NE 551 Final Exam Fall 2016

Name:

Due 5 p.m., December 8, 2016.

Email your completed exam to lheilbro@utk.edu.

Include a copy of this cover sheet along with your signature that affirms all of the work on this exam was your own and that you consulted no one while working on your answers.

The BULK II Excel toolkit referred to in question # 9 is in the materials section, resources folder, of the course blackboard site.

140 points total.

Good luck!

Problem 1.

A flux of 100 eV neutrons (1×10^9 neutrons/cm^2/s) is incident on a ^{197}Au disk that is 1 cm in diameter and is 2.54×10^{-3} cm thick.

(a) Calculate the absorption cross section using a thermal cross section of 98.7 b. Compare with the 100 eV absorption cross section from the NNDC ENDF/B-VII.1 evaluation and explain why the two values are different.

(b) Calculate the Q-value for the absorption reaction: n+^{197}Au$\rightarrow$ ^{198}Au (mass excesses are: n=8.071 MeV/c^2, ^{197}Au=-31.141 MeV/c^2, ^{198}Au=-29.582 MeV/c^2).

(c) The kinetic energy of the ^{198}Au immediately after the absorption of the neutron and before the emission of any gamma rays from the compound ^{198}Au nucleus.

(d) The activity of ^{198}Au after 7 days of irradiation. Use the NNDC ENDF/B-VII.1 value of the absorption cross section at 100 eV.

Solution

Part (a)

For low energies, $\sigma_a \propto v^{-1} \propto \sqrt{\frac{1}{T}}$.

$$\begin{aligned}\sigma_a(E) &\approx \sigma_a^{Th}\sqrt{\frac{E_{Th}}{E}}\\ &\approx 98.7\,\mathrm{b}\sqrt{\frac{0.025\,\mathrm{eV}}{100\,\mathrm{eV}}}\\ &\approx 1.56\,\mathrm{b}\end{aligned}$$

From NNDC ENDF/B-VII.1 we use the values of (100.246 eV, 3.996 48 b) and (99.579 eV, 3.906 69 b) to interpolate and arrive at a value of 3.963 36 b. This is much larger than the calculated value. From the plot of cross sections (figure 1) we see that at this point we have entered the resonance region for this nucleus. Since the A number is relatively high for gold, there are many available energy states–and the resonance region starts at comparatively low energies.

Part (b)

$$Q = (M_X + M_{\mathrm{n}^0} - M_Y)\,c^2$$

Use mass excess in lieu of mass

$$\begin{aligned}&= -31.141\,\mathrm{MeV} + 8.071\,\mathrm{MeV} - -29.582\,\mathrm{MeV}\\ &= 6.512\,\mathrm{MeV}\end{aligned}$$

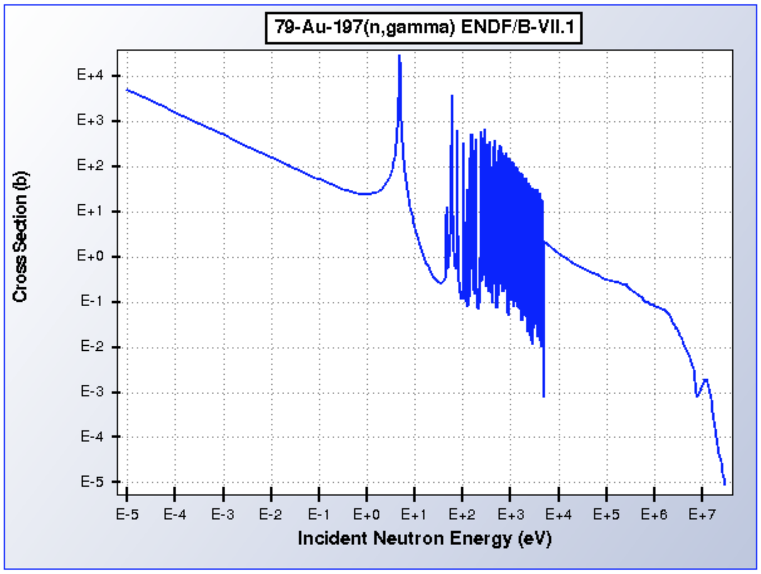

Figure 1: Absorption cross section for ^{197}Au

Part (c)

Since the incident neutron energy is much less than its mass, we can ignore relativistic effects and calculate the kinetic energy of the ^{198}Au using conservation of momentum.

$$\begin{aligned}
\sqrt{2m_{\mathrm{n}^0}E_{\mathrm{n}^0}} &= \sqrt{2m_{^{198}\mathrm{Au}}E_{^{198}\mathrm{Au}}} \\
E_{^{198}\mathrm{Au}} &= E_{\mathrm{n}^0}\frac{m_{\mathrm{n}^0}}{m_{^{198}\mathrm{Au}}} \\
&= 100\,\mathrm{eV}\frac{1.008\,66\,\mathrm{u}}{197.968\,\mathrm{u}} \\
&= 0.510\,\mathrm{MeV}
\end{aligned}$$

Part (d)

Since NNDC gives no cross section for ^{198}Au, assume it does not absorb neutrons.

$$\begin{aligned}
\dot{N}_{197} &= -N_{197}\sigma_a\phi \\
\dot{N}_{198} &= N_{197}\sigma_a\phi - \lambda_{198}N_{198}
\end{aligned}$$

Solving these gives:

$$
\begin{aligned}
A(t) &= N_{197}\sigma_a\phi\left(1-\exp\{-\lambda t\}\right)\\
N_{197} &= \frac{A_V}{m_{197}}m_{Au}\\
m_{Au} &= \rho V = \rho\times\pi(0.5\,\mathrm{cm})^2\left(2.54\times10^{-3}\,\mathrm{cm}\right) = 19.32\,\mathrm{g/cm^3}\times1.96\times10^{-3}\,\mathrm{cm^3}\\
&= 3.79\times10^{-2}\,\mathrm{g}\\
N_{197} &= \frac{6.022\times10^{23}}{196.966\,\mathrm{g}}3.79\times10^{-2}\,\mathrm{g} = 1.16\times10^{20}\\
\lambda &= \frac{\log 2}{2.695\,17\,\mathrm{d}} = 0.2572\,\mathrm{d^{-1}}\\
A(7\,\mathrm{d}) &= 1.16\times10^{20}\times3.963\,36\times10^{-24}\,/\mathrm{cm^2}\times1\times10^{9}\,\mathrm{neutrons/cm^2}\times\left(1-\exp\left\{-0.2572\,\mathrm{d^{-1}}\times7\,\mathrm{d}\right\}\right)\\
&= 3.84\times10^{5}\,\mathrm{Bq}
\end{aligned}
$$

Problem 2.

A beam of 1 MeV neutrons is incident upon a ^{12}C graphite target, density=2.2 g/cm^3. Using the CENDL elastic scattering cross section, determine the mass-energy transfer coefficient for 1 MeV $n+{}^{12}$C s-wave elastic scattering, assuming all of the scattering is elastic at this energy.

Solution

Anderson equation 8.45:

$$\begin{aligned}\left(\frac{\mu_{tr}}{\rho}\right)_{es} &= \left(\frac{\mu}{\rho}\right)_{es}\left[\frac{\langle T_A\rangle}{T}\right] \\ &= \left(\frac{\mu}{\rho}\right)_{es}\left[\frac{T-\langle T'\rangle}{T}\right] \\ \left(\frac{\mu}{\rho}\right)_{es} &= n_m\sigma_{es}\end{aligned}$$

From CENDL, $\sigma_{es} = 2.490\,78\,\mathrm{b}$

$$\begin{aligned}\left(\frac{\mu_{tr}}{\rho}\right)_{es} &= n_m\sigma_{es}\left[\frac{1-\alpha}{2}\right] \\ \alpha &= \left[\frac{A-1}{A+1}\right]^2 \approx 0.716 \\ \left(\frac{\mu_{tr}}{\rho}\right)_{es} &= \frac{\rho A_V}{m_m}\sigma_{es}\left[\frac{1-\alpha}{2}\right] \\ &= \frac{2.2\,\mathrm{g/cm^3}\times 6.022\times 10^{23}}{12\,\mathrm{g}}\times 2.490\,78\times 10^{-24}\,\mathrm{cm^2}\times\left[\frac{1-0.716}{2}\right] \\ &= 0.039\,\mathrm{cm^{-1}}\end{aligned}$$

Problem 3.

Define what is meant by stochastic biological effects due to radiation and list two such effects in humans.

Solution

Stochastic biological effects are effects from radiation which grow more likely as exposure increases, but do not become more severe. The symptoms may occur long after the exposure. Any radiation exposure increases the probability of occurence (according to the linear-no-threshold theory), which leads to the ALARA principle. Two examples are solid tumor formation and leukemia.

Problem 4.

Define what is meant by deterministic biological effects due to radiation and list two such effects in humans.

Solution

Deterministic effects are a direct result of damage from the radiation exposure. The severity of the effects increases with the amount of exposure, and different effects occur at different minimum threshold levels. Examples include hair loss and nausea/vomiting.

Problem 5.

Explain the importance of cell cycle in regards to the deleterious biological effects from radiation.

Solution

Different cells divide at different rates. Since the ongoing biological damage (stochastic effects) depends on damage to cell DNA being propogated through cell generations, the time relative to that cell dividing is relevant to the resultant damage. If the cell has just been born, DNA damage is likely to be repaired or detected before the cell can reproduce, preventing the propogation of the mutation/damage. If the cell is damaged just before it reproduces, then that alteration is more likely to spread to its daughter cells before it can be repaired.

Problem 6.

A 144 Ci point source of ^{24}Na is to be stored at the bottom of a pool of water. ^{24}Na decays by beta emission to stable ^{24}Mg. In the decay process it emits two gammas per disintegration with energies 2.75 MeV and 1.37 MeV. How deep must the water be if the exposure rate at a point 6 m above the source is not to exceed $20\,\mathrm{mR\,h^{-1}}$?

Solution

Assume that for these photon energies, the dose rate equation 1 is valid to calculate unshielded dose. From Turner figure 8.9, we find that the mass attenuation coefficients are $0.043\,\mathrm{cm^{-1}}$ (γ_1) and $0.061\,\mathrm{cm^{-1}}$ (γ_2). First we focus on γ_1.

$$\dot{X} = \frac{0.5 \times C \times E}{r^2} \tag{1}$$

Equation 1 gives an exposure of $5500\,\mathrm{mR\,h^{-1}}$. To determine the shielding needed to reduce this to $20\,\mathrm{mR\,h^{-1}}$, we use equation 2 to arrive at 5.62. This does not account for the buildup factor, so we must interpolate from Turner figure 15.2 to deterimine $B_1^{(1)} \approx 5.8$. To shield against this additional factor we calculate $(\mu t)^{(1)} = (\mu t)^{(0)} + \log B_1^{(1)} \approx 7.41$. Repeating this process iteratively, we arrive at a final buildup factor of 7.62.

$$\begin{aligned} \dot{X}_{shielded} &= \dot{X}_{unshielded}\mathrm{e}^{-\mu t} \\ 20\,\mathrm{mR\,h^{-1}} &= 5500\,\mathrm{mR\,h^{-1}}\mathrm{e}^{-\mu t} \\ \mu t &= 5.62 \end{aligned} \tag{2}$$

We then multiply equation 2 by this factor to determine a value given that $\mu_1 t = 7.65$:

$$\begin{aligned} \dot{X}_{shielded} &= 7.62 \times 5500 \times \mathrm{e}^{-7.65} \\ &= 19.93 \end{aligned}$$

Next we determine, given this shielding, the dose contribution from the 1.37 MeV gamma.

$$\begin{aligned} \mu_2 t &= \mu_1 t \times \frac{E_2}{E_1} \\ &= 7.65 \times \frac{0.061}{0.043} \\ &= 10.85 \end{aligned}$$

This is off of figure 15.2, so instead I interpolated in the tables found at `http://www.nucleonica.net/Application/Help/Helpfiles/Appendix4.htm`. Not following the same iterative procedure as before (since μt is now fixed), I arrived at a buildup factor of 21.54 (see attached code). This resulted in a dose contribution of $0.0011\,\mathrm{mR\,h^{-1}}$

$$\begin{aligned}\dot{X}_{sh} &= 21.54 \times \frac{0.5 \times 144\,\mathrm{Ci} \times 1.37\,\mathrm{MeV}}{6\,\mathrm{m}^2} \times \mathrm{e}^{-10.85} \\ &= 0.0011\,\mathrm{mR\,h^{-1}}\end{aligned}$$

This gives a total exposure rate of $19.93\,\mathrm{mR\,h^{-1}}$. To determine the thickness of water, then, we use $\mu t/\mu = 7.65/0.043\,\mathrm{cm^{-1}} = 177.9\,\mathrm{cm}$

Problem 7.

Calculate the whole body effective dose for an individual who has simultaneously received all of the following exposures:

(a) 2 mGy alpha to the lung

(b) 5 mGy thermal neutrons to the whole body

(c) 5 mGy gamma, whole body

(d) 40 mGy beta to the thyroid

(e) 2 Gy beta to the skin

Which, if any, of the recommended annual exposure limits from NCRP 116 (see `https://www.ceessentials.net/article6#section5_5`) for occupational exposures are exceeded?

Solution

To determine dose we use equation 3, where D is the exposure and Q is the quality factor for the radiation in question. For quality factors we refer to the ICRP 103 specifications (`https://www.euronuclear.org/info/encyclopedia/r/radiation-weight-factor.htm`) and arrive at 1 for betas and photons, 20 for alphas, and 2.5 for thermal neutrons (calculated using equation 4).

$$H_T = Q \times D \tag{3}$$

$$Q(E) = 2.5 + 18.2\mathrm{e}^{-[\log(E)]^2/6} \tag{4}$$

To convert this value into a whole body dose equivalent, we use equation 5 which uses a tissue weighting factor to convert dose applied to a specific organ into the dose to the whole body that would cause the equivalent stochastic biological damage. Tissue weighting factors were obtained from the 2007 ICRP report, accessed from `http://www.icrp.org/docs/David%20Brenner%20Effective%20Dose%20a%20Flawed%20Concept.pdf`. The values used were 0.12 for lungs, 0.04 for thyroid, and 0.01 for skin.

$$E = \sum_T w_T H_T \tag{5}$$

Exposure (mGy)	Radiation	Target	Dose	Whole-Body Equivalent
2	alpha	lung	40.0	4.8
5	thermal neutron	whole body	12.5	12.5
5	gamma	whole body	5.0	5.0
40	beta	thyroid	40.0	1.6
2000	beta	skin	2000.0	20.0

The total dose was determined to be 43.9 mGy. This does not exceed the NRCP 116 annual exposure limit of 50 mGy per year, nor does it exceed the dose limit to the skin.

Assuming that the worker is an adult, lifetime dose limits are also likely not a problem. That said, it is possible that local control limits may have been violated.

Problem 8.

Suppose that a 1 mCi soft beta emitter is distributed uniformly through your body. Calculate

(a) the effective dose after 1 year,

(b) the effective dose after 5 years, and

(c) the committed effective doses

for an occupational worker based on the following:

- The physical half life is 2 years and the biological half life is 1 year
- The energy deposited per decay is 10 keV
- The body has a mass of 74 kg

Solution

Dose as a function of time can be calculated using equation 6. In this case, since the energy deposited per decay is given, we know that $w = 10\,\mathrm{keV}$. We also assume that all energy is contained in the body, so $\phi_{af} = 1$.

$$D(t) = \frac{C_{max}\Delta}{(\lambda_p + \lambda_b)}\phi_{af}\left[1 - \mathrm{e}^{-(\lambda_p+\lambda_b)t}\right] \tag{6}$$

Part (a)

$$D(1\,\mathrm{yr}) = \frac{5 \times 10^5\,\mathrm{Bq\,kg^{-1}} \times 1.6 \times 10^{-15}\,\mathrm{J/decay}}{3.29 \times 10^{-8}\,\mathrm{s^{-1}}} \times \mathrm{e}^{-3.29 \times 10^{8}\,\mathrm{s^{-1}} \times 3.16 \times 10^{7}\,\mathrm{s}} = 15.7\,\mathrm{mSv}$$

Part (b)

$$D(5\,\mathrm{yr}) = \frac{5 \times 10^5\,\mathrm{Bq\,kg^{-1}} \times 1.6 \times 10^{-15}\,\mathrm{J/decay}}{3.29 \times 10^{-8}\,\mathrm{s^{-1}}} \times \mathrm{e}^{-3.29 \times 10^{8}\,\mathrm{s^{-1}} \times 5 \times 3.16 \times 10^{7}\,\mathrm{s}} = 24.1\,\mathrm{mSv}$$

Part (c)

$$CDE = \frac{5 \times 10^5\,\mathrm{Bq\,kg^{-1}} \times 1.6 \times 10^{-15}\,\mathrm{J/decay}}{3.29 \times 10^{-8}\,\mathrm{s^{-1}}} = 24.3\,\mathrm{mSv}$$

Problem 9.

Using the BULK II tool discussed in class, calculate the thickness of concrete shielding needed to reduce the dose at points A, B, and C to 100 mSv in a year for a worker standing 50 cm away from the outside of the concrete shielding. Assume the worker is there 20 hours per week, and works 40 weeks per year treating patients. The distance from the target (patient) center to the concrete walls is 250 cm in the +Y direction, 200 cm in the -Y direction, 250 cm in the +X direction, 200 cm in the -X direction, 250 cm in the +Z direction, and 100 cm in the -Z direction, with the target defining isocenter. Use a density of 2.3 for the concrete. Use water for the target. Beam moves along the y-axis in the +Y direction. Points A, B, C lie in the Z=0 plane. Design the shielding for 230 MeV proton facility with a beam current of 10 nA ($1 \times 10^{-8}\,\mathrm{C\,s^{-1}}$) on target. Either copy the input and output tabs from your results, or email the entire work book along with the rest of your exam.

Solution

The allowed exposure per week is $100\,\mathrm{mSv\,yr^{-1}}/40\,\mathrm{wk\,yr^{-1}} = 2.5\,\mathrm{mSv\,wk^{-1}} = 2.5 \times 10^{3}\,\mathrm{\mu Sv\,wk^{-1}}$

A spreadsheet is included in this email with final values. The +X wall is 106 cm thick; the -X wall is 114 cm thick; and the +Y wall is 291 cm thick. This gave final exposure values of $2.45\,\mathrm{mSv\,wk^{-1}}$, $2.43\,\mathrm{mSv\,wk^{-1}}$, and $2.37\,\mathrm{mSv\,wk^{-1}}$ at points A, B, and C respectively.