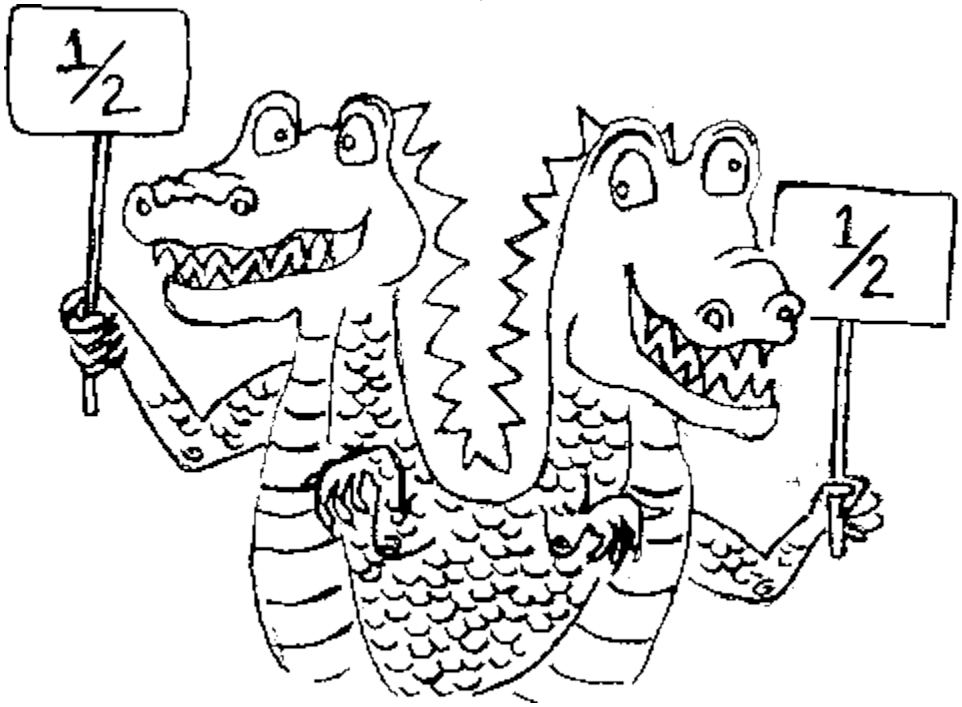
Oct 16 Homework: Pages 33 to 36, all. Read below for Fraction Notes:

What is a Fraction?

A fraction is a number that expresses part of a group.

Fractions are written in the form $\frac{a}{b}$ or *a*/*b*, where a and b are whole numbers, and the number b is not 0. For the purposes of these web pages, we will denote fractions using the notation *a*/*b*, though the preferred notation is generally $\frac{a}{b}$.

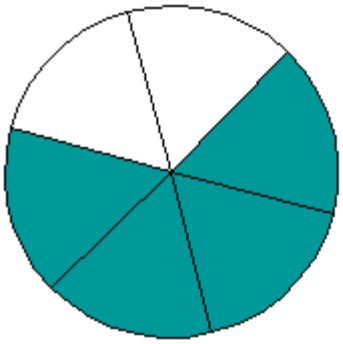
The number *a* is called the numerator, and the number *b* is called the denominator.



Examples:

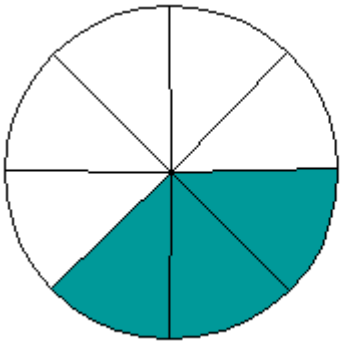
The following numbers are all fractions
1/2, 3/7, 6/10, 4/99

Example:
The fraction 4/6 represents the shaded portion of the circle below. There are 6 pieces in the group, and 4 of them are shaded.



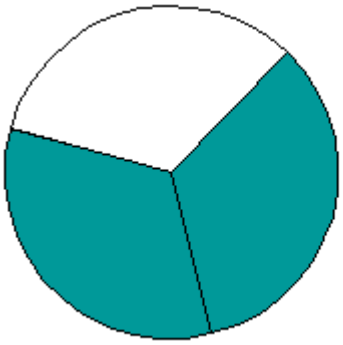
Example:

The fraction 3/8 represents the shaded portion of the circle below. There are 8 pieces in the group, and 3 of them are shaded.



Example:

The fraction 2/3 represents the shaded portion of the circle below. There are 3 pieces in the group, and 2 of them are shaded.



Equivalent Fractions

Equivalent fractions are different fractions which name the same amount.

Examples:

The fractions 1/2, 2/4, 3/6, 100/200, and 521/1042 are all equivalent fractions.
The fractions 3/7, 6/14, and 24/56 are all equivalent fractions.
We can test if two fractions are equivalent by cross-multiplying their numerators and denominators. This is also called taking the cross-product.

Example:

Test if 3/7 and 18/42 are equivalent fractions.
The first cross-product is the product of the first numerator and the second denominator: $3 \times 42 = 126$.
The second cross-product is the product of the second numerator and the first denominator: $18 \times 7 = 126$.
Since the cross-products are the same, the fractions are equivalent.

Example:

Test if 2/4 and 13/20 are equivalent fractions.
The first cross-product is the product of the first numerator and the second denominator: $2 \times 20 = 40$.
The second cross-product is the product of the second numerator and the first denominator: $4 \times 13 = 52$.
Since the cross-products are different, the fractions are not equivalent. Since the second cross-product is larger than the first, the second fraction is larger than the first.

Comparing Fractions

1. To compare fractions with the same denominator, look at their numerators. The larger fraction is the one with the larger numerator.
2. To compare fractions with different denominators, take the cross product. The first cross-product is the product of the first numerator and the second denominator. The second cross-product is the product of the second numerator and the first denominator. Compare the cross products using the following rules:
a. If the cross-products are equal, the fractions are equivalent.
b. If the first cross product is larger, the first fraction is larger.
c. If the second cross product is larger, the second fraction is larger.

Example:

Compare the fractions 3/7 and 1/2.
The first cross-product is the product of the first numerator and the second denominator: $3 \times 2 = 6$.
The second cross-product is the product of the second numerator and the first denominator: $7 \times 1 = 7$.
Since the second cross-product is larger, the second fraction is larger.

Example:

Compare the fractions 13/20 and 3/5.
The first cross-product is the product of the first numerator and the second denominator: $5 \times 13 = 65$.
The second cross-product is the product of the second numerator and the first denominator: $20 \times 3 = 60$.
Since the first cross-product is larger, the first fraction is larger.

Converting and Reducing Fractions

For any fraction, multiplying the numerator and denominator by the same nonzero number gives an equivalent fraction. We can convert one fraction to an equivalent fraction by using this method.

Examples:

$1/2 = (1 \times 3)/(2 \times 3) = 3/6$

$2/3 = (2 \times 2)/(3 \times 2) = 4/6$

$3/5 = (3 \times 4)/(5 \times 4) = 12/20$

Another method of converting one fraction to an equivalent fraction is by dividing the numerator and denominator by a common factor of the numerator and denominator.

Examples:

$20/42 = (20 \div 2)/(42 \div 2) = 10/21$

$36/72 = (36 \div 3)/(72 \div 3) = 12/24$

$9/27 = (9 \div 3)/(27 \div 3) = 3/9$

When we divide the numerator and denominator of a fraction by their greatest common factor, the resulting fraction is an equivalent fraction in lowest terms.

Lowest Terms

A fraction is in lowest terms when the greatest common factor of its numerator and denominator is 1. There are two methods of reducing a fraction to lowest terms.

Method 1:

Divide the numerator and denominator by their greatest common factor.

$12/30 = (12 \div 6)/(30 \div 6) = 2/5$

Method 2:

Divide the numerator and denominator by any common factor. Keep dividing until there are no more common factors.

$12/30 = (12 \div 2)/(30 \div 2) = 6/15 = (6 \div 3)/(15 \div 3) = 2/5$

Improper Fractions

Improper fractions have numerators that are larger than or equal to their denominators.

Examples:

11/4, 5/5, and 13/2 are improper fractions.

Mixed Numbers

Mixed numbers have a whole number part and a fraction part.

Examples:

$2\frac{3}{4}$ and $6\frac{1}{2}$ are mixed numbers also written as 2 3/4 and 6 1/2. In these web pages, we denote mixed numbers in the form *a b*/*c*.

Converting Mixed Numbers to Improper Fractions

To change a mixed number into an improper fraction, multiply the whole number by the denominator and add it to the numerator of the fractional part.

Examples:

2 3/4 = ((2 × 4) + 3)/4 =11/4

6 1/2 = ((6 × 2) + 1)/2 = 13/2

Converting Improper Fractions to Mixed Numbers

To change an improper fraction into a mixed number, divide the numerator by the denominator. The remainder is the numerator of the fractional part.

Examples:

11/4 = 11 ÷ 4 = 2 *r*3 = 2 3/4

13/2 = 13 ÷ 2 = 6 *r*1 = 6 1/2