Application of Wavelet in Computer Graphics: Wavelet Radiosity

Pinkesh Barsopia (123079006)

Guide: Prof. V. M. Gadre
Department of Electrical Engineering
Indian Institute of Technology Bombay

May 26, 2015

Outline

Direct and Indirect Illumination

- Rendering-Image synthesis from closed scene given
  - Geometry of scene
  - Optical properties of surface (BRDF)
- Types of renderer
  - Direct Illumination
  - Indirect Illumination (global Illumination)
- Types of global Illumination renderer
  - Ray-Tracing
  - Radiosity (Lambertian BRDF)

Direct and Indirect Illumination

Figure : Ideal Scalable Bitstream[1]

[1] **www.wikipedia.org/wiki/Radiosity(computer graphics)**.

Radiosity Integral Equation (RIE)

Radiosity $\rightarrow$ Power per unit area

$$B_i = E_i + \sum_j K_{i,j} B_j, \quad j \in \{Allsurfaces\} \tag{1}$$

$$K_{i,j} = \frac{\text{Radiosity received by i from j}}{\text{Radiosity leaving from j}}$$

$E_i \rightarrow$ Non zero for light source

$p_i \rightarrow$ BRDF of surface

Domain, $M^2 \rightarrow L^2(\mathbb{R}^2)$, finite support

$$B(x) = B(x) + p(x) \int_{M^2} K(x,y) B(y) \quad \forall x \in M^2 \tag{2}$$

Literature Survey

- Cohen et al. solved with n patches (constant radiosity) and n^2 from factors, $K_{i,j}, \quad i,j = 1,2,...,n.$
- Kajia $\rightarrow$ radiosity integral equation $\rightarrow$ Projection methods (solve IE in finite space).
- Finite space
  - Heckbert $\rightarrow$ Piecewise linear
  - Zatz $\rightarrow$ Piecewise polynomial of higher order
  - Hanarahan et al. $\rightarrow$ Hierarchy of basis (Haar wavelet basis)
- Gortler et al. $\rightarrow$ used wavelet basis $\rightarrow$ wavelet radiosity

Test Scenes: 3 Dimensional

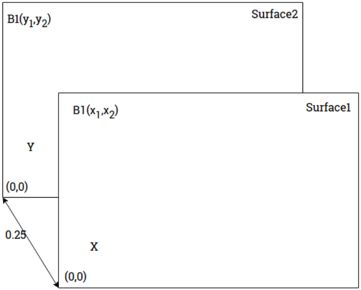

Figure : 3D scene: Two parallel surfaces

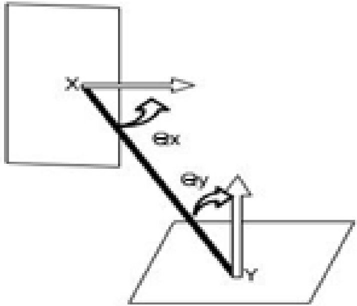

Figure : 3D Geometry: Kernel calculation

$$K(x,y) = \frac{\cos\theta_x \cos\theta_y}{\pi r_{xy}^2} V(x,y) \tag{3}$$

Test Scenes: 2 Dimensional

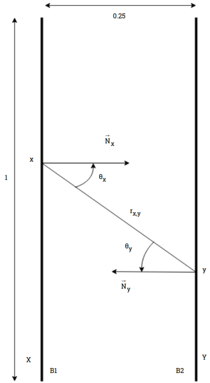

Figure : Flatland scene 1: parallel segments

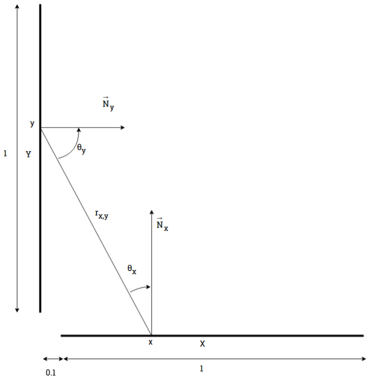

Figure : Flatland scene 2: perpendicular segments

Kernel Calculation: 2 Dimensional

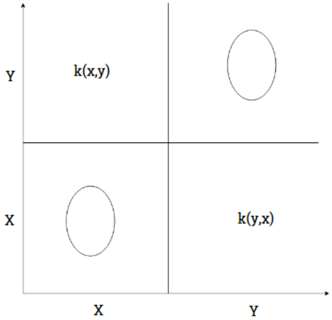

Figure : Kernel for two line segments

$$K(x,y) = \frac{\cos\theta_x \cos\theta_y}{2r_{xy}} V(x,y) \quad (4)$$

Solution of Flatland Scenes 1

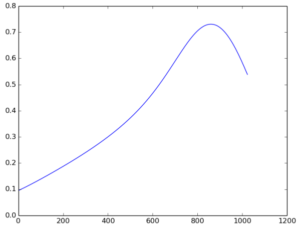

Figure : Radiosity $B_1(x)$

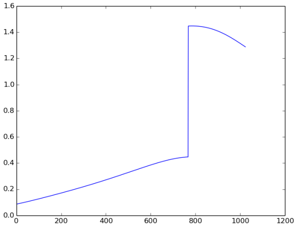

Figure : Radiosity $B_2(y)$

Figure : Image of B_1

Figure : Image of B_2

Solution of Flatland Scenes 2

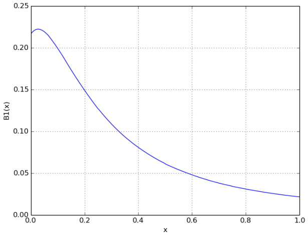

Figure : Image of B_1

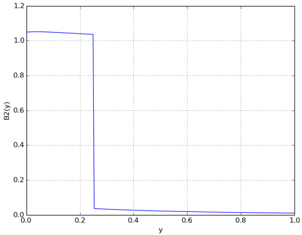

Figure : Image of B_2

Figure : Radiosity $B_1(x)$

Figure : Radiosity $B_2(y)$

Metric for Comparison

Relative error(L^2 norm) in K for comparison

$$\text{relative error} = \frac{\int\int (\, K(x,y) - \hat{K}(x,y)\,)^2\, dy\, dx}{\int\int\, K(x,y)^2\, dy\, dx} \tag{5}$$

Relative error (L^2 norm) in B for comparison

$$\text{relative error} = \frac{\int (\, B(x) - \hat{B}(x)\,)^2\, dx}{\int\, B(x)^2\, dx} \tag{6}$$

Projection Methods for Radiosity

Choice of Finite space

Space of Piecewise polynomial function of order m, over standard interval ($\frac{1}{n}$)

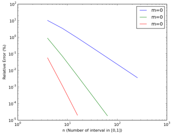

Figure : Error in projection of K(x,y) for Flatland scene 1

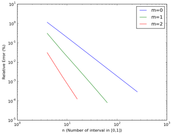

Figure : Error in projection of K(x,y) for Flatland scene 2

Analytical Solution

$$B(x) = 1 + \int K(x,y)B(y)dy \tag{7}$$

$$K(x,y) = x^2 + xy, \quad x, y \in [0,1]$$

$$\text{Degenerate kernel, } K(x,y) = \sum_{i=1}^{n} a_i(x) b_i(y)$$

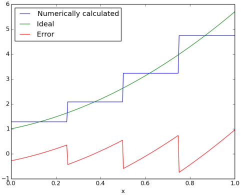

Figure : Solution using $n = 4, m = 0$

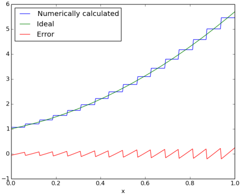

Figure : Solution using $n = 16, m = 0$

Analytical Solution for $m = 1, 2$

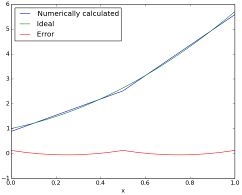

Figure : Solution using $n = 2, m = 1$

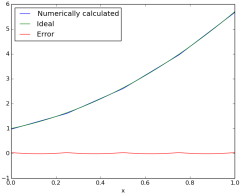

Figure : Solution using $n = 4, m = 1$

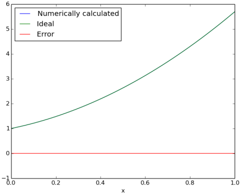

Figure : Solution using $n = 1, m = 2$

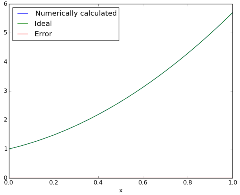

Figure : Solution using $n = 2, m = 2$

Choice of Basis: Wavelets

Choice of alternative basis for chosen space

- Haar
- Linear Legendre Multi-Wavelet
- Quadratic Legendre Multi-Wavelet

Advantages

- Vanishing Moments
- Negligible coefficients
- Sparse System of equation
- Faster Solution

Haar Wavelet: Alternate Basis

Haar Wavelet

$$\phi(x) = \begin{cases} 1 and \text{if } 0 \leq x < 1 \\ 0 and elsewhere \end{cases}$$

$$\psi^0(x) = \begin{cases} 1 and \text{if } 0 \leq x < \frac{1}{2} \\ -1 and \text{if } \frac{1}{2} \leq x < 1 \\ 0 and elsewhere \end{cases}$$

Basis: Haar scaling function

$$\{\phi_{\frac{1}{n},k}(x)\},$$

k=0,1,...,n-1

where, $\phi_{\frac{1}{n},k}(x) = \frac{1}{\sqrt{n}}\phi(nx-k)$

Basis: Haar wavelet

$$\{\phi(x), \psi(x), \phi_{\frac{1}{L},k}(x)\},$$

k=0,1,...,n-1 and k=0,1,...,L-1

where, $\psi_{\frac{1}{n},k}(x) = \frac{1}{\sqrt{n}}\psi(nx-k)$

Error vs n

Projection Methods for Radiosity