On the nature of jump risk premia*

Piotr Orłowski
HEC Montréal
piotr.orlowski@hec.ca

Paul Schneider
USI Lugano
Swiss Finance Institute
paul.schneider@usi.ch

Fabio Trojani
Université de Genève
Swiss Finance Institute
fabio.trojani@unige.ch

May 16, 2019

Abstract

We shed light on the nature of jump risk compensation by studying the profits from a trading strategy that bets on high-frequency jump skew of S&P 500 returns. Earlier evidence suggests the jump premium is large and positive. We find it to be concentrated in periods when the index option market is closed, and investors cannot trade options. Whenever skew can be traded continuously, the premium vanishes. We conclude that the jump skew premium in index options is not compensation for the risk of occasional, large returns, but for the investors' inability to adjust their nonlinear risk exposure.

Keywords: rare events, jump risk premium, options, high-frequency data

JEL codes: G10, G12, C61

---

*This paper was previously presented as "Big Risk?". Trojani and Orłowski acknowledge the financial support from the Swiss National Science Foundation (Project 150198, "Higher order robust resampling and multiple testing methods") and the Swiss Finance Institute (Project "Term structures and cross-sections of asset risk premia"). Schneider acknowledges the financial support form the Swiss National Science Foundation (Project 169582, "Model-free asset pricing"). We would like to thank Viktor Todorov, Alexandre Jeanneret, Valeri Sokolovski, Mathieu Fournier, Francesca Carrieri, and the participants of the Quantitative Finance Seminars at Kellogg School of Management for valuable comments and suggestions.

Piotr Orłowski:

I have nothing to disclose.

Paul Schneider:

I have nothing to disclose.

Fabio Trojani:

I have nothing to disclose.

I. Introduction

Jumps – occasional, large, but not disastrous returns are essential ingredients to most models attempting to capture the salient features of the time series of index returns. A high market price of jump risk is also often seen as crucial for explaining low index option returns.[1] Asset pricing theory states that equilibrium asset prices can vary discretely over time, i.e., they can jump at times where information is not revealed continuously (Huang, 1985a,b). Accordingly, priced jump risk reflects systematic compensations for particular sources of information discreteness in asset markets.

The magnitude and nature of the jump risk premium are crucial research questions, both among theorists (Drechsler, 2013) and empiricists (see e.g. Bollerslev and Todorov, 2011; Andersen et al., 2015b). Available evidence suggests that compensation for negative jump risk is large, persistent, and distinct from compensation for variance risk (Andersen et al., 2015b). Further evidence suggests the jump premium is the dominant component of both equity and variance risk premia (Bollerslev and Todorov, 2011; Bollerslev et al., 2016). A pronounced role of negative jump premia implies that investors are particularly averse to asymmetry, or skewness of jump risks. The goal of this paper is to answer the question of whether nonlinear jump risk does indeed command risk compensation, i.e. whether trading assets that are exposed solely to jump skew offer premia that cannot be explained by exposure to other systematic risk factors.

We shed direct light on the exact nature of the premium for jump skew risk with a novel, model-free approach. Our starting motivation is that an investor in the index option market holding nonlinear exposure to underlying returns may face two structurally disparate market regimes: during market hours, and after market hours, i.e. overnight. During market hours, an investor can hedge by trading in the option market. This is impossible overnight, when the option market is closed and delta-hedging in the electronic market for the underlying (the only dynamic hedging strategy available) is insufficient. . We characterize the premia associated with market hours jump skew risk and overnight skew risk by means of a new trading strategy. During market hours, the strategy allows one to separate the small returns from the large returns over very short time spans, hence it is a pure play on jumps.

[1] Fear of rare disasters, i.e. once-in-a-century, very large decreases of the aggregate endowment allows for explaining a plethora of asset pricing puzzles (Gabaix, 2012; Wachter, 2011). However, Backus et al. (2011) and Du (2011) show that the skew patterns in index options are not compatible with the rare disasters paradigm.

In contrast, after market hours – when active option trading is no longer possible – the investor is exposed to the skew of the overnight return by means of a static option position. Leading option pricing models are unable to discern between these two market regimes and thus predict that the strategy should be profitable both during and after market hours. In turn, we find a positive skew premium only when holding the strategy throughout the after-hours overnight period, until trading resumes on the following day. This premium is not explained by exposures to the market and intermediary risk factors. We interpret this model-independent finding as evidence that what the literature identifies as premium for jump risk is actually rather a premium for a reduced ability to hedge one's position with nonlinear payoffs.

We analyze the price of jump skew risk in the market for S&P 500 Friday Weeklys.[2] We focus on options with seven and fewer days to maturity because these are the most effective instruments to investors for hedging time-varying skewness risk (Bollerslev and Todorov, 2011; Andersen et al., 2017). Whenever options can be actively traded, our strategy allows us to trade the skew of the underlying returns over arbitrarily short periods, as well as to accumulate the traded skews during any time horizon shorter than the maturity of the options. In principle, our strategy can trade the skew of daily returns over the course of a week, or the skew of five-minute returns over the course of a lunch break. With sufficiently frequent trading, the strategy's payoff gains the interpretation of a premium for jump skewness. In contrast, when trading is less frequent, the strategy becomes a bet on a different notion of skewness, first described by Schneider and Trojani (2018), that is unable to separate jump skewness from other sources of skewness risk. It naturally follows that trading frequently during market hours becomes a direct test of whether the exposure to the risk of jump skewness in high-frequency returns is priced. Trading the options less frequently is, instead, a direct test of whether a broader exposure to skewness requires compensation.[3] Inspired by recent evidence of risk premia differentials in the stock market between daytime and overnight periods (Lou et al., 2018; Hendershott et al., 2018), we examine premia differentials between infrequently rebalanced skewness positions during

[2]SPX Friday Weeklys are options with weekly expirations. They were the first weekly options introduced by the CBOE. They attract a major share of trading activity in index options. More details are available at http://www.cboe.com/products/stock-index-options-spx-rut-msci-ftse/s-p-500-index-options/spx-weeklys-options-spxw.

[3]This notion of skewness is driven both by jumps and continuous increments in asset prices. Without our results on high-frequency skewness trading, it would be impossible to exclude that jumps carry a risk premium if a positive premium for this notion of skewness existed in market hours trading.

and after market hours. In all cases, we incorporate transaction costs into our analysis to examine whether the premia are indeed available to investors.

Existence of a non-zero excess payoff for jump skew is a necessary condition for the existence of a jump skewness risk premium. To study whether jump skew risk excess payoffs can be explained by exposures to systematic risk factors, we complement our analysis with more formal asset pricing tests. We consider three well-known systematic risk factors that could explain the profitability of overnight skewness exposures: the return on the forward on the S&P 500 index, the return on the value-weighted portfolio of financial intermediaries (He et al., 2017), and the gains from shorting variance (Ang et al., 2006; Carr and Wu, 2009). We find the intermediary factor has virtually no explanatory power for the gains from our skew option strategies, while the return on the forward on the S&P 500 index explains up to 50% of the gains available to overnight and weekend skewness traders. Subsequently, we examine whether the exposure to skewness in non-trading periods is distinct from the exposure to variance. Note that in our sample, short variance positions are only profitable on weekends – not overnight nor during the day. Nevertheless, we check whether a portfolio containing a position in the market index and a short position in variance can mimic the profitability of the skew strategy. We conclude this is not the case, and that investors exposed to skew outside of option trading hours require appropriate compensation for jump skew, beyond their compensation for variance.

Our jump skew strategy extends the ideas first brought forward by Schneider and Trojani (2018), who introduce the family of divergence strategies to trade general notions of realized risks. It swaps realized jump skewness for a combination of a fixed payment and the cumulative changes in the price of variance risk.[4] We give an explicit characterization of the strategy in terms of all the positions an investor has to take to trade jump skewness for any trading frequency and horizon, based on a dynamic trading strategy in variance swaps and a single higher-order option portfolio. The strategy can be implemented in option markets with discrete strike grids and sparse maturity calendars. Under no-arbitrage (our main assumption), it can be interpreted as a strategy generating a premium for skewness. Under the assumption of a semi-martingale world with continuous trading, it gains a clear-cut interpretation: it is a strategy replicating a premium for the realized skew of only jump

[4]At its fundamental level, the strategy can be understood as buying option portfolios, selling the same options later, and replacing them with new, adjusted portfolios, just like in a managed stock strategy.

returns, perfectly separating big risks from small, Brownian risks, and it is the first such model-free strategy.

A. Literature review

The importance of jumps and the associated risk premia in finance derives from three lines of empirical research. The presence of jumps in returns was first proposed by, among others, Press (1967) and Merton (1976), but sharp differentiation between jumps and non-jumps is only possible since the advent of high-frequency data, and methods for the study thereof. These methods first allowed the documentation of a significant contribution of jump realizations to asset returns (Huang, 2005; Bollerslev et al., 2008). However, a renewed debate of the subject emerged recently, suggesting that standard methods of jump detection tend to produce too many false positives (Christensen et al., 2014; Bajgrowicz et al., 2016), hence the role of jump risk in asset returns may be less pronounced than previously thought. The observation that the premia for jump returns are distinct from those for regular returns is based on parametric and semi-parametric studies of index option prices. Including non-Gaussian (e.g. Bates, 2000, 2006; Babaoglu et al., 2015; Christoffersen et al., 2015) and specifically negative jumps, whose frequency is detached from continuous volatility (Andersen et al., 2015b,a, 2017), yields important improvements in the fit of continuous-time, no-arbitrage option pricing models. These studies, however, either do not use high-frequency data, or use it indirectly in the estimation process, in the sense that the option payoffs are not strictly based thereon and do not differentiate between market opening and closing periods. This implicitly treats closing periods as jumps. Short-maturity, deep out-of-the-money options' prices are determined predominantly by jump risk. Bollerslev and Todorov (2011) couple this insight with a semi-parametric approach to modeling the tail behavior of returns and its impact on option prices to infer the premium for jump risk. Their results, again, suggest a large premium for negative jumps, albeit they do not take a stand on what these jumps are and when they occur. Bollerslev et al. (2016) examine the importance of jump risk via its impact on the cross-section of expected returns within the APT paradigm. They find that only overnight index returns and index returns identified as jumps are priced in the cross-section of stocks. Their study is the closest to ours in spirit, as it focuses on a non-parametric scrutiny of trading strategies' exposure to jump risk. Bollerslev et al. focus on the role of jump and overnight market risk for the cross-section of stocks. In contrast, the analysis of trading profits from our trading strategy directly addresses a different question: whether time-varying jump risk in index returns is priced as a systematic risk factor. Our main conclusion is that time-varying jump skewness risk does not generate a risk premium during periods in which option markets are open. This finding does not contradict those in Bollerslev et al.. Indeed, they show that exposure to linear market risk generated by jumps can explain the cross-section of stock returns. To this end, market risk generated by jumps has to pay a risk premium. Such features are consistent, e.g., with a conditional CAPM setting with priced jump risk and constant return skewness, in which, by construction, no skew risk premium arises. Finally, Muravyev and Ni (2016) find patterns of positive (negative) returns in index, equity and ETF option returns during the day (overnight), and attribute them to behavioral biases of investors who ignore intraday and overnight volatility patterns, but does not explicitly consider the role of continuous and jump risks.

II. Measuring and trading realized jump skewness

This section introduces in Section II.A our approach to measuring and trading realized jump skewness introduced. Section II.B further details the replicating strategy for realized jump skewness, section II.C contrasts our strategy with other skew trading strategies present in the literature, and Section II.D develops the appropriate notion of a model-free risk premium.

A. From variance to skewness trading

The key insight for measuring the risk premium attached to nonlinear return variations in a model-free manner is that such variations may be replicated by means of appropriate delta-hedged option strategies. In this context, various measures of second- and higher-order return variation can be introduced, which involve replicating portfolios of different complexity.

Consider a sequence of forward prices $F_0, F_1, \dots, F_n$, for a fixed maturity T and trading times $t =:$

$t_0 < t_1 < \ldots < t_{n-1} < t_n := T$, with corresponding log returns $r_i(n) := \ln(F_i/F_{i-1})$.[5]

Bondarenko (2014) and Schneider and Trojani (2018), among others, show that the following second-order return variation measure

$$D_t^{(n)} := \sum_i D(r_i(n)) \; , \tag{1}$$

with

$$D(r) := e^r - 1 - r \; , \tag{2}$$

is replicable with a single, dynamically delta-hedged portfolio of out-of-the-money options; see also Section II.B.1 below. In equation (1), $D_t^{(n)}$ simply accumulates the variation of subsequent log returns, as measured by $D(r)$ from equation (2). Since $D(r) = \frac{r^2}{2} + \frac{r^3}{6} + O(|r|^4)$, it indeed defines a measure of second-order return variation.[6] Moreover, as the price of the replicating portfolio for $D_t^{(n)}$ is proportional to one half the square of the well-known VIX implied volatility index, it is natural to call $D_t^{(n)}$ a VIX–type realized variance of (forward) returns. In contrast to other measures of second-order return variation, such as squared return variation, it is easy to replicate realized variance defined in this way with a single (static) delta-hedged option portfolio.

While the leading term in $D_t^{(n)}$ reflects second-order return variations, $D_t^{(n)}$ also loads on higher-order variations, which across infinitesimal time intervals can only be generated by jumps. This insight allows us to measure and to trade higher-order return variations that primarily depend on realized return skewness, by considering differences of distinct second-order return variations. While, in general, replicating higher-order return variations by differences of second-order variations always

[5] For simplicity, we write F_0 with F_t and F_n with F_T whenever this has no impact on clarity.

[6] Under a semimartingale assumption of the form

$$dF_t = \mu_t F_{t-} dt + \sigma_{t-} F_{t-} dW_t + F_{t-} \int_{\mathbb{R}\ \{0\}} (e^x - 1)\nu_t(dx, dt) \; ,$$

Bondarenko (2014) and Schneider and Trojani (2018), among others, obtain the continuous-time limit

$$D_t^{(n)} \to \frac{1}{2} \int_t^T \sigma_{u-}^2 du + \int_t^T \int_{\mathbb{R}\ \{0\}} D(x)\nu_u(dx, du) \; ,$$

as sup $|t_i - t_{i-1}| \to 0$. Therefore, in absence of jumps $2D_t^{(n)}$ converges to the integrated variance of returns.

entails dynamic option trading, having VIX–type realized variance as one of the legs in this difference helps keep the dynamic option portfolio component manageable and transparent. This intuition motivates our definition of realized big risks.

Definition 1 (Realized jump skewness)**.** Realized jump skewness $S_t^{(n)}$ is defined by

$$S_t^{(n)} := \sum_i S(r_i(n)) \,, \tag{3}$$

where

$$S(r) := e^r - 1 - r - \frac{r^2}{2} \,.$$

Realized skewness $S_t^{(n)}$ in Definition 1 simply accumulates the variation of subsequent log returns measured by function $S(r)$. Since $S(r) = \frac{r^3}{6} + O(|r|^4)$, equation (3) defines a measure of third-order return variation that is interpretable as a proxy of realized return skewness generated by big risks. Formally, this interpretation follows from the limit of continuous-time trading because in such settings $S_t^{(n)}$ converges to a measure of return skewness purely due to jump risk.

Result 1. *Under a continuous-time semi-martingale assumption of the form*

$$dF_t = \mu_t F_{t-} dt + \sigma_{t-} F_{t-} dW_t + F_{t-} \int_{\mathbb{R}\ \{0\}} (e^x - 1)\nu_t(dx, dt) \,, \tag{4}$$

we obtain the limit $S_t^{(n)} \longrightarrow \int_t^T \int_{\mathbb{R}\ \{0\}} S(x)\nu_u(dx, du)$ *as* $\sup |t_i - t_{i-1}| \to 0$*.*

Note that together with the realized variance equation (1), Definition 1 yields

$$S_t^{(n)} = D_t^{(n)} - \frac{1}{2} \sum_i r_i^2(n) \,. \tag{5}$$

Therefore, given that $D_t^{(n)}$ is tradeable with static option portfolios, the replication of $S_t^{(n)}$ is transparent: it just amounts to hedging away one half the realized variance of log returns $\frac{1}{2}\sum_i r_i^2(n)$ from the strategy replicating realized variance. This hedging component is necessary in order to isolate the higher-order return variation due to big risk. It complements the standard dynamic delta-hedging required to replicate realized variance and is based on dynamic option trading.

B. *Trading realized jump skew*

By definition, realized variance and realized jump skew accumulate second- and third-order variation of subsequent log returns through a suitable nonlinear transformation of returns. Therefore, the trading of these return variations has to rely on corresponding replicating portfolios of options on the underlying risk.

B.1. Trading variance

The first challenge in replicating realized return variation is the fact that it is not usually possible to trade options for maturities corresponding to all trading dates $t = t_0, t_1, \dots, t_n = T$ underlying a given variation measure. Therefore, one has to replicate return variation using option portfolios with a fixed maturity T, making dynamic option trading unavoidable in almost all cases. Due to a particular aggregation property induced by the logarithmic function (2), the realized variance in equation (1) is the exception to this rule:[7]

$$D_t^{(n)} = \underbrace{\left[-\ln(F_T/F_t) + \frac{1}{F_t}(F_T - F_t)\right]}_{\text{Static option payoff}} + \underbrace{\sum_i \left(\frac{1}{F_{i-1}} - \frac{1}{F_t}\right)(F_i - F_{i-1})}_{\text{dynamic } \Delta\text{-hedging payoff}} . \qquad (6)$$

Here, the second term with the sum is replicable by a dynamic delta-hedging strategy in the forward market. In contrast, the first term in the square parentheses is replicable with a single static option portfolio constructed at time t, provided that options with maturity T are traded over a sufficiently wide range of strike prices. Tradability of this term follows with function $\phi(x) = -\ln(x)$ from the Carr and Madan (2001) identity[8]

$$D_\phi(F_T, F_t) := \phi(F_T) - \phi(F_t) - \phi'(F_t)(F_T - F_t) = \int_0^\infty \phi''(K) O_t(K, F_T) dK , \qquad (7)$$

[7] To obtain this identity, note that $\sum_i r_i(n) = -\ln(F_T/F_t)$ and $e^{r_i(n)} - 1 = F_i/F_{i-1} - 1$. As mentioned, an example of a second-order return variation measure that does not aggregate is the sum of squared log returns $\sum_i (r_i(n))^2$; see equation (5) and Remark 1 below.

[8] Indeed, for $\phi(x) = -\ln(x)$ we have $D_\phi(F_T, F_t) = -\ln(F_T) + \ln(F_t) - \ln'(F_t)(F_t - F_t)$, where $\ln'(F_t) = 1/F_t$, i.e., equation (6).

where $O_t(K, F_T)$ denotes the payoff at T of an option of strike K that is out-of-the-money at time t. As the price of variance becomes observable from the price of the option replicating portfolio in identity (7), the excess returns for realized variance risk are quantifiable with a model-free approach based on a static option replication.

B.2. Trading realized skewness

The main challenge in trading return variations that are different from the realized variance (1) is that they do not aggregate in the same way, which makes a dynamic option replication approach unavoidable for realized skew. However, the tight link between $D_t^{(n)}$ and $S_t^{(n)}$ in equation (5) implies that such a replicating strategy is relatively transparent, as it amounts to adding a delta-hedged dynamic option strategy to the static VIX-type option portfolio in the replicating strategy for realized variance.

Remark 1. To understand the key issues in replicating realized skewness, write[9]

$$S_t^{(n)} - D_t^{(n)} = \underbrace{-\frac{1}{2}\left[(\ln F_T - \ln F_t)^2\right]}_{\text{static option payoff}} + \underbrace{\sum_i [\ln(F_{i-1}/F_t)(\ln F_i - \ln F_{i-1})]}_{\text{dynamic option } + \, \Delta\text{-hedging payoff}} . \tag{8}$$

As in expression (6) for realized variance, the first term on the right hand side of this identity is tradable with a single option portfolio because it can be written as in equation (7) using the function $\phi(x) = -\frac{1}{2}[\ln(x/F_t)]^2$.[10] In contrast, the second term with the sum is not replicable by a dynamic delta-hedging strategy in the forward market alone and it needs a delta-hedged dynamic option portfolio replicating the stochastically-weighted sum of log returns $\sum_i \ln(F_{i-1}/F_t)(\ln F_i - \ln F_{i-1})$.

Given the form of the nonlinearity in the sum of equation (8), it turns out that the dynamic option replicating strategy actually consists of a weighted sequence of static option portfolios. All these portfolios statically replicate realized variance on a sequence of shrinking trading periods with common final date T. Hence, realized skew is replicable with a weighted sequence of option portfolio

[9] To obtain this identity, note that $\sum_i r_i^2(n) = \sum_i[(\ln(F_i/F_t))^2 - (\ln(F_{i-1}/F_t))^2 - 2\ln(F_{i-1}/F_t)(\ln F_i - \ln F_{i-1})]$.

[10] Indeed, for $\phi(x) = -\frac{1}{2}[\ln(x/F_0)]^2$ it follows that $D_\phi(F_T, F_t) = -\frac{1}{2}(\ln F_T - \ln F_t)^2$, using the fact that $\phi(F_t) = \phi'(F_t) = 0$.

payoffs (7) having identical maturity T. More explicitly,

$$S_t^{(n)} = D_{\phi_0}(F_T, F_t) + \sum_{i=1}^{n-1} [\Gamma_i D_{\phi_i}(F_T, F_i) + \Delta_i (F_T - F_i)] \,, \tag{9}$$

for corresponding functions $\phi_0, \dots, \phi_n$ in identity (7) and coefficients $\Gamma_1, \dots, \Gamma_{n-1}$ and $\Delta_1, \dots, \Delta_{n-1}$. We detail the constituents of this trading strategy in the next result.

Result 2 (Replicating strategy for realized skew)**.** *The replicating strategy* (9) *requires* $\phi_0(x) = -[\ln(x) + \frac{1}{2}(\ln(x/F_0))^2]$ *and* $\phi_i(x) = -\ln x$ *for* $i = 1, \dots, n-1$*. The corresponding delta and gamma coefficients are*

$$\Delta_i = \frac{1 + \ln(F_i/F_{i-1})}{F_i} - \frac{1}{F_{i-1}} \; ; \; \Gamma_i = \ln(F_{i-1}/F_i) \,. \tag{10}$$

The Γ_i coefficients reproduce the dynamic weights for the sequence of static VIX–type option payoffs needed to replicate the dynamic nonlinearity in equation (8). These coefficients are decreasing in lagged forward returns. Coefficients Δ_i extend the standard delta-hedging in the replication of realized variance by a term that is necessary to compensate the difference between the dynamic nonlinearity in equation (8) and the weighted sequence of VIX-type payoffs $D_{\phi_i}(F_T, F_i)$ in equation (9). Contrary to standard delta-hedging in the replication of realized variance, the delta-hedging coefficients for realized skew increase in lagged forward returns.

In summary, the dynamic replicating strategy underlying equation (9) works as described in Table V of Appendix B. At the inception date t, the investor takes a static long position in a single skew portfolio of maturity T, with option portfolio weights $\phi_0''(K) = \frac{\ln(K/F_t)}{K^2}$ reported in the left panel of Figure 1.[11] As intuitively expected, these weights translate to long positions in out-of-the-money calls and short positions in out-of-the-money puts, which replicate the static payoff $S(\ln F_T/F_t)$ plotted in the right panel of Figure 1.

At trading dates t_i after the inception date, the investor also trades in a sequence of VIX-type variance payoffs $D(\ln(F_T/F_{t_i}))$, with weights $\Gamma_i \phi_i''(K) = -\frac{\ln(F_i/F_{i-1})}{K^2}$ and common target maturity T, and in a sequence of delta-hedging forward positions Δ_i. These option and delta-hedging posi-

[11]These weights directly follow from identity (7).

tions are designed to maintain the desired exposure to the realized jump skew $S_t^{(n)}$ in Definition 1. In particular, it turns out that when the lagged return is negative (positive) the investor buys (sells) both out-of-the-money calls and puts in VIX-type portfolios having absolute option exposures that decrease with the strike price.

From Result 2, the first option portfolio payoff in replicating strategy (9) can be written as $D_{\phi_0}(F_T, F_t) = S(\ln(F_T/F_t))$, where S is the skew function in Definition 1. When coupled with a delta-hedging strategy with weights $\Delta_i^{\phi_0} := \frac{1+\ln(F_i/F_t)}{F_i} - \frac{1+\ln(F_{i-1}/F_t)}{F_{i-1}}$, this static option portfolio becomes a skew strategy in itself, as shown by Schneider and Trojani (2018).

Definition 2 (Realized static skew (Schneider and Trojani, 2018))**.** Static skew $US_t^{(n)}$ is defined as the payoff of the strategy which at inception replicates the function $S(\ln F_T/F_0)$, where S is given in Definition 1 and delta-hedges the payoff with coefficients $\Delta_i^{\phi_0} = \frac{1+\ln(F_i/F_t)}{F_i} - \frac{1+\ln(F_{i-1}/F_t)}{F_{i-1}}$. This yields the gross payoff at settlement equal to

$$US_t^{(n)} = \sum_i \ln \frac{F_{i-1}}{F_t} D(r_i(n)) \tag{11}$$

Result 3 (Static and realized jump skew)**.** *The realized jump skew replication strategy in Result 2 can be decomposed into a strategy paying off the static skew from Definition 2, and dynamic trading in forwards and in variance swaps.*

$$S_t^{(n)} = US_t^{(n)} + \sum_{i=1}^{n-1} [\Gamma_i D_{\phi_i}(F_T, F_i) + \Delta_i (F_T - F_i)] - \sum_i \left(\Delta_i^{\phi_0} - \left(\frac{1}{F_i} - \frac{1}{F_{i-1}} \right) \right) (F_T - F_i), \tag{12}$$

where the coefficients Γ_i and Δ_i are given in Result 2, and $\Delta_i^{\phi_0} = \frac{1+\ln(F_i/F_t)}{F_i} - \frac{1+\ln(F_{i-1}/F_t)}{F_{i-1}}$.

Additional properties of this strategy and its brief comparison with jump skew and other skew strategies from the literature are given in Section II.C.

C. Comparison with other Skewness Measures

In this section, we confront our realized jump skewness (OST) with the skewness measures proposed by Kozhan et al. (2013) (KNS) and Schneider and Trojani (2018) (ST).

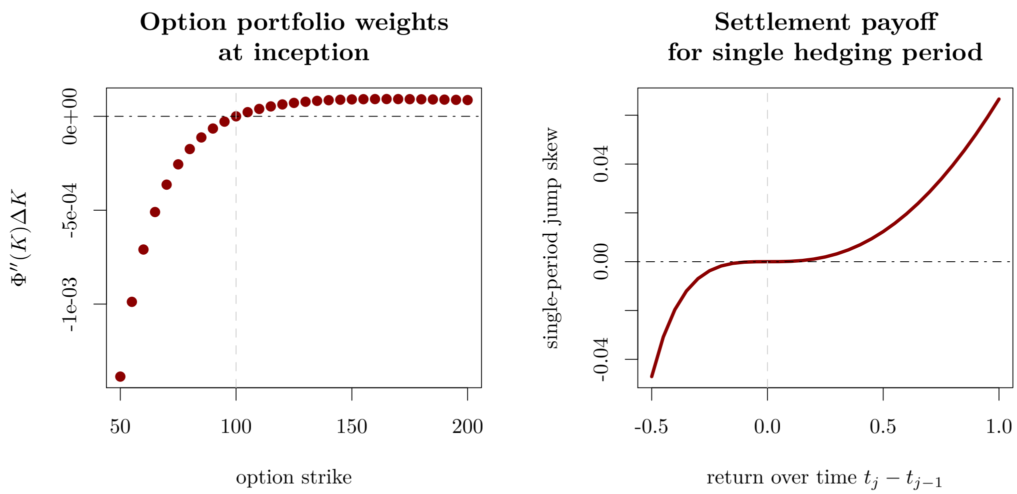

Figure 1: Weights of initial option portfolio in the skew trading strategy, for $F_0 = 100\$$.

ST introduce a family of skewness contracts based on simple trading strategies, consisting of a static option portfolio position and subsequent delta-hedging. The static skew from Definition 2 is a member of this family, with the payoff (given explicitly in equation (11)) converging to

$$US_t^{(n)} \to \int_t^T \frac{1}{6} \ln \frac{F_s}{F_t} \sigma_s^2 ds + \sum_{t<s\leq T} \frac{1}{6} \ln \frac{F_{s-}}{F_t} \left(e^{\Delta X_s} - 1 - \Delta X_s\right), \tag{13}$$

in the continuous-time limit with $\Delta X_s := \ln F_s / F_{s-}$. In a diffusion world, static skew is generally different from zero as Equation (13) reduces to integrated covariation between expanding log returns $\ln F_s/F_t$ and variance σ_s^2. In Result 3 we show explicitly the relation between static skew and realized jump skewness in terms of hedging coefficients and forward and option positions.

The KNS skew is engineered to pay realized covariation between index returns and changes in implied variance, which has the interpretation of a realized leverage effect. To replicate this realized covariation, the KNS skew necessitates dynamic option trading at the highest available frequency like the OST skew. Unlike the ST and OST strategies, the KNS delta-hedge ratio depends on the implied volatility surface and, hence, on the cross sections of option prices at the same frequency at which the portfolios are traded. Since the KNS skew is constructed to pay realized covariation between index returns and index implied variance in a diffusion world, it has a non-zero payoff in the absence of jumps. Therefore, the KNS skew also captures a clearly distinct notion of realized skew than the OST jump skew.

D. *Properties of tradable realized skewness and skewness risk premia*

The tradability of realized jump skewness in Definition 1 naturally opens the possibility to quantify the excess payoffs for realized jump skewness risk under a model-free approach. However, in contrast to the replicating portfolio for the realized variance (1), the dynamic option component in replicating strategy (9) does not allow one to observe the forward price of realized jump skewness at the inception date. Indeed, while the forward price of realized variance is simply the forward price of its static option replicating portfolio, the forward replication cost of realized jump skewness contains a stochastic ongoing running cost generated by the dynamic option replication. In the sequel, we discuss in more detail the properties of the price of tradable jump skew and the excess returns for exposure to realized jump skewness.

D.1. The price of tradable skewness

From the replication identity (9) and Result 2, the total forward costs at maturity T for replicating the realized jump skewness $S_t^{(n)}$ is given by

$$\mathcal{S}_t^{(n)} := \underbrace{\mathbb{E}_t^{\mathbb{Q}}[S(\ln(F_T/F_t))]}_{\text{Price of static skewness}} + \underbrace{\sum_i \Gamma_i \mathbb{E}_{t_i}^{\mathbb{Q}}[D(\ln(F_T/F_{t_i}))]}_{\text{Runing cost component}}\,, \tag{14}$$

where $\mathbb{Q}$ is the $T-$forward measure.[12] The first component in equation (14) is the forward price of the static skewness payoff $S(\ln(F_T/F_t))$ component, which is observable at the inception date. The second component in equation (14) is a running cost given by a weighted sum of forward prices for the static variance payoffs $D(\ln(F_T/F_{t_i}))$, which are observable only at trading dates t_i strictly after the inception date. From an inception date's perspective, this running cost is stochastic both because

[12] By definition, the forward price of the delta-hedging component in replicating portfolio (9) is zero.

of the uncertainty about the future price of variance payoffs and because of the random weights Γ_i in the replicating strategy, which depend on return realizations at trading dates strictly after the inception date. Hence, in order to replicate the realized skewness after the inception date, the investor has to accumulate forward the costs or proceeds for subsequently replicating a sequence of variance payoffs with common maturity T, while learning about their actual prices and underlying returns at the moment of executing the trades.

D.2. Excess returns for tradable jump skewness

Given that the forward costs (14) for replicating realized jump skewness are paid only at maturity T, the replicating strategy (9) for realized skewness is self-financed and yields the excess payoff

$$SP_t^{(n)} := S_t^{(n)} - \mathcal{S}_t^{(n)} \, . \tag{15}$$

$SP_t^{(n)}$ can be understood as the net payoff a trader can obtain for accumulating exposures to jump skew in underlying returns between the inception date and a fixed maturity date. In this sense, it is an excess payoff for a cumulative exposure to jump skew in returns. Note that this net payoff depends on the chosen trading frequency because both $S_t^{(n)}$ and $\mathcal{S}_t^{(n)}$ do. Therefore, in our empirical analysis, we study the risk premia for trading realized skewness using various choices for the trading frequency. Such risk premia are naturally defined by the expected net payoff

$$\mathbb{E}_t^{\mathbb{P}}[SP_t^{(n)}] = \mathbb{E}_t^{\mathbb{P}}[S_t^{(n)}] - \mathbb{E}_t^{\mathbb{P}}[\mathcal{S}_t^{(n)}] \, . \tag{16}$$

Consistent with the replication identity (9), the excess payoff (15) is the sum of (i) an excess payoff for a static skewness exposure over the whole trading period and (ii) an excess payoff generated by a delta-hedged time-varying exposure to a sequence of variance payoffs

$$SP_t^{(n)} = \underbrace{S(\ln(F_T/F_t)) - \mathbb{E}_t^{\mathbb{Q}}[S(\ln(F_T/F_t))]}_{\text{Static component of skewness excess payoff}} \tag{17}$$

$$+ \underbrace{\sum_i \left\{ \Gamma_i \left[D(\ln(F_T/F_{t_i})) - \mathbb{E}_{t_i}^{\mathbb{Q}}[D(\ln(F_T/F_{t_i}))] \right] + \Delta_i (F_T - F_i) \right\}}_{\text{Dynamic component of skewness excess payoff}} \, . \tag{18}$$

Therefore, in our empirical analysis, we also quantify the contribution of each excess payoff component to the risk premium (16) for realized jump skewness risk in returns. Note that the total forward replication costs in definition (15) may depend in general on the chosen option maturity in the corresponding replicating portfolio whenever the option maturity and the terminal date T in Definition 1 do not coincide. In our empirical analysis, we avoid this issue by working with weekly options replicating realized skewness on weekly intervals.

Finally, under a semimartingale assumption (4), we obtain in the limit of continuous-time trading a more formal interpretation of excess payoff (15) as a net payoff for exposure to cumulated jump skewness in returns.

Result 4. *Under a semi-martingale assumption* (4)*, the following limit holds as* $\sup|t_i - t_{i-1}| \to 0$

$$SP_t^{(n)} \to \int_0^T \int_{\mathbb{R}\setminus\{0\}} S(x)\nu_u(dx,du) - \left\{ \mathbb{E}_t^{\mathbb{Q}}[S(\ln(F_T/F_t))] + \int_0^T \ln(F_u/F_t) d\mathbb{E}_u^{\mathbb{Q}}[D(\ln F_T/F_u)] \right\} . \tag{19}$$

In Result 4, the limit of $SP_t^{(n)}$ is the difference between a continuous-time measure of pure-jump realized skew and a corresponding limit measuring the forward replication costs for realized jump skewness under continuous-time trading. The replication cost limit is itself the sum of the forward price of the static skewness payoff $S(\ln F_T/F_t)$ and the limit of the ongoing running costs, given by the last integral on the right hand side of equation (19). Here, the continuous-time running costs cumulate with subsequent changes in the forward price of tradable variance ($d\mathbb{E}_u^{\mathbb{Q}}[D(\ln F_T/F_u)]$) proportionally to a dynamic weight given by the past log return $\ln(F_u/F_t)$.[13]

D.3. The price of jump skewness in benchmark jump-diffusion settings

The forward costs (14) of trading the realized jump skewness in Definition 1 depend on the joint dynamics of returns and the price of variance. Therefore, realized jump skewness cannot be typically traded at the inception date with a single delta-hedged forward position in the option market, and its forward price is not observable at inception date. These features motivate the following natural

[13] In a model-based setting, $\mathbb{E}_u^{\mathbb{Q}}[D(\ln F_T/F_u)]$ is often specified as a function of some underlying state variables, such as the level of the the volatility or the jump intensity in an affine framework (Duffie et al., 2000).

decomposition of tradable risk premium (16) for realized skewness risk

$$\underbrace{\mathbb{E}_t^{\mathbb{P}}[SP_t^{(n)}]}_{\text{Tradable risk premium}} = \underbrace{\left\{\mathbb{E}_t^{\mathbb{P}}[S_t^{(n)}] - \mathbb{E}_t^{\mathbb{Q}}[\mathcal{S}_t^{(n)}]\right\}}_{\text{Untradable risk premium}} - \underbrace{\left\{\mathbb{E}_t^{\mathbb{P}}[\mathcal{S}_t^{(n)}] - \mathbb{E}_t^{\mathbb{Q}}[\mathcal{S}_t^{(n)}]\right\}}_{\text{Residual}} . \tag{20}$$

In decomposition (20), the tradable jump skewness risk premium on the left hand side is the difference between an untradable risk premium and a residual. The untradable risk premium on the right hand side is the premium for exposure to realized jump skewness risk in an economy with computable forward price of $E_t^{\mathbb{Q}}[\mathcal{S}_t^{(n)}]$.[14] It is not observable in a model-free way because the realized skewness is not replicable with static option portfolios. However, it could in principle be quantified under additional model-based assumptions. The residual on the right hand side of equation (20) measures the discrepancy between the forward price of realized skewness in the fictitious economy and the expected running costs for replicating realized skewness.

To gain a better understanding of the differences between tradable and non-tradable jump skewness risk premia in decomposition (20), we can assume a benchmark iid jump-diffusion process for returns, such as Merton (1976), with equally spaced trading dates, such that $t_i - t_{i-1} = (T-t)/n$. In such a setting, it follows from equation (14)

$$\mathcal{S}_t^{(n)} = \underbrace{-\,n\mathbb{M}_{\mathbb{Q}}^{(1)}(n) - \frac{n^2}{2}\mathbb{M}_{\mathbb{Q}}^{(2)}(n)}_{\text{Price of static skewness}} + \underbrace{\mathbb{M}_{\mathbb{Q}}^{(1)}(n)\sum_i(-\Gamma_i)(n-i)}_{\text{Running cost component}} , \tag{21}$$

where $\mathbb{M}_{\mathbb{Q}}^{(k)}(n) := \mathbb{E}^{\mathbb{Q}}[(\ln(F_i/F_{i-1}))^k]$ is the $k-$th forward moment of iid log returns. The resulting model-based expected running cost and forward price of realized skewness at the inception date are given explicitly in the next result.

Result 5. *Let $\mathbb{M}_{\mathbb{P}}^{(1)}(n) := \mathbb{E}^{\mathbb{P}}[\ln(F_i/F_{i-1})]$ be the physical expected log return in an iid jump-diffusion model for forward returns with equally-spaced trading dates such that $t_i - t_{i-1} = (T-t)/n$. The*

[14] Since by the law of iterated expectations $\mathbb{E}_t^{\mathbb{Q}}\left[S_t^{(n)}\right] = \mathbb{E}_t^{\mathbb{Q}}\left[\mathcal{S}_t^{(n)}\right]$, expectation $\mathbb{E}_t^{\mathbb{Q}}\left[\mathcal{S}_t^{(n)}\right]$ is indeed the forward price of realized skewness at the inception date t in such an economy.

model-based expected running costs of realized skewness read[15]

$$\mathbb{E}_t^{\mathbb{P}}\left[\mathcal{S}_t^{(n)}\right] = \underbrace{-n\mathbb{M}_{\mathbb{Q}}^{(1)}(n) - \frac{n^2}{2}\mathbb{M}_{\mathbb{Q}}^{(2)}(n)}_{\text{Price of static skewness}} + \underbrace{n(n-1)\mathbb{M}_{\mathbb{P}}^{(1)}(n)\mathbb{M}_{\mathbb{Q}}^{(1)}(n)}_{\text{Expected running cost}} . \tag{22}$$

Similarly, the model-based forward price of realized skewness at the inception date t is

$$\mathbb{E}_t^{\mathbb{Q}}\left[\mathcal{S}_t^{(n)}\right] = \underbrace{-n\mathbb{M}_{\mathbb{Q}}^{(1)}(n) - \frac{n^2}{2}\mathbb{M}_{\mathbb{Q}}^{(2)}(n)}_{\text{Price of static skewness}} + \underbrace{n(n-1)(\mathbb{M}_{\mathbb{Q}}^{(1)}(n))^2}_{\text{Forward price of running cost}} . \tag{23}$$

From Result 5, it is useful to note that the difference between the expected running costs and the forward price for realized jump skewness in benchmark jump-diffusion models is decreasing rapidly at short horizons. Indeed,

$$n(n-1)\mathbb{M}_{\mathbb{Q}}^{(1)}(n)\left[\mathbb{M}_{\mathbb{P}}^{(1)}(n) - \mathbb{M}_{\mathbb{Q}}^{(1)}(n)\right] = (T-t)^2\mathbb{M}_{\mathbb{Q}}^{(1)}\left[\mathbb{M}_{\mathbb{P}}^{(1)} - \mathbb{M}_{\mathbb{Q}}^{(1)}\right] + O\left(\left(\frac{T-t}{n}\right)^2\right) ,$$

where $\mathbb{M}_{\mathbb{Q}}^{(1)} := \mathbb{E}_t^{\mathbb{Q}}[d\ln F_t]$ and $\mathbb{M}_{\mathbb{P}}^{(1)} := \mathbb{E}_t^{\mathbb{P}}[d\ln F_t]$ are the first forward and physical moments of log returns under the jump-diffusion model. Hence, the difference $(\mathbb{E}_t^{\mathbb{P}} - \mathbb{E}_t^{\mathbb{Q}})[\mathcal{S}_t^{(n)}]$ in the tradable skewness risk premium of equation (20) tends to vanish rapidly at short investment horizons.

Consistent with intuition, the tradable skewness risk premium in equation (20) has a number of expected properties under the benchmark jump-diffusion setting. For instance, it is zero if and only if the jump diffusion parameters are identical under both the physical and the forward probabilities, i.e., if, and only if, there are no jump risk premia in the first place. Second, under a more complete option market where realized skewness may become tradable with static portfolios of exotic derivatives, the difference between the realized skewness S_t^n and the forward costs $\mathcal{S}_t^{(n)}$ would become tradable

[15]To see this, note that for measure $\mathbb{M} = \mathbb{P}, \mathbb{Q}$, it follows

$$\mathbb{E}_t^{\mathbb{N}}\left[\mathbb{M}_{\mathbb{Q}}^{(1)}(n)\sum_i(-\Gamma_i)(n-i)\right] = \mathbb{M}_{\mathbb{N}}^{(1)}(n)\mathbb{M}_{\mathbb{Q}}^{(1)}(n)\sum_i(n-i) = \mathbb{M}_{\mathbb{N}}^{(1)}(n)\mathbb{M}_{\mathbb{Q}}^{(1)}(n)n(n-1) .$$

itself. Usually, however, this tradability is not granted and one needs additional-model based assumptions to quantify empirically the nontradable component of the skewness risk premium in equation (20).

III. Trading realized jump skewness in the CBOE SPX Weeklys market

In this section we implement our trading strategies in the market for short-maturity options on the S&P 500 index. We give an overview of our dataset in Section III.A, and then report our main results in Section III.B. First, we analyze the unconditional premia for the strategies across different trading regimes – during and after option market hours, and at the weekly horizon. Then, we investigate our strategies' exposure to systematic risk factors: the market return and the variance premium. The summary statistics of realized variance, jump skew and skew – the floating legs of our strategies – are reported in Appendix E. Finally, in Section III.C we provide an overview of how close the strategies' floating legs are to the true realized quantities calculated from high frequency data, and we further examine the strategies' exposure to the intermediary risk factor of He et al. (2017).

A. Data on Friday Weeklys Trades and Quotes

Our dataset "MDR" contains "[...] quotes and trades captured by Cboe's internal data retrieval systems."[16] The records are time-stamped with one-second resolution. For each option symbol, maturity and strike, quote records consist of bid and ask prices and quantities available for trade on each side, while trade records contain only the execution price and the number of options that change hands.

Friday Weeklys began trading in September 2007 and our sample contains data up until the end of 2015. The data becomes rich enough for the purpose of our high-frequency analysis from 2011 onward, and we present supporting evidence in the Internet Appendix A.

[16] https://datashop.cboe.com/mdr-quotes-trades-data

The dataset contains predominantly quotes (99.7% of all records). While there is significant trading activity in most options, basing our analysis solely on trade data would not allow us to study the significant impact of transaction costs on the results. We create our dataset by combining quote and trade data as follows. First, we divide the trading day into five-minute periods. Then, within each five-minute period we select options with the nearest Friday maturity, and for each option type and execution price we select the best bid and ask prices among the latest recorded quotes. Based on these quotes and synchronized data on the underlying S&P 500 index, we calculate the implied forward price of the index for the option maturity, the direct counterpart of F_t in Section II. In order to avoid stale quotes, we match each quote with a preceding or subsequent trade occurring within a 30-minute window of it. Additionally, we require that the recorded trade price is not smaller (not greater) than the quote's bid (ask) price.

With the implied forwards at hand, we apply the following additional filters to the data:

- retain only out of the money forward options;
- remove all quotes with bid prices equal to zero;
- remove all quotes where the ask price is more than five times the bid price;
- remove all quotes where $k/(\sigma_{IV,t}\sqrt{\tau}) \notin [-12, 4]$, where $k := \ln K/F_t$ and $\sigma_{IV,t}$ is the implied volatility of the at-the-money option mid-price recorded in the same 5-minute window.

This leaves us with over 4.6 million data points covering the period from December 31, 2010 to December 31, 2015, i.e. 261 weeks of trading.

B. Premia for realized jump skew risk

This section presents our empirical framework for inference on the nature of jump skew risk premia in the market for short-maturity S&P 500 index options.

Leading option pricing models attribute many empirical regularities of index option prices to jumps in index returns and to distinct jump risk premia, in particular for asymmetric jump risk, or jump skew. These models do not account for the fact that investors who trade options face two structurally disparate market regimes. During option market hours, they can trade almost continuously in the options and in the underlying index, but after option market hours, they can only hedge in the electronic market for index futures. This implies that during market hours, the investors can hedge many nonlinear risks, notably also jump skew, with the use of the strategy presented in Section II.B. If investors indeed demand compensation for the risk of asymmetry specific to jump returns, our realized jump skew trading strategy should, on average be profitable during the option market hours, and these profits should not be spanned by exposures to known systematic risk factors. After option market hours, the investors cannot actively hedge with dynamic option trading strategies, hence they cannot be exposed uniquely to jump risk. While this fact does not preclude the profitability of overnight skew trading strategies such as the static skew swap in Schneider and Trojani (2018), it does prevent us from attributing the premia to any single type of risk.

Based on this motivation, we study average profits from trading jump skew $S_t^{(n)}$ across multiple trading regimes and frequencies, in the sense defined in Section III.B.1 below. In addition to trading during option market hours (to which we refer as the "daytime regime"), and after option market hours (to which we refer as the "overnight regime"), we analyze premia associated with trading jump skew risk starting after the expiration of a given series of Friday Weeklys and trading actively in the following series (i.e. at the weekly regime), in order to quantify the relative importance of the types of premia that we can uncover. Once we establish that, at a given trading regime, the realized (jump) skew strategy carries a risk premium, we further investigate whether the premium can be attributed to known systematic risk factors: the market return and the short market variance swap payoff (see Carr and Wu, 2009) in formal APT-like tests.

In order to establish a broader picture of nonlinear risk premia in the S&P 500 index option market across different trading regimes, we present additional evidence on trading variance risk during hours, after market hours, and over the weekly trading regime. In this way, we add to existing evidence about the intraday and overnight behaviour of option prices presented by Muravyev and Ni (2016), the key difference being the fact that we explicitly concentrate on model-free variance and skew trading. We employ the variance strategy associated with the VIX index. It measures the variance of high-frequency returns as in equation (1). The replicating strategy does not require dynamic option trading, but does require active delta-hedging in the underlying index. Our results presented in Appendix A show how to adapt the strategy to arbitrary trading regimes.

B.1. Measurement of risk premia across multiple trading regimes

We measure unconditional risk premia available to investors exposed to variance and jump skew risks as time-series averages of trading profits after transaction costs. Equation (6) and Result 2 indicate how to implement the variance and jump skew trading strategies from some initial point in time until option maturity, while maintaining strictly controlled risk exposures. In Appendix A, equation (A4), we extend Result 2 to show how our strategies can be generalized so that an investor can close her position at an arbitrary time before option maturity, while still earning payoffs $D_t^{(n)}$ or $S_t^{(n)}$, respectively, over the subperiod when she managed her forward and option exposures. This result allows us to investigate the existence and magnitude of premia for jump risk across different trading regimes. We start with the weekly trading regime that naturally follows from the Friday Weeklys' settlement calendar. Trading in options with next Friday's maturity commences immediately after the preceding maturity settles, i.e. at 9 a.m. on Friday or in the afternoon (or the following Monday if there is not enough data). Applying position corrections from equation (A4) in Appendix A allows us to examine trading at the following regimes:

1. daytime trading between 9:00 a.m. and 3:00 p.m. when both the option and stock markets are open;
2. overnight trading between 3:00 p.m. and 9:00 a.m. the following day, holding the options over the period when the option market is closed;
3. weekend/holiday trading between 3:00 p.m. on a Friday and 9:00 a.m. the following Monday, or across any periods when the option market is closed for longer than a single night;
4. weekly trading, starting immediately after the settlement of the previous series of Friday Weeklies, and rebalancing the positions actively whenever the option market is open, until settlement.

Daytime trading is equivalent to opening the option position at 9 a.m., adjusting forward and option positions throughout the day at constant time intervals, and taking a final offsetting position at 3 p.m.[17] As a result, between the end of the trading day and maturity, the investor holds an offsetting

[17] In our implementation the exact opening and closing times depend on the availability of sufficiently rich trade and

portfolio of options and forwards whose total payoff no longer changes. In other words, by the end of the day the investor sells (repurchases) all the options and forward contracts that she buys (sells) during the day.

In overnight and weekend/holiday trading, the positions are not dynamically rebalanced or delta-hedged between the opening and closing trades, which take place at 3 p.m. and at 9 a.m. the following trading day. In this case, the opening and closing trades can be seen as the sale (repurchase) of exactly the same option portfolio that was bought (sold) the preceding afternoon. Note that in our sample period, which spans from the beginning of 2011 until the end of 2015, no electronic trading takes place in index options outside the opening hours of the CBOE market. Finally, note that by construction in all settings where intermediate option and forward hedging are eliminated (i.e. in overnight trades, and in daytime trades at the lowest – "daily" – trading frequency), there is no difference between the jump skew and static skew trading strategies: their floating legs and aggregate costs become the same.

The high frequency payoffs $D_t^{(n)}$ and $S_t^{(n)}$ measure realized variance and skew of the (log) return, i.e. of a position of \$1 notional value. For the sake of readability, we rescale all quantities associated with trading variance (skew) by 10^4 (10^5), so that they measure variance (skew) of a \$10,000 (\$100,000) position. The economic significance of such option positions can be better understood in terms of the margin requirement imposed on the investor. The CBOE requires that investors only post margins for shorted options. An investor who shorts variance, sells options with weights K^{-2}, while an investor taking a long position in skew initially sells puts and buys calls with weights $K^{-2} \ln(K/F)$. According to the information available on CBOE's website,[18] and taking into account the typical range of available strikes such that $\ln(K/F) \in [-0.4, 0.15]$, the margin requirement net of received premia of a long skew trader would be 85% lower than the one of a short variance trader.

quote records. In almost all cases the first trade of the day takes place no later than 9:30 a.m. and the last one no sooner than 2:30 p.m.

[18] http://www.cboe.com/trading-tools/calculators/margin-calculator

B.2. Long/short positions and transaction costs

Throughout our analysis, we explicitly take transaction costs in the option market into account. In order to do so, we first define the notion of a long or short position in our (dynamic) option trading strategies, and then clarify which long or short positions we investigate.

Definition 3 (Long and short positions)**.** Let $Z_t^{(n)}$ denote a replicable realized variation measure and $\mathcal{Z}_t^{(n)}$ the aggregate option premiums paid or received at maturity T in order to replicate $Z_t^{(n)}$. We say that the investor takes a long (short) position in the replicating option strategy if at the settlement of the options her net payoff is $Z_t^{(n)} - \mathcal{Z}_t^{(n)}$ ($\mathcal{Z}_t^{(n)} - Z_t^{(n)}$).

In what follows, we analyze the profits from short variance positions and long static/jump skew positions. This choice is motivated by the following facts. First, the literature on the variance risk premium typically finds that the premium is negative when trading options with maturities of one month, i.e. short monthly market variance positions are profitable.[19] Second, both in a short variance position and a long static/jump skew position the agent sells out-of-the-money put options, which are typically much more expensive than out-of-the-money call options. Third, in our analysis of trading profits before transaction costs, we record a negative variance risk premium and a positive jump risk premium at the weekly trading regime. By taking this side of the trades, we implicitly choose to analyze the risk exposures from the point of view of an investor who would earn a positive excess return in absence of transaction costs.

We calculate transaction costs for option trading based on adjusted bid and ask prices. In the sequel, we first discuss the nature of the adjustment needed to account for transaction costs and provide details of how to calculate transaction costs when dynamically trading option portfolios.

We note that our sample transactions often take place inside the bid-ask spread, far away from quoted prices. This finding is true for all option moneyness groups defined in terms of $\kappa = k/(\sigma_{IV,t}\sqrt{\tau})$, first defined in Section III.A, with group j containing options between κ_j and κ_{j+1}, $\kappa_j \in \{-12, -10, -8, -6, -4, -2, 0, 2, 4\}$. This implies that some investors are able to trade under much better terms than others. Throughout our analysis we assume an investor would, on

[19]Thus, including transaction costs reduces the average gain of the investor taking short positions (selling variance) and increases the average loss of the investor taking long positions (buying variance).

average, be able to trade under such conditions, paying the half-spread for moneyness group j, $HS_j := \frac{1}{N_j}\sum_{k=1}^{N_j} \left|p_{k,\text{mid}} - p_{k,\text{trade}}\right|$, where HS_j is calculated from options that trade strictly within the spread. For quotes where $p_{\text{trade}} = p_{\text{bid}}$ or $p_{\text{trade}} = p_{\text{ask}}$, we modify these records by replacing the bid price with $p_{\text{mid}} - HS_j$, and the ask price with $p_{\text{mid}} + HS_j$.

Consider a general dynamic option trading strategy of the form in equation (9). The strategy's position at each point in time t_i in an individual option depends on two quantities: the weight of each option $\phi_i''(K)dK$ in a static replicating portfolio for the period from time t_i to maturity T, which follows from equation (7), and the coefficient Γ_i, which depends on evolution of the forward price up to time t_i. Importantly, the option weight also depends on whether the investor takes a long or short position in the strategy. Therefore, the option position at time t_i of an investor that is long trading strategy (9) equals $\omega_{i,K} = \Gamma_i\phi_i''(K)dK$. An investor that is short trading strategy (9) would instead hold an option position $\omega_{i,K} = -\Gamma_i\phi_i''(K)dK$. Thus, when calculating the aggregate cost of replication following equation (14), all options whose weights are positive (negative) are recorded at ask (bid) prices, adjusted as described above.

B.3. Average profits and their systematic risk exposures

Our main results are presented in Tables I through III. Table I contains the estimates of premia (and their respective 95% confidence intervals) for trading realized jump skew, realized static skew, and realized variance across all trading regimes and hedging frequencies, after adjusting for transaction costs. Tables II through III, contain the estimates of CAPM and APT-like regressions. In Appendix F, we present Tables VII through IX which are counterparts to Tables I through III, but which do not take into consideration transaction costs . When interpreting the premia and asset pricing regression estimates, recall that we incorporate transaction costs directly into the calculation of trading profits. For instance, in Section III.B.2 we commit to analyzing the risk exposures of short realized variance positions and long realized jump skew positions because the literature documents that such strategies earn positive returns at longer trading horizons. As a consequence, in Table I, positive estimates of premia imply that an investor can make money by taking the aforementioned short and long positions. Yet, negative premium estimates do not imply that an investor can make money, net of transaction costs, by taking the opposite side of the trades. Similarly, in Tables II through III, positive alphas indicate that a given strategy outperforms the corresponding factor portfolio. Yet again, negative alphas do not indicate that reversing the trade direction would bring abnormal profits, net of transaction costs. Finally, the coefficients from factor regressions containing variance profits cannot be used for hedging factor risks away from a given strategy. This is because doing so would require an opposite trading position (e.g. a long instead of a short variance position) whose factor coefficient could potentially be very different.

Our sample period contains the “Mini Flash Crash”, which occurred in US equity markets before the start of trading on Monday, August 24th, 2015, and which renders the available data unreliable. On that day, irregularities in the final period of pre-market-hours electronic trading led to a large, and later quickly reversed, downward spike in recorded levels of the S&P 500 index.[20] The opening of the market was staggered; 15% of the S&P 500 by market capitalization did not start trading until 9:40 am. Thus, the value of the index could not be calculated for a significant period of time. This led to the breach of arbitrage relations between the equity market and the index option market, where the CBOE failed to publish the VIX index before 10:00 am. Some sources attribute this failure to the fact that trading in Friday Weeklys, then recently included in the calculation of the VIX index, was particularly erratic, and liquidity was “evaporating”.[21] Once the VIX index was published, it opened at a value of 53 points - a phenomenon unseen since the Great Financial Crisis. Therefore, when estimating risk premia associated with our strategies in Section B.3.1, we discard the data from the first two hours of trading. When estimating spanning regressions in Section B.3.2, we discard observations that include trading on August 24th, 2015.

B.3.1. Average profits

Directly significant and model-free evidence on the nature of the premia for jump skew risk is contained in Panels “Daytime-”, “Overnight-”, and “Weekend trading” of Table I. First, recall that if investors were indeed averse to the skew of realized jumps, we should observe a positive premium for our jump skew strategy during the option market hours, when jump returns can indeed be separated

[20] It also led to significant valuation gaps between Exchange Traded Products and their underlying portfolios.

[21] See Barrons at https://www.barrons.com/articles/the-odds-of-an-interest-rate-cut-just-got-higher-51557936993 and https://www.barrons.com/articles/what-broke-the-vix-on-aug-24-1444154936.

from continuous returns. We find that this is not the case, as trading realized jump skew risk during the opening hours of the index option market does not carry a premium significantly different from zero. This observation is not driven by the fact that we correct trading profits for transaction costs: in the "Daytime trading" panel of Table VII in Appendix F we also record insignificant jump skew risk premia when working with mid-quote prices.

Next, we analyze after-hours skew trades separately for weekday overnight periods, and for holidays and weekends. Over holidays and weekends, the periods when an investor's hedging ability is reduced are 24 or 48 hours longer than they would be during weekday overnight periods and the risk associated with holding the positions without being able to hedge them scales nonlinearly with time.[22] We find that the profits from trading $S_t^{(n)}$ are positive both after market hours and when trading over weekends and holidays, albeit the premium estimate is significantly greater than zero only in the former case. Together, these observations imply that investors are averse to exposure to skew risk when the option market is closed. However, the resulting skew risk premium cannot be interpreted as a risk premium for jump risk, understood as the risk of occasional, abnormally low returns.

Further evidence corroborates our main findings. First, one may wonder whether the skew risk premium arising when the option market is closed is not simply a risk premium for a different type of skewness risk, i.e., realized static skewness risk, as, across the after-hours periods, realized jump skewness $S_t^{(n)}$ and static realized skewness $US_t^{(n)}$ coincide. Second, investors may require compensation for holding nonlinear positions over longer regimes, independently of whether they can actively hedge them or not. We reject each of these hypotheses based on evidence contained in Table I. Indeed, we find that trading trading static skew $US_t^{(n)}$ is not profitable during market hours trading for any delta-hedging frequency. Once more, these results do not rely on the adjustment for transaction costs, as is documented by the skew risk premia estimates in Table VII in Appendix F.

Another important finding is that, in the overnight trading regime, the risk premium for short variance exposures is not positive. This feature likely reflects the distinct contributions of the out-of-the money put options in the replicating portfolios for realized skewness and realized variance. Indeed,

[22] The "Weekend" trading regime also contains positions held over one-day holidays, i.e. New Year's Day, Independence Day, Thanksgiving Day and Christmas.

the replicating option portfolios for realized skewness are naturally tilted more towards out-of-the-money options in order to replicate the more nonlinear skew payoffs. According to this evidence, exposure to variance risk cannot be an explanation for a positive skew risk premium, as SectionB.3.2 below shows more systematically.[23]

Finally, we study how skew and variance risk premia aggregate at the weekly horizon. We do this by holding and actively hedging the positions in the replicating strategies for realized variance and realized skewness, whenever possible, until their Friday settlement. Note that, for short variance and long static skew positions, an investor who follows the weekly settlement calendar would benefit from a significant reduction in transaction costs as she has to to execute one instead of two option trades. This is a key difference to the strategy replicating weekly realized jump skew. Indeed, we find positive premia associated with shorting variance and long static skew positions held in the weekly regime. In contrast, while the estimated risk premium for trading weekly realized skew of intraday jump and overnight returns is positive, the confidence intervals of the estimate contain zero at every regime. The important impact of transaction costs is again evident from comparing the weekly trading results with Table VII, where the premium for trading $S_t^{(n)}$ at high frequencies is not distinguishable from the premium for trading $US_t^{(n)}$. In summary, we conclude that the nature of the premia for tradable realized risks in S&P 500 option markets crucially depends on the practicability of dynamic option hedging and the degree of nonlinearity of these risks.

B.3.2. Systematic risk exposures

In this section, we study whether the risk premia found in the previous section are compensation for exposures to different systematic risks. We analyze our trading strategies' exposure to the market return, and – in the case of our skew replicating strategies $S_t^{(n)}$ – to the excess returns of short variance positions. Tables II and III report an APT-like analysis of the risk exposures and systematic premia for trading realized skew and short variance after market hours and following the weekly settlement

[23]In the case of overnight variance trade, we can explicitly see by comparing the results contained in Table VII in Appendix F that transaction costs play an important role in eliminating the premium for shorting variance overnight. This is contrary to our previous observations.

calendar, respectively. We estimate regressions of the form

$$S_{t,h}^{(n)} - \mathcal{S}_{t,h}^{(n)} = \alpha_h + \beta_{m,h} r_{mt,h} + \beta_{D,h} \left(D_{t,h}^{(n)} - \mathcal{D}_{t,h}^{(n)} \right) + \varepsilon_{t,h}$$

and

$$D_{t,h}^{(n)} - \mathcal{D}_{t,h}^{(n)} = \alpha_h + \beta_{m,h} r_{mt,h} + \varepsilon_{t,h}$$

for each regime h of interest. In these regressions, all systematic risk factors and explained payoffs are net payoffs from tradable investment strategies. Hence, in order to determine whether a given strategy carries a distinct risk premium, we test whether $\alpha_h > 0$. Since our analysis incorporates transaction costs, the right-hand side of APT-like regressions can be understood as the factor portfolio that most closely replicates the profits from a given trading strategy. Whenever $\alpha_h > 0$, this portfolio underperforms and the strategy offers a priced exposure to a risk factor not present in the replicating portfolio. Whenever $\alpha_h \leq 0$, the strategy's risk premium is subsumed by its exposure to systematic factors.[24]

We start by addressing the question of whether the overnight and weekend trading profits are spanned by the given systematic risk factors. Our CAPM and APT-like regression results for the strategies are collected in Table II. The top panel presents results for weeknight short variance positions and long skew positions, while the bottom panel contains results for weekend trades. Recall that on weeknights, shorting variance does not carry a premium. In a CAPM regression we find that a short variance position indeed has a positive exposure to the market return, but transaction costs eat away potential profits and an investor would be left with a negative α_h (model (1) in the top panel of Table II). Different evidence emerges for weekends, as here the intercept is positive and significant. Moving to after-hours skew trading, we find that holding the skew option portfolio over the weekend generates premia that cannot be spanned by market risk (model (4) in the bottom panel of Table II). Engaging in the skew strategy on weeknights is less lucrative, as the weekend α_h is almost seven times the weeknight α_h. While the latter is not statistically significant, it is economically large. Indeed, the exposure of the weeknight skew excess payoffs to market risk

[24] Whenever $\alpha_h < 0$, trade direction in these strategies cannot simply be reversed on order to flip losses into profits: transaction costs would eat into any potential gains because these opposite positions are not profitable at mid prices in the first place.

explains only 50% of the strategy's average profits, with an R^2 of 28%. Finally, we study whether the skew strategy offers risk exposure beyond shorting variance. The estimates of model (7) in the top and bottom panels of Table II show that this is indeed the case. Both on weeknights and weekends, a factor portfolio that is long market risk and short variance risk underperforms long positions in after-hours skew.

Recall that in the weekly trading regime, we found the average gains from trading jump skew are positive, but not statistically different from zero. In contrast, trading weekly variance or static skew is profitable. These variance and static skew profits are not spanned by the market returns, as shown by the estimates of models (1) and (4) in Table III, but the static skew profits are spanned by variance profits (as seen in model (7)).

The estimates of models (1) and (4) show that shorting variance and taking a long position in static skew both have positive loadings on the market index. Model (1) and average profitability of short variance positions thus indicate the existence of a negative weekly variance risk premium. At the same time, the estimates of model (7) indicate that while trading static skew at the weekly regime provides a non-zero risk premium, an investor shorting variance is exposed to the same risk factors. Hence, the premia for trading static skew can be explained by exposure to variance risk. This evidence is consistent with the findings of Kozhan et al. (2013) for monthly gains of a skew swap replicating the realized covariation of return and implied volatility.

To better understand the properties of jump skew trading profits over the weekly regime, we further study whether transaction costs, or their exposure to variance risk, are responsible for the low profitability of the strategy. APT-style tests in Table IX of Appendix F, performed at mid-quote prices, confirm that the positive profits from trading weekly jump skew are explained by their exposure to tradable variance risk. Taken together, these results imply that at the weekly trading regime, investors do not gain a premium for isolating jump skew in the market for short-maturity index options. In summary, this emphasizes the important role of variance compensation over holidays and weekends, but does not preclude the existence of the premium for after-hours skew documented above.

The key implication of the above findings is that the profitability of short variance and long skew positions in weekly trading is due to the compensation for holding them overnight, when active option trading is impossible. One consequence of not actively hedging overnight is that the jump and static skew strategies become identical during such periods, if entered into simultaneously before markets close and exited from simultaneously after markets open. The situation is different for the positions held when these strategies are executed in the weekly regime. In the latter case, as evident from Result 2 and Definition 2, for overnight returns, the jump skew strategy pays a third-power function of returns while the static skew strategy's payoff is identical to that of a position in a variance swap of size $\ln F_{t_i}/F_0$.

In summary, we find that trading skew overnight carries a distinct risk premium which is not spanned by market risk or short variance risk. Skew profits accumulated over five nights and the weekend are greater than profits from holding a static skew position over the course of the week, even if they are subject to much greater transaction costs. Weekend skew profits are double those available over weekday nights. Shorting variance on weekday nights is not profitable when transaction costs are taken into account, but would be significantly lucrative at mid prices (see the third panel of Table VII). In contrast, shorting variance over weekends remains lucrative even after transaction costs.

Based on the evidence presented in this section, we draw a number of important conclusions. First, for option investors, the nature of premia for nonlinear risk exposures is crucially determined by the inability to actively adjust these exposures. Second, skew exposures in after-hours periods are distinct from variance exposures. Finally, jumps in high-frequency returns on the S&P 500 index do not require an extra risk premium. The difference between shorting variance and taking long skew positions overnight lies in the relative weights of out-of-the-money options in the replicating portfolios: skew strategies load more on very deep out-of-the-money puts and calls, and short put positions contribute more to the profits. As such, the model-free evidence in this paper adds to the findings in e.g. **?** who find that parametric models with time-invariant jump distributions are unable to explain the implied skew properties of weekly options.

C. Robustness checks

C.1. Jump skew risk and financial intermediaries

Constantinides et al. (2009) make the case for the importance of intermediaries' financial health for option prices and returns. He et al. (2017) find that monthly returns on index option portfolios constructed by matching options on moneyness and maturity as in Constantinides et al. (2013) are largely explained by the intermediary pricing factor. As our analysis is based on returns recorded at weekly, overnight and daytime frequencies, we can't use He et al.'s factor directly to study the implications of intermediary risk for skew excess returns. Instead, we download from CRSP the data on returns on publicly traded primary dealers of the NY Fed – financial intermediaries in the definition of He et al. and construct the value-weighted index of intermediary returns. For intermediaries not listed on US exchanges, we use US listings of corresponding ADRs. We construct the overnight and daytime intermediary returns based on the opening and closing prices reported in CRSP. When we aggregate our index from this high frequency to close-to-close daily returns, we obtain a correlation of 94% with the value-weighted index provided by He et al.

We find that including the intermediary index in APT tests does not help explain the profitability of variance and skew trading across overnight and weekend/holiday periods. Whenever we add the return on the intermediary index to the APT regressions of Tables II and III, neither the changes in the estimated loadings on other factors, nor the change in the regression's intercept are statistically significant.

C.2. Replication accuracy

Our realized jump skew trading strategy relies on Result 2 to construct a dynamically adjusted portfolio of options and forwards whose payoff at settlement replicates the realized skew as in Definition 1. There are three distinct potential sources of replication error in doing so. The strike grid available to investors is discrete, truncated, and changes over time. In this section we show that our dataset is sufficiently rich to replicate with reasonable accuracy all realized measures considered in this work at intraday frequencies.

Our implementation of the spanning identity (7) is specifically designed for dynamic option trading in incomplete option markets, where strike grid discretization problems may be exacerbated by the fact that option quotes might be observed for different strike grids over time. Therefore, we formulate the option portfolio weight choice problem as the problem of minimising expected squared replication error under a suitable probability measure. The details of the procedure are available in Appendix C.

In Figure 2 we give an overview of how well our strategy's option and forward payoff at settlement replicates realized jump skewness. At the weekly trading horizon (panel A in Figure 2), as well as at the overnight / weekend trading horizons (panel C in Figure 2) we obtain a virtually perfect replication, regardless of the chosen trading frequency. In trading during market opening hours, replication quality suffers at the highest (five-minute) frequency (top-left sub-panel of panel B in Figure 2), but it rapidly improves as the trading frequency drops to 15 and 30 minutes. Moreover, it is useful to note that relatively large replication errors are observed only in 2011, and disappear later due to improved option trading activity. Indeed, in the 2012-2015 subsample the correlation between true realized jump skew and its tradable counterpart is 79% at the 15-minute trading frequency and 93% at the 30-minute trading frequency.

Importantly, results of our empirical analysis in Section III.B.1 are not sensitive to the choice between five-, 15- and 30- minute trading frequencies, nor to restricting our sample to the years 2012-2015. The results remain unchanged also if in the calculation of the jump skew premium we replace the settlement payoff from Result 2 with its true realized counterpart calculated from equation (3) with high-frequency returns on the S&P 500 index. This robustness of our findings is explained by the fact that the scale of daytime realized jump skewness, $|S_t^{(n)}|$, is typically much smaller than the aggregate premium received or paid from dynamic option rebalancing.

C.3. $S_t^{(n)}$ and other measures of jump variation

The floating leg of a trading strategy which pays off whenever jumps occur should be correlated with other measures of jump variation that one can calculate from high-frequency data. In Appendix D we show that at our highest trading frequencies $S_t^{(n)}$ contains a dominant contribution from jump movements. In this section we demonstrate that $S_t^{(n)}$ indeed covaries with jump variation measured from ultra-high frequency data.

At the trading horizons we consider, the question of whether the strategy indeed captures high-frequency jump skew arises first and foremost for daytime trading. In weekly trading, there is a large time span without option rebalancing or delta-hedging when the position is held overnight.

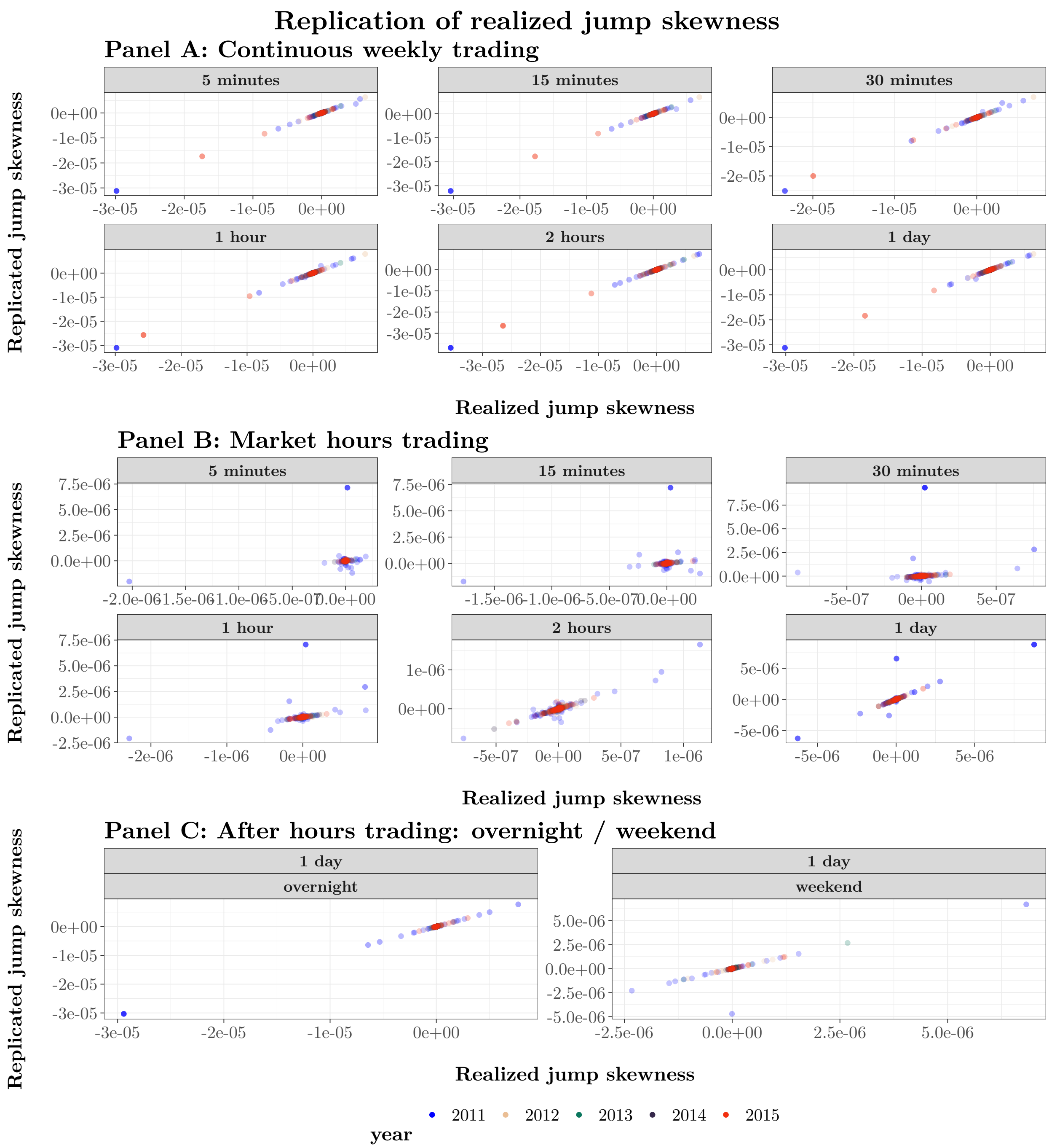

Figure 2: Replication accuracy of realized skew across all trading horizons and frequencies.

Hence, almost by construction, these periods contribute with large returns to the calculation of $S_t^{(n)}$, to the extent that they are its dominant component, as can be inferred from Section III.C.2. Overall, our trading strategies are implemented at frequencies ranging from over 70 trades per day (every five minutes) to one trade per day. Next, we turn to very high frequency data on the S&P 500 futures, demonstrating that $S_t^{(n)}$ indeed captures jump variation empirically.

The most widely-used measure of realized jump variation is the difference of realized variance and bipower variation:

$$JV_t^{(n)} := RV_t^{(n)} - BV_t^{(n)}.$$

Barndorff-Nielsen and Shephard (2006) show that in the absence of microstructure noise, $JV_t^{(n)} \to \int\int x^2 \nu_t(dx, dt)$ as $n \to \infty$. Note that contrary to $S_t^{(n)}$, $JV_t^{(n)}$ is not directly tradable and does not differentiate between positive and negative jumps.

In what follows, we study the relationship between daily measurements of $JV_t^{(m)}$ and $S_t^{(n)}$ when m is large and n is the daily number of trades in our sample, from $n = 1$ for once-per-day trading to $n = 78$ for five-minute trading. The sampling frequency for $JV_t^{(m)}$ varies from day to day and ranges from five-second to one-minute intervals, depending on the quality of available data. At such high frequencies, estimates are biased by microstructure noise in general, and the bid-ask bounce phenomenon in particular. Hence, we employ the pre-averaging estimators of $RV_t^{(m)}$ and $BV_t^{(m)}$ described in Christensen et al. (2014).

We find that positive (negative) realizations of $S_t^{(n)}$ are positively (negatively) correlated with $JV_t^{(m)}$, and that the association becomes stronger as the trading frequency increases. Figure 3 presents scatterplots of $S_t^{(n)}$ and $JV_t^{(m)}$ at all trading frequencies, with axes rescaled for improved readability. Correlations of the rescaled measures are reported in Table IV and range from 30% to 40%. In light of how difficult jump estimation is from a statistical point of view (Christensen et al., 2014; Bajgrowicz et al., 2016), we use these results to support our claim that $S_t^{(n)}$ is indeed a measure of realized jump skew with desirable empirical properties.

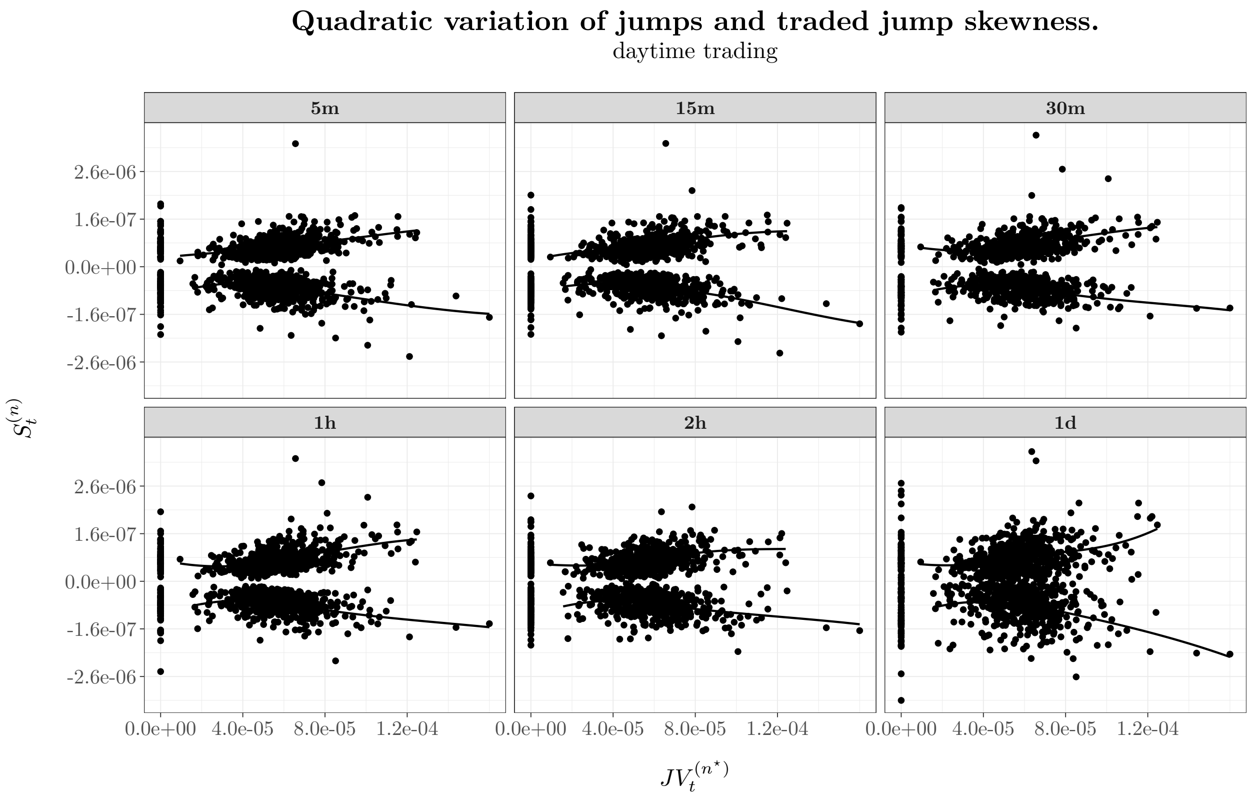

Figure 3: Scatterplots of daytime jump variation and traded $S_t^{(n)}$. Jump variation is measured as $JV_t = \max(RV_t - BV_t, 0)$, where both quantities are calculated from high frequency (5-second to 1-minute) data from the market for S&P 500 futures. For both quantities we use estimators robust to microstructure noise, described in COP (2014). In the plots, axes are rescaled for improved legibility; the following transformations are applied: $JV_t \rightarrow JV_t^{0.25}$, and $S_t^{(n)} \rightarrow \text{sign}(S_t^{(n)})|S_t^{(n)}|^{0.25}$. Regression lines obtained with the LOESS (see Cleveland et al., 2017) method for positive and negative skew realizations are plotted in black.

Table I: Profits (after transaction costs) from shorting realized variance and going long skew strategies with Friday Weeklys in 2011-2015 at all trading horizons and trading frequencies ranging from 5 minutes (72 trades per day) to 1 day (i.e. 1 trade per day, or just opening and closing position in the case of daytime trading). All reported statistics are scaled by 10^4 for variance trading and by 10^5 for skew trading. All statistics and confidence intervals calculated with time-series bootstrap of Politis and Romano (1994). Calculations performed with package boot version 1.3.20 in R version 3.5.2 (2018-12-20).

	jump skew, $S_t^{(n)}$			static skew, $US_t^{(n)}$			variance, $D_t^{(n)}$		
trade freq	μ_b	$q_{\mu_b}^{0.05}$	$q_{\mu_b}^{0.95}$	μ_b	$q_{\mu_b}^{0.05}$	$q_{\mu_b}^{0.95}$	μ_b	$q_{\mu_b}^{0.05}$	$q_{\mu_b}^{0.95}$
Weekly trading									
5m	0.041	-0.098	0.269	0.250	0.086	0.374	0.521	0.238	0.788
15m	0.069	-0.099	0.310	0.261	0.096	0.392	0.532	0.235	0.834
30m	0.079	-0.114	0.329	0.275	0.079	0.417	0.557	0.206	0.902
1h	0.073	-0.113	0.354	0.277	0.092	0.432	0.518	0.192	0.863
2h	0.109	-0.122	0.413	0.291	0.075	0.431	0.513	0.187	0.811
1d	0.110	-0.074	0.386	—	—	—	0.543	0.231	0.816
Daytime trading									
5m	-0.111	-0.154	-0.043	-0.084	-0.120	-0.023	-0.164	-0.218	-0.098
15m	-0.098	-0.136	-0.042	-0.081	-0.117	-0.033	-0.162	-0.209	-0.098
30m	-0.092	-0.129	-0.042	-0.079	-0.109	-0.030	-0.154	-0.194	-0.096
1h	-0.086	-0.121	-0.048	-0.076	-0.106	-0.032	-0.153	-0.203	-0.100
2h	-0.059	-0.077	-0.042	-0.053	-0.071	-0.034	-0.131	-0.157	-0.104
1d	-0.086	-0.122	-0.037	—	—	—	-0.160	-0.202	-0.099
Overnight trading									
–	0.048	0.006	0.081	—	—	—	-0.029	-0.097	0.058
Weekend trading									
–	0.093	-0.023	0.216	—	—	—	0.176	0.038	0.326
Monthly trading									
5m	-1.970	-5.507	3.010	4.649	0.875	7.219	5.129	2.962	6.959

Table II: **Spanning tests with transaction costs: overnight and weekend/holiday horizon**. In this table we report results of spanning tests of short variance, long static skew, and long dynamic skew profits by the market risk factor (r_m), the intermediary risk factor (r_d), and appropriate lower-order strategy trading profits. The returns on the intermediary factor are calculated as returns on the value-weighted portfolio of NY Fed primary dealers (see He et al. (2017)). If profits from a given strategy are spanned by tradable risk factors, the presence of positive alpha indicates that unspanned risk compensation is available by trading the strategy. Models (1)-(6) are estimated with OLS. Model (7) is estimated with IV in order to alleviate EIV issues related to strategies which, due to the characteristics of the option market, only approximately trade the underlying (variance, skew) risk factors, and whose replication errors are likely correlated. We use powers of the market and intermediary returs as instruments. We calculate Newey-West standard errors with pre-whitening with the use of package `sandwich` version 2.5.0 in R version 3.5.2 (2018-12-20).

Overnight trading

	$D_t^{(n)} - \mathcal{D}_t^{(n)}$			$S_t^{(n)} - \mathcal{S}_t^{(n)}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
r_m	0.63***		0.31	0.49***		0.35	0.39***
	(0.21)		(0.27)	(0.13)		(0.24)	(0.10)
r_d		0.45***	0.28*		0.32***	0.13	
		(0.12)	(0.14)		(0.07)	(0.12)	
$D_t^{(n)} - \mathcal{D}_t^{(n)}$							0.17
							(0.10)
Constant	-0.06	-0.07	-0.07	0.03	0.02	0.02	0.04**
	(0.05)	(0.05)	(0.05)	(0.03)	(0.02)	(0.02)	(0.02)
Adjusted R^2	0.15	0.16	0.17	0.28	0.24	0.29	0.51

Weekend / holiday trading

	$D_t^{(n)} - \mathcal{D}_t^{(n)}$			$S_t^{(n)} - \mathcal{S}_t^{(n)}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
r_m	0.83***		0.69***	0.75***		0.44***	0.68***
	(0.26)		(0.23)	(0.18)		(0.12)	(0.23)
r_d		0.51***	0.11		0.49***	0.24*	
		(0.18)	(0.24)		(0.11)	(0.14)	
$D_t^{(n)} - \mathcal{D}_t^{(n)}$							0.08
							(0.15)
Constant	0.23***	0.25***	0.23***	0.14***	0.17***	0.15***	0.12**
	(0.06)	(0.07)	(0.07)	(0.04)	(0.04)	(0.04)	(0.06)
Adjusted R^2	0.27	0.23	0.27	0.45	0.43	0.47	0.55

Notes: ***Significant at the 1 percent level.
**Significant at the 5 percent level.
*Significant at the 10 percent level.
r^k scaled by 10^{2k}, $D_t^{(n)} - \mathcal{D}_t^{(n)}$ scaled by 10^4, $S_t^{(n)} - \mathcal{S}_t^{(n)}$ scaled by 10^5.

Table III: **Spanning tests with transaction costs: weekly horizon**. For detailed legend see Table II. Models (1)-(6) and (8)-(10) are estimated with OLS. Models (7) and (11) are estimated with IV.

	$D_t^{(n)} - \mathcal{D}_t^{(n)}$			$US_t^{(n)} - \mathcal{US}_t^{(n)}$				$S_t^{(n)} - \mathcal{S}_t^{(n)}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
r_m	0.37**		0.39**	0.20***		0.23***	−0.03	0.28***		0.28***	0.05
	(0.15)		(0.16)	(0.07)		(0.07)	(0.06)	(0.07)		(0.09)	(0.06)
r_d		0.19**	−0.02		0.11**	−0.02			0.15***	−0.00	
		(0.10)	(0.08)		(0.04)	(0.03)			(0.05)	(0.06)	
$D_t^{(n)} - \mathcal{D}_t^{(n)}$							0.64***				0.62***
							(0.17)				(0.12)
Constant	0.47***	0.50***	0.47***	0.23***	0.24***	0.23***	−0.07	0.02	0.05	0.02	−0.27***
	(0.13)	(0.14)	(0.13)	(0.06)	(0.07)	(0.07)	(0.11)	(0.11)	(0.11)	(0.11)	(0.10)
Adjusted R^2	0.10	0.07	0.09	0.14	0.09	0.13	0.56	0.06	0.04	0.06	0.59

Notes: ***Significant at the 1 percent level.
**Significant at the 5 percent level.
*Significant at the 10 percent level.
r^k scaled by 10^{2k}, $D_t^{(n)} - \mathcal{D}_t^{(n)}$ scaled by 10^4, $S_t^{(n)} - \mathcal{S}_t^{(n)}$ scaled by 10^5.

Table IV: Correlation of $S_t^{(n)}$ and sign$(S_t^{(n)})JV_t^{(n^\star)}$ with daytime measurement at a range of frequencies, after rescaling $JV_t^{(n^\star)} \to (JV_t^{(n^\star)})^{0.25}$, and $S_t^{(n)} \to \text{sign}(S_t^{(n)})|S_t^{(n)}|^{0.25}$.

	trading frequency					
sign$(S_t^{(n)})$	5m	15m	30m	1h	2h	1d
negative	-0.37	-0.39	-0.33	-0.33	-0.28	-0.29
positive	0.40	0.40	0.41	0.46	0.38	0.31

IV. Conclusions

A large literature attributes a number of asset pricing phenomena to the risk of jumps, and the investors' aversion thereto, from the extreme skew in the S&P 500 option implied volatility surface to the magnitude of the equity premium. Recent evidence, which to a large extent is based on stochastic jump-diffusion processes, underlines the importance of asymmetry in the compensation for jump risk. We add to this literature model-free evidence based on a trading strategy that bets on the realized skewness of index return jumps and is unaffected by the skewness of small, regular returns, whenever options can be actively traded. By comparing the strategy's average profits across periods when the option markets are open and when they are closed, we discover that jump skewness in high-frequency market hours index returns does not carry a risk premium, while the skewness of overnight returns does. Based on the empirical evidence, we conclude that a major share of the compensation for nonlinear risk associated with trading options is related to the investors (in)ability to actively hedge their nonlinear risks.

Bets on realized variance, realized skewness and realized jump skewness of returns are not profitable during the opening hours of the option market, with or without transaction costs. However, nonlinear return bets that are tradable with options are highly profitable when the index option market is closed and investors are only able to, at best, delta-hedge in electronic futures markets for the underlying. Premia for after-hours skewness are distinct from the premia for after-hours variance. Trading skewness requires taking positions tilted towards deeper out-of-the-money options than trading variance. This, together with aforementioned evidence on average profitability of these strategies, implies that investors are indeed more averse to particularly large returns occurring in periods when they cannot actively trade options. Such returns need not be, and very likely are not, jumps within the framework of jump-diffusion processes.

Our results suggest that models for index returns and for options should explicitly take into account the institutional features of markets, with particular emphasis on market opening hours and the investors' ability to hedge their risks. A possible avenue to reconcile the highly non-Gaussian features of the implied volatility surface and the sample path of the S&P 500 at high-frequency, could be lower-frequency models concentrating on market frictions and market design.

Appendices

Appendix A. Replication of realized return variation

The replication of payoffs that are non-linear functions of the future price of an underlying asset relies on results in Breeden and Litzenberger (1978) and Carr and Madan (2001). More precisely, given a twice-differentiable function $\phi : \mathbb{R}_+ \to \mathbb{R}$, the following identity holds:

$$\begin{aligned} D_\phi(F_T, F_t) &:= \phi(F_T) - \phi(F_t) - \phi'(F_t)(F_T - F_t) \\ &= \int_0^\infty \left[\phi''(K)(K - F_T)\mathbf{1}_{\{F_T \le F_t\}} + \phi''(K)(F_T - K)\mathbf{1}_{\{F_T \ge F_t\}}\right] dK \,, \end{aligned} \tag{A1}$$

where $\mathbf{1}_A$ denotes the indicator function of event A. As a consequence, in a setting of complete option markets allowing the trading at time $t < T$ of options with arbitrary strike price $K > 0$ for maturity T, it is possible to replicate payoff $D_\phi(F_T, F_t)$ using a portfolio of out-of-the-money options with portfolio weights $\phi''(K)$. In this way, the price of payoff $D_\phi(F_T, F_t)$ becomes observable in a model-free way from the price of the option portfolio replicating $D_\phi(F_T, F_t)$.

Static option replication is not enough to trade payoffs that depend on intermediate values of the forward price strictly between times t and T. Therefore, realized return variation is not in general replicable using only static option portfolios. For such payoffs, it is necessary to implement a dynamic option replicating strategy that consists of a sequence of option portfolio payoffs of the form (A1), which are implemented sequentially at the trading dates $t =: t_0 < t_1 < \ldots < t_{n-1} < t_n := T$. Realized return variation measures that can be replicated by a sequence of such option portfolio payoffs are hence tradable.

Definition 4 (Tradable realized return variation)**.** A realized return variation measure $V_t^{(n)}$ is tradable if it can be written as

$$V_t^{(n)} = D_{\phi_0}(F_T, F_t) + \sum_i \left(\Gamma_i D_{\phi_i}(F_T, F_i) + \Delta_i (F_T - F_i)\right) \,, \tag{A2}$$

with functions ϕ_i and coefficients Γ_i, Δ_i that are known at time t_i.

Whenever weights Γ_i in Definition 4 are nonzero for some $i > 0$, the replication of realized return variation entails a dynamic option replication. In the opposite case, a static option replication based on a single, dynamically delta-hedged, option portfolio of the form (A1) is possible.

Remark 2. Realized return variation measures of the form $\sum_i W_{i-1} D_\phi(F_i, F_{i-1})$ are tradable. Indeed,

$$\sum_i W_{i-1} D_\phi(F_i, F_{i-1}) = \sum_i W_{i-1}[D_\phi(F_T, F_{i-1}) - D_\phi(F_T, F_i) - (\phi'(F_i) - \phi'(F_{i-1}))(F_T - F_i)] \, ,$$

which is a weighted sum of option portfolio and forward payoffs for the fixed maturity T. More explicitly, for the case where the option maturity T and the horizon of the return variation measure coincide ($t_n = T$), it follows

$$V_t^{(n)} = W_0 D_\phi(F_T, F_t) + \sum_i (W_i - W_{i-1}) D_\phi(F_T, F_i) - \sum_i W_{i-1}(\phi'(F_i) - \phi'(F_{i-1}))(F_T - F_i) \, , \tag{A3}$$

i.e., the tradable replication identity (A2) with $\phi_0 = W_0 \phi$, $\phi_1 = \phi_2 = \ldots = \phi_{n-1} = \phi$, $\Gamma_i = W_i - W_{i-1}$ and $\Delta_i = W_{i-1}(\phi'(F_{i-1}) - \phi'(F_i))$.

Remark 3. An extension of equation (A3) applies in the case where the option maturity is larger than the horizon of the return variation measure ($t_n < T$). In such a setting, we obtain

$$\begin{aligned} V_t^{(n)} &= W_0 D_\phi(F_T, F_t) + \sum_i (W_i - W_{i-1}) D_\phi(F_T, F_i) - W_n D_\phi(F_T, F_n) \\ &\quad - \sum_i W_{i-1}(\phi'(F_i) - \phi'(F_{i-1}))(F_T - F_i) \, . \end{aligned} \tag{A4}$$

Compared to equation (A3), the additional term $-W_n D_\phi(F_T, F_n)$ in equation (A4) compensates the mismatch between the option maturity and the replication horizon. Therefore, the tradability of return variation measure in Remark 2 is granted also when the maturity of the tradable options is longer than the horizon of the return variation measure that needs to be replicated. Our benchmark empirical results are based on weekly replication horizons with weekly option maturities. However, we also study daily/overnight replication horizons with weekly option maturities to better study the implications of within-the-day vs. overnight trading.

Appendix B. Replicating strategy for realized return skewness $S_t^{(n)}$

Proof of Result 2. To prove Result 2, start from

$$S_t^{(n)} = D_t^{(n)} + (S_t^{(n)} - D_t^{(n)})\,.$$

From equalities (6) and (7), we have with $\phi(x) = -\ln(x)$:

$$D_t^{(n)} = D_\phi(F_T, F_t) + \sum_i \left[\left(\frac{1}{F_{i-1}} - \frac{1}{F_t}\right)(F_i - F_{i-1})\right] = D_\phi(F_T, F_t) + \sum_i \left(\frac{1}{F_i} - \frac{1}{F_{i-1}}\right)(F_T - F_i)\,.$$

Moreover, from equalities (7) and (8) we also have, with $\psi(x) = -\frac{1}{2}[\ln(x/F_t)]^2$:

$$\begin{aligned} S_t^{(n)} - D_t^{(n)} &= D_\psi(F_T, F_t) + \sum_i \ln(F_{i-1}/F_t)(\ln F_i - \ln F_{i-1}) \\ &= D_\psi(F_T, F_t) + \sum_i \ln(F_{i-1}/F_t)(D_\phi(F_T, F_i) - D_\phi(F_T, F_{i-1})) \\ &\quad + \sum_i \ln(F_{i-1}/F_t)\left[\frac{1}{F_{i-1}}(F_T - F_{i-1}) - \frac{1}{F_i}(F_T - F_i)\right]\,. \end{aligned}$$

Rearranging the last two sums, we obtain

$$\begin{aligned} \sum_i \ln(F_{i-1}/F_t)\left[\frac{1}{F_{i-1}}(F_T - F_{i-1}) - \frac{1}{F_i}(F_T - F_i)\right] &= \sum_i \ln(F_i/F_{i-1})\frac{1}{F_i}(F_T - F_i)\,, \\ \sum_i \ln(F_{i-1}/F_t)(D_\phi(F_T, F_i) - D_\phi(F_T, F_{i-1})) &= \sum_i \ln(F_{i-1}/F_i) D_\phi(F_T, F_i)\,. \end{aligned}$$

[25] Therefore,

$$S_t^{(n)} - D_t^{(n)} = D_\psi(F_T, F_t) + \sum_i \ln(F_{i-1}/F_i)[D_\phi(F_T, F_i) - \frac{1}{F_i}(F_T - F_i)]\,. \tag{B1}$$

[25] Note that for $Z_0 = 0$ and $W_n = W_T$, $\sum_{i=1}^n Z_{i-1}(W_i - W_{i-1}) = \sum_{i=1}^n (Z_i - Z_{i-1})(W_T - W_i)$. If $W_n \neq W_T$, $\sum_{i=1}^n Z_{i-1}(W_i - W_{i-1}) = \sum_{i=1}^n (Z_i - Z_{i-1})(W_T - W_i) - Z_n(W_T - W_n)$.

Putting together the replicating expression for $D_t^{(n)}$ and $S_t^{(n)}$ above, it follows

$$S_t^{(n)} = D_{\phi+\psi}(F_T, F_t) + \sum_i \ln(F_{i-1}/F_i) D_\phi(F_T, F_i) + \sum_i \left[\frac{1 + \ln(F_i/F_{i-1})}{F_i} - \frac{1}{F_{i-1}}\right](F_T - F_i)\,.$$

Therefore, we can define in equation (9) of the main text $\phi_0 := \phi + \psi$ and $\phi_i := \phi$ for any other $i = 1, \dots, n-1$. Similarly, we obtain

$$\begin{aligned}
\Gamma_i &= \ln(F_{i-1}/F_i)\,, \\
\Delta_i &= \frac{1 - \ln(F_{i-1}/F_i)}{F_i} - \frac{1}{F_{i-1}}\,.
\end{aligned}$$

This concludes the proof.

$\square$

Table V: **Replicating strategy for realized skewness.** Generating functions ϕ_i and weights Γ_i, Δ_i in the dynamic replicating strategy for realized skewness $S_t^{(n)}$. The strategy inception and maturity date are t and T, respectively. The replicating portfolio in options and forwards is rebalanced at times $t < t_i < T$, with for brevity index i denoting time t_i.

time	payoff	position	generating function	option weight
t	$S(\ln(F_T/F_t))$	1	$-\left[\ln(x) + \frac{1}{2}(\ln(x/F_t))^2\right]$	$\frac{\ln(K/F_t)}{K^2}$
i	$F_T - F_i$	$\Delta_i = \frac{1-\ln(F_{i-1}/F_i)}{F_i} - \frac{1}{F_{i-1}}$	$-$	$-$
	$D(\ln(F_T/F_i))$	$\Gamma_i = \ln(F_{i-1}/F_i)$	$-\ln x$	$\frac{1}{K^2}$

Appendix C. Optimal option portfolio weights in incomplete option markets

In incomplete option markets, the exact replication identity (7) can hold only approximately, so that the option weights $\phi''(K)$ in the corresponding option replicating portfolio may lead to a suboptimal replication accuracy relative to other option portfolio compositions. Therefore, we introduce in this section a procedure for computing optimal portfolio weights that minimize replication accuracy under an incomplete option market.

In order to replicate realized skewness, we need to replicate at trading times $t =: t_0, t_1, \dots, t_{n-1}$ a sequence of payoffs of the form $D_{\phi_i}(F_T, F_i)$ implied by corresponding generating functions $\phi_0, \phi_1, \dots, \phi_{n-1}$. Therefore, we can focus without loss of generality on the problem of replicating payoff $g(F_T) := D_{\phi_i}(F_T, F_i)$ using a discrete set of out-of-the-money options at time t_i, with payoffs $O(F_T, K_j)$ parameterized by a corresponding vector of strike prices $\mathbf{K}' = (K_1, K_2, \dots, K_J)$. We denote by $\mathbf{O}(F_T, \mathbf{K})' := (O(F_T, K_1), \dots, O(F_T, K_J))$ the corresponding vector of option payoffs.

For any admissible vector of option weights $\mathbf{w}' := (w_1, \dots, w_J)$ in the replicating portfolio, the portfolio payoff is

$$\mathbf{w}'\mathbf{O}(F_T, \mathbf{K}) = \sum_j w_j O(F_T, K_j) , \tag{C1}$$

so that the replication error of a weight vector $\mathbf{w}$ is

$$e(F_T, \mathbf{w}) = g(F_T) - \mathbf{w}'\mathbf{O}(F_T, \mathbf{K}) . \tag{C2}$$

We propose to determine optimal portfolio weight vector $\mathbf{w}^\star$ in the replicating portfolio by minimizing a weighted L_2—replication error metric with integrable density $\eta : \mathbb{R}_+ \to \mathbb{R}_+$,

$$\mathbf{w}^\star := \arg\min_{\mathbf{w}} L_\eta(\mathbf{w}) := \arg\min_{\mathbf{w}} \int_0^\infty \eta(x) e^2(x, \mathbf{w}) dx . \tag{C3}$$

This defines a quadratic programming problem based on the loss function

$$L_\eta(\mathbf{w}) = C + \mathbf{w}'\mathbf{Q}\mathbf{w} - \mathbf{q}'\mathbf{w} , \tag{C4}$$

with a scalar $C := \int_0^\infty \eta(x) g^2(x) dx$, a J—dimensional vector

$$\mathbf{q} := 2 \int_0^\infty \eta(x) g(x) \mathbf{O}(x, \mathbf{K}) dx , \tag{C5}$$

and a symmetric positive definite $J \times J$ matrix

$$\mathbf{Q} := \int_0^\infty \eta(x)g(x)\mathbf{O}(x,\mathbf{K})\mathbf{O}'(x,\mathbf{K})dx \,. \tag{C6}$$

The components of vector $\mathbf{q}' = (q_1, \dots, q_J)$ read explicitly

$$q_j = \begin{cases} 2\int_0^{K_j} \eta(x)g(x)(K_j - x)dx & \text{if } K_j \leq F_i \\ 2\int_{K_j}^\infty \eta(x)g(x)(x - K_j)dx & \text{if } K_j > F_i \end{cases} .$$

The components of matrix $\mathbf{Q} = [Q_{kj}]_{1\leq k,j\leq J}$ read explicitly

$$Q_{kj} = \begin{cases} 0 & \text{if } K_k < F < K_j \text{ or } K_j < F < K_k \\ \int_0^{\min(K_k,K_j)} \eta(x)(K_k - x)(K_j - x)dx & \text{if } K_k, K_j \leq F \\ \int_{\max(K_k,K_j)}^\infty \eta(x)(x - K_k)(x - K_j)dx & \text{if } K_k, K_j > F \end{cases} ,$$

Therefore, symmetric positive definite matrix $\mathbf{Q}$ is block-diagonal, with two blocks that correspond to second moment matrices of out-of-the-money call and put payoffs, respectively, under density η. Such a block diagonal structure allows to equivalently decompose the optimal replication problem (C3) into two quadratic programming problems for the optimal portfolio weights of out-of-the-money put and call options, respectively, which improves computational tractability.

Note that all integrals in the definition of vector $\mathbf{q}$ and matrix $\mathbf{Q}$ are easily computed numerically, even in presence of a large number of option strikes. In some interesting special cases, e.g., for a uniform density η of the form $\eta(x) \equiv \mathbf{1}_{\{K_1-\varepsilon\leq x\leq K_J+\varepsilon\}}$ for some $\varepsilon > 0$, the optimal replication problem can be even solved analytically. In our empirical analysis, we worked with uniform densities and found a satisfactory replication accuracy over realistic supports for returns.

Appendix D. Properties of tradable skewness and variance in discrete trading

As shown above, at the high-frequency limit $S_t^{(n)}$ and $D_t^{(n)}$ ehxibit very distinct behaviour, the former becoming a pure jump measure, while the latter collecting quadratic variation of jumps and continuous increments. At a finite trading frequency the Brownian component of price variation can dominate in both $D_t^{(n)}$ and $S_t^{(n)}$ when volatility is high relative to jump variation. In this section, we show in a stylized setting the effect that departing from continuous trading has on the "jump content" of the jump skew measure $S_t^{(n)}$.

We consider measuring $D_t^{(n)}$ and $S_t^{(n)}$ over the course of one trading day in a setting when precisely one jump of size $k\sigma$ arrives, where σ is the annualized volatility level of continuous increments. The literature on Threshold Realized Volatility estimators (Mancini, 2001) determines whether a jump occurred or not by comparing each return observation with a threshold, typically equal to $M\sqrt{\widehat{\sigma}}/\sqrt{252n}$, where $\widehat{\sigma}$ is an annualized estimate of Brownian quadratic variation, and n is the number of intraday observations used in calculating $\widehat{\sigma}$. With typical values of M between 3 and 5, and 72 observations in a trading day (i.e. using 5-minute data between 9:00 and 15:00), this translates to using k between 0.023 and 0.038 in our stylized setting, i.e. jumps ranging between 0.4% and 0.6% for $\sigma = 16\%$ or between 0.92% and 1.52% when $\sigma = 40\%$. Below we show that at trading frequencies of 5 and 15 minutes, even barely detectable jumps would contrbute at least 50% to realized jump skew $S_t^{(n)}$, and that lower bound is only achieved if all returns during the day are of the same sign. Jumps twice that size would contribute no less than 75% and 87%, respectively.

Discretizing a semi-martingale log-price process, we can write the realized measures in terms of Brownian and jump increments that drive log-returns r_t:

$$D_t^{(n)} = \sum_{s=1}^{n} \left[\left(\sigma_s \sqrt{\Delta} Z_s + X_s \mathbf{1}_{\{N_{s+\Delta} - N_s = 1\}} \right)^2 + O(|r_s|^3) \right]$$

$$S_t^{(n)} = \sum_{s=1}^{n} \left[\left(\sigma_s \sqrt{\Delta} Z_s + X_s \mathbf{1}_{\{N_{s+\Delta} - N_s = 1\}} \right)^3 + O(|r_s|^4) \right].$$

where $\Delta := t_s - t_{s-1}$, $Z_s \sim N(0,1)$, N_s is the jump counting process, X_s is the jump size at time

s, and for the sake of clarity assume that the volatility is constant during the day, $\sigma_t = \sigma$.[26] Assume that one jump of magnitude $k\sigma$ occurrs during the day and note that $n = T/\Delta$, so that

$$\mathbb{E}[D_t^{(n)}] = T\sigma^2 + k^2\sigma^2 + O(\mathbb{E}|r_s|^3)$$
$$\mathbb{E}[|S_t^{(n)}|] \leq T\frac{2^{3/2}}{\pi^{1/2}}\sigma^3\Delta^{1/2} + k^3\sigma^3 + O(\mathbb{E}|r_s|^4).$$

For $|S_t^{(n)}|$ the above inequality holds (approximately) as an equality if all returns during the day are of the same sign. For both measures we can calculate measures of relative contribution of jump variation for a range of trading frequencies Δ and relative jump magnitudes k. In the case of $D_t^{(n)}$ the contribution will is exact while in the case of $S_t^{(n)}$ it is a lower bound on the impact of jump variation. The measures are defined as

$$\rho_{D_t^{(n)}}(k, \Delta) := \frac{k^2}{T + k^2} \tag{D1}$$
$$\rho_{|S_t^{(n)}|}(k, \Delta) := \frac{k^3}{T\Delta^{1/2}\frac{2^{3/2}}{\pi^{1/2}} + k^3}, \tag{D2}$$

and their graphs are presented in Figure 4. Clearly, in the case of realized volatility, the contribution from the jump and continuous part does not depend on the trading frequency. In the case of realized jump skew, the contribution of the jump part increases quickly as the relative jump size increases. If $k = 0.1$, which for the annualized volatility of the market index of 16% translates to a jump size of 1.6%, then the contribution of continuous variation to realized skewness is capped at approximately 1% at the 1-minute trading frequency and at 2.5% at the 5-minute trading frequency. For a smaller jump, say $k = 0.05$, the cap $\rho_{|S_t^{(n)}|}$ rises to 9% and 18% at the respective frequencies.

In this simplified analytical setting J_t does not necessarily have to be treated as a jump in the strict sense, but can be looked at as a movement resulting from an abrupt and transient increase in volatility or an explosion of the drift term (see e.g. Christensen et al. (2014, 2016)). As long as the effects of such an event are not reversed within the time window Δ, it will have an outsized impact on both $D_t^{(n)}$ and $S_t^{(n)}$, but a significantly larger one on the latter – a measure of big risk.

[26] An analogous reasoning can be derived for $\sigma = \sup_t \sigma_t$.

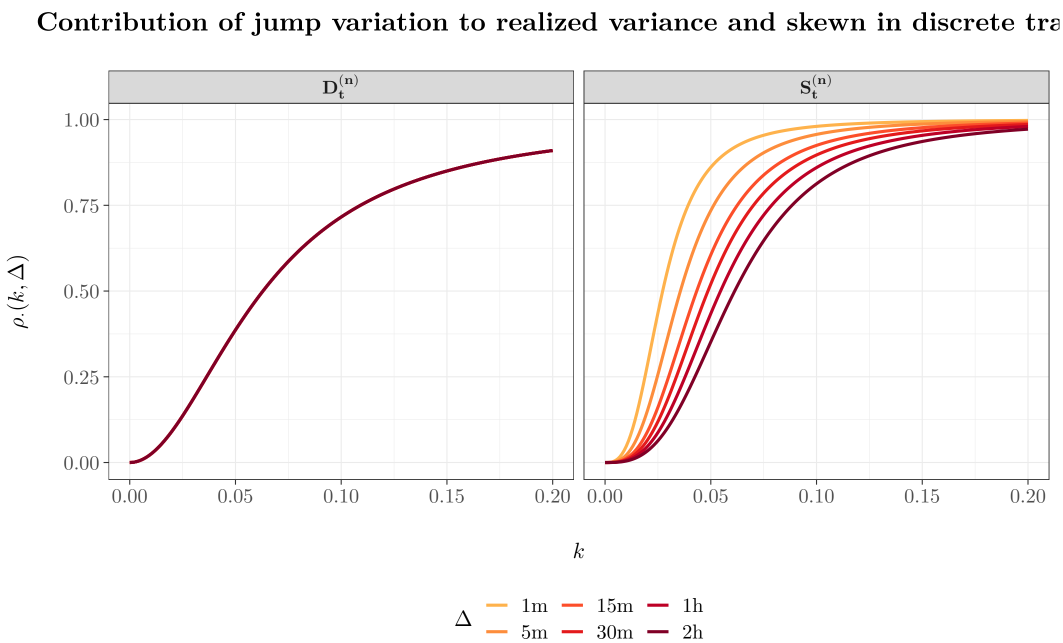

Figure 4: Relative contribution of jump variation to $D_t^{(n)}$ (equation (D1)) and $|S_t^{(n)}|$ (equation (D2)) for trading frequencies between 1 minute and 2 hours, for occurrence of a single jump of magnitude $k\sigma_t$.

Appendix E. Realized variance and skew payoffs: descriptive statistics

Table VI shows summary statistics of settlement payoffs from different option trading strategies. As we show in Section III.C.2, these option payoffs are very close to realized variance and skew measures calculated directly from returns on index forwards. For weekly maturities, variance and jump skew are largely independent of the trading frequency varying between 5 minutes and one day. While the two constituents of jump skewness, static skew and dynamic rebalancing do appear to be dependent on the trading frequency, the two effects cancel out. The reason for this is apparent from comparing the corresponding hedging formulae in Section II.B and Table V in Appendix B. Jump skew and static skew are negative on average, but with confidence bands containing zero.

Both static skew and big risk skew are measures of asymmetry in returns. At the daytime and overnight trading horizons, when only a single trade is undertaken, they are identical. The fact that static skew is negative on average is consistent with the evidence on the leverage effect, negative correlation of returns and their variance. Over the weekend, static and jump skew are positive on average, but again with confidence bands containing zero.

Due to the difference between jump skew and static skew induced from the rebalancing terms, they are not perfectly correlated: 0.59 in daytime and 0.80 in weekly trading.

Table VI: Settlement payoffs from trading realized realized variance and realized skewness with Friday Weeklys in 2011-2015. Multiple trading frequencies for daytime and weekly trading. Results for variance trading scaled by 10^4. Results for realized skewness trading scaled by 10^6. All statistics and confidence intervals calculated with time-series bootstrap of Politis and Romano (1994). Calculations performed with package `boot` version 1.3.20 in R version 3.5.2 (2018-12-20).

	jump skew, $S_t^{(n)}$			static skew, $US_t^{(n)}$			variance, $D_t^{(n)}$		
trade freq	μ_b	$q_{\mu_b}^{0.05}$	$q_{\mu_b}^{0.95}$	μ_b	$q_{\mu_b}^{0.05}$	$q_{\mu_b}^{0.95}$	μ_b	$q_{\mu_b}^{0.05}$	$q_{\mu_b}^{0.95}$
Weekly trading									
5m	-0.021	-0.048	0.019	-0.131	-0.246	-0.004	1.875	0.943	2.501
15m	-0.021	-0.051	0.016	-0.121	-0.229	0.013	1.865	0.928	2.528
30m	-0.018	-0.044	0.016	-0.106	-0.200	0.010	1.840	0.940	2.466
1h	-0.024	-0.055	0.019	-0.105	-0.201	0.015	1.878	1.019	2.531
2h	-0.027	-0.062	0.012	-0.090	-0.168	0.019	1.883	0.950	2.476
1d	-0.021	-0.047	0.014	—	—	—	1.853	0.910	2.473
Daytime trading									
5m	0.000	-0.001	0.001	-0.001	-0.003	0.000	0.162	0.109	0.205
15m	0.000	-0.001	0.001	-0.001	-0.003	0.001	0.159	0.107	0.198
30m	0.001	-0.001	0.002	0.000	-0.001	0.001	0.146	0.109	0.176
1h	0.001	-0.001	0.002	0.000	-0.002	0.000	0.136	0.102	0.164
2h	0.000	0.000	0.000	0.000	-0.001	0.000	0.105	0.080	0.123
1d	0.000	-0.001	0.002	—	—	—	0.157	0.109	0.193
Overnight trading									
–	-0.002	-0.008	0.007	—	—	—	0.244	0.103	0.335
Weekend trading									
–	-0.001	-0.007	0.006	—	—	—	0.296	0.138	0.418
Monthly trading									
5m	-0.062	-0.135	0.023	-1.686	-3.183	0.592	7.963	3.228	11.402

Appendix F. Supplementary Tables

This Appendix contains tables with descriptive statistics and results of APT-like tests calculated from trading at mid prices.

Internet Appendix

Appendix A. Trading activity in CBOE Weeklys

Friday Weeklys are European options on the S&P500 index that are traded at the CBOE which follow a weekly expiration calendar. Since their introduction in September 2007, these short-maturity options have become the most widely traded group of derivatives at the CBOE. As noted by Andersen et al. (2017), their popularity stems mainly from the fact that the at-the-money/out-of-the money short-maturity options are to a large extent differentially influenced by small and big risks, and thus allow investors to hedge their positions appropriately.

Two kinds of price observations are available in the Market Data Express feed: quotes and trades. A quote data point contains information on the bid and ask prices for a given option and the numbers of options available at those prices. A trade data point contains information on the execution price and the number of options that changed hands. The dataset provides a complete record of all quotes and trades on SPX options. Quote records constitute 99.6% of the dataset, yet most of them are not informative for a number of reasons, thus not all of them are used in our analysis. Market makers quote prices for very deep out of the money options. These quotes' spreads, however, are such that these quotes are rarely acted upon by traders, as seen in Figure 5. Furthermore, with such wide spreads, the market makers do not update these quotes for large stretches of time. On the other hand, it is not possible to base the analysis on trade data only: even though some options are traded very frequently, overall the trades are not regular enough to serve as the only source of option portfolio replication data. Furthermore, trade data suffers from the bid-ask bounce phenomenon, which introduces artificial variability into observed prices.

In our analysis we combine quote and trade data in the following way. In each five minute period we include only quote data on options which have been traded in a thirty-minute window around the time of the quote, and for which the nearest trade happenen inside the bid-ask spread. In this way, we ensure that we do not use stale quotes or quotes with unrealistic prices, where a market for a given contract essentially does not exist. Besides taking these measures, we drop all quotes with negative spreads, with 0$ bid prices and those where $p_{\text{ask}}/p_{\text{bid}} > 5$.

Before the beginning of 2011 there was little trading in the market for Weeklys. Since 2011, the market boomed. Between 2011 and 2016 the width of quoted and actively traded option strikes dou-

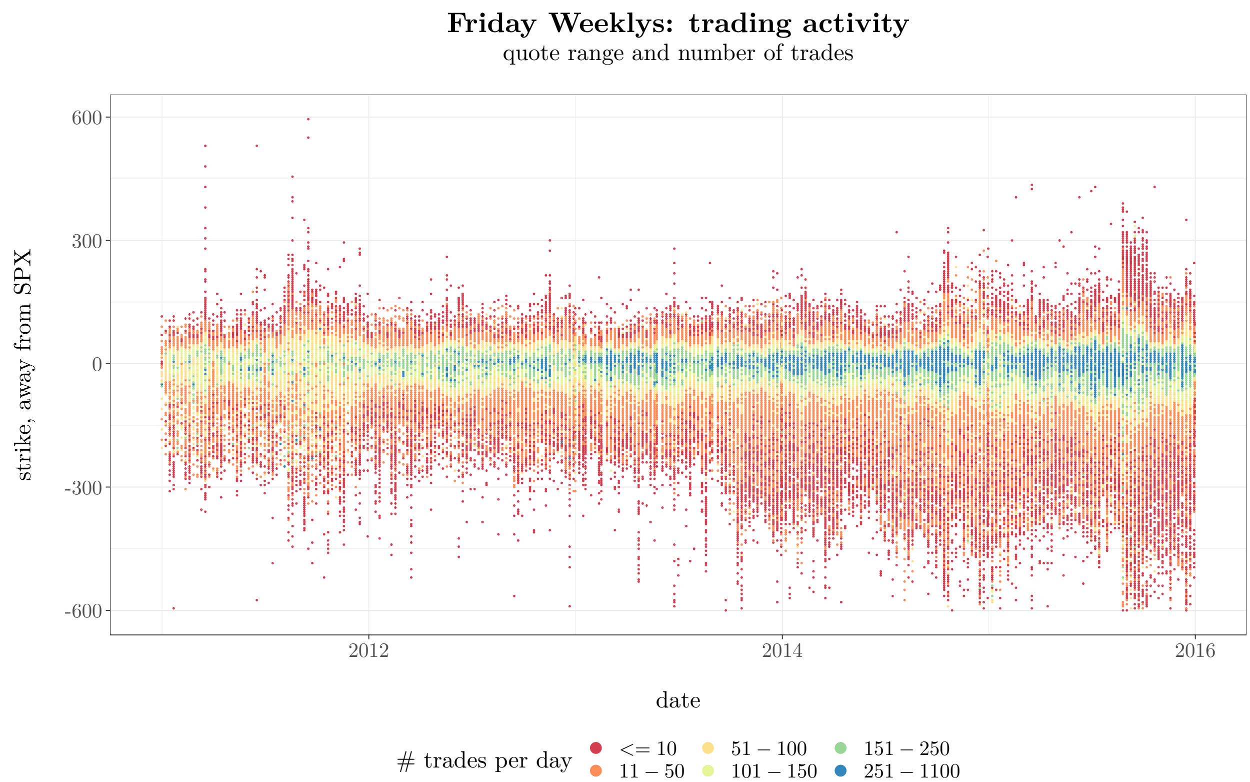

Figure 5: CBOE Friday Weeklys quoting and trading activity. Each dot represents whether the out of the money option (call for positive strike, put for negative strike) was quoted at least once in a given week. The colour of the dot represents how many times the option was traded per day, on average, in a given week.

bled, and the number of trades at each traded strike grew substantially, with many options traded more than 150 times per day (see Figure 5). The range of traded and quoted option strikes changes over time in response to market conditions, too. The traded options, however, are but a small subset of the quoted options. Moreover, this aggregate picture does not contain any information about the intraday distribution of the trades. In what follows, we provide an analysis of seasonal, trend and market-conditions factors impacting Friday Weeklys trading activity. We focus on the number of trades (trading intensity) rather that on trading volume as we interpret trades as a reality check on posted quotes, which we then use in our analysis to calculate trading profits. For this purpose, observing many small trades over a period of time is more informative than observing a single big trade.

The properties of option contracts change over time. We describe the contracts in terms of measures which take these changes into account: first, in subsequent analysis we control for option maturity; second, we use Black-Scholes deltas in order to standardize the options. The Black-Scholes delta approximates the sensitivity of an option's price to the changes in underlying asset, given market volatility and time to maturity. Intuitively, at two points in time in different market conditions, an investor will consider that two call options with deltas equal 0.2 are far more similar than two call options struck at 100$ above the underlying. Thus, for the purpose of estimating trend, seasonal and market condition factors, we calculate Black-Scholes deltas for all the options and divide calls and puts into deciles based on the absolute value of delta, $|\Delta_{BS}|$. The first decile of $|\Delta_{BS}|$ puts (calls) groups the deepest out of the money contracts, while the last decile – the at the money contracts. We group the option trades by Black-Scholes delta and by time of day, with hourly granularity, for a total of 186'615 type-delta-hour groups. In incomplete hours (8:30-9:00, 15:00-15:15) the number of trades is multiplied by 2 and 4, respectively, in order to give a picture of trading intensity. We then fit a generalized additive model to $\log_{10}(\text{\# trades})$. We estimate intraday seasonality with hourly fixed effects per option type (call/put), and other effects (weekly seasonality, time trend and sensitivities to $\log_{10}(\sigma_{BS})$) with smoothing splines. After data cleaning we are left with 4'628'043 observations.

With trading intensity corrections, this amounts to 31.5 transactions per hour of trading, per option type and delta decile, on average. Table X gives a more detailed breakdown of the results. Out of the money puts are traded more often than calls. In both calls and puts, there is little difference in trading activity across option deltas, with a 10% uptick for deep out of the money contracts. The seasonal and other smoothing terms are plotted in Figure 6. The terms should be interpreted as follows: when relative response to a given term is e.g. -0.1, the average number of trades in the given group of options changes by a factor of $10^{-0.1}$ from the baseline presented in Table X.

Average trading intensity in Friday Weeklies increases by a factor of 3.5 during the course our sample, with fastest growth in 2012 (top-left panel of Figure 6). The weekly seasonal patterns in trading intensity are similar for both call and put options (middle row of Figure 6), albeit with marked differences across option moneyness, as measured by $|\Delta_{BS}|$. There is a strong seasonal pattern for deep out of the money options, which are heavily traded (up to 70% more than the average number of trades) with 7 and 1 days to maturity. Options closer to the prevailing level of the S&P500 index see increases in trading activity of 20-25% in these periods. While weekly seasonal patterns are very similar for puts and calls, the intraday patterns are surprisingly disparate (top-right panel of Figure 6). The two lines should be interpreted as giving the trading intensity in a given option type, at a given time, relative to the trading intensity of call options between 8:00 and 8:30. Trading in calls is mostly concentrated in the morning hours, drops by up to 70% at around lunchtime, and then doubles (to 65% of the morning activity level) before the market closes. Puts are traded actively during the whole day, and the trading activity is increasing up until the closing hours.

Finally, in the bottom two rows of figure 6 we give the trading activity's response to changes in at the money Black Scholes implied volatility. For all options, there is more trading when volatility is higher, more so for deep out of the money options. In turbulent periods, deep OTM calls can be traded up to 3.8 times more frequently than in the calmest periods. For puts, the response of deep OTM option trading is somewhat more muted, mostly because put trading activity does not decrease as much as call trading activity in benign market periods. Response patterns for at the money and mid-out of the money options are similar across both groups.

Overall, we conclude that the trading activity in Friday Weeklys is sufficiently rich that the market makers' price quotes for actively traded options are reliably informative of prevailing market conditions.

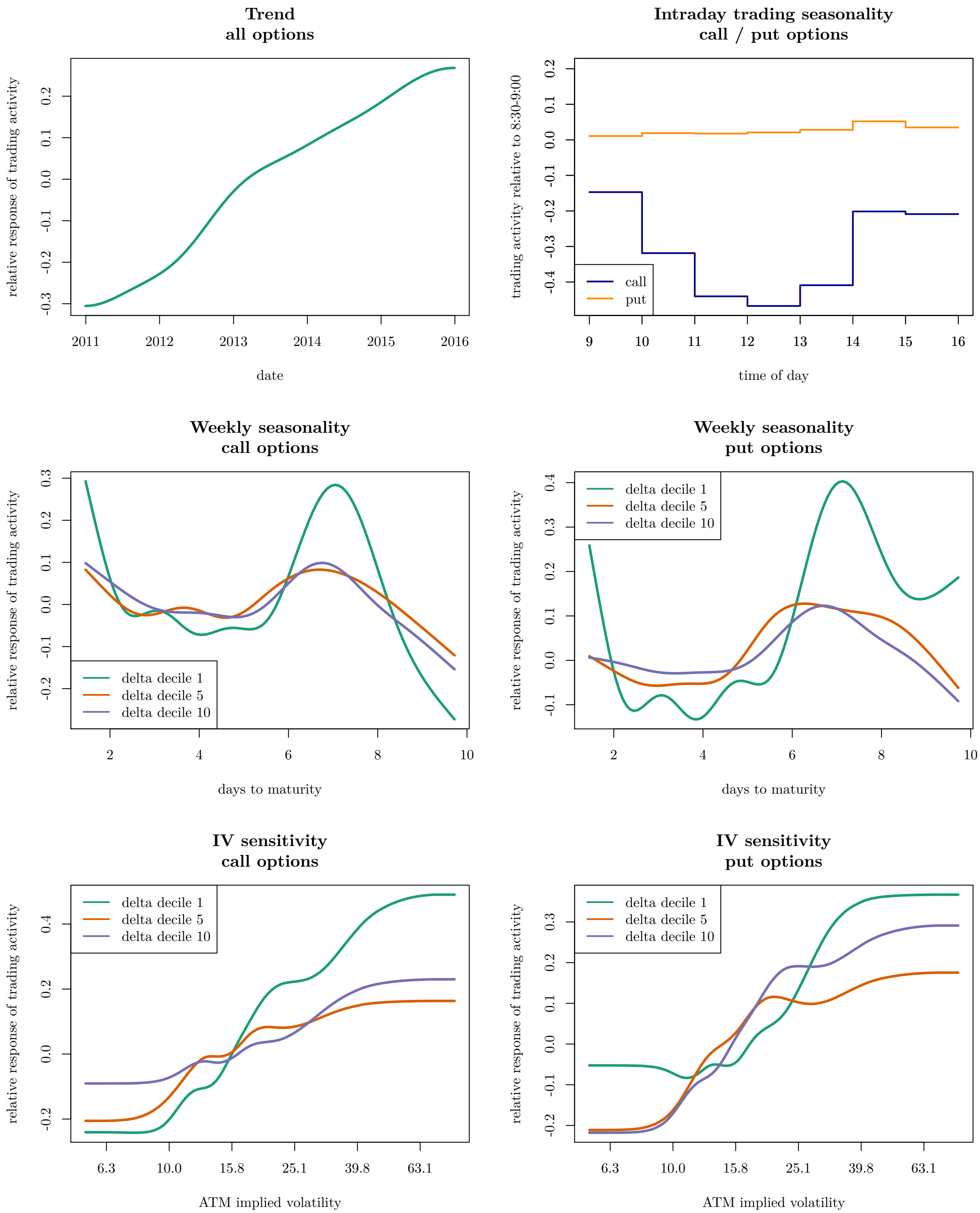

Figure 6: Seasonal and market condition factors in Friday Weeklys trading activity. Smooth terms / fixed effects from generalized additive model fitted to $\log_{10}$(# trades).

REFERENCES

Andersen, T., N. Fusari, and V. Todorov
2017. Short-Term Market Risks Implied by Weekly Options. *The Journal of Finance*, 72(3):1335–1386.

Andersen, T. G., N. Fusari, and V. Todorov
2015a. Parametric Inference and Dynamic State Recovery From Option Panels. *Econometrica*, 83(3):1081–1145.

Andersen, T. G., V. Todorov, and N. Fusari
2015b. The risk premia embedded in index options. *Journal of Financial Economics*, 117(3):558–584.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang
2006. The Cross-Section of Volatility and Expected Returns. *The Journal of Finance*, 61(1):259–299.

Babaoglu, K., P. Christoffersen, S. L. Heston, and K. Jacobs
2015. Option Valuation with Volatility Components, Fat Tails, and Non-Monotonic Pricing Kernels.

Backus, D., M. Chernov, and I. Martin
2011. Disasters Implied by Equity Index Options. *The Journal of Finance*, 66(6):1969–2012.

Bajgrowicz, P., O. Scaillet, and A. Treccani
2016. Jumps in High-Frequency Data: Spurious Detections, Dynamics, and News. *Management Science*, 62(8):2198–2217.

Barndorff-Nielsen, O. E. and N. Shephard
2006. Econometrics of Testing for Jumps in Financial Economics Using Bipower Variation. *Journal of Financial Econometrics*, 4:1–30.

Bates, D. S.
2000. Post-'87 Crash Fears in S&P500 Futures Options. *Journal of Econometrics*, 94(1-2):181–238.

Bates, D. S.
2006. Maximum Likelihood Estimation of Latent Affine Processes. *Review of Financial Studies*, 19 (3)(319):909–965.

Bollerslev, T., T. H. Law, and G. Tauchen
2008. Risk, jumps, and diversification. *Journal of Econometrics*.

Bollerslev, T., S. Z. Li, and V. Todorov
2016. Roughing up Beta: Continuous versus Discontinuous Betas and the Cross Section of Expected Stock Returns. *Journal of Financial Economics*, 120(3):464–490.

Bollerslev, T. and V. Todorov
2011. Tails, Fears, and Risk Premia. *The Journal of Finance*, 66(6):2165–2211.

Bondarenko, O.
2014. Variance Trading and Market Price of Variance Risk. *Journal of Econometrics*, 180(1):81–97.

Breeden, D. T. and R. H. Litzenberger
1978. Prices of State-contingent Claims Implicit in Option Prices. *The Journal of Bussiness*, 51(4):621–651.

Carr, P. and D. Madan
2001. Optimal Positioning in Derivative Securities. *Quantitative Finance*, 1(1):19–37.

Carr, P. and L. Wu
2009. Variance Risk Premiums. *Review of Financial Studies*, 22(3):1311–1341.

Christensen, K., R. C. Oomen, and M. Podolskij
2014. Fact or Friction: Jumps at Ultra High Frequency. *Journal of Financial Economics*, 114(3):576–599.

Christensen, K., R. C. A. Oomen, and R. Renò
2016. The Drift Burst Hypothesis.

Christoffersen, P., B. Feunou, and Y. Jeon
2015. Option valuation with observable volatility and jump dynamics. *Journal of Banking and Finance*.

Cleveland, W. S., E. Grosse, and W. M. Shyu
2017. Local Regression Models. In *Statistical Models in S*.

Constantinides, G. M., J. C. Jackwerth, and S. Perrakis
2009. Mispricing of S&P 500 index options. *Review of Financial Studies*.

Constantinides, G. M., J. C. Jackwerth, and A. Savov
2013. The Puzzle of Index Option Returns. *Review of Asset Pricing Studies*, 3(2):229–257.

Drechsler, I.
2013. Uncertainty, Time-Varying Fear, and Asset Prices. *The Journal of Finance*, 68(5):1843–1889.

Du, D.
2011. General equilibrium pricing of options with habit formation and event risks. *Journal of Financial Economics*, 99(2):400–426.

Duffie, D., J. Pan, and K. Singleton

2000. Transform Analysis and Asset Pricing for Affine Jump-Diffusions. *Econometrica*, 68(6):1343–1376.

Gabaix, X.
2012. Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance. *The Quarterly Journal of Economics*, 127(2):645–700.

He, Z., B. Kelly, and A. Manela
2017. Intermediary Asset Pricing: New Evidence from Many Asset Classes. *Journal of Financial Economics*, 126(1):1–35.

Hendershott, T., D. Livdan, and D. Rösch
2018. Asset Pricing: A Tale of Night and Day.

Huang, C. F.
1985a. Information structure and equilibrium asset prices. *Journal of Economic Theory*.

Huang, C. F.
1985b. Information structures and viable price systems. *Journal of Mathematical Economics*.

Huang, X.
2005. The Relative Contribution of Jumps to Total Price Variance. *Journal of Financial Econometrics*, 3(4):456–499.

Kozhan, R., A. Neuberger, and P. Schneider
2013. The Skew Risk Premium in the Equity Index Market. *Review of Financial Studies*, 26(9):2174–2203.

Lou, D., C. Polk, and S. Skouras
2018. A tug of war: overnight versus intraday expected returns. *Journal of Financial Economics*.

Mancini, C.
2001. Disentangling the jumps of the diffusion in a geometric jumping brownian motion. *Giornale dell'Istituto Italiano degli Attuari*, LXIV:pp. 19–47.

Merton, R. C.
1976. Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics*, 3(1-2):125–144.

Muravyev, D. and X. Ni
2016. Why Do Option Returns Change Sign from Day to Night? SSRN Scholarly Paper ID 2820264, Social Science Research Network, Rochester, NY.

Politis, D. N. and J. P. Romano
1994. The Stationary Bootstrap. *Journal of the American Statistical Association*, 89:1303–1313.

Press, S. J.
1967. A Compound Events Model for Security Prices. *The Journal of Business*, 40(3):317–335.

Schneider, P. and F. Trojani
2018. Divergence and the Price of Uncertainty. *Journal of Financial Econometrics*.

Wachter, J. A.
2011. Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility? *SSRN eLibrary*.

Table VII: Profits (without transasction costs) from trading realized variance and realized skewness with Friday Weeklys in 2011-2015. Multiple trading frequencies for daytime and weekly trading. Results are scaled by 10^4. All statistics and confidence intervals calculated with time-series bootstrap of Politis and Romano (1994). Calculations performed with package `boot` version 1.3.20 in R version 3.5.2 (2018-12-20).

	jump skew, $S_t^{(n)}$			static skew, $US_t^{(n)}$			variance, $D_t^{(n)}$		
trade freq	μ_b	$q_{\mu_b}^{0.05}$	$q_{\mu_b}^{0.95}$	μ_b	$q_{\mu_b}^{0.05}$	$q_{\mu_b}^{0.95}$	μ_b	$q_{\mu_b}^{0.05}$	$q_{\mu_b}^{0.95}$
Weekly trading									
5m	0.230	0.019	0.462	0.279	0.136	0.412	0.596	0.287	0.857
15m	0.202	0.003	0.427	0.289	0.115	0.423	0.606	0.277	0.878
30m	0.189	-0.026	0.425	0.304	0.118	0.451	0.631	0.244	0.950
1h	0.170	-0.062	0.445	0.305	0.104	0.447	0.593	0.255	0.958
2h	0.188	-0.053	0.464	0.320	0.052	0.469	0.588	0.253	0.931
1d	0.184	-0.008	0.421	—	—	—	0.618	0.273	0.886
Daytime trading									
5m	-0.010	-0.032	0.019	-0.020	-0.046	0.013	0.005	-0.031	0.050
15m	-0.013	-0.032	0.012	-0.018	-0.041	0.011	0.009	-0.027	0.054
30m	-0.013	-0.036	0.015	-0.015	-0.038	0.019	0.017	-0.019	0.051
1h	-0.010	-0.030	0.014	-0.011	-0.033	0.013	0.018	-0.026	0.053
2h	0.010	-0.014	0.027	0.010	-0.013	0.029	0.039	-0.004	0.074
1d	-0.019	-0.045	0.017	—	—	—	0.010	-0.034	0.049
Overnight trading									
–	0.089	0.034	0.130	—	—	—	0.109	0.041	0.176
Weekend trading									
–	0.204	0.064	0.333	—	—	—	0.424	0.227	0.597
Monthly trading									
5m	4.018	-0.580	7.938	5.426	1.409	8.274	5.622	3.142	7.628

Table VIII: **Spanning tests without transaction costs: overnight and weekend/holiday horizon**. For detailed legend see Table II in the main text. Models (1)-(6) are estimated with OLS. Model (7) is estimated with IV.

Overnight trading

	$D_t^{(n)} - \mathcal{D}_t^{(n)}$			$S_t^{(n)} - \mathcal{S}_t^{(n)}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
r_m	0.65***		0.29	0.48***		0.32	0.40***
	(0.22)		(0.27)	(0.13)		(0.24)	(0.10)
r_d		0.47***	0.30**		0.32***	0.14	
		(0.12)	(0.14)		(0.06)	(0.13)	
$D_t^{(n)} - \mathcal{D}_t^{(n)}$							0.12
							(0.12)
Constant	0.08*	0.07	0.07	0.07**	0.06**	0.06**	0.06*
	(0.04)	(0.04)	(0.04)	(0.03)	(0.02)	(0.03)	(0.03)
Adjusted R^2	0.15	0.17	0.18	0.25	0.23	0.27	0.44

Weekend / holiday trading

	$D_t^{(n)} - \mathcal{D}_t^{(n)}$			$S_t^{(n)} - \mathcal{S}_t^{(n)}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
r_m	0.81***		0.62**	0.71***		0.39***	0.71**
	(0.25)		(0.30)	(0.19)		(0.13)	(0.28)
r_d		0.50***	0.14		0.48***	0.25**	
		(0.18)	(0.28)		(0.12)	(0.11)	
$D_t^{(n)} - \mathcal{D}_t^{(n)}$							0.00
							(0.18)
Constant	0.47***	0.49***	0.48***	0.24***	0.26***	0.25***	0.24**
	(0.08)	(0.09)	(0.08)	(0.06)	(0.07)	(0.07)	(0.12)
Adjusted R^2	0.22	0.19	0.22	0.38	0.37	0.40	0.38

Notes: ***Significant at the 1 percent level.
**Significant at the 5 percent level.
*Significant at the 10 percent level.
r^k scaled by 10^{2k}, $D_t^{(n)} - \mathcal{D}_t^{(n)}$ scaled by 10^4, $S_t^{(n)} - \mathcal{S}_t^{(n)}$ scaled by 10^5.

Table IX: **Spanning tests without transaction costs: weekly horizon**. For detailed legend see Table II in the main text.

	$D_t^{(n)} - \mathcal{D}_t^{(n)}$			$US_t^{(n)} - \mathcal{US}_t^{(n)}$				$S_t^{(n)} - \mathcal{S}_t^{(n)}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
r_m	0.37**		0.40**	0.20***		0.23***	−0.04	0.26***		0.29***	0.03
	(0.15)		(0.16)	(0.07)		(0.07)	(0.06)	(0.06)		(0.08)	(0.07)
r_d		0.19**	−0.03		0.11**	−0.02			0.14***	−0.02	
		(0.10)	(0.07)		(0.04)	(0.03)			(0.05)	(0.05)	
$D_t^{(n)} - \mathcal{D}_t^{(n)}$							0.66***				0.65***
							(0.18)				(0.14)
Constant	0.55***	0.58***	0.54***	0.26***	0.27***	0.25***	−0.10	0.16	0.18	0.16	−0.19*
	(0.14)	(0.14)	(0.14)	(0.07)	(0.08)	(0.08)	(0.12)	(0.10)	(0.11)	(0.11)	(0.11)
Adjusted R^2	0.09	0.06	0.09	0.13	0.09	0.13	0.55	0.07	0.05	0.07	0.66

Notes: ***Significant at the 1 percent level.
**Significant at the 5 percent level.
*Significant at the 10 percent level.
r^k scaled by 10^{2k}, $D_t^{(n)} - \mathcal{D}_t^{(n)}$ scaled by 10^4, $S_t^{(n)} - \mathcal{S}_t^{(n)}$ scaled by 10^5.

Table X: Average number of trades per hour in Friday Weeklys by option type and delta (moneyness).

		Delta decile									
type	all	1	2	3	4	5	6	7	8	9	10
call	25.9	28.6	25.6	25.4	25.4	25.5	25.6	25.8	25.7	25.6	25.8
put	37.0	40.9	38.7	37.6	36.7	36.1	35.9	35.8	36.1	36.3	36.2