

Chapter 2

Drawing a Picture or Diagram

2.1 Sample Problem & Solution

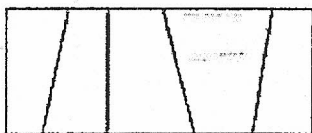
Four distinct straight lines are drawn across a rectangle thus subdividing the interior of the rectangle into regions which do not overlap. What is the greatest number of regions that the four lines can produce?

Draw a diagram of the rectangle and experiment with the placement of the four lines across the interior of the rectangle.

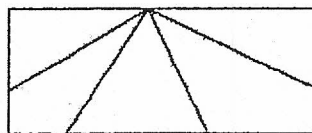
Solution: Draw four lines across the rectangle for the following cases:

1. The four lines do not intersect in an interior point,
2. The four lines intersect in one common point on a side of the rectangle, and
3. The four lines intersect in one common point in the interior of the rectangle

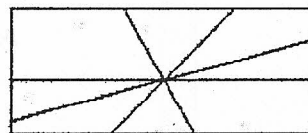
There are 5 regions in Case 1, 5 regions in Case 2, and 8 regions in Case 3.



Case 1

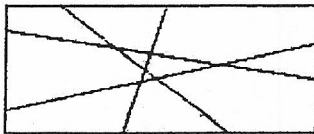


Case 2



Case 3

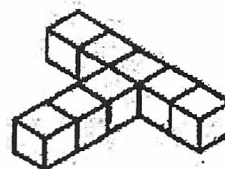
The greatest number of non-overlapping regions occurs when each of the four lines drawn across the rectangle intersects the other three lines in different points located in the interior of the rectangle.



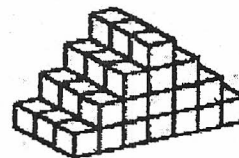
The greatest number of regions into which the interior of a rectangle can be subdivided by four straight lines is eleven.

6. If 24 gallons of water are poured into an empty tank, then $\frac{3}{4}$ of the tank is filled. How many gallons does a full tank hold?

7. Eight one-inch cubes are put together to form the T-figure shown. The complete outside of the T figure is painted red and then separated into one-inch cubes. How many of the cubes have exactly four red faces?

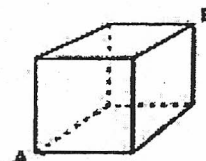


8. The set of stairs shown is constructed by placing layers of cubes on top of each other. What is the total number of cubes contained in the staircase?



9. A train traveling at 30 miles per hour reaches a tunnel which is 9 times as long as the train. If the train takes 2 minutes to completely clear the tunnel, how long is the train?
(1 mile = 5,280 feet)
10. Six people participated in a checker tournament. Each participant played exactly three games with each of the other participants. How many games were played in all?

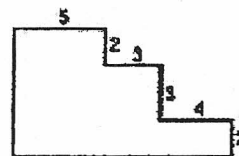
16. The length of the shortest trip from A to B along the edges of the cube shown is the length of 3 edges. How many different 3-edge trips are there from A to B?



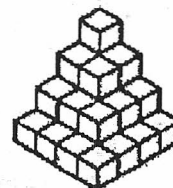
17. The figure shown is a "magic square" with missing entries. When complete, the sum of the four entries in each column, each row, and each diagonal is the same. Find the value of A and the value of B.

A		7	12
	4	9	
	5	16	
8	11		B

18. In the figure shown, each number represents the length of the segment which is nearest it. How many square units are in the area of the figure if there is a right angle at each corner of the figure?



19. The tower of unit cubes shown has no gaps. Suppose it is painted red on all exterior sides including the bottom, and then cut into cubes along the indicated lines. How many unit cubes will each have red paint on just three faces?



20. Consecutive numbers are whole numbers that follow in order such as 3, 4, 5. Find the smallest of the five consecutive numbers whose sum is 100.