

Problem 1. 2-1

What target isotope must be used for forming the compound nucleus ${}_{11}^{24}\text{Na}$ when the incident projectile is:

- (a) a neutron
- (b) a proton
- (c) an alpha particle?

Solution**Part (a)**

A neutron will increase the mass number, A , by one, but leave the element number, Z , unchanged. Therefore, the answer is a lighter isotope of Neon: ${}_{10}^{23}\text{Ne}$

Part (b)

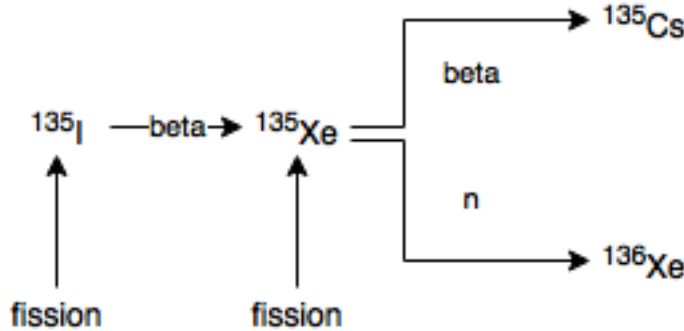
Capturing a proton increases both the mass number and element number by one: ${}_{10}^{23}\text{Ne}$

Part (c)

Capturing an α particle increases the mass number by four and the element number by two: ${}_{8}^{20}\text{O}$

Problem 2. 2-4

A fission product of very considerable importance in thermal reactor operation is ^{135}Xe , which has an enormous thermal absorption cross section of $2 * 10^6 b$. This nuclide can be produced either directly as a fission product or by beta decay of ^{135}I , as indicated by the radioactive chains below:



Write the rate equations describing the concentration of ^{135}I and ^{135}Xe in a nuclear reactor. Then assuming a constant production rate of these isotopes from fission and transmutation rate by neutron capture, determine the steady-state or saturated concentration of ^{135}Xe .

Solution

^{135}I has one production path (production as a fission daughter) and one decay path (β decay into ^{135}I). The rate equation is:

$$\dot{N}_{^{135}\text{I}} = \phi \Sigma_{fission} \gamma_{^{135}\text{I}} - \lambda_{^{135}\text{I}} N_{^{135}\text{I}}$$

^{135}Xe is also produced by fission, as well as by the β decay of ^{135}I . It is removed by β decay as well as by neutron absorption:

$$\dot{N}_{^{135}\text{Xe}} = \phi \Sigma_{fission} \gamma_{^{135}\text{Xe}} + \lambda_{^{135}\text{I}} N_{^{135}\text{I}} - \lambda_{^{135}\text{Xe}} N_{^{135}\text{Xe}} - \sigma_a \phi N_{^{135}\text{Xe}}$$

In steady state, set $\dot{N} = 0$:

Iodine:

$$\begin{aligned} \dot{N}_{^{135}\text{I}} &= 0 \rightarrow \\ \lambda_I N_I &= \phi \Sigma_f \gamma_I \\ N_{^{135}\text{I}} &= \frac{\phi \Sigma_f \gamma_I}{\lambda_I} \end{aligned}$$

Xenon:

$$\begin{aligned}\dot{N}_{Xe} &= 0 \rightarrow \\ N_{Xe}(\lambda_{Xe} + \sigma_{Xe}\phi) &= \phi\Sigma_f\gamma_{Xe} + \lambda_I N_I \\ N_{Xe} &= \frac{\phi\Sigma_f\gamma_{Xe} + \lambda_I N_I}{\lambda_{Xe} + \sigma_{Xe}\phi} \\ &= \frac{\phi\Sigma_f\gamma_{Xe} + \phi\Sigma_f\gamma_I}{\lambda_{Xe} + \sigma_{Xe}\phi} \\ N_{^{135}_{54}\text{Xe}} &= \frac{\phi\Sigma_f(\gamma_{Xe} + \gamma_I)}{\lambda_{Xe} + \sigma_{Xe}\phi}\end{aligned}$$

Problem 3. 2-6

Boron is a common material used to shield against thermal neutrons. Estimate the thickness of boron required to attenuate an incident thermal neutron beam to 0.1% of its intensity. (Use the thermal cross section data in Appendix A.)

Solution

From *Duderstadt* Appendix A, $\Sigma_t = 104\text{cm}^{-1}$ for Boron.

$$\begin{aligned}\left(\frac{1}{e}\right)^n &= \frac{1}{1000} \\ e^{-n} &= 1000^{-1} \\ e^n &= 1000 \\ n &= \ln(1000)\end{aligned}$$

Dividing this by the macroscopic cross section Σ_t gives:

$$\begin{aligned}\frac{n}{\Sigma_t} &= \frac{\ln(1000)}{\Sigma_t} \\ &= 0.0664\text{cm}\end{aligned}$$

Problem 4. 2-8

A free neutron is unstable against beta decay with a half-life of 11.7m. Determine the relative probability that a neutron will undergo beta-decay before being absorbed in an infinite medium. Estimate this probability for a thermal neutron in H_2O .

Solution

We know that:

$$\begin{aligned} P_{absorption}(x) &= \Sigma_a \int_0^x e^{-\Sigma_a x} dx \\ &= \sigma_a N \int_0^x e^{-\sigma_a x} dx \end{aligned}$$

Assume that the incident particle is a thermal neutron with speed $\dot{x} = 2.2 * 10^5 cm/s$. In this case, the neutron should travel a distance of $\dot{x}\lambda$ before decaying:

$$\begin{aligned} &= \sigma_a N \int_0^{\dot{x}\lambda} e^{-\sigma_a x} dx \\ &= 1 - e^{-\sigma_a N \dot{x}\lambda} \end{aligned}$$

The probability of decaying instead of being absorbed (assuming that these are the only two interaction modes:

$$\begin{aligned} 1 &= P_{absorption} + P_{decay} \\ P_{decay} &= 1 - P_{absorption} \\ &= e^{-\sigma_a N \dot{x}\lambda} \end{aligned}$$

Substituting our speed $\dot{x} = 2.2 * 10^5 cm/s$, half life $\lambda = 11.7m = 702s$, and $\sigma_a N = 0.022$ for a thermal neutron in H_2O :

$$= 2.11 * 10^{-1475594}$$

To any reasonable approximation this may be treated as equal to zero.

Problem 5. 2-10

How many mean free paths thick must a shield be designed in order to attenuate an incident neutron beam by a factor of 1000?

Solution

We know that for every mean free path, Σ , travelled, the incident beam attenuates by a factor of $\frac{1}{e}$. Therefore:

$$\begin{aligned}\left(\frac{1}{e}\right)^n &= \frac{1}{1000} \\ e^{-n} &= 1000^{-1} \\ e^n &= 1000 \\ n &= \ln(1000) \\ &= 6.91\end{aligned}$$