Homework 6: Optimization

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1 Analytic Solution

To find the minima, first calculate the first derivative:

$$
\begin{aligned}
f(x) &= \left(x^2 + x\right)\cos(x) \\
u &= x^2 + x \\
u' &= 2x + 1 \\
v &= \cos(x) \\
v' &= -\sin(x) \\
f'(x) &= uv' + vu' \\
f'(x) &= (2x + 1)\cos(x) - \left(x^2 + x\right)\sin(x)
\end{aligned}
$$

To distinguish between local maxima and minima, take the second derivative:

$$
\begin{aligned}
f''(x) &= 2\cos(x) - (2x + 1)\sin(x) - (2x + 1)\sin(x) + (x^2 + x)\cos(x) \\
&= 2\cos(x) - (4x + 2)\sin(x) + (x^2 + x)\cos(x)
\end{aligned}
$$

The first derivative was set equal to zero in order to find the maxima and minima. Since that equation does not have an easy analytical solution, the following matlab code was used to find the solutions to $f'(x) = 0$:

```
syms x;
s = [];
for i = -10:0.1:10
m = vpasolve((2*x+1)*cos(x)-(x^2+x)*sin(x)==0,x,i);
s = [s m];
end
s = s(s>-10 & s<10);
```

This produced nine groups of solutions (groups were unique but within machine precision of each other). A plot of (x, f(x)) (figure 1) for each solution was generated to visually identify the smallest value of the plot, which was determined to be $x \approx 9.6204$.

2 Optimization Toolbox

The same function was entered into Matlab's optimization toolbox to determine the minimum value. When initially directed to seek the minimum in the whole range with an initial guess of 0, it found the local minimum value at $x = -0.442$. The initial guess was then altered in steps of 1 through the range of values to try to find the absolute minimum in the range. Interestingly, with an initial guess of 6, the minimum value found was at $x = -9.641$. When the initial value was set to 7, however, the true minimum was found at $x = 9.62$

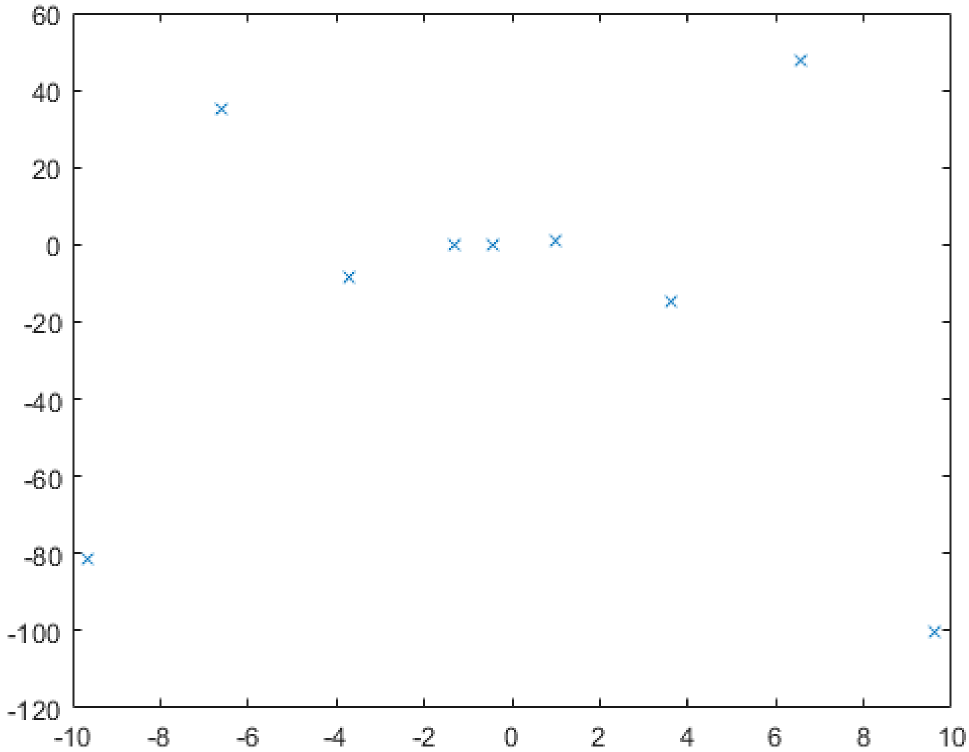

Figure 1: Plot of values for minima candidates

3 More complex optimization

An attempt was made to find the minimum value for a more complex function, $f(x, y) = y\sin(4x) + 1.1x\sin(2y)$ in the range $0 \leq x, y \leq 10$. As before, the optimization tool box was used, with configuration shown in figure 2. Similar to before, a plot was generated (figure 3) in order to find starting values for the problem. Based on the figure, starting values of (10,10) were used, and experimentation with other values indicated that these provided the function's overall minimum at $x, y = (9.04, 8.665), f(x, y) = -18.59$. Other values produced local minima which evaluated with higher values than the above point.

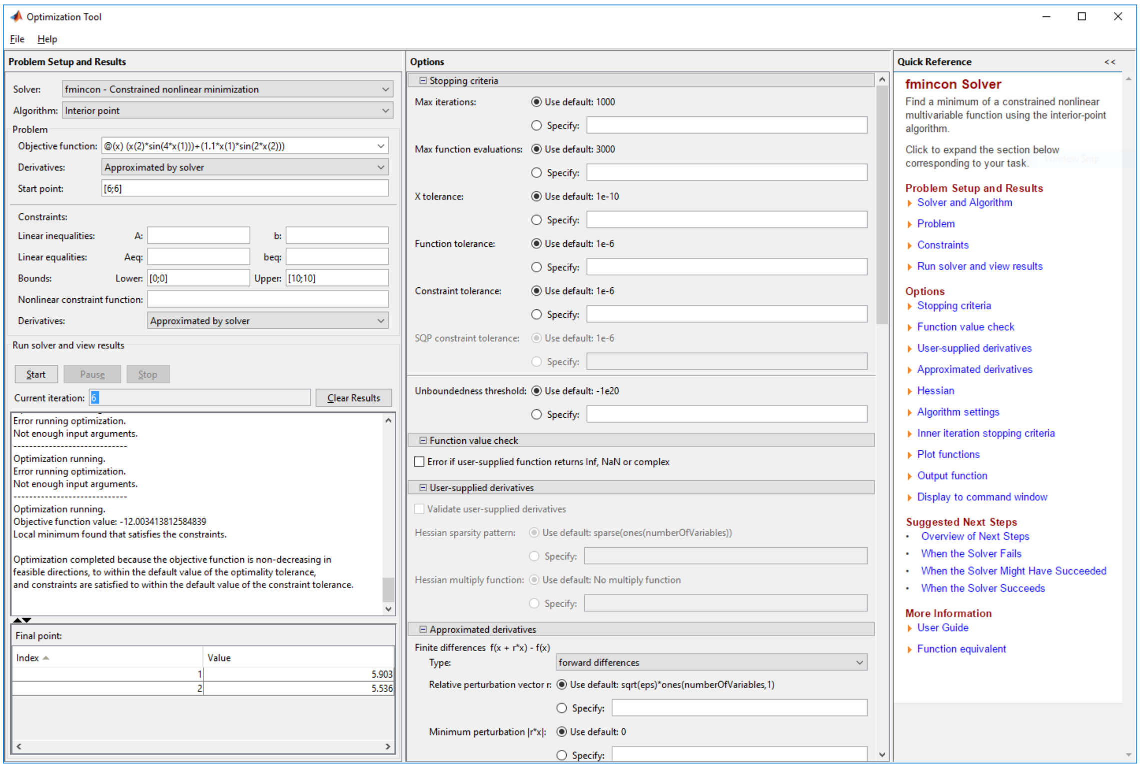

Figure 2: Optimization toolbox for more complex function

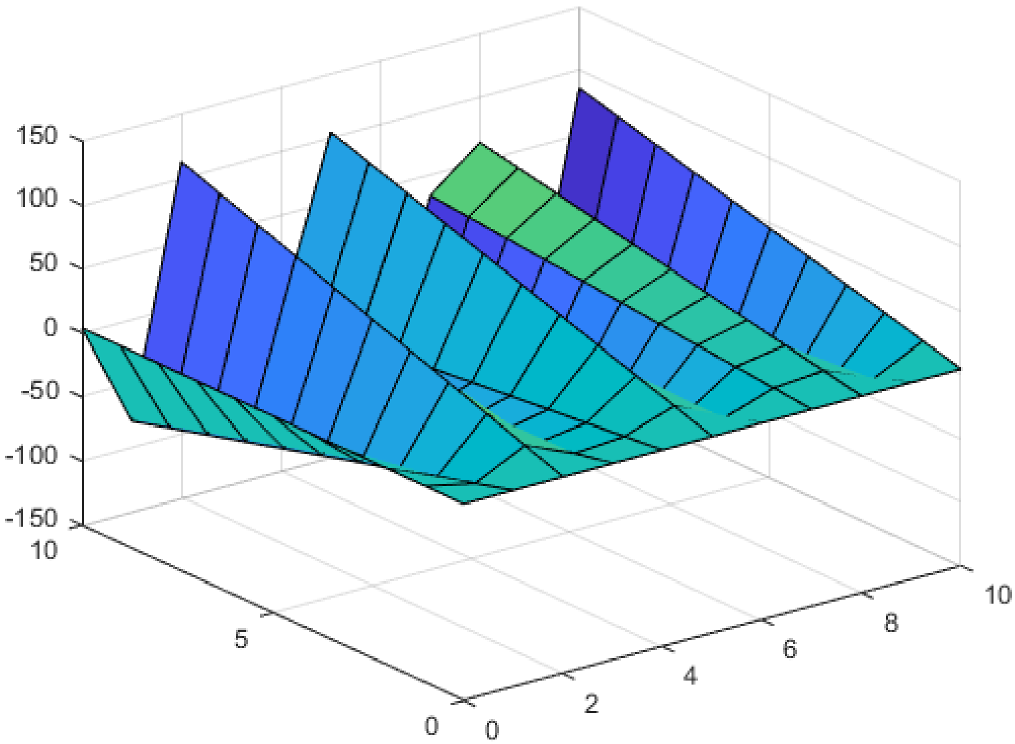

Figure 3: Plot of more complex function