NE470 Homework 4 Addendum

October 24, 2016

1 NE470 Homework No. 4

1.1 Problem 3

This is an attempt to solve the for the function $f(x)$ by discretizing it into N nodes such that

$$f_n \approx f(n * \Delta x), \Delta x = W/n, n \in \{0, 1, \ldots, N-1\}$$

For this problem, $W = 4$. Our boundary conditions are set as

$$f(0) = 2$$

and

$$f(4) = 54.61647$$

```
In [2]: # numpy contains useful array/matrix tools
        import numpy as np
```

The equation we are trying to solve is:

$$f'' - f = 0$$

Using the following approximation of the second derivative:

$$f'' \approx \frac{f_{n-1} - 2f_n + f_{n+1}}{\Delta x^2}$$

we arrive at an operator that approximately solves for f:

$$\left(\frac{1}{\Delta x^2}\right) f_{i-1} + \left(\frac{-2}{\Delta x^2} - 1\right) f_i + \left(\frac{1}{\Delta x^2}\right) f_{i+1} = b_i$$

Now we solve for

$$\mathbf{A} f = b$$

```
In [3]: def solution(N):
            # define our interval length
            deltaX = 4.0 / (N-1)
            deltaXsquared = deltaX ** 2

            # define our matrix
```

```
        diag = -2.0 / deltaXsquared - 1.0
        offdiag = 1.0 / deltaXsquared

        # instantiate matrix and fill diagonal values
        A = np.eye(N-2) * diag
        A = A + offdiag * np.diagflat(np.resize(np.array([1.0]), (N-3)), 1)
        A = A + offdiag * np.diagflat(np.resize(np.array([1.0]), (N-3)), -1)

        # define our output vector, b
        b = np.zeros(N-2)
        b[0] = -2.0 / deltaXsquared
        b[N-3] = -54.61647 / deltaXsquared

        # compute solution
        f = np.linalg.solve(A, b)

        return f
```

```
In [4]: def analytic(N):
            """ The analytic solution to the equation """
            x = np.linspace(0, 4, num=N)
            x = x[1:-1]
            return np.exp(x) + np.exp(-x)
```

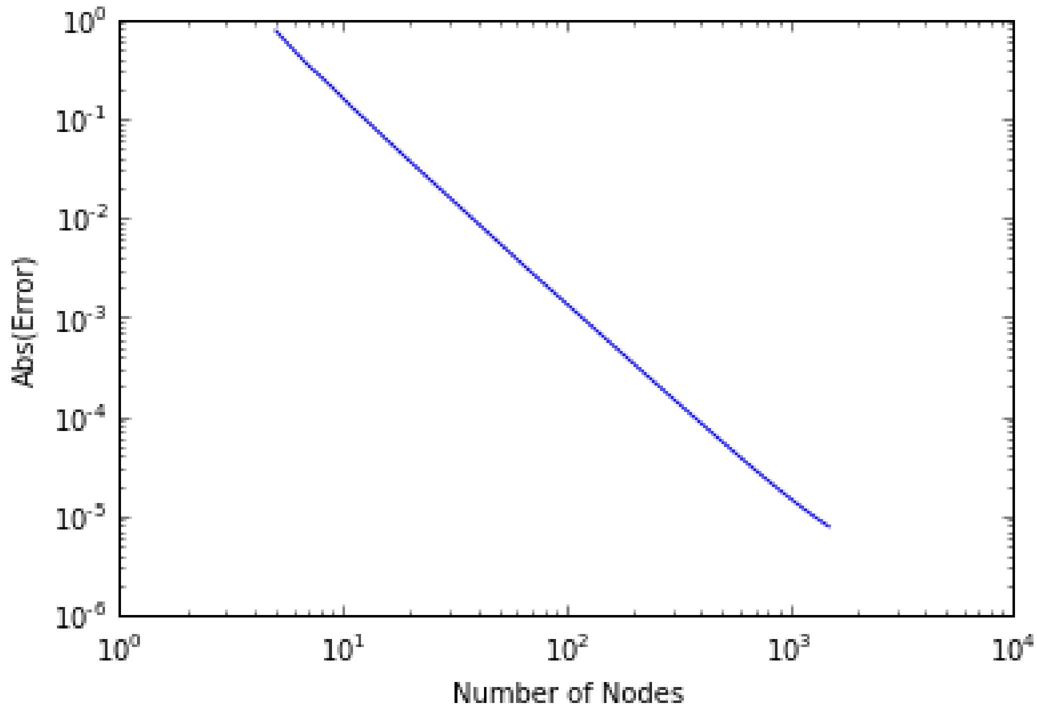
```
In [5]: def error(n):
            return np.amax(np.abs(analytic(n) - solution(n)))
```

```
In [6]: %%timeit
        e_min = 10**-4
        e=1.0

        n_max = 10**5
        n = 5

        while e>e_min and n<n_max:
            n += 1
            e = error(n)
        print "{0}\t\t{1}".format(n, e)
```

```
370             9.97073790181e-05
370             9.97073790181e-05
370             9.97073790181e-05
370             9.97073790181e-05
1 loop, best of 3: 605 ms per loop
```

```
In [9]: %matplotlib inline
        import matplotlib.pyplot as plt
        n = np.arange(5, 1500)
```

```
plt.plot(n, [error(i) for i in n])
plt.yscale('log')
plt.xscale('log')
plt.xlabel('Number of Nodes')
plt.ylabel('Abs(Error)')
plt.show()
```



1.2 Problem 5

```
In [10]: def sigma_tr(sigma_t, mu_0, sigma_s):
             # mu_0 is actually 1-mu_0 in my table
             mu_0 = 1 - mu_0

             return sigma_t - mu_0 * sigma_s

In [22]: def diffusion_coefficient(sigma_t, mu_0, sigma_s):
             return (1.0 / 3.0) / sigma_tr(sigma_t, mu_0, sigma_s)

In [11]: def z_0(sigma_t, mu_0, sigma_s):
             return 0.7104 / sigma_tr(sigma_t, mu_0, sigma_s)
```

Light water

```
In [12]: sigma_tr(3.45, 0.676, 3.45)
```

```
Out[12]: 2.3322000000000003

In [23]: diffusion_coefficient(3.45, 0.676, 3.45)

Out[23]: 0.1429265643312466

In [13]: z_0(3.45, 0.676, 3.45)

Out[13]: 0.30460509390275275
```

Heavy water

```
In [14]: sigma_tr(0.449, 0.884, 0.449)

Out[14]: 0.396916

In [25]: diffusion_coefficient(0.449, 0.884, 0.449)

Out[25]: 0.8398082549792231

In [15]: z_0(0.449, 0.884, 0.449)

Out[15]: 1.7897993530117204
```

Graphite

```
In [16]: sigma_tr(0.385, 0.9444, 0.385)

Out[16]: 0.36359400000000003

In [24]: diffusion_coefficient(0.385, 0.9444, 0.385)

Out[24]: 0.9167734707760119

In [17]: z_0(0.385, 0.9444, 0.385)

Out[17]: 1.953827620917837
```

Uranium

```
In [18]: sigma_tr(0.765, 0.9972, 0.397)

Out[18]: 0.7638884

In [26]: diffusion_coefficient(0.765, 0.9972, 0.397)

Out[26]: 0.4363639156365423

In [19]: z_0(0.765, 0.9972, 0.397)

Out[19]: 0.9299787770045991

In [ ]:
```