① $y'' - y' - 2y = 16xe^{3x}$ BRAYAN MALDONADO CARRASCO

$y'' - y' - 2y = 0 \Rightarrow$
$$\boxed{\begin{aligned} y &= e^{rx} \\ y' &= re^{rx} \\ y'' &= r^2e^{rx} \end{aligned}}$$

luego : $y_H = C_1e^{-x} + C_2e^{2x}$

$\left.\begin{aligned} y_1 &= e^{-x} \\ y_2 &= e^{2x} \end{aligned}\right\} \quad y_p = V_1y_1 + V_2y_2$

luego:

$$W = \begin{vmatrix} e^{-x} & e^{2x} \\ -e^{-x} & 2e^{2x} \end{vmatrix} = (e^{-x})(2e^{2x}) - [e^{2x}(-e^{-x})] = 2e^{x} + e^{x} \Rightarrow \boxed{3e^{x} \neq 0}$$

$$V_1' = \frac{\begin{vmatrix} 0 & e^{2x} \\ 16xe^{3x} & 2e^{2x} \end{vmatrix}}{3e^{x}} = \frac{-[e^{2x} \cdot 16xe^{3x}]}{3e^{x}} = \frac{-16xe^{5x}}{3e^{x}} \Rightarrow$$

$$\Rightarrow \boxed{\frac{-16xe^{4x}}{3} = V_1'}$$

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$$V_2' = \frac{\begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & 16xe^{3x} \end{vmatrix}}{3e^{x}} \Rightarrow \frac{e^{-x}(16xe^{3x})}{3e^{x}} = \frac{16xe^{2x}}{3e^{x}} = \boxed{\frac{16xe^{x}}{3}}$$

Luego

$$V_1 = \frac{-16}{3}\int x e^{4x}\,dx \Rightarrow \begin{matrix} u = x & dv = e^{4x} \\ du = dx & v = 4e^{4x} \end{matrix} \quad \left\{ \begin{array}{l} uv - \int v\,du \\ 4xe^{4x} - \int 4e^{4x}\,dx \end{array} \right.$$

$$\Rightarrow 4\int e^{4x}dx \Rightarrow 16e^{4x} \Rightarrow 4xe^{4x} - 16e^{4x} = 4e^{4x}(x-16) \Rightarrow$$

$$\Rightarrow -\frac{16}{3}\left(4e^{4x}(x-16)\right)$$

$$V_2 = \frac{16}{3}\int xe^{x} \Rightarrow \begin{matrix} u = x \\ du = dx \\ dv = e^{x} \\ v = e^{x} \end{matrix} \Rightarrow \left\{ \begin{array}{l} uv - \int v\,du \\ xe^{x} - \int e^{x}dx \Rightarrow xe^{x} - e^{x} \Rightarrow \\ \Rightarrow e^{x}(x-1) \Rightarrow \frac{16}{3}e^{x}(x-1) \end{array} \right.$$

$$\therefore\ y_p = -\frac{64}{3}e^{4x}(x-16) + \frac{16}{3}e^{3x}(x-1) \quad //$$

Fin !

(3) $y'' + 9y = 9\sec(3x)\,tg(3x)$ BRAYAN MALDONADO CARRASCO.

$r^2 + 9 = 0$

$r = 0 \pm 3i$

Luego: $y = e^{0x}(C_1\cos(3x) + C_2\,sen(3x))$

$\therefore\ y_H = C_1\cos(3x) + C_2\,sen(3x)$

Luego: $y_p = u_1\cos(3x) + u_2\,sen(3x)$

$y_1 = sen(3x)$ | $f(x) = 9\sec(3x)\,Tg(3x)$

$y_2 = \cos(3x)$

Luego: $W = \begin{vmatrix} sen(3x) & \cos(3x) \\ 3\cos(3x) & -3\,sen(3x) \end{vmatrix} \Rightarrow -3\,sen^2(3x) - 3\cos^2(3x)$

$\Rightarrow \boxed{-3}$

$u_1 = -\int \left[\dfrac{\cos(3x)\,9\sec(3x)\,tg(3x)}{-3}\right] dx$

$= -\int \cos(3x) \cdot -3\sec(3x)\,Tg(3x)\,dx$

$\rightarrow -\int \left[-\cancel{\cos(3x)} \cdot \dfrac{3}{\cancel{\cos(3x)}} \cdot Tg(3x)\right] dx$

$\Rightarrow -\int \left[-3\,\dfrac{sen(3x)}{\cos(3x)}\right] dx \Rightarrow 3\int \dfrac{sen(3x)}{\cos(3x)}\,dx$

Luego $= v = \cos(3x)$

$\Rightarrow \cancel{3}\int -\dfrac{1}{\cancel{3}v}\,dv \Rightarrow -\int \dfrac{1}{v}\,dv = \ln(v)$

$\therefore\ u_1 = \ln(\cos(3x))$

Luego: $u_2 = -\int sen(3x) \cdot -3\sec(3x) Tg(3x)\,dx$ Brayan MALDONADO CARRASCO.

$$u_2 = -3 \int \frac{1}{\cos(3x)} sen(3x)\, Tg(3x)\,dx$$

$$= -3 \int \frac{sen(3x)}{\cos(3x)} tg(3x)\,dx$$

$$= -3 \int Tg^2(3x)\,dx$$

Identidad $\Rightarrow -3 \int -1 + \sec^2(3x)\,dx$

$$\Rightarrow -3 \left(-\int dx + \int \sec^2(3x) \right)$$

$$\Rightarrow -3 \left(-x + \tfrac{1}{3} Tg(3x) \right)$$

$$u_2 = 3x - Tg(3x)$$

$$y_p = \ln|\cos(3x)| \cos(3x) + (3x - Tg(3x))\, sen(3x)$$

Luego sol. general

$$Y(x) = C_1 \cos(3x) + C_2 sen(3x) + \cos(3x) \ln|\cos(3x)| + \ldots$$

$$\ldots [3x - Tg(3x)]\, sen(3x)$$

Fin 3 //

Brayan MALDONADO CARRASCO.

(4) $x^2y'' - xy' - 3y = -30\sqrt{x}$

$y = x^m$

$y' = m x^{m-1}$

$y'' = m(m-1)x^{m-2}$

$x^2(m(m-1)x^{m-2} - x(mx^{m-1}) - 3x^m = 0 \to y_H$

$x^m(m(m-1)) - (mx^m) - 3x^m = 0$

$x^m(m^2 - m) - x^m \cdot m - 3x^m = 0$

$\cancel{x^m}^{0}(m^2 - m - m - 3) = 0$

$\Rightarrow m^2 - 2m - 3 = 0$

$(m-3)(m+1) = 0$

$m_1 = 3$

$m_2 = -1$

$\therefore y_H = c_1 x^3 + c_2 x^{-1}$

luego $y_1 = x^3$

$y_2 = x^{-1}$

$W = \begin{vmatrix} x^3 & \frac{1}{x} \\ 3x^2 & -x^{-2} \end{vmatrix} \neq 0$

$W = (x^3)(-x^{-2}) - \left[3x^2 \cdot \frac{1}{x}\right]$

$W = x^{\cancel{3}} \cdot \frac{-1}{x^{\cancel{2}}} - \left[3x^{\cancel{2}} \cdot \frac{1}{\cancel{x}}\right]$

$W = -x - 3x$

$\boxed{W = -4x}$

luego $\Rightarrow x^2y'' - xy' - 3y = -30\sqrt{x} \;/\; \frac{1}{x^2}$

$y'' - \ldots = \frac{-30x^{1/2}}{x^2} \Rightarrow y'' - \ldots = -30/\sqrt{x^3}$

$$v_1' = \frac{\begin{vmatrix} 0 & x^{-1} \\ \frac{-30}{\sqrt{x^3}} & -x^{-2} \end{vmatrix}}{-4x} \Rightarrow \frac{-\left[\frac{-30}{\sqrt{x^3}} \cdot \frac{1}{x}\right]}{-4x} \Rightarrow \frac{\frac{30}{\sqrt{x^5}}}{-4x}$$

[illegible]

$$\Rightarrow \frac{30}{\sqrt{x^5}} \cdot \frac{1}{-4x} = \frac{30}{-4x^{7/2}} \Rightarrow \boxed{v_1' = \frac{-30}{4x^{7/2}}}$$

$$v_2' = \frac{\begin{vmatrix} x^3 & 0 \\ 3x^2 & \frac{-30}{\sqrt{x^3}} \end{vmatrix}}{-4x} \Rightarrow \frac{x^3 \cdot \frac{-30}{x^{3/2}}}{-4x} \Rightarrow \frac{-30\sqrt{x^3}}{-4x}$$

$$\Rightarrow \boxed{v_2' = \frac{30}{4}\sqrt{x}}$$

luego: $v_1 = \int \frac{-30}{4x^{7/2}} \Rightarrow -\frac{30}{4}\int x^{-7/2}\,dx \Rightarrow \frac{-30}{4} x^{-5/2} \cdot \frac{2}{5}$

$$\Rightarrow \frac{-30}{\cancel{4}2} \cdot \frac{-\cancel{2}}{5x^{5/2}} \Rightarrow \frac{30}{10x^{5/2}} \Rightarrow \boxed{3x^{-5/2}}$$

$$\Rightarrow v_2 = \int \frac{30x^{1/2}}{4} \Rightarrow \frac{30}{4}\int x^{1/2}\,dx \Rightarrow \frac{30}{\cancel{4}2} x^{3/2} \cdot \frac{\cancel{2}}{3} \Rightarrow 5x^{3/2}$$

luego: $y_p = y_1 v_1 + y_2 v_2 \Rightarrow y_p = x^3(3x^{-5/2}) + \frac{1}{x}(5x^{3/2})$

$y_p = 3x^{1/2} + 5x^{1/2}$

$\underline{y_p = 8x^{1/2}}$ | fin 4.