Dec 18 Homework: Reviews D,E, and F, pages-83 to 96. Pages 60 & 67 (if you want more percentage practice problems)
Happy New Year. See you in Jan 2012!

Percentages

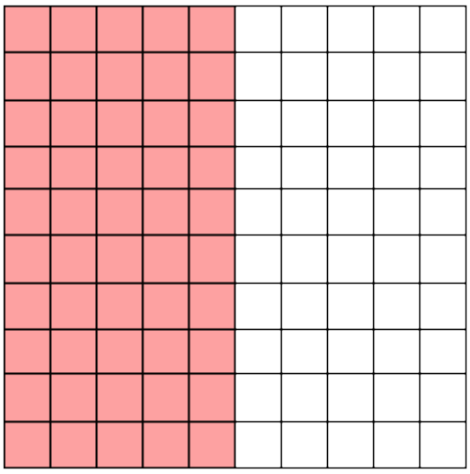
A **percentage** is a fraction that is written out of 100.

For example, 15 per cent means 15 parts out of 100. The term **per cent** means per one hundred. The symbol used for per cent is %.

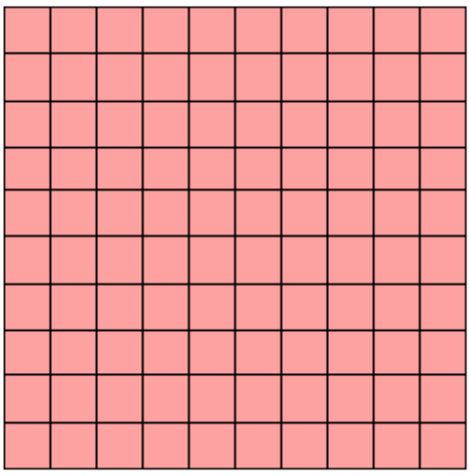
$$\therefore 15 \text{ per cent} = 15\% = \frac{15}{100}$$

Note:

- $\frac{50}{100}$ means 50% and it can be represented by the shaded part of the following grid.



- $\frac{100}{100}$ means 100% and it can be represented by the shaded part of the following grid.



- 100% means the whole.

From the preceding discussion, we notice that:

A **percentage** is the numerator of a fraction with a denominator of 100.

For example, $\frac{1}{2} = \frac{50}{100}$ is a fraction equivalent to 50%.

The % (per cent) symbol is used in place of 'over 100'.

Thus, $100\% = \frac{100}{100} = 1$

Converting a Percentage to a Fraction

To convert a percentage to a fraction, write it as a fraction with a denominator of 100 and then simplify the fraction if possible.

Example 1

Express 65% as a fraction.

Solution:

$$65\% = \frac{65}{100} \qquad \text{(Divide the numerator and denominator by 5)}$$
$$= \frac{13}{20}$$

Mixed Number Percentage

To convert a **mixed number percentage** to a fraction, change the mixed number to an improper fraction, write the percentage as a fraction with a denominator of 100 and simplify the fraction if possible.

Example 2

Express $4\frac{2}{7}\%$ as a fraction.

Solution:

$$4\frac{2}{7}\% = \frac{\frac{30}{7}}{100} \qquad \text{(Write the percentage as a fraction out of 100)}$$

$$= \frac{\frac{3}{\cancel{30}}}{7} \times \frac{1}{\frac{\cancel{100}}{10}}$$

$$= \frac{3}{70}$$

Key Terms

percentage, per cent, mixed number percentag

Converting a Percentage to a Decimal

To convert a percentage to a decimal, first write the percentage as a fraction out of 100 and then move the decimal point two places to the left.

Example 3

Express the following percentages as decimal numbers:

a. 37%

b. 24.6%

c. $7\frac{3}{4}\%$

Solution:

a. $37\% = \frac{37}{100}$ (Move the decimal point two places to the left)

$= 0.37$

Note:

Place a zero in the units column.

b. $24.6\% = \frac{24.6}{100}$ (Move the decimal point two places to the left)

$= 0.246$

c. $7\frac{3}{4}\% = 7.75\%$

$= \frac{7.75}{100}$ (Move the decimal point two places to the left)

$= 0.0775$

Converting a Fraction or Decimal to a Percentage

To convert a fraction or decimal to a percentage, multiply it by 100%. Then simplify.

Example 4

Convert each of the following fractions to a percentage:

a. $\frac{3}{100}$

b. $\frac{9}{10}$

c. $\frac{3}{5}$

d. $\frac{1}{8}$

Solution:

a. $\frac{3}{100} = \frac{3}{100} \times 100\% \qquad \left\{\because 100\% = \frac{100}{100} = 1\right\}$

$= \frac{3}{\cancel{100}^{1}} \times \frac{\cancel{100}^{1}}{1}\%$

$= \frac{3}{1}\%$

$= 3\%$

b. $\frac{9}{10} = \frac{9}{10} \times 100\% \qquad \left\{\because 100\% = \frac{100}{100} = 1\right\}$

$= \frac{9}{\cancel{10}^{1}} \times \frac{\cancel{100}^{10}}{1}\%$

$= \frac{90}{1}\%$

$= 90\%$

c. $\frac{3}{5} = \frac{3}{5} \times 100\% \qquad \left\{\because 100\% = \frac{100}{100} = 1\right\}$

$= \frac{3}{\cancel{5}^{1}} \times \frac{\cancel{100}^{20}}{1}\%$

$= \frac{60}{1}\%$

$= 60\%$

d. $\frac{1}{8} = \frac{1}{8} \times 100\% \qquad \left\{\because 100\% = \frac{100}{100} = 1\right\}$

$= \frac{1}{\cancel{8}_{2}} \times \frac{\overset{25}{\cancel{100}}}{1}\%$

$= \frac{25}{2}\%$

$= 12\frac{1}{2}\%$

Example 5

Express 0.25 as a percentage.

Solution:

$0.25 = 0.25 \times 100\% \qquad \left\{\because 100\% = \frac{100}{100} = 1\right\}$

$= 25\%$

Finding a Percentage of a Quantity

To find a certain percentage of a given quantity, write the percentage as a fraction out of 100 and change the 'of' to a ×. Then simplify.

Example 6

Find:

a. 20% of 65

b. $4\frac{1}{2}\%$ of \$40

Solution:

a. $20\% \text{ of } 65 = 20\% \times 65 \qquad \{\text{Change the 'of' to } \times\}$

$= \frac{\overset{1}{\cancel{20}}}{\underset{5}{\cancel{100}}} \times \frac{\overset{13}{\cancel{65}}}{1}$

$= \frac{13}{1}$

$= 13$

b. $4\frac{1}{2}\%$ of $\$40 = \frac{9}{2}\% \times \$40$ (Change the 'of' to $\times$)

$$= \$\frac{9}{\cancel{200}_{5}} \times \frac{\overset{1}{\cancel{40}}}{1} \qquad \left\{ \because \frac{\frac{9}{2}}{100} = \frac{\frac{9}{2}}{\frac{100}{1}} = \frac{9}{2} \times \frac{1}{100} = \frac{9}{200} \right\}$$

$$= \$\frac{9}{5}$$

$$= \$1.80$$

Example 7

If 5% of Australians play tennis, how many people would you expect to play tennis out of a group of 320 people?

Solution:

$$\text{Number of tennis players} = 5\% \text{ of } 320$$

$$= 5\% \times 320$$

$$= \frac{\overset{1}{\cancel{5}}}{\cancel{100}_{\cancel{10}_{1}}} \times \frac{\overset{16}{\cancel{32}0}}{1}$$

$$= \frac{16}{1}$$

$$= 16$$

16 people would be expected to play tennis.

Expressing One Quantity as a Percentage of Another Quantity

To express one quantity as a percentage of another, make sure that both quantities are expressed in the same units. Write the given quantity as a fraction of the total and multiply it by 100%. Then simplify.

Example 8

I obtained 40 marks out of 60 in a test. Convert this test mark to a percentage. Round your answer to the nearest per cent.

Solution:

$$\text{Percentage mark} = \frac{40}{60} \times 100\%$$
$$= \frac{\overset{2}{\cancel{40}}}{\underset{3}{\cancel{60}}} \times \frac{100}{1}\%$$
$$= \frac{200}{3}\%$$
$$= 66\frac{2}{3}\%$$
$$= 67\% \qquad \text{(Rounded to the nearest per cent)}$$

So, the percentage test mark is 67%.

Note:

Write 40 as a fraction of 60. Then multiply it by 100%.

Example 9

What percentage of \$5 is 42¢?

Solution:

$$\$1 = 100¢$$
$$\therefore \$5 = 5 \times 100¢ = 500¢$$

$$\therefore \text{Required percentage} = \frac{42}{\underset{5}{\cancel{500}}} \times \frac{\overset{1}{\cancel{100}}}{1}\%$$
$$= \frac{42}{5}\%$$
$$= 8.4\%$$

Note:

To express one quantity as a percentage of another, we must have both quantities in the same units.

Application of Percentages

Percentages are often used for calculations involving money in daily life. The prices of most quantities increase (or decrease) by a percentage over time. The changes in prices are often reported in newspapers as a percentage increase (or decrease). Profit, loss, discount, commission and rates of interest can all be expressed as percentages.

Percentage Increase

Example 10

The price of a calculator increases from \$15 to \$18. Express this increase as a percentage of the old price.

Solution:

$$\begin{aligned}\text{Increase} &= \text{New Price} - \text{Old price}\\ &= \$18 - \$15\\ &= \$3\end{aligned}$$

$$\begin{aligned}\therefore \%\ \text{Increase} &= \frac{\text{Increase}}{\text{Old Price}} \times 100\%\\ &= \frac{3}{15} \times 100\%\\ &= \frac{\overset{1}{\cancel{3}}}{\underset{\cancel{5}}{\cancel{15}}_{1}} \times \frac{\overset{20}{\cancel{100}}}{1}\%\\ &= \frac{20}{1}\%\\ &= 20\%\end{aligned}$$

Percentage Decrease

Example 11

The price of a book decreases from \$25 to \$21. Express the decrease as a percentage of the old price.

Solution:

$$\begin{aligned}\text{Decrease} &= \text{Old price} - \text{New Price}\\ &= \$25 - \$21\\ &= \$4\end{aligned}$$

$$\therefore \%\ \text{Decrease} = \frac{\text{Decrease}}{\text{Old Price}} \times 100\%$$
$$= \frac{4}{25} \times 100\%$$
$$= \frac{4}{\cancel{25}_{1}} \times \frac{\overset{4}{\cancel{100}}}{1}\%$$
$$= \frac{16}{1}\%$$
$$= 16\%$$

Discount

The **marked price** of an article that is offered for sale is the amount written on the price tag.

Shopkeepers sometimes allow the customers to have the article for less than the marked price. For example, if the marked price is \$40, the shopkeeper may sell the article for \$30. The \$10 that has been deducted from the marked price is called a **discount**.

Thus, Selling Price = Marked Price − Discount on Marked Price

In symbolic form,

$SP = MP - \text{Discount}$

Discounts are often expressed as percentages, and then always as a percentage of the marked price.

So, if an article marked at \$40 is sold for \$30, then:

$$\text{Discount} = \text{Marked price} - \text{Selling price}$$
$$= \$40 - \$30$$
$$= \$10$$

$$\therefore \%\ \text{Discount} = \frac{\text{Discount}}{MP} \times 100\%$$
$$= \frac{\overset{1}{\cancel{10}}}{\underset{\cancel{4}}{\cancel{40}}_{1}} \times \frac{\overset{25}{\cancel{100}}}{1}\%$$
$$= \frac{25}{1}\%$$
$$= 25\%$$

We say that the article has been discounted by twenty-five per cent.

Example 14

A jeweller discounts a watch marked at $120 by 25%.
a. How much is the discount?
b. What will a customer pay for the watch?

Solution:

a. Marked price = $120

Discount = 25% of MP

$$= 25\% \times 120$$

$$= \frac{\cancel{25}^{1}}{\cancel{100}_{\cancel{4}_{1}}} \times \frac{\cancel{120}^{30}}{1}$$

$$= \frac{30}{1}$$

$$= 30$$

So, the discount is $30.

b. Selling price = Marked price − Discount on MP

$$= 120 - 30$$

$$= 90$$

So, the customer pays $90.

Key Terms

discount, marked pric

Basic "Percent of" Word Problems *(page 1 of 3)*

Sections: Basic percentage exercises, Markup / markdown, General increase / decrease

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When you learned how to translate simple English statements into mathematical expressions, you learned that "of" can indicate "times". This frequently comes up when using percentages.

If you need to find 16% of 1400, you first convert the percentage "16%" to its decimal form; namely, the number "0.16". (When you are doing actual math, you need to use actual numbers. Always convert the percentages to decimals!) Then, since "sixteen percent OF fourteen hundred" tells you to multiply the 0.16 and the 1400, you get: (0.16)(1400) = 224. This says that 224 is sixteen percent of 1400.

Percentage problems usually work off of some version of the sentence "(this) is (some percentage) of (that)", which translates to "(this) = (some decimal) × (that)". You will be given two of the values, or at least enough information that you can figure two of them out. Then you'll need to pick a variable for the value you don't have, write an equation, and solve for that variable.

- **What percent of 20 is 30?**

We have the original number (20) and the comparative number (30). The unknown in this problem is the rate or percentage. Since the statement is "(thirty) is (some percentage) of (twenty)", then the variable stands for the percentage, and the equation is:

$$30 = (x)(20)$$

$$30 \div 20 = x = 1.5$$

Since x stands for a percentage, I need to remember to convert this decimal back into a percentage:

$$1.5 = 150\%$$

Thirty is 150% of 20.

- **What is 35% of 80?**

  Here we have the rate (35%) and the original number (80); the unknown is the comparative number which constitutes 35% of 80. Since the exercise statement is "(some number) is (thirty-five percent) of (eighty)", then the variable stands for a number and the equation is:

  $$x = (0.35)(80)$$

  $$x = 28$$

  Twenty-eight is 35% of 80.

- **45% of what is 9?**

  Here we have the rate (45%) and the comparative number (9); the unknown is the original number that 9 is 45% of. The statement is "(nine) is (forty-five percent) of (some number)", so the variable stands for a number, and the equation is:

  $$9 = (0.45)(x)$$

  $$9 \div 0.45 = x = 20$$

  Nine is 45% of 20.

The format displayed above, "(this number) is (some percent) of (that number)", *always* holds true for percents. In any given problem, you plug your known values into this equation, and then you solve for whatever is left.

- **Suppose you bought something that was priced at $\$6.95$, and the total bill was $\$7.61$. What is the sales tax rate in this city? (Round answer to one decimal place.)**

  The sales tax is a certain percentage of the price, so I first have to figure what the actual tax was. The tax was:

  $$7.61 - 6.95 = 0.66$$

  Then (the sales tax) is (some percentage) of (the price), or, in mathematical terms:

  $$0.66 = (x)(6.95)$$

  Solving for x, I get:

$$0.66 \div 6.95 = x = 0.094964028... = 9.4964028...\%$$

The sales tax rate is 9.5%.

In the above example, I first had to figure out what the actual tax was. Many percentage problems are really "two-part-ers" like this: they involve some kind of increase or decrease relative to some original value. Warning: Always figure the percentage of change relative to the *original* value.

- **Suppose a certain item used to sell for seventy-five cents a pound, you see that it's been marked up to eighty-one cents a pound. What is the percent increase?**

  First, I have to find the absolute increase:

  $$81 - 75 = 6$$

  The price has gone up six cents. Now I can find the percentage increase over the original price.

Note this language, "increase/decrease *over* the original", and use it to your advantage: it will remind you to put the increase or decrease *over* the original value, and then divide.

This percentage increase is the relative change:

$$^{6}/_{75} = 0.08$$

...or **an 8% increase in price per pound.**

An important category of percentage exercises is markup and markdown problems. For these, you calculate the markup or markdown in absolute terms (you find by how much the quantity changed), and then you calculate the percent change relative to the original value. So they're really just another form of "increase - decrease" exercises.

- **A computer software retailer used a markup rate of 40%. Find the selling price of a computer game that cost the retailer $\$25$.**

  The markup is 40% of the $\$25$ cost, so the markup is:

  $$(0.40)(25) = 10$$

  Then the selling price, being the cost plus markup, is:

  $$25 + 10 = 35$$

  The item sold for $\$35$.

- **A golf shop pays its wholesaler $\$40$ for a certain club, and then sells it to a golfer for $\$75$. What is the markup rate?**

  First, I'll calculate the markup in absolute terms:

  $$75 - 40 = 35$$

  Then I'll find the relative markup over the original price, or the markup rate: ($\$35$) is (some percent) of ($\$40$), or:

  $$35 = (x)(40)$$

...so the relative markup over the original price is:

$$35 \div 40 = x = 0.875$$

Since x stands for a percentage, I need to remember to convert this decimal value to the corresponding percentage.

The markup rate is 87.5%.

- **A shoe store uses a 40% markup on cost. Find the cost of a pair of shoes that sells for $\$63$.**

This problem is somewhat backwards. They gave me the selling price, which is cost plus markup, and they gave me the markup rate, but they didn't tell me the actual cost or markup. So I have to be clever to solve this.

I will let "x" be the cost. Then the markup, being 40% of the cost, is $0.40x$. And the selling price of $\$63$ is the sum of the cost and markup, so:

$$63 = x + 0.40x$$
$$63 = 1x + 0.40x$$
$$63 = 1.40x$$
$$63 \div 1.40 = x = 45$$

The shoes cost the store $\$45$.

- **An item originally priced at $\$55$ is marked 25% off. What is the sale price?**

First, I'll find the markdown. The markdown is 25% of the original price of $\$55$, so:

$$x = (0.25)(55) = 13.75$$

By subtracting this markdown from the original price, I can find the sale price:

$$55 - 13.75 = 41.25$$

The sale price is $\$41.25$.

- **An item that regularly sells for $\$425$ is marked down to $\$318.75$. What is the discount rate?**

First, I'll find the amount of the markdown:

$$425 - 318.75 = 106.25$$

Then I'll calculate "the markdown over the original price", or the markdown rate: ($\$106.25$) is (some percent) of ($\$425$), so:

$$106.25 = (x)(425)$$

...and the relative markdown over the original price is:

$$x = 106.25 \div 425 = 0.25$$

Since the "x" stands for a percentage, I need to remember to convert this decimal to percentage form.

The markdown rate is 25%.

- **An item is marked down 15%; the sale price is $127.46. What was the original price?**

  This problem is backwards. They gave me the sale price ($127.46) and the markdown rate (15%), but neither the markdown amount nor the original price. I will let "x" stand for the original price. Then the markdown, being 15% of this price, was $0.15x$. And the sale price is the original price, less the markdown, so I get:

  $$x - 0.15x = 127.46$$
  $$1x - 0.15x = 127.46$$
  $$0.85x = 127.46$$
  $$x = 127.46 \div 0.85 = 149.952941176...$$

  This problem didn't state how to round the final answer, but dollars-and-cents is always written with two decimal places, so:

  The original price was $149.95.

Note in this last problem that I ended up, in the third line of calculations, with an equation that said "eighty-five percent of the original price is $127.46". You can save yourself some time if you think of discounts in this way: if the price is 15% off, then you're only actually paying 85%. Similarly, if the price is 25% off, then you're paying 75%; if the price is 30% off, then you're paying 70%; and so on.

Note that, while the values below do not refer to money, the procedures used to solve these problems are otherwise identical to the markup - markdown examples on the previous page.

- **Growing up, you lived in a tiny country village. When you left for college, the population was 840. You recently heard that the population has grown by 5%. What is the present population?**

  First, I'll find the actual amount of the increase. Since the increase is five percent of the original, then the increase is:

  $$(0.05)(840) = 42$$

  The new population is the old population plus the increase, or:

  $$840 + 42 = 882$$

  The population is now 882.

- **Your friend diets and goes from 125 pounds to 110 pounds. What was her percentage weight loss?**

  First, I'll find the absolute weight loss:

  $$125 - 110 = 15$$

  This fifteen-pound decrease is some percentage of the original, since the rate of change is always with respect to the original value. So the percentage is "change over original", or:

  $$15 = (x)(125)$$

  $$15 \div 125 = x$$ (See? The change, 15, is over the original, 125.)

$$15 \div 125 = 0.12$$

The change is a percentage, so I need to convert this decimal to percentage form:

She lowered her weight by 12%.

- **Your boss says that his wife has put an 18×51 foot garden in along the whole back end of their back yard. He says that this has reduced the back-yard lawn area by 24%. What are the total dimensions of his back yard? What are the dimensions of the remaining lawn area?**

Since no suburban lot is going to be only eighteen feet wide (because then the house couldn't fit along the street frontage), the width of the lot must be the 51-foot dimension. Now I need to figure out the length of the back yard. The area of the garden is:

$$(18)(51) = 918$$

This represents 24% of the total yard area; that is, 24% of the original lawn area. This says that (918 square feet) is (twenty-four percent) of (the original), so:

$$918 = 0.24x$$

$$918 \div 0.24 = x = 3825$$

The total back yard area is 3825 square feet. Since the width is 51 feet, then:

$$3825 \div 51 = 75$$

The length then is 75 feet. Since 18 feet are taken up by the garden, then the lawn area is $75 - 18 = 57$ feet deep.

The back yard measures $51' \times 75'$ and the lawn measures $51' \times 57'$.