??

$$M\ddot{U}(t)+C\dot{U}(t)+KU(t)=F(t) \quad (1)$$

??

$$U(0)={}^0U\dot{U}(0)={}^0\dot{U} \quad (2)$$

??β**?**θ**???**

?
?
?
??
?

??$M,C,KF(t)tU(t)(\cdot)\dot{U}(t),\ddot{U}(t)$
$\dot{U}(t)=$
$V(t)$**??**

$$\dot{U}(t)\dot{V}(t)=\mathbf{0}\mathbf{I}-M^{-1}K-M^{-1}CU(t)V(t)+\mathbf{0}M^{-1}F(t) \quad (3)$$

$$U(0)={}^0UV(0)={}^0\dot{U} \quad (4)$$

$\mathbf{0}\mathbf{I}^1$**????**
???

$$\ddot{u}(t)+2\xi\omega\dot{u}(t)+\omega^2u(t)=f(t) \quad (5)$$

$\xi,\omega f(t)u(t)$
?
?

$$\{\dot{y}_1(t)=\lambda_2y_1(t)+f(t)\dot{y}_2(t)=\lambda_1y_2(t)+f(t) \quad (6)$$

$$\lambda_{1,2}=-\xi\omega\pm j\sqrt{1-\xi^2}\omega \quad (7)$$

$j=\sqrt{-1}0\leq\xi<1u(t)y_1(t),y_2(t)$

$$\{y_1(t)=\dot{u}(t)-\lambda_1u(t)y_2(t)=\dot{u}(t)-\lambda_2u(t) \quad (8)$$

$$u(t)=-\frac{y_1(t)-y_2(t)}{\lambda_1-\lambda_2} \quad (9)$$

$\lambda_{1,2}y_{1,2}(t)u(t)$
$y(0)=y(t_0=0)$

$$\dot{y}(t)=f(y,t) \quad (10)$$

$t\in[0,T][0,T]$

$$[0,T]=\bigcup_{n=0}^{N-1}[t_n,t_{n+1}] \quad (11)$$

$\Delta t=t_{n+1}-t_nt_nt_{n+1}$
??k**?**
?

$$\alpha_0{}^{t+\Delta t}y+\alpha_1{}^ty+\cdots+\alpha_k{}^{t+(1-k)\Delta t}y=\Delta t[\beta_0f({}^{t+\Delta t}y,t_{n+1}+\beta_1f({}^ty,t_n)+\cdots+\beta_kf({}^{t+(1-k)\Delta t}y,t_{n+1-k})] \quad (12)$$

α_i,β_i

$$\sum_{i=0}^{k}\{\alpha_i{}^{t+(1-i)\Delta t}y+\Delta t\beta_if({}^{t+(1-i)\Delta t}y,t_{n+1-i})\}=0 \quad (13)$$

$\beta_0=0kk$

$$\ddot{y}=f(\dot{y}(t),y(t),t)=g_0y(t)+g_1\dot{y}(t)+h(t) \quad (14)$$

$g_0,g_1\in$

$$\frac{^{n+1}y - {^n}y}{\Delta t} = {^n}\dot{y}$$
(19)
$$^{n+1}y = {^n}y + \Delta t {^n}\dot{y} = {^n}y + \Delta t f({^n}y, t_n)$$
(20)
t_n ${^n}y$ t_{n+1} ${^{n+1}}y$
$\dot{y}(t)$

$$^{n+1}y = {^n}y + \Delta t {^{n+1}}\dot{y} = {^n}y + \Delta t f({^{n+1}}y, t_n)$$
(21)
${^{n+1}}y$ $f(y,t)$
${^n}\dot{y}$ ${^{n+1}}\dot{y}$ **?**
? $(0 \leq \alpha \leq 1)$

$$^{n+\alpha}\dot{y} = (1-\alpha){^n}\dot{y} + \alpha {^{n+1}}\dot{y}$$
(22)
$$^{n+1}y - {^n}y = \Delta t {^{n+\alpha}}\dot{y}$$
(23)
$$^{n+1}y = {^n}y + \Delta t[(1-\alpha){^n}\dot{y} + \alpha {^{n+1}}\dot{y}]$$
(24)
$\alpha = 0$ $\alpha = 1$ $\alpha = \frac{1}{2}$

$$^{n+1}y = {^n}y + \frac{\Delta t}{2}({^n}\dot{y} + {^{n+1}}\dot{y})$$
(25)

$$M\dot{d} + Kd = F$$
(26)
${^{n+1}}\dot{d} + K {^{n+1}}d = {^{n+1}}F$
${^{n+1}}d = {^n}d + \Delta t {^{n+\alpha}}\dot{d}$
${^{n+\alpha}}\dot{d} = {^n}\dot{d}(1-\alpha) + {^{n+1}}\dot{d}\alpha$
dF

$$(M + \alpha \Delta t K){^{n+1}}d = [M - (1-\alpha)\Delta t K]{^n}d + \Delta t[{^{n+1}}F\alpha + {^n}F(1-\alpha)]$$
(27)
$\alpha = \theta$ θ **?**
y α

$$^{n+\alpha}y = {^{n+1}}y\alpha + {^n}y(1-\alpha)$$
(28)
t β

$$t_{n+\beta} = \beta t_{n+1} + (1-\beta)t_n$$
(29)
$y[t_n, t_{n+1}]$ ${^{n+1/2}}y$

$$^{n+1/2}y = \frac{^{n+1}y + {^n}y}{2}$$
(30)
$$t_{n+\frac{1}{2}} = \frac{t_{n+1} + t_n}{2}$$
(31)

$$^{n+1/2}y \neq y(t_{n+\frac{1}{2}})$$
(32)
${^{n+1/2}}y$
[?]
${^{n+1}}y = {^n}y + \Delta t {^{n+1/2}}\dot{y} = {^n}y + \Delta t f({^{n+1/2}}y, t_{n+\frac{1}{2}})$
${^{n+1/2}}y =$
${^n}y +$