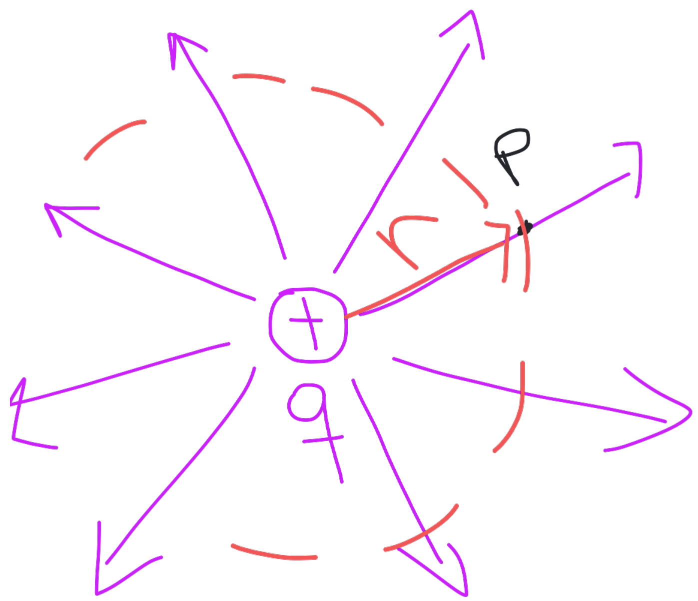
Potencial para una carga discreta



$$V_P = -\int_{\infty}^{P} E \cdot dl$$

<u>cálculo E</u> : Sup. Gauss con r.

$$Q_{enc} = q$$

$$\oint \vec{E} \cdot ds = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{4\pi r^2 \varepsilon_0}.$$

$$V_P = -\int_{\infty}^{P=r} \frac{q}{4\pi r'^2 \varepsilon_0} \cdot dr'$$

$$= -\frac{q}{4\pi\varepsilon_0}\int_{\infty}^{P=r} \frac{dr'}{r'^2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r'}\right]_{\infty}^{r}$$

$$= \frac{q}{4\pi\varepsilon_0 r} \quad , \text{ pero } K = \frac{1}{4\pi\varepsilon_0}$$

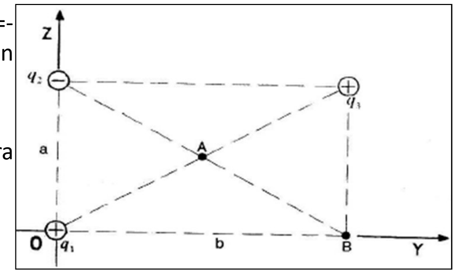
$$\boxed{V_P = \frac{Kq}{r}} \rightarrow \text{Potencial para cargas discretas.}$$

Ejemplo

- La figura muestra 3 cargas puntuales, q_1=40 [uC]; q_2=-50[uC] y q_3=30 [uC] ubicadas en los vértices de un rectángulo de lados a=30[cm] y b=40[cm], calcule:

a) La diferencia de potencial entre los puntos A y B.

b) El trabajo que debe realizar un agente externo para trasladar una carga q_0= 0,2 [uC] desde A hasta B.



a) $\Delta V = V_b - V_a$

$V_b = V_1 + V_2 + V_3$

$V_p = K \dfrac{q}{|\vec{r}_p - \vec{r}|}$

$V_b = 9\times10^9 \cdot \left[\dfrac{40\times10^{-6}}{0{,}4} + \dfrac{-50\times10^{-6}}{0{,}5} + \dfrac{30\times10^{-6}}{0{,}3} \right]$

$V_b = 900.000 \,(V)$

$V_a = V_1 + V_2 + V_3$

$V_a = 9\times10^9 \left(\dfrac{40\times10^{-6}}{0{,}25} + \dfrac{-50\times10^{-6}}{0{,}25} + \dfrac{30\times10^{-6}}{0{,}25} \right)$

$= 720.000 \,(V)$

$\Delta V = 180.000 \,(V)$

b) $\Delta V = \dfrac{W}{q_0}$

$W = \Delta V \, q_0$

$W = 180.000 \times 0{,}2 \times 10^{-6}$

$W = 0{,}036 \,(J)$

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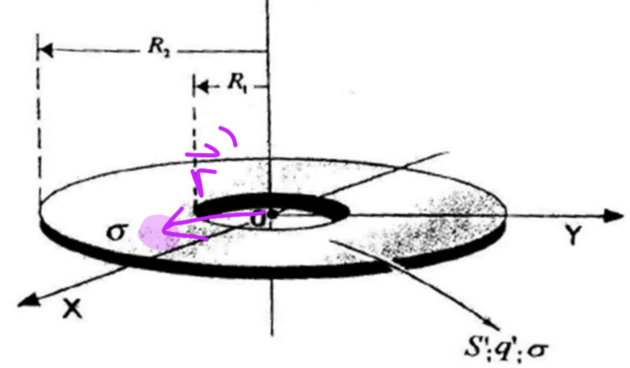
Si la corona de la figura de radio interior R1 y exterior R2 tiene una densidad superficial de carga σ =cte, calcular el potencial eléctrico en el centro de la corona



$$V_0 = K \int \frac{dq}{|\vec{r}-\vec{r}'|}$$

$$\vec{r} = \vec{0}$$

$$\vec{r}' = r\cos\theta\,\hat{\imath} + r\,\text{sen}\theta\,\hat{\jmath}$$

$$\vec{r}-\vec{r}' = -r\cos\theta\,\hat{\imath} - r\,\text{sen}\theta\,\hat{\jmath}$$

$$|\vec{r}-\vec{r}'| = r$$

$$dq = \sigma \cdot ds$$

$$ds = r\,d\theta\,dr$$

$$R_1 \leq r \leq R_2$$

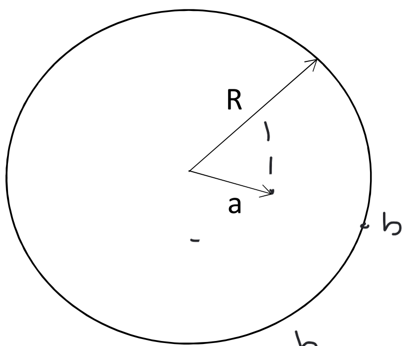
$$0 \leq \theta \leq 2\pi$$

$$V_p = K \int_{R_1}^{R_2}\int_0^{2\pi} \frac{\sigma \cdot \cancel{r}\,d\theta\,dr}{\cancel{r}}$$

$$\boxed{V_p = K\sigma \cdot 2\pi \cdot (R_2 - R_1)\ [V]}$$

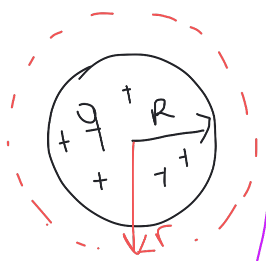
Una carga q se distribuye uniformemente en un volumen esférico no conductor de radio R. Demostrar que el potencial a una distancia a del centro, siendo a<R esta dado por:

$$V_a = \frac{q(3R^2-a^2)}{8\pi\varepsilon_0 R^3}\,[V]$$



$V_a = V_{\infty b} + V_{ba}$
(1) (2)

(1) $V_{\infty b} = -\int_{\infty}^{b} E\cdot dl = -\int_{\infty}^{b} E_{ext}\, dr$



E_{ext}: Sup. Gauss con r>R

$\oint \vec{E}\cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon_0}$ S.G

$Q_{enc} = q$

$E(4\pi r^2) = \frac{q}{\varepsilon_0}$

$$\boxed{E_{ext} = \frac{q}{4\pi r^2\varepsilon_0}} \quad r>b$$

$V_{\infty b} = -\int_{\infty}^{b=R} \frac{q}{4\pi r^2 \varepsilon_0}\cdot dr$

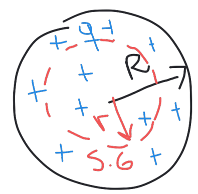
$= -\frac{q}{4\pi\varepsilon_0}\int_{\infty}^{R}\frac{dr}{r^2}$

$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{\infty}^{R}$

$$\boxed{V_{\infty b} = \frac{q}{4\pi\varepsilon_0 R}\,[V]}$$

(2) $V_{ba} = -\int_{b}^{a} E_{int}\, dr$

E_{int}: Sup. Gauss con r<R



$\oint \vec{E}\cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon_0}$

Q_{enc}: $\rho = cte$

$\rho_T = \rho_{enc}$

$\frac{Q_T}{V_T} = \frac{Q_{enc}}{V_{enc}} \rightarrow Q_{enc} = \frac{Q_T\cdot V_{enc}}{V_T}$

$Q_{enc} = \frac{q\cdot\frac{4}{3}\pi\cdot r^3}{\frac{4}{3}\pi\cdot R^3}$

$Q_{enc} = \frac{q\, r^3}{R^3}$

$\oint \vec{E}\cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon_0}$

$E(4\pi \cancel{r^2}) = \frac{q\, r^{\cancel{3}}}{R^3\cdot\varepsilon_0} \rightarrow \boxed{E_{int} = \frac{q\, r}{4\pi R^3\varepsilon_0}\ (N/C)}$ r<b

$V_{ba} = -\int_{b}^{a} E_{int}\cdot dr$

$= -\int_{b=R}^{a} \frac{q\, r}{4\pi R^3\varepsilon_0}\, dr$

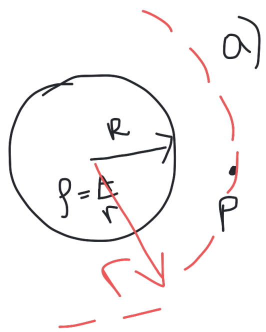
$= -\frac{q}{4\pi R^3\varepsilon_0}\int_{R}^{a} r\cdot dr$

$= -\frac{q}{4\pi R^3\varepsilon_0}\left[\frac{r^2}{2}\right]_{R}^{a}$

$$\boxed{V_{ba} = \frac{q}{8\pi R^3\varepsilon_0}(R^2-a^2)\,[V]}$$

$V_a = V_{\infty b} + V_{ab}$

Una esfera de radio R esta cargada con una densidad de carga volumétrica dada por $\rho = \frac{A}{r}$,
con A contante y r la distancia radial, Determinar:
a) El potencial en el exterior de la distribución
b) El potencial en el interior de la distribución



a) $V_P = -\int_{\infty}^{P} E_{ext}\cdot dr$

E_{ext}: Sup. Gauss con $r > R$

$$\oint \vec{E}\cdot\vec{ds} = \frac{Q_{enc}}{\varepsilon_0}$$

Q_{enc}: $\rho \neq cte$

$$Q_{enc} = \int \rho \cdot dV$$

$$= \int_0^R\int_0^\pi\int_0^{2\pi} \frac{A}{\cancel{r}}\, r^{\cancel{2}} \sin\theta\, d\phi\, d\theta\, dr$$

$0\le\phi\le 2\pi$
$0\le\theta\le\pi$
$0\le r\le R$

$$= A\cdot 2\pi\cdot \cancel{2}\cdot \frac{R^2}{\cancel{2}}$$

$$Q_{enc} = 2\pi A R^2$$

$$\oint \vec{E}\cdot \vec{ds} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{2\pi A R^2}{\varepsilon_0}$$

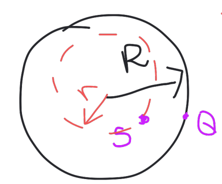
$$\boxed{E_{ext} = \frac{AR^2}{2r^2\varepsilon_0} \quad (N/C)}$$

$$V_{\infty P} = -\int_{\infty}^{P=r} \frac{AR^2}{2r'^2\varepsilon_0}\cdot dr'$$

$$= -\frac{AR^2}{2\varepsilon_0}\int_\infty^r \frac{dr'}{r'^2} = \frac{AR^2}{2\varepsilon_0}\left[\frac{1}{r'}\right]_\infty^r$$

$$\boxed{V_{\infty P} = \frac{AR^2}{2\varepsilon_0 r}}$$ V_{ext} $r \geq R$.

b) V_{int}: $V_S = V_{\infty Q} + V_{QS}$
(1) (2)



(1) $V_{\infty Q} = -\int_\infty^{Q=R} E_{ext}\cdot dr$

$$V_{\infty Q} = \frac{AR^2}{2\varepsilon_0 R}$$

$$\boxed{V_{\infty Q} = \frac{AR}{2\varepsilon_0}\quad (V)}$$

(2) $V_{QS} = -\int_Q^S E_{int}\cdot dr$

E_{int}: Sup. Gauss con $r<R$

$$\oint \vec{E}\cdot\vec{ds} = \frac{Q_{enc}}{\varepsilon_0}$$

Q_{enc}:

$$Q_{enc} = \int\rho\cdot dV$$

$$= \int_0^r\int_0^\pi\int_0^{2\pi}\frac{A}{\cancel{r'}}\, r'^{\cancel{2}}\sin\theta\,d\phi\,d\theta\,dr'$$

$0\le\phi\le2\pi$
$0\le\theta\le\pi$
$0\le r\le r$

$$Q_{enc} = 2\pi A r^2$$

$$\oint \vec{E}_{int}\cdot\vec{ds} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E_{int}(4\cancel{\pi}\cancel{r^2}) = \frac{2\cancel{\pi}A\cancel{r^2}}{\varepsilon_0}$$

$$\boxed{E_{int} = \frac{A}{2\varepsilon_0}\quad (N/C)}$$

$$V_{QS} = -\int_{Q=R}^{S=r}\frac{A}{2\varepsilon_0}\,dr'$$

$$\boxed{V_{QS} = \frac{A}{2\varepsilon_0}(R-r)\quad (V)}$$

$$V_S = \left(\frac{A}{2\varepsilon_0}(R-r) + \frac{AR}{2\varepsilon_0}\right)(V)$$

Se tiene un cilindro solido de radio a, con carga +Q, coaxial a él hay un casquete cilíndrico de radio b y con carga –Q. Determinar la diferencia de potencial entre a y b