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Application of probabilistic tools to extend load test design of bridges prior to opening

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ABSTRACT

Load tests of bridges are widely performed in a large number of countries. Deterministic comparison of measurement results to the theoretical, FEM (finite element method)-based outcomes with possible further calibration is mostly applied. Sometimes, the data collected in the tests are also used to calibrate the reliability factors of bridge structures or their components. This work proposes to complement the stage of the load test design with the use of probabilistic tools. This approach is intended to provide a reliable and trustworthy limit range of measured values (e.g. displacements) instead of restrictive single values, streamlining the performance of *in-situ* tests. The proposed procedure is supported by an arch bridge example with the following uncertainty sources: random imperfections of the arch girder, random stiffness of the deck and random total weight of the applied load trucks. The presented calculations refer to global structural stiffness assessment. Both point estimate method (PEM) and response surface method (RSM) are applied here. It has been shown that the proposed procedure effectively supplements the deterministic approach, thus the suggested extension of application of probabilistic tools to bridge load test design is innovative and justified.

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Load tests; probabilistic methods; arch bridges; structural response; random variables; finite element method; uncertainty principles; point estimate method; response surface method

Introduction

Independently from recent rapid development of measurement techniques and design procedures, the main objective of bridge load tests still concerns collecting important information on the performance of the structure or its components (Lantsoght, van der Veen, de Boer, & Hordijk, 2017). These tests are performed in different stages of the life cycle of the structure:

- during the construction – to control the behaviour of certain components of the structure (e.g. load tests of foundation piles),
- directly prior to opening – within the scope of obligatory acceptance tests,
- during service life – to evaluate the load-bearing capacity of the object before a repair, when a capacity increase is planned, or when its technical state is investigated.

The presented domain may be divided into static and dynamic load tests. The first type covers the evaluation of structural stiffness (displacements and strains measurement), assessment of co-operation between individual elements of the object (e.g. lateral distribution), analysis of deformations and cracking of girders, settlement of supports, deformations of bearings, etc. (Filar, Kałuża, & Wazowski, 2017; Lantsoght, van der Veen, Hordijk, & de Boer, 2017; Łaziński, 2009; Łaziński & Salamak, 2015). Dynamic tests, in turn, are directed to observation of variations of accelerations, displacements, deformations, damping, vibration modes, etc.

The procedure of preparing and conducting load tests of bridges depends on the standards and specifications valid in a given country. Differences are related to the type and span length of the objects classified for testing, types of physical quantities measured during the test, aggravating load types and masses, etc. These are the topics of current technical and scientific discussions and analysis in the context of their importance and profitability (Hester, Brownjohn, Bocian, & Xu, 2017). In most cases, the deterministic criterion of structural response assessment is chosen in static tests (Lantsoght, van der Veen, de Boer, et al., 2017; Łaziński & Salamak, 2015; PN-S-10040, 1999). It translates into a direct comparison of the measurements results with the outcomes of FEM model, the latter do not directly take any uncertainties into account.

Uncertainties in bridge load tests and current ways of their consideration

Yet, the traditional design of bridge load tests often takes random uncertainties into account. Establishing a proper uncertainty sources selection with regard to a bridge type is the basic issue of load test design. Most often, the uncertainties associated with the occurrence of imperfections are

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indicated as having the crucial impact. These are the geometric imperfections (indicated, e.g. in inventory reports on geometry of individual precast elements prepared by steel structure manufacturers or in a geodetic survey), material imperfections (associated with, e.g. Young's modulus variation of concrete with regard to appropriate cement mix used and its age during tests) or load imperfections (suggested, e.g. in reports of total vehicle mass variation from numerous prior load tests, supplemented with the outline of their distribution and wheelprints). The data on uncertainties are often gathered by site visits and reports on the extent of the construction's completion or related to failures.

The uncertainties are also associated with the test itself, e.g. with the precision and resolution of the measuring devices (Owerko & Honkisz, 2017) or with the variation of the temperature of the structure during the test. Finally, the uncertainties are also related to the type of finite elements applied to represent the structure (Bień, Kużawa, & Kamiński, 2015). Apart from these crucial uncertainties, other potential variability sources are distinguished, yet they are detected to have a much lower impact on structural response. The major sources, which are considered important by the Authors, are collectively presented in (Table A1 in Appendix 1).

Frequently, statistical analysis is required, performed on a basis of a collective report database on load tests of bridges of a selected type. Not only concerning the structural responses but also the data on types and sources of uncertainties encountered and identified during the tests. A valuable supplement to the inquiry can be found in the regulations provided by the Joint Committee on Structural Safety (JCSS) (e.g. Diamantidis, 2001; 'Probabilistic Model Code – JCSS'; Vrouwenvelder, 1997). Especially in the case when specific, particular structural data are inaccessible or inexistent and a need exists to find a more general uncertainty data set with sufficient theoretical and experimental background.

Despite the complexity and the randomness of the uncertainties occurrence issue, in most cases, they are solely reflected by load and resistance factors (first-level random approach), mainly in scope of determining target proof loads. However, these factors are often calibrated using probabilistic (second-level) methods. For example, probabilistic calculations are the basis of factors used to find the target proof load in the AASHTO LRFR Manual for Bridge Evaluation MBE (AASHTO, 2016).

In some cases, bridge models are updated (deterministically too) upon data collected by technicians during the *in-situ* tests. It should, however, be pointed out that only some uncertainty types, due to their origin, can be accurately verified during these tests (e.g. the presence of asphalt pavement, edge beams, barrier walls that increase the stiffness of the bridge deck). Unfortunately, significant uncertainty sources remain unknown to some extent. Both approaches presented above tend towards the adjustment of a limit value only (using global factors), set for a chosen type of structural response. They do not lead to calculation of probabilistic response ranges, determined on the basis of a comprehensive analysis of multiple uncertainty sources.

In addition to the engineering practice and guidelines, there are many literature researches regarding consideration of uncertainties in bridge load tests. Most of them do not discard the randomness of uncertainties and they incorporate the probabilistic approach. The use of load test or monitoring data in reliability factor calibration for computational models of bridges is presented by Cho, Choi, and Sho (1998), Frangopol, Strauss, and Kim (2008) and Nowak and Tharmabala (1988). The paper (Wiśniewski, Casas, & Ghosn, 2009) highlights the need to develop simple engineering probabilistic computational methods to define further use conditions of existing railway bridges. The response surface method (RSM) and the Latin hypercube sampling (LHS) technique were proposed to assess structural reliability. Additionally, some advanced probabilistic models of real-life loads of bridge structures are currently developed (Yan, Luo, Yuan, & Lu, 2017). The distribution data of structural and load parameters for bridges are also processed (Estes & Frangopol, 2005; Faber, Val, & Stewart, 2000). The probabilistic analysis is also introduced to assessment phase of load-bearing capacity and reliability of existing structures (Lantsoght, van der Veen, de Boer, et al., 2017) or to the representation of real-life traffic loads in dynamic analysis (Chang, 2014).

However, neither the works above nor many others, even if their investigations apply more complex probabilistic techniques than those used in this paper, direct the presented techniques to design stage of load tests prior to opening. This fact has become the motivation for the creation of the proposed extended probability-based load test design procedure.

Aim of the study

This work proposes an extended use of selected, widely used probabilistic tools already at the earliest stages of bridge engineering process, namely in the load test design of newly executed structures, prior to their opening. Exhaustive literature overview and patent base study performed by the Authors indicates that nearly all investigations focusing on random approach to *in-situ* uncertainties conducted up to date refer to the post-test design stages only. Thus, this work, directed to such an early stage is one of the very few which deal with the problem. Moreover, it uniquely approaches the entire scope of the problem.

In the paper, a firm suggestion is made to determine expected probable ranges of measured quantities (e.g. displacements) regarding important uncertainty sources, e.g. random geometrical imperfections, random material parameters, etc. Appropriate ranges of measured quantities provide a reliable source of structural response evaluation during the load test, in contrary to their single, deterministic values set by the appropriate standards, even precisely computed. This can, in turn, facilitate the work of technicians and experts performing the field tests and improve the bridge assessment procedure.

A supporting example is provided, covering the probabilistic calculations of vertical displacements of the model of a real arch bridge built recently in southern Poland. The calculations are based on global structural stiffness analysis. Analytical estimation was provided here along with two second-level probabilistic methods: point estimate method (PEM) and RSM supplemented by Monte Carlo (MC) simulation.

The paper is intended to prove that the proposed procedure can substantially reduce the effort of the entire relevant load test design preparation course, making it easy to analyse and interpret. Furthermore, its simple and straightforward form can be successfully applied on an engineering level by load test designers and the technical staff performing measurements, even if they are unfamiliar with probabilistic methodology.

Alternative proposal of the preparation procedure of bridge load tests

Short description of the existing deterministic procedure – an example of Polish standard

Acceptance load tests, necessary to allow the object for service are attributed past or present obligatory status, e.g. in Italy, Spain, Slovenia, Switzerland (until 1999) and Poland (Łagoda, 2013). According to Polish Standard (PN-S-10040, 1999) acceptance load tests of bridges have to be performed on objects with a theoretical span length exceeding 20 m.

Structural stiffness determination is an important aspect of the test. This is conducted by measuring vertical displacements in chosen points on the spans as well as, e.g. support settlements and bearing deformations. This allows calculating theoretical deflections of the object. According to requirements and testing standard (PN-S-10040, 1999) – masses, quantities, types and layout schemes of loading vehicles are selected accordingly to generate a 75–100% level of internal forces (or stresses) in selected cross-sections of key structural members, in relation to the same effects calculated on the basis of a characteristic combination (partial safety factors equal to 1.0) of live loads supported by an appropriate bridge design standard. In case of standard ('PN-85-S-10030', 1985), a single 800 kN vehicle load and 4 kN/m^2 of uniformly distributed load are used for the highest road class. The dead load effects are omitted in these calculations as they are already present in structure during the *in-situ* tests.

Next, deterministically, theoretical deflections are computed under the set of loading vehicles in their predefined layout. The stiffness criterion is verified by direct comparison of the measured and theoretical deflections (accounting for long-term displacements). If the first ones are smaller, there is no reason to reject the object from service due to stiffness criterion. In practice, this sometimes leads to the acceptance of uncalibrated, deterministic FEM (finite element method) models with stiffness different from the real one by up to 50% (Łaziński, 2009).

Usually only while the theoretical deflections are exceeded, remedial actions are undertaken, e.g. preparing an additional FEM model of a different class or assuming different model parameters. In such a case, the Polish Standard does not define the action course precisely. Instead, the standard only advises that if a bridge exhibits such anomalies, it can be conditionally approved, but appropriate long-term observations and monitoring must be ensured. Furthermore, according to cited standard, in case of response exceedance, the detailed procedure has to be individually determined by a designated research unit. This clearly shows the need to unify the acceptance criterion by building a strict procedure, as performed in this paper.

New proposal for modifying the acceptance criteria definition

The work proposes not to base the acceptance criteria on specific deterministic values (e.g. deflections, strains, etc.) but on adequately selected ranges of these values defined upon probabilistic analysis. This analysis should take into account the sensitivity of a structural model to the variation of particular parameters regarding given uncertainties. The load test designer should select these parameters adequately and assume their relevant random models. As it was previously mentioned, specific items may be helpful in a well-motivated choice and description of random parameter variation, such as imperfection reports, concrete laboratory results, data from previous load tests, etc. The acceptable randomness of the parameters may also be specified directly in a form of contract demands imposed on the bridge contractor.

The most important step is to ensure a correct identification of these key uncertainty sources action on the structural response and their rational employment in the acceptance criteria using selected probabilistic tools. On this basis a unique, probabilistic load test acceptance criterion can be given in a form of a range of expected values, taking the random nature of the response into full account. Owing to the proposed approach, not only the need to calibrate the model would be unjustified if the theoretical values are marginally exceeded, but also every object more rigid than its FEM model would not be unconditionally accepted.

Application example of the proposed approach

Introduction to the considered arch bridge

A steel–concrete arch bridge in southern Poland, spanning over 109 m, was the inspiration for the performed analysis. The bridge is composed of two steel arch girders (each one consisting of three pipes arranged in a quasi-triangular shape), connected with post-tensioned girders (arch tie-beams) to the concrete deck with a set of steel hangers in a mesh system (see Figures 1 and 2). Moreover, Figure 2 presents key sections and dimensions of the structure (additional details are presented in Appendix 2). As a result of passing the load tests, in which no structural response irregularities were observed, the bridge was approved for use under real-life traffic load of 'A' class (the highest one), according to the Polish Standard ('PN-85-S-10030', 1985).

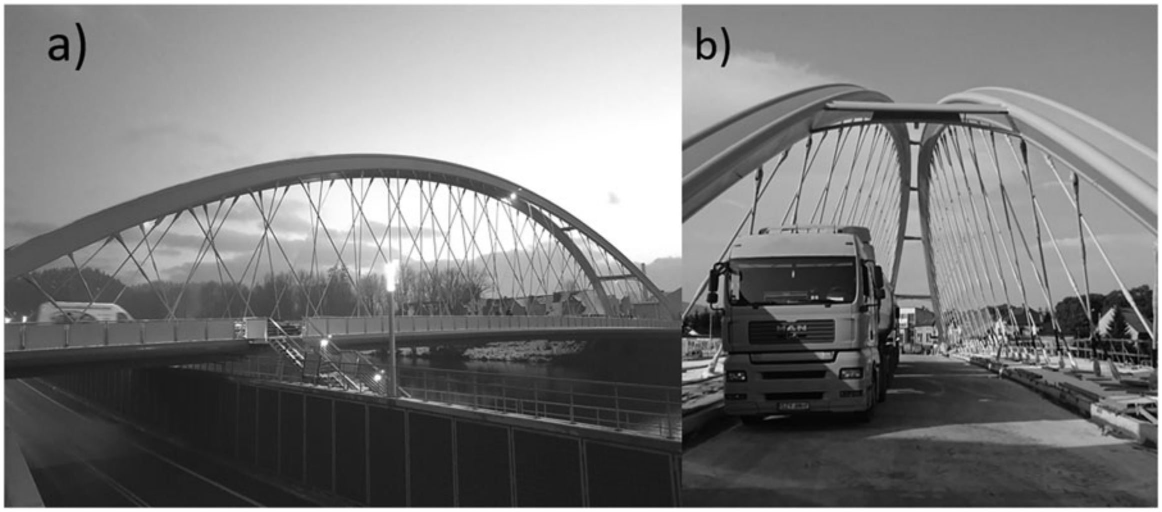

Figure 1. The considered bridge. (a) Side view, (b) front view during load tests (photos taken by the first Author of this paper while setting the vehicles).

This allows vehicles with a total mass of up to 50 tons per a single vehicle to use the bridge.

FEM model of the bridge

The exemplary study was restricted to global structural stiffness, so a typical linear-elastic 3D model of the bridge was adopted. The elements are further distinguished on the basis of their FEM model physical geometry, and not upon the number of nodal degrees of freedom (DOFs), to remain independent of the elements' parameterisation. The bridge deck elements, the post-tensioned beams (with appropriate parts of the cooperating deck slab) and cross-beams, are modelled using two-noded 1D frame elements (with 6 DOFs per node), embedded in a 3D space. This same element type is also used to represent arch girders and hangers.

The connection between the arch and the bridge deck (the skewback) has been modelled in a standard manner using rigid 2D shell elements that occupy the area corresponding to the size of the skewback, between the arc girder and the deck girder and have the appropriate thickness parameter. Mostly quadrinodal elements with 5 DOFs per node (no warping of the element is possible) embedded in a 3D space are used. Thus, according to classification proposed by Bień (2011) and Bień et al. (2015), it is a e1s3 + e2s3 class model, where 'e' denotes the element type and 's' denotes the space type (Figure 3). Detailed information on key cross-sectional parameters used in model are given in Table 1, with additional information on transverse members given in Appendix 2. The total number of 1054 finite elements is used, spanned on 873 nodes.

Additionally, 2D horizontal faces were used to ensure a proper transformation of the aggravating vehicle loads (represented as point forces) to an appropriate set of distributed loads acting directly on the deck grating (Figure 3(a)), in accordance with the envelope layout. This transition is performed automatically by the used FEA software. The described model derives directly from the original, deterministic design of the load test made by the first Author of this paper. Its relatively simple form is aimed at matching the complexity level of the models actually considered by design offices and engineers carrying out load tests of bridges prior to opening. Sufficient compliance was proven between the model response and the *in-situ* tests result in each loading layout (scheme) and in each measurement point.

Analogous simple models have been successfully applied in complex reliability studies of arch bridges, this fact is confirmed for instance by Cheng and Li (2009). In the present study, on the basis of this initial model, several similar models were further created for the purpose of representing appropriate random variables (e.g. geometrical imperfections) in probabilistic analysis. Independently, a similar model was developed by the designer of the presented bridge in the bridge design phase. Certainly, general design analysis of the structure, compared to this regarding the load tests design, was more complex, in order to check all the limit states. Furthermore, a complex spatial buckling analysis concerning the geometrical imperfections was carried out to ensure that they do not threaten the structural safety.

Original *in-situ* load tests – concise description and key results

In the course of original, deterministic load test design, the first Author of this paper concluded that the required internal forces and stress levels would be induced by a properly arranged set of tractors with three-axis trailers, assuming a 40 t mass of each single vehicle (Figure 4). This first load scheme 'S I' is represented by FEM model illustratively shown in Figure 3(b). Figure 5 presents all three load schemes under investigation (both lanes of the bridge were loaded at the same time). 'S I' and 'S III' schemes were designed to induce a proper level of internal forces in the cross-sections of main structural members located in ¼ and ¾ of bridge span, respectively, while the 'S II' scheme was intended in regard to mid-span cross-sections.

Table 2 presents the *in-situ* deflection results due to static tests in 'S I', 'S II' and 'S III' load schemes respectively. The

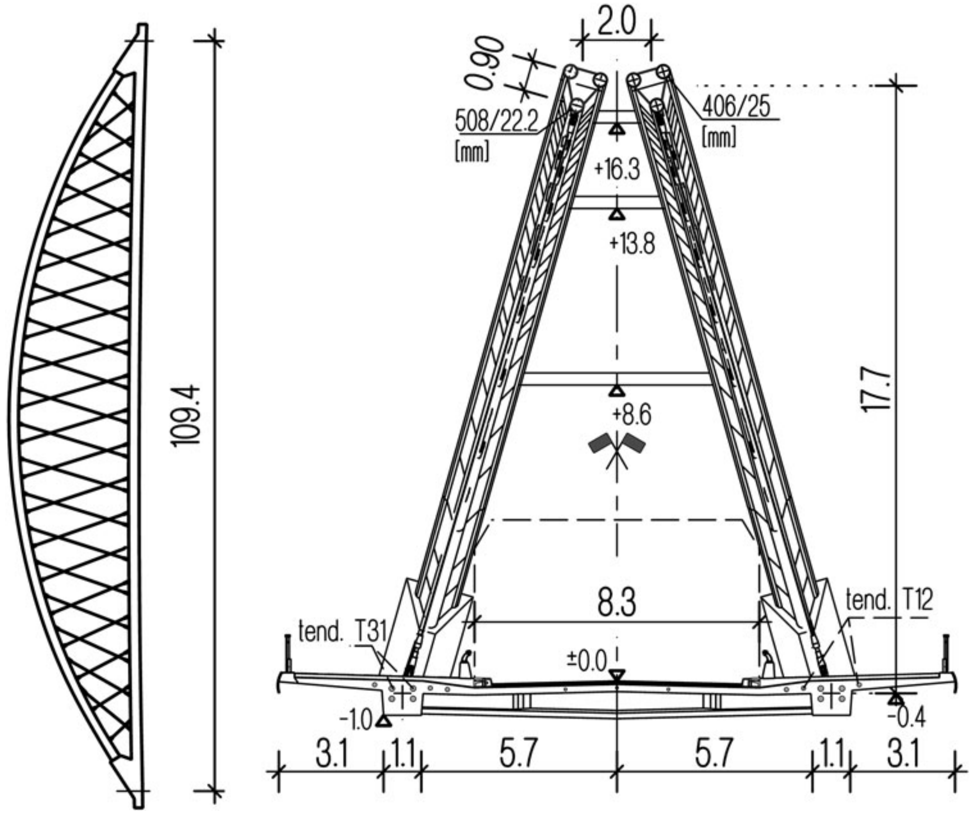

Figure 2. Geometry overview and the cross-sectional details of the bridge under consideration.

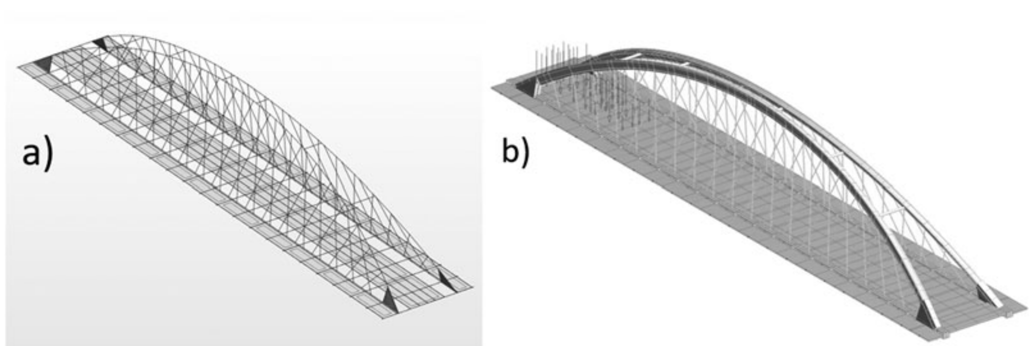

Figure 3. The FEM model of the analysed bridge. (a) Mathematical model, (b) visualisation.

mechanical response of the bridge in all three cases was almost symmetrical (cross-section-wise). For example, vertical deflections of both deck girders under the real-life load in first load scheme were both ca. 10.90 mm. The residual to the total deflection ratio was relatively low (<10% in all schemes), this value meets the national requirements (PN-S-10040, 1999). During the load tests, neither mechanical response anomalies nor structural failures were noted. Due to very safe design assumptions regarding the foundations (e.g. the use of large diameter piles), the maximum measured settlements during the tests were equal to 0.10 mm (they were observed for the 'S III' scheme, on the abutment closest to the area of loading). These settlements have been included in the deflections results presented in Table 2.

However, the theoretical deflection of 10.75 mm calculated in the initial, deterministic FEM model was marginally lower than the measurement results obtained in the 'S I' scheme (10.90 mm), whereas the total vehicle mass complied with theoretical assumptions. For this reason, a new calibrated FEM model was created by modifying the Young's modulus of finite elements representing the post-tensioned deck components. This is the reason why an alternative approach is proposed in the following paragraphs to properly estimate the probable ranges of bridge response concerning its random nature, to contrast the single deterministic value approach that urged the calibration.

Considered uncertainty types

Following the remarks outlined in the chapter concerning the uncertainties in bridge load tests, the identification of crucial types of important uncertainties and their influence on the structural response was conducted. The final choice

Table 1. Cross-sectional properties of key structural elements of the FEM model of the bridge under consideration.

Member[a]	Number of FE	Cross-sectional area (m^2)	Torsional moment of inertia (m^4)	Moment of inertia vs. local horizontal axis (m^4)	Moment of inertia vs. local vertical axis (m^4)	Material
Deck post-tensioned beams (with appropriate deck slab)	$2 \times 31 = 62$	3.11	0.175	0.1968	17.18	C 35/45
External arch bracing	4	0.0166	0.0001	0.0001	0.0013	S355 steel
Arch bracing	4	0.0223	0.0001	0.0005	0.0013	S355 steel
Hangers with D = 80 mm	$2 \times 28 = 56$	0.005	0	0	0	S355 steel
Hangers with D = 90 mm	$2 \times 10 = 20$	0.0064	0	0	0	S355 steel
Arch girder	$2 \times 39 = 78$	0.1573	0.015	0.0293	0.0213	S355 steel
Arch skewbacks	$4 \times 150 = 600$		Shells with associated thickness of 1.25 m			C 35/45

[a]Deck transverse elements (230 elements) not showcased (see Appendix 2).

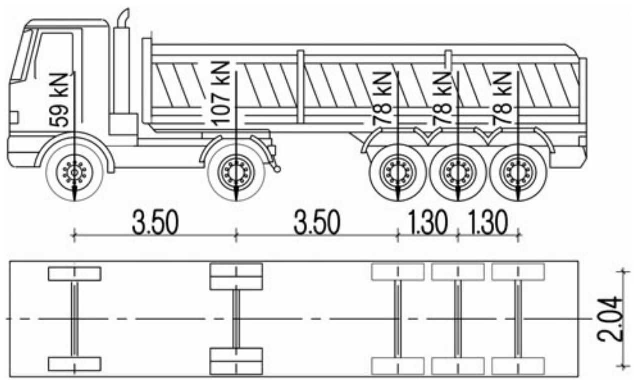

Figure 4. A single vehicle used in the load test, with details on its wheelprints, dimensions and axle loads given.

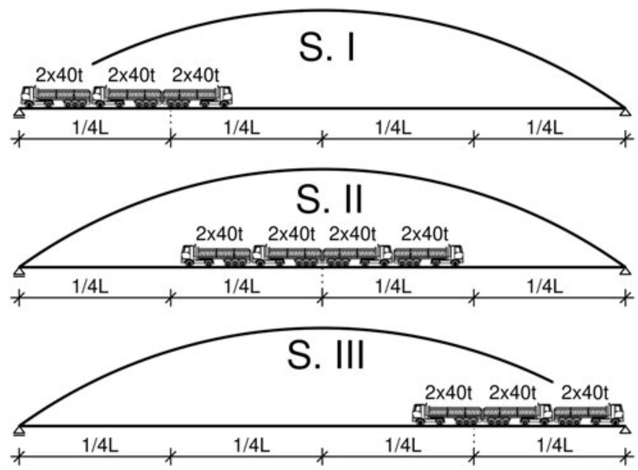

Figure 5. Load schemes 'S I', 'S II' and 'S III' carried out during the *in-situ* tests.

was made upon thorough investigation of bridge design criteria, executive documentation and analysis of previous load tests of similar structures. Of all possible uncertainty sources, three main types were chosen and represented by four random variables. The full motivation of acceptance/rejection of key sources is given, alongside other data, in (Table A1 in Appendix 1).

The following uncertainties were taken into account in the analysed example:

- random total weight of the applied aggravating load – variable $X1$,
- random deck stiffness (secant modulus of elasticity of concrete) – variable $X2$,
- random geometric imperfections of the arch girder – variables $X3$ and $X4$.

As the selection of uncertainty types to be considered is arbitrary and specific for the analysed bridge, it should be emphasised that the example presented in the paper is solely to be viewed as a practical illustration of the proposed approach. Other bridges and response types may imply a different choice; however, the outline of the proposed approach remains the same. Hence, the presented analysis may be considered a template to prepare the load test design of a tested bridge.

The description of the adopted random variables

Due to the fact that the presented example refers to the stage of load test design preparation, total weight of the applied load was assumed an important uncertainty source, and thus adopted as the first random variable of the task, $X1$. Its real value may not be equal to the assumed one due to a limited accuracy of the truck loading process, such as the presence of precipitation (affecting the aggregate mass even of non-cohesive materials) or even the number and mass of drivers (it is not always possible for the driver to leave the vehicle during the test).

These uncertainties may be verified on the day of the *in-situ* test and on the spot, for example by performing appropriate weight in motion measurements (COST323, 2002; 'OIML R 134', 2006). However, they should remain probabilistic in the load test design, so no need of the model re-calculation in *in-situ* conditions is imposed. Hence, following this experience, the standard deviation (SD) of the random variable $X1$ was assumed in the described example as 1.5% of the total vehicle mass, in accordance with common engineering practice and professional work experience of the first Author of the paper. Gaussian-type variable was assumed with its expected value equal to $\mu_{X1,I} = 240$t (6×40 t in 'S I' and 'S III') or $\mu_{X1,II} = 320$t (8×40 t in 'S II').

The second random variable $X2$ representing the secant modulus of elasticity of the deck concrete is adopted in compliance with JCSS guidelines ('Probabilistic Model Code – JCSS'), EC2 regulations ('EN-1992-1-1', 2004) and the design documentation. In accordance with these instructions, a log-normal distribution was used to describe its variability, with an arithmetic mean value of $\mu_{X2} = 34$GPa and an arithmetic SD of $\sigma_{X2} = 5.2$GPa. If the load test

Table 2. Deflection results obtained in S I, S II and S III load test schemes.

Longitudinal location	¼ L		½ L		¾ L	
	Girder 1	Girder 2	Girder 1	Girder 2	Girder 1	Girder 2
Vertical deflections (mm) – Scheme S I, support settlements taken into account						
Total defl.	11.35	11.55	1.75	2.05	−2.40	−2.45
Residual defl.	0.50	0.65	0.25	0.35	−0.40	−0.35
Elastic defl.	10.85	10.90	1.50	1.70	−2.00	−2.10
Vertical deflections (mm) – Scheme S II, support settlements taken into account						
Total defl.	5.30	4.95	15.00	14.70	5.80	6.20
Residual defl.	0.70	0.50	1.30	0.95	0.50	0.65
Elastic defl.	4.60	4.45	13.70	13.75	5.30	5.55
Vertical deflections (mm) – Scheme S III, support settlements taken into account						
Total defl.	−2.30	−2.55	1.70	1.45	11.40	11.50
Residual defl.	−0.35	−0.40	0.20	0.10	0.90	0.80
Elastic defl.	−1.95	−2.15	1.50	1.35	10.50	10.70

design is performed after the concrete has been placed and consolidated, it is recommended to modify these parameters based on the laboratory data and the remarks on the time variability of its elastic modulus given in (Łaziński, 2009) and (Knoppik-Wróbel & Klemczak, 2015). It is worth adding that because the model reflects standard engineering practice, it does not exhibit a complex character, and the precise determination of the contact between the beams and the slab of the deck is neglected in this analysis. Moreover, the girders and the deck were originally casted at the same time in one long uniform process. This fact, and the general intention to present a random-based approach applicable at a daily basis, prompted the adoption of one variable for the whole concrete structure.

Finally, two variables associated with imperfections of arch girders were introduced, designated as $X3$ and $X4$, according to Figure 6. The sum or difference of these random variables results in diverse shapes of arch axis deformation. In the described example, it was assumed that imperfections occur in the arch plane only, showing symmetric distribution with respect to the longitudinal bridge axis. The same imperfection value is implemented in both arches at once. Although such a simplification is strong, it is popularly adjusted to arch imperfections of bridges in standard engineering practice. It was assumed that the expected values of random imperfections $X3$ and $X4$ would be $\mu_{X3} = \mu_{X4} = 0\text{mm}$ (perfect state of girders), their SD equal to $\sigma_{X2} = \sigma_{X2} = 30\text{mm}$.

The values and modes of possible imperfections of arch girders were in this case based on inventory geometry reports of individual precast elements prepared by steel structure manufacturers. The imperfections are surveyed during the production of individual components of the arch, prior to their assembly at the construction site. The surveys concern the mapping inaccuracy of the theoretical longitudinal axis of individual components. Next, specified values of these imperfections were submitted to the Shapiro–Wilk statistical test (Shapiro & Wilk, 1965) of a 95% statistical significance. Based on its results, one may not discard the thesis that these imperfections come from a population of normal distribution.

To sum up, it should be emphasised that in the analysed example, the Authors' assumptions upon the basic variables and their corresponding parameters are all based on actual real-life data concerning the specific considered bridge and its type. The documentation on related, similar structures was available too, further supporting the choice of adopted probabilistic parameters. Introduction of random model with given parameters makes it possible to sufficiently and accurately reflect the random nature of mechanical response of the considered arch bridge.

Sensitivity analysis

Sensitivity analysis addresses the relationship between the vertical displacements of the deck and the change of the assumed uncertainty sources Xi. The requested vertical displacements were computed in appropriate new FEM models incorporating the subsequently chosen random variables. All these models were based on the initial one (Figure 3), applied in deterministic procedures, yet properly adjusted in its pre-processor in order to allow a quick modification of successive, discrete values of random variates.

The diagrams in Figure 7 present the sensitivity analysis results. The relative changes of the considered displacements in ¼ and in ¾ of the structure span are given in Figure 7(a) (regarding 'S I' and 'S III'), whereas Figure 7(b) (regarding 'S II') give the results in ½ of the span. Full insight into the abovementioned displacement results is given in Table 3. Regarding the random response approximation, linear relationship pattern appears between the changes in these relative displacements and the changes of the values of random variables $X1$, $X3$ and $X4$ (analogous, linear relationships were also obtained in other load schemes). The $X2$ variable is the only one to exhibit a slightly probabilistic non-linear behaviour. Although it may be approximated with a linear function without a significant error, non-linear approximations were also taken into consideration later on, for comparative purposes.

Approximation of random structural response with appropriate functions

The first, most accessible approach is based on the hypothesis that the random response variable U describing vertical displacement of the deck in the analysed point and under a given load scheme is a linear function of the input random variables, given as:

$$U = a_0 + \sum_{i=1}^{4} [a_i \cdot (X_i - \mu_{Xi})] \tag{1}$$

where $a_0 = U(\mu_{Xi})$ is the mean value of the response variable U, computed assuming all input variables X_i are set in their respective mean values.

The probabilistic linear combination coefficients a_i were determined directly by means of sensitivity analysis assuming single sources of uncertainty (Figure 7, Table 3). For example, the coefficient of displacement variability with respect to deviation of vehicle mass, with the mean of 240 t, can be calculated as:

$$a_1 = \frac{-11.19 + 10.53}{4 \cdot 1.5\% \cdot 240} = -0.0458 \quad (2)$$

Consequently, the response function U given by Equation (1) reads:

$$\begin{aligned} U = &- 10.86 - 0.0458 \cdot (X1 - \mu_{X1}) + 0.0358 \cdot (X2 - \mu_{X2}) \\ &+ 0.0010 \cdot X3 - 0.0007 \cdot X4 \end{aligned} \quad (3)$$

In the case of uncorrelated component random variables, SD of the random response variable U can be calculated directly as:

$$\sigma_U = \sqrt{\sum_{i=1}^{4} (a_i^2 \cdot \sigma_{X_i}^2)} \quad (4)$$

Hence, the calculated SD of random response variable U was equal to $\sigma_U = 0.253$ mm. Assuming the proposed hypothesis is proved, a stiffness-based acceptance criterion can be formulated. The Authors propose to assume a range covering 95% of possible result population, which is in a normal variable-only case approximately equal to a quadrupled SD. In the presented case the U range cannot be determined by this approximation, as the $X2$ variable is log-normal. In turn, the range was computed numerically on the basis of the MC simulation technique using NumPy© (Jones, Oliphant, & Peterson, 2001), a scientific package for Python 3©. This led to definition of the following acceptable range of the *in-situ* test results (neglecting the measurement uncertainties) as:

$$10.39 \leq U \leq 11.38 \; [mm] \quad (5)$$

However, it is worth noting that the range computation using a quadrupled SD would return a very similar result: 10.40–11.41 mm. This range, derived for first load scheme 'S I' is also valid for the third scheme 'S III', as the latter is symmetrical to 'S I'. Analogous calculations were provided for the second load scheme 'S II'. For this case, the random deck displacements in ½ of the structure's span ranged between 13.51 mm and 15.07 mm.

As can be seen in Figure 7, the FEM model response to the change in the elasticity modulus can be described more accurately. If a second-order polynomial approximation is used, the correlation coefficient R increases visibly. In 'S I' and 'S III' cases the R^2 value increases from 0.981 to 0.999, while in 'S II', it increases from 0.974 to 0.999. However, it should be noted that higher-order approximations trigger more complex forms of U functions, which are very often difficult to process. The interval corresponding to 95% of the results population can be determined using Taylor series expansions or by MC simulations. However, the formula for SD (4) can no longer be used. With this in mind, alternative probabilistic non-linear approximation ranges were obtained – 10.42–11.38 mm for 'S I' and 'S III' and 13.59–15.09 mm in the case of 'S II'. As it may be observed in Table 4, these values do not strongly vary from the previous results.

Analysis with point estimate method

These ranges can be also computed by means of PEM, described in the '$2K+1$' version by Rosenblueth (1975). It is relatively easy to implement in engineering calculations even if not all input variables are Gaussian (Nowak & Collins, 2000) and in many cases, it can be as effective as other versions of PEM (Che-Hao, Yeou-Koung, & Jinn-Chuang, 1995). Due to this approach, the values of random response variable are calculated for $N = 2K + 1$ samples, where K is the number of input random variables. In the discussed example these values were: $N = 9$ and $K = 4$.

Following that assumption, discrete values of the vertical displacement response function were investigated, based on the FEM calculations. The values of input parameters are chosen as follows:

$$U'_0 = f(\mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_i}, \ldots, \mu_{X_K}) \quad (6)$$

$$U'^{+}_i = f(\mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_i} + \sigma_{X_i}, \ldots, \mu_{X_K}) \quad (7)$$

$$U'^{-}_i = f(\mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_i} - \sigma_{X_i}, \ldots, \mu_{X_K}) \quad (8)$$

Subsequently, the parameters of mean value and coefficient of variation of the requested displacement function were determined due to each input random variable Xi with the following equations (Nowak & Collins, 2000):

$$\overline{U} = U'_0 \prod_{i=1}^{K} \left(\frac{\overline{U}'_i}{U'_0} \right) \quad (9)$$

$$V_U = \sqrt[2]{\left\{ \prod_{i=1}^{K} (1 + V_{U'_i}{}^2) \right\} - 1} \quad (10)$$

where:

$$\overline{U}'_i = \frac{U'^{+}_i + U'^{-}_i}{2} \quad (11)$$

$$V_{U'_i} = \frac{U'^{+}_i - U'^{-}_i}{U'^{+}_i + U'^{-}_i} \quad (12)$$

The values in Equations (6)–(12) regarding load schemes 'S I' and 'S III' are collected in Table 5. Finally, following the previous subsection, SD of the response function U was computed and the required function range was defined and related to the *in-situ* loading test in schemes 'S I' and 'S III' (see also Table 4):

$$10.37 \leq U \leq 11.35 \; [mm] \quad (13)$$

It can be observed that this range is very similar to the range determined in the previous section, expressed by interval (5), showing the change of the expected value of response variable U equal to approx. 9%. Same conclusions were obtained in the analysis of load scheme 'S II'. The appropriate range was 13.48–15.02 mm. It is worth emphasising that the actual bridge displacement results from the original tests are in the ranges calculated utilising the probabilistic approach proposed in the paper for all three load schemes (Tables 2 and 4).

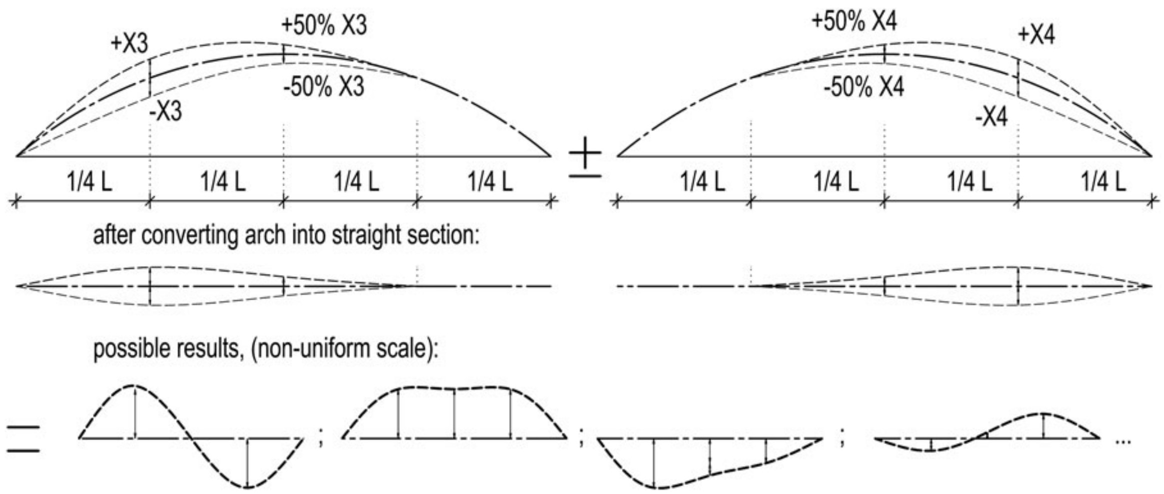

Figure 6. Depiction of executive geometric imperfections of arch girders – variables $X3$ and $X4$.

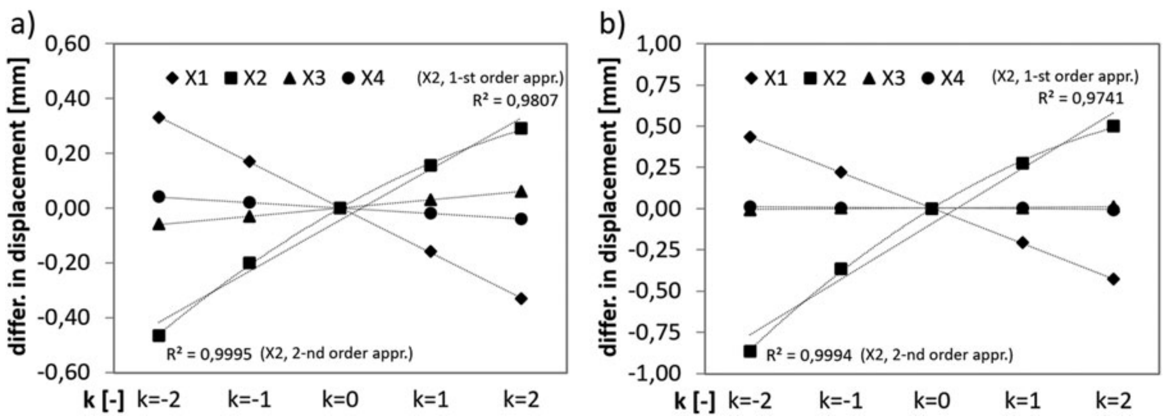

Figure 7. Results of the sensitivity analysis: (a) regarding 'S I' and 'S III' and (b) regarding 'S II'.

Analysis with response surface method

The most advanced third approach was based on the RSM incorporating polynomial approximation. The method was introduced in its original form in the 1950s (Box & Wilson, 1951), but it is still applied in many engineering domains. Its use is documented in structural reliability studies (Winkelmann & Górski, 2014) and in FEM-based bridge modelling (Nowak & Cho, 2007; Zong, Lin, & Niu, 2015).

The approximated structural response takes a general form of a measurement function of the response dependence on the basic random variables assumed in the task:

$$\overline{U} \equiv \overline{U}(\boldsymbol{x}) = f(x_1;\ x_2; x_3; \ldots; x_n) + \varepsilon \tag{14}$$

where $x_1,\ x_2, x_3, \ldots, x_n$ are the realisations of basic random variables, $\overline{U}$ is the approximate structural response to given variables and ε is the zero-mean experimental estimation error of real-life structural response.

If the structural response approximation is regarded only in a subspace of the global sample space or the skewness of the response surface is low, it can be approximated with the first-order (main effects) model, i.e. linear function of independent random variables (first-order polynomial), given by the equation:

$$\overline{U}(\boldsymbol{x}) = \ b_0 + \sum_{i=1}^{n} b_i x_i + \varepsilon \tag{15}$$

However, if a significant curvature of the actual response surface proves the first-order model inaccurate, higher-order models are used. The second-order model is the next, described with the second-order polynomial with cross-terms, defined by the equation:

$$\overline{U}(\boldsymbol{x}) = \ b_0 + \sum_{i=1}^{n} b_i \mathrm{x}_i + \sum_{i=1}^{n} b_{ii} x_i^2 + \sum_{i<j}^{n} \sum_{j=2}^{n} b_{ij} x_i x_j + \varepsilon \tag{16}$$

Most structural engineering cases mark the second-order model with a high fit-ratio to the initial surface, however, a large input database is required to approximate the surface properly.

In this paper, slope factors of the response surface are estimated for the approximation models by means of original dedicated software RSM-Win® (Winkelmann, 2013) coded in Fortran 90©. However, they may be easily calculated in any other computer-aided design software. The core of the software employs the approximation matrix transformations joint with the least-square method. In the process, the ANOVA (ANalysis Of VAriance) table is created (Anscombe, 1948) to give values of estimated model slope factors and the ratio of their fitting to the real-life structural model response determined in FEM calculations.

In the considered task, the performed sensitivity analysis marks the probabilistic linear (first-order) model, shown in

Equation (15), almost relevant. Nevertheless, the probabilistic non-linear behaviour of $X2$ variable shows that the quadratic approximation given in Equation (16) is rather suggested. The procedure proposed in the paper addresses such a possibility. In the flowcharts presented in Figures 8 and 9, there is a clear distinction of the suggested action patterns dependent on whether the structural response indicates the need of using first-order or second-order approximation. It should be noted that despite the first-order approach was primarily used in this paper and is the one fully described, several elements of the second-order approach were also applied and calculated, to verify the correctness of the second possible action course of the procedure depicted in the flowcharts.

In order to perform the RSM approximation, a set number of samples was assumed. It should be clarified that a sample is defined here as an appropriate FEM model incorporating the given values of random input parameters (variates), alongside with the key result of the model (in this case, the vertical displacement of the deck U). In the considered example, an optimal sampling approach intended for RSM is proposed. Despite the application of the sampling approach depends slightly on the preferred response surface approximation model and the sensitivity analysis results, the approach is universal.

The application of the abovementioned sampling scheme starts with the establishing of a central sample (labelled as 'MEAN' in Figure 10, considering all variables set on their mean values, as shown in Table 3) is necessary. This does not require additional calculations as this model represents the natural case scenario, for which the initial calculations are performed.

In the second step of the proposed sampling scheme, the response of the sensitivity/PEM-based samples applying the one SD modification (labelled as 'SIG' in Figure 10, data from Table 3) were added ($2 \times 4 = 8$ samples in the given example) to the mean sample data. It is worth noting that these essential data are already gathered in the process of sensitivity assessment, so these samples benefit both PEM and RSM at the same time. The $2N$ data (N being the number of random variables) are added in one step only. Thus, a nine-sample information becomes the starting data for the RSM analysis in the presented example.

It should be noted here that for structures showing a non-negligible non-linear probabilistic behaviour, using the eight samples with two SDs modification instead of one (labelled as 'SIG*2' in Figure 10, data from Table 3), or adding them to the eight 'SIG' samples, might be more beneficial. First of all, they cover a broader range of the structural response, so they facilitate the assessment of the probabilistic non-linearity. Moreover, they provide a greater database for the ANOVA table computations, which frees the table of its initial high approximation error. Taking the SIG/SIG*2 samples into account may not be sufficient, so random samples may be used in the performed approximation procedure as well.

In the third final step of the sampling scheme, the response of randomly generated samples is added (labelled as 'RAND' in Figure 10, data from Table 6). In the presented analysis, 10 random computations were performed on samples generated according to the MC method by means of the random number generator in MATLAB®, considering a given random variable type. A total number of 19 samples are finally achieved in the approximation.

However, the random samples number was assumed a priori, for illustrative purposes only. A proper number of the random samples should be determined individually – it should be big enough to provide a proper convergence of

Table 3. Vertical displacements (mm) obtained in the FEM models used in the sensitivity analysis and PEM (the 'MEAN', 'SIG' and 'SIG*2' samples and their respective results).

$U(\bar{x})$ (SI and SIII)[a]	$U(\bar{x})$ (S II)[b]	X_1 (SI and SIII)	X_1 (SII)	X_2	X_3	X_4
'MEAN' sample – $X_i = E(X_i)$						
−10.86	−14.25	2400	3200	34	0	0
'SIG' samples – $X_i = E(X_i) + k\sigma(X_i)$; k = +1/ − 1; $X_j = E(X_j)$; $j \neq i$						
−11.02	−14.46	2436	3248	34	0	0
−10.69	−14.03	2364	3152	34	0	0
−10.71	−13.98	2400	3200	39.2	0	0
−11.06	−14.62	2400	3200	28.8	0	0
−10.83	−14.25	2400	3200	34	30	0
−10.89	−14.25	2400	3200	34	−30	0
−10.88	−14.25	2400	3200	34	0	30
−10.84	−14.25	2400	3200	34	0	−30
'SIG*2' samples – $X_i = E(X_i) + k\sigma(X_i)$; k = +2/ − 2; $X_j = E(X_j)$; $j \neq i$						
−11.19	−14.68	2472	3296	34.0	0	0
−10.53	−13.82	2328	3104	34.0	0	0
−10.57	−13.75	2400	3200	44.4	0	0
−11.33	−15.12	2400	3200	23.6	0	0
−10.80	−14.24	2400	3200	34.0	60	0
−10.92	−14.26	2400	3200	34.0	−60	0
−10.90	−14.26	2400	3200	34.0	0	60
−10.82	−14.24	2400	3200	34.0	0	−60

[a]S I and S III denote the displacements calculated in 1/4 and 3/4 of the span, respectively.
[b]SII denotes the displacements calculated in 1/2 of the span.

Table 4. Comparison of theoretical deflections and the measurements results.

	Vertical deflections (mm)					
	S I[a]		S II[a]		S III[a]	
Max. longitudinal location Measured elastic deflections	Girder 1 −10.85	Girder 2 −10.90	Girder 1 −13.70	Girder 2 −13.75	Girder 1 −10.50	Girder 2 −10.70
Initial FEM model	−10.75		−14.05		−10.75	
Calibrated FEM model	−10.86		−14.25		−10.86	
Probabilistic range, linearisation	−10.39 ÷ −11.38		−13.51 ÷ −15.07		−10.39 ÷ −11.38	
Probabilistic range, quadratic approx.	−10.42 ÷ −11.38		−13.59 ÷ −15.09		−10.42 ÷ −11.38	
Point estimate method	−10.37 ÷ −11.35		−13.48 ÷ −15.02		−10.37 ÷ −11.35	
Response surface method, first-order approx.	−10.37 ÷ −11.35		−13.45 ÷ −15.01		−10.37 ÷ −11.35	
Response surface method, second-order approx.	−10.41 ÷ −11.38		−13.59 ÷ −15.10		−10.41 ÷ −11.38	

[a]S I, S II and S III denote the displacements calculated in 1/4, 1/2 and 3/4 of the span, respectively.

Table 5. Results of calculations using PEM.

Load sch.	U_i' (mm)	$\bar{U}_i'$ (mm)	$V_{U_i'}$ (−)	$\bar{U}$	V_U
SI and SIII[a]	$U_0' = -10.86$			-10.87	0.0225
	$U_1'^{+} = -11.02$	$\bar{U}_1' = -10.86$	$V_{U_1'} = 0.0152$		
	$U_1'^{-} = -10.69$				
	$U_2'^{+} = -10.71$	$\bar{U}_2' = -10.88$	$V_{U_2'} = -0.0163$		
	$U_2'^{-} = -11.06$				
	$U_3'^{+} = -10.83$	$\bar{U}_3' = -10.86$	$V_{U_3'} = -0.0028$		
	$U_3'^{-} = -10.89$				
	$U_4'^{+} = -10.88$	$\bar{U}_4' = -10.86$	$V_{U_4'} = 0.0018$		
	$U_4'^{-} = -10.84$				
SII[b]	$U_0' = -14.25$			-14.28	0.0269
	$U_1'^{+} = -14.46$	$\bar{U}_1' = -14.25$	$V_{U_1'} = 0.0150$		
	$U_1'^{-} = -14.03$				
	$U_2'^{+} = -13.98$	$\bar{U}_2' = -14.30$	$V_{U_2'} = -0.0223$		
	$U_2'^{-} = -14.62$				
	$U_3'^{+} = -14.25$	$\bar{U}_3' = -14.25$	$V_{U_3'} = 0.00$		
	$U_4'^{-} = -14.25$				
	$U_4'^{+} = -14.25$	$\bar{U}_4' = -14.25$	$V_{U_4'} = 0.00$		
	$U_4'^{-} = -14.25$				

[a]S I and S III denote the displacements calculated in 1/4 and 3/4 of the span, respectively.
[b]SII denotes the displacements calculated in 1/2 of the span.

both PEM/linearisation and RSM approaches. A 3% difference is proposed as the limit discrepancy of results of both methods, as indicated in the flowchart given in Figure 9. This is the reason why checking the RSM approximation convergence after each subsequent single random sample addition is advised by the Authors.

Convergence of the approximation quality indicator is presented in Figure 10. The indicator is defined as the total least-squares approximation error per number of samples used, with respect to the number of response surface approximation samples. It should be pointed out that in the presented example, a sufficient approximation quality was achieved for just one randomly chosen sample – in a linear problem with four variables only $4+1=5$ samples are required for the ANOVA table to fill its degrees of freedom, ensuring a proper approximation. It should, however, be noted that this is not a sufficient condition, as even some linear cases may require a greater number of random samples calculation, according to the Authors' experience.

Conclusively, RSM is proven to be sufficient for the formulation of structural response acceptable range during the *in-situ* bridge load tests. The first-order RSM analytical case proved the linearisation of the discussed task was reasonable, showed a good approximation quality of the response surface for the proposed sampling technique. Following, the obtained acceptable range of *in-situ* test results of the vertical displacements are identical to those indicated in the range defined with Equation (13). Afterwards, as already mentioned before, the second-order approximation (using the same sampling technique) was also conducted. Final results of this analysis are given in Table 4.

On the basis of the data given in Tables 2 and 4, an insight may be formulated on the relation of the deflection values determined using the *in-situ* measurements and the chosen probabilistic approaches. It is easy to notice that all real-life surveys fall within the determined probabilistic structural response expected ranges (intervals), regardless of the undertaken random method. All first-order methods (linearisation, PEM and first-order RSM) give almost identical ranges. In 'S I' and 'S III', the respective interval span is ~0.99 mm, and differs only by 0.01 mm in regard to different methods. In 'S II' the respective interval is ~1.55 mm and differs only by 0.02 mm. The closed-form methods (PEM, RSM) shift the interval boundaries by 0.03 mm for 'S I' and 'S III' and by 0.06 mm in 'S II'. As can be noted, the values of the discrepancies are very low in relation to medium values of displacement computed in each loading scheme. In 'S I' and 'S III', the shift of 0.03 mm is only ~0.3% of 10.86 mm medium displacement. Accordingly, in 'S II', the shift of 0.06 mm in only ~0.4% of 14.25 mm.

Although a linear random response approximation is proven satisfactory, a second-order approach was also tested for comparative purposes. Due to a probabilistic non-linear sensitivity of *X*2, the estimated interval boundaries are shifted up to 0.04 mm for 'S I' and 'S III', and up to 0.14 mm for 'S II' in regard to results of the first-order approach. Nevertheless, the quadratisation and second-order RSM give interval values of an almost perfect compliance to each other (their spans vary by 0.01 mm only). Nevertheless, it should be pointed out that the results of the probabilistic methods may exhibit large dispersions, especially in complex, non-linear phenomena. This is not the case in the analysed example; however, a necessity of control calculations may come. An additional cross-check and multi-method analysis (PEM-RSM-MC) may be advantageous in selected problems.

Authors firmly propose performing parallel computations of both: linearisation formula given in Equation (2) or the PEM samples-based formula given in Equation (9) and the RSM assessment presented in Equations (15) and (16), dependent on the approximation model. This action, while unnecessary for a linear probabilistic problem, proves correctness of probable computational ranges in strongly non-linear probabilistic cases. Moreover, in both probabilistic linear and non-linear cases, it provides a comprehensive information on structural response relation on key input variables and on the optimal number of necessary model calculations.

Initial procedure of load test design conduct and assessment prior to opening

In the view of previous considerations, an initial draft of a procedure of implementing probability-based elements to the load test design was created in the form of flowcharts presented in Figures 8 and 9.

The proposed procedure features five general segments (stages), the segments number 2 and 3 represent the innovative, probabilistic part of the analysis. This approach is an essential issue of this paper, the necessary steps to obtain the acceptable response range in random terms are presented in detail in Figure 9. The remaining segments (1, 4 and 5) are associated with a specific load test type. Various circumstances or different bridge structural types may trigger their alterations, expansions and generalisations, strongly interconnected with design uniqueness or the purpose of the bridge under future consideration. Furthermore, these

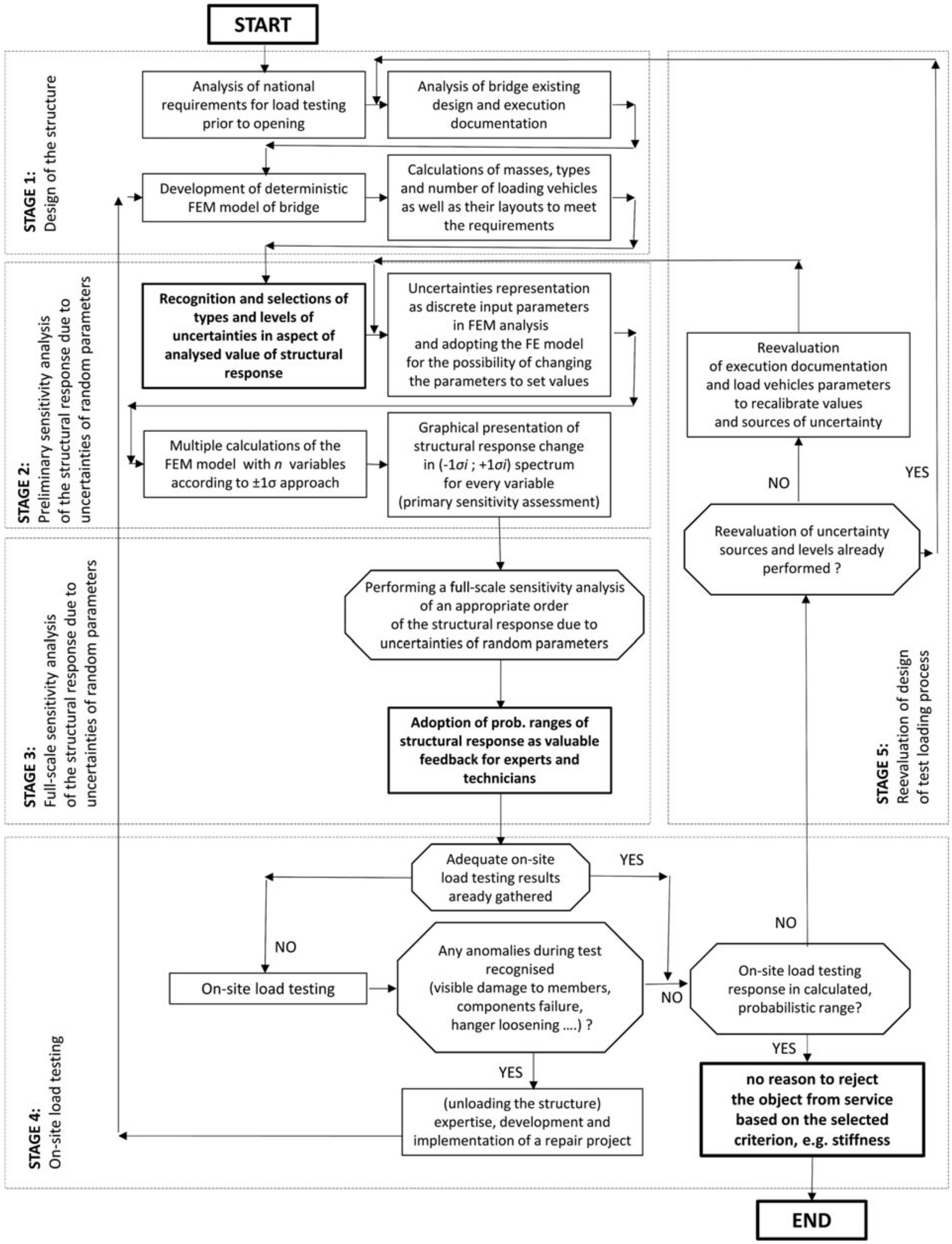

Figure 8. The proposed initial procedure of extending load test design with probability-based elements – a general flowchart.

charts (Figures 8 and 9) are a concept to extend the design and results assessment of load tests prior to opening using probabilistic methodology. This is an idea open for future scientific discussions.

Two major advantages of the proposed approach may be pointed out. First, the probability-based expected ranges of acceptable responses are easy to justify on the basis of preliminary uncertainty analysis. Owing to this fact, the calibrations would no longer be required when the structural response marginally exceeds the deterministic FE theoretical value, but no damage to the structure occurred, all uncertainties are explicable, and the response is in the computed probabilistic

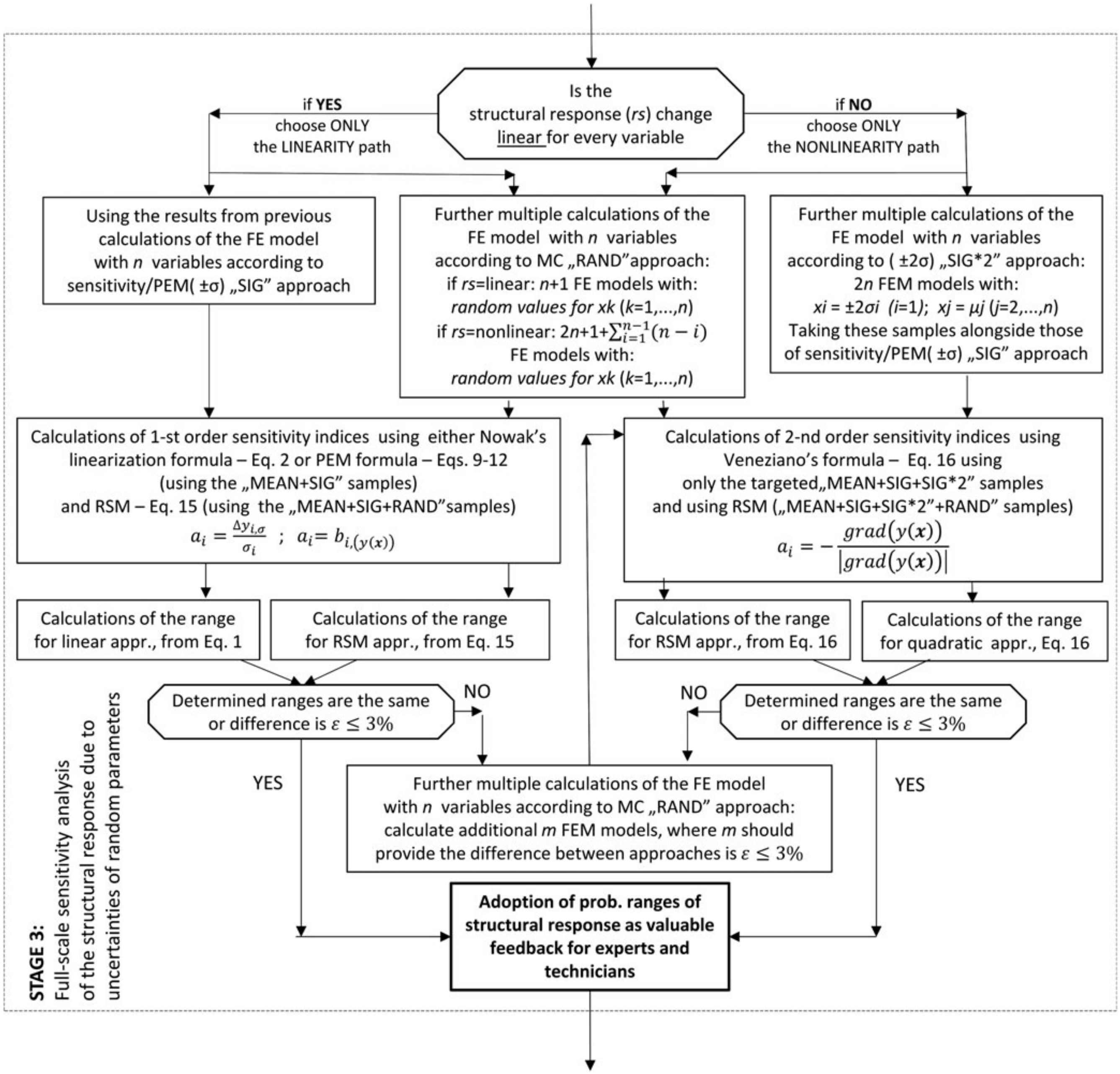

Figure 9. The proposed initial procedure of extending load test design with probability-based elements – a detailed flowchart of the segments based on probabilistic methodology.

range. This situation is explained in Example 1 of Appendix 3. Second, it negates the possible unreliable FEM modelling or erroneous *in-situ* measurements.

For example, when the *in-situ* deflections are significantly lower than their theoretical FE values (e.g. the measured bridge deflection is 60% of the numerically estimated value), the proposed probabilistic procedure deems such results unacceptable. Furthermore, it specifies the necessity of further analysis, including, e.g. FEM model calibration. In contrast, due to the current deterministic procedure, such a result would be identified as correct. This is clarified in Example 2 of Appendix 3.

Summary

The paper presents an idea to extend the use of common probabilistic methods (RSM supplemented with PEM and random samples) to complement load test design of newly built bridges prior to opening. The main goal of the presented paper is to formulate a procedure aimed at estimating the expected probable structural response ranges and their adoption to formulate a double-sided assessment criterion for *in-situ* load tests. In the Authors' opinion, the computed ranges hold a great advantage over the designated single deterministic limit values, the latter widely applied nowadays, as recognised on a basis of a broad literature study performed by the Authors. The proposed procedure states the limit ranges and checks their determination, as parallel multi-method computations are introduced.

It is shown that after a precise and customised selection of possible uncertainties, important for the load test design, all calculated ranges include the results of *in-situ*

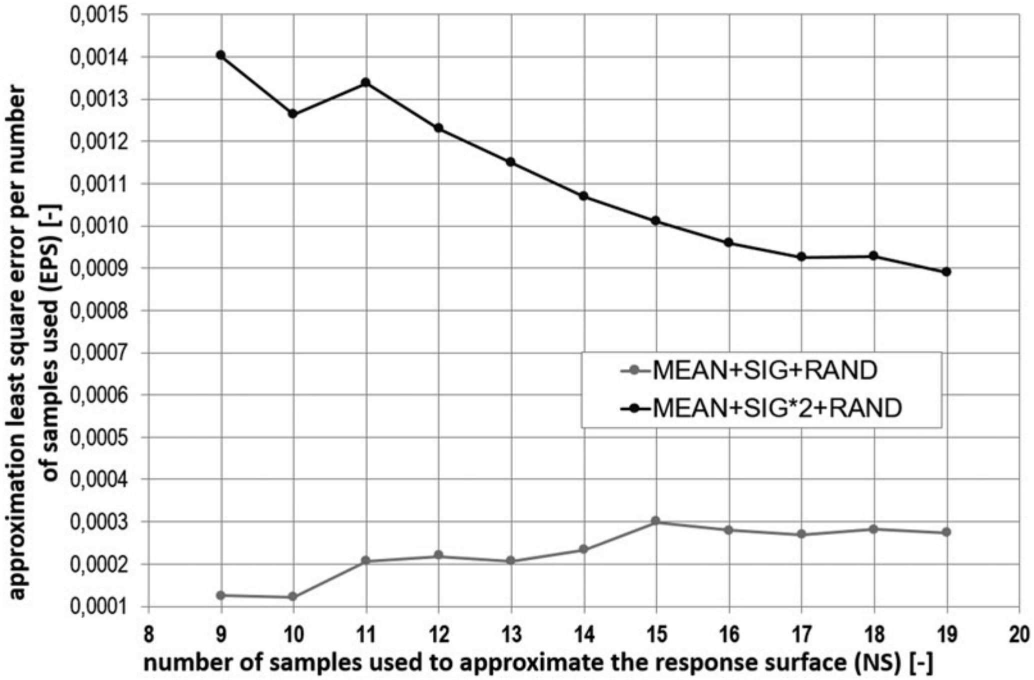

Figure 10. Total least-squares approximation error per sample quantity versus the number of samples used for the response surface approximation.

measurements, regardless of the applied load scheme. The paper presents parameter selection and description for geometric and material imperfections of a structural model and load variability based on preliminary documentation, engineering practice and the professional work experience of the first Author of this paper. Four variables were chosen in the paper, however, the number of random parameters is dependent on the load test designer (see Table A1 in Appendix 1). The process initially evaluates the impact of random variables on the response function results. In this case, the SD of the deck vertical displacements estimation was a key concern.

The proposed random approach makes it easier and faster to numerically verify the measurements in the course of load tests. Although the work analyses the dispersion of allowable deflections, an identical procedure may be derived with regard to any structural response parameter, e.g. stresses or strains, due to the flowcharts illustrating the proposed procedure, given in Figures 8 and 9.

The final variation ranges of the structural response function obtained in the presented analysis make it worthy to represent the important uncertainty sources in probabilistic terms already at the stage of preparation of load test design documentation for any newly executed structure. A more complex and precise information on possible bridge performance during the load test may be easily achieved. Such a knowledge can be valuable for technicians and experts taking measurements at the site and for future structure users.

In general, the results of probabilistic methods are proven to need a thorough check. The paper addresses this issue by proposing parallel application of several random methods: PEM, RSM and MC. The discussion was made upon sample selection and the minimum number of samples required to precisely reflect a linear response random function. It was proven that the PEM samples may be entered into RSM, thus no additional numerical model computations are required for a check. The example confirms the Authors' viewpoint that a parallelly computed RSM variant based on a proposed sampling outline allows any (random) sample to be investigated in the analysis (taking variable type into account), even the ones not following any specialised methodology. In the meantime, it optimises numerical time and cost of the analysis, as it re-uses samples calculated for different purposes (the sensitivity analysis, in this case). The RSM does not require broad probabilistic fundamentals to be simply applied, no need for specialised software is triggered (the RSM may be easily incorporated in open-access spreadsheets). Therefore, its use is accessible for engineers involved in bridge load testing, with the aim to enhance the standard design procedures, even if the engineers are unfamiliar with probabilistic methodology.

To summarise, in countries holding purely deterministic acceptance criteria for bridges, it is necessary to introduce other criteria into load test standards, as suggested by numerous sources, e.g. the JCSS Probabilistic Model Code. Nevertheless, the improvements should be introduced step by step, starting from the lowest expenditure variants. In the Authors' opinion, the proposed step may replace a set deterministic limit value by a double-sided limit (interval) derived upon uncertainty analysis.

The presented approach, supported by an example of its applicability, can be also used in the analysis of similar cases. Also, upgrades may be further introduced in the proposed procedure. Further research and discussion are required in several fields, notably due to the possible choice of input variables statistical moments, the nature of input variables and the type of the considered bridge structure. The possibility of variability reduction of the considered random variables should also be further investigated. This

Table 6. Vertical displacements (mm) obtained in the FEM models generated using random variates (the 'RAND' samples and their respective results).

$U(\bar{x})$ (SI and SIII)[a]	$U(\bar{x})$ (S II)[b]	X_1 (SI and SIII)	X_1 (SII)	X_2	X_3	X_4
		'RAND' samples – $X_i = random(X_i)$				
−10.81	−14.16	2355	3141	31.23	−43.41	−3.26
−10.58	−13.80	2397	3196	42.75	23.80	0.27
−11.01	−14.61	2407	3210	29.45	37.80	−35.94
−10.86	−14.33	2386	3181	31.48	−0.55	−46.13
−10.98	−14.38	2379	3172	30.30	−58.63	10.48
−11.07	−14.64	2392	3189	27.89	−0.46	−6.15
−10.95	−14.37	2409	3212	33.00	2.00	7.88
−10.88	−14.35	2401	3201	32.57	31.44	−10.04
−10.59	−13.85	2399	3199	41.96	29.10	−22.24
−10.69	−14.04	2351	3135	32.71	−0.71	−0.12

[a]S I and S III denote the displacements calculated in 1/4 and 3/4 of the span respectively.
[b]SII denotes the displacements calculated in 1/2 of the span.

may be achieved by e.g. creating guidelines that will force the load test designers to carry out the tests after the bridge construction (so that using the data from the *in-situ* laboratories will be possible) or to specify the requirements regarding the aggravating vehicles, e.g. their loading processes and their subsequent weighing.

The performed numerical experiments initiate the planned series of tests aimed at checking validity and efficiency of the proposed approach. It is also planned to develop recommendations not only for the scope of load testing prior to opening but also to consider and analyse the applicability of the presented approach in its modified version, e.g. adjusted to proof load testing of existing structures.

Conclusions

Based on the presented methodology application and discussion the following conclusions can be drawn:

- Upon a thorough study of literature and national standards for proof load testing of bridges prior to opening, the implementation of probabilistic methods to complement deterministic load test design is the main goal of this paper. Such a goal seems justified and innovative.
- The paper succeeds in creating an assessment criterion for *in-situ* load tests in a form of an allowance interval, proposed to become a replacement for the designated single restrictive deterministic limit values, as given by the load test conductance standards.
- A precise and customised analysis of uncertainties, which is also shown in the paper, is paired with common probabilistic methods (MC + PEM + RSM) to achieve this goal. It was proven that a proposed simplified approach to key points of second-level methods prove standard software is sufficient for a proper probability-based calculation of postulated response ranges, in many cases. This makes the presented approach available for practicing engineers and not just scientists.
- Exemplary calculations show that the proposed approach is able to provide a reliable and trustworthy limit range of measured values (e.g. displacements) – the ranges calculated with the presented approach include the real-life results of *in-situ* measurements. In turn, the approach can streamline the performance of *in-situ* tests.
- The proposed innovative procedure may be related and applied to any bridge structure, with regard to any structural response parameter, owing to the illustrative step-by-step overview of the proposed approach, ensured by clear and easy-to-understand proprietary flowcharts.

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No potential conflict of interest was reported by the authors.

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Appendix 1

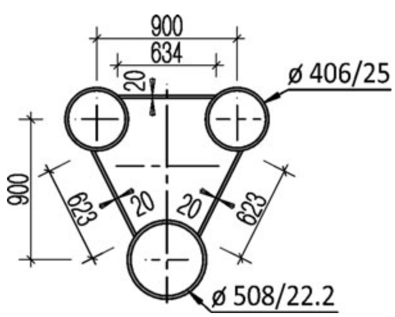
Table A1. Remarks on the majority of typically considered uncertainty sources, the reasoning behind the acceptance/rejection of given uncertainties.

Source of uncertainty	Remarks Reasoning in undertaken example explained in brackets	Taken into acc. in the example
Geometrical imperfections of arch girders: • projection of longitudinal axis, • projection of cross-section shape and dimensions	• especially important in case cross-sections which are not typical and difficult in execution, • requires analysis of the reports of the precast elements production and inventory [in the example: cross-sectional imperfections on negligible level].	Yes (only axial)
Stiffness of the deck with respect to the assumed E modulus	• according to observations in (Łaziński, 2009) it can be a very important factor, • application of deterministic values directly from the design standards seems to be a wrong approach, • sometimes an approach to introduce 90% of the Young's modulus for the initial stiffness is implemented	Yes
Stiffness of the deck – structural imperfections	• depend on the construction technology and formwork, • can have big values, e.g. when formwork of the deck is suspended on the arch [in the example: imperfections on negligible level]	No
Stiffness of the deck – transition of sections into cracked state	• requires the analysis of prestress level in the deck girders in relation to the thrust forces in the arch [in the example: high level of prestress in the analysed object]	No
Stiffness of the deck – the presence of deck equipment (e.g. pedestrian and traffic railing, edge beams, etc.)	• no presence of additional equipment during the test was assumed, • no layer of asphalt pavement was assumed (tests performed directly on the deck slab) • [in the example: decision based on executive reports and on-site control visit]	no
Uniform/non-uniform/gradient heating or cooling of the structural elements during the measurement	• very important factor in case of some types of structures, • a requirement to perform the tests in late night hours was formulated, • possible deterministic correction of the model was assumed in case when temperature variations are registered during the load test. [in the example: all tests performed in late night hours]	No
Uncertainties connected with measuring devices and measure process	[in the example: rejected because of the assumed measuring technique with high precision and high confidence level]	No
Mass of vehicles, its distribution and accuracy as well as location on the deck	• such parameters as precise mass of the loaded material, actual spacing of vehicle's axes and their layout on the object can be verified (with certain precision and confidence level) just before or during the measurement, • the collected information should be used if there is a need to calibrate the model and perform deeper analyses. [in the example: vehicles axes spaces expected to be consistent with the assumed values]	Yes (only total mass)
Uncertainty connected with the choice of FEM model class	• application of oversimplified model can have significant importance (Bień et al., 2015), • the applied model should be of appropriate class based on the recommendations and statistical database from previous tests of structures of a given type. [in the example: chosen model class proved to be sufficient in numerous prior cases of analogical structures]	No
Uncertainty connected with the possibility of incorrect structural behaviour of the elements of the structure	• e.g. incorrect behaviour of hangers or bearings, uneven settlement of supports, cracks, damages, etc. • one of the fundamental aims of load tests verified during the *in-situ* tests by technicians. [in the example: these uncertainties are taken into account by general procedure]	No

Appendix 2

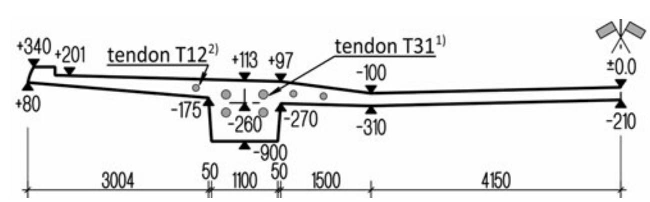
Additional detailed information on geometrical properties of the bridge and its FEM model

(a) Additional dimensions of the arch girder:



Length of one arc (in the axis of inertia): 117.5 m, each arc is represented by 39 finite elements

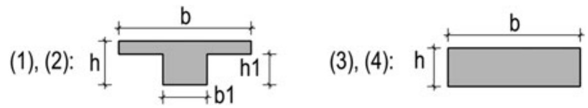
(b) Additional information on deck longitudinal girder (beam element in FEM model):



Length of one girder (in the axis of inertia): 109.4 m, each girder is represented by 31 finite elements

1)31-strand tendon; area of single strand: 150 mm^2, characteristic tensile strength: 1860 MPa; diameter of sheath: 130 mm, straight-line layout (with exception for the anchorage zone), double-sided tension, [2)]12-strand tendon; area of single strand: 150 mm^2, characteristic tensile strength: 1860 MPa; diameter of sheath: 80 mm, straight-line layout, double-sided tension.

(c) Cross-sectional properties of representative* transverse elements of the FEM model



Member (and dimensions [m])	Number of FE	Cross-sectional area (m^2)	Torsional moment of inertia (m^4)	Moment of inertia vs. local hor. axis (m^4)	Moment of inertia vs. local vertical axis (m^4)	Material
(1) Transverse deck beams (internal) L** = 11.4; b = 5.0; h = 0.73; h1 = 0.52; b1 = 0.47	84	1.29	0.0283	0.0358	2.192	C 35/45
(2) Transverse deck beams (external – in line of abutments) L** = 12.5; b = 4.7; h = 1.20; h1 = 0.90; b1 = 2.00	8	3.21	0.7419	0.417	3.195	C 35/45
(3) Transverse representation of deck's slab (cantilever) L = 2.93; b = 5.0; h = 0.21	46	1.05	0.0150	0.00386	2.188	C 35/45
(4) Transverse representation of deck's main girder L** = 1.15; b = 5; h = 1.013	92	5.065	1.460	0.433	10.55	C 35/45

*Element height slightly fluctuates along the transverse axis due to deck thickness.
**L – the sum of the length of collinear elements.

Appendix 3

Examples indicating the advantage of the proposed probabilistic approach in the analysis of load test results in the assessment of the tested bridge and its theoretical model

U – structural defection measured in the *in-situ* tests (mm)
U_{DETER} – theoretical deflection taken from a deterministic FEM model (mm)
U_{MIN} – the lower boundary of the range calculated using the probabilistic approach (mm)
U_{MAX} – the upper boundary of the range calculated using the probabilistic approach (mm)

(a) Example 1 (the values are taken directly from 'S I' loading scheme)

	Current deterministic approach	Proposed probabilistic approach
Vertical deflection measured during the *in-situ* test (mm)	$U = 10.85$	
Acceptance criterion	$U < U_{DETER}$ $10.85 > 10.75$	$U \in U_{MIN}; U_{MAX}$ $10.85 \in 10.39; 11.38$
Acceptance criterion fulfilled?	no	yes
Damage, local failure or incorrect behaviour in tests?	No	
Conclusions and remarks	Rejection of a sufficiently correct FEM model, only due to showing deflections different from *in-situ* measurements by less than 1%	Acceptance of a sufficiently correct FEM model, based on proper uncertainty analysis. Complete decision-making schemes included in the paper
Aftermath	Time-consuming redundant necessity to calibrate the FEM model without strict guidelines on calibration factors and their levels, often leading to arbitrary decisions	Allowance of the bridge structure for service

(b) Example 2 (the *in-situ* value is hypothetical, others are taken from 'S I' loading scheme), illustrating a situation encountered very often, e.g. during tests of precast beam bridges

	Current deterministic approach	Proposed probabilistic approach
Vertical deflection measured during the *in-situ* test (mm)	$U = 6.15$	
Acceptance criterion	$U < U_{DETER}$ $6.15 < 10.75$	$U \in U_{MIN}; U_{MAX}$ $6.15 \notin 10.39; 11.38$
Acceptance criterion fulfilled?	yes	no
Damage, local failure or incorrect behaviour in tests?	No	
Conclusions, remarks	Acceptance of a completely uncalibrated FEM model, showing deflections different from the *in-situ* measurements by more than 40%	Properly justified rejection of an uncalibrated or erroneous FEM model or faulty uncertainty sources assumptions
Aftermath	Allowance of the bridge structure for service, despite the fact that the decision is based on a probably incorrect deterministic FEM model	Repetition of uncertainty analysis based on *in-situ* measurements, to recalculate probability ranges, implementation of a higher-class FEM model or other decisions consistent with the schemes included in the paper