Do Now (A,B,E only)

3A *Time: 4 minutes*
The four-digit number 489A is divisible by 11.
What digit does A represent?

3B *Time: 5 minutes*
Emma has $1.85 in her pocket, all in quarters and nickels. Her father gives her another quarter. She now has the same number of quarters as nickels. How many nickels does Emma have in all?

3C *Time: 5 minutes*
Tony has an 8 cm by 12 cm paper rectangle. He folds it in half three times, each time making a smaller rectangle. What is the least possible perimeter of the rectangle after the third fold, in cm?

3D *Time: 5 minutes*
If 4 people can paint 2 fences in 5 hours, how many hours in all will it take for 8 people to paint 8 fences?

3E *Time: 7 minutes*
7^2 means 7×7 and its product is 49, which has a units digit of 9.
7^3 means $7 \times 7 \times 7$ and its product is 343, which has a units digit of 3.
7^4 means $7 \times 7 \times 7 \times 7$ and its product is 2401, which has a units digit of 1.
What is the units digit in the product of 7^{50}?

Elementary Math Olympiad
Chapter 5: Guess and Check

5.1 Simple Divisibility Theorems

If a number ends in 0,2,4,6,8, it is divisible by 2

If a number ends in 0 or 5, it is divisible by 5

If a number ends in 25, 50, or 75, it is divisible by 25

If a number ends in 00, it is divisible by 4 and 25 or by 2^2 and 5^2

If a number ends in 000, it is divisible by 8 and 125 or by 2^3 and 5^3

If a number ends in 0000, it is divisible by 16 and 625 or by 2^4 and 5^4

Note: 2^3 or 5^2 simply means 2x2x2 or 5x5. The larger number(base) multiplied by the number of times indicated by the small number(exponent).

5.2 Distributive principles:

Ax(B+C) = AxB + AxC

Ax(B-C) = AxB – AxC

Examples:

224 = 210+14 = 7x30+7x2 = 7x(30+2) = 7x32

196 = 210-14 = 7x30-7x2 = 7x(30-2) = 7x28

5.3 Divisibility by 2^n

Example: Is 5,472,381,539,672 divisible by 8?

a) Break the number into 5,472,381,539,000 + 672

b) Find factors of 8 in 672 (if any). In this example, 672 = 8x84 so 672 is divisible by 8 or 2x2x2 or 2^3

c) By the Distributive principle above,
5,472,381,539,672 =
5,472,381,539,000 + 672 =
5,472,381,539,000 (divisible by 8) + 8x84 =
8x(factor + 84) ⇔ 5,472,381,539,672 is divisible by 8

Divisibility Rules

- Divisibility by 3, 9

Remember that if the digit-sum of a number is a multiple of 9, then the number itself is a multiple of 9.

Same for 3.

- Divisibility by 5, 10?
- Divisibility by 4, 6, 8?
- Divisibility by 7?

Can you write 2 examples in each of the following categories that are divisible by 3, 4, 6, 8, 9, and 11?

a) A 2- digit number
b) A 3- digit number
c) A 4-digit number
d) A 5-digit number
e) A 6-digit number

- Divisibility by 11

You learned that to find products of 11x(2 digit-number), you add the sum of the 2-digit number and then insert it in the middle of the number.

To find if a number such as 1452 is divisible by 11, add the odd-place digits and the even-place digits. If the difference between them is 0 or a multiple of 11, then the number is divisible by 11. Example: since 1+5 = 6 and 4+2 = 6. Difference is 0. 1452 is divisible by 11. Check: 1452/11=132.

Another Example:

853691 divisible by 11? Since 8+3+9=20 and 5+6+1 = 12. Difference is neither 11 nor 0. 853691 is NOT divisible by 11. Check: 853691/11 = 77608 R 3

Chapter 5, Problems 10 and 20

- I have 10 coins whose total value is $1. If three of the coins are quarters, what are the remaining coins? (hint: for #10 & #19, use 1=penny, 5=nickel, 10=dime, 25=quarter and 100=dollar)

- A and B represent different digits. If AB is a 2-digit number and BBB is a 3-digit number, what 2-digit number does AB represent?

$$\begin{array}{r} A\ B \\ \times \quad 6 \\ \hline B\ B\ B \end{array}$$

Chapter 5, Problem 6 and 16

- Each letter stands for a digit & different letters stand for different digits. What digits do H,E, and A represent?

```
  HE
  HE
  HE
+ HE
----
  AH
```

- There are 3 two-digit numbers in which different letters represent different digits. What digits do A,B, C represent?

```
  A A
  B B
+ C C
-----
  B A C
```

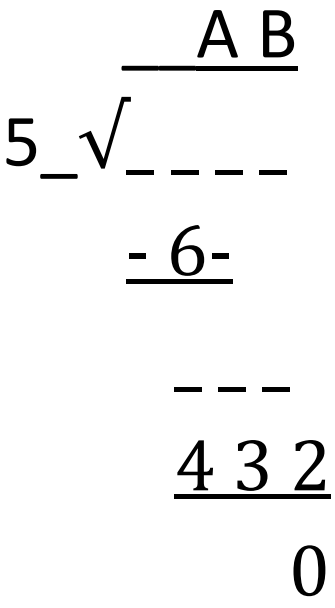
Chapter 5, Problems 13 & 15

A4273B is a 6-digit number in which A & B are different digits. The number is divisible by 72 without any remainder. Find the values of A and B.

- The 4-digit number A55B is divisible by 36. What is the sum of A and B?

Chapter 5, Problem 5

- The blanks represent missing digits in the division problem shown. If A & B are the digits of the quotient, what are the values of A and B?



Homework (Due 11/14)

- Math III

1) Finish DO NOW
2) From Chapter 5 of Elementary Math Olympiad booklet, #6,8,9,10,16-20
3) Write 2 examples in each of the following categories that are divisible by 3, 4, 6, 8, 9, and 11?

a) A 2- digit number
b) A 3- digit number
c) A 4-digit number
d) A 5-digit number
e) A 6-digit number

NOTE: Nov 14 is Math Olympiad at SCCS. Please arrive on time at 2:15pm in C134 for the Div E test. Parents: Please volunteer as graders and proctors.

- Math IV

1) Finish DO NOW
2) From Chapter 5 of Elementary Math Olympiad section of your reader, #1-20 skip 3 and 7.
3) Write 2 examples in each of the following categories that are divisible by 3, 4, 6, 8, 9, and 11?

a) A 2- digit number
b) A 3- digit number
c) A 4-digit number
d) A 5-digit number
e) A 6-digit number

NOTE: Nov 14 is Math Olympiad at SCCS. Please arrive on time at 2:15pm in C134 for the Div E test. Parents: please volunteer as graders and proctors.