BIG RISK[1]

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I show how to replicate a large family of high-frequency measures of realised return variation using dynamic option portfolios. With this technology investors can generate optimal hedging payoffs for realised variance and several measures of realised jump variation in incomplete option markets. These trading strategies induce excess payoffs that are direct compensation for second- and higher order risk exposure in financial markets. Sample averages of these excess payoffs are natural estimates of risk premia associated with second- and higher order risk exposures. In an application to the market for short-maturity European options on the S&P500 index I obtain new important evidence about the pricing of variance and jump risk. I find that the variance risk premium is positive during daytime, when the hedging frequency is high enough, and negative during night-time. Similarly, daytime profits are grater in absolute value than night-time losses. Compensation for big risk is mostly available overnight. The premium for jump skewness risk is positive, while the premium for jump quarticity is negative (contrary to variance, also during the trading day). The risk premium for big risk is also concentrated in states with large past big risk realisations.

Keywords: High-frequency trading, Jump risk, VIX, Divergence, Variance risk premium.

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1. INTRODUCTION

The financial distinction between run-of-the-mill price variation and "disastrous events" is usually made by introducing risks that scale with time (small risks) and risks that are significant even at minuscule decision horizons (big risks). Aït-Sahalia (2004) succinctly states that "...the ability to disentangle jumps from volatility is the essence of risk management, which should focus on controlling large risks leaving aside the day-to-day Brownian fluctuations". In this context, jumps are naturally identified as big risk, and diffusive variation as small risk. While it is widely accepted in the literature that investors exhibit different attitudes to small and big risk, the possibility of disentangling them for risk sharing purposes has not yet been fully explored.

In this paper I introduce a general trading technology which allows for optimal hedging of a family of realised risk measures in incomplete option markets. The technology introduces two core innovations. First, it improves upon commonly employed techniques of hedging non-linear payoffs by providing a convenient optimal option portfolio for a given set of observed price quotes. Second, through a dynamic generalization, it renders many important measures of realised risk directly available to investors, as hedgeable payoffs.[1] Equipped with this technology, I analyse the traded properties of higher order risk in the market for one-week maturity European index options at the CBOE.[2] The CBOE is one of the world's most busy derivative trading venues, and the available data has enormous information content. I find that the compensation for variance risk is positive at such short horizons, and that in disaggregated results it is positive during market opening hours and negative during overnight periods, the daytime effect being larger in magnitude. Furthermore, I document the existence of compensation for directional jump risk, which is similarly concentrated in overnight periods, and increases substantially after the S&P500 index is subject to a large shock.

The market for very short maturity index options remains severely under-studied at a time when its importance is soaring. Despite the availability of data, most research to-date treated it with distrust and often discarded it. To a large extent that is because, option markets have been analysed through the lens of models, and the short end of the term structure turned out to be, to put it mildly, problematic to fit. Jump risk, however, has been considered necessary for explaining many key observations about the option market, and in the extant mathematical framework it is even more so at short maturities. The prices of weekly options, analysed with the use of my technology offer the sharpest insights into investors' perception of jump risk, unobscured by modelling assumptions.

[1] I give the exact sense in which the resulting trading strategies provide hedges in Section 2.

[2] At the time of writing, options with the weekly Friday settlement calendar constituted between 15% and 40% of trading volume in all SPX options. In 2016, the CBOE followed up on their success and introduced weekly options settled on Mondays and Wednesdays.

I exploit the availability of high-frequency records of trades and quotes and show how the intuition about realised variation measures can be translated to analysing option data.

Realised variation measures were quickly embraced by the asset pricing literature, which treated them as risk factors in attempts to resolve a number of asset pricing puzzles. Tradability of the newly introduced risk factors remained an open question. In cases like variance swaps, where a tradable representation is available, issues arise about the discrepancy between the trade's complete-market form, and its feasible incomplete-market implementation. This study is built on the premise of ensuring that the hedging strategies are the *exact* representations of the realised variation measures used for measuring risk premia for small and big risk in the sense that the payoffs of the strategies at settlement should be as close as possible to the realised variation calculated directly from high-frequency price records. Those settlement payoffs, taken together with the accumulated cost of establishing option positions, form excess payoffs.

I will illustrate the issues with an example based on the ubiquitous synthetic variance swap. In such a hedge, a single option position is established at its inception, and an associated trading strategy is implemented in the forward market for the underlying. At maturity, the accumulated forward and option positions are settled and the payoffs should in principle be equal to realised variance calculated directly from the forward prices. Once the cost of the initial option portfolio is subtracted, the analyst has an excess payoff on the table. I form the swaps with the approach prevailing in the literature and compare the resulting swap rates and payoffs with those yielded by my technology.[3] The left panel of Figure 1 shows the comparison of hedging errors, measured as the absolute value of the difference between realised variance and the traded payoff. In annualised terms, the often used approach yields absolute hedging errors ranging up to 1 percentage point in variance terms, while my technology reduces them by an order of magnitude. The right panel of Figure 1 plots the difference between the commonly calculated variance swap rate, and my approach. The commonly calculated swap rates are lower, on average. As a result, an unconditional estimate of the variance risk premium is 10% off when the standard technology is used. This is not innocuous: the unconditional estimate of the premium is biased. The main conclusion is that in the common approach the trading strategy *does not hedge realised variance*, but also a noise component which should be outside of the researcher's focus. The noise is uncorrelated with realised variance and the market return, but its magnitude increases with realised variance. Ensuring that the settlement payoff is as close as possible to the hedged realised variation measure gives the researcher confidence that she is indeed measuring a *meaningful* risk premium. In this paper I show that this question

[3]An exact description of the procedures is available in Section 4.

becomes crucial when measuring the premia for higher order risks, where issues related to replication accuracy are a first-order effect.

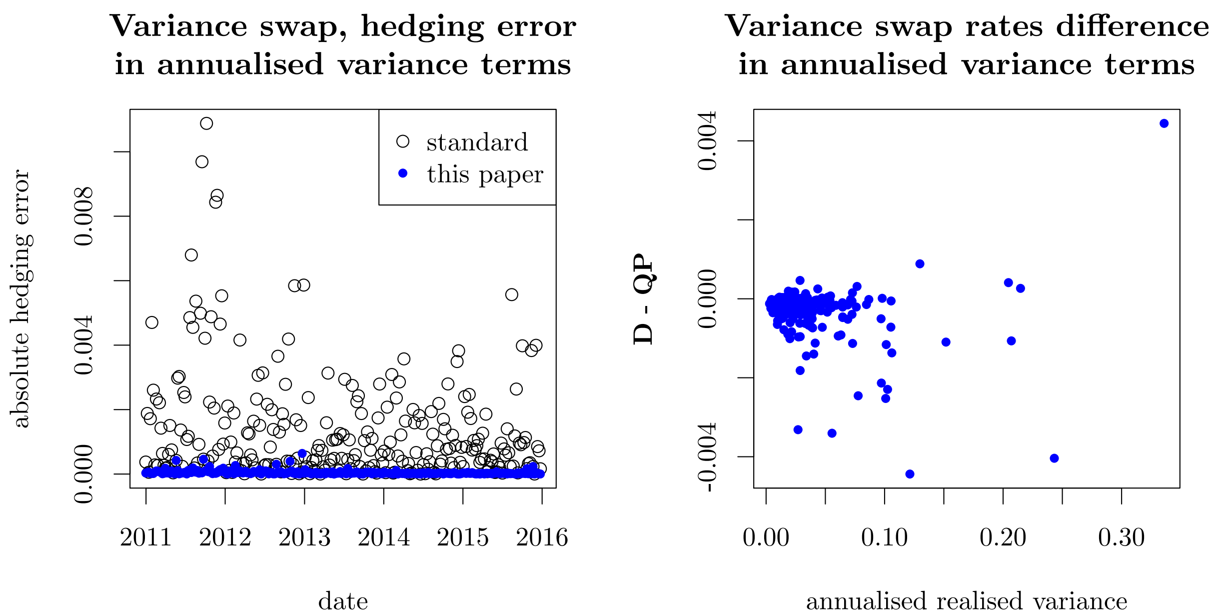

Figure 1: Trading technology: consequences of inadequate option hedging.

Jump variation proves especially significant when explaining the features of both stock and derivative markets, and many recent studies suggest that a premium for jump variation is the dominant component of both the equity and variance risk premia. In the literature, the standard approach to isolating a jump premium is via holding a delta-hedged, short-maturity, deep out-of-the-money option. The intrinsic difficulty of estimating such a risk premium lies in the fact that such strategies rarely pay off, yielding extremely skewed return distributions. The single option has to be sufficiently far out of the money as to avoid payoffs in cases of increased market volatility. Furthermore, this strategy does not hedge the investor against jump movements, that are subsequently reversed, so that ultimately the option expires worthless. To the contrary, the technology introduced in this paper allows the investor to hedge against adverse events on a continuous basis, through sequential rebalancing of the initial swap portfolio. In an event such as the Flash Crash on May 6$^{\text{th}}$ 2010, the holder of an out-of-the-money, delta-hedged put would incur losses stemming from the delta-hedging error, and her hedge of the final asset value would expire worthless. An investor following the hedging strategy described in this paper would accrue a payoff when the market was collapsing, and possibly when it was recovering, depending on her hedging frequency.

The hedging technology has one additional advantage over a standard approach, exemplified before with variance swaps. Allowing for sequential option trades makes it possible to hedge realised variation measures over arbitrary periods before option maturity while maintaining the benefit of exact hedging. I exploit this feature of the technology in order to investigate the difference in risk pricing between daytime and overnight trading. Investors holding positions in realised variation measures overnight are exposed to variance and higher order risk differently. While in daytime trading they can dynamically hedge or close the position altogether, overnight they face far more important constraints to trading, and much less efficient price discovery, if the market is open at all. The patterns that I find are striking. Long variance positions[4] are profitable, on average, during daytime trading, but in night-time trades they entail losses. Evidence on aggregate variance risk premia for maturities longer than two weeks showed that the premia were uniformly negative, which gave the variance swap the interpretation of an insurance contract. Not only is the insurance feature present exclusively in night-time trading, but also the daytime profits from long positions are greater than potential overnight losses: the aggregate variance risk premium over weekly horizons is positive. Where premia for jumps – big risk – are considered, similar patterns arise. Excess payoffs from long position in jump skewness and short in jump quarticity are significantly higher in overnight trades than in daytime trades. Additionally, the excess payoffs from such trades greatly increase after large stock price movements, even if subsequent big risks do not materialise. The effect of large innovations to the price of the index is thus clearly seen in a persistently elevated level of premia for big risk hedges.

In this paper I only consider hedging realised variation measures which are calculated indiscriminately for positive and negative returns. The technology is flexible enough, however, to extend the study towards exactly hedging variation measures such as realised semi-variances or third power of only negative jumps, for example. Such quantities merit extra attention, as evidenced by many studies of "good" and "bad" volatility.

1.1. *Literature review*

This paper is a direct off-shoot fo the literature on model-free hedging strategies. Since Neuberger (1994) posited the creation of a log-return derivative, the literature gathered steam, culminating

[4]A long position in a realised variation measure is a hedging strategy which at the settlement of the option and forward contracts pays the investor the accrued realised measure, if the latter is positive, and is paid by the investor otherwise.

in several important studies. Jiang and Tian (2005) implemented Britten-Jones and Neuberger (2000)'s option-based estimator of model-free implied volatility, i.e. essentially the VIX^2 index. The most comprehensive, model-independent study of variance risk premia came from Carr and Wu (2009a), who established the negative sign of the index variance risk premium, however used the standard approach to forming option portfolios, and calculated the floating leg of the swaps directly from the price process, instead of from the instrument payoffs; furthermore they noted that their floating leg corresponds only approximately to the theoretical option payoff in the presence of jump risk. Martin (2012) proposed a change in definition of the option portfolio, and of the floating leg, this time calculated from payoffs, in order to obtain a variance swap whose swap leg exactly priced the floating leg even in the presence of jumps. Further on, Bondarenko (2014) combined the previous approaches to develop a fully tradable setting, which also paves the way towards fully hedgeable skewness measures. All these approaches are generalised by the concept of realised divergence, and associated higher-order measures, introduced by Schneider and Trojani (2015a). This paper takes the latter as a foundation to build upon, taking the realised divergence measures, recalled in Section 3.1, as primitives. A separate branch of the literature focused on providing pricing bounds for the prices of certain realised variation hedges, for example Hobson and Klimmek (2012).

The semi-martingale model of asset prices has been the fundamental tool for defining and identifying small and big risks, and – in various guises – it was central for establishing the importance of big risks. The observation that stock returns are leptokurtic (e.g. Fama, 1965, who reports earlier findings by Moore (1962)) encouraged the first forays into jump modelling (see e.g. Press (1967)), based on estimation of simple semi-martingale models on low frequency data. With time, the frequency of returns and complexity of models and methods increased (Aït-Sahalia, 2004; Bates, 2012), and more model-agnostic methods for intra-day data, based on estimating quadratic variation, were developed (Barndorff-Nielsen and Shephard, 2006; Huang and Tauchen, 2005; Andersen et al., 2011).[5] Option price data has been used rarely in non-parametric studies (Li, 2011; Bollerslev and Todorov, 2011b,a) and often in model-based investigations (Eraker et al., 2003; Andersen et al., 2015a,b). All of these studies concluded that big risk is a significant contributor to the variation of asset prices (between 5 and 20% in most studies), and that big risk goes a long way towards explaining the dynamics of option prices. Lately, evidence arose from the analysis of intra-day data at tick frequencies which indicates that those estimates were inflated by an order of magnitude because of model mis-specification or erroneous classification of market movements as jumps, as in

[5] The high-frequency methods do not posit parametric models for the stochastic volatility process or the jump distribution, but are built upon the foundation of the semi-martingale model.

Christensen et al. (2014) and Bajgrowicz et al. (2015). Nevertheless, Christensen et al. write that such erroneous classification is the consequence of *bursts in volatility*, when volatility suddenly rises and falls so fast, that on a given time scale continuous, yet abrupt price movements are indistinguishable from jumps. Thus, even though the latest evidence jump variation markedly diminishes its significance, it also shows that the *perception* of what constitutes a jump movement depends on the investors' profile. For a dynamic investor with a trading horizon of minutes or hours bursts in the drift or volatility of the underlying diffusion still constitute big risks. Furthermore, Liu et al. and Christensen et al. point out the relation of big risk realizations to drops in liquidity, further emphasizing the inability to hedge them at the time adverse events hit.

A broad, yet more recent body of work investigates the pricing of big risk, and its importance in portfolio formation. Liu et al. (2003) underline the infinitesimal impact of big risk on investor wealth, and the potential importance (and difficulty) of hedging jump risk with options. In studies of the cross section of stock returns, Bollerslev et al. (2016) concluded that the exposure to small risks is essentially not compensated, while the exposure to big risk – of jump returns and overnight returns – is. Evidence from the aforementioned option pricing literature also indicates that jump risk carries significant premia.

While those studies offer a way of measuring big risk and shed some light on how investors price it, they offer little in way of hedging it separately from small risk. The access to sharing big risk is restricted in financial markets. Investors can purchase individual out-of-the-money, short-maturity options, however those do not protect against the *immediate* consequences of jumps. They can purchase variance swaps, which offer protection from immediate events, but do not discriminate between small and big risk. Through the dynamic option trading strategies one can achieve both objectives, at the price of exposing oneself to cumulative weighted risk of the price of volatility.

The remainder of this paper is laid out as follows. Section 2 makes precise the idea of tradable realised variation measures. In Section 3 I describe a family of tradable realised variation measures. In Section 4 I discuss the choice of strategies, the methods for rendering the exercise feasible, and the properties of the data set, as well as replication accuracy. I describe the empirical results in Section 5, and summarize the conclusions in Section 6.

2. REALIZED VARIATION IN ASSET PRICING

In this section I stress the importance of obtaining *tradable* counterparts of the realized variation measures in order to ensure that the resulting estimates of higher order risk premia are meaningful.

With log returns $r_{t_j} : \ln F_j/F_{j-1}$ realized variance is defined as,

$$RV_{t,t+1} := \sum_{t \le t_j \le t+1} \ln^2 r_{t_j} \to \int_t^{t+1} \sigma_s^2 ds + \sum_{t \le s \le t+1} J_s^2 =: QV_{t,t+1}. \quad (2.1)$$

RV was initially a concept confined to the domain of financial econometrics. Over the course of twenty years of development, it gained importance in empirical asset pricing. Many studies have observed that the *variance risk premium*,

$$VRP_t := \mathbb{E}_t^{\mathbb{P}} [QV_{t,t+1}] - \mathbb{E}_t^{\mathbb{Q}} [QV_{t,t+1}]$$

has important predictability properties for both future volatility and index stock returns, and that it is important for the pricing of the cross-section of stocks. The rise of VRP to its current prominence was only possible because researchers found a way of approximately hedging the $QV_{t,t+1}$ payoff with extant financial instruments. The price of realized variance, $\mathbb{E}_t^{\mathbb{Q}} [QV_{t,t+1}]$, can be approximately calculated as the price of an option portfolio with readily calculable weights. The final payoff combines the option settlement with this of a series of positions in the underlying market. The VRP is typically available to investors through *variance swaps*, defined as paying realized daily squared log returns in exchange for a fixed amount, paid by the buyer of the swap upon inception. This approach is not exact, as explained in Carr and Wu (2009b), and as I show below in Example 1, in the sense that there exists a discrepancy between what the seller of the swap is contractually obliged to pay the buyer, and what she can hedge in the market.

Nevertheless, since the success of realized variance as an asset pricing concept, many researchers shifted their interest towards studying other realized variation measures. At the same time, after Carr and Madan (2001) showed how to replicate non-linear payoffs with European options, another strand of literature sprang up, investigating the relation between the prices of various non-linear replicating portfolios and the cross-section of stock returns (ANG et al., 2006; Chang et al., 2009; Goyal and Saretto, 2009; Amaya et al., 2011, , among others). These two branches of the literature should be meeting in the middle: finding prices that correspond to selected realized variation measures allows to calculate excess payoffs and estimate risk premia. This is because in empirical asset pricing, tradable quantities are of special importance, from the point of view of the financial theory (they allow for a direct representation of risk factors in the market). The concept of *alpha* as extraordinary returns beyond the fundamental compensation for risk is firmly based upon the idea that all risk factors under consideration are actually tradable. Unfortunately, it is not straightforward to engineer a trading strategy that replicates an arbitrary realized variation measure.

Even in the case of variance swaps, the literature relied heavily on approximations. The simplest – and most widely known – example of a variation-replicating strategy is the VIX^2-based variance swap. Typically, researchers interested in the variance risk premium calculate the payoffs of such swaps as

$$RV_{t,T} - VIX^2_{t,T}. \tag{2.2}$$

In pure diffusion models the $VIX^2_{t,T}$ is the exact price of $RV_{t,T}$. However, in the presence of jumps there is a wedge between what the option portfolio prices, and the limit of the floating leg of the swap. The wedge can be eliminated by slightly tweaking the definition of the floating leg, or equivalently, of the hedged realised variation measure. In order to put the VIX^2-based variance swap firmer on asset pricing footing, I present Schneider and Trojani's example of how the floating leg, or equivalently, realized measure, should be defined so that the variance swap is *exactly* priced by the VIX^2 option portfolio.

EXAMPLE 1 (The exact VIX^2 variance swap) Schneider and Trojani (2015a) show that an option portfolio with long positions $\phi(K)$ in *out of the money* options (with prices $O_T(K)$ at strike K):

$$\phi(K) = \frac{2}{K^2}, \text{ price } \int_0^\infty \phi(K) O_T(K) dK =: VIX^2 \text{ and payoff } -2\left(\ln F_T - \ln F_0 - \frac{1}{F_0}(F_T - F_0)\right), \tag{2.3}$$

coupled with the dynamic trading strategy:

$$\sum_{j=1}^{n} \underbrace{\left(\frac{2}{F_{j-1}} - \frac{2}{F_0}\right)}_{\text{trading weights}} \underbrace{(F_j - F_{j-1})}_{\text{forward position}}, \tag{2.4}$$

pays at maturity the difference between the Itakura and Saito (1968) divergence of the stock price the price of the initial option portfolio:

$$\underbrace{\sum_{j=1}^{n} 2\left[-\ln F_j/F_{j-1} + (F_j/F_{j-1} - 1)\right]}_{\text{realized divergence}} - \int_0^\infty \phi(K) O_T(K) dK. \tag{2.5}$$

The continuous-time limit of the divergence inside the summation in (2.5) is

$$\int_0^T \sigma_s^2 ds + \sum_{0 \le s \le T} 2\left[-\ln F_s/F_{s-} + (F_s/F_{s-} - 1)\right],$$

and the jump terms are of leading order $\ln^2 F_s/F_{s-}$. The divergence in the summation (2.5) is the sum of the option payoff in (2.3) and accumulated forward payoffs (2.4). The strategy defined by (2.3)-(2.4) is *model independent*: the replication argument is valid for any no-arbitrage market set-up and the continuous-time limit is valid in a semi-martingale setting. Equation (2.3) is the definition of the square of the VIX index (CBOE (2000)) and as a consequence VIX-based variance swaps offer exact replication of realised divergence.

Example 1 is firmly grounded in a complete option market setting, i.e. it assumes that a continuum of options is purchased or sold at the inception of the hedging strategy. In reality, an investor at the CBOE has to form a replicating portfolio for the non-linear payoff with a finite (albeit large) number of options. In the introduction I already hinted at the fact that the choice of the incomplete market replicating option portfolio is not trivial, and may have significant consequences for the measured risk premium. I defer further consideration of this subject to Section 4.

2.1. *Dynamic tradability*

The trading strategy in Example 1 consists of a single option trade, at inception, accompanied by sequential trading in the (forward on the) underlying asset. Although this framework offers many interesting extensions, in this paper I advocate the use of strategies involving *both* dynamic option and forward trading. I start from two fundamental results. First, Carr and Madan's replication formula, which is essential to understand how the derivatives are traded when the investor wants to replicate a realized variation measure. Second, from the observation by Schneider and Trojani (2015a) that Carr and Madan's approach is strongly related to the information-theoretic concept of *divergence*.

At time t_j in a complete option market[6] with maturity T, for an arbitrary twice-differentiable function $g : \mathbb{R}_+ \to \mathbb{R}$, it is possible to form an option portfolio paying *at maturity* the Bregman (1967) divergence of g:

$$\begin{aligned}(2.6)\qquad G(F_T, F_{t_j}) :&= \int_0^\infty \left[g''(K)(K - F_T)\mathbf{1}_{\{F_T \le F_{t_j}\}} + g''(K)(F_T - K)\mathbf{1}_{\{F_T \ge F_{t_j}\}} \right] dK \\ &= g(F_T) - g(F_{t_j}) - g'(F_{t_j})(F_T - F_{t_j}).\end{aligned}$$

The option portfolio is comprised of positions of size $g''(K)dK$ in put (call) options for $K \le F_{t_j}$ ($K > F_{t_j}$). The price of the non-linear payoff is calculated simply as a weighted average of observed option prices:

$$(2.7)\qquad \mathcal{G}_{t_j} := \int_0^\infty g''(K) O_{t_j}(K) dK.$$

[6] If a continuum of options on the forward is available with strikes ranging from 0 to ∞.

A long position (i.e. a purchase of options with positive weights, and sale of options with negative weights) in the option portfolio pays exactly (2.6) at maturity.

Equipped with this concept of an option trade, I define the dynamic strategy. Let δ, ϕ be sequences of real numbers, potentially depending on F_{t_j} or other information available at time t_j.[7] With these tools one can form *dynamic option trading strategies*

$$\sum_{j=0}^{n-1} \phi_{t_j} \left[G(F_T, F_{t_j}) - \mathcal{G}_{t_j} \right] + \delta_{t_j}(F_T - F_i). \tag{2.8}$$

$\mathcal{G}_{t_j}$ was defined as the (t_j, T) forward price of $G(F_T, F_{t_j})$, and under no-arbitrage[8] there exists a forward-neutral measure $\mathbb{Q}_T$ such that $\mathcal{G}_{t_j} = \mathbb{E}^{\mathbb{Q}_T}\left[G(F_T, F_{t_j}) | \mathcal{F}_{t_j} \right]$. I decompose the payoff of (2.8) into two components. The *settlement payoff*

$$\sum_{j=0}^{n-1} \phi_{t_j} G(F_T, F_{t_j}) + \delta_{t_j}(F_T - F_{t_j}), \tag{2.9}$$

consists of the trading gains in the forward market and the settlement payoffs of the option positions. The *aggregate cost*

$$\sum_{j=0}^{n-1} \phi_{t_j} \mathcal{G}_{t_j} \tag{2.10}$$

represents the total financial outlay, uncertain at time t_0, that the investor has to make in order to obtain the settlement payoff. The distinction is important: I design strategies whose settlement payoff corresponds exactly to realized variation measures in the following sense:

$$\sum_{j=0}^{n-1} \phi_{t_j} G(F_T, F_{t_j}) + \delta_{t_j}(F_T - F_{t_j}) = \sum_{j=0}^{n-1} G(F_{t_{j+1}}, F_{t_j}) \tag{2.11}$$

The VIX^2-based variance swap is a special case of strategy (2.8) with $\phi_{t_j} = 0$ for $j > 0$. The price for such a generalization of option trading is the blurring of the distinction between the fixed and floating legs of the variance swap contract. The strategy is, however, the *only* currently known way of obtaining equality in (2.11) with the use of two categories of financial instruments: European options and forwards. The concept of strategy costs returns into focus: it's the financial outlay necessary for obtaining the requisite settlement payoff.

By construction, the replication cost is not known at time t_0, but instead is stochastic. There are two sources of risk determining the aggregate option cost: the changes in weights ϕ_{t_j} and the

[7] Further in the text I consider continuous-time limits of the trading strategies. In order for them to exist, technical conditions have to be imposed on δ and ϕ, such as adaptedness and boundedness.

[8] Acciaio et al. (2013) FTAP reference.

changes in option prices. I give an interpretation of both in Section 3.2. Thus overall, dynamic option trading strategies can be understood as exchanges of the risk of the settlement payoff for the risk of the aggregate replication cost. In this setting I take the natural definition of the risk premium associated with replicating $\sum_{j=0}^{n-1} G(F_{t_{j+1}}, F_{t_j})$ at settlement to be:

$$
\begin{aligned}
(2.12)\qquad & \mathbb{E}\left[\sum_{j=0}^{n-1} G(F_{t_{j+1}}, F_{t_j}) - \sum_{j=0}^{n-1} \phi_{t_j}\mathcal{G}_{t_j}\right] \\
&= \mathbb{E}\left[\sum_{j=0}^{n-1} \phi_{t_j} G(F_T, F_{t_j}) + \delta_{t_j}(F_T - F_{t_j}) - \sum_{j=0}^{n-1} \phi_{t_j}\mathbb{E}^{\mathbb{Q}_T}\left[G(F_T, F_{t_j})|\mathcal{F}_{t_j}\right]\right],
\end{aligned}
$$

i.e. the final settlement payoff of the aggregate derivative position less the cost of "producing" the payoff. Such excess payoffs can indeed be useful for evaluating asset pricing models. Write:

$$
(2.13)\qquad \mathbb{E}^{\mathbb{Q}_T}\left[\sum_{j=0}^{n-1} \phi_{t_j}\mathbb{E}^{\mathbb{Q}_T}\left[G(F_T, F_{t_j})|\mathcal{F}_{t_j}\right]\right] = \mathbb{E}\left[\sum_{j=0}^{n-1} \phi_{t_j}\mathbb{E}\left[\prod_{k=0}^{j-1}\frac{M_{t_{k+1}}}{M_{t_k}} G(F_T, F_{t_j})|\mathcal{F}_{t_j}\right]\right],
$$

and note that in order to evaluate this expression non-parametrically it would suffice that at time t_0 options for each maturity t_j were quoted. Even in absence of such an abundance of data, (2.12) or (2.13) can be of use for asset pricers willing to characterise the higher moments of returns on assets such as hedge fund portfolios. Both the settlement payoff and the aggregate cost components of such strategies reflect all the risk factors to which the investors are exposed, contrary to resorting to using a realised variation measure as the driving force of the stochastic discount factor.

3. TRADABLE REALIZED VARIATION MEASURES

In the previous Section I introduced the general concept of dynamic option trading, with the objective of replicating certain realised variation measures. That framework can be generalized even further, however I defer this discussion to the end of this Section. The level of generality is already sufficient to consider trading big risk separately from small risk. To fix ideas, before I move to defining the exactly replicable realized variation measures, I start with considering those types of realized variation measures that in a semi-martingale model separate jumps from Brownian increments, and those that do not.

If the asset price indeed follows a semi-martingale with finite-activity jumps, it is relatively easy to estimate hypothetical jump and Brownian variation separately. Realized variance, as defined in (2.1), is an example of a measure that aggregates both kind of risk. On the pure-jump side, the literature offers both indirect measures, such as a difference between realised variance and multi-power variation (Barndorff-Nielsen et al., 2006), truncated measures (Mancini, 2001) or direct measures, such as 3rd or 4th power variation of log returns (Jacod and Protter, 2012):

$$RV_{t,t+1} := \sum_{t\leq t_j\leq t+1} \ln^2 r_{t_j} \longrightarrow \int_t^{t+1} \sigma_s^2 ds + \sum_{t\leq s\leq t+1} J_s^2 \tag{3.1}$$

$$RV_{t,t+1} - BV_{t,t+1} := \sum_{t\leq t_j\leq t+1} \ln^2 r_{t_j} - \sum_{t\leq t_j\leq t+1} \left|\ln r_{t_j}\right| \left|\ln r_{t_{j-1}}\right| \longrightarrow \sum_{t\leq s\leq t+1} J_s^2 \tag{3.2}$$

$$RPV^3 := \sum_{j=1}^{n} \ln^3 r_{t_j} \longrightarrow \sum_{t_0\leq s<t_n} J_s^3 \tag{3.3}$$

$$RPV^4 := \sum_{j=1}^{n} \ln^4 r_{t_j} \longrightarrow \sum_{t_0\leq s<t_n} J_s^4. \tag{3.4}$$

These realized measures are not directly replicable, i.e. there does not exist a strategy of type (2.8), whose settlement payoff is equal to any of the quantities on the left-hand side of the limits in equations (3.1) through (3.4).

This Section is devoted to describing the family of realized variation measures which is replicable as settlement payoffs of dynamic option trading strategies introduced in the previous Section, and to a closer look at the behaviour of the aggregate option cost of the most important of such strategies.

3.1. *Realised (jump) divergence*

The starting point is the construction of realised weighted power divergence, a concept which generalises realised variance. Realised power divergence and associated power divergence swaps were introduced and comprehensively described in Schneider and Trojani (2015a). Hereby I recall the basic definitions and properties.

Definition 1 (Realised power divergence) *Let F_t denote the forward price of the underlying asset at time t and maturing at time T and let γ_t define an adapted process. Set a grid of $n+1$ times, $0 = t_0 \leq t_1 \leq \ldots \leq t_n \leq T$. For function $\phi_p(x) := \frac{x^p-1}{p(p-1)}$ and $p \in \mathbb{R}$,*[9] *define realised power divergence as:*

$$D_\gamma^{n,p}(F) := \sum_{j=1}^{n} \gamma_{t_{j-1}} D_p(F_{t_j}, F_{t_{j-1}}) :=$$

$$\sum_{j=1}^{n} \gamma_{t_{j-1}} \frac{F_{t_j}^p - F_{t_{j-1}}^p}{p(p-1)} - \gamma_{t_{j-1}} \frac{F_{t_{j-1}}^{p-1}}{p-1} \left(F_{t_j} - F_{t_{j-1}}\right).$$

[9] For $p = 0$ and $p = 1$ limits of all involved expressions exist.

The weighting process γ plays an important role. Throughout this paper I concentrate on the fundamental case $\gamma_t := F_t^{-p}$, so that

$$\gamma_s D_p(F_t, F_s) = \frac{e^{p\ln(F_t/F_s)} - 1}{p(p-1)} - \frac{e^{\ln(F_t/F_s)} - 1}{p-1} = \frac{\ln^2(F_t/F_s)}{2} + O\left(\ln^3(F_t/F_s)\right). \tag{3.5}$$

Power divergence, for the appropriate γ scaling process, is locally quadratic in log returns, and insensitive to the *level* of F. Furthermore, the leading order of the Taylor expansion in (3.5) does not depend on p. Taking p-derivatives of (3.5) eliminates the leading order terms and yields higher-order realised measures.

Definition 2 (Realised jump divergence) *Let $\gamma^J = \gamma_t F_t^{-p}$ and γ_t does not depend on F nor p. For $p \in \mathbb{R}$ define realised jump skewness*

$$S^{n,p}_{\gamma^J}(F) := \frac{\partial D^{n,p}_{\gamma^J}(F)}{\partial p} = \sum_{j=1}^{n} \gamma_{t_{j-1}} \frac{\partial}{\partial p} \frac{D_p(F_{t_j}, F_{t_{j-1}})}{F^p_{t_{j-1}}} = \sum_{j=1}^{n} \gamma_{t_j} \frac{\ln^3(F_t/F_s)}{6} + O\left(\ln^4(F_t/F_s)\right),$$

and realised jump quarticity:[10]

$$Q^{n,p}_{\gamma^J}(F) := \frac{\partial^2 D^{n,p}_{\gamma^J}(F)}{\partial p^2} = \sum_{j=1}^{n} \gamma_{t_{j-1}} \frac{\partial^2}{\partial p^2} \frac{D_p(F_{t_j}, F_{t_{j-1}})}{F^p_{t_{j-1}}} = \sum_{j=1}^{n} \gamma_{t_j} \frac{\ln^4(F_t/F_s)}{12} + O\left(\ln^5(F_t/F_s)\right)$$

Khajavi et al. (2016) provide the theoretical framework for inference about realised (jump) divergence in a semi-martingale framework. Realised jump divergence measures in Definition 2 have well-defined high-frequency limits and their estimators based on discretized observations obey central limit theorems. Finally, I fix the idea of what the former realised variation measures represent in a semi-martingale framework.

Interpretation (1) (Continuous time limits) Let F follow a general semi-martingale process with finite activity jumps:

$$\frac{dF_s}{F_{s-}} = \mu_s ds + \sigma_s dW_s + \int_{\mathbb{R}\setminus\{0\}} (e^x - 1)\, \nu_s(dx, dt). \tag{3.6}$$

Let $\mathbb{F}$ be the filtration to which the forward price process F is adapted, potentially greater than the one generated by F itself. Let γ be adapted to $\mathbb{F}$ and bounded. Let the maturity T be fixed and let $n \to \infty$ such that $\max_{i\in\{0\ldots n-1\}} |t_{i+1} - t_i| \to 0$. The following limit holds under technical assumptions in Khajavi et al. (2016):

$$\lim_{n\to\infty} D^{n,p}_{\gamma}(F) = \frac{1}{2}\int_0^T \gamma_s F_s^p \sigma_s^2 ds + \sum_{0\le s\le T} \gamma_{s-} F^p_{s-} D_p(F_s, F_{s-}) \tag{3.7}$$

$$\lim_{n\to\infty} D^{n,p}_{F_s^{-p}} = \frac{1}{2}\int_0^T \sigma_s^2 ds + \sum_{0\le s\le T} \frac{D_p(F_s, F_{s-})}{F^p_{s-}}. \tag{3.8}$$

[10] Full formulae are given in equations (A.1) through (A.6) in Appendix A.I.

The measures $S^{n,p}_{\gamma^J}(F)$ and $Q^{n,p}_{\gamma^J}(F)$ have the following pure jump limits:

$$\lim_{n\to\infty} S^{n,p}_{\gamma F_s^{-p}}(F) = \sum_{t_0\leq s<T} \gamma_{s-} S_{F_{s-}^{-p}}(F_s, F_{s-}) \tag{3.9}$$

$$\lim_{n\to\infty} Q^{n,p}_{\gamma F_s^{-p}}(F) = \sum_{t_0\leq s<T} \gamma_{s-} Q_{F_{s-}^{-p}}(F_s, F_{s-}) \tag{3.10}$$

In Figure 2 I illustrate how the divergence, skewness and quarticity functions behave for $p \in \{0, 1/2, 1\}$, when setting $F_0 = 1$ and varying F_T (the first argument). The bottom row of the Figure replicates the top row for a narrower range of F values. Power divergences clearly exhibit locally quadratic behaviour, skewness – locally cubic, while quarticity – locally quartic. The differences between payoff function values become apparent mostly for large deviations of F_T from 1, that is for large returns. In Figure 3 I demonstrate how realised skewness $S^{n,1/2}_{F_t^{-1/2}}(F)$ picks up jumps in a simulated data set. The leftmost panel presents a high-frequency simulated price path. The top right panel presents log returns calculated from the path and three return jumps (marked with dashed orange lines) are clearly seen in the sample. The bottom right panel shows the cumulative skewness divergence during the trading period: the measure markedly decreases at each jump movement.

It is possible to design trading strategies that replicate realised jump divergence measures for arbitrary γ weighting strategies thanks to three fundamental results. First, Schneider and Trojani (2015a) show that the difference $D_p(F_T, F_s) - D_p(F_T, F_t)$ is an affine function of $F_T - F_t$, for $t > s$. Second, by linearity of differentiation, so are the p-derivatives of the difference, which define realised jump divergence measures. Third, Carr and Madan (2001) show how to construct option portfolios replicating non-linear payoffs such as $D_p(F_T, F_s)$, $S_p(F_T, F_s)$ and $Q_p(F_T, F_s)$.

The exact replication of realised divergence measures associated with function g and its Bregman divergence G is possible if, as noted before, the difference between divergences is affine in $F_T - F_t$. Schneider and Trojani (2015a) showed that this is indeed the case for power divergences generated by function $\phi_p(x)$, $D^{n,p}_{\gamma}(F)$. For any realised divergence G meeting that condition, one can rewrite it – or the corresponding settlement payoff – as:

$$
\begin{aligned}
(3.11)\qquad G^n_\gamma(F) &= \sum_{j=1}^{n} \gamma_{t_{j-1}} G\left(F_{t_j}, F_{t_{j-1}}\right) \\
&= \sum_{j=1}^{n} \gamma_{t_{j-1}} \left[G\left(F_T, F_{t_{j-1}}\right) - G\left(F_T, F_{t_j}\right)\right] + \sum_{j=1}^{n} \gamma_{t_{j-1}} \left[g'_p(F_{t_{j-1}}) - g'_p(F_{t_j})\right]\left(F_T - F_{t_j}\right) \\
&= \gamma_{t_0} G\left(F_T, F_{t_0}\right) + \sum_{j=1}^{n} \left[\gamma_{t_j} - \gamma_{t_{j-1}}\right] G(F_T, F_{t_j}) - \gamma_{t_n} G\left(F_T, F_{t_n}\right) \\
&+ \sum_{j=1}^{n} \gamma_{t_{j-1}} \left[g'_p(F_{t_{j-1}}) - g'_p(F_{t_j})\right]\left(F_T - F_{t_j}\right).
\end{aligned}
$$

The dynamic trading strategy (2.8) whose *settlement payoff* is equal to (3.11) is sonsequently defined by ϕ such that $\phi_{t_0} = \gamma_{t_0}$, $\phi_{t_j} = \gamma_{t_j} - \gamma_{t_{j-1}}$ and $\phi_{t_n} = -\gamma_{t_n}$, and δ such that $\delta_{t_0} = 0$ and $\delta_{t_j} = \gamma_{t_{j-1}}\left[g'_p(F_{t_{j-1}}) - g'_p(F_{t_j})\right]$. The strategy can be implemented at an arbitrary frequency and its *exactness* for all considered realised measures is purely a consequence of the fact that equation (3.11) is an algebraic transformation of a high-frequency realised variation measure into expressions which can be obtained as settlement payoffs of option and forward positions assumed over time. A straightforward application of this basic strategy allows to trade weigthted realised power divergence. An extension to jump divergences requires some tedious algebra. An inspection of formulae (A.1) through (A.6) in Appendix A.I indicates that the requisite strategies are more complex in two dimensions: first, these realised measures are combinations of four components: $F_T/F_s - 1$ (a forward position), $F_s^{-p} D_p(F_T, F_s)$ (power divergence weighted by F_s^{-p}), $K^{(1)}_{s,p} \ln(F_T/F_s)(F_T/F_s)^p$ and $K^{(2)}_{s,p} \ln^2(F_T/F_s)(F_T/F_s)^p$; second, equation (3.11) is not directly applicable to the two latter components. Some manipulation is required to express them as *portfolios* of divergences with additional weighting. I defer full expressions to Appendix A.II, equations (A.7) through (A.12).

The variation measures $D^{n,p}_{\gamma^J}(F)$, $S^{n,p}_{\gamma^J}(F)$ and $Q^{n,p}_{\gamma^J}(F)$ are replicable as settlement payoffs (2.9) of portfolios of dynamic option trading strategies (2.8). The strategy position processes (ϕ, δ) can be calculated by applying (3.11) to equations (A.7) through (A.12). An investor who enters such a strategy faces outlays or proceeds from dynamic option trading (2.10) which are uncertain at time t_0. The investor can decide to replicate the payoff until time $t_n < T$, i.e. she can finish accruing the settlement payoff before the option maturity. In this case, from time t_n to time T she *holds an option position* and a *forward position* such that their payoffs at settlement will exactly offset each other, and while the payoff of any single instrument in the portfolio is not known before T, the aggregate payoff *is known.*

The theory of realized divergence provides realized variation measures whose interpretation is –

to first order – identical will familiar power variation measures. The theory exploits the small-time difference between small and big risk to develop measures which isolate big risk. The great benefit of these measures is that – as shown above – there exist dynamic trading strategies (2.8), which yield a settlement payoff equivalent to the realised measure in the sense defined in equation (2.11).

3.2. *Costs of dynamic replication*

In the dynamic strategy (2.8) the uncertain option cost component (2.10) merits a closer look. The uncertainty means that an investor willing to obtain at settlement one of the payoffs under consideration is exposed to risks of changing position weights and changing option prices. In this section I investigate the most salient features of the aggregate cost risks for weights $\gamma = F_t^{-p}$ (scale-free divergence). The most important findings are that *a*) the option rebalancing costs for $S^{n,p}_{\gamma^J}(F)$ and $Q^{n,p}_{\gamma^J}(F)$ are not 0 even if the realised measures are almost surely equal to 0, i.e. no jumps are allowed in the model, *b*) the option rebalancing costs for $S^{n,p}_{\gamma^J}(F)$ are not 0 even if the price of a skewness swap is 0, and *c*) the rebalancing costs in each period are of the order $\ln(F_{t_i}/F_{t_{i-1}})\mathcal{D}_{p,t_j}$, i.e. they depend on return realisations, the price of divergence swaps, and the remaining maturity of the strategy.

The aggregate cost of implementing the dynamic trading strategy (2.8) is easiest to infer from the second line of (3.11):

$$\mathcal{C}[G^n_\gamma] := \sum_{j=1}^{n} \gamma_{j-1} \left[\mathbb{E}^{\mathbb{Q}_T}_{t_{j-1}} \left[G(F_T, F_{t_{j-1}}) \right] - \mathbb{E}^{\mathbb{Q}_T}_{t_j} \left[G(F_T, F_{t_j}) \right] \right] \tag{3.12}$$
$$\xrightarrow{\mathbb{P}} \int_{t_0}^{t_n} -\gamma_s d\mathcal{G}_s,$$

with $\mathcal{G}_t$ defined in equation (2.7). The option trading costs (or proceeds) are an integral of the weighting function with respect to the changes in the price of the option portfolio that replicates the payoff $G(F_T, F_t)$. The option portfolio price $\mathcal{G}_t$ changes, firstly, because F_t changes (G itself is not necessarily scale-free), secondly, because of changes in relevant state variables that drive the forward price F (3.6) and directly influence option prices (e.g. stochastic volatility), and thirdly, because of the shrinking maturity of the traded contracts.

In this section I analyse the trading costs of a long strategy[11] in the payoff $D^{n,p}_\gamma$ with weighting process $\gamma = F_t^{-p}$ (see Definition 1), and I defer $S^{n,p}_{\gamma^J}$ and $Q^{n,p}_{\gamma^J}$ to the Appendix. The time-t swap rate for power divergence $D_p(F_T, F_t)$ can be conveniently expressed as:

$$\mathcal{D}_{p,t} = \mathbb{E}^{\mathbb{Q}_T}_t \left[D_p(F_T, F_t) \right] = \frac{F_t^p}{p(p-1)} \mathbb{E}^{\mathbb{Q}_T}_t \left[e^{p \log F_T/F_t} - 1 \right].$$

[11]The trader *receives* the realised settlement payoff.

Define $\varphi_t(p, T-t) := \mathbb{E}_t^{\mathbb{Q}_T}\left[e^{p \log F_T/F_t} - 1\right]$ and assume that all requisite moments exist. Such a representation with the use of the moment-generating function of the log-return is widely popular in Finance, mostly because of the developments in the literature of affine jump-diffusion models (Duffie et al., 2000). The MGF completely characterizes the conditional distribution of the return and under the reasonable assumption that the return is independent of the level of the stock price prevailing at time t, only depends on the values of the (potentially latent) state variables. The total option costs/proceeds from settlement-replicating $D_\gamma^{n,p}$ are thus:

$$\mathcal{C}\left[D_p\right] = -\int_{t_0}^{t_n} \frac{F_s^{-p}}{p(p-1)} dF_s^p \varphi_s\left(p, T-s\right) .$$

By applying Itō's formula to the function $f(x, y) = xy$, we can rewrite the integrator as:

$$dF_s^p \varphi_s\left(p, T-s\right) = \varphi_s\left(p, T-s\right) dF_s^p + F_s^p d\varphi_s\left(p, T-s\right) + \frac{1}{2} d\left[F^p, \varphi\left(p, T-s\right)\right]_s^c ,$$

and plug it back into the cost expression.

$$\begin{aligned} \mathcal{C}\left[D_p\right] = & \overbrace{\frac{1}{p(p-1)}\left(\varphi_{t_0}(p, T-t_0) - \varphi_{t_n}(p, T-t_n)\right)}^{\text{Starting and ending swap rate}} \\ & - \underbrace{\int_{t_0}^{t_n} \frac{\varphi_s\left(p, T-s\right)}{p(p-1)} \frac{dF_s^p}{F_s^p}}_{\text{Rebalancing of the divergence position}} \underbrace{- \frac{1}{2}\int_{t_0}^{t_n} \frac{F_s^{-p}}{p(p-1)} d\left[F^p, \varphi\left(p, T-s\right)\right]_s^c}_{\text{Covariation between } F \text{ and state variables}} \end{aligned} \tag{3.13}$$

The costs of the dynamic option trading have three sources. First, establishing the initial static swap position, the first term in the above equation. Second, two types of costs associated with the rebalancing of the divergence position, the first of which corrects for changes of scaling required so that the portfolio payoff at maturity tracks the divergence of the log return, second, a term resulting from the fact that returns can be instantaneously correlated with changes in the price of divergence. The second term in equation (3.13) indicates that whenever a trader replicates a strategy that insures her against variance (i.e. she receives $D_\gamma^{n,p}$ at settlement), a positive return *decreases* the replication cost, while for a trader providing the payoff (i.e. short variance, paying $D_\gamma^{n,p}$ at settlement) a negative return generates proceeds from the option trading. Moreover, as noted before, the costs and proceeds from the rebalancing depend on the maturity of the options used for replicating $D_p(F_T, F_t)$; when replicating short-term payoffs, for example $D_\gamma^{n,p}$ over the course of a week, a trader might prefer to use options in the "weekly" CBOE series with the nearest maturity, rather than the standard monthly-calendar instruments, especially because longer-maturity instruments are exposed to risk long beyond the target maturity. The third term in (3.13) is related to the leverage effect: the most important driver of $\varphi_s(p, T-s)$ is the time-varying volatility.

In Appendix D I derive the distributions of cost or proceeds of high frequency option replication of $D^{n,p}_{\gamma}$ and associated jump measures $S^{n,p}_{\gamma^J}$ and $Q^{n,p}_{\gamma^J}$ in the Black-Scholes model. In this setting the asset price variance is constant and the limiting divergence – non-stochastic, yet option trading costs are not zero. Similarly, even though in the Black-Scholes settings the price paths are almost surely continuous, the option strategy replicating zero at maturity does entail stochastic rebalancing flows. This is most instructive: in order to analyse the risk factors driving the premia associated with such dynamic option trading strategies, the Black-Scholes model remains an intuitive benchmark. The most important takeaway from this analysis is that the dynamic strategies are not *purely* exposed to jump risk, but they are *the only* model-free method of receiving the pure jump settlement payoffs. Thus I interpret average profits from such strategies as *premia for big risk* only because the settlement profits time series does indeed contain realisations of big, non-scalable risk.

The investor willing to obtain realized variation payoffs at settlement enters into an exchange of risks. The option cost component, described here in detail, exposes him to an interaction of return and price-of-variance risk. In essence, and investor gains from the incremental, single-period trade whenever the replicated power of the return is greater than the same return times the prevailing price of variance.

4. FEASIBILITY OF HIGH-FREQUENCY TRADING AT THE CBOE

In recent years most high frequency studies of stock returns concentrated on the five minute frequency. It has been argued that at this frequency the return records are sufficiently dense to separate jump from Brownian innovations while keeping contamination from microstructure noise negligible. I investigate the results of dynamic option trading at two frequencies: in addition to the base five-minute frequency, I consider investors who rebalance every 1 hour during the trading day. I choose this frequency in order to compare some trading results with other studies of option payoffs, such as Muravyev and Ni (2016). These two choices anchor my results in a wider context and allow to form a bridge between the high-frequency econometrics and the asset pricing literatures.

Before I move to the description of how I implement the trading technology, I briefly describe the intra-day trading patterns in the market for short-maturity options at the CBOE. Then I discuss the technical details of obtaining sufficiently accurate replication, and finally I demonstrate the feasibility of accurate realized variation trading.

4.1. *Short-maturity option trading at the CBOE*

[TBA: summary of trading/quoting frequency over time and by strike and moneyness, and maturity, for the nearest maturity]

I analyse a subset of the complete set of CBOE's trade and quote records for S&P 500 index options, purchased from Market Data Express. The complete database holds records from January 2001 until December 2015, a total of 41 billion observations, 99.6% of which are quotes. My object of interest is the trading in weekly SPX options, between January 2011 and December 2015, over 261 option maturity cycles. Even though weeklies have been introduced in September 2007, they became sufficiently richly traded at the beginning of 2011. See Figure 5 for an illustration of how the number of strikes for which quotes are available, changed over time.

I aggregate the data to an hourly frequency, and a five-minute, using the "previous tick" rule, prevalent in the high-frequency literature. For each hour of records, for each option type (call or put), and for each strike, I pick the quotes with the latest time stamp available.[12] If multiple quotes share the same time stamp, I pick the lowest available ask price and the highest available bid price. I set the forward price of the underlying asset as the median of forward prices implied by put-call parity in the 5-minute interval. Within each period, I discard in-the-money options, as well as quotes for which $p_{ask}/p_{bid} > 5$. Furthermore, I discard quotes for which $log(K/F_t)/(\sigma_t^{IV}\sqrt{\tau_t})$ is smaller than -12 and greater than 8.[13] Finally, in order to alleviate the problem of using stale quotes, I use the procedure described in Appendix I to ensure that at each time stamp the resulting option price system does not allow for static arbitrage.

4.2. *Replication technology*

At every time t_j the trader takes a position in the option market according to equation (3.11): she replicates (a linear combination of) payoffs $D_p(F_T, F_{t_j})$ and $\Psi_{p,k}(F_T, F_{t_j})$, say $M_{p,\gamma}(F_T, F_{t_j})$, generated by function $m : \mathbb{R}_+ \to \mathbb{R}$. If at each time options with expiry date T and strikes $K \in [0, \infty)$ were quoted in the market, she would take the following position in put $(P(K, T))$ and call $(C(K, T))$ options, following Carr and Madan:

$$\int_0^{F_i} P(K,T)m''(K)dK + \int_{F_i}^{\infty} C(K,T)m''(K)dK. \tag{4.1}$$

[12]The quotes for many deep out of the money options are updated very infrequently because the market makers post very wide bid-ask spreads. The previous-tick rule means that often option quotes from many hours beforehand are used.

[13]This method of standardizing the strike range of analysed option data was used, among others, by Andersen et al. (2015a)

The most widely used method of forming an approximating portfolio resorts to a basic discretization of the integrals in the above formula, with options quoted for J strikes:

$$\sum_{k=1}^{\max k \text{ s.t. } K_k \le F_{t_j}} P(K_k,T)m''(K_k)(K_k - K_{k-1}) + \sum_{\min k \text{ s.t. } K_k > F_{t_j}}^{J} C(K_k,T)m''(K_k)(K_k - K_{k-1}) \,. \tag{4.2}$$

A similar technique with a special treatment of K_0 and K_J is used in the calculation of the VIX index (CBOE (2000)). The result is a portfolio of options with weights $w_k = m''(K_k)(K_k - K_{k-1})$. Such an approach has been considered sufficiently accurate in semi-static strategies. In dynamic option trading, such as settlement-replicating the payoff $D_\gamma^{n,p}$, it yields approximately 5% errors (MAPE). Recall that the higher order measures $S_\gamma^{n,p}$ and $Q_\gamma^{n,p}$ accumulate quantities orders of magnitude smaller than individual small time increments of $D_\gamma^{n,p}$.[14] Moreover, in, for example, a week of trading the replication has to be repeated up to 400 times: the accumulation of individual replication inaccuracies renders the method inadequate.

I improve on (4.2) by implementing the following optimisation problem. At time t_j the trader chooses an importance function $\eta : \mathbb{R}_+ \to R_+$ and then option portfolio weights $w \in \mathbb{R}^J$ which minimise the following objective:

$$\min_{w \in \mathbb{R}^J} \int_0^\infty \eta(F_T) \left(M_{p,\gamma}(F_T, F_{t_j}) - \sum_{k=1}^{J} w_k O(F_T, K_k) \right)^2 dF_T, \tag{4.3}$$

optionally s.t.

$$\operatorname{sign}(w_k) = \operatorname{sign}(m''(K_k)) \tag{4.4}$$

where

$$O(F_T, K_k) \equiv \begin{cases} \max(F_T - K_k, 0) & \text{if} \quad F_T > F_{t_j} \\ \max(K_k - F_T, 0) & \text{if} \quad F_T \le F_{t_j} \end{cases}.$$

The most extreme available strikes are $0 < K_1 < K_J < \infty$, hence for many $M_{p,\gamma}$ criterion (4.3) will be unbounded unless η decreases sufficiently fast for $\log F_T \to \pm\infty$. The importance function η determines where on the support of at-maturity asset prices, payoff replication error is considered important. Natural choices for η are, for example $\eta(F_T) = f_0^{\mathbb{P}}(F_T|F_{t_0})$ and $\eta(F_T) = f_0^{\mathbb{Q}}(F_T|F_{t_0})$, the statistical and risk-neutral conditional distributions of F_T, if one assumes that sufficiently high moments of both exist. Likewise, simple truncation, i.e. $\eta(F_T) = \mathbf{1}_{\{K_0 - \varepsilon \le F_T \le K_J + \varepsilon\}}$, yields sufficiently accurate results, because the terminal asset price is almost never outside $[K_0, K_J]$. In

[14] $S_\gamma^{n,p}$ and $Q_\gamma^{n,p}$ can be though of as limits of first (second) differences of $D_{p_0 \pm h}$ around some p_0, scaled by h (h^2), and any inaccuracy in replicating D_p will be significantly augmented.

practice, the choice of η is not crucial, however choices which put more emphasis on accuracy around F_{t_0} than far away from that point are preferred, because they improve the relative replication error.

The problem in (4.3) is quadratic in w, irrespective of the choice of η. The details on the formulation and division into convex sub-problems are given in Appendix B. If no extra constraints such as (4.4) are required, (4.3) is equivalent to the L^2 projection of $M_{p,\gamma}(F_T, F_{t_i})$ on the space of piecewise-linear functions, weighted by $\eta(F_T)$, and the weights can also be obtained as solutions of a system of linear equations, if an adequate orthogonal basis is constructed from option payoffs. The convex quadratic programming formulation allows for very efficient numerical implementations.

This technology allows the investor who wishes to hedge a non-linear payoff in an incomplete market to choose their hedge optimally, and to minimise the expected discrepancy between the hedged quantity, and the received payoff.

4.3. *Replication accuracy*

In addition to the example presented in Section 1, I demonstrate the superiority of my trading technology over standard discretization techniques on a challenging data sample. While the core of my analysis is devoted to trading weekly options in the years 2011-2015, the sample of standard monthly-expiry CBOE SPX options from 2004 to 2011 raises more challenges. The basic reason is the availability of option quotes sufficiently out of the money, especially in the early part of the sample. I compare two implementations of the trading technology. In the first, option portfolios are formed with the **D**iscretization technique (4.2), while in the second one via the solution of the **Q**uadratic **P**rogramming problem (4.3).

Clearly, the **D** method does not yield accurately replicated payoffs beyond trading realized divergence, i.e. the strategy most resembling variance swaps. Key evidence is gathered in Tables I and II. For both trading technologies, I report the summary statistics for the resulting settlement payoffs, and compare them with the realized (jump) divergence measures calculated directly from price data, as in Definitions 1 and 2. I use the Kolmogorov-Smirnov test to determine whether the trading settlement payoffs come from the same distribution as the true realized measures (which is the null hypothesis).

In the case of $D_p(1/2)$ trade the test does not reject the null neither for the **D** technology, nor for the **QP** technology. The descriptive statistics – sample minima, maxima, quartiles, mean and standard deviation – still indicate that the **QP** method is superior and offers lower hedging errors. The **QP** technology is undeniably superior when it comes to accurately representing higher order measures as payoffs: in both the $S_p(1/2)$ and $Q_p(1/2)$ cases the relatively weak K-S test rejects the

D trading technology: the resulting payoffs are far removed from the true realized measures. The **QP** method recovers the properties of the payoffs much better. One minor caveat is that the **QP** method can yield negative estimates of $Q_p(1/2)$ when big risk events do not materialize, and the resulting realized measure is almost 0 – in the order of magnitude of $1e-10$. Those negative payoffs are negligibly small, and the more accurate reflection of other summary statistics speaks in favour the **QP** trading technology.

In order to strengthen the message even further, consider the hedging errors for trading $S_p(1/2)$ with both technologies, presented in Figure 4. I plot trading settlement payoffs against true realized measure values for trading $S_p(1/2)$ with the use of the **D** and **QP** technologies (in the left and right panels, respectively). When using **QP**, the points in the scatter-plot are aligned along the $y = x$ line, while in the case of **D**, hedging errors are often greater in magnitude than the true realized payoffs, rendering the exercise mute.

The results of applying the **QP** technology in trading weekly options from the beginning on 2011 until the end of 2015 are presented in Figures 6 for the one-hour hedging interval, and in Figure 7 for the five-minute hedging interval. Hedging errors are completely negligible when trading the divergence of returns $D_\gamma(1/2)$ at both frequencies. Hedging errors are significantly smaller at the five-minute frequency when replicating higher-order payoffs. Such erroneous observations are removed from the final statistical analysis.

Overall, I find that, when using the **QP** technology, trading profits follow realised variation measure values closely enough to allow for an analysis of premia for big risk through the lens of dynamic option trading strategies. In certain cases, the observations (i.e. individual options) that cause significant replication error, can be identified and purged from the data.

5. TRADING BIG RISK

In this section I present empirical evidence about the compensation for big risk in the market for short-maturity S&P 500 index options. I reiterate that even though at times I made use of the semi-martingale continuous-time approach, mostly for the purpose of interpreting certain quantities, the evidence for the significance of pricing of big risks, presented below, does not depend on using that approach. The purely *algebraic* transformation of the realized measure into a trading strategy in equation (3.11) is the foundation of all the results.

I consider a risk exposure to carry significant risk compensation, if the corresponding total payoff is persistently different from 0. While the theory of Finance defines a risk premium as $-cov(M, R)$, where M is the stochastic discount factor and R is the total payoff, M is unobservable and is not measurable in a purely non-parametric setting. The hurdle of non-zero profitability is a difficult one to clear from a statistical point of view, given the high variation of trading profits and settlement payoffs. In some cases, it is cleared even *after transaction costs*, i.e. when the bid-ask spread in the option market is taken into account. If a big risk strategy delivers non-zero profits after transaction costs, and the settlement payoff exhibits extreme variation, it is evidence that a premium is paid for the underlying exchange of risks.

I first briefly report on the statistical properties of settlement payoffs – or equivalently, realised divergence measures. Then I break down the analysis of trading profits into two parts. I report the unconditional descriptive statistics to provide an overview of the properties of engaging in big risk trading. Further on, I attempt to determine *when*, in what market conditions big risk trading is profitable. To that end, I accompany the standard (i.e. weekly) trading results with disaggregated evidence.

The analysis starts with trading options along the weekly settlement calendar. At the settlement of an option series,[15] I start the trading strategy in options expiring at the end of the following week,[16] and I rebalance the position at fixed time intervals, except for overnight trades (i.e. from 15^{15} to 08^{30} the following day, Chicago time) when hedging is not available.

The construction of my dynamic strategies allows for trading over arbitrary periods and at arbitrary frequencies. In the disaggregate analysis I take the following vantage points. First, I trade big risk from the opening to the closing of the Chicago option market; such an implementation offers exposure to big risk on single days, when active hedging is possible in many markets. Second, I trade close-to-open positions, which reduce to an opening trade around 15^{15} and a closing trade at 08^{30}, without intermittent option rebalancing nor trading in the forward contracts.[17] In the light of the findings in Muravyev and Ni (2016), the daytime and night-time trading periods are significantly different in the option market to merit a more careful treatment. From the point of view of an actively trading investor, the overnight period is when she is forcefully exposed to big risk due to the inability to incrementally hedge the accruing variation.

Settlement payoffs from trading strategies with γ^J weighting are – by construction – of extremely small magnitude. Not only are they difficult to compare against each other, but also in the dimension of varying trading horizons. In order to facilitate the interpretation of trading profits, I rescale the results of trading $2S_\gamma^{1/2}(F)$ and $2Q_\gamma^{1/2}(F)$ by the sample averages of the prices of the corresponding

[15] Most SPX options are settled on Friday morning or afternoon.

[16] Unless there exists a contract which matures within one day of the default weekly option and is far more traded. This occurs for some quarterly option settlements in the early part of the sample.

[17] While CBOE now offers extended trading hours for option contracts, this is not the case from the start of my sample. Furthermore, outside regular trading hours liquidity virtually dries up.

static absolute variation swaps of corresponding maturity. Thus $S_\gamma^{1/2}(F_{0,T})$ is rescaled by

$$2\mathbb{E}_0^{\mathbb{Q}}\left[\left|S_{F_0^{-p}}(F)\right|\right], \tag{5.1}$$

and $Q_\gamma^{1/2}(F_{0,T})$ by

$$2\mathbb{E}_0^{\mathbb{Q}}\left[Q_{F_0^{-p}}(F)\right]. \tag{5.2}$$

The strategies $\left|S_{F_0^{-p}}(F)\right|$ and $Q_{F_0^{-p}}(F)$ are constructed by taking p-derivatives of $\gamma_s D_p$ with constant $\gamma = F_0^{-p}$, as in Definitions 1 and 2. They correspond to simple absolute third order variation, and kurtosis swaps with payoffs:

$$\frac{1}{2}\int_0^T \left|\ln(F_s/F_0)\frac{F_s^p}{F_0^p}\right|\sigma_s^2 ds + \sum_{0\le s\le T}\frac{\partial}{\partial p}\frac{F_{s-}^p}{F_0^p}D_p(F_s, F_{s-}) \tag{5.3}$$

$$\frac{1}{2}\int_0^T \ln^2(F_s/F_0)\frac{F_s^p}{F_0^p}\sigma_s^2 ds + \sum_{0\le s\le T}\frac{\partial^2}{\partial p^2}\frac{F_{s-}^p}{F_0^p}D_p(F_s, F_{s-}). \tag{5.4}$$

These payoffs arise naturally as a generalisation of Simple Variance Swaps (Martin, 2012) to higher order measures, and are studied, among others, in Schneider and Trojani (2015b). They can be seen as *weighted* realised variance measures, with more weight put on variation far away from the forward price at inception due to the factor $\ln^k(F_s/F_0)F_s^p/F_0^p$. Rescaling the payoffs by sample averages of these quantities renders the results easier to interpret. $\mathbb{E}_0^{\mathbb{Q}}\left[\left|S_{F_0^{-p}}(F)\right|\right]$ is typically about 1/8th of the value of a variance swap. Thus trading results are given in *multiples* of the prices of static higher order variation swaps.

The extreme nature of both settlement and total payoffs makes the estimation of their distributions' location parameter challenging. When evaluating the profitability of the trading strategies, I turn not only to the averages of the payoffs, but also to medians. I form confidence intervals for both estimates of location by bootstrap methods.

5.1. *Weekly trading*

I implement the weekly trading strategies at two hedging frequencies: with hourly and five-minute rebalancing. The summary statistics of realized variation measures, aggregate option costs, and trading profits are collected in Tables III and IV, respectively. I first briefly describe the time-series properties of the settlement payoffs, and then move to reporting on risk compensation.

5.1.1. *Time-series properties of settlement payoffs*

An overview of the time series of settlement payoffs is available in Figures 8 (for hourly hedging) and 9 (for five-minute hedging). In both plots, the top-left panel represents realised divergence, which does not have a big risk interpretation, but is given as a reference whose time-series properties are well-known, an analogue of realised variance. The top right figure plots the time series for realized big skewness $S_{\gamma^J(1/2)}$, while the bottom plot presents realized big quarticity, $Q_{\gamma^J(1/2)}$. This layout and colour coding – orange for $D_{\gamma^J(1/2)}$, blue for $Q_{\gamma^J(1/2)}$ and pink for $Q_{\gamma^J(1/2)}$ are retained through the remainder of the paper.

It is immediately visible that the big risk measures have time series properties that are very different from those of realised divergence. As the hedging interval shrinks to 5 minutes, the clustering of large movements becomes less pronounced, and two big risk periods stand out. These periods are of different character. The first, in the second half of 2011, coincides with a prolonged period of raised uncertainty due to the European debt crisis. Over the course of six months realisations of big risk are of greater magnitude than in all of the ensuing sample, except for the second stand-out moment. The latter is related to the Asian markets sell-off in August 2015, and the extreme realisation of all jump variation measures happens on 2015-08-24, the Monday when Asian markets dropped by 7%, and in the US the S&P500 was reflecting that movement in early trading. This event, also known as the "mini flash-crash" did not cause a prolonged period of large market movements.

At the higher hedging frequency the core properties of the time series of settlement payoffs are the same as in Section 5.1.1. The key difference – as seen in Figure 9 – . The higher hedging frequency allows to capture the dip and recovery in the price of the S&P500 index, which lasted around 30 minutes. The replicated skewness is strongly negative, which implies that the downward movement in the "mini flash-crash" was much more abrupt than the following recovery. The magnitude of settlement payoffs in other periods is lower, but similar to levels recorded at the one-hour trading frequency.

Contrary to measures of realised volatility, the measures of big risk variation are neither persistent nor predictable, which can be seen in Figure 15. While the reported plots present results for weekly trading at the 1-hour frequency, the results at shorter trading horizons and higher frequencies are not unlike those presented. These results are in line with other studies of jump variation.

5.1.2. *Weekly trading profits*

Trading profits for both trading frequencies are summarised in Tables III and IV. Table III contains summary statistics for the hourly hedging frequency, while tables IV contain summary statistics for the five-minute hedging frequency. In each table, data on trading $D^p_\gamma(F)$ are contained in panel A, on $S^{1/2}_\gamma(F)$ in panel B, and on $Q^{1/2}_\gamma(F)$ in Panel C.

The summary statistics are reported at both mid-quote and bid-ask option cost calculation. The mid-quote profits, denoted $\mathcal{P}[\cdot]$, are reported for *long* strategies, in which the investor receives the settlement payoff at maturity. The transaction-cost profits, denoted $\mathcal{T}[\cdot]$, are reported for *long* strategies if average $\mathcal{P}[\cdot]$ for the corresponding strategy is positive, and from *short* strategies otherwise. This convention makes evaluating the final profitability of the strategies easier. Thus, for example, in panel A of Table III, the average profits for trading divergence at 1-hour frequency are negative at mid-prices, while the profits after transaction costs reported in the following line, are of a short strategy, and are positive; furthermore, the bootstrapped confidence interval does not contain 0. Whenever the after-transaction-cost profits $\mathcal{T}[\cdot]$ are *negative*, it implies that the transaction costs render profiting from the strategy infeasible in the CBOE option market.

First, I investigate the γ^J pure-jump strategies and also comment on divergence trading profits. At both trading frequencies I find that mid-price profits from trading big skewness, $\mathcal{P}[S_\gamma^{1/2}(F)]$ are, on average, positive, and of similar size. The average profits are between 0.14 (median) and 0.27 of the corresponding absolute third order variation swap rates. The standard deviation of the profits is much higher than in the case of divergence (variance) trading. Furthermore, the skewness strategies offer a very different higher moment properties, than divergence strategies (exposed to non-directional, small risk). Average $\mathcal{P}[S_\gamma^{1/2}(F)]$ are significantly different from 0 at both the 1-hour trading frequency, and at the 5-minute trading frequency. In both cases, median profits are of the same size, positive, and significantly different from 0. After transaction costs are taken into account, in both cases the confidence intervals for average trading profits contain 0. It is not the case for the median profits, however.

Trading big quarticity at both frequencies is "insurance", similarly to the results reported in the literature on trading variance over longer horizons (e.g. Carr and Wu (2009b)). $\mathcal{P}[Q_\gamma^{1/2}(F)]$ is significantly negative at both frequencies. Including transaction costs in the calculation erases all potential profitability at the high trading frequency, but allows for positive average profits from a short strategy at the 1-hour frequency; the latter quantity, however, is not statistically greater than 0. Median estimates of location indicate in both cases that mid-price average profits are indeed negative, and the median profits from the short strategy pass the statistical significance hurdle after transaction costs.

An interesting finding is that the sign of the profits from trading divergence with the use of short-maturity options, changes when the frequency increases to 5-minutes. That is, the short-maturity variance risk premium in the period under consideration is positive, if hedging is frequent enough. This result is not driven by rebalancing in the option market, because it prevails if the

$D^0_\gamma(F)$ strategy is considered, the VIX^2-based variance swap.[18] The profits from that strategy are virtually identical, as can be seen in the lower part of panel A in Table IV. As a matter of fact, the higher divergence trading profits at the higher frequency are a result of an increase in the settlement payoff. Simply put, it is due to a more precise measurement of realized return divergence, which is fully reflected in the payoff.

To the contrary, the average and median quarticity strategy profits do not change sign with a change in the hedging frequency. At a high frequency and with short-maturity options, that price events only in the nearest future, investors are compensated for taking on extreme risks such as in the quarticity and skewness strategies. Compensation for the magnitude of small risks, which are the main driver of realized return divergence, is positive in the sample and horizon under consideration.

The time series of trading profits can be inspected in Figures 12 and 13. When viewed alongside Figures 10 and 11, which present the aggregate option costs defined in equation (2.10). The fundamental observation is that total payoffs for the big risk strategies are mostly generated by aggregate option costs, and not by the settlement payoffs. In the case of skewness trading, the settlement payoff is of greater magnitude than option rebalancing costs under 7% of the time, while for quarticity there are only 2 such instances in weekly trading. This is hardly surprising in light of the analysis in Section 3.2: the rebalancing costs accrue proportionally to return-weighted price of *divergence*, which is a magnitude greater than skewness or quarticity before rescaling is considered. This observation is important: it renders how difficult it is to isolate big risk in extant financial markets. That some of these strategies are profitable, on average, at mid-prices is indicative enough of the fact that big risk is a distinct feature of the S&P500 index option market.

5.2. *Disaggregated trading*

One of the most interesting properties of the trading strategies I consider is that they allow the investor to accrue the settlement payoff over arbitrary periods. I exploit this feature in order to study the differences in trading small and big risk during daytime (market opening hours), and overnight (from the time the market closes, to the time it reopens). This investigation is directly motivated by Muravyev and Ni (2016), who found a striking pattern in option returns across all maturities (longer than two weeks) and wide strike ranges. Returns on delta-hedged option positions held from market open to market close are positive, while overnight returns are negative and so big in magnitude that overall returns on options are negative. I present results from trading during the day and overnight separately in Tables V and VI.

[18] Recall that this strategy corresponds to $\gamma^J_t := F^0_t \equiv 1$, so it belongs to the family of semi-static strategies.

5.2.1. *Time series properties of settlement payoffs*

The general properties of the time series of daytime and night-time settlement payoffs are identical with those already reported for weekly trading. It is most instructive, however, to compare the two time series side by side (Figure 18). In the case of non-directional strategies, the overnight variation captured by the settlement payoffs is of significantly smaller magnitude than variation during daytime. In the extreme case of trading $Q_\gamma(1/2)$, all payoffs except for the "mini-flash-crash" are virtually equal to 0. In the case of a directional trade – big skewness – the daytime variation is smaller than overnight variation due to the fact that during daytime, frequent measurement will allow for cancelling out potential upwards and downwards jump movements.

5.2.2. *Daily and overnight trading profits*

In daytime trading, the average mid-price profits for trading big skewness are negative, but not significantly different from 0. The median mid-price profits remain positive. Transaction costs are, however, high enough to erase all potential profitability. Daytime average mid-price quarticity profits are positive, however median mid-price quarticity profits are not significantly greater from 0. Both average and median post-transaction cost quarticity profits are negative. These results imply that during daytime trading there is scant unconditional evidence for compensation for big risk.

The compensation seems to be present, however, in overnight trading. Average mid-price profits for trading big skewness are positive, as are median mid-price profits. Both location estimates are significantly greater than 0. The bid-ask spread, however, seems an obstacle to actually cash in on those payoffs – average transaction-price profits from a long strategy are negative, but median profits remain significantly greater than 0. Trading big quarticity is – unlike during the day – "insurance", in the sense that it carries a negative premium overnight. Both average and median profits are significantly negative, and the compensation is high enough so that the average profits remain positive after transaction costs are taken into account. When one takes into account how small overnight quarticity is on all days except around 2015-08-24, it is clear that the strategy is a hedge against really crippling events.

Overnight profits from trading big risk are also much greater (in absolute value) than daytime profits. After scaling by the $1/\tau^k$ factor, they are also greater than aggregate week-time profits, which is another indicator that compensation for big risks is present – unconditionally – in overnight trading.

The data on the divergence trading strategy $D_\gamma(1/2)$ offers additional insights into the question of the sign of the divergence premium. In Section 5.1.2 I noted that at the 5-minute trading frequency divergence profits over the weekly trading horizon are positive, on average. After disaggregation into daytime and night-time profits, a clear patter emerges: during daytime, the high-frequency divergence trade earns a positive risk premium, but overnight the premium is negative. The daytime premium is, however, of greater magnitude than the negative night-time premium.

The difference in premia when trading at various frequencies is, as noted before, mostly due to obtaining more precise "measurements" of the realized measures in the settlement payoffs. It remains an open question as to what hedging frequencies investors have in mind when engaging in such trades. One hypothesis is that there are many kinds of agents in the financial market, heterogeneous in their decision horizons. In that case the explanation of the option prices being "insufficiently high" to bring around a negative divergence risk premium – compatible with most to-date evidence – would lie in the interplay between the two groups of traders in the financial market.

Overall, I conclude that the compensation for big risk strategies in extant financial markets comes mostly through overnight trading. Thus, it is related to the fact that in these periods agents cannot engage in actively hedging their positions.

5.2.3. *Conditioning*

A careful look at the time-series plots of realised big variation measures (Figure 9) and trading profits (Figure 13 indicates that trading big risk might become highly profitable *after* such an event is actually observed. Such outcomes prevail in weekly trading for the big skewness strategy $S^{1/2}_{\gamma^J}(F)]$ and the short big quarticity strategy $Q^{1/2}_{\gamma^J}(F)]$ (Figure 13b and 13c, respectively). In disaggregate trading results more than 1200 daytime and overnight observations are available for a more detailed analysis.

I present more evidence backing up this claim in Figure 17. In all four panels, median mid-price profits from big risk trading strategies are plotted, conditional on the realisations of the previous period's corresponding realised big variation measure. For daytime trading (in the left two panels), the preceding period is from the previous market close to the current day's market opening. For night-time trading, the preceding period is from the current day's market opening, until market closing, when the position is taken. The top two plots present results for trading big skewness, while the lower two – big quarticity.

During daytime trading, there is no evidence for the dependence between median profits and realised variation. The patterns are striking for overnight trading. For both strategies, there exists a non-linear relation between the magnitude of preceding period's realised big variation, and the current period's trading profits. Compensation for skewness increases dramatically after a big risk event in the 10^{th} decile, and so does the price of insurance against quarticity. Both these strategies load heavily on out of the money options. The increase in skewness compensation (or in the price of insurance against quarticity) reflects an increase in put prices after a truly large market movement. This result is consistent with evidence presented by Andersen et al. (2015a), who parametrically model option prices and postulate the existence of a "tail risk" factor, which mostly influences deep out of the money put option prices. This factor is very persistent under the risk-neutral measure, but such extreme jump movements very rarely occur under the statistical probability measure.

6. CONCLUSIONS

In this work I introduced a general concept of dynamic option and forward trading, which opens new avenues for the asset pricing literature. The trading strategies' settlement payoffs can exactly hedge a family of realized return variation measures, that until now could only be calculated from asset price data. Rendering such specialized payoffs tradable allows researchers to put a price on precise concepts of asset price variation, including the possibility of separating big (jump) risk from small (diffusive). The strategies are based on active rebalancing of positions in currently known synthetic derivatives similar to variance swaps.

The feasibility of implementing such trading strategies in the option data hinges on using option portfolio formation methods that improve upon simple discretization, as has been used by practitioners and researchers alike. I show how to implement a conceptually simple and computationally fast method for obtaining optimal replicating option portfolios in incomplete option markets.

I expect the strategies to be useful for expanding the knowledge about the contribution of higher-order risks to the cross section of stock returns. The excess payoffs of the strategies, that is the difference between the settlement payoff and the aggregate option costs, reflect accumulated information about the investors' perception of higher order risk, and their reward for engaging into trading it. In hereby unreported results I find that the excess payoffs are not related to currently fundamental risk factors, such as Market, Size, Value (Fama and French, 1993) and Momentum (Carhart, 1997). Their response to investor perception of the price of variance can be useful in capturing yet unexplained cross-sectional expected return puzzles, especially in fields such as returns on hedge fund investments, which have relatively disorderly higher moment properties.

The properties of the trading strategies allow me to provide evidence about compensation for divergence (variance) and jump risk on an aggregate (over the weekly settlement horizon) and disaggregate (separately for daily and overnight trading) basis. There is a number of important findings. First, the divergence (variance) risk premium in weekly trading is negative at the hourly trading frequency, and positive at the very high 5-minute trading frequency. The change comes from the fact that in the latter case the realized measure is higher: an increased measurement frequency better captures small Brownian variation, which otherwise washes away. This effect for the divergence premium prevails also for the VIX^2-based divergence strategy, which does not involve active option between opening and closing the position. The finding complements a large body of literature about the term structure of variance risk premia. Almost all of this literature is based on longer-maturity contracts, and reports a negative risk premium for index variance.

The premia for big risk do not change that dramatically with an increase in the hedging frequency. I consider two contracts, paying of "big skewness" and "big quarticity". The fundamental difference between them is that the former is directional, i.e. a long position brings losses if negative jumps occur, while the latter is convex and similar to a variance swap: large positive and negative jumps count positively towards the settlement payoff. The compensation for big skewness risk is positive in the market for very short maturity options, albeit the profits are smaller than in the case of divergence trading after similar financing capacity of the investor is considered. The compensation for big quarticity risk is negative, i.e. the strategy serves as a hedge against the magnitude of jumps. The profitability of the strategy on a weekly basis is, however very low, especially with five-minute hedging. If transaction costs (bid-ask spread in the option market) are taken into account, the average profits from the strategies are not statistically different from 0 anymore. It is worth noting, however that median profits are different from zero, and that transaction costs in the CBOE SPX option market are considered high among practitioners. Overall, I find evidence that investors require a premium for taking on directional jump risk, and are willing to pay to hedge against the magnitude of jump risk. The premium should be understood as this for an exchange of two risk profiles, however: of the pure realized variation measure for appropriately weighted price of divergence (variance).

The disaggregate analysis brings further insights into the nature of dynamic option trading. I report three important findings. One is about trading the divergence of stock returns, i.e. a contract exposed to both small and big risk. The latter two are about pure big risk trading. First, in trading the divergence of returns, I obtain a result similar in spirit to Muravyev and Ni: excess payoffs from trading divergence are positive during the day, and negative in the overnight periods. The daytime profits are, on average, much higher than night-time losses associated with a long position. The difference in excess payoffs from trading return divergence can mostly be attributed to the fact that overnight return divergence is on average smaller than daytime return divergence. Second, in big risk trading the compensation for risk is earned mostly in the overnight period. Profits from trading big skewness during daytime are not significantly different from zero, but they are considerable during night-time: profits from trading big skewness are of the same magnitude as profits from trading return divergence. Compensation for selling protection against big overnight quarticity is smaller than in the case of overnight return divergence. The third finding is about the conditional magnitudes of premia for trading big risk. I find that in night-time trading profits from big skewness and big quarticity strategies are, respectively, significantly higher and lower, if the preceding daytime-measured jump variation (in any direction) falls in its top decile. In those strategies, the aggregate option cost is the dominant component of the final excess payoff. This, together with the fact that I do not observe clusterings of big risk events, suggests that after a "jump" investors hike their expectations of another jump occurring, which is reflected in option prices. The effect is persistent. This evidence is similar to findings of Andersen et al. (2015a) who find that put options become – and stay – more expensive after a negative jump in returns.

Overall, my empirical investigation brings around a new method of direct measurement of compensation for big risk, often identified with jump risk. Through this lens I report novel evidence on risk premia available to investors in the market for European index options. The evidence raises new challenges for theoretical asset pricing models, as well as for the option pricing literature.

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APPENDIX A: DIVERGENCE: USEFUL EXPRESSIONS

I. *Higher-order measures*

The following equations express the derivatives of $F_s^{-p} G_p(F_t, F_s)$ with respect to p expressed in terms of $y := \ln F_t/F_s$.

$$\frac{\partial}{\partial p}\frac{G_p(F_t,F_s)}{F_s^p} = \frac{ye^{py}-(e^y-1)}{p(p-1)} - \left(\frac{1}{p^2(p-1)}+\frac{1}{p(p-1)^2}\right)\left(e^{py}-1-p\left(e^y-1\right)\right), \tag{A.1}$$

$$\lim_{p\to 0}\frac{\partial}{\partial p}\frac{G_p(F_t,F_s)}{F_s^p} = -\frac{y^2}{2} - y + (e^y-1), \tag{A.2}$$

$$\lim_{p\to 1}\frac{\partial}{\partial p}\frac{G_p(F_t,F_s)}{F_s^p} = \frac{y^2e^y}{2} - ye^y + (e^y-1), \tag{A.3}$$

$$\begin{aligned}\frac{\partial^2}{\partial p^2}\frac{G_p(F_t,F_s)}{F_s^p} &= \frac{y^2e^{py}}{p(p-1)} - 2\left(\frac{1}{p^2(p-1)}+\frac{1}{p(p-1)^2}\right)\left(ye^{py}-(e^y-1)\right)\\ &+\left(\frac{2}{p^3(p-1)}+\frac{2}{p^2(p-1)^2}+\frac{2}{p(p-1)^3}\right)\left(e^{py}-1-p(e^y-1)\right),\end{aligned} \tag{A.4}$$

$$\lim_{p\to 0}\frac{\partial^2}{\partial p^2}\frac{G_p(F_t,F_s)}{F_s^p} = -\frac{y^3}{3} - y^2 - 2y + 2(e^y-1), \tag{A.5}$$

$$\lim_{p\to 0}\frac{\partial^2}{\partial p^2}\frac{G_p(F_t,F_s)}{F_s^p} = \frac{y^3e^3}{3} - y^2e^y + 2ye^y - 2(e^y-1). \tag{A.6}$$

II. *Higher-order tradability*

The following equations express the quantities required for jump replication as portfolios of weighted divergences. I start with defining additional functions to facilitate notation.

$$\psi_{p,k}(x) := \frac{x^p\ln^k x}{p(p-1)} \qquad \Psi_{p,k}(y,x) := \psi_{p,k}(y) - \psi_{p,k}(x) - \psi'_{p,k}(x)(y-x)$$

The function Ψ is simply the Bregman divergence of ψ. The following equations express the scaled divergence derivatives as divergences to which equation (3.11) is applicable.

$$\frac{\partial}{\partial p}\frac{G_p(F_t,F_s)}{F_s^p} = \frac{\Psi_{p,1}(F_t,F_s)}{F_s^p} - \frac{\ln F_s + \left(\frac{1}{p}+\frac{1}{p-1}\right)}{F_s^p}G_p(F_t,F_s) \tag{A.7}$$

$$\lim_{p\to 0}\frac{\partial}{\partial p}\frac{G_p(F_t,F_s)}{F_s^p} = -\frac{\Psi_{0,2}(F_t,F_s)}{2} - (\ln F_s - 1)\,G_0(F_t,F_s) \tag{A.8}$$

$$\lim_{p\to 1}\frac{\partial}{\partial p}\frac{G_p(F_t,F_s)}{F_s^p} = \frac{\Psi_{1,2}(F_t,F_s)}{2F_s} - \frac{\ln F_s - 1}{F_s}G_1(F_t,F_s) \tag{A.9}$$

$$\begin{aligned}\frac{\partial^2}{\partial p^2}\frac{G_p(F_t,F_s)}{F_s^p} &= \frac{\Psi_{p,2}(F_t,F-s)}{F_s^p} - 2\frac{\ln F_s + \left(\frac{1}{p}+\frac{1}{p-1}\right)}{F_s^p}\Psi_{p,1}(F_t,F_s)\\ &+\frac{\ln^2 F_s + 2\left(\frac{1}{p}+\frac{1}{p-1}\right)\left(\ln F_s+\frac{1}{p}+\frac{1}{p-1}\right)-\frac{2}{p(p-1)}}{F_s^p}G_p(F_t,F_s)\end{aligned} \tag{A.10}$$

$$\begin{aligned}\lim_{p\to 0}\frac{\partial^2}{\partial p^2}\frac{G_p(F_t,F_s)}{F_s^p} &= -\frac{\Psi_{0,3}(F_t,F_s)}{3} + (\ln F_s - 1)\,\Psi_{0,2}(F_t,F_s)\\ &+\left(\ln^2 F_s - 2\left(\ln F_s - 1\right)\right)G_0(F_t,F_s)\end{aligned} \tag{A.11}$$

$$\begin{aligned}\lim_{p\to 1}\frac{\partial^2}{\partial p^2}\frac{G_p(F_t,F_s)}{F_s^p} &= \frac{\Psi_{0,3}\left(F_t,F_s\right)}{3F_s} - \frac{\ln F_s + 1}{F_s}\Psi_{0,2}(F_t,F_s)\\ &+\frac{\ln^2 F_s + 2\left(\ln F_s + 1\right)}{F_s}G_1(F_t,F_s)\end{aligned} \tag{A.12}$$

APPENDIX B: OPTION PORTFOLIO WEIGHTS

In this section I give a detailed description of the quadratic programming problem of optimal static replication. Rewrite the optimization criterion in equation (4.3) more compactly and separate the terms, with $g(x)$ the function to be replicated, $\eta(x)$ a weighting function and $O(x, K_j)$ the payoff of an out-of-the money option:

$$\begin{aligned}
&\int_0^\infty \eta(x)\left(g(x) - \sum_{j=1}^J w_j O(x,K_j)\right)^2 dx \\
&\qquad = \int_0^\infty \eta(x)\left(g(x)^2 + \left(\sum_{j=1}^J w_j O(x,K_j)\right)^2 - 2g(x)\sum_{j=1}^J w_j O(x,K_j)\right) dx \\
&\qquad = C + \int_0^\infty \eta(x)\left(\sum_{j=1}^J \sum_{k=1}^J w_j w_k O(x,K_j)O(x,K_k) - 2g(x)\sum_{j=1}^J w_j O(x,K_j)\right) dx \\
&\qquad = C + w^T \mathbf{Q} w - \mathbf{q}^T w
\end{aligned}$$

Above, $\mathbf{Q}$ is a square $J \times J$ matrix with entry Q_{ij}:

$$\begin{aligned}
Q_{ii} &= \int_0^\infty \eta(x) O(x,K_i)^2 dx = \begin{cases} \int_0^{K_i} \eta(x)(K_i - x)^2 dx & \text{if } K_i \le F \\ \int_{K_i}^\infty \eta(x)(x-K_i)^2 dx & \text{if } K_i > F \end{cases} \\
Q_{ij} &= \int_0^\infty \eta(x) O(x,K_i) O(x,K_j) dx \\
&= \begin{cases} 0 & \text{if } K_i < F < K_j \text{ or } K_j < F < K_i \\ \int_0^{\min(K_i,K_j)} \eta(x)(x-K_i)(x-K_j)dx & \text{if } K_i, K_j \le F \\ \int_{\max(K_i,K_j)}^\infty \eta(x)(K_i - x)(K_j - x)dx & \text{if } K_i, K_j > F \end{cases},
\end{aligned}$$

$\mathbf{q} \in \mathbb{R}^J$ with entries q_j:

$$q_j = \int_0^\infty 2\eta(x) g(x) O(x,K_j) dx = \begin{cases} \int_0^{K_j} 2\eta(x)g(x)(x-K_j)dx & \text{if } K_j \le F \\ \int_{K_j}^\infty 2\eta(x)g(x)(K_j - x)dx & \text{if } K_j > F \end{cases},$$

while $C = \int_0^\infty \eta(x)g(x)^2 dx$ is a constant that can be ignored in the optimization. All integrals can be easily evaluated numerically, and in special cases, for example with $\eta(x) \equiv \mathbf{1}_{\{K_1 - \varepsilon \le x \le K_J + \varepsilon\}}$, analytically.

The matrix $\mathbf{Q}$ is block-diagonal and block-wise positive definite. Thus for computational efficiency and without loss of replication accuracy, it's possible to split the above problem into two convex subproblems for replicating the payoff function $g(x)$ separately for $x \le F$ and $x > F$.

APPENDIX C: DATA FILTERS

I remove the following records from my dataset of all SPX option quotes and trades:

1. Options which are not those with monthly settlements effective on the morning of the third Friday of the month (e.g. weeklies and LEAPS), and which are not the closest maturity;
2. Quotes with zero bid prices;
3. Quotes with non-positive bid-ask spreads;
4. Quotes with bid-ask spread greater than 500% of the bid price;
5. Options with strikes whose standardised moneyness, $\log(K/F)/(\sigma_{IV}^{ATM}\sqrt{T-t})$ is lower than -12 or higher than 6;
6. Quotes whose mid-prices are not contained in the bid-ask spread after imposing the no-arbitrage mid-price system of the following subsection I.

I. *No-arbitrage option price system*

The relative price of a put option is the integral of the risk-neutral distribution function of the return on the underlying asset:

$$p_{K,T} \equiv \frac{P_{K,T}}{F} = \int_0^K P(F_T/F \leq x)dx, \tag{C.1}$$

and as such, for $K_j > K_i$, I should have $p_{K_j,T} - p_{K_i,t} > 0$. Based on this relation I construct an L^1 correction method for observed relative option prices which ensures that the resulting mid-prices do not allow for arbitrage, and that does not rely on parametric assumptions. Let J denote the number of option price quotes prevailing at a given point in time. Let $\hat{\pi}_{K,T}$ denote the price which ensures there is no arbitrage in the system. I find $\hat{\pi} = [\hat{\pi}_{K_1,T}, \ldots, \hat{\pi}_{K_J,T}]^T$ as:

$$\hat{\pi} = \underset{\pi \in \mathbb{R}^N_+}{\operatorname{argmin}} \sum_{j=1}^{J} \left(p_{K_j,T} - X_{(j,\cdot)}\pi\right)\left(\frac{1}{2} - \mathbf{1}_{\{p_{K_j,T} - X_{(j,\cdot)}\pi \leq 0\}}\right)$$

with

$$X_{(\cdot,1)} = \mathbf{1}_{J\times 1}$$

$$X_{(i,j)} = K_i - K_{j-1} \text{ for } i \in 2,\ldots,J \text{ and } j \in 2,\ldots,i\,.$$

Applying the above to mid-prices $p_{K,T}$ and subsequently discarding these records for which $\hat{\pi}_{K_j,T}$ is outside the bid-ask spread allows us to find a set of no-arbitrage "mid" prices which would not allow for arbitrage in a market free of transaction profits.

APPENDIX D: HIGH FREQUENCY OPTION TRADING IN THE BLACK-SCHOLES MODEL

The simplest Brownian-driven model of stock price behavimy allows us to learn about risk compensation in benchmark cases. First, the model is put-call symmetric with constant volatility. Second, asset price paths are continuous. Hence, at the high-frequency trading limit the realized divergence strategies will pay exactly $\sigma^2(t_n - t_0)$ while the skewness and quarticity strategies have zero payoffs. Furthermore, in the known case of the log contract, i.e. $D_n^0(F)$, where divergence weights are constant, the risk premium for divergence is 0.

Assume the following dynamics of the forward price of a stock index under $\mathbb{P}$:

$$dF_s = \mu F_s ds + \sigma F_s dW_s\,, \tag{D.1}$$

and under $\mathbb{Q}$ (where F is a martingale):

$$dF_s = \sigma F_s dW_s\,. \tag{D.2}$$

The statistical and risk-neutral characteristic functions of the log-return on F are, respectively:

$$\mathbb{E}_s^{\mathbb{P}}\left[e^{u\log F_t/F_s}\right] = e^{(t-s)\left(\frac{1}{2}\sigma^2(u^2-u)+\mu u\right)} \text{ and } \mathbb{E}_s^{\mathbb{Q}}\left[e^{u\log F_t/F_s}\right] = e^{\frac{1}{2}\sigma^2(t-s)(u^2-u)}\,. \tag{D.3}$$

I assume that the trading strategy is executed on a period $[0,T]$, where T is the maturity of the options and forward contracts.

I. *Dynamics of some functions of the underlying in the Black-Scholes model*

Here I gather expressions that are useful in the following sections when studying the trading profits in the Black-Scholes model:

$$\phi_s(p, T-s) \equiv \mathbb{E}_s^{\mathbb{Q}}\left[e^{u\log F_t/F_s}\right] - 1, \tag{D.4}$$

$$\frac{dF_s^p}{F_s^p} = \left(p\mu + \frac{p(p-1)}{2}\sigma^2\right)ds + p\sigma dW_s, \tag{D.5}$$

$$\frac{dF_s^p \log F_s}{F_s^p} = \left(p\mu\log F_s + \mu + \frac{1}{2}\sigma^2\left[p(p-1)\log F_s + (2p-1)\right]\right)ds + \sigma\left(p\log F_s + 1\right)dW_s \tag{D.6}$$

II. *Realized divergence trading*

I proceed with deriving the expressions for the payoff, trading profits, expected tradings profits, and finally the premium for trading realized divergence in the Black-Scholes model. First recall that equation (3.8) implies that the payoff $D_n^p(F)$ of the options and forwards at maturity reaches the limit of integrated divergence:

$$D_n^p(F) \xrightarrow{\mathbb{P}} \frac{1}{2}\sigma^2 T \equiv D_\infty^p(F). \tag{D.7}$$

From equation (3.13) I can decompose the costs of obtaining the payoff, $\mathcal{C}\left[D_\infty^p\right]$, as:

$$\mathcal{C}\left[D_\infty^p(F)\right] = \frac{\phi_0(p, T-s)}{p(p-1)} - \int_0^T \frac{\phi_s(p, T-s)}{p(p-1)} \frac{dF_s^p}{F_s^p}.$$

The conditional premium for trading is $DP_{\mathbb{Q}_T}(p) \equiv \mathbb{E}_0^{\mathbb{P}}\left[D_\infty^p(F)\right] - \mathbb{E}_0^{\mathbb{Q}_T}\left[\mathcal{C}\left[D_\infty^p(F)\right]\right]$ if the market allows for contingent contracts such that the profits of rebalancing can be contracted at time 0. The conditional premium in case where the profits of rebalancing have to be borne along the trading path, is: $DP_{\mathbb{P}}(p) \equiv \mathbb{E}_0^{\mathbb{P}}\left[D_\infty^p(F)\right] - \mathbb{E}_0^{\mathbb{P}}\left[\mathcal{C}\left[D_\infty^p(F)\right]\right]$. $DP_{\mathbb{Q}_T}(p)$ is not identically 0:

$$DP_{\mathbb{Q}_T}(p) = \sigma^2 T - 2\frac{\phi_0(p, T)}{p(p-1)}, \quad \lim_{p\to 0} DP_{\mathbb{Q}_T}(p) = \lim_{p \to 1} DP_{\mathbb{Q}_T}(p) = 0. \tag{D.8}$$

$DP_{\mathbb{Q}_T}(p) > 0$ for $p \in (0,1)$, $DP_{\mathbb{Q}_T}(p) < 0$ for $p \notin [0,1]$. The convexity adjustment for $p \notin \{0,1\}$ arises because the second derivative of the price of divergence at $p = 0$ and $p = 1$ is polynomial in σ^2, but has an exponential term for $p \notin \{0,1\}$. The conditional premium in case where rebalancing cannot be contracted at time 0, depends on the risk premium μ:

$$DP_{\mathbb{P}}(p) = \sigma^2 T + \frac{\mu T}{p-1} - \frac{2\mu\phi_0(p,T)}{p(p-1)^2\sigma^2} - \frac{2\phi_0(p,T)}{p(p-1)}, \tag{D.9}$$

$$\lim_{p\to 0} DP_{\mathbb{P}}(p) = 0 \text{ and } \lim_{p\to 1} DP_{\mathbb{P}}(p) = -\frac{\mu\sigma^2 T^2}{4}. \tag{D.10}$$

From (D.5) and (D.7) I infer that $\mathcal{C}\left[D_\infty^p(F)\right]$ is normally distributed, with mean and variance given below:

$$\mathcal{C}\left[D_\infty^p(F)\right] \sim \mathcal{N}\left(-\frac{\left(2\mu + (p-1)\sigma^2\right)\left(\frac{2\phi_0(p,T)}{(p-1)p\sigma^2} - T\right)}{2(p-1)}, \left(-2\frac{\phi_0(p,T)}{p(p-1)^2\sigma} + \frac{T\sigma}{p-1}\right)^2\right). \tag{D.11}$$

On $p \in [0,1]$ the convexity adjustment profit is an order of magnitude smaller than the expected rebalancing profit.

III. *Realized skewness trading*

Trading jump skewness in the Black-Scholes model reveals more peculiar features of dynamic option trading. Recall that from equation (3.9), the payoff $S_n^p(F)$ of the options and forwards at maturity reaches the limit:

$$S_n^p(F) \xrightarrow{\mathbb{P}} 0 \equiv S_\infty^p(F), \tag{D.12}$$

because the path of F is continuous. Analogously to equation (3.13), I can decompose the cost of obtaining the payoff, $\mathcal{C}\left[S^p_\infty\right]$, as:

$$\begin{aligned}\mathcal{C}\left[S^p_\infty\right] &= \frac{\phi_0'(p,T)}{p(p-1)} - \frac{2p-1}{(p(p-1))^2}\phi_0(p,T) - \int_0^T \frac{\phi_s(p,T-s)}{p(p-1)}\frac{dF_s^p \log F_s}{F_s^p} - \int_0^T \frac{\phi_s'(p,T-s)}{p(p-1)}\frac{dF_s^p}{F_s^p} \\ &+ \int_0^T \frac{\log F_s \phi_s(p,T-s)}{p(p-1)}\frac{dF_s^p}{F_s^p} + \int_0^T \frac{(2p-1)\phi_s(p,T-s)}{(p(p-1))^2}\frac{dF_s^p}{F_s^p} \\ &= \frac{\phi_0'(p,T)}{p(p-1)} - \frac{2p-1}{(p(p-1))^2}\phi_0(p,T) \\ &- \int_0^T \frac{\phi_s'(p,T)}{p(p-1)}\left(p\mu + \frac{p(p-1)}{2}\sigma^2\right)ds - \int_0^T \frac{\phi_s'(p,T)}{p(p-1)}p\sigma dW_s, \\ &- \int_0^T \frac{\phi_s(p,T-s)}{p(p-1)}\left(\mu + \frac{2p-1}{2}\sigma^2\right)ds - \int_0^T \frac{\phi_s(p,T-s)}{p(p-1)}\sigma dW_s \\ &+ \int_0^T \frac{(2p-1)\phi_s(p,T-s)}{(p(p-1))^2}\left(p\mu + \frac{p(p-1)}{2}\sigma^2\right)ds + \int_0^T \frac{(2p-1)\phi_s(p,T-s)}{(p(p-1))^2}p\sigma dW_s\end{aligned}$$

The conditional premium for trading is $SP_{\mathbb{Q}_T}(p) \equiv \mathbb{E}_0^{\mathbb{P}}\left[S^p_\infty(F)\right] - \mathbb{E}_0^{\mathbb{Q}_T}\left[\mathcal{C}\left[S^p_\infty(F)\right]\right]$ if the market allows for contingent contracts such that the profits of rebalancing can be contracted at time 0. The conditional premium in case where the profits of rebalancing have to be borne along the trading path, is: $SP_{\mathbb{P}}(p) \equiv \mathbb{E}_0^{\mathbb{P}}\left[S^p_\infty(F)\right] - \mathbb{E}_0^{\mathbb{P}}\left[\mathcal{C}\left[S^p_\infty(F)\right]\right]$. Note that the swap rate for skewness is zero if and only if $p = 1/2$. Similarly to the previous section, I can derive closed-form expressions for $SP_{\mathbb{Q}_T}(p)$ and $SP_{\mathbb{P}}(p)$, but in order to save space, I only present the limiting cases:

$$SP_{\mathbb{Q}_T}(p): \quad \lim_{p\to 0} SP_{\mathbb{Q}_T}(p) = \frac{\sigma^4 T^2}{4} \text{ and } \lim_{p\to 1} SP_{\mathbb{Q}_T}(p) = -\frac{\sigma^4 T^2}{4} \tag{D.13}$$

$$SP_{\mathbb{P}}(p): \quad \lim_{p\to 0} SP_{\mathbb{P}}(p) = \frac{\sigma^2\left(\sigma^2-\mu\right)T^2}{4} \text{ and } \lim_{p\to 1} SP_{\mathbb{P}}(p) = -\frac{\sigma^2\left(\sigma^2+\mu\right)T^2}{4} - \frac{\mu\sigma^4 T^3}{24}. \tag{D.14}$$

IV. *Quarticity trading*

[This section is not complete]

TABLE I

Trading and realised variation measures: replication accuracy. Weekly settlements.

RM	Min	$q_{0.25}$	Med	Mean	$q_{0.75}$	Max	SD	MAD	MAE	ME	p-val
					Panel A: divergence						
$D_{1/2}$	0.00106	0.00423	0.00714	0.0119	0.0132	0.178	0.0158	0.00591			
$D_{1/2}$	-0.00215	0.004	0.00706	0.0117	0.0132	0.178	0.0157	0.00599	0.000266	0.000175	0.999
					Panel B: skewness						
$S_{\gamma^J}(1/2)$	-472	-3.02	0.825	-1.14	5.21	392	57.8	5.99			
$S_{\gamma^J}(1/2)$	-472	-3.18	1.18	-1.19	7.97	363	61.3	7.85	5.52	0.0478	0.808
					Panel C: quarticity						
$Q_{\gamma^J}(1/2)$	0.0872	1	3.84	32	15.2	1.73e+03	127	5.07			
$Q_{\gamma^J}(1/2)$	-1.09e+03	0.871	3.82	29.4	15.6	1.73e+03	177	5.2	21.2	2.59	0.808

Replication accuracy summary for divergence (Panel A), skewness (Panel B) and quarticity (Panel C) strategies associated with weighting γ^J. The first and third rows of each panel report the true realised measures calculated from forward prices. q_x denotes $100x$th percentile; Med denotes median; SD . denotes standard deviation; MAD denotes median absolute deviation; MAE denotes mean absolute error; ME denotes mean error.

TABLE II

Trading and realised variation measures: replication accuracy. Weekly settlements, 5-min hedging.

RM	Min	$q_{0.25}$	Med	Mean	$q_{0.75}$	Max	SD	MAD	MAE	ME	p-val
Panel A: divergence											
$D_{1/2}$	0.00106	0.00423	0.00714	0.0119	0.0132	0.178	0.0158	0.00591			
$D_{1/2}$	-0.00215	0.004	0.00706	0.0117	0.0132	0.178	0.0157	0.00599	0.000266	0.000175	0.999
Panel B: skewness											
$S_{\gamma^J}(1/2)$	-236	-1.51	0.413	-0.572	2.61	196	28.9	2.99			
$S_{\gamma^J}(1/2)$	-236	-1.59	0.591	-0.596	3.98	182	30.7	3.93	2.76	0.0239	0.808
Panel C: quarticity											
$Q_{\gamma^J}(1/2)$	0.0436	0.5	1.92	16	7.58	865	63.7	2.54			
$Q_{\gamma^J}(1/2)$	-546	0.435	1.91	14.7	7.82	865	88.6	2.6	10.6	1.29	0.808

Replication accuracy summary for divergence (Panel A), skewness (Panel B) and quarticity (Panel C) strategies associated with weighting γ^J. The first and third rows of each panel report the true realised measures calculated from forward prices. q_x denotes $100x$th percentile; Med denotes median; SD . denotes standard deviation; MAD denotes median absolute deviation; MAE denotes mean absolute error; ME denotes mean error.

TABLE III

Summary statistics of divergence trading with weeklies at one hour frequency

qty	Min	$q_{0.25}$	Med	CI (median) 90%		$q_{0.75}$	Max	Mean	SD	CI (mean) 90%	
Panel A: divergence											
$D_{\gamma^J}(1/2)$	0.001029	0.003974	0.006537	0.00562	0.007515	0.01272	0.1779	0.01125	0.01625	0.009885	0.01336
$\mathcal{C}[D_{\gamma^J}(1/2)]$	0.002626	0.006924	0.00997	0.009466	0.01068	0.01614	0.1258	0.01512	0.01586	0.01371	0.01705
$\mathcal{P}[D_{\gamma^J}(1/2)]$	-0.09832	-0.006246	-0.003358	-0.003731	-0.002765	-0.0006309	0.09479	-0.003864	0.01264	-0.00512	-0.002534
$\mathcal{T}[D_{\gamma^J}(1/2)]$	-0.1099	-0.0005735	0.002374	0.001949	0.00276	0.005057	0.09161	0.002348	0.01248	0.0008954	0.00347
Panel B: skewness											
$S_{\gamma^J}(1/2)$	-0.6292	-0.003806	0.0014	0.0006428	0.001934	0.006875	0.3067	-0.003219	0.06929	-0.01207	0.002467
$\mathcal{C}[S_{\gamma^J}(1/2)]$	-7.418	-0.2774	-0.1417	-0.1596	-0.1258	-0.0662	7.162	-0.239	0.8755	-0.3281	-0.1468
$\mathcal{P}[S_{\gamma^J}(1/2)]$	-7.791	0.06375	0.1479	0.1227	0.1641	0.2733	7.471	0.2357	0.9036	0.1401	0.3223
$\mathcal{T}[S_{\gamma^J}(1/2)]$	-10.75	-0.01135	0.08051	0.06109	0.09702	0.1746	5.907	0.0724	0.9869	-0.06306	0.1535
Panel C: quarticity											
$Q_{\gamma^J}(1/2)$	5.727e-06	0.0001766	0.0006414	0.0005001	0.0007549	0.002488	0.32	0.005494	0.02463	0.003719	0.00985
$\mathcal{C}[Q_{\gamma^J}(1/2)]$	-14.25	0.08055	0.1394	0.1273	0.1526	0.2984	14.43	0.303	1.675	0.1186	0.463
$\mathcal{P}[Q_{\gamma^J}(1/2)]$	-14.42	-0.2975	-0.1373	-0.1537	-0.1274	-0.08031	14.38	-0.2975	1.686	-0.4615	-0.1036
$\mathcal{T}[Q_{\gamma^J}(1/2)]$	-24.25	0.03526	0.07528	0.06325	0.08464	0.1364	10.31	0.05263	2.037	-0.2669	0.2065

Descriptive statistics of hourly frequency divergence trading profits with weekly option settlement for strategy γ^J. Trading period: from 2010-12-31 to 2015-12-24 with a total of 261 weeks. $\mathcal{C}[\cdot]$ denotes the integrated option cost at mid prices; $\mathcal{P}[\cdot]$ denotes total trading profits at mid prices; $\mathcal{T}[\cdot]$ denotes total trading profits after the transaction costs (bid-ask spread) are taken into account. $\mathcal{P}[\cdot]$ and $\mathcal{T}[\cdot]$ are reported in the following way: if mean $\mathcal{P}[\cdot]$ from . a *long* strategy is negative, $\mathcal{P}[\cdot]$ and $\mathcal{T}[\cdot]$ from a *short* strategy are reported. q_x denotes 100xth percentile; Med denotes median; SD denotes standard deviation; MAD denotes median absolute deviation.

TABLE IV

Summary statistics of divergence trading with weeklies at five minute frequency

qty	Min	$q_{0.25}$	Med	CI (median) 90%		$q_{0.75}$	Max	Mean	SD	CI (mean) 90%	
Panel A: divergence											
$D_{\gamma^J}(1/2)$	0.001459	0.007246	0.0121	0.01065	0.01294	0.01905	0.2688	0.01818	0.02479	0.01613	0.02139
$\mathcal{C}[D_{\gamma^J}(1/2)]$	0.002819	0.00717	0.01031	0.009538	0.01064	0.016	0.1387	0.01541	0.01641	0.01395	0.01735
$\mathcal{P}[D_{\gamma^J}(1/2)]$	-0.05686	-0.003522	0.0009186	0.0003282	0.00158	0.005803	0.228	0.002774	0.01852	0.001339	0.005425
$\mathcal{T}[D_{\gamma^J}(1/2)]$	-0.06671	-0.00482	0.0004762	-0.0006722	0.001155	0.004921	0.2198	0.001267	0.01832	-0.0001882	0.003765
D_{VIX}	0.001459	0.007246	0.01209	0.01072	0.01294	0.01904	0.2703	0.01819	0.02485	0.01611	0.0213
$\mathcal{C}[D_{VIX}]$	0.00282	0.007213	0.01024	0.009548	0.01078	0.01607	0.1425	0.01551	0.0166	0.01406	0.01747
$\mathcal{P}[D_{VIX}]$	-0.05852	-0.003677	0.00087	0.0002947	0.001558	0.005733	0.2393	0.00268	0.01919	0.001179	0.00546
$\mathcal{T}[D_{VIX}]$	-0.06775	-0.004908	0.0004272	-0.0007096	0.001099	0.00492	0.2378	0.001265	0.01932	-0.0002345	0.004018
Panel B: skewness											
$S_{\gamma^J}(1/2)$	-0.2547	-0.004106	0.000539	0.0001603	0.0009623	0.004029	0.2361	-0.002871	0.04359	-0.007489	0.001221
$\mathcal{C}[S_{\gamma^J}(1/2)]$	-8.172	-0.2847	-0.1569	-0.1759	-0.1458	-0.07935	7.269	-0.2746	0.9273	-0.3714	-0.1834
$\mathcal{P}[S_{\gamma^J}(1/2)]$	-7.033	0.07723	0.158	0.1445	0.1717	0.2874	8.221	0.2717	0.9307	0.1773	0.3688
$\mathcal{T}[S_{\gamma^J}(1/2)]$	-13.82	-0.06591	0.04021	0.02469	0.05654	0.1201	4.915	-0.05125	1.179	-0.2232	0.0356
Panel C: quarticity											
$Q_{\gamma^J}(1/2)$	4.672e-06	0.0001729	0.0005	0.0003709	0.000607	0.00159	0.1813	0.003479	0.01398	0.002399	0.005714
$\mathcal{C}[Q_{\gamma^J}(1/2)]$	-22.91	0.08741	0.1436	0.1234	0.1537	0.2823	15.87	0.3187	1.986	0.06931	0.4879
$\mathcal{P}[Q_{\gamma^J}(1/2)]$	-15.86	-0.2819	-0.1431	-0.154	-0.1281	-0.08728	23.09	-0.3153	1.994	-0.4868	-0.06533
$\mathcal{T}[Q_{\gamma^J}(1/2)]$	-48.68	-0.003656	0.03855	0.0327	0.04237	0.09346	8.409	-0.154	3.283	-0.8162	0.06882

Descriptive statistics of 5-minute frequency divergence trading profits with weekly option settlement for strategy γ^J. Trading period: from 2010-12-31 to 2015-12-24 with a total of 261 weeks. $\mathcal{C}[\cdot]$ denotes the integrated option cost at mid prices; $\mathcal{P}[\cdot]$ denotes total trading profits at mid prices; $\mathcal{T}[\cdot]$ denotes total trading profits after the transaction costs (bid-ask spread) are taken into account. $\mathcal{P}[\cdot]$ and $\mathcal{T}[\cdot]$ are reported in the following way: if mean $\mathcal{P}[\cdot]$ from a *long* strategy is negative, $\mathcal{P}[\cdot]$ and $\mathcal{T}[\cdot]$ from a *short* strategy are reported. q_x denotes 100xth percentile; Med denotes median; SD denotes standard deviation; MAD denotes median absolute deviation.

TABLE V

Summary statistics of open-to-close divergence trading with weeklies at five minute frequency

qty	Min	$q_{0.25}$	Med	CI (median) 90%		$q_{0.75}$	Max	Mean	SD	CI (mean) 90%	
Panel A: divergence											
$D_{\gamma^J}(1/2)$	0.0001151	0.002495	0.005041	0.004664	0.005358	0.01163	0.7204	0.0117	0.02824	0.01064	0.01344
$\mathcal{C}[D_{\gamma^J}(1/2)]$	-0.4276	0.001219	0.004199	0.003946	0.004414	0.008529	0.4149	0.006361	0.02443	0.005168	0.007492
$\mathcal{P}[D_{\gamma^J}(1/2)]$	-0.2294	-0.003282	0.0007545	0.0004215	0.001214	0.008023	0.6754	0.005334	0.03417	0.003959	0.00722
$\mathcal{T}[D_{\gamma^J}(1/2)]$	-0.3177	-0.009363	-0.003626	-0.004279	-0.003216	0.00263	0.5354	-0.003355	0.03102	-0.004775	-0.001795
D_{VIX}	0.0001155	0.002493	0.005041	0.004672	0.005365	0.01163	0.7201	0.0117	0.02823	0.01066	0.01342
$\mathcal{C}[D_{VIX}]$	-0.4516	0.001125	0.00416	0.003936	0.00437	0.008577	0.4274	0.006352	0.02529	0.005148	0.007567
$\mathcal{P}[D_{VIX}]$	-0.2418	-0.003288	0.0007952	0.0003824	0.001168	0.007879	0.6746	0.005344	0.03478	0.003961	0.007314
$\mathcal{T}[D_{VIX}]$	-0.3285	-0.009429	-0.003648	-0.004302	-0.003251	0.002673	0.5543	-0.003344	0.03196	-0.004756	-0.00171
Panel B: skewness											
$S_{\gamma^J}(1/2)$	-0.06286	-0.0001898	4.38e-06	-3.979e-06	1.304e-05	0.0002412	0.1284	5.945e-06	0.004474	-0.0001434	0.0003139
$\mathcal{C}[S_{\gamma^J}(1/2)]$	-5.67	-0.03245	-0.005487	-0.00708	-0.003578	0.02172	11	0.004335	0.4408	-0.0125	0.02978
$\mathcal{P}[S_{\gamma^J}(1/2)]$	-11.01	-0.02218	0.005413	0.00354	0.007231	0.03271	5.673	-0.004329	0.4419	-0.02954	0.01217
$\mathcal{T}[S_{\gamma^J}(1/2)]$	-12.69	-0.1389	-0.05752	-0.06211	-0.05372	-0.02084	8.186	-0.1527	0.6282	-0.187	-0.1275
Panel C: quarticity											
$Q_{\gamma^J}(1/2)$	1.71e-09	3.616e-06	1.306e-05	1.09e-05	1.564e-05	5.551e-05	0.06128	0.0002529	0.002172	0.0001807	0.0004353
$\mathcal{C}[Q_{\gamma^J}(1/2)]$	-12.05	-0.01531	-0.0004048	-0.001003	0.0001241	0.007236	6.577	-0.0233	0.5288	-0.05544	-0.00193
$\mathcal{P}[Q_{\gamma^J}(1/2)]$	-6.569	-0.007201	0.0004722	-0.0001067	0.00101	0.01531	12.05	0.02356	0.5285	0.001757	0.05589
$\mathcal{T}[Q_{\gamma^J}(1/2)]$	-18.81	-0.1136	-0.03534	-0.04063	-0.0317	-0.0109	8.237	-0.2049	1.009	-0.2702	-0.1642

Descriptive statistics of open-to-close (8:30 to 15:15) 5-minute frequency divergence trading profits with weekly option settlement for strategy γ^J. Trading period: from 2010-12-30 to 2015-12-30 with a total of 1258 days. $\mathcal{C}[\cdot]$ denotes the integrated option cost at mid prices; $\mathcal{P}[\cdot]$ denotes total trading profits at mid prices; $\mathcal{T}[\cdot]$ denotes total trading profits after the transaction costs (bid-ask spread) are taken into account. $\mathcal{P}[\cdot]$ and $\mathcal{T}[\cdot]$ are reported in the following way: if mean $\mathcal{P}[\cdot]$ from a *long* strategy is negative, $\mathcal{P}[\cdot]$ and $\mathcal{T}[\cdot]$ from a *short* strategy are reported. q_x denotes 100xth percentile; Med denotes median; SD denotes standard deviation; MAD denotes median absolute deviation.

TABLE VI

Summary statistics of close-to-open divergence trading with weeklies

qty	Min	$q_{0.25}$	Med	CI (median) 90%		$q_{0.75}$	Max	Mean	SD	CI (mean) 90%	
Panel A: divergence											
$D_{\gamma^J}(1/2)$	7.392e-08	0.0003514	0.00155	0.001441	0.001774	0.005503	0.1386	0.005493	0.01169	0.00497	0.006158
$\mathcal{C}[D_{\gamma^J}(1/2)]$	-0.07597	0.002953	0.00555	0.005194	0.005915	0.01093	0.2602	0.009274	0.01782	0.008477	0.01033
$\mathcal{P}[D_{\gamma^J}(1/2)]$	-0.2285	-0.006935	-0.003535	-0.003756	-0.003305	-0.0009259	0.2034	-0.003781	0.01885	-0.004744	-0.002836
$\mathcal{T}[D_{\gamma^J}(1/2)]$	-0.3184	-0.005829	-0.0007378	-0.001117	-0.000436	0.001779	0.166	-0.005119	0.02181	-0.006384	-0.004137
D_{VIX}	8.525e-07	0.002267	0.01095	0.007766	0.01307	0.0272	0.3241	0.02312	0.03849	0.01933	0.02882
$\mathcal{C}[D_{VIX}]$	0.01637	0.02971	0.04494	0.04062	0.04792	0.06418	0.5256	0.06586	0.07513	0.05824	0.07647
$\mathcal{P}[D_{VIX}]$	-0.4315	-0.04626	-0.02772	-0.03227	-0.02542	-0.01633	0.06974	-0.04273	0.06378	-0.05136	-0.03611
$\mathcal{T}[D_{VIX}]$	-0.1043	0.01143	0.0226	0.01969	0.02521	0.0381	0.3883	0.03297	0.05601	0.02705	0.04059
Panel B: skewness											
$S_{\gamma^J}(1/2)$	-0.7087	-0.0002245	2.721e-06	2.293e-07	6.231e-06	0.0005384	0.5688	-0.0001522	0.03928	-0.002119	0.001611
$\mathcal{C}[S_{\gamma^J}(1/2)]$	-4.45	-0.06159	-0.0239	-0.02629	-0.02239	-0.007998	2.09	-0.05744	0.2575	-0.07111	-0.04652
$\mathcal{P}[S_{\gamma^J}(1/2)]$	-2.799	0.00737	0.02474	0.02285	0.02745	0.0643	4.483	0.05728	0.2774	0.04472	0.07122
$\mathcal{T}[S_{\gamma^J}(1/2)]$	-3.262	-0.02571	0.001545	0.0002621	0.002658	0.01861	3.208	-0.01626	0.2439	-0.0278	-0.004963
Panel C: quarticity											
$Q_{\gamma^J}(1/2)$	8.806e-15	1.118e-07	9.929e-07	8.095e-07	1.3e-06	8.611e-06	0.004213	4.876e-05	0.0002752	3.677e-05	6.683e-05
$\mathcal{C}[Q_{\gamma^J}(1/2)]$	-0.2191	0.0006619	0.002072	0.0019	0.002289	0.006414	0.7328	0.008294	0.03882	0.006689	0.01088
$\mathcal{P}[Q_{\gamma^J}(1/2)]$	-0.7325	-0.006362	-0.002072	-0.002278	-0.001896	-0.0006347	0.2221	-0.008245	0.03883	-0.01079	-0.006484
$\mathcal{T}[Q_{\gamma^J}(1/2)]$	-0.3063	-0.0002681	0.0003993	0.0003391	0.0004561	0.00174	0.4742	-0.0001248	0.02795	-0.001597	0.001335

Descriptive statistics of close-to-open (15:15 to 8:30) divergence trading profits with weekly option settlement for strategy γ^J. 99% of payoffs with smallest absolute replication error are retained. Trading period: from 2010-12-31 to 2015-12-24 with a total of 261 days $\mathcal{C}[\cdot]$ denotes the integrated option cost at mid prices; $\mathcal{P}[\cdot]$ denotes total trading profits at mid prices; $\mathcal{T}[\cdot]$ denotes total trading profits after the transaction costs (bid-ask spread) are . taken into account. $\mathcal{P}[\cdot]$ and $\mathcal{T}[\cdot]$ are reported in the following way: if mean $\mathcal{P}[\cdot]$ from a *long* strategy is negative, $\mathcal{P}[\cdot]$ and $\mathcal{T}[\cdot]$ from a *short* strategy are reported. q_x denotes 100xth percentile; Med denotes median; SD denotes standard deviation; MAD denotes median absolute deviation.

APPENDIX E: FIGURES

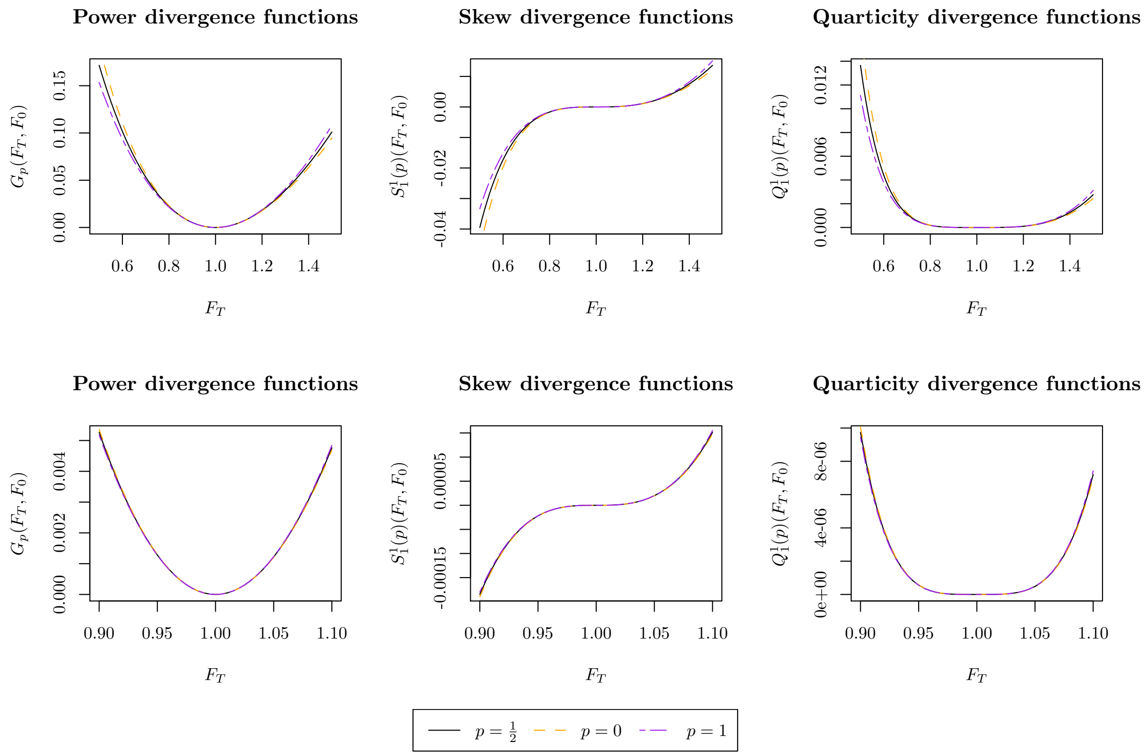


Figure 2: Payoffs: power divergence, skewness divergence, and quarticity divergence. Power divergence G_p plotted for $p \in \{1/2, 0, 1\}$. Skew and quarticity divergences formed around the respective power divergence function.

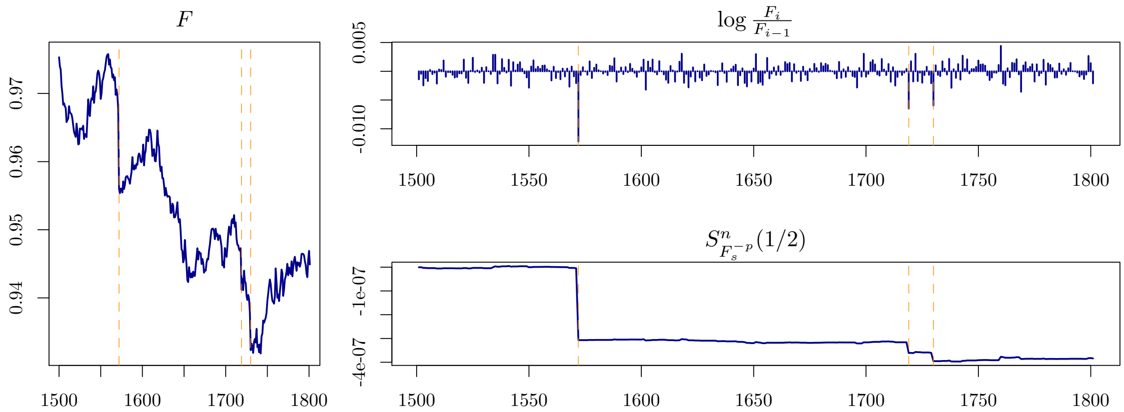


Figure 3: $S^n_{F_t^{-p}}(1/2)$: simulated data. The figure plots data from a simulated trading day at a 5-minute frequency. Three jumps (marked by vertical dashed orange lines) occur within the day. The left panel presents the evolution of the underlying price F. The top right panel plots the logarithmic returns. The bottom right panel plots cumulative Hellinger skewness calculated over the course of the trading day.

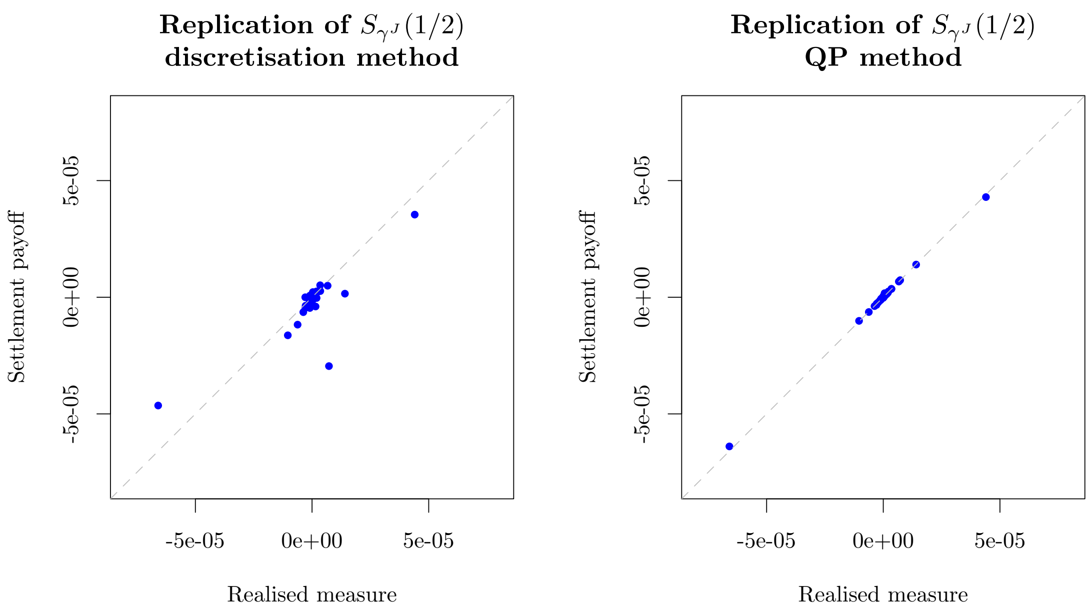


Figure 4: Comparison of replication accuracy of $S_{\gamma^J}(1/2)$ with the discretisation method (left panel) and the quadratic programming method (right panel).

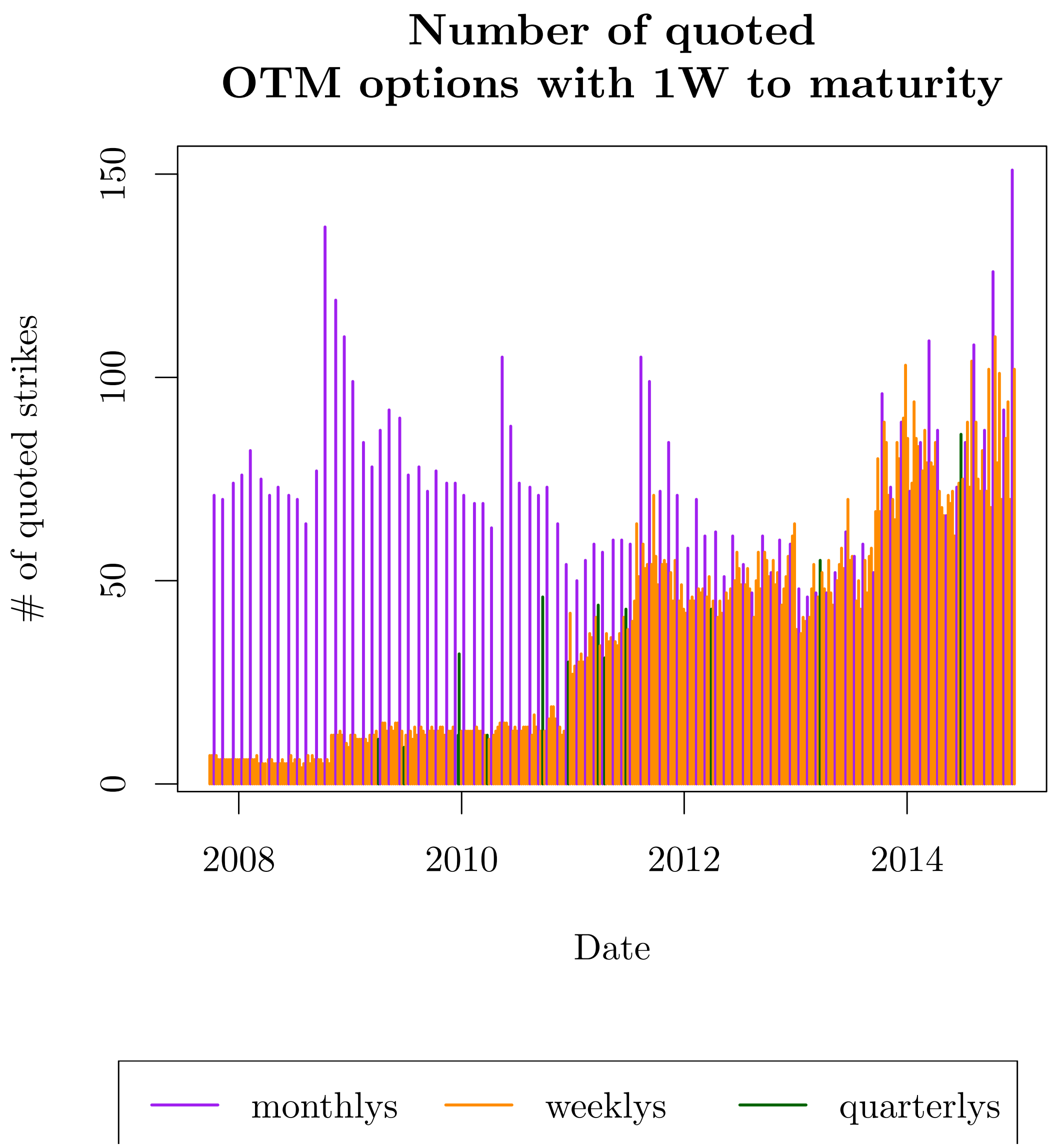


Figure 5: Number strikes at which out of the money options are quoted at the end of each day since the weekly options appear in the MDR sample, i.e. with maturity on 2007-09-28. Whenever a quarterly maturity is no more than 1 day away from the weekly maturity, and more quarterly options are quoted than weeklies, the number of quarterlys strikes is taken.

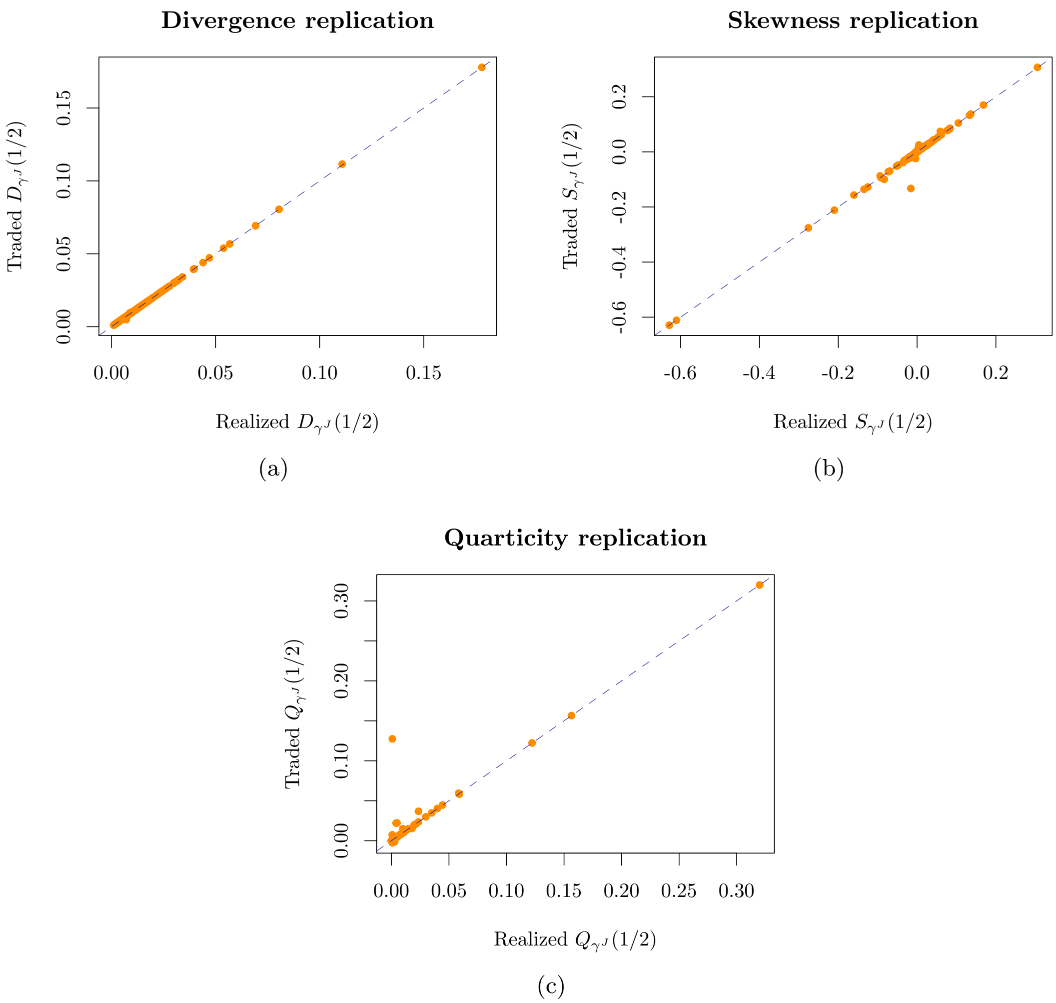
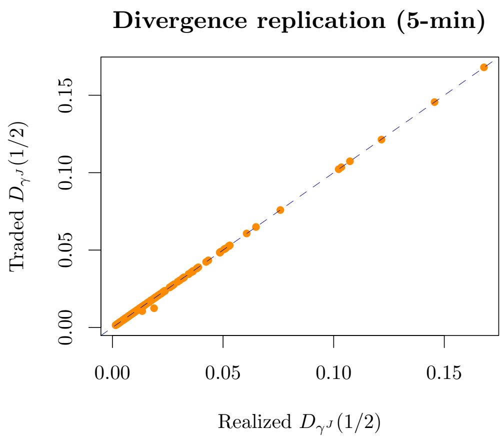
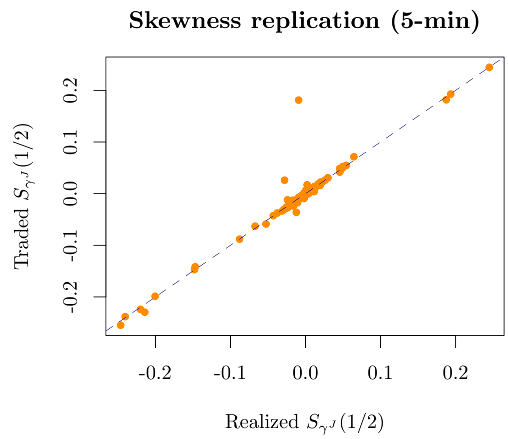


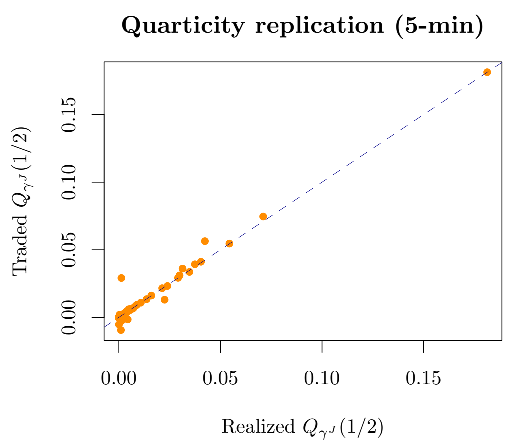
Figure 6: Scatterplots of realized divergence, skewness and quarticity with weighting γ^J, and dynamically replicated settlement payoffs, with weekly option settlement and 1-hmy hedging interval. Trading sample from 2010-12-31 to 2015-12-24.



(a)

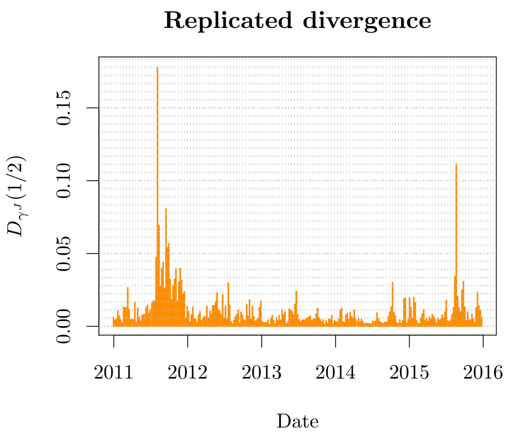


(b)

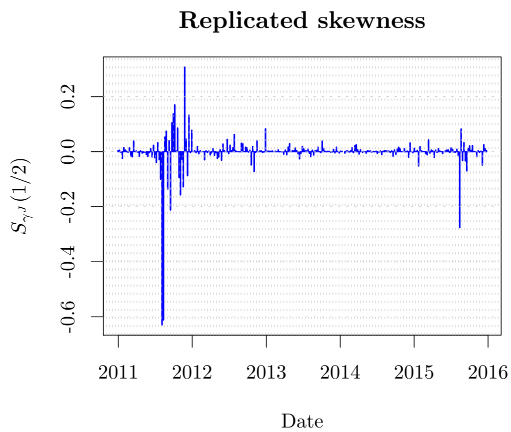


(c)

Figure 7: Scatterplots of realized divergence, skewness and quarticity with weighting γ^J and dynamically replicated settlement payoffs, with weekly option settlement and 5-minute hedging interval. Trading sample from 2010-12-31 to 2015-12-24.



(a)



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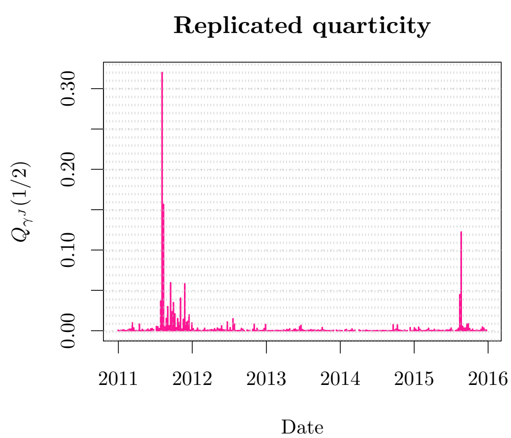


(c)

Figure 8: Time series plots of replicated divergence, skewness and quarticity trading strategies with weighting γ^J, with weekly option settlement and one-hmy hedging frequency. Trading sample from 2010-12-31 to 2015-12-24.

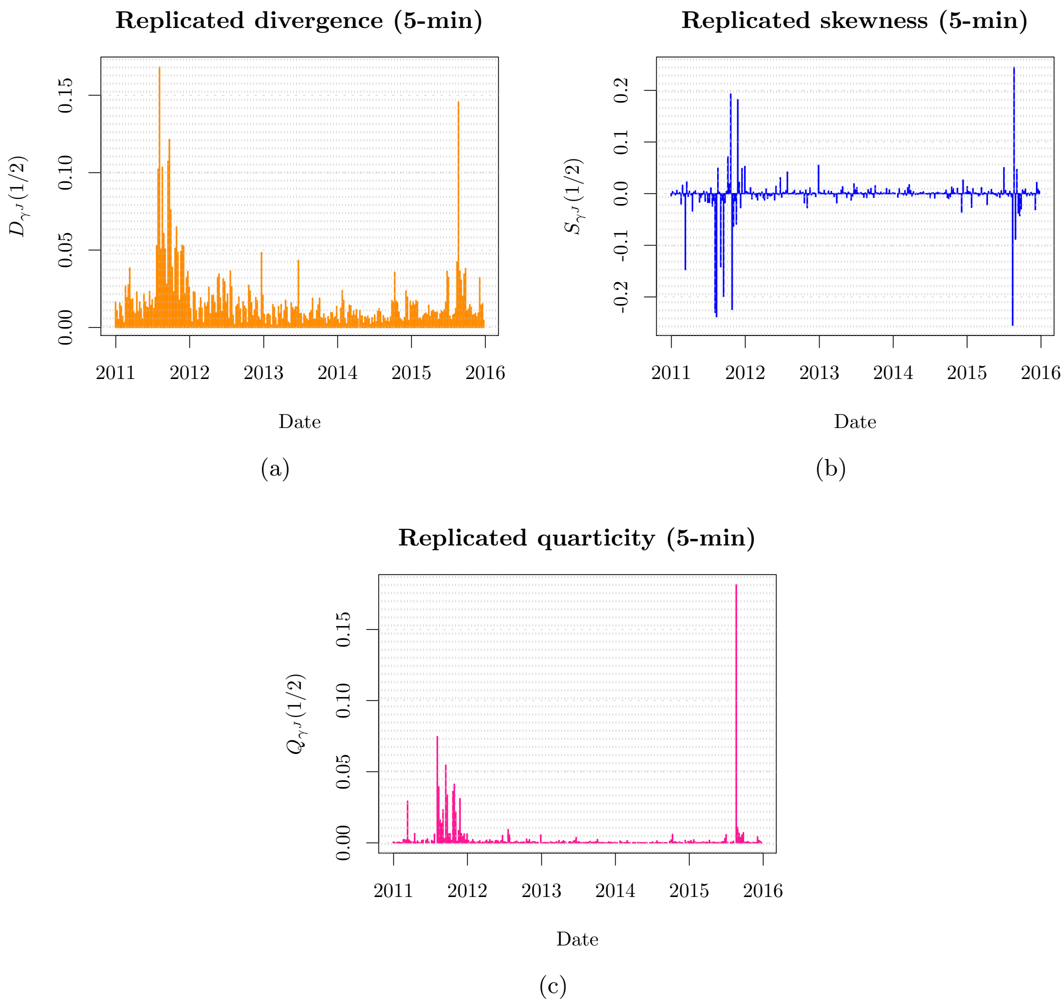
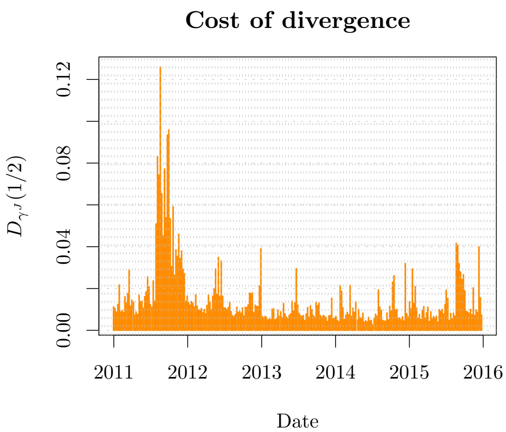
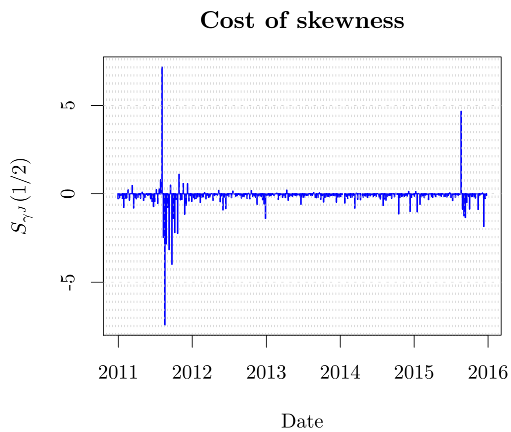


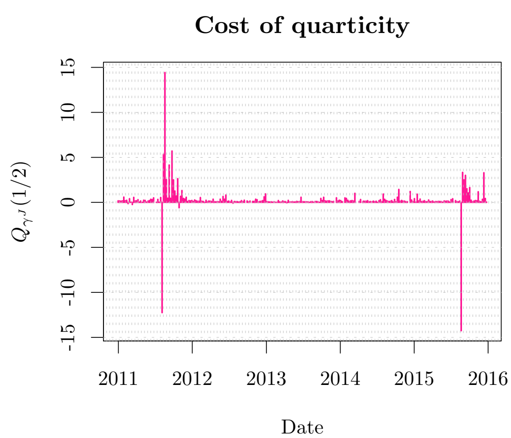
Figure 9: Time series plots of replicated divergence, skewness and quarticity trading strategies with weighting γ^J, with weekly option settlement and 5-minute hedging frequency. Trading sample from 2010-12-31 to 2015-12-24.



(a)

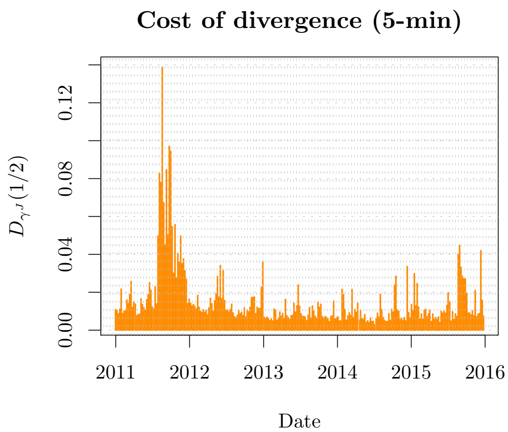


(b)



(c)

Figure 10: Time series plots of cost of replicating divergence, skewness and quarticity trading strategies with weighting γ^J, with weekly option settlement and one-hmy hedging frequency. Trading sample from 2010-12-31 to 2015-12-24.



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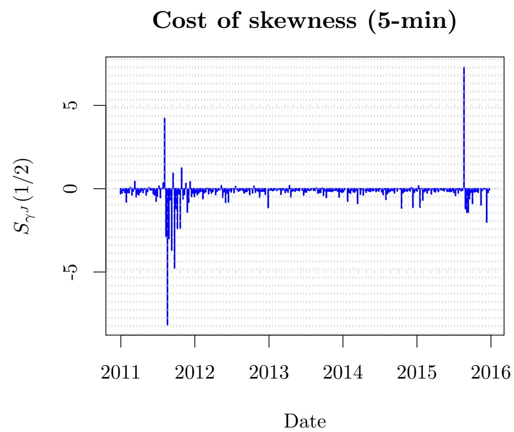

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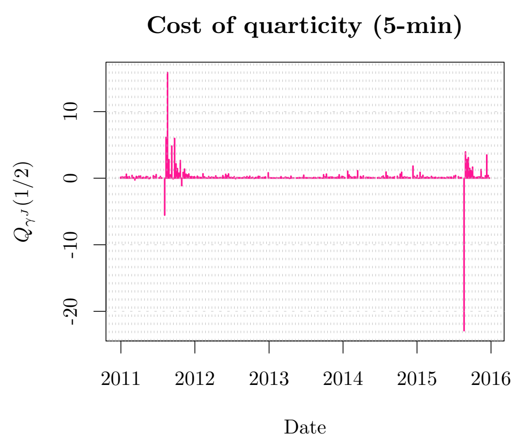
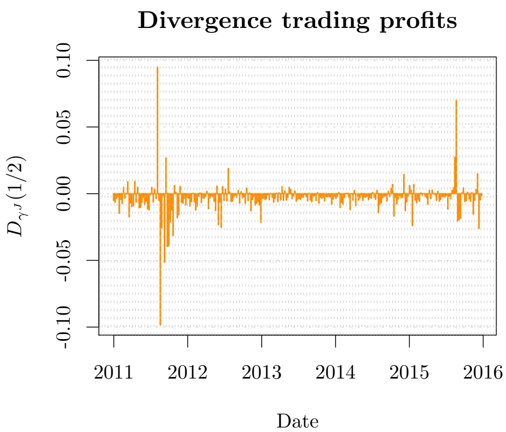


(c)

Figure 11: Time series plots of cost of replicating divergence, skewness and quarticity trading strategies with weighting γ^J, with weekly option settlement and five minute hedging frequency. Trading sample from 2010-12-31 to 2015-12-24.



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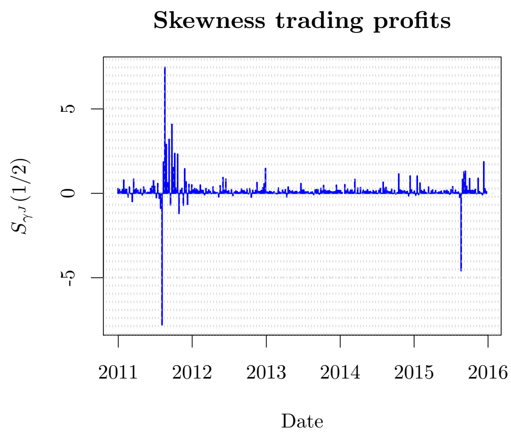

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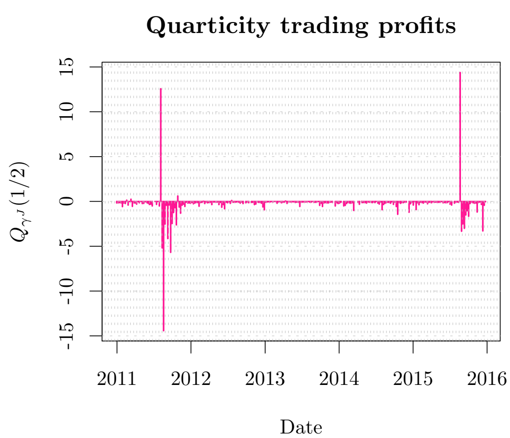
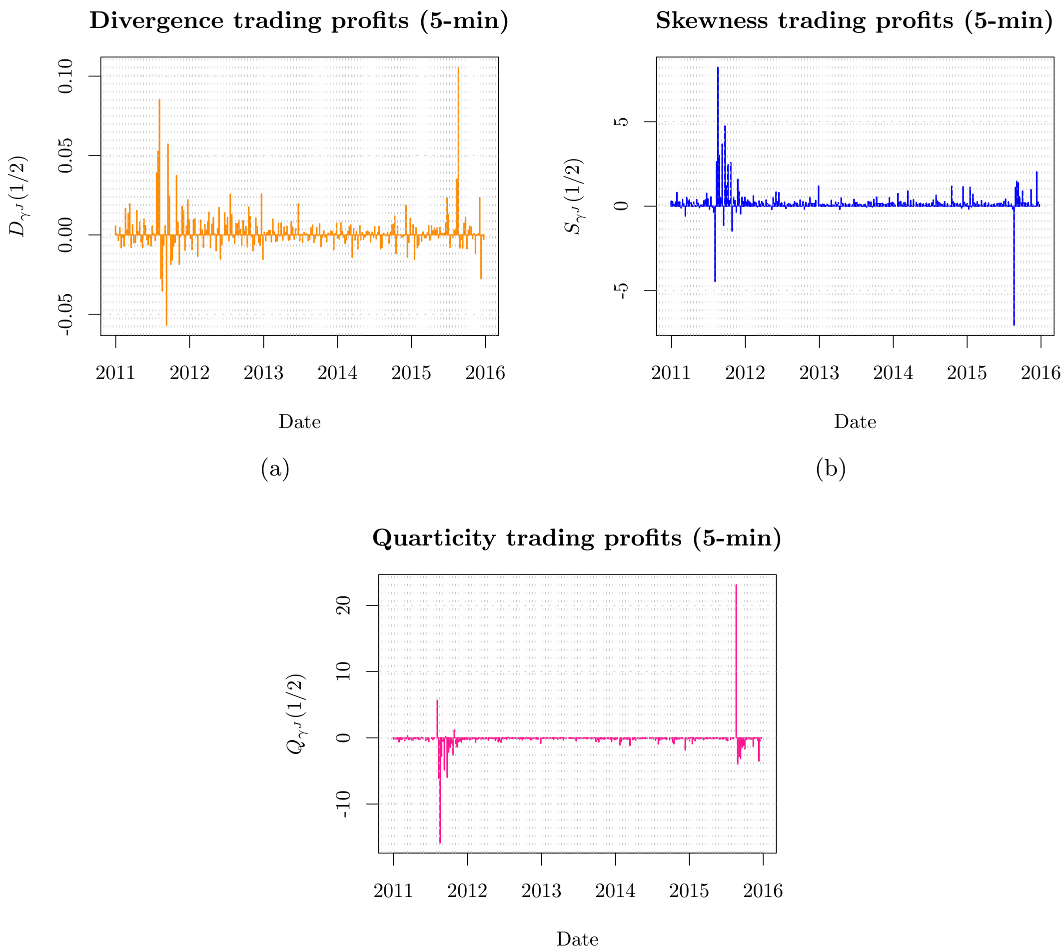


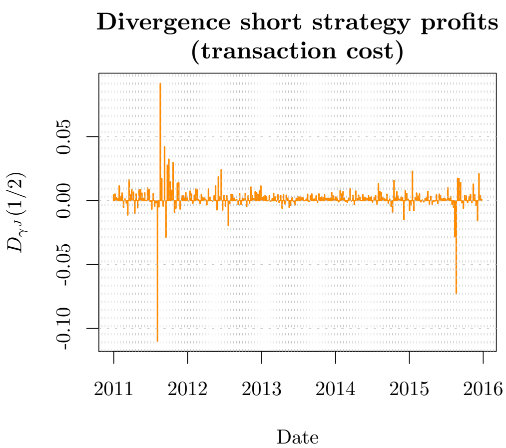
(c)

Figure 12: Time series plots of profits from replicating divergence, skewness and quarticity trading strategies with weighting γ^J, with weekly option settlement and one-hmy hedging frequency. Trading sample from 2010-12-31 to 2015-12-24.

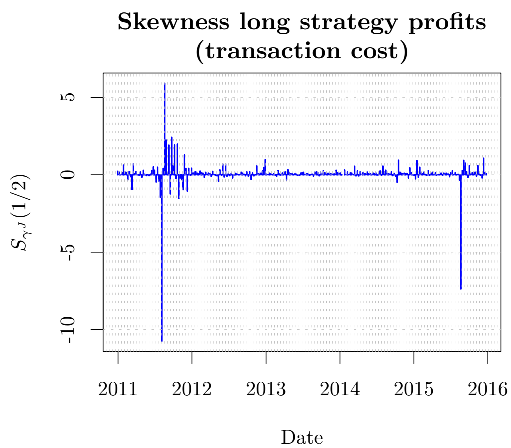


(a) (b) (c)

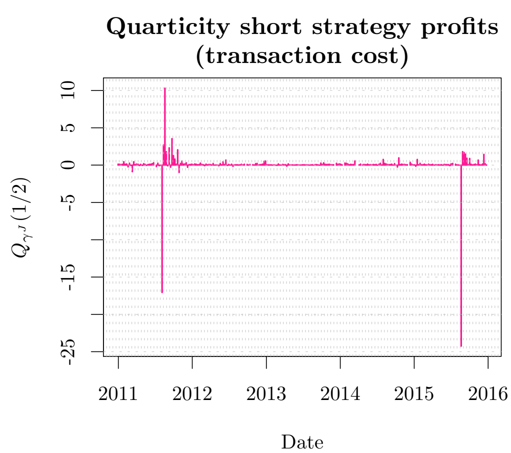
Figure 13: Time series plots of profits from replicating divergence, skewness and quarticity trading strategies with weighting γ^J, with weekly option settlement and five-minute hedging frequency. Trading sample from 2010-12-31 to 2015-12-24.



(a)

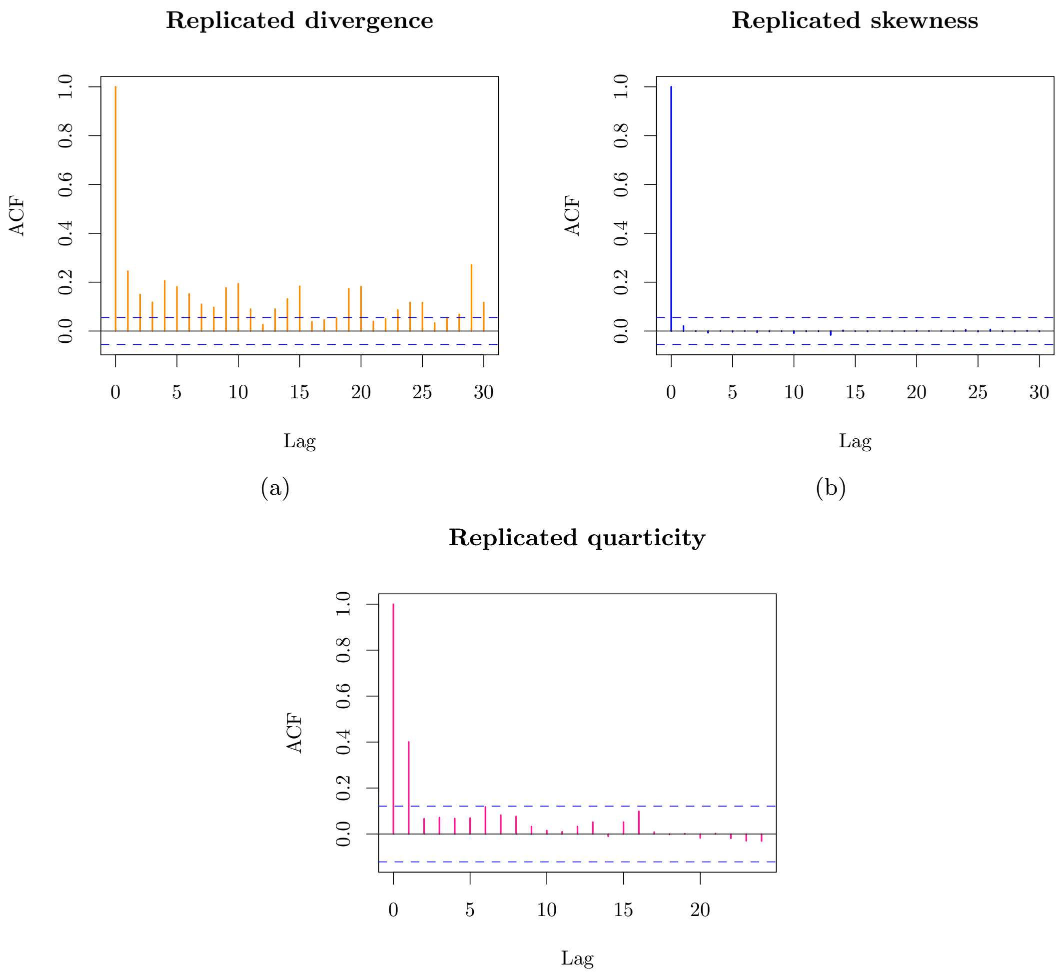


(b)



(c)

Figure 14: Time series plots of profits (including transaction costs) from replicating divergence, skewness and quarticity trading strategies with weighting γ^J, with weekly option settlement and one-hmy hedging frequency. Trading sample from 2010-12-31 to 2015-12-24.



(a) (b) (c)

Figure 15: Autocorrelation function plots of traded divergence, skewness and quarticity with weighting γ^J (left-hand plots) and γ^{JV}. Trading sample from 2010-12-31 to 2015-12-24.

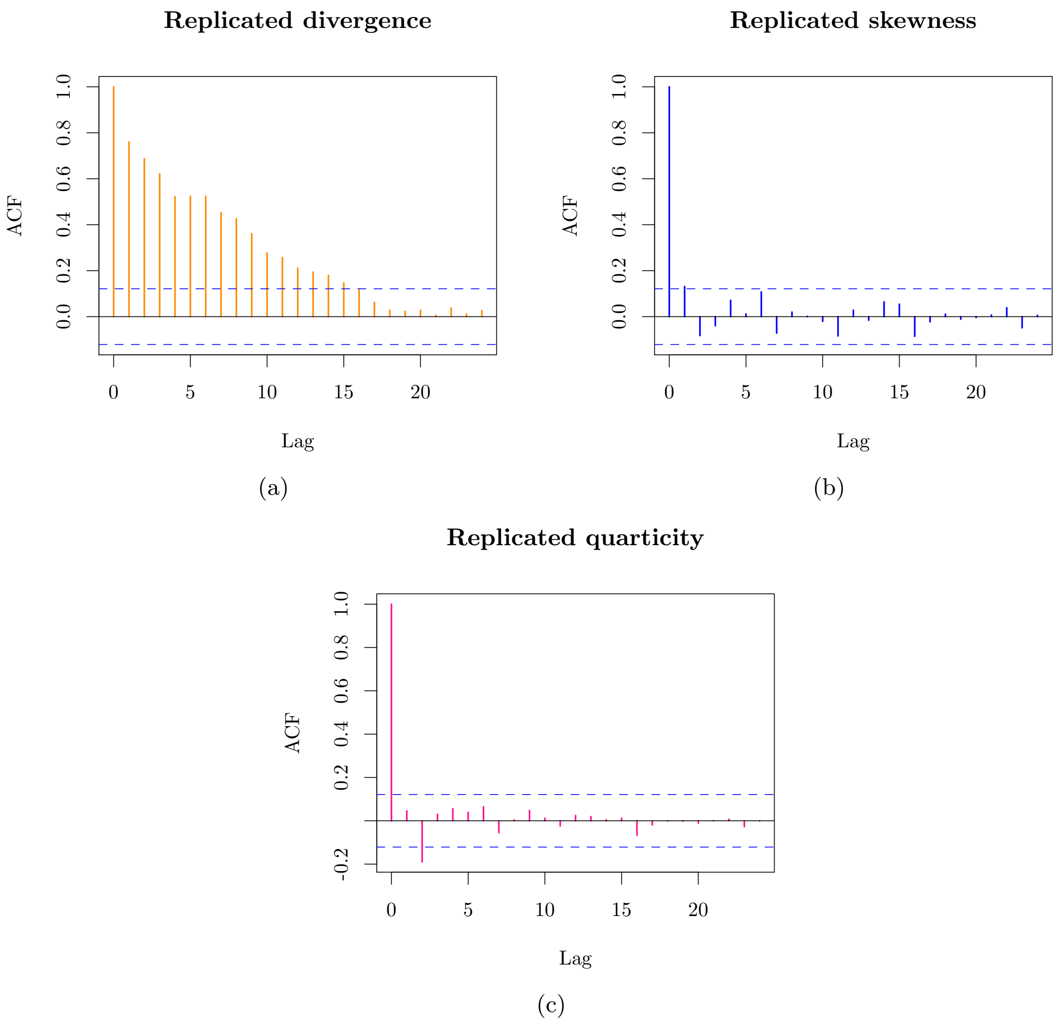


Figure 16: Autocorrelation function plots of integrated cost of trading divergence, skewness and quarticity with weighting γ^J with weekly settlements at the 1-hmy trading frequency. Trading sample from 2010-12-31 to 2015-12-24.

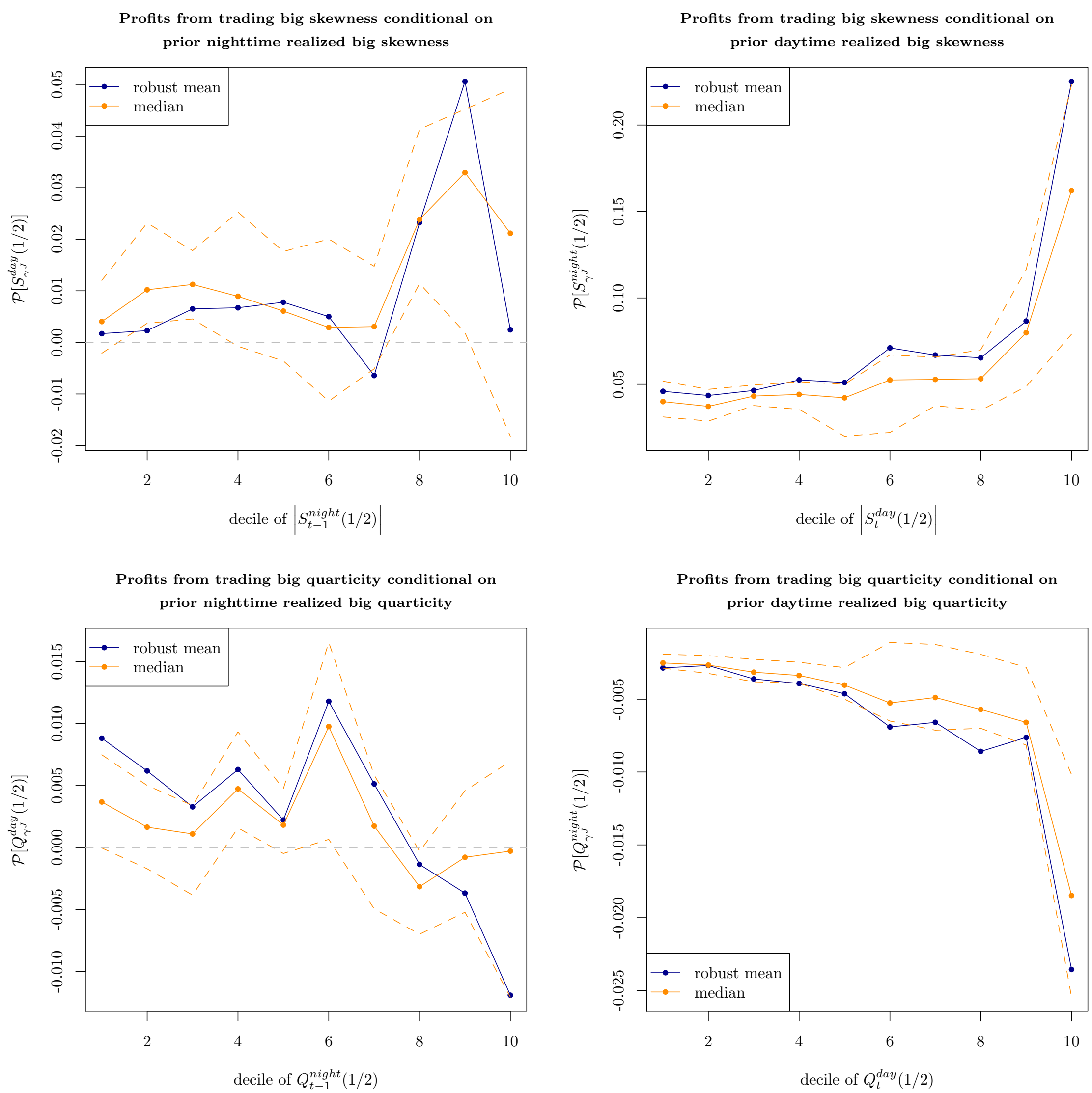


Figure 17: Profits from trading big skewness and big quarticity with weekly options, conditionally on realised big variation over the preceding period. For daytime trading, the preceding period is from previous closing until opening of markets on day of trading. For nighttime trading, the preceding period is from the day's market opening until market closing on night of trading. During the day hedging is at 5-minute frequency.

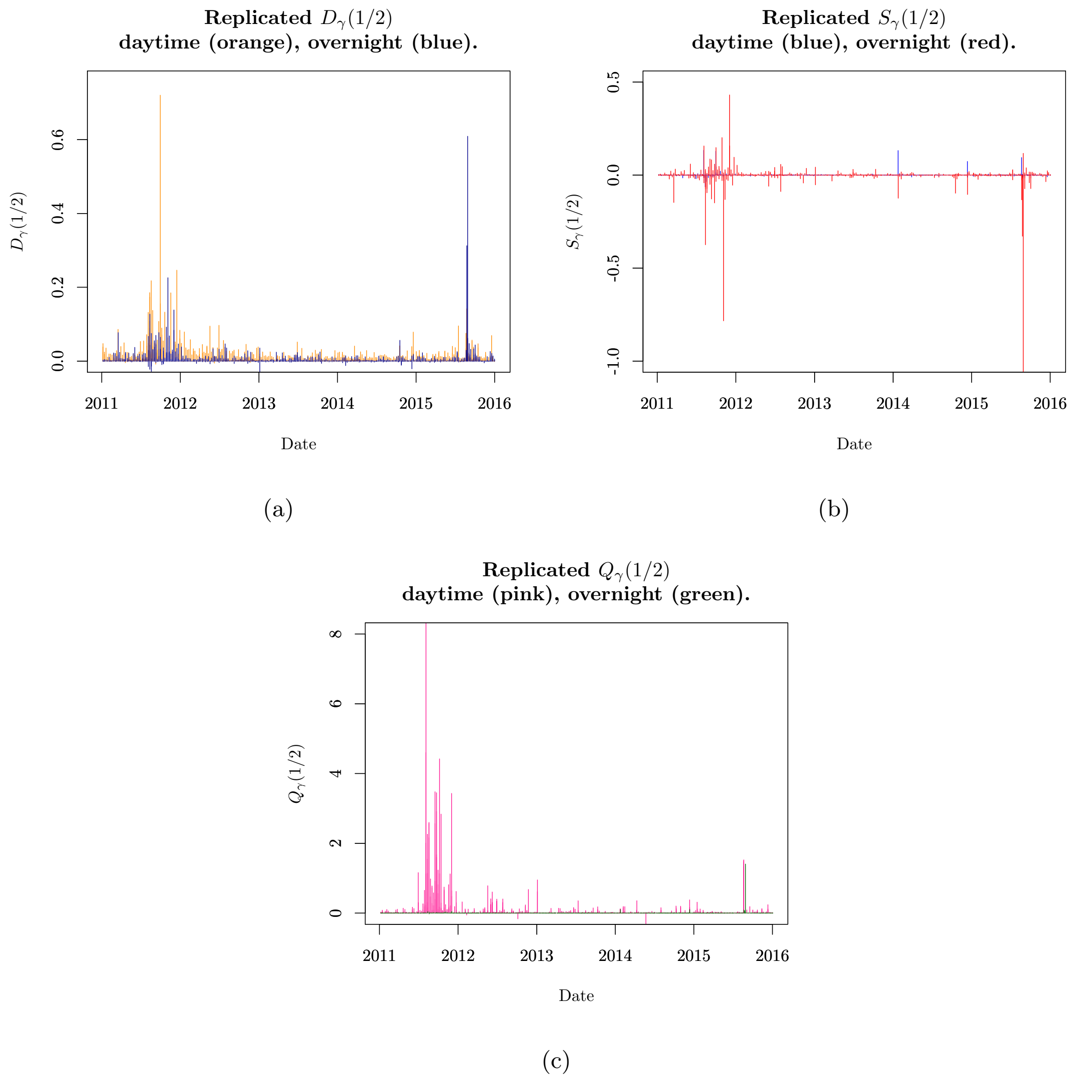


Figure 18: Time series plots of profits from replicating divergence, skewness and quarticity trading strategies with weighting γ^J, with daily and overnight option settlement and five-minute hedging frequency. Trading sample from 2010-12-31 to 2015-12-24.