zespolone A (3R)

1. $\forall z\in\mathbb{C}, n\in\mathbb{N}: |z^n| = |z|^n$

D: $L = |z^n| = \left|\,|z|^n(\cos n\varphi + i\sin n\varphi)\right| = \left||z|^n\right|\cdot|\cos n\varphi + i\sin n\varphi| =$

$= |z|^n \cdot \underbrace{\sqrt{\cos^2 n\varphi + \sin^2 n\varphi}}_{=1} = |z|^n = P$ □

2. $A = \{z\in\mathbb{C} : |z-1-2i| \le 3$ $\qquad B = \{z\in\mathbb{C} : 0 \le \mathrm{Re}(iz) \le 1\}$

Niech $z = x+iy,\ x,y\in\mathbb{R}$

A:

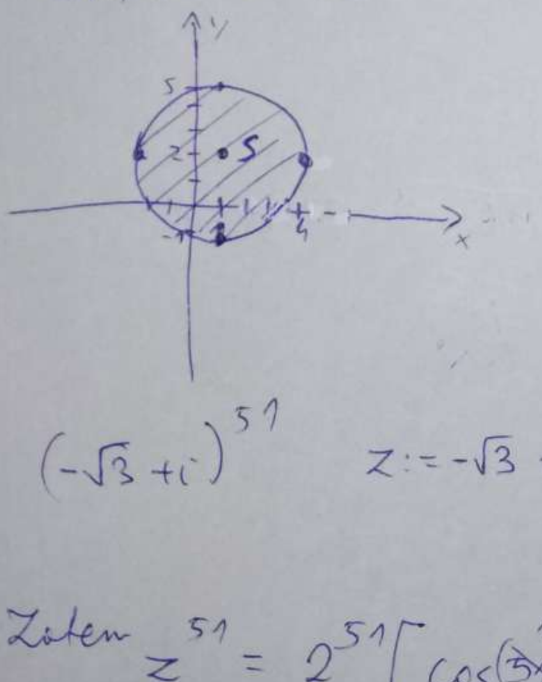
$|x+iy-1-2i| \le 3$

$|x-1+(y-2)i| \le 3$

$\sqrt{(x-1)^2+(y-2)^2} \le 3 \quad /^2$

$(x-1)^2+(y-2)^2 \le 9$

$S(1,2) \quad r=3$



B:

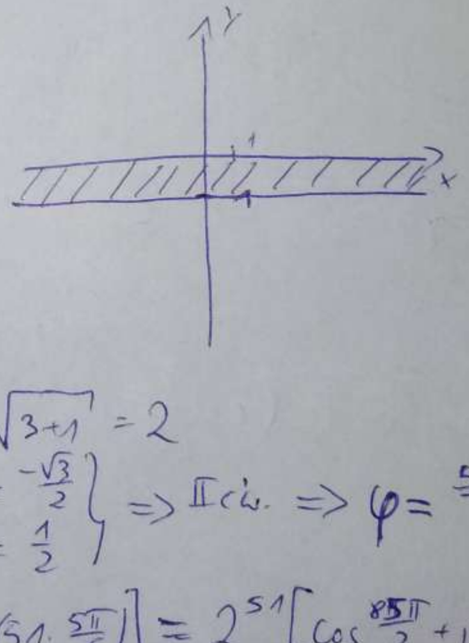
$0 \le \mathrm{Re}(i(x+iy)) \le 1$

$0 \le \mathrm{Re}(ix-y) \le 1$

$0 \le -y \le 1 \quad /\cdot(-1)$

$0 \ge y \ge -1$

$y \in \langle -1, 0\rangle$



3. $(-\sqrt{3}+i)^{51}$ $\qquad z := -\sqrt{3}+i \qquad |z| = \sqrt{3+1} = 2$

$\left.\begin{array}{l}\cos\varphi = \frac{-\sqrt{3}}{2}\\ \sin\varphi = \frac{1}{2}\end{array}\right\} \Rightarrow$ II ćw. $\Rightarrow \varphi = \frac{5\pi}{6}$

Zatem $z^{51} = 2^{51}\left[\cos\left(\overset{17}{\cancel{51}}\cdot\frac{5\pi}{\underset{2}{\cancel{6}}}\right) + i\sin\left(51\cdot\frac{5\pi}{6}\right)\right] = 2^{51}\left[\cos\frac{85\pi}{2} + i\sin\frac{85\pi}{2}\right] =$

$= 2^{51}\left(\cos\left(42\pi+\frac{\pi}{2}\right) + i\sin\left(42\pi+\frac{\pi}{2}\right)\right) = 2^{51}\left(\underset{0}{\cos\frac{\pi}{2}} + i\underset{1}{\sin\frac{\pi}{2}}\right) =$

$= \underline{i\cdot 2^{51}}$

4. $z^2+z+1=0$ $\quad \Delta = 1-4=-3 \quad \sqrt{\Delta} = \{-\sqrt{3}i;\ \sqrt{3}i\}$

$z_1 = \frac{-1-\sqrt{3}i}{2} \quad z_2 = \frac{-1+\sqrt{3}i}{2}$

$$\underline{z \in \left\{-\frac{1}{2}-\frac{\sqrt{3}}{2}i;\ -\frac{1}{2}+\frac{\sqrt{3}}{2}i\right\}}$$

5. $\sqrt[3]{8+8i}$ $\quad z := 8+8i \quad |z| = \sqrt{64+64} = 8\sqrt{2}$

$$\left.\begin{aligned} \cos\varphi &= \frac{8}{8\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \sin\varphi &= \frac{8}{8\sqrt{2}} = \frac{\sqrt{2}}{2} \end{aligned}\right\} \Rightarrow \varphi = \frac{\pi}{4}$$

$w_0 = \sqrt[3]{8\sqrt{2}}\left(\cos\frac{\frac{\pi}{4}}{3} + i\sin\frac{\frac{\pi}{4}}{3}\right) = 2\sqrt[6]{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

$w_1 = 2\sqrt[6]{2}\left(\cos\frac{\frac{\pi}{4}+2\pi}{3} + i\sin\frac{\frac{\pi}{4}+2\pi}{3}\right) = 2\sqrt[6]{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) =$

$= 2\sqrt[6]{2}\left(\cos\left(\pi-\frac{\pi}{4}\right) + i\sin\left(\pi-\frac{\pi}{4}\right)\right) = 2\sqrt[6]{2}\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = \sqrt[6]{2}\left(-\sqrt{2}+\sqrt{2}i\right)$

$w_2 = 2\sqrt[6]{2}\left(\cos\frac{\frac{\pi}{4}+4\pi}{3} + i\sin\frac{\frac{\pi}{4}+4\pi}{3}\right) = 2\sqrt[6]{2}\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right)$

$$\underline{\sqrt[3]{8+8i} = \{w_0, w_1, w_2\}}$$