PIOTR MAGIERA

$n \to 3 \bmod 5 + 1 = 4$

$\Phi = u \qquad \Omega = (0,3)$

$u'' = 4\pi G \rho \quad / \cdot v \quad / \int dx$

$\int_0^3 u'' v \, dx = \int_0^3 4\pi G \rho v \, dx$

$\underbrace{[u'v]_0^3}_{=0} - \int_0^3 u'v' \, dx = 4\pi G \int_0^3 \rho v \, dx$

(bo $v \in V$ $\Rightarrow v(0) = v(3) = 0$)

$\Downarrow$

$-\int_0^3 u'v' \, dx = 4\pi G \int_0^3 \rho v \, dx$

0

$u = \bar{u} + w$

$v, w \in \mathcal{U}$

$\mathcal{U} \subset H_1$

$\mathcal{U} = \{f \in H^1 : f(0) = f(3) = 0\}$

$\begin{cases} \bar{u}(0) = 5 \\ \bar{u}(3) = 4 \end{cases}$

4.4 Potencjał grawitacyjny

$$\frac{d^2\Phi}{dx^2} = 4\pi G\rho(x)$$

$\Phi(0) = 5 \quad u(0) = 5$

$\Phi(3) = 4 \quad u(3) = 4$

$$\rho(x) = \begin{cases} 0 & \text{dla } x \in [0,1] \\ 1 & \text{dla } x \in (1,2] \\ 0 & \text{dla } x \in (2,3] \end{cases}$$

zatem niech $\bar{u}(x) = 5 - \frac{1}{3}x$ (*)

$g(x) \neq 0 \iff x \in (1;2\rangle$ zatem równanie ma postać

$$-\int_0^3 u'v'\,dx = 4\pi G \int_1^2 v\,dx \qquad \left(B(u,v) = L(v) \right)$$

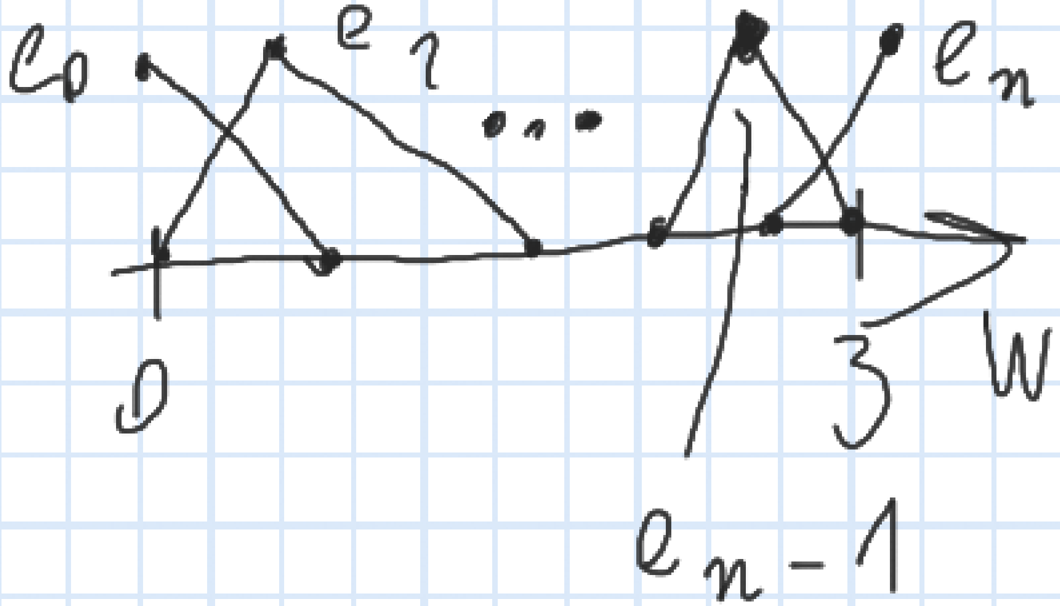
$$u = \bar{u} + w \overset{(*)}{\Longrightarrow} u = 5 - \frac{1}{3}x + w \quad \text{stąd} \quad u' = -\frac{1}{3} + w'$$

$$\int_0^3 \left(\frac{1}{3} - w'\right) v'\,dx = 4\pi G \int_1^2 v\,dx$$

$$-\int_0^3 w'v'\,dx = 4\pi G \int_1^2 v\,dx - \frac{1}{3}\int_0^3 v'\,dx \quad \left(B(w,v) = \tilde{L}(v) \right)$$

FORMA WARIACYJNA

n - dowolne



$$w \approx w_h = \alpha_0 e_0 + \alpha_1 e_1 + \dots + \alpha_{n-1} e_{n-1} + \alpha_n e_n$$

$e_0, e_n \notin \mathcal{U}$ (bo $e_0(0) \neq 0$, $e_n(3) \neq 0$)

zatem $\alpha_0 = \alpha_n = 0$ czyli

$$w \approx w_h = \alpha_1 e_1 + \dots \alpha_{n-1} e_{n-1}$$

zatem należy rozwiązać układ równań o postaci macierzowej

$$\begin{bmatrix} B(e_1, e_1) & \cdots & B(e_{n-1}, e_1) \\ \vdots & \ddots & \vdots \\ B(e_1, e_{n-1}) & \cdots & B(e_{n-1}, e_{n-1}) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_{n-1} \end{bmatrix} = \begin{bmatrix} \tilde{L}(e_1) \\ \vdots \\ \tilde{L}(e_{n-1}) \end{bmatrix}$$

$$e_i = \begin{cases} \frac{x-x_{i-1}}{x_i-x_{i-1}} & \text{dla } x \in (x_{i-1}, x_i) \\ \frac{x_{i+1}-x}{x_{i+1}-x_i} & \text{dla } x \in (x_i, x_{i+1}) \end{cases}$$

$$x_i = \frac{3}{n} \cdot i \qquad h = \frac{3}{n}$$

$$e_i' = \begin{cases} \frac{1}{h} & \text{dla } x \in (x_{i-1}, x_i) \\ -\frac{1}{h} & \text{dla } x \in (x_i, x_{i+1}) \\ 0 & \text{wpp.} \end{cases}$$

gdzie $$B(w,v) = -\int_0^3 w'v'\,dx$$

$$\tilde{L}(v) = 4\pi G \int_1^2 v\,dx - \frac{1}{3}\int_0^3 v'\,dx = \boxed{0}$$

dla $\tilde{L}(e_k)$ gdzie $k = 1, \dots, n-1$

(patrz wzór e_i')

a następnie obliczyć u korzystając ze wzorów

$$w \approx w_h = \alpha_1 e_1 + \dots + \alpha_{n-1} e_{n-1} \quad \wedge \quad \bar{u} = 5 - \frac{1}{3}x$$

$$u = \bar{u} + w$$

można łatwo zauważyć, że $B(e_i, e_j) \neq 0 \iff |i-j| \leqslant 1$

dla $i+1=j$ $\quad B(e_i,e_j) = -\int_{x_i}^{x_{i+1}} -\frac{1}{h^2}\,dx = \int_{x_i}^{x_{i+1}} \frac{1}{h^2}\,dx$

dla $i=j$ $\quad B(e_i,e_j) = \int_{x_{i-1}}^{x_{i+1}} \frac{1}{h^2}\,dx$

dla $i=j+1$ $\quad B(e_i,e_j) = -\int_{x_{i-1}}^{x_i} -\frac{1}{h^2}\,dx = \int_{x_{i-1}}^{x_i} \frac{1}{h^2}\,dx$

wpp $\quad B(e_i,e_j)=0$