21.4.3 In each part below, find the fraction that is equal to the given decimal.

(a) $0.\overline{4}$ (b) $0.\overline{273}$ (c) $0.63\overline{5}$ (d) $0.8\overline{81}$

21.4.4 Each term in the sequence $a_1 = 1, a_2 = 0.2, a_3 = 0.04, a_4 = 0.008, \ldots$ is obtained by doubling the previous term and then shifting the decimal point one place to the left. What is the sum of all the terms in the sequence? *(Source: Mandelbrot)*

21.4.5★ Find all values of x that satisfy $x = 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots$. *(Source: HMMT)*

22.1.1 Suppose $a + \frac{1}{a} = 3$.

(a) Find $a^2 + \frac{1}{a^2}$. (b) Find $a^4 + \frac{1}{a^4}$.

22.1.2 I'm thinking of two numbers. The sum of my numbers is 14 and the product of my numbers is 46. What is the sum of the squares of my numbers?

22.1.3 Simplify $\sqrt{7 - \sqrt{13}} - \sqrt{7 + \sqrt{13}}$.

Problem 22.7: Orion, Amadea, and Atlas are each thinking of a positive number. The product of Orion's number and Amadea's number is 27. The product of Amadea's number and Atlas's number is 72. The product of Orion's number and Atlas's number is 6. Find the number each person is thinking of.

22.2.3 Solve the equation $x + \cfrac{1}{x + \cfrac{1}{x + \cdots}} = 2x$.

22.2.4★ Evaluate $1 + \cfrac{1}{1 + \cfrac{2}{1 + \cfrac{2}{1 + \cfrac{2}{1 + \cfrac{2}{1 + \cdots}}}}}$. **Hints:** 128

22.2.1 Evaluate $\sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \cdots}}}}$.

22.2.2 Evaluate $3 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \cdots}}}}}$.

22.8 If $\sqrt{r} + \dfrac{2}{\sqrt{r}} = 6$, what is $r + \dfrac{4}{r}$?

22.9 Find the value of $\sqrt{12 - \sqrt{12 - \sqrt{12 - \cdots}}}$.

22.10 Evaluate $3 + \cfrac{10}{3 + \cfrac{10}{3 + \cfrac{10}{3 + \cfrac{10}{3 + \cdots}}}}$.

22.11 Find p, q, r, and s if

$$\begin{aligned} p + q + r - s &= 32, \\ p + q - r + s &= 13, \\ p - q + r + s &= -14, \\ -p + q + r + s &= 21. \end{aligned}$$

22.12 Simplify $\sqrt{14 - 5\sqrt{3}} + \sqrt{14 + 5\sqrt{3}}$.