Geom. analityczna 3S poprawa

K1.

1. $A(-3,1)$ $B(5,-3)$ $C(6,4)$

$(x-a)^2+(y-b)^2=r^2$ $S(a,b)$ - środek

$$\begin{cases}(-3-a)^2+(1-b)^2=r^2\\(5-a)^2+(-3-b)^2=r^2\\(6-a)^2+(4-b)^2=r^2\end{cases}\qquad \begin{array}{l}\mathrm{I}\\\mathrm{II}\\\mathrm{III}\end{array}\begin{cases}9+6a+a^2+1-2b+b^2=r^2\\25-10a+a^2+9+6b+b^2=r^2\\36-12a+a^2+16-8b+b^2=r^2\end{cases}$$

$\mathrm{I}\wedge\mathrm{II}$:

$$10+6a-2b+\cancel{a^2}+\cancel{b^2}=34-10a+6b+\cancel{a^2}+\cancel{b^2}$$

$$16a-8b=24\ /:8$$

$$\underline{2a-b=3}$$

$\mathrm{I}\wedge\mathrm{III}$

$$10+6a-2b+\cancel{a^2}+\cancel{b^2}=52-12a-8b+\cancel{a^2}+\cancel{b^2}$$

$$18a+6b=42\ /:6$$

$$\underline{3a+b=7}$$

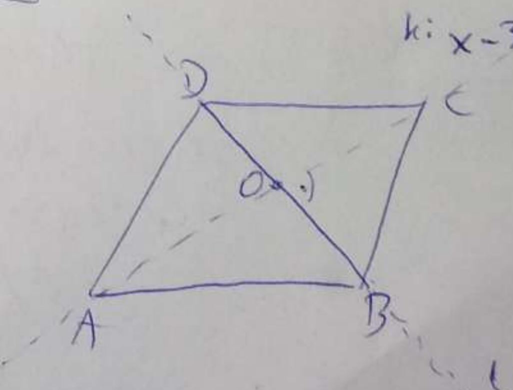
$$+\begin{cases}2a-b=3\\3a+b=7\end{cases}$$

$$5a=10\Rightarrow \underline{a=2}\Rightarrow b=2a-3=4-3=1\Rightarrow S(2,1)$$

$$r^2=10+12-2+4+1=25\Rightarrow r=5$$

$$\underline{o:(x-2)^2+(y-1)^2=25}$$

2. $A(-2,-1)$ $D(2,2)$




pr. BD $\perp$ pr. AC

$l: -3x-y+C=0$

$D\in l \Rightarrow -6-2+C=0\Rightarrow C=8$

$$-3x-y+8=0\ /\cdot(-1)$$

$$3x+y-8=0$$

$$O=k\cap l \Rightarrow \begin{cases}x-3y-1=0\\3x+y-8=0\ /\cdot 3\end{cases}\quad +\begin{cases}x-3y-1=0\\9x+3y-24=0\end{cases}$$

$$10x=25\Rightarrow x=\frac{5}{2}$$

Zatem $y=8-3x = 8-\frac{15}{2}=\frac{1}{2} \Rightarrow O\left(\frac{5}{2},\frac{1}{2}\right)$

• $O = S_{DB}$

$\left(\frac{5}{2},\frac{1}{2}\right)=\left(\frac{2+x_B}{2},\frac{2+y_B}{2}\right) \Rightarrow 5=2+x_B \wedge 1=2+y_B$

$x_B=3 \wedge y_B=-1$

$\boxed{B(3,-1)}$

b) pr. AB

$y=ax+b$

$a=\frac{-1+1}{3+2}=0$

$y=b$

$-1=b \Rightarrow \underline{y=-1}$

$\Updownarrow$

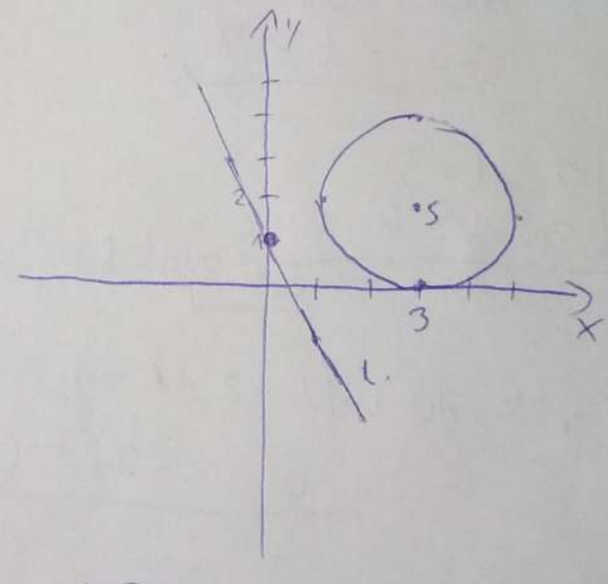
$y+1=0$

$d(pr.AB, pr.CD) = d(D, pr.AB) = \frac{|2+1|}{\sqrt{1}} = 3$

3. $o_1: (x-3)^2+(y-2)^2=4 \quad r=2 \quad S(3,2)$

$l: \cancel{y=} 2x+y-1=0$

$y=-2x+1$




Brak punktów wspólnych.

b) $k: 2x-y+c=0$

$d(S,k)=r$

$\frac{|6-2+c|}{\sqrt{4+1}}=2 \Rightarrow |4+c| = 2\sqrt{5}$

$4+c=2\sqrt{5} \vee 4+c=-2\sqrt{5}$

$c=2\sqrt{5}-4 \vee c=-2\sqrt{5}-4$

$\underline{c\in\{2\sqrt{5}-4, -2\sqrt{5}-4\}}$

c) $k: 2x-y+2=0 \; (\Leftrightarrow y=2x+2)$

Trzeba środek S odbić symetrycznie względem prostej k.

2)

• pr. $SS'$

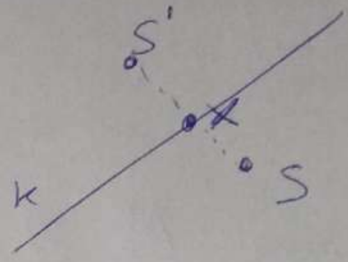
$y = ax + b$

$a = -\frac{1}{2}$

$y = -\frac{1}{2}x + b$

$2 = -\frac{3}{2} + b \Rightarrow b = \frac{7}{2}$

$y = -\frac{1}{2}x + \frac{7}{2}$




$X = \text{pr. } SS' \cap k$

$\begin{cases} y = -\frac{1}{2}x + \frac{7}{2} \\ y = 2x + 2 \end{cases} \Rightarrow \quad -\frac{1}{2}x + \frac{7}{2} = 2x + 2 \quad / \cdot 2$

$-x + 7 = 4x + 4$

$3 = 5x$

$x = \frac{3}{5} \Rightarrow y = 2 \cdot \frac{3}{5} + 2 = \frac{6}{5} + 2 = \frac{16}{5}$

$\underline{X\left(\frac{3}{5}, \frac{16}{5}\right)}$

$X = S_{SS'}$

$\left(\frac{3}{5}, \frac{16}{5}\right) = \left(\frac{3 + x_1}{2}, \frac{2 + y_1}{2}\right)$

$\frac{3}{5} = \frac{3 + x_1}{2} \quad \wedge \quad \frac{16}{5} = \frac{2 + y_1}{2} \Big/ \cdot 10$

$6 = 15 + 5x_1 \quad \wedge \quad 32 = 10 + 5y_1 \quad \Rightarrow S'\left(-\frac{9}{5}, \frac{22}{5}\right)$

$x_1 = -\frac{9}{5} \qquad y_1 = \frac{22}{5}$

$o_2: \left(x + \frac{9}{5}\right)^2 + \left(y - \frac{22}{5}\right)^2 = 4$

4. $A(-3,4) \quad B(1,0) \quad C(6,1)$

$\vec{v} = \vec{AC} + \vec{BC} = [9, -3] + [5, 1] = [14, -2] \qquad |\vec{v}| = \sqrt{196 + 4} = \sqrt{200} = \underline{10\sqrt{2}}$

$\vec{AS} = 3 \cdot \vec{AB}$

$[x_S + 3; y_S - 4] = 3 \cdot [4, -4]$

$\begin{cases} x_S + 3 = 12 \\ y_S - 4 = -12 \end{cases} \quad \begin{cases} x_S = 9 \\ y_S = -8 \end{cases} \Rightarrow \underline{S(9, -8)}$

4. (II termin)

$P((x,y)) = (y+4, -x+6)$

a)

D: Bierzemy $A(x_A, y_A)$, $B(x_B, y_B)$. Pokażemy, że $|AB| = |A'B'|$, gdzie $P(A)=A'$, $P(B)=B'$.

$A' = (y_A+4; -x_A+6)$ $B' = (y_B+4, -x_B+6)$

$|A'B'| = \sqrt{(y_B+4-y_A-4)^2 + (-x_B+6+x_A-6)^2} = \sqrt{(y_B-y_A)^2+(-x_B+x_A)^2} =$

$= \sqrt{(y_B-y_A)^2+(x_B-x_A)^2} = |AB|$ $\square$

b) $o: x^2+y^2-4x+6y+12=0$ $S(2,-3)$ $r=\sqrt{4+9-12}=1$

$S' = P(S) = (-3+4, -2+6) = (1,4)$ nowy środek.

$o_1: (x-1)^2+(y-4)^2 = 1$