Math 4 Homework: (due 12/18)pages 71 to 82 all

Ratios

Sections: Ratios, Proportions, Checking proportionality, Solving proportions

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Proportions are built from ratios. A "ratio" is just a comparison between two different things. For instance, someone can look at a group of people, count noses, and refer to the "ratio of men to women" in the group. Suppose there are thirty-five people, fifteen of whom are men. Then the ratio of men to women is 15 to 20.

Notice that, in the expression "the ratio of men to women", "men" came first. This order is very important, and must be respected: whichever word came first, its number must come first. If the expression had been "the ratio of women to men", then the numbers would have been "20 to 15".

Expressing the ratio of men to women as "15 to 20" is expressing the ratio in words. There are two other notations for this "15 to 20" ratio:

> odds notation: $15 : 20$
>
> fractional notation: ${}^{15}/_{20}$

You should be able to recognize all three notations; you will probably be expected to know them for your test.

Given a pair of numbers, you should be able to write down the ratios. For example:

- **There are 16 ducks and 9 geese in a certain park. Express the ratio of ducks to geese in all three formats.**

  $16 : 9,\ \frac{16}{9},\ 16 \text{ to } 9$

- **Consider the above park. Express the ratio of geese to ducks in all three formats.**

  $9 : 16,\ \frac{9}{16},\ 9 \text{ to } 16$

The numbers were the same in each of the above exercises, but the *order* in which they were listed differed, varying according to the order in which the elements of the ratio were expressed. In ratios, order is very important.

Let's return to the 15 men and 20 women in our original group. I had expressed the ratio as a fraction, namely, ${}^{15}/_{20}$. This fraction reduces to ${}^{3}/_{4}$. This means that you can also express the ratio of men to women as ${}^{3}/_{4}$, $3 : 4$, or "3 to 4".

This points out something important about ratios: the numbers used in the ratio might not be the *absolute* measured values. The ratio "15 to 20" refers to the *absolute* numbers of men and women, respectively, in the group of thirty-five people. The simplified or reduced ratio "3 to 4" tells you only that, for every three men, there are four women. The simplified ratio also tells you that, in any representative set of seven people ($3 + 4 = 7$) from this group, three will be men. In other words, the men comprise ${}^{3}/_{7}$ of the people in the group. These relationships and reasoning are what you use to solve many word problems:

- **In a certain class, the ratio of passing grades to failing grades is 7 to 5. How many of the 36 students failed the course?**

  The ratio, "7 to 5" (or $7 : 5$ or $^7/_5$), tells me that, of every $7 + 5 = 12$ students, five failed. That is, $^5/_{12}$ of the class flunked. Then ($^5/_{12}$)(36) = **15 students failed.**

- **In the park mentioned above, the ratio of ducks to geese is 16 to 9. How many of the 300 birds are geese?**

  The ratio tells me that, of every $16 + 9 = 25$ birds, 9 are geese. That is, $^9/_{25}$ of the birds are geese. Then there are ($^9/_{25}$)(300) = **108 geese.**

Generally, ratio problems will just be a matter of stating ratios or simplifying them. For instance:

- **Express the ratio in simplest form: $10 to $45**

  This exercise wants me to write the ratio as a *reduced* fraction:

  $$^{10}/_{45} = \mathbf{^2/_9}.$$

This reduced fraction is the ratio's expression in simplest fractional form. Note that the units (the "dollar" signs) "canceled" on the fraction, since the units, "$", were the same on both values. When both values in a ratio have the same unit, there should generally be no unit on the reduced form.

- **Express the ratio in simplest form: 240 miles to 8 gallons**

  When I simplify, I get (240 miles) / (8 gallons) = (30 miles) / (1 gallon), or, in more common language, **30 miles per gallon.**

In contrast to the answer to the previous exercise, this exercise's answer did need to have units on it, since the units on the two parts of the ratio, the "miles" and the "gallons", do not "cancel" with each other.

Conversion factors are simplified ratios, so they might be covered around the same time that you're studying ratios and proportions. For instance, suppose you are asked how many feet long an American football field is. You know that its length is 100 yards. You would then use the relationship of 3 feet to 1 yard, and multiply by 3 to get 300 feet. For more on this topic, look at the "Cancelling / Converting Units" lesson.

Ratios are the comparison of one thing to another (miles to gallons, feet to yards, ducks to geese, et cetera). But their true usefulness comes in the setting up and solving of proportions....