3A ***Time: 3 minutes***

Let p, q, and r represent the missing digits in the subtraction shown.
Find the sum p + q + r.

$$\begin{array}{r} 8\ 4\ p \\ -\ q\ 6\ 8 \\ \hline 4\ r\ 3 \end{array}$$

3B ***Time: 5 minutes***

For how many positive integer values of x is the expression $\sqrt{50 - x}$ also a positive integer?

3C ***Time: 6 minutes***

Both P and $(98 - P)$ are prime numbers. What is the least possible value of P ?

3D ***Time: 5 minutes***

Line segment $\overline{AB}$ has endpoints $A(-5,4)$ and $B(7,13)$. Point C lies on $\overline{AB}$ and is two-thirds of the way from point A to point B. Find the coordinates (x,y) of point C.

3E ***Time: 6 minutes***

The symbol 3! means 3×2×1, which equals 6. Similarly, 4! means 4×3×2×1, which equals 24.
Find the greatest prime factor of the sum 5! + 7!

Please fold over on line. Write answers on back.

Division **M**

Mathematical Olympiads for Elementary and Middle Schools

JANUARY 11, 2012

Contest **3**

3A *Student Name and Answer*

$p + q + r =$

3B *Student Name and Answer*

values

3C *Student Name and Answer*

3D *Student Name and Answer*

(,)

3E *Student Name and Answer*

Please fold over on line. Write answers in these boxes.

Division M

Mathematical Olympiads for Elementary and Middle Schools

January 11, 2012

Contest 3

SOLUTIONS AND ANSWERS

3A

3A *Strategy: Write the problem as an addition.*

$$\begin{array}{r} 4\ r\ 3 \\ +\ q\ 6\ 8 \\ \hline 8\ 4\ p \end{array}$$

3 + 8 ends in 1 so p = 1. 1 + r + 6 ends in 4, so r = 7. 1 + 4 + q = 8 so q = 3. Then **p + q + r = 11.**

Follow-Up: *(1) Find w + y + z, if 4z x y7 = 1w02* [16]

11

3B

3B *Strategy: Determine possible values of the radicand.*

If $\sqrt{50 - x}$ is a positive integer and x is positive, (50 – x) must be a perfect square less than 50. (50 – x) can be any of 49, 36, 25, 16, 9, 4, or 1 leading to x = 1, 14, 25, 34, 41, 46, or 49. **There are 7 positive integer values of x for which $\sqrt{50 - x}$ is a whole number.**

Follow-Up: *For how many whole number values of x is $\sqrt{108 - 3x}$ a whole number?* [3]

7

3C

3C *Strategy: List values of P in increasing order.*

P	2	3	5	7	11	13	17	**19**
98 – P	96	95	93	91	87	85	81	**79**
Is 98 – P prime?	No; (÷2)	No; (÷5)	No; (÷3)	No; (÷7)	No; (÷3)	No; (÷5)	No; (÷3)	**Prime**

Therefore, **the least value of P is 19.**

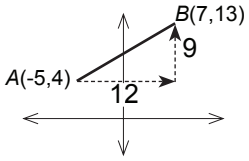
Follow-Up: *Find a whole number value for N such that the value of $N^2 + N + 41$ is not prime.* [The least is 40; the most readily found is 41.]

19

3D

3D *Strategy: Determine the x- and y-coordinates separately.*

C is 2/3 of the way from *A* to *B*. Then the x-coordinate of *C* is 2/3 of the way from the *x*-coordinate of *A* to the x-coordinate of *B*. The *x* (horizontal) distance from *A* to *B* is 7 – (–5) = 12 and 2/3 of 12 = 8. The *x*-coordinate of *C* is (–5) + 8 = 3.



Similarly, the *y*-coordinate of *C* is 4 + 2/3 × (13-4) = 10. **The coordinates of point *C* are (3,10).**

(3,10)

3E

43

3E **METHOD 1:** *Strategy: Factor the given expression.*

7! = 1 × 2 × 3 × 4 × 5 × 6 × 7 = 5! × (6 × 7).

Then 5! + 7! = 5! + (5! × 6 × 7)

= (5! × 1) + (5! × 6 × 7)

= 5! × (1 + 6 × 7) [by the distributive property]

= (1 × 2 × 3 × 4 × 5) × 43.

43 is larger than any prime factor of 5!.

The largest prime factor of 5! + 7! is 43.

METHOD 2: *Strategy: Perform the indicated operations and then factor.*

5! + 7! = 120 + 5,040 = 5,160. Start by factoring out as many small primes as you can.

$5{,}160 = 2^3 \times 3 \times 5 \times 43$. The largest prime factor of 5! + 7! is 43.

FOLLOW-UP: *Find the value of N for which N! × 4! = (N+1)!* [23]

NOTE: *Other FOLLOW-UP problems related to some of the above can be found in our books "Math Olympiad Contest Problems for Elementary and Middle Schools" and "Creative Problem Solving in School Mathematics."*