## Chapter 4

# alpr: **Alp Rider**

The Alp Rider example was originally proposed by Stephen Campbell to describe the path of a terrain following aircraft. The "peaks" are modeled as simple exponential spikes, and the differential equations used to model the dynamics are stiff. The example was constructed to illustrate the behavior of a specific mesh refinement algorithm described in reference [13, Sect. 4.7.6].

**Example 4.1** alpr01: Stiff ODE, Terrain Following.

Phase 1 ..... Phase 1

Independent Variable: $(t)$ .....

$t = 0$ $\qquad$ $0 \le t \le 20$ $\qquad$ $t = 20$

Differential Variables: $(y_1, y_2, y_3, y_4)$ .....

$y_1 = 2$ $\qquad$ $y_1 = 2$
$y_2 = 1$ $\qquad$ $y_2 = 3$
$y_3 = 2$ $\qquad$ $y_3 = 1$
$y_4 = 1$ $\qquad$ $y_4 = -2$

Algebraic Variables: $(u_1, u_2)$ .....
Differential-Algebraic Equations .....

$$\dot{y}_1 = -10y_1 + u_1 + u_2 \tag{4.1}$$
$$\dot{y}_2 = -2y_2 + u_1 + 2u_2 \tag{4.2}$$
$$\dot{y}_3 = -3y_3 + 5y_4 + u_1 - u_2 \tag{4.3}$$
$$\dot{y}_4 = 5y_3 - 3y_4 + u_1 + 3u_2 \tag{4.4}$$
$$y_1^2 + y_2^2 + y_3^2 + y_4^2 \ge 3p(t, 3, 12) + 3p(t, 6, 10) + 3p(t, 10, 6) + 8p(t, 15, 4) + 0.01 \tag{4.5}$$

where $p(t, a, b) = e^{-b(t-a)^2}$.

Objective .....

$$\text{Minimize} \qquad J = \int_0^{20} 10^2(y_1^2 + y_2^2 + y_3^2 + y_4^2) + 10^{-2}(u_1^2 + u_2^2)dt$$

$$J^* = 2030.85609$$