4. What Every Young Mathlete Should Know

I. VOCABULARY AND LANGUAGE

The following explains, defines, or lists some of the terms in **bold** that may be used in Olympiad problems. An answer is acceptable on an Olympiad contest only if it is consistent with both this section and the wording of the related problem.

1. Basic Terms

Sum, difference, product, quotient, remainder, ratio, square of a number (also, **perfect square**), **factors of a number.** The **value** of a number is the simplest name for that number. "**Or**" is inclusive: "*a* or *b*" means "*a* or *b* or both."

⟹ **DIVISION M: Square root** of a number, **cube** of a number (also, **perfect cube**).

2. Reading Sums

An ellipsis (…) means "and so on":

Read "1 + 2 + 3 + …" as "one plus two plus three and so on, without end."

Read "1 + 2 + 3 + … + 10" as "one plus two plus three and so on up to ten."

3. Writing Whole Numbers

The **standard form of a number** refers to the form in which we usually write numbers (also called Hindu-Arabic numerals or positional notation).

A **digit** is any one of the ten numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. All numerals are written by assigning place values to combinations of these digits. A number may be described by the number of digits it contains: 358 is a three-digit number. The "**lead-digit**" (leftmost digit) of a number is not counted as a digit if it is 0: i.e., 0358 is a three-digit number. **Terminal zeroes** of a number are the zeroes to the right of the last nonzero digit: i.e., 30,500 has two terminal zeroes because to the right of the digit 5 there are two zeroes.

4. Sets of Numbers

a. **Whole Numbers** = {0, 1, 2, 3, …}.

b. **Counting Numbers** = {1, 2, 3, …}.

Note: The term natural numbers will not be used in Olympiad contests because in some fields of mathematics, the definition of natural numbers is the same as counting numbers while in other fields it is the same as whole numbers.

c. ⟹ **DIVISION M: Integers** = {…, −3, −2, −1, 0, +1, +2, +3, …};

Positive numbers, negative numbers, nonnegative numbers, and nonpositive numbers.

7. Fractions

a. A **common** (or **simple**) **fraction** is a fraction (of the form $\frac{a}{b}$) where the **numerator** and **denominator** are whole numbers, except that the denominator cannot be 0. Its operational meaning is that the numerator is divided by the denominator.

b. A **unit fraction** is a common fraction with numerator 1.

c. A **proper fraction** is a common fraction in which the numerator is less than the denominator. Its value is more than 0 and less than 1.

d. An **improper fraction** is a common fraction in which the numerator is equal to or more than the denominator. Its value is 1 or greater than 1. A fraction whose denominator is 1 is equivalent to an integer.

e. A **complex fraction** is a fraction whose numerator or denominator contains a fraction.

Examples: $\dfrac{\frac{2}{3}}{5}$, $\dfrac{2}{\frac{3}{5}}$, $\dfrac{\frac{2}{3}}{\frac{5}{7}}$, $\dfrac{2+\frac{3}{5}}{5-\frac{1}{2}}$

Complex fractions are often simplified by using the operational meaning.

Example: $\dfrac{\frac{2}{3}}{5} = \frac{2}{3} \div 5 = \frac{2}{3} \times \frac{1}{5} = \frac{1}{15}$.

f. The fraction is **simplified** ("**in lowest terms**") if a and b have no common factor other than 1 [GCF$(a,b) = 1$].

g. A **decimal** or **decimal fraction** is a fraction whose denominator is a power of ten. The decimal is written using decimal point notation. *Examples*: $0.07 = \frac{7}{100}$, $0.153 = \frac{153}{1000}$, $6.4 = 6\frac{4}{10}$ or $\frac{64}{10}$

h. ⟹ DIVISION M: A **percent** or **percent fraction** is a fraction whose denominator is 100. The percent sign represents the division by 100. *Examples*: $9\% = \frac{9}{100}$, $125\% = \frac{125}{100}$, $0.3\% = \frac{0.3}{100}$ or $\frac{3}{1000}$

8. Statistics and Probability

The **average** (**arithmetic mean**) of a set of N numbers is the sum of all N numbers divided by N. The **mode** of a set of numbers is the number listed most often. A set with every number listed once is said to have no mode. The **median** of an ordered set of numbers is the middle number if N is odd or it is the mean of the two middle numbers if N is even.

The **probability of an event** is a value between 0 and 1 inclusive that expresses how likely an event is to occur. It is often found by dividing the number of times an event does occur by the total number of times the event can possibly occur. *Example*: The probability of rolling an odd number on a die is $\frac{3}{6}$ or $\frac{1}{2}$. Either $\frac{3}{6}$ or $\frac{1}{2}$ will be accepted as a correct probability on an Olympiad contest.

d. **Consecutive Numbers** are counting numbers that differ by 1, such as 83, 84, 85, 86, and 87. **Consecutive Even Numbers** are multiples of 2 that differ by 2, such as 36, 38, 40, and 42. **Consecutive Odd Numbers** are nonmultiples of 2 that differ by 2, such as 57, 59, 61, and 63.

5. Multiples, Divisibility, and Factors

The product of any two whole numbers is called a **multiple** of each of the whole numbers. Zero is considered a multiple of every whole number. *Examples*: The multiples of 6 = {0, 6, 12, 18, ...}. *Note: many but not all authorities expand the definition of multiples to include all integers. To them, –24 is a multiple of 6. For Olympiad problems, no multiples will be negative.*

A whole number, ***a***, is said to be **divisible** by a natural number, ***b***, if the remainder is zero upon division. In this case: (1) their quotient is also a whole number, (2) b is called a factor of a, and (3) a is called a multiple of b.

6. Number Theory

a. A **prime number** (also, **prime**) is a counting number which has exactly ***two*** **different** factors, namely the number itself and the number 1. *Examples*: 2, 3, 5, 7, 11, 13, ...

b. A **composite number** is a counting number which has *at least three* different factors, namely number itself, the number 1, and at least one other factor. *Examples*: 4, 6, 8, 9, 10, 12, ...

c. The number 1 is neither prime nor composite since it has *exactly one* factor, namely the number 1. Thus, there are 3 separate categories of counting numbers: prime, composite, and the number 1.

d. A number is **factored completely** when it is expressed as a product of only prime numbers. *Example*: $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$. It may also be written as $144 = 2^4 \times 3^2$.

e. The **Greatest Common Factor (GCF)** of two counting numbers is the largest counting number that divides each of the two given numbers with zero remainder. *Example*: GCF (12,18) = 6.

f. If the GCF of two numbers is 1, then we say the numbers are **relatively prime** or **co-prime**.

g. The **Least Common Multiple (LCM)** of two counting numbers is the smallest number that each of the given numbers divides with zero remainder. *Example*: LCM (12,18) = 36.

h. **Order of Operations**. When computing the value of expressions involving two or more operations, the following priorities must be observed from left to right:

1) do operations in parentheses, braces, or brackets first, working from the inside out,
2) do multiplication and division from left to right, and then
3) do addition and subtraction from left to right.

Example:
$$\begin{aligned} & 3 + 4 \times 5 - 8 \div (9 - 7) \\ &= 3 + 4 \times 5 - 8 \div 2 \\ &= 3 + 20 - 4 \\ &= 19 \end{aligned}$$

9. Geometry

a. Angles: **degree-measure, vertex, congruent; acute, right, obtuse, straight, reflex.**

b. **Congruent segments** are two **line segments** of equal length.

c. **Polygons and circles:**

i. Parts: **side, angle, vertex, diagonal; interior region, exterior region; diameter, radius, chord.**

ii. **Triangles: acute, right, obtuse; scalene, isosceles, equilateral.**
Note: all equilateral triangles are isosceles, but only some isosceles triangles are equilateral.

iii. **Quadrilaterals: parallelogram, rectangle, square, trapezoid, rhombus.**
Note: a square is a rectangle with all sides congruent. It is also a rhombus with all angles congruent.

iv. Other **polygons: pentagon, hexagon, octagon, decagon, dodecagon, icosagon.**

v. **Perimeter**: the number of unit lengths in the boundary of a **plane figure.**

vi. **Area**: the number of **unit squares** (also, **one-unit squares**) contained in the interior of a region. A unit square is a square each of whose sides measures 1 unit. The area of each unit square is **one square unit**. *Example*: The area of a **5-cm square** is 25 sq cm.

vii. **Circumference**: the perimeter of a circular region.

viii. **Congruent figures**: two or more plane figures whose **corresponding** pairs of sides are congruent and whose **corresponding** pairs of angles are congruent.

ix. **Similar figures**: two or more plane figures whose size may be different but whose shape is the same. *Note: all squares are similar; all circles are similar.*

d. **Geometric Solids**:

i. **Cube, rectangular solid; face, edge.**
⇨ **DIVISION M: cylinder** (right circular only).

ii. ⇨ **DIVISION M: volume**: the number of **unit cubes** contained in the interior of a solid. A **one-unit cube** is a cube each of whose edges measures 1 unit. The volume of each 1-unit cube is **1 cubic unit**. *Example*: The volume of a **5-km cube** is 125 cu km.

iii. ⇨ **DIVISION M: surface area**: the sum of the areas of all the faces of a geometric solid.
Example: The surface area of a **5-cm cube** is 150 square cm.