ISSN 1749-3889 (print), 1749-3897 (online)
International Journal of Nonlinear Science
Vol.13(2012) No.4,pp.387-395

Numerical Solution of Fokker-Planck Equation Using the Flatlet Oblique Multiwavelets

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(Received 10 December 2011, accepted 15 February 2012)

Abstract: A numerical technique is presented for the solution of Fokker- Planck equation. This method uses the the flatlet oblique multiwavelets. The method consists of expanding the required approximate solution as the elements of the flatlet oblique multiwavelets scaling and wavelet bases. Using the operational matrix of derivative, we reduce the problem to a set of algebraic equations. Some numerical example is included to demonstrate the validity and applicability of the technique. The method is easy to implement and produces very accurate results.

Keywords: Fokker-Planck equation; Flatlet oblique multiwavelets; Scaling functions

1 Introduction

The Fokker-Planck equation has wide applications in several branches of physics, chemistry and biology including solid-state physics, quantum optics, chemical physics, theoretical biology and circuit theory. A Fokker-Planck equation describes the change of probability of a random function in space and time; hence it is naturally used to describe solute transport. The Fokker-Planck equation was first used by Fokker and Planck (for instance, see [1]) to describe the Brownian motion of particles. If a small particle of mass m is immersed in a fluid, the equation of motion for the distribution function W(x, t) is given by:

$$\frac{\partial W}{\partial t} = \gamma \frac{\partial vW}{\partial v} + \gamma \frac{KT}{m} \frac{\partial^2 W}{\partial v^2} \tag{1.1}$$

where v is the velocity for the Brownian motion of a small particle, t is the time,γ is the fraction constant, K is Boltzmanns constant and T is the temperature of fluid [1]. Eq. (1.1) is one of the simplest type of Fokker-Planck equations. By solving Eq.(1.1) starting with distributation function $W(x,t)$ for $t = 0$ and subject to the appropriate boundary conditions, one can obtain the distributation function $W(x,t)$ for $t > 0$.

The general Fokker-Planck equation for the motion of a concentration field $u(x,t)$ of one space variable x at time t has the form [1-5]

$$\frac{\partial u}{\partial t} = \left[-\frac{\partial}{\partial x} A(x) + \frac{\partial^2}{\partial x^2} B(x) \right] u, \tag{1.2}$$

with the initial condition

$$u(x,0) = f(x), \qquad x \in \mathbb{R} \tag{1.3}$$

where $u(x,t)$ is unknown, $B(x) > 0$ is the diffusion coefficient and $A(x) > 0$ is the drift coefficient. The drift and diffusion coefficients may also depend on time,i.e.

$$\frac{\partial u}{\partial t} = \left[-\frac{\partial}{\partial x} A(x,t) + \frac{\partial^2}{\partial x^2} B(x,t) \right] u. \tag{1.4}$$

Eq.(1.1) is seen to be a special case of the Fokker-Planck equation where the drift coefficient is linear and the diffusion coefficient is constant. Eq.(1.2) is an equation of motion for the distribution function $u(x,t)$. Mathematically, this equation is a linear second order partial differential equation of parabolic type. Roughly speaking, it is a diffusion equation

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IJNS.2012.06.30/619

with an additional first order derivative with respect to x. In the mathematical literature, Eq.(1.2) is also called forward Kolmogorov equation. The similar partial differential equation is a backward Kolmogorov equation that is in the form: [1]

$$\frac{\partial u}{\partial t} = \left[-A(x,t)\frac{\partial}{\partial x} + B(x,t)\frac{\partial^2}{\partial x^2} \right] u. \tag{1.5}$$

A generalization of Eq. (1.2) to N variables $x_1, \ldots, x_N$ has the form:

$$\frac{\partial u}{\partial t} = \left[-\sum_{i=1}^{N} \frac{\partial}{\partial x_i} A_i(X) + \sum_{i,j=1}^{N} \frac{\partial^2}{\partial x_i \partial x_j} B_{i,j}(X) \right] u, \tag{1.6}$$

with the initial condition

$$u(X,0) = f(X), \qquad X \in \mathbb{R}^N, \tag{1.7}$$

where $X = (x_1, \ldots, x_N)$. The drift vector A_i and diffusion tensor $B_{i,j}$ generally depend on the N variables $x_1, ..., x_N$.

One may find analytical solutions of the Fokker-Planck equation. Generally, however, it is difficult to obtain solutions, especially if no separation of variables is possible or if the number of variables is large.

Various methods of solution are: simulation methods, transformation of a Fokker-Planck equation to a Schrödinger equation, numerical integration methods and etc. [1].

There is a more general form of Fokker-Planck equation. Nonlinear Fokker-Planck equation has important applications in various areas such as plasma physics, surface physics, population dynamics, biophysics, engineering, neurosciences, nonlinear hydrodynamics, polymer physics, laser physics, pattern formation, psychology and marketing (see [4-5] and references therein). In one variable case the nonlinear FokkerPlanck equation is written in the following form:

$$\frac{\partial u}{\partial t} = \left[-\frac{\partial}{\partial x} A(x,t,u) + \frac{\partial^2}{\partial x^2} B(x,t,u) \right] u. \tag{1.8}$$

For N variables $x_1, ..., x_N$, it has the form:

$$\frac{\partial u}{\partial t} = \left[-\sum_{i=1}^{N} \frac{\partial}{\partial x} A_i(X,t,u) + \sum_{i,j=1}^{N} \frac{\partial^2}{\partial x_i \partial x_j} B_{i,j}(X,t,u) \right] u, \tag{1.9}$$

where $X = (x_1, ..., x_N)$. Notice that when $A_i(X,t,u) = A_i(X)$ and $B_{i,j}(X,t,u) = B_{i,j}(X)$ the nonlinear Fokker-Planck equation (1.9) reduces to the linear Fokker-Planck equation (1.6).

In this work we reduce the problem to a set of algebraic equations by expanding the unknown function as the flatlet oblique multiwavelets, with unknown coefficients. The operational matrix of derivative are given. This matrix together the the flatlet oblique multiwavelets are then utilized to evaluate the unknown coefficients.

This paper is organized as follows: In Section 2, we describe the formulation of the flatlet oblique multiwavelets on $[0, 1]$, and construct dual functions and then derive the operational matrices of derivative and integral required for our subsequent development. In Section 3 the proposed method is used to approximate the solution of the problem in interval [0,1] for variables x and t. As a result a set of algebraic equations are formed and a solution of the considered problem is introduced. In Section 4, we report our computational results and demonstrate the accuracy of the proposed numerical scheme by presenting numerical examples. Section 5, ends this paper with a brief conclusion. Note that we have computed the numerical results by Maple programming.

2 Flatlet oblique multiwavelets System

A flatlet multiwavelet system [7-8] with multiplicity $m+1$ consists of $m+1$ scaling functions and $m+1$ wavelets defined on $[0, 1]$. The simplest example ($m = 0$) for the flatlet family is identical to the Haar wavelets. To construct higher order flatlet multiwavelet system, we can follow the same procedures as Haar wavelets. The scaling functions in this system are defined as a set of m + 1 unit constant functions $\phi_0(x), \ldots, \phi_m(x)$ divided equally into $m + 1$ intervals on $[0, 1]$ by

$$\phi_i(x) = \begin{cases} 1, & \frac{i}{m+1} < x < \frac{i+1}{m+1}, \\ 0, & \text{otherwise.} \end{cases} \qquad i = 0, 1, \ldots, m. \tag{2.10}$$

Let $m+1$ functions $\psi_0(x),\ldots,\psi_m(x)$ be flatlet wavelets corresponding to flatlet scaling functions defined on $[0,1]$. We construct corresponding wavelets by using a two-scale relation which will be introduced next. First for simplicity, we put flatlet scaling functions and wavelets into two vector functions

$$\Phi(x)=\begin{bmatrix}\phi_0(x)\\ \vdots\\ \phi_m(x)\end{bmatrix},\ \ \Psi(x)=\begin{bmatrix}\psi_0(x)\\ \vdots\\ \psi_m(x)\end{bmatrix}. \tag{2.11}$$

Now the two-scale relations for the flatlet multiwavelet system may be expressed as

$$\Phi(x)=\mathbf{P}\begin{bmatrix}\Phi(2x)\\ \Phi(2x-1)\end{bmatrix},\ \ \Psi(x)=\mathbf{Q}\begin{bmatrix}\Phi(2x)\\ \Phi(2x-1)\end{bmatrix}, \tag{2.12}$$

where $\mathbf{P}$ and $\mathbf{Q}$ are $(m+1)\times 2(m+1)$ matrices. Rewriting the two-scale relations (2.12) in the matrix form, yields

$$\begin{bmatrix}\Phi(x)\\ \Psi(x)\end{bmatrix}=\begin{bmatrix}\mathbf{P}\\ \mathbf{Q}\end{bmatrix}\begin{bmatrix}\Phi(2x)\\ \Phi(2x-1)\end{bmatrix}, \tag{2.13}$$

which is called the reconstruction relation. Also the coefficients matrix in (2.13) is called reconstruction matrix(RCM) which is invertible. Because of the simplicity of the flatlet scaling functions, the matrix $\mathbf{P}$ in the two-scale relations (2.12) is obtained as

$$\mathbf{P}=\begin{bmatrix}1 & 1 & & & & 0\\ & & 1 & 1 & & \\ & & & & \ddots & \\ 0 & & & & 1 & 1\end{bmatrix}, \tag{2.14}$$

For computing the $2(m+1)^2$ entries of matrix $\mathbf{Q}$ we need $2(m+1)^2$ independent conditions. There are many possibilities in choosing the conditions to be used that result different flatlet multiwavelet systems with different properties. In this sequel, we use the $\frac{(m+1)(m+2)}{2}$ orthonormality conditions

$$\int_0^1 \psi_i(x)\psi_j(x)dx=\delta_{i,j},\quad i,j=0,1,\ldots,m. \tag{2.15}$$

and also $\frac{(m+1)(m+2)}{2}$ vanishing moment conditions

$$\int_{-\infty}^{\infty} \psi_i(x)x^j dx=0,\quad i=0,1,\ldots,m,\ \ j=0,1,\ldots,m+i. \tag{2.16}$$

By using (2.10) and (2.12), equation (2.16) can be written to the following system of linear equations

$$\sum_{k=0}^{2(m+1)}\{(k+1)^{j+1}-(k)^{j+1}\}q_{j,k}=0,\quad j=0,1,\ldots,m+i. \tag{2.17}$$

By solving (2.15) and (2.17), the unknown matrix $\mathbf{Q}$ and so $\Psi(x)$ are obtained.(for first and second flatlet basis functions see [7]).

2.1 Biorthogonal Flatlet Multiwavelet System

Let $\tilde{\Phi}(x)$ and $\tilde{\Psi}(x)$ be dual scaling and wavelet vector functions in biorthogonal flatlet multiwavelet system(BFMS), respectively as

$$\tilde{\Phi}(x)=\begin{bmatrix}\tilde{\phi}_0(x)\\ \vdots\\ \tilde{\phi}_m(x)\end{bmatrix},\ \ \tilde{\Psi}(x)=\begin{bmatrix}\tilde{\psi}_0(x)\\ \vdots\\ \tilde{\psi}_m(x)\end{bmatrix}. \tag{2.18}$$

Note that according to the biorthogonality conditions in BFMS we must have

$$\prec \tilde{\phi}_i, \phi_j \succ = \int_0^1 \tilde{\phi}_i(x)\phi_j(x)dx = \delta_{i,j}, \tag{2.19}$$

$$\prec \tilde{\psi}_i, \psi_j \succ = \int_0^1 \tilde{\psi}_i(x)\psi_j(x)dx = \delta_{i,j}, \tag{2.20}$$

$$\prec \tilde{\psi}_i, \phi_j \succ = \int_0^1 \tilde{\psi}_i(x)\phi_j(x)dx = 0, \quad i,j = 0,1,\ldots,m. \tag{2.21}$$

Now we define $\tilde{\phi}_i(x)$ and $\tilde{\psi}_i(x),\;\; i = 0,1,\ldots,m$ as polynomials and piecewise polynomials of degree m and $m+1$ respectively, by

$$\tilde{\phi}_i(x) = \begin{cases} a_{i1} + a_{i2}x + \ldots + a_{i,m+1}x^m, & 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases} \tag{2.22}$$

$$\tilde{\psi}_i(x) = \begin{cases} b^1_{i1} + b^1_{i2}x + \ldots + b^1_{i,m+1}x^m, & 0 \le x < \frac{1}{2}, \\ b^2_{i1} + b^2_{i2}x + \ldots + b^2_{i,m+1}x^m, & \frac{1}{2} \le x < 1, \\ 0, & \text{otherwise.} \end{cases} \tag{2.23}$$

Based on biorthogonality conditions (2.19), we show that coefficients $a_{i,j},\;\; i = 0,1,\ldots,m,\;\; j = 1,\ldots,m+1$, $b^1_{i,j}$ and $b^2_{i,j}, i = 0,1,\ldots,m,\;\; j = 1,\ldots,m+1$, in (2.22) and (2.23) are uniquely determined.

Lemma 1 *(See lemma 3.1 in [7]) Let $A = [a_{i,j}]_{n\times n}$ be a square matrix with $a_{i,j} = p_{i-1}(j)$, a polynomial of exact degree $i-1$, then A is invertible.*

Theorem 2 *(See theorem 3.2 in [7]) For oblique flatlet multiwavelets, The dual functions defined in (2.22)-(2.23) are uniquely determined. (For first and second flatlet basis dual functions see [7].)*

2.2 The Operational Matrix of Derivative

A function $f(x)$ defined in $[0,1]$ may be approximated by the flatlet multiwavelets [8] as

$$f(x) \simeq \Theta^T.f, \tag{2.24}$$

or

$$f(x) \simeq \tilde{\Theta}^T.\tilde{f}, \tag{2.25}$$

where

$$\Theta(x) = \begin{bmatrix} \phi_0(x) \\ \vdots \\ \phi_m(x) \\ \psi_0(x) \\ \vdots \\ \psi_i(2^J x - k) \\ \vdots \\ \psi_m(2^J x - 2^J + 1) \end{bmatrix}, \;\; \tilde{\Theta}(x) = \begin{bmatrix} \tilde{\phi}_0(x) \\ \vdots \\ \tilde{\phi}_m(x) \\ \tilde{\psi}_0(x) \\ \vdots \\ \tilde{\psi}_i(2^J x - k) \\ \vdots \\ \tilde{\psi}_m(2^J x - 2^J + 1) \end{bmatrix}. \tag{2.26}$$

and $f,\;\; \tilde{f}$ are N-vectors as

$$f = [c'_0, \ldots, c'_m, d_{0,0,0}, \ldots, d_{i,l,k}, \ldots, d_{m,J,2^J-1}]$$

$$\tilde{f} = [\tilde{c}'_0, \ldots, \tilde{c}'_m, \tilde{d}_{0,0,0}, \ldots, \tilde{d}_{i,l,k}, \ldots, \tilde{d}_{m,J,2^J-1}]$$

in which $N = 2^J(m+1)$. Also, a two variable function $g(x,z)$ can be approximated by flatlet multiwavelets [8] as

$$g(x,t) \simeq \tilde{\Theta}^T(t).\,\mathbf{G}.\Theta(x), \tag{2.27}$$

where

$$[\mathbf{G}]_{i,j} = \int_0^1 \int_0^1 g(x,z)\tilde{\theta}_i(x)\theta_j(z)dxdz, \;\; i,j = 1,2,\ldots,N, \tag{2.28}$$

Note that $\theta_i(x)$ and $\tilde{\theta}_i(x), i = 1,2,...,N$, are ith component of $\Theta(x)$ and $\tilde{\Theta}(x)$, respectively.

Next, we use representation (2.25) for approximating the unknown function. In fact, these basis functions use piecewise polynomials of degree m and have higher order approximations comparing with flatlet multiwavelets.

2.2.1 The OMD

In this method, it is crucial to express the expansion of $f'(x)$ in terms of coefficients of the expansion of $f(x)$. This can be done by using the OMD. By considering Equation (2.25) let

$$f'(x) \simeq \tilde{\Theta}^T.\mathbf{D}.f. \tag{2.29}$$

So, by using OMD, we can obtain the coefficients of expansion $f'(x)$ from the coefficients of expansion $f(x)$. Let $d_{i,j}$ be the i,jth entry of $\mathbf{D}$. If so, by using Equations (2.29), we have

$$d_{i,j} = \int_0^1 \theta_i(x)(\frac{d}{dx}\tilde{\theta}_j(x))dx, \;\; i,j = 1,2,\ldots,N. \tag{2.30}$$

Also, by using Equations (2.29) we can obtain the kth derivative of f in the expansion (2.25) as

$$f(x) \simeq \tilde{\Theta}^T.\mathbf{D}^{\mathbf{k}}.f, \tag{2.31}$$

According to Equation (2.26),

$$\theta_i(x) = \begin{cases} \phi_{i-1}(x), & i = 1,2,\ldots,m+1, \\ \psi_{j'}(2^n x - l), & i = m+2,\ldots,N. \end{cases}$$
$$\tilde{\theta}_i(x) = \begin{cases} \tilde{\phi}_{i-1}(x), & i = 1,2,\ldots,m+1, \\ \tilde{\psi}_{j'}(2^n x - l), & i = m+2,\ldots,N. \end{cases} \tag{2.32}$$

where

$$\begin{aligned} & j' = (i-1)\text{mod}(m+1) \\ & n = [log_2 k], \\ & k = [\frac{i-1}{m+1}], \\ & l = k - 2^n, \end{aligned} \tag{2.33}$$

and $[x]$ is the greatest integer less than or equal to x. Note that $\theta_i(x)$'s and $\tilde{\theta}_i(x)$'s are piecewise differentiable functions. For obtaining the elements of OMD, we consider two following cases.

Case 1 $i = 1,\ldots,m+1$. In this case from Equation (2.32) we have

$$\theta_i(x) = \phi_{i-1}(x). \tag{2.34}$$

It is shown [7] that $d_{i,j}$ is the form

$$d_{i,j} = \tilde{\phi}_j(x)|_{\Omega_{j,1}}, \quad i = 1,2,\ldots,m+1, \tag{2.35}$$

where

$$\Omega_{j,1} = [\frac{i-1}{m+1}, \frac{i}{m+1}] \bigcap E_j \tag{2.36}$$

and

$$E_j = \mathbf{supp}\, \tilde{\theta}_j(x), \quad j = 1,2,\ldots,N \tag{2.37}$$

Case 2 $i = m+2,\ldots,N$. In this case from Equation (2.32) we have

$$\theta_i(x) = \psi_{j'}(2^n x - l). \tag{2.38}$$

It is shown [7] that $d_{i,j}$ is the form

$$d_{i,j} = \sum_{u=1}^{m+1} q_{j'+1,u}(\tilde{\theta}_j(x)|_{\Omega_{j,2}}) + \sum_{u=m+1}^{2m+2} q_{j'+1,u}(\tilde{\theta}_j(x)|_{\Omega_{j,3}}), \quad i = m+2, \dots, N, \tag{2.39}$$

where $j = 1, 2, \dots, N$,

$$\begin{aligned} \Omega_{j,2} &= [S(u-1, n, 2l), S(u, n, 2l)] \bigcap E_j \\ \Omega_{j,3} &= [S(u-1, n, 2l+1), S(u, n, 2l+1)] \bigcap E_j \end{aligned} \tag{2.40}$$

and $S(u, n, l) = 2^{-(n+1)}(u/(m+1) + l)$. In the above relations we use this notation

$$f(x)|_{[a,b]} = f(b) - f(a). \tag{2.41}$$

3 Description of Numerical Method

Consider the Fokker-Planck equation (1.8),this equation can be rewritten as

$$\frac{\partial u}{\partial t} = -u\frac{\partial}{\partial x}A - A\frac{\partial u}{\partial x} + u\frac{\partial^2}{\partial x^2}B + 2\frac{\partial u}{\partial x}\frac{\partial}{\partial x}B + B\frac{\partial^2 u}{\partial x^2}. \tag{3.42}$$

where $A = A(x, t, u)$ and $B = B(x, t, u)$. Also using chained rule we can write

$$\frac{\partial}{\partial x}A(x, t, u) = \frac{\partial A}{\partial x} + \frac{\partial A}{\partial u}.\frac{\partial u}{\partial x}, \tag{3.43}$$

$$\frac{\partial}{\partial x}B(x, t, u) = \frac{\partial B}{\partial x} + \frac{\partial B}{\partial u}.\frac{\partial u}{\partial x}, \tag{3.44}$$

and

$$\frac{\partial^2}{\partial x^2}B = \frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial x \partial u}.\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2}.\frac{\partial B}{\partial u} + \frac{\partial^2 B}{\partial u^2}.\left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial^2 B}{\partial u \partial x}.\frac{\partial u}{\partial x}. \tag{3.45}$$

Using Eqs.(2.27) and (2.28) we have:

$$\frac{\partial u}{\partial t} = \tilde{\Theta}^T(t)\mathbf{D^T U}\Theta(x), \tag{3.46}$$

$$\frac{\partial u}{\partial x} = \tilde{\Theta}^T(t)\mathbf{UD}\Theta(x), \tag{3.47}$$

$$\frac{\partial^2 u}{\partial x^2} = \tilde{\Theta}^T(t)\mathbf{UD^2}\Theta(x), \tag{3.48}$$

Using Eqs.(3.43)-(3.48) in Eq. (3.42), we get:

$$\begin{gathered} \tilde{\Theta}^T(t)\mathbf{D^T A}\Theta(x) = -u\left[\partial A/\partial x + \partial A/\partial u.\tilde{\Theta}^T(t)\mathbf{UD}\Theta(x)\right] \\ -A\tilde{\Theta}^T(t)\mathbf{UD}\Theta(x) + u[\partial^2 B/\partial x^2 + \partial^2 B/\partial x \partial u.\tilde{\Theta}^T(t)\mathbf{UD}\Theta(x) \\ +\tilde{\Theta}^T(t)\mathbf{UD^2}\Theta(x).\partial B/\partial u + \partial^2 B/\partial u^2.\left(\tilde{\Theta}^T(t)\mathbf{UD}\Theta(x)\right)^2 \\ +\partial^2 B/\partial u \partial x.\tilde{\Theta}^T(t)\mathbf{UD}\Theta(x)] \\ +2\tilde{\Theta}^T(t)\mathbf{UD}\Theta(x)\partial B/\partial x + \partial B/\partial u.\tilde{\Theta}^T(t)\mathbf{UD}\Theta(x) \\ +B\tilde{\Theta}^T(t)\mathbf{UD^2}\Theta(x). \end{gathered} \tag{3.49}$$

Also applying (2.27) in the initial condition (1.3) we get

$$\tilde{\Theta}^T(0)\mathbf{U}\Theta(x) = f(x) \tag{3.50}$$

By collocating Eq.(3.49) in $N \times N - 1$ points $(x_i, t_j), i = 1, 2, \dots, N, j = 1, 2, \dots, N-1$ on $[0, 1] \times [0, 1]$ and Eq.(3.50) in N points $x_i, i = 1, 2, \dots, N$ on $[0, 1]$, we get an algebraic system of $N \times N$ equations and unknowns, that can be solved for $u_{i,j}, i, j = 1, 2, \dots, N$. So the unknown function $u(x, t)$ can be found.

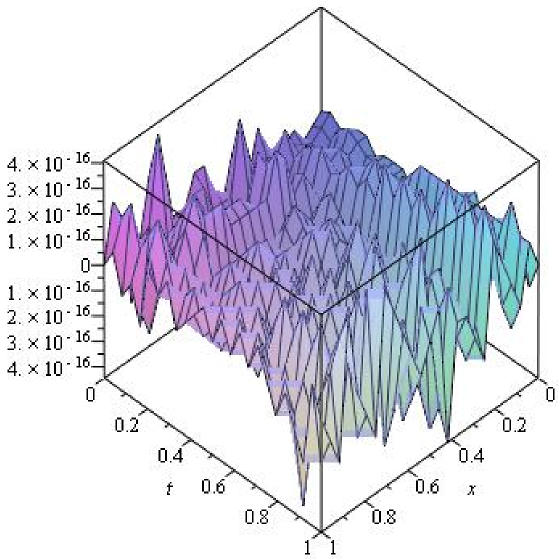

Figure 1: plots of error for Example 1 with m=2, J=1.

Table 1: Absolute values of errors for $u(x,t)$ for example 2 with m=3, J=1

t	$x = 0.2$	$x = 0.4$	$x = 0.6$	$x = 0.8$	$x = 1.0$
0.001	5.3×10^{-9}	1.3×10^{-8}	2.9×10^{-8}	5.7×10^{-8}	9.2×10^{-8}
0.005	2.7×10^{-8}	7.4×10^{-8}	1.5×10^{-7}	2.8×10^{-7}	4.6×10^{-7}
0.01	5.8×10^{-8}	9.9×10^{-7}	3.2×10^{-7}	5.7×10^{-7}	9.4×10^{-7}
0.05	4.0×10^{-7}	2.0×10^{-7}	1.8×10^{-6}	3.1×10^{-6}	4.8×10^{-6}
0.10	8.6×10^{-7}	2.6×10^{-6}	3.6×10^{-6}	6.0×10^{-6}	9.1×10^{-6}
0.15	1.1×10^{-6}	2.0×10^{-6}	4.8×10^{-6}	8.0×10^{-6}	1.2×10^{-5}
1.00	8.6×10^{-7}	1.2×10^{-6}	3.6×10^{-6}	6.0×10^{-6}	9.1×10^{-6}

4 Numerical Examples

In this section we give some computational results of numerical experiments with method based on preceding sections, to support our theoretical discussion. The nonlinear systems obtained by methods are solved by newton method and the collocation points are considered with equal space on the interval.

Example 1 *Consider (1.3) with:*

$$f(x) = x, \qquad x \in [0,1].$$

Let in Eq.(1.2) $A(x) = -1$, and $B(x) = 1$. The exact solution of this problem is $u(x,t) = x + t$. Figure 1 shows the plot of the error of the method presented $m = 2, J = 1$.

Example 2 *Consider (1.2) with $A(x) = x$, $B(x) = x^2/2$ and $f(x) = x$. The exact solution of this problem is $u(x,t) = xexp(t)$. Table 1 shows the absolute error using the method presented in previous section for $m = 3,\ J = 1$.*

Example 3 *consider the backward Kolmogorov equation (1.5) with drift and diffusion coefficients given respectively by:*

$$A(x,t) = -(x+1),$$

$$B(x,t) = x^2 exp(t).$$

Let the initial condition in (1.3) be given by:

$$f(x) = x + 1, \quad x \in [0, 1].$$

The exact solution of this problem is $u(x,t) = (x+1)exp(t)$. *Table 2 shows the absolute error using the method presented in previous section for* $m = 3,\ J = 1$.

Table 2: Absolute values of errors for $u(x,t)$ for example 3 with m=4, J=1

t	$x = 0.2$	$x = 0.4$	$x = 0.6$	$x = 0.8$	$x = 1.0$
0.001	6.0×10^{-9}	8.0×10^{-9}	4.0×10^{-9}	9.0×10^{-9}	6.1×10^{-7}
0.005	4.8×10^{-8}	5.5×10^{-8}	5.1×10^{-8}	6.0×10^{-8}	6.1×10^{-7}
0.01	1.0×10^{-7}	1.1×10^{-7}	1.1×10^{-7}	1.2×10^{-7}	6.2×10^{-7}
0.05	4.3×10^{-8}	4.8×10^{-7}	4.8×10^{-7}	5.1×10^{-7}	7.0×10^{-8}
0.10	7.0×10^{-7}	7.6×10^{-7}	7.9×10^{-7}	8.2×10^{-7}	8.8×10^{-7}
0.15	8.7×10^{-7}	9.4×10^{-7}	9.7×10^{-7}	1.5×10^{-6}	1.0×10^{-6}
1.00	7.0×10^{-7}	7.6×10^{-7}	7.9×10^{-7}	8.2×10^{-7}	8.8×10^{-7}

Example 4 *Consider the nonlinear Fokker-Planck equation(1.8) with:*

$$A(x,t,u) = 4\frac{u}{x} - \frac{x}{3}$$

$$B(x,t,u) = u,$$

and

$$f(x) = x^2.$$

The exact solution of this problem is $u(x,t) = x^2 exp(t)$. *Table 3 shows the absolute error using the method presented in previous section for* $m = 5,\ J = 2$.

Table 3: Absolute values of errors for $u(x,t)$ for example 4 with m=5, J=2

t	$x = 0.2$	$x = 0.4$	$x = 0.6$	$x = 0.8$	$x = 1.0$
0.001	1.9×10^{-11}	1.6×10^{-10}	9.2×10^{-10}	1.5×10^{-9}	2.0×10^{-9}
0.005	2.2×10^{-10}	1.5×10^{-9}	3.4×10^{-9}	6.5×10^{-9}	1.8×10^{-9}
0.01	6.0×10^{-10}	1.1×10^{-8}	4.3×10^{-9}	1.6×10^{-8}	1.0×10^{-8}
0.05	2.2×10^{-11}	9.3×10^{-10}	4.2×10^{-10}	1.8×10^{-9}	2.6×10^{-9}
0.10	5.0×10^{-10}	2.4×10^{-9}	3.1×10^{-8}	1.3×10^{-8}	6.6×10^{-8}
0.15	3.5×10^{-10}	1.2×10^{-8}	1.8×10^{-9}	1.7×10^{-8}	8.7×10^{-8}
1.00	1.2×10^{-9}	8.1×10^{-9}	2.0×10^{-7}	8.9×10^{-8}	9.2×10^{-8}

5 Conclusion

In this paper we presented a numerical scheme for solving the Fokker– Planck equation. The the flatlet oblique multi-wavelets together boundary scaling and wavelet functions on interval $[0, 1]$ was employed. The obtained results showed that this approach can solve the problem effectively.

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