GLOBAL ESTIMATION AND SCENARIO-BASED PROJECTIONS OF SEX RATIO AT BIRTH AND MISSING FEMALE BIRTHS USING A BAYESIAN HIERARCHICAL TIME SERIES MIXTURE MODEL*

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The sex ratio at birth (SRB) is defined as the ratio of male to female live births. The SRB imbalance in parts of the world over the past several decades is a direct consequence of sex-selective abortion, driven by the co-existence of son preference, readily available technology of prenatal sex determination, and fertility decline. Estimation and projection of the degree of SRB imbalance is complicated because of variability in SRB reference levels and because of the uncertainty associated with SRB observations.

We develop Bayesian hierarchical time series mixture models for SRB estimation and scenario-based projections for all countries from 1950 to 2100. We model the SRB regional and national reference levels, and the fluctuation around national reference levels. We identify countries at risk of SRB imbalances and model both (i) the absence or presence of sex ratio transitions in such countries and, if present, (ii) the transition process. The transition model of SRB imbalance captures three stages (increase, stagnation and convergence back to SRB baselines). The model identifies countries with statistical evidence of SRB inflation in a fully Bayesian approach. The scenario-based SRB projections are based on the sex ratio transition model with varying assumptions regarding the occurrence of a sex ratio transition in at-risk countries. Projections are used to quantify the future burden of missing female births due to sex-selective abortions under different scenarios.

^{*}This work was supported by a research grant from the National University of Singapore.

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Keywords and phrases: Bayesian hierarchical model, probabilistic scenario-based projection, time series analysis, sex-selective abortion, sex ratio transition, missing female births

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1. Introduction. Under normal circumstances, the sex ratio at birth (SRB; defined as the ratio of male to female live births) falls within a narrow range of 1.03 to 1.07 and varies slightly by ethnicity [Chahnazarian (1988); Chao et al. (2019a); Dubuc and Coleman (2007); Garenne (2002, 2008); Graffelman and Hoekstra (2000); James (1984, 1985, 1987); Kaba (2008); Ruder (1985); Marcus et al. (1998); Mathews and Hamilton (2005); Visaria (1967)]. For most of human history, SRB remained within that natural range. However, over recent decades, SRBs have risen in a number of Asian countries and in Eastern Europe [Basten and Verropoulou (2013); Bongaarts (2013); Bongaarts and Guilmoto (2015); Chao et al. (2019a); Chao and Yadav (2019); Chen et al. (2020); Choi and Hwang (2020); Das Gupta et al. (2003); Duthé et al. (2012); Goodkind (2011); Attané and Guilmoto (2007); Guilmoto, Hoàng and Van (2009); Guilmoto (2009); Guilmoto and Ren (2011); Guilmoto (2012a,b,c); Hudson and Den Boer (2004); Lin (2009); Meslé, Vallin and Badurashvili (2007); Park and Cho (1995); Tafuro and Guilmoto (2020); Vu and Yamada (2020)]. SRB imbalance results from the interaction of three main factors [Guilmoto (2009, 2012a)]; first, prolonged strong son preference, offering the motivation; second, fertility decreases leading to fewer children per family, inducing the willingness; third, accessible affordable sex-selection technology, providing the means. As a result, couples seeks abortion based on the knowledge of the sex of the pregnancy to obtain sons while maintaining a small family size.

Estimation of the degree of SRB imbalance is challenging because of (i) variation in baseline SRB levels and (ii) uncertainty associated with SRB observations. In prior work [Chao et al. (2019a)], we developed a model to estimate SRB and imbalances for 212 countries from 1950 to 2017. The SRB estimation model accounts for the difference in the SRB reference levels across regions and varying uncertainty associated with SRB observations. We identified 29 countries where SRB imbalance may have happened in the past or may happen in the future, which we refer to as countries at risk of SRB inflation. We fitted a model for SRB levels and trends in country-years without risk of sex-selective abortion and obtained estimates for national and regional SRB baseline values. Subsequently, we estimated sex imbalances using a sex ratio transition model to capture periods of increasing, constant and decreasing sex ratio imbalances.

Constructing SRB projections is challenging for countries with ongoing SRB imbalances [Gupta, Chung and Shuzhuo (2009)] and even more so for countries with normal SRB levels and trends but with potential of rising SRB in the future [Bongaarts and Guilmoto (2015)]. While prior work has assessed the potential future SRB imbalances for selected countries with ongoing transitions [Bongaarts and Guilmoto (2015); Chao et al. (2020)], no work to date has quantified the possible additional SRB imbalances for countries where sex ratio transitions may start in the future. Efforts to date to project the SRB globally include those by the United Nations (UN) Population Division [UN, DESA (2019)]. The UN Population Division publishes projections of demographic indicators including the SRB for all countries in the World Population Prospects (WPP) every 2 years. While the UN WPP projections of fertility, mortality and populations are probabilistic, SRB projections are deterministic and based on expert-based opinions [UN, DESA (2019)]. Specifically, WPP methods are based on the assumption that future SRB outcomes either remain at the same level as most recently observed or converge to 1.05 within the next 10–40 years, and future SRB imbalance is not assessed.

In this study, we extend the SRB estimation model [Chao et al. (2019a)] to produce scenario-based SRB projections, and associated SRB imbalances, till 2100 for all countries. The update in the SRB inflation model allows for identification of countries with past and ongoing SRB inflation in a fully Bayesian approach. We use the updated model to estimate the sex ratio transition for country-years with risk of SRB inflation. Subsequently, we construct scenario-based SRB projections that are based on the sex ratio transition model with varying assumptions regarding the occurrence of a sex ratio transition in at-risk countries.

This paper is organized as follows: we first summarize the data used for estimating the SRB in Section 2. Section 3 introduces the models for estimating the SRB, followed by the approaches used to produce scenario-based SRB projections based on model. In Section 4, we present results regarding SRB baselines, past SRB imbalances, and scenario-based projections, as well as validation results. Finally, in Section 5, we end with a discussion of implications of our model and results, limitations and possible future research.

2. Data. We produce SRB estimates for 212 countries¹ with total population size greater than 90,000 as of 2017. An overview of observations by data source type is in Table 1. There are 10,835 data points available from 202 countries.

TABLE 1

SRB observations by source type. DHS: Demographic and Health Surveys. Other DHS refer to non-standard DHS, including Special, Interim and National DHS, Malaria Indicator Surveys, AIDS Indicator Surveys, World Fertility Surveys, Reproductive Health Survey, Multiple Indicator Cluster Surveys, Pan Arab Project for Child Development and Pan Arab Project for Family Health. CRVS: civil registration vital statistics. SRS: sampling registration system.

Data Source Type	# Observations	# Country-Years
Census	48	48
DHS	2,257	5,413
Other DHS	1,392	3,662
Other	142	222
CRVS/SRS	6,996	7,257
Total	10,835	16,602

Data availability is summarized by source type in Table 1. We compile civil registration vital statistics (CRVS) data from the UN Demographic Yearbook and the Human Mortality Database, and sampling registration system (SRS) data for India, Pakistan and Bangladesh from annual reports. CRVS and SRS typically provide data on an annual basis, based on government administrative record from birth certificates. International survey data (Demographic and Health Surveys, World Fertility Surveys, Reproductive Health Survey, Multiple Indicator Cluster Surveys, Pan Arab Project for Child Development and Pan Arab Project for Family Health) were compiled from microdata when possible, and obtained from reports otherwise. Census and national-level survey data were obtained from reports. Censuses usually provide information on SRB for a retrospective period of 12 or 24 months prior to the survey date. Surveys collect data on recent births or full birth histories from women of reproductive ages for longer retrospective periods of 5 to 20 years before the survey date. Details on data series by country and preprocessing are given in Chao et al. (2019a,b).

3. Methods.

Notation. We use lowercase Greek letters for unknown parameters and uppercase Greek letters for variables which are functions of unknown parameters. Where relevant for communicating model assumptions, we add hyper parameters associated with parameters in parentheses. I.e. for some model parameter ϕ , the notation $\phi(\zeta)$ implies that the distribution for ϕ is parametrized using hyper parameter ζ . Roman letters indicate variables that are known or fixed, including data (in lowercase) and estimates provided by other sources or the literature (in uppercase).

¹We use the term "country" to refer to populations that are considered as countries or areas in the UN classification.

 $\Theta_{c,t}$ denotes the main outcome of interest to be modeled, which is the SRB for country c in year t. Observations are combined across countries overtime and indexed by $i \in \{1, \cdots, n\}$; c[i] refers to the country the i-th observation belongs to, t[i] to the calendar year of the observation, and s[i] to the data source type (Table 1) of the observation. r[c] refers to the region that country c belongs to.

3.1. SRB model summary. We assume the i-th observed SRB y_i follows a normal distribution on the log-scale:

(1)
$$\log(y_i)|\Theta_{c[i],t[i]},\omega_{s[i]} \sim \mathcal{N}\left(\log(\Theta_{c[i],t[i]}),\omega_{s[i]}^2 + v_i^2\right),$$

The variance for the i-th log-scaled SRB observation $\log(y_i)$ is the sum of known stochastic/sampling variance v_i^2 and unknown non-sampling error variance $\omega_{s[i]}^2$ for data source type s[i] (Section 3.2).

In its general form, the process model for SBR $\Theta_{c,t}$ is defined as follows:

(2)
$$\Theta_{c,t} = \beta_c \eta_{c,t}(\boldsymbol{\phi}) + \delta_c \Omega_{c,t}(\boldsymbol{\zeta}),$$

where β_c refers to country-specific time-invariant baseline SRB, and $\eta_{c,t}$ the country-year-specific fluctuations around the baseline. δ_c indicates the absence or presence of SRB inflations in country c, and for countries with inflations, $\Omega_{c,t}$ captures the SRB imbalance in year t. The vector of hyper parameters related to $\eta_{c,t}$ is denoted as ϕ , and we use ζ for the vector of hyper parameters related to $\Omega_{c,t}$.

We identify a set of countries where SRB inflation may have happened in the past or may happen in the future, i.e. countries with $\Pr(\delta > 0) \neq 0$, which we refer to as "countries at risk of SRB inflation" (see Section 3.3). The estimation of baseline SRB β_c and fluctuations around baseline $\eta_{c,t}$ is described in Section 3.4. The sex transition model $\delta_c \Omega_{c,t}$ is given in Section 3.5. Finally, in Section 3.6, we introduce the approach for constructing scenario-based projections, using the models developed in the preceding sections. The full model specification, priors and details on computation are in the Appendix 5.

3.2. Error variances. We account for differences in error variance across observations from civil registration vital statistics systems (CRVS), surveys and censuses. Errors—and hence the error variance—associated with non-CRVS data tend to be larger than errors associated with CRVS data and this is reflected in the model fitting, as the weight assigned to a data point increases as its error variance decreases. Resulting model-based estimates are more strongly weighted by observations with smaller errors, and uncertainty ranges are narrower for country-periods with more observations with smaller error variance.

As per Equation 1, the variance for the i-th log-scaled observed SRB $\log(y_i)$ is the sum of known stochastic/sampling variance v_i^2 and unknown non-sampling error variance $\omega_{s[i]}^2$ for data source type s[i] as listed in Table 1. For CRVS observations, v_i is the stochastic error and is pre-calculated as described elsewhere [Chao et al. (2019a,b)], and we assume that non-sampling error is zero: $\omega_s = 0$ when s = CRVS/SRS. For observations from surveys or censuses, v_i is the sampling error and is pre-calculated using a jackknife method as explained in Chao et al. (2019a,b), to reflect the survey sampling design. Non-sampling variance term ω_s^2 captures random errors that may occur during the data collection process. This variance parameter is estimated and assigned a vague prior:

(3)
$$\omega_s \sim \mathcal{U}(0, 0.5)$$
, for $s \in \{\text{Census, DHS, Other DHS, Other}\}$.

3.3. Selection of countries at risk of SRB inflation. The model for natural fluctuations in the SRB is fitted to the global database after excluding data from country-years that may have been affected by masculinisation of the SRB. We use inclusive criteria to identify such country-years, based on a combination of qualitative and quantitative approaches. We select countries with at least one of the following manifestations of son preference: (i) a high level of desired sex ratio at birth (DSRB), or (ii) a high level of sex ratio at last birth (SRLB), or (iii) strong son preference or inflated SRB suggested by a literature review [Chao et al. (2019a,b)]. DSRB is calculated as the ratio of the reported number of desired male births to desired female births, as reported by women during survey interviews. The DSRB reflects the desired sex composition at the time of survey interview. The SRLB quantifies the sex ratio among births that are the latest births to women who desire no more children.

A total of 29 countries satisfy at least one of the aforementioned criteria, and hence are considered at risk of SRB inflation: Afghanistan; Albania; Armenia; Azerbaijan; China; Egypt; Gambia; Georgia; Hong Kong, SAR of China; India; Jordan; Pakistan; Republic of Korea; Mali; Mauritania; Montenegro; Morocco; Nepal; Nigeria; Senegal; Singapore; Taiwan, Province of China; Tajikistan; Tanzania; Tunisia; Uganda; Vietnam.

3.4. Estimating the SRB in country-years without SRB inflation. In the model for country-years not affected by sex-selective abortion, we assume $\delta_c = 0$ in Equation 2. The SRB is thus given by a product of two components:

(4)
$$\Theta_{c,t} = \beta_c \eta_{c,t}(\boldsymbol{\phi}),$$

where β_c is a national baseline for country c, which is assumed to be constant over time and $\eta_{c,t}(\phi)$ is a country-year-specific multiplier that captures the natural fluctuation of the country-specific SRB around its respective baseline value over time, and (ϕ) is the vector of the hyper parameters related to $\eta_{c,t}$. In this step of modeling, parameters that are not related to prenatal gender discrimination and sex-selective abortion are estimated. In order to do that, we use a reduced SRB database by excluding SRB observations that may be affected by prenatal sex discrimination and sex-selective abortion. The excluded data points are from the 29 countries at risk of SRB inflation (listed in Section 3.3) from reference year 1970 onward because the sex-selective abortion technology became more accessible and affordable since then [Allahbadia (2002); George (2002); Goodkind (1997); Oomman and Ganatra (2002); Tandon and Sharma (2006)].

The national baseline β_c follows a hierarchical distribution with mean at its corresponding regional baseline $\beta_{r[c]}^{(\text{region})}$:

(5)
$$\log(\beta_c)|\beta_{r[c]}^{(\text{region})}, \sigma_{\beta} \sim \mathcal{N}\left(\log(\beta_{r[c]}^{(\text{region})}), \sigma_{\beta}^2\right).$$

National baselines are pooled toward regional baseline $\beta_r^{(\text{region})}$ to capture SRB differences due to ethnic origin [Chahnazarian (1988); Dubuc and Coleman (2007); Garenne (2002, 2008); Graffelman and Hoekstra (2000); James (1984, 1985, 1987); Kaba (2008); Ruder (1985); Marcus et al. (1998); Mathews and Hamilton (2005); Visaria (1967)]. For example, we group countries in Europe, North America, Australia, and New Zealand to refer to the regional grouping of countries with a majority of Caucasians. We assume that the national baseline β_c and regional baseline $\beta_r^{(\text{region})}$ are constant over time. We assign independent uniform priors to each $\beta_r^{(\text{region})}$ and a vague prior to σ_β .

The country-year-specific multiplier $\eta_{c,t}$ is modeled on the log-scale with an autoregressive AR(1) time series process within a country. For countries without any data or with very limited information, $\eta_{c,t}$ is shrunk towards 1, such that the estimated SRBs without prenatal sex discrimination are close to their corresponding national baselines β_c . For countries

where the data suggest different levels or trends, $\eta_{c,t}$ captures these deviations from β_c . Let $\phi = \{\rho, \sigma_{\epsilon}\}$ be the vector of hyper parameters related to $\eta_{c,t}$. We assume:

(6)
$$\log(\eta_{c,t})|\phi \sim \mathcal{N}(0,(1-\rho^2)/\sigma_{\epsilon}^2), \text{ for } t = 1950,$$

(7)
$$\log(\eta_{c,t}) = \rho \log(\eta_{c,t-1}) + \epsilon_{c,t}, \text{ for } t \in \{1951, \dots, 2100\},$$

(8)
$$\epsilon_{c,t} | \sigma_{\epsilon} \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_{\epsilon}^2).$$

Vague priors are assigned to σ_{β} , ρ and σ_{ϵ} .

3.5. Model of country-years at risk of SRB inflation. We model SRB $\Theta_{c,t}$ for the 29 countries at risk of SRB inflation (listed in Section 3.3) as the sum of two parts: (i) the inflation-free SRB level, given by the model of country-years without SRB inflation as described in Section 3.4; and (ii) the SRB imbalanced level with probability. Specifically, $\Theta_{c,t}$ for country c, year t is modeled as:

(9)
$$\Theta_{c,t} = \hat{\beta}_c \eta_{c,t}(\hat{\phi}) + \delta_c \Omega_{c,t}(\zeta),$$

where $\hat{\beta}_c$ is the posterior median for the national baseline and $\hat{\phi} = \{\hat{\rho}, \hat{\sigma}_\epsilon\}$ the vector of posterior medians of ϕ obtained from the inflation-free model fit described in Section 3.4. The product $\delta_c \Omega_{c,t}(\zeta)$ captures SRB imbalance. It is the product of a binary indicator indicating presence or absence of SRB inflation δ_c for country c, and a non-negative SRB inflation $\Omega_{c,t}(\zeta)$ where ζ is the vector of hyper parameters related to $\Omega_{c,t}$.

The country-specific binary factor δ_c detects the existence of SRB inflation, with values either 0 (no inflation) or 1 (with inflation). δ_c is modeled with a Bernoulli distribution with country-specific probability of having inflation π_c :

(10)
$$\delta_c | \pi_c \sim \mathcal{B}(\pi_c).$$

Logit-transformed π_c follows a hierarchical normal distribution with a global mean at μ_{π} and a global variance at σ_{π}^2 :

(11)
$$\operatorname{logit}(\pi_c)|\mu_{\pi}, \sigma_{\pi} \sim \mathcal{N}(\mu_{\pi}, \sigma_{\pi}^2),$$

Vague priors are assigned to σ_{π} and μ_{π} .

 $\Omega_{c,t}$ is the upward SRB inflation factor for country c in year t to capture higher SRB levels that may be due to sex-selective abortion. We parameterize the sex ratio transition using a trapezoid function to represent consecutive phases of increase, stagnation and decrease back to zero (Figure 1). The inflation factor $\Omega_{c,t}$ is modeled as:

(12)
$$\Omega_{c,t} = \begin{cases} \xi_c(t - \gamma_{0,c})/\lambda_{1,c}, & \gamma_{0,c} < t < \gamma_{1,c} \\ \xi_c, & \gamma_{1,c} < t < \gamma_{2,c} \\ \xi_c - \xi_c(t - \gamma_{2,c})/\lambda_{3,c}, & \gamma_{2,c} < t < \gamma_{3,c} \\ 0, & t < \gamma_{0,c} \text{ or } t > \gamma_{3,c} \end{cases}, \text{ where }$$

$$\gamma_{1,c} = \gamma_{0,c} + \lambda_{1,c},$$

$$\gamma_{2,c} = \gamma_{1,c} + \lambda_{2,c},$$

$$\gamma_{3,c} = \gamma_{2,c} + \lambda_{3,c},$$

with sex ratio transition parameters $\gamma_{0,c}$, the start year of the inflation, ξ_c , the maximum inflation, and $\lambda_{1,c}$, $\lambda_{2,c}$ and $\lambda_{3,c}$, the lengths of the inflation period during the three phases.

The model for $\Omega_{c,t}$ makes use of fertility as an external covariate related to SRB inflation to better capture the sex ratio transition process. Specifically, information related to the "fertility squeeze" effect (fertility decreases leading to fewer children per family, inducing the

Sex ratio transition model

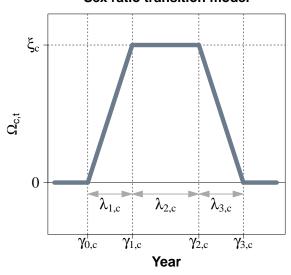


FIG 1. *Illustration of sex ratio transition model*. All parameters are for country c. $\gamma_{0,c}$: start year of SRB inflation. $\lambda_{1,c}$, $\lambda_{2,c}$ and $\lambda_{3,c}$: period lengths of the SRB inflation phases of increase, stagnation and decrease back to zero. ξ_c : maximum value of the inflation.

willingness to sex-selective abortion) is incorporated into the model through the parametrization of the start year $\gamma_{0,c}$ (see Section 3.5.1). Despite the acknowledged role of son preference (offering the motivation of sex-selective abortion) or availability of sex-selection technology (providing the means of sex selection) in driving SRB imbalance [Guilmoto (2009, 2012a)], the model for $\Omega_{c,t}$ does not make use of covariates related to these two factors because detailed information is general unavailable for estimation and available candidate predictors do not explain variability across countries [Chao et al. (2019a)]. Instead, sex ratio transition parameters related to the lengths of the transition phases and its maximum are estimated with Bayesian hierarchical models [Gelman et al. (2013); Lindley and Smith (1972)]:

(13)
$$\xi_c|\mu_{\xi}, \sigma_{\xi} \sim \mathcal{N}(\mu_{\xi}, \sigma_{\xi}^2)T(0,),$$

(14)
$$\lambda_{1,c}|\mu_{\lambda 1}, \sigma_{\lambda 1} \sim \mathcal{N}(\mu_{\lambda 1}, \sigma_{\lambda 1}^2)T(0,),$$

(15)
$$\lambda_{2,c}|\mu_{\lambda 2}, \sigma_{\lambda 2} \sim \mathcal{N}(\mu_{\lambda 2}, \sigma_{\lambda 2}^2)T(0,),$$

(16)
$$\lambda_{3,c}|\mu_{\lambda 3}, \sigma_{\lambda 3} \sim \mathcal{N}(\mu_{\lambda 3}, \sigma_{\lambda 3}^2)T(0,),$$

where $\mathcal{N}(\cdot)T(0,)$ refers to a truncated normal distribution with lower truncation at zero. We assign vague priors to the mean and standard deviation of these truncated distributions (Appendix 5).

3.5.1. Inflation start year. We model the SRB inflation start year $\gamma_{0,c}$ as:

(17)
$$\gamma_{0,c}|\sigma_{\gamma} \sim t_3(x_c, \sigma_{\gamma}^2)T(z_c,),$$

referring to a Student- t_3 distribution with lower truncation in year z_c , location x_c and scale σ_γ . The fertility squeeze effect is incorporated into the model for $\gamma_{0,c}$ using country-year estimates of the total fertility rate $F_{c,t}$ from the UN World Population Prospects (WPP) 2019 [UN, DESA (2019)]. The total fertility rate (TFR) approximates the number of children that

would be born to a woman who survives throughout her reproductive ages. The lower truncation z_c refers to the year that the TFR in country c decreased to 6 or the year 1970, whichever occurred later:

$$(18) z_c = \max\{1970, f_{c,6}\},\$$

where $f_{c,6}$ is the first year in which the TFR in country c declines to 6. The location indicator x_c is the year in which the TFR in country c declines to the pre-calculated value of 2.9 (the TFR value 2.9 is determined by the median of fertility levels when the observed sex ratio transition started among countries with high-quality CRVS data [Chao et al. (2019a,b)]):

(19)
$$x_c = \max\{1970, f_{c,2.9}\}.$$

3.6. Scenario-based projections. We construct scenario-based SRB projections that are based on the sex ratio transition model in Equation 2 with varying assumptions regarding the occurrence of a sex ratio transition in at-risk countries. The three scenarios are: (S1) SRB inflation continues in countries with strong statistical evidence of past inflations only, (S2) SRB inflations are projected to occur in all at-risk countries with country-specific inflation probabilities, and (S3) SRB inflations are projected to occur in all at-risk countries with probability 1. The scenarios are labeled in terms of increasing sex imbalance in future years, with S1 including the sex imbalance due to ongoing transitions in countries with strong statistical evidence of such inflations only, and S3 including sex imbalance for at-risk countries without current evidence of inflation.

The projections are based on estimates from different model fits. Model fits M1 and M2 were defined in previous sections, with:

- M1: Model without inflation term, $\Theta_{c,t} = \beta_c \eta_{c,t}(\phi)$, fit to dataset $\boldsymbol{y}^{(\text{risk-free})}$, as described in Section 3.4;
- M2: Model with inflation term, $\Theta_{c,t} = \hat{\beta}_c \eta_{c,t}(\hat{\phi}) + \delta_c \Omega_{c,t}(\zeta)$, fit to all data from at-risk countries $\boldsymbol{y}^{(\text{at-risk})}$, as described in Section 3.5.

We identify countries with strong statistical evidence of SRB inflation in a fully Bayesian approach, using ψ_c , the posterior mean of the inflation indicator δ_c for country c from M2: $\psi_c = \mathbb{E}(\delta_c|\boldsymbol{y}^{(\text{at-risk})})$. ψ_c represents the relative inclusion of sex ratio inflation in country c. If $\psi_c \geq 95\%$, then we consider the country to have strong statistical evidence of SRB inflation. We use $\mathcal{C}^{\text{inflation}}$ to denote the set of country indices c with $\psi_c \geq 95\%$. For $c \in \mathcal{C}^{\text{inflation}}$, the projections under scenarios S1, S2 and S3 are identical and based on posterior estimates of the model parameters from M2, combined with additional uncertainty related to the country baseline β_c as obtained from M1:

(20)
$$\Theta_{c,t}^{S} = \beta_c^{(\text{M1})} \eta_{c,t}^{(\text{M2})} + \delta_c^{(\text{M2})} \Omega_{c,t}^{(\text{M2})}$$
, for scenario $S \in \{S1, S2, S3\}, c \in \mathcal{C}^{\text{inflation}}$

where superscript (M) refers to the model fit used to obtain the estimates for that parameter. For countries that are not at risk of SRB inflation, with $c \in \mathcal{C}^{\text{base}}$, projections are based on the country-specific baseline estimates and deviations away from that baseline and are the same across scenarios, obtained from M1:

(21)
$$\Theta_{c,t}^S = \beta_c^{(\text{M1})} \eta_{c,t}^{(\text{M1})}, \text{ for scenario } S \in \{S1, S2, S3\}, c \in \mathcal{C}^{\text{base}}.$$

The construction of the scenario-based projections for the remaining countries, those at risk of inflation but without strong evidence of past or ongoing inflations, denoted by set C^{risk} , differs between scenarios. In S2, the country-specific probability of having inflation as obtained in M2, is used for constructing S2 projections $\Theta_{c,t}^{S2}$:

(22)
$$\Theta_{c,t}^{S2} = \beta_c^{(M1)} \eta_{c,t}^{(M2)} + \delta_c^{(M2)} \Omega_{c,t}^{(M2)}, \ c \in \mathcal{C}^{risk}.$$

For scenarios S1 and S3 for at-risk countries with $c \in C^{risk}$, additional model fits are introduced to obtain projections based on estimates without inflation (S1) and with 100% chance of inflation (S3). The additional model fits are as follows:

- M3 (used for S1): Model without inflation term, $\Theta_{c,t} = \hat{\beta}_c \eta_{c,t}(\phi)$, fit to country-specific dataset $y^{(c)}$;
- M4 (used for S3): Model with inflation term and $\delta_c = 1$, $\Theta_{c,t} = \hat{\beta}_c \eta_{c,t}(\hat{\phi}) + \Omega_{c,t}(\hat{\zeta})$, fit to country-specific dataset $y^{(c)}$, where $\hat{\zeta}$ refers to point estimates of the vector of hyper parameters for the inflation factor from M2 ($\zeta = \{\mu_{\xi}, \sigma_{\xi}, \mu_{\lambda 1}, \sigma_{\lambda 1}, \mu_{\lambda 2}, \sigma_{\lambda 2}, \mu_{\lambda 3}, \sigma_{\lambda 3}, \sigma_{\gamma}\}$).

With these additional models, projections are defined as follows:

(23)
$$\Theta_{c,t}^{S1} = \beta_c^{(M1)} \eta_{c,t}^{(M3)}, \ c \in \mathcal{C}^{risk},$$

(24)
$$\Theta_{c,t}^{S3} = \beta_c^{(\text{M1})} \eta_{c,t}^{(\text{M4})} + \Omega_{c,t}^{(\text{M4})}, \ c \in \mathcal{C}^{\text{risk}}.$$

We incorporate the uncertainty associated with the start year of the inflation, due to uncertainty in TFR projections, into the SRB projections for S2 and S3 in the projection result for the set of countries C^{risk} . For trajectories with posterior samples of start year $\gamma_{0,c}$ beyond the most recent SRB observation, we add in additional uncertainty of x_c , the year in which TFR in country c declines to 2.9, based on TFR trajectories. We use 1,000 TFR trajectories for associated uncertainty for each country-year based on projections from the bayesTFR R-package [Ševčíková, Alkema and Raftery (2011); Ševčíková et al. (2019)].

- 3.7. *Model validation*. We assess model performance via validation exercises focused on: 1) predicting left-out recent SRB observations and associated inflation estimates, and, 2) predicting SRB inflation transitions.
- 3.7.1. Predicting left-out SRB observations and associated inflation estimates. We assess model performance by leaving out recent data. Specifically, we leave out 20% of the data points that were collected after a certain survey year [Alkema, Wong and Seah (2012); Alkema et al. (2014)] for the out-of-sample validation. After leaving out data, we fitted the model to the training data set, and obtain median estimates and prediction intervals for the SRB and SRB inflation that would have been constructed based on available data set in the survey year selected. We also assess the model performance by leaving out data at random, i.e. leaving out 20% of the data randomly, and repeat this exercise 30 times.

We calculate median errors and median absolute errors for the left-out SRB observations, where errors are defined as: $e_j = y_j - \mathbb{E}(y_j|\boldsymbol{y}^{\text{train}})$, where $\mathbb{E}(y_j|\boldsymbol{y}^{\text{train}})$ refers to the posterior median of the predictive distribution based on the training dataset $\boldsymbol{y}^{\text{train}}$ for the j-th left-out observation y_j . Coverage for 95% prediction intervals for left-out observations is given by $1/J \cdot \sum_{j=1}^J \mathbb{I}_{\mathbf{A}}(y_j, l_j) \cdot \mathbb{I}_{\mathbf{B}}(y_j, u_j)$, where J is the total number of left-out observations, l_j and u_j correspond to the 2.5th and 97.5th percentiles of the posterior predictive distribution (PPD) for the j-th left-out observation y_j , and sets $\mathbf{A} = \{(a,b) \in \mathbb{R}^2 : a > b\}$ and $\mathbf{B} = \{(a,b) \in \mathbb{R}^2 : a < b\}$. For the 80% prediction interval coverage, l_j and u_j refer to the 10th and 90th percentiles of the PPD respectively. The validation measures are calculated for 1000 permutations of left-out observations, where each permutation consists one randomly selected left-out observation from each country with data left out. The reported validation results are based on the mean of the outcomes from the 1000 permuted sets of left-out observations.

For the median estimates based on full data set and training data set, errors are defined as $e(\Theta)_{c,t} = \mathbb{E}(\Theta_{c,t}|\boldsymbol{y}^{\text{full}}) - \mathbb{E}(\Theta_{c,t}|\boldsymbol{y}^{\text{train}})$, where $\mathbb{E}(\Theta_{c,t}|\boldsymbol{y}^{\text{full}})$ is the posterior median for country c in year t based on the full dataset $\boldsymbol{y}^{\text{full}}$, and $\mathbb{E}(\Theta_{c,t}|\boldsymbol{y}^{\text{train}})$ is the posterior median

for the same country-year based on the training dataset. Similarly, the error for the sex ratio transition process with probability is defined as $e(\delta\Omega)_{c,t} = \mathbb{E}(\delta_c\Omega_{c,t}|\boldsymbol{y}^{\text{full}}) - \mathbb{E}(\delta_c\Omega_{c,t}|\boldsymbol{y}^{\text{train}})$. Coverage is computed in a similar manner as for the left-out observations, based on the lower and upper bounds of the equal-tail 95% credible interval of $\mathbb{E}(\Theta_{c,t}|\boldsymbol{y}^{\text{train}})$ from the training dataset.

3.7.2. Predicting sex ratio transitions. We assess the predictive performance of the inflation model by simulating for each at-risk country (from the set of countries $C^{\text{inflation}} \cup C^{\text{risk}}$) its SRB inflation. In this exercise, country data after 1970 is not used to directly inform the transition parameter median estimates. Instead, we use (i) median estimates of the non-inflation terms based on country-specific data prior to 1970, and (ii) simulate the non-inflation and inflation components as follows:

$$\widetilde{\Theta}_{c,t} = \beta_c^{(M1)} \widetilde{\eta}_{c,t} + \widetilde{\delta}_c \widetilde{\Omega}_{c,t},$$

where $\widetilde{\eta}_{c,t}$ are given by M2 prior to 1970 and simulated for t>1970, and $\widetilde{\delta}_c$ and $\widetilde{\Omega}_{c,t}$ are simulated without taking into account any country-specific data, following the model specification for these parameters. After generating the simulated values, we calculate the same set of results as described in Section 3.7.1.

3.8. Estimates of missing female births. The realization of SRB inflation due to sex-selective abortion is quantified with the number of "missing female births", which refers to the additional number of female births that would have been born if the inflation were absent. It is calculated as the difference between the number of female births under normal circumstance (referred here as the "inflation-free" number of female births) and the actual number under the observed SRB, and the computation was first introduced by Dréze and Sen (1990).

The estimated and expected inflation-free number of female live births for a country-year, denoted by $\Psi_{c,t}$ and $\Psi_{c,t}^{(\text{inflation-free})}$, are computed as:

$$\begin{split} \Psi_{c,t} &= B_{c,t}/(1+\Theta_{c,t}), \text{ and} \\ \Psi_{c,t}^{(\text{inflation-free})} &= (B_{c,t}-\Psi_{c,t})/\Theta_{c,t}^{(\text{inflation-free})}, \end{split}$$

where $B_{c,t}$ is the total number of births for a certain country-year, obtained from the UN WPP 2019 [UN, DESA (2019)]. The number of inflation-free female births $\Psi_{c,t}^{(\text{inflation-free})}$ is obtained from the estimated number of male births $(B_{c,t} - \Psi_{c,t})$, and the inflation-free SRB $\Theta_{c,t}^{(\text{inflation-free})} = \beta_c \eta_{c,t}$ for the respective country-year [Dréze and Sen (1990); Guilmoto, Chao and Kulkarni (2020)].

The annual number of missing female births (AMFB) for country c in year t is defined as:

$$\Psi_{c,t}^{(\mathrm{missing})} = \Psi_{c,t}^{(\mathrm{inflation-free})} - \Psi_{c,t}.$$

The cumulative number of missing female births (CMFB) for period t_1 to t_2 in country c is defined as the sum of AMFB from the year t_1 up to the year t_2 :

$$\Lambda_{c,[t_1,t_2]}^{(\text{missing})} = \sum_{t=t_1}^{t_2} \Psi_{c,t}^{(\text{missing})}.$$

4. Results. We first summarize findings related to SRB baselines and natural deviations for country-years without risk of inflation. Secondly, the sex ratio transition model results are presented, followed by projections. Finally, validation results are presented.

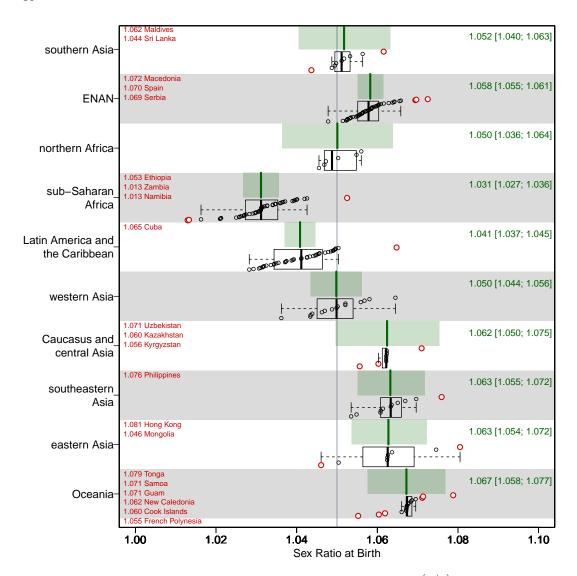


FIG 2. SRB regional and national baseline. Regional baseline median estimates $\beta_{\tau}^{(region)}$ (dark green line), with 95% credible intervals (shaded area) and printed values (green). Box plots summarize the distributions of national baselines β_c within each region. National median estimates are shown (dots) with those outside the range of [25th percentile - 1.5IQR; 75th percentile+1.5IQR] highlighted in red. The countries with outlying national baselines are listed in the legend with the median estimates reported (ordered by median estimate). ENAN: the combination of countries in Europe, North America, Australia, and New Zealand.

4.1. SRB baselines and country-year variations in years without risk of inflation. The median estimates of regional SRB baselines $\beta_r^{(\text{region})}$ and national baselines β_c are illustrated in Figure 2. The lowest regional baseline is estimated in sub-Saharan Africa at 1.031 (95% credible interval [1.027; 1.036]) and the highest is in the Oceania at 1.067 [1.058; 1.077]. Among the ten regions, the probability that baseline is smaller than 1.05 (the widely assumed historical norm) is greater than 97.5% in two regions (sub-Saharan Africa, Latin America and the Caribbean) and the probability that baseline is larger than 1.05 is bigger than 97.5% in four regions (ENAN, southeastern Asia, eastern Asia, and Oceania). The national baselines are estimated to range from 1.013 [1.000; 1.026] in Zambia and 1.013 [0.997; 1.028] in Namibia to 1.081 [1.068; 1.093] in Hong Kong.

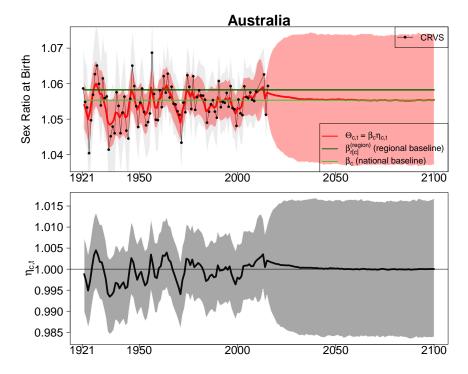


FIG 3. SRB estimates and projections for Australia. Top row: SRB median estimates $\Theta_{c,t}$ (red curve) and 95% credible intervals (red shades), median estimates of the regional baselines $\beta_{r[c]}^{(region)}$ (dark green horizontal line), median estimates of the national baselines β_c (light green horizontal line). SRB observations are displayed with dots and observations are connected with lines when obtained from the same source. Shaded areas around observation series represent the sampling variability in the series (quantified by two times the stochastic/sampling standard errors). Bottom row: median estimates of natural deviation $\eta_{c,t}$ (solid line) and 95% credible intervals (shades).

Australia is an example of a country without risk of SRB inflation. Its estimates and projections are displayed in Figure 3. It typifies countries with high quality annual CRVS data, here available from 1921 to 2015. SRB estimates follow the CRVS data trend and the uncertainty assessment takes into account the stochastic uncertainty associated with the CRVS data. Its national baseline β_c is estimated at 1.055 [1.049; 1.062]. The national baseline of Australia differs from its regional baseline $\beta_{r[c]}^{(region)}$ at 1.058 [1.055; 1.061] for the region ENAN (the combination of countries in Europe, North America, Australia, and New Zealand) since the national baseline is informed by the CRVS data available in Australia. The estimated SRB for Australia $\Theta_{c,t}$ ranges from 1.048 [1.042; 1.055] in 1935 to 1.060 [1.054; 1.067] in 1928. As Australia is not identified to have SRB inflation risk, all the deviations of $\Theta_{c,t}$ from the national baseline of SRB β_c are considered as natural fluctuation and are captured with $\eta_{c,t}$ (Figure 3, bottom row). Given that Australia is considered as not having risk of SRB inflation, its projected SRB is based on the model of country-years without SRB inflation. The SRB projection for Australia is approximately constant and given by its national baseline β_c , with the projection for 2100 given by 1.055 [1.037; 1.074].

4.2. Sex ratio transition model. Table 2 summarizes model results for the sex ratio transition for the 29 countries with risk of SRB inflation. In total, 12 countries are identified as having strong statistical evidence of SRB inflation: Albania; Armenia; Azerbaijan; China; Georgia; Hong Kong, SAR of China; India; Republic of Korea; Montenegro; Taiwan, Province of China; Tunisia; Vietnam.

Figure 4 presents the SRB inflation $\Omega_{c,t}$ (rather than SRB itself) for the 12 countries with strong evidence of SRB inflation. For the 12 countries, the median estimate of start year of the sex ratio transition $\gamma_{0,c}$ is estimated to be the year in which TFR declines to 5.2 in India to 1.0 in Hong Kong (SAR of China). The estimated duration of the sex ratio transition $(\lambda_{1,c} + \lambda_{2,c} + \lambda_{3,c})$ varies greatly across the 12 countries and ranges from relatively fast transitions that last 11, 25 and 26 years for Hong Kong (SAR of China), Georgia, and the Republic of Korea respectively, to 45, 51 and 58 years in Montenegro, China and India. The country-specific maximum value of SRB imbalance ξ_c is estimated to be higher than 0.100 in China at 0.112 [0.078; 0.151], in Armenia at 0.108 [0.088; 0.129] and in Azerbaijan at 0.104 [0.087; 0.121]. The maximum imbalance is lower than 0.050 in Montenegro at 0.045 [0.022; 0.068], in Tunisia at 0.035 [0.019; 0.050] and in Taiwan (Province of China) at 0.030 [0.016; 0.043]. For the 12 countries, the average projection period from the most recent observation year to end of transition is 8.8 years, corresponding to an average of 20.1% of country-specific duration of the sex ratio inflation.

Figure 5 presents the projected SRB inflation $\Omega_{c,t}$ for a country prior to observing SBR data. Given the hierarchical structure of the SRB inflation model, the sex ratio transition in Figure 5 represents the average experience of SRB imbalance. The median projected SRB inflation process $(\lambda_{1,c} + \lambda_{2,c} + \lambda_{3,c})$ has a span of 37 [15; 64] years. The maximum SRB inflation ξ_c for a new country has a median at 0.032 with 95% credible interval [0.000; 0.132]. The inflation maximum is reached around 11 [1; 28] years after the country's TFR declines to 2.9 in the year x_c .

Table 2: Sex ratio transition model results for the 29 countries at risk of SRB inflation. Numbers above the brackets are posterior median estimates. Numbers inside the brackets are 95% credible intervals. ψ_c : the relative inclusion of SRB inflation. $\gamma_{0,c}$: the start year of SRB inflation. $\lambda_{1,c}$, $\lambda_{2,c}$, $\lambda_{3,c}$: period lengths for increase, stagnation and decrease for the sex ratio transition process. $\gamma_{3,c} = \gamma_{0,c} + \lambda_{1,c} + \lambda_{2,c} + \lambda_{3,c}$: the end year of SRB inflation. ξ_c : maximum SRB inflation. 2100+: indicates the year is beyond 2100. Countries with strong statistical evidence of SRB inflation are in boldface.

Country	ψ_c	TFR in	$\gamma_{0,c}$	$\gamma_{3,c}$	ξ_c	$\lambda_{1,c}$	$\lambda_{2,c}$	$\lambda_{3,c}$
	(in %)	$\gamma_{0,c}$	(start year)	(end year)				
Albania	100	3.1	1988	2024	0.059	15	3	15
			[1973; 1997]	[2016; 2043]	[0.038; 0.079]	[3; 31]	[0; 26]	[6; 33]
Armenia	100	2.5	1992	2029	0.109	7	5	26
			[1990; 1993]	[2020; 2042]	[0.088; 0.13]	[5; 9]	[0; 19]	[7; 39]
Azerbaijan	100	3.0	1991	2031	0.104	10	9	19
			[1988; 1993]	[2019; 2049]	[0.087; 0.122]	[7; 15]	[2; 23]	[3; 39]
China	100	2.6	1980	2030	0.114	20	11	18
			[1972; 1988]	[2017; 2051]	[0.08; 0.156]	[8; 32]	[1; 28]	[3; 39]
Georgia	100	2.1	1992	2016	0.054	3	9	12
			[1979; 1994]	[2008; 2027]	[0.039; 0.07]	[0; 20]	[1; 13]	[0; 27]
Hong Kong,	100	1.0	2004	2013	0.076	6	2	2
SAR of			[2002; 2005]	[2012; 2014]	[0.059; 0.096]	[4; 9]	[0; 4]	[0; 3]
China								
India	100	5.2	1975	2033	0.056	19	16	24
			[1970; 1981]	[2021; 2050]	[0.042; 0.072]	[7; 29]	[2; 34]	[3; 44]
Republic of	100	2.4	1982	2006	0.072	8	4	13
Korea								
			[1978; 1984]	[1997; 2011]	[0.058; 0.087]	[6; 12]	[2; 5]	[2; 18]
Tunisia	100	4.9	1982	2021	0.036	13	10	15
			[1976; 1989]	[2012; 2039]	[0.021; 0.052]	[3; 25]	[1; 21]	[2; 36]

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Table 2 – continued from previous page

			Tubic 2 Cont	mueu mom previ	lous puge			
Country	ψ_c	TFR in	γ0,c	γ3,c	ξ_c	$\lambda_{1,c}$	$\lambda_{2,c}$	$\lambda_{3,c}$
¥7° 4	(in %)	$\gamma_{0,c}$	(start year)	(end year)	0.066	10	0	1.6
Vietnam	100	2.0	2001	2036	0.066	10	8	16
	400		[1991; 2005]	[2017; 2061]	[0.035; 0.131]	[1; 26]	[0; 25]	[2; 37]
Montenegro	100	2.3	1980	2024	0.048	14	10	19
			[1971; 1990]	[2014; 2043]	[0.026; 0.076]	[2; 29]	[1; 26]	[3; 39]
Taiwan,	99.7	2.4	1982	2023	0.031	10	12	20
Province of China			[1972; 1987]	[2012; 2041]	[0.018; 0.045]	[3; 27]	[1; 22]	[5; 39]
Mauritania	63.8	2.9	2065	2100+	0.064	11	8	16
			[2038; 2093]	[2068; 2100+]	[0.005; 0.163]	[1; 28]	[0; 25]	[1; 37]
Mali	63.4	2.9	2061	2099	0.064	11	8	16
			[2036; 2089]	[2066; 2100+]	[0.005; 0.16]	[1; 28]	[0; 25]	[1; 37]
Afghanistan	63.3	2.9	2033	2071	0.063	11	8	16
			[2013; 2063]	[2041; 2100+]	[0.005; 0.16]	[1; 28]	[0; 25]	[1; 37]
Nigeria	63.2	2.9	2065	2100+	0.064	11	8	16
_			[2038; 2093]	[2069; 2100+]	[0.005; 0.163]	[1; 28]	[0; 25]	[1; 37]
Gambia	63.1	2.9	2053	2091	0.064	11	8	16
			[2027; 2082]	[2057; 2100+]	[0.005; 0.161]	[1; 28]	[0; 25]	[1; 37]
Pakistan	63.0	2.9	2030	2068	0.062	11	8	16
			[1995; 2058]	[2027; 2100+]	[0.005; 0.16]	[1; 28]	[0; 25]	[1; 37]
Senegal	63.0	2.9	2061	2099	0.064	11	8	16
			[2034; 2089]	[2064; 2100+]	[0.005; 0.161]	[1; 28]	[0; 25]	[1; 37]
Tanzania	62.7	2.9	2068	2100+	0.064	11	8	16
			[2041; 2096]	[2071; 2100+]	[0.005; 0.162]	[1; 28]	[0; 25]	[1; 37]
Uganda	62.6	2.9	2042	2080	0.064	11	8	16
			[2020; 2070]	[2049; 2100+]	[0.005; 0.162]	[1; 28]	[0; 25]	[2; 37]
Nepal	62.6	2.6	2009	2047	0.058	12	8	16
			[1989; 2036]	[2018; 2083]	[0.005; 0.155]	[1; 28]	[0; 25]	[1; 37]
Tajikistan	62.2	2.9	2038	2076	0.063	11	8	16
			[2016; 2067]	[2044; 2100+]	[0.005; 0.162]	[1; 28]	[0; 25]	[1; 37]
Egypt	61.9	2.9	2030	2068	0.063	11	8	16
			[2006; 2058]	[2034; 2100+]	[0.005; 0.161]	[1; 28]	[0; 25]	[1; 37]
Jordan	56.3	2.7	2019	2057	0.061	11	8	16
			[2000; 2048]	[2028; 2095]	[0.004; 0.158]	[1; 28]	[0; 25]	[1; 37]
Singapore	44.9	2.3	1975	2014	0.029	10	8	17
			[1970; 2012]	[1990; 2055]	[0.003; 0.145]	[1; 27]	[0; 25]	[2; 37]
Morocco	40.8	2.6	2003	2041	0.061	11	8	16
			[1982; 2033]	[2010; 2080]	[0.003; 0.156]	[1; 28]	[0; 25]	[1; 37]
Bangladesh	37.4	2.4	2009	2046	0.061	11	8	16
-			[1987; 2038]	[2015; 2085]	[0.003; 0.162]	[1; 28]	[0; 25]	[1; 37]
Turkey	34.9	2.9	1993	2031	0.046	11	7	16
•			[1974; 2028]	[2001; 2074]	[0.003; 0.151]	[1; 28]	[0; 25]	[2; 37]

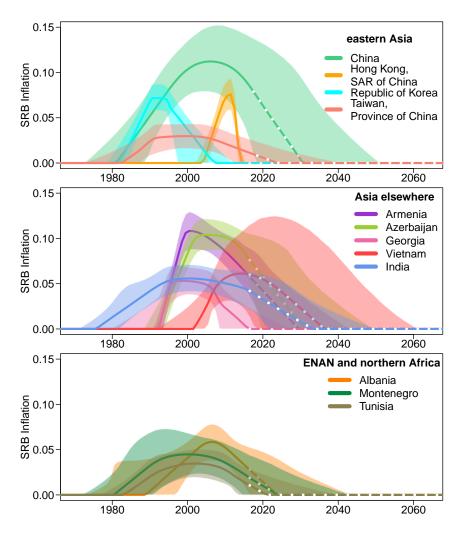


FIG 4. SRB inflation estimates and projections for countries with strong statistical evidence of SRB inflation. SRB inflation median estimates during periods with data (solid lines), median projections (dashed lines), and 95% credible intervals (shades). ENAN: the combination of countries in Europe, North America, Australia, and New Zealand.

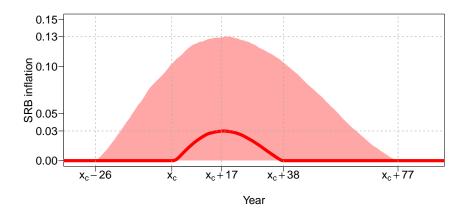


FIG 5. SRB inflation projection for a country with future SRB inflation/prior to observing data. Median projection (solid line) with 95% credible intervals (shades). x_c refers to the year in which the total fertility rate (TFR) in a country declines to 2.9.

4.3. SRB estimates and projections for countries at risk of SRB inflation. SRB and resulting missing female births projections by scenario for all countries at risk of SRB inflation are given in the Supplementary figure A 9. Here we illustrate projections for selected countries.

Countries with strong statistical evidence of SRB inflation: China. China is identified with strong statistical evidence of SRB inflation as listed in Table 2. Its SRB inflation is still ongoing at the start of the projection period as shown in Figure 6. The start year of the inflation is estimates at 1980 [1972; 1988] when the TFR declined to 2.6. We estimate that the SRB in China peaked in 2005 at 1.179 [1.141; 1.220], with associated SRB inflation of 0.112 [0.079; 0.150]. In 2017, the SRB in China is estimated at 1.144 [1.079; 1.206]. We project that the SRB will converge back to the range of natural fluctuations around its national baseline value of 1.063 [1.044; 1.082] in 2030 [2017; 2051].

The annual number of missing female births (AMFB) in China peaked in 2007 at 0.8 [0.6; 1.1] million female births per year. With the inflation projected to decrease to zero by 2030, the resulting AMFB is also projected to decrease to zero at that time. The cumulative number of missing female births (CMFB) for China since 1970 is projected to be 27.9 [18.6; 41.4] million by 2100.

Countries at risk but without strong evidence of SRB inflation: Afghanistan and Senegal. 17 countries are classified with risk of SRB inflation without strong statistical evidence of SRB inflation as listed in Table 2. The SRB in these countries are projected under three scenarios based on different assumptions on the occurrence of a sex ratio transition. We use Afghanistan and Senegal as examples to illustrate the scenario-based projections of SRB for such country (Figure 7 and Figure 8).

During the observation period for Afghanistan (Figure 7), data series for Afghanistan do not imply SRB inflation and hence all the fluctuations and deviations of $\Theta_{c,t}$ away from national baseline β_c are captured with $\eta_{c,t}$. For Afghanistan, $\eta_{c,t}$ is close to one throughout the estimation period.

The three SRB projection scenarios result in substantively different future SRB and associated missing births in Afghanistan. The projection under S1 without future inflation, $\Theta_{c,t}^{S1}$, remains at its national baseline β_c . The S2 projection $\Theta_{c,t}^{S2}$ include a sex ratio transition in $\psi_c = 63\%$ of all future trajectories while S3 projection $\Theta_{c,t}^{S3}$ includes transitions for all trajectories. For future trajectories with sex imbalances in Afghanistan under scenarios S2 and S3, the transition is projected to start in the 2030s ($\gamma_{0,c}$ is projected in the year 2033 [2013; 2063]) and ends in 2071 [2041; beyond 2100]. Given that we incorporate the uncertainty in the TFR for S2 and S3 projections $\Theta_{c,t}^{S2}$ and $\Theta_{c,t}^{S3}$, the SRB inflation process is more flattened than the one shown in Figure 5. Under scenario S2, the average annual number of missing female births (AMFB) during 2018–2100 is projected to be 4 [0; 22] thousand. The corresponding cumulative number of missing female births (CMFB) during 2018–2100 is projected to be 303 [0; 1787] thousand. Under scenario S3, the average AMFB is projected at 8 [0; 22] thousand during 2018–2100 and end up with 624 [36; 1848] thousand missing female births cumulatively.

For Senegal (Figure 8), the model detects no SRB inflation during the data period and projects the SRB imbalance to start in 2061 [2034; 2089] when the TFR declines to 2.9 and to end in 2099 [2064; beyond 2100]. The timing of the inflation is projected to be later than the sex ratio transition in Afghanistan due to differences in TFR projections, with Senegal's TFR projections suggesting a slower decline than expected in Afghanistan. For Senegal under scenario S2, the average AMFB from 2018 to 2100 is projected to be 1 [0; 16] thousand and corresponding CMFB is projected to be 104 [0; 1,298] thousand. The average AMFB and resulting CMFB for scenario S3 are 5 [0; 17] thousand and 399 [0; 1,422] thousand.

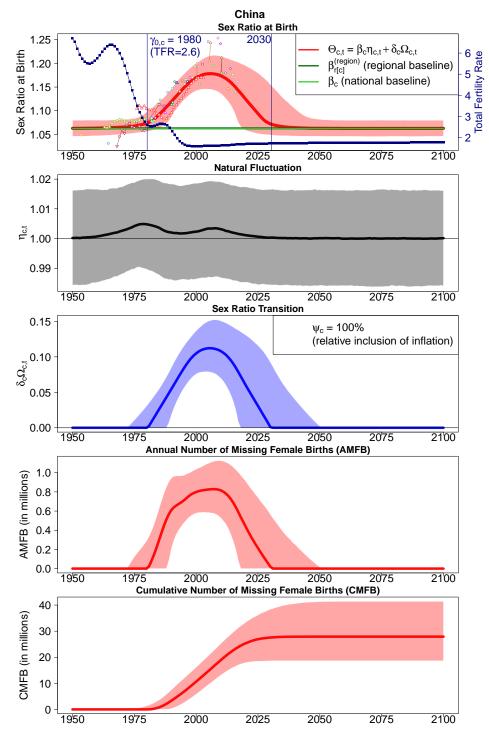


FIG 6. SRB and missing female births estimates and projections for China. Row 1: SRB median estimates $\Theta_{c,t}$ (red curve) and 95% credible intervals (red shades), median estimates of the regional baselines $\beta_{r[c]}^{(region)}$ (dark green horizontal line), median estimates of the national baselines β_c (light green horizontal line). SRB observations are displayed with dots and observations are connected with lines when obtained from the same source. Shaded areas around observation series represent the sampling variability in the series (quantified by two times the stochastic/sampling standard errors). The median estimates and projections of total fertility rate (TFR) from the UN WPP 2019 are added (blue squared dots). The median estimates of inflation start year $\gamma_{0,c}$ and end year $\gamma_{3,c}$ are the vertical lines. The TFR value in the year $\gamma_{0,c}$ is shown. Row 2: median estimates of natural deviation $\eta_{c,t}$ (solid line) and 95% credible intervals (shades). Row 3: median estimates of SRB inflation $\delta_c \Omega_{c,t}$ (curves) and 95% credible intervals (shades). Row 4: annual number of missing female births (AMFB) estimates and projections. Row 5: cumulative number of missing female births (CMFB) estimates and projections.

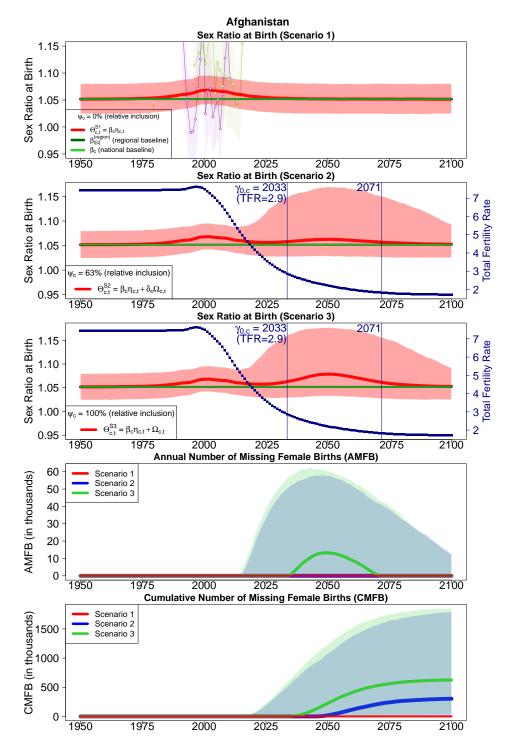


FIG 7. SRB and missing female births estimates and scenario-based projections for Afghanistan. Row 1: SRB median estimates/projections and 95% credible intervals for Scenario 1 $\Theta_{c,t}^{S1}$ (red curve and shades), median estimates of the regional baselines $\beta_{r[c]}^{(region)}$ (dark green horizontal line), median estimates of the national baselines β_c (light green horizontal line). SRB observations are displayed with dots and observations are connected with lines when obtained from the same source. Shaded areas around observation series represent the sampling variability in the series (quantified by two times the stochastic/sampling standard errors). Row 2: SRB median estimates/projections and 95% credible intervals for Scenario 2 $\Theta_{c,t}^{S2}$ (curve and shades). Row 3: SRB median estimates/projections of total fertility rate (TFR) from the UN WPP 2019 are added (blue squared dots). The median estimates of inflation start year $\gamma_{0,c}$ and end year $\gamma_{3,c}$ are the vertical lines. The TFR value in the year $\gamma_{0,c}$ is shown. Row 4: annual number of missing female births (AMFB) estimates and projections by scenario. Row 5: cumulative number of missing female births (CMFB) estimates and projections by scenario.

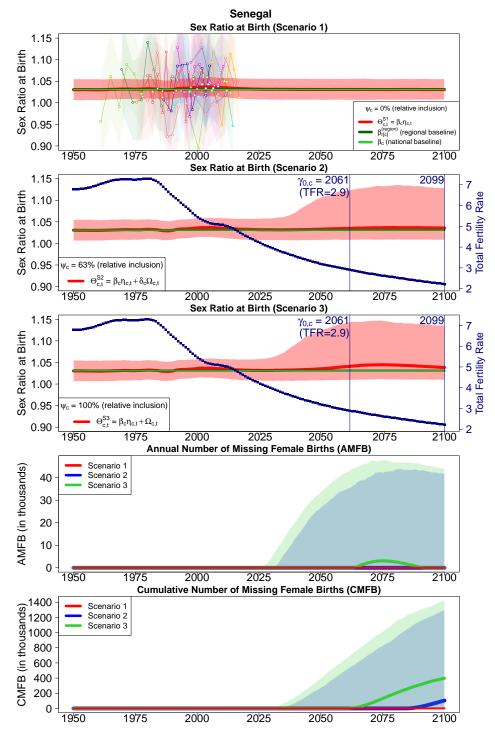


FIG 8. SRB and missing female births estimates and scenario-based projections for Senegal. Row 1: SRB median estimates/projections and 95% credible intervals for Scenario 1 $\Theta_{c,t}^{S1}$ (red curve and shades), median estimates of the regional baselines $\beta_{r[c]}^{(region)}$ (dark green horizontal line), median estimates of the national baselines β_c (light green horizontal line). SRB observations are displayed with dots and observations are connected with lines when obtained from the same source. Shaded areas around observation series represent the sampling variability in the series (quantified by two times the stochastic/sampling standard errors). Row 2: SRB median estimates/projections and 95% credible intervals for Scenario 2 $\Theta_{c,t}^{S2}$ (curve and shades). Row 3: SRB median estimates/projections of total fertility rate (TFR) from the UN WPP 2019 are added (blue squared dots). The median estimates of inflation start year $\gamma_{0,c}$ and end year $\gamma_{3,c}$ are the vertical lines. The TFR value in the year $\gamma_{0,c}$ is shown. Row 4: annual number of missing female births (AMFB) estimates and projections by scenario. Row 5: cumulative number of missing female births (CMFB) estimates and projections by scenario.

4.4. Validation and simulation results. Table 3 summarizes the results related to the left-out SRB observations for: (i) M1 (model without inflation as described in Section 3.4) by leaving out observations obtained from the year 2005 onward, consisting 20.3% of all observations with no risk of SRB inflation $y^{(\text{risk-free})}$; (ii) M1 by randomly leaving out 20% observations (repeated 30 times); (iii) M2 (model with inflation term as described in Section 3.5) by leaving out observations obtained from the year 2010 onward, consisting 19.8% of the total observations at risk of inflation $y^{(\text{at-risk})}$; and (iv) prediction of sex ratio transitions based on the simulation setup as described in Section 3.7.2. Median errors and median absolute errors are close to zero for left-out observations. The coverage of 95% and 80% prediction intervals are as expected and symmetrical.

Table 4 shows results for the comparison between model estimates obtained based on the full dataset and based on the training set for the out-of-sample validation exercises in (i) M1: for the SRB $\Theta_{c,t}$, and (ii) M2: for the SRB $\Theta_{c,t}$ and the inflation $\delta_c\Omega_{c,t}$. Median errors and the median absolute errors are close to zero. The proportions of updated estimates that fall above or below their respective credible intervals constructed based on the training set are reasonable and mostly within the expected range, with at most two countries' estimates falling outside their respective bounds.

TABLE 3

Validation results for left-out observations. Error is defined as the difference between a left-out observation and the posterior median of its predictive distribution.M1: model with inflation term (Section 3.5). M2: model with inflation term (Section 3.5). Simulation from 1970: predicting sex ratio transition based on simulation setup (Section 3.7.2).

Left-out observations	M1	[M2	
y_j	Recent obs.	Random	Recent obs.	Simulation from 1970
# Country in training dataset	176	184	29	29
# Country in test dataset	143	169	28	29
Median error	0.000	-0.001	-0.003	0.010
Median absolute error	0.015	0.012	0.020	0.027
Below 95% prediction interval (%)	2.7	3.0	4.6	1.4
Above 95% prediction interval (%)	3.6	2.3	1.7	3.6
Expected (%)	2.5	2.5	2.5	2.5
Below 80% prediction interval (%)	9.7	9.8	11.3	7.0
Above 80% prediction interval (%)	10.2	8.2	8.6	14.0
Expected (%)	10	10	10	10

TABLE 4

Validation results for $\Theta_{c,t}$ estimates based on training set for M1, and for $\Theta_{c,t}$ and $\Omega_{c,t}\delta_c$ estimates based on training set for M2. Error is defined as the differences between an estimate based on full dataset and training set. The percentages (%) of countries in which the SRB median estimates based on the full dataset fall below or above their respective 95% and 80% credible intervals based on the training set are reported. Numbers in the parentheses indicate the number of countries with median estimates based on the full dataset that fall below or above their respective 95% and 80% credible intervals based on the training set. M1: model with inflation (Section 3.5). M2: model with inflation (Section 3.5).

Validation		M1		M2					
(Out-of-Sample)		$\Theta_{c,t}$			$\Theta_{c,t}$			$\delta_c\Omega_{c,t}$	
	1995	2005	2015	1995	2005	2015	1995	2005	2015
Median error	0.001	0.000	0.001	0.000	0.000	-0.002	0.000	0.000	0.000
Median absolute error	0.003	0.004	0.004	0.001	0.000	0.003	0.000	0.000	0.000
Below 95% credible interval (%)	2.4	2.4	0.9	0.0	0.0	3.4(1)	0.0	0.0	3.4(1)
Above 95% credible interval (%)	2.4	2.8	2.8	0.0	0.0	0.0	0.0	0.0	0.0
Expected proportions (%)	≤2.5	\leq 2.5	\leq 2.5	≤2.5	≤2.5	\leq 2.5	≤2.5	\leq 2.5	≤2.5
Below 80% credible interval (%)	9.0	9.0	4.7	0.0	0.0	6.9 (2)	0.0	0.0	3.4(1)
Above 80% credible interval (%)	8.0	9.4	8.0	0.0	3.4(1)	3.4(1)	0.0	0.0	3.4(1)
Expected proportions (%)	≤10	≤10	≤10	≤10	≤10	≤10	≤10	≤10	≤10

5. Discussion. We described a Bayesian hierarchical time series model for producing SRB estimates and scenario-based projections for all countries from 1950 to 2100. The model produces estimates of SRB baselines by country and region, capturing variations by ethnicity. The model also captures SRB variations under normal circumstance and in settings with sex-selective abortion. The SRB inflation model includes a country-specific indicator to estimate the probability of SRB inflation and identifies countries with strong statistical evidence of SRB inflation in a fully Bayesian approach. Furthermore, the inflation model provides a reproducible way to project SRB inflation based on three scenarios that are data-driven and model-based. Model validation exercises suggest that the SRB models are reasonably well calibrated and have satisfactory predictive performance.

We provide scenario-based projections that quantify the SRB inflation if high fertility countries with a son preference would also experience sex ratio transitions, similar to the ones observed so far. In the 17 countries at risk of sex ratio imbalance but without strong statistical evidence of SRB inflation, scenario-based projections S2 and S3 indicate that future sex ratio transitions may happen later this century, when fertility in at-risk countries reaches low levels.

Our scenario-based projections of SRB are based on several model assumptions and are subject to limitations as discussed in detail in our prior study [Chao et al. (2019a)]. We summarize the main ones here. Firstly, the SRB baselines are modeled according to regional groupings and do not depend on external indicators. Secondly, aside from the fertility squeeze effect, we are not able to incorporate additional factors in the SRB inflation model that may affect SRB imbalance. Thirdly, out of the 212 countries included in the study, some do not have information to indicate whether they are at risk of SRB imbalance (contributing 3.2% of the global births in 1970–2017). We assumed that these countries have no risk of SRB inflation which is a limitation for monitoring in those specific countries. Lastly, an additional limitation related to the scenario-based projections is that the calculation of missing female births is based on the UN WPP projections of the number of total births, as opposed to projections of births that take account of lower or higher SRBs resulting in larger or smaller female cohorts, and subsequently, larger or smaller number of births.

This study focuses on modeling national-level SRB estimation and projection. However, insights from national-level analyses may not be sufficient in national populations with great demographic heterogeneity. Several studies have shown that the SRB can differ greatly across geographic locations or other subpopulations within a country [Chao and Yadav (2019); Guilmoto (2015); Jiang, Ge and Tai (2019)]. For these countries, SRB imbalances may start to decline in some regions while increasing in others, i.e. in higher fertility regions. Projections on the national level may mask variability in the sex ratio imbalance at birth across sub-regions within countries. Constructing subnational scenario-based projections in such settings will provide additional important insights into future missing female births.

While the assumptions made for the scenario-based projections presented in this study may be hypothetical, the associated projections of missing girls are important illustrations of the potential burden of future prenatal sex discrimination and the need to monitor SRBs in countries with son preference. The Sustainable Development Goals (see http://www.un.org/sustainabledevelopment/sustainable-development-goals/) include the goal to "achieve gender equality and empower all women and girls" by 2030. Monitoring and projecting the sex ratio at birth is an essential part in protecting the gender equality at the prenatal stage. Our scenario-based projections underscore the importance of the monitoring of the sex ratio at birth over time, especially in countries with ongoing inflations and countries where future sex imbalances may occur, to avoid future aborting of girls in favor of male offspring.

APPENDIX: MODEL SPECIFICATION AND PRIORS

We fit different variations of the general SRB process model equation

$$\Theta_{c,t} = \beta_c \eta_{c,t}(\boldsymbol{\phi}) + \delta_c \Omega_{c,t}(\boldsymbol{\zeta}),$$

summarized as follows (with details provided in the remainder of this section):

- M1: Model without inflation term, $\Theta_{c,t} = \beta_c \eta_{c,t}(\phi)$, fit to dataset $\boldsymbol{y}^{(\text{risk-free})}$, as described in Section 3.4;
- M2: Model with inflation term, $\Theta_{c,t} = \hat{\beta}_c \eta_{c,t}(\hat{\phi}) + \delta_c \Omega_{c,t}(\zeta)$, fit to all data from at-risk countries $\boldsymbol{y}^{(\text{at-risk})}$, as described in Section 3.5.
- M3: Model without inflation term, $\Theta_{c,t} = \hat{\beta}_c \eta_{c,t}(\hat{\phi})$, fit to country-specific dataset $y^{(c)}$, used for scenario S1;
- M4: Model with inflation term, $\Theta_{c,t} = \hat{\beta}_c \eta_{c,t}(\hat{\phi}) + \Omega_{c,t}(\hat{\zeta})$, fit to country-specific dataset $y^{(c)}$, used for scenario S3.

[M1] The model for the SRB in country-years without SRB inflation:

$$\begin{split} \Theta_{c,t} &= \beta_c \eta_{c,t}(\boldsymbol{\phi}), \text{ for } c \in \mathcal{C}^{\text{base}}, \text{ for } t = 1950, \cdots, 2100, \\ \log(\beta_c) |\beta_{r[c]}^{(\text{region})}, \sigma_{\beta} &\sim \mathcal{N}\left(\log(\beta_{r[c]}^{(\text{region})}), \sigma_{\beta}^2\right), \text{ for } c \in \mathcal{C}, \\ \beta_r^{(\text{region})} &\stackrel{\text{i.i.d.}}{\sim} \mathcal{U}(1, 1.1), \text{ for } r \in \mathcal{R}, \\ \log(\eta_{c,t}) |\boldsymbol{\phi} &\sim \mathcal{N}(0, (1-\rho^2)/\sigma_{\epsilon}^2), \text{ for } t = 1950, \\ \log(\eta_{c,t}) &= \rho \log(\eta_{c,t-1}) + \epsilon_{c,t}, \text{ for } t \in \{1951, \cdots, 2100\}, \\ \epsilon_{c,t} |\sigma_{\epsilon} &\stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_{\epsilon}^2), \\ \sigma_{\beta} &\sim \mathcal{U}(0, 0.05), \\ \rho &\sim \mathcal{U}(0, 1), \\ \sigma_{\epsilon} &\sim \mathcal{U}(0, 0.05). \end{split}$$

where C^{base} refers to the set of countries not identified with risk of SRB inflation and C is the set of all the 212 countries. \mathcal{R} is the set of all the 10 regions. $\phi = \{\rho, \sigma_{\epsilon}\}$ is the vector of hyper parameters related to $\eta_{c,t}$. $\mathcal{U}(a,b)$ refers to a continuous uniform distribution with lower and upper bounds at a and b respectively.

[M2] The SRB model for country-years with potential SRB inflation is:

$$\begin{split} \Theta_{c,t} &= \hat{\beta}_c \eta_{c,t}(\hat{\phi}) + \delta_c \Omega_{c,t}(\zeta), \text{ for } c \in \mathcal{C}^{\text{inflation}} \cup \mathcal{C}^{\text{risk}}, \text{ for } t = 1950, \cdots, 2100, \\ \Omega_{c,t} &= \begin{cases} \xi_c(t - \gamma_{0,c})/\lambda_{1,c}, & \gamma_{0,c} < t < \gamma_{1,c} \\ \xi_c, & \gamma_{1,c} < t < \gamma_{2,c} \\ \xi_c - \xi_c(t - \gamma_{2,c})/\lambda_{3,c}, & \gamma_{2,c} < t < \gamma_{3,c} \\ 0, & t < \gamma_{0,c} \text{ or } t > \gamma_{3,c} \end{cases}, \text{ where} \\ \gamma_{1,c} &= \gamma_{0,c} + \lambda_{1,c}, \\ \gamma_{2,c} &= \gamma_{1,c} + \lambda_{2,c}, \\ \gamma_{3,c} &= \gamma_{2,c} + \lambda_{3,c}, \\ \xi_c | \mu_{\xi}, \sigma_{\xi} \sim \mathcal{N}(\mu_{\xi}, \sigma_{\xi}^2) T(0,), \text{ for } c \in \mathcal{C}^{\text{inflation}} \cup \mathcal{C}^{\text{risk}}, \\ \lambda_{1,c} | \mu_{\lambda 1}, \sigma_{\lambda 1} \sim \mathcal{N}(\mu_{\lambda 1}, \sigma_{\lambda 1}^2) T(0,), \text{ for } c \in \mathcal{C}^{\text{inflation}} \cup \mathcal{C}^{\text{risk}}, \\ \lambda_{2,c} | \mu_{\lambda 2}, \sigma_{\lambda 2} \sim \mathcal{N}(\mu_{\lambda 2}, \sigma_{\lambda 2}^2) T(0,), \text{ for } c \in \mathcal{C}^{\text{inflation}} \cup \mathcal{C}^{\text{risk}}, \end{cases}$$

$$\begin{split} \lambda_{3,c} | \mu_{\lambda 3}, \sigma_{\lambda 3} &\sim \mathcal{N}(\mu_{\lambda 3}, \sigma_{\lambda 3}^2) T(0,), \text{ for } c \in \mathcal{C}^{\text{inflation}} \cup \mathcal{C}^{\text{risk}}, \\ \gamma_{0,c} | \sigma_{\gamma} \sim t_{3}(x_{c}, \sigma_{\gamma}^{2}, \nu = 3) T(z_{c},), \text{ for } c \in \mathcal{C}^{\text{inflation}} \cup \mathcal{C}^{\text{risk}}, \\ \delta_{c} | \pi_{c} \sim \mathcal{B}(\pi_{c}), \text{ for } c \in \mathcal{C}^{\text{inflation}} \cup \mathcal{C}^{\text{risk}}, \\ \log \operatorname{it}(\pi_{c}) | \mu_{\pi}, \sigma_{\pi} \sim \mathcal{N}(\mu_{\pi}, \sigma_{\pi}^{2}), \text{ for } c \in \mathcal{C}^{\text{inflation}} \cup \mathcal{C}^{\text{risk}}, \\ \mu_{\xi} \sim \mathcal{U}(0, 2), \\ \mu_{\lambda 1} \sim \mathcal{U}(0, 40), \\ \mu_{\lambda 2} \sim \mathcal{U}(0, 40), \\ \mu_{\lambda 3} \sim \mathcal{U}(0, 40), \\ \operatorname{inverse-logit}(\mu_{\pi}) \sim \mathcal{U}(0, 1), \\ \sigma_{\lambda 1} \sim \mathcal{U}(1, 10), \\ \sigma_{\lambda 2} \sim \mathcal{U}(1, 10), \\ \sigma_{\lambda 3} \sim \mathcal{U}(1, 10), \\ \sigma_{\tau} \sim \mathcal{U}(0, 2), \\ \sigma_{\tau} \sim \mathcal{U}(0, 2). \end{split}$$

 $\hat{\beta}_c$ is the posterior median for the national baseline and $\hat{\phi} = \{\hat{\rho}, \hat{\sigma}_{\epsilon}\}$ the vector of posterior medians of ϕ . $\zeta = \{\mu_{\xi}, \sigma_{\xi}, \mu_{\lambda 1}, \sigma_{\lambda 1}, \mu_{\lambda 2}, \sigma_{\lambda 2}, \mu_{\lambda 3}, \sigma_{\lambda 3}, \sigma_{\gamma}\}$ is the vector of hyper parameters related to $\Omega_{c,t}$.

The data quality model for SRB observations is:

$$\begin{split} \log(y_i)|\Theta_{c[i],t[i]},\omega_{s[i]} \sim & \mathcal{N}\left(\log(\Theta_{c[i],t[i]}),\omega_{s[i]}^2 + v_i^2\right), \text{ for } i \in \{1,\cdots,n\}, \\ \omega_s &= 0 \text{ for } s = \text{CRVS/SRS}, \\ \omega_s &\sim & \mathcal{U}(0,0.5), \text{ for } s \in \{\text{Census, DHS, Other DHS, Other}\} \end{split}$$

Scenario-based projections for countries with strong statistical evidence of SRB inflation $C^{\text{inflation}}$:

$$\Theta^S_{c,t} = \beta^{(\text{M1})}_c \eta^{(\text{M2})}_{c,t} + \delta^{(\text{M2})}_c \Omega^{(\text{M2})}_{c,t}, \text{ for scenario } S \in \{S1,S2,S3\}, c \in \mathcal{C}^{\text{inflation}}.$$

Scenario-based projections for countries without risk of SRB inflation C^{base} :

$$\Theta_{c,t}^S = \beta_c^{(\text{M1})} \eta_{c,t}^{(\text{M1})}$$
, for scenario $S \in \{S1, S2, S3\}, c \in \mathcal{C}^{\text{base}}$.

Scenario-based projections for countries at risk of SRB inflation but without strong statistical evidence of SRB inflation C^{risk} :

$$\begin{split} \Theta_{c,t}^{S1} &= \beta_c^{(\text{M1})} \eta_{c,t}^{(\text{M3})}, \text{ for } c \in \mathcal{C}^{\text{risk}}, \\ \Theta_{c,t}^{S2} &= \beta_c^{(\text{M1})} \eta_{c,t}^{(\text{M2})} + \delta_c^{(\text{M2})} \Omega_{c,t}^{(\text{M2})}, \text{ for } c \in \mathcal{C}^{\text{risk}}, \\ \Theta_{c,t}^{S3} &= \beta_c^{(\text{M1})} \eta_{c,t}^{(\text{M4})} + \Omega_{c,t}^{(\text{M4})}, \text{ for } c \in \mathcal{C}^{\text{risk}}. \end{split}$$

TABLE 5

MCMC specifications.

MCMC Specifications		Normal Mode	Inflation Model		
	Full	Validation		Full	Validation
		Out-of-Sample	In-Sample		Out-of-Sample
# Chains	8	8	8	14	8
# Burn-in	8,000	8,000	8,000	7,600	1,000
# Thinning	20	20	20	10	2
# Posterior samples per parameter	4,000	4,000	4,000	28,000	11,000

APPENDIX: COMPUTATION

We obtain posterior samples using a Markov chain Monte Carlo (MCMC) algorithm, implemented in the open source software R 3·6·1 [R Core Team (2019)] and JAGS 4·0·1 [Plummer (2003)], using R-packages rjags [Plummer (2011)], R2jags [Su and Yajima (2011)] and MCMCpack [Martin, Quinn and Park (2011)]. Convergence of the MCMC algorithm and the sufficiency of the number of samples obtained are checked through visual inspection of trace plots and convergence diagnostics of Gelman and Rubin [Gelman and Rubin (1992)], implemented in the coda R-package [Plummer et al. (2006)]. Due to the multimodal nature of the posterior distributions for start year parameters in Albania and Republic of Korea, we apply the *Stacking* approach as discussed in [Yao, Vehtari and Gelman (2020)] to obtain representative samples from the posterior distribution. Table 5 summarizes the MCMC specifications for model runs.

Acknowledgements. The authors are grateful to Christophe Z. Guilmoto for helpful discussions. This work was supported by a research grant from the National University of Singapore. The study described is solely the responsibility of the authors and does not necessarily represent the official views of the United Nations.

SUPPLEMENTARY MATERIAL

Supplement figure A: Scenario-based SRB projection during 1950–2100, by country (Page 28–38; .pdf). SRB median estimates/projections (curve) and 95% credible intervals (shades) for Scenario 1 $\Theta_{c,t}^{S1}$ (in red), Scenario 2 $\Theta_{c,t}^{S2}$ (in blue) and Scenario 3 $\Theta_{c,t}^{S3}$ (in green), and median estimates of the national baselines β_c (black dashed horizontal line). The median estimates and projections of total fertility rate (TFR) from the UN WPP 2019 are added (blue squared dots). The median estimates of inflation start year $\gamma_{0,c}$ and end year $\gamma_{3,c}$ are the vertical lines. The TFR value in the year $\gamma_{0,c}$ is shown.

Supplement figure B: Scenario 1 SRB projection during 1950–2100 for all countries (Page 39–110; .pdf). SRB median estimates/projections and 95% credible intervals for Scenario 1 $\Theta_{c,t}^{S1}$ (red curve and shades), median estimates of the regional baselines $\beta_{r[c]}^{(\text{region})}$ (dark green horizontal line), median estimates of the national baselines β_c (light green horizontal line). SRB observations are displayed with dots and observations are connected with lines when obtained from the same source. Shaded areas around observation series represent the sampling variability in the series (quantified by two times the stochastic/sampling standard errors). Model results are shown before 1950 if observations are available prior 1950.

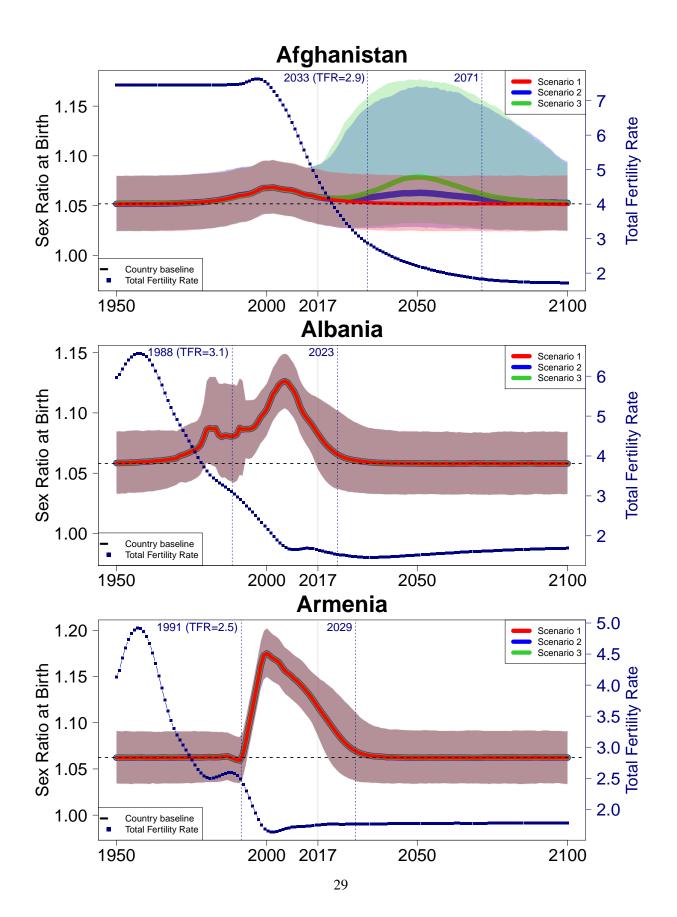
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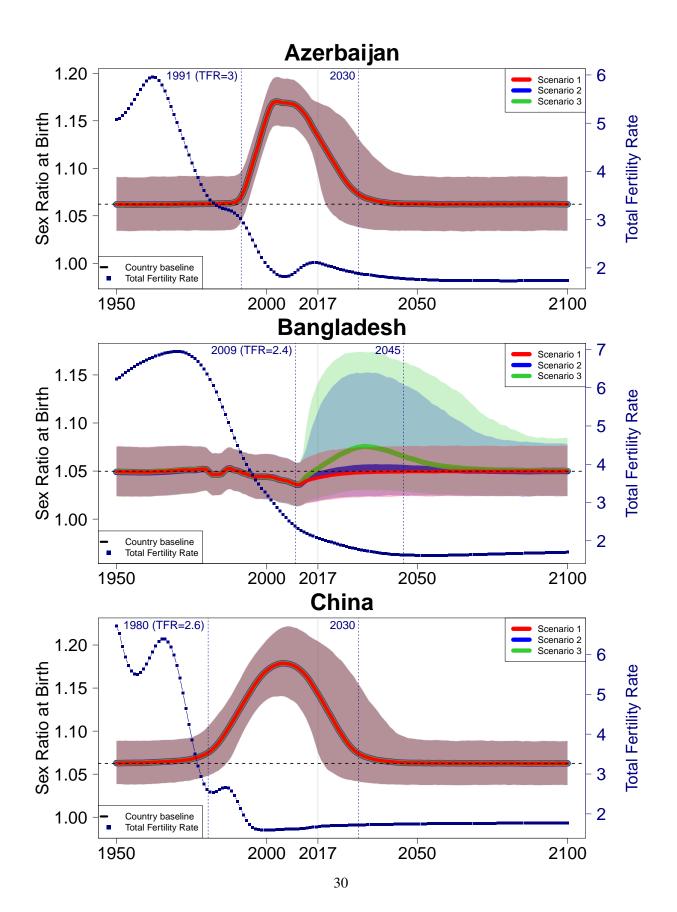
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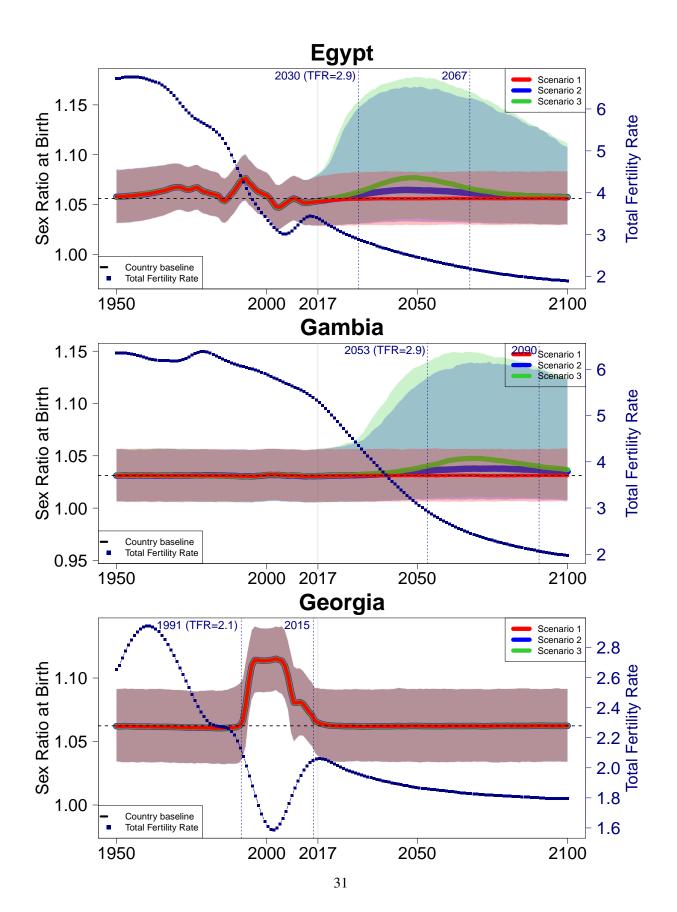
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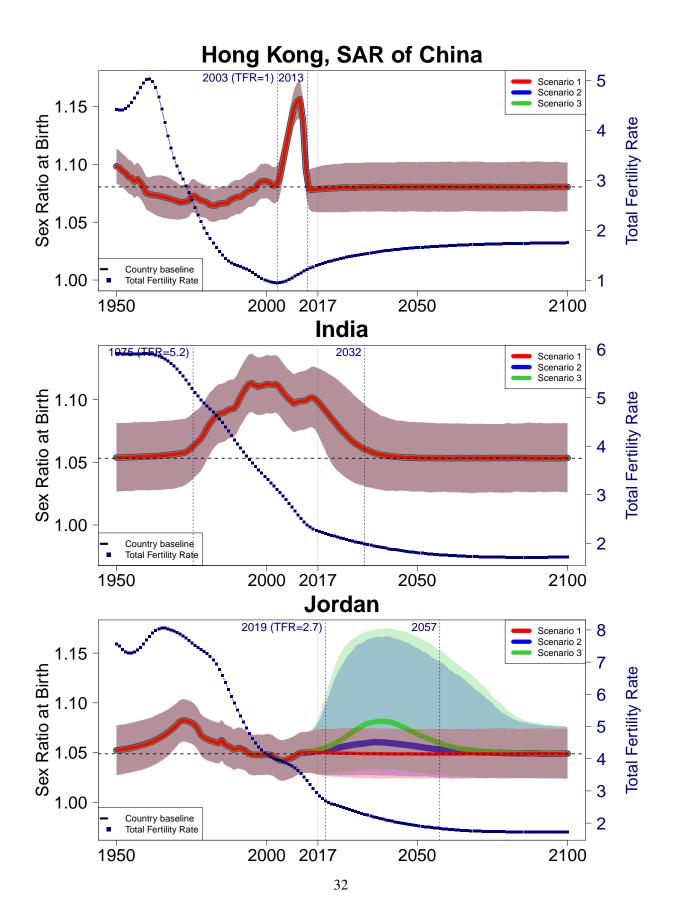
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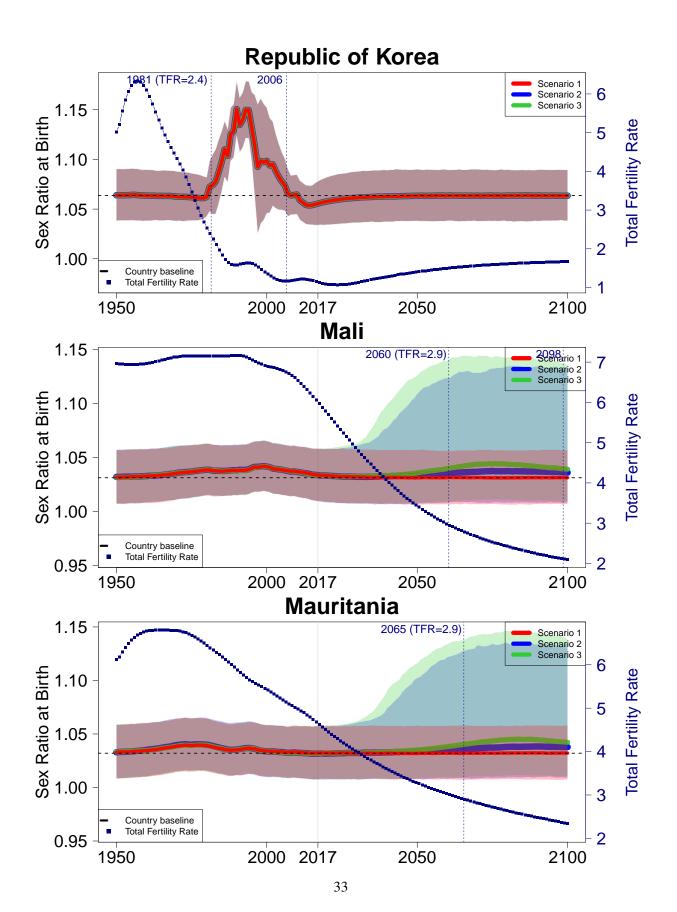
FIG 9. Scenario-based SRB projection during 1950–2100, by country. SRB median estimates/projections (curve) and 95% credible intervals (shades) for Scenario 1 $\Theta_{c,t}^{S1}$ (in red), Scenario 2 $\Theta_{c,t}^{S2}$ (in blue) and Scenario 3 $\Theta_{c,t}^{S3}$ (in green), and median estimates of the national baselines β_c (black dashed horizontal line). The median estimates and projections of total fertility rate (TFR) from the UN WPP 2019 are added (blue squared dots). The median estimates of inflation start year $\gamma_{0,c}$ and end year $\gamma_{3,c}$ are the vertical lines. The TFR value in the year $\gamma_{0,c}$ is shown.

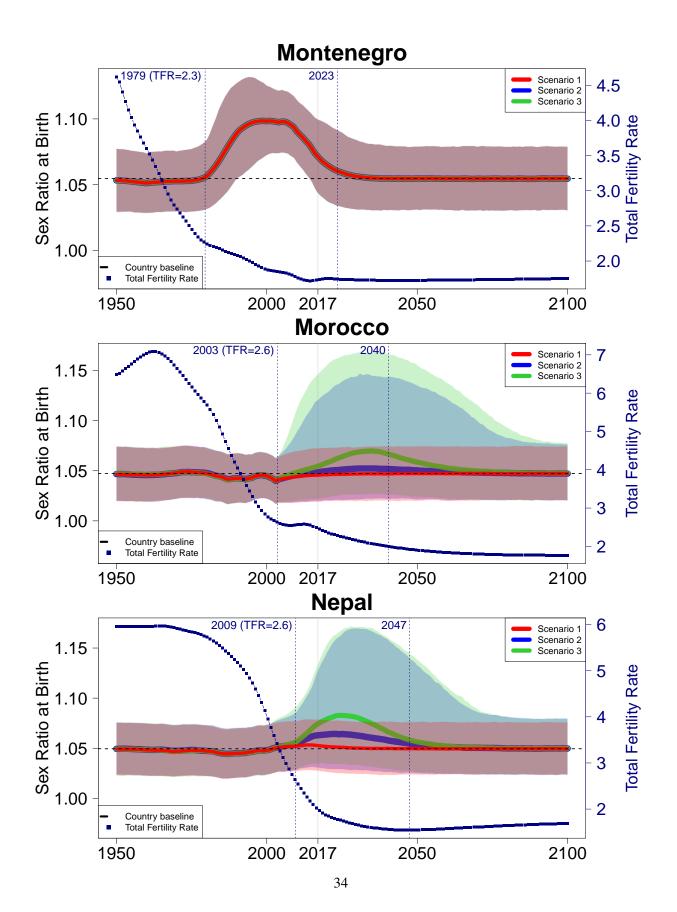


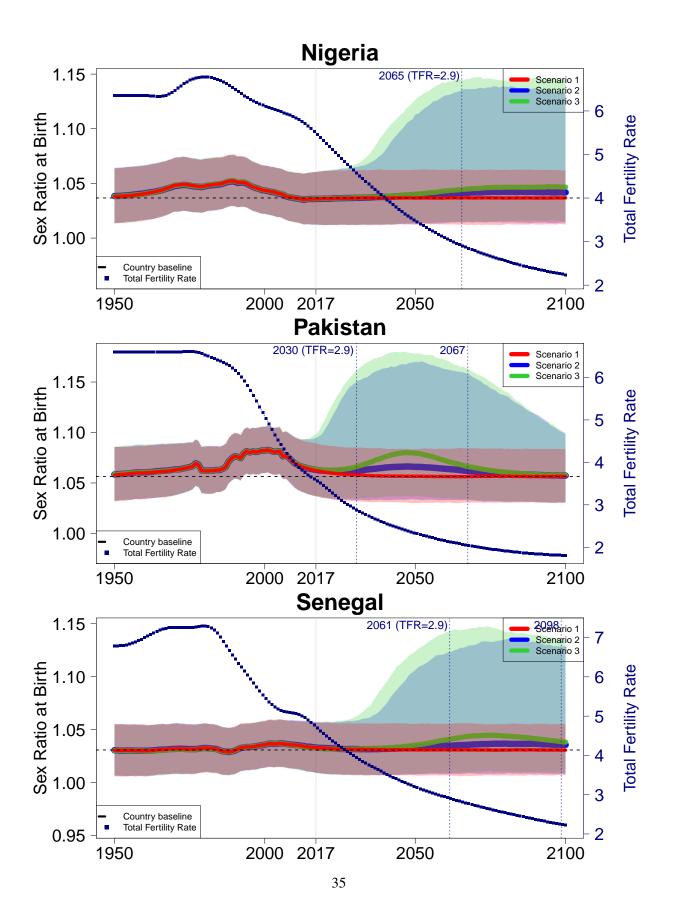


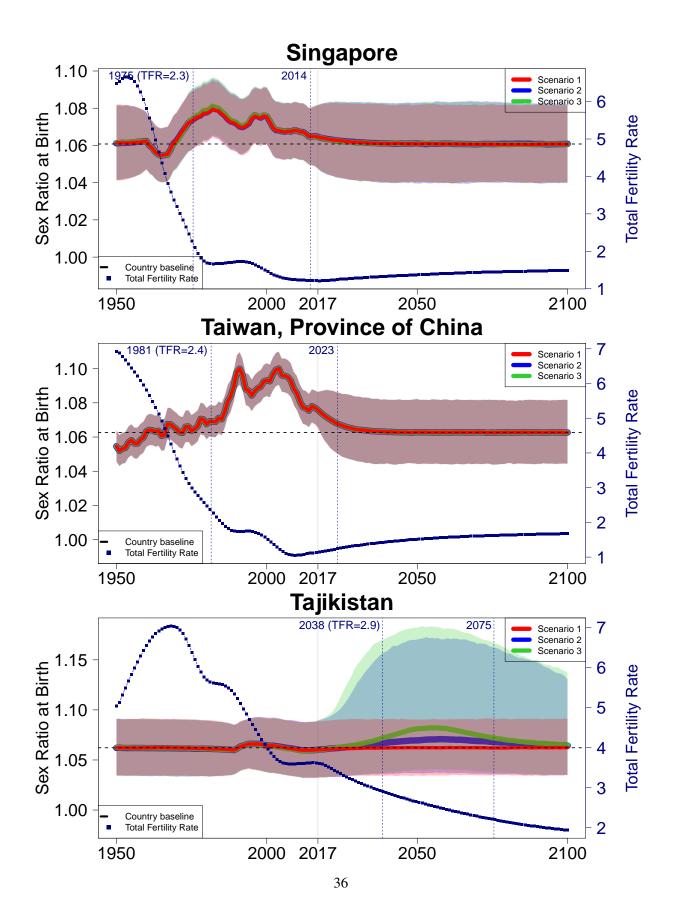


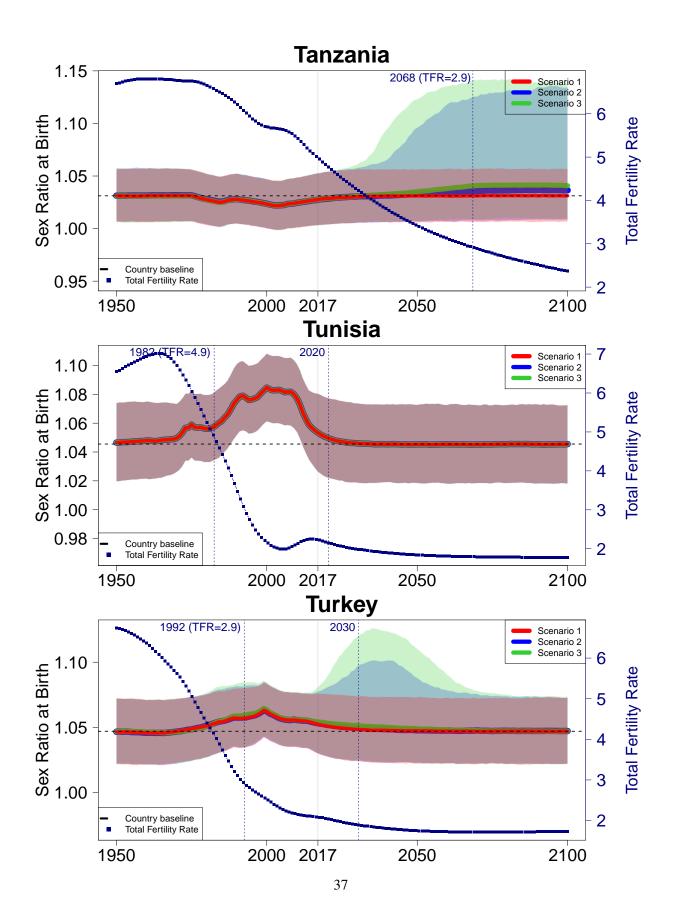












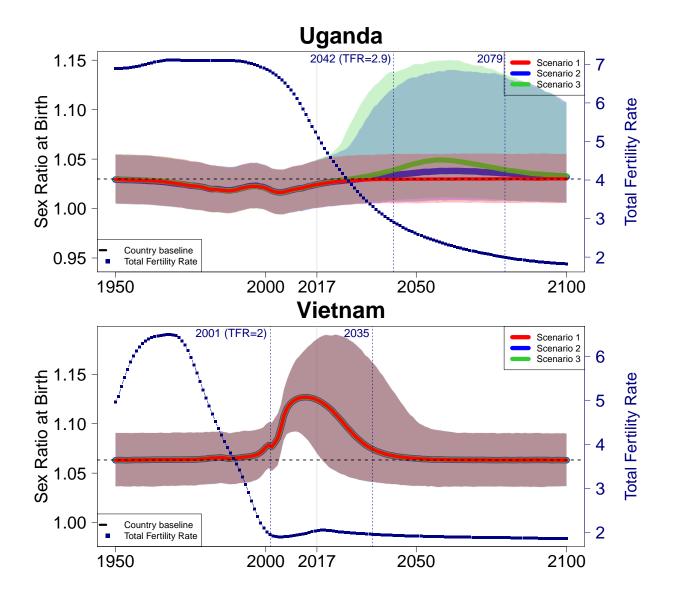
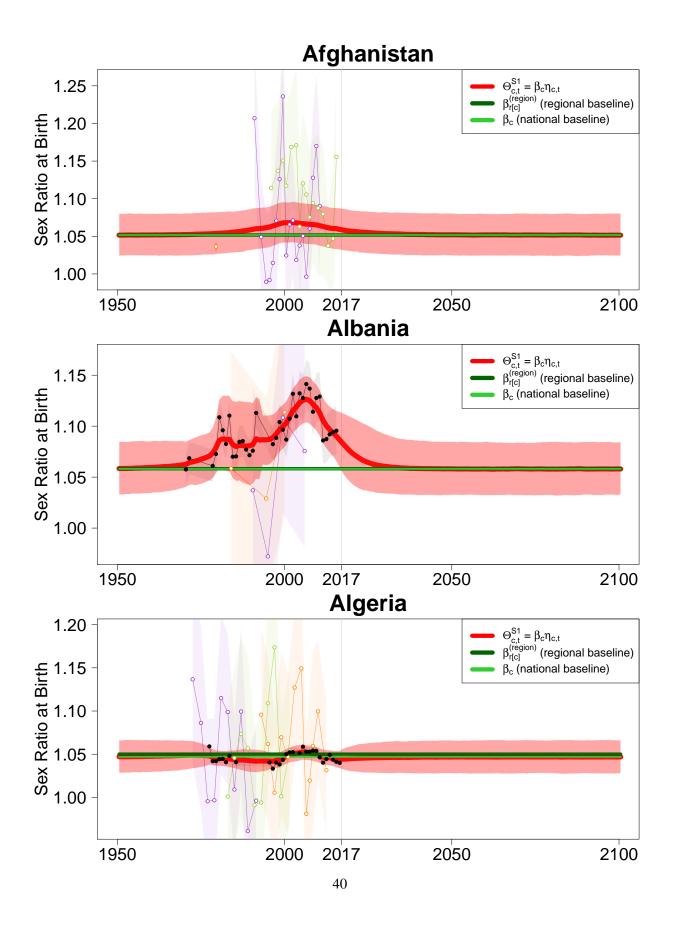
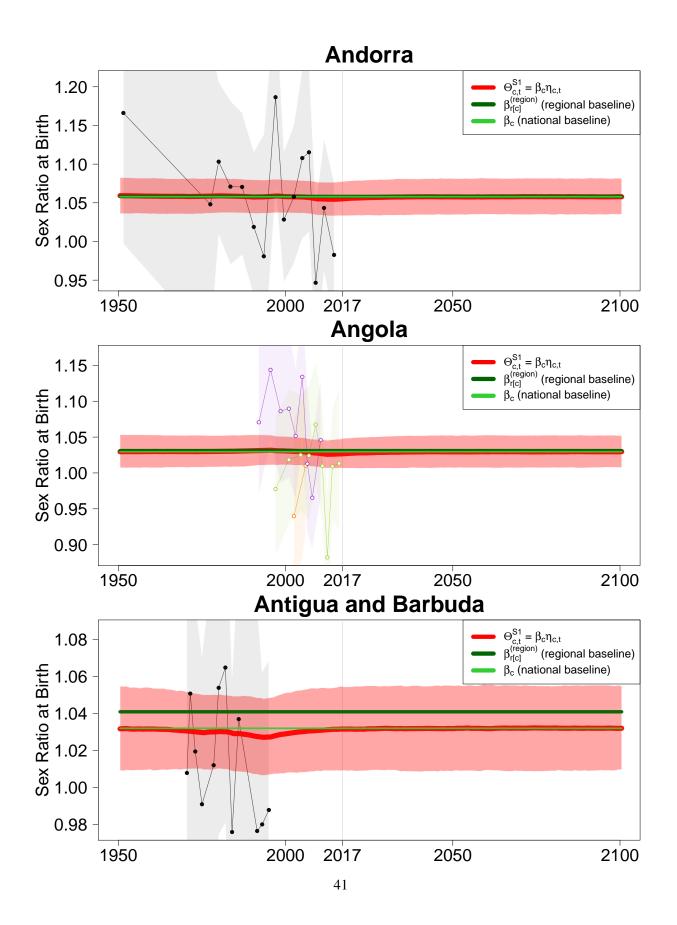
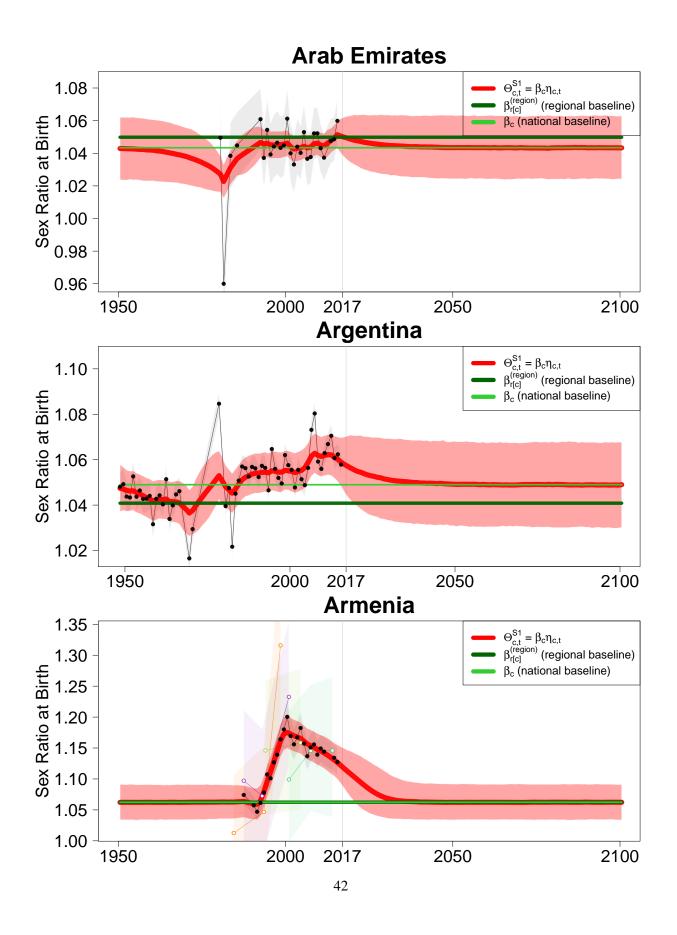
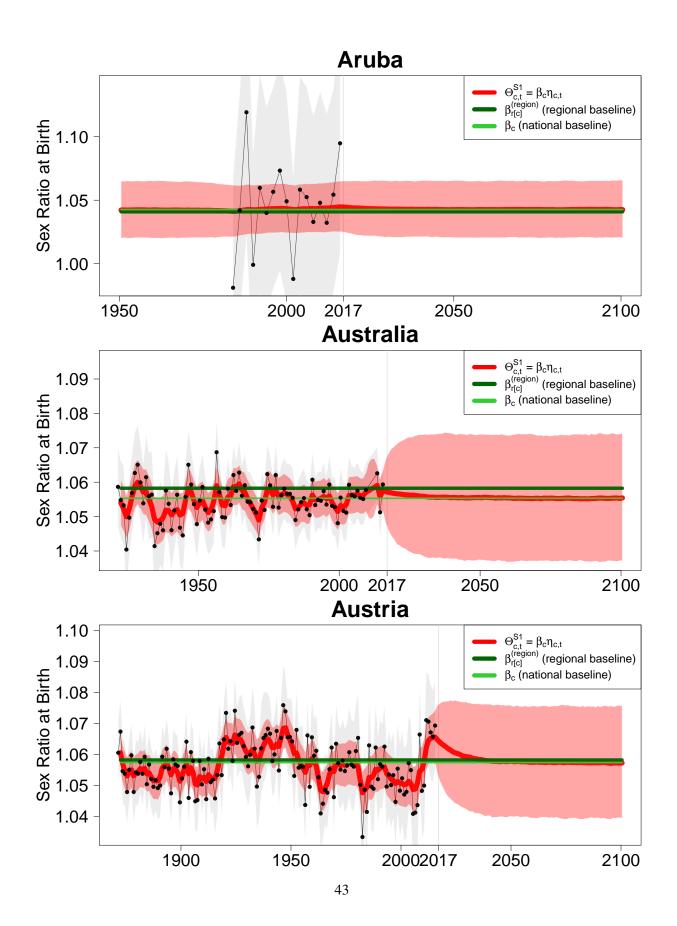


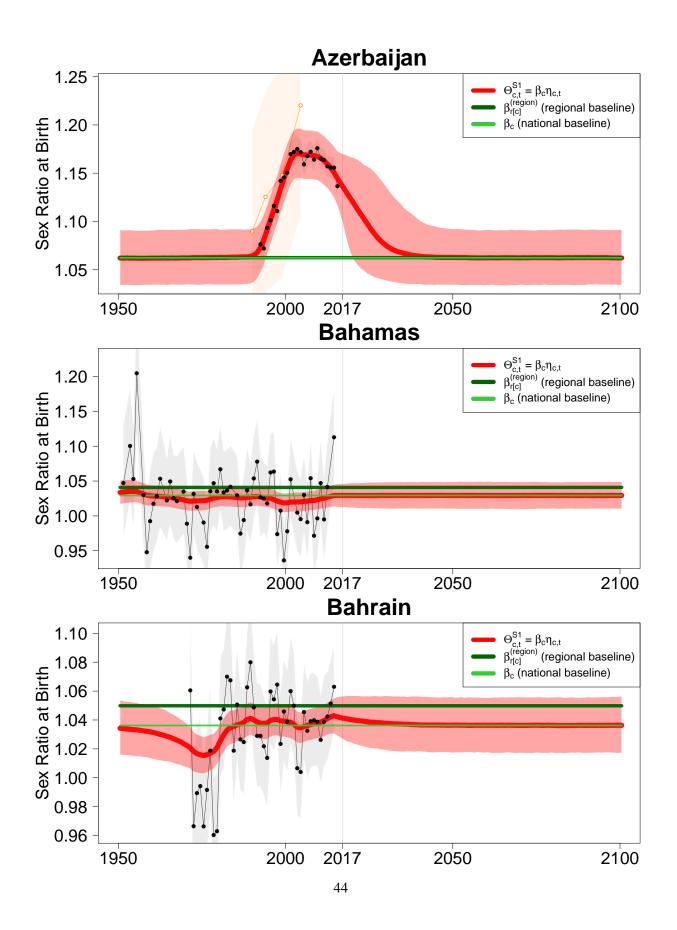
FIG 10. Scenario 1 SRB projection during 1950–2100 for all countries. SRB median estimates/projections and 95% credible intervals for Scenario 1 $\Theta_{c,t}^{S1}$ (red curve and shades), median estimates of the regional baselines $\beta_{r[c]}^{(region)}$ (dark green horizontal line), median estimates of the national baselines β_c (light green horizontal line). SRB observations are displayed with dots and observations are connected with lines when obtained from the same source. Shaded areas around observation series represent the sampling variability in the series (quantified by two times the stochastic/sampling standard errors). Model results are shown before 1950 if observations are available prior 1950.

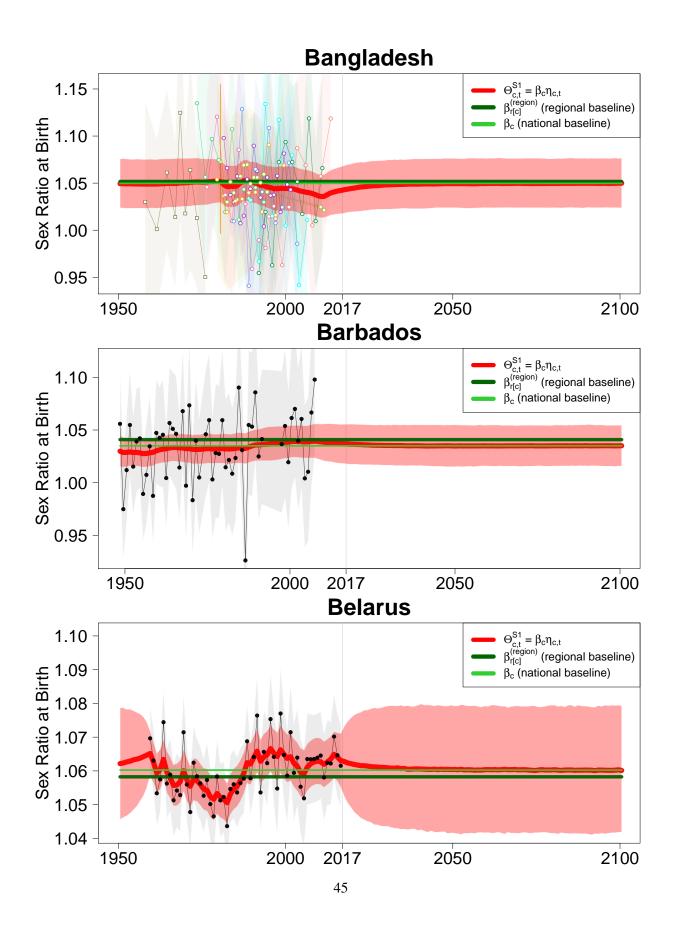


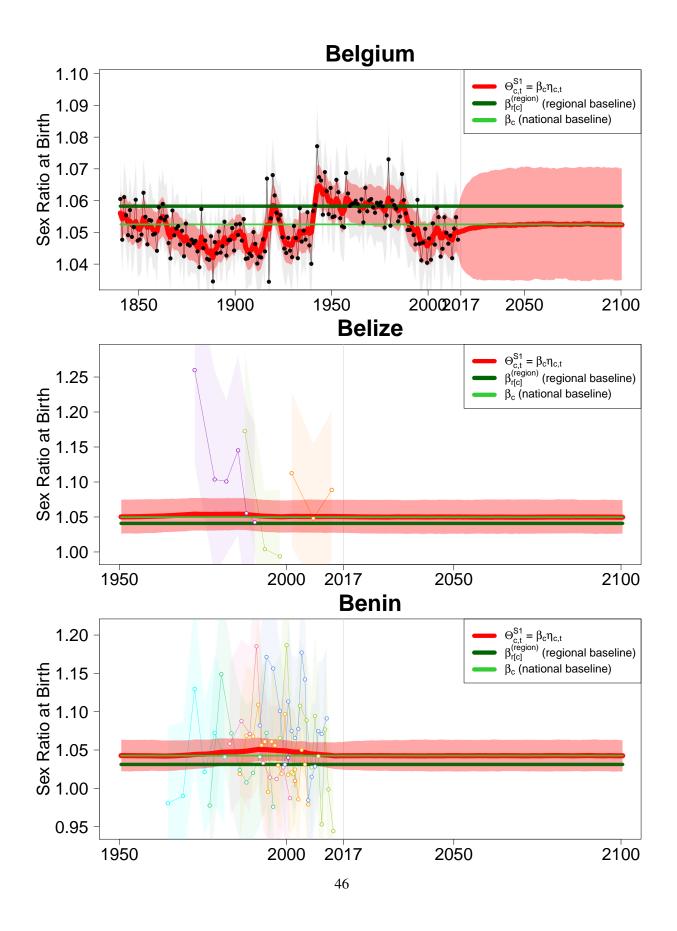


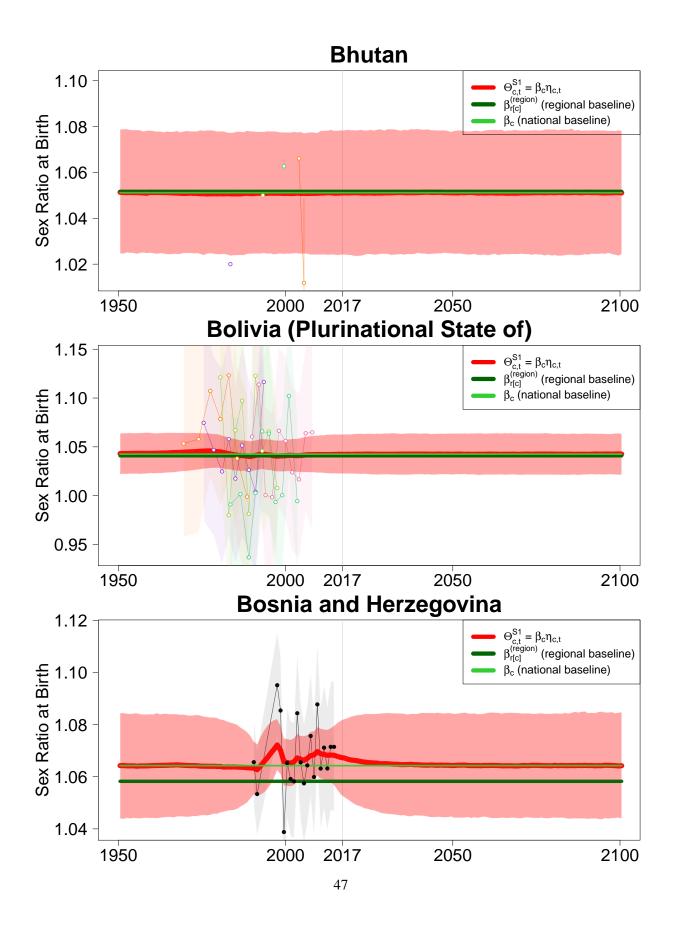


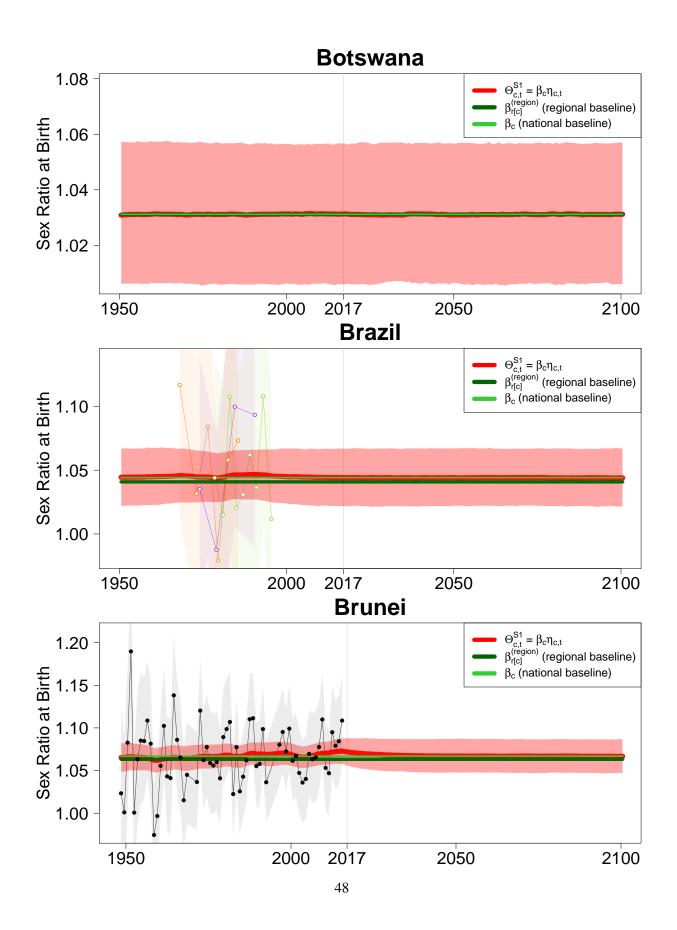


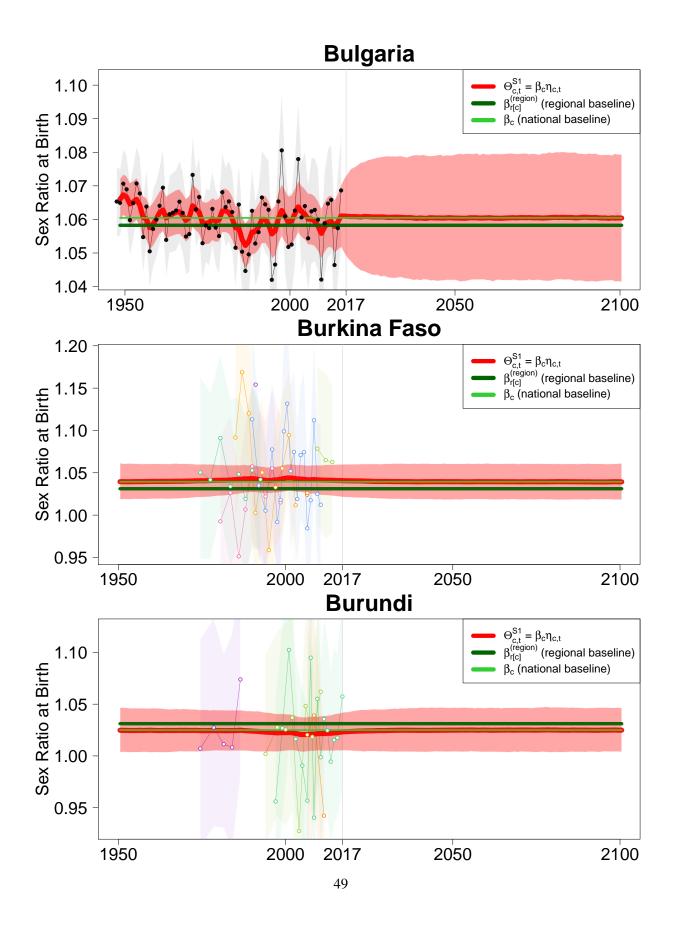


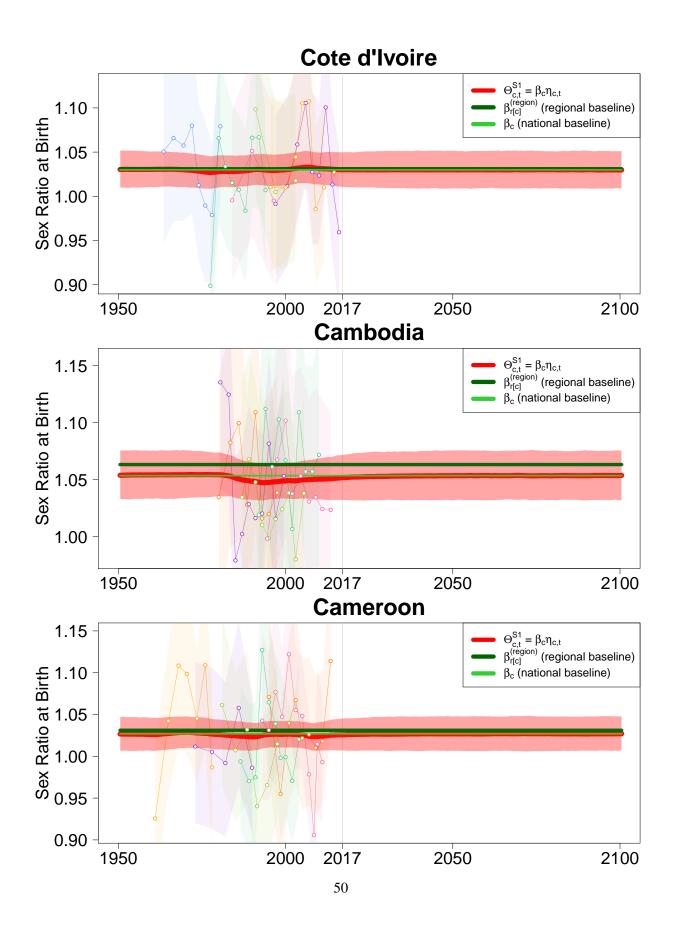


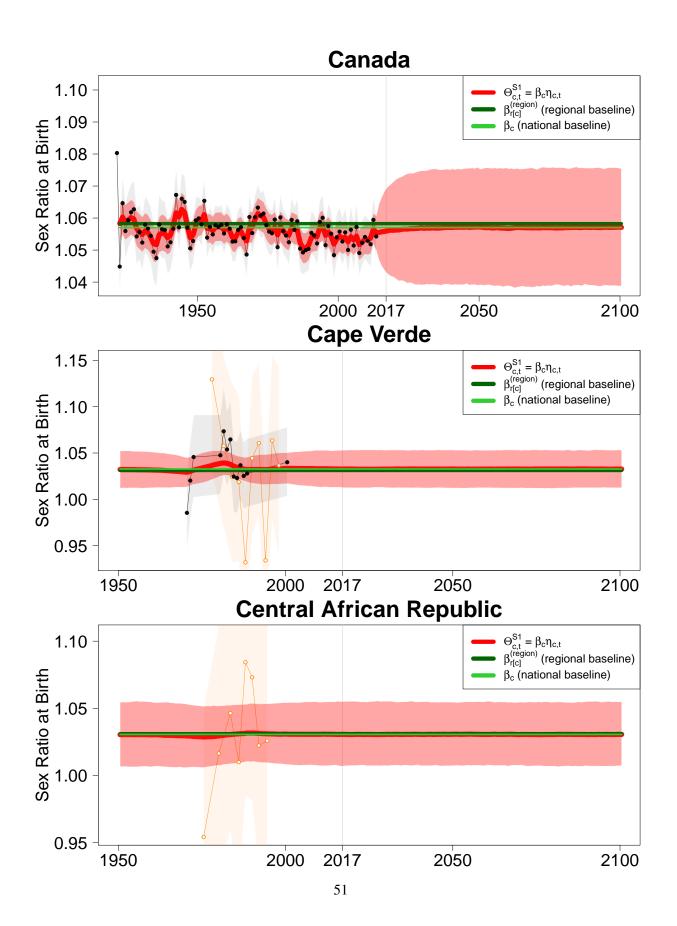


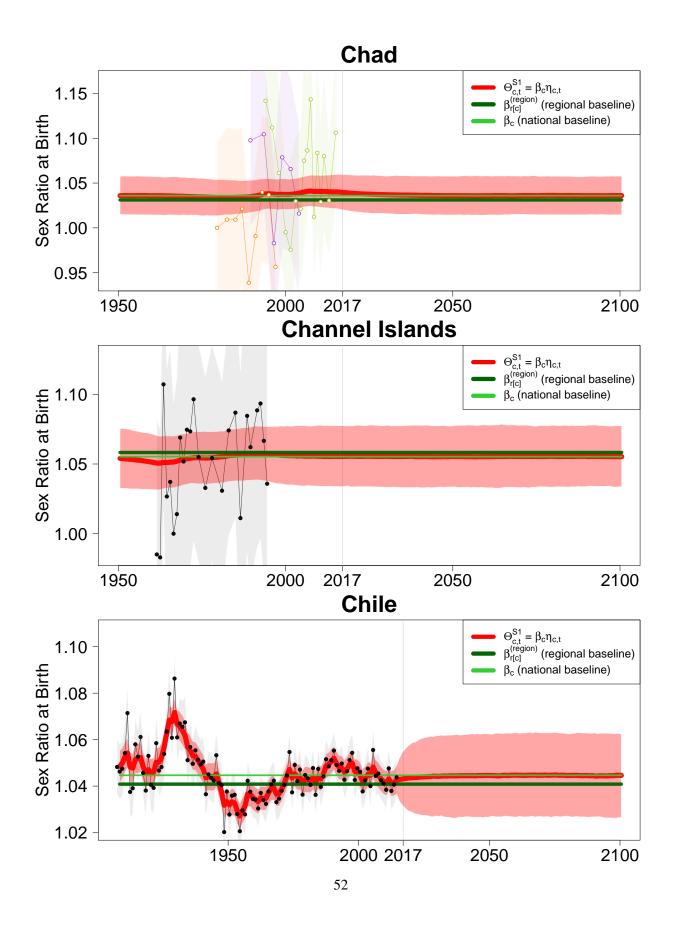


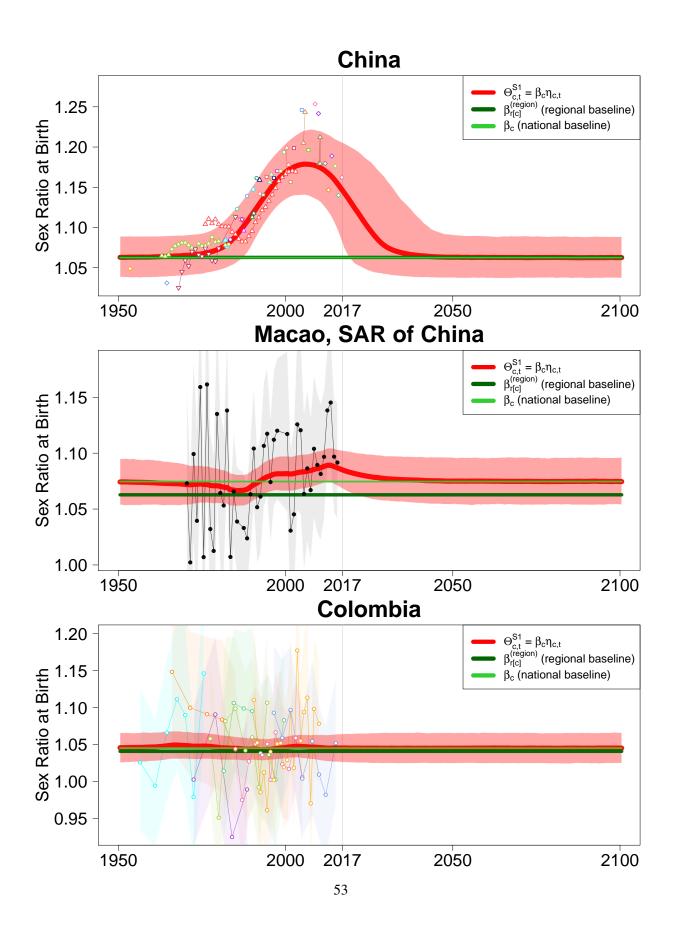


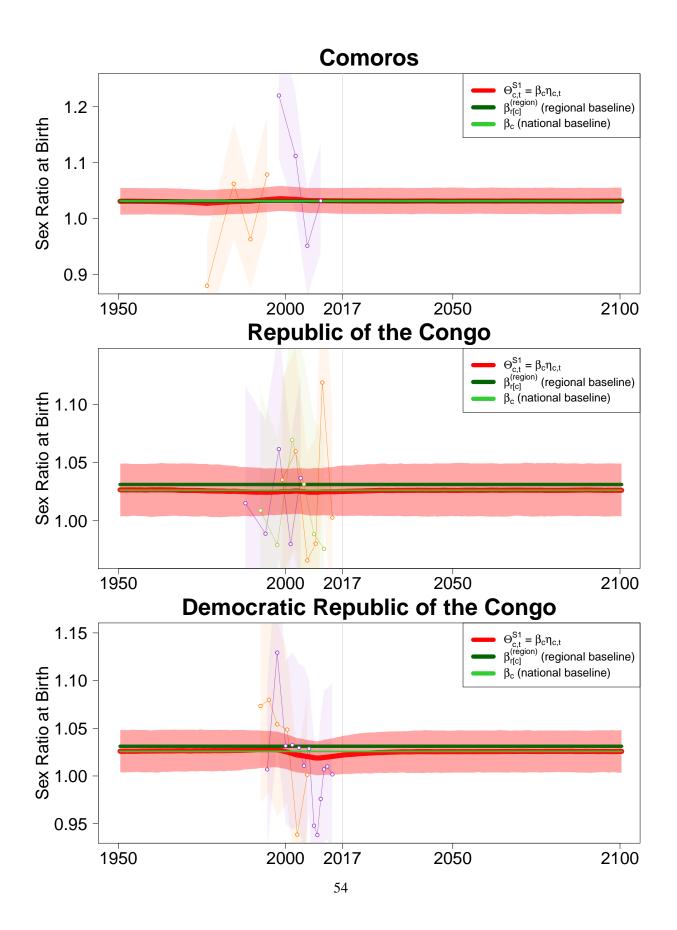


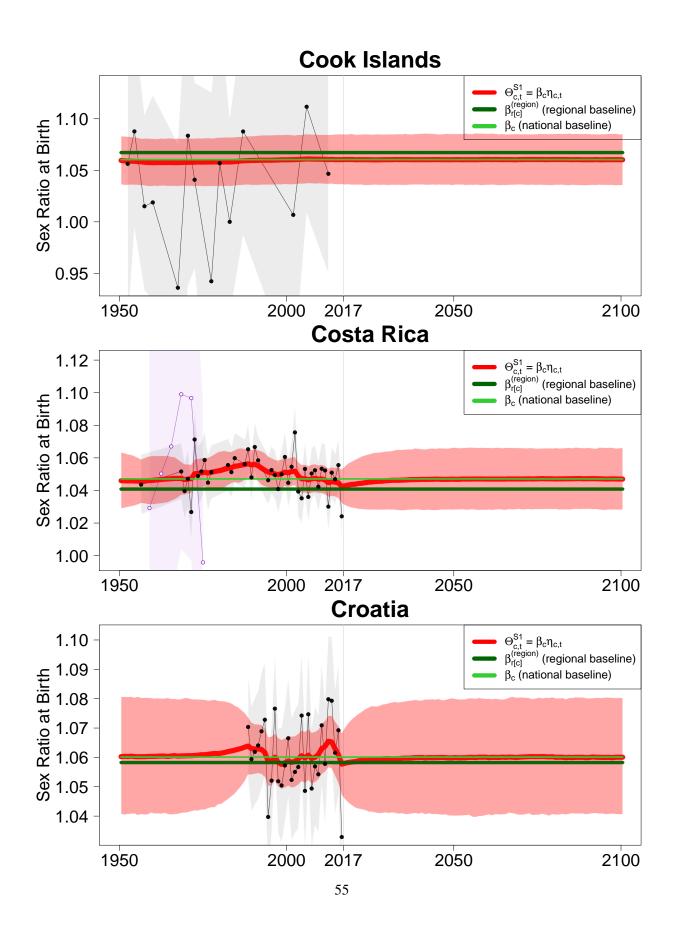


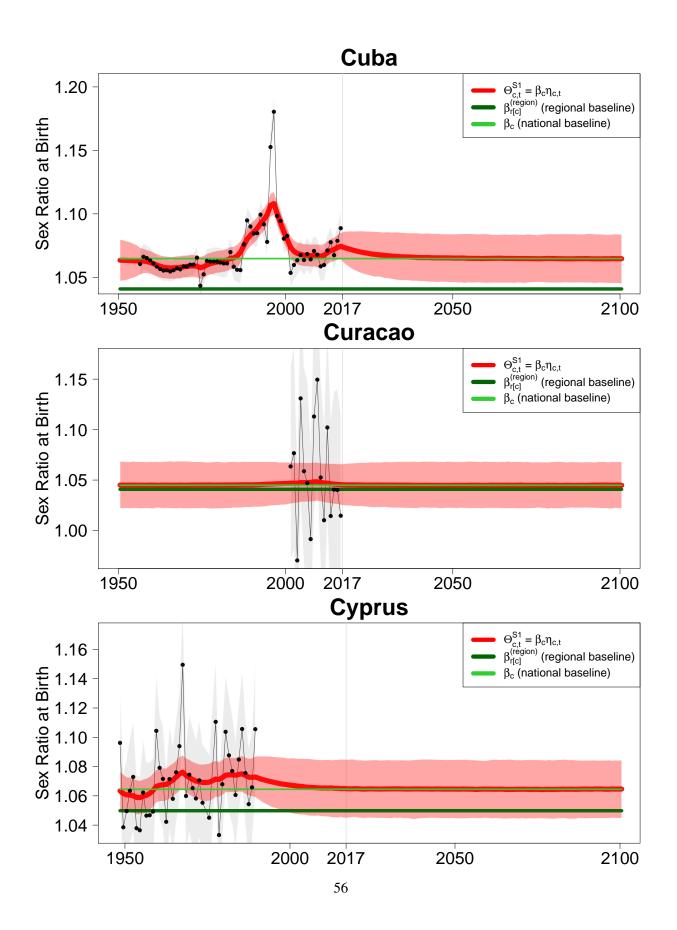


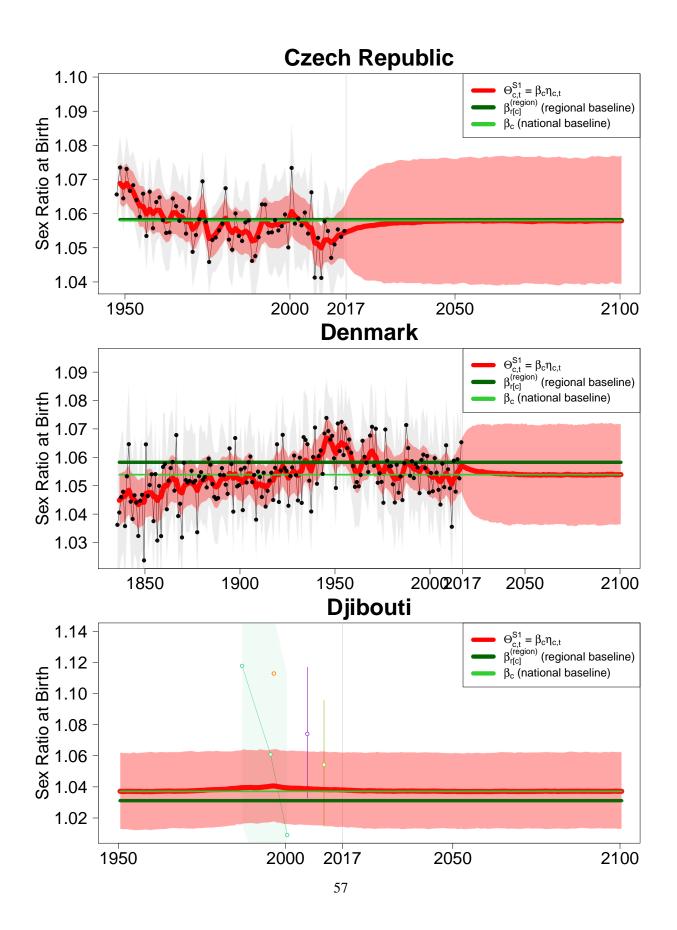


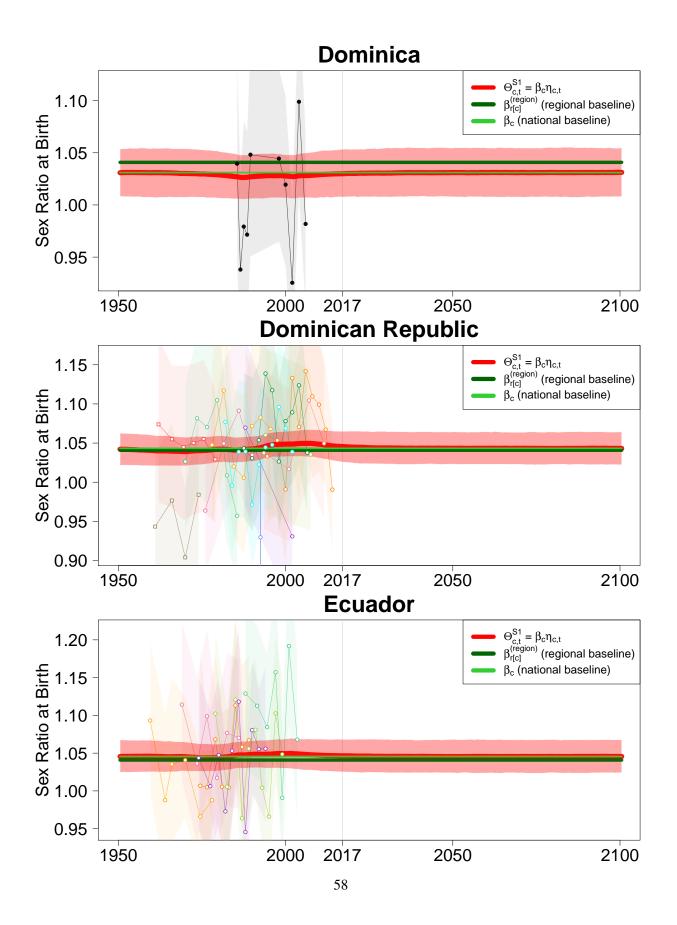


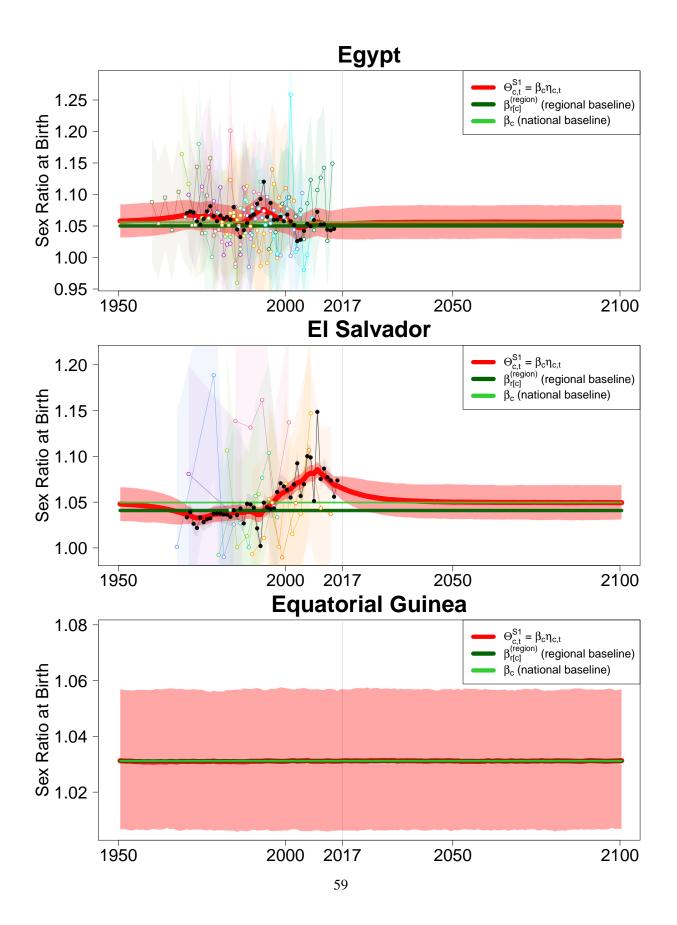


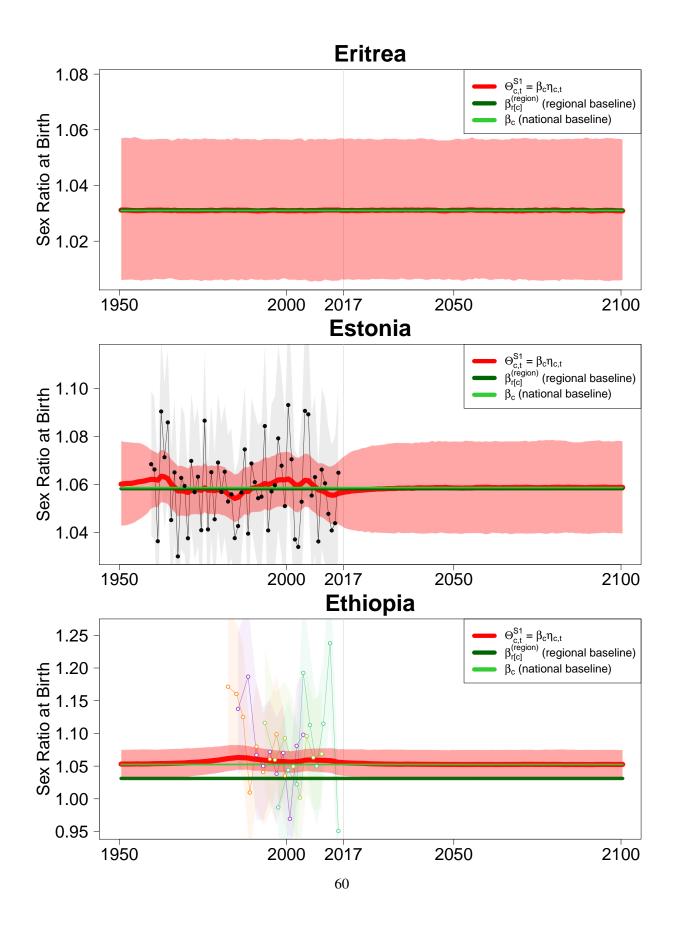


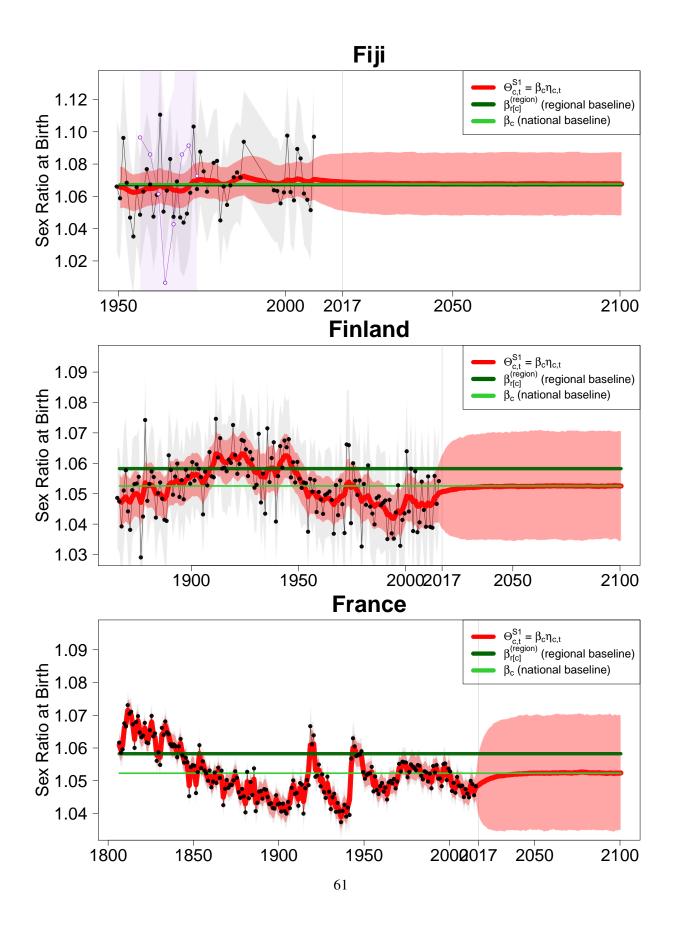


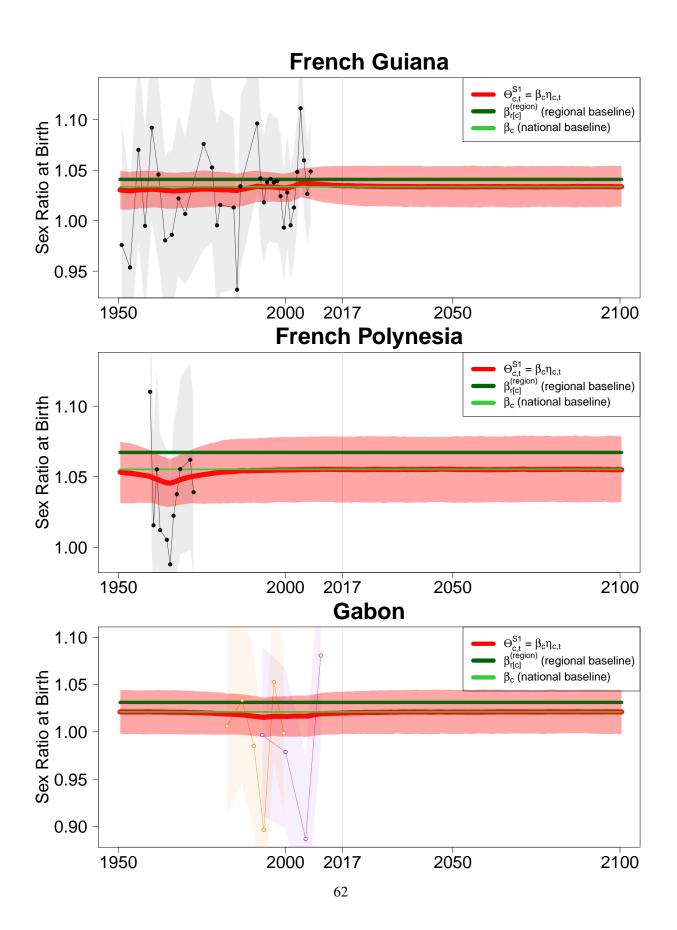


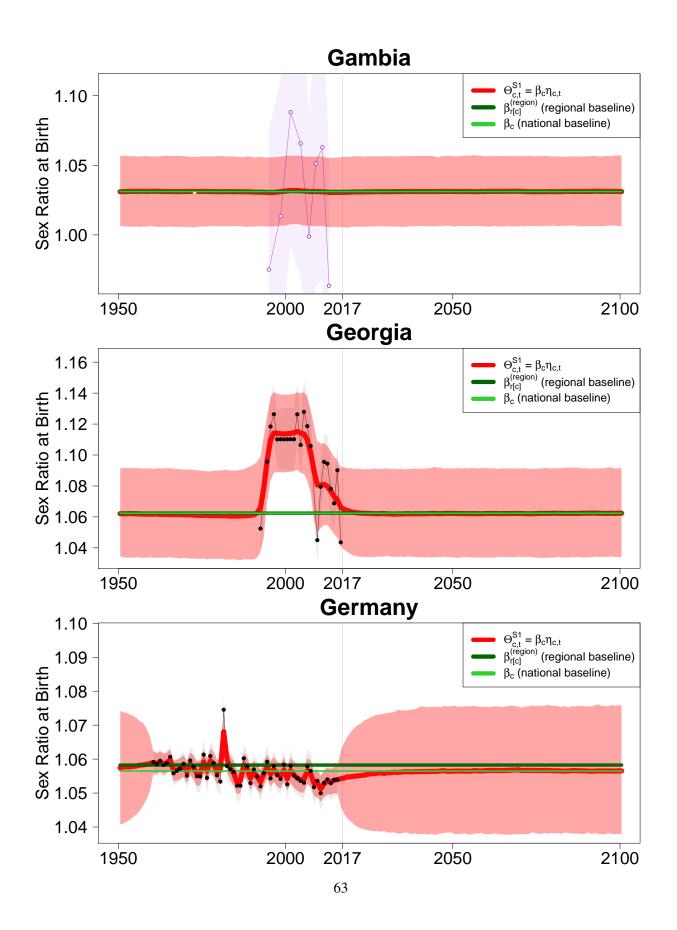


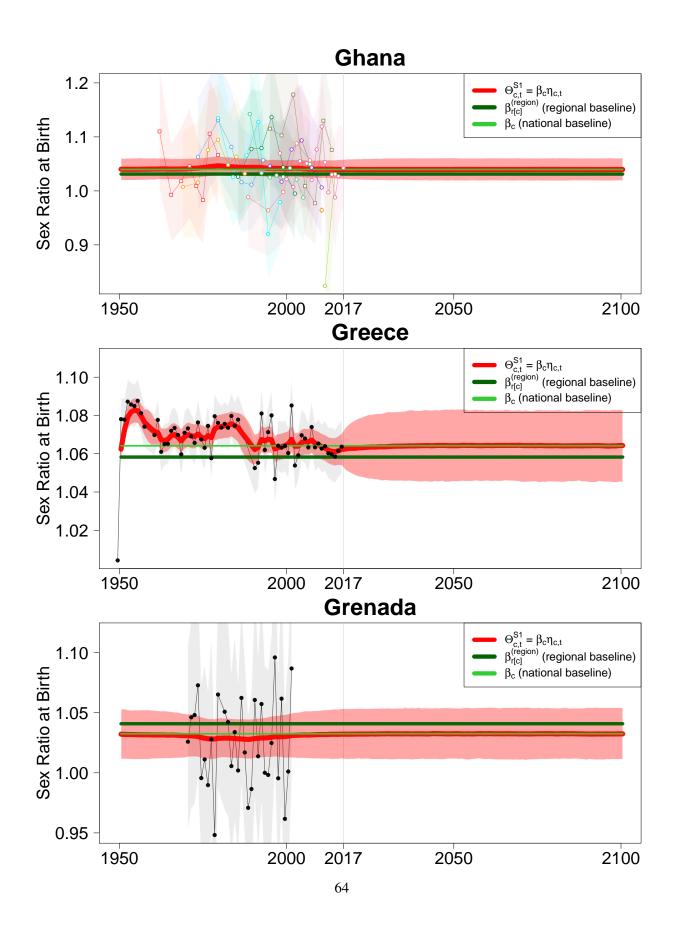


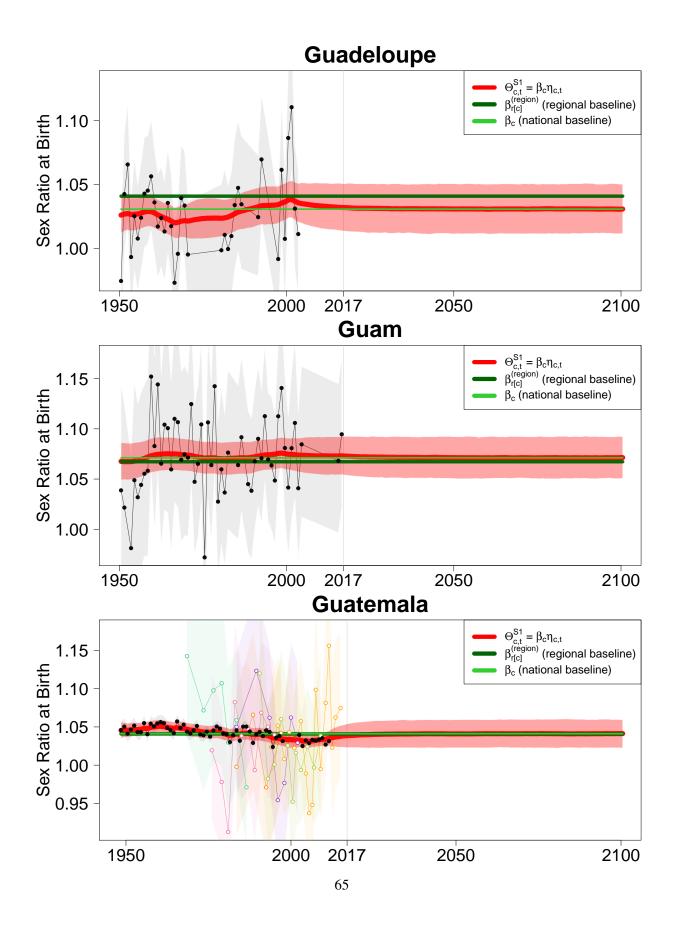


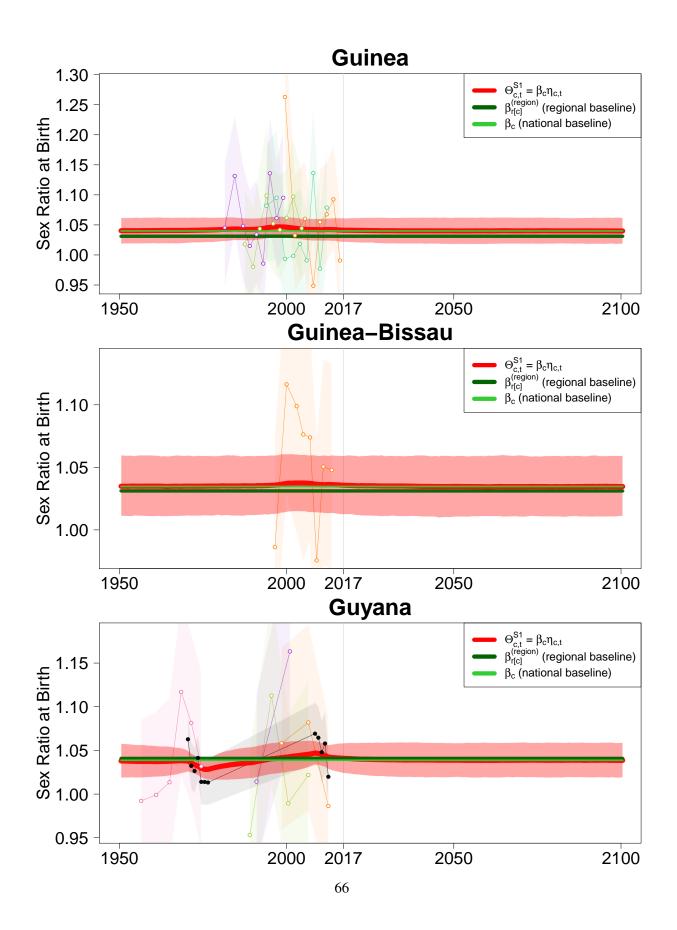


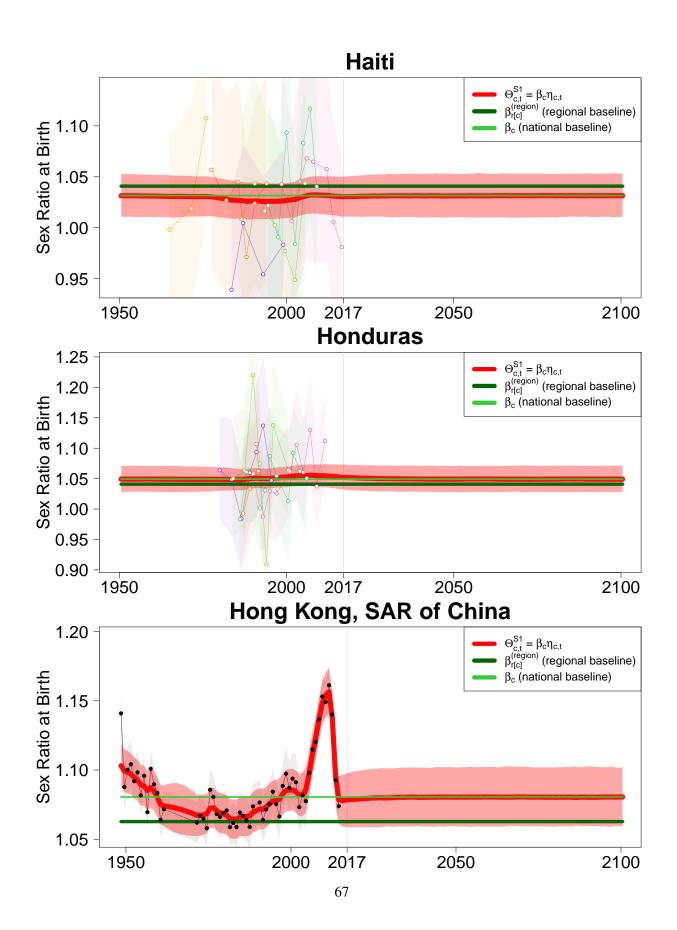


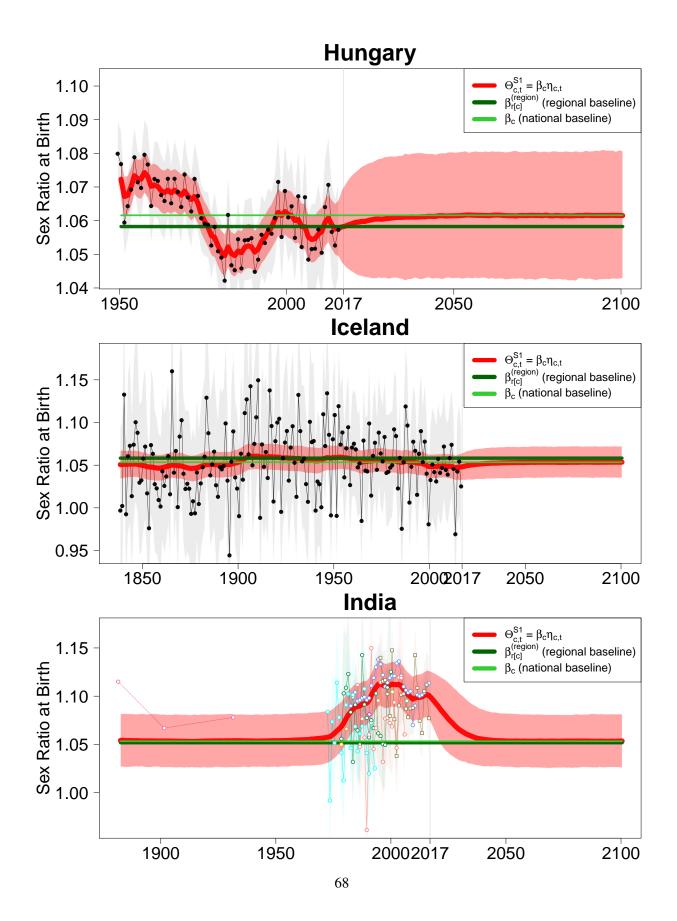


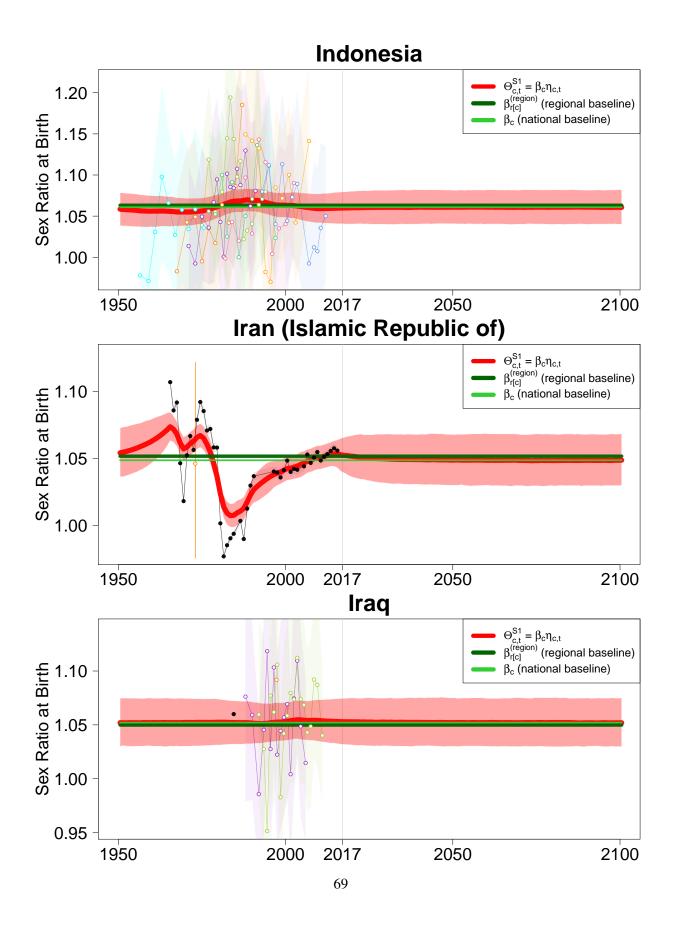


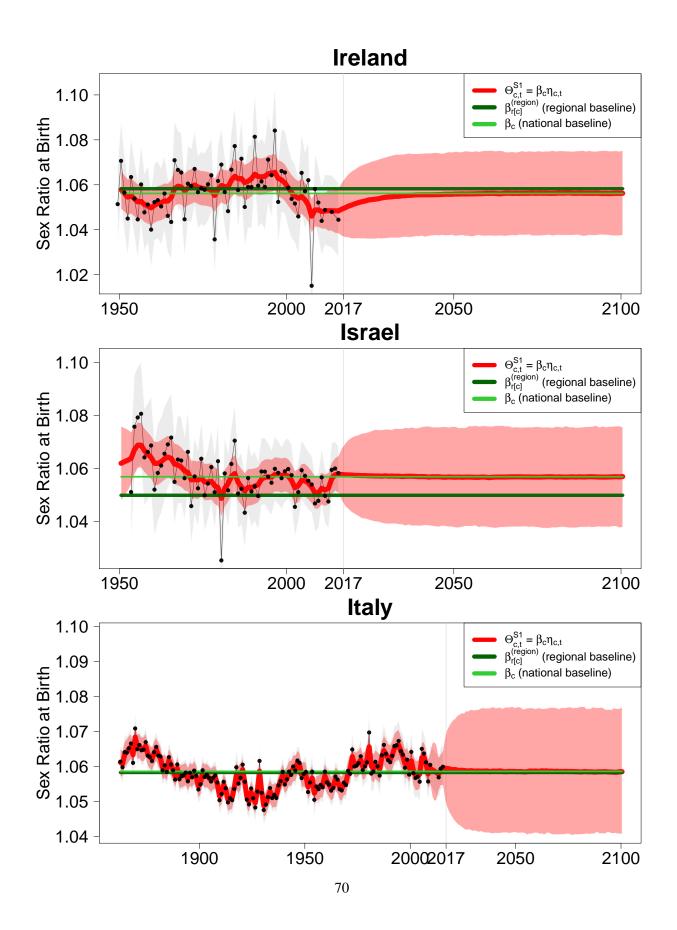


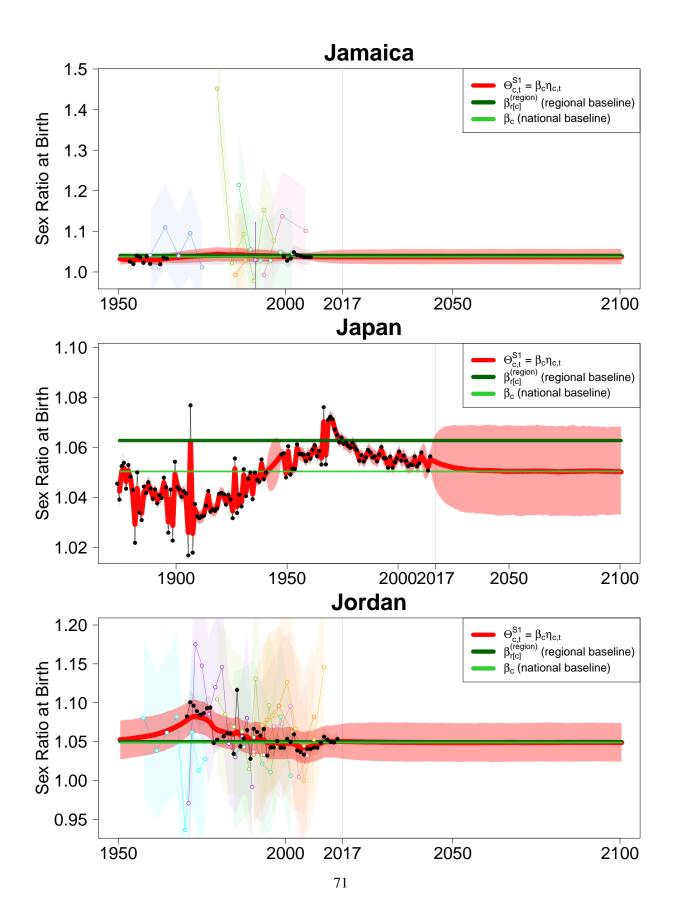


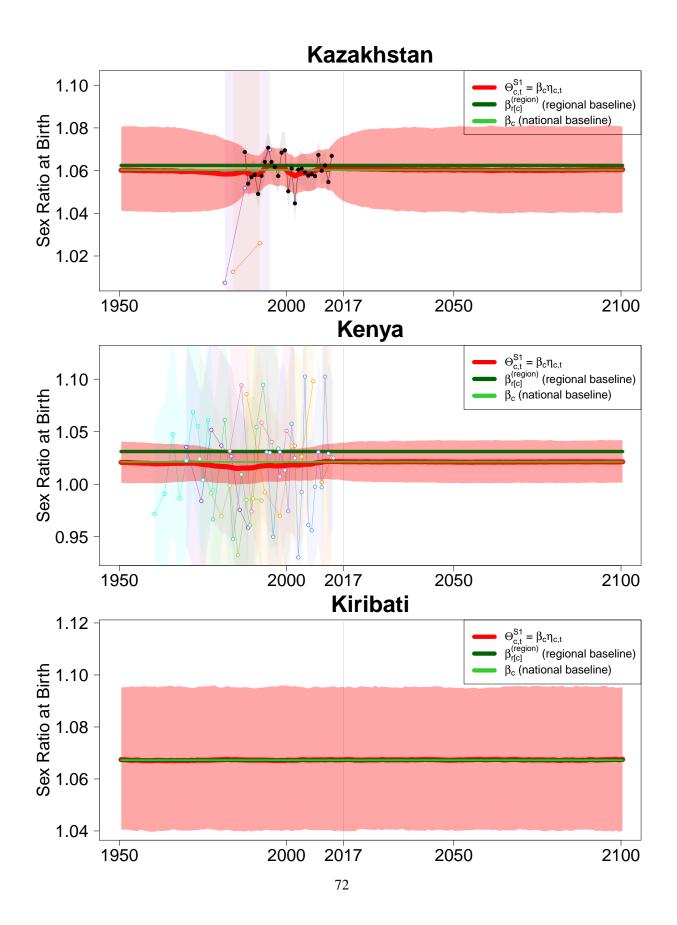


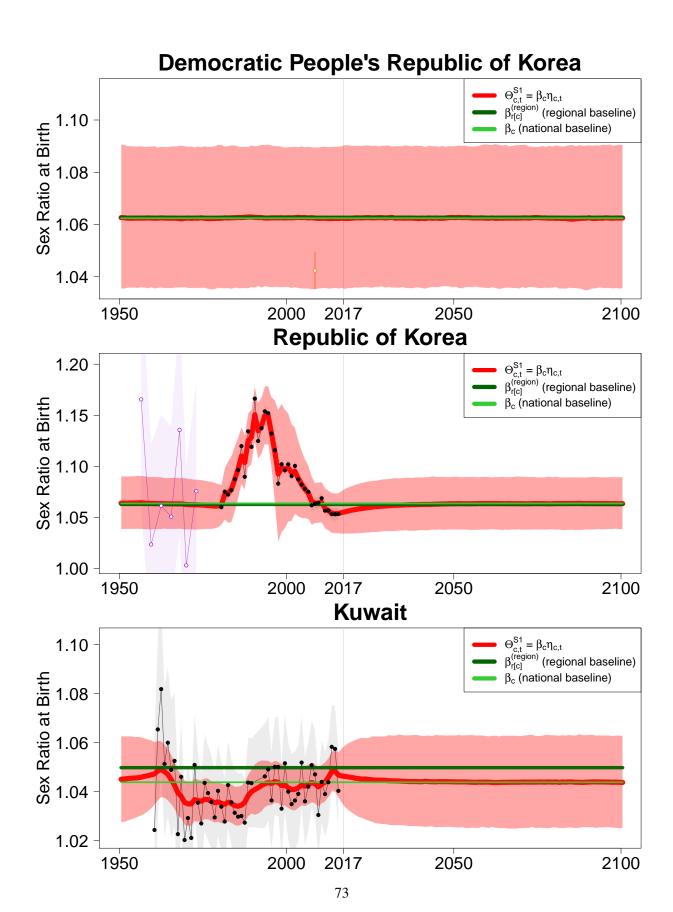


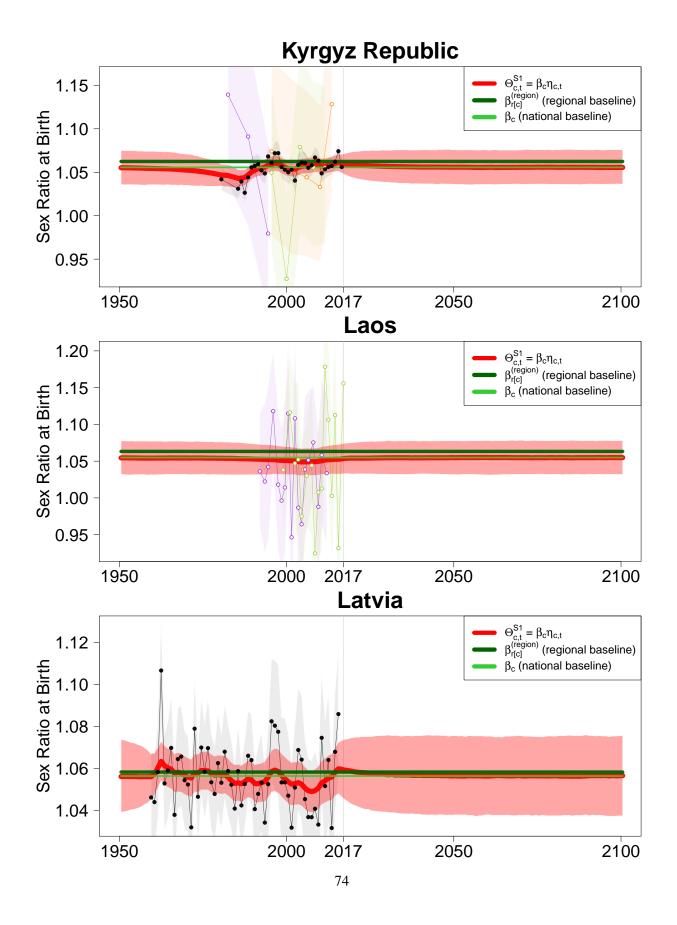


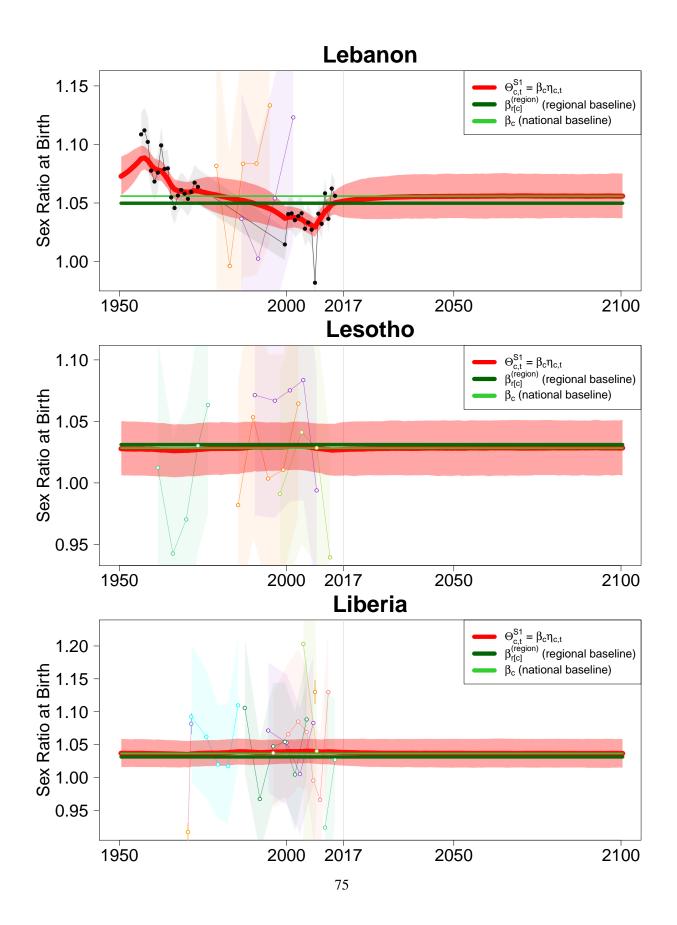


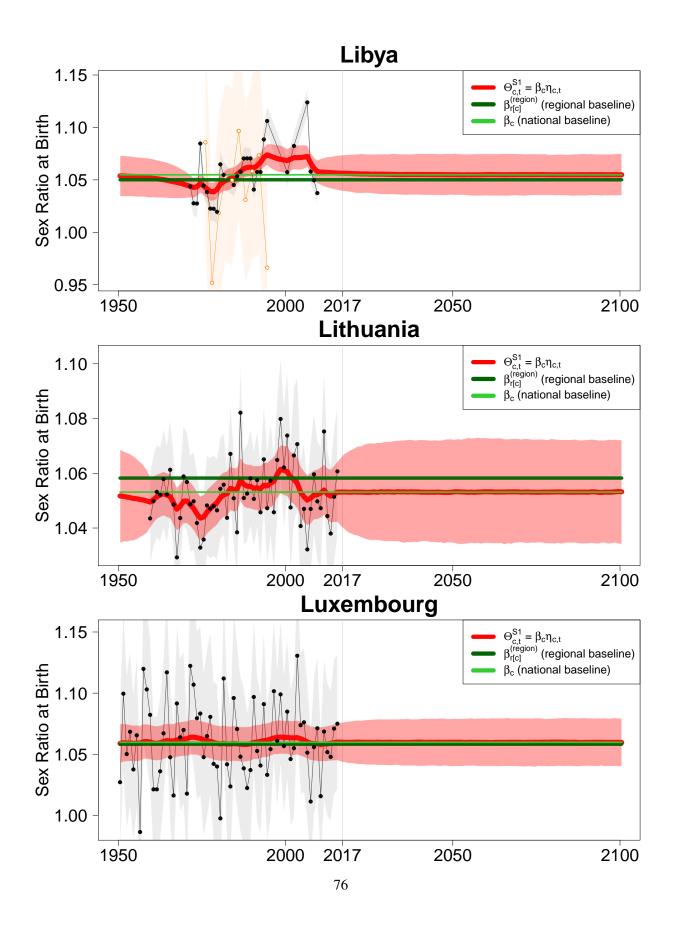


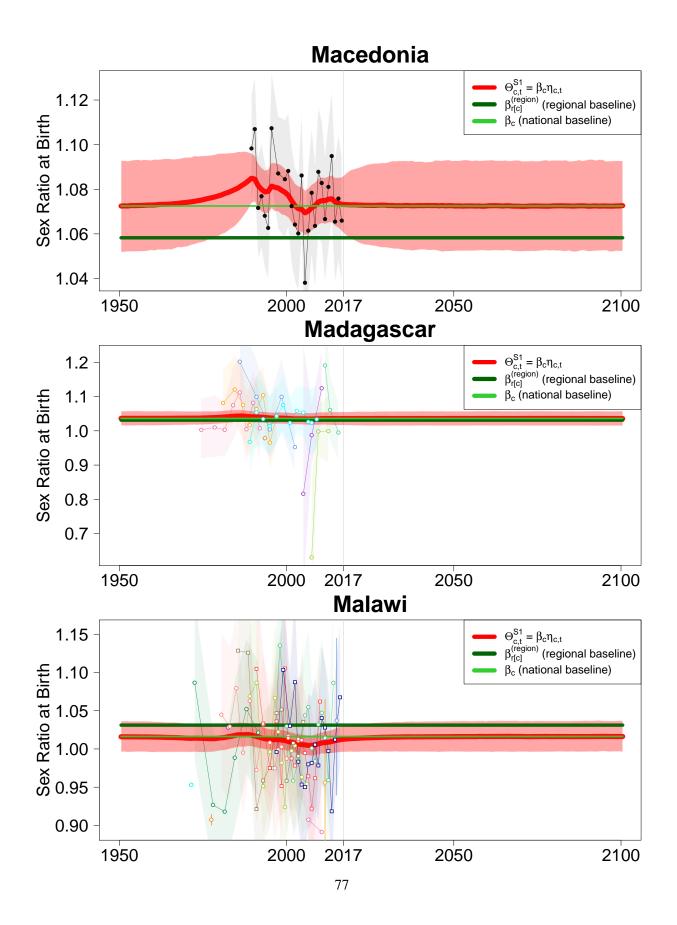


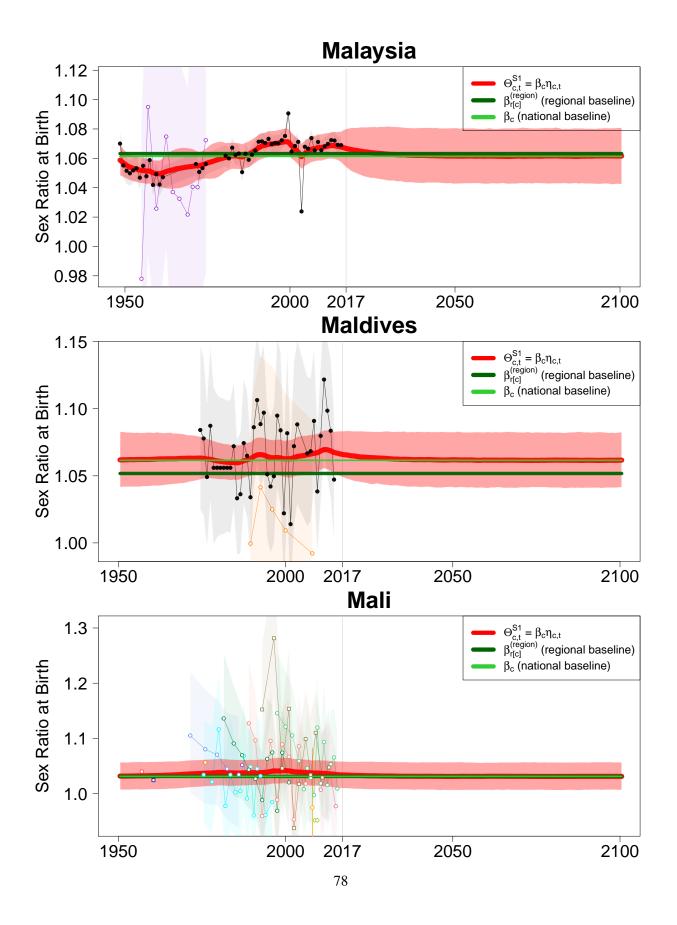


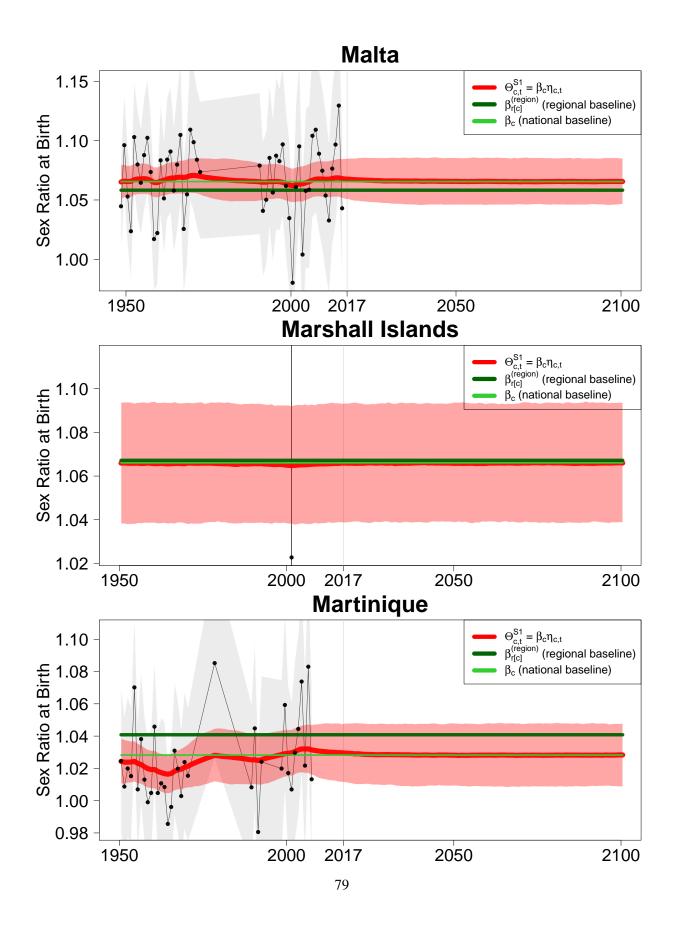


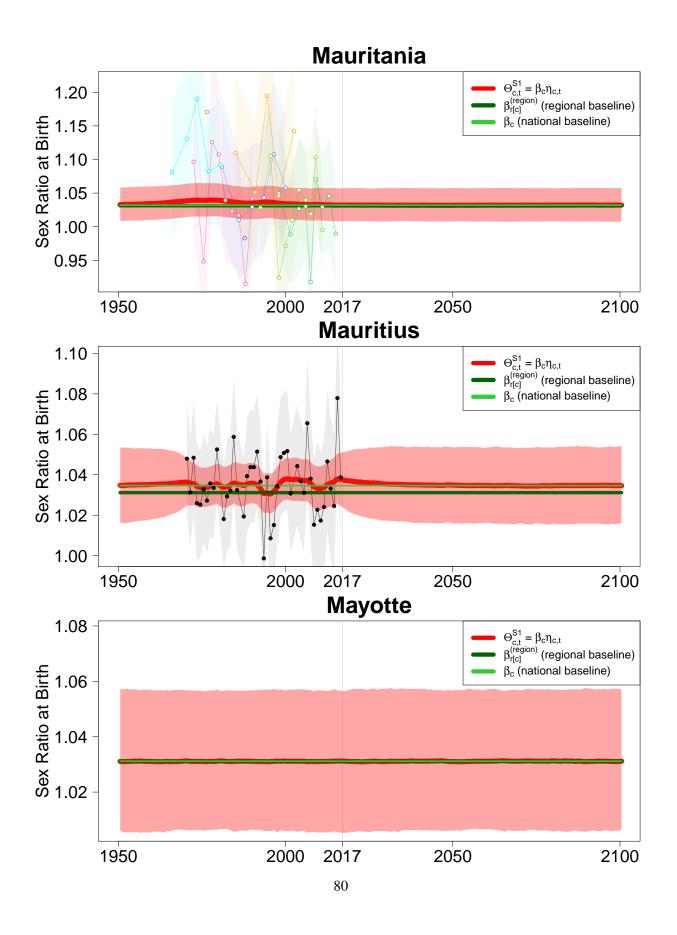


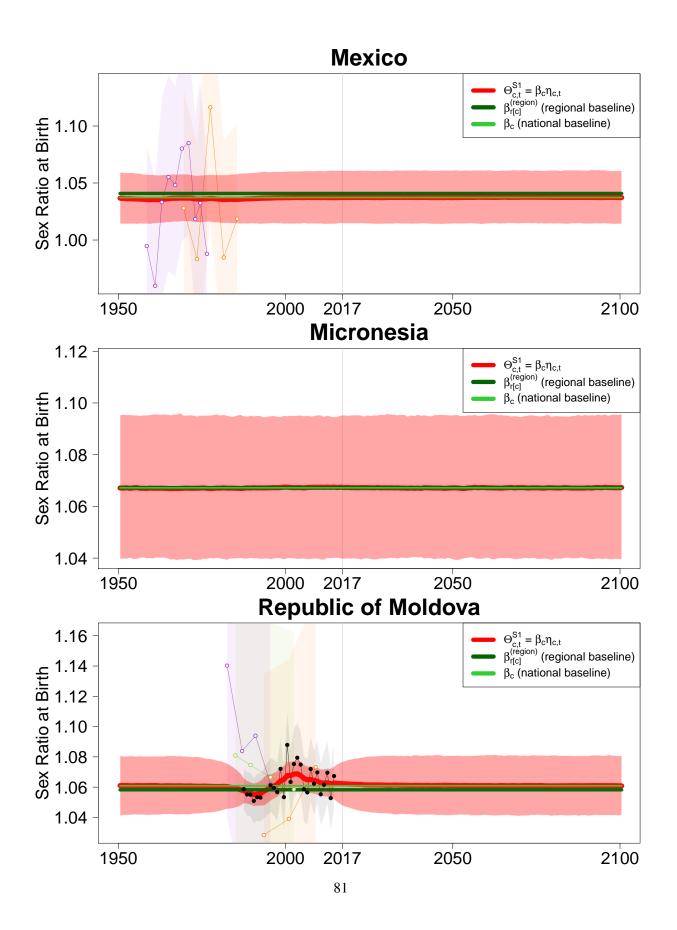


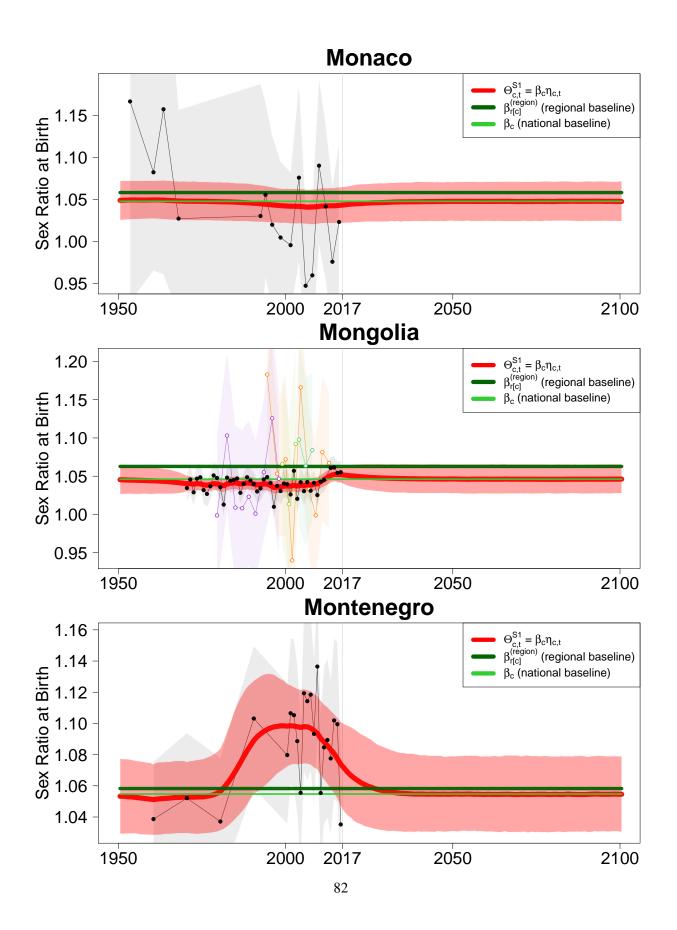


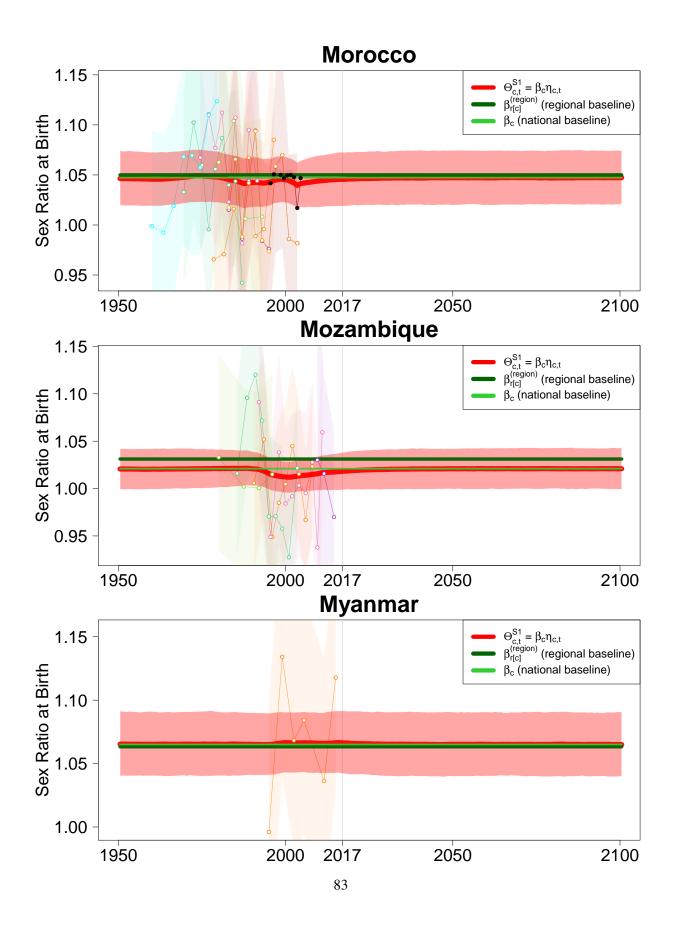


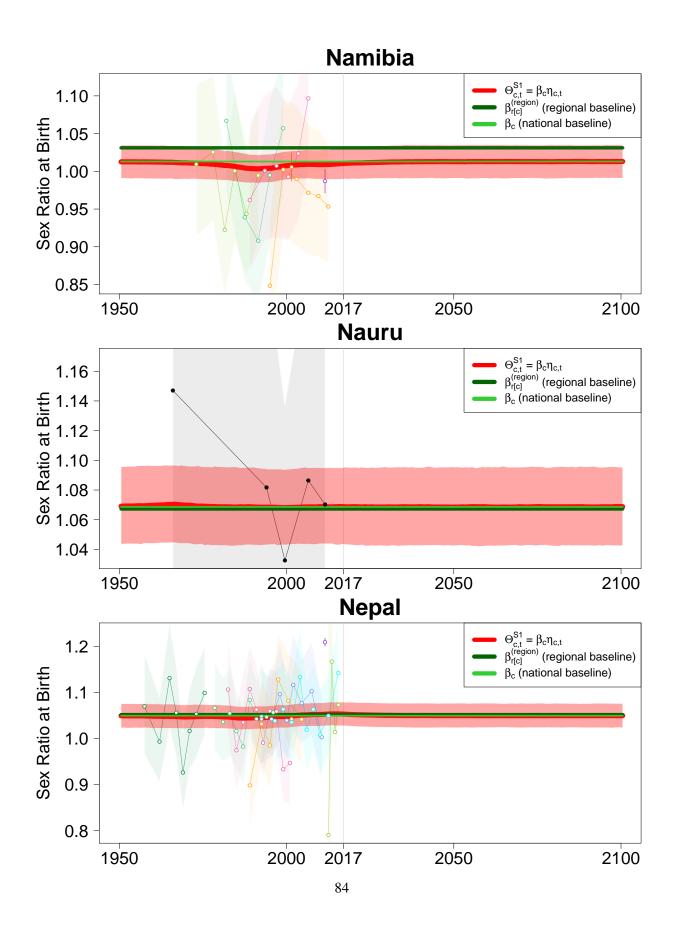


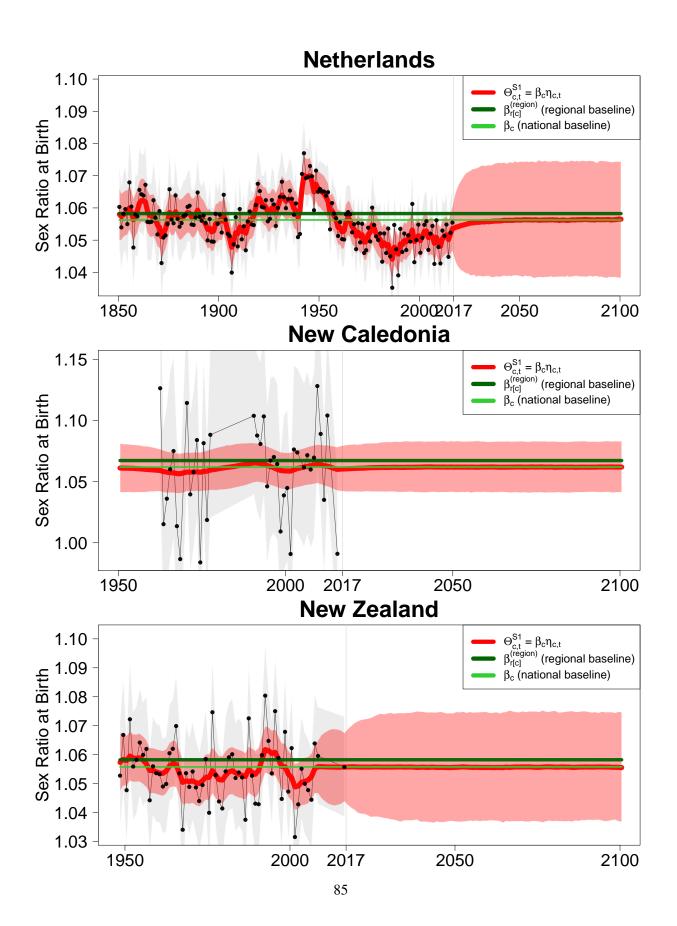


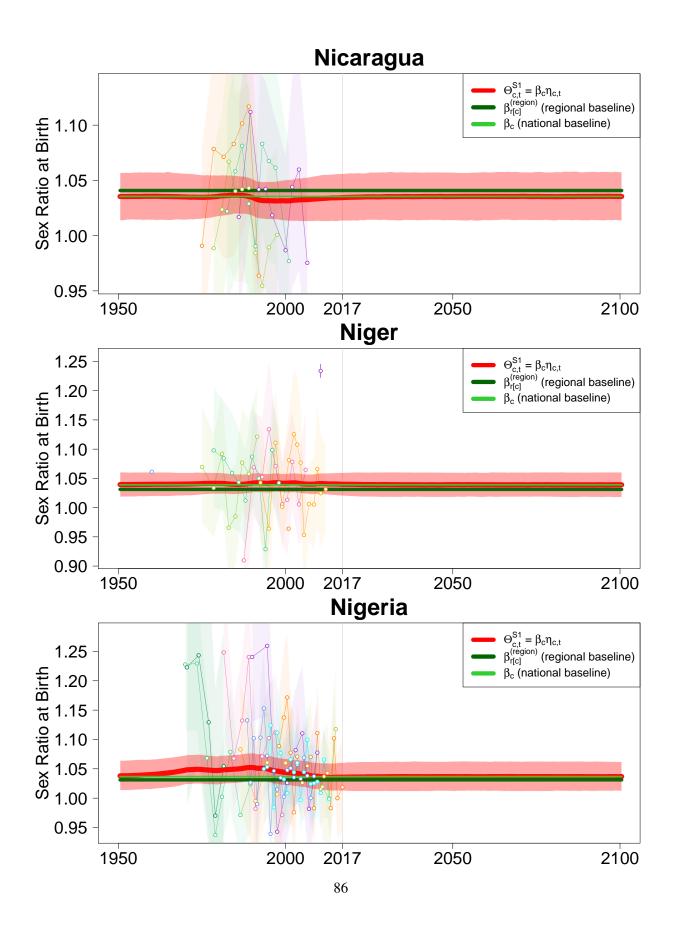


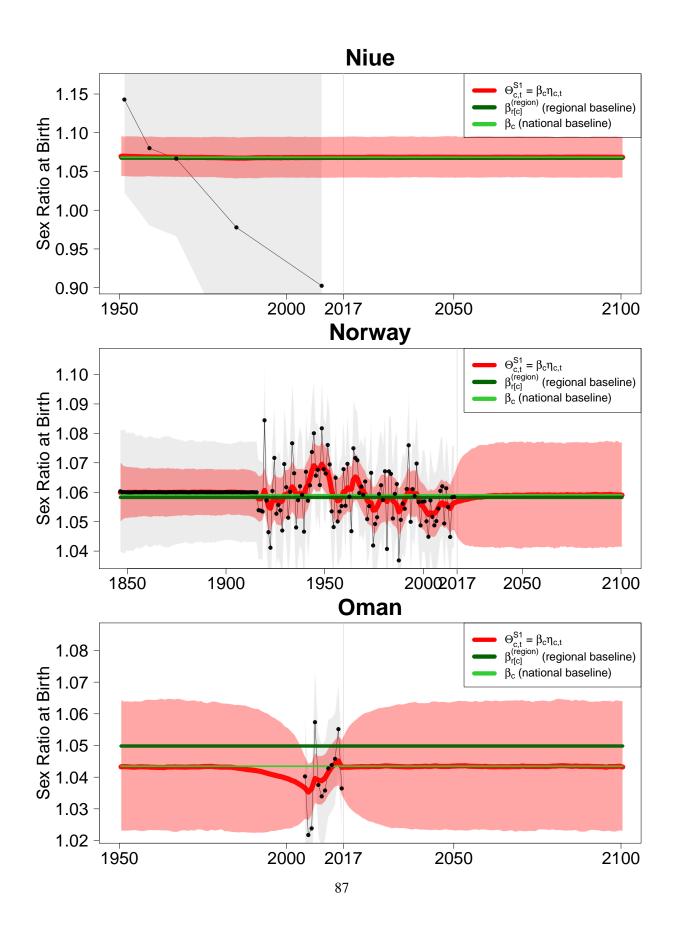


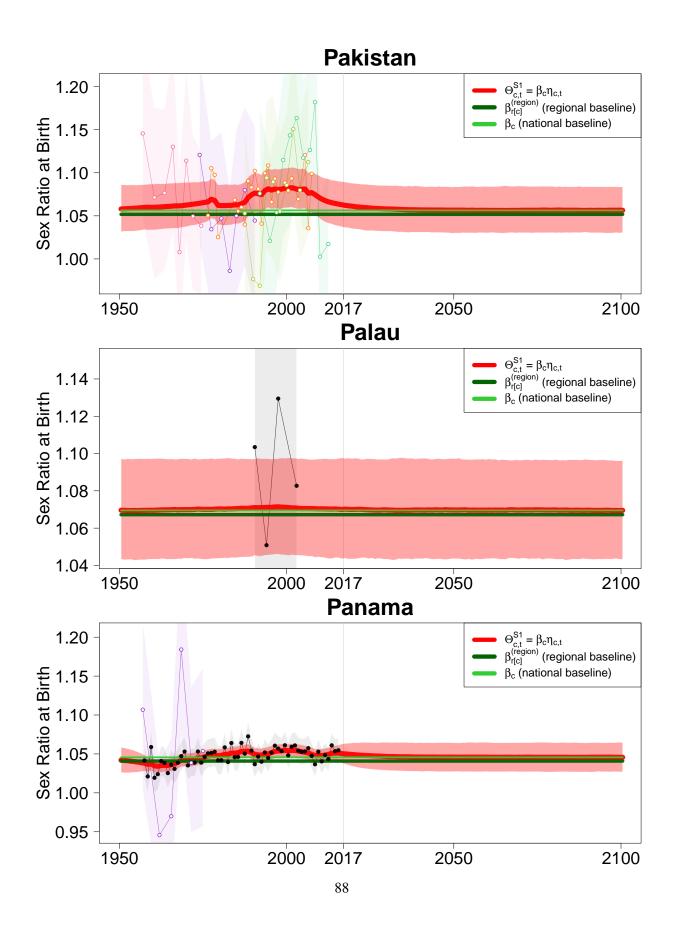


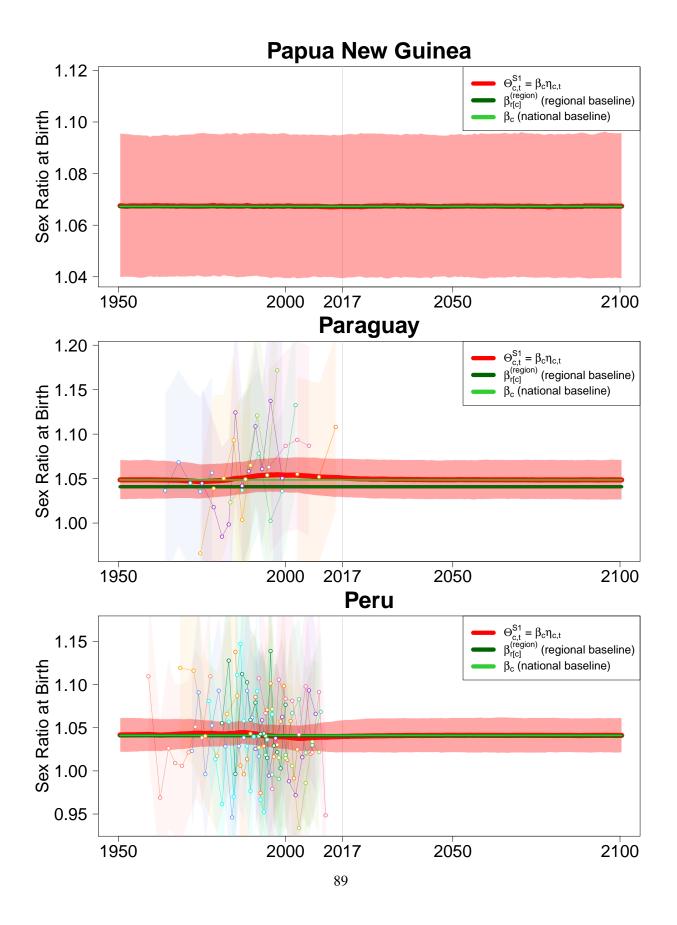


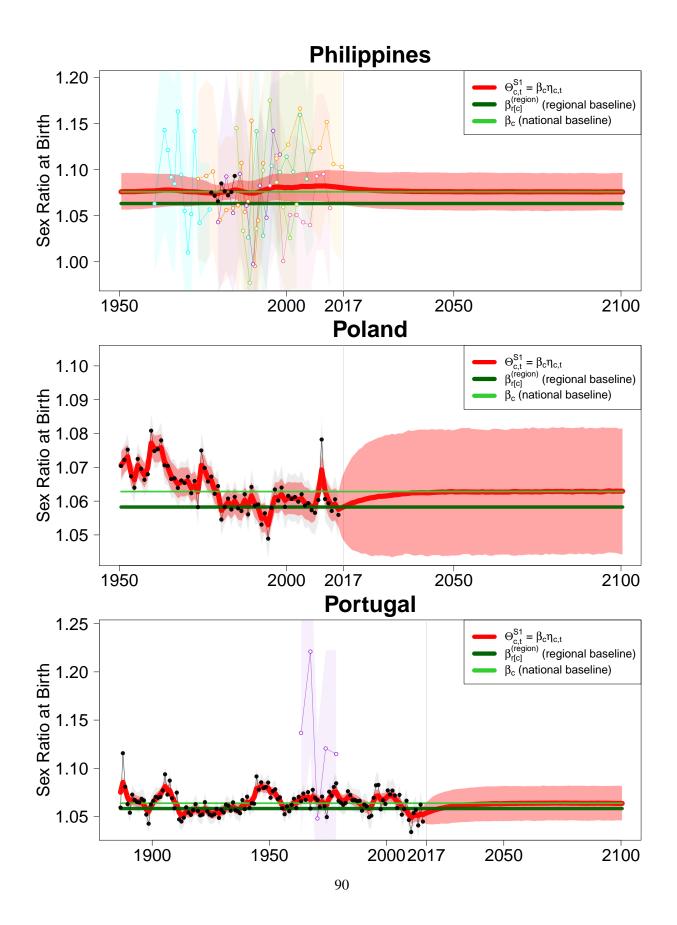


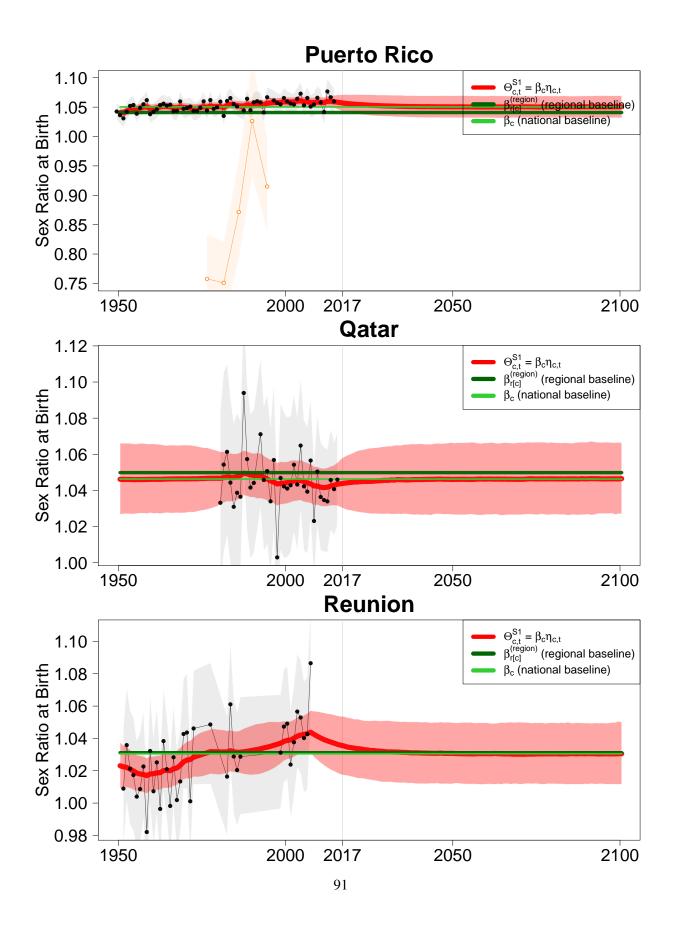


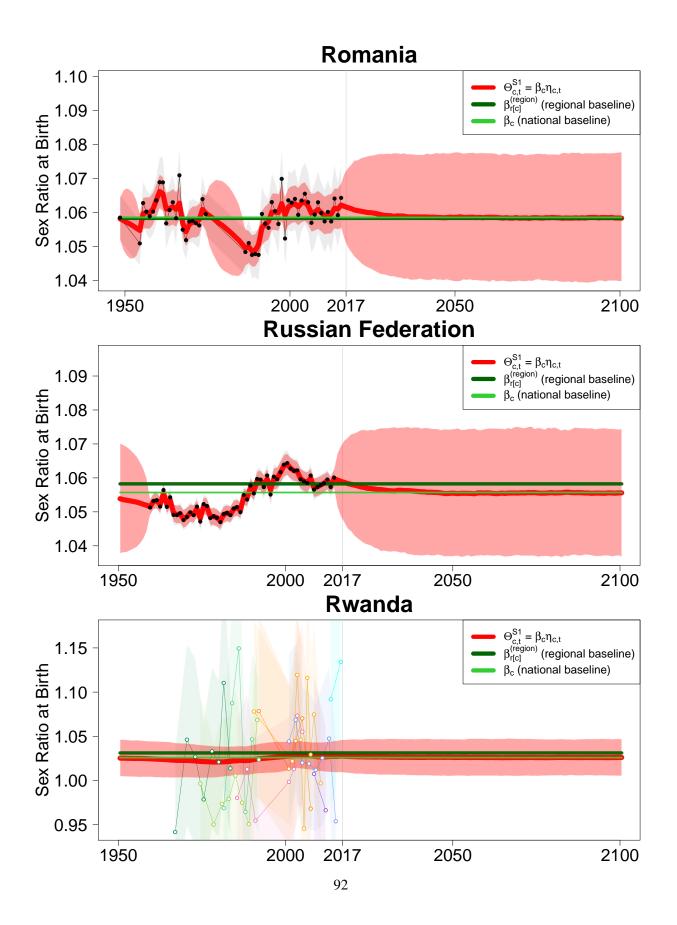


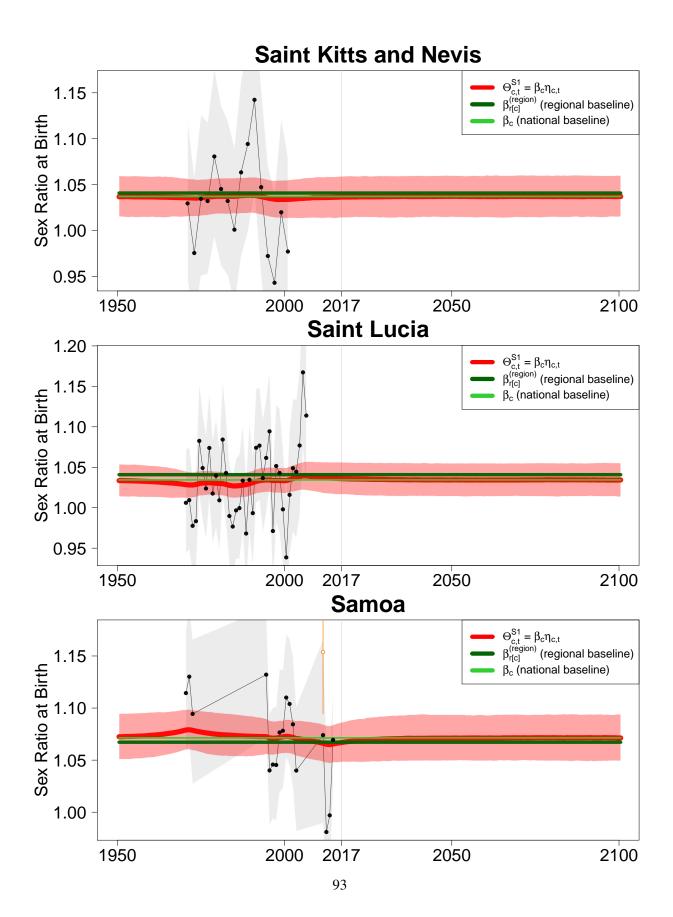


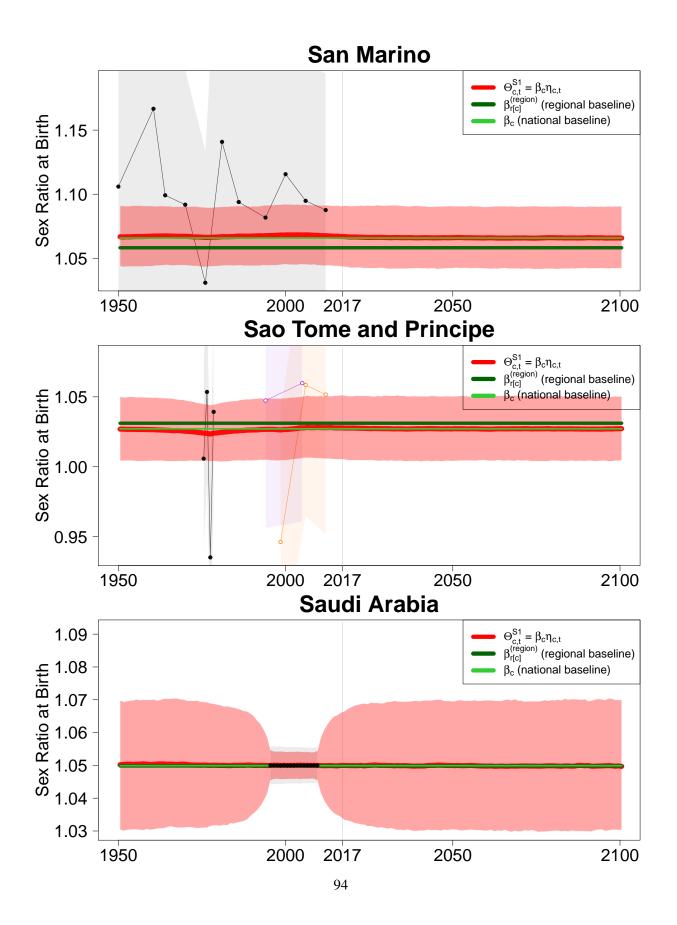


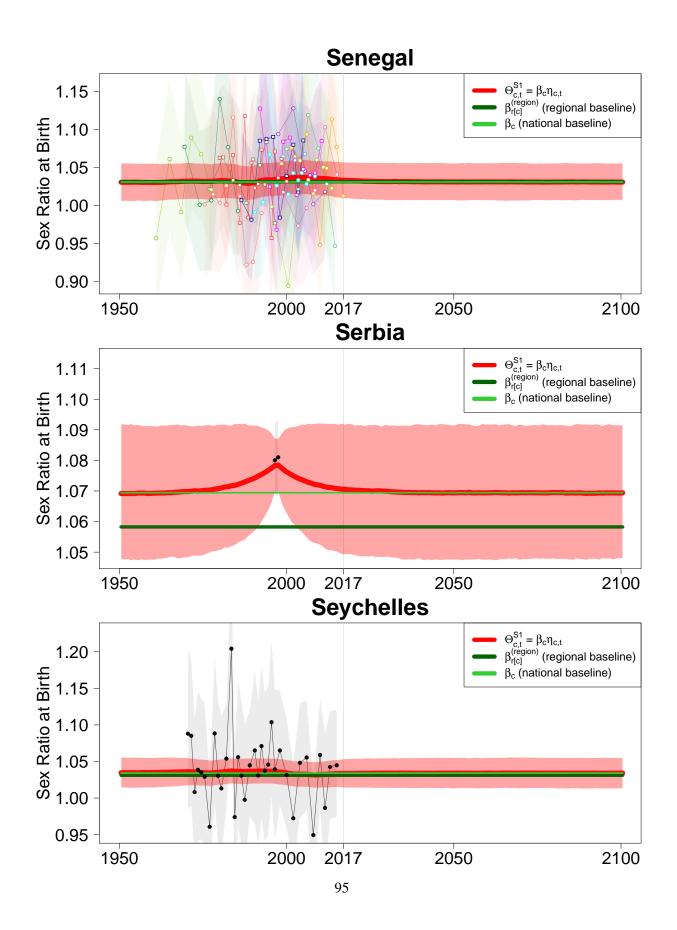


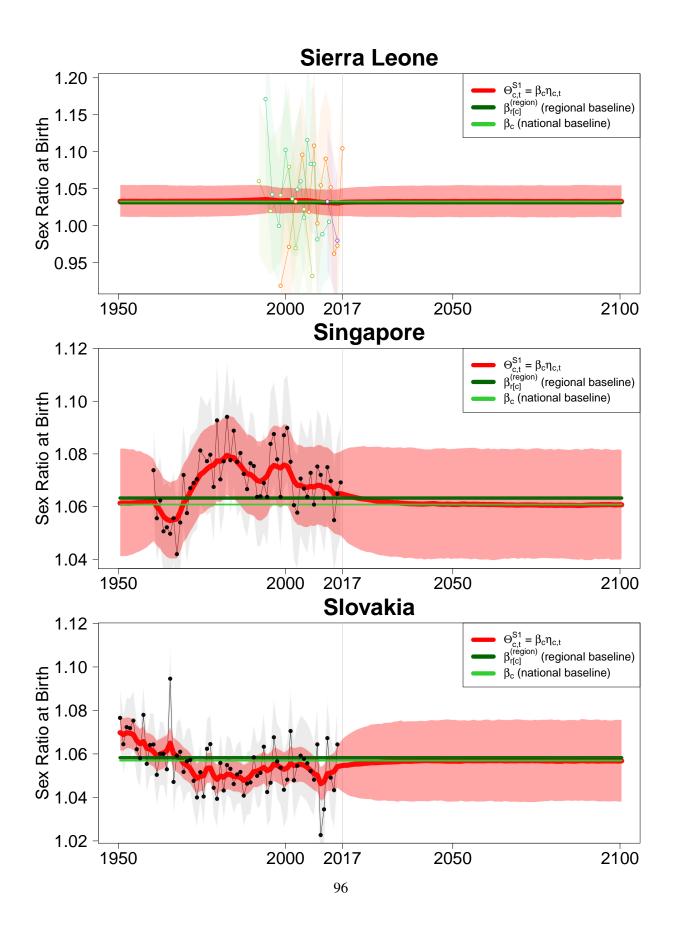


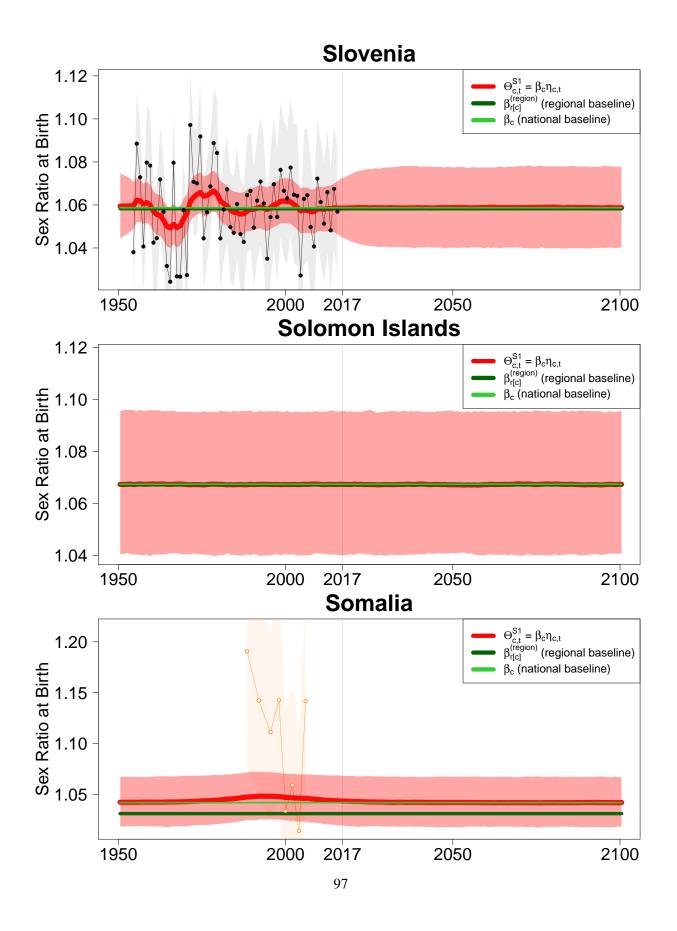


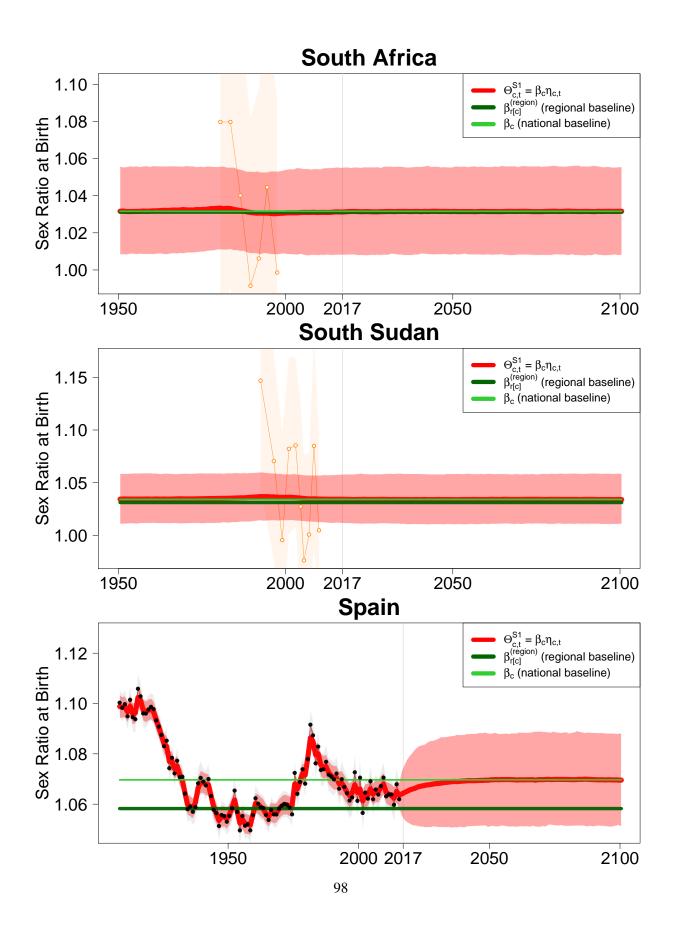


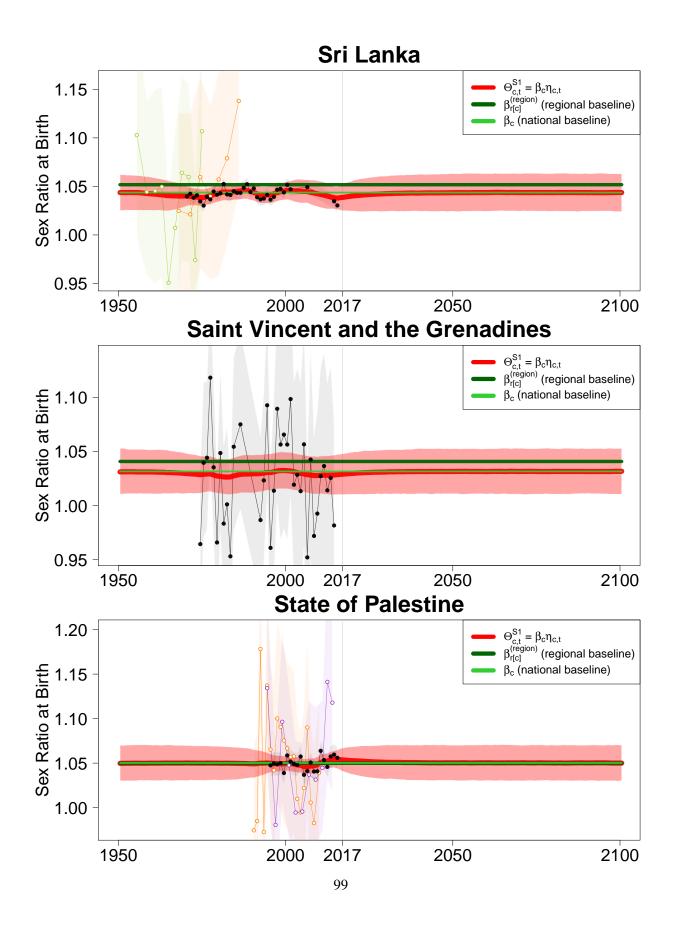


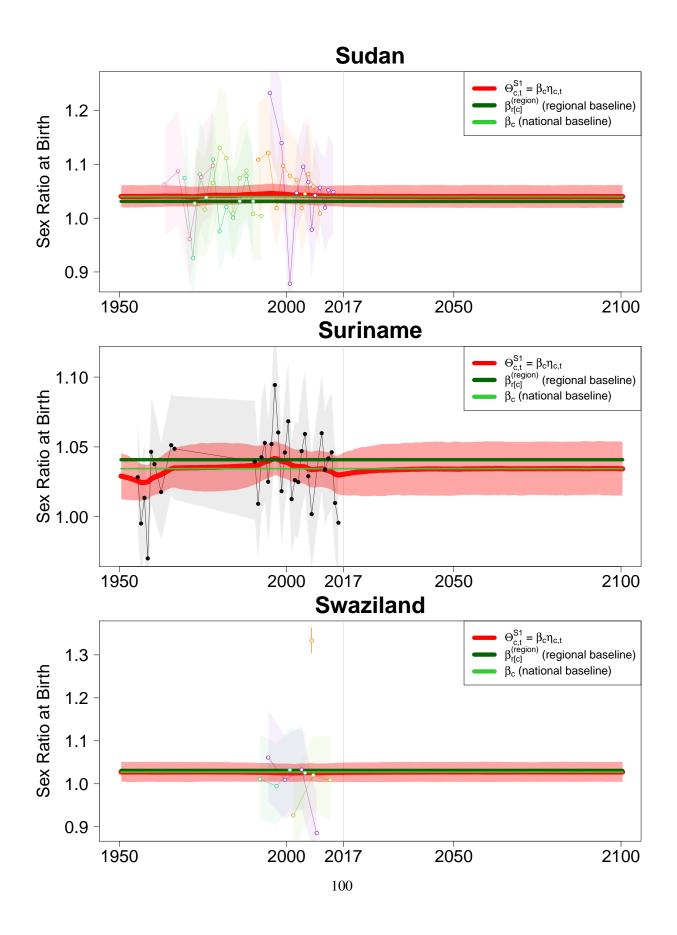


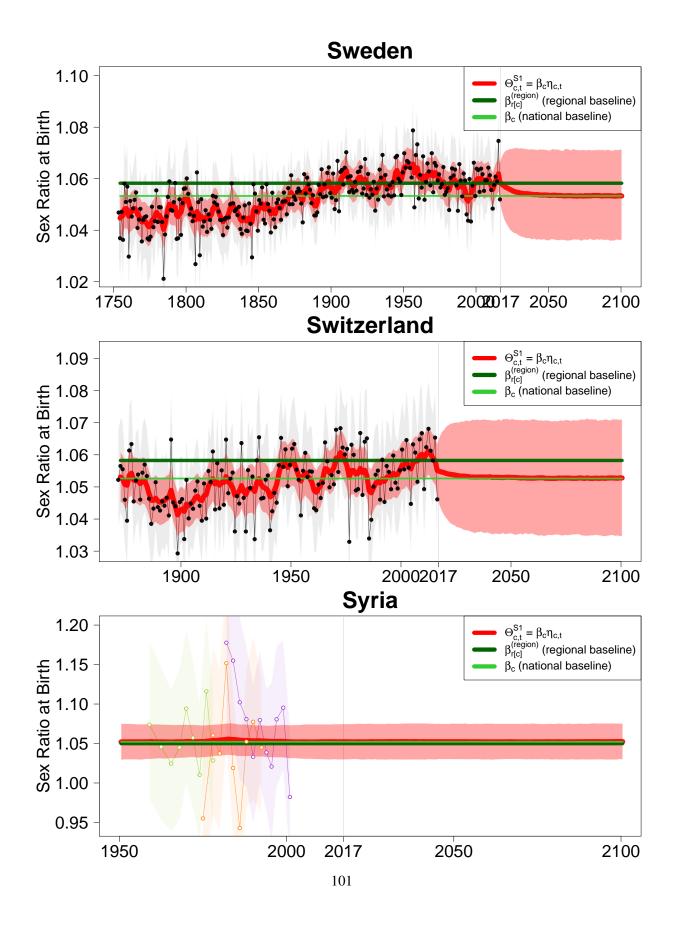


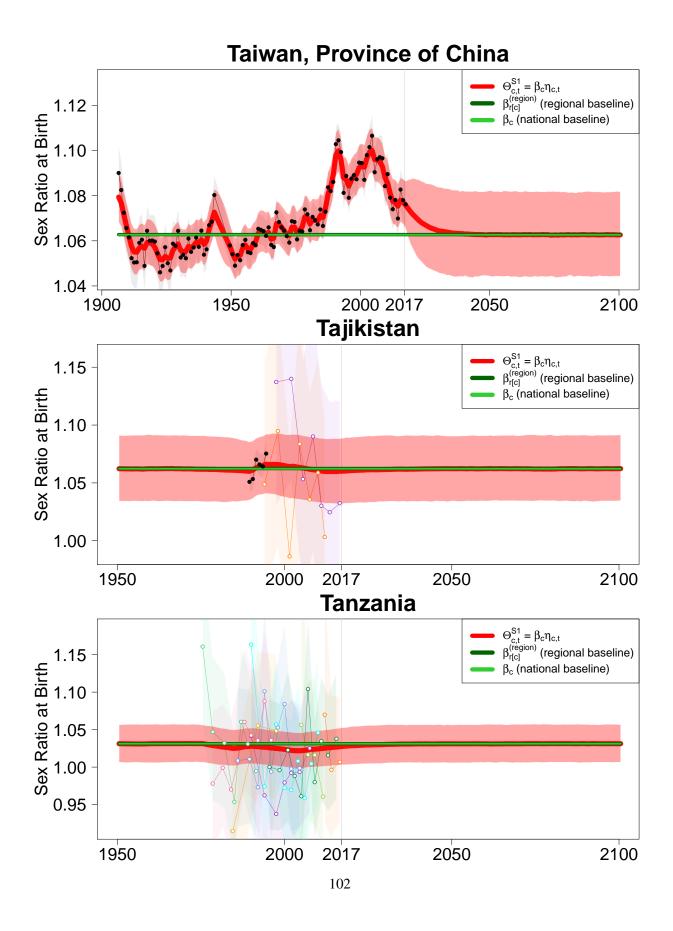


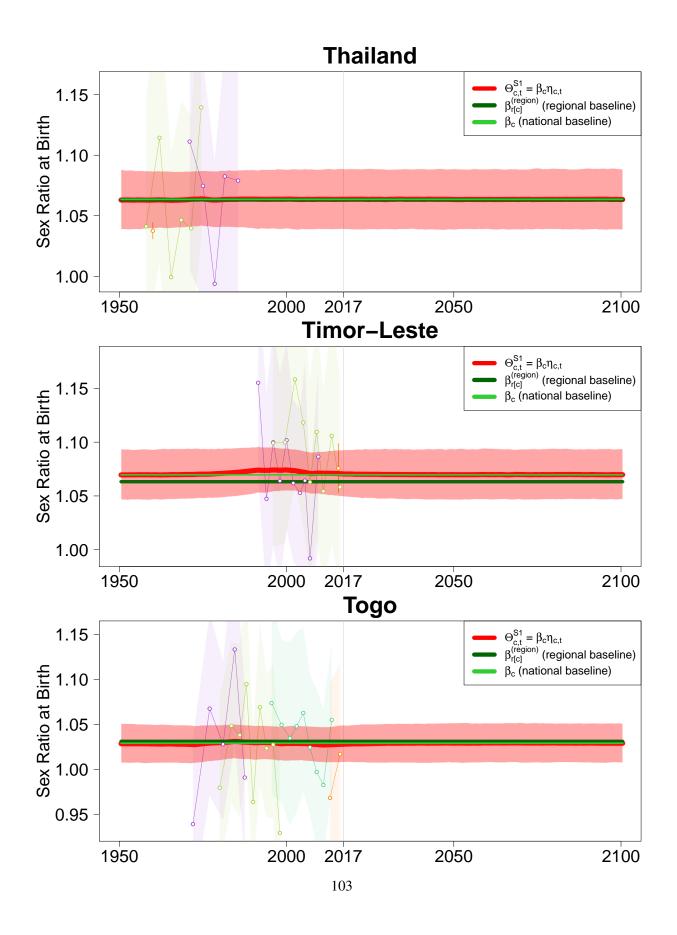


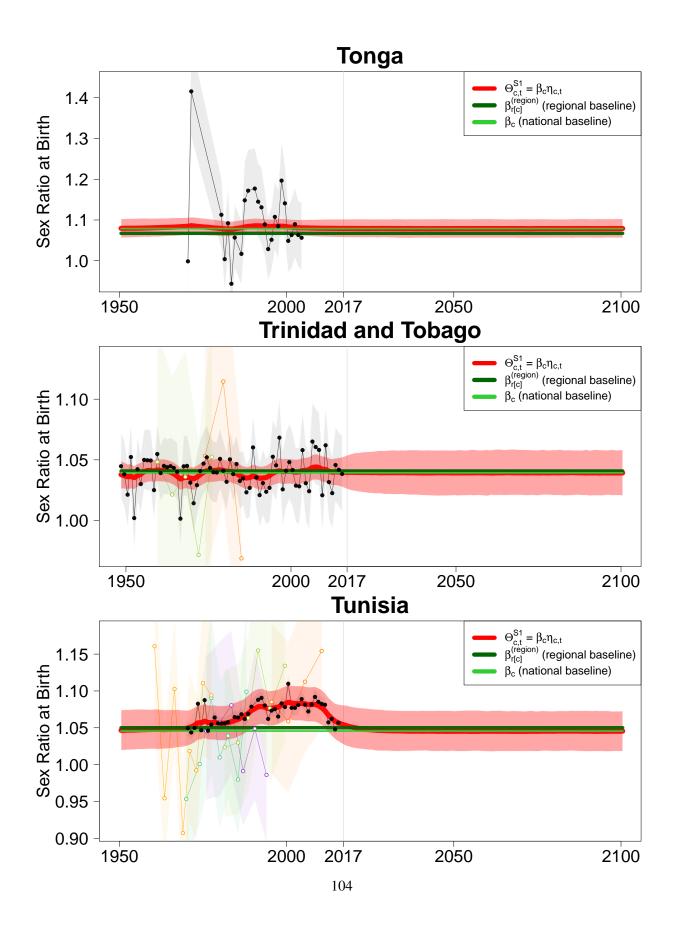


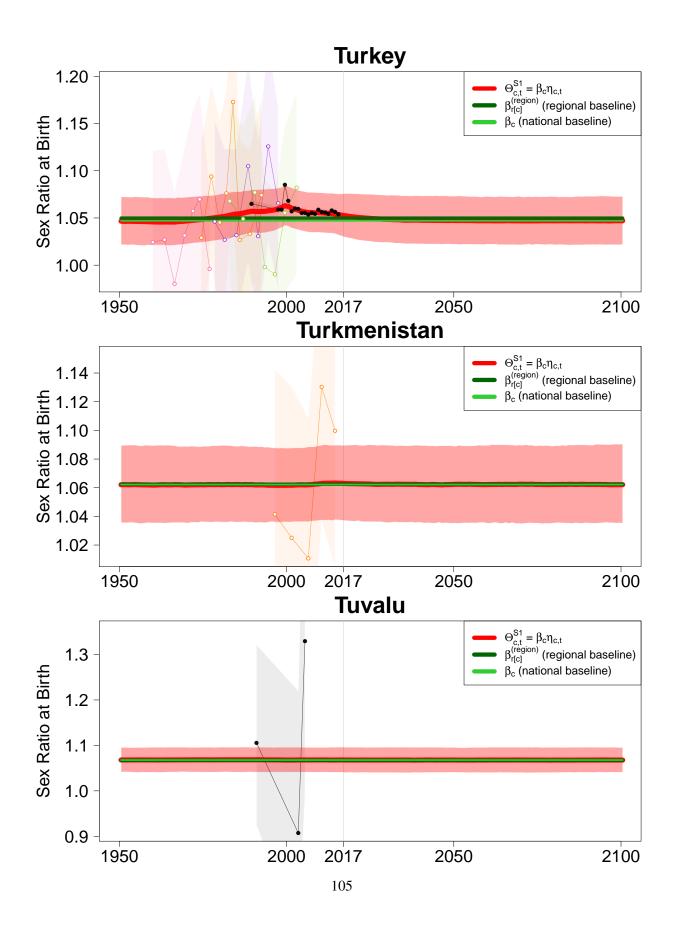


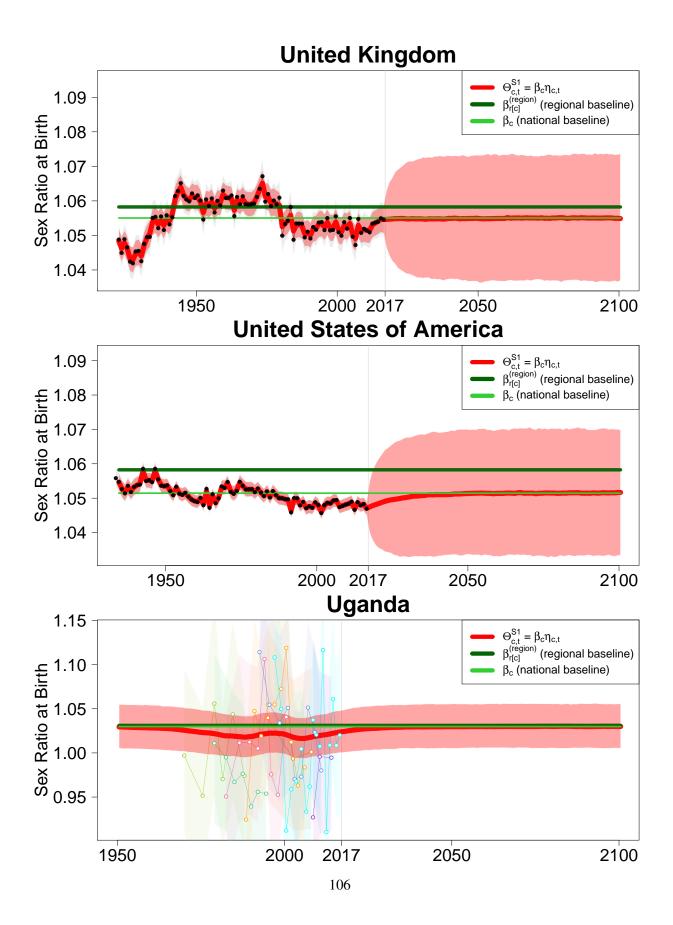


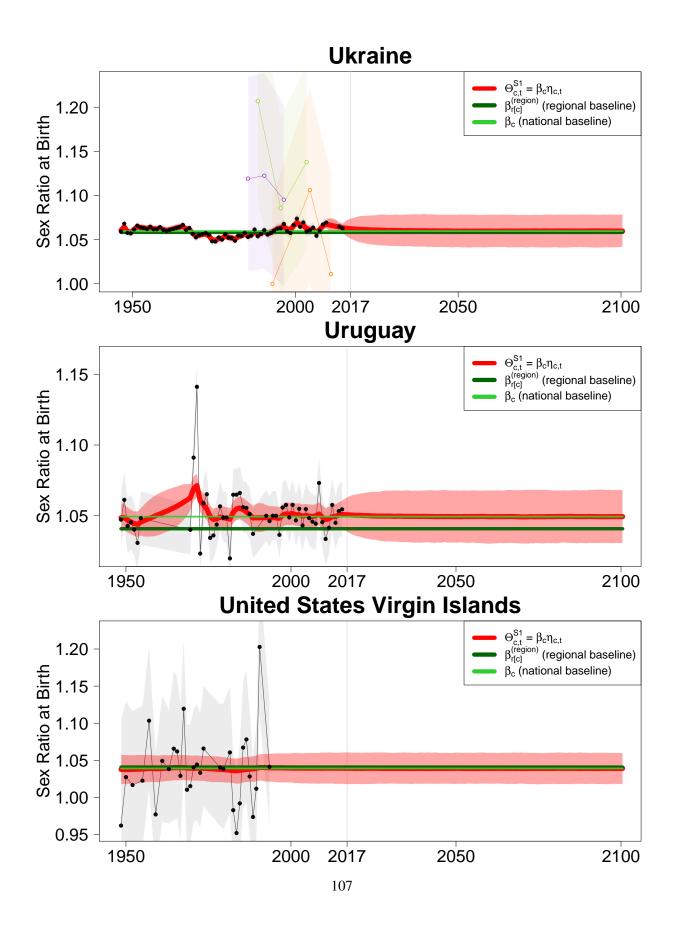


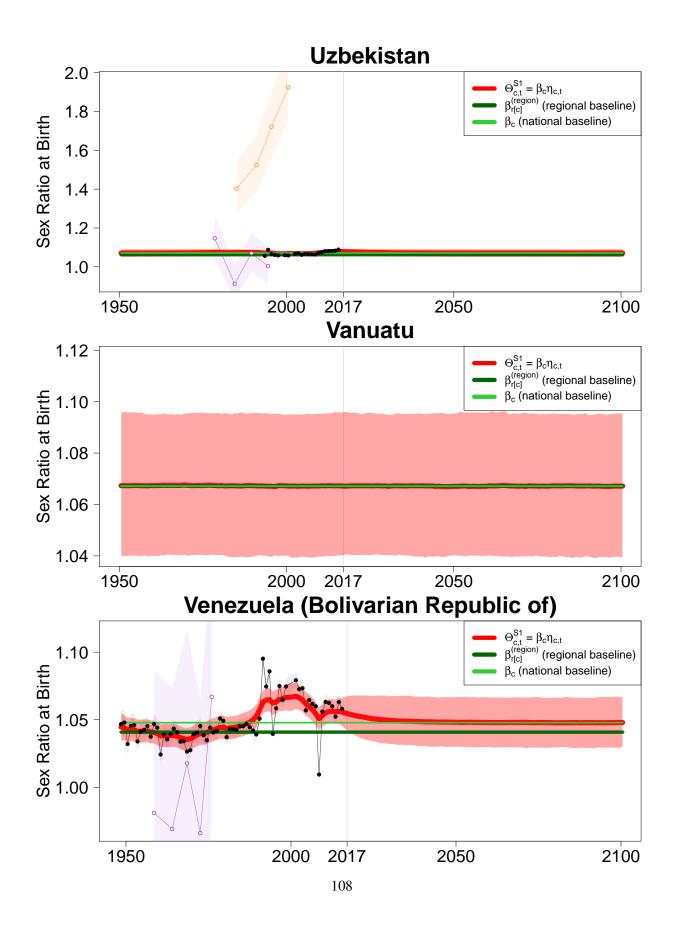


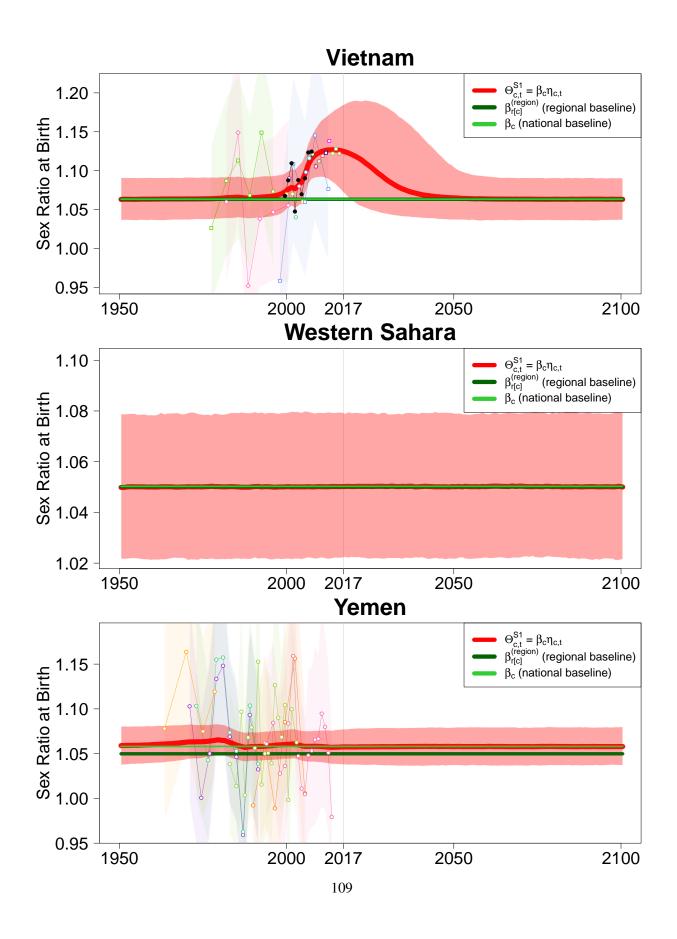


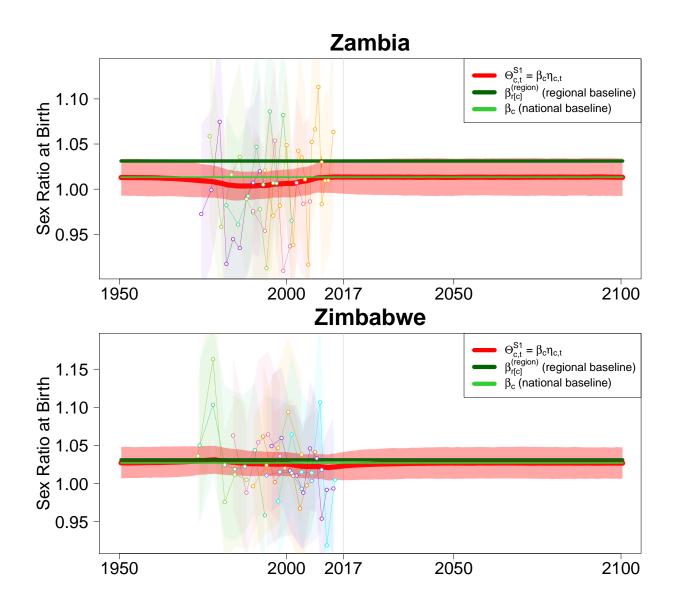












— The End —