

Question 5

$$(i) \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right) = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$= \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{array} \right) \rightarrow \underline{0+0 \neq -1} \rightarrow \boxed{\text{No solution}}$$

$$(ii) \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 2 & 2 & 4 & 4 \end{array} \right) = \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 \end{array} \right) = \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \rightarrow \begin{matrix} u_1 + u_3 = 1 \\ u_2 + u_3 = 1 \end{matrix}$$

Say $u_3 = c$: $u_1 = 1 - c$, $u_2 = 1 - c$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c \cdot \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \quad \text{where } c \text{ is any real number}$$