ISyE 6739 — Summer 2017

Homework #10 Solutions (Covers Module 5 — Hypothesis Testing)

1. (Hines et al., 11–1.) The breaking strength of a fiber used in manufacturing cloth is required to be at least 160 psi. Past experience has indicated that the standard deviation of breaking strength is 3 psi. A random sample of four specimens is tested and the average breaking strength is 158 psi.

   (a) Should the fiber be judged acceptable with level of significance $\alpha = 0.05$?

   (b) What is the probability of accepting $H_0\colon \mu \le 160$ if the fiber has a true breaking strength of 165 psi?

   Solution: We'll only judge the sample acceptable if we judge that its mean is $>$ 160. Therefore,

   (a) H_0: $\mu \le 160$ $\quad$ H_1: $\mu > 160$ $\qquad Z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{158 - 160}{3/2} = -1.333$

   The fiber is acceptable if $Z_0 > z_\alpha = z_{0.05} = 1.645$. Since $Z_0 = -1.333 < 1.645$, the fiber is not acceptable. $\square$

   (b) From first principles,

$$
\begin{aligned}
&\mathsf{Pr}(\text{Accept } H_0 \mid \mu = 165) \\
&= \mathsf{Pr}(Z_0 \le z_{0.05} \,|\, \mu = 165) \\
&= \mathsf{Pr}\left(\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \le z_{0.05} \,\Big|\, \mu = 165 \right) \\
&= \mathsf{Pr}\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} + \frac{\mu - \mu_0}{\sigma/\sqrt{n}} \le 1.645 \,\Big|\, \mu = 165 \right) \\
&= \mathsf{Pr}\left(\text{Nor}(0,1) + \frac{5}{3/2} \le 1.645 \right) \\
&= \mathsf{Pr}\left(\text{Nor}(0,1) \le -1.688 \right) \doteq 0.046. \quad \square
\end{aligned}
$$

   We could also do this problem using Operating Characteristic (OC) curves, as described in the text. To do so, note that $d = (\mu - \mu_0)/\sigma = (165 - 160)/3 = 1.67$; so if $n = 4$, then the OC curves yield $\beta \simeq 0.05$. $\square$

2. (Hines et al., 11–2.) The yield of a chemical process is being studied. The variance of the yield is known from previous experience with this process to be 5 (the units of σ^2 = percentage^2). The past five days of plant operation have resulted in the following yields (percentages): 91.60, 88.75, 90.80, 89.95, 91.30.

(a) Is there reason to believe the yield is less than 90%? Use a level of significance of $\alpha = 0.05$.

(b) What sample size would be required to detect a true mean yield of 85% with probability 0.95?

Solution:

(a) H_0: $\mu \geq 90$
H_1: $\mu < 90$

$$Z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{90.48 - 90}{\sqrt{5/5}} = 0.48$$

Since $Z_0 > -z_{0.05} = -1.645$, do not reject H_0. There is no evidence that mean yield is less than 90 percent. □

(b) Using the nice equation from class, we have

$$n = (z_\alpha + z_\beta)^2\sigma^2/\delta^2 = (1.645 + 1.645)^2 5/(5)^2 = 2.16 \simeq 3. \quad \square$$

Alternatively, you could use the OC curves, with $d = (\mu_0 - \mu)/\sigma = (90 - 85)/\sqrt{5} = 2.24$ and $\beta = 0.05$, also giving $n = 3$. □

3. (Hines et al., 11–7.) Two machines are used for filling plastic bottles with a net volume of 16.0 ounces. The filling processes can be assumed to be normally distributed, with standard deviations $\sigma_1 = 0.015$ and $\sigma_2 = 0.018$, respectively. Quality engineering suspects that both machines fill to the same net volume, whether or not this volume is 16.0 ounces. A random sample is taken from the output of each machine.

Machine 1	16.03	16.04	16.05	16.05	16.02	16.01	15.96	15.98	16.02	15.99
Machine 2	16.02	15.97	15.96	16.01	15.99	16.03	16.04	16.02	16.01	16.00

Do you think that quality engineering is correct? Use $\alpha = 0.05$.

Solution:
H_0: $\mu_1 = \mu_2$
H_1: $\mu_1 \neq \mu_2$

$$Z_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = 1.349$$

Since $|Z_0| < z_{\alpha/2} = 1.96$, do not reject H_0. □

4. (Hines et al., 11–12.) The lateral deviation in yards of a certain type of mortar shell is being investigated by the propellant manufacturer. The following data have been observed.

11.28	−10.42	−8.51	1.95	6.47
−9.48	6.25	10.11	−8.65	−0.68

Test the hypothesis that the mean lateral deviation of these mortar shells is zero. Assume that lateral deviation is normally distributed.

Solution:

H_0: $\mu = 0$ $\quad t_0 = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{-0.168 - 0}{8.5638/\sqrt{10}} = -0.062$

H_1: $\mu \neq 0$ $\quad |t_0| = 0.062 < t_{0.025,9} = 2.2622,$ do not reject H_0. □

5. (Hines et al., 11–15.) An article in the *Journal of Construction Engineering and Management* (1999, p. 39) presents some data on the number of work hours lost per day on a construction project due to weather-related incidents. Over 11 workdays, the following lost work hours were recorded.

8.8	12.5	5.4	12.8	9.1	14.7
8.8	12.2	13.3	6.9	2.2	

Assuming work hours are normally distributed, is there any evidence to conclude that the mean number of work hours lost per day is greater than 8 hours?

Solution: We'll test $H_0 : \mu \leq 8$ vs. $H_1 : \mu > 8$ under the assumption of unknown variance. The test statistic is

$$t_0 = \frac{\bar{x} - \mu_0}{\sqrt{s^2/n}} = \frac{9.7 - 8}{\sqrt{14.62/11}} = 1.47,$$

and we will reject H_0 if $t_0 > t_{\alpha,n-1}$. Note that $t_{0.05,10} = 1.812$ and $t_{0.10,10} = 1.372$. Therefore, we do not reject at level of significance $\alpha = 0.05$; but we do reject H_0 at level $\alpha = 0.10$. □

6. (Hines et al., 11–23.) Suppose that two random samples were drawn from normal populations with equal variances. The sample data yields $\bar{x} = 20.0$, $n = 10$, $\sum_{i=1}^{n}(x_i - \bar{x})^2 = 1480$ and $\bar{y} = 15.8$, $m = 10$, $\sum_{i=1}^{m}(y_i - \bar{y})^2 = 1425$. Test the hypothesis that the two means are equal. Use $\alpha = 0.01$.

Solution: We'll test $H_0 : \mu_x = \mu_y$ vs. $H_1 : \mu_x \neq \mu_y$, and we'll use a pooled variance estimator (since we've been told that the variances are equal, but unknown). Thus,

$$s_p = \sqrt{\frac{(n-1)s_x^2 + (m-1)s_y^2}{n+m-2}} = \sqrt{\frac{1480 + 1425}{18}} = 12.704$$

and

$$t_0 = \frac{\bar{x} - \bar{y}}{s_p\sqrt{\frac{1}{n} + \frac{1}{m}}} = \frac{20.0 - 15.8}{12.704\sqrt{\frac{1}{10} + \frac{1}{10}}} = 0.74.$$

Since $|t_0| < t_{0.005,18} = 2.878$, we do not reject H_0. □

7. (Hines et al., 11–32.) Two machines produce metal parts. The following data have been collected on the weights of the parts.

$$\begin{array}{ccc} n = 25 & \bar{x} = 0.984 & s_x^2 = 13.46 \\ m = 30 & \bar{y} = 0.907 & s_y^2 = 9.65 \end{array}$$

Test the hypothesis that the two machines produce parts having the same mean weight. Use $\alpha = 0.05$.

Solution: We'll test $H_0 : \mu_x = \mu_y$ vs. $H_1 : \mu_x \neq \mu_y$, assuming that the unknown variances are unequal. First, we'll need to calculate the approximate degrees of freedom,

$$\nu \equiv \frac{\left(\frac{s_x^2}{n} + \frac{s_y^2}{m}\right)^2}{\frac{(s_x^2/n)^2}{n+1} + \frac{(s_y^2/m)^2}{m+1}} - 2 = \frac{\left(\frac{13.46}{25} + \frac{9.65}{30}\right)^2}{\frac{(13.46/25)^2}{26} + \frac{(9.65/30)^2}{31}} - 2 = 49.06 \to 49.$$

Then the test statistic is

$$t_0^\star = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{s_x^2}{n} + \frac{s_y^2}{m}}} = \frac{0.077}{\sqrt{\frac{13.46}{25} + \frac{9.65}{30}}} = 0.083.$$

Since $|t_0^\star| \leq t_{\alpha/2,\nu} = t_{0.025,49} = 2.01$, we do not reject H_0. □

8. (Hines et al., 11–34.) Two types of exercise equipment, A and B, for handicapped individuals are often used to determine the effect of the particular exercise on heart rate (in beats per minute). Seven subjects participated in a study to determine whether the two types of equipment have the same effect on heart rate. The results are given in the table below.

Subject	A	B
1	162	161
2	163	187
3	140	199
4	191	206
5	160	161
6	158	160
7	155	162

Conduct an appropriate hypothesis test to determine whether there is a significant difference in heart rate due to the type of equipment used.

Solution: Since the observations occur in natural pairs, we will use a paired t-test to test $H_0 : \mu_A = \mu_B$ vs. $H_1 : \mu_A \neq \mu_B$, or equivalently, $H_0 : \mu_D = 0$ vs. $H_1 : \mu_D \neq 0$, where $\mu_D = \mu_A - \mu_B$. The paired t-test is conducted using the *differences* of the observations, as calculated in the D_i column of the following enhanced table.

Subject	A	B	D_i
1	162	161	1
2	163	187	-24
3	140	199	-59
4	191	206	-15
5	160	161	-1
6	158	160	-2
7	155	162	-7

From the table, we find that $\bar{D} = -15.29$ and $S_D^2 = 450.2$. Thus,

$$t_0 \;=\; \frac{\bar{D}}{\sqrt{S_D^2/n}} \;=\; \frac{-15.29}{\sqrt{450.2/7}} \;=\; -1.907.$$

Lets take $\alpha = 0.05$. Since $|t_0| \leq t_{\alpha/2,n-1} = t_{0.025,6} = 2.447$, we fail to reject H_0. □

9. (Hines et al., 11–27.) A manufacturer of precision measuring instruments claims that the standard deviation in the use of the instrument is 0.00002 inch. An analyst, who is unaware of the claim, uses the instrument eight times and obtains a sample standard deviation of 0.00005 inch. Using $\alpha = 0.01$, is the claim justified?

Solution: H_0: $\sigma \leq 0.00002$ $\quad$ $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2} = \dfrac{7(0.00005)^2}{(0.00002)^2} = 43.75$

H_1: $\sigma > 0.00002$ $\quad$ Since $\chi_0^2 > \chi_{0.01,7}^2 = 18.475$, we reject H_0. The claim is unjustified. □

10. (Hines et al., 11–31.) Consider the following two samples drawn from two normal populations.

Sample 1	4.34	5.00	4.97	4.25	5.55	6.55	6.37	5.55	3.76		
Sample 2	1.87	2.00	2.00	1.85	2.11	2.31	2.28	2.07	1.76	1.91	2.00

Is there evidence to conclude that the variance of population 1 is greater than that of population 2? Use $\alpha = 0.01$.

Solution: We need to use an F-test on the ratio of two variances.

H_0: $\sigma_1^2 = \sigma_2^2$ $\quad$ $F_0 = s_1^2/s_2^2 = 0.9027/0.0294 = 30.69$
H_1: $\sigma_1^2 > \sigma_2^2$ $\quad$ $F_0 > F_{\alpha,n-1,m-1} = F_{0.01,8,10} = 5.06$, so reject H_0. □

11. (Hines et al., 11–37.) Of 400 randomly selected motorists, 48 were found to be uninsured. Test the hypothesis that the actual uninsured rate is at most 10%. Use $\alpha = 0.05$.

Solution: We'll test H_0: $p \leq 0.10$ vs. H_1: $p > 0.10$. Since the sample size is large, we can use the usual Central Limit Theorem normal approximation to the binomial. In this case, we obtain

$$Z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{48 - 400(0.1)}{\sqrt{400(0.1)(0.9)}} = 1.333.$$

Since $Z_0 < z_\alpha = 1.645$, we do not reject H_0. □