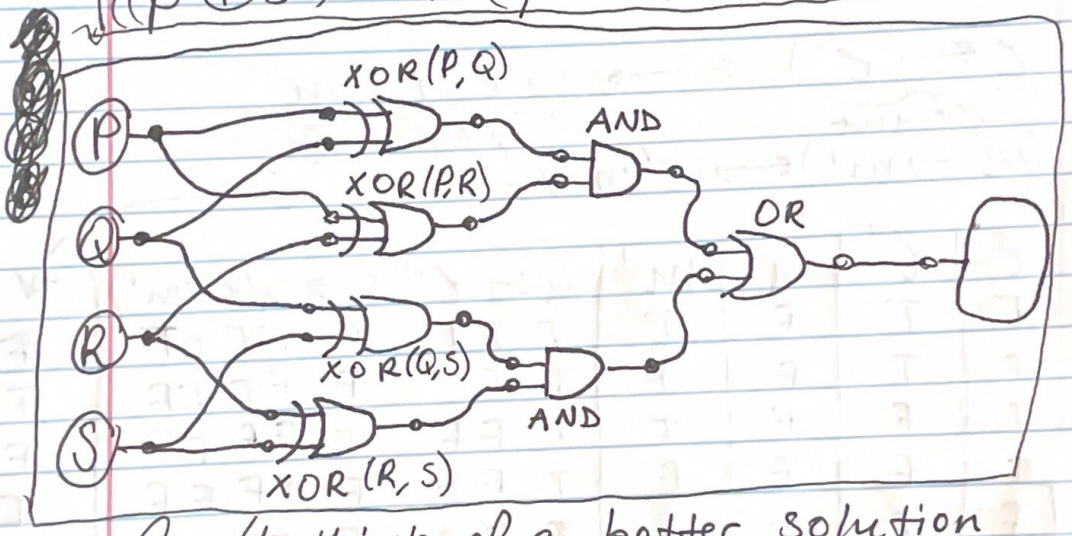
Homework 2

Not, And, Or, XOR, NAND, Nor

1) p, q, r, s | Exactly 2 true = true

$(p \oplus q) \wedge (s \oplus r)) \vee ((p \oplus s) \wedge (q \oplus r))$

$(p \wedge s) \vee (p \wedge r) \vee (q \wedge s) \vee (q \wedge r)$



Can't think of a better solution

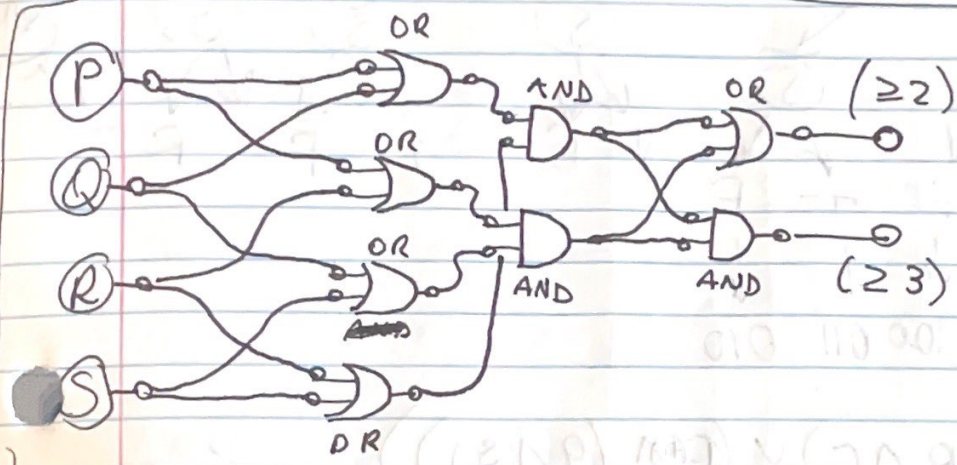
2) 2 outputs (00, 01, 11) 2 true, 3 true

$((p \oplus q) \wedge (s \oplus r)) \vee ((p \oplus s) \wedge (q \oplus r))$ } 2

$((p \oplus q) \wedge (s \oplus r)) \vee ((p \wedge q) \oplus (s \wedge r))$ } ≥ 2

$((p \vee q) \wedge (r \vee s)) \vee ((p \vee r) \wedge (q \vee s))$ } ≥ 2

$(((p \vee q) \wedge (r \vee s)) \wedge ((p \vee r) \wedge (q \vee s)))$ } ≥ 3



(3) $1011 = 10 + 11 = 2 + 3 = 101 = 5$

(4 2 1)

if q and s T → 010 if pqrs = T → 110

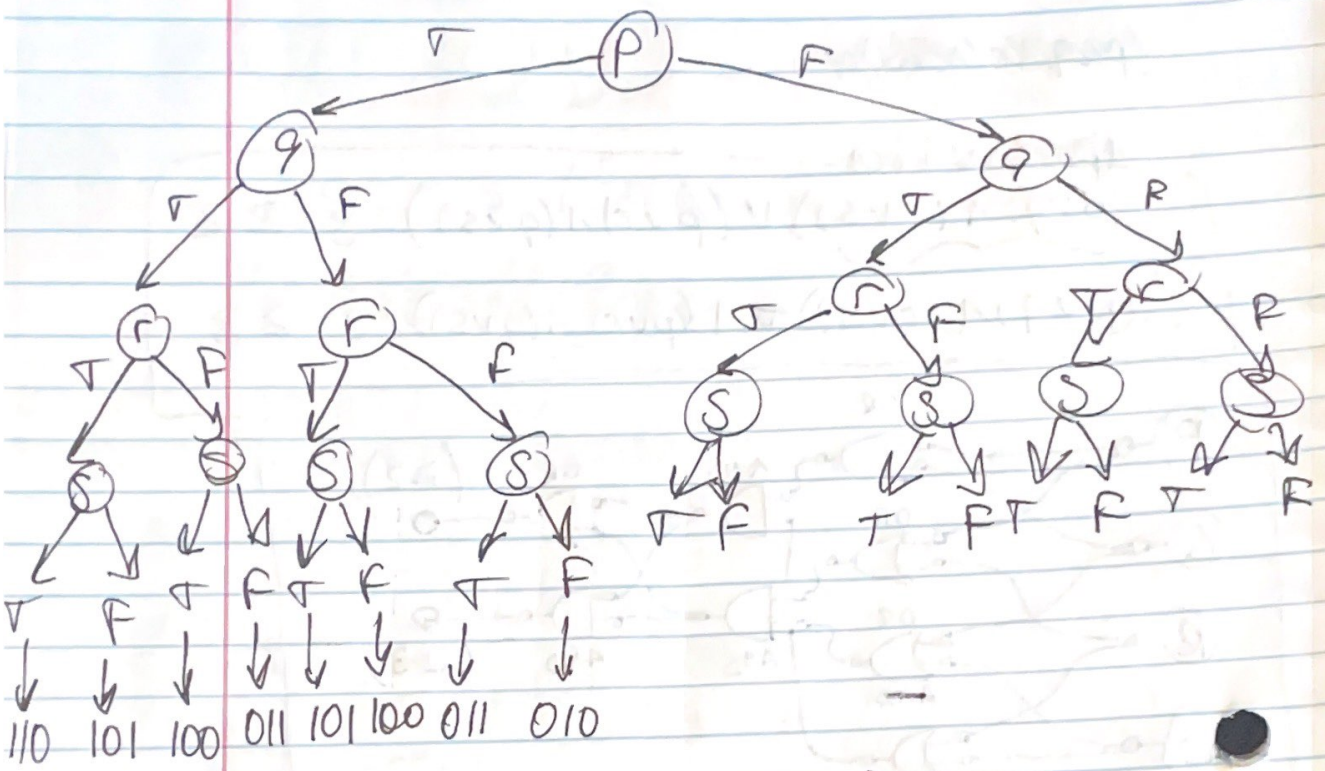
if p and r T → 100 if pqr = T → 101

if p ⊕ r T → 010 if qrs = T → 100

if p ⊕ s T → 001

q ⊕ s → 1 box

PQ, RS (00,00)



$\bigcirc\!\!-\!-\!- \equiv ((p \wedge r) \vee (p \wedge (q \wedge s)))$

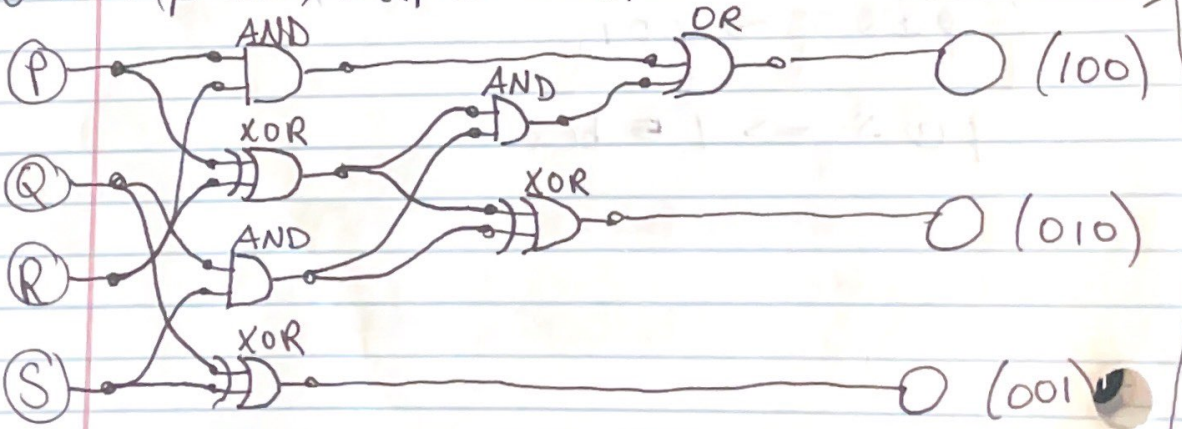
$-\!\bigcirc\!\!-\!- \equiv (p \oplus r) \oplus (q \wedge s)$

$--\!\bigcirc \equiv (q \oplus s)$

$001 \equiv (q \oplus s)$

$010 \equiv ((p \oplus r) \oplus (q \wedge s))$

$100 \equiv \neg((p \wedge r) \vee ((p \oplus r) \wedge (q \wedge s)))$



(4) Single digit → input if 1 is true, carry over if both are true

out ≡ (p ⊕ q)

carry over ≡ (p ∧ q)

Next addition is previous carry over + inputs. if inputs ∧ carry over, carry over to next set to T

out ≡ (p ⊕ q) ⊕ carry

carry over ≡ (p ∧ q) ∨ (carry ∧ (p ⊕ q))

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two digit addition, start at smallest (P Q R S) test, p=T, q=T, r=F, s=T
1, 0, 1 0

out ≡ (q ⊕ s) ≡ F 000

carry over ≡ (q ∧ s) ≡ T

Next nums

out ≡ (p ⊕ r) ⊕ T ≡ T ⊕ T = F

carry over ≡ ((p ∧ r) ∨ (T ∧ (p ⊕ r)))

≡ (T) ∨ (F) ≡ T

0 0 0

0 0 0 = carry over r = T

final = 100 ✓

Threedigot

000 000
p q r | s t u

test (p = 1, q = 0, r = 1, s = 0, t = 1, u = 0)

- out $\equiv (r \oplus u) \equiv T$, carry $\equiv (p \wedge u) \equiv F$ } 0001
- out $\equiv (q \oplus t) \oplus carry(F) \equiv T$, carry $\equiv (q \wedge t) \vee (F \wedge (q \oplus t)) \equiv F$ } 0011
  - $(q \wedge t)$: F; $(F \wedge (q \oplus t))$: F
- out $\equiv (p \oplus s) \oplus F \equiv T$, carry $\equiv (p \wedge s) \vee (F \wedge (p \oplus s)) \equiv F$ } 0111
  - $(p \wedge s)$: F; $(F \wedge (p \oplus s))$: F
- final = carry $\equiv F$

0111 ✓

000<u>0</u> $\equiv (lowest_1 \oplus lowest_2)$

00<u>0</u>0 $\equiv (current_1 \oplus current_2) \oplus (previous_1 \wedge previous_2)$

0<u>0</u>00 $\equiv (current_1 \oplus current_2) \oplus (previous_1 \wedge previous_2)$

<u>0</u>000 $\equiv (previous_1 \wedge previous_2)$

proof using 2 3digit binary #'s

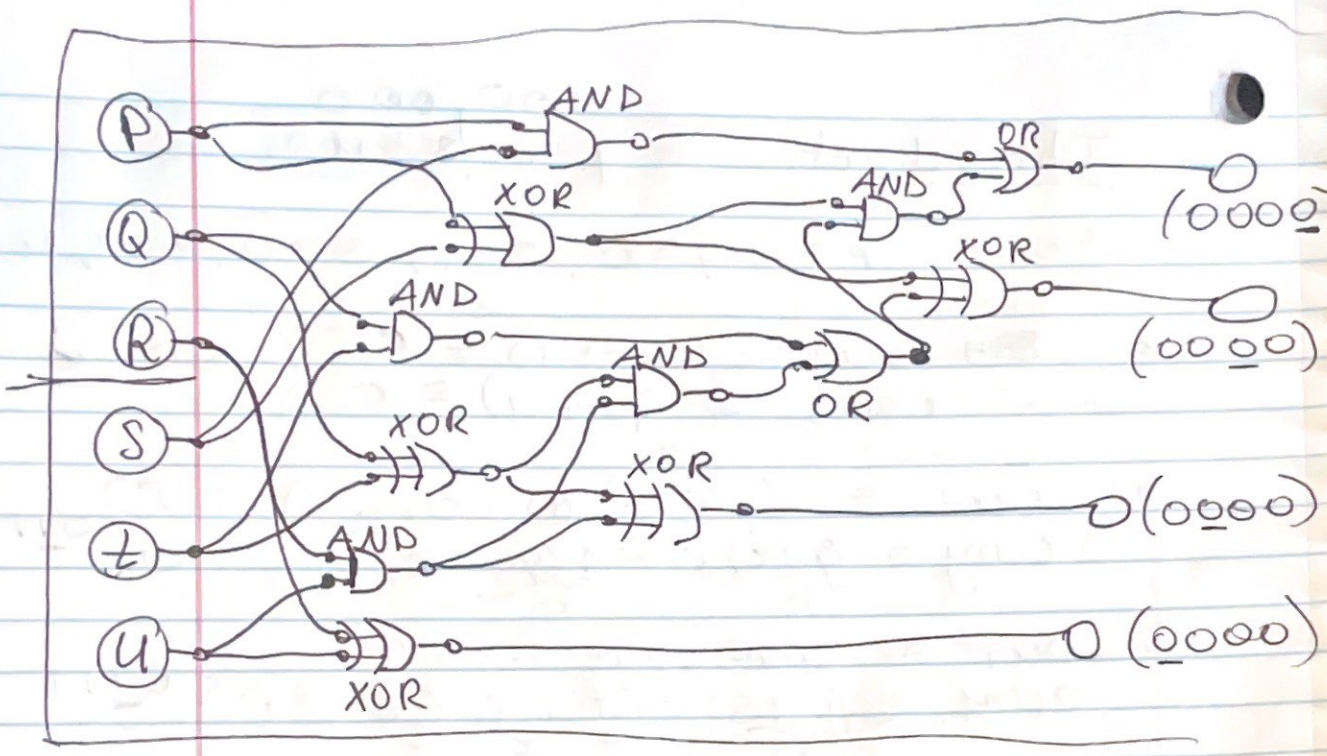
(pqrstu) ⟶

000<u>0</u> $\equiv (r \oplus u)$

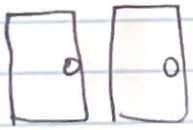
00<u>0</u>0 $\equiv (q \oplus t) \oplus (r \wedge u)$

0<u>0</u>00 $\equiv (p \oplus s) \oplus ((q \wedge t) \vee ((r \wedge u) \wedge (q \oplus t)))$

<u>0</u>000 $\equiv ((p \wedge s) \vee ((q \wedge t) \wedge (p \oplus s)))$



(5)



tiger or marriage
each door has signs either true or false

door	tiger	marriage
truth	tiger	marriage
lie	marriage	tiger

a) Door 1) There is a lady behind at least one of these doors
Door 2) There is a tiger behind door 1

[redacted] can be two tigers, two women, or 1 of each

a, b ⇒ a ⟷ a' = [redacted] door 1 is true and woman

5

(a) a = woman behind first door
b = woman behind second door

A statement
→ Woman behind at least one door
B statement (ψ)
→ A is tiger ← (θ)

$\psi \leftrightarrow \theta$

$\equiv (a \vee b) \leftrightarrow \neg a$

a	b	$(a \vee b) \leftrightarrow \neg a$
T	T	T T T F F T
T	F	T T F F F T
F	T	F T T T F F
F	F	F F F T T F

$a = F$, $b = T$ only solution

$((a \vee b) \leftrightarrow \neg a)$

a is a tiger, b is a woman,
both signs are truth

(b) prisoner knows one sign true one isn't

Door 1 statement → There is a lady behind the second door (Ψ)
Door 2 statement → There is one lady and one tiger (Θ)

$\Psi \oplus \Theta$

a = Door 1 is lady
b = Door 2 is lady

$b \oplus (a \oplus b)$

a	b	$b \oplus (a \oplus b)$				
T	T	T	T	T	F	T
T	F	F	T	T	T	F
F	T	T	F	F	T	T
F	F	F	F	F	F	F

a is lady, b is lady... a said the truth, b lied ✓

a is lady, b is tiger... a lied, b said the truth ✓

(a) $a \equiv T$, $b \equiv T$ OR $a \equiv F$, $b \equiv F$