

Comparision of the exponential distribution with the Central Limit Theorem

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Overview

This project runs simulations of exponential distribution, compares it with predictions of Central Limit Theorem. All the simulations use means of 40 samples and $\lambda = 0.2$. All the simulations are using R language.

Simulations

Below you can see a distribution of the mean of 40 exponentials.

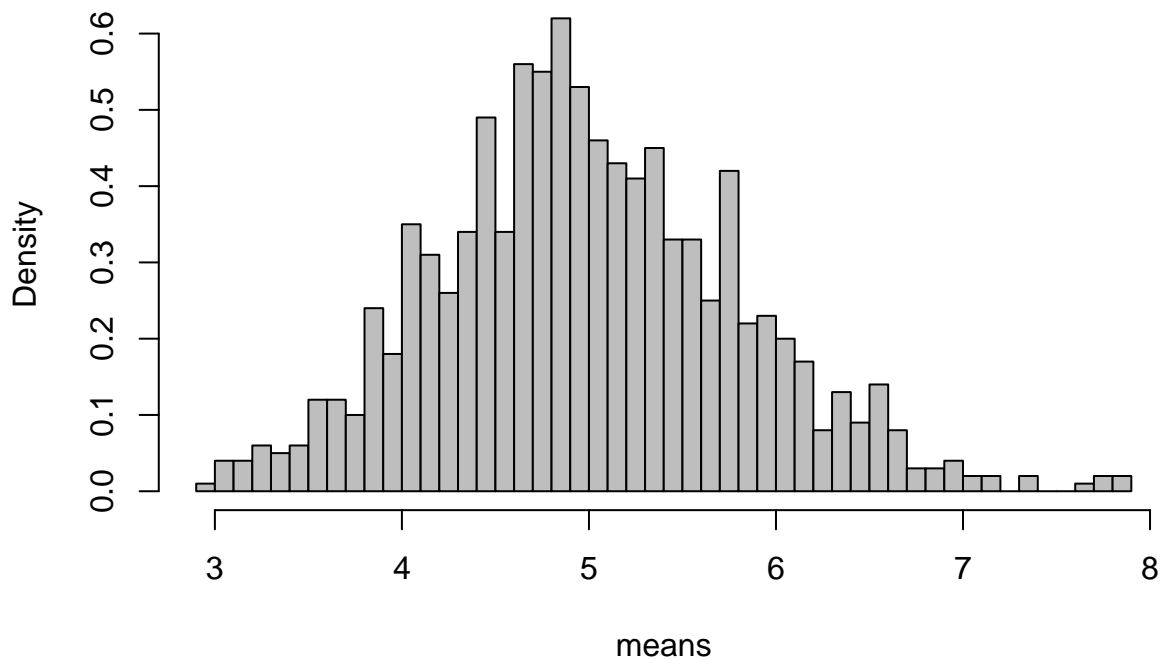
```
set.seed(0)

lambda <- 0.2
sampleSize <- 40
simulationsNumber <- 1000

simulate <- function(n) {rowMeans(matrix(rexp(sampleSize * n, lambda), nrow=n))}

means <- simulate(simulationsNumber)
hist(means, breaks = 50, freq = F, col = "grey")
```

Histogram of means



Sample mean vs Theoretical mean

Theoretical mean is equal to the population mean:

```
(sampleTheoreticalMean <- populationMean <- 1/lambda)
```

```
## [1] 5
```

Actual sample mean is:

```
mean(means)
```

```
## [1] 4.989678
```

Sample variance vs Theoretical variance

Theoretical variance is:

```
populationSd <- 1/lambda  
populationVar <- populationSd^2  
(sampleTheoreticalVar <- populationVar/sampleSize)
```

```
## [1] 0.625
```

Actual sample variance is:

```
var(means)
```

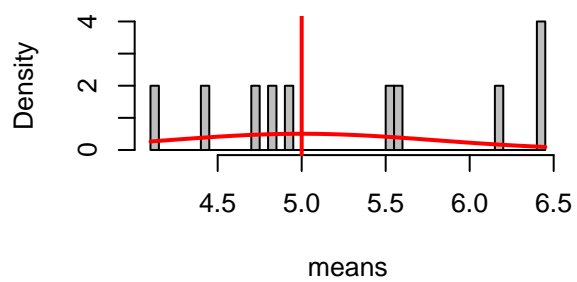
```
## [1] 0.6638725
```

Aproximation of sample mean distribution

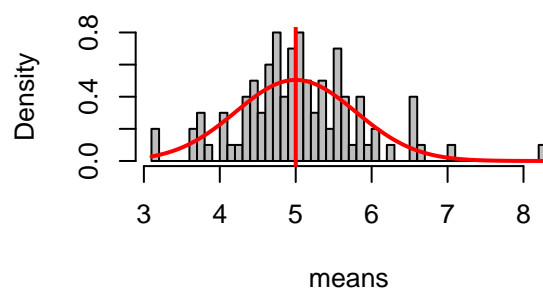
To see that the distribution is approximately normal let's see how it changes with number of simulations. To better see the effect, each histogram contains theoretical normal distribution with marked mean.

```
sampleTheoreticalSd <- sqrt(sampleTheoreticalVar)  
  
par(mfrow=c(2,2))  
for (n in 10^(1:4)) {  
  hist(simulate(n), freq = F, breaks=50,  
       col = "grey", main = paste (n, 'simulations'), xlab = 'means')  
  curve(dnorm(x, mean=sampleTheoreticalMean, sd=sampleTheoreticalSd),  
       col = 'red', add = TRUE, lwd=2)  
  abline(v=sampleTheoreticalMean, col="red", lwd=2)  
}
```

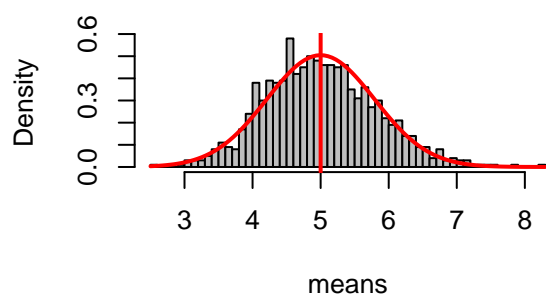
10 simulations



100 simulations



1000 simulations



10000 simulations

