

## Pseudo-highpass filters based on semi-distributed balanced composite right/left-handed unit cells

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In this study, the design of high selectivity, pseudo-highpass filters is presented. The proposed circuits utilize a novel semi-distributed-element composite right/left-handed unit cell (UC) composed of sections of transmission lines and a lumped capacitor. By proper balancing the structure, a very broad operation band can be obtained. Moreover, a single-layer microstrip realization is possible making the UC well suitable for low-cost filter realization. The proposed concept has been experimentally verified by the design and measurements of an exemplary pseudo-highpass filter.

**Keywords:** composite right/left-handed metamaterials; unit cell; filters

### 1. Introduction

In recent years, metamaterial structures have gained a significant interest for microwave applications due to their properties. An effectively homogenous metamaterial transmission lines composed of a finite number of elementary unit cells (UCs) has been proposed and their applications are explored since. In [1,2], a composite right/left-handed (CRLH) UC has been described and characterized. The structure features left-handed and right-handed frequency region, and when properly balanced (i.e. there is no stopband between these two passbands), a broadband operation can be obtained. Such structures are of interest for realization of microwave components, e.g. broadband filters. When parasitic elements responsible for right-handed band are extracted/designed and included into the model, one can tune their value to provide the desired properties. In [3–7], the authors have proposed many CRLH UC models and presented their application in filters' design.

In this study, we present pseudo-highpass filters utilizing a novel semi-distributed-element UC featuring a CRLH character. A UC composed of transmission-line sections and a series capacitor is introduced and analysed. Since the UC features a periodic very wide passband character with narrow stopbands, it can be treated as pseudo-highpass. Such filters can find applications in systems where high frequency filtering and very sharp slope of attenuation as well as broad operation is required. It is then important to have an appropriate circuit model valid for broad frequency range to describe the behaviour of the circuit. The UC is modelled mostly using distributed elements, in contrary to e.g. [3,5–8], which allows to keep the model simple and to include all important effects occurring in the structure. The discussed UC is theoretically investigated and its behaviour and properties are described and illustrated in Section 2. Furthermore,

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formulas for UC element values' calculations are formulated and applied for the design of an exemplary pseudo-highpass filter presented in Section 3.

## 2. Analysis and design of a CRLH UC

A generic lumped-element composite right/left-handed symmetric UC [1] is shown in Figure 1(a). It is modelled using ideal lumped capacitors and inductors. Inductors  $L_L$  and capacitor  $C_L$  are related to left-handed behaviour while  $L_R$  and  $C_R$  partially model parasitics which occur in a physically realized circuit and correspond to right-handed behaviour. By the appropriate selection of elements' values, one can realize a broad-band structure having a composite character with smooth transition between the two regions.

In this article, we propose a semi-distributed-elements approach with the use of transmission-line sections as it has been shown in Figure 1(b). It can be noted that a series transmission line can model very well real series inductance and shunt capacitance related to right-handed band, whereas a short-ended transmission-line models shunt capacitance, as well as shunt inductance. Therefore, one can directly translate the calculated circuit elements (i.e. characteristic impedance and electrical length of transmission-line sections) into physical structure, in opposite to the lumped-element structure where ideal elements are used and circuit extraction is required to incorporate parasitics into the model.

The important properties of the UCs, i.e. propagation constant, pass band limits and Bloch impedance, can be determined using the even- and odd-mode excitation method as presented in [8] when a UC is symmetrical. Such an approach significantly simplifies the analysis when the UC is more complex since the structure is decomposed into two sub-networks described by even  $Z_e$  and odd  $Z_o$  impedances, which are less complex than the initial network.

The proposed UC has been decomposed into even- and odd-mode circuits (see Figure 1(c)) and the appropriate impedances are equal:

$$Z_e = j \frac{Z_L \operatorname{ctg}(\theta_L) Z_S \operatorname{tg}(\theta_S)}{Z_L \operatorname{ctg}(\theta_L) - Z_S \operatorname{tg}(\theta_S)} \quad (1)$$

$$Z_o = j \frac{Z_S \operatorname{tg}(\theta_S) Z_L [Z_L \operatorname{tg}(\theta_L - X_C)]}{Z_S \operatorname{tg}(\theta_S) [Z_L + X_C \operatorname{tg}(\theta_L)] + Z_L [Z_L \operatorname{tg}(\theta_L) - X_C]} \quad (2)$$

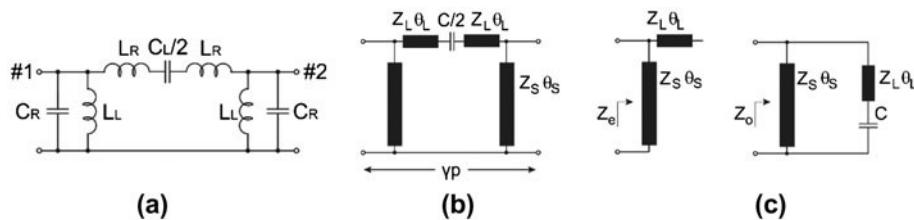


Figure 1. A lumped-element CRLH UC (a) and the proposed semi-distributed-element approach where transmission-line sections are used (b) and its decomposition into even- and odd-mode circuits (c).

where  $\theta_S = (f/f_T)\theta_{0S}$  and  $\theta_{0S}$  is the electrical length of a short-ended transmission-line section,  $\theta_L = (f/f_T)\theta_{0L}$  and  $\theta_{0L}$  is the electrical length of a series transmission-line section, all defined at  $f_T$  and  $f_T$  is the transition frequency between left-handed and right-handed region for which  $\gamma = 0$ ,  $X_C = 1/(2\pi f C)$ .

Bloch impedance of the proposed structure equals:

$$Z_B = \sqrt{-\frac{Z_L ctg(\theta_L) Z_{Stg}(\theta_S)}{Z_L ctg(\theta_L) - Z_{Stg}(\theta_S)} \cdot \frac{Z_{Stg}(\theta_S) Z_L [Z_L tg(\theta_L - X_C)]}{Z_{Stg}(\theta_S) [Z_L + X_C tg(\theta_L)] + Z_L [Z_L tg(\theta_L) - X_C]}} \quad (3)$$

When even and odd impedances are of different signs, Bloch impedance of the UC is purely real and passband occurs

The structure is called balanced when there is no stopband between left-handed and right-handed region. To obtain that, in case of even- and odd-mode analysis, the balancing condition is given by  $\text{Im}\{Z_e(f_T)\} \rightarrow \infty$  and  $\text{Im}\{Z_o(f_T)\} = 0$ .<sup>[8]</sup> Thus, the denominator of  $Z_e$  and the numerator of  $Z_o$  must both be equal to zero at the transition frequency  $f_T$ , hence:

$$Z_L ctg(\theta_{0S}) - Z_{Stg}(\theta_{0S}) = 0 \quad (4)$$

$$Z_{Stg}(\theta_{0S}) Z_L \left[ Z_L tg(\theta_{0L}) - \frac{1}{2\pi f_T C} \right] = 0 \quad (5)$$

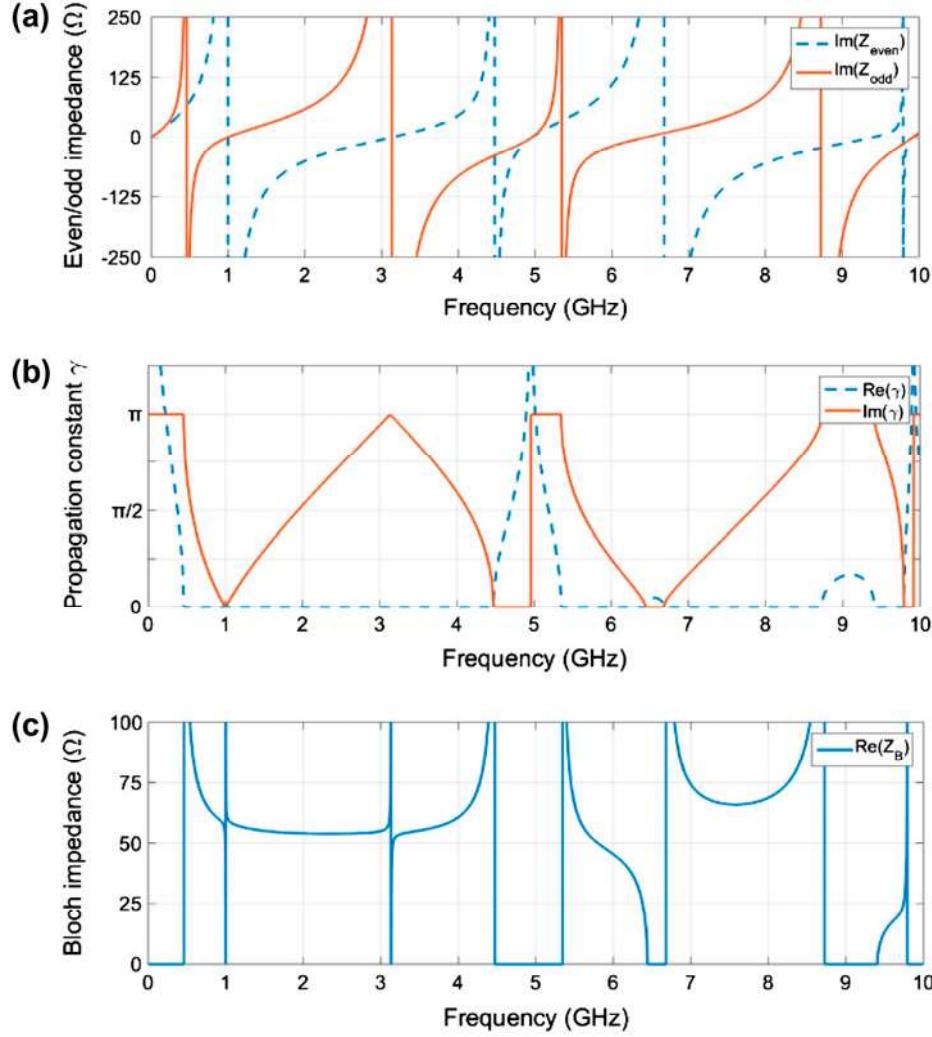
The lower and upper cut-off band limit for a lowest order band composed of left-handed band with continuous transition to right-handed band appear for  $Z_o(f_{co}) \rightarrow \infty$  which means that the denominator of  $Z_o$  must be equal 0 for  $f_{co}$ , where  $f_{co}$  is either lower or higher cut-off frequency:

$$Z_{Stg} \left( \frac{f_{co}}{f_T} \theta_{0S} \right) \left[ Z_L + \frac{1}{2\pi f_{co} C} tg \left( \frac{f_{co}}{f_T} \theta_{0L} \right) \right] + Z_L \left[ Z_L tg \left( \frac{f_{co}}{f_T} \theta_{0L} \right) - \frac{1}{2\pi f_{co} C} \right] = 0 \quad (6)$$

$$Z_L cot \left( \frac{f_{co2}}{f_T} \theta_{0L} \right) - Z_L tan \left( \frac{f_{co2}}{f_T} \theta_{0S} \right) = 0 \quad (7)$$

However, when the design is focused on realization of pseudo-highpass structure, one can broaden the continuous band by proper engineering of second-order band to start at the edge of first-order right-handed band (see Figure 2). Then, lower cut-off frequency  $f_{co1}$  can be found based on (5) to be at first pole of  $Z_o$  while upper cut-off frequency  $f_{co2}$  can be found to be at second pole of  $Z_e$ , i.e.  $Z_o(f_{co1}) \rightarrow \infty$  and  $Z_e(f_{co2}) \rightarrow \infty$ . Moreover, by proper selection of cut-off frequencies, one can minimize narrow stop band between higher order passbands to obtain pseudo-highpass character. However, it is not possible to suppress appearing stopband to broaden the fundamental passband without creating a stopband at lower frequencies.

The UC element values can be calculated based on (3)–(7) for the desired operating frequency range of the UC, i.e.  $f_B$ ,  $f_{co1}$ ,  $f_{co2}$  and for  $Z_B$  (defined at  $f_B$  different than  $f_T$  since at  $f_T$  the function has its discontinuity). However, relations allowing to calculate  $\theta_{0L}$  and  $\theta_{0S}$  cannot be solved analytically and numerical calculation need to be performed. Solution is a set of electrical lengths that satisfies simultaneously (8) and (9) ( $0^\circ < \theta_{0L}, \theta_{0S} < 90^\circ$ ):



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Figure 2. Properties of the proposed balanced CRLH UC: imaginary parts of even- and odd-mode impedances (a) real and imaginary part of propagation constant (b) and real part of Bloch impedance.

$$\frac{\tan\left(\frac{f_{o1}}{f_r}\theta_{0S}\right)}{\tan(\theta_{0S})\tan(\theta_{0L})} \left[ 1 + \tan(\theta_{0L}) \frac{f_T}{f_{co1}} \tan\left(\frac{f_{co1}}{f_T}\theta_{0L}\right) \right] + \left[ \tan\left(\frac{f_{co1}}{f_T}\theta_{0L}\right) - \tan(\theta_{0L}) \frac{f_T}{f_{co1}} \right] = 0 \quad (8)$$

$$\frac{1}{\tan\left(\frac{f_{co2}}{f_r}\theta_{0L}\right)} - \frac{\tan\left(\frac{f_{co2}}{f_r}\theta_{0S}\right)}{\tan(\theta_{0S})\tan(\theta_{0L})} = 0 \quad (9)$$

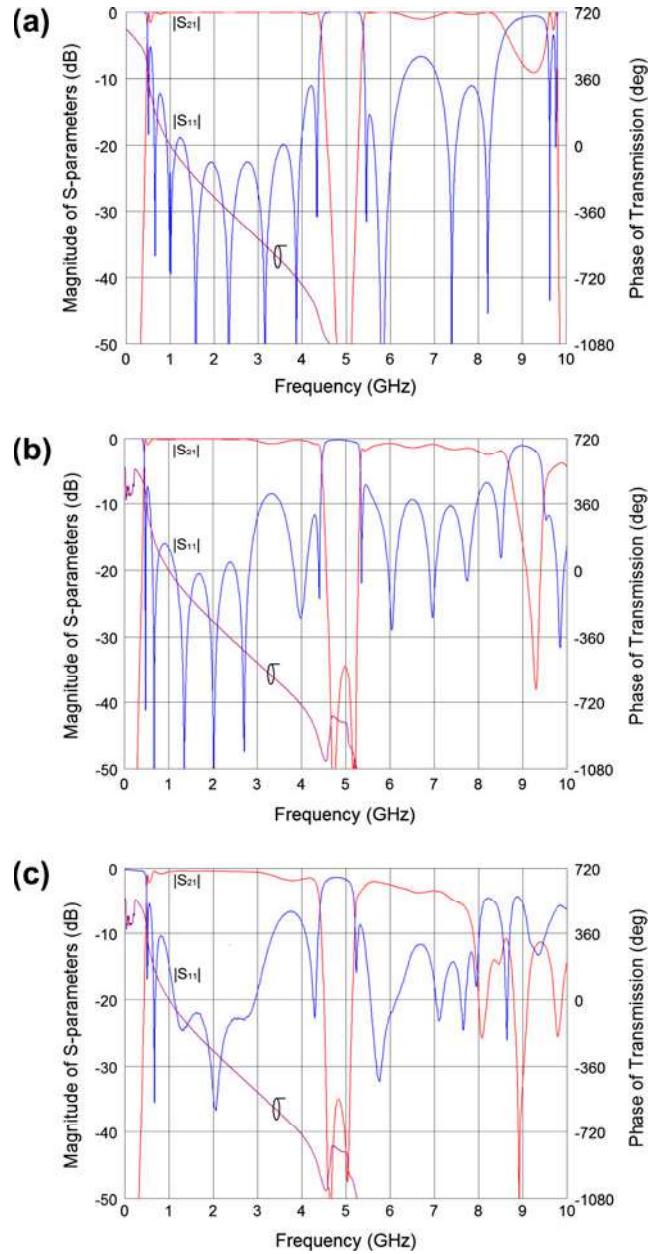


Figure 3. S-parameters of the designed pseudo-highpass filter composed of  $n = 3$  proposed UCs: ideal elements' circuit (a) EM calculated (b) and measured ones (c).

Having found electrical length, one can calculate values of the remaining elements:

$$A = \frac{\tan\left(\frac{f_p}{f_T}\theta_{0S}\right)}{\tan(\theta_{0S})\tan(\theta_{0L})}. \quad (10)$$

$$Z_L = \frac{Z_B}{A \sqrt{-\left\langle \cot\left(\frac{f_B}{f_T} \theta_{0L}\right) - A \right\rangle \left\langle A \left[ 1 + \frac{f_T}{f_B} \tan(\theta_{0L}) \tan\left(\frac{f_B}{f_T} \theta_{0S}\right) \right] + \left[ \cot\left(\frac{f_B}{f_T} \theta_{0L}\right) - \frac{f_T}{f_B} \tan(\theta_{0L}) \right] \right\rangle}}. \quad (11)$$

$$Z_S = Z_L \frac{1}{\tan(\theta_{0L}) \tan(\theta_{0S})} \quad (12)$$

$$C = \frac{1}{Z_L \tan(\theta_{0L}) 2\pi f_T} \quad (13)$$

The properties of an exemplary UC utilized for the pseudo-highpass filter design having  $f_T = 1$  GHz,  $f_{co1} = 0.43$  GHz,  $f_{co2} = 4.48$  GHz,  $f_B = 1.85$  GHz,  $Z_B@f_B = 50 \Omega$ , what yields  $\theta_{0L} = 28.7^\circ$ ,  $Z_L = 71.9 \Omega$ ,  $\theta_{0S} = 36.3^\circ$ ,  $Z_S = 178.8 \Omega$ ,  $C/2 = 2.02 \text{ pF}$  are shown in Figure 2.

### 3. Pseudo-highpass filter realization and experimental results

To verify the presented theoretical analysis, a pseudo-highpass filter utilizing the proposed balanced CRLH UC has been designed, manufactured and measured. The proposed filter is composed of  $n = 3$  UCs (having elements' values as shown in previous section), and it has to be underlined that even for such a small number of UCs, a very sharp slope of attenuation can be obtained (see Figure 3(a)). The designed circuit has been electromagnetically investigated with the use of *AWR Design Environment*, and the results of simulations are presented in Figure 3(b). The pseudo-highpass filter has been manufactured on a single-layer laminate (Arlon 25 N) having thickness of  $h = 1.524$  mm and permittivity  $\epsilon_r = 3.38$ . Two SMD 0402 1 pF capacitors per UC have been used to realize series capacitance. For the required value of capacitance the SMD components are more suitable in comparison to distributed planar capacitors, e.g. interdigital, since the dimensions of the former would be incomparably larger and would introduce parasitics significantly deteriorating the filter's performance. Two of such elements applied in parallel allowed for more even distribution of capacitance across the width of the series transmission line. Furthermore, two adjacent short-ended transmission lines have been

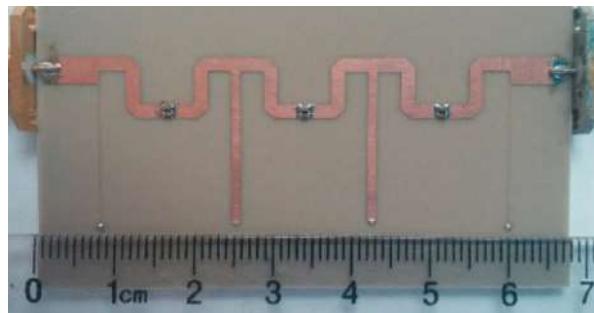


Figure 4. A picture of the manufactured pseudo-highpass filter. Short-ended transmission lines of adjacent UCs have been integrated and realized as one having impedance of  $Z_S/2$  to obtain a more compact layout.

integrated to obtain more compact size. The measurements result and a picture of the manufactured circuit are shown in Figures 3(b) and 4, respectively.

As it can be seen based on measurements, the manufactured exemplary filter provides useful pseudo-highpass characteristic within very broad frequency range of 0.43–4.48 GHz. It is worth underlining that the lower cut-off slope is very sharp providing high attenuation near passband below  $f_{c01} = 0.43$  GHz. Also the second passband and narrow stopband between first and second passbands are clearly visible.

The deterioration of performance within higher frequencies may be caused by lumped capacitors' parasitics as well as losses; however, it clearly appears in the second passband and above, and does not influence significantly the fundamental passband which is of interest in practical applications.

#### 4. Conclusion

The pseudo-highpass filter utilizing a novel semi-distributed-element CRLH UC has been presented. The theoretical analysis of the UC has been conducted and appropriate design formulas ensuring balancing of the structure have been formulated. The proposed approach has been verified by an exemplary realization of a compact filter in single-layer microstrip structure. The obtained measurement results proved the applicability and good performance of the circuit.

#### Disclosure statement

No potential conflict of interest was reported by the authors.

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