Problem 1.

For each of the assumptions listed on slide 1-9, give a physical situation for which the assumption may not be a good one.

- Particles are points
- Particles travel in straight lines, unaccelerated until they interact
- Particles don't hit other particles
- Collisions are resolved instantaneously
- Material properties are the same no matter what direction a particle approaches
- Composition, configuration, and material properties are known and constant in time
- Only the expected (mean) values of reaction rates are needed

Solution

- In particle accelerators where the beam energy is high, the wavelength can be comparable to the radius of the target particles. At this point it is necessary to treat the nucleons as having volume.
- In the presence of large masses (e.g. neutron stars), spacetime is curved. This distorts the path of particles.
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- There is a small but measurable delay between the absorption of a neutron and subsequent emission or fission event. There is a further delay before the decay of fission fragments produces decay neutrons.
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- In a PWR, the macroscopic cross section of the moderator material changes with temperature, altering the material properties.
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Problem 2.

Use integration by parts and l'Hopital's rule to show that:

$$\lambda = \frac{\int_0^\infty x\sigma_t I(x)dx}{\int_0^\infty \sigma_t I(x)dx} = \frac{\int_0^\infty x\mathrm{e}^{-\sigma_t x}dx}{\int_0^\infty \mathrm{e}^{-\sigma_t x}dx} = \ldots = \frac{\frac{1}{\sigma_t^2}}{\frac{1}{\sigma_t}} = \frac{1}{\sigma_t}$$

Solution

$$\begin{aligned}
\lambda &= \frac{\int_0^\infty x\mathrm{e}^{-\sigma_t x}dx}{\int_0^\infty \mathrm{e}^{-\sigma_t x}dx} \\
\lambda \int_0^\infty \mathrm{e}^{-\sigma_t x}dx &= \int_0^\infty x\mathrm{e}^{-\sigma_t x}dx \\
u &= x \\
\frac{du}{dx} &= 1 \\
v &= -\frac{1}{\sigma_t}\mathrm{e}^{-\sigma_t x} \\
\frac{dv}{dx} &= \mathrm{e}^{-\sigma_t x} \\
\lambda \left(-\frac{1}{\sigma_t}\mathrm{e}^{-\sigma_t x}\Big|_0^\infty \right) &= -\frac{1}{\sigma_t}x\mathrm{e}^{-\sigma_t x}\Big|_0^\infty - \frac{1}{\sigma_t^2}\mathrm{e}^{-\sigma_t x}\Big|_0^\infty \\
\lambda \frac{-1}{\sigma_t}(0-1) &= -\frac{1}{\sigma_t}x\mathrm{e}^{-\sigma_t x}\Big|_0^\infty - \frac{1}{\sigma_t^2}(0-1) \\
\lambda \frac{1}{\sigma_t} &= \frac{1}{\sigma_t}\left(\frac{1}{\sigma_t} - \frac{x}{\mathrm{e}^{\sigma_t x}}\Big|_0^\infty \right) \\
\lambda \frac{1}{\sigma_t} &= \frac{1}{\sigma_t}\left(\frac{1}{\sigma_t} - \left[\lim_{x\to\infty} \frac{x}{\mathrm{e}^{\sigma_t x}} - \lim_{x\to 0} \frac{x}{\mathrm{e}^{\sigma_t x}} \right] \right) \\
\lambda \frac{1}{\sigma_t} &= \frac{1}{\sigma_t}\left(\frac{1}{\sigma_t} - \left[\lim_{x\to\infty} \frac{x}{\mathrm{e}^{\sigma_t x}} - 0 \right] \right)
\end{aligned}$$

To evaluate $\lim_{x\to\infty} \frac{x}{\mathrm{e}^{\sigma_t x}}$, we use l'Hopital's rule.

$$\begin{aligned}
\lim_{x\to\infty} \frac{x}{\mathrm{e}^{\sigma_t x}} &= \lim_{x\to\infty} \frac{\frac{d}{dx}x}{\frac{d}{dx}\mathrm{e}^{\sigma_t x}} \\
&= \lim_{x\to\infty} \frac{1}{\sigma_t \mathrm{e}^{\sigma_t x}} \\
&= 0
\end{aligned}$$

Plugging this back in, we get:

$$
\begin{aligned}
\lambda \frac{1}{\sigma_t} &= \frac{1}{\sigma_t}\left(\frac{1}{\sigma_t} - [0 - 0]\right) \\
\lambda \frac{1}{\sigma_t} &= \frac{1}{\sigma_t^2} \\
\lambda &= \frac{1}{\sigma_t^2} / \frac{1}{\sigma_t} \\
\lambda &= \frac{1}{\sigma_t}
\end{aligned}
$$