

## Math V (5<sup>th</sup> Grade) Homework due 10/9:

From Singapore Math Textbook, pages 12-14, all

Answers to Chapter 1 will be posted next week.

Please review the following useful information on variables/expressions/equations/solution/simplification...

### **Variables**

A variable is a symbol that represents a number. Usually we use letters such as  $n$ ,  $t$ , or  $x$  for variables. For example, we might say that  $s$  stands for the side-length of a square. We now treat  $s$  as if it were a number we could use. The perimeter of the square is given by  $4 \times s$ . The area of the square is given by  $s \times s$ . When working with variables, it can be helpful to use a letter that will remind you of what the variable stands for: let  $n$  be the number of people in a movie theater; let  $t$  be the time it takes to travel somewhere; let  $d$  be the distance from my house to the park.

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### **Expressions**

An expression is a mathematical term or a sum or difference of mathematical terms that may use numbers, variables, or both.

#### **Example:**

The following are examples of expressions:

$$2$$

$$x$$

$$3 + 7$$

$$2 \times y + 5$$

$$2 + 6 \times (4 - 2)$$

$$z + 3 \times (8 - z)$$

#### **Example:**

Roland weighs 70 kilograms, and Mark weighs  $k$  kilograms. Write an expression for their combined weight. The combined weight in kilograms of these two people is the sum of their weights, which is  $70 + k$ .

#### **Example:**

A car travels down the freeway at 55 kilometers per hour. Write an expression for the distance the car will have traveled after  $h$  hours. Distance equals rate times time, so the distance traveled is equal to  $55 \times h$ .

#### **Example:**

There are 2000 liters of water in a swimming pool. Water is filling the pool at the rate of 100 liters per minute. Write an expression for the amount of water, in liters, in the swimming pool after  $m$  minutes. The amount of water added to the pool after  $m$  minutes will be 100 liters per minute times  $m$ , or  $100 \times m$ . Since we started with 2000 liters of water in the pool, we add this to the amount of water added to the pool to get the expression  $100 \times m + 2000$ .

To evaluate an expression at some number means we replace a variable in an expression with the number, and simplify the expression.

**Example:**

Evaluate the expression  $4 \times z + 12$  when  $z = 15$ .

We replace each occurrence of  $z$  with the number 15, and simplify using the usual rules: parentheses first, then exponents, multiplication and division, then addition and subtraction.

$4 \times z + 12$  becomes

$$4 \times 15 + 12 =$$

$$60 + 12 =$$

$$72$$

**Example:**

Evaluate the expression  $(1 + z) \times 2 + 12 \div 3 - z$  when  $z = 4$ .

We replace each occurrence of  $z$  with the number 4, and simplify using the usual rules: parentheses first, then exponents, multiplication and division, then addition and subtraction.

$(1 + z) \times 2 + 12 \div 3 - z$  becomes

$$(1 + 4) \times 2 + 12 \div 3 - 4 =$$

$$5 \times 2 + 12 \div 3 - 4 =$$

$$10 + 4 - 4 =$$

$$10.$$

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## Equations

An equation is a statement that two numbers or expressions are equal. Equations are useful for relating variables and numbers. Many word problems can easily be written down as equations with a little practice. Many simple rules exist for simplifying equations.

**Example:**

The following are examples of equations:

$$2 = 2$$

$$17 = 2 + 15$$

$$x = 7$$

$$7 = x$$

$$t + 3 = 8$$

$$3 \times n + 12 = 100$$

$$w + 4 = 12 - w$$

$$y - 1 - 2 - 9.3 = 34$$

$$3 \times (d + 4) - 11 = 321 - 2^3$$

Example:

Translate the following word problem into an equation:

My age in years  $y$  plus 20 is equal to four times my age, minus 10.

The first expression stands for "my age in years plus 20", which is  $y + 20$ .

This is equal to the second expression for "four times my age, minus 10", which is  $4 \times y - 10$ .

Setting these two expressions equal to one another gives us the equation:

$$y + 20 = 4 \times y - 10$$

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## Solution of an Equation

When an equation has a variable, the solution to the equation is the number that makes the equation true when we replace the variable with its value.

Example:

We say  $y = 3$  is a solution to the equation  $4 \times y + 7 = 19$ , for replacing each occurrence of  $y$  with 3 gives us

$$4 \times 3 + 7 = 19 \implies$$

$$12 + 7 = 19 \implies$$

$$19 = 19 \text{ which is true.}$$

### Examples:

$x = 100$  is a solution to the equation  $x \div 2 - 40 = 10$

$z = 12$  is a solution to the equation  $5 \times (z - 6) = 30$

### Counterexample:

$y = 10$  is NOT a solution to the equation  $4 \times y + 7 = 19$ . When we replace each  $y$  with 10, we get

$$4 \times 10 + 7 = 19 \implies$$

$$40 + 7 = 19 \implies$$

$$47 = 19 \text{ not true!}$$

### Counterexamples:

$x = 200$  is NOT a solution to the equation  $x \div 2 - 40 = 10$

$z = 20$  is NOT a solution to the equation  $5 \times (z - 6) = 30$

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## Simplifying Equations

To find a solution for an equation, we can use the basic rules of simplifying equations. These are as follows:

- 1) You may evaluate any parentheses, exponents, multiplications, divisions, additions, and subtractions in the usual order of operations. When evaluating expressions, be careful to use the associative and distributive properties properly.
- 2) You may combine like terms. This means adding or subtracting variables of the same kind. The expression  $2x + 4x$  simplifies to  $6x$ . The expression  $13 - 7 + 3$  simplifies to 9.
- 3) You may add any value to both sides of the equation.
- 4) You may subtract any value from both sides of the equation. This is best done by adding a negative value to each side of the equation.
- 5) You may multiply both sides of the equation by any number except 0.
- 6) You may divide both sides of the equation by any number except 0.

Hint: Since subtracting any number is the same as adding its negative, it can be helpful to replace subtractions with additions of a negative number.

### Example:

This problem illustrates grouping like terms and dealing with subtraction in an equation.

Solve  $x - 12 + 20 = 37$ .

Replacing the  $-12$  with a  $+(-12)$ , we get

$$x + (-12) + 20 = 37.$$

Since addition is associative, the two like terms (the integers) may be combined.

$$(12) + 20 = 8$$

The left side of the equation becomes

$$x + 8 = 37.$$

Now we may subtract 8 from each side of the equation, (we will actually add a  $-8$  to each side).

$$x + 8 + (-8) = 37 + (-8)$$

$$x + 0 = 29$$

$$x = 29$$

We can check this solution in the original equation:

$$29 - 12 + 20 = 37$$

$$17 + 20 = 37$$

$37 = 37$  so our solution is correct.

### Example:

This problem illustrates the proper use of the distributive property.

$$\text{Solve } 2 \times (x + 1 + 4) = 20.$$

Grouping like terms in the parentheses, the left side of the equation becomes

$$2 \times (x + 1 + 4) \implies 2 \times (x + 5).$$

Using the distributive property,

$$2 \times (x + 5) \implies 2 \times x + 2 \times 5.$$

Carrying out multiplications,

$$2 \times x + 2 \times 5 \implies 2x + 10.$$

The equation now becomes

$$2x + 10 = 20.$$

Subtracting a 10 (adding a -10) to each side gives us

$$2x + 10 + (-10) = 20 + (-10) \implies$$

$$2x + (10 + (-10)) = 20 - 10 \implies$$

$$2x + 0 = 10 \implies$$

$$2x = 10.$$

Since the  $x$  is multiplied by 2, we divide both sides by 2 to solve for  $x$ :

$$2x = 10 \implies$$

$$2x \div 2 = 10 \div 2 \implies$$

$$(2x)/2 = 5 \implies$$

$$x = 5.$$

We can check this solution in the original equation:

$$2 \times (5 + 1 + 4) = 20 \implies$$

$$2 \times 10 = 20 \implies$$

$$20 = 20 \text{ so our solution is correct.}$$

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## Combining like terms

One of the most common ways to simplify an expression is to combine like terms. Numeric terms may be combined, and any terms with the same variable part may be combined.

### Example:

Consider the expression  $2 + 7x + 12 - 3x - 5$ . The numeric like terms are the numbers 2, 12, and 5. The variable like terms are  $7x$  and  $3x$ . Combining the numeric like terms, we have  $2 + 12 - 5 = 14 - 5 = 9$ . Combining the variable like terms, we have  $7x - 3x = 4x$ , so the expression  $2 + 7x + 12 - 3x - 5$  simplifies to  $9 + 4x$ .

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## Simplifying with addition and subtraction

We can use addition and subtraction to get all the terms with variables on one side of an equation, and all the numeric terms on the other.

The equations  $3x = 17$ ,  $21 = y$ , and  $z/12 = 24$  each have a variable term on one side of the  $=$  sign, and a number on the other.

The equations  $x + 3 = 12$ ,  $21 = 30 - y$ , and  $(z + 2) \times 4 = 10$  do not.

We usually do this after simplifying each side using the distributive rules, eliminating parentheses, and combining like terms. Since addition is associative, it can be helpful to add a negative number to each side instead of subtracting to avoid mistakes.

### Examples:

For the equation  $3x + 4 = 12$ , we can isolate the variable term on the left by subtracting a 4 from both sides:

$$3x + 4 - 4 = 12 - 4 \implies$$

$$3x = 8.$$

For the equation  $7y - 200 = 10$ , subtracting the 200 on the left side is the same as adding a -200:

$$7y + (-200) = 10.$$

If we add 200 to both sides of the equation, the 200 and -200 will cancel each other:

$$7y + (-200) + 200 = 10 + 200 \implies$$

$$7y = 210.$$

For the equation  $8 = 20 - z$ , we can add  $z$  to both sides to get  $8 + z = 20 - z + z \implies 8 + z = 20$ . Now subtracting 8 from both sides,

$$8 + z - 8 = 20 - 8 \implies$$

$$z = 12, \text{ so we get a solution for } z.$$

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## Simplifying by multiplication

When solving for a variable, we want to get a solution like  $x = 3$  or  $z = 2001$ . When a variable is divided by some number, we can use multiplication on both sides to solve for the variable.

### Example:

Solve for  $x$  in the equation  $x \div 12 = 5$ .

Since the  $x$  on the left side is being divided by 12, the equation is the same as  $x \times 1/12 = 5$ . Multiplying both sides by 12 will cancel the  $1/12$  on the left side:

$$x \times 1/12 \times 12 = 5 \times 12 \implies$$

$$x \times 1 = 60 \implies$$

$$x = 60.$$

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## Simplifying by division

When solving for a variable, we want to get a solution like  $x = 3$  or  $z = 2001$ . When a variable is multiplied by some number, we can use division on both sides to solve for the variable.

### Example:

Solve for  $x$  in the equation  $7x = 133$ . Since the  $x$  on the left side is being multiplied by 7, we can divide both sides by 7 to solve for  $x$ :

$$7x \div 7 = 133 \div 7 \implies$$

$$(7x)/7 = 133 \div 7 \implies$$

$$x/1 = 19 \implies$$

$$x = 19.$$

Note that dividing by 7 is the same as multiplying both sides by  $1/7$ .

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## Word problems as equations

When converting word problems to equations, certain "key" words tell you what kind of operations to use: addition, multiplication, subtraction, and division. The table below shows some common phrases and the operation to use.

Word	Operation	Example	As an equation
sum	addition	The sum of my age and 10 equals 27.	$y + 10 = 27$
difference	subtraction	The difference between my age and my younger sister's age, who is 11 years old, is 5 years.	$y - 11 = 5$
product	multiplication	The product of my age and 14 is 168.	$y \times 14 = 168$
times	multiplication	Three times my age is 60.	$3 \times y = 60$
less than	subtraction	Seven less than my age equals 32.	$y - 7 = 32$
total	addition	The total of my pocket change and 20 dollars is \$22.43.	$y + 20 = 22.43$
more than	addition	Eleven more than my age equals 43.	$11 + y = 43$

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## Sequences

A sequence is a list of items. We can specify any item in the list by its place in the list: first, second, third, fourth, and so on. Many useful lists have patterns so we know what items occur in each place in the list. There are 2 kinds of sequences. A finite sequence is a list made up of a finite number of items. An infinite sequence is a list that continues without end.



### Examples:

The following are examples of finite sequences.

The sequence 1, 3, 5, 7, 9, 11, 13, 15, 17, 19 is the sequence of the first 10 odd numbers.

The sequence *a, e, i, o, u*, is the sequence of vowels in the alphabet.

The sequence *m, m, m, m, m, m* is the sequence of 6 *m*'s.

The sequence 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0 is the sequence of 12 alternating 1's and 0's.

The sequence 1, 2, 3, 4, ..., 9998, 9999, 10000 is the sequence of the first ten thousand integers.

The sequence 0, 1, 4, 9, 16, 25, 36, 49 is the sequence of the squares of the first 8 whole numbers.

### Examples:

The following are examples of infinite sequences.

The sequence 2, 4, 6, 8, 10, 12, 14, 16, ... is the sequence of even whole numbers. The 100th place in this sequence is the number 200.

The sequence *a, b, c, a, b, c, a, b, c, a, b, ...* is the sequence of the letters a, b, c, repeating in this pattern forever.

The 100th place in this sequence is the letter *a*. The 300th place in this sequence is the letter *c*.

The sequence -1, 2, -3, 4, -5, 6, -7, 8, -9, ... is the sequence of integers with alternating signs. The 10th place in this sequence is 10. The 100th place in this sequence is 100. The 101st place in this sequence is -101.

The sequence 1, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, ... is a sequence of 1's separated by 1 zero, then 2 zeros, then 3 zeros, and so on. The 100th place in this sequence is a 0. The 105th place in this sequence is a 1.

The sequence 1, 3, 6, 10, 15, 21, 28, 36, 45, ... is the sequence of places the 1 occurs in the sequence of 1's and 0's above! If this sequence seems strange, note the difference between pairs of numbers next to one another:

$$3 - 1 = 2$$

$$6 - 3 = 3$$

$$10 - 6 = 4$$

$$15 - 10 = 5$$

$$21 - 15 = 6$$

$$28 - 21 = 7$$

Checking these differences makes the pattern clearer.

1, 1, 1, 1, 1, 1, ... is the sequence where every item in the list is the number 1.

1, 2, 3, 4, 5, 6, 7, ... is the sequence of counting numbers. Each item in the list is its place number in the list.

a, b, a, b, a, b, a, b, ... is the sequence of alternating letters a and b. The a's occur in odd-numbered places, and the b's occur in the even-numbered places.

$1/1$ ,  $1/2$ ,  $1/3$ ,  $1/4$ ,  $1/5$ ,  $1/6$ ,  $1/7$ , ... is the sequence of reciprocals of the whole numbers.

1, 4, 9, 16, 25, 36, 49, 64, 81, ... is the sequence of squares of the whole numbers.

a, e, i, o, u, a, e, i, o, u, a, e, ... is the repeating sequence of vowels in the alphabet.

4, 7, 10, 13, 16, 19, 22, 25, ... is the sequence of numbers beginning with the number 4, and each number in the list is 3 more than the number before it.