Problem 1.

For a total cross section given by the equation

$$\sigma_t(E) = 5 + 0.5E - 0.1E^2, E \texttt{ in } \mathrm{keV}$$

find the total group cross section for a group that spans from 2 keV to 3 keV. Assume flux is $1/E$.

Solution

$$\begin{aligned}
\sigma_g &= \frac{\int_{E_{g-1}}^{E_g} \sigma_t(E)\phi(E)\,\mathrm{d}E}{\int_{E_{g-1}}^{E_g} \phi(E)\,\mathrm{d}E} \\
&= \frac{\int_2^3 (5 + 0.5E - 0.1E^2)\frac{1}{E}\,\mathrm{d}E}{\int_2^3 \frac{1}{E}\,\mathrm{d}E} \\
&= \frac{5\int_2^3 \frac{1}{E}\,\mathrm{d}E + 0.5\int_2^3 \mathrm{d}E - 0.1\int_2^3 E\,\mathrm{d}E}{\ln 3 - \ln 2} \\
&= \frac{5(\ln 3 - \ln 2) + 0.5(3-2) - 0.1(\frac{3^2}{2} - \frac{2^2}{2})}{\ln 3 - \ln 2} \\
\sigma_g &= 5.62\,\mathrm{b}
\end{aligned}$$

Problem 2.

Find the isotropic elastic scatter cross section for Carbon-12 (A=12) from an energy group that spans from 0.6 keV to 0.7 keV to a group that spans from 0.4 keV to 0.5 keV. Assume the flux spectrum is $1/E$ and that the scattering cross section is a constant 5 b.

Solution

First, calculate α

$$\alpha = \frac{(A-1)^2}{(A+1)^2} = 0.715$$

Then calculate cross sections

$$\begin{aligned}
\sigma_{g'\to g} &= \int_{0.6\alpha}^{0.5} \mathrm{d}E \int_{0.6}^{0.5/\alpha} \mathrm{d}E' \, \frac{\sigma}{(1-\alpha)E'} \psi(E') \\
&= \frac{\sigma}{1-\alpha} \int_{0.6\alpha}^{0.5} \mathrm{d}E \int_{0.6}^{0.5/\alpha} \frac{\mathrm{d}E'}{E'^2} \\
&= \frac{\sigma}{1-\alpha}(0.5 - 0.6\alpha)(\frac{1}{0.6} - \frac{1}{0.5/\alpha}) \\
&= \frac{5}{1-0.715}(0.5 - 0.6 * 0.715)(\frac{1}{0.6} - \frac{1}{0.5/0.715}) \\
\sigma_{g'\to g} &= 0.29\,\mathrm{b}
\end{aligned}$$

Problem 3.

For the same physical situation as in the previous problem, find the within-group scattering cross sections for the energy group that spans from 0.6 keV to 0.7 keV.

Solution

$$\sigma_{g\to g} = \int_{0.6\alpha}^{0.7} \mathrm{d}E \int_{0.6}^{0.7/\alpha} \mathrm{d}E' \, \frac{\sigma}{(1-\alpha)E'}\psi(E')$$

Since $0.6\alpha = 0.43$ and $0.7/\alpha = 0.98$ are outside the bounds of this group, instead use the bounds of integration:

$$\begin{aligned}
\sigma_{g\to g} &= \int_{0.6}^{0.7} \mathrm{d}E \int_{0.6}^{0.7} \mathrm{d}E' \, \frac{\sigma}{(1-\alpha)E'}\psi(E') \\
&= \frac{\sigma}{(1-\alpha)}(0.7-0.6)(\frac{1}{0.6}-\frac{1}{0.7}) \\
\sigma_{g\to g} &= 0.42\,\mathrm{b}
\end{aligned}$$