

Nuclear Reactor Theory Project #1

Group #3

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Material	$\Sigma_{tr}(\text{cm}^{-1})$	$\Sigma_a(\text{cm}^{-1})$	$\nu\Sigma_f(\text{cm}^{-1})$	Relative Absorption
H	1.79×10^{-2}	8.08×10^{-3}	0	0.053
O	7.16×10^{-3}	4.90×10^{-6}	0	0
Zr	2.91×10^{-3}	7.01×10^{-4}	0	0.005
Fe	9.46×10^{-4}	3.99×10^{-3}	0	0.026
^{235}U	3.08×10^{-4}	9.24×10^{-2}	0.145	0.602
^{238}U	6.95×10^{-3}	1.39×10^{-2}	1.20×10^{-2}	0.091
^{10}B	8.77×10^{-6}	3.41×10^{-2}	0	0.223
	3.62×10^{-2}	0.1532	0.1570	1.000

TABLE I
MACROSCOPIC CROSS SECTIONS

Abstract

THIS IS THE ABSTRACT

I. INTRODUCTION & BACKGROUND

Proving the capabilities and safety of a reactor design requires effective modeling of the neutron flux in the core (expressed in equation 1). For real cores, however, this is impossible, and must be first simplified, then discretized to provide the solution for a representative mesh.

For this project we have analyzed a simplified, monoenergetic, non-multiplying medium in one dimension. The flux originates from a single source at $x = 0$ with a strength of $S = 1 \times 10^8 \text{ s}^{-1}$. These assumptions simplify the transport equation to that presented in equation 2.

In the following sections, we will first describe the terms in equation 2, then provide both an analytical and a discrete solution. We will also provide an analysis of the accuracy of the analysis as a function of the number of nodes. Finally, we will analyze the solution for different coordinate systems to equation 2.

$$\frac{\partial n}{\partial t} + v\hat{\Omega} \cdot \nabla n + v\Sigma_t n(\mathbf{r}, E', \hat{\Omega}, t) = \int_{4\pi} d\hat{\Omega}' \int_0^\infty dE' v' \Sigma_s(E' \rightarrow E, \hat{\Omega}' \rightarrow \hat{\Omega}) n(\mathbf{r}, E', \hat{\Omega}', t) + s(\mathbf{r}, E, \hat{\Omega}, t) \quad (1)$$

$$-D_m \frac{d^2 \phi}{dx^2} + \Sigma_a^m \phi = \begin{cases} S & (x = 0) \\ 0 & (x > 0) \end{cases} \quad (2)$$

II. METHODOLOGY

Equation 2 is a simplified description of neutron diffusion through a finite medium, similar to a point source travelling through a shielding material to a detector. The flux, therefore, depends on the transport cross section. This is accounted for in the term D_m , which is related to the transport coefficient by $D_m = 3\Sigma_{tr}^{-1}$. Values for Σ_{tr} for typical reactor materials are found in table I.

A. Analytic Solution

In the slab, equation 2 is equal to 0, $-D_m \frac{\partial^2 \phi}{\partial x^2} + \Sigma_a^m \phi = 0$. In order to better group constants, specify a diffusion length, $L = \sqrt{D_m/\Sigma_a}$. We can then solve for $\phi(x)$:

$$\begin{aligned} \frac{\partial^2 \phi}{\partial x^2} - \frac{\phi}{L} &= 0 \\ \phi(x) &= Ae^{-x/L} + Ce^{x/L} \end{aligned} \quad (3)$$

First use the boundary condition $\phi(w) = 0$ to solve for C

$$\begin{aligned} 0 &= Ae^{-w/L} + Ce^{w/L} \\ C &= -Ae^{2w/L} \end{aligned}$$

Next, use the fact that $-D_m \phi'(0) = J(0) = S/2$ to solve for A

$$\begin{aligned} J(0) &= \frac{S}{2} = -\frac{A}{L} (1 + e^{-2w/L}) \\ A &= \frac{SL}{2D_m} (1 + e^{-2w/L})^{-1} \end{aligned}$$

Substituting this in to equation 3, we get:

$$\phi(x) = \frac{SL}{2D_m} \left(\frac{e^{-x/L} - e^{(x-2w)/L}}{1 - e^{-2w/L}} \right) \quad (4)$$

B. Numerical Approximation

It is rare to be faced with a design that allows for an analytical solution. Fortunately, numerical analysis methods exist that allow for approximation of the analytical solution. By dividing our hypothetical medium into discrete sections with nodes at the boundaries between these sections, it is possible to express the flux vector with equation 5.

$$\mathbf{A}\vec{\phi} = \vec{S} \quad (5)$$

III. RESULTS

Numerical Approximation of Flux for n Nodes

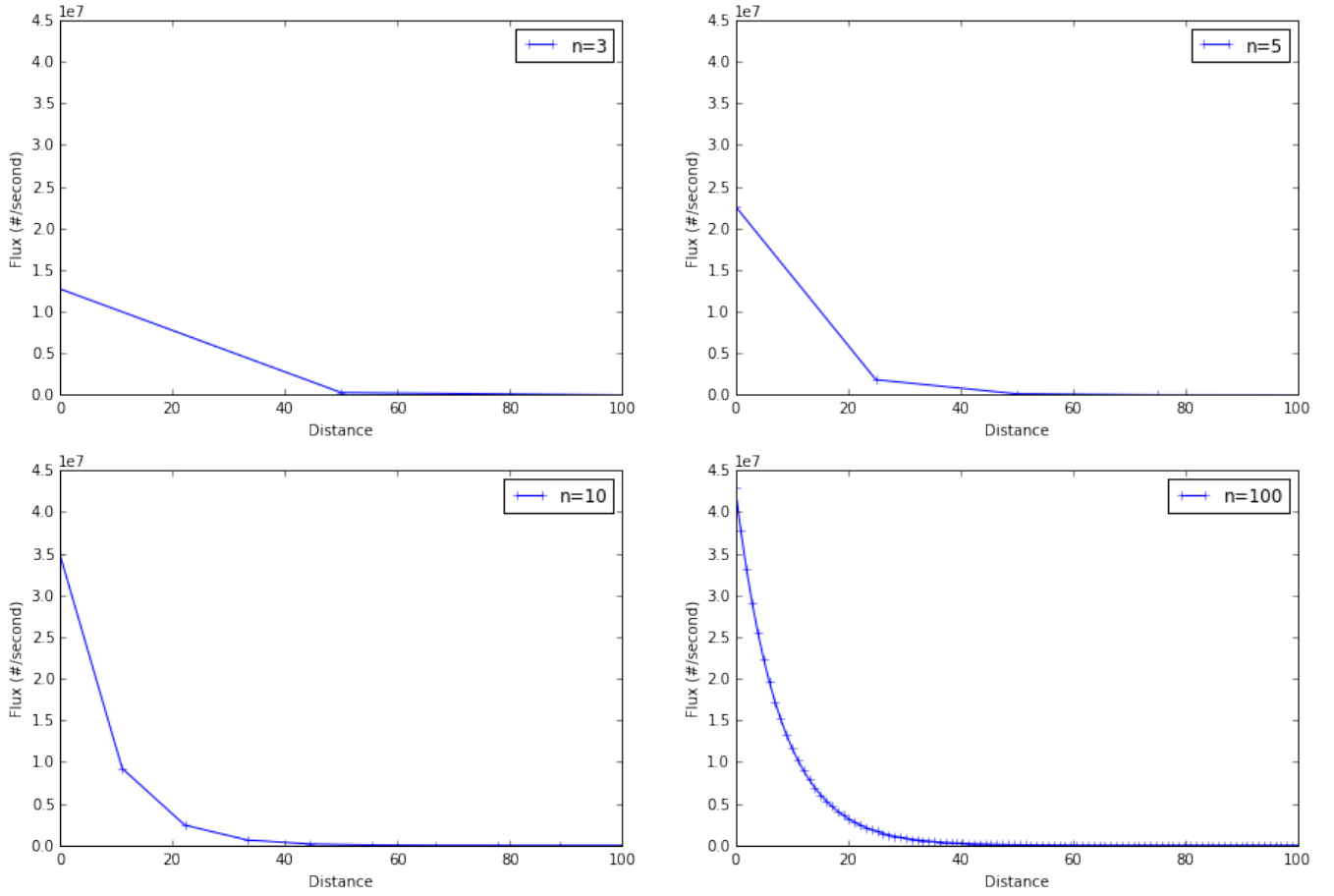


Fig. 1. Numerical Solution for 3, 5, 10, and 100 Nodes

IV. CONCLUSIONS

The conclusions go here

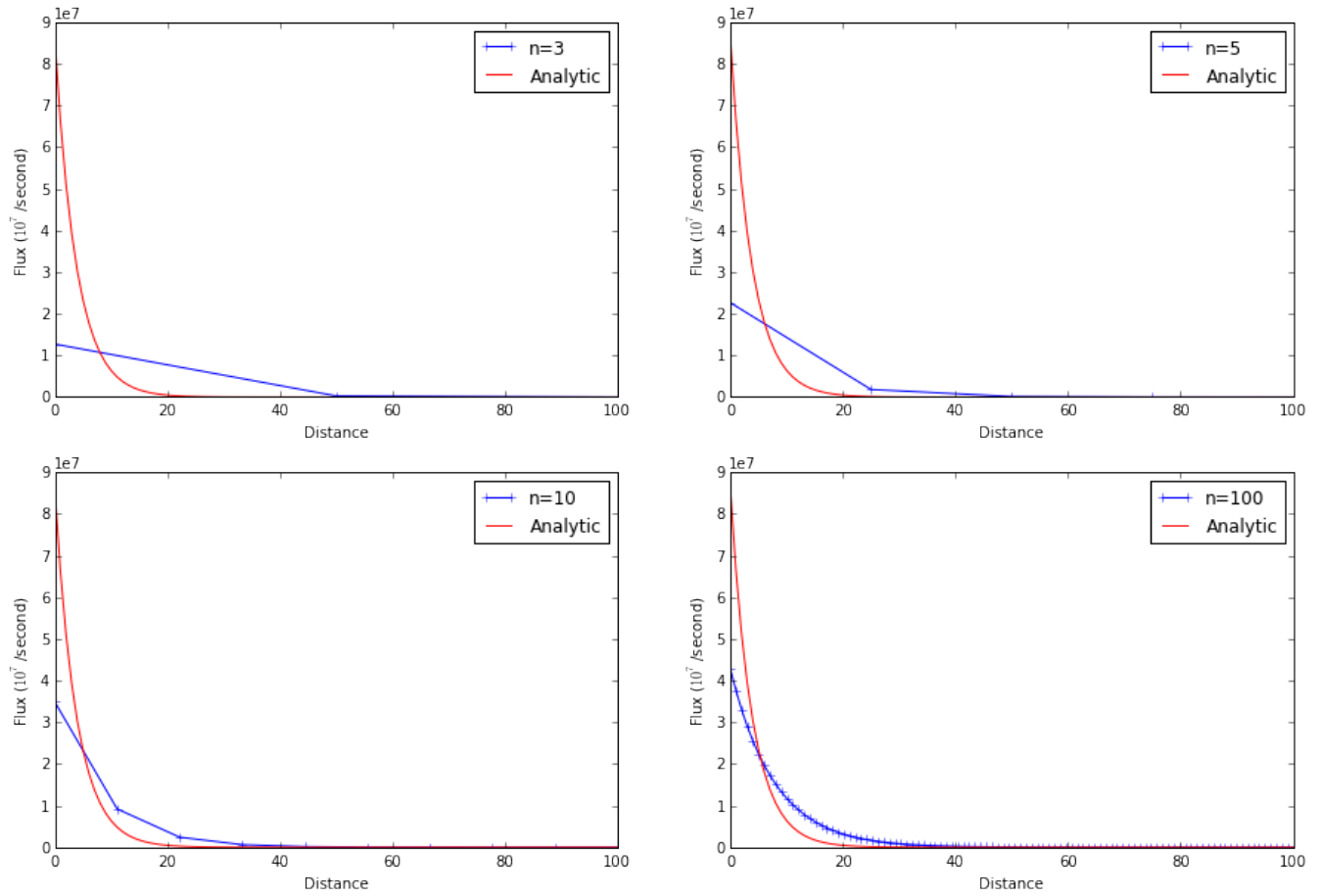
Flux vs. Distance for n Nodes

Fig. 2. Comparison of Numerical and Analytical Solutions