Problem 1.

Use the product differentiation rule to show that the conservative and non-conservative forms of the 1D spherical equation are identical.

Solution

Non-conservative form:

$$\mu\frac{\partial\Psi(r,\mu,E)}{\partial r}+\frac{1-\mu^2}{r}\frac{\partial\Psi(r,\mu,E)}{\partial\mu}+\sigma_t(r,E)\Psi(r,\mu,E)=q(r,\mu,E)$$

Conservative form:

$$\frac{\mu}{r^2}\frac{\partial\left[r^2\Psi(r,\mu,E)\right]}{\partial r}+\frac{\partial}{\partial\mu}\left[\frac{\left(1-\mu^2\right)\Psi(r,\mu,E)}{r}\right]\sigma_t(r,E)\Psi(r,\mu,E)=q(r,\mu,E)$$

$$\frac{\mu}{r^2}\left[2r\Psi(r,\mu,E)+r^2\frac{\partial\Psi(r,\mu,E)}{\partial r}\right]+\frac{1}{r}\left[-2\mu\Psi(r,\mu,E)+\left(1-\mu^2\right)\frac{\partial\Psi(r,\mu,E)}{\partial\mu}\right]+\sigma_t\Psi(r,\mu,E)=q(r,\mu,E)$$

$$\cancel{\frac{2\mu}{r}\Psi(r,\mu,E)}+\mu\frac{\partial\Psi(r,\mu,E)}{\partial r}-\cancel{\frac{2\mu}{r}\Psi(r,\mu,E)}+\frac{1-\mu^2}{r}\frac{\partial\Psi(r,\mu,E)}{\partial\mu}+\sigma_t\Psi(r,\mu,E)=q(r,\mu,E)$$

$$\mu\frac{\partial\Psi(r,\mu,E)}{\partial r}+\frac{1-\mu^2}{r}\frac{\partial\Psi(r,\mu,E)}{\partial\mu}+\sigma_t(r,E)\Psi(r,\mu,E)=q(r,\mu,E)$$

Problem 2.

Use the product differentiation rule to show that the conservative and non-conservative forms of the 1D cylindrical equation are identical.

Solution

Non-conservative form:

$$\mu\frac{\partial\Psi(r,\omega,\xi,E)}{\partial r}-\frac{\eta}{r}\frac{\partial\Psi(r,\omega,\xi,E)}{\partial\omega}+\sigma_t(r,E)\Psi(r,\omega,\xi,E)=q(r,\omega,\xi,E)$$

Conservative form:

$$\frac{\mu}{r}\frac{\partial\left[r\Psi(r,\omega,\xi,E)\right]}{\partial r}-\frac{1}{r}\frac{\partial\left[\eta\Psi(r,\omega,\xi,E)\right]}{\partial\omega}+\sigma_t(r,E)\Psi(r,\omega,\xi,E)=q(r,\omega,\xi,E)$$

$$\begin{aligned}
\xi &= \cos\theta\\
\mu &= \cos\theta\cos\omega\\
\nu &= \cos\theta\sin\omega\\
\frac{\partial\nu}{\partial\omega} &= \cos\theta\cos\omega\\
\frac{\mu}{r}\left[\Psi+r\frac{\partial\Psi}{\partial r}\right]-\frac{1}{r}\left[\frac{\partial\eta}{\partial\omega}\Psi+\eta\frac{\partial\Psi}{\partial\omega}\right]+\sigma_t\Psi &= q\\
\frac{\mu}{r}\Psi+\mu\frac{\partial\Psi}{\partial r}-\frac{1}{r}\frac{\partial\eta}{\partial\omega}\Psi+\frac{\eta}{r}\frac{\partial\Psi}{\partial\omega}+\sigma_t\Psi &= q\\
\mu\frac{\partial\Psi}{\partial r}-\frac{\eta}{r}\frac{\partial\Psi}{\partial\omega}+\sigma_t\Psi+\frac{\mu}{r}\Psi-\frac{1}{r}\Psi\frac{\partial}{\partial\omega}\sin\omega &= q\\
\mu\frac{\partial\Psi}{\partial r}-\frac{\eta}{r}\frac{\partial\Psi}{\partial\omega}+\sigma_t\Psi+\frac{\mu}{r}\Psi-\frac{1}{r}\Psi\cos\omega &= q\\
\mu\frac{\partial\Psi}{\partial r}-\frac{\eta}{r}\frac{\partial\Psi}{\partial\omega}+\sigma_t\Psi+\cancel{\frac{\mu}{r}\Psi}-\cancel{\frac{1}{r}\Psi}\mu &= q\\
\mu\frac{\partial\Psi(r,\omega,\xi,E)}{\partial r}-\frac{\eta}{r}\frac{\partial\Psi(r,\omega,\xi,E)}{\partial\omega}+\sigma_t(r,E)\Psi(r,\omega,\xi,E) &= q(r,\omega,\xi,E)
\end{aligned}$$

Problem 3.

Show that the white boundary condition is given by:

$$\Psi(\vec{r}_s, \hat{\Omega}, E, t) = 4J_n^+(\vec{r}_s, E, t)$$

Solution

For every neutron leaving the volume ($\hat{\Omega} \cdot \hat{n} > 0$), a neutron must return isotropically. That neutron is therefore averaged over the area of a half sphere.

$$\int_0^{\pi/2} \int_0^{2\pi} \frac{\Psi}{\mathrm{d}\theta \mathrm{d}\phi}$$