Characterizing the Neurocognitive Mechanisms of Arithmetic

Pedro Pinheiro-Chagas
INSERM-CEA Cognitive Neuroimaging Unit – NeuroSpin Center, France

Université Pierre et Marie Curie

École Doctorale Cerveau Cognition et Comportment
Inserm-CEA Unicog, NeuroSpin

Characterizing the Neurocognitive Mechanisms of Arithmetic

Pedro PINHEIRO-CHAGAS

Dissertation submitted for the degree of
Doctor of Philosophy

Supervised by Stanislas DEHAENE
Co-supervised by Manuela PIAZZA

Dror DOTAN
Josef PARVIZI
Amy DAITCH

Jury:

Wim FIAS	Ghent University	Reviewer
Christian BÉNAR	Aix-Marseille University	Reviewer
Lionel NACCACHE	Pierre et Marie Curie University	Examiner
Elizabeth SPELKE	Harvard University	Examiner
Manuela PIAZZA	Trento Univeristy	Co-supervisor
Stanislas DEHAENE	Collège de France	Supervisor

Introduction

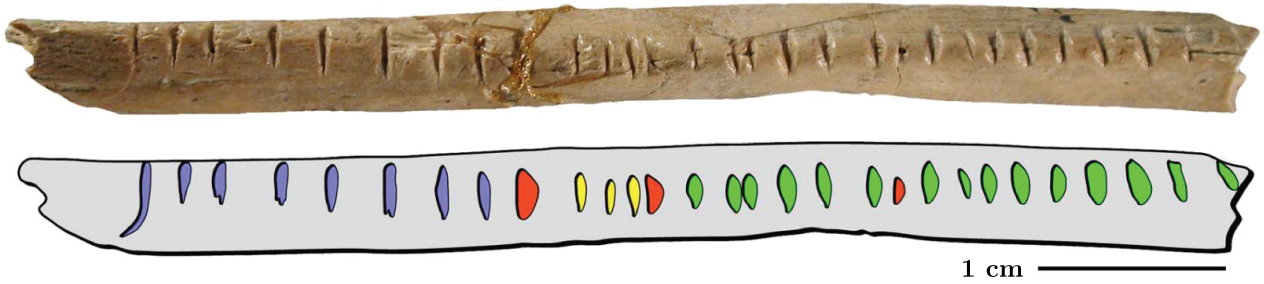
- Mathematics is among the most remarkable human inventions.
- The fundamental language of science.
- Arithmetic is the most elementary branch of mathematics.

- How are two numbers combined into a third? i.e. **3+5=8**.
- Sophisticated abstract concepts and complex network of brain regions.

Paleolithic Arithmetic

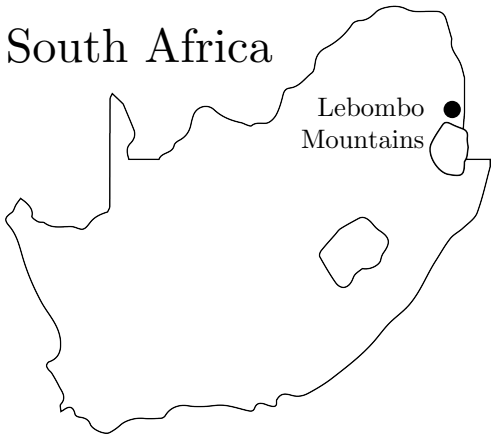
Artificial Memory Systems

Lebombo Bone (baboon fibula)



Addition

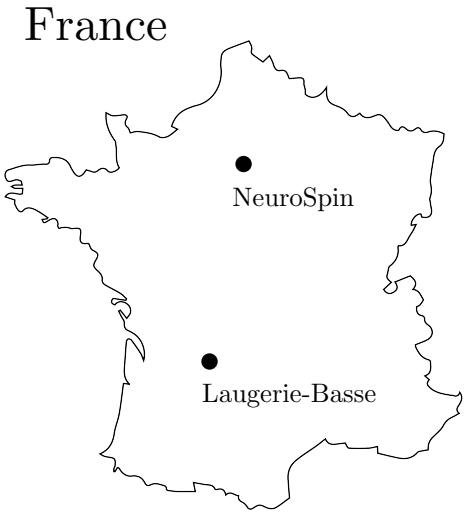
44,000 BP



Spatula (rib)

Subtraction

17,000 - 12,000 BP



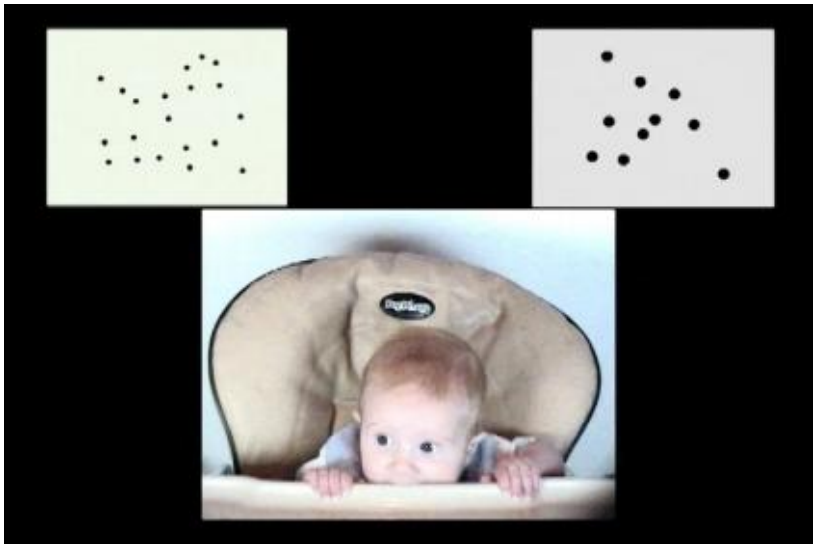
d'Errico (1998); d'Errico et al. (2012)

The origins of arithmetic thinking

- **The Number Sense** *(Dantzig, 1967; Dehaene, 1999)*
  - Approximate Number System - obeys Weber's law.
  - Object-tracking system – *subitizing* 1-4 items.

- Evolutionary origin: protect cubs, select preys, mating, etc.

Number discrimination in babies
(Starr et al., 2013; Izard et al., 2009)

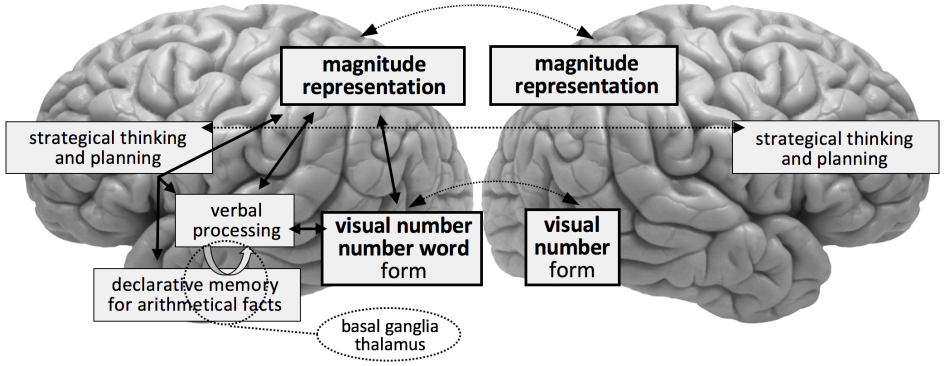


Democratic decisions in baboons
(Piantadosi & Cantlon, 2017)

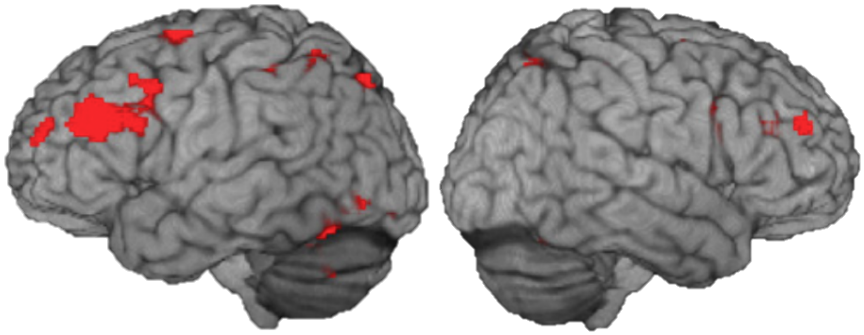
- ANS allows for approximate calculation (babies, children & monkeys).
  (McCrink & Wynn, 2004, Gilmore & Spelke, 2007; Livingstone et al. 2014)

Brain networks for arithmetic processing

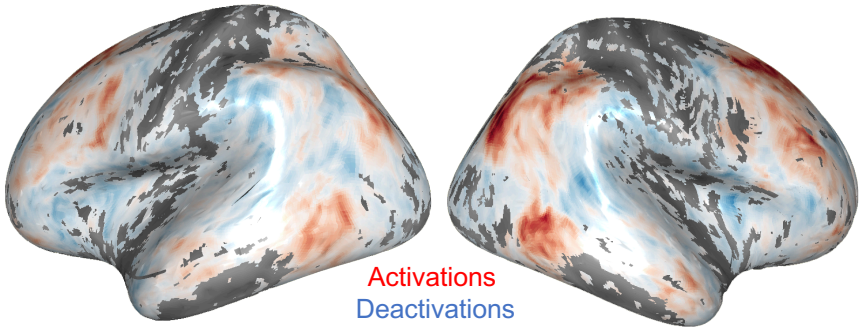
Putative network - neuropsychology
(Dehaene & Cohen, 1995)



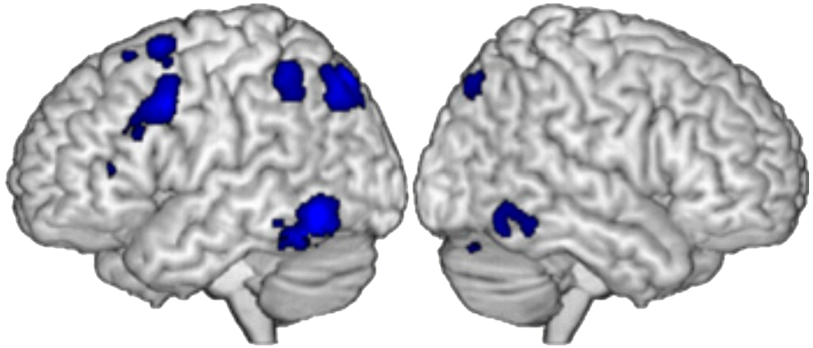
fMRI meta-analysis
(Arsalidou & Taylor, 2011)



fMRI activations for number words in natural speech
boldpredictions.gallantlab.org



Similar network engaged in high-level mathematics
(Amalric & Dehaene, 2016)



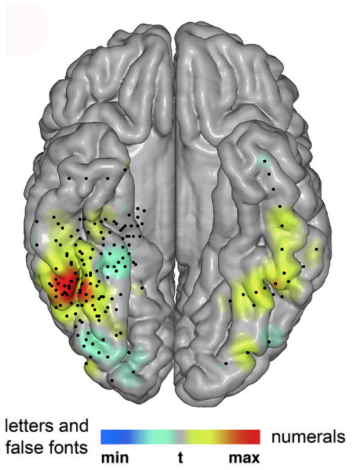
- **Lateral parietal cortex**: main hubs for magnitude processing and calculation.
  BOLD activity in the **IPS** and **SPL increase** as a function of **problem-size.**
  (Dehaene et al., 1999; Kanjlia et al., 2016, etc.)

- **Ventral temporal cortex:** visual recognition of numerals.

Arithmetic processing in the *ventral* stream

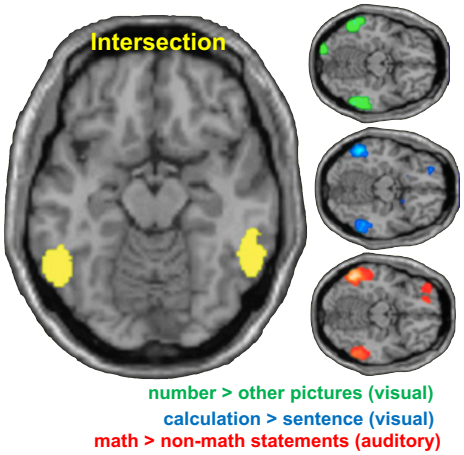
'Number Form Area' in the pITG

(Shum et al. 2013)



Activation during high-level math

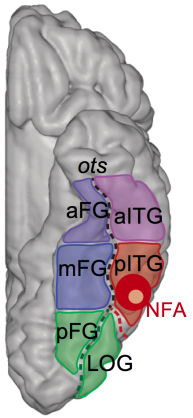
(Amalric & Dehaene, 2016)

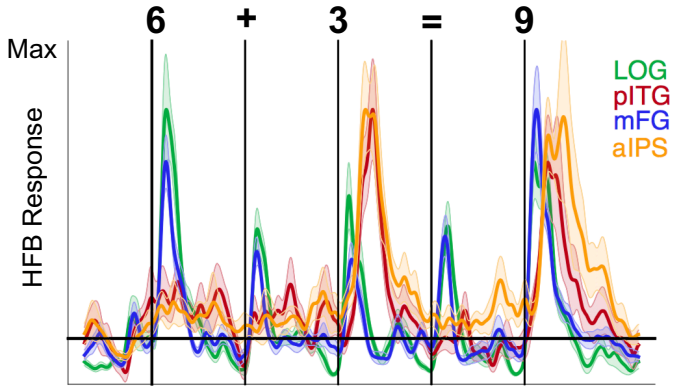


Possible role of the pITG in arithmetic

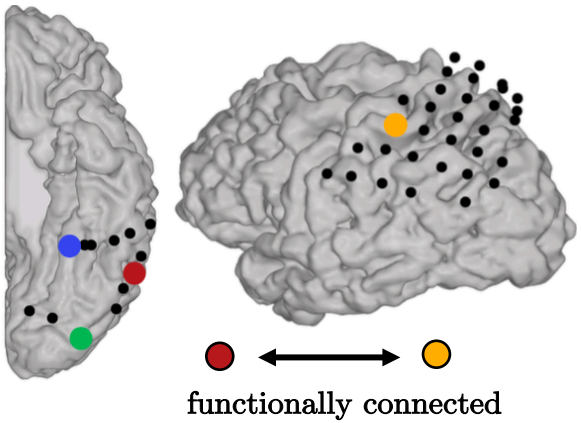
(Daitch et al., 2016)

NFA: response to numerals is *context independent*





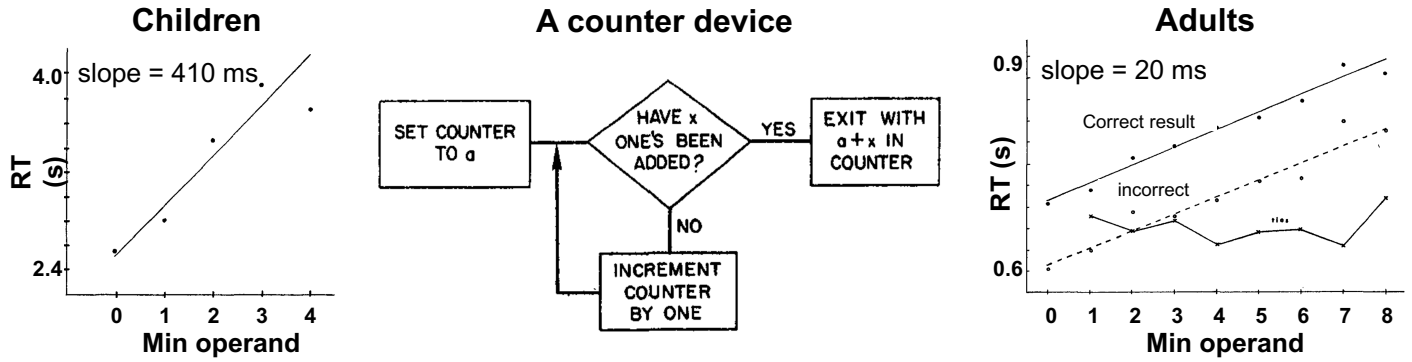
Adjacent population – $pITG_{math}$: response to numerals depends on *calculation*



Cognitive models of symbolic arithmetic

- **Problem-size effect**: RTs increase as a function the size of the operands.

- **The '*min*' counting model** *(Groen & Parkman, 1972).*



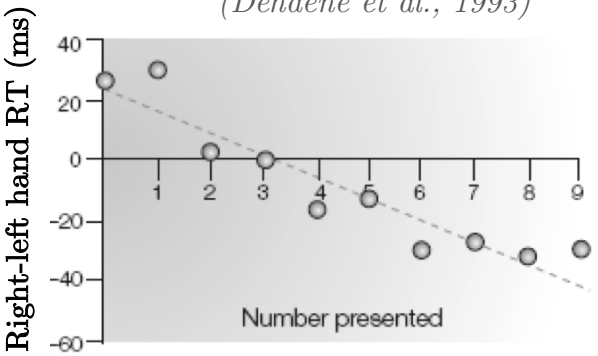
- **Fact retrieval models**
  - Acquisition history determines strategy *(Siegler & Shrager, 1984).*
  - Tabular representation of the addition facts *(Ashcraft, 1992).*
  - Only half of the table is memorized (larger+smaller) *(Butterworth et al., 2001).*

- **Compact counting procedures**
  - Linear increase in RT even in very small additions *(Barrouillet & Thevenot, 2013).*
  - Operator-priming effect in additions & subtractions *(Fayol & Thevenot, 2012).* Procedures can be pre-activated by the operation sign.

Calculation as *'movement'* along the mental number line

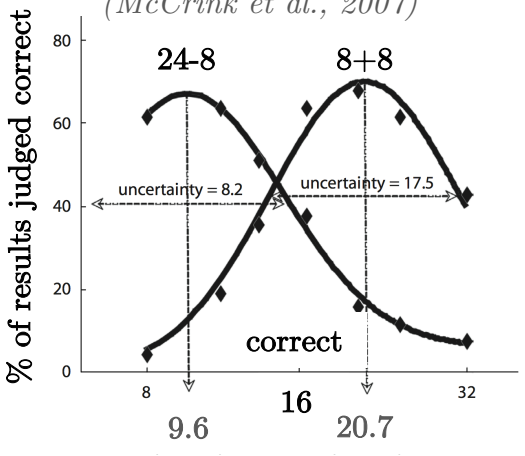
SNARC effect

(Dehaene et al., 1993)



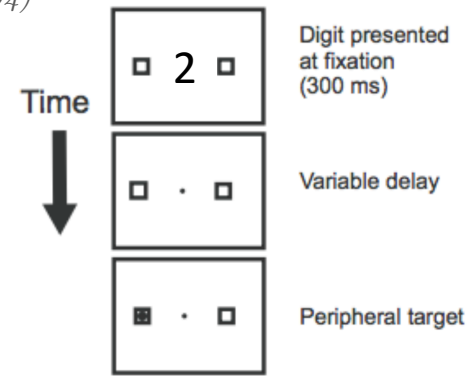
Operational Momentum effect

(McCrink et al., 2007)



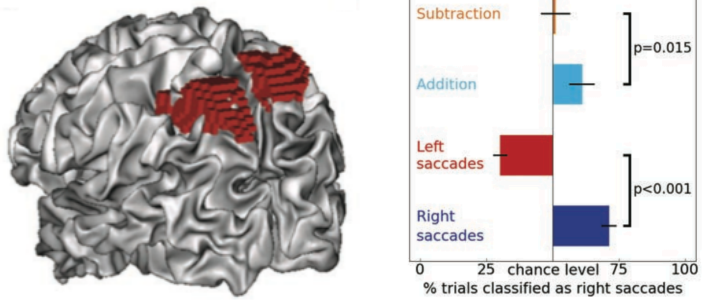
Perceiving numbers shift spatial attention

(Fischer et al, 2004)



Decoding right vs. left saccades generalizes to add vs. sub in pSPL

(Knops et al, 2009)



VIP-LIP vector addition for eye movement control

(Pouget, et al., 2002)

- Compatible with *min* and tabular fact retrieval models.
  Precise mechanism still unknown.

Introduction

- **Simple additions** and **subtractions**: direct retrieval from long-term memory.
  Recent challenges: **Operational momentum** & **operator priming** effects.
  - Compact counting procedures: 'movement' along the mental number line.

- **LPC**: magnitude processing and calculation; **VTC**: visual recognition of numerals.
  Recent challenges: 2 distinct neuronal populations in VTC.
  - **NFA**: selective response to numerals **irrespectively of the context**.
  - **Adjacent pITG$_{math}$**: response to numerals **only during calculation**.

- **Methodological barrier** – summary measures of the entire processing chain.
  - **Mental chronometry**: blind to the absolute timing and order of the stages.
  - **fMRI**: low temporal resolution & relatively coarse functional specificity.

- Approach to parse and characterize the processing stages of calculation:
  - **Trajectory-tracking**, **iEEG** and **time-resolved MVPA** applied to **MEG**.

Index of the presentation

- **Chapter 2. Pinheiro-Chagas, P.**, Dotan, D., Piazza, M., Dehaene, S. (2017). Finger tracking reveals the covert stages of mental arithmetic. *Open Mind: Discoveries in Cognitive Science,* 1(1), 30-41.

- **Chapter 3. Pinheiro-Chagas, P.***, Daitch, A.*, Parvizi, J., Dehaene, S. (*under review*). Brain mechanisms of arithmetic: a crucial role for ventral temporal cortex.

- **Chapter 4. Pinheiro-Chagas, P.**, Piazza, M., Dehaene, S. (*under review*). Decoding the processing stages of mental arithmetic with magnetoencephalography.

- **General discussion** and perspectives.

Chapter 2. **Finger-tracking reveals the covert stages of mental arithmetic**

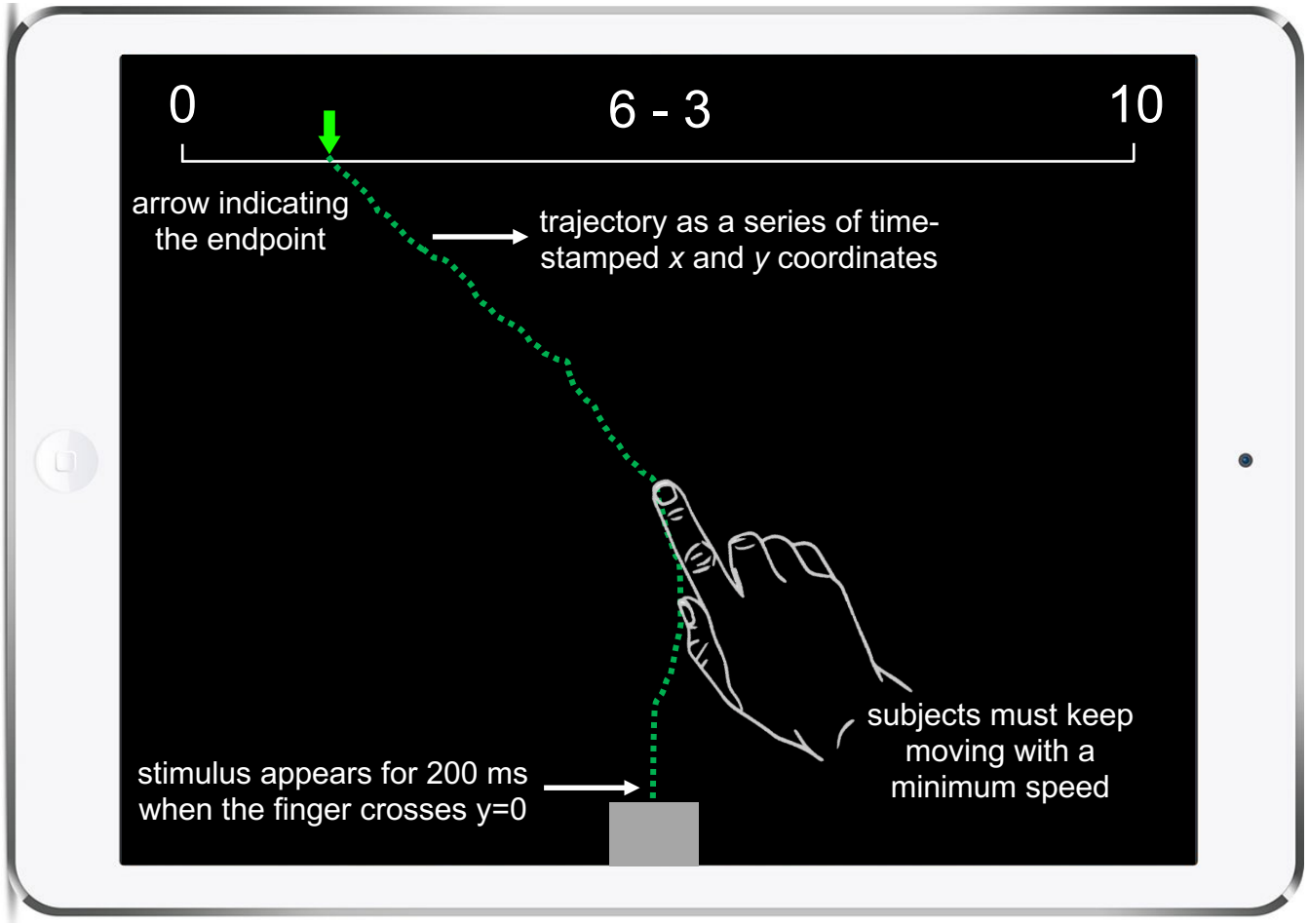
- Decompose the covert stages of mental arithmetic.
- Are the two operands processed serially or in parallel?
- Is there a stage whose duration increases linearly with the size of the numerical quantities, as implied by counting models?
- Can we visualize a reordering of the two operands when solving additions, as predicted by the COMP model?
- Can we determine the moment when the visuospatial biases underlying addition and subtraction occur?

Pinheiro-Chagas, Dotan, Piazza & Dehaene (2017) Open Mind.

Dror Dotan
Lecturer & **finger-tracker**
Tel Aviv University

Experimental Setup

30 adults, students, right-handed



Assumption

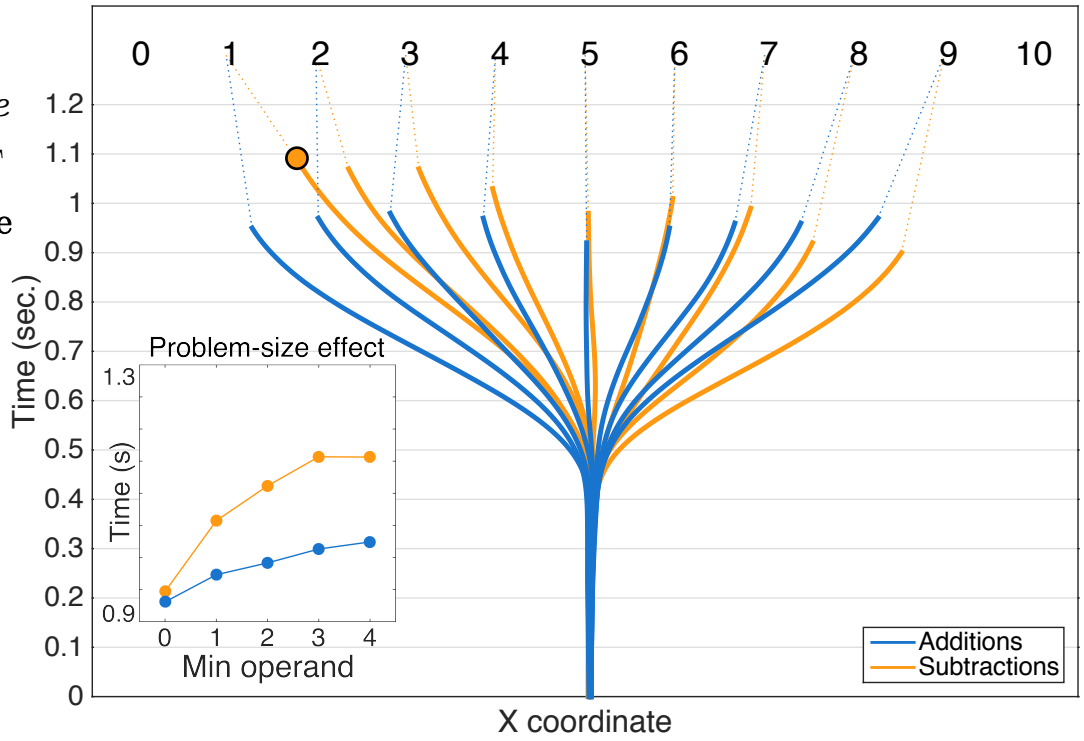
finger trajectories track the ongoing decision process underlying a cognitive task

Averaged trajectories by result

movement time

equivalent to RT

summary measure

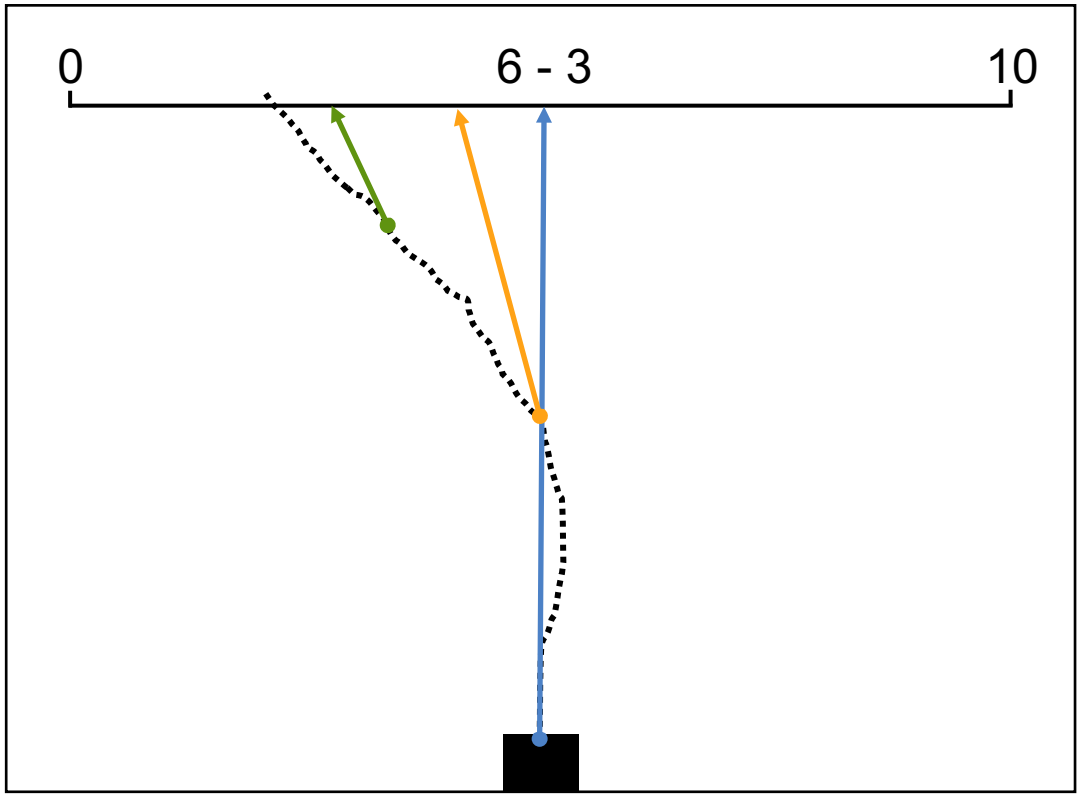


- The ***min*** operand is the best predictor of MT in additions & subtractions slope: 55 ms *sub*; 25 ms *add; Replicates Groen & Parkman (1972), etc.*

Research in the past 40 years would stop at this stage to formulate cognitive models of arithmetic

Decomposing the calculation task

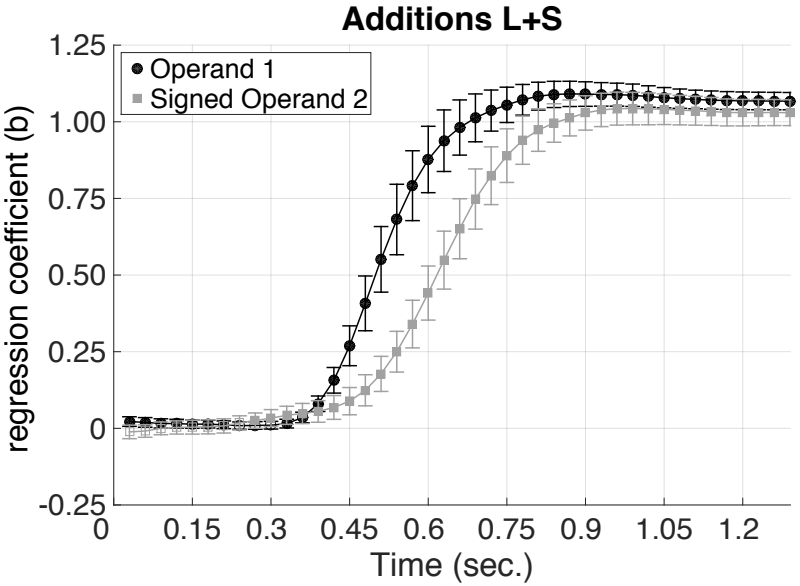
Time-resolved multiple regression

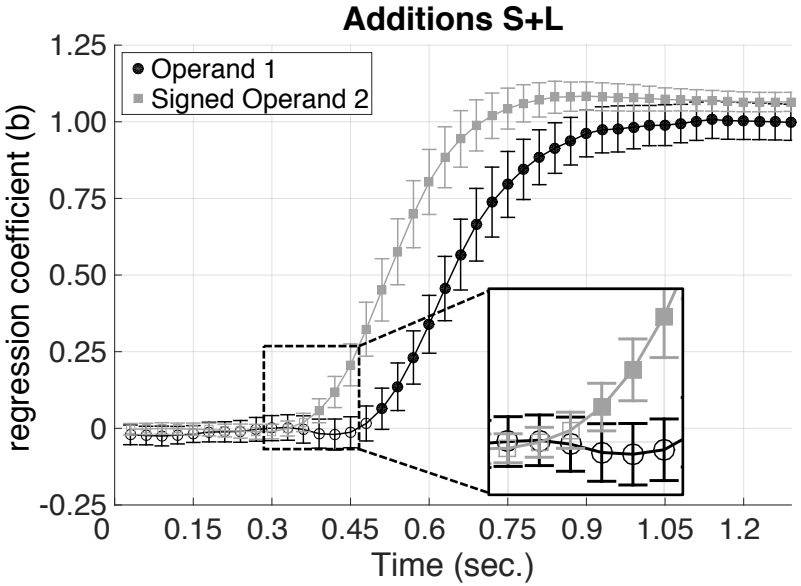


Implied endpoints: where the subject is aiming at each time t

- **Multiple Regression** at each time sample *(30 ms), per subject*
  - ***Dependent variable****: implied endpoint*
  - ***Predictors****: operand 1, operand 2, etc.*

Serial processing of the operands in additions

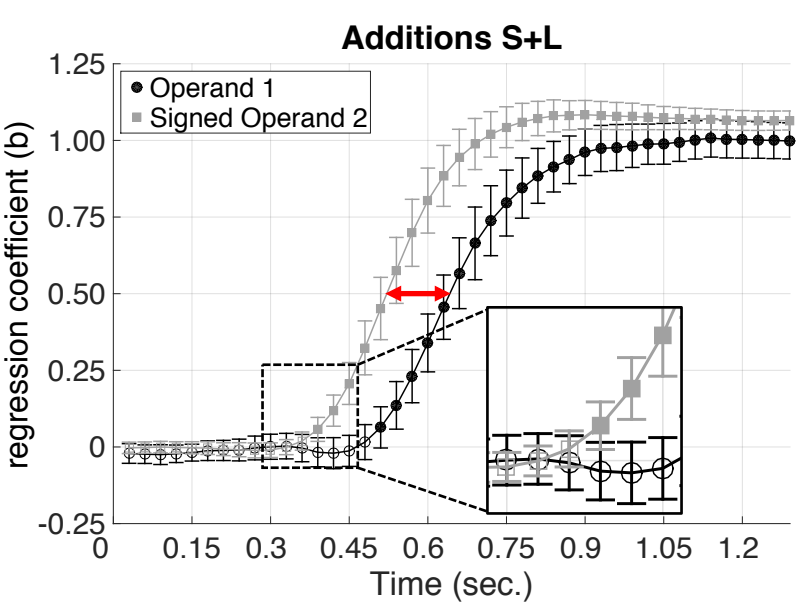


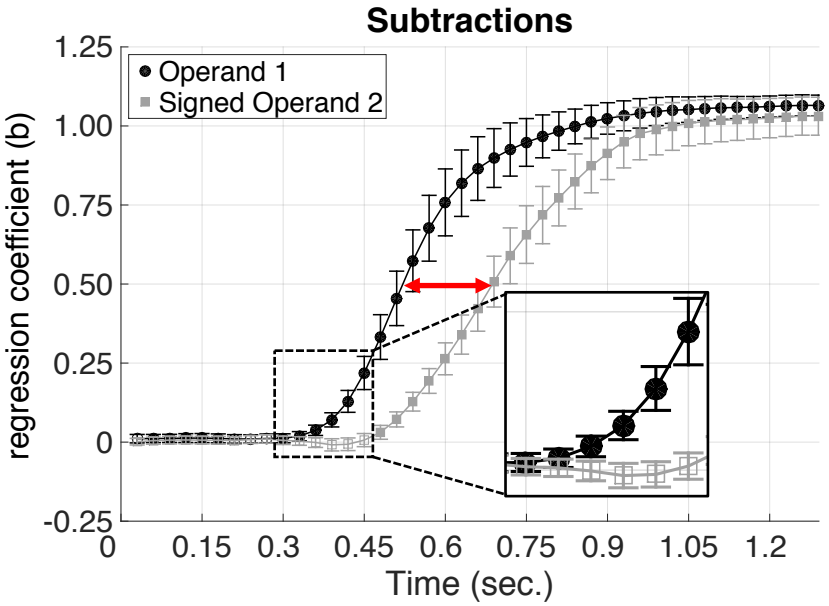


- **Reordering of the operands**: subjects first point to the larger operand irrespectively of the its location.

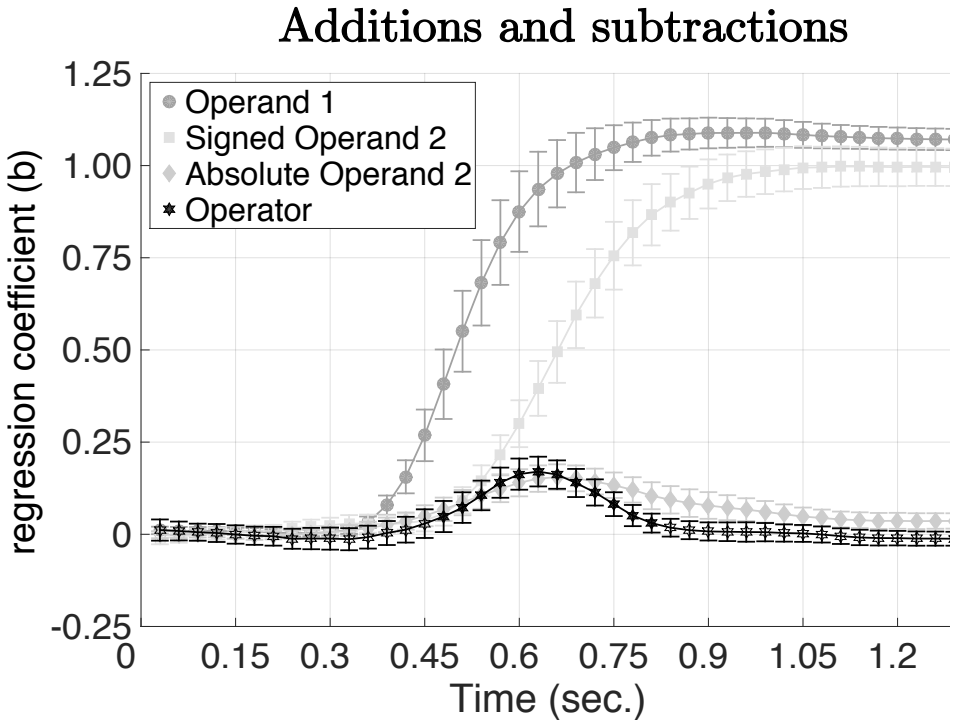
- Cost in MT of 14 ms.

Higher serial processing of the operands in subtractions



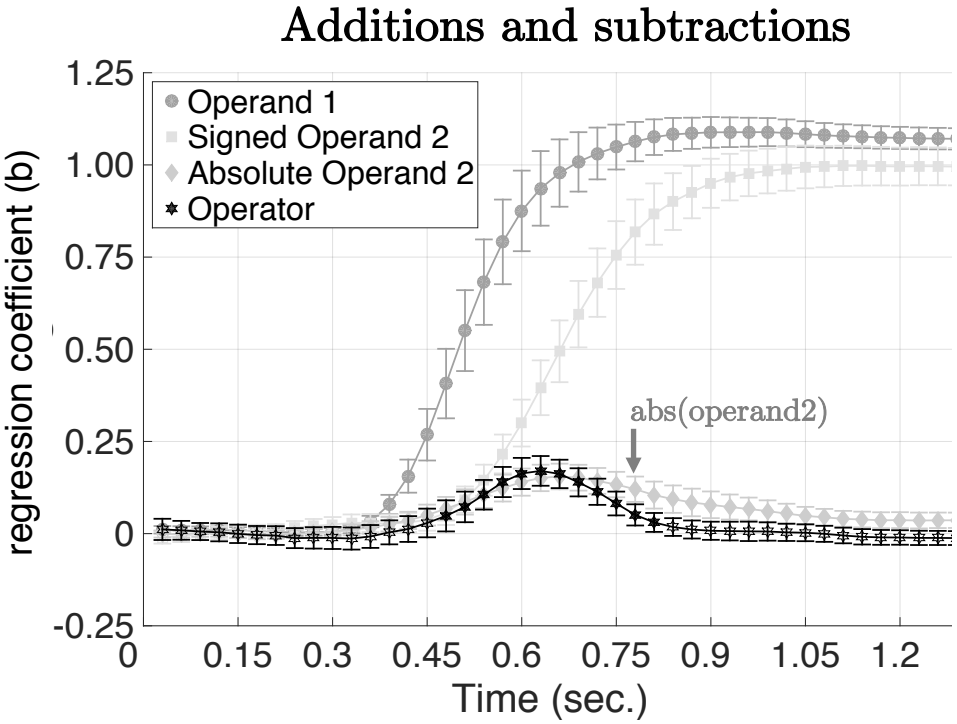


Transient OM effect at the time of processing the *min* operand



- **OM effect**: + and – signs distorts the finger to the right & left sides

Transient activation of the absolute value of the subtrahend



- **Additional stage in subtraction**: discard the absolute value of the subtrahend

Chapter 2. **Discussion and conclusions**

- The **operands** are **processed serially**: larger operand first independent of its location: **direct visualization** of the **reordering effect** for additions S+L

- The deviation from the larger operand to the correct result unfolds serially and **proportionally** to the **size** of the ***min* operand** (intermediate stages?)

- **Transient OM effect** at the time of the **integration of the *min***: visuospatial attention system actively engaged during the calculation.

- **Transient activation** of the **absolute value** of the **subtrahend**. Potentially explain why subtractions are generally slower.

- **Support for a model in which single-digit additions and subtractions are computed by a stepwise displacement on the mental number line**
  Compatible with compact counting procedures & retrieval by tabular search

Pinheiro-Chagas, Dotan, Piazza & Dehaene (2017) Open Mind.

Chapter 3. **Brain mechanisms of arithmetic: a crucial role for ventral temporal cortex**

- **Traditional view:**
  - **LPC**: main hubs for magnitude processing and calculation.
  - **VTC**: visual recognition of numerals.

- Recent findings suggest that the **VTC engagement** is **more complex**.

- Fine-grained re-evaluation of the LPC and VTC roles in arithmetic. processing, by recording electrophysiological activity directly from the brain.

- Test if, how and when numerical features *(problem-size)* modulate the activity in calculation-selective neuronal populations in the LPC and VTC.

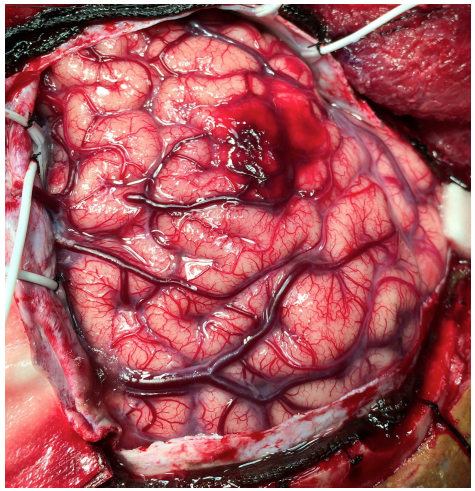
Pinheiro-Chagas, Daitch*, Parvizi & Dehaene (under review)*

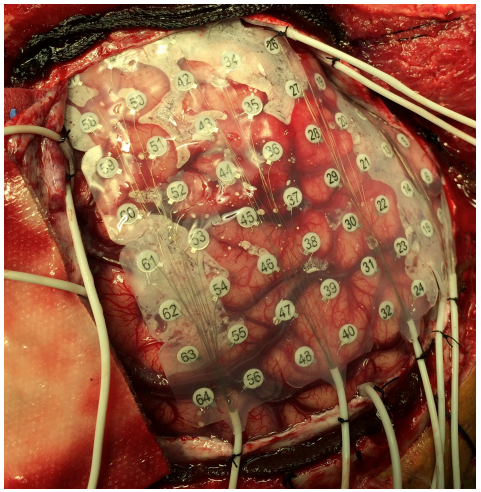
Josef Parvizi

Amy Daitch

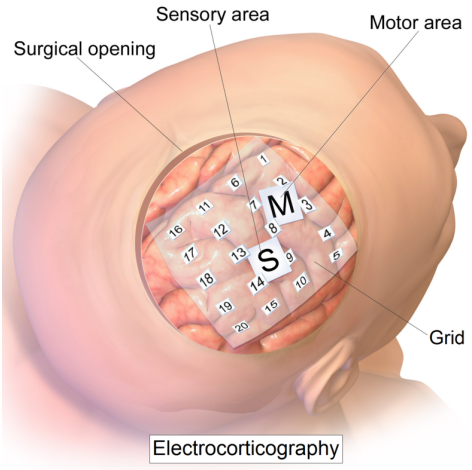
Stanford University

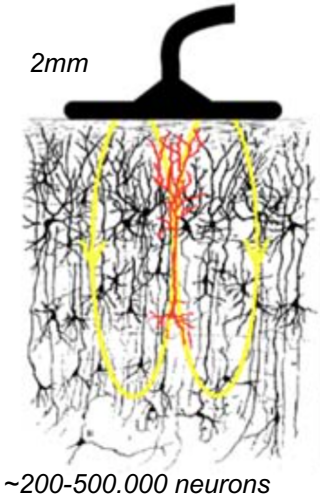
Electrocorticography (ECoG)



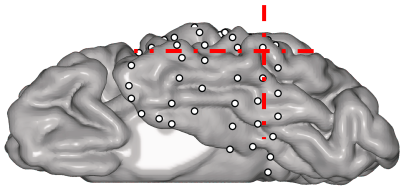


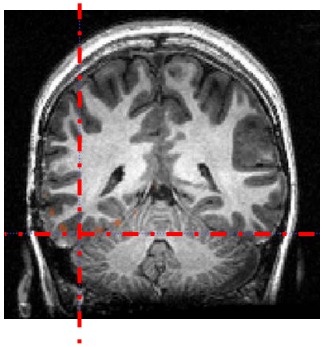
Electrocorticography (ECoG)



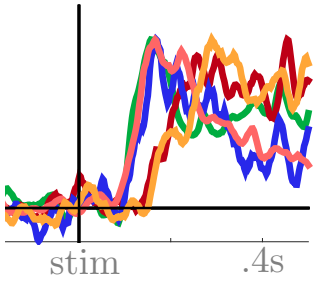


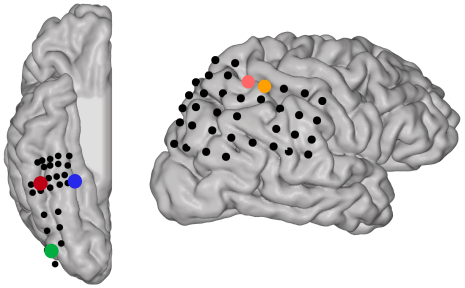
High spatial precision



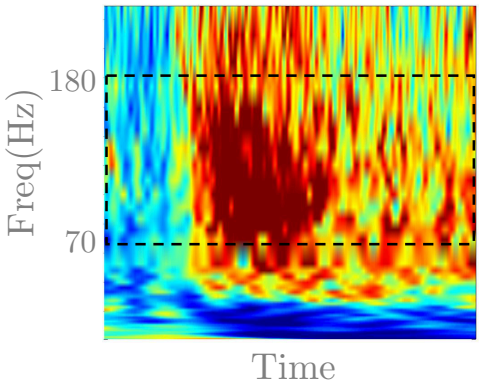


High temporal resolution





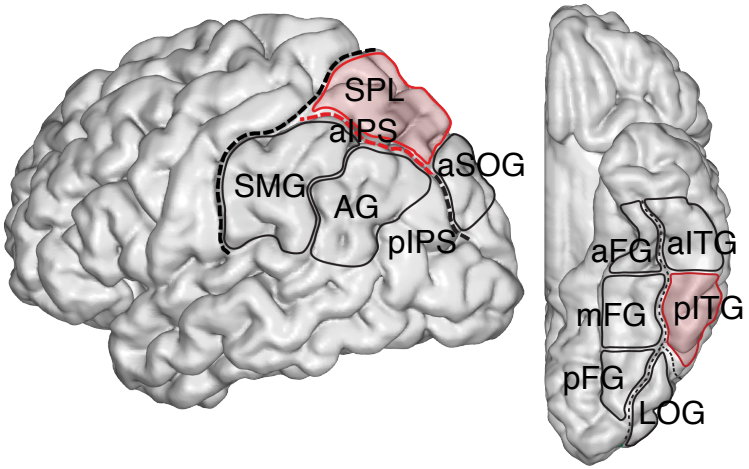
Rich frequency content



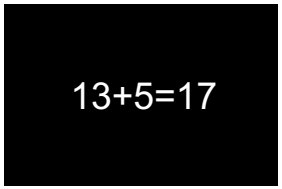
Task, ROIs, selectivity and problem-size effect

10 subjects with coverage in VTC and LPC

ROIs



Math

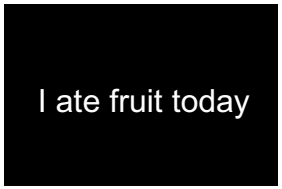


80 trials

Min op: 1:9

Max op: 10:87

Memory



control

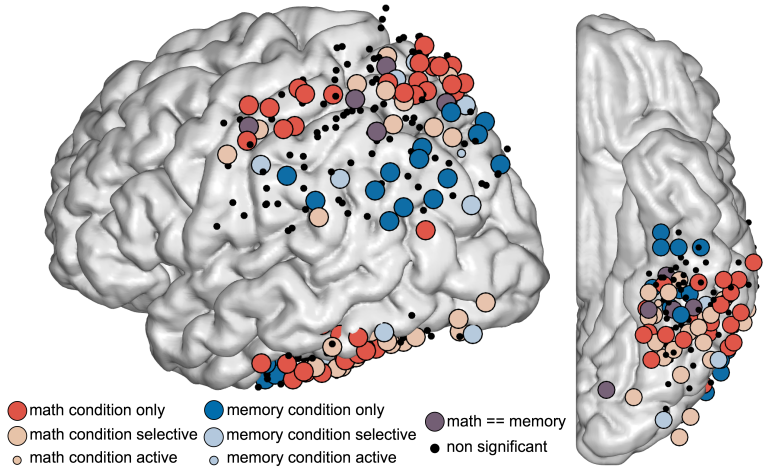
sentence comprehension

50 trials

Self-paced

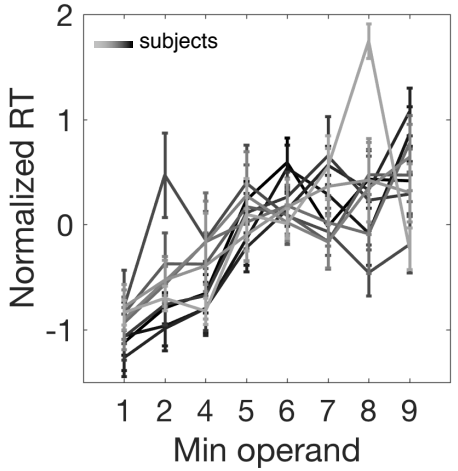
ITI 200 ms

Channel selectivity



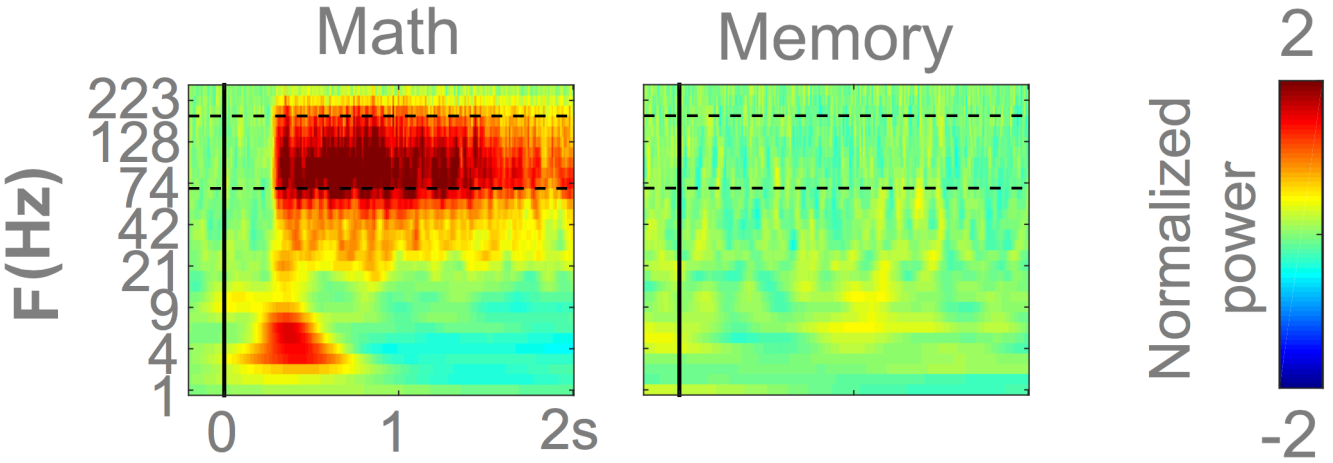
All sites from all subjects projected into a single left hemisphere

Problem-size effect



Best predictor of RT in 9/10 subjects

High frequency broadband (HFB): 70–180 Hz

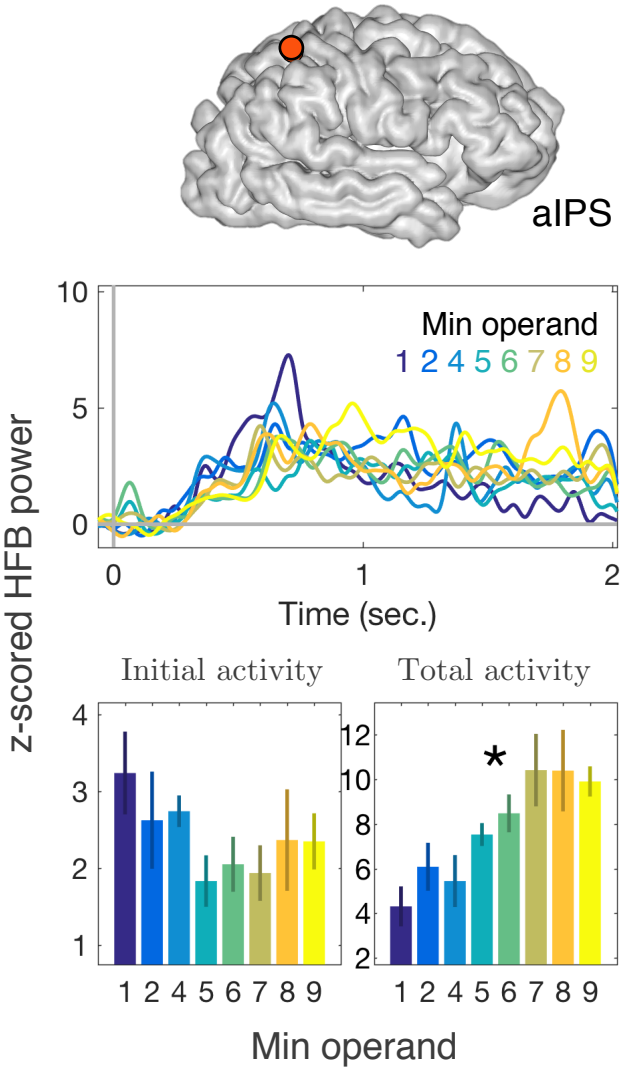


Exemplar aIPS channel

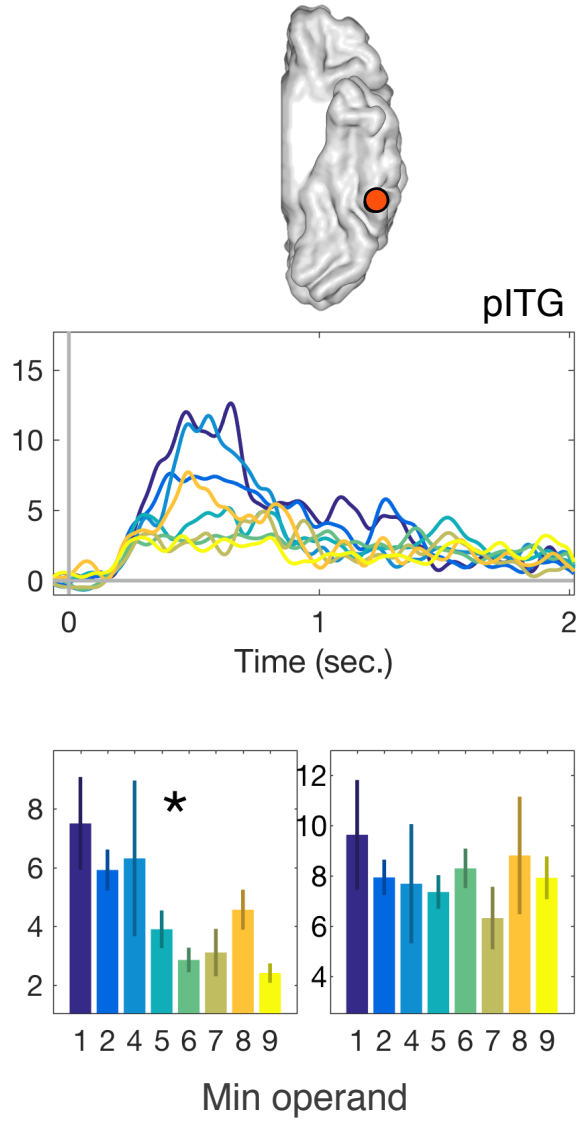
- **initial activity : 1st second**
  averaged over the 1st second, when greatest increase in activity occurs

- **total activity: integral**
  integral of the activity from trial onset to response: equivalent to BOLD

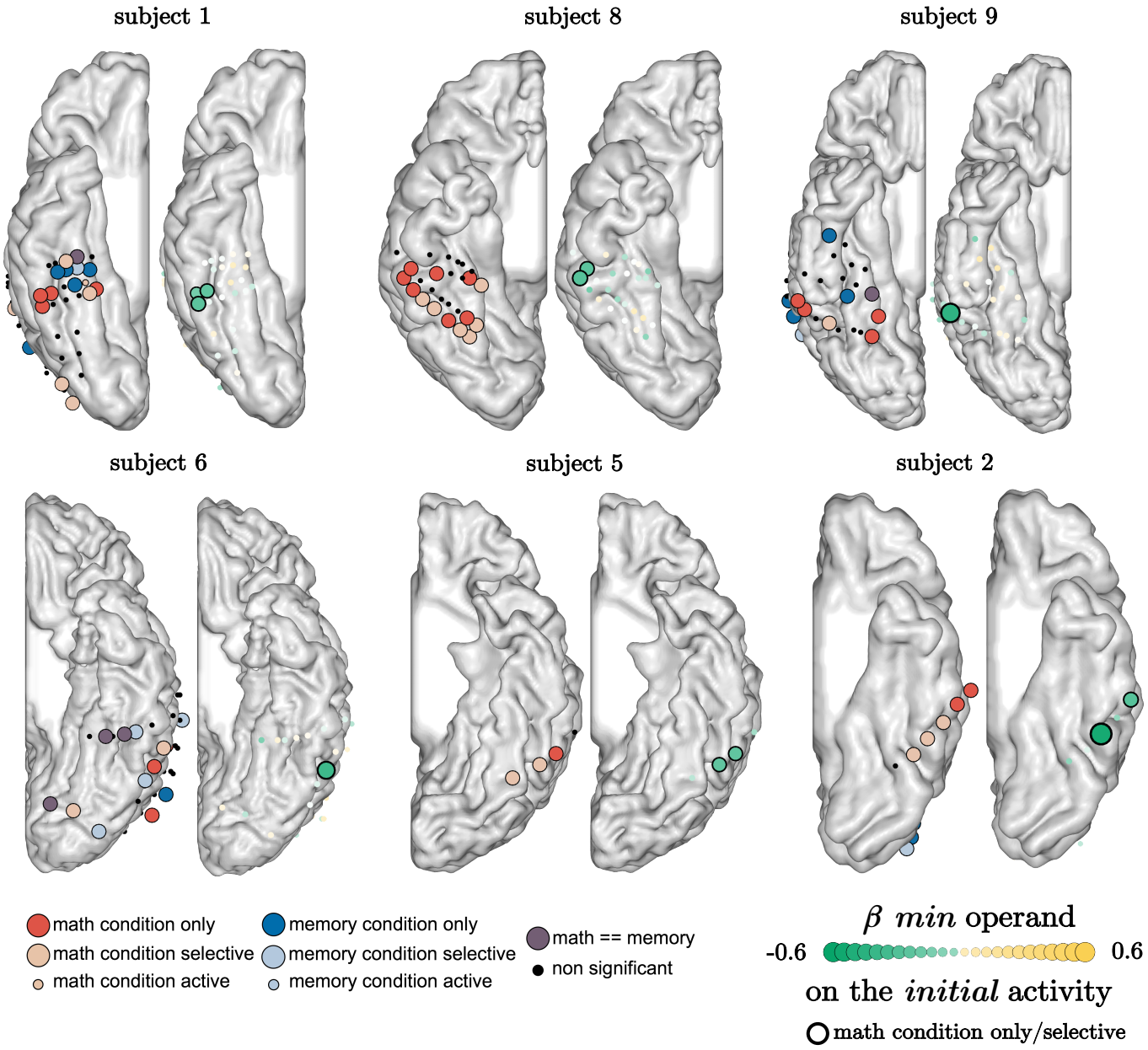
Increased *total* activity by *min* operand in aIPS and SPL



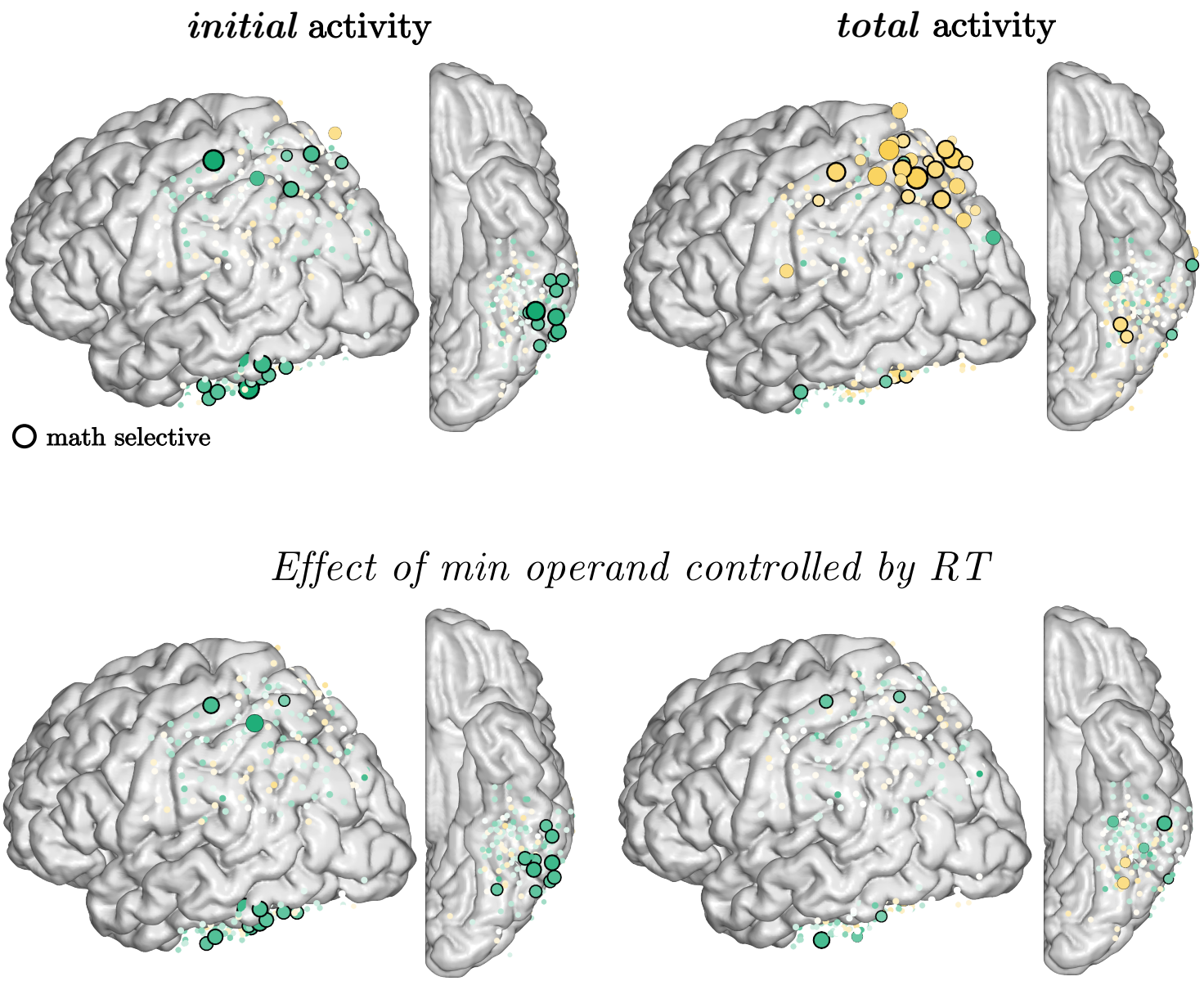
Decreased *initial* activity by *min* operand in pITG



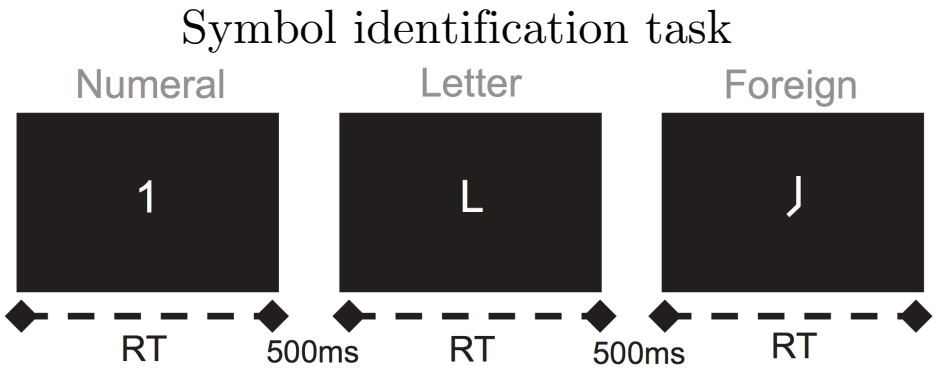
Anatomical and functional precision of the pITG modulation



Effect of the *min* operand on the:



Modulation of pITG does not reflect tuning to digits



Subject	Hemi	Region	Numeral active	Numeral selective	Modulation by numeral
S1	R	pITG	✓	-	-
S1	R	pITG	✓	-	-
S1	R	pITG	-	-	-
S2	L	pITG	✓	-	-
S2	L	aITG	-	-	-
S5	L	pITG	✓	-	-
S5	L	pITG	✓	-	-
S6	L	pITG	-	-	-
S8	R	pITG	✓	✓	-
S8	R	pITG	✓	-	-
S9	R	pITG	-	-	-

Hemi, hemisphere; Numeral active (relative to baseline); Numeral selective (relative to baseline, Latin letters, and foreign letters). (✓) Statistically significant at $P < 0.05$, FDR corrected.

- **9/10 pITG** sites are **not NFA.**
- **pITG modulation** is exclusive to the **calculation** task.

Chapter 3. **Discussion and conclusions**

- **Modulation of aIPS and SPL corroborates previous fMRI findings.**
  Increased total activity as a function of problem-size.
  Slow and sustained – highly correlated with RT.
  Index the accumulation of evidence to achieve a decision.

- **Surprising role of pITG in mental arithmetic beyond digit recognition.**
  Decreased initial activity as a function of problem-size: blind to fMRI.
  Fast - not correlated with RT.
  Early identification of problem difficulty (amount of evidence?).

- Re-evaluation of neurocognitive models of arithmetic and dyscalculia.
  pITG as an **important hub** for **calculation**.

- **Expands the classical view of the VTC**: contains regions specifically involved in sophisticated symbolic forms of reasoning, such as mental arithmetic.

Pinheiro-Chagas, Daitch*, Parvizi & Dehaene (under review)*

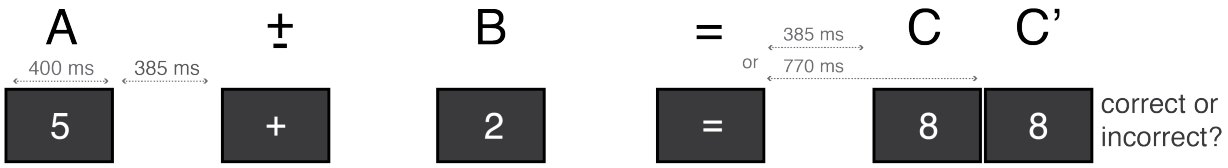
Chapter 4. **Decoding the processing stages of mental arithmetic with MEG**

- New insights on the neurocognitive mechanisms of mental calculations discovered in the previous studies.
- Still lacking a comprehensive picture of the organization of brain processes.
- Time-resolved multivariate pattern analysis applied to MEG.
- Can we decode the identity of the operands?
- Can we track in time the emergence of the internally computed result?
- Are the brain processes completely serial or do they partially overlap in a form of a cascade of computations that can be simultaneously decoded?

Pinheiro-Chagas, Piazza & Dehaene (under review)

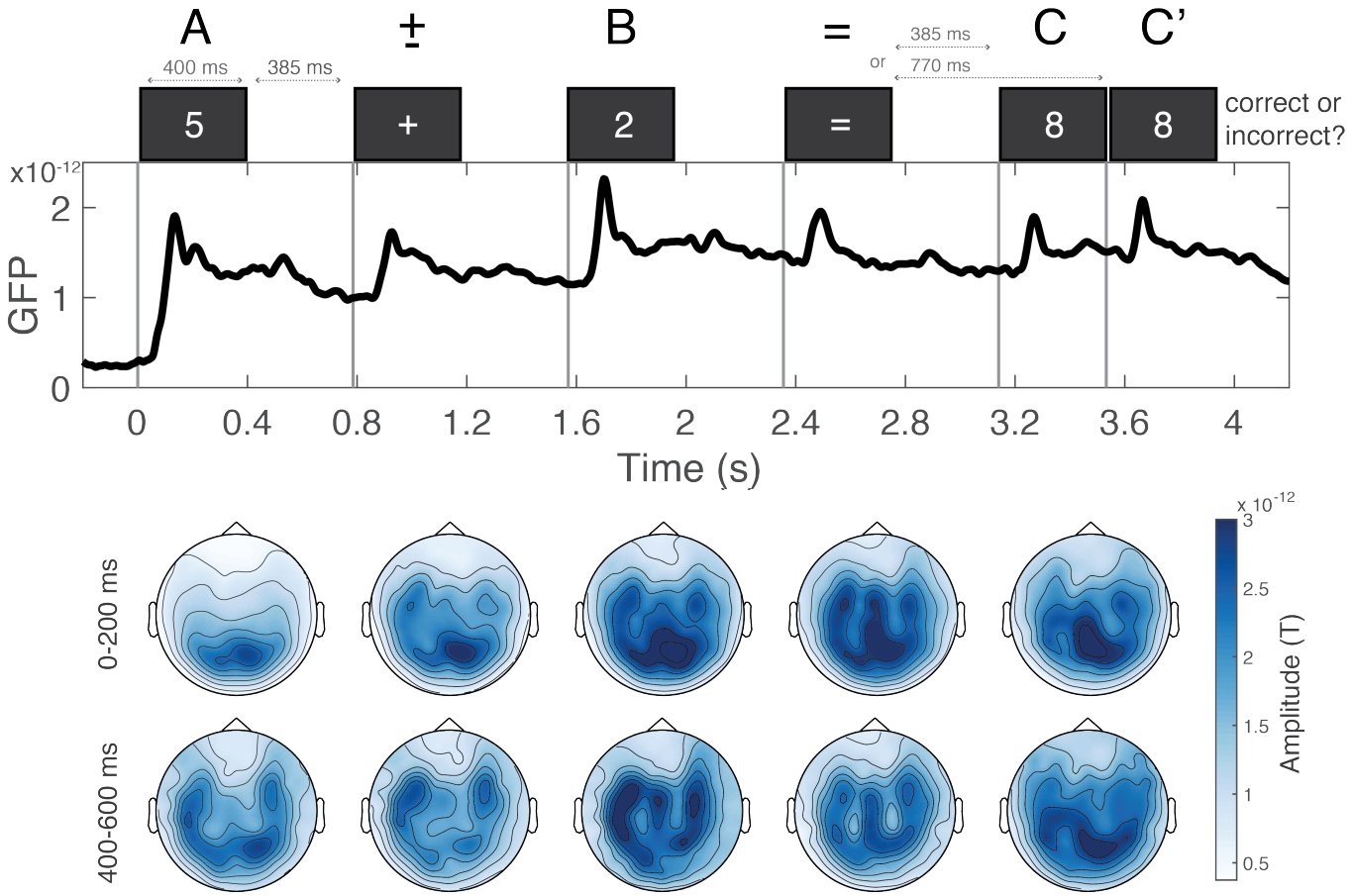
Experimental Design

20 adults, students, right-handed

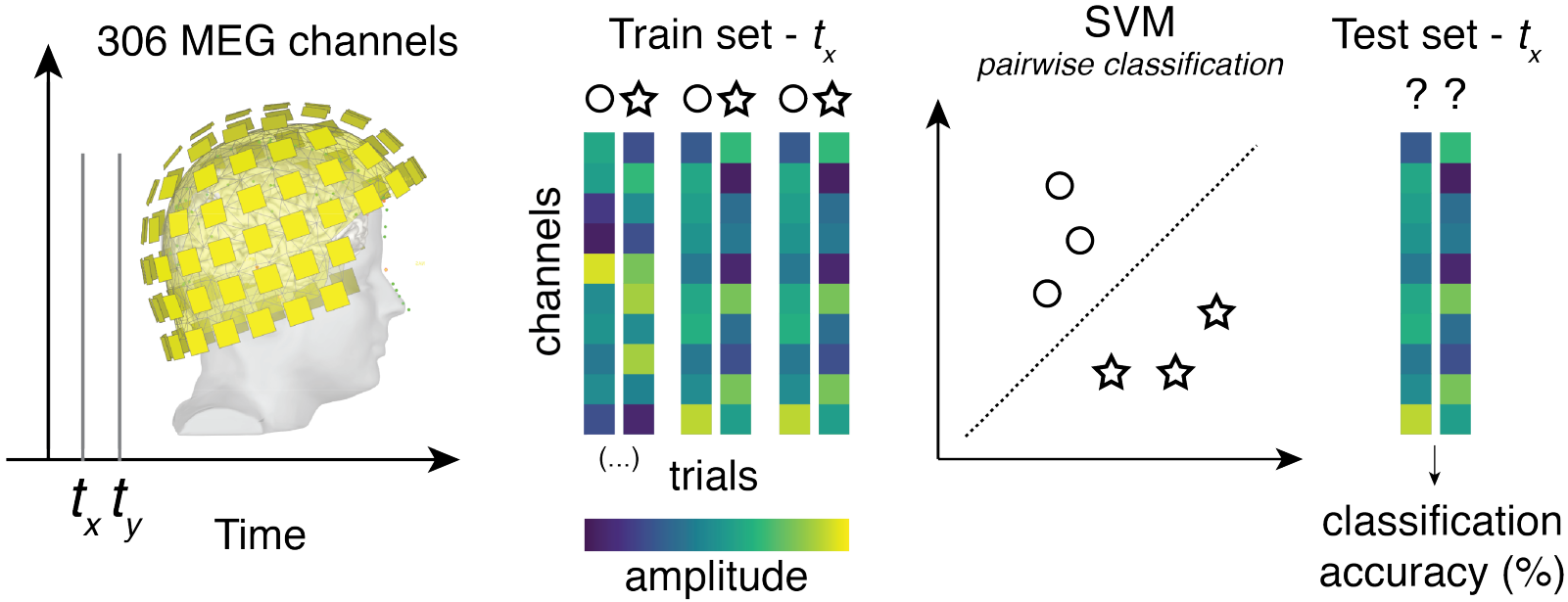


- **A -** *max* operand = **3, 4, 5, 6**
- **B -** *min* operand = **0, 1, 2, 3**
- **Result = 0 – 9** (**3, 4, 5, 6 -** 50 trials each)
- **C** – proposed result = **0 – 9**
  50% incorrect (absolute distance: 1, 2, 3, 4)

Sustained activation from posterior to anterior sensors



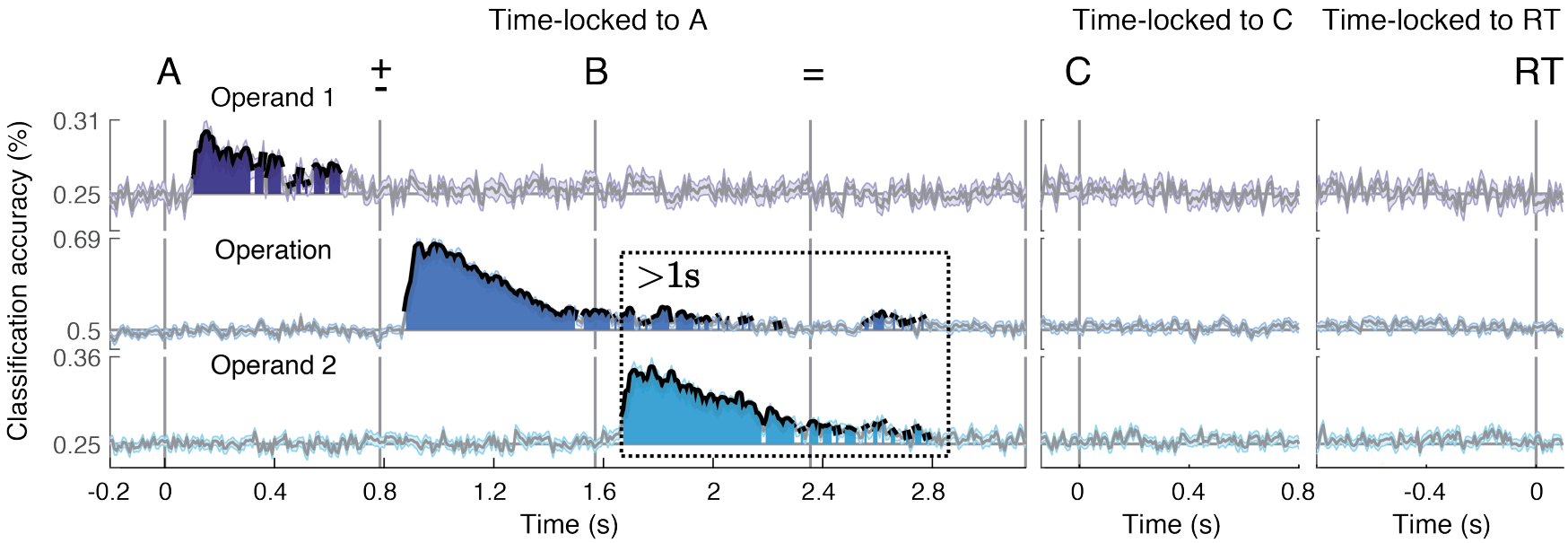
Time-resolved decoding



- The classifier is applied at each time sample $\boldsymbol{t_x}$, $\boldsymbol{t_y}$ ... , *per subject.*
- Generalization across time: can the classifier trained at $\boldsymbol{t_x}$ generalize to $\boldsymbol{t_y}$?
- Generalization across conditions: test the existence of possible common codes.

(King & Dehaene, 2014)

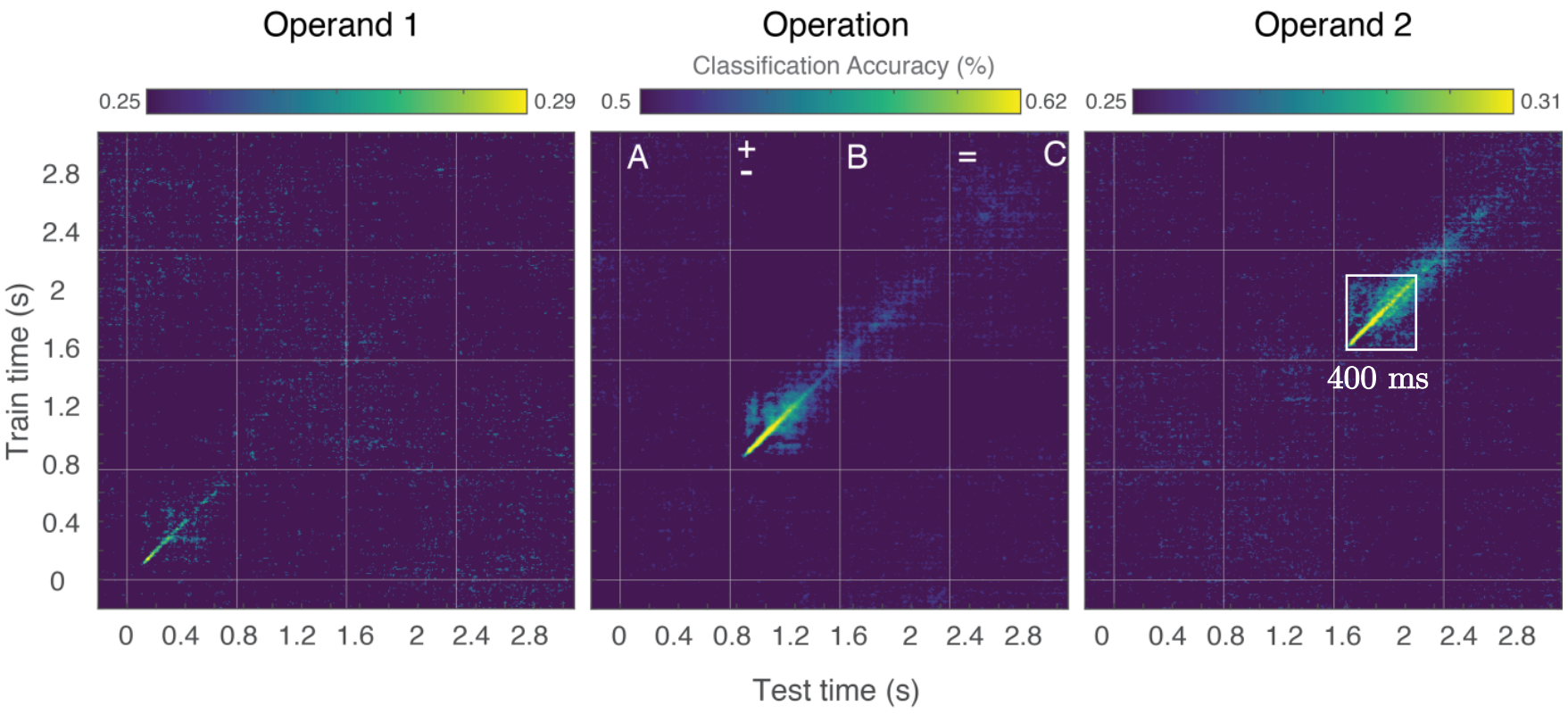
Decoding operands and operation



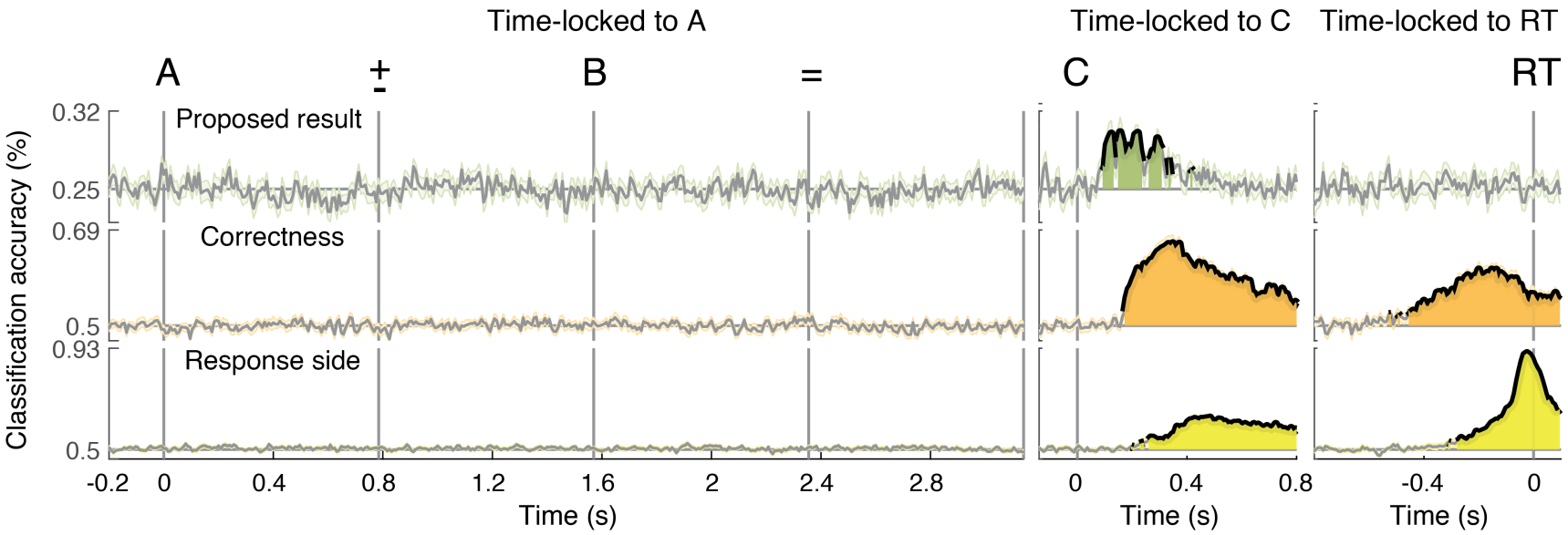
- **Operand 1**: **transient**, rapidly decreases after stim offset.
- **Operand 2: higher** accuracy and for a **longer** period – 1s.
- **Operation** *(sub vs. add)*: **sustained** for 2s, rebound after equal sign.
- Long **overlap** between **operation** and **operand 2** – 1s.

- Classifier trained on **sub** vs. **add** generalized to **operand 2** (**0-1** vs. **2-3**), not to operand 1.

Short time generalization: highly dynamic coding scheme

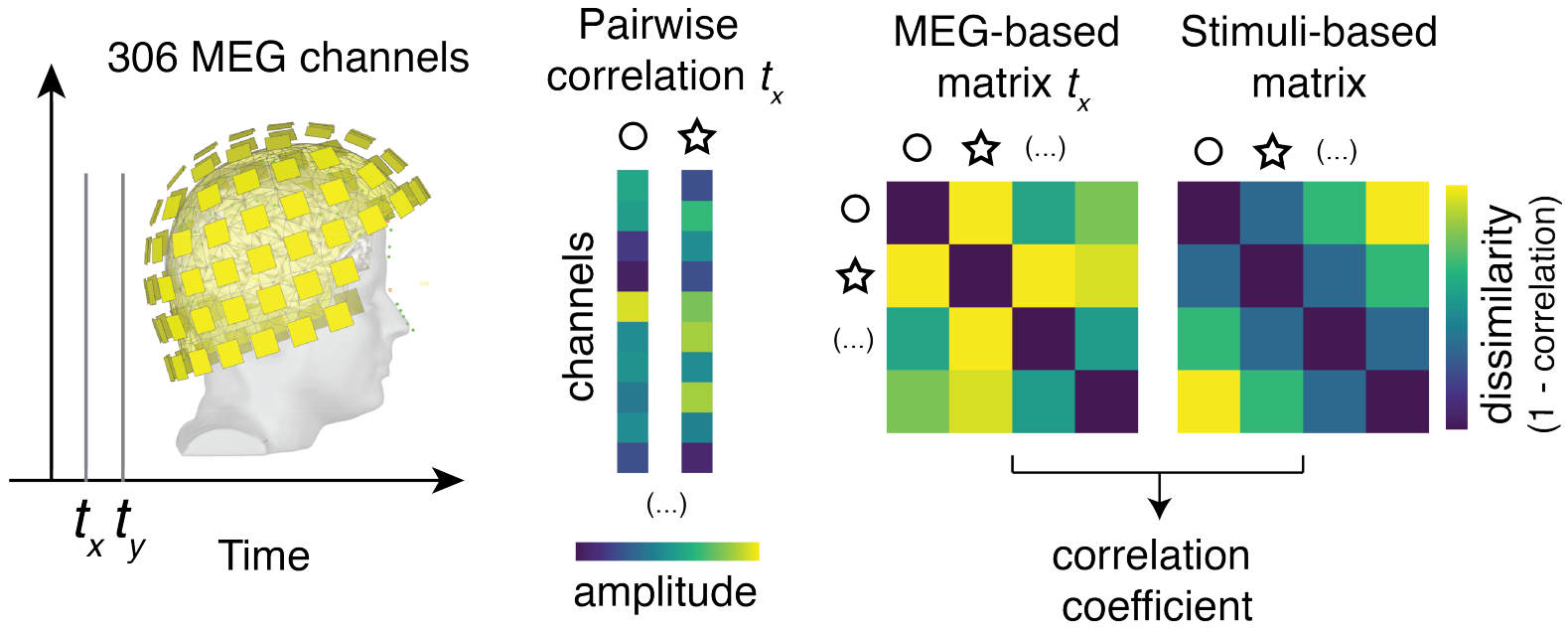


Highly overlapping dynamics at the decision stage



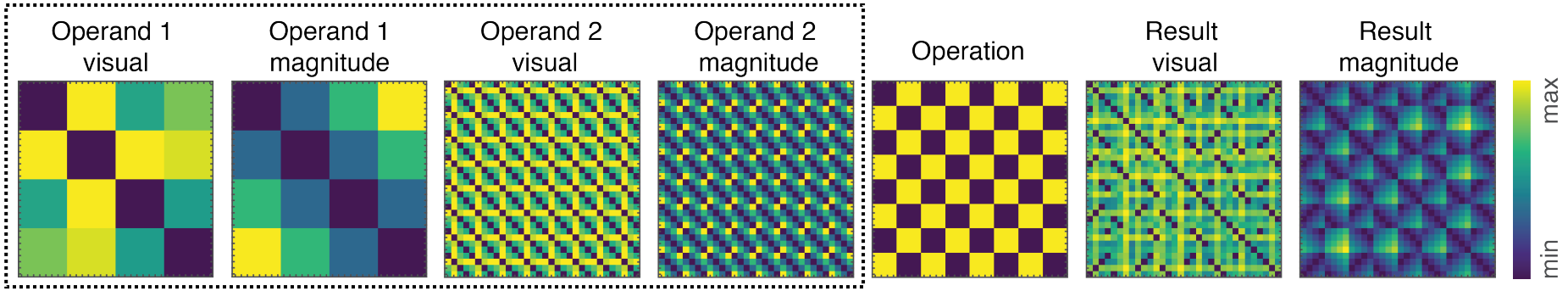
1. Identify the proposed result.
2. Judge whether it is correct or incorrect.
3. Press the response button.

Time-resolved Representational Similarity Analysis

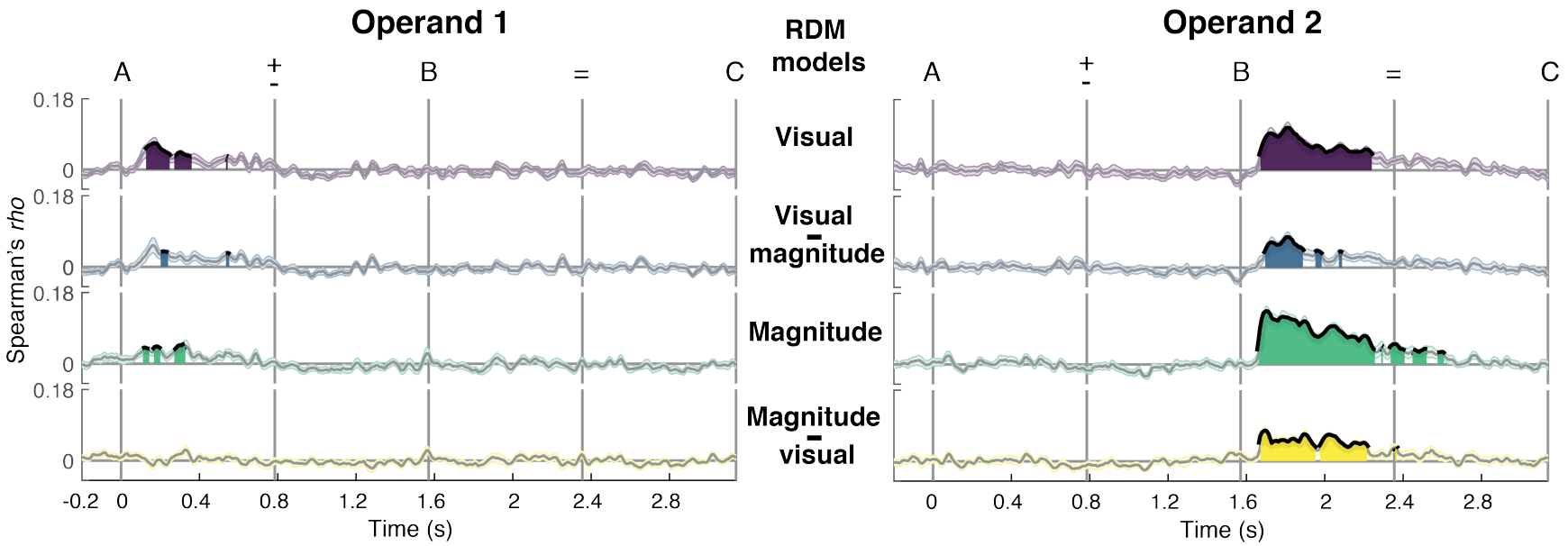


- The RSA is computed at each time sample $\boldsymbol{t_x}$, $\boldsymbol{t_y}$ ... , *per subject.*
- Explicitly partial-out the effect of other stimuli-based models.

RDM models:



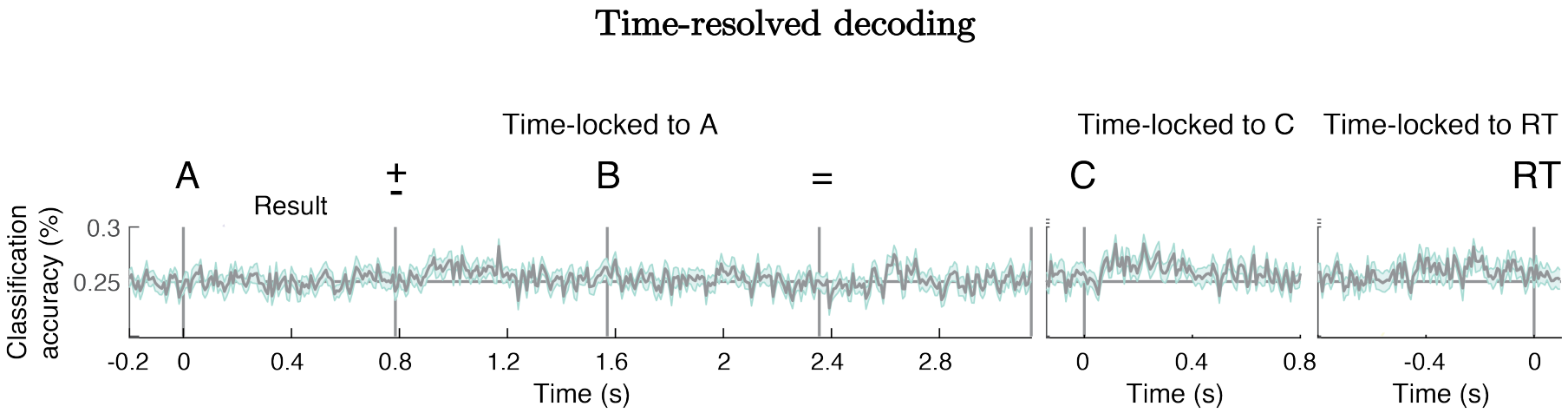
Representational geometries of the operands



- **Operand 1**: **visual dimension** is dominant.
- **Operand 2**: both visual and magnitude dimensions, **magnitude** dominant.
  No precedence of the visual dimension.

Searching for a neural signature of the internally computed result

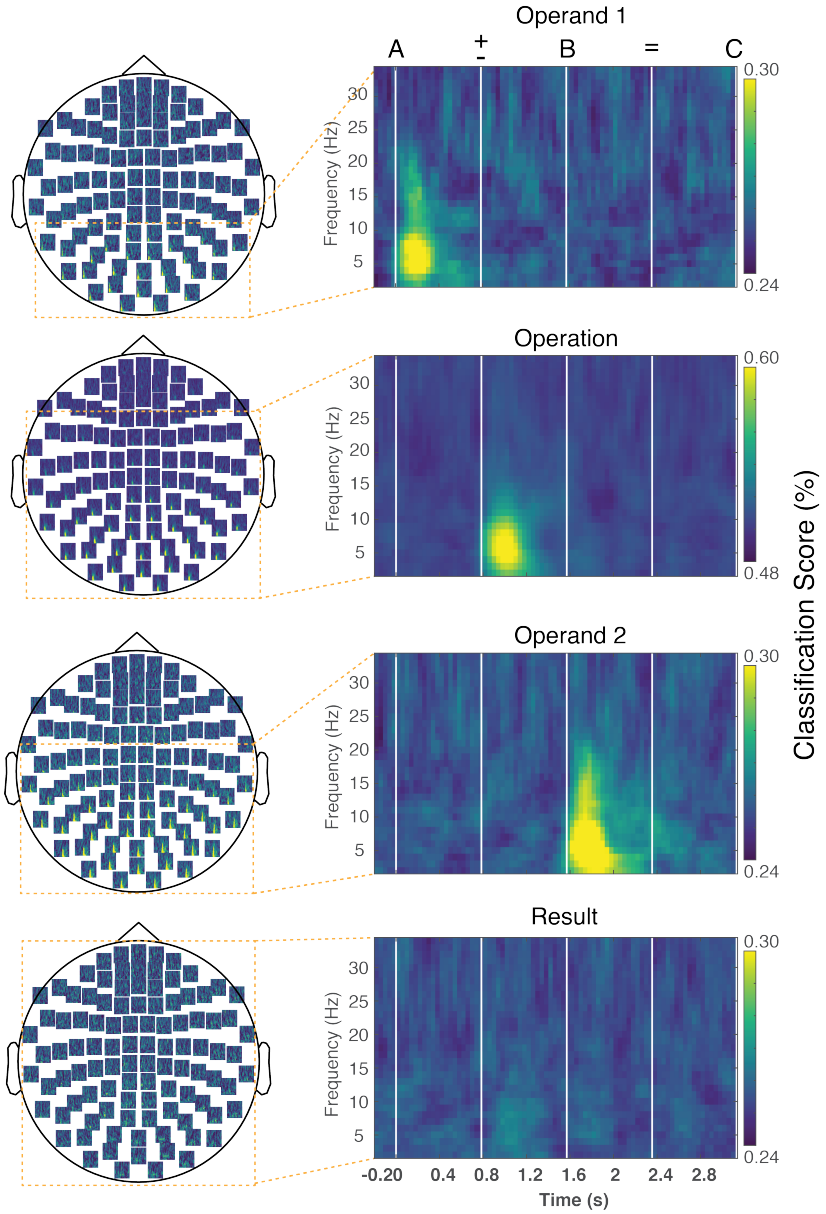
Several attempts, no success



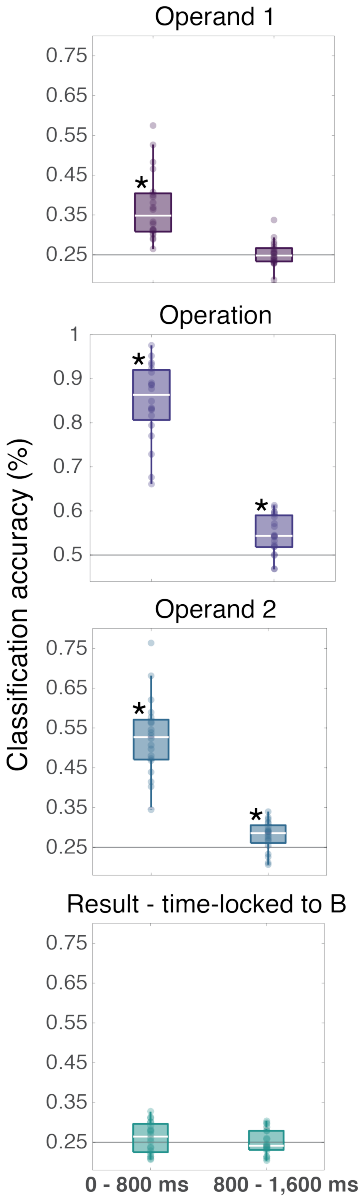
Searching for a neural signature of the internally computed result

Several attempts, no success

Searchlight LDA (sensor, time and frequency)



Riemannian Geometry (integrated over time)



Chapter 4. **Discussion and conclusions**

- First **comprehensive picture** of the **unfolding processing stages** underlying **arithmetic** calculations at a single-trial level.

- Decoding of **operand 1**: **visual dimension.**

- Decoding **operand 2**: visual and **magnitude** dimensions; possibly sharing a common code with operation type: link between SNARC and OM effects?

- **Fast** and **highly overlapping dynamics** at the **decision stage:**
  1. Identify the proposed result.
  2. Judge whether it is correct or incorrect.
  3. Press the response button.

- **Inability to decode the internally computed result**
  - Externally and internally generated codes have different neural substrates.
  - Internally generated: *'silent states'*, rare gamma bursts, sparse code? *Hard to capture with MEG.*
  - Limitations of the design (n_{trials}, no time-stamping, etc.).

Pinheiro-Chagas, Piazza & Dehaene (under review)

General conclusions and future directions

- **Single-digit addition and subtraction rely on quantity manipulation:**
  - Stepwise displacement on the mental number line.
  - Possible common code between *min* operand and operation type.
  - *How does it actually works? Computational modelling.*

- **Arithmetic is implemented in the dorsal and ventral pathways:**
  - **IPS** and **SPL**: arithmetic computations and decision-making.
  - **pITG**: digit recognition and early identification of problem difficulty.
  - *How they communicate? iEEG – postdoc.*

- **Decoding the processing stages of mental calculation:**
  - Cascade of highly dynamic and partially overlapping brain states.
  - **Operand 1: visual**; **Operand 2**: visual and **magnitude** codes.

- **Searching for neural signatures of the internally computed result.**
  - Inability to decode with MEG.
  - *When and where is it generated? iEEG – postoc.*

- **Beyond numbers: the syntactic structure of arithmetic expressions.**

Stay tuned for the next articles

Pinheiro-Chagas, P., Dotan, D., Piazza, M., Dehaene, S. (*in preparation*). Decomposing the syntactic structure of arithmetic expressions.

Dotan, D., **Pinheiro-Chagas, P.**, Dehaene, S. (*in preparation*) Track it to crack it: revealing the succession of processing stages with pointing trajectories.

Baek, S., Daitch, A., **Pinheiro-Chagas, P.**, Parvizi, J. (*under revision*). Neuronal population responses in the human ventral temporal and lateral parietal cortex during arithmetic processing with digits and number words.

Other articles produced during the dissertation

Dresler, T., Bugden, S., Gouet, C., Lallier, M., Oliveira, D., **Pinheiro-Chagas, P.**, Pires, A., Wang, Y., Zugarramurdi, C., Weissheimer, J. (*under review*). Translational research in learning disabilities: the place of neuroimaging.

Borghesani*, V., de Hevia*, L., Viarouge*, A., **Pinheiro-Chagas, P.**, Eger, E., Piazza, M. (*under review*). Processing number and length in the parietal cortex: sharing resources, not a common code.

Pinheiro-Chagas, P.*, Dinino, D.*, Haase, V. G., Wood, G., Knops, A. (*in preparation*) The developmental trajectory of the operational momentum effect.

Borghesani*, V., de Hevia*, L., Viarouge*, A., **Pinheiro-Chagas, P.**, Eger, E., Piazza, M. (2016). Comparing magnitudes across dimensions: a univariate and multivariate approach. International Workshop on Pattern Recognition in Neuroimaging, 1-4.

Pinheiro-Chagas, P. Wood, G., Knops, A., Krinzinger, H., Lonnemann, J., Starling-Alves, I., Willmes, K., Haase, V. G. (2014). In how many ways is the approximate number system associated with exact calculation? *PLoS One*, 19, 9(11), e111155.

Carvalho, M. R., Vianna, G., Oliveira, L., Costa, A. J., **Pinheiro-Chagas, P.**, Sturzenecker, R., Zen, P. R., Rosa, R. F., de Aguiar, M. J., Haase, V. G. (2014). Are 22q11.2 distal deletions associated with math difficulties? *American Journal of Medical Genetics Part A*, 164A(9), 2256-62.

Haase, V. G., Júlio-Costa, A., Lopes-Silva, J. B., Starling-Alves, I., Antunes, A. M., **Pinheiro-Chagas, P.**, Wood, G. (2014). Contributions from specific and general factors to unique deficits: two cases of mathematics learning difficulties. *Frontiers in Psychology*, 13, 5-102.

Moura, R., Wood, G., **Pinheiro-Chagas, P.**, Lonnemann, J., Krinzinger, H., Willmes, K., Haase, V. G. (2013). Transcoding abilities in typical and atypical mathematics achievers: the role of working memory and procedural and lexical competencies. *Journal of Experimental Child Psychology*, 116(3), 707-27

* The authors equally contributed to the work

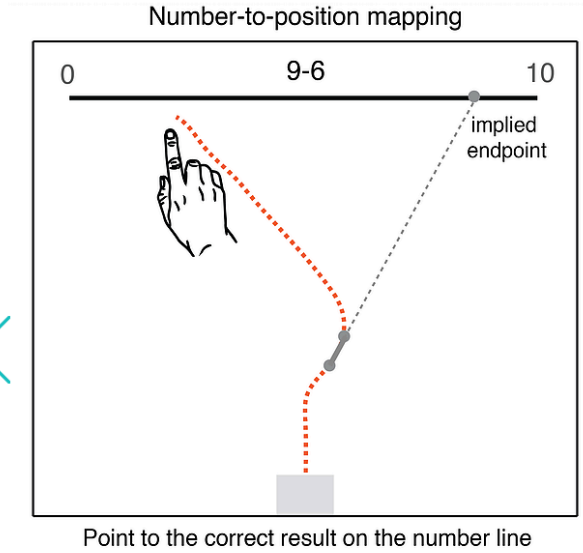
TrajTracker: track it to crack it

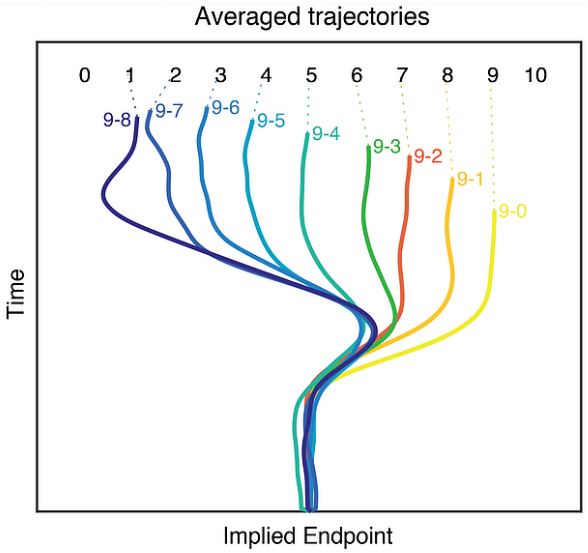
Platform for trajectory tracking experiments & data analysis

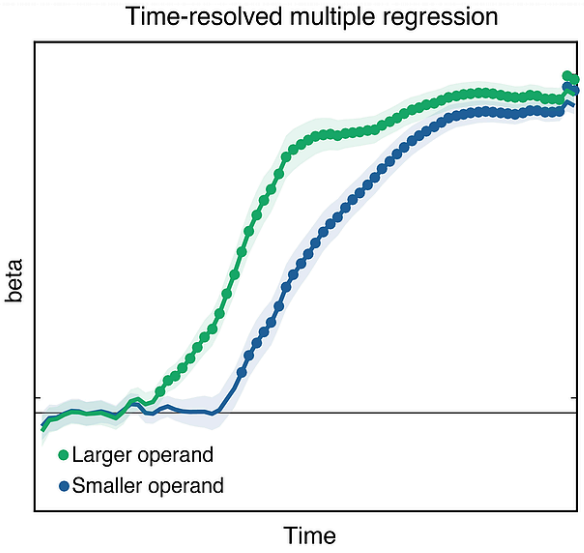
Home | Experiment | Paradigms | Analyze | About

TrajTracker Experiment 1.0 (beta version) is now released!

Android support will be available with the next release of Expyriment for Android.







tracking mental calculation online

TrajTracker is a software package for running and analyzing psychology experiments, focused on mouse/finger tracking experiments.

New to trajectory tracking? Read about it here.



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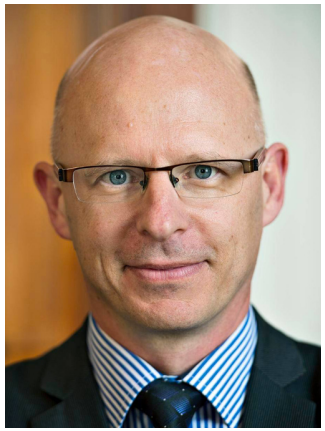

Stan Dehaene
Collège de France; NeuroSpin

Manuela Piazza
University of Trento

Dror Dotan
Tel Aviv University

Amy Daitch
Stanford University

Josef Parvizi
Stanford University