Wyrażenie alg. 2

A)

1. $\frac{1}{x^2-1}+\frac{1}{x^2+x}=\frac{1}{x^2-x}$

zał: $x^2-1\neq 0 \wedge x^2+x\neq 0 \wedge x^2-x\neq 0$

$x\neq 1 \wedge x\neq -1 \wedge x(x+1)\neq 0 \wedge x(x-1)\neq 0$

$x\neq 0 \wedge x\neq -1 \wedge x\neq 0 \wedge x\neq 1$

$\frac{1}{(x-1)(x+1)}+\frac{1}{x(x+1)}=\frac{1}{x(x-1)} \quad /\cdot x(x-1)(x+1)$ $\quad \underline{D=\mathbb{R}\setminus\{-1,0,1\}}$

$\cancel{x}+x-1=\cancel{x}+1$

$x-1=1$

$\underline{x=2}$

2. $\frac{-4x-3}{x+1}\leq 4x \quad /\cdot(x+1)^2$ $\quad \underline{D=\mathbb{R}\setminus\{-1\}}$

$(-4x-3)(x+1)\leq 4x(x+1)^2$

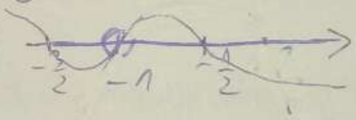
$(x+1)\left[-4x-3-4x(x+1)\right]\leq 0$

$(x+1)(-4x^2-8x-3)\leq 0$

$\Delta=64-48=16 \quad \sqrt{\Delta}=4$

$x_1=\frac{8-4}{-8}=-\frac{1}{2}$

$x_2=\frac{8+4}{-8}=-\frac{3}{2}$



$\underline{x\in\left[-\frac{3}{2},-1\right)\cup\left[-\frac{1}{2},+\infty\right)}$

3. $f(x)=\frac{x+1}{x+2}=\frac{x+2-1}{x+2}=1-\frac{1}{x+2}$

$D=\mathbb{R}\setminus\{-2\}$

$ZW_f=\mathbb{R}\setminus\{1\}$

f rośnie w przedziałach: $(-\infty,-2)$; $(-2,+\infty)$

4. Z: $x\neq 0$, T: $x^4+\frac{12}{x^2}\geq 9$

D: $x^4+\frac{12}{x^2}=x^4+\frac{3}{x^2}+\frac{9}{x^2}$

z nierówności między średnimi (A-G)

$\frac{x^4+\frac{3}{x^2}+\frac{9}{x^2}}{3}\geq\sqrt[3]{x^4\cdot\frac{3}{x^2}\cdot\frac{9}{x^2}} \quad /\cdot 3$

$x^4 + \frac{12}{x^2} \geq 3 \cdot \sqrt[3]{27}$

$x^4 + \frac{12}{x^2} \geq 3 \cdot 3 = 9$

$\square$

5. $(x+3)y = 10$

$10 = 1 \cdot 10$
$10 \cdot 1$
$2 \cdot 5$
$5 \cdot 2$
$-1 \cdot (-10)$
$-10 \cdot (-1)$
$-2 \cdot (-5)$
$-5 \cdot (-2)$

- $x+3 = 1 \wedge y = 10$
- $x+3 = 10 \wedge y = 1$
- $x+3 = 2 \wedge y = 5$
- $x+3 = 5 \wedge y = 2$
- $x+3 = -1 \wedge y = -10$
- $x+3 = -10 \wedge y = -1$
- $x+3 = -2 \wedge y = -5$
- $x+3 = -5 \wedge y = -2$

$\Rightarrow$ Te pary to:
$(-2, 10)$; $(7, 1)$; $(-1, 3)$; $(2, 2)$;
$(-4, -10)$; $(-13, -1)$; $(-5, -5)$; $(-8, -2)$

6. Nie jest ponieważ nie spełnia definicji funkcji malejącej.

np. $x_1 = -1$, $x_2 = 1$

$x_1 < x_2$, ale $f(x_1) = -1 < 1 = f(x_2)$

B

1. $\frac{1}{x^2-9} + \frac{1}{x^2+3x} = \frac{1}{x^2-3x}$

zał:
$x^2 - 9 \neq 0 \wedge x^2+3x \neq 0 \wedge x^2-3x \neq 0$
$x^2 \neq 9$ $\quad x(x+3) \neq 0$ $\quad x(x-3) \neq 0$
$|x| \neq 3$ $\quad x \neq 0 \wedge x \neq -3$ $\quad x \neq 0 \wedge x \neq 3$
$x \neq 3 \wedge x \neq -3$

$D = \mathbb{R} \setminus \{-3, 0, 3\}$

$\frac{1}{(x-3)(x+3)} + \frac{1}{x(x+3)} = \frac{1}{x(x-3)} \quad / \cdot x(x-3)(x+3)$

$x + x - 3 = x + 3$

$x = 6$

4. To samo co w grupie A

2

2. $\frac{4x-3}{x-1} \geq 4x \quad / \cdot (x-1)^2$

zał: $x-1 \neq 0$
$x \neq 1$ $\quad \boxed{D=\mathbb{R}\setminus\{1\}}$

$(4x-3)(x-1) \geq 4x(x-1)^2$

$(x-1)[4x-3-4x(x-1)] \geq 0$

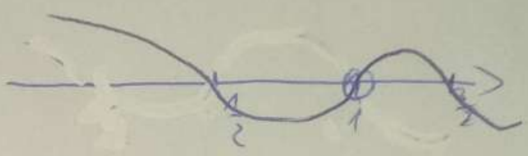
$(x-1)[4x-3-4x^2+4x] \geq 0$

$(x-1)(-4x^2+8x-3) \geq 0$

$\Delta = 64-48=16 \quad \sqrt{\Delta}=4$

$x_1 = \frac{-8-4}{-8} = \frac{3}{2}$

$x_2 = \frac{-8+4}{-8} = \frac{1}{2}$



$x \in \left(-\infty, \frac{1}{2}\right] \cup \left(1, \frac{3}{2}\right]$

3. $f(x) = \frac{x+1}{x-2} = \frac{x-2+3}{x-2} = 1 + \frac{3}{x-2}$

$D = \mathbb{R}\setminus\{2\}$

$ZW_f = \mathbb{R}\setminus\{1\}$

f maleje w przedziałach: $(-\infty, 2)$; $(2, +\infty)$

5. $x(y+3)=10$

$10 = 1\cdot 10$
$10\cdot 1$
$2\cdot 5$
$5\cdot 2$
$-1\cdot(-10)$
$-10\cdot(-1)$
$-2\cdot(-5)$
$-5\cdot(-2)$

$x=1 \wedge y+3=10$
$x=10 \wedge y+3=1$
$x=2 \wedge y+3=5$
$x=5 \wedge y+3=2$
$x=-1 \wedge y+3=-10$
$x=-10 \wedge y+3=-1$
$x=-2 \wedge y+3=-5$
$x=-5 \wedge y+3=-2$

$\Rightarrow$ Te pary to:

$(1,7)$; $(10,-2)$; $(2,2)$; $(5,-1)$
$(-1,-13)$; $(-10,-4)$; $(-2,-8)$; $(-5,-5)$

6. Nie, bo nie spełnia definicji funkcji malejącej.

np. $x_1=-1$, $x_2=1$

$x_1 < x_2$, ale $f(x_1) = -2 < 2 = f(x_2)$