Question 1: 5 points

Under the assumption that the unobservable product characteristics are independent of both product characteristics and price ($E[\xi_j \mid x_j, g_j, p_j] = 0$), estimate this model using the Berry inversion and OLS. Use all the data together (i.e. data from all the markets). Are your estimates reasonable?

Estimate the using the Berry Inversion and OLS.

$$ln\left(\frac{s_j}{s_0}\right) = \beta_0 + \beta_1 x_j + \beta_2 g_j + \alpha p_j + \xi_j$$

Table 1: Estimation results : OLS

Variable	**Coefficient**	**(Std. Err.)**
x	0.778	(0.027)
g	0.062	(0.045)
p	0.711	(0.215)
Intercept	-3.038	(0.421)

The estimates are not very reasonable. In particular the coefficient on price is positive implying that an increase in price will increase the market share of product j. We would expect an opposite effect which would imply a negative value for α, the coefficient on price.

Question 2: 5 points

Argue why we shouldn't assume that price is uncorrelated with unobservable product characteristics and how this might bias the above estimates. Consider two instruments: 1) a marginal cost shifter w_j, and 2) z_j - the total of the number of products in the same group as you. Should these new variables be correlated with p_j? Why? Might they be correlated with ξ_j? Argue both sides of this last question.

Correlation and Bias: We should not assume that price is uncorrelated with unobservable product characteristics since such characteristics are likely to directly effect the price of the good. For example, measures of quality or prestige that are unobserved to the econometrician will directly effect the price that is charged for a good. Omitting a variable such as quality will give an upward bias to the coefficient since higher quality would be positively correlated with price and have a positive effect on the share of the market a product has.

Marginal Cost Shifter: A shift in the marginal cost of product j should should be correlated with the price of product j for the simple reason that we believe that firms regard costs as an important factor when setting price.

The possibility does exist for cost shifters to be correlated with price. This is saying a firm would change the characteristics of the product because of the increased cost in producing the product. For example, if marginal cost increased for one input for a product, a firm may choose to use a cheaper, lower quality part which would affect the unobserved characteristic of quality.

However, it can be plausibly argued that such changes in product characteristics may be inflexible in the short term. Thus, depending on the time frame of the data we might be quite certain that such correlation does not exist. In any case, it seems reasonable to assume that firms will opt for the ease of adjusting the price of product rather than changing the product characteristics.

Number of Products: The number of products in a group should be correlated with the price of a product. If a product faces a larger number of competitors in a market we would expect its price to be higher than if it face few or no competitors. Thus we would expect a negative correlation between price and the total number of products in a group.

We might argue that the number of products in a market are uncorrelated with unobserved characteristics of a product for the same reasons as above. To adapt to a changing market, it is more likely that a firm will change price for their product rather than use the more costly and inflexible alternative of changing product characteristics.

On the other hand, if a firm changes its product characteristics as a market becomes more crowded in order to keep a competitive edge, this would lead to correlation between the number of products and product characteristics. However, this change would need to be systematic, and not random, in its effect. That is firms would always have changed characteristics to decrease price in response to more competition or viceversa. Since the change in characteristics would depend greatly on the details of the situation, it seems likely that the change in characteristics would be random rather than systematic.

Question 3: 5 points

Assuming that the marginal cost shifter is an appropriate instrument (you don't need to use z_j yet), use IV to re-estimate the model. Consider observation 77,78,79,80. Create a matrix of own and cross price elasticities for these four products. To compute these, use the true *market share - not the* predicted *market share at the parameter vector. Note that this implicitly uses your estimated ξ_j (the residual), not $\xi_j = 0$ (its expectation). Using these own price elasticities, compute the implied price-cost markups for the 4 markets (i.e. $p_j - mc_j$). Are these results intuitively appealing? How do these elasticities and markups relate to the characteristics of the 4 products and the characteristics of the market that the four products are in?*

Berry Inversion and 2SLS

Table 2: Estimation results : 2SLS

Variable	**Coefficient**	**(Std. Err.)**
p	-0.714	(0.430)
x	0.799	(0.031)
g	0.066	(0.050)
Intercept	-0.259	(0.841)

Price Elasticities from 2SLS Logit:

$$\begin{array}{c} \\ s_{77} \\ s_{78} \\ s_{79} \\ s_{80} \end{array} \begin{array}{c} \begin{array}{cccc} s_{77} & s_{78} & s_{79} & s_{80} \end{array} \\ \begin{pmatrix} -1.3857 & 0.0494 & 0.0641 & 0.0462 \\ 0.0872 & -1.2572 & 0.0641 & 0.0462 \\ 0.0872 & 0.0494 & -1.3849 & 0.0462 \\ 0.0872 & 0.0494 & 0.0641 & -1.2625 \end{pmatrix} \end{array}$$

Table 3: Price markups: 4 select products

Product	**Markup**	**Char**	**Price**	**Num**	**Group**
77	1.4896	0.4636	2.0716	3	0
78	1.4565	0.5146	1.8377	17	1
79	1.4664	0.0587	2.0380	3	0
80	1.4528	0.4473	1.8406	17	1

The elasticity results are not intuitively appealing. As the elasticity formulation works out $\frac{\partial s_j}{\partial p_k} = -\alpha s_k p_k$. This implies that the substitution patterns are the same across all products when we change the price of one product. We would expect there to be more substitution of products that are closer substitutes to the product whose price has changed.

The markups for the products relate well to the market characteristics. The products in the market with more competitors have a lower markup than the products in the market with relatively few competitors.

Question 4: 5 points

Now consider the nested logit model:

$$U_{ij} = \beta_0 + \beta_1 x_j + \beta_2 g_j + \alpha p_j + \xi_j + \rho\zeta_{ig} + (1-\rho)\epsilon_{ij}$$

where there are 3 groups: the outside alternative, $g = 0$, and $g = 1$ (Recall that the formulas derived in class assume that the outside alternative is its "own" group). Assume that the error term $\rho\zeta_{ig} + (1-\rho)\epsilon_{ij}$ satisfies the nested logit assumptions discussed in class. Estimate this model using the Nested Logit Berry inversion. Do we need another instrument here? Why? If so use z_j as an *additional instrument. Compare your estimated parameters to above. Discuss the significance or insignificance of ρ. Can you think of any other potential instruments?*

Nested Logit Berry Inversion

Table 4: Estimation results : 2SLS

Variable	**Coefficient**	**(Std. Err.)**
p	-0.510	(0.282)
lnsjg	0.463	(0.039)
x	0.647	(0.023)
g	0.081	(0.035)
Intercept	0.460	(0.600)

We will need another instrument for the share with in the group. It is quite likely that the within group share is correlated with unobserved characteristics for the same reasons that the unobserved characteristics are correlated with price. All other things equal, better unobserved characteristics will lead to higher market share for a given product.

The results from the nested logit are similar to those in the logit specification in most aspects. There are no sign reversals and the parameters are of the same order of magnitude. However, the standard error on the price coefficient is quite a bit smaller than in the logit model.

In the nested logit model ρ is statistically significant. If ρ was not significantly different from 0 we could have surmised that there was no added benefit to adding the complexity of a nested logit as our specification. Since ρ is significantly different from 0 we cannot make that conclusion.

Question 5: 5 points

As in 3), create a table of own and cross-price elasticities and compute price-cost markups for the 4 products. You will need to work out these formulas for the nested logit model (Hint: start by examining $\frac{\partial ln(s_j)}{\partial \delta_k}$ rather than $\frac{\partial s_j}{\partial p_k}$. You should be able to reduce this to a formula including $s_j, s_k, s_{k|g}$). Again, use the true market shares for computing elasticities. Importantly, note that the cross price elasticity formulas will depend on whether or not j and k are in the same group. Are your estimated substitution patterns different than from the logit model? Why Do they make more sense?

$$\frac{\partial s_j}{\partial p_j} = \frac{\partial s_j}{\partial \delta_j}\frac{\partial \delta_j}{\partial p_j} = \frac{\partial s_j}{\partial \delta_j}\alpha = \frac{\partial lns_j}{\partial \delta_j} s_j \alpha$$

$$s_j = s_{j|g}s_g = \left(\frac{exp(\frac{\delta_j}{1-\sigma})}{\sum_{k\in g_j} exp(\frac{\delta_k}{1-\sigma})} \right) \left(\frac{\left[\sum_{k\in g_j} exp(\frac{\delta_k}{1-\sigma})\right]^{1-\sigma}}{\sum_{g=1,\ldots,G}\left[\sum_{k\in g} exp(\frac{\delta_k}{1-\sigma})\right]^{1-\sigma}} \right)$$

$$lns_j = lns_{j|g} + lns_g$$

$$\frac{\partial lns_j}{\partial \delta_j} = \frac{\partial lns_{j|g}}{\partial \delta_j} + \frac{\partial lns_g}{\partial \delta_j}$$

$$\frac{\partial lns_{j|g}}{\partial \delta_j} = \frac{\partial}{\partial \delta_j}\left(\frac{\delta_j}{1-\sigma} - ln\left(\sum_{k\in g_j} exp(\frac{\delta_k}{1-\sigma})\right)\right)$$

$$\frac{\partial lns_{j|g}}{\partial \delta_j} = \frac{\partial}{\partial \delta_j}\left(\frac{1}{1-\sigma} - \frac{exp(\frac{\delta_j}{1-\sigma})}{\sum_{k\in g_j} exp(\frac{\delta_k}{1-\sigma})}\frac{1}{1-\sigma}\right) = \left(\frac{1-s_{j|g}}{1-\sigma}\right)$$

$$\frac{\partial lns_g}{\partial \delta_j}$$

Own Price Derivative:

$$\frac{\partial s_j}{\partial p_j} = \alpha s_j\left(\frac{1}{1-\rho}(1-s_{j|g}) + s_{j|g}(1-s_g)\right)$$

Within Group Cross Price Derivative:

$$\frac{\partial s_j}{\partial p_k} = -\alpha s_j\left(\frac{1}{1-\rho}s_{k|g} + s_{k|g}(1-s_g)\right)$$

Between Group Cross Price Derivative:

$$\frac{\partial s_j}{\partial p_k} = -\alpha s_j s_{k|g_k} s_{g_k}$$

Price Elasticities:

$$\left(\begin{array}{ccccc} & s_{77} & s_{78} & s_{79} & s_{80} \\ s_{77} & -1.5604 & 0.0354 & 0.7909 & 0.0331 \\ s_{78} & 0.0625 & -1.6608 & 0.0460 & 0.1140 \\ s_{79} & 1.0749 & 0.0354 & -1.6355 & 0.0331 \\ s_{80} & 0.0625 & 0.1219 & 0.0460 & -1.6689 \end{array}\right)$$

Table 5: Price markups: 4 select products

Product	**Markup**	**Char**	**Price**	**Num**	**Group**
77	1.3276	0.4636	2.0716	3	0
78	1.1065	0.5146	1.8377	17	1
79	1.2461	0.0587	2.0380	3	0
80	1.1029	0.4473	1.8406	17	1

Unlike the logit model, the substitution patterns in the nested logit show a higher rate of substitution to products within in the group than between groups as we would expect. This is due to the nesting restrictions that we have place on the model.

Question 6: 5 points

How does the random coefficient specification in this model compare to the Nested Logit model above (other than where the ξ_j enters)?

The random coefficients model is as follows:

$$U_{ij} = \beta_0 + \beta_1 x_j + \beta_{2i} gj + \alpha p_j + \epsilon_{ij}$$

In general the random coefficient model is more general than the nested logit model because it puts no restrictions a-priori on substitution patterns. However, the random coefficients model does not have the nice analytical solution that the nested logit has. Also it is more demanding on the data so you need more data points. This particular random coefficient is different than the general coefficient model in that only the group coefficient is allowed to b random. That is there are no individual differences in price sensitivity or product characteristic preference. In this case it is not clear that this random coefficients model is a generalization of the previous nested logit model.

Question 7: 5 points

Assuming that the unobserved product characteristic ξ_j is uncorrelated with a constant term and the exogenous variables (x_j, g_j, z_j, w_j), estimate this model with the moment condition:

$$G(\theta) = E\left[\xi_j \otimes \begin{pmatrix} 1 \\ x_j \\ g_j \\ z_j \\ w_j \end{pmatrix}\right] = 0 \text{ at the true theta}$$

See Table 6 below.

Question 8: 5 points

Compute standard errors of your estimates (ignoring error induced by the fact that you are simulating). The asymptotic variance matrix of a GMM estimate of θ is given by:

$$Var(\theta^*) = (\Gamma' A \Gamma)^{-1} \Gamma' A V A \Gamma (\Gamma' A \Gamma)^{-1}$$

where

$$\Gamma = E \frac{\partial g_j(\theta^*)}{\partial \theta'}$$

Γ should be approximated with:

$$\Gamma = \frac{1}{N} \sum_j \frac{\partial g_j(\theta^*)}{\partial \theta'} = \frac{G_N(\theta^*)}{\partial \theta'}$$

where these derivatives are numerically obtained.

Assuming A to be the identity matrix the results for the random coefficient model are as follows:

Table 6: Estimation results : Random Coefficients

Variable	**Coefficient**	**(Std. Err.)**
σ	2.9547	(0.068)
α	-0.7903	(0.062)
β_2	0.3845	(0.015)
β_1	0.9821	(0.011)
β_0	-.0041	(0.125)

1 Matlab Code

1.1 Main Program

1.2 Function mom

1.3 Function momj