

# Semi-Distributed Approach to Dual-Composite Right/Left-Handed Transmission Lines and Their Application to Bandstop Filters

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**Abstract**—In this letter, an innovative approach for realization of artificial transmission lines featuring a dual-composite right/left-handed character is presented. The circuits utilize a novel semi-distributed unit cell which is composed of sections of conventional transmission lines and a lumped capacitor. Such an approach allows for efficient modeling and convenient translation of model parameters (characteristic impedance and electrical length of transmission lines) into physical realization. Moreover, a single-layer microstrip realization is possible, making the unit cell attractive in practical applications. The proposed concept has been experimentally verified by the design and measurements of an exemplary transmission line section, and discussed in a context of possible applications in bandstop filters.

**Index Terms**—Artificial transmission lines, bandstop filters, dual-composite right/left-handed transmission lines, unit cells.

## I. INTRODUCTION

**M**ETAMATERIAL structures have gained recently a significant interest of microwave community due to their attractive properties. Various types of effectively homogenous metamaterial transmission lines composed of a finite number of elementary unit cells have been proposed and their applications are investigated. In [1] dual-composite right/left-handed (D-CRLH) unit cells (UC) have been introduced and characterized. The structures exhibit left-handed (LH) band at high frequencies, while the right-handed (RH) band is seen at low frequencies, moreover, they have inherently stop-band nature, i.e. high attenuation between LH and RH bands exists. Therefore, D-CRLH unit cells can be of interest for realization of microwave components, e.g., bandstop filters. Different D-CRLH unit cells together with physical dimensions of artificial transmission lines composed of such cells have been previously proposed [1]–[3]. However, the presented unit cells are modeled using ideal LC elements being easy to analyze on one hand, but featuring difficulties in straightforward translation into physical realization on the other.

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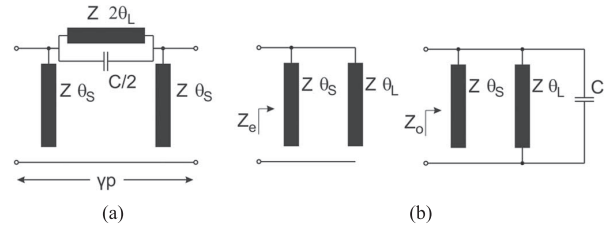


Fig. 1. Circuit schematic of the proposed finite-length, semi-distributed D-CRLH unit cell composed of transmission-line sections and a lumped capacitor (a) and its decomposition into even and odd mode circuits (b).

In this letter, we present a novel semi-distributed unit cell composed of sections of transmission lines and a series capacitor which features the D-CRLH character together with its appropriate design approach. In contrary to, e.g., [1]–[3] the unit cell is modeled mostly using distributed elements, which allows to keep the relation between model and physical realization simple. The proposed approach allows also to include all effects occurring in the structure related to the physical dimensions of the elements. Such an approach has been successfully applied in, e.g., [4] for realization of standard CRLH artificial transmission lines. The discussed unit cell is theoretically investigated and its behavior and properties are described and illustrated in Section II. Furthermore, formulas for calculation of unit cell element values are formulated and applied for the design of an exemplary D-CRLH transmission-line section. Since transmission lines composed of such unit cells are of bandstop nature, their application to bandstop filter realization is also considered.

## II. ANALYSIS AND DESIGN OF D-CRLH UNIT CELL

A schematic diagram of the proposed symmetrical semi-distributed D-CRLH unit cell composed of sections of transmission lines and a lumped capacitor is shown in Fig. 1(a). In the proposed unit cell the open-ended transmission-line stubs replace the shunt series LC resonant circuits, while series transmission line in parallel with a capacitor replace the series parallel LC resonant circuits of the D-CRLH unit cells investigated in, e.g., [1]. It has to be noted that the left-handed band of an ideal lumped-element unit cell shown in [1] can theoretically extend to infinity, whereas, in real implementation the LH band is limited due to finite unit cell dimensions. In contrary, this phenomenon is taken into account in the proposed unit cell, and the upper frequency limit can be predicted from the proposed model.

Since the unit cell is symmetrical, the even and odd mode excitation method presented in [5] can be used to for the analysis.

Such an approach significantly simplifies the analysis since the structure is decomposed into two less complex sub-networks described by even  $Z_e$  and odd  $Z_o$  impedances. Even and odd mode sub-circuits for the proposed unit cell are shown in Fig. 1(b) and the corresponding impedances are

$$Z_e = -j \frac{Z}{\tan(\theta_S) + tg(\theta_L)} \quad (1)$$

$$Z_o = -j \frac{Z \tan(\theta_L)}{\tan(\theta_L) \tan(\theta_S) - 1 + 2\pi f C Z \tan(\theta_L)} \quad (2)$$

where  $\theta_S = (f/f_0)\theta_{0S}$ ,  $\theta_L = (f/f_0)\theta_{0L}$ ,  $\theta_{0S}, \theta_{0L}$  are the electrical lengths defined at the frequency  $f_0$ , and  $f_0$  is the center frequency of the fundamental stopband between right-handed and left-handed regions. Having calculated  $Z_e$  and  $Z_o$ , one can determine basic properties of the unit cell [5], i.e., Bloch impedance and propagation constant as (3) and (4) as shown at the bottom of the page. By analyzing (1)–(3) it is possible to determine and engineer cut-off frequencies and characteristic (Bloch) impedance of the proposed unit cell.

The properties of an exemplary unit cell having  $f_0 = 2$  GHz,  $f_{co1} = 1.5$  GHz,  $f_{co2} = 2.5$  GHz,  $f_{B1} = 0.8$  GHz,  $f_{B2} = 3.1$  GHz,  $Z_B @ f_{B1,2} = 50 \Omega$  are shown in Fig. 2. Element values are:  $Z = 135 \Omega$ ,  $C/2 = 2.2$  pF,  $\theta_{0S} = 81.07^\circ$ ,  $2\theta_{0L} = 20.5^\circ$ . As seen from Fig. 2(a) the unit cell has a stopband between assumed frequencies.

The D-CRLH character of the proposed unit cell can be determined and verified based on phase and group velocities presented in Fig. 2(b). The unit cell features a left-handed character when phase and group velocities have opposite signs while for a right-handed character these velocities have the same sign [6]. It can be seen that an exemplary ideal unit cell features the right-handed character from 0 to 1.5 GHz, bandstop from 1.5 to 2.5 GHz and the left-handed character from 2.5 to 4 GHz. The Bloch impedance equals the assumed one  $Z_B = 50 \Omega$  [see Fig. 2(c)].

### III. D-CRLH TRANSMISSION LINE REALIZATION AND BANDSTOP FILTER APPLICATION CONSIDERATIONS

To verify the presented theoretical analysis and validate the properties of the proposed D-CRLH unit cell, a transmission-line section composed of four such cells ( $n = 4$ ), having parameters and element values as shown in the previous Section, has been designed, manufactured and measured. The circuit has been designed with the use of *AWR Design Environment* and manufactured on a single-layer laminate (*Arlon 25N*) having thickness of  $h = 0.508$  mm and permittivity  $\epsilon_r = 3.38$ . Moreover, a single SMD 0402 2.2 pF capacitor in each unit cell has

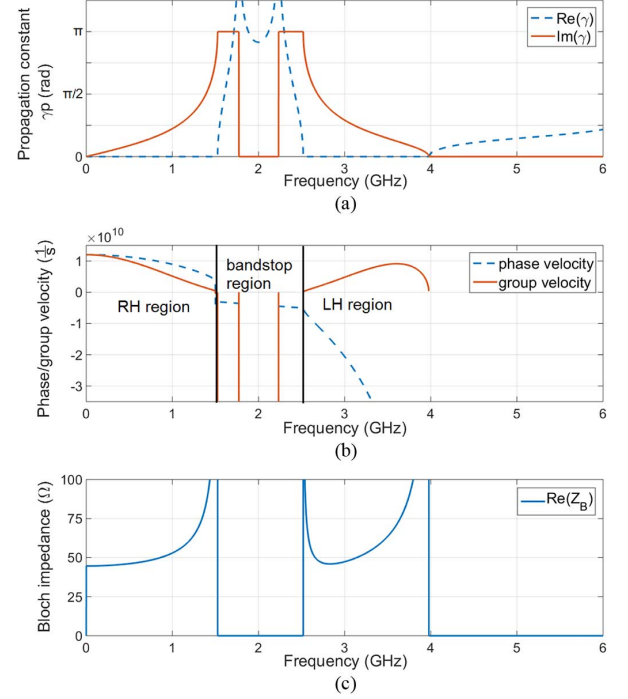


Fig. 2. Properties of an exemplary D-CRLH unit cell: (a) real and imaginary parts of propagation constant, (b) phase and group velocities (calculated from  $\beta p$  described by  $Im$  (Eq. (4))), and (c) real part of Bloch impedance.

been used to realize series capacitance. Dimensions of transmission lines have been found using *TXline* tool. Transitions to 50  $\Omega$  input/output lines have been incorporated during EM calculations, and electrical length of first and last stubs have been slightly tuned to cancel their negative influence. The measurement results vs. the characteristics of an ideal circuit are shown in Fig. 3.

As it can be seen, fundamental stopband as well as both right- and left-handed passbands of the measured transmission line section correspond to the results of the circuit calculations and feature good electrical performance. The discrepancy between ideal and measured circuits is due to conductor and dielectric losses. The difference between measured phase shift and the one calculated for an ideal circuit at higher frequencies is caused by parasitics of the utilized SMD capacitors. This has been confirmed by EM calculations along with circuit models of the SMD capacitors as presented in Fig. 3(b). The picture of the manufactured circuit is presented in Fig. 4.

The proposed D-CRLH transmission line can also be considered as a bandstop filter. It is worth noting that a very sharp slope of attenuation can be obtained at both edges of stopband even for such a small number of unit cells. Moreover, a metamaterial approach, i.e., utilization and chaining in series the same

$$Z_B = Z \sqrt{\frac{-\tan(\theta_L)}{[\tan(\theta_L) + tg(\theta_S)] [\tan(\theta_L)tg(\theta_S) - 1 + 2\pi f C Z \tan(\theta_L)]}} \quad (3)$$

$$\gamma p = \text{arccosh} \left( \frac{[\tan(\theta_L)tg(\theta_S) - 1 + 2\pi f C Z \tan(\theta_L)] + \tan(\theta_L) [\tan(\theta_S) + \tan(\theta_L)]}{[\tan(\theta_L) \tan(\theta_S) - 1 + 2\pi f C Z \tan(\theta_L)] - \tan(\theta_L) [\tan(\theta_S) + \tan(\theta_L)]} \right) \quad (4)$$

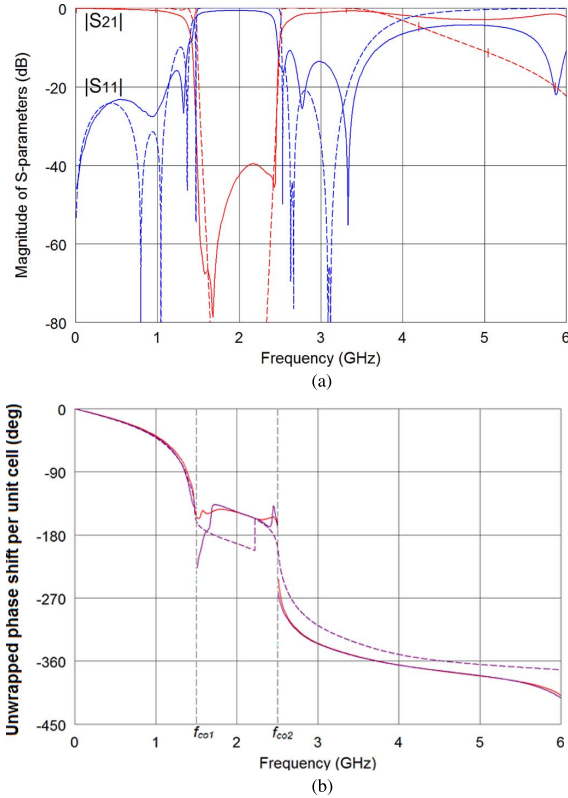


Fig. 3. S-parameters of the designed D-CRLH transmission line section composed of four proposed unit cells,  $n = 4$ : (a) magnitude, and (b) effective phase shift per unit cell. Dashed lines: ideal circuit calculation. Solid lines: measurements. Red line shows the EM calculated phase response for a circuit with included circuit models of SMD capacitors.

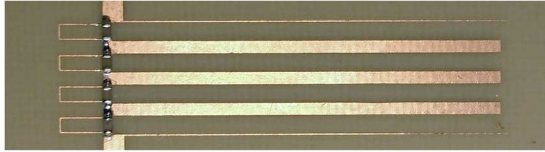


Fig. 4. A picture of the manufactured transmission line featuring D-CRLH character. Open-ended transmission lines of adjacent unit cells have been integrated and realized as one having impedance of  $Z_S/2$  to obtain a more compact layout. The size of the D-CRLH TL equals  $24.6 \times 6.3$  mm.

basic unit cells, for realization of such filters significantly simplifies its analysis and design process for practical applications, in comparison to the classic approach [6]. The provided design equations for realization of the proposed unit cell allow for fast and easy synthesis of the assumed filter transfer characteristics while the semi-distributed model itself allows for simple translation of constituting elements into physical realization. The proposed D-CRLH unit cell has been compared with alternative circuits in Table I. As seen the proposed unit cell requires three times and six times smaller area in comparison, respectively, to [1] and [7] with comparable insertion losses per unit cell. On the other hand the unit cell presented in [3] occupies smaller area

TABLE I  
PERFORMANCE COMPARISON OF THE PROPOSED UNIT CELL WITH OTHER REALIZATIONS, AND PROPOSED BANDSTOP FILTER WITH ALTERNATIVES

	Caloz [1]	Senior [3]	Song [7]	This paper
Element models	Lumped	Lumped	N/A	Semi-distr.
Size of unit cell $\lambda_g @ f_0$ for 50 $\Omega$ TL	.06 x .15	.012 x .06	.084 x .23	.013 x .233
Number of unit cells $n$	5	5	1	4
Stopband (GHz)	$\sim 3 - 4.5$	$2.9 - 5.1$	$2.1 - 4.2$	$1.4 - 2.5$
Insertion loss (dB)	$< 1$	$< 2.5$	$< 0.8$	$< 1.4$
for freq. (GHz)	DC to 2	DC to 2.5	DC to 1.8	DC to 1.2
Stopband rejection [dB]	-38	-30	-10	-40

due to the MMIC realization at the expense of larger insertion losses. Moreover, the rejection of the proposed filter is superior. It can be added that a further miniaturization can be achieved by shorting the stubs with additional lumped capacitors, e.g., by application of capacitors having  $C = 0.6$  pF a reduction of stubs' length would be up to 50%.

#### IV. CONCLUSION

A semi-distributed element approach to realization of a unit cell featuring a dual-composite right/left-handed character has been proposed. Theoretical analysis of the presented novel unit cell has been conducted and appropriate formulas allowing for fast and efficient synthesis of the unit cell featuring assumed properties have been formulated. The proposed approach has been verified by realization of an exemplary D-CRLH transmission line section composed of four such cells. Moreover, possible bandstop filter application has been discussed showing the compactness in longitudinal direction. The obtained measurement results have shown a good agreement with theoretical predictions and confirmed the applicability of the proposed approach.

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