3.2.1 - Expected Value and Variance of a Discrete Random Variable

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By continuing with example 3-1, what value should we expect to get? What would be the average value?

We can answer this question by finding the **expected value (or mean)**.

Expected Value (or mean) of a Discrete Random Variable

For a discrete random variable, the expected value, usually denoted as μ or $E(X)$, is calculated using:

$$\mu = E(X) = \sum x_i f(x_i)$$

The formula means that we multiply each value, x, in the support by its respective probability, $f(x)$, and then add them all together. It can be seen as an average value but weighted by the likelihood of the value.

Example 3-2: Expected Value

In Example 3-1 we were given the following discrete probability distribution:

x	0	1	2	3	4
$f(x)$	1/5	1/5	1/5	1/5	1/5

What is the expected value?

Answer

$$\begin{aligned}\mu = E(X) = \sum xf(x) &= 0\left(\frac{1}{5}\right) + 1\left(\frac{1}{5}\right) + 2\left(\frac{1}{5}\right) + 3\left(\frac{1}{5}\right) + 4\left(\frac{1}{5}\right) \\ &= 2\end{aligned}$$

For this example, the expected value was equal to a possible value of X. This may not always be the case. For example, if we flip a fair coin 9 times, how many heads should we expect? We will explain how to find this later but we should expect 4.5 heads. The expected value in this case is not a valid number of heads.

Now that we can find what value we should expect, (i.e. the expected value), it is also of interest to give a measure of the variability.

Variance of a Discrete Random Variable

The variance of a discrete random variable is given by:

$$\sigma^2 = \text{Var}(X) = \sum (x_i - \mu)^2 f(x_i)$$

The formula means that we take each value of x, subtract the expected value, square that value and multiply that value by its probability. Then sum all of those values.

There is an easier form of this formula we can use.

$$\sigma^2 = \text{Var}(X) = \sum x_i^2 f(x_i) - E(X)^2 = \sum x_i^2 f(x_i) - \mu^2$$

The formula means that first, we sum the square of each value times its probability then subtract the square of the mean. We will use this form of the formula in all of our examples.

Standard Deviation of a Discrete Random Variable

The standard deviation of a random variable, X, is the square root of the variance.

$$\sigma = \text{SD}(X) = \sqrt{\text{Var}}(X) = \sqrt{\sigma^2}$$

Example 3-3: Standard Deviation

Consider the first example where we had the values 0, 1, 2, 3, 4. The PMF in tabular form was:

x	0	1	2	3	4
$f(x)$	1/5	1/5	1/5	1/5	1/5

Find the variance and the standard deviation of X.

Answer

$$\text{Var}(X) = \left[0^2\left(\frac{1}{5}\right) + 1^2\left(\frac{1}{5}\right) + 2^2\left(\frac{1}{5}\right) + 3^2\left(\frac{1}{5}\right) + 4^2\left(\frac{1}{5}\right)\right] - 2^2 = 6 - 4 = 2$$

$$\text{SD}(X) = \sqrt{2} \approx 1.4142$$

Example 3-4: Prior Convictions

Click on the tab headings to see how to find the expected value, standard deviation, and variance. The last tab is a chance for you to try it.