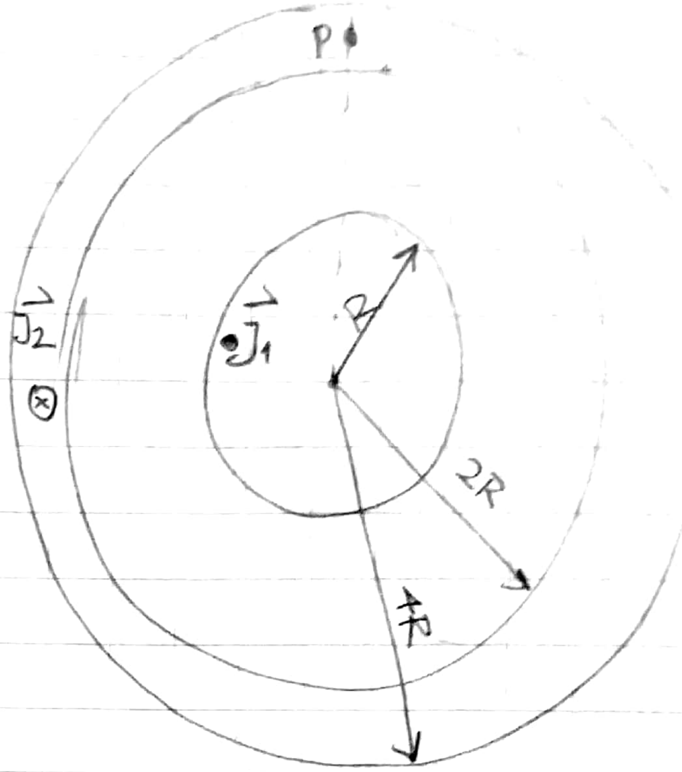
Problema 2



Para un rango

$a < R$

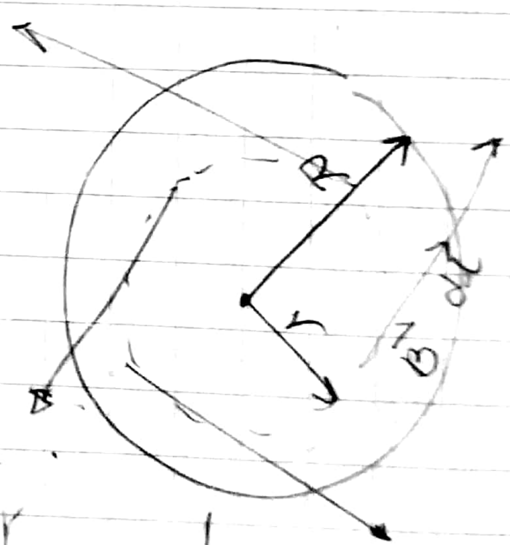
$\Rightarrow \oint \vec{B} \cdot \vec{dl} = \mu_0 \cdot I_{enc}$

I_{enc}:

$J \neq cte$

$J_1 = Ar^2 \, (\hat{k}$

$$I_{enc} = \int_0^r \int_0^{2\pi} Ar^2 \cdot ds$$



$$\Rightarrow A \int_0^r r^2 \cdot 2\pi$$

$$\Rightarrow A \cdot 2\pi \int_0^r r^2 \Rightarrow A \cdot 2\pi \frac{r^3}{3} = I_{enc}$$

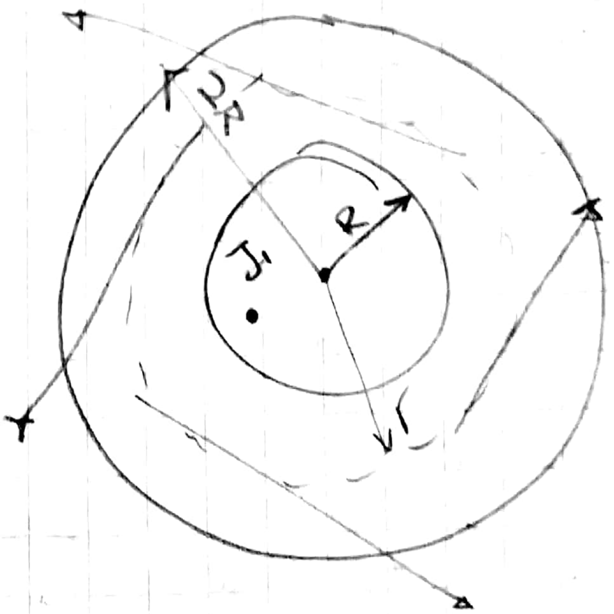
$$\Rightarrow \frac{I_{enc}}{A_{enc}} = \frac{I_T}{A_T} \Rightarrow I_{enc} = \frac{I_T \cdot A_{enc}}{A_T}$$

$\Rightarrow \oint \vec{B} \cdot \vec{dl} = \mu_0 \cdot I_{enc}$

$$B = \frac{\mu_0 \cdot I_{enc}}{dl} = \frac{\mu_0 \cdot A \cancel{2\pi} \frac{r^{\cancel{3}}}{3}}{\cancel{2\pi r}} \Rightarrow \frac{\mu_0 \cdot A r^2}{3}$$

* Con Región $R < r < 2R$ $(I_0 = I_{enc})$

$\oint \vec{B} \cdot \vec{dl} = \mu_0 \; I_{enc}$



I_{enc} :

$\Rightarrow I_{enc} = \iint J_1 \cdot ds$

$\Rightarrow \int_0^r \int_0^{2\pi} A r^2 \cdot ds \Rightarrow I_{enc}$

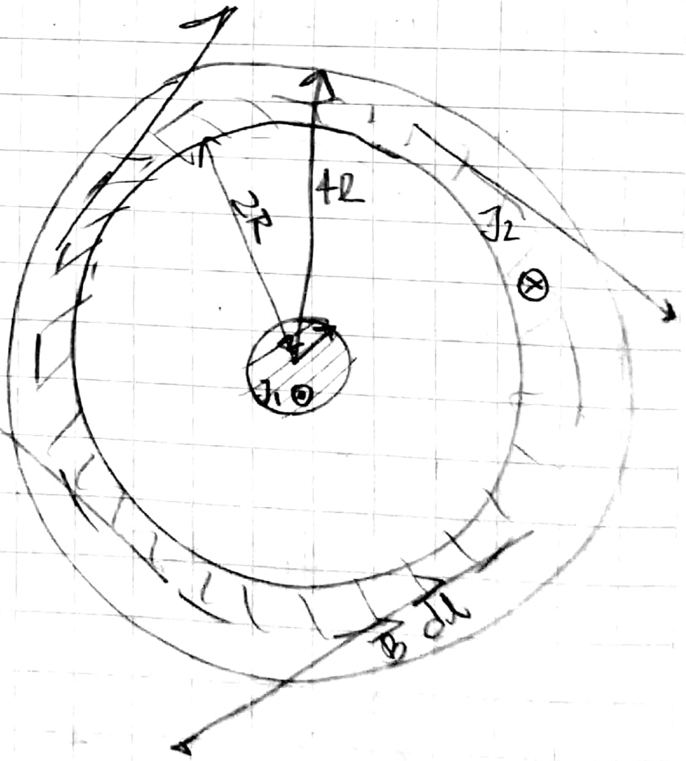
$\Rightarrow A \int_0^r r^2 \int_0^{2\pi} ds \Rightarrow A 2\pi \int_0^r r^2 \Rightarrow A\, 2\pi \cdot \frac{r^3}{3}$

$\Rightarrow \oint \vec{B}\, \vec{dl} = \mu_0\, I_{enc}$ | $I_0 = I_{enc}$

$\rightarrow B = \dfrac{\mu_0 \cdot I_{enc}}{dl} = \dfrac{\mu_0 \cdot A\, \cancel{2\pi}\, \frac{r^{\cancel{3}\,2}}{3}}{\cancel{2\pi}\, \cancel{r}}$

$\Rightarrow B = \dfrac{\mu_0 A r^2}{3}\ (T)$ con $a \leq r < b$

region $2R < r < 4R$



Ienc:

$J_2 = \dfrac{B}{r} - \hat{k}$

$J_2 \neq cte \Rightarrow I_{enc} = \iint J_2 \cdot ds$

$\Rightarrow \int_0^r \int_0^{2\pi} \dfrac{B}{r} \cdot ds$

$\Rightarrow B \cdot 2\pi \int_0^r \dfrac{1}{r}\, dr \Rightarrow \boxed{B\, 2\pi \ln|r| = I_{enc}}$

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$I_{TOTAL} = I_{\odot} - I_{\otimes} \Rightarrow I_{TOTAL} = A 2\pi \frac{r^3}{3} - B 2\pi \ln|r|$

Luego

$\oint \vec{B} \cdot \vec{dl} = \mu_0 \cdot I_{enc} \Rightarrow B = \frac{\mu_0 \cdot I_{enc}}{dl}$

$\Rightarrow B = \frac{\mu_0 \cdot B \cancel{2\pi} \ln|r|}{\cancel{2\pi} r} = \boxed{\frac{\mu_0 B \ln|r| (+)}{r}}$

Luego

$I_0 = I_{enc} \Rightarrow I_0 = B 2\pi \ln|r|$

$\boxed{\Rightarrow B = \frac{I_0}{2\pi \ln|r|}}$ //

Luego volviendo a $R < r < 2R$

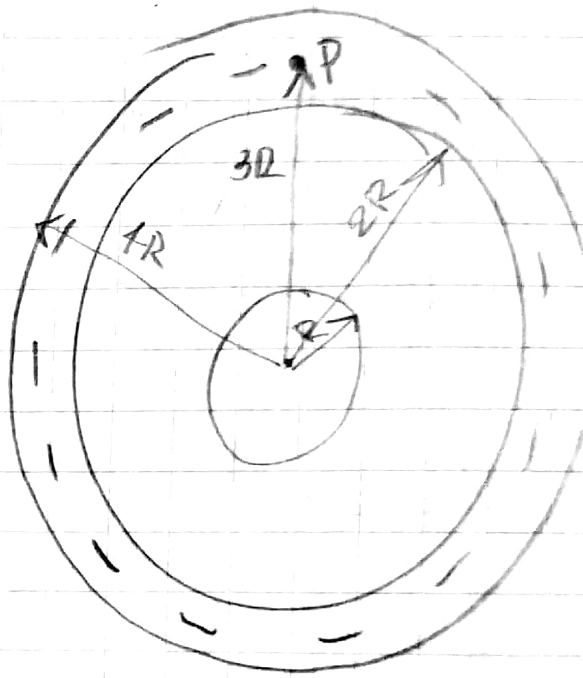
$\Rightarrow I_0 = I_{enc}$

$\Rightarrow I_0 = A 2\pi \frac{r^3}{3}$

$3 I_0 = A 2\pi r^3$

$\boxed{A = \frac{3 I_0}{2\pi r^3}}$ //

(B) Luego $\vec{B}_p$, region $r = 3R$



$$\oint \vec{B}_p \cdot d\vec{l} = \mu_0 \cdot I_{enc}$$

$J_2 \neq cte \Rightarrow I_{enc} = \iint J_2 \cdot ds$

$$\Rightarrow \int_0^r \int_0^{2\pi} \frac{B}{r} \, ds \Rightarrow B 2\pi \int_0^r \frac{dr}{r}$$

$$\boxed{\Rightarrow I_{enc} = B 2\pi \ln|r|}$$

$I_T = I_1 - I_2'$

$I_1 = I_{enc}$ de J_1

$$I_{Total} = \frac{A 2\pi r_1^3}{3} - B 2\pi \ln|r|$$

Luego:

$\Rightarrow \oint B_p \, dl = \mu_0 \cdot I_{enc}$

$\Rightarrow B_p = \dfrac{\mu_0 \cdot I_{enc}}{dl}$

$$\boxed{\Rightarrow B_p = \frac{\mu_0 \cdot}{6\pi R}\left[\frac{A\, 2\pi r^3}{3} - 2B\pi \ln|r|\right]}$$

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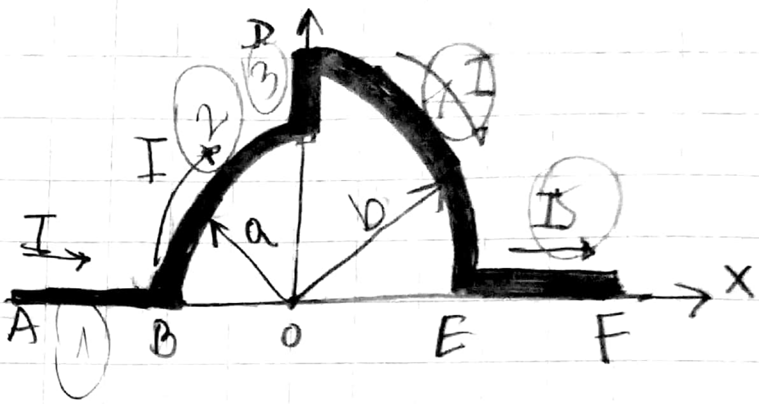
05 August 2020

Problema 1

Formula

a) $B_1 = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl} \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$



i) Vect. posi.

$\vec{r} = 0\hat{i} + 0\hat{j}$

$\vec{r}' = x\hat{i}$

$-x\hat{i}$

$|\vec{r} - \vec{r}'| = (x^2)^{3/2}$

ii) $\vec{dl} = dx\hat{i}$

$B_1 = \frac{\mu_0 I}{4\pi} \int \frac{dx\hat{i} \times (-x\hat{i})}{(x^2)^{3/2}} = 0$

$\vec{B}_2 = \frac{\mu_0 I}{} \int \frac{dl \times (\vec{r} - \vec{r}')}{}$

i) Vector posi.

$\vec{r} = 0$

$\vec{r}' = a\cos\theta\,\hat{i} + a\,\text{sen}\,\theta\,\hat{j}$

$|r - \vec{r}'| = -a\cos\theta\,\hat{i} - a\,\text{sen}\,\theta\,\hat{j}$

$\quad " \quad = -a\,\cancel{\cos\theta\,\hat{i} + \text{sen}\,\theta\,\hat{j}}^{\,1}$

$\quad " \quad = a^{3}$

ii) $dl = a\,d\theta$

$\vec{dl} = a\,d\theta = (\cos(\theta - \pi/2)\,\hat{i} - \text{sen}(\theta - \pi/2)\,\hat{j})$

$\vec{dl} = a\,d\theta(\sin\theta\,\hat{i} - \cos\theta\,\hat{j})$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a\sin\theta\,d\theta & -a\cos\theta\,d\theta & 0 \\ -a\cos\theta & -a\,\text{sen}\,\theta & 0 \end{vmatrix} \Rightarrow \hat{k}\,(-a^2\,\text{sen}^2\theta\,d\theta - a^2\cos^2\theta\,d\theta$$

$$\Rightarrow -a^2\,d\theta\,\hat{k}$$

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luego

$$\vec{B}_2 = \frac{\mu_0 I}{4\pi}\int_{\pi/2}^{\pi} -\frac{a^{\cancel{2}}\, d\theta\, \hat{n}}{a^{\cancel{3}}} = \frac{-\mu_0 I\, \hat{k}}{4\pi a}\left[\pi - \pi/2\right]$$

$$\boxed{\Rightarrow \frac{-\mu_0 I}{8a}\, \hat{k}(T)}$$

c) $\vec{B}_3 = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l} \times (\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}|^3}$

i) vector pos.

$\vec{r} = 0$ $\quad (\vec{r}-\vec{r}\,') = y^3$

$\vec{r}\,' = y\hat{j}$

ii) $dl = dy\hat{j}$

luego = $\quad b < y \leq c$

$$B_3 = \frac{\mu_0 I}{4\pi} \int_b^c \frac{dy\,\hat{j} \times (-y\,\hat{j})}{y^3} = 0$$

B_5 igual que B_1 con signo contrario :.

$B_5 = 0$.

B_4 :

i vector posic.

$\Rightarrow r = 0\hat{i} + 0\hat{j}$

$\vec{r}' = b(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$

$|r - \vec{r}'| = -b\cos\theta\,\hat{i} - b\sin\theta\,\hat{j}$

$\quad " \quad = b^3$

ii) $dl = b\,d\theta(\sin\theta\,\hat{i} - \cos\theta\,\hat{j}) = dl = \vec{dl} \times \vec{r} - \vec{r}'$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ b\sin\theta\,d\theta & -b\cos\theta\,\hat{j} & 0 \\ -b\cos\theta & -b\sin\theta & 0 \end{vmatrix} \Rightarrow -(2b^2)\,\hat{k}$$

$$B_4 = \frac{\mu_0 I}{4\pi} \int \frac{-2b^2 d\theta \hat{k}}{b^3} \Rightarrow \frac{-2\mu_0 I}{4\pi b} \int_0^{\pi/2} d\theta (\hat{k})$$

$$\Rightarrow \frac{-\mu_0 I}{4b} \hat{k}(T)$$