Problem 3

November 18, 2018

1 Problem 3

```
In [1]: from scipy.integrate import trapz
        from scipy.special import legendre
        from scipy.optimize.zeros import newton
        from numpy.polynomial.legendre import leggauss
        import numpy as np
        import matplotlib.pyplot as plt
```

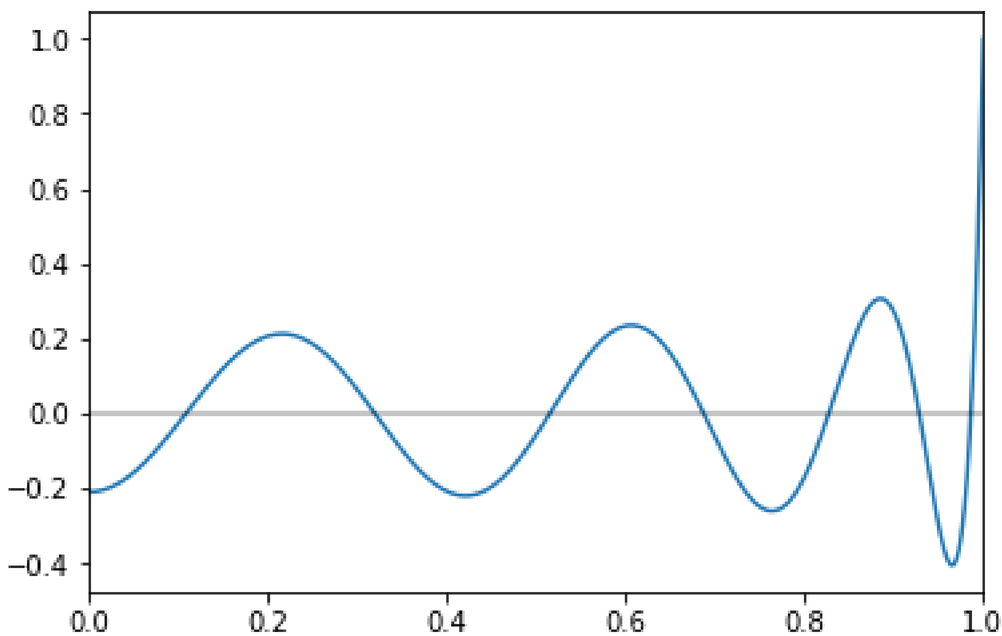
Plot the 14th Legendre polynomial to eyeball the starting guesses for the zeros

```
In [2]: x = np.linspace(-1, 1, 10000)
```

```
In [3]: l = legendre(14)
```

```
In [4]: plt.plot(x, l(x))
        plt.xlim(0, 1)
        plt.axhline(y=0, alpha=0.3, color='black')
```

```
Out[4]: <matplotlib.lines.Line2D at 0x115eb6940>
```



This has seven positive and seven negative zeros

```
In [5]: guesses = [
            0.1,
            0.33,
            0.52,
            0.7,
            0.8,
            0.9,
            1.0
        ]
```

`newton` uses the Newton-Raphson method to find zeros of a function

```
In [6]: zeros = np.array([newton(l, g) for g in guesses])
        print(zeros)

[ 0.10805495  0.31911237  0.51524864  0.6872929   0.82720132  0.92843488
  0.98628381]
```

Now I can construct the matrix of integrals for x^n
Calculate the numerical integral of x^n for even n's

```
In [7]: integrals = np.array([0.5*trapz(x**n, x) for n in range(15)[::2]])
        print(integrals)

[ 1.          0.33333334  0.20000001  0.14285716  0.11111114  0.09090912
  0.07692312  0.06666671]
```

Create a matrix where each column j and row i is μ_j^{2i}

```
In [8]: functions = np.array([zeros**n for n in range(15)[::2]])

In [9]: np.set_printoptions(precision=1)
        print(functions)

[[  1.0e+00   1.0e+00   1.0e+00   1.0e+00   1.0e+00   1.0e+00   1.0e+00]
 [  1.2e-02   1.0e-01   2.7e-01   4.7e-01   6.8e-01   8.6e-01   9.7e-01]
 [  1.4e-04   1.0e-02   7.0e-02   2.2e-01   4.7e-01   7.4e-01   9.5e-01]
 [  1.6e-06   1.1e-03   1.9e-02   1.1e-01   3.2e-01   6.4e-01   9.2e-01]
 [  1.9e-08   1.1e-04   5.0e-03   5.0e-02   2.2e-01   5.5e-01   9.0e-01]
 [  2.2e-10   1.1e-05   1.3e-03   2.4e-02   1.5e-01   4.8e-01   8.7e-01]
 [  2.5e-12   1.1e-06   3.5e-04   1.1e-02   1.0e-01   4.1e-01   8.5e-01]
 [  3.0e-14   1.1e-07   9.3e-05   5.2e-03   7.0e-02   3.5e-01   8.2e-01]]
```

```
In [10]: np.set_printoptions(precision=8)
```

Get the official values to compare with

```
In [11]: mus, wts = leggauss(14)
```

Compare the official μ values (stored in the variable `mus`) to my calculated values (stored in `zeros`)

```
In [12]: mus[7:]
```

```
Out[12]: array([ 0.10805495,  0.31911237,  0.51524864,  0.6872929 ,  0.82720132,
                 0.92843488,  0.98628381])
```

```
In [13]: zeros
```

```
Out[13]: array([ 0.10805495,  0.31911237,  0.51524864,  0.6872929 ,  0.82720132,
                 0.92843488,  0.98628381])
```

Compare the official weights (stored in `wts`) to my calculated values (stored in `weights`)

```
In [14]: wts[7:]
```

```
Out[14]: array([ 0.21526385,  0.20519846,  0.1855384 ,  0.15720317,  0.12151857,
                 0.08015809,  0.03511946])
```

```
In [15]: weights = np.linalg.inv(functions.T @ functions) @ functions.T @ integrals
         weights
```

```
Out[15]: array([ 0.2152639 ,  0.20519833,  0.18553861,  0.15720292,  0.12151885,
                 0.08015776,  0.03511963])
```

Calculate the fractional error between my calculated weights and the official ones

```
In [16]: np.abs(weights - wts[7:]) / wts[7:]
```

```
Out[16]: array([  2.08311775e-07,   6.70460299e-07,   1.12400692e-06,
                  1.56257606e-06,   2.33478854e-06,   4.08320243e-06,
                  4.96419886e-06])
```

Pretty close! Within ~10^{-4}%