

Nuclear Reactor Theory Project #1

Group #3

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Material	$\Sigma_{tr}(\text{cm}^{-1})$	$\Sigma_a(\text{cm}^{-1})$	$\nu\Sigma_f(\text{cm}^{-1})$	Relative Absorption
H	1.79×10^{-2}	8.08×10^{-3}	0	0.053
O	7.16×10^{-3}	4.90×10^{-6}	0	0
Zr	2.91×10^{-3}	7.01×10^{-4}	0	0.005
Fe	9.46×10^{-4}	3.99×10^{-3}	0	0.026
^{235}U	3.08×10^{-4}	9.24×10^{-2}	0.145	0.602
^{238}U	6.95×10^{-3}	1.39×10^{-2}	1.20×10^{-2}	0.091
^{10}B	8.77×10^{-6}	3.41×10^{-2}	0	0.223
	3.62×10^{-2}	0.1532	0.1570	1.000

TABLE I: Macroscopic Cross Sections

I. INTRODUCTION & BACKGROUND

Proving the capabilities and safety of a reactor design requires effective modeling of the neutron flux in the core (expressed in equation 1). For real cores, however, this is impossible, and must be first simplified, then discretized to provide the solution for a representative mesh. Simplifications that can be applied are:

- 1) **Isotropic assumption:** ignoring the direction of the incoming neutron. This effectively drops all terms involving $\hat{\Omega}$
- 2) **Monoenergetic assumption:** ignoring the energy variance of the neutrons. This means that a single set of cross sections can be used and allows us to ignore neutrons scattering into and out of the domain of interest
- 3) **Homogeneity assumption:** assuming that all core materials are evenly mixed throughout the volume of interest. This allows us to ignore discrete boundaries between materials
- 4) **Steady-state assumption:** assuming that the system has been in this state for a long period and that no transients occur. This allows us to remove all the time dependent terms.

For this project we have analyzed a simplified, monoenergetic, non-multiplying medium in one dimension. The flux originates from a single source at $x = 0$ with a strength of $S = 1 \times 10^8 \text{ s}^{-1}$. These assumptions simplify the transport equation to that presented in equation 2.

In the following sections, we will first describe the terms in equation 2, then provide both an analytical and a discrete solution. We will also provide an analysis of the accuracy of the analysis as a function of the number of nodes. Finally, we will analyze the solution to equation 2 for different coordinate systems.

$$\frac{\partial n}{\partial t} + v\hat{\Omega} \cdot \nabla n + v\Sigma_t n(\mathbf{r}, E', \hat{\Omega}, t) = \int_{4\pi} d\hat{\Omega}' \int_0^\infty dE' v' \Sigma_s(E' \rightarrow E, \hat{\Omega}' \rightarrow \hat{\Omega}) n(\mathbf{r}, E', \hat{\Omega}', t) + s(\mathbf{r}, E, \hat{\Omega}, t) \quad (1)$$

$$-D_m \frac{d^2 \phi}{dx^2} + \Sigma_a^m \phi = \begin{cases} S & (x = 0) \\ 0 & (x > 0) \end{cases} \quad (2)$$

II. METHODOLOGY

Equation 2 is a simplified description of neutron diffusion through a finite medium, similar to a point source travelling through a shielding material to a detector. The flux, therefore, depends on the transport cross section. This is accounted for in the term D_m , which is related to the transport coefficient by $D_m = 3\Sigma_{tr}^{-1}$. Values for Σ_{tr} for typical reactor materials are found in table ??.

A. Analytic Solution

In the slab, equation 2 is equal to 0, $-D_m \frac{\partial^2 \phi}{\partial x^2} + \Sigma_a^m \phi = 0$. In order to better group constants, specify a diffusion length, $L = \sqrt{D_m/\Sigma_a}$. We can then solve for $\phi(x)$:

$$\begin{aligned} \frac{\partial^2 \phi}{\partial x^2} - \frac{\phi}{L} &= 0 \\ \phi(x) &= Ae^{-x/L} + Ce^{x/L} \end{aligned} \quad (3)$$

First use the boundary condition $\phi(w) = 0$ to solve for C

$$\begin{aligned} 0 &= Ae^{-w/L} + Ce^{w/L} \\ C &= -Ae^{2w/L} \end{aligned}$$

Divide by Δx on both sides:

$$\begin{aligned} -D_m \frac{\phi_1 - \phi_0}{\Delta x^2} - \frac{S}{2\Delta x} + \frac{1}{2}\Sigma_a \phi_0 &= 0 \\ -\frac{D_m}{\Delta x^2} \phi_1 + \left(\frac{D_m}{\Delta x^2} + \frac{1}{2}\Sigma_a \right) \phi_0 &= 0 \end{aligned}$$

This gives a final matrix \mathbf{A} (for $n = 5$ nodes):

$$\begin{bmatrix} \frac{D_m}{\Delta x^2} + \frac{1}{2}\Sigma_a & -\frac{D_m}{\Delta x^2} & 0 & 0 \\ -\frac{D_m}{\Delta x^2} & \frac{2D_m}{\Delta x^2} + \Sigma_a & -\frac{D_m}{\Delta x^2} & 0 \\ 0 & -\frac{D_m}{\Delta x^2} & \frac{2D_m}{\Delta x^2} + \Sigma_a & -\frac{D_m}{\Delta x^2} \\ 0 & 0 & -\frac{D_m}{\Delta x^2} & \frac{2D_m}{\Delta x^2} + \Sigma_a \end{bmatrix}$$

Additional analysis was performed assuming the presence of a source of strength $f \times S$ located at $x = W$. This produced an operator with matrix values (this time for the $n = 4$ case):

$$\begin{bmatrix} \frac{D_m}{\Delta x^2} + \frac{1}{2}\Sigma_a & -\frac{D_m}{\Delta x^2} & 0 & 0 \\ -\frac{D_m}{\Delta x^2} & \frac{2D_m}{\Delta x^2} + \Sigma_a & -\frac{D_m}{\Delta x^2} & 0 \\ 0 & -\frac{D_m}{\Delta x^2} & \frac{2D_m}{\Delta x^2} + \Sigma_a & -\frac{D_m}{\Delta x^2} \\ 0 & 0 & -\frac{D_m}{\Delta x^2} & \frac{D_m}{\Delta x^2} + \frac{1}{2}\Sigma_a \end{bmatrix}$$

In this case the source vector, \vec{S} , would follow the form:

$$\begin{bmatrix} \frac{S}{2\Delta x} & 0 & \dots & 0 & f \frac{S}{2\Delta x} \end{bmatrix}$$

Similar derivations (attached) were performed for cylindrical and spherical coordinate systems. The operators for these (in the $n = 4$ case) were, for cylindrical:

$$\begin{bmatrix} \frac{D_m}{\Delta x^2} + \frac{1}{2}\Sigma_a & -\frac{D_m}{\Delta x^2} \left[1 + \frac{1}{2*i+1} \right] & 0 & 0 \\ -\frac{D_m}{\Delta x^2} \left[1 - \frac{1}{2*i-1} \right] & \frac{2D_m}{\Delta x^2} + \Sigma_a & -\frac{D_m}{\Delta x^2} \left[1 + \frac{1}{2*i+1} \right] & 0 \\ 0 & -\frac{D_m}{\Delta x^2} \left[1 - \frac{1}{2*i-1} \right] & \frac{2D_m}{\Delta x^2} + \Sigma_a & -\frac{D_m}{\Delta x^2} \left[1 + \frac{1}{2*i+1} \right] \\ 0 & 0 & -\frac{D_m}{\Delta x^2} \left[1 - \frac{1}{2*i-1} \right] & \frac{D_m}{\Delta x^2} + \frac{1}{2}\Sigma_a \end{bmatrix}$$

and for spherical:

$$\begin{bmatrix} \frac{D_m}{\Delta x^2} + \frac{1}{2}\Sigma_a & -\frac{D_m}{\Delta x^2} \left[1 + \frac{2}{2*i+1} \right] & 0 & 0 \\ -\frac{D_m}{\Delta x^2} \left[1 - \frac{2}{2*i-1} \right] & \frac{2D_m}{\Delta x^2} + \Sigma_a & -\frac{D_m}{\Delta x^2} \left[1 + \frac{2}{2*i+1} \right] & 0 \\ 0 & -\frac{D_m}{\Delta x^2} \left[1 - \frac{2}{2*i-1} \right] & \frac{2D_m}{\Delta x^2} + \Sigma_a & -\frac{D_m}{\Delta x^2} \left[1 + \frac{2}{2*i+1} \right] \\ 0 & 0 & -\frac{D_m}{\Delta x^2} \left[1 - \frac{2}{2*i-1} \right] & \frac{D_m}{\Delta x^2} + \frac{1}{2}\Sigma_a \end{bmatrix}$$

III. RESULTS

A Fortran program was developed to implement the discrete solution to the transport equation from section II-B. The source code is included as an attachment to this report. The numerical solution produced values that converged as the number of nodes increased, showing clearly exponential behavior as $n \rightarrow \infty$. This behavior can be seen in figures 1 and 2.

We have also examined this solution analytically in different coordinate systems, specifically cartesian, cylindrical, and spherical. The output of these solutions is easily seen in figure 3. Of note, the derivation for spherical coordinates did not allow for incorporation of the boundary condition $\phi(W) = 0$, which may account for the difference in that curve.

IV. CONCLUSIONS

We provide an analysis of a simplified one-group, one-dimensional neutron diffusion in a non-multiplying medium. Solutions are compared in assorted coordinate systems and for numerical approximation of the analytic solution. The various approaches to this problem produced similar but slightly different solutions, implying that careful choice of methodology is necessary in solving similar problems.

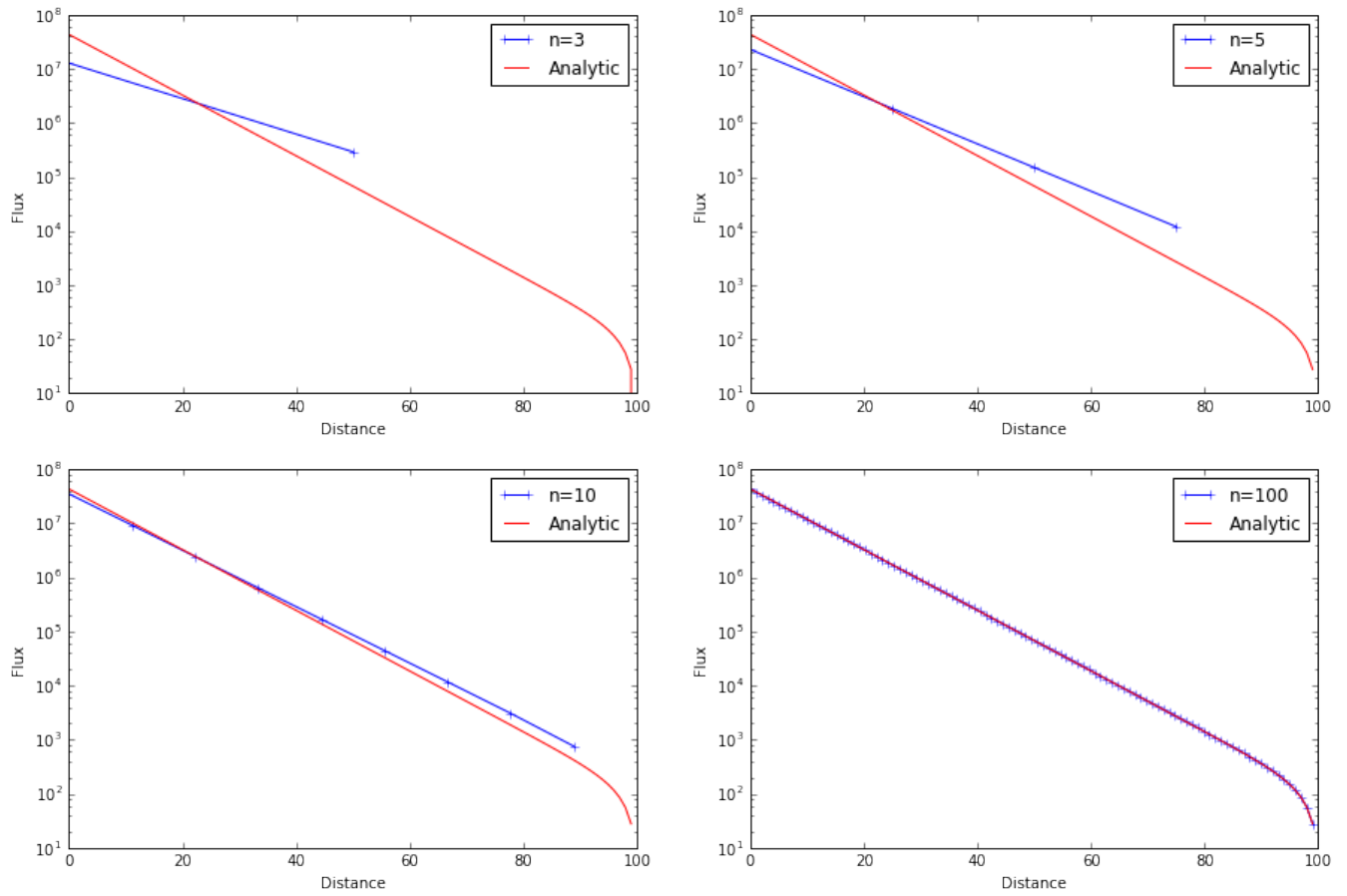


Fig. 1: Comparison of Numerical and Analytical Solutions

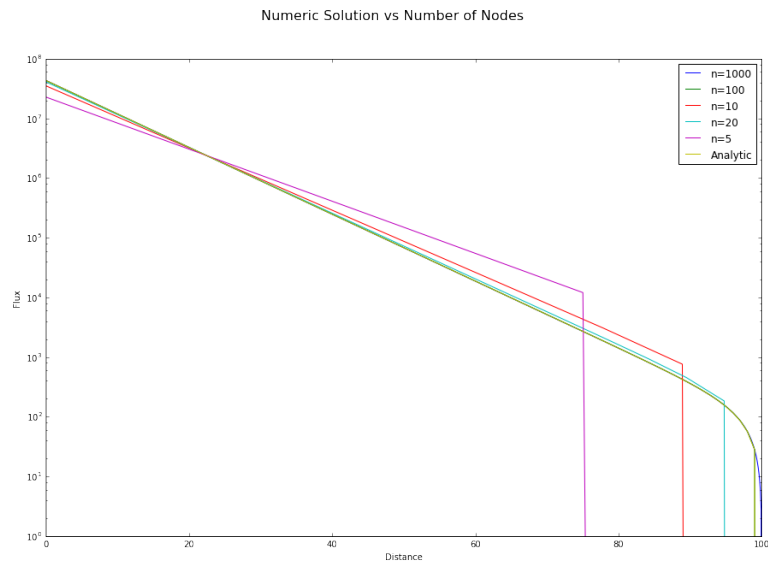


Fig. 2: Change in solution based on number of nodes

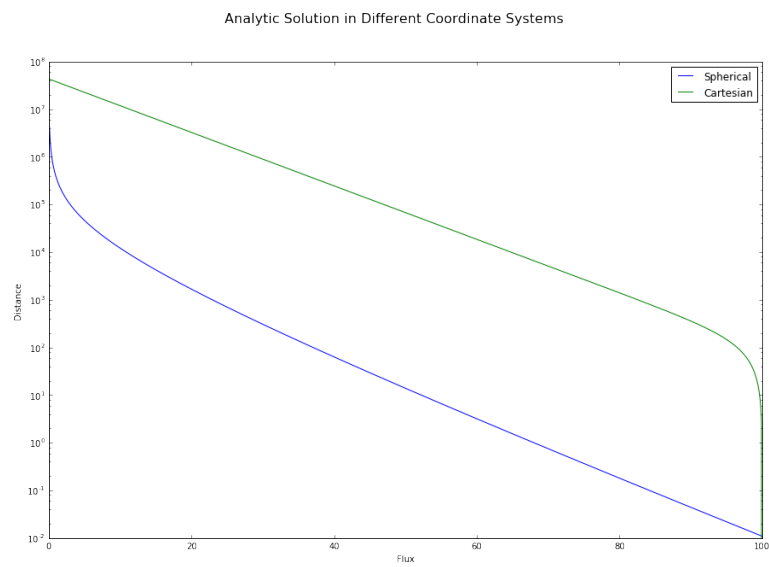


Fig. 3: Analytical Solution for Cartesian and Spherical Coordinates