Problem 1.

Use the product differentiation rule to show that the conservative and non-conservative forms of the 1D spherical equation are identical.

Solution

Non-conservative form:

$$\mu \frac{\partial \Psi(r,\mu,E)}{\partial r} + \frac{1-\mu^2}{r} \frac{\partial \Psi(r,\mu,E)}{\partial \mu} + \sigma_t(r,E)\Psi(r,\mu,E) = q(r,\mu,E)$$

Conservative form:

$$\frac{\mu}{r^2} \frac{\partial \left[r^2 \Psi(r,\mu,E)\right]}{\partial r} + \frac{\partial}{\partial \mu} \left[\frac{\left(1-\mu^2\right)\Psi(r,\mu,E)}{r}\right] \sigma_t(r,E)\Psi(r,\mu,E) = q(r,\mu,E)$$

$$\frac{\mu}{r^2}\left[2r\Psi(r,\mu,E) r^2 \Psi(r,\mu,E)\right] + \frac{1}{r}\left[-2\mu\Psi(r,\mu,E) + \left(1-\mu^2\right)\frac{\partial \Psi(r,\mu,E)}{\partial \mu}\right] + \sigma_t \Psi(r,\mu,E) = q(r,\mu,E)$$

$$\cancel{\frac{2\mu}{r}\Psi(r,\mu,E)} + \mu \frac{\partial \Psi(r,\mu,E)}{\partial r} - \cancel{\frac{2\mu}{r}\Psi(r,\mu,E)} + \frac{1-\mu^2}{r}\frac{\partial \Psi(r,\mu,E)}{\partial \mu} + \sigma_t \Psi(r,\mu,E) = q(r,\mu,E)$$

$$\mu \frac{\partial \Psi(r,\mu,E)}{\partial r} + \frac{1-\mu^2}{r} \frac{\partial \Psi(r,\mu,E)}{\partial \mu} + \sigma_t(r,E)\Psi(r,\mu,E) = q(r,\mu,E)$$

Problem 2.

Use the product differentiation rule to show that the conservative and non-conservative forms of the 1D cylindrical equation are identical.

Solution

Non-conservative form:

$$\mu\frac{\partial\Psi(r,\omega,\xi,E)}{\partial r}-\frac{\eta}{r}\frac{\partial\Psi(r,\omega,\xi,E)}{\partial\omega}+\sigma_t(r,E)\Psi(r,\omega,\xi,E)=q(r,\omega,\xi,E)$$

Conservative form:

$$\frac{\mu}{r}\frac{\partial\left[r\Psi(r,\omega,\xi,E)\right]}{\partial r}-\frac{1}{r}\frac{\partial\left[\eta\Psi(r,\omega,\xi,E)\right]}{\partial\omega}+\sigma_t(r,E)\Psi(r,\omega,\xi,E)=q(r,\omega,\xi,E)$$

Problem 3.

Show that the white boundary condition is given by:

$$\Psi(\vec{r}_s, \hat{\Omega}, E, t) = 4J_n^+(\vec{r}_s, E, t)$$

Solution