BRAYAN MALDONADO CARRASCO.

1a)
$$\int_0^3\int_0^x x^2 e^{xy}\,dy\,dx \Rightarrow \int_0^x x^2 e^{xy}\,dy \Rightarrow x^2\int_0^x e^{xy}\,dy$$

$$\Rightarrow x^2\int_0^x e^{xy}\,dy \quad \begin{array}{l} u = xy \\ du = x\,dy \\ \frac{du}{x} = dy \end{array} \Rightarrow x^2\int_0^x e^{u}\,\frac{du}{x} \Rightarrow x\int_0^x e^{u}\,du$$

$$\Rightarrow x\left(e^{xy}\Big|_0^x\right) \Rightarrow x\left(e^{x^2}-1\right) \Rightarrow \left(xe^{x^2}-x\right)$$

$$\Rightarrow \int_0^3 \left(xe^{x^2}-x\right)dx \Rightarrow \underbrace{\int_0^3 xe^{x^2}\,dx}_{I_1} - \underbrace{\int_0^3 x\,dx}_{I_2}$$

$$I_1 = \begin{array}{l} u = x^2 \\ du = 2x\,dx \\ \frac{du}{2} = x\,dx \end{array} \Rightarrow \int_0^3 e^{u}\,\frac{du}{2} \Rightarrow \frac{1}{2}\int_0^3 e^{u}\,du$$

$$\Rightarrow \frac{1}{2}\left(e^{u}\Big|_0^3\right) \Rightarrow \frac{1}{2}\left(e^{x^2}\Big|_0^3\right) \Rightarrow \boxed{\frac{1}{2}\left(e^{9}-1\right)}$$

$$\Rightarrow I_2 = -\int_0^3 x\,dx \Rightarrow -\left(\frac{x^2}{2}\Big|_0^3\right) \Rightarrow -\frac{9}{2} \Rightarrow \quad I_1+I_2 \Rightarrow \frac{e^{9}-10}{2}$$

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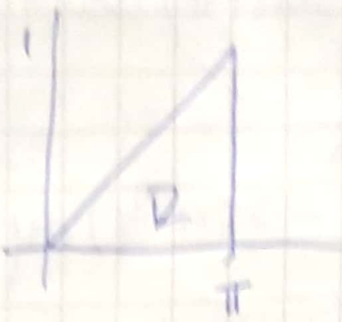
1 b)

$$\iint_R x\cos(x+y)\,dA$$



$0 \leq x \leq \pi$

$0 \leq y \leq \pi$

$$\to \int_0^{\pi}\left[\int_0^{\pi} x\cos(x+y)\,dx\right]dy \to \int_0^{\pi} x\cos(x+y)\,dx$$

$$\Rightarrow uv - \int du\,v \qquad \begin{array}{l} u = x \\ du = dx \end{array} \qquad \begin{array}{l} dv = \cos(x+y) \\ v = \operatorname{sen}(x+y) \end{array}$$

$$\Rightarrow x\operatorname{sen}(x+y) - \int \operatorname{sen}(x+y)\,dx$$

$$\to \left[x\operatorname{sen}(x+y) + \cos(x+y)\right]\Big|_0^{\pi}$$

$$\to \left(\pi\operatorname{sen}(\pi+y) + \cos(\pi+y)\right) - \left(0 + \cos(0+y)\right)$$

$$\to \left(\pi\operatorname{sen}(\pi+y) + \cos(\pi+y) - \cos(y)\right)$$

$$\Rightarrow \int_0^{\pi}\left(\pi\operatorname{sen}(\pi+y) + \cos(\pi+y) - \cos(y)\right)dy$$

$$\to \underbrace{\int_0^{\pi} \pi\operatorname{sen}(\pi+y)}_{I_1} + \underbrace{\int_0^{\pi} \cos(\pi+y)}_{I_2} - \underbrace{\int_0^{\pi} \cos y}_{I_3}$$

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$I_1)\ \pi \int_0^{\pi} (\mathrm{sen}(\pi)\cos(y) + \cos(\pi)\mathrm{sen}(y))\, dy$

$\Rightarrow \pi \int_0^{\pi} -\mathrm{sen}(y)\, dy \Rightarrow \pi(\cos(y)\Big|_0^{\pi} \Rightarrow$

$\Rightarrow \pi\left(\cos(\pi) - \cos(0)\right) \Rightarrow \pi(-1-1)$

$\Rightarrow \boxed{-2\pi}$

$I_2 = \int_0^{\pi} \cos(\pi)\cos(y) - \mathrm{sen}(\pi)\,\mathrm{sen}(y))\, dy$

$\Rightarrow -\mathrm{sen}(y)\Big|_0^{\pi} \Rightarrow -\mathrm{sen}\,\pi + \mathrm{sen}(0) = \boxed{0}$

$I_3 = \int_0^{\pi} \cos(y)\, dy \Rightarrow \mathrm{sen}(y)\Big|_0^{\pi} \Rightarrow \mathrm{sen}(\pi) - \mathrm{sen}(0)$

$\Rightarrow \boxed{0}$

∴ El valor de la integral es

$-2\pi u^3$ //

Fin 1b.

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(2) $f(x,y)\ \frac{1}{x}-\frac{64}{y}+xy$

$f_x=-\frac{1}{x^2}+y \qquad f_y=\frac{64}{y^2}+x$

$f_{xx}=\frac{2}{x^3} \qquad f_{yy}=-\frac{128}{y^3}$

luego:

$\rightarrow y=\frac{1}{x^2}\Rightarrow x=-\frac{1}{4}\rightarrow y=1/\left(\frac{-1}{4}\right)^2\Rightarrow \boxed{y=16}$

$f_x=0\rightarrow -\frac{1}{x^2}+y=0 \quad ; \quad f_y=0\rightarrow\frac{64}{y^2}+x=0$

$\Rightarrow x=-64x^4$

$64x^4+x=0$

$x(64x^3+1)=0$

$x=0 \vee 64x^3+1=0$

$64x^3=-1\Rightarrow x=\sqrt[3]{-\frac{1}{64}}\Rightarrow\boxed{x=-\frac{1}{4}}$

$x=-\frac{64}{y^2}$

$x=-\frac{64}{\left(\frac{1}{x^2}\right)^2}=\frac{-64}{\frac{1}{x^4}}$

Punto C $\Rightarrow\left(-\frac{1}{4},16\right)\Rightarrow D=\frac{2}{x^3}\cdot\frac{-128}{y^3}-1$

$\Rightarrow\frac{2}{\frac{-1}{64}}\cdot\frac{-128}{4096}-1\Rightarrow -128\cdot\frac{-128}{4096}-1=3$

$f_{xx}\left(-\frac{1}{4},16\right)<0 \ \therefore\left(-\frac{1}{4},16\right)$ //

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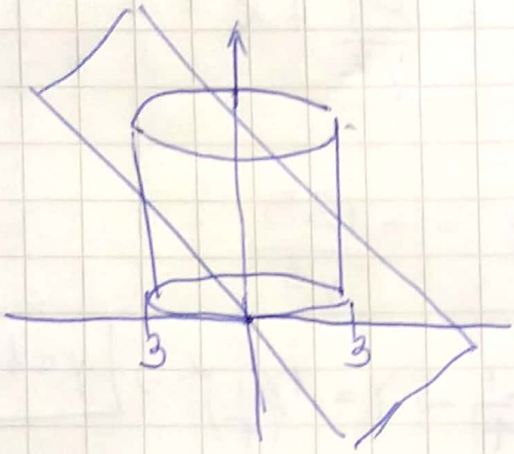
(3) Obtenga V. del solido

$z = 3x \quad \wedge \quad x^2+y^2=9$



$$V = \iint_D (3x)\, dA$$

$$D = \left\{ (x,y) \in \mathbb{R}^2 \;/\; \begin{matrix} -1 \leq x \leq 1, \\ -3 \leq y \leq 3 \end{matrix} \right\}$$

$$\left. \begin{matrix} -3 \leq x \leq 3 \\ -3 \leq y \leq 3 \end{matrix} \right\} \text{ B. vertical.}$$

$$\Rightarrow 4\int_0^3 \int_0^3 3x\, dy\, dx \Rightarrow \int_0^3 3x\, dy = 3xy \Big|_0^3$$

$$\Rightarrow (3x \cdot 3 - 3x \cdot 0) \Rightarrow \underline{9x} =$$

$$\Rightarrow 4\int_0^3 9x\, dx \Rightarrow 4 \cdot 9 \int_0^3 x\, dx \Rightarrow 4 \cdot 9 \left(\frac{x^2}{2} \Big|_0^3 \right)$$

$$\Rightarrow 36\left(9/2 - 0 \right) \Rightarrow 162\, u^3$$

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(4) $x = r\cos\theta$ $\quad \left((r\cos\theta)^2 + (r\,sen)^2\right)^2$

$y = r\,sen\,\theta$

$r^2\cos^2\theta + r^2 sen^2\theta$

$\left(r^2(\cos^2\theta + sen^2\theta)^1\right)^2$

$0 \leq \theta \leq 2\pi$ $\qquad r^4$

$1 \leq r \leq 4$

$$\int_0^{2\pi}\int_0^{2} \frac{\cancel{r^4}\cos^2\theta\, sen^2\theta}{\cancel{r^4}}\, dr\, d\theta \Rightarrow \int_1^2 \cos^2\theta\, sen^2\theta\, dr$$

$$\Rightarrow \cos^2\theta\, sen^2\theta \cdot r \Big|_1^2 \Rightarrow \cos^2\theta\, sen^2\theta(4-1) \Rightarrow$$

$$\Rightarrow \underline{\cos^2\theta \cdot sen^2\theta}$$

$$\Rightarrow \int_0^{2\pi} \cos^2\theta\, sen^2\theta\, d\theta \Rightarrow \int_0^{2\pi} \cos^2\theta\, sen^2\theta\, d\theta$$

luego Ident. $\Rightarrow \cos^2\theta\, sen^2\theta = \dfrac{1-\cos(4\theta)}{8}$

$$\Rightarrow \int_0^{2\pi} \frac{1-\cos(4\theta)}{8} \Rightarrow \frac{1}{8}\int_0^{2\pi} 1-\cos(4\theta)$$

$$\Rightarrow \frac{1}{8}\left(\int_0^{2\pi} d\theta - \int_0^{2\pi} \cos(4\theta)\, d\theta\right)$$

$$\hookrightarrow \theta \Big|_0^{2\pi} \Rightarrow 2\pi - 0 \Rightarrow \boxed{2\pi}$$

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Luego: $\int_0^{2\pi} \cos(4\theta)\, d\theta$ $\quad u = 4\theta$

$du = 4d\theta$

$\frac{du}{4} = d\theta$

$\Rightarrow \frac{1}{4}\int_0^{2\pi} \cos(u)\, du \Rightarrow \frac{1}{4}\left(\operatorname{sen}(4\theta)\right|_0^{2\pi}$

$\Rightarrow \frac{1}{4}\left(\operatorname{sen}(8\pi) - \operatorname{sen}(0)\right) = 0$

$\therefore \frac{2\pi}{8} \Rightarrow \pi/4$ //. fin4