ISyE 6739 — Summer 2017

Homework #9 Solutions (Module 4.4 — Confidence Intervals)

Most of the following problems are from Hines, et al.

10–40(a). The life in hours of a 75-W light bulb is known to be approximately normally distributed, with a standard deviation of $\sigma = 25$ hours. A random sample of 20 bulbs has a mean life of $\bar{x} = 1014$ hours. Construct a 95% two-sided confidence interval on the mean life.

Solution: Since σ is known, we use

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}.$$

Since $z_{0.025} = 1.96$, we have $1003.04 \le \mu \le 1024.96$. $\square$

10–42. Suppose that in Exercise 10–40 we wanted to be 95% confident that the error in estimating the mean life is less than 5 hours. What sample size should be used?

Solution: $n = (z_{\alpha/2}\sigma/\epsilon)^2 = [(1.96)25/5]^2 = 96.04 \simeq 97$. $\square$

10–46. The burning rates of two different solid-fuel rocket propellants are being studied. It is known that both propellants have approximately the same standard deviation of burning rate, $\sigma_1 = \sigma_2 = 3$ cm/s. Two random samples of $n_1 = 20$ and $n_2 = 20$ specimens are tested, and the sample mean burning rates are $\bar{x}_1 = 18$ and $\bar{x}_2 = 24$ cm/s. Construct a 99% confidence interval on the mean difference in burning rate.

Solution: Since both variances are known, we use

$$\bar{x}_2 - \bar{x}_1 - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{x}_2 - \bar{x}_1 + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}.$$

Since $z_{0.005} = 2.576$, we have $3.56 \le \mu_2 - \mu_1 \le 8.44$. $\square$

10–48(a). The compressive strength of concrete is being tested by a civil engineer. He tests 16 specimens and obtains the following data:

2216	2237	2249	2204
2225	2301	2281	2263
2318	2255	2275	2295
2250	2238	2300	2217

Construct a 95% two-sided confidence interval on the mean strength.

Solution: Since σ is unknown, we use

$$\bar{x} - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}.$$

We can easily calculate

$$\bar{x} = 2257.75 \quad \text{and} \quad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = (34.51)^2.$$

Since $t_{0.025,15} = 2.13$, we have $2239.4 \leq \mu \leq 2276.1$. □

10–49. An article in *Annual Reviews Material Research* (2001, p. 291) presents bond strengths for various energetic materials (explosives, propellants, and pyrotechnics). Bond strengths for 15 such materials are shown below. Construct a two-sided 95% confidence interval on the mean bond strength.

323, 312, 300, 284, 283, 261, 207, 183
180, 179, 174, 167, 167, 157, 120

Solution: Since σ is unknown, we use

$$\bar{x} - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}.$$

We can easily calculate

$$\bar{x} = 219.80 \quad \text{and} \quad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = (66.41)^2.$$

Since $t_{0.025,14} = 2.14$, we have $183.1 \leq \mu \leq 256.5$. □

10–50. The wall thickness of 25 glass 2-liter bottles was measured by a quality-control engineer. The sample mean was $\bar{x} = 4.05$ mm, and the sample standard deviation was $s = 0.08$ mm. Find a 90% lower confidence interval on the mean wall thickness.

Solution: The confidence interval will have the form

$$\bar{x} - t_{\alpha,n-1}(s/\sqrt{n}) \leq \mu$$

Since $t_{0.10,24} = 1.32$, we have $4.05 - t_{0.10,24}(0.08/\sqrt{25}) \leq \mu$. In other words, $4.029 \leq \mu$. □

10–56(a). Random samples of size 20 were drawn from two independent normal populations. The sample means and standard deviations were $\bar{x}_1 = 22.0$, $s_1 = 1.8$, $\bar{x}_2 = 21.5$, and $s_2 = 1.5$. Assuming that $\sigma_1^2 = \sigma_2^2$, find a 95% two-sided confidence interval on $\mu_1 - \mu_2$.

Solution: Since both variances are *unknown but assumed equal*, we use

$$\bar{x}_1 - \bar{x}_2 - t_{\alpha/2, n_1+n_2-2}\, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le \bar{x}_1 - \bar{x}_2 + t_{\alpha/2, n_1+n_2-2}\, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}},$$

where $n_1 = n_2 = 20$ and the pooled variance is

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = 2.745.$$

Since $t_{0.025,38} = 2.024$, we have $-0.561 \le \mu_1 - \mu_2 \le 1.561$. □

10–57. The diameter of steel rods manufactured on two different extrusion machines is being investigated. Two random samples of sizes $n_1 = 15$ and $n_2 = 18$ are selected, and the sample means and sample variances are $\bar{x}_1 = 8.73$, $s_1^2 = 0.30$, $\bar{x}_2 = 8.68$, and $s_2^2 = 0.34$. Assuming that $\sigma_1^2 = \sigma_2^2$, construct a 95% two-sided confidence interval on the difference in mean rod diameter.

Solution: Using the same equations as in the solution to Question 10–56(a), we obtain $-0.355 \le \mu_1 - \mu_2 \le 0.455$. (Note that the answer in the back of the book was wrong.) □

10–59(a). Consider the data in Exercise 10–48. Construct a 95% two-sided confidence interval on σ^2.

Solution: The desired confidence interval is of the form

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2, n-1}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\alpha/2, n-1}}.$$

From the solution to Exercise 10–48, we know that $s^2 = (34.51)^2$. Further, $\chi^2_{0.975,15} = 6.26$ and $\chi^2_{0.025,15} = 27.49$. Thus, the c.i. is $649.84 \le \sigma^2 \le 2853.69$. □

10–63. Consider the data in Exercise 10–56. Construct a 95% two-sided confidence interval on the ratio of the population variances σ_1^2/σ_2^2.

Solution: The desired confidence interval is of the form

$$\frac{s_1^2}{s_2^2} \frac{1}{F_{\alpha/2, n_1-1, n_2-1}} \ \le \ \frac{\sigma_1^2}{\sigma_2^2} \ \le \ \frac{s_1^2}{s_2^2} F_{\alpha/2, n_2-1, n_1-1}.$$

In other words, we want

$$\frac{(1.8)^2}{(1.5)^2}\,\frac{1}{F_{0.025,19,19}} \;\le\; \frac{\sigma_1^2}{\sigma_2^2} \;\le\; \frac{(1.8)^2}{(1.5)^2}\,F_{0.025,19,19}.$$

Since $F_{0.025,19,19} = 2.526$, we obtain the c.i. $0.57 \le \sigma_1^2/\sigma_2^2 \le 3.64$. $\square$

Bernoulli Question. A pollster asked a sample of 2000 people whether or not they were in favor of a particular proposal. Exactly 1200 people answered yes. Find a 95% confidence interval for the percentage of the population in favor of the proposal.

Solution: We are looking for a c.i. for the proportion p of favorable responses, i.e., the Bernnoulli parameter. Thus, the solution is of the form

$$\bar{x} - z_{\alpha/2}\sqrt{\frac{\bar{x}(1-\bar{x})}{n}} \;\le\; p \;\le\; \bar{x} + z_{\alpha/2}\sqrt{\frac{\bar{x}(1-\bar{x})}{n}}.$$

That is,

$$0.6 - 1.96\sqrt{\frac{0.6(0.4)}{2000}} \;\le\; p \;\le\; 0.6 + 1.96\sqrt{\frac{0.6(0.4)}{2000}},$$

or $0.579 \le p \le 0.621$. $\square$

BONUS: What do Stiller and Meara, Lou Reed, Suzanne Pleshette, and 44 have in common?

Solution: Syracuse University. GO ORANGE! $\square$