Problem 1.

We would like to take an asorption spectrum of a sample that we believe has its absorption maximum at 200 nm. Which of the following materials should our cuvette be made of: borosilicate glass, polystyrene, or quartz? Explain why your choice is the best option.

Solution

In order to measure the absorption in our sample, we need to ensure that our cuvette does not absorb our signal. Borosilicate has a very low transmission fraction below 300 nm or so, so it is a poor choice. Similarly, polystyrene has a strong, broad absorption peak at 250 nm, making it unsuitable for this cuvette.

Problem 2.

Why are mirrors preferred over lenses for imaging in many spectroscopic instruments that must cover multiple wavelengths?

Solution

Lenses may absorb wavelengths of interest, depending on their materials. Mirrors cover a broad wavelength range, e.g. 250 nm to 20 µm in the case of Alumninum-backed mirrors.

Problem 3.

What performance characteristics of a monochromator are affected when only the grating groove density is changed?

Solution

This affects the resolution of the monochromator. More groove density means that the spacing between the grooves is smaller. This means that the resolution, $R = \frac{\lambda}{\Delta\lambda} = nN$, increases.

Problem 4.

A ray in air (n=1.33) is incident on a block of sapphire (n=1.77) at a 40° angle from the normal to the glass surface. At what angle relative to the normal will the ray be transmitted through the glass?

Solution

$$\begin{aligned} n_1 \sin\theta_1 &= n_2 \sin\theta_2 \\ \theta_2 &= \arcsin\frac{n_1 \sin\theta_1}{n_2} \\ &= \arcsin\frac{1.33 \times \sin 40^\circ}{1.77} \\ \theta_2 &= 28.9^\circ \end{aligned}$$

Problem 5.

What is a birefringent crystal? How does a birefringent crystal work if you send light through it? What is an example of a birefringent crystal?

Solution

A birefringent crystal is a crystal made of a material whose index of refraction depends on the polarization of the incident light. When light is sent through the crystal, it splits into two different beam paths and **double refracts**. An observer on the other side will see two images, offset by a fixed amount. An example of a birefringent crystal is calcite.

Problem 6.

List the four types of lenses and identify them as either converging (C) or diverging (D).

Solution

There are six types of lenses:

- Biconvex: converging
- Plano-convex: converging
- Positive meniscus: converguing
- Negative meniscus: diverging
- Plano-concave: diverging
- Biconcave: diverging

Problem 7.

A thin biconvex lens of refractive index 1.47 and diameter of 50.8 mm has radii of curvature of $R_1 = 1\,\text{cm}$ and $R_2 = 0.5\,\text{cm}$.

(a) Find the focal point of the lens

(b) If the object is placed 2 cm from the lens, where is the image?

(c) What is the f/# of the lens?

(d) Calculate the NA of the lens

Solution

Part (a)

$$\begin{aligned}
\frac{1}{f} &= (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \\
&= (1.47-1)\left(\frac{1}{1\,\text{cm}} - \frac{-1}{0.5\,\text{cm}}\right) \\
\frac{1}{f} &= 1.41\,\text{cm}^{-1} \\
f &= 7.09\,\text{mm}
\end{aligned}$$

Part (b)

$$\begin{aligned}
\frac{1}{f} &= \frac{1}{s_1} + \frac{1}{s_2} \\
\frac{1}{s_2} &= \frac{1}{f} - \frac{1}{s_1} \\
s_2 &= \left(\frac{1}{7.09\,\text{mm}} - \frac{1}{20\,\text{mm}}\right)^{-1} \\
s_2 &= 1.10\,\text{cm}
\end{aligned}$$

The image will be 1.10 cm behind the lens.

Part (c)

$$\begin{aligned}
f/\# &= \frac{f}{D} \\
&= 7.09\,\text{mm}/50.8\,\text{mm} \\
&= 0.140
\end{aligned}$$

Part (d)

$$
\begin{aligned}
f/\# &\approx \frac{1}{2N.A.} \\
N.A. &\approx \frac{1}{2f/\#} \\
&\approx (2 * 0.140) \\
&\approx 3.58
\end{aligned}
$$

Problem 8.

What is the definition of an optical aberration? Also name the two types of optical aberrations.

Solution

An optical aberration is a deviation in the behavior of optical elements from the ideal. Two types are:

- chromatic: when broadband light is passed through an optical component, the different wavelengths follow different paths. This results in a spreading of the colors
- monochromatic: aberration in the lens that affects monochromatic light. These result from non-ideally shaped lenses and include coma and astigmatism

Problem 9.

A grating has a groove density of 3600 grooves per mm. If the incident beam strikes the grating at an angle of 30°,

(a) What diffraction angle will the first order of 240 nm appear?

(b) What diffraction angle will the first order of 350 nm appear?

(c) What can we conclude about the relationship between diffraction angle and incident wavelength from the answers you calculated in a and b?

(d) What wavelength in the 2$^{\text{nd}}$ order overlaps with the 350 nm 1$^{\text{st}}$ order beam?

(e) What is the free spectral range for the 1$^{\text{st}}$ order at 600 nm?

Solution

Part (a)

$$\begin{aligned}
m\lambda &= d\left(\sin\theta + \sin\phi\right) \\
\phi &= \arcsin\left(\frac{m\lambda}{d} - \sin\theta\right) \\
&= \arcsin\left(\frac{240\,\text{nm}}{3600\,\text{grooves}/\text{mm}^{-1}} - \sin 30^\circ\right) \\
&= \arcsin 0.364 \\
&= 21.3^\circ
\end{aligned}$$

Part (b)

$$\begin{aligned}
\phi &= \arcsin\left(\frac{350\,\text{nm}}{3600\,\text{grooves}/\text{mm}^{-1}} - \sin 30^\circ\right) \\
&= 49.5^\circ
\end{aligned}$$

Part (c)

To a point, longer wavelengths will be diffracted to larger angles in the first mode.

Part (d)

$$\begin{aligned}
\phi_2 &= \phi_1 \\
\arcsin\left(\frac{m_2\lambda_2}{d} - \sin\theta\right) &= \arcsin\left(\frac{m_1\lambda_1}{d} - \sin\theta\right) \\
2 \times \lambda_2 &= 1 \times \lambda_1 \\
\lambda_2 &= \frac{1}{2} 350\,\mathrm{nm} \\
&= 175\,\mathrm{nm}
\end{aligned}$$

Part (e)

$$\begin{aligned}
\Delta\lambda &= \frac{\lambda}{m} \\
\lambda + \Delta\lambda &= 600\,\mathrm{nm} \\
\lambda &= \frac{1}{2} 600\,\mathrm{nm} = 300\,\mathrm{nm} \\
\Delta\lambda &= 300\,\mathrm{nm}
\end{aligned}$$

Problem 10.

For a fiber optic probe with core and cladding refractive indices of 1.50 and 1.48, respectively, and $\theta_i = 28°$, calculate:

(a) θ_r

(b) NA

Solution

Part (a)

$$\begin{aligned}
\sin\theta_r &= \frac{n_1}{n_2}\sin\theta_i \\
\theta_r &= \arcsin\left(\frac{n_1}{n_2}\sin\theta_i\right) \\
&= \arcsin\left(\frac{1.50}{1.48}\sin 28°\right) \\
\theta_r &= 28.4°
\end{aligned}$$

Part (b)

$$\begin{aligned}
\text{N.A.} &= \sqrt{n_1^2 - n_2^2} \\
&= \sqrt{1.50^2 - 1.48^2} \\
\text{N.A.} &= 0.244
\end{aligned}$$

Problem 11.

A monochromator has the following specifications:

- reciprocal linear dispersion = $1.5\,\mathrm{nm\,mm^{-1}}$
- focal length = $320\,\mathrm{mm}$
- f/# = 4.6
- grating size: $68 \times 68\,\mathrm{mm}$
- groove density = $1800\,\mathrm{grooves/mm}$

Calculate the following at $633\,\mathrm{nm}$ assuming the 1st order is used:

(a) Angular dispersion

(b) Linear dispersion

(c) Slit width to obtain a $5\,\mathrm{nm}$ geometric spectral bandpass

Solution

Part (a)

$$
\begin{aligned}
D_l &= f \times D_a \\
R_d^{-1} &= f \times D_a \\
D_a &= \frac{1}{R_d \times f} \\
&= \frac{1}{320\,\mathrm{mm} \times 1.5\,\mathrm{nm\,mm^{-1}}} \\
&= 0.0021\,\mathrm{rad\,nm^{-1}} \\
&= 0.12\,^\circ\,\mathrm{nm^{-1}}
\end{aligned}
$$

Part (b)

$$
\begin{aligned}
&\left(1.5\,\mathrm{nm\,mm^{-1}}\right)^{-1} \\
&0.67\,\mathrm{mm\,nm^{-1}}
\end{aligned}
$$

Part (c)

$$
\begin{aligned}
S_g &= R_d W \\
W &= \frac{S_g}{R_d} \\
&= \frac{5\,\mathrm{nm}}{1.5\,\mathrm{nm}\,\mathrm{mm}^{-1}} \\
&= 3.33\,\mathrm{mm}
\end{aligned}
$$