X $\nearrow$ input

y $\nearrow$ output

θ $\nearrow$ parameters

$X \rightarrow$ dice topface.

$1/6, 1/6, 1/6, \frac{1}{6}, 1/6, \frac{1}{6}$

$P(X=1) = 1/6$

$p(1) = 1/6$

$X -$ dice $\qquad$ $Y -$ coin

$P(X, Y)$

$P(X=1, Y=h)$

$P(X=1, Y=t)$

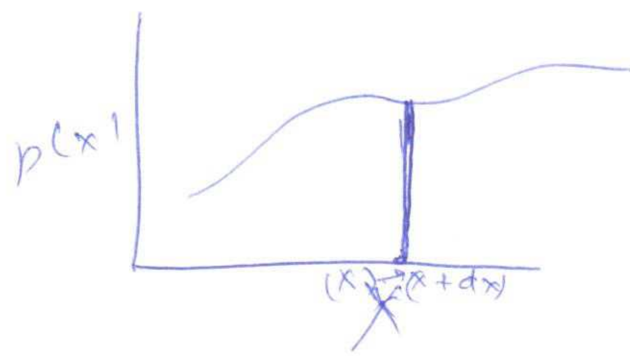
$\vdots$

$$P(Y=1 \mid X=1) = \frac{P(X=1 \mid Y=1)\,P(Y=1)}{\left(P(X=1 \mid Y=0)\,P(Y=0) + P(X=1 \mid Y=1)\,P(Y=1)\right)}$$

have cancer   tested +ve

$$P(Y=0 \mid X=1) = \frac{P(X=1 \mid Y=0)\,P(Y=0)}{P(X=1 \mid Y=0)\,P(Y=0) + P(X=1 \mid Y=1)\,P(Y=1)}$$

$p(x)$



X — {red, blue}

$P(X=\text{red}) = 0.7 \quad P(X=\text{blue}) = 0.3$

$$g(x) = \begin{cases} 8 & \text{if } x = \text{red} \\ 9 & \text{if } x = \text{blue} \end{cases}$$

$$\boxed{E[g(x)] = 8 * 0.7 + 9 * 0.3}$$

$X \in \mathcal{X}$ x

$$\boxed{P(X=x)}$$

$|\mathcal{X}|$: Finite

X - Discrete

p.m.f.

$P(X=x) = pmf(x)$

$|\mathcal{X}| = \infty$

X - Continuous

p.d.f.

$P(x \leq X \leq x+dx) = pdf(x)$

$p(x)$

$p_X(x)$

$p_Y(x)$

$$\sum_{x \in \mathcal{X}} p(x) = 1$$

$$\int_{x=-\infty}^{x=\infty} p(x)\,dx = 1$$

$E[X]$

$$\sum_{x \in \mathcal{X}} x\,p(x)$$

$$\int_{-\infty}^{\infty} x\,p(x)\,dx$$

$\mathcal{X} =$ 1, 2, 3, 4, 5, 6

$\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$

$E[X] = 1\times\frac{1}{6} + 2\times\frac{1}{6} + 3\times\frac{1}{6} + 4\times\frac{1}{6} + 5\times\frac{1}{6} + 6\times\frac{1}{6} = 3.5$

$f(x)$

$$E[f(x)] = \sum_{x \in X} f(x)\, p(x) \qquad \Big| \qquad \int_{-\infty}^{\infty} f(x)\, p(x)\, dx$$

e.g $f(x) = 2x$

$$E[f(x)] = 2 \times \frac{1}{6} + 4 \times \frac{1}{6} + \cdots = \_\_\_$$

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① $E[x] \rightarrow$ mean of $X \rightarrow \mu$

② $E[(X-\mu)^2] = E[(X - E[x])^2]$

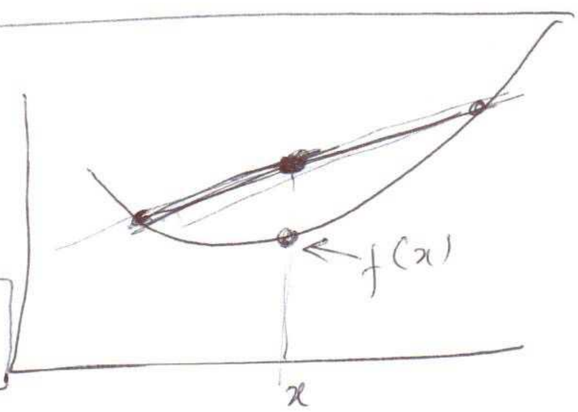
$\rightarrow$ variance of X

$var(X)$

$X \quad \varphi(x)$

$\rightarrow$ Convex

$$\boxed{\varphi(E[X]) \leq E[\varphi(x)]}$$



Binomial.

$$p(k) = Bin(k|n, \theta)$$
$$= \binom{n}{k} \theta^k (1-\theta)^{n-k}$$

E.g $n = 4$, $\theta = 0.2$

$$p(0) = \binom{4}{0} 0.2^0 (1-0.2)^{4-0}$$
$$= (0.8)^4$$
$$p(1) = \binom{4}{1} 0.2^1 (1-0.2)^{4-1}$$

Bernoulli. $n = 1$

$$p(k) = \theta^k (1-\theta)^{1-k}$$
$$\cancel{p(0) = \theta^0}$$
$$p(0) = 1-\theta$$
$$p(1) = \theta$$