



Division

M

Mathematical Olympiads

December 11, 2013

for Elementary & Middle Schools

Contest

2

2A Time: 4 minutes

Lynda can run a mile in 8 minutes, while Noelle can run a mile in 10 minutes. At these rates, how many minutes will it take Noelle to complete a race that takes Lynda 20 minutes to complete?

2B Time: 4 minutes

A rectangle is partitioned into four smaller rectangles of areas 6, 10, 21, and 35 square inches by drawing only two straight lines. Find the number of inches in the perimeter of the original rectangle.

2C Time: 5 minutes

Pencils cost 15 cents each and erasers cost 8 cents each. Stephanie buys some pencils and some erasers for a total cost of \$1.53. How many erasers did she purchase?

2D Time: 6 minutes

Using scientific notation, the difference $(2.3 \times 10^{12}) - (1.7 \times 10^{10}) = a \times 10^N$, where $1 \leq a < 10$ and N is an integer. Find the value of a .

2E Time: 7 minutes

The improper fraction $\frac{N}{6}$ is expressed as the sum of several distinct proper fractions each in lowest terms and each with denominator 6 or less. What is the greatest possible value of the whole number N ?

Please fold over on line. Write answers on back.

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2A

Student Name and Answer

2B

Student Name and Answer

2C

Student Name and Answer

2D

Student Name and Answer

2E

Student Name and Answer

Please fold over on line. Write answers in these boxes.



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2**SOLUTIONS AND ANSWERS**

2A

2A METHOD 1: *Strategy: Set up a proportion.*

If Noelle can run the race in n minutes then $20/8 = n/10$. Multiply both sides of the equation by 10 to get $n = 200/8 = 25$. **It takes 25 minutes for Noelle to finish the race.**

METHOD 2: *Strategy: Simplify the given rates.*

Running one-half mile takes Lynda 4 minutes and Noelle 5 minutes. At the same rates, in 20 minutes, Lynda can run 5 half-miles. To run 5 half-miles, Noelle needs 25 minutes.

FOLLOW-UP: A rectangular solid is 8 cm by 10 cm by 12 cm. Another rectangular solid having the same volume is x cm by 15 cm by 16 cm. Which solid has the larger surface area and by how much? [$4 \times 15 \times 16$, 136 sq cm]

25

2B

2B *Strategy: Use the fact that areas that have common factors will share a side.*

Construct a rectangle and partition as given.

Areas of 6 and 10 have a common factor of 2.

Areas of 6 and 21 have a common factor of 3.

Areas of 21 and 35 have a common factor of 7.

Areas of 10 and 35 have a common factor of 5.

The perimeter of the rectangle is $2(8 + 9) = 34$

FOLLOW-UP: What is the difference between the largest and smallest possible areas of a rectangle with integral sides if the rectangle has a perimeter of 60 inches? [196 sq in]

	2	7
3	6	21
5	10	35

34

2C

2C METHOD 1: *Strategy: Use the fact that every multiple of 15 ends in 0 or 5.*

If Stephanie buys 1 eraser, she then would spend \$1.45 for pencils. However, \$1.45 is not a multiple of 3¢ and therefore not a multiple of 15¢. At 8¢ each for erasers, the cost of the pencils will always be odd, ending in 5. Buying 6 erasers leaves $\$1.53 - \$0.48 = \$1.05$ for pencils, which is a multiple of 15¢. **Stephanie purchased 6 erasers.**

METHOD 2: *Strategy: Use a table to find multiples of 15 and 8 whose sum is 153.*

Multiples of 15	15	30	45	60	75	90	105
153 minus top row	138	123	108	93	78	63	48
Is the middle row a multiple of 8?	N	N	N	N	N	N	Y

Stephanie purchased $48\text{¢} \div 8\text{¢} = 6$ erasers.

FOLLOW-UP: Harry purchased some 12¢ stamps and some 39¢ stamps for a total of \$3.69. What are all of his possible purchases? [(21@12¢ & 3@39¢) or (8@12¢ & 7@39¢)]

2D

2.283

2E

33

Olympiad 2, Continued

2D METHOD 1: Strategy: Factor out the GCF and simplify.

$$\begin{aligned}(2.3 \times 10^{12}) - (1.7 \times 10^{10}) &= 10^{10}(2.3 \times 10^2 - 1.7) \\ &= 10^{10}(230 - 1.7) \\ &= 10^{10}(228.3) \\ &= 10^{12}(2.283)\end{aligned}$$

Since $1 \leq a < 10$ and $a \times 10^N = 2.283 \times 10^{12}$, $a = 2.283$.

METHOD 2: Strategy: Do the indicated arithmetic.

$$2.3 \times 10^{12} = 2.3 \times 1,000,000,000,000 = 2,300,000,000,000$$

$$1.7 \times 10^{10} = 1.7 \times 10,000,000,000 = 17,000,000,000$$

Subtract to get $2,283,000,000,000 = 2.283 \times 10^{12}$, thus $a = 2.283$.

FOLLOW-UP: What is the value of $5^2 \times 5^6 \times 5^{10} \times 5^{14} \div 5^{16} \div 5^{18}$? [1/25]

2E Strategy: List pairs of fractions whose denominators are 2, 3, 4, 5, and 6.

Let N represent the sum of the numerators when the denominator is 6.

$$\begin{aligned}\frac{N}{6} &= \frac{1}{2} + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{3}{4}\right) + \left(\frac{1}{5} + \frac{4}{5}\right) + \left(\frac{2}{5} + \frac{3}{5}\right) + \left(\frac{1}{6} + \frac{5}{6}\right) \\ &= 5\frac{1}{2} = \frac{11}{2} = \frac{33}{6}\end{aligned}$$

Therefore the largest value of the numerator, N , is 33.

FOLLOW-UP: If $\frac{1}{x} + \frac{1}{y} = \frac{2}{5}$ and $\frac{1}{x} - \frac{1}{y} = \frac{1}{5}$ find the value of x divided by y . [1/3]

NOTE: Other FOLLOW-UP problems related to some of the above can be found in our two contest problem books and in "Creative Problem Solving in School Mathematics."
Visit www.moems.org for details and to order.