??

$$M\ddot{U}(t)+C\dot{U}(t)+KU(t)=F(t)$$ (1)

??

$$U(0)={}^0U\dot{U}(0)={}^0\dot{U}$$ (2)

??β**?**θ**???**

?
?
?
??
?

??$M,C,KF(t)tU(t)(\cdot)\dot{U}(t),\ddot{U}(t)$
$\dot{U}(t)=$
$V(t)$**??**

$$\dot{U}(t)\dot{V}(t)=\mathbf{0}\mathbf{I}-M^{-1}K-M^{-1}CU(t)V(t)+\mathbf{0}M^{-1}F(t)$$ (3)

$$U(0)={}^0UV(0)={}^0\dot{U}$$ (4)

$\mathbf{0}\mathbf{I}^1$**????**
???

$$\ddot{u}(t)+2\xi\omega\dot{u}(t)+\omega^2u(t)=f(t)$$ (5)

$\xi,\omega f(t)u(t)$
?
?

$$\{\dot{y}_1(t)=\lambda_2y_1(t)+f(t)\dot{y}_2(t)=\lambda_1y_2(t)+f(t)$$ (6)

$$\lambda_{1,2}=-\xi\omega\pm j\sqrt{1-\xi^2}\omega$$ (7)

$j=\sqrt{-1}0\leq\xi<1u(t)y_1(t),y_2(t)$

$$\{y_1(t)=\dot{u}(t)-\lambda_1u(t)y_2(t)=\dot{u}(t)-\lambda_2u(t)$$ (8)

$$u(t)=-\frac{y_1(t)-y_2(t)}{\lambda_1-\lambda_2}$$ (9)

$\lambda_{1,2}y_{1,2}(t)u(t)$
$y(0)=y(t_0=0)$

$$\dot{y}(t)=f(y,t)$$ (10)

$t\in[0,T][0,T]$

$$[0,T]=\bigcup_{n=0}^{N-1}[t_n,t_{n+1}]$$ (11)

$\Delta t=t_{n+1}-t_nt_nt_{n+1}$
??k**?**
?

$$\alpha_0{}^{t+\Delta t}y+\alpha_1{}^ty+\cdots+\alpha_k{}^{t+(1-k)\Delta t}y=\Delta t[\beta_0f({}^{t+\Delta t}y,t_{n+1}+\beta_1f({}^ty,t_n)+\cdots+\beta_kf({}^{t+(1-k)\Delta t}y,t_{n+1-k})]$$ (12)

α_i,β_i

$$\sum_{i=0}^{k}\{\alpha_i{}^{t+(1-i)\Delta t}y+\Delta t\beta_if({}^{t+(1-i)\Delta t}y,t_{n+1-i})\}=0$$ (13)

$\beta_0=0kk$

$$\ddot{y}=f(\dot{y}(t),y(t),t)=g_0y(t)+g_1\dot{y}(t)+h(t)$$ (14)

$g_0,g_1\in$

$$\frac{^{n+1}y - {}^{n}y}{\Delta t} = {}^{n}\dot{y}$$
(19)
$$^{n+1}y = {}^{n}y + \Delta t\, {}^{n}\dot{y} = {}^{n}y + \Delta t f({}^{n}y, t_n)$$
(20)
$t_n\, {}^{n}y\, t_{n+1}\, {}^{n+1}y$
$\dot{y}(t)$

$$^{n+1}y = {}^{n}y + \Delta t\, {}^{n+1}\dot{y} = {}^{n}y + \Delta t f({}^{n+1}y, t_n)$$
(21)
$^{n+1}y f(y,t)$
$^{n}\dot{y}\, {}^{n+1}\dot{y}$ **?**
? $(0 \leq \alpha \leq 1)$

$$^{n+\alpha}\dot{y} = (1-\alpha)\,{}^{n}\dot{y} + \alpha\, {}^{n+1}\dot{y}$$
(22)
$$^{n+1}y - {}^{n}y = \Delta t\, {}^{n+\alpha}\dot{y}$$
(23)
$$^{n+1}y = {}^{n}y + \Delta t[(1-\alpha)\,{}^{n}\dot{y} + \alpha\, {}^{n+1}\dot{y}]$$
(24)
$\alpha = 0$ $\alpha = 1$ $\alpha = \frac{1}{2}$

$$^{n+1}y = {}^{n}y + \frac{\Delta t}{2}({}^{n}\dot{y} + {}^{n+1}\dot{y})$$
(25)

$$M\dot{d} + Kd = F$$
(26)
$^{n+1}\dot{d} + K\, {}^{n+1}d = {}^{n+1}F$
$^{n+1}d = {}^{n}d + \Delta t\, {}^{n+\alpha}\dot{d}$
$^{n+\alpha}\dot{d} = {}^{n}\dot{d}(1-\alpha) + {}^{n+1}\dot{d}\alpha$
dF

$$(M + \alpha \Delta t K)\, {}^{n+1}d = [M - (1-\alpha)\Delta t K]\, {}^{n}d + \Delta t[{}^{n+1}F\alpha + {}^{n}F(1-\alpha)]$$
(27)
$\alpha = \theta\theta$ **?**
$y\alpha$

$$^{n+\alpha}y = {}^{n+1}y\alpha + {}^{n}y(1-\alpha)$$
(28)
$t\beta$

$$t_{n+\beta} = \beta t_{n+1} + (1-\beta)t_n$$
(29)
$y[t_n, t_{n+1}]\, {}^{n+1/2}y$

$$^{n+1/2}y = \frac{^{n+1}y + {}^{n}y}{2}$$
(30)
$$t_{n+\frac{1}{2}} = \frac{t_{n+1} + t_n}{2}$$
(31)

$$^{n+1/2}y \neq y(t_{n+\frac{1}{2}})$$
(32)
$^{n+1/2}y$
[?]
$^{n+1}y = {}^{n}y + \Delta t\, {}^{n+1/2}\dot{y} = {}^{n}y + \Delta t f({}^{n+1/2}y, t_{n+\frac{1}{2}})$
$^{n+1/2}y =$
$^{n}y+$

‡

$^{t+\Delta t}\ddot{U} t+ \Delta t$

$^{t+\Delta t}\ddot{U} +$

$C^{t+\Delta t}\dot{\widetilde{U}} +$

$K^{t+\Delta t}\widetilde{U} =$

$^{t+\theta_0}F$

$^{t+\Delta t}\dot{\widetilde{U}} = {}^{t}\dot{U} + (2\theta_1 - \frac{3}{2}){}^{t}\ddot{U}\Delta t + (\frac{3}{2} - \theta_1){}^{t+\Delta t}\ddot{U}\Delta t$

$^{t+\Delta t}\widetilde{U} = {}^{t}U + \theta_1{}^{t}\dot{U}\Delta t + \frac{1}{4}(2 - \frac{1}{\theta_1^2}){}^{t}\ddot{U}\Delta t^2 + \frac{1}{4\theta_1^2}{}^{t+\Delta t}\ddot{U}\Delta t^2$

$^{t+\Delta t}\widetilde{U}\,{}^{t+\Delta t}\dot{\widetilde{U}}\,{}^{t+\Delta t}\ddot{U}$

$^{t+\Delta t}\dot{U} = {}^{t}\dot{U} + (\theta_1 - \frac{1}{2}){}^{t}\ddot{U}\Delta t + (\frac{3}{2} - \theta_1){}^{t+\Delta t}\ddot{U}\Delta t$

$^{t+\Delta t}U = {}^{t}U + {}^{t}\dot{U}\Delta t + (\frac{1}{2} - \frac{1}{4\theta_1^2}){}^{t}\ddot{U}\Delta t^2 + \frac{1}{4\theta_1^2}{}^{t}\ddot{U}\Delta t^2$

θ_1

?

^{3}?a

$$(\Lambda_6 W_1 M + \Lambda_5 W_2 C\Delta t + \Lambda_3 W_3 K\Delta t^2)\Delta\ddot{U} = -M{}^{t}\ddot{U} - C({}^{t}\dot{U} + \Lambda_4 W_1{}^{t}\ddot{U}\Delta t) - K({}^{t}U + \Lambda_1 W_1{}^{t}\dot{U}\Delta t + \Lambda_2 W_2{}^{t}\ddot{U}\Delta t^2) + (1 - W_1){}^{t}F + W \tag{90}$$

$^{t+\Delta t}U = {}^{t}U + \lambda_1{}^{t}\dot{U}\Delta t + \lambda_2{}^{t}\ddot{U}\Delta t^2 + \lambda_3\Delta\ddot{U}\Delta t^2$

$^{t+\Delta t}\dot{U} = {}^{t}\dot{U} + \lambda_4{}^{t}\ddot{U}\Delta t + \lambda_5\Delta\ddot{U}\Delta t$

$^{t+\Delta t}\ddot{U} = {}^{t}\ddot{U} + \Delta\ddot{U}$

‡

$\Lambda_i = \lambda_i, i = 1, 2, \cdots, 5$

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$^{t+\Delta t}U = {}^{t}\dot{U} + \frac{\Delta t^2}{2}{}^{t+\Delta t}\ddot{U}$

$^{t+\Delta t}\dot{U} = \frac{3}{2\Delta t}{}^{t+\Delta t}U - \frac{2}{\Delta t}{}^{t}U + \frac{1}{2\Delta t}{}^{t-\Delta t}U$

$t + \Delta t$??

$$M^{t+\Delta t}\ddot{U} + C^{t+\Delta t}\dot{U} + K^{t+\Delta t}U = {}^{t+\Delta t}F \tag{91}$$

$$^{\Delta t}\dot{U} = \frac{3}{2\Delta t}({}^{\Delta t}U - {}^{0}U) - \frac{1}{2}{}^{0}\dot{U} + \frac{\Delta t}{4}{}^{0}\ddot{U} \tag{92}$$

?$\beta =$

$\gamma =$