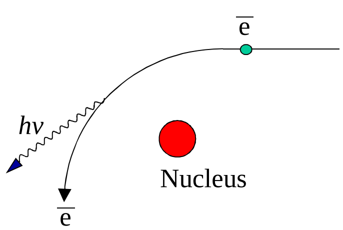
Interactions of Ionizing Radiation

Bremsstrahlung



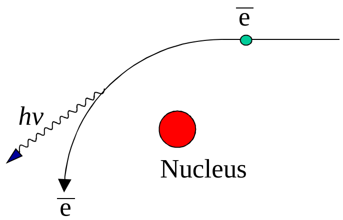
$$-\left(\frac{dE}{dx}\right)_r = \frac{NEZ(Z+1)e^4}{137m_0^2c^4}\left(4\ln\frac{2E}{m_0c^2} - \frac{4}{3}\right)$$

m = mass
N = Number Density
E = Kinetic Energy
Z = Atomic Density
e = Electric Charge
c = Lightspeed

Braking Radiation

- The process of bremsstrahlung is the result of the interaction between a high speed electron and a nucleus
- An electron passing near a nucleus may be deflected from its path by the action of Coulomb forces
- This causes the electron to loose energy according to Maxwell’s general theory of electromagnetic radiation

Bremsstrahlung



$$-\left(\frac{dE}{dx}\right)_r = \frac{NEZ(Z+1)e^4}{137 m_0^2 c^4}\left(4 \ln \frac{2E}{m_0 c^2} - \frac{4}{3}\right)$$

m = mass
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Braking Radiation

- According to this theory, energy is propagated through space be electromagnetic fields
- As the electron (*with its associated magnetic field*) passes in the vicinity of the nucleus, it suffers a sudden deflection and acceleration
- As a result, all or part of its energy is dissociated from the electron and propagates through space as electromagnetic radiation

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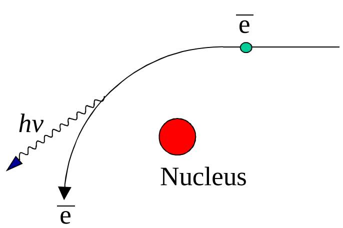
Bremsstrahlung



$$-\left(\frac{dE}{dx}\right)_r = \frac{NEZ(Z+1)e^4}{137 m_0^2 c^4}\left(4 \ln \frac{2E}{m_0 c^2} - \frac{4}{3}\right)$$

m = mass
N = Number Density
E = Kinetic Energy
Z = Atomic Density

e = Electric Charge
c = Lightspeed

Braking Radiation

- In other words, any charge must radiate energy when accelerated
- The linear specific energy loss through this radiative process is given by $-\left(\frac{dE}{dx}\right)$
- The factors E and Z^2 in the numerator show that radiative losses are most important for high energy electrons and for materials of large atomic numbers

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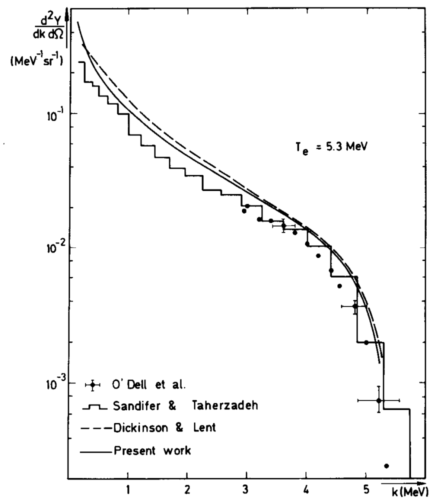
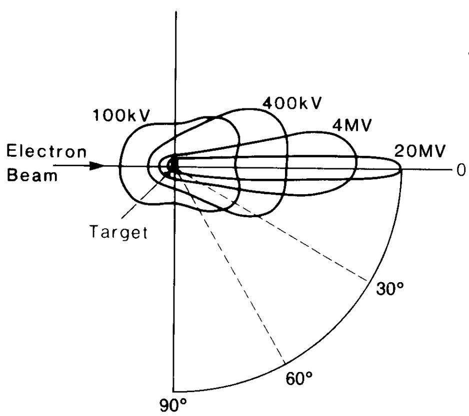
Bremsstrahlung



Braking Radiation

- An electron will have one or more bremsstrahlung interactions in the material
- An electron may loose all or part of its energy in each collision
- As such, the photon may have any energy up to the initial energy of the electron
- The emission of low energy photons predominates, and the average photon energy is a small fraction of the incident electron energy

Bremsstrahlung

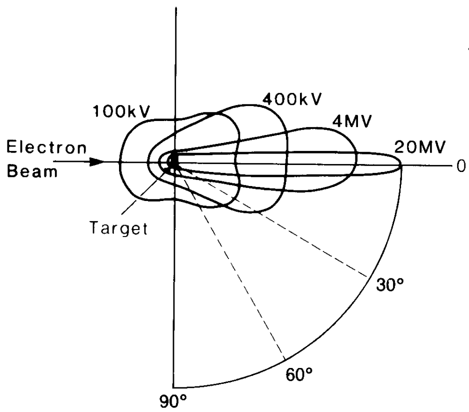


Efficiency = 9×10^{-10} ZV

Braking Radiation

- The direction of emission of bremsstrahlung photons depends on the initial energy of the incident electron
- At electron energies below 100-keV, x-rays are emitted essentially equally in all directions
- As the kinetic energy of the electrons increases, the direction of x-ray emission become more forward directed

Bremsstrahlung



Efficiency = 9 x 10^{-10} ZV

Braking Radiation

- Linear accelerators use *transmission-type* targets in which the electron beam bombards the target from one side, and the photon beam is emitted from the opposite side
- Diagnostic x-ray tubes (<*200-keV*) use 90-degree targets
- The efficiency of x-ray production depends on the atomic number (*Z*) and the voltage applied to the tube (*V*) in volts

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Bremsstrahlung

Sample Question: How much energy is deposited as heat for a 100kV electron beam incident on a tungsten target?

Efficiency = 9 x 10^{-10} ZV
= 9 x 10^{-10} (74) (100,000)
= 0.7% Photons
= 99.3% Heat

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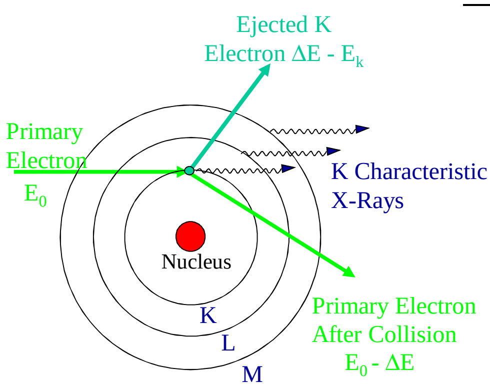
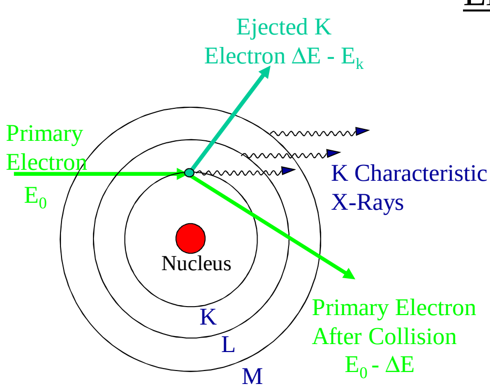
Characteristic X-Rays



Electron Shells

- Electron incident on a target also produce characteristic x-rays
- An electron with kinetic energy (E_0) can interact with the atom by ejecting an orbital electron
- When K, L, M, and N electrons are ejected, the atom becomes ionized
- The original electron will be deflected with an energy of $E_0 - \Delta E$
- A part of ΔE is spent in overcoming the binding energy of the electron and the rest is carried away by the ejected electron

Characteristic X-Rays



Electron Shells

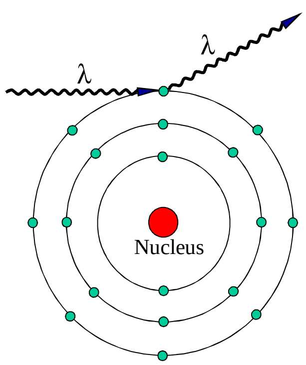
- This creates a vacancy in an inner orbit, and an outer orbital electron falls down to fill the vacancy
- As the electrons decreases its orbit, electromagnetic radiation is emitted
- These x-rays are characteristic of the atoms and the shells between which the transitions take place
- Characteristic x-rays are emitted at discrete energies: $h\nu = E_K - E_L$

Characteristic X-Rays

Principal Characteristic X-Ray Energies for Tungsten

	Lines	Transition	Energy (keV)
K Series	$K\beta_2$	$N_{III} - K$	69.09
	$K\beta_1$	$M_{III} - K$	67.23
	$K\alpha_1$	$L_{III} - K$	59.31
	$K\alpha_2$	$L_{II} - K$	57.97
L Series	$L\gamma_1$	$N_{IV} - L_{II}$	11.28
	$L\beta_2$	$N_V - L_{III}$	9.96
	$L\beta_1$	$M_{IV} - L_{II}$	9.67
	$L\alpha_1$	$M_V - L_{III}$	8.40
	$L\alpha_2$	$M_{IV} - L_{III}$	8.33

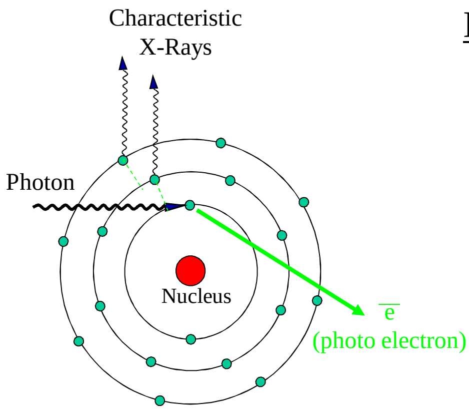
Coherent Scattering



Classical Scattering

- Coherent scattering is a classical scattering where the an electromagnetic wave passes near an electron
- The electron begins oscillating, and then reradiates the energy at the same frequency as the incident electromagnetic wave
- No energy is transferred, and the only effect is the scattering of the photon at small angles
- This process is only of academic interest in radiation therapy

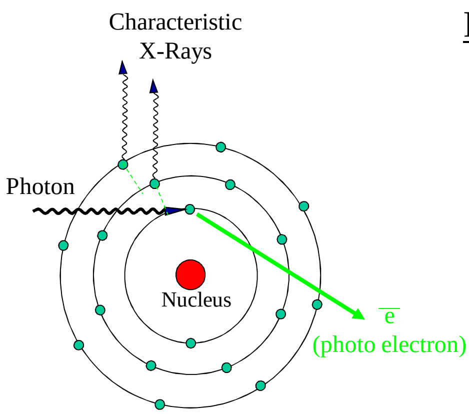
Photoelectric Effect



Electron Ejection

- The photoelectric effect is a phenomenon in which a photon interacts with an atom and ejects one of the orbital electrons
- The entire energy of the photon (hv) is transferred to the atomic electron
- The kinetic energy of the ejected electron is equal to $hv - E_B$, where E_B is the binding energy of the electron
- Interactions of this type take place in the K, L, M, or N shells

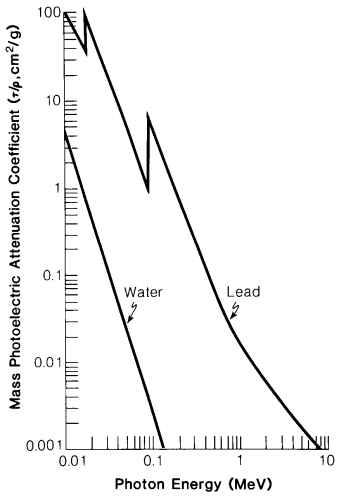
Photoelectric Effect



Photoelectron Ejection

- A vacancy is created in the shell after the electron is ejected
- This leaves the atom in an excited state
- The vacancy can be filled by an outer orbital electron and the emission of characteristic x-rays
- There is also the probability of the emission of Auger electrons
- The energy from Auger electrons and characteristic x-rays are typically deposited locally in biological material

Photoelectric Effect



Photoelectron Ejection

- The probability of photoelectric absorption (τ/ρ) depends on the incident photon energy and the density of the scattering material
- The relationship between τ/ρ and photon energy is approximately $\tau/\rho \propto 1 / E^3$
- The discontinuities in lead at 15 and 88 keV are absorption edges, which correspond to the binding energies of the K and L shells
- An electron with less than 15 keV does not have enough energy to eject an L electron

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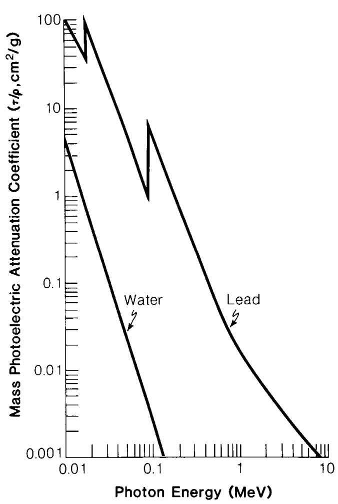
Photoelectric Effect



Photoelectron Ejection

- An photon with less than 15 keV is limited to interactions with the M or higher shells
- When the photon energy just matches the binding energy of the L shell, resonance occurs and the probability of photoelectric absorption becomes high
- Beyond this point, the probablity decreases by $1 / E^3$ until the K absorption edge
- The K edge for water occurs at very low energies (~ *0.5 keV*)

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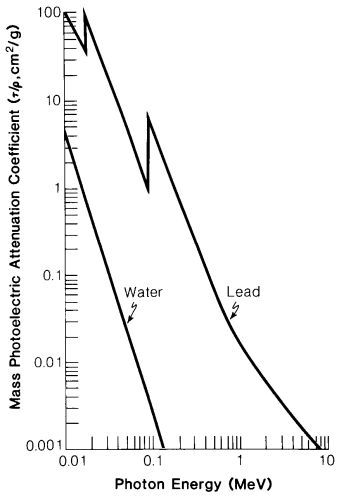
Photoelectric Effect



Photoelectron Ejection

- Photoelectric attenuation depends strongly on the atomic number of the absorbing material
  $\tau/\rho \propto 1 / Z^3$
- The difference is Z for various tissues (*bone, muscle, fat, lung, etc…*) amplified the differences in x-ray absorption
- This is the basis for diagnostic radiology, which uses 15 – 200 kV x-rays

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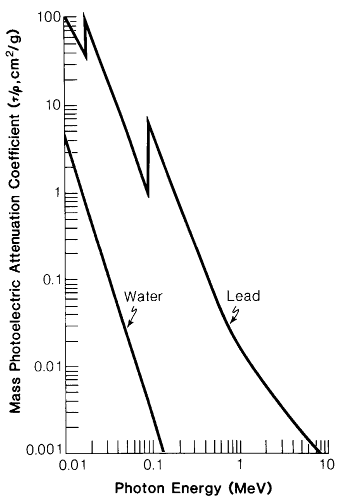
Photoelectric Effect



Photoelectron Ejection

- The Z^3 dependence is exploited in radiology using contrast materials such as $BaSO_4$
- The angular dependence of ejected electrons depends on he incident photon energy
- For a low-energy photon, the photoelectron will be emitted at 90-degrees relative to the direction of the incident photon
- As the photon energy increases, the photoelectrons are more forward directed

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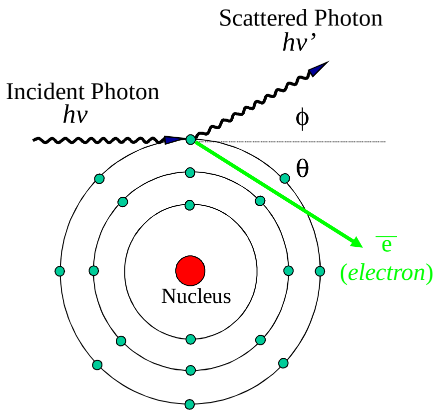
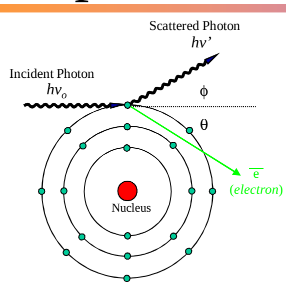
Compton Effect



Compton Scattering

- The interaction process of Compton Scattering takes place between the incident photon and an electron in the absorbing material
- This is the most predominate interaction mechanism in radiation therapy
- The incoming photon transfers energy to an orbital electron
- The photon looses energy and is deflected through an angle ϕ with respect to its original direction
- The electron is emitted at angle θ

Compton Effect



$$E = hv_o \frac{\alpha(1 - \cos\phi)}{1 - \alpha(1 - \cos\phi)}$$

$$hv' = hv_o \frac{1}{1 - \alpha(1 - \cos\phi)}$$

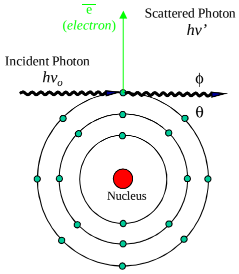
$$\cos\theta = (1 + \alpha)\tan\phi/2$$

$$\alpha = hv_o / m_o c^2$$

Compton Scattering

- All angles of scattering are possible, and the energy transferred to the electron can vary from zero to large fractions of the photon energy
- The Compton process can be analyzed in terms of a collision between two particles
- The scattered photon energy (*hv'*), and the scattered electron energy (*E*) can be calculated by applying the laws of conservation of energy and momentum

Compton Effect



$$E = hv_o \frac{\alpha(1 - \cos 0)}{1 - \alpha(1 - \cos 0)} = 0$$

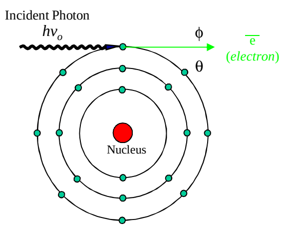
$$hv' = hv_o \frac{1}{1 - \alpha(1 - \cos 0)} = hv_o$$

$$\alpha = hv_o / m_o c^2$$

Compton: Grazing Hit

- If the photon makes a grazing hit with the electron, then the electron will be emitted at right angles
- The scattered photon will continue in the forward direction
- The electron energy would be zero
- In reality, a grazing hit would not eject an electron because there would not be enough energy transferred to the electron to overcome the shell binding energy

Compton Effect



$$E = hv_o \frac{\alpha(1 - \cos 180)}{1 - \alpha(1 - \cos 180)} = hv_o \frac{2\alpha}{1 + 2\alpha}$$

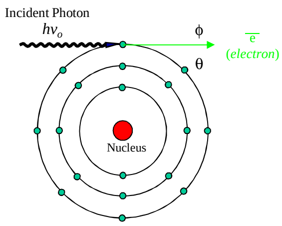
$$hv' = hv_o \frac{1}{1 - \alpha(1 - \cos 180)} = hv_o \frac{1}{1 + 2\alpha}$$

$$\alpha = hv_o / 0.511 \text{ MeV}$$

Compton: Direct Hit

- If the photon makes a direct hit with the electron, then the electron will travel forward (θ = *0-degrees*)
- The scattered photon will then travel backward (ϕ= *180-degrees*) after the collision
- The electron will receive the maximum possible energy
- The photon will be left with the minimum possible energy
- cos ϕ= cos (180) = -1

Compton Effect



$$E = hv_o \frac{2\alpha}{1+2\alpha} = 51.1 \frac{2(0.1)}{1+2(0.1)} = 8.52 \text{ keV}$$

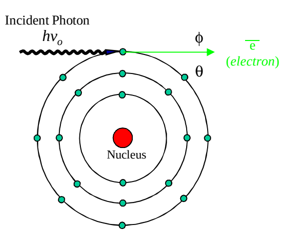
$$hv' = hv_o \frac{1}{1+2\alpha} = 51.1 \frac{1}{1+2(0.1)} = 42.58 \text{ keV}$$

$$\alpha = 0.0511 \text{ MeV} / 0.511 \text{ MeV} = 0.1$$

Compton: Direct Hit

- If the incident photon energy is much less than the rest energy of the electron, only a small part of the energy will be transferred to the electron
- Assume an incident photon energy of 51.1 keV
- The scattered photon has only slightly less energy than the incident photon

Compton Effect



$$E = hv_o \frac{2\alpha}{1+2\alpha} = 5.11 \frac{2(10)}{1+2(10)} = 4.87 \text{ MeV}$$

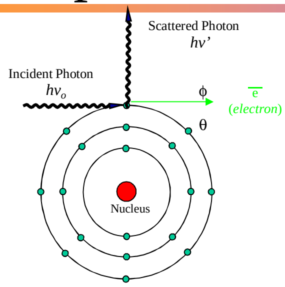
$$hv' = hv_o \frac{1}{1+2\alpha} = 5.11 \frac{1}{1+2(10)} = 0.24 \text{ MeV}$$

$$\alpha = 5.11 \text{ MeV} / 0.511 \text{ MeV} = 10$$

Compton: Direct Hit

- If the incident photon energy is much greater than the rest energy of the electron, the photon looses most of its energy to the electron
- Assume an incident photon energy of 5.11 MeV
- In contrast to low energy incident photons, interactions in the MeV range result in a substantial transfer of energy to the electron

Compton Effect



$$E = hv_o \frac{\alpha(1 - \cos 90)}{1 - \alpha(1 - \cos 90)} = hv_o \frac{\alpha}{1+\alpha}$$

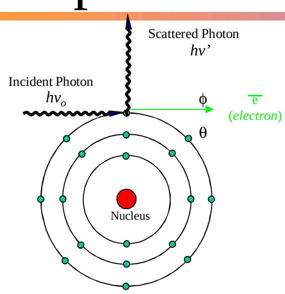
$$hv' = hv_o \frac{1}{1 - \alpha(1 - \cos 90)} = hv_o \frac{1}{1+\alpha}$$

$$\alpha = hv_o / 0.511\ MeV$$

Compton: 90-Degree Photon

- One needs to know the energy of the scattered photons when designing radiation protection shielding walls
- The energy of photons scattered at 90-degrees is used to calculate the wall thickness
- If the photon is scattered at right angles relative to its original direction (*ϕ= 90-degrees*), the electron emission will depend on α
- cos ϕ= cos (90) = 0

Compton Effect



$$E = hv_o \frac{\alpha(1 - \cos 90)}{1 - \alpha(1 - \cos 90)} = hv_o \frac{\alpha}{1+\alpha}$$

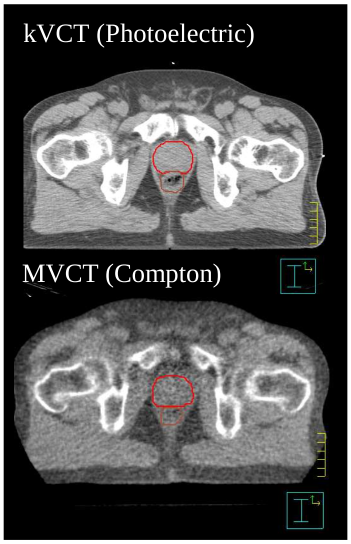
$$hv' = hv_o \frac{1}{1 - \alpha(1 - \cos 90)} = hv_o \frac{1}{1+\alpha}$$

$$\alpha = hv_o / 0.511\ MeV$$

Compton: 90-Degree Photon

- For high energy photons, α is much less than 1
- The photon scatter equation then reduces to hv_o/α, or 0.511 MeV
- If the energy of the incident photon is high, then the radiation scattered at right angles is independent of the incident energy and has a maximum value of 0.511 MeV
- Similarly, the radiation scatter backwards is independent of the incident energy and has a maximum value of 0.255 MeV

Compton Effect



Compton: Density Effect

- Because Compton scatter involves essentially free electrons, it is independent of atomic number
- The Compton mass attenuation coefficient (τ/ρ) depends on the number of electrons per gram, which is nearly the same for all materials
- For a Cobalt-60 beam, the attenuation per g/cm^2 is the same for bone and soft tissue
- However, 1-cm of bone will attenuate more that soft tissue because of the increased density

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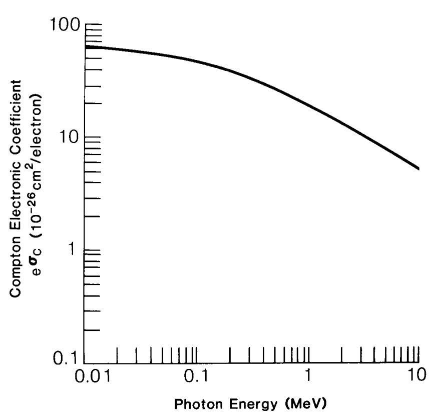
Compton Effect



Compton: Energy Effect

- In Compton scattering, the energy of the incident photon must be large compared with the electron binding energy
- As the photon energy increases beyond the binding energy of the K electron, the photoelectric effect decreases rapidly with energy and the Compton Effect becomes more and more important
- However, the Compton Effect does decrease with increasing photon energy

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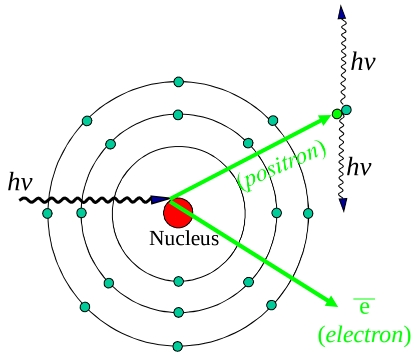
Pair Production



Pair Production (E=cm^2)

- If the energy of the incident photon is greater than 1.02 MeV, then the photon may interact with the atom through the mechanism of pair production
- In this process, the photon interacts strongly with the electromagnetic field of the nucleus and gives up all of its energy in the process of creating a pair consisting of an electron and a positron
- Since the rest mass of the electron is 0.511 MeV, the threshold energy for pair production is 1.02 MeV

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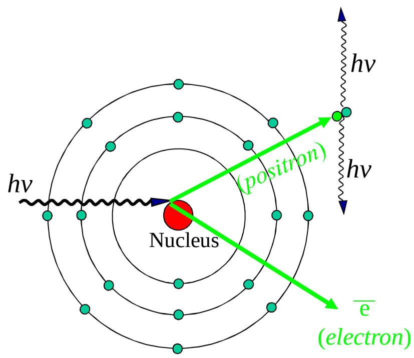
Pair Production



Pair Production (E=cm^2)

- The total kinetic energy available for the pair is (hv – 1.02 MeV)
- The most probable energy distribution is for each particles to acquire half the kinetic energy
- The two particles tend to be emitted in the forward direction
- The electron and positron lose energy as they traverse matter by ionization, excitation, and bremsstrahlung

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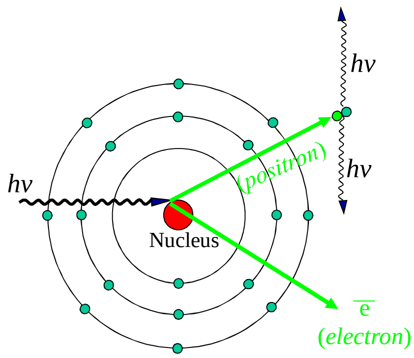
Pair Production

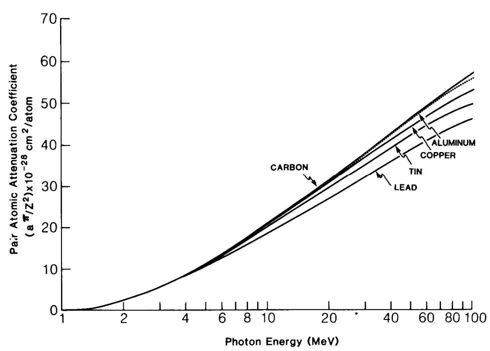
Pair Production (E=cm^2)



- Near the end of the positron's range, it slows and combines with one of the free electrons in its vicinity
- This gives rise to two annihilation photons, each having 0.51 MeV in energy
- This is an example in which mass is converted into energy: E = mc^2
- Since the momentum is conserved, the photons are ejected in opposite directions

Pair Production

Pair Production: Energy Effect

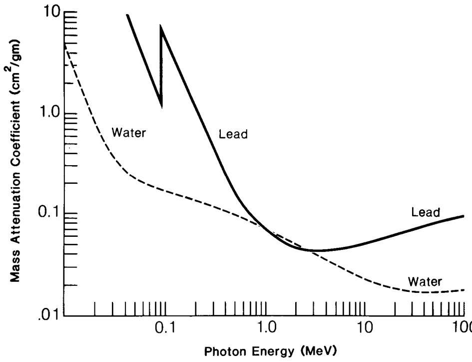


- Since pair production results from an interaction with the electromagnetic field of the nucleus, the probability of this process increases rapidly with atomic number

The attenuation coefficient for pair production varies by Z2 per atom

The likelihood of this interaction increases as the logarithm of the incident photon energy (*above the threshold energy*)

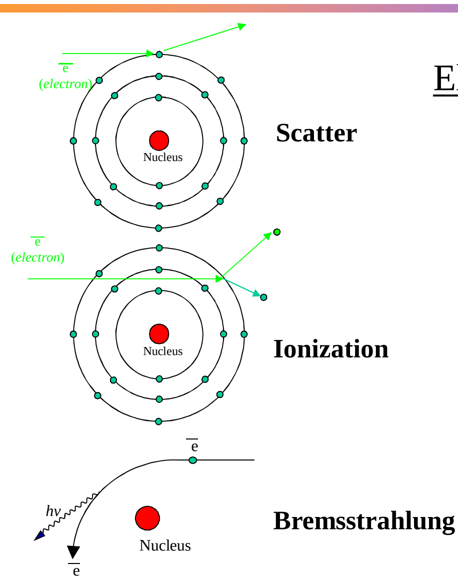
Total Attenuation Coefficient



Importance of Interactions

- Coherent scattering is only important for very low photon energies
- The attenuation coefficients decrease rapidly with energy until the photon energy far exceeds the binding energies and the Compton Effect becomes the predominate mode of interaction
- The dominance of Pair Production occurs at energies much greater than the threshold energy

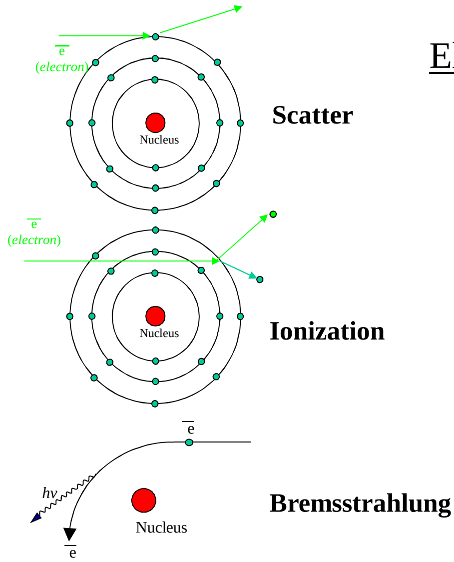
Electron Interactions



Electron Energy Loss

- Electrons lose energy predominately by ionization and excitation, which results in the deposition of energy in the medium
- Ionization consists of stripping electrons from the atoms
- The stripped electron can potentially receive enough energy to produce an ionization track of its own
- The ejected electron is called a delta-ray

Electron Interactions



Electron Energy Loss

- If the energy transfer is not sufficient to overcome the binding energy, then the orbital electron is displaced from its stable position
- The excited electron then returns to its stable position
- The electron may also lose energy through bremsstrahlung interactions with the nucleus
- Overall, electrons undergo a great deal of scattering and changes in direction because of their small mass