

**Problem 1.**

Compute the macroscopic fission cross section of 1, 2, 3, 4, and 5 wt% enriched  $\text{UO}_2$  with a density of  $10.5 \text{ g/cm}^3$ . Use the  $2200 \text{ m/s}$  microscopic cross sections provided in Appendix A (pp. 606-610 in D&H). What is the probability that a neutron will be absorbed in  $^{238}\text{U}$  (relative to all absorptions) in these mixtures?

**Solution**

$$\Sigma_f = \cancel{N_{238}\sigma_f^{238}} + N_{235}\sigma_f^{235} + \cancel{N_{238}\sigma_f^{238}}$$

N is a number percentage—we must convert this from the provided weight percentage.

$$M(U) = \frac{M(U - 235) * M(U - 238)}{wM(U - 238) + (1 - w)M(U - 235)}$$

$$N(U - 235) = \rho \frac{\frac{M(U)}{M(U-235)}w}{M(U - 235)} A_V$$

For the second part we must calculate  $N(^{235}\text{U})$ .

w	N	$\Sigma_f$
0.01	2.37135e+20	0.136827
0.02	4.74263e+20	0.27365
0.03	7.11384e+20	0.410469
0.04	9.48498e+20	0.547283
0.05	1.1856e+21	0.684094

$$N(U - 238) = \rho \frac{\frac{M(U)}{M(U-238)}(1 - w)}{M(U - 238)} A_V$$

$$N(\text{UO}_2) = \rho \frac{A_V}{M(\text{U})_2}$$

Results:

w	Probability
0.01	0.313432
0.02	0.310222
0.03	0.307013
0.04	0.303805
0.05	0.300598

**Problem 2.**

Using the Table of Nuclides at: <http://atom.kaeri.re.kr/ton> or that found at <http://www.nndc.bnl.gov/sigm> find the following information:

- (a) What is the total fission cross section of U-233, U-235, Pu-239, and Pu-241 at 0.0253 eV?
- (b) What is the accumulated fission yield of  $^{90}\text{Sr}$  from the thermal fission of  $^{235}\text{U}$ ?
- (c) Obtain a plot of the  $^{241}\text{Pu}$  total absorption cross section at 300K over the range  $10^{-9}$  to  $20\text{MeV}$ . Print out the plot and label the major features of the cross section behavior. Turn in the labeled plot.

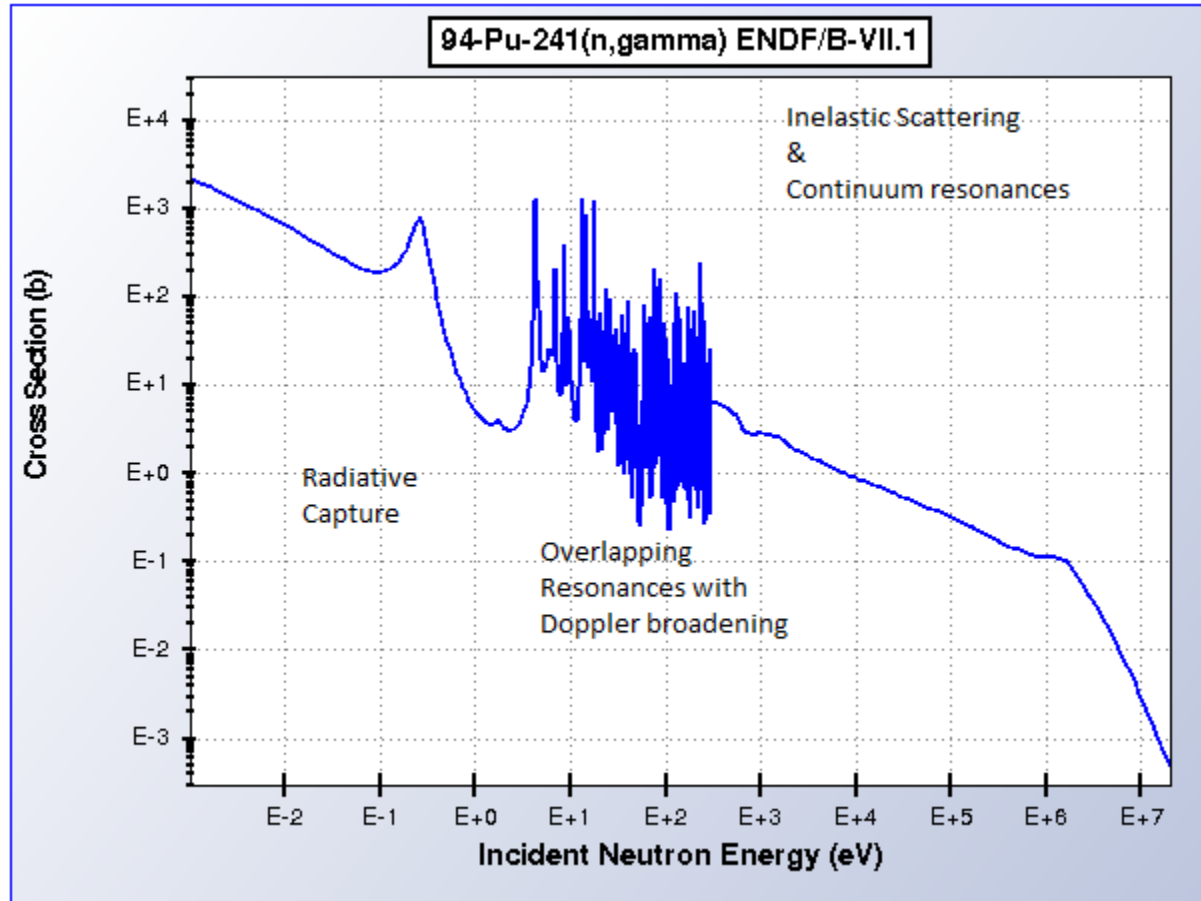
**Solution****Part (a)**

Isotope	$\sigma_f$
$^{233}\text{U}$	532.707b
$^{233}\text{U}$	585.472b
$^{233}\text{U}$	751.322b
$^{233}\text{U}$	1013.11b

**Part (b)**

$$0.0578194 \pm 5.78194 * 10^{-4}$$

## Part (c)



**Problem 3.**

How many collisions are required to slow down a neutron from 2MeV to 0.025 eV in

- (a) Hydrogen,
- (b) Deuterium,
- (c) Graphite, and
- (d) Lead?

**Solution**

From Mragheb equation 37, we know that the average number of collisions required to slow a neutron from  $E$  to  $E'$  is:

$$N = \ln \left( \frac{E}{E'} \right) / \xi$$

With values for  $\xi$  from Appendix A:

Material	$\xi$	N
Hydrogen	1.000	18.20
Deuterium	0.725	25.1
Graphite	0.158	115
Lead	0.0096	1896

### Problem 4. Duderstadt & Hamilton, problem 3-8

Consider an infinitely large homogeneous mixture of  $^{235}\text{U}$  and a moderating material. Determine the ratio of fuel-to-moderator density that will render this system critical for the following moderators: (a) graphite, (b) beryllium [*sic*], (c) water ( $\text{H}_2\text{O}$ ), and (d) heavy water ( $\text{D}_2\text{O}$ ). Use the thermal cross section data given in Appendix A.

#### Solution

From Appendix A: From Deuderstadt pg. 84,  $\eta$  and  $\epsilon$  change primarily with fuel selection

Moderator	$\sigma_s$ (b)	$\sigma_a$ (b)
<i>Graphite</i>	4.8	0.004
<i>Beryllium</i>	7.0	0.010
<i>Water</i>	3.45	0.66
<i>Heavywater</i>	0.449	0.001

and should be constant. For a homogeneous core, the fast fission factor,  $\epsilon$  is approximately 1<sup>1</sup>. To calculate a value for  $\eta$  (neglecting neutron re-emission reactions) we use the equation:

$$\begin{aligned}\eta_{^{235}\text{U}} &= v * \frac{\sigma_f}{\sigma_f + \sigma_\gamma} \\ &= 2.068\end{aligned}$$

We can also say that all neutrons are successfully thermalized, meaning  $p = 1$ . We then set Deuderstadt equation 3-13 to 1.0:

$$\begin{aligned}k_{\text{inf}} &= \eta_{^{235}\text{U}} f = 1.0 \\ 1/\eta_{^{235}\text{U}} &= f \\ 1/\eta_{^{235}\text{U}} &= \frac{N_F \sigma_a^F}{N_F \sigma_a^F + N_M \sigma_a^M} \\ 1/\eta_{^{235}\text{U}} * (N_F \sigma_a^F + N_M \sigma_a^M) &= N_F \sigma_a^F \\ \frac{N_M}{N_F} &= (\eta - 1) \frac{\sigma_a^F}{\sigma_a^M}\end{aligned}$$

Moderator	Moderator/Fuel Number Ratio
Water	12.43
Heavy Water	8202
Graphite (C)	2050
Beryllium	820.2

<sup>1</sup><http://www.nuclear-power.net/nuclear-power/reactor-physics/nuclear-fission-chain-reaction/fast-fission-factor/>

### Problem 5. Duderstadt & Hamilton, problem 3-1

What is the maximum value of the multiplication factor that can be achieved in any conceivable reactor design?

#### Solution

We know that the multiplication factor for a thermal reactor core is:

$$k_{eff} = \epsilon p f \eta P_{FNL} P_{TNL}$$

We can assume that an ideal core would be large enough to allow for no leakage, so  $P_{fastnonleakage} = P_{thermalnonleakage} = 1$ . If we assume 100% enrichment,  $p = \epsilon = 1.0$ . Finally we should maximize  $\eta$ . For thermal fuels, from the appendix A data: This table assumes

Fuel	$\sigma_\gamma$	$\sigma_f$	$v$	$\eta$
U-233	49	524	2.4968	2.2832
U-235	101	577	2.4355	2.0726
Pu-239	274	741	2.88	2.1025
Pu-241	425	950	2.9479	2.0367

only thermal fuels, so we will need a moderator. Our concern here is to avoid absorption by the fuel, so we should select a moderator with a high  $\Sigma_s$  and low  $\Sigma_a$ . We would also want a large  $\xi$  to minimize the number of possibilities for absorption. This is captured in the moderating ratio, for which heavy water has the highest value (5670).

This only considers thermal fuels. A fast fuel would not require a moderator, meaning  $f = 1$ , but might have a poor value of  $\eta$ , making it unsuitable.

Actual reactor design would require values of  $k$  very close to 1 in order to allow for reactor control. If we assume a core has a neutron generation time on the order of a millisecond, a  $k$  value of 1.0093 means power will go from 1% to exceeding design capacity in less than 0.5 seconds. This provides very little time for human feedback. Even if we assume that automated controls are an order of magnitude faster, this still only allows for a  $k$  value of 1.096 before exceeding design capacity.