Welcome to Gummi 0.6.5

Alexander van der Mey
Wei-Ning Huang
Dion Timmermann

1 Before you start

Radiosity methods have been shown to be an effective means to solve the global illumination problem in Lambertian diffuse environments. These methods approximate the radiosity integral equation by projecting the unknown radiosity function into a set of basis functions with limited support resulting in a set of n linear equations where n is the number of discrete elements in the scene. Classical radiosity methods required the evaluation of n 2 interaction coefficients. Efforts to reduce the number of required coefficients without compromising error bounds have focused on raising the order of the basis functions, meshing, accounting for discontinuities, and on developing hierarchical approaches, which have been shown to reduce the required interactions to O(n). In this paper we show that the hierarchical radiosity formulation is an instance of a more general set of methods based on wavelet theory. This general framework offers a unified view of both higher order element approaches to radiosity and the hierarchical radiosity methods. After a discussion of the relevant theory, we discuss a new set of linear time hierarchical algorithms based on wavelets such as the multiwavelet family and a flatlet basis which we introduce. Initial results of experimentation with these basis sets are demonstrated and discussed.

2 Feedback

In computer graphics, radiosity methods have been used to solve the global illumination problem in environments consisting en- tirely of Lambertian (diffuse) reflectors and emitters. The solution is a radiosity function over the domain of the surfaces in the scene. Classical radiosity [9, 6] (CR), derived from the radia- tive heat transfer literature, approximates the radiosity function as piecewise constant. An energy balance argument gives rise to a linear system. This system has n 2 coefficients called form factors. Here n is the number of discrete areas, or elements, over which the radiosity function has been assumed to be constant. The form factor describes the fraction of the energy leaving one element and arriving at another. Typically, an iterative algorithm such as Gauss-Seidel iteration [22] or progressive radiosity [5, 10] is used to solve the system of linear equations for the radiosities.

3 One more thing

Recently Beylkin et al. [3] made the observation that integral operators satisfying very general smoothness conditions can be approximated to any finite precision with only O(n) coefficients when projected into a wavelet basis instead of the usual O(n 2). This remarkable result means that, in practice, integral equations governed by smooth kernels lead to sparse matrices that can be solved in linear time. Since the radiosity kernel is, in general, a smooth function of the type required by this theorem, wavelet methods can be used to obtain O(n) complexity radiosity algo- rithms. We call this wavelet radiosity. Hierarchical basis functions have been used before with finite- element methods [24] and applied to problems such as surface interpolation [23]. In those instances, hierarchical basis functions were used to improve the condition number of the matrix. In our context, the hierarchical basis functions (wavelets) are used because many of the resulting matrix coefficients are small enough to be ignored while still allowing for an accurate answer. In some sense we are regarding the matrix as an image on which we are able to perform lossy compression. Coefficients are negligible because over many regions the kernel can be well approximated by a low order polynomial. The mathematical tools of wavelet analysis provide a general framework offering a unified view of both higher order element approaches to radiosity, and the hierarchical radiosity methods. Figure 1 places earlier algorithms plus the new methods we inves- tigate here into a matrix relating hierarchy versus the order of the underlying basis. CR uses zero order polynomials, while GR uses higher order polynomials (indicated by the arrow). The vertical axis represents the sparseness obtained by exploiting smoothness of some order in the kernel. HR exploits constant smoothness in the kernel. Within this context, we recognize HR as a first order wavelet. Higher order wavelets can be used that result in an even sparser matrix. One such family of higher order wavelets is the multiwavelet family of [1] (M 2,3 in Figure 1). We will also introduce a new family of wavelets, which we have dubbed flatlets (F 2,3 in Figure 1) that require only low order quadrature methods while maintaining most of the benefits of other wavelet sets. This paper proceeds with a review of projection methods for solving integral equations followed by a discussion of re- cent advances concerning the solution of integral equations using wavelets. Finally we discuss our implementation and report ex- perimental findings. Some of the more technical details of wavelet projections, as well as a detailed analysis of