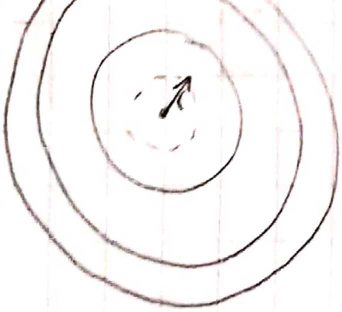
1) Una Esfera de Radio 2R tiene una carga uniformemente distribuida en todo su volumen. Rodeada por corteza esferica conductora DE radios 3R y 4R. Si la carga en la sup. externa de la corteza conductora es $Q_{ext} = 18Q$ (C) y se mide el campo electrico en $r = R$, siendo $E = \frac{Q}{5\pi \varepsilon_0 R^2}$ (N/C). determine

a) la carga inducida en la superficie interna de la corteza conductora

b) Potencial electrico en r con $3R < r < 4R$.



$r < 2R$ Sup. gauss.

$$E = \frac{Q}{5\pi \varepsilon_0 R^2}$$

a) $\oint \vec{E}\, \vec{dr} = \frac{Q_{enc}}{\varepsilon_0} \Rightarrow E(4\pi r^2) = \frac{Q_{enc}}{\varepsilon_0}$

$$\Rightarrow \frac{Q}{5\pi \varepsilon_0 R^2} \cdot (4\pi r^2) = \frac{Q_{enc}}{\varepsilon_0}$$

$$\Rightarrow \varepsilon_0 \cdot \frac{Q}{5\cancel{\pi} \cancel{\varepsilon_0} R^2} \cdot 4\cancel{\pi} r^2 = Q_{enc}$$

$$\Rightarrow Q_{enc} = \frac{4 Q r^2}{5 R^2} (C)$$

Luego: ρ - cte

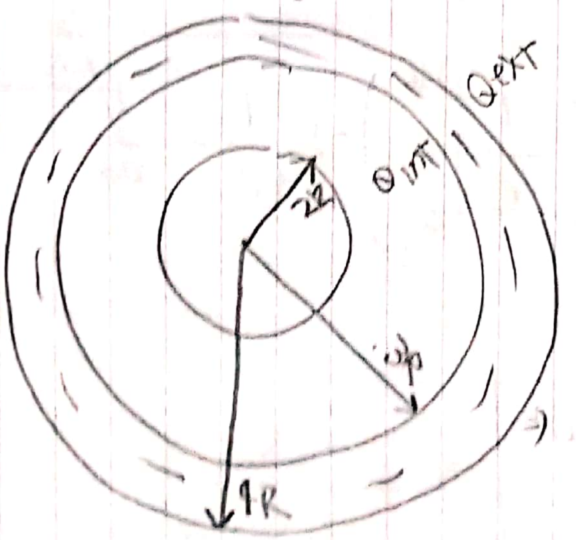
$$\frac{Q_T}{V_T} = \frac{Q_{enc}}{V_{enc}} \Rightarrow Q_T = \frac{Q_{enc}}{V_{enc}} \cdot V_T$$

$$\Rightarrow Q_T = \frac{\frac{4Qr^2}{5R^2}}{\frac{4}{3}\pi r^3} \cdot V_T \Rightarrow \frac{4Qr^2}{5R^2 \cdot \cancel{\frac{4}{3}}\pi r^3} \cdot \cancel{\frac{4}{3}}\pi (2R)^3$$

$$\Rightarrow \frac{4Qr^2 \cdot 8R^3}{5R^2 \cdot r^3}$$ luego. $r = R$ $\therefore$ $$\frac{4Q\cancel{r^2}\, 8\cancel{R^3}}{5\cancel{R^2}\,\cancel{r^3}}$$

$$\Rightarrow \frac{32}{5}Q = Q_T$$ → carga interna

b) Luego Región 3. con $3R < r < 4R$



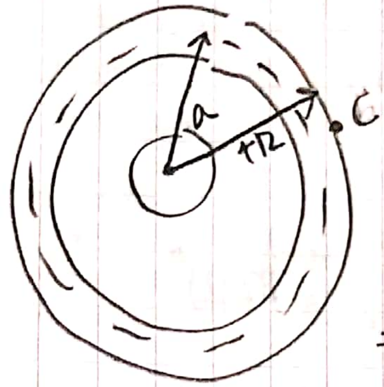
$Q_{int} + Q_{ext} = Q_{neta}$

$$-\frac{32}{5} + 18 = \frac{58}{5}$$

$$\Rightarrow \oint \vec{E} \cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon_0} \Rightarrow E(4\pi r^2) = \frac{Q_{enc}}{\varepsilon_0}$$

$$\Rightarrow E_{int} = \frac{\frac{58}{5}}{\varepsilon_0 4\pi r^2} \Rightarrow \boxed{\frac{58}{20\pi r^2 \varepsilon_0}\ (N/C)}$$

Formula $V_A = -\int \vec{E}\cdot\vec{dL}$



$$V_a = V_{\infty c} + V_{ca}$$

$$\Rightarrow V_{\infty c} \Rightarrow -\int_{\infty}^{c} \vec{E}_{EXT}\cdot\vec{dr}$$

i) Sup. gauss con $r > 4R$

$$\Rightarrow E(4\pi r^2) = \frac{18Q}{\varepsilon_0} \Rightarrow E = \frac{18Q}{4\pi r^2 \varepsilon_0}$$

Luego $\Rightarrow -\int_{\infty}^{c} \vec{E}_{EXT}\cdot\vec{dr} = \int_{\infty}^{c} \frac{18Q}{4\pi r^2 \varepsilon_0}\cdot dr$

$$\Rightarrow \frac{18Q}{4\pi\varepsilon_0}\int_{\infty}^{4R}\frac{dr}{r^2} \Rightarrow \frac{18Q}{4\pi\varepsilon_0}\cdot\left(\frac{1}{r}\Big|_{\infty}^{4R}\right) \Rightarrow \frac{1}{4R} - \cancel{\frac{1}{\infty}}^{\,0}$$

$$\Rightarrow \frac{18Q}{4\pi\varepsilon_0 \cdot 4R} \Rightarrow \frac{18Q}{16\pi\varepsilon_0 R} = V_{\infty c}$$

Luego V_{ca} sup. gauss con $r<4R$

$$\Rightarrow -\int_c^a \vec{E}_{int} \cdot \vec{dr} = \int_c^a \frac{5\delta}{20\pi r^2 \varepsilon_0} \cdot dr$$

$$\Rightarrow \frac{5\delta}{20\pi\varepsilon_0} \int_c^a \frac{dr}{r^2} \Rightarrow \frac{1}{r}\Bigg|_{4R}^{a} \Rightarrow \frac{1}{a} - \frac{1}{4R}$$

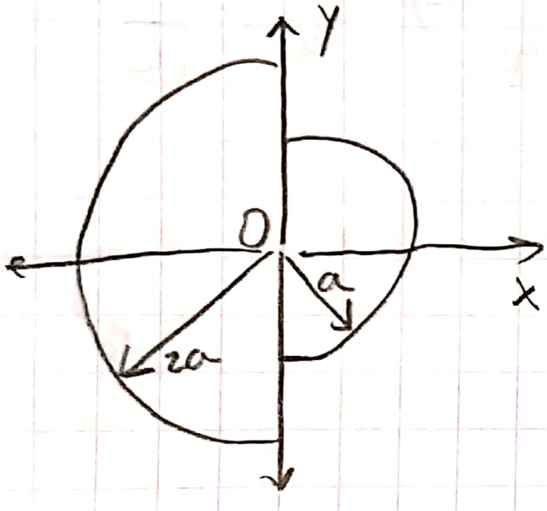
$$\Rightarrow -\frac{5\delta}{20\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{4R}\right)$$

Luego $V_a = V_{\infty c} + V_{ca}$

$$\Rightarrow -\frac{18Q}{16\pi\varepsilon_0 R} - \frac{5\delta}{20\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{4R}\right) [V]$$

Brayan Maldonado Carrasco.

(2) Considere dos alambres concéntricos doblados de forma semicircunferencial como F2. el primero tiene λ_1 y el segundo una densidad lineal de carga positiva λ_2. Calcule para conf.(a) el campo electrico resultante en el punto "O" y (b) la fuerza electrica si se coloca una carga puntual "q" en el mismo punto.



i) Campo electrico en "O"

ii) fuerza electrica en q.

i) E_0.

$$\vec{r} = 0\hat{i} + 0\hat{j}$$

$$\vec{r}' = a\cos\theta\hat{i} + a\,\text{sen}\hat{j}$$

$$(r - r') = -a\cos\theta\hat{i} - a\,\text{sen}\hat{j}$$

$$|r - r'| = a^3$$

Luego: $dq = \lambda ds \longrightarrow dl = a d\theta$

$$dq = \lambda_1 \cos\theta \cdot R\, d\theta$$

Luego: $\frac{3\pi}{2} \leq \theta \leq \pi/2$

$$\vec{E}_o = k\int \lambda_1 a\cos\theta\, d\theta \frac{\left(-\cancel{a}\overset{(A)}{\cos\theta\,\hat{\imath}} - \cancel{a}\overset{(B)}{\sin\theta\,\hat{\jmath}}\right)}{\cancel{a^3}}$$

$$\Rightarrow \frac{-k\lambda_1}{a}\int \left(\overset{(A)}{\cos\theta\,\hat{\imath}} + \overset{(B)}{\sin\theta\,\hat{\jmath}}\right)\cos\theta$$

(A) $\displaystyle\int_{3\pi/2}^{\pi/2} \cos^2\theta \, d\theta\, \hat{\imath}$ $\qquad \cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

$$\Rightarrow \frac{1}{2}\left[\overset{(1)}{\int \cos(2\theta)\, d\theta} + \overset{(2)}{\int d\theta}\right]$$

(1) $\Rightarrow \dfrac{1}{2}\left[\begin{array}{l} u = 2\theta \\ du = 2\,d\theta \\ \dfrac{du}{2} = d\theta \end{array}\right. \Rightarrow \displaystyle\int \cos(u)\frac{du}{2} \Rightarrow \frac{1}{2}\sin u \Big|_{u_1}^{u_2}$

$$\Rightarrow \frac{1}{2}\left[\sin 2\theta\right]\Big|_{3\pi/2}^{\pi/2} \Rightarrow \frac{1}{2}\left(\sin \cancel{2}\frac{\pi}{\cancel{2}} - \sin \cancel{2}\cdot\frac{3\pi}{\cancel{2}}\right)^{0}$$

$\rightarrow \boxed{0}$

(2) $\displaystyle\int_{3\pi/2}^{\pi/2} d\theta \Rightarrow \theta\Big|_{3\pi/2}^{\pi/2} \Rightarrow \boxed{-\pi}$

Scanned with CamScanner

Luego B

$$\int_{\frac{3\pi}{2}}^{\pi/2} \text{sen}\theta \cos\theta \, d\theta \, \hat{j}$$

$u = \text{sen}$
$du = \cos\theta \, d\theta$

$$\int u \, du = \frac{u^2}{2} \Rightarrow \frac{\text{sen}\,\theta^2}{2} \Bigg|_{3\pi/2}^{\pi/2} \rightarrow 0 \, \hat{j}$$

$$\therefore \; E_0 = \frac{+K\lambda_1}{a} \pi$$

Luego Para λ_2

$\vec{r} = 0\hat{i} + 0\hat{j}$

$\vec{r}' = 2a\cos\theta\hat{i} + 2a\,\text{sen}\theta\hat{j}$

$|r - r'| = 8a^3$

Luego: $dq = \lambda_2 \, 2a \cdot d\theta \; ; \; \pi/2 \leq \theta \leq 3\pi/2$

$$\rightarrow E_1 = K \int \lambda_2 \, 2a \cdot d\theta \, \frac{(-2a\cos\theta\hat{i} - 2a\,\text{sen}\theta\hat{j})}{8a^3}$$

$$E_1 = \frac{-K\lambda_2}{4a} \int \underset{(A)}{\cos\theta\hat{i}} + \underset{(B)}{\text{sen}\theta\hat{j}}$$

Scanned with CamScanner

Ⓐ

$$\int_{\pi/2}^{3\pi/2} \cos\theta \, d\theta \, \hat{\imath} \Rightarrow \operatorname{sen}\theta \Big|_{-\pi/2}^{3\pi/2} = 2\hat{\imath}$$

Ⓑ $$\int_{\pi/2}^{3\pi/2} \operatorname{sen}\theta \, \hat{\jmath} \Rightarrow -\cos \Big|_{\pi/2}^{3\pi/2} = 0\hat{\jmath}$$

$$\rightarrow E_1 = -\frac{K\lambda_2}{\cancel{4}a} \cdot \cancel{2} \Rightarrow \boxed{\frac{-K\lambda_2}{2a}\hat{\imath}}$$

Brayan Maldonado Carrasco.