

Exercise 8.4

Consider the filtration data of Data Set G. Open the Excel workbook **Exe8.4G.xlsx** which contains these data from the Exercises folder.

Assuming the data to be suitably distributed, complete a two-tailed test of whether the population mean impurity differs between the two filtration agents, and interpret your findings.

Results

t-Test: Paired Two Sample for Means

	<i>Agent1</i>	<i>Agent2</i>
Mean	8,25	8,683333333
Variance	1,059090909	1,077878788
Observations	12	12
Pearson Correlation	0,901055812	
Hypothesised Mean Difference	0	
df	11	
t Stat	-3,263938591	
P(T<=t) one-tail	0,003772997	
t Critical one-tail	1,795884819	
P(T<=t) two-tail	0,007545995	
t Critical two-tail	2,20098516	

Difference in Means -0,433333333

The variances are approximately 1.06 and 1.08 for Agent 1 and Agent 2 respectively, indicating a relatively consistent spread of scores around the mean within each group.

The t statistic of -3.26 is the test statistic calculated for this t-test. A negative value indicates that the mean of Agent 1 is lower than the mean of Agent 2.

Two-Tailed Test:

The p-value for the two-tailed test is 0.0075, which is suggesting a statistically significant difference in the means at 99% confidence level

The critical t-value for the two-tailed test is 2.20, and again the absolute value of the t statistic (3.26) is greater than this critical value, further confirming the significance of the difference.

Difference in Means:

The difference in means is -0.433, which indicates that on average, Agent 1 scored 0.433 points lower in impurity than Agent 2, therefore Agent 1 should be preferred.

Exercise 8.5

Recall that in Exercise 8.4, a two-tailed test was undertaken of whether the population mean impurity differs between the two filtration agents in Data Set G.

Suppose instead a one-tailed test had been conducted to determine whether Filter Agent 1 was the more effective. What would your conclusions have been?

One-Tailed Test:

The p-value for the one-tailed test is 0.0038, this suggests a statistically significant difference in the means at a 99.5% confidence level.

The critical t-value for the one-tailed test is 1.80, and the absolute value of the t statistic (3.26) is greater than this critical value, further confirming the significance of the difference.

Therefore the conclusions after one-tailed test only enhance the previous conclusions that Agent 1 is the preferred choice as it contains lower impurity levels.

Exercise 8.6

Consider the bank cardholder data of Data Set C. Open the Excel workbook **Exe8.6C.xlsx** which contains this data from the Exercises folder.

Assuming the data to be suitably distributed, complete an appropriate test of whether the population mean income for males exceeds that of females and interpret your findings. What assumptions underpin the validity of your analysis, and how could you validate them?

F-Test Two-Sample for
Variances

	<i>Male</i>	<i>Female</i>
Mean	52,91333333	44,23333333
Variance	233,1289718	190,1758192
Observations	60	60

df	59	59
F	1,225860221	
P(F<=f) one-tail	0,21824624	
F Critical one-tail	1,539956607	

p2 0,43649248

Given the $F = 1.226$ with 59 degrees of freedom but $p\text{-value} = 0.43$ the F ratio result is not statistically significant, therefore we can proceed with the equal variances t test.

t-Test: Two-Sample Assuming Equal Variances

	<i>Male</i>	<i>Female</i>
Mean	52,91333333	44,23333333
Variance	233,1289718	190,1758192
Observations	60	60
Pooled Variance	211,6523955	
Hypothesized Mean Difference	0	
df	118	
t Stat	3,267900001	
P(T<=t) one-tail	0,000709735	
t Critical one-tail	1,657869522	
P(T<=t) two-tail	0,00141947	
t Critical two-tail	1,980272249	

Difference in Means 8,68

This test has a t Stat of 3.27 with 118 degrees of freedom.

One-Tailed Test:

The p-value for the one-tailed test is 0.0007, which is significantly less than 0.05. This indicates a statistically significant difference in means in one direction (Male earn more than Female).

The critical t-value for the one-tailed test is 1.66, and the t statistic 3.27 is greater than this critical value, further confirming the significance of the difference.

Two-Tailed Test:

The p-value for the two-tailed test is 0.0014, which is also significantly less than 0.05, indicating a statistically significant difference in means.

The critical t-value for the two-tailed test is 1.98, and again the t statistic 3.27 is greater than this critical value, further confirming the significance of the difference.

Difference in Means:

The difference in means is 8.68, which suggests that, on average, the Male's salary is 8.68 currency units higher than the Females.

In conclusion, there is a statistically significant difference between the mean salary of Male and Female, with Male having a higher mean. This is evident from both the one-tailed and two-tailed tests as the p-values are below the significance threshold of 0.05, and the t statistic is greater than the critical t-value in both cases.