#2

ⓒ weighted diamond difference $\bar{\Psi}_i = (1-\alpha)\psi_{i-1/2} + (\alpha)\psi_{i+1/2}$

$$\psi_c = 2([1-\alpha]\psi_{n,i-1/2} + \alpha\,\psi_{n,i+1/2}) - \tfrac{1}{2}(\psi_{n,i-1/2} + \psi_{n,i+1/2})$$

$$= (2-2\alpha)\psi_{n,i-1/2} + 2\alpha\,\psi_{n,i+1/2} - \tfrac{1}{2}\psi_{n,i-1/2} - \tfrac{1}{2}\psi_{n,i+1/2}$$

$$= \left(\tfrac{3}{2} - 2\alpha\right)\psi_{n,i-1/2} + \left(2\alpha - \tfrac{1}{2}\right)\psi_{n,i+1/2}$$

$$= \left(\tfrac{3}{2} - 2\alpha\right)\psi_{n,i-1/2} + \left(2\alpha - \tfrac{1}{2}\right)\left[\frac{\bar{S}_{n,i} + \left(\mu_n/\Delta x_i - (1-\alpha)\sigma_{ti}\right)\psi_{n,i-1/2}}{\mu_n/\Delta x_i + \alpha\sigma_{ti}}\right]$$

ⓑ again assuming $\bar{S}_{n,i} = 0$

$$\left(\tfrac{3}{2} - 2\alpha\right) + \left(2\alpha - \tfrac{1}{2}\right)\frac{\mu/\Delta - (1-\alpha)\sigma}{\mu/\Delta + \alpha\sigma} > 0$$

1. if $\alpha < 0.5$, $2\alpha - \tfrac{1}{2} < 0$
2. if $\alpha > 0.75$, $\left(3/2 - 2\alpha\right) < 0$
3. if $\alpha > 1 - \mu/\Delta\sigma$, $\mu/\Delta - (1-\alpha)\sigma < 0$

#3 is limiting for ψ_R and $\bar{\Psi}$ — $\Delta\sigma$ is the width of the cell in mean free paths, which must be larger than μ in every case

#2

$$\bar{\psi} = \frac{\psi_\ell + 2\psi_c + \psi_r}{4} \Rightarrow \psi_c = 2\bar{\psi} - \tfrac{1}{2}(\psi_\ell + \psi_r)$$

ⓐ Step: $\bar{\psi} = \psi_r$

$$\psi_{n,i+1/2} = \frac{\bar{S} + \mu/\Delta x\, \psi_{n,i-1/2}}{\mu/\Delta x + \sigma}$$

$$\psi_c = 2\psi_r - \tfrac{1}{2}(\psi_\ell + \psi_r)$$

$$= (2 - \tfrac{1}{2})\psi_r - \tfrac{1}{2}\psi_\ell$$

$$= \frac{3}{2}\,\frac{\bar{S}_{n,i} + (\mu^n/\Delta x_i)\psi_{n,i-1/2}}{\mu^n/\Delta x_i + \sigma_{ti}} - \frac{1}{2}\psi_{n,i-1/2}$$

ⓑ limits — ψ_c must be >0. The most limiting case would be a cell with no source, implying:

$$\frac{3}{2}\,\frac{\mu/\Delta}{\mu/\Delta + \sigma_{ti}} - \frac{1}{2} > 0$$

$$3\,\frac{\mu/\Delta}{\mu/\Delta + \sigma} > 1$$

$$3\mu/\Delta > \mu/\Delta + \sigma$$

$$2\mu/\Delta > \sigma$$

$\boxed{2\mu > \sigma\Delta}$ in order for $\psi_c > 0$

meaning cell must be $> 2\mu$ mean free paths wide

2

(b) Diamond Difference $\rightarrow$ $\bar{\Psi} = \frac{1}{2}(\Psi_{in} + \Psi_{o})$

$$\Psi_o = \Psi_{n,i+1/2} = \frac{\bar{S}_{n,i} + \left(\mu_n/\Delta x_i - \sigma_{ti}/2\right)\Psi_{n,i-1/2}}{\mu_n/\Delta x_i + \sigma_{ti}/2}$$

$$\Psi_c = 2\bar{\Psi}_{n,i} - \frac{1}{2}\left(\Psi_{n,i+1/2} + \Psi_{n,i-1/2}\right)$$

$$\Psi_c = \frac{2}{2}\left[\Psi_{n,i-1/2} + \Psi_{n,i+1/2}\right] - \frac{1}{2}\left[\Psi_{n,i+1/2} + \Psi_{n,i-1/2}\right]$$

$$= \frac{1}{2}\left[\Psi_{n,i-1/2} + \frac{\bar{S}_{n,i} + \left(\mu_n/\Delta x_i - \sigma_{ti}/2\right)\Psi_{n,i-1/2}}{\mu_n/\Delta x_i + \sigma_{ti}/2}\right]$$

(b) limits

$$\Psi_c > 0 \rightarrow -1 > \mu/\Delta - \sigma/2 \Rightarrow (\sigma/2 - 1)\Delta > \mu$$

the more limiting case is $\Psi_R\,(\Psi_{i+1/2}) > 0$

$$\Psi_R > 0 \rightarrow \mu/\Delta - \sigma/2 > 0 \rightarrow \mu/\Delta > \sigma/2 \rightarrow 2\mu > \sigma\Delta \rightarrow$$

the same condition as in the step

this means that Δ/μ must be at least 2 mean free paths