Application of Wavelets in Computer Graphics: Wavelet Radiosity

Pinkesh Barsopia (123079006)

Guide: Prof. V. M. Gadre
Department of Electrical Engineering
Indian Institute of Technology Bombay

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Outline

Direct and Indirect Illumination

- Rendering-image synthesis from closed scene given,
  - Geometry of scene
  - Optical properties of surfaces (BRDF)
  - Location of light source
- Types of renderer
  - Direct illumination
  - Indirect illumination (global illumination)
- Types of indirect illumination renderer
  - Ray-tracing
  - Radiosity (lambertian BRDF)

Direct and Indirect Illumination

Figure : Direct and Indirect Illumination[1]

[1] **www.wikipedia.org/wiki/Radiosity(Computer Graphics)**.

Radiosity Integral Equation (RIE)

Radiosity $\rightarrow$ Power (light) per unit area

$$B_i = E_i + \sum_j K_{i,j} B_j, \quad j \in \{All\ Surfaces\} \tag{1}$$

$$K_{i,j} = \frac{\text{Radiosity received by i from j}}{\text{Radiosity leaving from j}}$$

$E_i \rightarrow$ Non zero for light source

$p_i \rightarrow$ BRDF of surface i

Domain of surfaces (M^2) is $L^2(\mathbb{R}^2)$ with finite support

$$B(x) = E(x) + p(x) \int_{M^2} K(x,y) B(y) dy \quad \forall\ x \in M^2 \tag{2}$$

Literature Survey

- Cohen et al. solved with n patches (constant radiosity) and n^2 from factors, $K_{i,j}, \quad i,j = 1,2,...,n$.
- Kajia et al. $\rightarrow$ radiosity integral equation $\rightarrow$ Projection methods (solve RIE in finite dimensional space).
- Finite Dimensional space
  - Heckbert $\rightarrow$ Piecewise linear
  - Zatz $\rightarrow$ Piecewise polynomial of higher order
  - Hanarahan et al. $\rightarrow$ Hierarchy of basis (Haar wavelet basis)
- Gortler et al. $\rightarrow$ used wavelet basis $\rightarrow$ wavelet radiosity

Test Scenes: 3 Dimensional

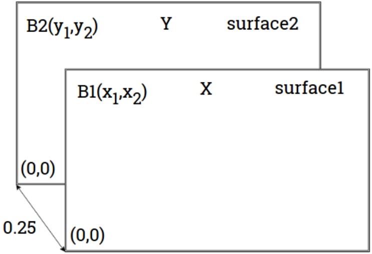

Figure : 3D scene: Two Parallel Surfaces

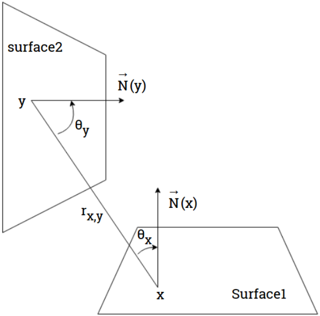

Figure : 3D Geometry: Kernel Calculation

$$K(x,y) = \frac{\cos\theta_x \cos\theta_y}{\pi r_{xy}^2} V(x,y) \tag{3}$$

Test Scenes: 2 Dimensional

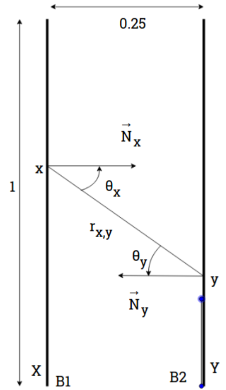

Figure : Flatland Scene 1: Parallel Segments

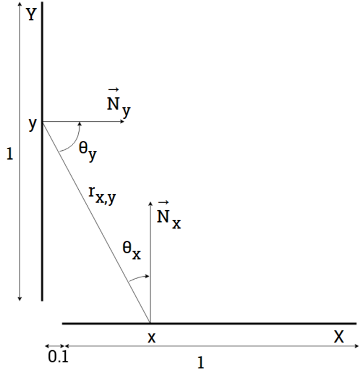

Figure : Flatland Scene 2: Perpendicular Segments

Kernel Calculation: 2 Dimensional

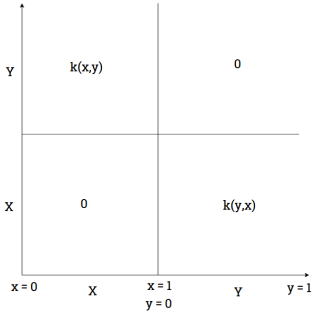

Figure : Kernel for Any Two Line Segments X and Y

$$K(x,y) = \frac{\cos\theta_x \cos\theta_y}{2r_{xy}} V(x,y) \tag{4}$$

Solution for Flatland Scene 1

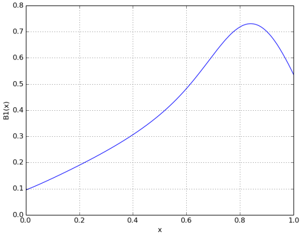

Figure : Radiosity $B_1(x)$

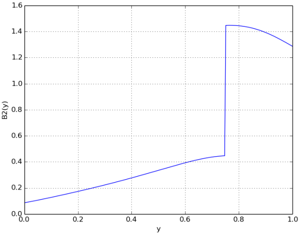

Figure : Radiosity $B_2(y)$

Figure : Image of B_1

Figure : Image of B_2

Solution of Flatland Scene 2

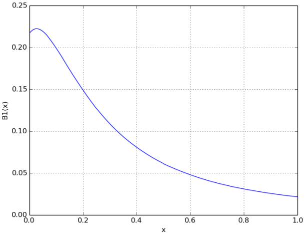

Figure : Radiosity $B_1(x)$

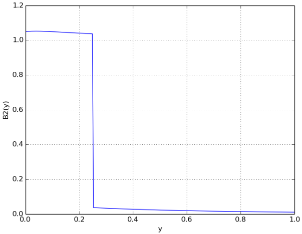

Figure : Radiosity $B_2(y)$

Figure : Image of B_1

Figure : Image of B_2

Metric for Comparison

Relative Error in Projection of K

$$\text{Relative Error} = \frac{\int\int (\, K(x,y) - \hat{K}(x,y)\,)^2 \, dy \, dx}{\int\int \; K(x,y)^2 \, dy \, dx} \tag{5}$$

Relative Error in Projection of B

$$\text{Relative Error} = \frac{\int (\, B(x) - \hat{B}(x)\,)^2 \, dx}{\int \; B(x)^2 \, dx} \tag{6}$$

Projection Methods for Radiosity

Discretization of Radiosity Integral Equation, using Basis, $\{N_i(x)\}_{i \in 0,1,2,\ldots}$,of the infinite dimensional space L^2

$$B_p(x) = \sum_{i=0}^{\infty} \langle B(x), \tilde{N}_i(x) \rangle N_i(x) \quad (7)$$

$$B_p(x) = E_p(x) + \int K(x,y) B_p(y) dy \quad (8)$$

$$B_p(x) = E_p(x) + K B_p(x) \quad (9)$$

where, $KB_p(x) = \int K(x,y) B_p(y) dy$

$$KB_p(x) = \sum_{i=0}^{\infty} \langle KB_p(x), \tilde{N}_i(x) \rangle N_i(x) \quad (10)$$

Projection Methods for Radiosity

$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} & \cdots \\ k_{21} & k_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \end{bmatrix} \tag{11}$$

$$\mathbf{B} = \mathbf{E} + \mathbf{KB} \tag{12}$$

$$K_{i,j} = \langle\, \langle K(x,y), N_j(y) \rangle, \tilde{N}_i(x) \,\rangle = \int\int K(x,y) N_j(y) \tilde{N}_i(x) dy\, dx$$

- Discrete, but countably infinite no. of coefficients
- Project in finite dimensional space $\rightarrow$ Finite no. of coefficients
- Error in Projection $\rightarrow$ Choice of finite dimensional space
- Speed of solution $\rightarrow$ Choice of a basis for the chosen space

Implementation of Algorithm

- Select finite dimensional function space
- Select Basis for that space
- Project $E(x)$ and $K(x)$ into chosen space $\rightarrow$ inner product using Gauss-Legendre Quadrature rule for integration
- Solve System of linear equation using Jacobi iteration

Radiosity for scene with RGB colors can be calculated by solving this algorithm for 3 wavelength separately.

Choice of a Finite Dimensional Space

Space of piece-wise polynomial functions of order m, over standard interval $(1/n)$, using set of shifted Legendre polynomial up to order m

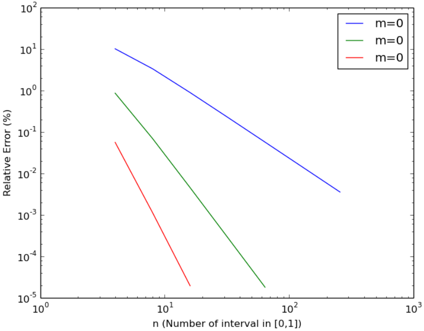

Figure : Error in Projection of

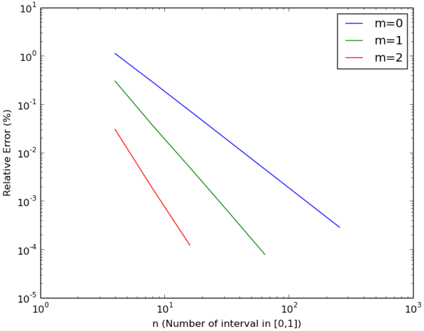

Figure : Error in Projection of

Analytical Solution

$$B(x) = 1 + \int K(x,y)B(y)dy \tag{13}$$

$$K(x,y) = x^2 + xy, \quad x,y \in [0,1]$$

Degenerate kernel, $K(x,y) = \sum_{i=1}^{n} a_i(x)b_i(y)$

Numerical Solution with $m = 0$

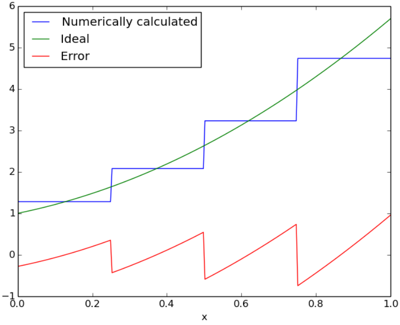

Figure : Solution Using $n = 4, m = 0$

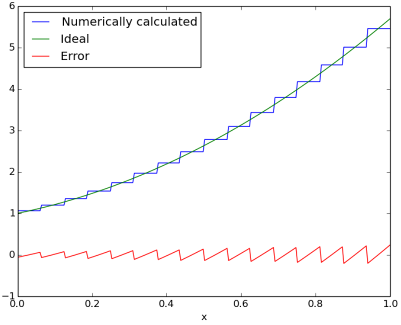

Figure : Solution Using $n = 16, m = 0$

Numerical Solution with $m = 1$

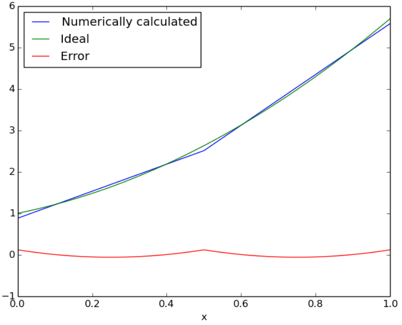

Figure : Solution Using $n = 2, m = 1$

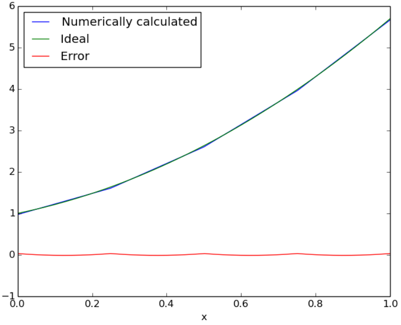

Figure : Solution Using $n = 4, m = 1$

Numerical Solution with $m = 2$

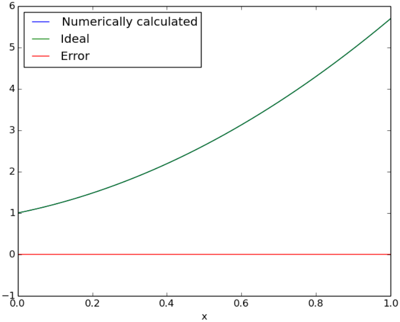

Figure : Solution Using $n = 1, m = 2$

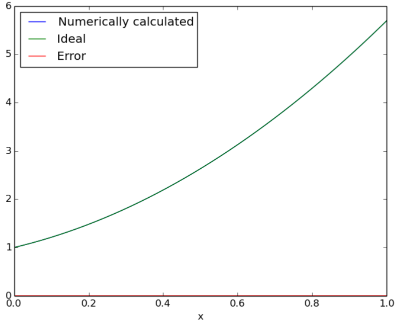

Figure : Solution Using $n = 2, m = 2$

Choice of a Basis: Wavelets

Choice of alternative basis for chosen space

- Haar
- Linear Legendre Multi-Wavelet
- Quadratic Legendre Multi-Wavelet

Advantages

- Vanishing moments
- Negligible coefficients
- Sparse system of equation
- Faster solution

Haar Wavelet: As a Alternate Basis

$$\phi_0(x) = \begin{cases} 1 & \text{if } 0 \leq x < 1 \\ 0 & \textit{elsewhere} \end{cases}$$

$$\psi_0(x) = \begin{cases} 1 & \text{if } 0 \leq x < \frac{1}{2} \\ -1 & \text{if } \frac{1}{2} \leq x < 1 \\ 0 & \textit{elsewhere} \end{cases}$$

Basis: Haar scaling function

$$\{\phi_{\frac{1}{n},k}(x)\},$$

$$k = 0, 1, ..., n-1$$

where, $\phi_{\frac{1}{n},k}(x) = \sqrt{n}\phi_0(nx - k)$

Basis: Haar wavelet

$$\{\phi_0(x), \psi_0(x), \psi_{\frac{1}{L},k}(x)\},$$

$$L = 2, 4, 8, ..., 2^{n-1} \text{ and}$$
$$k = 0, 1, ..., L-1$$

where, $\psi_{\frac{1}{n},k}(x) = \sqrt{n}\psi_0(nx - k)$

Sparsity of Kernel Projection Matrix

Flatland 1 using Haar wavelet and scaling functions. Radius of circle is proportional to absolute value of coefficient

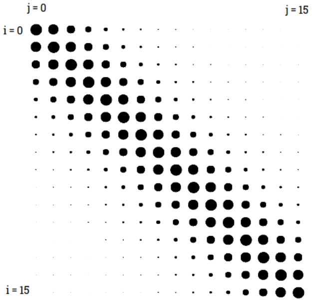

Figure : Scaling Functions as basis

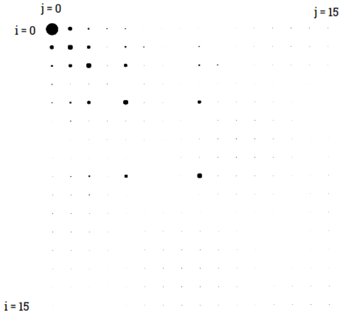

Figure : Wavelet Functions as basis

Linear Legendre Multi-wavelet (LLMW)

Two mother wavelet and scaling function

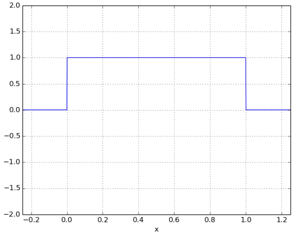

Figure : LLMW: $\phi^0(x)$

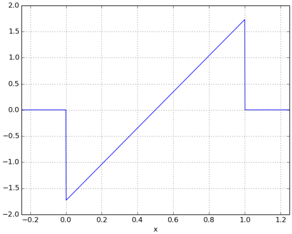

Figure : LLMW: $\phi^1(x)$

$$\phi^0(x) = \begin{cases} 1 & \text{if } 0 \leq x < 1 \\ 0 & \textit{elsewhere} \end{cases}$$

$$\phi^1(x) = \begin{cases} \sqrt{3}(2x-1) & \text{if } 0 \leq x < 1 \\ 0 & \textit{elsewhere} \end{cases}$$

Linear Legendre Multi-wavelet (LLMW)

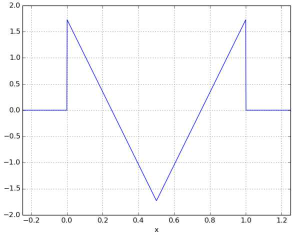

Figure : LLMW: $\psi^0(x)$

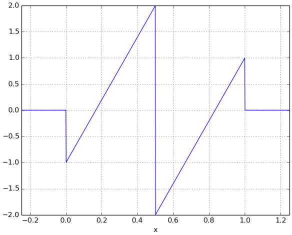

Figure : LLMW: $\psi^1(x)$

$$\psi^0(x) = \begin{cases} -\sqrt{3}(4x-1) & \text{if } 0 \leq x < \frac{1}{2} \\ \sqrt{3}(4x-1) & \text{if } \frac{1}{2} \leq x < 1 \\ 0 & \textit{elsewhere} \end{cases}$$

$$\psi^1(x) = \begin{cases} (6x-1) & \text{if } 0 \leq x < \frac{1}{2} \\ (6x-5) & \text{if } \frac{1}{2} \leq x < 1 \\ 0 & \textit{elsewhere} \end{cases}$$

Quadratic Legendre Multi-wavelet (QLMW)

Three mother wavelet and scaling function

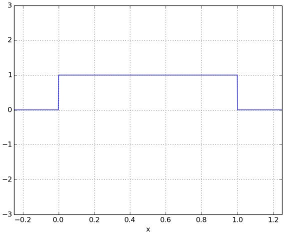

Figure : QLMW: $\phi^0(x)$

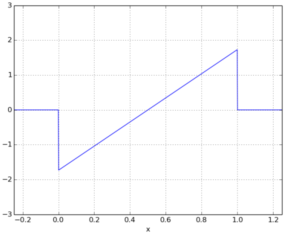

Figure : QLMW: $\phi^1(x)$

$$\phi^0(x) = \begin{cases} 1 & \text{if } 0 \leq x < 1 \\ 0 & \textit{elsewhere} \end{cases}$$

$$\phi^1(x) = \begin{cases} \sqrt{3}(2x-1) & \text{if } 0 \leq x < 1 \\ 0 & \textit{elsewhere} \end{cases}$$

Quadratic Legendre Multi-wavelet (QLMW)

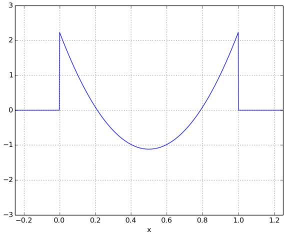

Figure : QLMW: $\phi^2(x)$

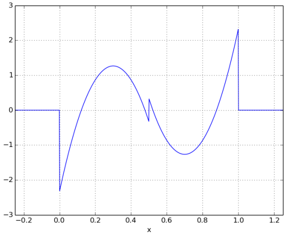

Figure : QLMW: $\psi^0(x)$

$$\phi^2(x) = \begin{cases} \sqrt{5}(6x^2 - 6x + 1), & \text{if } 0 \le x < 1 \\ 0 & \textit{elsewhere} \end{cases}$$

$$\psi^0(x) = \begin{cases} -\frac{1}{3}(120x^2 - 72x + 7) & \text{if } 0 \le x < \frac{1}{2} \\ \frac{1}{3}(120x^2 - 168x + 55) & \text{if } \frac{1}{2} \le x < 1 \\ 0 & \textit{elsewhere} \end{cases}$$

Quadratic Legendre Multi-wavelet (QLMW)

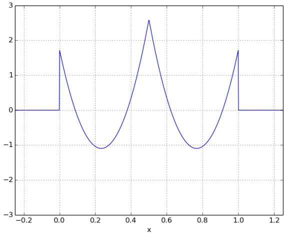

Figure : QLMW: $\psi^1(x)$

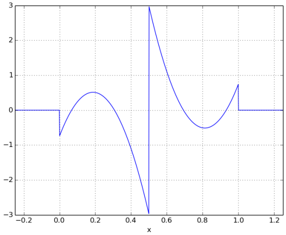

Figure : QLMW: $\psi^2(x)$

$$\psi^1(x) = \begin{cases} \sqrt{3}(30x^2 - 14x + 1) & \text{if } 0 \leq x < \frac{1}{2} \\ \sqrt{3}(30x^2 - 46x + 17) & \text{if } \frac{1}{2} \leq x < 1 \\ 0 & \textit{elsewhere} \end{cases}$$

$$\psi^2(x) = \begin{cases} -\frac{\sqrt{5}}{3}(48x^2 - 18x + 1) & \text{if } 0 \leq x < \frac{1}{2} \\ \frac{\sqrt{5}}{3}(48x^2 - 78x + 31) & \text{if } \frac{1}{2} \leq x < 1 \\ 0 & \textit{elsewhere} \end{cases}$$

Sparsity of Kernel Projection Matrix

Flatland 1 using Haar wavelet and scaling functions

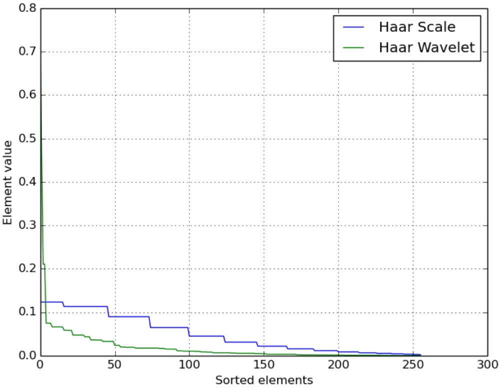

Figure : Sorted Coefficients Based on Magnitude

Sparsity of Kernel Projection Matrix

Flatland 1 using LLMW wavelet and scaling functions

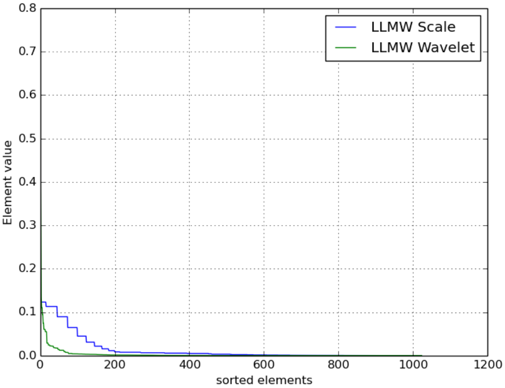

Figure : Sorted Coefficients Based on Magnitude

Sparsity of Kernel Projection Matrix

Flatland 1 using LLMW wavelet and scaling functions

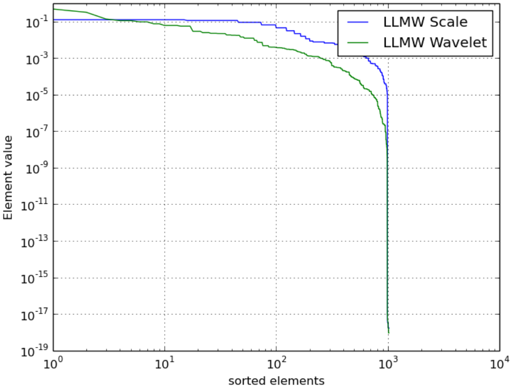

Figure : Sorted Coefficients Based on Magnitude (Log Scale)

Sparsity of Kernel Projection Matrix

Flatland 1 using QLMW wavelet and scaling functions

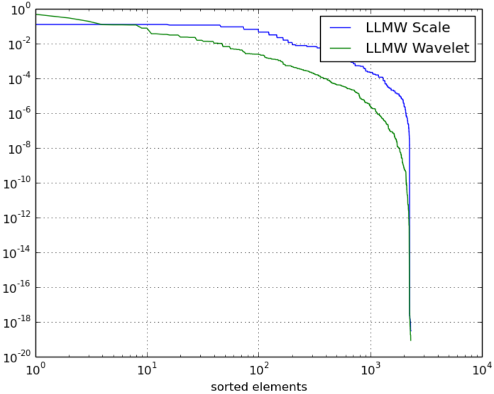

Figure : Sorted Coefficients Based on Magnitude (Log Scale)

Error in Projection

Keeping top n coefficients (rest of the coefficient are set to zero)

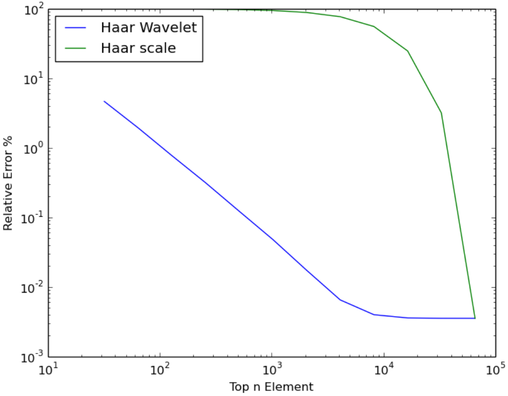

Figure : Haar, $n = 256$

Error in Projection

Keeping top n coefficients (rest of the coefficient are set to zero)

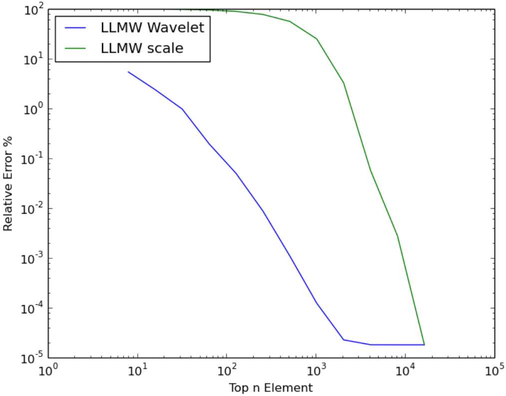

Figure : LLMW, $n = 64$

Image Synthesis from 3D scene: Haar Wavelet

Solution of 3D scene with Haar wavelet and $n = 4$

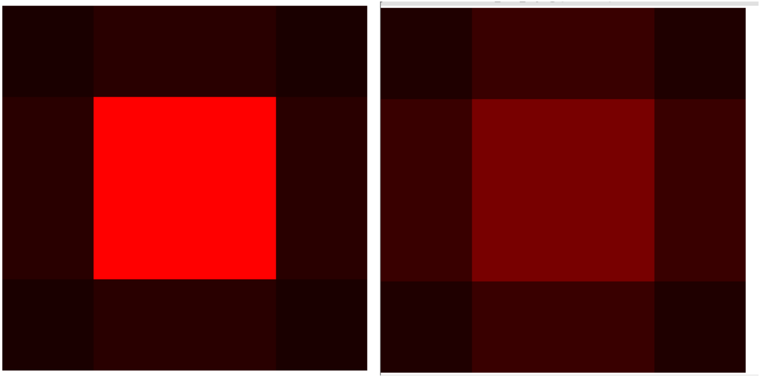

Figure : Illumination of 3D scene (2 parallel surfaces) by square light source shown near center of one surface (left image)

Image Synthesis from 3D scene: Haar Wavelet

Solution of 3D scene with Haar wavelet and $n = 16$

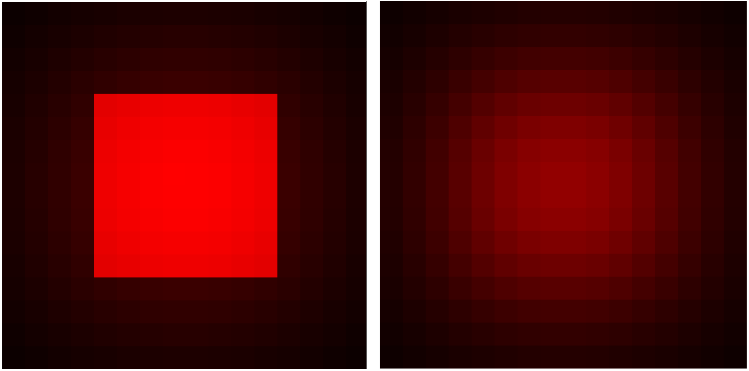

Figure : Illumination of 3D scene (2 parallel surfaces) by square light source shown near center of one surface (left image)

Image Synthesis from 3D scene: Haar Wavelet

Solution of 3D scene with Haar wavelet, $n = 16$, retaining half of the 256 coefficients.

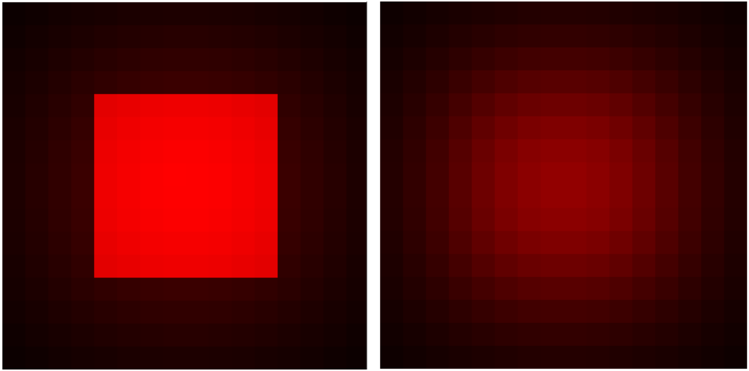

Figure : Illumination of 3D scene (2 parallel surfaces) by square light source shown near center of one surface (left image)

Image Synthesis from 3D scene: LLMW Wavelet

Solution of 3D scene with LLMW wavelet and $n = 16$

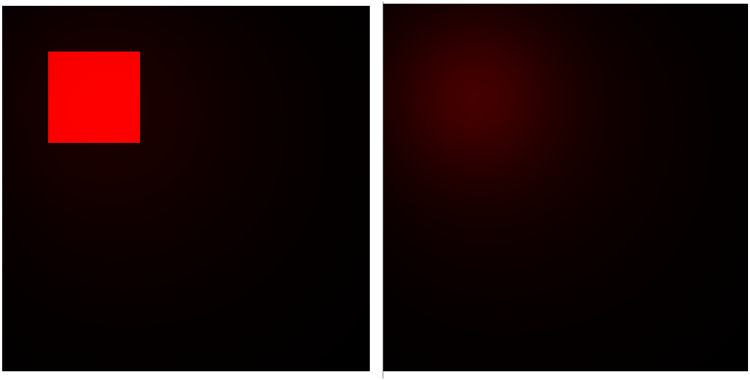

Figure : Illumination of 3D scene (2 parallel surfaces) by square light source shown near center of one surface (left image)

Image Synthesis from 3D scene: LLMW Wavelet

Solution of 3D scene with LLMW wavelet and $n = 16$, retaining quarter of the 1024 coefficients.

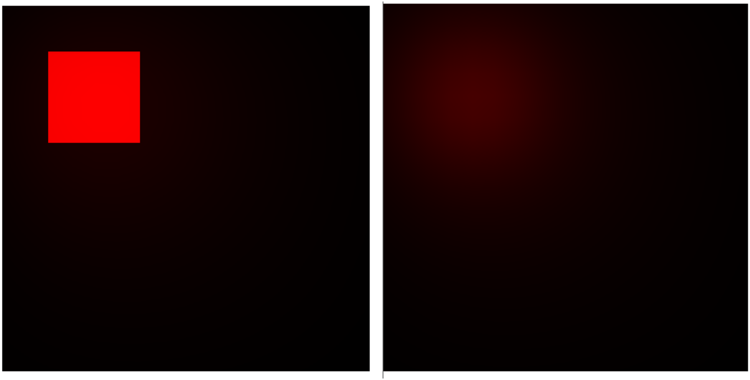

Figure : Illumination of 3D scene (2 parallel surfaces) by square light source shown near center of one surface (left image)

Image Synthesis from 3D scene: QLMW Wavelet

Solution of 3D scene with QLMW wavelet and $n = 4$.

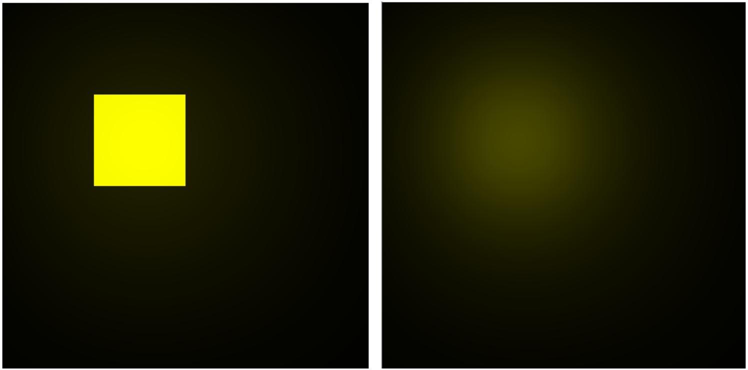

Figure : Illumination of 3D scene (2 parallel surfaces) by square light source shown near center of one surface (left image)

Image Synthesis from 3D scene: QLMW Wavelet

Solution of 3D scene with LLMW wavelet and $n = 16$, retaining half of the 144 coefficients.

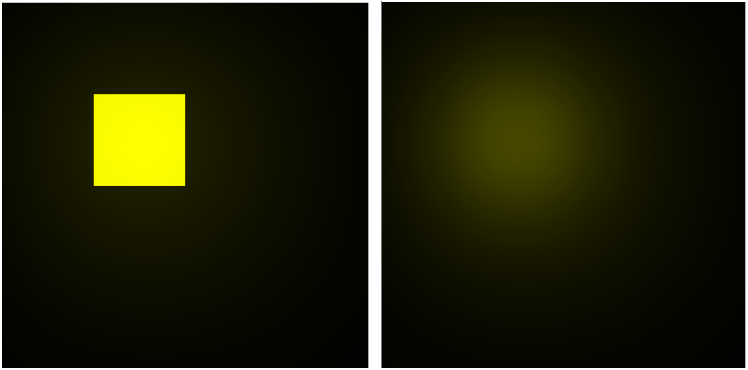

Figure : Illumination of 3D scene (2 parallel surfaces) by square light source shown near center of one surface (left image)

Conclusion and Future Work

- Projection methods are used to solve a radiosity integral equation.
- Different finite dimensional function space of piecewise polynomials of order $0, 1, 2$ has been used for projection.
- Projection error decreases either when order of polynomials is increased or when interval size is decreased.
- Different basis like Haar wavelet, shifted Legendre polynomials, Legendre multi-wavelet (linear and quadratic) has been used for Comparison. In future other wavelets like Chebyshev multi-wavelet and other polynomial order wavelets can be used.
- Wavelet basis gives very sparse system of linear equations. Thus, setting negligible coefficients to zero increases speed of the algorithm.

- Different test scenes, 2D and 3D, has been used to test the algorithm. In future we can use more test scenes to verify results.
- Test scene under consideration are without occlusion, in future, testing performance of wavelets with scenes having occlusion can be done

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