Problem 1.

Use the NIST ASTAR to find the range of a 10 MeV alpha particle, and then use the range values at lower energies to determine at what depth a 10 MeV/nucleon alpha particle has traveled 99% of its range. Do the same for a 20 MeV/nucleon alpha and 50 MeV/nucleon alpha. Use this information for your semester stopping power assignment to help guide you in determining a lower energy limit/cutoff when calculating ranges.

Solution

T	CSDA	1% CSDA	T′
10 MeV	1.13×10^{-2} cm	1.13×10^{-4} cm	7.00×10^{-2} MeV
40 MeV	1.240×10^{-1} cm	1.240×10^{-3} cm	2.25 MeV
80 MeV	4.287×10^{-1} cm	4.287×10^{-3} cm	5.5 MeV
200 MeV	2.240 cm	2.240×10^{-2} cm	15 MeV

Table 1: Estimated ranges after 99% energy loss

From ASTAR, a 10 MeV α particle travels 1.13×10^{-2} cm in water. At the time that it has travelled 99% of this range, it has an energy of 7×10^{-2} MeV. Similar results are summarized in table 1

Problem 2. Anderson 4.2

Use Eqution 4.13 to calculate the range of a 5 MeV alpha particle in N_2 gas at 760 mmHg pressure. Assume that the exponential integral at T_1 (low-energy limit) and $R_1(T_1)$ can be neglected.

$$\begin{aligned} R &= \frac{Mc^2 I^2}{32 z^2 \pi r_0^2 \left(m_e c^2\right)^3 N_A \left(Z/M_m\right) \rho} \int_{u_1}^{u_0} \frac{du}{\ln u} + R_1\left(T_1\right) \\ &= \frac{Mc^2 I^2}{32 z^2 \pi r_0^2 \left(m_e c^2\right)^3 N_A \left(Z/M_m\right) \rho} \left[\mathrm{Ei}\left(\ln u_0\right) - \mathrm{Ei}\left(\ln u_1\right)\right] + R_1\left(T_1\right) \end{aligned} \tag{4.13}$$

with

$$u = \left(\frac{4 m_e c^2 \tau}{I}\right)^2 = \left(\frac{4 m_e \cancel{c^2} T}{I M \cancel{c^2}}\right)^2 \tag{1}$$

Solution

See attached addendum for calculations.

$$\mathrm{R} = 3.04$$

Problem 3. Anderson 4.5

Calculate the ratio of the range of a 14 MeV $^{14}N^{+++}$ ion to the range of a 1 MeV proton. Use equation 4.18.

$$(R\rho)_b \approx \frac{(M/z^2)_b}{(M/z^2)_a} (R\rho)_a \tag{4.18}$$

Solution

We know that for particles with similar values of $\tau = T/mc^2$, equation 4.18 can be used to compare Range-density values. Therefore:

$$\begin{aligned}(R\rho)_b &\approx \frac{(M/z^2)_b}{(M/z^2)_a} (R\rho)_a \\ &\approx \frac{(14/3^2)_b}{(1/1^2)_a} (R\rho)_a \\ &\approx 1.56 (R\rho)_a\end{aligned}$$

The 14 MeV $^{14}N^{+++}$ ion will travel approximately 1.56 times further than the 1 MeV proton.

Problem 4.

Use the NIST PSTAR utility to do the following: produce a Bragg Curve for 250 MeV protons passing through a variable water column, as measured by two air ionization chambers, each 1 cm thick. Neglect energy loss in the ion chamber windows and in any air gaps between the ion chambers and water column. NIST states a 250 MeV proton ranges out in about 38 cm of water, so calculate the ratio of the ion chamber currents at 0 cm, 10 cm, 20 cm, 30 cm, 35 cm, 36 cm, 37 cm and 38 cm of water between the ion chambers.

Assume dry air in the ion chambers at a density of $0.0012\,\mathrm{g/cm^3}$. More details about this problem will be covered in class.

Solution

Calcluations are performed in the attached code. The curve produced is show in figure 1

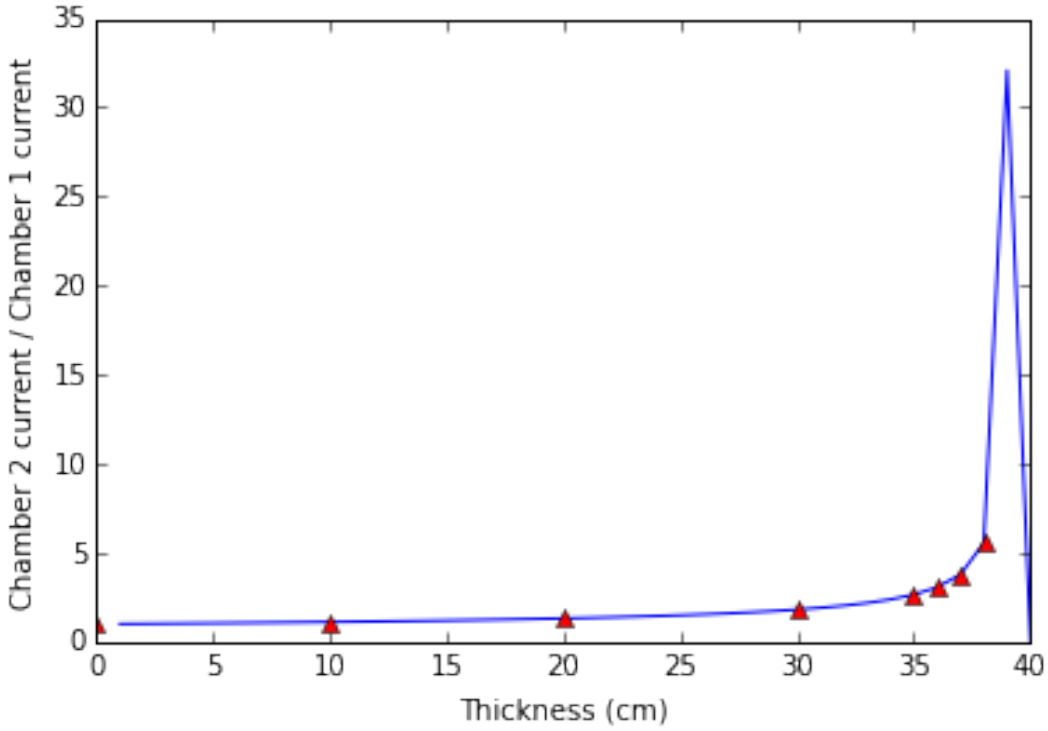


Figure 1: Bragg Curve for 250MeV proton in water

NE551 Homework 4 Addendum

J.R. Powers-Luhn
2016/09/28

Problem 2 (Anderson 4.2)

In [255]:
```
%%javascript
MathJax.Extension["TeX/cancel"]={version:"2.4.0",ALLOWED:{color:1,mathcolor:1,background:1,mathbackground:1,padding:1,thickness:1}};MathJax.Hub.Register.StartupHook("TeX Jax Ready",function(){var c=MathJax.InputJax.TeX,a=MathJax.ElementJax.mml,b=MathJax.Extension["TeX/cancel"];b.setAttributes=function(h,e){if(e!==""){e=e.replace(/ /g,"").split(/,/);for(var g=0,d=e.length;g<d;g++){var f=e[g].split(/[:=]/);if(b.ALLOWED[f[0]]){if(f[1]==="true"){f[1]=true}if(f[1]==="false"){f[1]=false}h[f[0]]=f[1]}}}return h};c.Definitions.Add({macros:{cancel:["Cancel",a.NOTATION.UPDIAGONALSTRIKE],bcancel:["Cancel",a.NOTATION.DOWNDIAGONALSTRIKE],xcancel:["Cancel",a.NOTATION.UPDIAGONALSTRIKE+" "+a.NOTATION.DOWNDIAGONALSTRIKE],cancelto:"CancelTo"}},null,true);c.Parse.Augment({Cancel:function(e,g){var d=this.GetBrackets(e,""),f=this.ParseArg(e);var h=b.setAttributes({notation:g},d);this.Push(a.menclose(f).With(h))},CancelTo:function(e,g){var i=this.ParseArg(e),d=this.GetBrackets(e,""),f=this.ParseArg(e);var h=b.setAttributes({notation:a.NOTATION.UPDIAGONALSTRIKE+" "+a.NOTATION.UPDIAGONALARROW},d);i=a.mpadded(i).With({depth:"-.1em",height:"+.1em",voffset:".1em"});this.Push(a.msup(a.menclose(f).With(h),i))}});MathJax.Hub.Startup.signal.Post("TeX cancel Ready")});MathJax.Ajax.loadComplete("[MathJax]/extensions/TeX/cancel.js");
```

$$R = \frac{M \cancel{c^2} I^2}{32 z^2 \pi r_0^2 \left(m_e \cancel{c^2} \right)^3 N_A \, (Z/M_m) \, \rho} \left[\mathrm{Ei}(\ln u_0) - \cancel{\mathrm{Ei}(\ln u_1)} \right] + \cancel{R_1(T_1)}$$

and

$$u = \left(\frac{4 m_e c^2 \tau}{I} \right)^2 = \left(\frac{4 m_e \cancel{c^2} T}{I M \cancel{c^2}} \right)^2$$

In [256]:
```
import scipy.constants as const
from scipy.special import expi # Exponential integral function, Ei
from math import log, pi
import numpy as np
from scipy.interpolate import interp1d
```

In [257]:
```
# Constants!
I = 82 * 10 ** -6 # MeV
M = const.physical_constants['alpha particle mass energy equivalent in MeV'][0] # 3727.379... MeV
z = 2 # incident alpha particle
m_e = const.physical_constants['electron mass energy equivalent in MeV'][0]
r_0 = const.physical_constants['classical electron radius'][0] * 100 # cm
N_A = const.physical_constants['Avogadro constant'][0]
Z = 14
M_m = 28.014 # g/mol
rho = 0.00125 # g/cm^3
```

In [258]:
```
def u(T):
    numerator = 4 * m_e * T
    denominator = I * M
    return (numerator / denominator) ** 2
```

In [259]:
```
u(5)
```

Out[259]:
```
1118.0599713539173
```

In [260]:
```
# Approximation of Exponential integral
u(5) / log(u(5))
```

Out[260]:
```
159.28254390194414
```

In [261]:
```
def R(T, approxEi=False):
    """ Gives predicted range
        Can optionally specify approxEi to use the log approximation for Ei(log(u))
    """
    numerator = M * I * I
    denominator = 32 * z ** 2 * pi * r_0 ** 2 * m_e ** 3 * N_A * Z * rho / M_m
    if not(approxEi):
        ei = expi(log(u(T)))
    else:
        ei = u(T) / log(u(T)) # Approximation as u >> 1
    return numerator * ei / denominator
```

In [262]:
```
print R(5, approxEi=True)
```

```
2.49058984845
```

In [263]:
```
print R(5, approxEi=False)
```

```
3.04222192314
```

Problem 4

In [279]:
```
r_e = r_0 # Copying code from scratch sheet
I_water = 75.0
I_air = 85.7
rho_water = 1.0
rho_air = 0.0012
```

In [280]:
```
def beta_squared(T, m):
    """ Gives value of beta^2 for a given Mass (MeV/c^2) and Kinetic E
nergy (MeV)
    """
    numerator = T * (T + 2 * m)
    denominator = (T + m) ** 2
    return np.divide(numerator, denominator)
```

In [281]:
```
def gamma_squared(T, m):
    """ Gives value of gamma^2 for a given mass (MeV/c^2) and Kinetic
Energy (MeV)
    """
    return 1.0 / (1 - beta_squared(T, m)) ** 0.5
```

In [282]:
```
def S_c(incident, target, T, M_b, density, I):
    """ Returns the stopping power in MeV/cm
        T in MeV
        density in g/cm^3
        Output in MeV/cm
    """
    # Currently the incident is specified in Z number. Incident is ass
umed to be fully ionized
    z = incident

    # Z is actually Z/A
    Z = target

    first = 4 * (z ** 2) * pi * (r_e ** 2) * m_e
    second = N_A * Z * density
    third = 1.0 / beta_squared(T, M_b)
    logpart = (2 * m_e * beta_squared(T, M_b) * gamma_squared(T, M_b))
/ (I * 10 ** -6)
    fourth = np.log(logpart) - beta_squared(T, M_b) + beta_squared(T,
M_b)

    result = first * second * third * fourth

    return result
```

In [283]:
```
S_c_chamber_one = S_c(1.0, 0.55, 250.0, 937.0, rho_air, I_air)
```

In [284]:
```
T_array = np.linspace(-10.0, 300.0, 260000)
```

In [285]:
```
interpolator = interp1d(
    T_array,
    np.piecewise(
        T_array,
        [T_array <= 0.001, T_array > 0],
        [
            lambda T_array: 0.0,
            lambda T_array: S_c(1.0, 0.55, T_array, 937.0, rho_water,
I_water)
        ]
    )
)
```

In [286]:
```
S_c(1.0, 0.55, 250.0, 937.0, 1.0, 75.0)
```

Out[286]: 3.934864903456555

In [287]:
```
interpolator(250)
```

Out[287]:
```
array(3.9348649034678584)
```

In [311]:
```
def EofT(E0, t, step=1):
    if t<step:
        return E0
    else:
        e = EofT(E0, t-step, step=step)
        if e > E0:
            return E0
        elif e>=0:
            temp = e - step * interpolator(e)
            if temp >= 0.0:
                return temp
            else:
                return 0.0
```

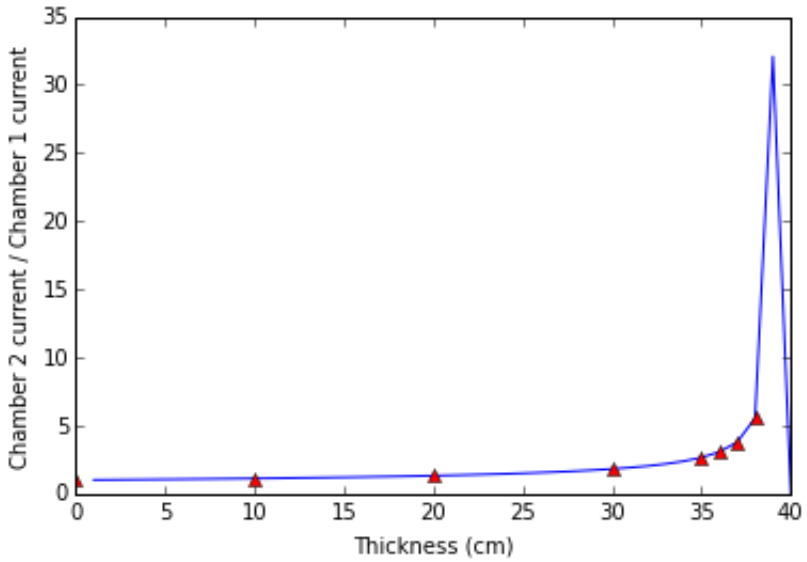
In [346]:
```
def bragg_curve(t, debug=False):
    S_c_chamber_one = S_c(1.0, 0.49918972273707857, 250.0, 937.0, rho_
air, I_air)
    E_after_chamber = EofT(250.0, t)
    if E_after_chamber > 0.0:
        S_c_chamber_two = S_c(1.0, 0.49918972273707857, E_after_chambe
r, 937.0, rho_air, I_air)
    else:
        S_c_chamber_two = 0.0
    if debug is True:
        print S_c_chamber_two
        print S_c_chamber_one
        print E_after_chamber
    return S_c_chamber_two / S_c_chamber_one
```

In [347]:
```
thickness = [0.0, 10.0, 20.0, 30.0, 35.0, 36.0, 37.0, 38.0]
```

In [348]:
```
%matplotlib inline
```

In [349]:
```
import matplotlib.pyplot as plt

plt.plot([i + 1.0 for i in range(40)], [bragg_curve(i + 1.0) for i in
range(40)], 'b',
         thickness, [bragg_curve(i) for i in thickness], 'r^'
        )
plt.xlabel("Thickness (cm)")
plt.ylabel("Chamber 2 current / Chamber 1 current")
plt.show()
```



In [ ]: