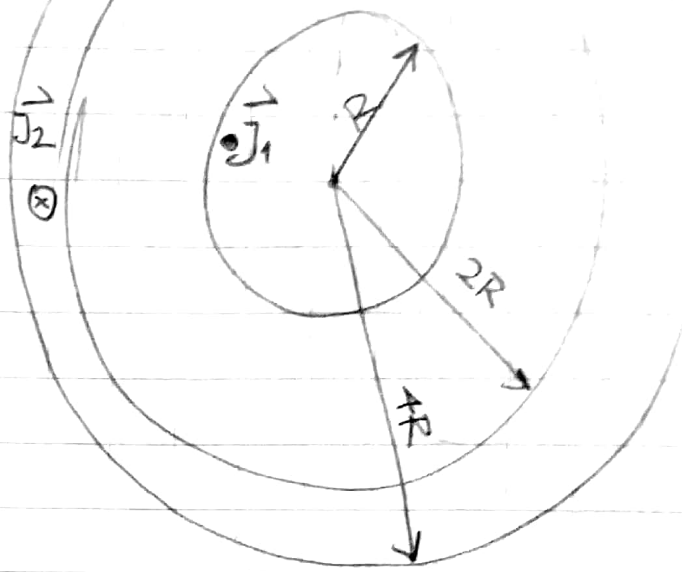
6 27/60 3,3

Problema 2



Para un rango

$a < R$

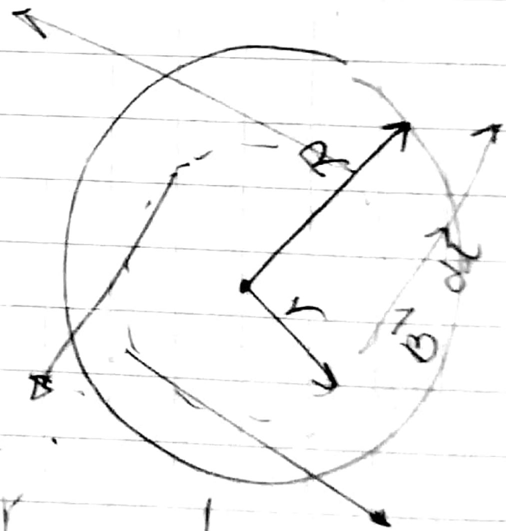
$$\rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 \cdot I_{enc}$$

I_{enc}:

$J \neq cte$

$$J_1 = A r^2 (\hat{k}$$

$$I_{enc} = \int_0^{r} \int_0^{2\pi} A r^2 \cdot ds$$



$$\rightarrow A \int_0^{r} r^2 \cdot 2\pi$$

$$\rightarrow A \cdot 2\pi \int_0^{r} r^2 \Rightarrow A \cdot 2\pi \, \frac{r^3}{3} = I_{enc}$$

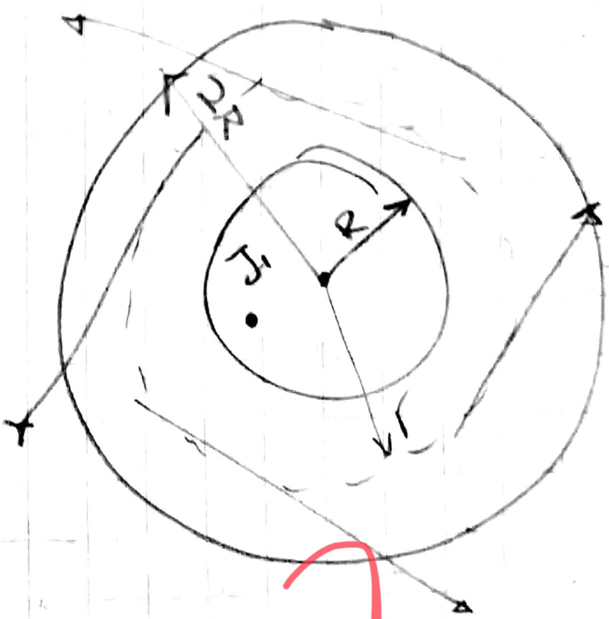
$$\Rightarrow \frac{I_{enc}}{A_{enc}} = \frac{I_T}{A_T} \Rightarrow I_{enc} = \frac{I_T \cdot \cancel{A_{enc}}}{A_T}$$

$$\Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0 \cdot I_{enc}$$

$$B = \frac{\mu_0 \cdot I_{enc}}{dl} = \frac{\mu_0 \cdot A \cancel{2\pi} \frac{r^{\cancel{3}}}{3}}{\cancel{2\pi r}} \Rightarrow \frac{\mu_0 \cdot A r^2}{3}$$

* Con Región $R < r < 2R$ $(I_0 = I_{enc})$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \; I_{enc}$$



I_{enc} :

$$\Rightarrow I_{enc} = \iint J_1 \cdot ds$$

$$\Rightarrow \int_0^r \int_0^{2\pi} A r^2 \cdot ds \Rightarrow I_{enc}$$

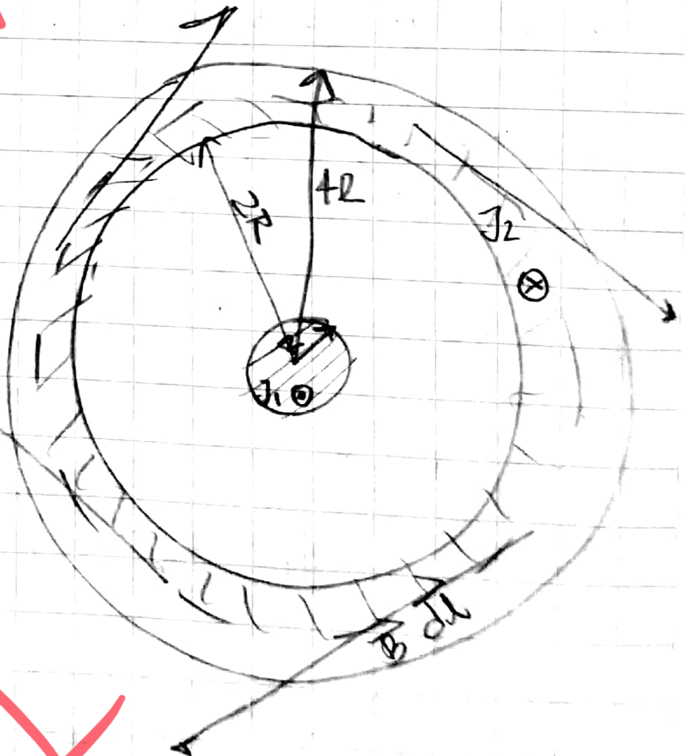
$$\Rightarrow A \int_0^r r^2 \int_0^{2\pi} ds \Rightarrow A\, 2\pi \int_0^r r^2 \Rightarrow A\, 2\pi \cdot \frac{r^3}{3}$$

$\Rightarrow \oint \vec{B}\, \vec{dl} = \mu_0 \, I_{enc}$ | $I_0 = I_{enc}$

$\rightarrow B = \dfrac{\mu_0 \cdot I_{enc}}{dl} = \dfrac{\mu_0 \cdot A \,\cancel{2\pi}\, \frac{r^{\cancel{3}\,2}}{3}}{\cancel{2\pi}\,\cancel{r}}$

$\Rightarrow B = \dfrac{\mu_0 A r^2}{3}$ (T) ~~con~~ $a \leq r < b$

region $2R < r < 4R$



$\underline{I_{enc}}$:

$J_2 = \dfrac{B}{r} \hat{k}$

$J_2 \neq cte \Rightarrow I_{enc} = \iint J_2 \cdot ds$

$\Rightarrow \int_0^r \int_0^{2\pi} \dfrac{B}{r} \cdot ds$

$\Rightarrow B \cdot 2\pi \int_0^r \dfrac{1}{r} dr \Rightarrow \boxed{B 2\pi \ln|r| = I_{enc}}$

$$I_{TOTAL} = I_{\odot} - I_{\otimes} \Rightarrow I_{TOTAL} = A2\pi\frac{r^3}{3} - B2\pi \ln|r|$$

Luego

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \cdot I_{enc} \Rightarrow B = \frac{\mu_0 \cdot I_{enc}}{dl}$$

$$\Rightarrow B = \frac{\mu_0 \cdot B\cancel{2\pi}\ln|r|}{\cancel{2\pi}r} = \boxed{\frac{\mu_0 B \ln|r|}{r} (+)}$$

2

Luego

$$I_0 = I_{enc} \Rightarrow I_0 = B2\pi\ln|r|$$

$$\boxed{\Rightarrow B = \frac{I_0}{2\pi\ln|r|}}$$

//

Luego volviendo a $R<r<2R$

$$\Rightarrow I_0 = I_{enc}$$

$$\Rightarrow I_0 = A2\pi\frac{r^3}{3}$$

$$3I_0 = A2\pi r^3$$

$$\boxed{A = \frac{3I_0}{2\pi r^3}}$$

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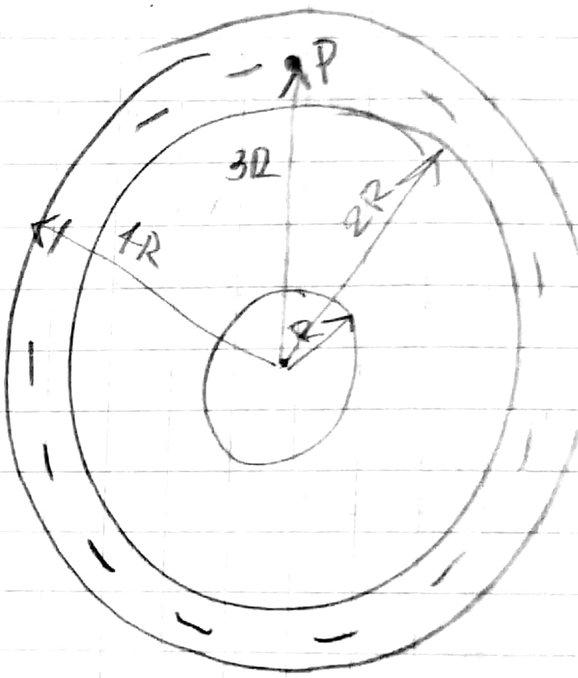
(B) Luego $\vec{B}_p$; región $r = 3R$



$$\oint \vec{B}_p \cdot d\vec{l} = \mu_0 \cdot I_{enc}$$

$$J_2 \neq cte \Rightarrow I_{enc} = \iint J_2 \cdot ds$$

$$\Rightarrow \int_0^{r}\int_0^{2\pi} \frac{B}{r}\, ds \Rightarrow B 2\pi \int_0^{r} \frac{dr}{r}$$

$$\Rightarrow I_{enc} = B 2\pi \ln|r|$$

$$I_T = I_1 - I_2'$$

$I_1 = I_{enc}$ de J_1

$$I_{Total} = \frac{A 2\pi r^3}{3} - B 2\pi \ln|r|$$

Luego

$$\Rightarrow \oint B_p \, dl = \mu_0 \cdot I_{enc}$$

$$\therefore B_p = \frac{\mu_0 \cdot I_{enc}}{dl}$$

$$\boxed{\rightarrow B_p = \frac{\mu_0 \cdot}{6\pi R}\left[\frac{A\,2\pi r^3}{3} - 2B\pi \ln|r|\right]}$$

//

Problema 1

05 August 2020

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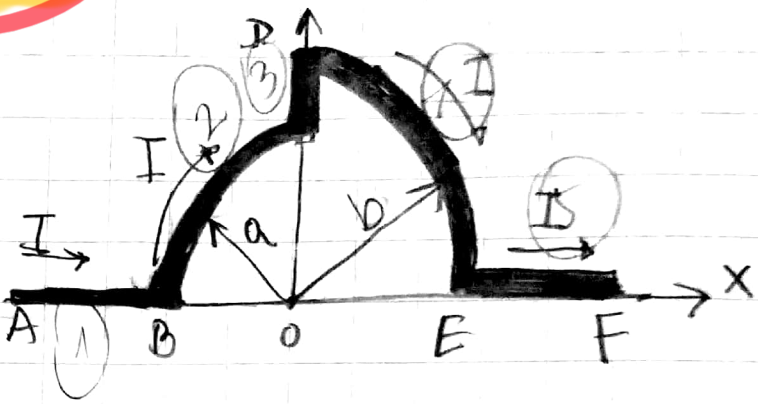
Formula

a/ $B_1 = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl} \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$



i) vect. posi.

$\vec{r} = 0\hat{\imath} + 0\hat{\jmath}$

$\vec{r}' = -x\hat{\imath}$ $\Big\} -x\hat{\imath}$

$|\vec{r} - \vec{r}'| = (x^2)^{3/2}$

ii) $\vec{dl} = dx\hat{\imath}$

$B_1 = \frac{\mu_0 I}{4\pi} \int \frac{dx\hat{\imath} \times (-x\hat{\imath})}{(x^2)^{3/2}} = 0$ 2

$\vec{B}_2 = \frac{\mu_0 I}{} \int \frac{dl \times (\vec{r} - \vec{r}')}{}$

i) Vector posi.

$\vec{r} = 0$

$\vec{r}\,' = a\cos\theta\,\hat{i} + a\,\mathrm{sen}\,\theta\,\hat{j}$

$|r - \vec{r}\,'| = -a\cos\theta\,\hat{i} - a\,\mathrm{sen}\,\theta\,\hat{j}$

$\quad '' \quad = -a\,\cancel{(\cos\theta\,\hat{i} + \mathrm{sen}\,\theta\,\hat{j})}^{1}$

$\quad '' \quad = a^3$

3

→ Desarrollo

ii) $dl = a\,d\theta$

$d\vec{l} = a\,d\theta = (\cos(\theta - \pi/2)\,\hat{i} - \mathrm{sen}(\theta - \pi/2)\,\hat{j})$

$d\vec{l} = a\,d\theta\,(\sin\theta\,\hat{i} - \cos\theta\,\hat{j})$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a\sin\theta\,d\theta & -a\cos\theta\,d\theta & 0 \\ -a\cos\theta & -a\,\mathrm{sen}\,\theta & 0 \end{vmatrix} \Rightarrow \hat{k}\,(-a^2\mathrm{sen}^2\theta\,d\theta - a^2\cos^2\theta\,d\theta$$

$\Rightarrow -a^2\,d\theta\,\hat{k}$

1

luego

$$\vec{B}_2 = \frac{\mu_0 I}{4\pi}\int_{\pi/2}^{\pi} -\frac{a\,d\theta\,\hat{k}}{a^2} = \frac{-\mu_0 I\,\hat{k}}{4\pi a}\left[\pi - \pi/2\right]$$

3

$$\Rightarrow \boxed{\frac{-\mu_0 I}{8a}\,\hat{k}\,(T)}$$

c) $\vec{B}_3 = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l} \times (\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$

i) vector pos.

$\vec{r} = 0$ $\quad (\vec{r}-\vec{r}\,') = y^3$

$\vec{r}\,' = y\hat{j}$

ii) $d\vec{l} = dy\,\hat{j}$

luego = $b < y \leq c$

3

$$B_3 = \frac{\mu_0 I}{4\pi} \int_b^c \frac{dy \hat{j} \times \cancel{(-y \hat{j})}}{y^3} = 0$$

B_5 igual que B_1 con ~~signo~~ contrario :.

3

$B_T = 0$.

B_4 :

i vector posic.

$\Rightarrow r = 0\hat{i} + 0\hat{j}$

$\vec{r}' = b(\cos\theta\hat{i} \cancel{+} \sin\theta\hat{j})$

3

$|r - \vec{r}'| = -b\cos\theta\hat{i} \cancel{-} b\sin\theta\hat{j}$

$u = b^3$

denominador

ii) $dl = b\, d\theta(\sin\theta\hat{i} - \cos\theta\hat{j}) = dl = \vec{dl} \times \vec{r} - \vec{r}'$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ b\sin\theta\, d\theta & -b\cos\theta\, d\theta & 0 \\ -b\cos\theta & -b\sin\theta & 0 \end{vmatrix} \Rightarrow \cancel{-}(2b^2)\hat{k}\, d\theta$$

$$B_4 = \frac{\mu_0 I}{4\pi}\int \frac{-2b^2\,d\theta\,\hat{n}}{b^3} \Rightarrow \frac{-2\mu_0 I}{4\pi b}\int_0^{\pi/2} d\theta\,(\hat{k})$$

$$\Rightarrow \frac{-\mu_0 I}{4b}\,\hat{k}\,(T)$$

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