EMSC3032 – Computing Assignment 4

Q1 (10 marks)

Parts of a least squares solution are available from the analysis of 14-days of GRACE spanning 1-15 September 2010 [i.e. $(A^tWA)^{-1}$ and A^tWb], providing information that permit the estimation of the temporal change in mass of 7527 mascons across the Earth. These matrices have been generated from the analysis of real GRACE data; therefore, they contain both signal and noise/error.

Your task is to invert the matrices and produce an estimate of the change in the temporal gravity field relative to the static gravity field used to model the satellite orbits (reference epoch 2009.0).

Each mascon parameter represents a change equivalent water height required over the entire mascon region. The units are in metres.

It may be necessary to apply regularisation constraints on the matrix inversion in order to stabilise numerically the solution and to mitigate the noise/error contained in the observations and modelling. You can choose how to apply the regularisation – try a number of different options to see what effect it has on the solution for the gravity field anomalies.

i) derive and plot the unregularised solution for the gravity field anomalies for this 15-day period (Day 2010-09-06 is not included in the normal equations)

ii) derive and plot your best global estimate of the mass changes
   a. using Tikhonov regularisation
   b. using region-specific regularisation. What values did you assign to each region?

iii) What is your best estimate for the gravity field anomalies and how did you regularise your solution to derive your best estimate? Justify your choice.

Note: Different mascons have different spatial areas but you can represent each mascon as a single dot when plotting your estimated values.

Information

Files provided on wattle:

AtWA.data.gz	: (gzipped) AtWA matrix
AtWb.data.gz	: (gzipped) AtWb matrix
mascons.latlon	: coordinates of the centres of mass of each mascon
mascons.regions	: information to associate each mascon with a specific geographical region (land, ocean, Antarctica, Alaska, Greenland)

Regularisation

A regularised least squares solution is found using:

$$\hat{x} = (A^t W A + C)^{-1} A^t W b$$

where C is the regularisation matrix.

C can take the form of

1. a "Tikhonov" regularisation, where the same number is used for each diagonal element (i.e. $C = \lambda I$), where λ is chosen to provide the best-possible solution for the parameter estimates.
2. n/σ^2 where n = 14 (the number of days in the solution) and σ can be assigned a different value for different geographical regions (e.g. regularise continental Antarctica differently from the global oceans)
3. Use a diagonal C matrix. That is, do NOT use non-zero values for the off-diagonal elements of the C matrix.
4. Only the mascon parameters need to be regularised. The C matrix diagonal elements should be left as 0 for the orbital parameters

Format of AtWA and AtWb:

1. There are 7527 mascon parameters and 336 orbital parameters (12 parameters per satellite per day) contained in the matrices, making 7863 parameters in total.
2. The AtWA.data file contains two header lines, then the full 7863x7863 normal equation matrix
3. The AtWb.data file contains a 7863x1 matrix