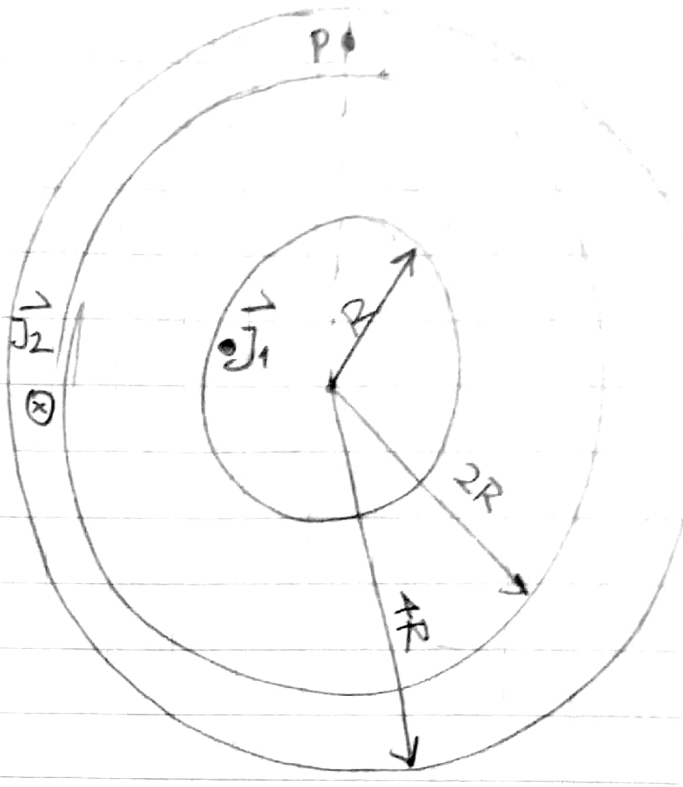
Problema 2



Para un rango

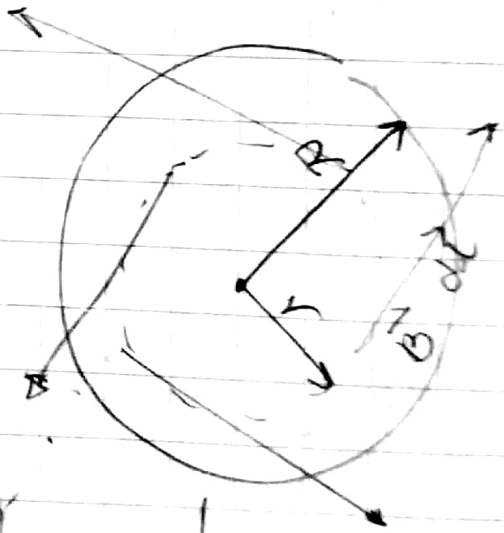
$a < R$

$$\rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 \cdot I_{enc}$$

I_{enc}:

$J \neq cte$

$$J_1 = A r^2 (\hat{k}$$



$$I_{enc} = \int_0^r \int_0^{2\pi} A r^2 \cdot ds$$

$$\rightarrow A \int_0^r r^2 \cdot 2\pi$$

$$\rightarrow A \cdot 2\pi \int_0^r r^2 \Rightarrow A \cdot 2\pi \frac{r^3}{3} = I_{enc}$$

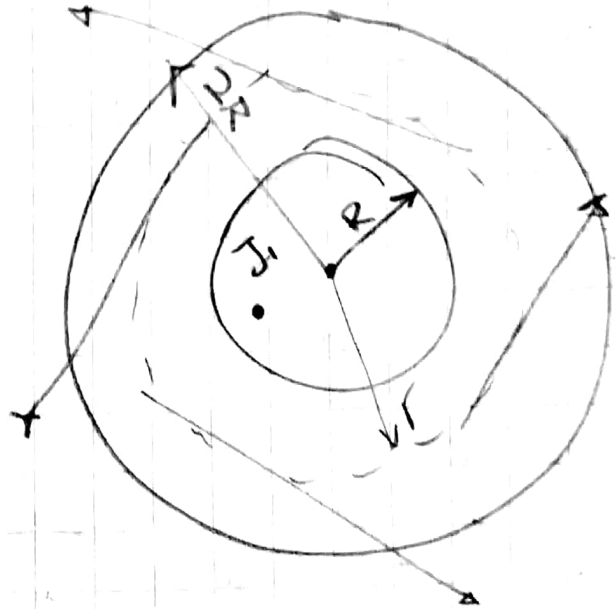
$$\Rightarrow \frac{I_{enc}}{A_{enc}} = \frac{I_T}{A_T} \Rightarrow I_{enc} = \frac{I_T \cdot A_{enc}}{A_T}$$

$$\Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 \cdot I_{enc}$$

$$B = \frac{\mu_0 \cdot I_{enc}}{dl} = \frac{\mu_0 \cdot A/2\pi \frac{r^3}{3}}{2\pi r} \Rightarrow \frac{\mu_0 \cdot A r^2}{3}$$

* Con Región $R < r < 2R$ $(I_0 = I_{enc})$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \; I_{enc}$$



I_{enc} :

$$\Rightarrow I_{enc} = \iint J_1 \cdot ds$$

$$\Rightarrow \int_0^r \int_0^{2\pi} A r^2 \cdot ds \Rightarrow I_{enc}$$

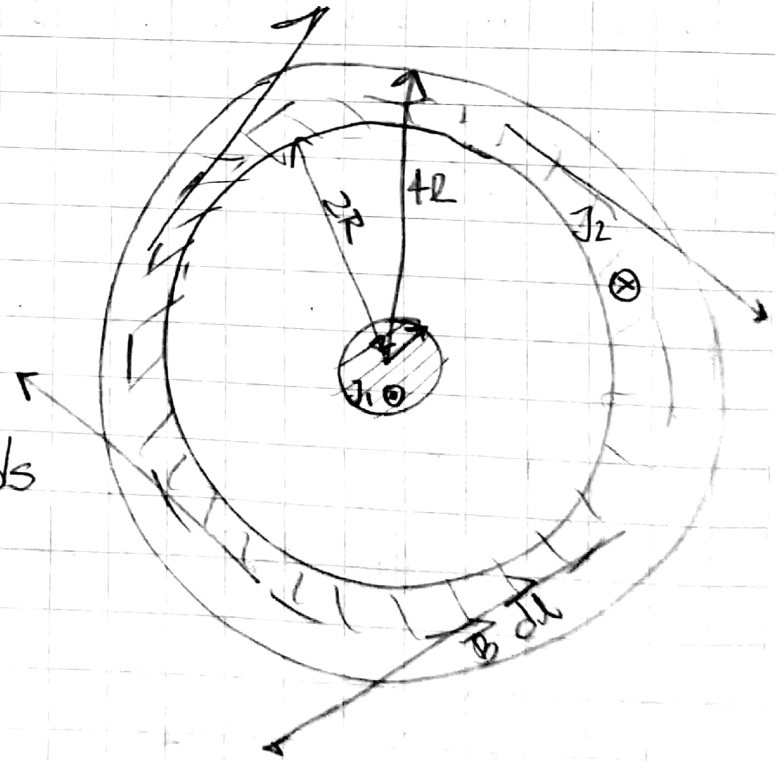
$$\Rightarrow A \int_0^r r^2 \int_0^{2\pi} ds \Rightarrow A 2\pi \int_0^r r^2 \Rightarrow A\, 2\pi \cdot \frac{r^3}{3}$$

$$\Rightarrow \oint \vec{B}\, d\vec{l} = \mu_0 \, I_{enc} \quad | \quad I_0 = I_{enc}$$

$$\rightarrow B = \frac{\mu_0 \cdot I_{enc}}{dl} = \frac{\mu_0 \cdot A \cancel{2\pi} \frac{r^3}{3}}{\cancel{2\pi} r}$$

$$\Rightarrow B = \frac{\mu_0 A r^2}{3} \ (T) \quad \text{con} \quad a \le r < b$$

region $2R < r < 4R$



I_{enc}:

$$J_2 = \frac{B}{r} - \hat{k}$$

$$J_2 \neq cte \rightarrow I_{enc} = \iint J_2 \cdot ds$$

$$\rightarrow \int_0^r \int_0^{2\pi} \frac{B}{r} \cdot ds$$

$$\Rightarrow B \cdot 2\pi \int_0^r \frac{1}{r} dr \Rightarrow \boxed{B 2\pi \ln|r| = I_{enc}}$$

$I_{TOTAL} = I_{\odot} - I_{\otimes} \Rightarrow I_{TOTAL} = A 2\pi \frac{r^3}{3} - B 2\pi \ln|r|$

Luego

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \cdot I_{enc} \Rightarrow B = \frac{\mu_0 I_{enc}}{dl}$$

$$\Rightarrow B = \frac{\mu_0 \cdot B 2\pi \ln|r|}{2\pi r} = \boxed{\frac{\mu_0 B \ln|r| \ (\top)}{r}}$$

Luego

$$I_0 = I_{enc} \Rightarrow I_0 = B 2\pi \ln|r|$$

$$\Rightarrow B = \frac{I_0}{2\pi \ln|r|} \quad //$$

Luego volviendo a $R < r < 2R$

$$\Rightarrow I_0 = I_{enc}$$

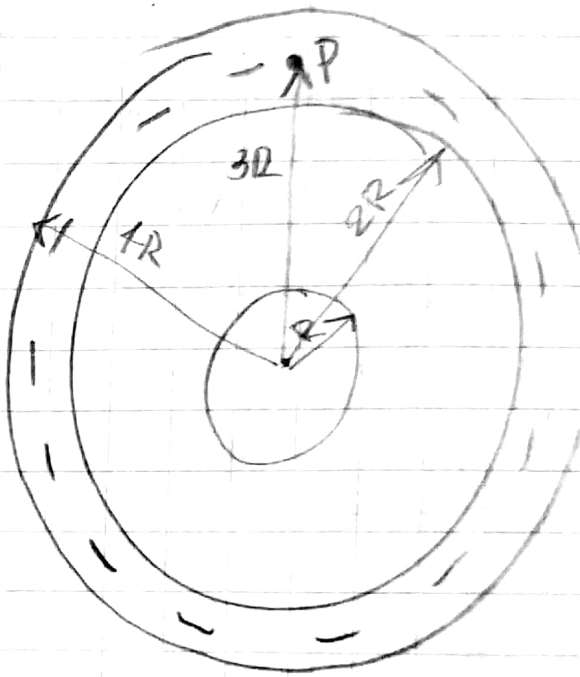
$$\Rightarrow I_0 = A 2\pi \frac{r^3}{3}$$

$$3I_0 = A 2\pi r^3$$

$$A = \frac{3 I_0}{2\pi r^3} \quad //$$

(B) Luego $\vec{B}_p$, región $r = 3R$

$$\oint \vec{B}_p \cdot d\vec{l} = \mu_0 \cdot I_{enc}$$



$$J_2 \neq cte \Rightarrow I_{enc} = \iint J_2 \cdot ds$$

$$\Rightarrow \int_0^r \int_0^{2\pi} \frac{B}{r}\, ds \Rightarrow B2\pi \int_0^r \frac{dr}{r}$$

$$\Rightarrow I_{enc} = B2\pi \ln|r|$$

$$I_T = I_1 - I_2'$$

$I_1 = I_{enc}$ de J_1

$$I_{Total} = \frac{A2\pi r^3}{3} - B2\pi \ln|r|$$

Luego

$$\Rightarrow \oint B_p \, dl = \mu_0 \cdot I_{enc}$$

$$\therefore B_p = \frac{\mu_0 \cdot I_{enc}}{dl}$$

$$\rightarrow B_p = \frac{\mu_0 \cdot}{6\pi R} \left[\frac{A\, 2\pi r^3}{3} - 2B\pi \ln|r| \right]$$

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