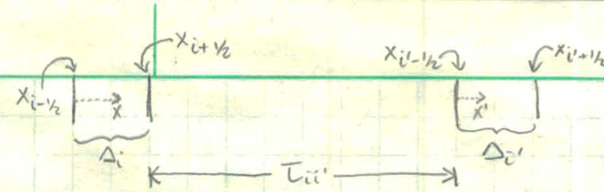
#5

$$P_{ii'} = \frac{\sigma_i}{\Delta_{i'}} \int_{x_{i-1/2}}^{x_{i+1/2}} dx \int_{x_{i'-1/2}}^{x_{i'+1/2}} dx' \, \tfrac{1}{2} E_1\left[\tau(x,x')\right]$$



$$\boxed{\tau(x,x') = \tau_{ii'} + (\sigma_i\Delta_i - \tau) + \tau'}$$

let $\tau = \sigma_i (x - x_{i-1/2})$ $\qquad$ $\tau' = \sigma_{i'}(x - x_{i'-1/2})$

$d\tau = \sigma_i \, dx$ $\qquad$ $d\tau' = \sigma_{i'} \, dx$

$$P_{ii'} = \frac{\cancel{\sigma_i}}{\Delta_{i'}} \int_0^{\sigma_i\Delta_i} \frac{d\tau}{\cancel{\sigma_i}} \int_0^{\sigma_{i'}\Delta_{i'}} \frac{1}{2} \frac{d\tau'}{\sigma_{i'}} E_1\left[\sigma_i\Delta_i - \tau + \tau_{ii'} + \tau'\right]$$

$$= \frac{1}{2\Delta_{i'}\sigma_{i'}} \int_0^{\sigma_i\Delta_i} d\tau \int_0^{\sigma_{i'}\Delta_{i'}} d\tau' \, E_1\left[\sigma_i\Delta_i - \tau + \tau_{ii'} + \tau'\right]$$

let $u = \sigma_i\Delta_i - \tau + \tau_{ii'} + \tau'$

$du = d\tau'$

$$= \frac{1}{2\Delta_{i'}\sigma_{i'}} \int_0^{\sigma_i\Delta_i} d\tau \int_{\sigma_i\Delta_i - \tau + \tau_{ii'}}^{\sigma_i\Delta_i - \tau + \tau_{ii'} + \sigma_{i'}\Delta_{i'}} du \, E_1[u]$$

$\Rightarrow$ since $\displaystyle\int_a^b f(x)\,dx = \int_a^\infty f(x)\,dx - \int_b^\infty f(x)\,dx \Rightarrow$

$$= \frac{1}{2\Delta_{i'}\sigma_{i'}} \int_0^{\sigma_i\Delta_i} d\tau \left[\int_{\sigma_i\Delta_i - \tau + \tau_{ii'}}^{\infty} du \, E_1[u] - \int_{\sigma_i\Delta_i - \tau + \tau_{ii'} + \sigma_{i'}\Delta_{i'}}^{\infty} du \, E_1[u]\right]$$

$\Rightarrow$ $\displaystyle E_{n+1}(\tau) = \int_\tau^\infty d\tau' \, E_n(\tau') \Rightarrow$

$$= \frac{1}{2\Delta_{i'}\sigma_{i'}} \int_0^{\sigma_i\Delta_i} d\tau \left[E_2\left[\sigma_i\Delta_i - \tau + \tau_{ii'}\right] - E_2\left[\sigma_i\Delta_i - \tau + \tau_{ii'} + \sigma_{i'}\Delta_{i'}\right] \right]$$

#5

$$= \frac{1}{2\Delta_{i'}\sigma_{i'}} \int_0^{\sigma_i\Delta_i} d\tau \, E_2[\sigma_i\Delta_i - \tau + \tau_{ii'}]$$

$$- \frac{1}{2\Delta_{i'}\sigma_{i'}} \int_0^{\sigma_i\Delta_i} d\tau \, E_2[\sigma_i\Delta_i - \tau + \tau_{ii'} + \sigma_{i'}\Delta_{i'}] \quad = \quad A - B$$

$$u = \sigma_i\Delta_i - \tau + \tau_{ii'}, \quad du = -d\tau$$

first part

$$A \Rightarrow \frac{-1}{2\Delta_{i'}\sigma_{i'}} \int_{\sigma_i\Delta_i + \tau_{ii'}}^{\tau_{ii'}} du \, E_2[u]$$

$$= \frac{-1}{2\Delta_{i'}\sigma_{i'}} \left[\int_{\sigma_i\Delta_i + \tau_{ii'}}^{\infty} du \, E_2[u] - \int_{\tau_{ii'}}^{\infty} du \, E_2[u] \right]$$

$$= \frac{-1}{2\Delta_{i'}\sigma_{i'}} \left[E_3[\sigma_i\Delta_i + \tau_{ii'}] - E_3[\tau_{ii'}] \right]$$

$$B \Rightarrow \quad u = \sigma_i\Delta_i - \tau + \tau_{ii'} + \sigma_{i'}\Delta_{i'}, \quad du = -d\tau$$

$$\Rightarrow \frac{1}{2\Delta_{i'}\sigma_{i'}} \int_{\sigma_i\Delta_i + \tau_{ii'} + \sigma_{i'}\Delta_{i'}}^{\tau_{ii'} + \sigma_{i'}\Delta_{i'}} E_2[u]$$

$$= \frac{1}{2\Delta_{i'}\sigma_{i'}} \left[\int_{\sigma_i\Delta_i + \tau_{ii'} + \sigma_{i'}\Delta_{i'}}^{\infty} E_2[u] - \int_{\tau_{ii'} + \sigma_{i'}\Delta_{i'}}^{\infty} E_2[u] \right]$$

$$= \frac{1}{2\Delta_{i'}\sigma_{i'}} \left[E_3[\sigma_i\Delta_i + \tau_{ii'} + \sigma_{i'}\Delta_{i'}] - E_3[\tau_{ii'} + \sigma_{i'}\Delta_{i'}] \right]$$

#5

A & B together: $A - B =$

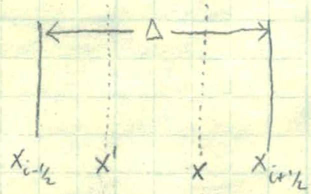
$$= -\frac{1}{2\Delta_{i'}\sigma_{i'}}\left[E_3(\tau_{ii'} + \sigma_i\Delta_i) - E_3(\tau_{ii'}) + E_3(\sigma_i\Delta_i + \tau_{ii'} + \sigma_{i'}\Delta_{i'}) - E_3(\sigma_{i'}\Delta_{i'} + \tau_{ii'})\right]$$

$$= \frac{1}{2\Delta_{i'}\sigma_{i'}}\left[E_3(\tau_{ii'}) - E_3(\tau_{ii'} + \sigma_i\Delta_i) - E_3(\tau_{ii'} + \sigma_{i'}\Delta_{i'}) + E_3(\tau_{ii'} + \sigma_i\Delta_i + \sigma_{i'}\Delta_{i'})\right]$$

this is equation 5-39

#5

$$\frac{\sigma}{\Delta}\int_{x_{i-1/2}}^{x_{i+1/2}} dx \int_{x_{i-1/2}}^{x_{i+1/2}} dx'\, E_1(\tau(x,x'))$$



$$\tau(x,x') = \tau - \tau'$$

$$\tau = \sigma(x - x_{i-1/2}) \qquad \tau' = \sigma(x' - x_{i-1/2})$$
$$d\tau = \sigma dx \qquad d\tau' = \sigma dx'$$

$$\frac{\cancel{\sigma}}{\Delta}\int_0^{\Delta\sigma} \frac{d\tau}{\cancel{\sigma}} \int_0^{\Delta\sigma} \frac{d\tau'}{\sigma}\, E_1(\tau-\tau')$$

Since we want $\tau - \tau' > 0$, change bounds of integration for second integral (will double

$$= \frac{2}{\Delta\sigma}\int_0^{\Delta\sigma} d\tau \int_0^{\tau} d\tau'\, E_1(\tau-\tau') \qquad u = \tau - \tau' \quad du = -d\tau'$$

$$= \frac{2}{\Delta\sigma}\int_0^{\Delta\sigma} d\tau \int_{\tau}^{0} du\, E_1(u) = \frac{-2}{\Delta\sigma}\int_0^{\Delta\sigma} d\tau \int_0^{\tau} du\, E_1(u)$$

$$= \frac{-2}{\Delta\sigma}\int_0^{\Delta\sigma} d\tau \left[\int_0^{\infty} du\, E_1(u) - \int_{\tau}^{\infty} du\, E_1(u)\right]$$

$$= \frac{-2}{\Delta\sigma}\int_0^{\Delta\sigma} d\tau \left(E_2(0) - E_2(\tau)\right) \qquad E_2(0) = 1$$

$$= \frac{-2}{\Delta\sigma}\int_0^{\Delta\sigma} d\tau + \frac{2}{\Delta\sigma}\int_0^{\Delta\sigma} d\tau\, E_2(\tau)$$

$$= -2 + \frac{2}{\Delta\sigma}\int_0^{\infty} d\tau\, E_2(\tau) - \frac{2}{\Delta\sigma}\int_{\Delta\sigma}^{\infty} d\tau\, E_2(\tau)$$

#5

$$= -2 + \frac{2}{\Delta\sigma} E_3(0) - \frac{2}{\Delta\sigma} E_3(\Delta\sigma) \qquad E_3(0) = 0.5$$

$$= -2 + \frac{2}{\Delta\sigma}\left(\frac{1}{2} - E_3(\Delta\sigma)\right)$$

man does this ever look close!

$$= -2 + \frac{1}{\Delta\sigma}\left(1 - 2E_3(\Delta\sigma)\right)$$

I am within a factor of −2 — and I added a 2x factor earlier

$$= 1 - \frac{1}{2\Delta\sigma}\left(1 - 2E_3(\Delta\sigma)\right)$$