Q-3 Curve $(x^2+y^2)x - ay^2 = 0$

Sol $x^3 + xy^2 - ay^2 = 0$

∥ to x Axis

Since Cofficient of highest power of x is Constant then Asymptote Does not Exits.

∥ to y Axis

$x - a = 0$

$\boxed{x = a}$ Ans

Q.4 $x^4 + y^4 = a^2(x^2 - y^2)$

Q.5 $x^2y^2 - a^2(x^2+y^2) - a^3(x+y) + a^4 = 0$

Q.6 $\frac{a^3}{x^3} - \frac{b^3}{y^3} = 1$

Sol 4 $x^4 + y^4 = a^2(x^2 - y^2)$

$x^4 + y^2 = a^2x^2 - a^2y^2$

$x^4 + y^2 - a^2x^2 + a^2y^2 = 0$

Since Cofficient of highest power of x & y is Constant the Asymptote Does not Exits.

Sol 5 $x^2y^2 - a^2x^2 - a^2y^2 - a^3x - a^3y + a^4 = 0$

Asymptote ∥ to x Axis

$y^2 - a^2 = 0$

$y^2 = (a^2)^{1/2}$

$\boxed{Y = \pm a}$

|| to y Axis

$x^2 - a^2 = 0$

$x^2 = a^2$

$\boxed{x = \pm a}$

Q.6 $\frac{a^3}{x^3} - \frac{b^3}{y^3} = 1$

$\frac{x^3a^3 - y^3b^3}{x^3y^3} = 1$

$x^3a^3 - y^3b^3 = x^3y^3$

$x^3a^3 - y^3b^3 - x^3y^3 = 0$

|| to x Axis

$a^3 - y^3 = 0$

$y^3 = a^3$

$y = (a^3)^{1/3} \Rightarrow \boxed{y = a}$ Ans

|| to y axis

$\cancel{x^3} - b^3 + x^3 = 0$

$b^3 + x^3 = 0$

$x = (b^3)^{1/3}$

$\boxed{x = b}$

(i) $\frac{a^3}{x^3} - \frac{b^3}{y^3} = 1$

$\frac{y^3a^3 - x^3b^3}{x^3y^3} = 1$

$y^3a^3 - x^3b^3 = x^3y^3$

$y^3a^3 - x^3b^3 - x^3y^3 = 0$

|| to x Axis

$+b^3 + y^3 = 0$

$+b^3 = y^3$

$+y = (b^3)^{1/3}$

$\boxed{y = b}$

|| to y Axis

$a^3 - x^3 = 0$

$x^3 = a^3$

$x = (a^3)^{1/3}$

$\boxed{x = a}$

or $x^3y^3 + x^3b^3 - y^3a^3 = 0$

|| to x

$y^3 + b^3 = 0$

$y^3 = -b^3$

$y = \sqrt[3]{-(b)^3}$

$y = -(b^3)^{1/3}$

$y = -b$

|| to y Axis

$x^3 - a^3 = 0$

$x^3 = a^3$

$\boxed{x = a}$

Oblique Asymptote :->

Finding of Asymptote :-

(n degree curve)

given Curve $f(x,y) = 0$

→ first we find Asymptote pll to the Axis

→ Put $x = 1$, $y = m$ in highest Power term (n degree)

then

$\phi_n(m) = 0$

its roots

$m = m_1, m_2$ ---

→ If all roots are different

$$c = -\frac{\phi_{n-1}(m)}{\phi'_n(m)}$$ differenciate

$c = c_1, c_2, \ldots$

then required Asymtotes

अभिलेख

$y = m_1 x + c_1$

$y = m_2 x + c_2$

$y = m_3 x + c_3$

...

Q Curve

$$\underbrace{4x^3 - x^2y - 4xy^2 + y^3}_{f_3} + \underbrace{3x^2 + 2xy - y^2}_{f_2} - 7 = 0$$

$\because$ Coefficient of highest power/degree of x and y are constant

Asymptote doesn't exit. $\parallel$ to the Axis.

Now $\phi_3 = 4(1)^3 - (1)^2 m - 4(1)(m)^2 + m^3 = 0$

$\Rightarrow 4 - m - 4m^2 + m^3 = 0$ ——— (1)

$\Rightarrow 4 - m - m^2(4 - m) = 0$

$\Rightarrow (4-m)(1-m^2) = 0$

$4 - m = 0$ | $1 - m^2 = 0$

$m = 4$ | $m = \pm 1$

And $\phi_2(m) = 3 + 2m - m^2$

$\phi_3'(m) = 0 - 1 - 4 \cdot 2m + 3m^2$

$$\because c = -\frac{\phi_2(m)}{\phi_3'(m)}$$

$$= \frac{3 + 2m - m^2}{-1 - 8m + 3m^2}$$

at $m = 4 \Rightarrow -\dfrac{3 + 8 - 16}{-1 - 32 + 48} \Rightarrow -\left(\dfrac{-5}{15}\right) \Rightarrow \dfrac{1}{3}$

at $m = 1 = -\dfrac{3 + 2 - 1}{-1 - 8 + 3} \Rightarrow -\left(\dfrac{4}{-6}\right) \Rightarrow \dfrac{2}{3}$

at $m = -1 = -\dfrac{3 - 2 - 1}{-1 + 8 + 3} \Rightarrow 0 \Rightarrow 0$

then Req. Asymptote

$y = mx + c \,,\quad y = mx + c \,,\quad y = mx + c$

$\cancel{y = 4x + c} \,,\quad y =$

<u>Curve</u>

$$x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 = 1$$

the Cofficient of higest power of x ~~is~~ & y are Constant so asymptote does not exist ~~parallel~~ ∥ to x & y Axis

~~Asymptote ∥ to y Axis.~~

$$-2y^2 = 0$$

$$y^2 = 2$$

$$y = \pm\sqrt{2}$$

Now ~~ϕ_3~~ $x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 - 1 = 0$

now $\phi_3 = 1 + 2m - m^2 - 2m^3 = 0$

$\Rightarrow 1 + 2m - m^2(1 + 2m) = 0$

$\Rightarrow (1 + 2m)(1 - m^2)$

$1 + 2m = 0$	$1 - m^2 = 0$
$2m = -1$	$m^2 = 1$
$m = -\frac{1}{2}$	$m = \pm 1$

Then $\phi_2(m) \Rightarrow m - m^2 + 1 = 0$

$\phi'_3 \Rightarrow 2 - 2m - 6m^2 = 0$

$\because c \Rightarrow -\dfrac{\phi_{(n-1)}m}{\phi'_n(m)}$

$c \Rightarrow -\dfrac{m - m^2 - 1}{2 - 2m - 6m^2}$

$m = -\frac{1}{2} \Rightarrow -\left(\dfrac{(-1/2) - (-1/2)^2 + 1}{2 - 2(-1/2) - 6(-1/2)^2}\right)$

$\Rightarrow -\left(\dfrac{-1/2 - 1/4 - 1}{2 + 1 - 6/4}\right) \Big\| -\left(\dfrac{-1/2 - 1/4}{2 + 1 - 6/4}\right)$

$\Rightarrow -\left(\dfrac{\frac{-2-1}{4}}{\frac{4+2-3}{2}}\right) - \left(\dfrac{-3/4}{-3/2}\right) \Rightarrow$

$\Rightarrow -\left(\cancel{\dfrac{+7/4}{+3/2}}\right) \Rightarrow -\cancel{\left(\frac{7}{4}\right) \times \frac{2}{3}} - \left[\frac{3}{4}\right] \times \left[\frac{2}{3}\right]$

$\Rightarrow \cancel{-\frac{7}{6}} \Rightarrow -\frac{1}{2}$

$\frac{7}{4} \times \frac{2}{3}$

2

$m = (-1) \Rightarrow -\left[\dfrac{(-1) - (-1)^2 \cancel{+m}}{2 - 2(-1) - 6(-1)^2}\right]$

$\Rightarrow -\left[\dfrac{-1 - 1 \cancel{+m}}{2 + 2 - 6}\right] \Rightarrow -\left[\dfrac{-2}{-2}\right] \Rightarrow \cancel{-\frac{3}{2}} \; -1$

$m = (1) \Rightarrow -\left[\dfrac{1 - (1)^2 - 1}{2 - 2(1) - 6(1)^2}\right]$

$\rightarrow -\left[\dfrac{1 - 1 + 1}{2 - 2 - 6}\right]$

$\rightarrow \cancel{\left[\dfrac{-1}{-6}\right] \Rightarrow \frac{1}{6}}$ 0

then by Asymptote

$y = m_1 x + c$, $y = m_2 x + c$, $y = m_3 x + c$

$y = -\frac{1}{2} x + c$, $y = -\frac{3}{2} x + c$, $y = -\frac{1}{6} x + c$

$y = -\frac{1}{2} x + c$ $\quad y = -1x + c$ $\quad y = \cancel{c}$

$y = -\frac{1}{2} x + \left(\frac{1}{-2}\right)$

$y = -\frac{1}{2} x - \frac{1}{2}$ | $y = -x - 1$ | $y = 0$

Note $m = m_2 = m_2$ two roots are same-

formulla

$$\frac{c^2}{2!}\phi_3''(m) + \frac{c^1}{1!}\phi_2'(m) + \frac{c^0}{0!}(m) = 0$$

$$\Rightarrow \frac{c^2}{2}\phi_3''(m) + c\phi_2'(m) + \phi_1(m) = 0$$

Question

curve $\Rightarrow y^3 - xy^2 - x^2y + x^3 + \underbrace{x^2 - y^2}_{} - 1 = 0$

the Cofficient of highiest power of x & y are constant so ∴ Asmptote of x and y does not exists

soln $\phi_3(m) = m^3 - m^2 - m + 1 = 0$

$\Rightarrow m^2(m-1) - (m-1) = 0$

$\Rightarrow (m-1)(m^2-1) = 0$

$\Rightarrow m-1 = 0 \quad | \quad m^2-1 = 0$

$m = 1 \quad | \quad m = \pm 1$

$m = 1, -1, 1$

As $\phi_2 = 1 - m^2$

$\phi_3'(m) = 3m^2 - 2m - 1 + 0$

$\because c = -\dfrac{\phi_2(m)}{\phi_3'(m)}$

$$= -\frac{1-m^2}{3m^2-2m+1}$$

$m = -1$ पर $c = 0$

$m = 1, 1$

$$\frac{c^2}{2}\phi_3''(m) + c\phi_2'(m) + \phi_1(m) = 0$$

$$\Rightarrow \frac{c^2}{2}[3.2m-2-0] + c[0-2m] + 0 = 0$$

$$\Rightarrow \frac{c^2}{2}[6-2] + c(-2) = 0$$

$$\Rightarrow 2c^2 - 2c = 0$$

$$\Rightarrow c^2 - c = 0$$

$$\Rightarrow c(c-1) = 0$$

$$\Rightarrow c = 0, \quad c - 1 = 0$$
$$c = 1$$

Required Asymptote

$$y = mx + c$$

~~at m = (+1) & c = 0~~

$$y = -x + 0$$

$y = x + 0$ & $y = x + c$

Q (1) $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$

the Cofficient of highest power of x & y are Constant so the ~~Asy~~ symptote ‖ to x & Y Axis doesnot exist.

$\phi_3 m \Rightarrow 1 + 3m - 4m^3 = 0$

$= 1 + m(3 - 4m^2) = 0$

$= m(3 - 4m^2) = 0$

UNIT → II

SETS

Sets :- Collection of well defined objects, Called sets

→ we represent sets by Capital letters
A, B, C . . .

→ Elements of sets are represented by small letters a, b, c, d . .

Types of sets :-

① **Roster form / tabular form :-**
→ all the member or Elements of a set are listed, the Elements are being seperated by Commas and are enclosed within curly braces, { }

Ex $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

② **Rule form :-**

a Rule, or the formula or the statement is written within the pair of brackets so that the set is well defined.

$A = \{x \mid \text{Rule for } x\}$

belongs to

Ex $A = \{1, 2, 3, 4, 5, \ldots n \mid n \in N\}$ Natural numbers

such that
(जहाँ)

Some Important set :-

1. Set of natural No.

$N = \{1, 2, 3, 4 --- n\}$

2. Set of whole no.

$W = \{0, 1, 2, --- n\}$

3. Set of Integer no.

$Z = \{ --- -3, -2, -1, 0, 1, 2, 3 --- \}$

4. Set of Rational no.

$Q = \{ p/q \mid q \neq 0, p, q \in Z \}$

5. set of irrational no.

$I = \{ \sqrt{2}, \sqrt{3}, \pi, e, --- \}$

6. Set of Real No.

$R = Q \vee I$

7. Set of Complex no.

$C = \{ x + y \mid x, y \in R \}$

$i = \sqrt{-1}$

* Union of set :-

$A = \{1, 2, 3, 4\}$, $B = \{5, 6\}$

then $A \cup B = \{1, 2, 3, 4, 5, 6\}$

* Intersection of set :-

$A = \{1, 2, 3, 4, 5\}$, $B = \{1, 4\}$

$A \cap B = \{1, 4\}$

* Subset :-

Let A is any set, set A is said ~~let~~ to be A subset of B, if all the element of set A are also present in set B.

$A = \{1, 2, 3, 4, 5\}$
$B = \{1, 3, 5\}$

then $B \subseteq A$
$\in \rightarrow$ belongs to

* Null / Empty set *

→ has no element in set
→ represent by $\phi = \{\}$

ex
$A = \{\}$

* Equal set *

Two set A and B are equal set

i) if $n(A) = n(B)$
ii) Every element of A and B are same.

Ex $A = \{1, 2, 3, 4\}$
$B = \{3, 1, 4, 2\}$

therefore, $\boxed{A = B}$

* Finite and Infinite set *

Set

↓ Finite set
(no of elements in a set is a finite no.)

Ex:-
$n(A) = 5$ (Finite)

$A = \{1, 2, 3, 4, 5\}$

↓ Infinite set
(no of elements in a set is a infinite no.)

Ex
$n(A) = \infty$

$A = \{1, 2 \text{ -- } \infty)$

* Singleton set *

Any set is singleton set if it contain only one element

Ex $A = \{5\}$

Question

write set in Roster form.

$B = \{x \mid x \in W, -1 \leq x < 5\}$

Ans $B = \{0, 1, 2, 3, 4\}$

* Disjoint set *

Two set A and B are disjoint set if
$A \cap B = \phi$

Ex

$A = \{1, 3, 5, 7 \ldots\}$
$B = \{2, 4, 6 \ldots\}$

Then $A \cap B = \phi = \{\}$

* Universal set *

All set under observation are subset of a fixed set U then U is called universal set.

$A = \{1, 2, 3\}$ $B = \{4, 5, 6, 7\}$
and
$C = \{x \mid 0 < x < 8\}$

C is universal set of A and B

* Power set *

The set of all subset of a set A is Called Power set.

Ex:

$A = \{1, 2, 3\}$ is any set

its subset $B = \{1, 2\}$
$C = \{1, 3\}$
$D = \{2, 3\}$
$\phi = \{\}$

then,
Power of A,

$P(A) = \{B, C, D, \phi, \{1\}, \{2\}, \{3\}, A\}$

No of element in $P(A) = 8$ or 2^3

Note :
1. No of element in power set $= 2^n$
   ~~Where~~, n = number of element in set.
2. Power set of empty set $P(\phi) = \{\phi\}$

Q- Prove that

$n\{P(P(P(\phi)))\} = 4$

Sol we Know that

$P(\phi) = \{\phi\}$

$\Rightarrow P\{P(\phi)\} = \{\phi, \{\phi\}\}$

and

$P(P(P\{\phi\})) = \{\phi, \{\phi\}, \{\phi, \{\phi\}\}, \{\{\phi\}\}\}$

then

$n\{P(P(P\{\phi\}))\} = 4$

Ex $A = \{2, 4\}$. find $P(A)$

Sol

$P(A) = \{\{2\}, \{4\}, \phi, \{2, 4\}\}$

$n(P(A)) = 2^2 = 4$

Q. Write the power set of $\{0, 2, 4\}$

Sol $A = \{0, 2, 4\}$

$n(A) = 3$ than $n\{P(A)\} = 2^3 = 8$

than

$P(A) = \{\phi, \{0\}, \{2\}, \{4\}, \{0,2\}, \{0,4\}, \{2,4\}, \{0,2,4\}\}$

Q. If $A \subset B$, $B \subset C$ and $C \subset A$

Prove that $A = C$

Sol $\because A \subset B \rightarrow$ Each element of set A exist in B

$\because B \subset C \rightarrow$ Each element of set B exist in C

$\therefore$ ~~Each~~ Each element of set A exist in C

$\Rightarrow A \subset C$ — (1)

According A.T.Q. the question: $C \subset A$ — (2)

by (1) and (2)

$\boxed{A = C}$

Hence Proved

Qn 96 A is any set and $A \subset \phi$ the prove $A = \phi$

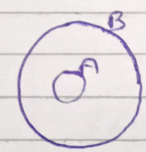
Sol Let A is any set
we know that
$\phi \subset A$ — (1)

A.T.Q $A \subset \phi$ — (2)

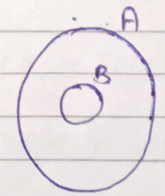
by (1) & 2
$A = \phi$

* Venn diagram * diagrams drawn to represent sets are called ~~Vehn~~ Venn diagrams.

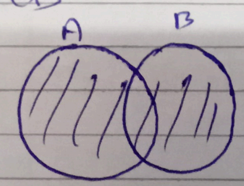
(1) $A \subseteq B$



(2) $B \subset A$



(3) Union of sets

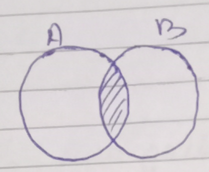


$A \cup B$ = total Elements of A and B

Ex: $A = \{1, 2, 4\}$ And $B = \{1, 3, 4, 5\}$

Then $A \cup B = \{1, 2, 3, 4, 5\}$

Intersection of sets $\Rightarrow$
Common



$A \cap B$

Ex: $A = \{1, 2, 4\}$. $B = \{1, 3, 4, 5\}$

then $A \cap B = \{1, 4\}$

Ex: If $A = \{2, 4, 6, 8, 7, 9, 10\}$
and
$B = \{x \mid x \text{ is prime number} < 10\} = \{2, 3, 5, 7\}$

then
$A \cap B = \{2, 7\}$

$\in$ = belons to

* Compliment of a set *

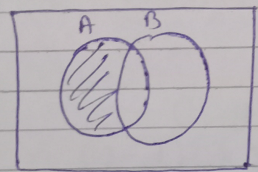
If A is any set and U is universal set then compliment of A is defined by

$U - A = A'$ (A^c compliment)

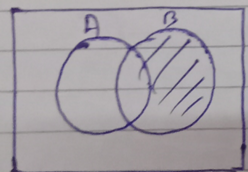
Ex $A = \{1,2,3,4\}$, $U = \{0,1,2,3,4,5,6,7,8\}$

$U - A \Rightarrow \{0,5,6,7,8\} = A'$ या A^c

* Difference of Sets *



$A - B = \{x \in A \mid x \notin A \cap B\}$



$B - A = \{x \in B \mid x \notin A \cap B\}$

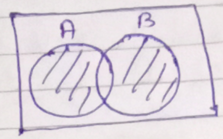
Ex Ex: $A = \{1,2,3,4,5\}$, $B = \{2,4,5,6\}$

then $A - B = \{1,3\}$ and $B - A = \{6\}$

Result

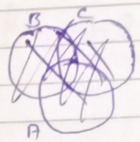
$A - B \neq B - A$

* $(A-B) \cup (B-A) = \{1, 3, 6\}$



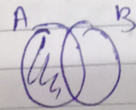
Q Prove that ~~A∩B~~ $A \cap (B-C) = (A \cap B) - C$
A, B, C are any sets

Sol L.H.S
~~$A \cap (B-C)$~~
~~then B - C~~



Let ~~$x \in A$~~ $x \in A \cap (B-C)$
$\Rightarrow x \in A$ and $x \in (B-C)$

$\Rightarrow x \in A$ and $x \in B$ and $x \notin C$ —— (1)



Again If $x \in (A \cap B) - C$

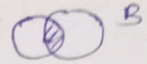
$\Rightarrow x \in A \cap B$ and $x \notin C$

$\Rightarrow x \in A$ and $x \in B$ and $x \notin C$ —— (2)

By (1) & (2)
$A \cap (B-C) = (A \cap B) - C$

H.P

~~Or~~ Imp



$* \begin{cases} x \in A \cap B \\ \Rightarrow x \in A \text{ and } x \in B \end{cases}$

A B

(1) ∪ (2)

(12)

$\begin{cases} x \in A \cup B \\ \Rightarrow x \in A \text{ or } x \in B \end{cases}$

* Factorial function *

$n! = n(n-1)(n-2)(n-3) \ldots 3.2.1$

Note

- $* \; 5! = 5.(5-1)! = 5.4!$
- $* \; n! = n.(n-1)!$
- $* \; (4+1)! = 5.4!$
- $* \; 4! * 5 = 5!$

Ex ⇒ $5! = 5 \times 4 \times \times 3 \times 2 \times 1$

$= 120$

Defind $0! = 1$

Combination → $^nC_r = \dfrac{n!}{(n-r)! \cdot r!}$

Ex $^6C_3 \Rightarrow \dfrac{6!}{(6-3)! \cdot 3!}$

$\Rightarrow \dfrac{\overset{2}{\cancel{6}} \times 5 \times \overset{2}{\cancel{4}} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{(\cancel{3} \times \cancel{2} \times \cancel{1}) \cdot (3 \times \cancel{2} \times 1)}$

⇒ 20 Ans

Prove that

$$^{n}C_{r} + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$$

L.H.S

$$^{n}C_{r} + {}^{n}C_{r+1} \Rightarrow \frac{n!}{(n-r)!\cdot r!} + \frac{n!}{(n-(r+1))!\cdot(r+1)!}$$

$$\Rightarrow \frac{n!}{(n-r)(n-r-1)!\cdot r!} + \frac{n!}{(n-r-1)!\cdot(r+1)(r+1-1)!}$$

expanded one's

ex. one's

$$\Rightarrow \frac{n!}{(n-r)(n-r-1)!\cdot r!} + \frac{n!}{(n-r-1)!\cdot(r+1)\cdot r!}$$

$$\rightarrow \frac{n!}{(n-r-1)!\cdot r!}\left[\frac{1}{(n-r)} + \frac{1}{r+1}\right]$$

$$\Rightarrow \frac{n!}{(n-r-1)!\cdot r!}\left\{\frac{r+1+n-r}{(n-r)(r+1)}\right\}$$

$$\Rightarrow \frac{n!}{(n-r-1)!\cdot r!}\left\{\frac{1+n}{(n-r)(r+1)}\right\}$$

$$\Rightarrow \frac{n!\cdot(n+1)}{(n-r-1)!\,r!\cdot(n-r)(r+1)} = \frac{(n+1)!}{(n-r)!\,(r+1)!} \quad \text{———} \; (1)$$

and

R.H.S