University of Tennessee
NE 470 Nuclear Reactor Theory

Group 3 Project 3

Multi-Group Diffusion Theory

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Content

- **Assumptions:**

  1) Isotropic assumption: ignoring the direction of the incoming neutron.

  2) Discretized energies: Neutrons are placed into energy bins or groups

  3) There is an interface between the moderator and reflector material.

  4) Steady-state assumption: the system has been in this state for a long period and that no transients occur

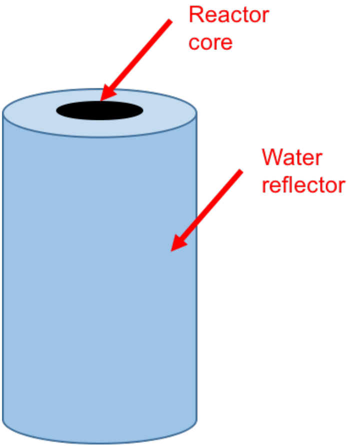
  5) There is no flux outside of the reflector ($r \geq W + \delta W$).

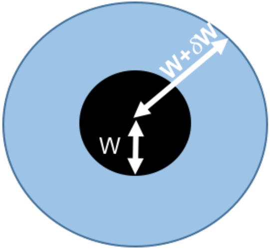
  6) All neutrons are born fast.

- **Multi-Group Diffusion Equation:**

$$-\nabla \cdot D_g \nabla \phi_g + v\Sigma_{Rg}\phi_g = \sum_{g'=1}^{g-1} \Sigma_{sg'g}\phi_{g'} + \frac{1}{k}\chi_g \nu_{g'} \Sigma_{tg'}\phi_{g'} \qquad (1)$$

- In this model, we consider a one-dimensional cylindrical reactor of radius W of a multiplying medium with a water reflector with infinite thickness.





- Using 2 and 4 group diffusion theory, the goal was to design a Fortran program that was able to determine the critical width of the reactor core with a typical PWR composition.

- 2 Group (fast and slow):

$$-\nabla \cdot D_1 \nabla \phi_1 + \Sigma_{R_1} \phi_1 = \frac{1}{k} [\nu_1 \Sigma_{f_1} \phi_1 + \nu_2 \Sigma_{f_2} \phi_2]$$

$$-\nabla \cdot D_2 \nabla \phi_2 + \Sigma_{a_2} \phi_2 = \Sigma_{s_{12}} \phi_1$$

- 4 Group:

$$-\nabla \cdot D_1 \nabla \phi_1 + \Sigma_{R_1} \phi_1 = \frac{\chi_1}{k} [\nu_1 \Sigma_{f_1} \phi_1 + \nu_2 \Sigma_{f_2} \phi_2 + \nu_3 \Sigma_{f_3} \phi_3 + \nu_4 \Sigma_{f_4} \phi_4]$$

$$-\nabla \cdot D_2 \nabla \phi_2 + \Sigma_{a_2} \phi_2 = \Sigma_{s_{12}} \phi_1$$

$$-\nabla \cdot D_3 \nabla \phi_3 + \Sigma_{a_3} \phi_3 = \Sigma_{s_{13}} \phi_1 + \Sigma_{s_{23}} \phi_2$$

$$-\nabla \cdot D_4 \nabla \phi_4 + \Sigma_{a_4} \phi_4 = \Sigma_{s_{14}} \phi_1 + \Sigma_{s_{24}} \phi_2 + \Sigma_{s_{34}} \phi_3$$

Key tools in solving for fininte-difference equations and matrix elements

- Cylindrical Gradient:

$$-D\frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr} + \Sigma_a\phi = S$$

Key tools in solving for fininte-difference equations and matrix elements

- Cylindrical Gradient:

$$-D\frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr} + \Sigma_a\phi = S$$

- Two Node Approximation:

$$\frac{d^2\phi}{dr^2} = \frac{\phi_{i-1} - 2\phi_i + \phi_{i+1}}{\Delta r^2}$$

Key tools in solving for finite-difference equations and matrix elements

- Boundary Conditions
  Mirror Boundary where $J = 0$ at $r = 0$
  Zero Flux Boundary $\phi = 0$ at $r = W + \delta W$

The following matrix element accounts for contributions from the reactor core:

$$\begin{bmatrix} \Sigma_{R,g}^{m}\Delta r + \frac{2D_g^m}{\Delta r} & -\frac{D_g^m}{\Delta r} & 0 & \dots \\ -\frac{D_g^m}{\Delta r}\left(1 - \frac{1}{2i-1}\right) & \frac{2D_g^m}{\Delta r} + \Sigma_{R,g}^{m}\Delta r & -\frac{D_g^m}{\Delta r}\left(1 + \frac{1}{2i-1}\right) & \dots \\ 0 & -\frac{D_g^m}{\Delta r}\left(1 - \frac{1}{2i-1}\right) & \frac{2D_g^m}{\Delta r} + \Sigma_{R,g}^{m}\Delta r & -\frac{D_g^m}{\Delta r}\left(1 + \frac{1}{2i-1}\right) \\ \dots & \dots & \frac{2D_g^m}{\Delta r} + \Sigma_{R,g}^{m}\Delta r & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

The following matrix element accounts for contributions from the reflector:

$$\begin{bmatrix} \cdots & \cdots & \cdots & \cdots \\ -\frac{D_g^R}{\Delta r_R}\left(1-\frac{1}{2i-1}\right) & \frac{2D_g^R}{\Delta r_R}+\Sigma_{a,g}^R\Delta r & -\frac{D_g^R}{\Delta r_R}\left(1+\frac{1}{2i-1}\right) & \cdots \\ \cdots & \frac{2D_g^R}{\Delta r_R}+\Sigma_{a,g}^R\Delta r_R & \cdots & \cdots \\ \cdots & 0 & -\frac{D_g^R}{\Delta r_R}\left(1-\frac{1}{2i-1}\right) & \frac{2D_g^R}{\Delta r_R}+\Sigma_{a,g}^R\Delta r_R \end{bmatrix}$$

The following matrix element accounts for the interface node:

$$\begin{bmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & -\frac{D_g^m}{\Delta r} & \frac{D_g^m}{\Delta r} + \frac{D_g^R}{\Delta r_R} + \frac{1}{2}\Sigma_{R,g}^m \Delta r + \frac{1}{2}\Sigma_{a,g}^R \Delta r_R & -\frac{D_g^R}{\Delta r_R} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

The following matrix element accounts for the fission source:

$$\begin{bmatrix} \frac{1}{2k}\chi_g \sum_{g=1}^{G} \nu_g \Sigma_{f,g}^{m} & 0 & \dots & \dots \\ 0 & \frac{1}{k}\chi_g \sum_{g=1}^{G} \nu_g \Sigma_{f,g}^{m} & 0 & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & 0 & \frac{1}{2k}\chi_g \sum_{g=1}^{G} \nu_g \Sigma_{f,g}^{m} \end{bmatrix}$$

The following matrix element accounts for inscattering from the moderator:

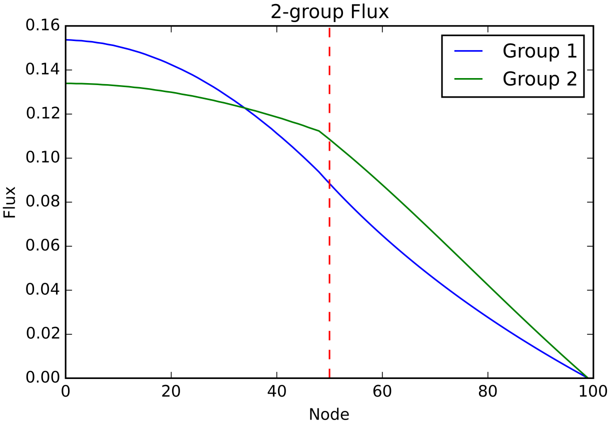
$$\begin{bmatrix} \frac{1}{2}\Sigma^{m}_{sg'g}\phi_{g'}\Delta r & 0 & \dots \\ 0 & \Sigma^{m}_{sg'g}\phi_{g'}\Delta r & 0 \\ \dots & \dots & \dots \end{bmatrix}$$

The following matrix element accounts for inscattering from the reflector and at the interface node:

$$\begin{bmatrix} \cdots & \cdots & \cdots \\ \frac{1}{2}\Sigma^{m}_{sg'g}\phi_{g'}\Delta r + \frac{1}{2}\Sigma^{R}_{sg'g}\phi_{g'}\Delta r_R & 0 & \cdots \\ 0 & \Sigma^{R}_{sg'g}\phi_{g'}\Delta r_R & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & 0 & \frac{1}{2}\Sigma^{R}_{sg'g}\phi_{g'}\Delta r_R \end{bmatrix}$$

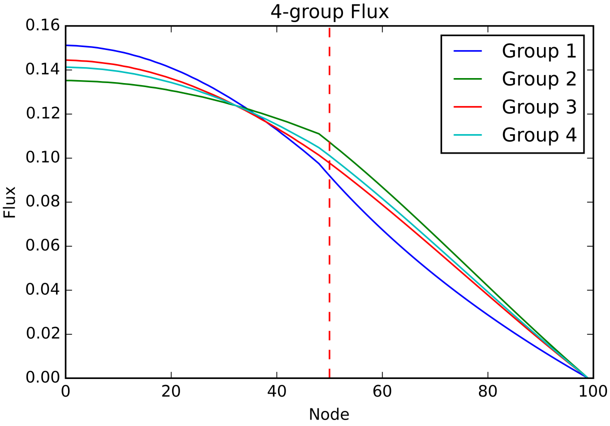
Because who doesn't like Christmas!
CHRISTMAS: CRIticality Search for neuTron Multi-group reActor Slab

- In spite of the (successful) corrections we made to the critical width search in Project 2, our code still did not successfully find this width
- The code converged on k values of 0.68 (2-group) and 0.48 (4-group)
- Result: core width was enlarged until we hit our maximum number of iterations, with no meaningful change in the k-value



2-group Flux
Flux
Node
Group 1
Group 2
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0
20
40
60
80
100

Results
Four Group



- Program uses command line prompts for parameters:
  1. Number of node in core
  2. Number of nodes in reflector
  3. ratio between the size of the regions (δW/W)
  4. Number of groups involved
- Outputs iterations required, profiles of group fluxes, and converged width and k-value

Improvements

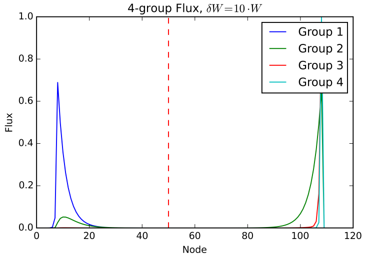
The following improvements are offered for future work:

- Criticality search
- Input file for group constants (generalize for n-groups)

Conclusion
Successes

- Group 1 flux falls off exponentially in reflector (no fast production in reflector)
- Reflector peaks appear in each of the other groups, but much more pronounced in Group 2
- Group 2 flux is the lowest at the center–perhaps because of low scattering production from Group 1 in core?
- Critical width sensitive to reflector size

- Final k value is reported as perfectly convergent (change between generations 0.0...). This is unexpected, and points to an error in our criticality search. That conclusion is supported by the lack of convergence in flux
- Similarly, critical width was determined to be 0.15 (2-group) and 8.44×10^{-2} (4-group). These ludicrously small values defy belief; further troubleshooting is called for
- Weird things happen when the reflector is much larger than the core...



Contributions

Lee S

- Derived matrix and source terms

Contributions

D Miller

- Derived matrix and source terms
- Graphics for presentation
- LaTeXfor presentation and report
- Programming

Contributions
A Payant

- Fortran coding and debugging
- Graphics generation

Contributions

J Powers-Luhn

- Generated the LaTeXtemplate[1]
- Presentation
- Fortran code debugging
- Graphics
- Brought candy

[1] modified from `https://www.overleaf.com/latex/templates/beamer-presentation-template-feather-theme/`, licensed under GPLv3

Questions?