**??**

$$M\ddot{U}(t)+C\dot{U}(t)+KU(t)=F(t)$$ (1)

**??**

$$U(0)={}^0U \quad \dot{U}(0)={}^0\dot{U}$$ (2)

**??**$\beta$**?**$\theta$**???**

**?**
**?**
**?**
**??**
**?**

**??**$M, C, K$ $F(t)$ $t$ $U(t)$ $(\cdot)$ $\dot{U}(t), \ddot{U}(t)$ $\dot{U}(t)=V(t)$**??**

$$\dot{U}(t) \dot{V}(t) = \mathbf{0} \mathbf{I} - M^{-1}K - M^{-1}C \, U(t) V(t) + \mathbf{0} M^{-1}F(t)$$ (3)

$$U(0)={}^0U \quad V(0)={}^0\dot{U}$$ (4)

$\mathbf{0}$ $\mathbf{I}$[1] **????**
**???**

$$\ddot{u}(t)+2\xi\omega\dot{u}(t)+\omega^2u(t)=f(t)$$ (5)

$\xi, \omega$ $f(t)$ $u(t)$
**?**
**?**

$$\{\dot{y}_1(t)=\lambda_2 y_1(t)+f(t) \quad \dot{y}_2(t)=\lambda_1 y_2(t)+f(t)$$ (6)

$$\lambda_{1,2}=-\xi\omega \pm j\sqrt{1-\xi^2}\omega$$ (7)

$j=\sqrt{-1}$ $0 \le \xi < 1$ $u(t)$ $y_1(t), y_2(t)$

$$\{y_1(t)=\dot{u}(t)-\lambda_1 u(t) \quad y_2(t)=\dot{u}(t)-\lambda_2 u(t)$$ (8)

$$u(t)=-\frac{y_1(t)-y_2(t)}{\lambda_1-\lambda_2}$$ (9)

$\lambda_{1,2}$ $y_{1,2}(t)$ $u(t)$
$y(0)=y(t_0=0)$

$$\dot{y}(t)=f(y,t)$$ (10)

$t \in [0,T]$ $[0,T]$

$$[0,T]=\bigcup_{n=0}^{N-1}[t_n,t_{n+1}]$$ (11)

$\Delta t = t_{n+1}-t_n$ $t_n$ $t_{n+1}$
**??**$k$**?**
**?**

$$\alpha_0{}^{t+\Delta t}y+\alpha_1{}^{t}y+\cdots+\alpha_k{}^{t+(1-k)\Delta t}y=\Delta t[\beta_0 f({}^{t+\Delta t}y,t_{n+1}+\beta_1 f({}^{t}y,t_n)+\cdots+\beta_k f({}^{t+(1-k)\Delta t}y,t_{n+1-k})]$$ (12)

$\alpha_i, \beta_i$

$$\sum_{i=0}^{k}\{\alpha_i{}^{t+(1-i)\Delta t}y+\Delta t\beta_i f({}^{t+(1-i)\Delta t}y,t_{n+1-i})\}=0$$ (13)

$\beta_0=0$ $k$ $k$

$$\ddot{y}=f(\dot{y}(t),y(t),t)=g_0 y(t)+g_1\dot{y}(t)+h(t)$$ (14)

$g_0, g_1 \in$

$$\frac{{}^{n+1}y - {}^{n}y}{\Delta t} = {}^{n}\dot{y}$$

(19)

$${}^{n+1}y = {}^{n}y + \Delta t\,{}^{n}\dot{y} = {}^{n}y + \Delta t f({}^{n}y, t_n)$$

(20)

$t_n\,{}^{n}y t_{n+1}\,{}^{n+1}y$
$\dot{y}(t)$

$${}^{n+1}y = {}^{n}y + \Delta t\,{}^{n+1}\dot{y} = {}^{n}y + \Delta t f({}^{n+1}y, t_n)$$

(21)

${}^{n+1}y f(y,t)$
${}^{n}\dot{y}\,{}^{n+1}\dot{y}$**?**
**?**$(0 \leq$
$\alpha \leq$
$1)$

$${}^{n+\alpha}\dot{y} = (1-\alpha)\,{}^{n}\dot{y} + \alpha\,{}^{n+1}\dot{y}$$

(22)

$${}^{n+1}y - {}^{n}y = \Delta t\,{}^{n+\alpha}\dot{y}$$

(23)

$${}^{n+1}y = {}^{n}y + \Delta t[(1-\alpha)\,{}^{n}\dot{y} + \alpha\,{}^{n+1}\dot{y}]$$

(24)

$\alpha =$
$0 \alpha =$
$1 \alpha =$
$\frac{1}{2}$

$${}^{n+1}y = {}^{n}y + \frac{\Delta t}{2}({}^{n}\dot{y} + {}^{n+1}\dot{y})$$

(25)

$$M\dot{d} + Kd = F$$

(26)

${}^{n+1}\dot{d}+$
$K\,{}^{n+1}d =$
${}^{n+1}F$
${}^{n+1}d =$
${}^{n}d+$
$\Delta t\,{}^{n+\alpha}\dot{d}$
${}^{n+\alpha}\dot{d} =$
${}^{n}\dot{d}(1-$
$\alpha)+$
${}^{n+1}\dot{d}\alpha$
$dF$

$$(M + \alpha\Delta t K)\,{}^{n+1}d = [M - (1-\alpha)\Delta t K]\,{}^{n}d + \Delta t[{}^{n+1}F\alpha + {}^{n}F(1-\alpha)]$$

(27)

$\alpha =$
$\theta\theta$**?**
$y\alpha$

$${}^{n+\alpha}y = {}^{n+1}y\alpha + {}^{n}y(1-\alpha)$$

(28)

$t\beta$

$$t_{n+\beta} = \beta t_{n+1} + (1-\beta)t_n$$

(29)

$y[t_n, t_{n+1}]\,{}^{n+1/2}y$

$${}^{n+1/2}y = \frac{{}^{n+1}y + {}^{n}y}{2}$$

(30)

$$t_{n+\frac{1}{2}} = \frac{t_{n+1} + t_n}{2}$$

(31)

$${}^{n+1/2}y \neq y(t_{n+\frac{1}{2}})$$

(32)

${}^{n+1/2}y$
[**?**]

${}^{n+1}y =$
${}^{n}y+$
$\Delta t\,{}^{n+1/2}\dot{y} =$
${}^{n}y+$
$\Delta t f({}^{n+1/2}y, t_{n+\frac{1}{2}})$
${}^{n+1/2}y =$