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NeuroDataReHack, Janelia Research Campus

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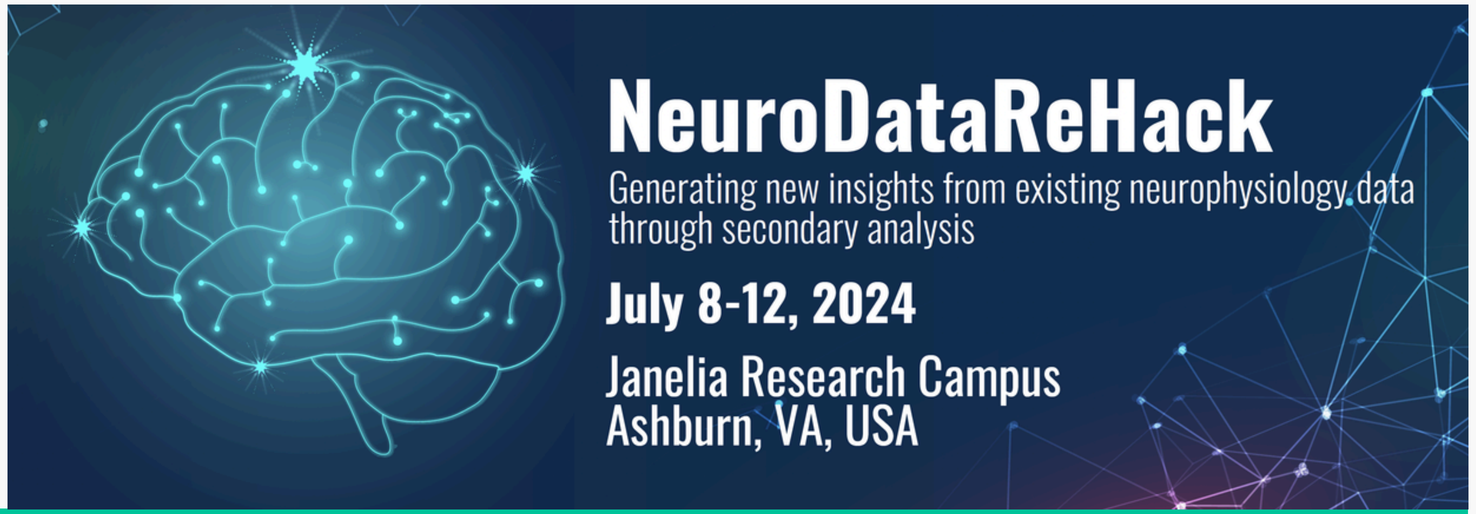
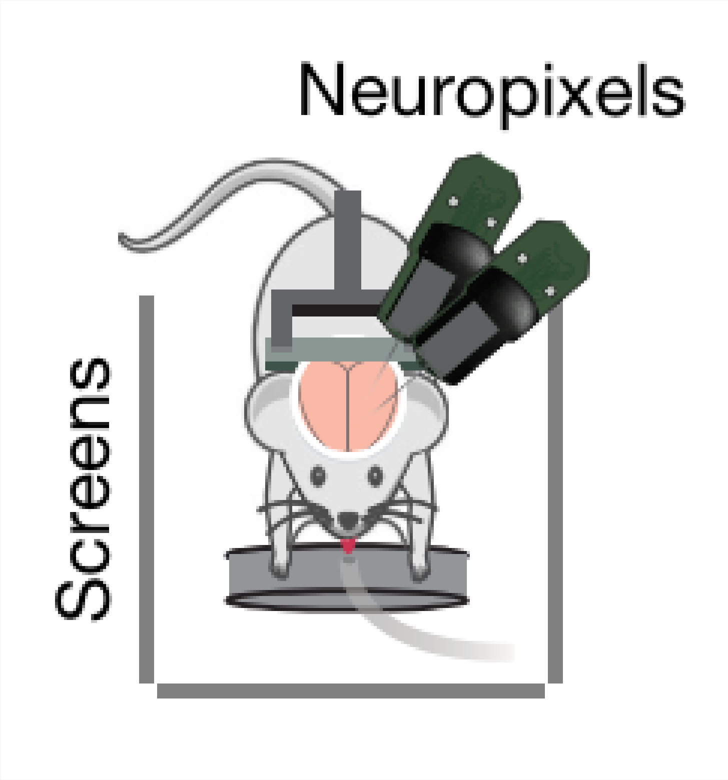
Estimating Partial Coherence Networks from Neural Spike Trains

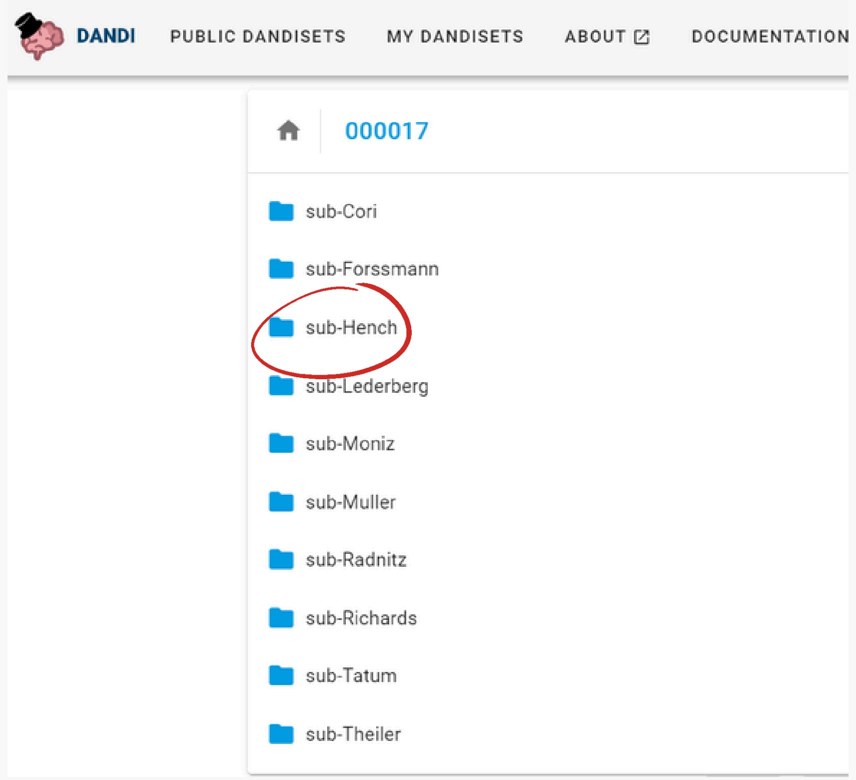


I DANDISET 000017 - Distributed coding of choice, action and engagement across the mouse brain

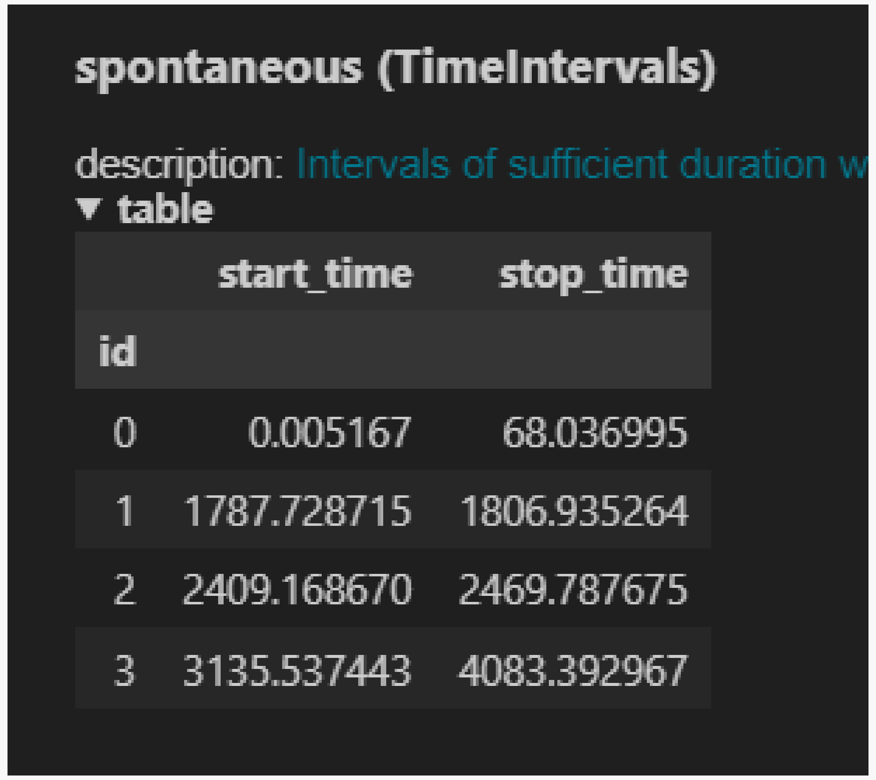
- Neuropixels probes were used to record from ~30,000 neurons from 42 brain regions in 10 mice



- We analyse data from a single mouse. Neuropixels probes were used to record units from 13 brain regions



- Firing times of individual neurons were identified using Kilosort and phy. We analyse spontaneous firing activity


spontaneous (TimeIntervals)

description: Intervals of sufficient duration w

▾ table

id	start_time	stop_time
0	0.005167	68.036995
1	1787.728715	1806.935264
2	2409.168670	2469.787675
3	3135.537443	4083.392967

<u>Question:</u> How can we estimate interactions between neural spike trains?

I Methods

II Spectral Analysis for Multivariate Point Processes

Neural spike trains can be represented by a multivariate point process

$$\mathbf{N}(t) := \{N_1(t), \ldots, N_p(t)\}.$$

Spike times in unit p

The first order properties of a multivariate point process are characterised by the intensity function

$$\boldsymbol{\Lambda}(t) := \frac{\mathbb{E}\{d\mathbf{N}(t)\}}{dt}.$$

The second order properties are captured by the covariance density matrix

$$\boldsymbol{\mu}(t,u) = \frac{\mathbb{E}\{d\mathbf{N}(u)d'\mathbf{N}(t)\}}{dtdu} - \boldsymbol{\Lambda}(u)\Lambda'(t).$$

The spectral density matrix is defined as the Fourier transform of the covariance density matrix

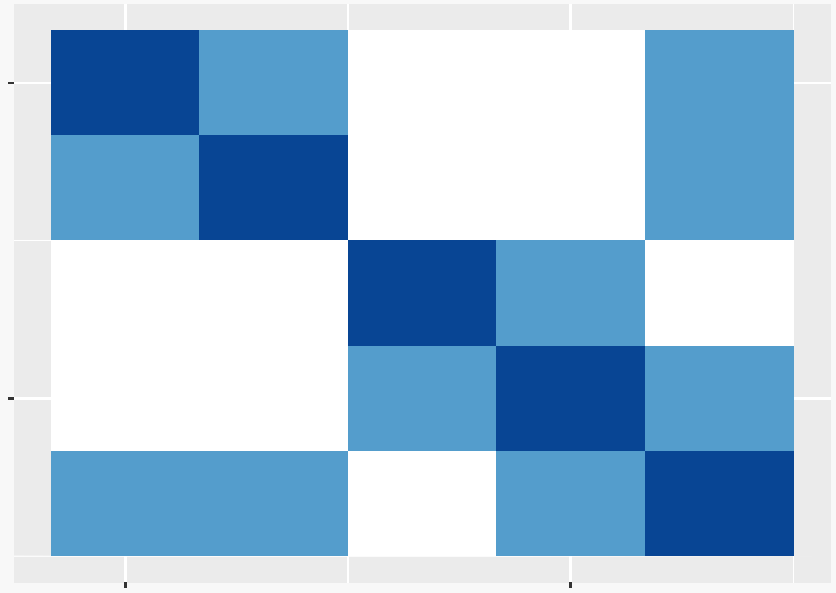
$$\mathbf{S}(\omega) = \tfrac{1}{2\pi}\{\mathrm{diag}(\boldsymbol{\Lambda}) + \textstyle\int_{-\infty}^{\infty} e^{-i\tau\omega}\boldsymbol{\mu}(\tau)d\tau\}.$$

Key Assumption

The multivariate point process is assumed to be second order stationary.

III Estimating Partial Coherence Networks

Interactions between components of the multivariate process can be captured by the inverse spectral density matrix $\boldsymbol{\Theta}(\omega) := \mathbf{S}^{-1}(\omega)$.



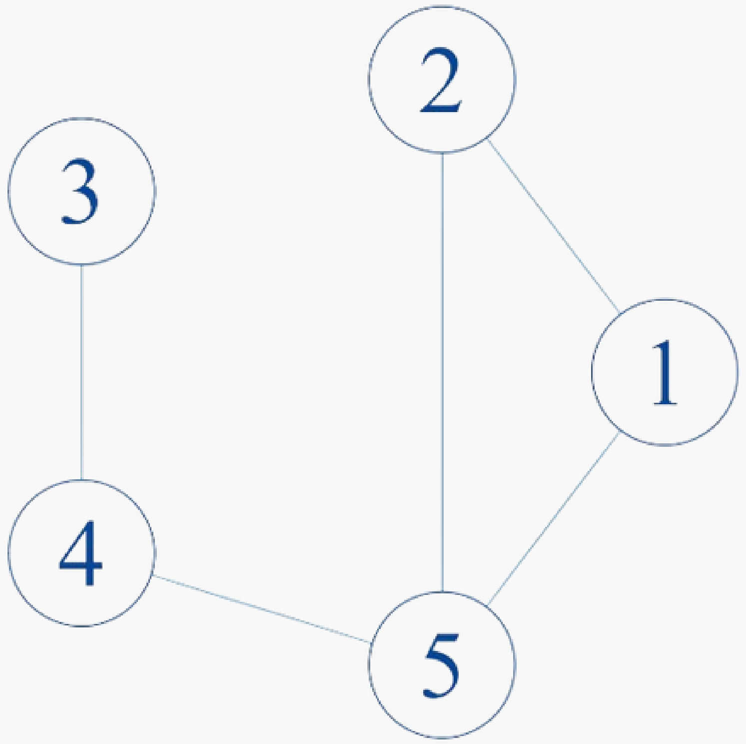
The partial coherence, defined as

$$\rho_{ij}(\omega) = \frac{|\Theta_{ij}(\omega)|^2}{\Theta_{ii}(\omega)\Theta_{jj}(\omega)},$$

provides a measure of the partial correlation structure between pairs of neural processes in the frequency domain.

We estimate the inverse spectral density matrix via

$$\hat{\Theta}(\omega) := \mathrm{argmin}_{\Theta(\omega)\in\mathcal{C}} \left\{ -\log\det(\Theta(\omega)) + Tr\{\hat{S}(\omega)\Theta(\omega)\} + \lambda\|\Theta(\omega)\|_1 \right\}.$$



- The regularisation parameter determines the level of sparsity
- Solve the above optimisation problem with the alternating direction method of multipliers (ADMM) algorithm

II Analysis

IV Spectral Estimation for Multivariate Point Processes

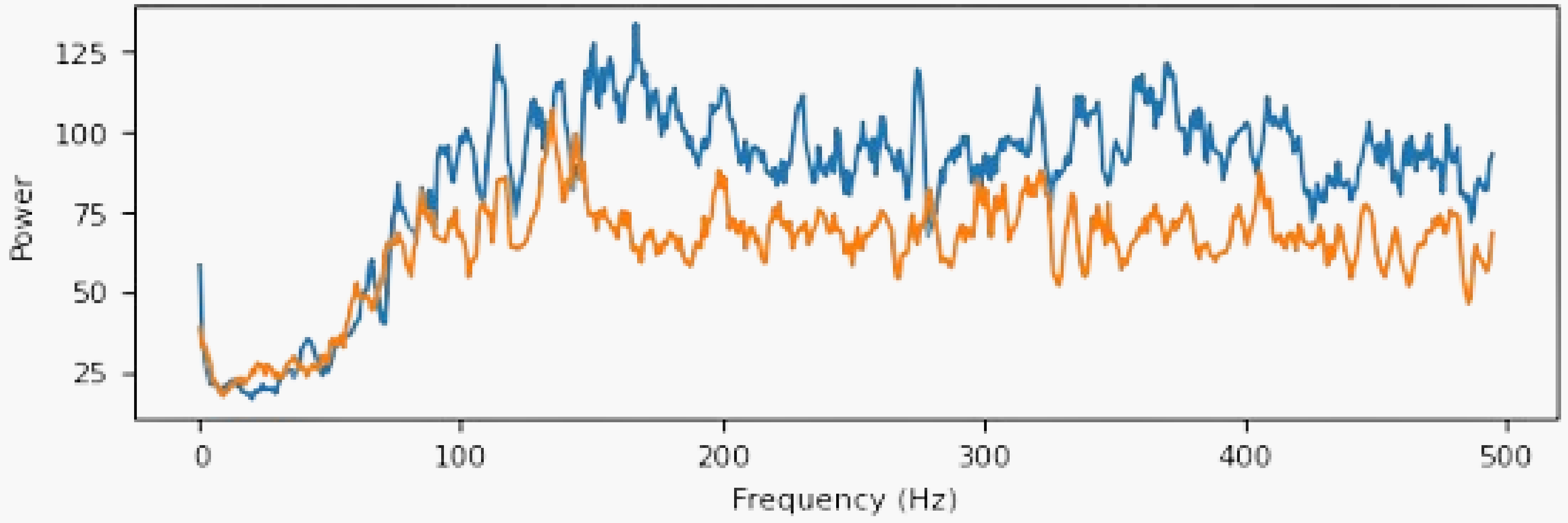
The Tapered Fourier transform of $N_j(t)$ for $t \in (0, T]$ is

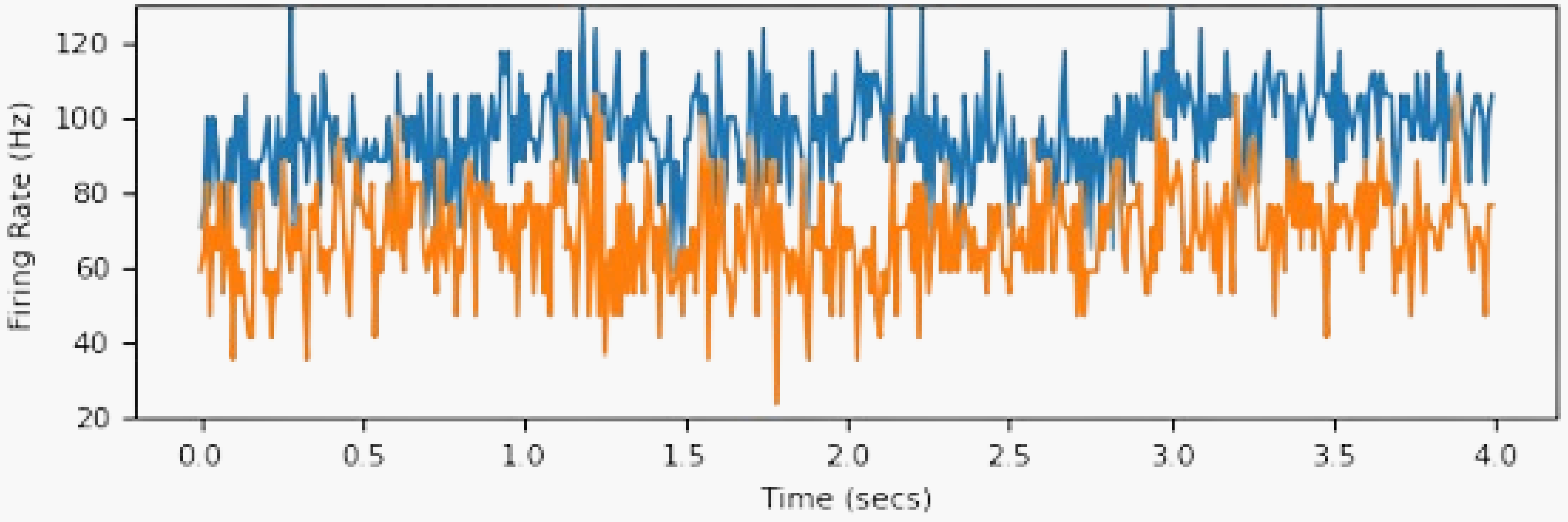
$$d_{l,j}(\omega) = \int_0^T h_l(t/T) e^{-i\omega t} dN_j(t),$$

for a set of $l = 1, \ldots, m$ taper functions $h_l(z) : (0, 1] \to \mathbb{R}$. We estimate the spectrum via

$$\hat{S}(\omega) = \frac{1}{m} \sum_{l=1}^{m} \bar{\mathbf{d}}_l(\omega) \bar{\mathbf{d}}_l^H(\omega),$$

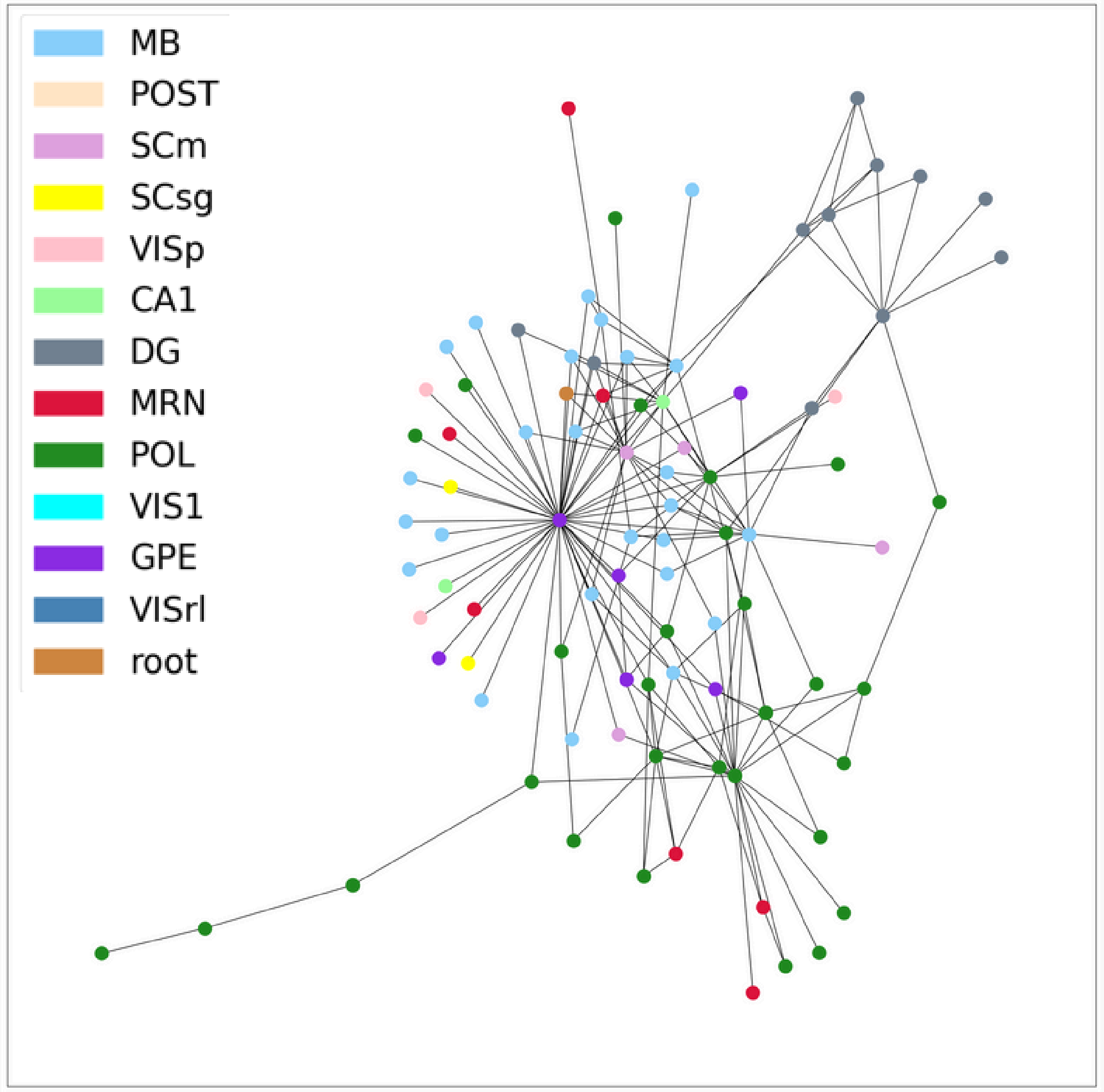
where $\bar{\mathbf{d}}_l(\omega) = (\bar{d}_{l,1}(\omega), \ldots, \bar{d}_{l,p}(\omega))$ are the mean corrected coefficients.



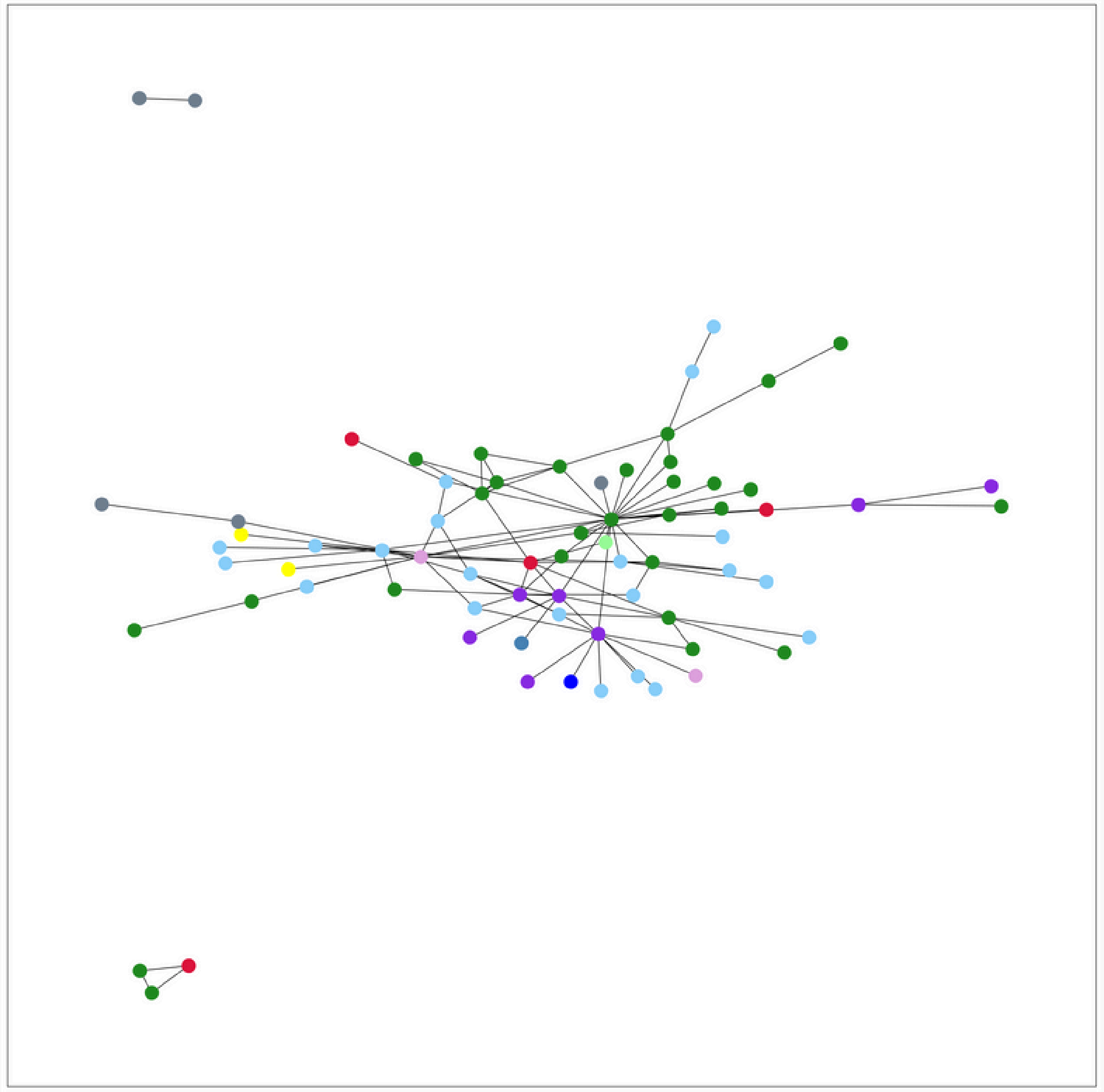


IV Preliminary Results I

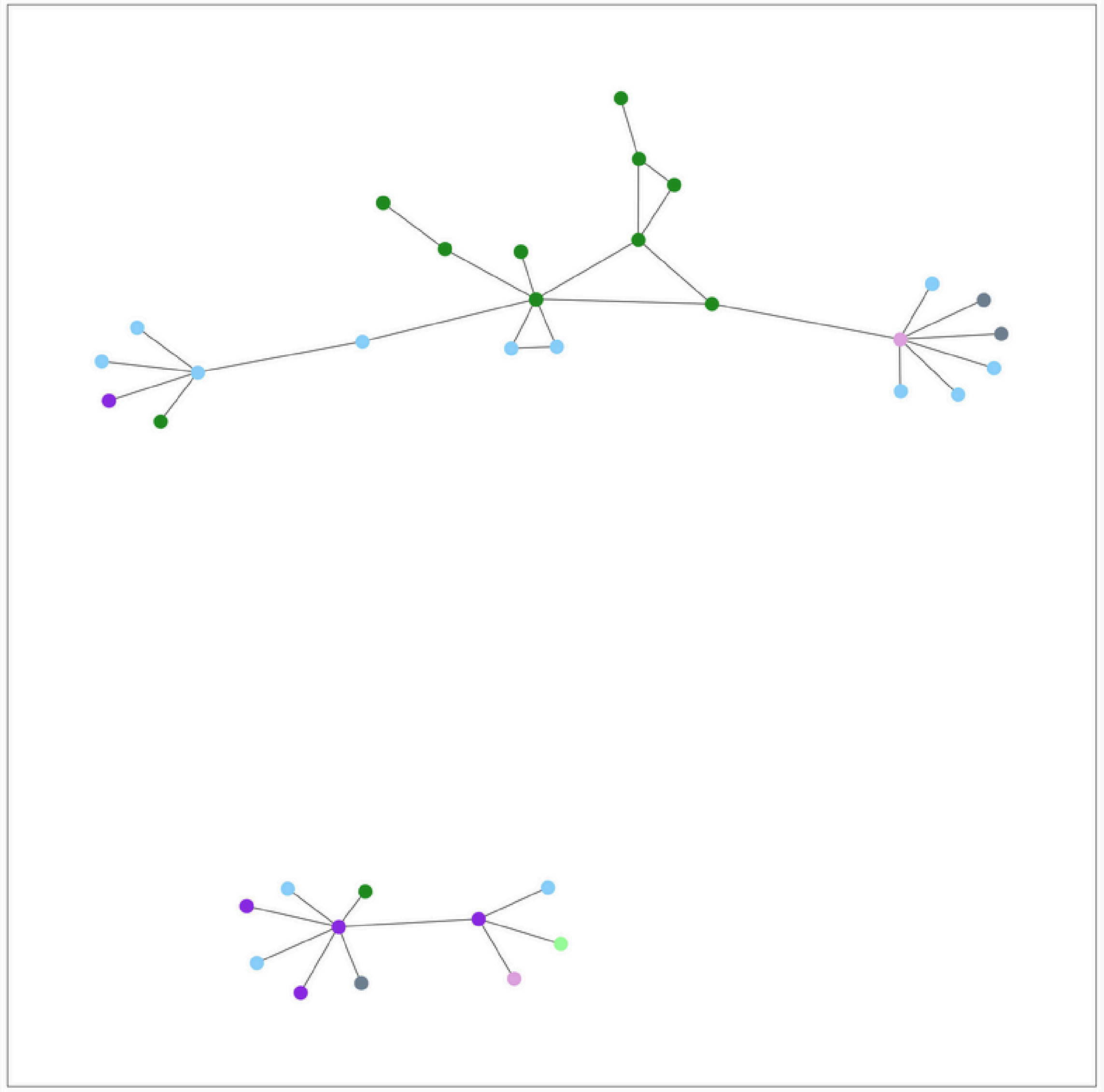
(0,25)Hz



(25,100)Hz

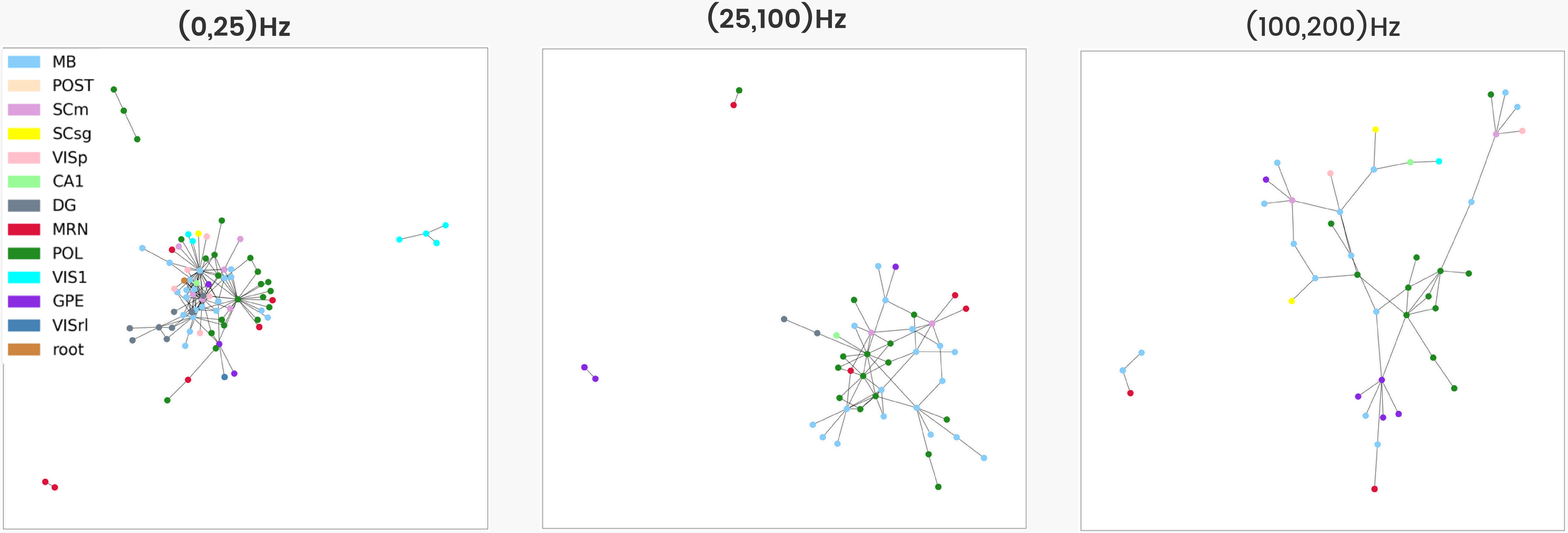


(100,200)Hz



Number of edges	174	105	36
Regularisation Parameter	0.42	0.22	0.24

IV Preliminary Results II



Number of edges	149	65	46
Regularisation Parameter	0.46	0.24	0.24

V Discussions and Future Work

- We have developed a tool to estimate high-dimensional inverse spectral density matrices in the point process framework which can be used to infer neural connectivity in the brain network
- Python package is currently under development and will hopefully be available very soon

References

- Pinkney, C., Euan, C., Gibberd, A. and Shojaie, A., 2024. Regularised Spectral Estimation for High Dimensional Point Processes. arXiv preprint arXiv:2403.12908
- Steinmetz, Nicholas; Zatka-Haas, Peter; Carandini, Matteo; Harris, Kenneth; Wang, Renee (2024) Distributed coding of choice, action and engagement across the mouse brain (Version 0.240329.1926) [Data set]. DANDI archive. https://doi.org/10.48324/dandi.000017/0.240329.1926

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Thank you for listening!

4. Synthetic Experiments

We evaluated the **performance** of the RSE on **synthetic data** where the **true spectrum** is known.

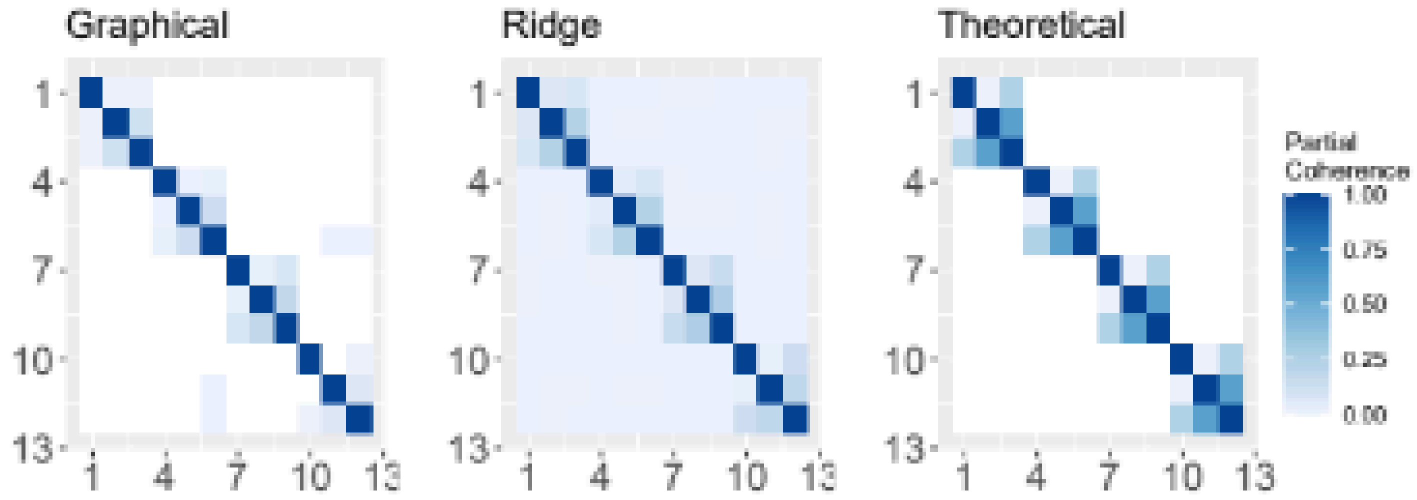

Figure 2. Simulation results for estimating the partial coherence matrix.

The **Graphical** estimator is preferable for **sparse estimation** of the inverse spectrum. Both estimators outperform existing methods which break down in **high dimensional** settings.

		Mean Squared Error				F_1 Score	
p	m	Inverted Periodogram	Ridge	G_1	G_2	G_1	G_2
12	10	-	2.36 (0.01)	1.86 (0.01)	4.37	0.32	0.81
	50	1.59	1.58 (0.01)	1.53	4.20	0.31	0.97
48	10	-	0.70	0.37	1.03	0.13	0.74
	50	6218.81 (151.89)	0.59	0.31	1.01	0.09	0.98
96	10	-	0.43	0.19	0.51	0.10	0.71
	50	-	0.28	0.14	0.50	0.05	0.96

Table 1. *Simulation results over 100 replications for estimating the inverse spectral density matrix. All results are recorded at a particular frequency $\omega = 0.0628$ and are in the form of mean (standard error). Standard errors of $< 10^{-2}$ are omitted for brevity. Hyphenated entries (-) denote that the multi-taper periodogram matrix could not be inverted. G_1 and G_2 refer to the Graphical estimator tuned using the MSE and F_1 score respectively.*