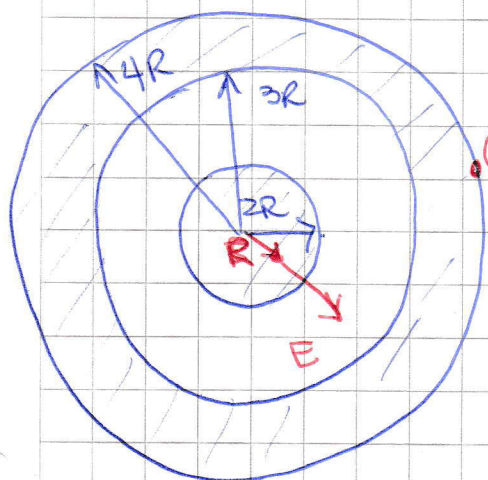
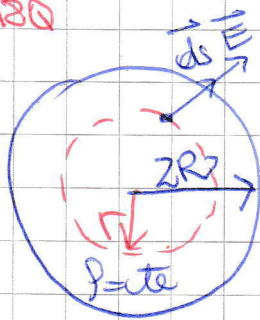


# Roura LPP. Electro (ejercicio nº 1)



$$Q_{ext} = 12Q$$

nos damos una sup Gauss  
con  $r = R \rightarrow E = \frac{Q}{5\pi\epsilon_0 R^2}$



$$\oint \vec{E} \cdot d\vec{S} = \frac{Q_{enc}}{\epsilon_0}$$

$$\int E dS_{Gauss} = \frac{Q_{enc}}{\epsilon_0} \quad (1)$$

$$E(4\pi r^2) = \frac{Q_{enc}}{\epsilon_0}, \text{ si } r = R$$

$$\frac{Q}{5\pi\epsilon_0 R^2} (4\pi \cdot R^2) = \frac{Q_{enc}}{\epsilon_0}$$

$$Q_{enc} = \frac{4Q}{5} \quad (r = R) \quad (3)$$

1 como  $\rho = \text{cte}$

$$\frac{Q_T}{V_T} = \frac{Q_{enc}}{V_{enc}} \rightarrow Q_T = V_T \cdot \frac{Q_{enc}}{V_{enc}} = \frac{\frac{4}{3}\pi \cdot (2R)^3 \cdot \frac{4Q}{5}}{\frac{4}{3}\pi R^3} \quad (1)$$

$$Q_T = \frac{32Q}{5} \quad (2) \quad (\text{carga total esfera de Radio } 2R)$$

$$a) Q_{int} = -\frac{32Q}{5} \quad (4) \quad (\text{carga interna del cascarón conductor})$$

$$b) V_r = V_{0,4R} + V_{4R,3R}$$

$$V_{0,4R} = - \int_0^{4R} E_{ext} dr \quad (1)$$

$E_{ext}$ : Sup. Gauss con  $r > 4R$

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{enc}}{\epsilon_0}$$

(1)

 $Q_{int} + Q_{ext}$ 

$$Q_{enc} = Q_{aislante} + Q_{cascaron}$$

$$Q_{enc} = \frac{32Q}{5} + \left( -\frac{32Q}{5} + 18Q \right)$$

$$Q_{enc} = 18Q$$

(2)

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{enc}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{18Q}{\epsilon_0}$$

(2)

$$\rightarrow E_{ext} = \frac{18Q}{4\pi r^2 \epsilon_0} = \frac{9Q}{2\pi r^2 \epsilon_0} \quad (N/C) \quad r > 4R$$

$$V_{\infty 4R} = - \int_{\infty}^{4R} \frac{9Q}{2\pi r^2 \epsilon_0} dr = - \frac{9Q}{2\pi \epsilon_0} \int_{\infty}^{4R} \frac{dr}{r^2} = \frac{9Q}{2\pi \epsilon_0 \cdot 4R}$$

$$V_{\infty 4R} = \frac{9Q}{8\pi \epsilon_0 R} \quad (V) \quad (3)$$

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$$V_{4R, 34} = - \int_{4R}^{34} \vec{E} \cdot d\vec{r} \quad (32R < r < 4R)$$

(1)

$E_{(32R < r < 4R)}$ : Sup. Gauss con  $3R < r < 4R$

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{enc}}{\epsilon_0}$$

$$Q_{enc} = Q_{aislante} + Q_{int} \text{ del cascaron}$$

$$Q_{enc} = \frac{32Q}{5} - \frac{32Q}{5}$$

(2)

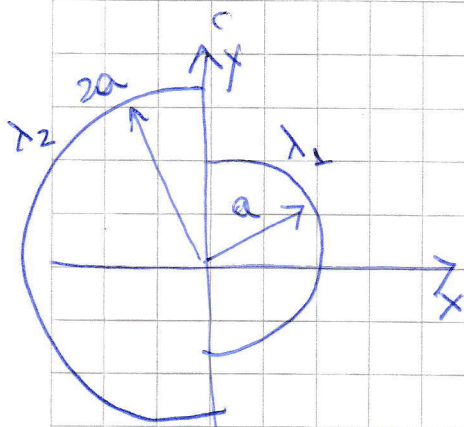
$$Q_{enc} = 0 \rightarrow E = 0 \rightarrow V_{4R, 34} = 0 \quad (V) \quad (3)$$

(conductor)



$$V_r = \frac{9Q}{8\pi\epsilon_0 R} \quad (V)$$

(32474R) (4)



$$a) \vec{E}_o = \vec{E}_{\lambda_1} + \vec{E}_{\lambda_2}$$

$E_{\lambda_1}$ :

$$\vec{r} = \vec{0} \quad (1)$$

$$\vec{r}' = a \cos \theta \hat{x} + a \sin \theta \hat{y} \quad (1)$$

$$\left. \begin{aligned} \vec{r} - \vec{r}' &= -a \cos \theta \hat{x} - a \sin \theta \hat{y} \\ |\vec{r} - \vec{r}'|^3 &= a^3 \end{aligned} \right\} \quad (1)$$

$$\left. \begin{aligned} dq &= \lambda_1 dl \\ dl &= a d\theta \end{aligned} \right\} dq = \lambda_1 a d\theta \quad (1)$$

$$3\pi/2 \leq \theta \leq \pi/2 \quad (1)$$

$$\vec{E}_{\lambda_1} = K \int_{3\pi/2}^{\pi/2} \frac{\lambda_1 a d\theta (-a \cos \theta \hat{x} - a \sin \theta \hat{y})}{a^3}$$

$$= \frac{K \lambda_1}{a} \int_{3\pi/2}^{\pi/2} (\cos \theta \hat{x} + \sin \theta \hat{y}) d\theta$$

$$\int_{3\pi/2}^{\pi/2} \cos \theta d\theta \hat{x} = \left( \sin \frac{\pi}{2} - \sin \frac{3\pi}{2} \right) \hat{x} = 2 \hat{x} \quad (1)$$

$$\int_{3\pi/2}^{\pi/2} \sin \theta d\theta \hat{y} = \left( -\cos \frac{\pi}{2} + \cos \frac{3\pi}{2} \right) \hat{y} = 0 \hat{y} \quad (1)$$

$$\boxed{\vec{E}_{\lambda_1} = \frac{-2K \lambda_1}{a} \hat{x} \quad (N/C)} \quad (3)$$

$E_{\lambda_2}$ :

$$\left. \begin{aligned} \vec{r}' &= 2a \cos \theta \hat{x} + 2a \sin \theta \hat{y} \\ \vec{r} - \vec{r}' &= -2a \cos \theta \hat{x} - 2a \sin \theta \hat{y} \end{aligned} \right\} \left. \begin{aligned} |\vec{r} - \vec{r}'|^3 &= (2a)^3 \end{aligned} \right\} \quad (1)$$

$$\left. \begin{aligned} dq &= \lambda_2 dl \\ dl &= 2a d\theta \end{aligned} \right\} dq = \lambda_2 \cdot 2a d\theta \quad (1)$$

$$\pi/2 \leq \theta \leq 3\pi/2 \quad (1)$$

$$\vec{E}_{\lambda_2} = K \int_{\pi/2}^{3\pi/2} \lambda_2 \cdot \frac{2a d\theta}{(2a)^2} (-2a \cos\theta \hat{i} - 2a \sin\theta \hat{j})$$

$$= - \frac{K \lambda_2}{2a} \int_{\pi/2}^{3\pi/2} (\cos\theta \hat{i} + \sin\theta \hat{j}) d\theta$$

$$\int_{\pi/2}^{3\pi/2} \cos\theta d\theta \hat{i} = \left( \sin \frac{3\pi}{2} - \sin \frac{\pi}{2} \right) \hat{i} = -2 \hat{i} \quad (1)$$

$$\int_{\pi/2}^{3\pi/2} \sin\theta d\theta \hat{j} = \left( -\cos \frac{3\pi}{2} + \cos \frac{\pi}{2} \right) \hat{j} = 0 \hat{j} \quad (2)$$

$$\vec{E}_{\lambda_2} = \frac{2K\lambda_2}{2a} \hat{i} \rightarrow \boxed{\vec{E}_{\lambda_2} = \frac{K\lambda_2}{a} \hat{i} \text{ (N/C)}} \quad (3)$$

$$\vec{E}_0 = \frac{K}{a} (-2\lambda_1 + \lambda_2) \hat{i} \text{ (N/C)} \quad (4)$$

b)  $\vec{F}_Q = \vec{E}_0 \cdot Q$

$$\vec{F}_Q = \frac{KQ}{a} (-2\lambda_1 + \lambda_2) \hat{i} \text{ (N)} \quad (5)$$