Division **M**

Mathematical Olympiads

November 13, 2013

for Elementary & Middle Schools

Contest **1**

1A *Time: 3 minutes*

A circle and a square are drawn on the same flat surface. What is the greatest number of points in their intersection?

1B *Time: 4 minutes*

The letters in each word on the sign "MY CANDY" are cycled separately as shown.

The next correct spelling of both words appears on line *n*. What is the smallest value of *n*?

1. MY CANDY
2. YM ANDYC
3. MY NDYCA

⋮

n. MY CANDY

1C *Time: 6 minutes*

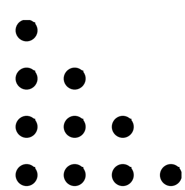
A palindrome is a whole number which reads the same forward as backwards. For example: 17571 is a palindrome, but 1234 is not a palindrome. Find the <u>least</u> sum of two palindromic years after the year 2013, such that this sum is <u>not</u> a palindrome.

1D *Time: 6 minutes*

The equation $8(Ax-7)-5(2x-1)=14x+B$ is true for every number that can replace the letter x. Find $A + B$.

1E *Time: 7 minutes*

In the figure shown, 10 points lie in four equally spaced columns, and four equally spaced rows. [Consecutive rows are 1 cm apart, and consecutive columns are 1 cm apart.] Find the probability, as a fraction in lowest terms, that three points selected at random lie in a straight line.



Please fold over on line. Write answers on back.

Division M — Mathematical Olympiads — Contest 1

November 13, 2013

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1A *Student Name and Answer*

1B *Student Name and Answer*

1C *Student Name and Answer*

1D *Student Name and Answer*

1E *Student Name and Answer*

Please fold over on line. Write answers in these boxes.

Division **M**

Mathematical Olympiads for Elementary & Middle Schools

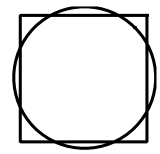
November 13, 2013

Contest **1**

SOLUTIONS AND ANSWERS

1A METHOD 1: *<u>Strategy</u>: Create a diagram.*

Count the intersections. **The diagram shows that the maximum number of intersections for a circle and a square is 8.**



METHOD 2: *<u>Strategy</u>: Consider intersections of a circle and a line.*

Each side of the square can intersect the circle in at most 2 points. Since there are 4 sides to the square the maximum number of intersections between the square and the circle is $2 \times 4 = 8$.

1B METHOD 1: *<u>Strategy</u>: Look for patterns.*

The word MY occurs in this form on every odd-numbered row. The word CANDY occurs in row 1 and then every 5 rows after that. Therefore it occurs in row 1, 6, 11, 16, and so on. The first time after row 1 that both occur in the correct order is row 11. **Thus $n = 11$.**

METHOD 2: *<u>Strategy</u>: Use the concept of LCM.*

MY has two letters and CANDY has 5 letters. The least common multiple (LCM) of 2 and 5 is 10. Therefore it will take 10 additional lines for the words to appear in the same order as the row 1. Therefore the row number when this occurs is $n = 11$.

> *FOLLOW-UP: The letters in the sign "FREE CHACHA LESSONS" are cycled separately within each word and placed in a numbered row as in the original problem. So row 2 reads "REEF HACHAC ESSONSL" and row 3 reads "EEFR ACHACH SSONSLE". What is the smallest row number after row 1 when the sign will once again correctly read "FREE CHACHA LESSONS"?* [LCM(4, 6, 7) + 1 = 85]

1C *<u>Strategy</u>: Observe the characteristics when adding two palindromic numbers together.*

The sum of two palindromic numbers is also palindromic when the sum of each pair of digits is less than 10. Therefore we are looking for two palindromic years after 2013 for which there will be a sum for two corresponding digits that is greater than 10. The next few palindromic years are: 2112, 2222, 2332, 2442, 2552, 2662, 2772, 2882, 2992, etc. Observe that 2112 + 2992 = 2222 + 2882 = 2332 + 2772 = 2442 + 2662 = 5104. **Hence, the least non-palindromic sum for two palindromic years after 2013 is 5104.**

> *FOLLOW-UP: What is the smallest sum, greater than 10,000, formed by adding a 3-digit palindromic number and a 4-digit palindromic number?* [10,001]

1A 8

1B 11

1C 5104

1D -48

1E $\frac{1}{8}$

1D *Strategy: Apply the distributive property and combine like terms.*

$8(Ax - 7) - 5(2x - 1) = 14x + B$

$8Ax - 56 - 10x + 5 = 14x + B$

$(8A - 10)x - 51 = 14x + B$

Since this last equation is true for all x, $8A - 10 = 14$ and $B = -51$. Solve for A by adding 10 to both sides of the equation and then divide both results by 14. This results in $8A = 24$ and $A = 3$.
Hence $A + B = 3 + (-51) = -48$.

FOLLOW-UP: The equation $2(x - A) + 3(x + B) = Cx$ is true for all values of x. If A, B, and C are positive integers, what is the least possible value for A + B + C? [10]

1E METHOD 1: *Strategy: Count the cases using columns, rows, and diagonals.*

Columns: The first column contains 4 dots and there are ${}_4C_3 = 4$ ways to choose three of the dots to form a line. In the second column there are only 3 dots so only one way to select them to form a line. In the third and fourth columns there are only 2 dots and 1 dot so no lines of 3 dots are possible. Thus there are 5 ways to form a 3 dotted line in the columns.
Rows: The exact same patterns exist in the 4 rows. The top 2 rows have no possible 3 dotted lines. The row with 3 dots has 1 and the row with 4 dots has 4 for a total of 5 more ways.
Diagonals: There are the same number of dots in the 4 diagonals as there were in the columns and the rows. Thus there are 5 more ways to connect 3 dots to form a line.
The total number of lines is 5 + 5 + 5 = 15. The total number of ways to choose 3 dots out of 10 dots is ${}_{10}C_3 = (10 \times 9 \times 8)/(3 \times 2 \times 1) = 120$ ways. **Therefore the probability that 3 chosen dots lie in a straight line is 15/120 = 1/8.**

METHOD 2: *Strategy: Replace the dots with letters and make a list of all possible 3-dot lines.*

A			
B	C		
D	E	F	
G	H	I	J

Use the diagram to verify this list of 15 sets of 3-dot lines:
Vertical Column: ABD, ABG, ADG, BDG, CEH
Horizontal Row: GHI, GHJ, GIJ, HIJ, DEF
Diagonal: ACF, ACJ, AFJ, CFJ, BEI
There are 120 groups of 3 dots calculated as follows:
Any one of the 10 can be the first dot, followed by any of 9 remaining dots, followed by one of the remaining 8 dots. Therefore $10 \times 9 \times 8 = 720$ might appear to be the correct number. However choosing dot A then B then C is the same as choosing B, A, and C. So we must divide 720 by 6, all the different arrangements of any 3 given dots. 720/6 = 120.
The probability of choosing 3 dots that form a line is 15/120 = 1/8.

FOLLOW-UP: Use the table provided to select a pair of numbers that are adjacent in any row, column or along any diagonal. What is the probability the sum of the two numbers is even? [18/42 = 3/7]

1	2	1	2
2	1	2	1
1	2	1	2
2	1	2	1

NOTE: Other FOLLOW-UP problems related to some of the above can be found in our two contest problem books and in "Creative Problem Solving in School Mathematics." Visit www.moems.org for details and to order.