C. Calculate uncertainty of random functions

1. $a_1 = 3.1 \pm 0.5$ $\quad a_2 = 5.7 \pm 0.8$ $\quad a_3 = 12 \pm 1$

1. $q = a_1 + a_2$ $\quad \delta q = \sqrt{\sum_i \left(\frac{\partial q}{\partial x_i}\delta q_i\right)^2}$

$q = 3.1 + 5.7$

$= 8.8 + \delta q$

$= \sqrt{\left(\frac{\partial}{\partial a_1}[a_1 + a_2]\delta a_1\right)^2 + \left(\frac{\partial}{\partial a_2}[a_1 + a_2]\delta a_2\right)^2}$

$= \sqrt{\delta a_1^2 + \delta a_2^2}$

$= \sqrt{0.5^2 + 0.8^2}$

$= 0.9$

$\boxed{q = 8.8 \pm 0.9}$

2. $q = a_1 * a_2$ $\quad \delta q = \sqrt{\left(\frac{\partial}{\partial a_1}[a_1 * a_2]\delta a_1\right)^2 + \left(\frac{\partial}{\partial a_2}[a_1 * a_2]\delta a_2\right)^2}$

$= 3.1 * 5.7 + \delta q$ $\quad = \sqrt{(a_2 \delta a_1)^2 + (a_1 \delta a_2)^2}$

$= 18 + \delta q$

$= \sqrt{(5.7 * 0.5)^2 + (3.1 * 0.8)^2}$

$= 4$

$\boxed{q = 18 \pm 4}$

C.

1.

3. $q = a_1 * a_2^2$

$q = 3.1 * (5.7)^2$

$= 100 \pm \delta_q$

$$\delta_q = \sqrt{\left(\frac{\partial}{\partial a_1}[a_1 a_2^2]\,\delta a_1\right)^2 + \left(\frac{\partial}{\partial a_2}[a_1 a_2^2]\,\delta a_2\right)^2}$$

$$= \sqrt{(a_2^2 \delta a_1)^2 + (2 a_1 \delta a_2)^2}$$

$$= \sqrt{(3.1^2 * 0.8)^2 + (2 * 3.1 * 0.5)^2}$$

$$= 8$$

$\boxed{q = 100 \pm 8}$

4. $q = a_1 / a_2$

$q = 3.1/5.7$

$= 0.54 \pm \delta_q$

$$\delta_q = \sqrt{\left(\frac{\partial}{\partial a_1}[a_1/a_2]\,\delta a_1\right)^2 + \left(\frac{\partial}{\partial a_2}[a_1/a_2]\,\delta a_2\right)^2}$$

$$= \sqrt{\left(\frac{\delta a_1}{a_2}\right)^2 + \left(-\frac{a_1 \delta a_2}{a_2^2}\right)^2}$$

$$= 0.1$$

$= \boxed{0.54 \pm 0.1}$