Africa	-1.29	-4.89
Hungary	3.17	-2.85
Russia	131.11	17.81
Turkey	20.91	3.71
Mexico	-4.99	-6.03
Bolivia	-2.02	-4.72
Brazil	6.08	-2.82
China	-4.20	69.65
Philippines	-3.86	-0.19
Indonesia	151.82	200.64
Thailand	8.09	64.03