Data Analysis Exercise 5: Comparison of detectors for small X-ray beams

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1 Introduction

Stereotactic radiotherapy is a very precise technique used for the treatment of small brain tumors and requires narrow X-ray beams. A study was carried out at the Western General Hospital in Edinburgh to examine the effect of using circular cylinders attached to the head of the accelerator (called collimators) to produce these beans. The collimators used in the study had internal diameters ranging from 5 to 40 mm. The aim of the study was to measure the output of the X-ray beans, for which 3 different types of detector (Diamond, EFD, PFD) were used to measure the X-ray dose, and then compare the precision of the three types of detector. The study compared doses that were recorded using the three types of detector over the range of collimator diameters. The data contains the values of the collimator diameter (mm) and the corresponding X-ray dose measurement with the type of detector used along with the day on which the measurement was recorded. For each day when the measurements were made, the experiment always started with a collimator diameter of 40 mm and a value of 1 for the X-ray measurement. This is because the reading at the largest diameter of 40 mm was thought to be the most precise and was treated as a reference measurement. All the X-ray doses measured on the same day were standardized by dividing by the X-ray dose measured with the largest collimator on the same day. Note that while it is possible to assess whether the day has an effect on the response, there is generally too few measurements per day for a potential difference to be significant. Furthermore, it is reasonable to think that the day would have no impact on the measurements as the measurements are undertaken under the same conditions (temperature, humidity and pressure, for instance). Two models are used to fit the data - the *Exponential* and *Hill* models respectively. In the models, the Greek letters are the unknowns to be estimated, Y is the X-ray dose and d the collimator diameter.

$$E(Y|d) = \alpha[1 - exp(-\beta - \gamma d)], \tag{1}$$

$$E(Y|d) = \epsilon d^\delta / (\eta + d^\delta). \tag{2}$$

We will first examine whether these models provide a good fit to the data and use the fitted models to compare the precision of the three types of detector. Then, we will choose which model offers a better fit. Finally, for this model, we will test whether the data support a common diameter-response (dose) relationship, in other words, we will test whether the type of detector has an effect on the dose measured.

2 Statistical analysis

2.1 Residual plots

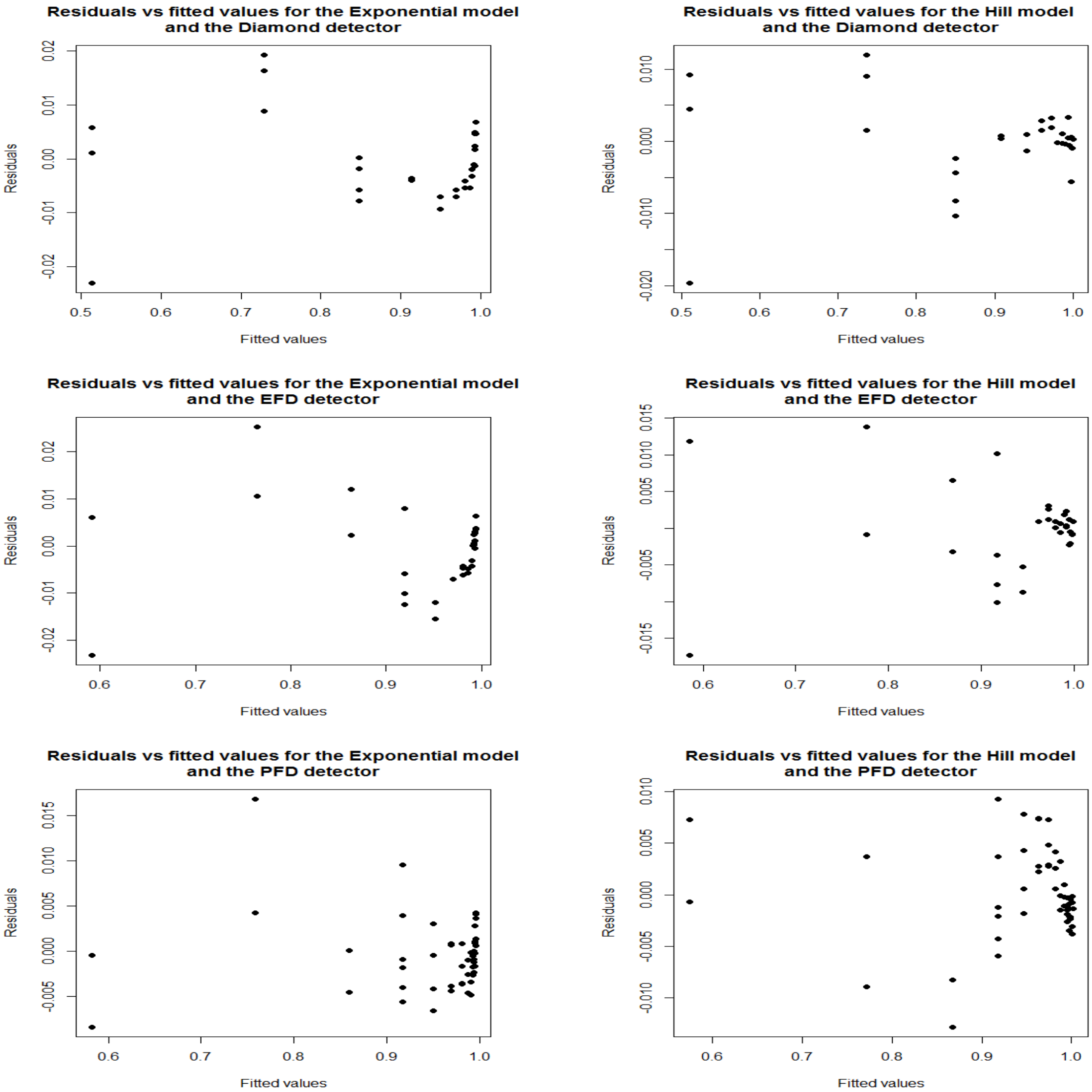

Figure 1: Plots of the residuals vs the fitted values for both the Exponential and Hill models for each type of detectors (Diamond, EFD, PFD).

For both models and each type of detectors, the variance seems larger for low values of the diameters (since small diameter results in small dose measurement) and gets smaller for larger values. Initially, the dose measurement increases rapidly before converging to 1 (the maximal value due to the normalization) which could cause a greater variance in the dose measurement for small diameters. Furthermore, there seems to be a quadratic pattern in the residuals for the Exponential model which is absent for the Hill model. Also note that the data is positively skewed and that there are very few observations with low dose measurements. This may be pointed out as a weakness in the design and analysis of the experiment as more measurements at a small diameter may result in a more accurate prediction. The residual plots indicate that the Hill model would be more adequate to fit the data. Let us now look at the plots of the data with the fitted curves of both models.

2.2 Data plots

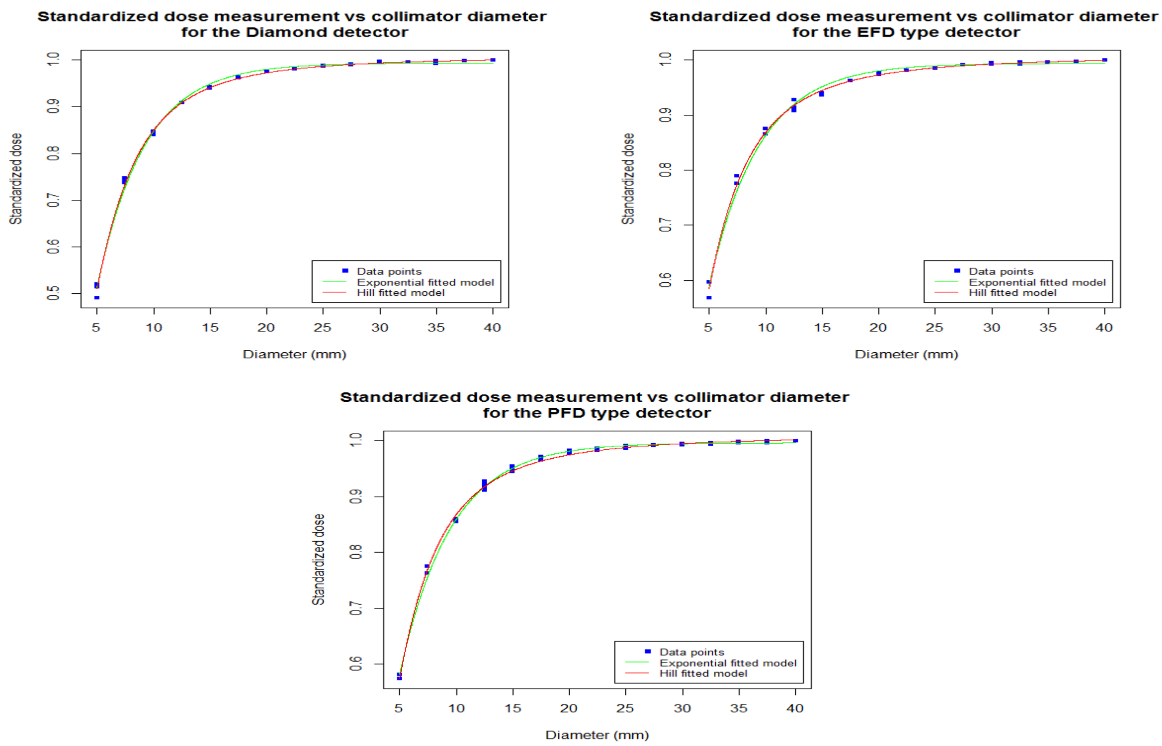

Figure 2: Plots of the data points for each type of detectors with the fitted lines plotted for the Exponential and Hill models.

By looking at the plots in Figure 2, we see that the fitted lines seem to give a good estimate of the relationship between dose and diameter. However it is hard to see which model fits the data better. By simply looking at the 3 plots, it seems that the Exponential model produces an overshoot when the diameter is between 10 and 25 mm. The Hill model seems to go through the data points smoothly. However, one needs to look at the residual standard error in order to statistically assess the quality of a model.

2.3 Residual standard errors

Table 1 below summarizes the residual standard errors for all the detector type and model combinations.

Model	Detector type	Residual standard error
Exponential	Diamond	0.00827
	EFD	0.00906
	PFD	0.00424
Hill	Diamond	0.00585
	EFD	0.00581
	PFD	0.00428

Table 1: Residual standard errors for both models and each type of detector.

As we can see from Table 1, the residual standard errors are small. Keeping in mind that the plots of residuals were poor, especially for the Exponential model, one can argue that the models offer a reasonable fit to the data. In order to compare the precision of the three types of detector, under the assumption that for each detector, the standard deviation does not depend on the collimator diameter, one can use the residual standard error. If a detector type is perfectly precise, then there is very little random effect in the dose measurement leading to a small variance. Therefore, for the same model, the best fit will happen where there is the least variation in the response. If the variance is big, then the standard residual error will be big, since it measures the distance between the fitted value and the corresponding observed value. Therefore from Table 1, we see that for both models, the smallest residual standard error happens for the PFD detector type. This means that the use of the PFD detector type leads to less variation in the dose measurements compared to the Diamond and EFD detectors. Therefore, the PFD detector type is the most precise one.

We have seen before that the residuals plots looked slightly better for the Hill model since it did not have a quadratic pattern. Furthermore, we observed in Figure 2 that the Exponential model seemed to produce an overshoot while the Hill model did not. Looking at the Table 1, we see that the residual standard errors for the Hill model are smaller than those of the Exponential model, for each detector type. Therefore, even though the difference is very small, it would appear that the Hill model fit the data better. We thus proceed using the Hill model. We now want to investigate whether the data is consistent with a common diameter-response relationship for the three types of detector, for the Hill model. Table 2 below summarizes the parameter estimates for the Hill model in (2). The null hypothesis H_0 correspond to the model where detector types have no effect on the diameter-dose relationship.

Detector type	Parameter	Estimate(standard error)
Diamond	δ	2.40(0.0360)
	ϵ	1.01(0.00194)
	η	46.3(2.98)
EFD	δ	2.18(0.0421)
	ϵ	1.01(0.00186)
	η	24.1(1.83)
PFD	δ	2.22(0.0270)
	ϵ	1.01(0.00114)
	η	26.8(1.33)
H_0	δ	2.29(0.0415)
	ϵ	1.01(0.00189)
	η	33.4(2.506)

Table 2: Estimate of the parameters and their standard errors for each of the detector type separately and for the null hypothesis H_0.

We want to test whether the data is compatible with H_0. For that, we define a new function that takes into account the effect of the detectors and perform an ANOVA test to compare it with the Hill model under H_0. We get an F-value of 73.723 and a p-value of less than $2.2 \cdot 10^{-16}$ which is extremely significant. Therefore, we reject H_0 and we conclude that the type of detector does have an effect on the diameter-response relationship.

3 Conclusion

We have fitted two models: the Exponential model (1) and the Hill model (2). While it was not easy to decide which model fitted the data better, the Hill model seemed to have the better residuals plots as well as the smaller residual standard error across all types of detector. Looking at the residual standard error for each detector type and for both model, we concluded that the PFD detector offered the best dose measurement precision. It is thus advised to use this type of detector to carry out X-ray dose measurements. Furthermore, for the Hill model, the detector type does have an effect on the X-ray dose measurement and the hypothesis of a common diameter-response relationship for the three types of detector is rejected. In order to improve the design and analysis of the experiment, it may be beneficial to make more measurements using a small diameter (between 5 and 20 mm) in order to obtain a better fitted line through the data points.In addition, one may question the standardization of the dose measurements. Indeed, one cannot know what the original dose measurements were before the scaling and it is not possible to retrieve them from the data recorded. It could have been of interest to know the magnitude of the X-ray dose measurements.