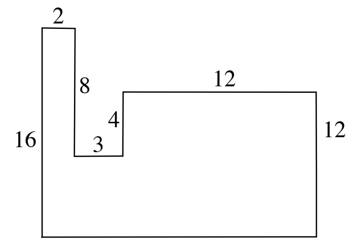
4A *Time: 4 minutes*

The accompanying figure is composed entirely of straight line segments meeting at right angles with selected lengths shown.

Find the perimeter of this figure.

Note: the figure is NOT drawn to scale.



4B *Time: 4 minutes*

Express as a fraction in lowest terms: $\dfrac{3-6+9-12+\ .....\ +27-30}{5-10+15-20+\ .....\ +45-50}$.

4C *Time: 6 minutes*

Each of the digits 3, 5, 6, and 9 is used exactly once to in the following expression: $\frac{\square}{\square}-\frac{\square}{\square}$. What is the numeric value of the least positive difference of these two fractions in lowest terms?

4D *Time: 6 minutes*

What is the probability that a randomly selected three-digit positive integer has no repeated digits? Express your answer in decimal form.

4E *Time: 7 minutes*

A wood cube is painted white and cut into smaller $1 \times 1 \times 1$ "cubies". There are forty-eight $1 \times 1 \times 1$ "cubies" with exactly two faces painted white. How many of the $1 \times 1 \times 1$ "cubies" have exactly one face painted white?

Please fold over on line. Write answers on back.

Division

M

Mathematical Olympiads

February 12, 2014

for Elementary & Middle Schools

Contest

4

4A *Student Name and Answer*

4B *Student Name and Answer*

4C *Student Name and Answer*

4D *Student Name and Answer*

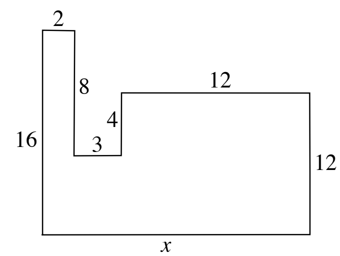
4E *Student Name and Answer*

Please fold over on line. Write answers in these boxes.

SOLUTIONS AND ANSWERS

4A *Strategy: Find the horizontal then vertical lengths.*

Label the missing length *x*. The top three horizontal lengths must be the same as the bottom length so $x = 2 + 3 + 12 = 17$.



The entire perimeter is 16 + 2 + 8 + 3 + 4 + 12 + 12 + 17 = 74.

4B METHOD 1: *Strategy: Factor and cancel.*

$$\frac{3-6+9-12+\ .....\ +27-30}{5-10+15-20+\ .....\ +45-50}=\frac{3\times(1-2+3-4+\ .....\ +9-10)}{5\times(1-2+3-4+\ .....\ +9-10)}=\frac{3}{5}.$$

The fraction in lowest terms is 3/5.

METHOD 2: *Strategy: Add efficiently.*

Group the numbers in both numerator and denominator two at a time. The result is nine –3's in the numerator and nine –5's in the denominator or $\frac{9(-3)}{9(-5)}=\frac{3}{5}$.

4C *Strategy: Find the LCD of all possible fractions.*

Write out the $4 \times 3 = 12$ possible fractions and then rewrite them with the same denominator: $\frac{3}{5},\frac{3}{6},\frac{3}{9},\frac{5}{6},\frac{5}{9},\frac{6}{9},\frac{9}{6},\frac{9}{5},\frac{6}{5},\frac{9}{3},\frac{6}{3},\frac{5}{3}$. These are the same as $\frac{54}{90},\frac{45}{90},\frac{30}{90},\frac{75}{90},\frac{50}{90},\frac{60}{90},\frac{135}{90},\frac{162}{90},\frac{108}{90},\frac{270}{90},\frac{180}{90},\frac{150}{90}$. When arranged in order we can see that $\frac{45}{90}$ and $\frac{50}{90}$ are the closest. **Therefore the least positive difference in simplest form is** $\mathbf{\frac{5}{90}=\frac{1}{18}}$. [$\frac{54}{90}-\frac{50}{90}=\frac{3}{5}-\frac{5}{9}$ violates the problem constraints.]

FOLLOW-UP: Use each of the numbers 3, 5, 6, and 9 exactly once to form the difference of two fractions as in the original problem, but find the largest possible difference and the largest possible product. [8/3 and 6]

4A 74

4B $\frac{3}{5}$

4C $\frac{1}{18}$

4D .72

4E 96

4D **METHOD 1**: *Strategy: Use multiplication to generate outcomes.*

There are $9 \times 10 \times 10 = 900$ three-digit positive integers. There are 9 choices for the hundreds digit (cannot be 0), 9 choices for the tens digit (cannot be the same digit chosen for the hundreds place but it can be 0), and 8 remaining choices for the ones digit. There are $9 \times 9 \times 8 = 648$ non-repeating three-digit positive integers. **The probability of selecting a 3-digit number without repeats is** $\mathbf{\frac{\cancel{9} \times 9 \times 8}{\cancel{9} \times 10 \times 10} = \frac{72}{100} = .72}$.

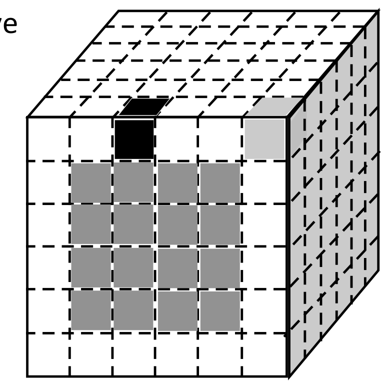
METHOD 2: *Strategy: Apply the fundamental counting principle.*

The hundreds-digit can be any digit except zero. The tens-digit can be any digit but it must be different from the hundreds-digit and the probability of that occurring is $\frac{9}{10}$. The probability that the units-digit is different from the previous two digits is $\frac{8}{10}$. The probability that all three are digits differ is $\frac{9}{10} \cdot \frac{8}{10} = \frac{72}{100} = .72$.

FOLLOW-UP: How many 5-digit codes can be created if the first digit must be a 3, the last digit must be prime and no digits can repeat? [504]

4E *Strategy: Use symmetry and then consider corner, edge, and face (middle) cubies.*

A $1 \times 1 \times 1$ "cubie" with exactly two faces painted white must have come from an edge of the original cube but not a corner (see the very dark colored cubie). Since a cube has 12 edges; $\frac{48}{12} = 4$ of the smaller $1 \times 1 \times 1$ "cubies" must lie on each edge. Then there are 2 more cubies at each corner, so the original cube is $6 \times 6 \times 6$. For each face of the large cube, there are exactly 16 cubies with one white face (colored gray in the diagram). **Since there are 6 faces, there are $6 \times 16 = 96$ cubies with one white face.**



NOTE: Other FOLLOW-UP problems related to some of the above can be found in our two contest problem books and in "Creative Problem Solving in School Mathematics." Visit www.moems.org for details and to order.