Problem 1. 2-1

What target isotope must be used for forming the compound nucleus ${}^{24}_{11}\mathrm{Na}$ when the incident projectile is:

(a) a neutron

(b) a proton

(c) an alpha particle?

Solution

Part (a)

A neutron will increase the mass number, A, by one, but leave the element number, Z, unchanged. Therefore, the answer is a lighter isotope of Neon: ${}^{23}_{11}\mathrm{Na}$

Part (b)

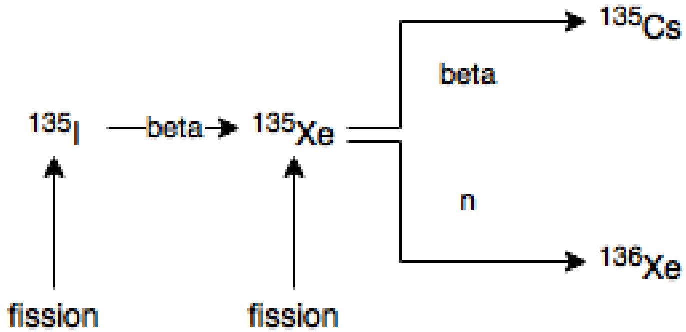
Capturing a proton increases both the mass number and element number by one: ${}^{23}_{10}\mathrm{Ne}$

Part (c)

Capturing an α particle increases the mass number by four and the element number by two: ${}^{20}_{8}\mathrm{O}$

Problem 2. 2-4

A fission product of very considerable importance in thermal reactor operation is ^{135}Xe, which has an enormous thermal absorption cross section of $2 * 10^6 b$. This nuclide can be produced either directly as a fission product or by beta decay of ^{135}I, as indicated by the radioactive chains below:



Write the rate equations describing the concentration of ^{135}I and ^{135}Xe in a nuclear reactor. Then assuming a constant production rate of these isotopes from fission and transmutation rate by neutron capture, determine the steady-state or saturated concentration of ^{135}Xe.

Solution

^{135}I has one production path (production as a fission daughter) and one decay path (β decay into ^{135}I). The rate equation is:

$$\dot{N}_{^{135}\mathrm{I}} = \phi\Sigma_{fission}\gamma_{^{135}\mathrm{I}} - \lambda_{^{135}\mathrm{I}}N_{^{135}\mathrm{I}}$$

^{135}Xe is also produced by fission, as well as by the β decay of ^{135}I. It is removed by β decay as well as by neutron absorption:

$$\dot{N}_{^{135}\mathrm{Xe}} = \phi\Sigma_{fission}\gamma_{^{135}\mathrm{Xe}} + \lambda_{^{135}\mathrm{I}}N_{^{135}\mathrm{I}} - \lambda_{^{135}\mathrm{Xe}}N_{^{135}\mathrm{Xe}} - \sigma_a\phi N_{^{135}\mathrm{Xe}}$$

In steady state, set $\dot{N} = 0$:

Iodine:

$$\dot{N}_{^{135}_{53}\mathrm{I}} = 0 \rightarrow$$
$$\lambda_I N_I = \phi\Sigma_f\gamma_I$$
$$N_{^{135}_{53}\mathrm{I}} = \frac{\phi\Sigma_f\gamma_I}{\lambda_I}$$

Xenon:

$$
\begin{aligned}
\dot{N}_{Xe} &= 0 \rightarrow \\
N_{Xe}\left(\lambda_{Xe} + \sigma_{Xe}\phi\right) &= \phi\Sigma_f\gamma_{Xe} + \lambda_I N_I \\
N_{Xe} &= \frac{\phi\Sigma_f\gamma_{Xe} + \lambda_I N_I}{\lambda Xe + \sigma_{Xe}\phi} \\
&= \frac{\phi\Sigma_f\gamma_{Xe} + \phi\Sigma_f\gamma_I}{\lambda_{Xe} + \sigma_{Xe}\phi} \\
N_{{}^{135}_{54}\mathrm{Xe}} &= \frac{\phi\Sigma_f\left(\gamma_{Xe} + \gamma_I\right)}{\lambda_{Xe} + \sigma_{Xe}\phi}
\end{aligned}
$$

Problem 3. 2-6

Boron is a common material used to shield against thermal neutrons. Estimate the thickness of boron required to attenuate an incident thermal neutron beam to 0.1% of its intensity. (Use the thermal cross section data in Appendix A.)

Solution

From *Duderstadt* Appendix A, $\Sigma_t = 104cm^{-1}$ for Boron.

$$\begin{aligned}\left(\frac{1}{e}\right)^n &= \frac{1}{1000}\\ e^{-n} &= 1000^{-1}\\ e^n &= 1000\\ n &= ln(1000)\end{aligned}$$

Dividing this by the macroscopic cross section Σ_t gives:

$$\begin{aligned}\frac{n}{\Sigma_t} &= \frac{ln(1000)}{\Sigma_t}\\ &= 0.0664cm\end{aligned}$$

Problem 4. 2-8

A free neutron is unstable against beta decay with a half-life of 11.7m. Determine the relative probability that a neutron will undergo beta-decay before being absorbed in an infinite medium. Estimate this probability for a thermal neutron in H_2O.

Solution

To determine the relative probability, we take a ratio of the mean free path for neutron absorption and divide it by the distance travelled by a thermal neutron before decaying:

$$\begin{aligned}\frac{x_{absorption}}{x_{decay}} &= \frac{{\Sigma_a}^{-1}}{\dot{x}t_{1/2}} \\ &= \left(\Sigma_a \dot{x} t_{1/2}\right)^{-1}\end{aligned}$$

Subsituting the values for a thermal ($\dot{x} = 2.2 * 10^5 cm/s$) neutron in water ($\Sigma_a = 0.022cm^{-1}$):

$$\left(\left(0.022cm^{-1}\right)\left(2.2 * 10^5 \frac{cm}{s}\right)\left(11.7minutes * \frac{60s}{minute}\right)\right)^{-1} = 2.94 * 10^{-7}$$

It is much more likely that the neutron will decay rather than be absorbed in water (once thermalized).

Problem 5. 2-10

How many mean free paths thick must a shield be designed in order to attenuate an incident neutron beam by a factor of 1000?

Solution

We know that for every mean free path, Σ, travelled, the incident beam attenuates by a factor of $\frac{1}{e}$. Therefore:

$$\begin{aligned}
\left(\frac{1}{e}\right)^n &= \frac{1}{1000} \\
e^{-n} &= 1000^{-1} \\
e^n &= 1000 \\
n &= ln(1000) \\
&= 6.91
\end{aligned}$$