5A ***Time: 3 minutes***

What is the greatest 4-digit number ABCD such that

$A = B \times C$ and $B = C \times D$?

(*The digits A, B, C, and D are not necessarily different.*)

5B ***Time: 4 minutes***

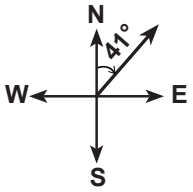
Find the mean of x and y, if $23 - x = y - 71$.

5C ***Time: 5 minutes***

If $2^{15} + 3^{10}$ is evaluated, what is the ones digit in the final result?

5D ***Time: 6 minutes***

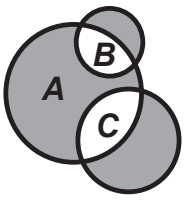
A sailboat changes course at a rate of 3° [3 degrees] per second. What is the least number of seconds the sailboat needs to change from a heading (direction) of 41° East of North (shown) to a new heading of 59° West of South?



5E ***Time: 7 minutes***

Three circles with areas 4π, 9π, and 16π intersect as shown. The largest circle is partitioned into shaded region *A* and unshaded regions *B* and *C*.

If the total area of the shaded regions is 17π, find the area of region *A* in terms of π.



Please fold over on line. Write answers on back.

Division **M**

Mathematical Olympiads for Elementary and Middle Schools

MATH OLYMPIAD

March 7, 2012

Contest **5**

5A *Student Name and Answer*

5B *Student Name and Answer*

5C *Student Name and Answer*

5D *Student Name and Answer*

seconds

5E *Student Name and Answer*

Please fold over on line. Write answers in these boxes.

Division M — Mathematical Olympiads for Elementary and Middle Schools — March 7, 2012 — Contest 5

SOLUTIONS AND ANSWERS

5A *Strategy: Maximize the digits, working from the left.*
The largest possible value for A is 9. Since A = B × C, B and C could be 3 and 3, or 9 and 1. Of these, the largest possible value of B is 9. Then C = 1. Since B = C × D and B = 9 and C = 1, then D must be 9. **The greatest 4-digit number ABCD is 9919.**

Follow-Up: *Find the largest four digit number, ABCD, so that $A \times B = C$ and $A \times C = D$.* [3139]

5A 9919

5B **METHOD 1:** *Strategy: Use algebra and the definition of mean.*
Rewrite $23 - x = y - 71$ as $x + y = 23 + 71 = 94$.
The mean of x and y is $\frac{1}{2}(x + y) = \frac{1}{2}(94) =$ **47.**

METHOD 2: *Strategy: Assign values to x .*
The wording of the question implies that there is a single answer no matter what value is assigned for x. Therefore assign any value to find the mean: suppose $x = 1$. Then $y = 93$ and the mean is $\frac{1}{2}(1 + 93) = 47$. To check, assign a very different value: If $x = 80$, then $y = 14$, and again the mean is 47. (In fact, the mean would be 47 for any value of x.)

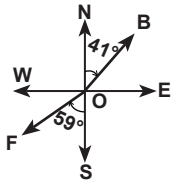
5B 47

5C *Strategy: Find a pattern in the successive powers of 2 and of 3.*
$2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$, and so on. The ones digits repeat in the pattern 2, 4, 8, 6, 2, 4, … The ones digit in 2^{16} is the same as the ones digit in 2^4, i. e. 6, and the ones digit in 2^{15} is the same as the ones digit in the term before 2^4, i.e. $2^3 = 8$.
Repeat the process on powers of 3. The successive ones digits are 3, 9, 7, 1, 3, 9, … The ones digit in 3^{10} is the same as the ones digit in $3^2 = 9$. **The ones digit in $2^{15} + 3^{10}$ is** the same as the ones digit in 8 + 9, which is **7.**

Follow-Ups: *(1) What is the ones digit in the product of* $2^{2012} \times 3^{2013} \times 5^{2014}$? [0]
(2) How many consecutive zeros appear at the end of the product? [2012]

5C 7

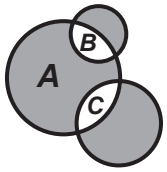
5D *Strategy: Draw a picture.*
$m\angle BOE = 90 - 41 = 49$ and $m\angle WOF = 90 - 59 = 31$. To begin at a heading of B and finish at a heading of F, the boat must turn either $41 + 90 + 31 = 162°$ counter-clockwise, or $49 + 90 + 59 = 198°$ clockwise. The lesser angle requires less time, and at 3° per second, **the least time required is** $162 \div 3 =$ **54 seconds.**



5D 54

5E 10π

5E *Strategy: Find the total of the unshaded areas.*



The sum of the areas of the 3 circles, $4\pi + 9\pi + 16\pi = 29\pi$, includes each interior region in the picture, but it includes regions *B* and *C* twice, since each is part of two circles. The sum of the areas of *B* and *C* is then $(29\pi - 17\pi) \div 2 = 6\pi$. The sum of the areas of *A*, *B*, and *C* is 16π, so **the area of region *A* is** $16\pi - 6\pi =$ **10π.**

NOTE: *Other FOLLOW-UP problems related to some of the above can be found in our books "Math Olympiad Contest Problems for Elementary and Middle Schools" and "Creative Problem Solving in School Mathematics."*