```python
# Keep dividing the points by half and call merge function recursively
# Time complexity of this divide and conquer algorithm would be O(nlogn) by the
Master Theorem
# Space complexity would be O(n)
def compute_hull(self, points_set):
    # Base case n == 2 or n == 3
    if len(points_set) == 2:
        return points_set
    # When n == 3, reorder the points in the list if needed.
    if len(points_set) == 3:
        return self.rearrange(points_set)

    # Dividing part of the algorithm
    left = self.compute_hull(points_set[:len(points_set) // 2])
    right = self.compute_hull(points_set[len(points_set) // 2:])

    # Merge
    return self.make_hull(left, right)

# Compare the two slopes from point[0], and change the order if the second slope
is bigger than the first slope
# in order to keep the order same throughout the merging part.
# Takes O(1) time and space complexity.
def rearrange(self, points):
    slope1 = self.get_slope(points[0], points[1])
    slope2 = self.get_slope(points[0], points[2])

    # Swap point[1] and point[2]
    if slope1 < slope2:
        temp = points[1]
        points[1] = points[2]
        points[2] = temp

    # Returning the reordered point set
    return points

# This part will take O(n) time / space complexity
def make_hull(self, left, right):

    # Find the rightmost index from left hull
    closest = 0
    rightmost_point = -300
    for i in range(len(left)):
        if left[i].x() > rightmost_point:
            rightmost_point = left[i].x()
            closest = i
    rightmost_index = closest

    # Find the leftmost index from right hull
    closest = 0
    leftmost_point = 300
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    for i in range(len(right)):
        if right[i].x() < leftmost_point:
            leftmost_point = right[i].x()
            closest = i
    leftmost_index = closest

    # Get upper / lower tangent (4 points)
    up_left_tangent, up_right_tangent = self.get_upper_tangent(left, right,
rightmost_index, leftmost_index)
    low_left_tangent, low_right_tangent = self.get_lower_tangent(left, right,
rightmost_index, leftmost_index)

    # Make temp list to return
    # should take O(n) time using cut and paste method
    temp_list = []
    for index1 in range(len(left)):
        temp_list.append(left[index1])

        if up_left_tangent == index1 % len(left):

            for index2 in range(up_right_tangent, up_right_tangent + len(right)):
                temp_list.append(right[index2 % len(right)])

                if low_right_tangent == index2 % len(right):

                    for index3 in range(low_left_tangent, low_left_tangent +
len(left)):

                        if (index3 % len(left)) != 0:
                            temp_list.append(left[index3 % len(left)])
                        else:
                            break
                    break
            break
    return temp_list

# Calculate slope from given two points
# Takes O(1) time and space
def get_slope(self, left, right):
    return (right.y() - left.y()) / (right.x() - left.x())

# Get upper tangent indices by calling get_low_right_tangent /
get_low_left_tangent function
# Keep comparing new left/right index and return when they are the same
# O(n) time / space complexity
def get_lower_tangent(self, left, right, rightmost_index, leftmost_index):

    index_left = rightmost_index
    index_right = leftmost_index
    while True:
        current_index_right = self.get_low_right_tangent(left, right, index_left,
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index_right)
        current_index_left = self.get_low_left_tangent(left, right, index_left,
index_right)

        # if new indices are not changed from the previous indices, return.
Otherwise, update the current indices.
        if index_left == current_index_left and index_right ==
current_index_right:
            break
        else:
            index_left = current_index_left
            index_right = current_index_right
    return index_left, index_right

# O(n) time / space complexity
def get_low_left_tangent(self, left, right, rightmost_index, leftmost_index):

    slope = self.get_slope(left[rightmost_index], right[leftmost_index])
    temp = 0

    while True:
        # Calculate slope between leftmost index and next clockwise point from
left hull
        next_slope = self.get_slope(left[(rightmost_index + 1 + temp) %
len(left)], right[leftmost_index])

        # if current slope is bigger than the next slope, we're done. Otherwise,
update the current slope.
        if slope > next_slope:
            break
        else:
            temp = temp + 1
            slope = next_slope
    return (rightmost_index + temp) % len(left)

# O(n) time / space complexity
def get_low_right_tangent(self, left, right, rightmost_index, leftmost_index):
    slope = self.get_slope(left[rightmost_index], right[leftmost_index])
    temp = 0

    while True:
        # Calculate slope between rightmost index and next counterclockwise point
from right hull
        next_slope = self.get_slope(left[rightmost_index], right[(leftmost_index
- temp - 1) % len(right)])

        # if current slope is smaller than the next slope, we're done. Otherwise,
update the current slope.
        if slope < next_slope:
            break
        else:
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            temp = temp + 1
            slope = next_slope
    return (leftmost_index - temp) % len(right)

# Get upper tangent indices by calling get_up_right_tangent / get_up_left_tangent
function
# Continue until finding the tangent line from both left and right hull
# O(n) time / space complexity
def get_upper_tangent(self, left, right, rightmost_index, leftmost_index):
    index_left = rightmost_index
    index_right = leftmost_index
    while True:
        current_index_right = self.get_up_right_tangent(left, right, index_left,
index_right)
        current_index_left = self.get_up_left_tangent(left, right, index_left,
index_right)

        # if new indices are not changed from the previous indices, return.
Otherwise, update the current indices
        if index_left == current_index_left and index_right ==
current_index_right:
            break
        else:
            index_right = current_index_right
            index_left = current_index_left
    return index_left, index_right

# O(n) time / space complexity
def get_up_right_tangent(self, left, right, rightmost_index, leftmost_index):
    slope = self.get_slope(left[rightmost_index], right[leftmost_index])
    temp = 0
    while True:
        # Calculate the slope between rightmost index and next clockwise point
from right hull
        next_slope = self.get_slope(left[rightmost_index], right[(leftmost_index
+ 1 + temp) % len(right)])
        # if current slope is bigger than the next slope, we're done. Otherwise,
update the current slope.
        if next_slope < slope:
            break
        else:
            temp = temp + 1
            slope = next_slope
    return leftmost_index + temp

# O(n) time / space complexity
def get_up_left_tangent(self, left, right, rightmost_index, leftmost_index):
    slope = self.get_slope(left[rightmost_index], right[leftmost_index])
    temp = 0
    while True:
        # Calculate the slope between leftmost index and next counterclockwise
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point from left hull
        next_slope = self.get_slope(left[(rightmost_index - temp - 1) %
len(left)], right[leftmost_index])

        # if current slope is smaller than the next slope, we're done. Otherwise,
update the current slope.
        if next_slope > slope:
            break
        else:
            temp = temp + 1
            slope = next_slope
    return (rightmost_index - temp) % len(left)

def run(self):
    assert( type(self.points) == list and type(self.points[0]) == QPointF )

    n = len(self.points)
    print( 'Computing Hull for set of {} points'.format(n) )

    t1 = time.time()
    # TODO: SORT THE POINTS BY INCREASING X-VALUE
    # Sorting the given set of points by using x value as key
    sorted_points = sorted(self.points, key=lambda p: p.x())

    t2 = time.time()
    print('Time Elapsed (Sorting): {:3.3f} sec'.format(t2-t1))

    t3 = time.time()
    # TODO: COMPUTE THE CONVEX HULL USING DIVIDE AND CONQUER
    # Calling the divide & conquer algorithm
    points = self.compute_hull(sorted_points)
    t4 = time.time()

    USE_DUMMY = False
    if USE_DUMMY:
        # This is a dummy polygon of the first 3 unsorted points
        polygon = [QLineF(self.points[i],self.points[(i+1)%3]) for i in range(3)]
        # When passing lines to the display, pass a list of QLineF objects.
        # Each QLineF object can be created with two QPointF objects
        # corresponding to the endpoints
        assert( type(polygon) == list and type(polygon[0]) == QLineF )

        # Send a signal to the GUI thread with the hull and its color
        self.show_hull.emit(polygon,(0,255,0))

    else:
        # TODO: PASS THE CONVEX HULL LINES BACK TO THE GUI FOR DISPLAY
        polygon = [QLineF(points[i], points[(i+1)%len(points)]) for i in
range(len(points))]
        assert( type(polygon) == list and type(polygon[0]) == QLineF)
        self.show_hull.emit(polygon, (0,255,0))
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    # Send a signal to the GUI thread with the time used to compute the
    # hull
    self.display_text.emit('Time Elapsed (Convex Hull): {:3.3f} sec'.format(t4-
t3))
    print('Time Elapsed (Convex Hull): {:3.3f} sec'.format(t4-t3))
```

2. Analysis of Algorithm

- The whole algorithm will take O(nlogn) by the Master Theorem. We divide into 2 subproblems, so a = 2, and size is n / 2, so b = 2. Since the merging part would take linear time, d = 1. Therefore, the Master Theorem gives O(nlogn) time complexity. For space complexity, the space needed grows linearly in the algorithm, mostly the lists to store the sub hulls. Therefore, it has O(n) space complexity.

3. Empirical analysis

1) n = 10 (0.000s, 0.000s, 0.001s, 0.001s, 0.001s) mean time required = 0.0006s

2) n = 100 (0.009s, 0.001s, 0.005s, 0.006s, 0.005) mean time required = 0.0052s

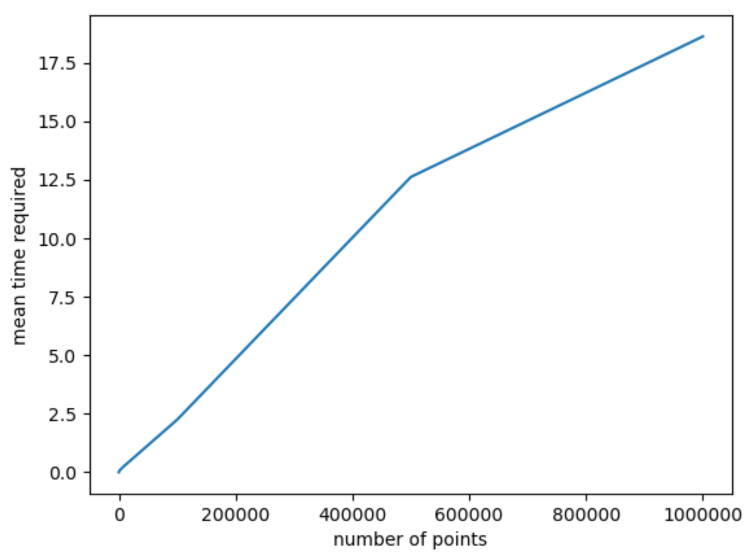
3) n = 1,000 (0.044s, 0.072s, 0.068s, 0.084s, 0.066s) mean time required = 0.0668s

4) n = 10,000 (0.272s, 0.262s, 0.267s, 0.337s, 0.289s) mean time required = 0.2854s

5) n = 100,000 (1.995s, 2.089s, 2.403s, 2.591s, 2.165s) mean time required = 2.2486s

6) n = 500,000 (13.177s, 12.533s, 14.023s, 14.375s, 8.996s) mean time required = 12.621s

7) n = 1,000,000 (18.397s, 18.208s, 19.678s, 18.631s, 18.22s) mean time required = 18.627s



- The graph is showing it has O(nlogn) time complexity. Since the number of inputs' scale is logarithmic and the graph increases linearly, it is O(nlogn).

1) n = 10, 0.0006 = k * 10 log 10, k = 0.00006
2) n = 100, 0.0052 = k * 100 log 100, k = 0.000026
3) n = 1000, 0.0668 = k * 1000 log 1000, k = 0.000022
4) n = 10000, 0.2854 = k * 10000 log 10000, k = 0.0000071
5) n = 100000, 2.2486 = k * 100000 log 100000, k = 0.0000045
6) n = 500000, 12.621 = k * 500000 log 500000, k = 0.0000044
7) n = 1000000, 18.627 = k * 1000000 log 1000000, k = 0.0000031

The values of constant proportionality k are tiny that it doesn't really affect the time. So, it confirms that nlogn best fits for g(n) in this case.