

Zhao Ping HW2

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Problem 1

(a) Gradient Function

```
#x=(x1,x2,x3,x4), and t os the parameter (theta)
grad <- function (x, t) {
  dl = x[1]/(2+t)-(x[2]+x[3])/(1-t)+x[4]/t
}
```

(b) Stop your algorithm when either of the following criteria is satisfied:

```
secant <- function(maxit, x, t0, t1, tolerr, tolgrad){
  tstar = -1657/7680+sqrt(3728689)/7680
  digits = -log10(abs(t1 - tstar))
  it=0
  stop=0

  theta=c()
  converage.rate=c()
  digit=c()
  iteration=c()
  modified.relerr=c()
  Gradients=c()
  while (it < maxit & stop==0){
    it = it+1
    dl.1 = grad(x,t1)
    dl.0 = grad(x,t0)

    tnew = t1-dl.1*(t1-t0)/(dl.1-dl.0)

    rate = abs(tnew - tstar)/abs(t1 - tstar) #check convergent rate
    digits = -log10(abs(tnew - tstar)/abs(tstar))
    mod.relerr = abs(t1 - tnew)/max(1,abs(tnew))
    dl.new <-grad(x,tnew)

    if (mod.relerr < tolerr & abs(dl.new) < tolgrad) stop=1 # Stop iteration condition
    #print(c(it, t, rate,digits))
    #print(sprintf('it = %2.0f    teta = %12.12f    Rate = %4.2f    digits = %2.1f',it,tnew,rate,digi
    #print(sprintf('                relerr = %4.1e,                grad = %4.1e',relerr,dl))
    t0 = t1 # Update t0
    t1 = tnew # Update and return

    theta[it]<- tnew
    converage.rate[it]<- rate
  }
```

```

        digit[it]<-digits
        iteration[it]<- it
        modified.relerr[it] <-mod.relerr
        Gradients[it]<-dl.1
    }
    conver.info<-as.data.frame(cbind(iteration, theta, converage.rate, digit, modified.relerr, Gradients))
    return(conver.info)
}

a=c()

```

(c) &(d)

$\theta^{(0)} = .02$ and $\theta^{(1)} = .01$, and use $\text{tolerr} = 1e-6$ and $\text{tolgrad}=1e-9$

The iteration information is in matrix m. A larger matrix m.index contains information of starting points.

```

d <- c(1997,907,904,32)

#secant(20,x=d,0.02,0.01,1e-6,1e-9 )

m<-secant(20,x=d,0.02,0.01,1e-6,1e-9)
m

```

##	iteration	theta	converage.rate	digit	modified.relerr
## 1	1	0.02456185	4.328318e-01	0.5065360	1.456185e-02
## 2	2	0.02782321	7.065220e-01	0.6574103	3.261365e-03
## 3	3	0.03335105	2.959467e-01	1.1861968	5.527834e-03
## 4	4	0.03519756	2.053261e-01	1.8737526	1.846511e-03
## 5	5	0.03564619	5.967468e-02	3.0979625	4.486269e-04
## 6	6	0.03567431	1.218047e-02	5.0122985	2.812386e-05
## 7	7	0.03567466	7.259626e-04	8.1513843	3.465340e-07
## 8	8	0.03567466	8.819921e-06	13.2059196	2.517513e-10

```

##      Gradients
## 1 2.364239e+03
## 2 4.326183e+02
## 3 2.720887e+02
## 4 6.813010e+01
## 5 1.331728e+01
## 6 7.855955e-01
## 7 9.562060e-03
## 8 6.941636e-06

```

The matrix above gives iteration information about secant method.

However, I need to add information of the starting point θ_0 and θ_1 to the convergence matrix. As we are applying secant method, the θ_2 is from the first iteration. (I don't think this index system is good.) I added an column of θ index to the matrix.

```

Index.Theta<-seq(0,nrow(m)+1)
tstar = -1657/7680+sqrt(3728689)/7680

#mod.relerr.0 = abs(0.02 - tnew)/max(1,abs(tnew))
mod.relerr.1 = abs(0.01 - 0.02)/max(1,abs(0.01))

```

```

rate.t1 = abs(0.01 - tstar)/abs(0.02 - tstar)

digits.1 = -log10(abs(0.01 - tstar)/abs(tstar))
digits.0 = -log10(abs(0.02 - tstar)/abs(tstar))

t0.info<-c(NA,0.02,NA,digits.0, NA, grad(d,0.02))
t1.info<-c(NA,0.01,rate.t1,digits.1,mod.relerr.1,grad(d,0.01))

m.starting<-rbind(t0.info,t1.info,m)
m.index<-cbind(Index.Theta,m.starting)

m.index

```

```

##      Index.Theta iteration      theta convergence.rate      digit
## 1           0         NA 0.02000000          NA 0.3571618
## 2           1         NA 0.01000000 1.637973e+00 0.1428552
## 3           2          1 0.02456185 4.328318e-01 0.5065360
## 4           3          2 0.02782321 7.065220e-01 0.6574103
## 5           4          3 0.03335105 2.959467e-01 1.1861968
## 6           5          4 0.03519756 2.053261e-01 1.8737526
## 7           6          5 0.03564619 5.967468e-02 3.0979625
## 8           7          6 0.03567431 1.218047e-02 5.0122985
## 9           8          7 0.03567466 7.259626e-04 8.1513843
## 10          9          8 0.03567466 8.819921e-06 13.2059196
##      modified.relerr      Gradients
## 1           NA 7.406547e+02
## 2      1.000000e-02 2.364239e+03
## 3      1.456185e-02 2.364239e+03
## 4      3.261365e-03 4.326183e+02
## 5      5.527834e-03 2.720887e+02
## 6      1.846511e-03 6.813010e+01
## 7      4.486269e-04 1.331728e+01
## 8      2.812386e-05 7.855955e-01
## 9      3.465340e-07 9.562060e-03
## 10     2.517513e-10 6.941636e-06

```