Problem 1.

For each of the assumptions listed on slide 1-9, give a physical situation for which the assumption may not be a good one.

- Particles are points
- Particles travel in straight lines, unaccelerated until they interact
- Particles don't hit other particles
- Collisions are resolved instantaneously
- Material properties are the same no matter what direction a particle approaches
- Composition, configuration, and material properties are known and constant in time
- Only the expected (mean) values of reaction rates are needed

Solution

- In particle accelerators where the beam energy is high, the wavelength can be comparable to the radius of the target particles. At this point it is necessary to treat the nucleons as having volume.
- In the presence of large masses (e.g. neutron stars), spacetime is curved. This distorts the path of particles.
- In the Large Hadron Collider, protons and anti-protons collide.
- There is a small but measurable delay between the absorption of a neutron and subsequent emission or fission event. There is a further delay before the decay of fission fragments produces decay neutrons.
- Some detector materials are made of long polymer strands that are (relatively) widely spaced, linked by small CH molecules. A particle incident on these detectors would interact differently if it entered parallel to the chain compared to perpendicular.
- In a PWR, the macroscopic cross section of the moderator material changes with temperature, altering the material properties.
- In a material that has been work hardened, interstitial nuclei with high cross sections would not be uniformly distributed throughout the material. The interstitial sites would interact with neutrons differently from the bulk lattice.

Problem 2.

Use integration by parts and l'Hopital's rule to show that:

$$\lambda = \frac{\int_0^\infty x\sigma_t I(x)dx}{\int_0^\infty \sigma_t I(x)dx} = \frac{\int_0^\infty x\mathrm{e}^{-\sigma_t x}dx}{\int_0^\infty \mathrm{e}^{-\sigma_t x}dx} = \ldots = \frac{\frac{1}{\sigma_t^2}}{\frac{1}{\sigma_t}} = \frac{1}{\sigma_t}$$

Solution

$$\begin{aligned}
\lambda &= \frac{\int_0^\infty x\mathrm{e}^{-\sigma_t x}dx}{\int_0^\infty \mathrm{e}^{-\sigma_t x}dx} \\
\lambda \int_0^\infty \mathrm{e}^{-\sigma_t x}dx &= \int_0^\infty x\mathrm{e}^{-\sigma_t x}dx \\
u &= x \\
\frac{du}{dx} &= 1 \\
v &= -\frac{1}{\sigma_t}\mathrm{e}^{-\sigma_t x} \\
\frac{dv}{dx} &= \mathrm{e}^{-\sigma_t x} \\
\lambda\left(-\frac{1}{\sigma_t}\mathrm{e}^{-\sigma_t x}\Big|_0^\infty\right) &= -\frac{1}{\sigma_t}x\mathrm{e}^{-\sigma_t x}\Big|_0^\infty - \frac{1}{\sigma_t^2}\mathrm{e}^{-\sigma_t x}\Big|_0^\infty \\
\lambda\frac{-1}{\sigma_t}(0-1) &= -\frac{1}{\sigma_t}x\mathrm{e}^{-\sigma_t x}\Big|_0^\infty - \frac{1}{\sigma_t^2}(0-1) \\
\lambda\frac{1}{\sigma_t} &= \frac{1}{\sigma_t}\left(\frac{1}{\sigma_t} - \frac{x}{\mathrm{e}^{\sigma_t x}}\Big|_0^\infty\right) \\
\lambda\frac{1}{\sigma_t} &= \frac{1}{\sigma_t}\left(\frac{1}{\sigma_t} - \left[\lim_{x\to\infty}\frac{x}{\mathrm{e}^{\sigma_t x}} - \lim_{x\to 0}\frac{x}{\mathrm{e}^{\sigma_t x}}\right]\right) \\
\lambda\frac{1}{\sigma_t} &= \frac{1}{\sigma_t}\left(\frac{1}{\sigma_t} - \left[\lim_{x\to\infty}\frac{x}{\mathrm{e}^{\sigma_t x}} - 0\right]\right)
\end{aligned}$$

To evaluate $\lim_{x\to\infty}\frac{x}{\mathrm{e}^{\sigma_t x}}$, we use l'Hopital's rule.

$$\begin{aligned}
\lim_{x\to\infty}\frac{x}{\mathrm{e}^{\sigma_t x}} &= \lim_{x\to\infty}\frac{\frac{d}{dx}x}{\frac{d}{dx}\mathrm{e}^{\sigma_t x}} \\
&= \lim_{x\to\infty}\frac{1}{\sigma_t\mathrm{e}^{\sigma_t x}} \\
&= 0
\end{aligned}$$

Plugging this back in, we get:

$$\begin{aligned}
\lambda\frac{1}{\sigma_t} &= \frac{1}{\sigma_t}\left(\frac{1}{\sigma_t} - [0-0]\right) \\
\lambda\frac{1}{\sigma_t} &= \frac{1}{\sigma_t^2} \\
\lambda &= \frac{1}{\sigma_t^2}/\frac{1}{\sigma_t} \\
\lambda &= \frac{1}{\sigma_t}
\end{aligned}$$