Basic Concepts for Young Mathletes

0.1 Reading Numbers, Sequences, and Series

Numbers: Use "and" before a fraction or decimal.
Read $305\frac{1}{4}$ as "three hundred five and one-fourth".
Read 10001.2 as "one thousand one and two-tenths.".

Sequences and Series: Use three dots $(\ldots)$ to mean "and so forth."
Read the sequence $2, 4, 6, \ldots$ as "two, four, six, and so forth."
Read the series $1 + 2 + 3 + \ldots$ as "one plus two plus three, and so forth."
Read the series $1 + 2 + 3 + \ldots + 10$ as "one plus two plus three, and so forth, up to ten."

0.2 Digits

A *digit* is any one of the ten numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The number 358 is a three-digit number; its *lead-digit* is 3. The lead digit of a counting number may not be 0; 0358 is considered to be a three-digit number.

0.3 Expanded Form of a Number

A counting number may be written in expanded form as follows:

$$358 = \begin{cases} 300 + 50 + 8 \\ 3 \cdot 100 + 5 \cdot 10 + 8 \cdot 1 \\ 3 \cdot 10^2 + 5 \cdot 10 + 8 \cdot 1 \end{cases}$$

0.4 Sets of Numbers

$\textit{Whole Numbers} = 0, 1, 2, 3, \ldots$

$\textit{Natural or Counting Numbers} = 1, 2, 3, \ldots$

Integers $= \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

Rational Numbers are numbers which can be written in the form $\frac{A}{B}$ where A and B are integers and B does not equal zero.

EXAMPLES: $\frac{7}{15}, 2\frac{3}{4} = \frac{11}{4}, .9 = \frac{9}{10}$ Note that all whole numbers are rational. For example, 3 is a rational number because $3 = \frac{3}{1}$ or $\frac{6}{2}$ and so forth; 0 is rational because $0 = \frac{0}{1}$ or $\frac{0}{2}$ and so forth.

0.5 Fractions

A *common* or *simple fraction* is a fraction in the form $\frac{A}{B}$ where A and B are whole numbers and B is not equal to zero. Note that all common fractions are rational numbers.

A *unit fraction* is a common fraction with numerator 1.

A *proper fraction* is a common fraction in which A is less than B.
EXAMPLES: $\frac{2}{5}$ and $\frac{1}{3}$

An *improper fraction* is a common fraction in which A is greater than or equal to B.
EXAMPLES: $\frac{5}{2}, \frac{3}{3}, \frac{7}{1}$

A *complex fraction* is a fraction whose numerator or denominator (or both) contains a fraction.
EXAMPLES:

$$\frac{\frac{2}{3}}{5}, \frac{5}{\frac{3}{4}}, \frac{\frac{3}{8}}{\frac{4}{5}}, \frac{3+\frac{1}{2}}{2+\frac{1}{3}}$$

The fraction $\frac{A}{B}$ is *simplified* or *reduced to lowest terms* if A and B have no common factor other than 1.

0.6 Order of Operations

When computing the value of an expression involving two or more operations, the following priorities must be observed in the order listed:

1. Do computations in parentheses, braces, and brackets, THEN
2. Do multiplications and divisions in order from left to right, THEN
3. Do additions and subtractions in order from left to right.

EXAMPLES:

1. $3 + 4 \cdot (8 - 6) \div 2 = 3 + 4 \cdot 2 \div 2 = 3 + 4 = 7$

2. $3 + 4 \cdot 8 - 6 \div 2 = 3 + 32 - 3 = 32$

3. $3 + (4 \cdot 8 - 6) \div 2 = 3 + (32 - 6) \div 2 = 3 + 26 \div 2 = 3 + 13 = 16$

0.7 Average

The *average* of N numbers is the sum of the N numbers divided by N.
EXAMPLE: The average of 3, 4, 8, and 9 is $(3 + 4 + 8 + 9) \div 4 = 24 \div 4 = 6$.

0.8 Basic Definitions for Number Theory

Let A and B be natural numbers. We say that *A divides B exactly* if the remainder is zero when the division $B \div A$ is performed.

A *prime* is a natural number greater than 1 that is divisible only by itself and by 1.
EXAMPLES: The first eight prime numbers are 2, 3, 4, 7, 11, 13, 17 and 19. Some larger primes are 43, 101, 20147, and 20149. Notice that 2 is the only even prime number, and that 2 and 3 are the only two consecutive numbers that are each prime.

A *composite* is a natural number that is the product of 2 or more primes.
EXAMPLES: $18 = 2 \cdot 3 \cdot 3$, $35 = 7 \cdot 5$, and $91 = 7 \cdot 13$. The first eight composite numbers are 4, 6, 8, 9, 10, 12, 14 and 15. All natural numbers which are not prime and not 1 are composites.

The number 1 is called a *unit*. It is neither prime nor composite, and is a factor of all natural numbers.

A natural number is said to be *factored completely* when it is expressed as a product of prime numbers.
EXAMPLES: Since $72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$, then $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$ represents the *complete factorization* or *prime factorization* of 72. This can also be written as $2^3 \cdot 3^2$.

The *Greatest Common Factor (GCF)* of two natural numbers A and B is the largest natural number that divides both A and B exactly.
EXAMPLE: GCF(12, 18) = 6.

Two natural numbers are *relatively prime* or *co-prime* if their GCF equals 1.

EXAMPLES: 8 and 15 are relatively prime because GCF(8, 15) = 1; 27 and 35 are relatively prime; 995 and 996 are relatively prime. Note that any two consecutive numbers are relatively prime.

The *Least Common Multiple (LCM)* of two natural numbers A and B is the smallest natural number that is divisible by both A and B. EXAMPLE: $\mathrm{LCM}(9, 12) = 36$.