Group Work Project 1

Kumar Shantanu
Shadreck Simumba
Olubukola Abiodun Kasumu

May 2, 2024

Contents

1 Markowitz Asset Allocation

I made a deliberate choice to do the last step first. Markowitz's model provides a theoretical basis for portfolio construction, emphasizing the importance of diversification and risk-return trade-offs. Therefore, before diving into factor models, which are built upon these fundamental concepts, it was necessary to thoroughly understand and implement the Markowitz Asset Allocation model.

1.1 Asset Selection

In constructing my portfolio, I've assembled these 5 assets (cryptocurrencies): Bitcoin (BTC), Cardano (ADA), Dogecoin (DOGE), Shiba Inu (SHIB), and USDT. BTC and ADA are considered good investments, while DOGE and SHIB are deemed "shitcoins," and USDT serves as a cash equivalent or risk-free asset. The decision to structure my portfolio in this manner was intentional. One of my professors once told me that thinking in limits and asymptotes helps you understand a system better.

Asset	Description
BTC	Gold standard of cryptocurrencies. Nowadays, it exhibits a lower volatility than bond markets. It has a beautiful Guassian-like return distribution but with a large Kurtosis.
ADA	An intriguing project that always seems poised for take-off but never quite achieves liftoff. It has a beautiful Gaussian-like return distribution but with large standard deviation.
DOGE	Elon Musk's favourite pet. The return distribution is characterized by long tails because Elon Musk enjoys playing with it.
SHIB	A meme coin that started as a joke but has become one of the best performers in the Crypto Market. The return distribution has very fat tails and is multi-modal.
USDT	A stablecoin tethered to the US Dollar. The value of 1 USDT must always equal $1 (theoretically), making it as good as cash. However, it occasionally experiences extreme fluctuations. One may wonder, how can a dollar cost more than a dollar? Well, in crypto, anything happens. The return distribution has a very low standard deviation as expected.

1.2 Visualisation of Return Distribution as Kernel Density Plots

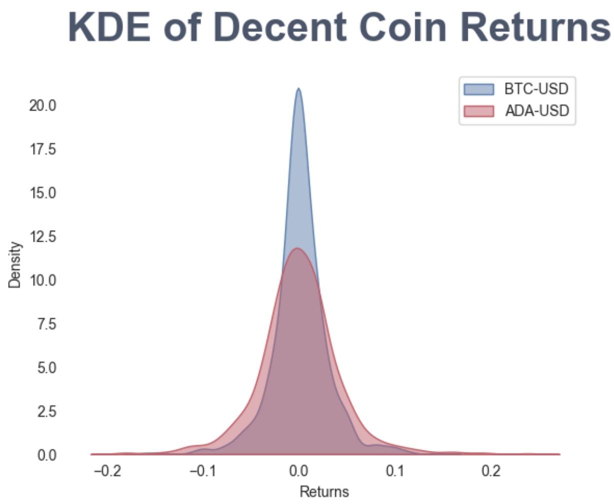

Figure 1: KDE Visualisation of Decent Coins

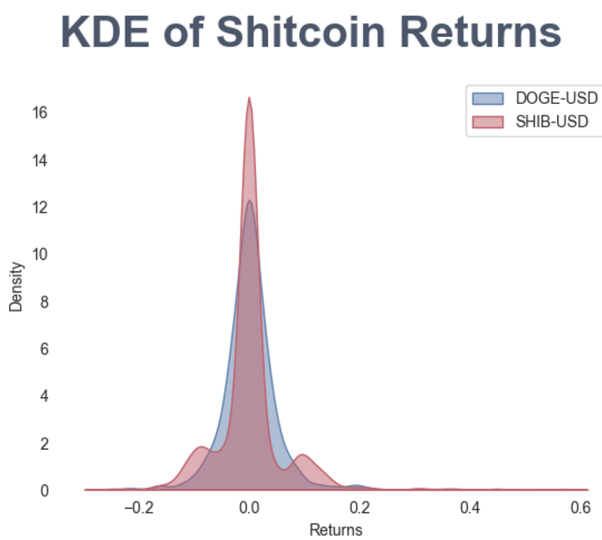

Figure 2: KDE Visualisation of Shit Coins

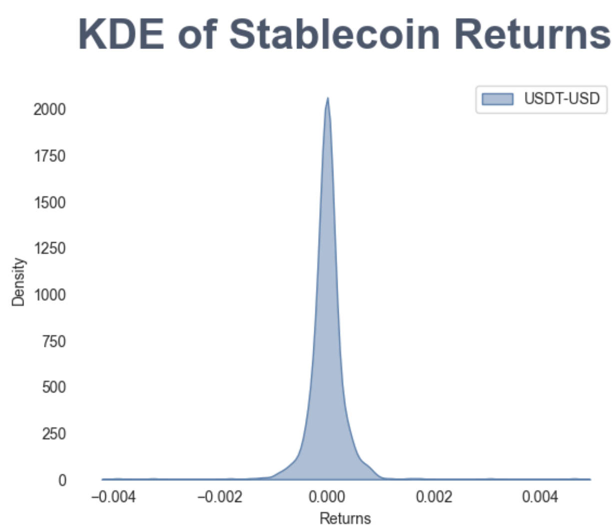

Figure 3: KDE Visualisation of Stable Coins

1.3 Quadratic Optimization Problem

A confession to start with: I opted to utilize PyTorch instead of NumPy for the implementation. The decision to do so was primarily driven by a personal selfish inclination to familiarize myself with the PyTorch API. Although, the library is neither as lightweight or as fast as NumPy (on CPU), however, it is talk of the town nowadays, especially with its GPU capabilities.

I commenced by generating 50,000 random portfolio allocations encompassing the specified assets. The allocations were generated by randomly selecting weights for each asset, ensuring that the sum of the weights equaled 1. In risk-return space, these allocations were then plotted and they appear as follows:

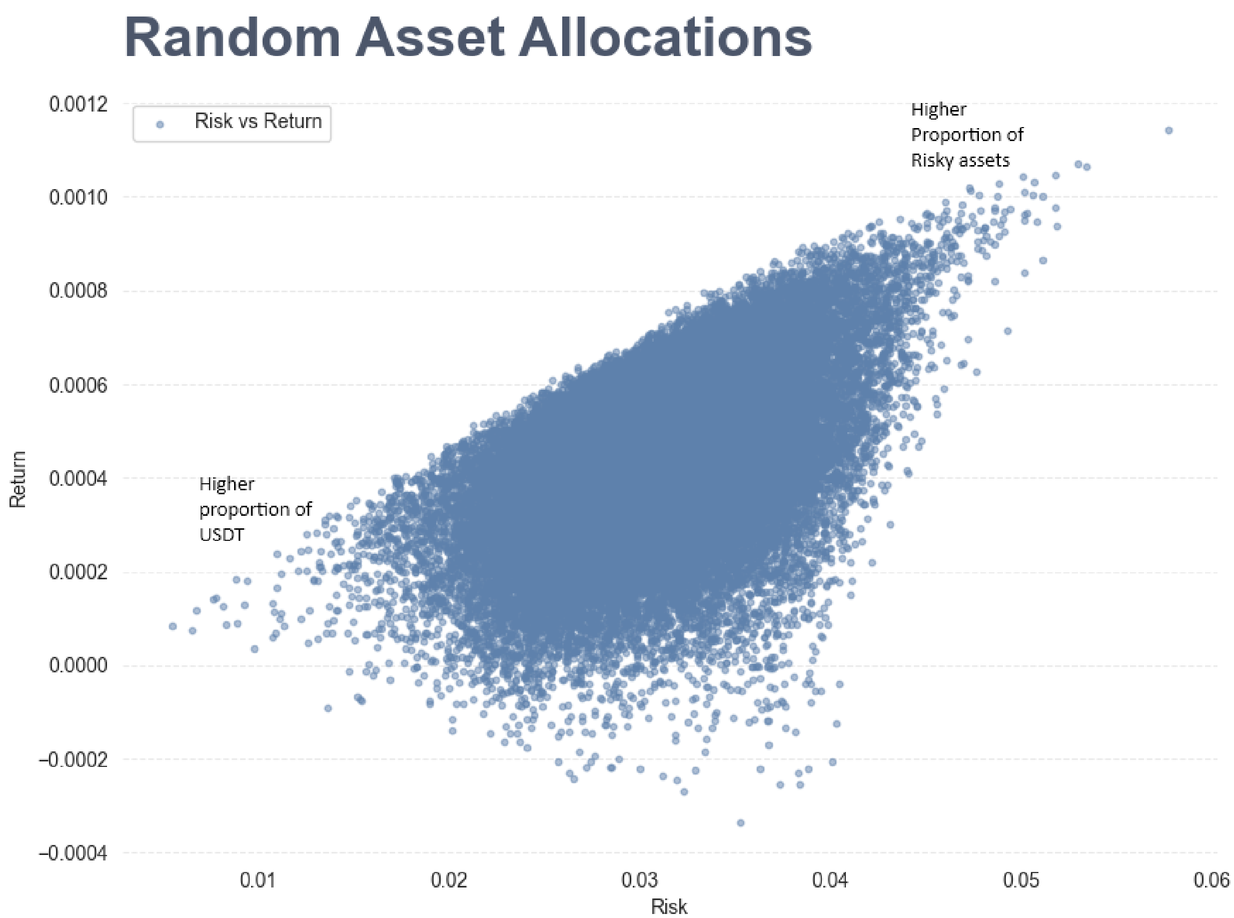

Figure 4: Risk and Reward from 50,000 Random Allocations

This visualisation is interesting primarily because it is slightly different from rightward-facing parabola. In my opinion this is primarily because of choice of my assets. On the lower right we have the We have the portfolios which have a larger proportion of usdt which I'm considering here as a risk free asset or a low risk asset. Then from there the correlation between risk and reward becomes stronger and we have a steep linear climb of reward for the per unit risk. This is interesting because the ensemble of all portfolios is triangular shaped rather than conic parabola.

The efficient frontier was then generated by setting up a quadratic optimization problem and solving it iteratively on Python, which is the Markowitz mean-variance optimization. The efficient frontier is the set of optimal portfolios that offer the highest expected return for a given level of risk or the lowest risk for a given level of expected return. The efficient frontier is plotted below:

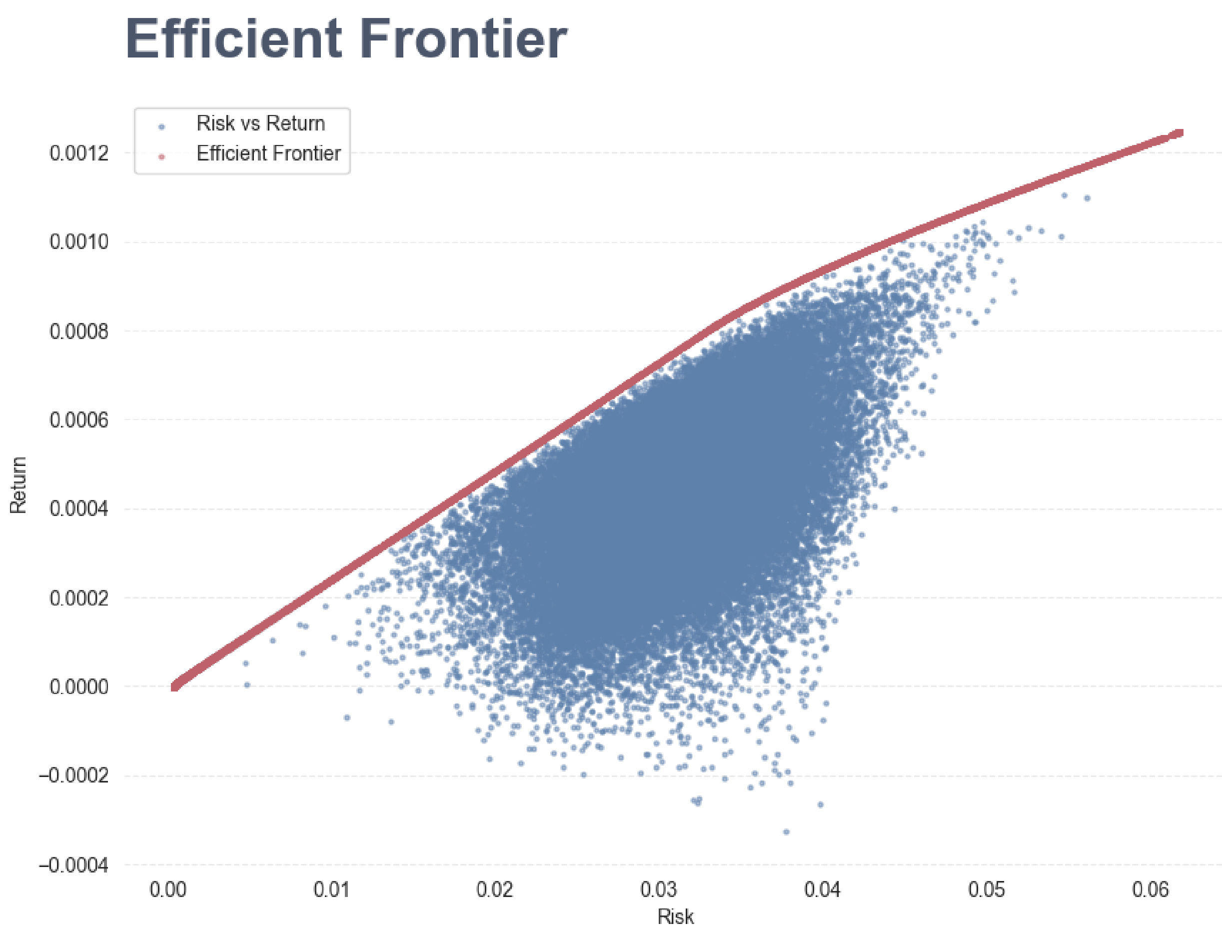

Figure 5: Efficient Frontier

Now, I brute-forced my way to find the Sharpe ratio maximizing portfolio, which is also the tangential portfolio. Then, I back calculated the Capital Market Line (CML). A bit dirty, but I did not know how else to do this.

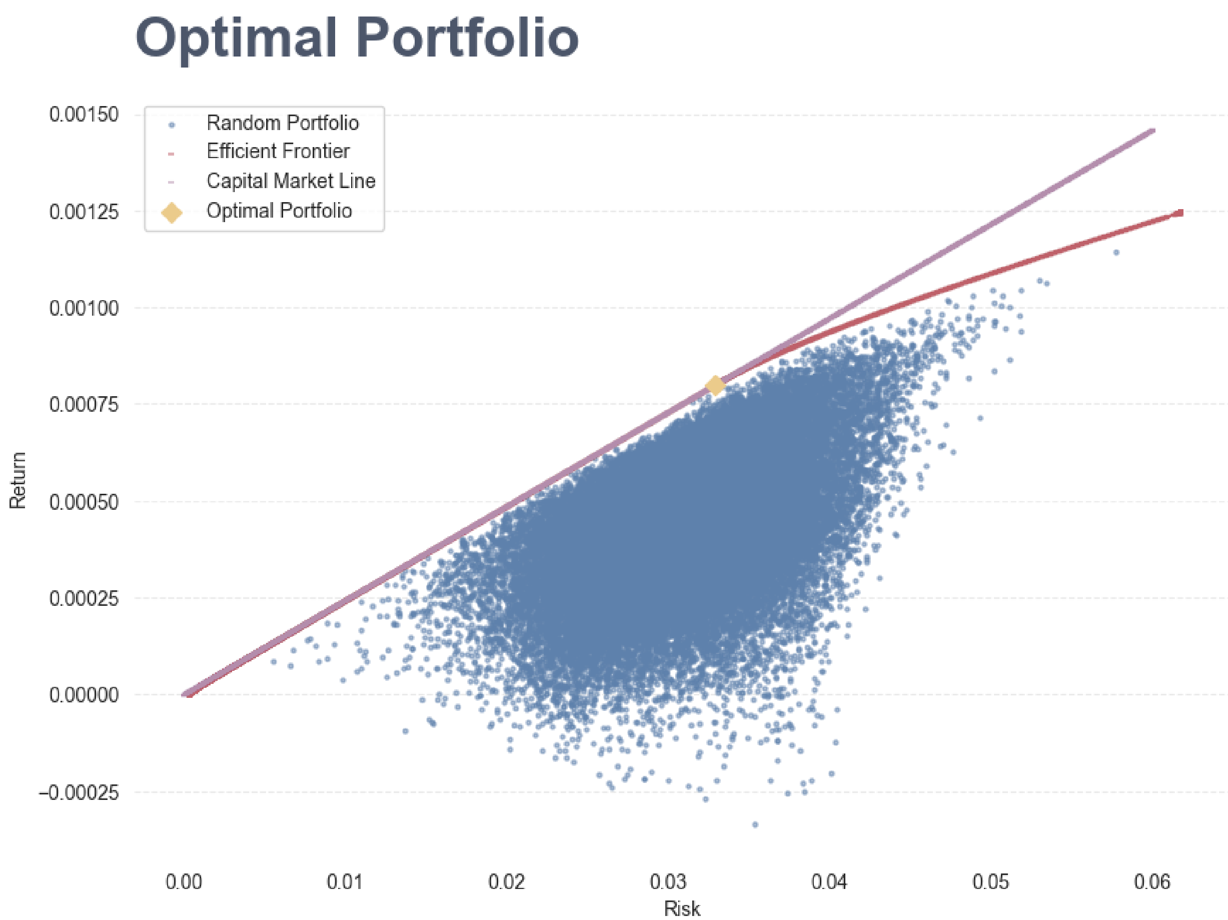

Figure 6: Efficient Frontier

1.4 Interpretation

The interpretation is crucial. Any other portfolio within the entire set will have a lower Sharpe ratio than this optimal portfolio. While it's possible to increase the reward, it will inevitably entail a higher level of risk. The optimal portfolio is the one that offers the highest reward for a given level of risk.

Therefore,

- The first key takeaway is that we now possess a theoretical basis for the relationship between risk and reward from first principles.

- The second key takeaway is that if all investors are mean-variance optimizers, any excess return will be arbitraged away.
- The third key takeaway is that a higher return must necessarily be accompanied by a higher level of risk.

1.5 A Note on Market Efficiency

It is important to hand-wave market efficiency before we jump onto the factor models.

In a perfectly efficient market, asset prices fully reflect all available information, leaving no room for investors to consistently outperform the market based on publicly available information. Therefore, the only factor that should explain returns is the systematic risk, also known as market risk.

Mathematically, the CAPM equation can be expressed as:

$$R_i = R_f + \beta_i(R_m - R_f)$$

Where:

- R_i is the expected return of asset i,
- R_f is the risk-free rate of return,
- R_m is the expected return of the market portfolio,
- β_i is the beta coefficient of asset i, representing its sensitivity to market movements (systematic risk).

In an efficient market, investors are compensated for taking on systematic risk through the risk premium $(R_m - R_f)$, which is the excess return of the market portfolio over the risk-free rate. The beta coefficient β_i captures the extent to which an asset's returns move in response to changes in the market.

Thus according to this framework, the only factor that should explain asset returns is the systematic risk captured by the beta coefficient. Any deviations from this relationship would suggest either market inefficiency or the presence of additional factors influencing returns beyond what is captured by systematic risk.

The debate on market efficiency is a longstanding and multifaceted discussion. Factor models, such as the Fama-French Three-Factor Model pose a challenge to the Efficient Market Hypothesis (EMH) by suggesting that there are systematic sources of risk and return beyond what is captured by market beta. These models introduce additional factors, such as size, value, profitability, or momentum, that have been shown to explain a significant portion of the variation in asset returns. If markets were truly efficient, asset prices would reflect all relevant information instantaneously, and there would be no systematic patterns or predictable returns left unaccounted for by market beta. However, the presence of factors like size or value suggests that certain characteristics of assets consistently outperform or underperform the market over time, contradicting the notion of market efficiency.

2 Three Factor Model

The following three factors are considered for FF3 model.

1. **Market Risk (Mkt-RF)**:
    - Market risk, also known as the excess market return, represents the return of the market portfolio (typically proxied by a broad market index like the S&P 500) minus the risk-free rate of return.
    - This factor captures the systematic risk inherent in investing in the overall market and is a measure of how much an asset's return is influenced by movements in the broader market.

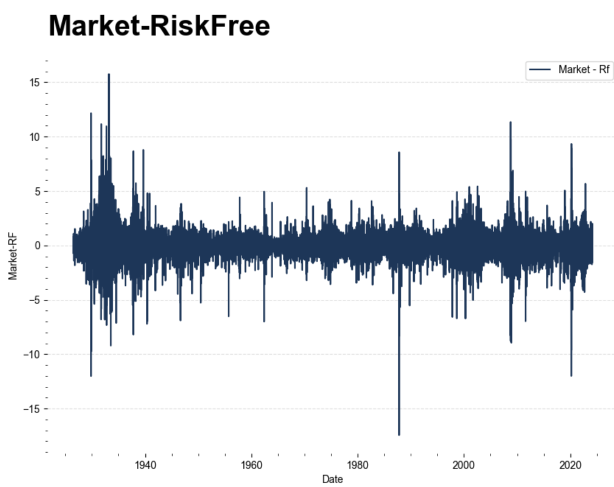

Figure 7: Market minus Risk Free Rate

2. **Size (SMB - Small Minus Big)**:

    - The size factor reflects the historical outperformance of small-cap stocks compared to large-cap stocks.
    - SMB is calculated by taking the difference in returns between a portfolio of small-cap stocks and a portfolio of large-cap stocks.
    - This factor suggests that small-cap stocks tend to have higher average returns than large-cap stocks over the long term, possibly due to higher levels of risk or less liquidity.

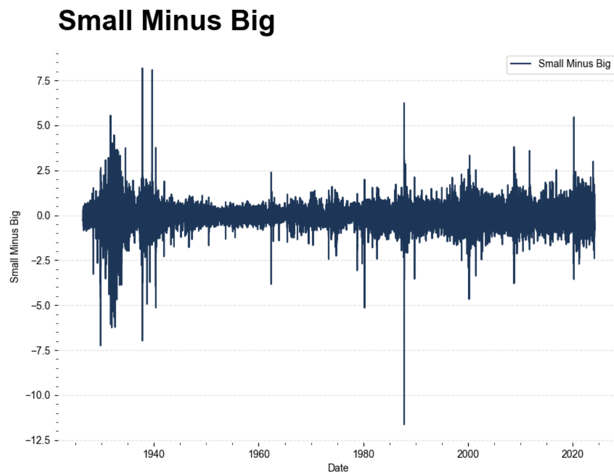

Figure 8: Small minus Big

3. **Value (HML - High Minus Low)**:
   - The value factor captures the historical outperformance of value stocks compared to growth stocks.
   - HML is calculated by taking the difference in returns between a portfolio of high book-to-market (value) stocks and a portfolio of low book-to-market (growth) stocks.
   - This factor suggests that stocks with low valuations (high book-to-market ratios) tend to outperform stocks with high valuations over the long term, possibly because they are undervalued by the market.

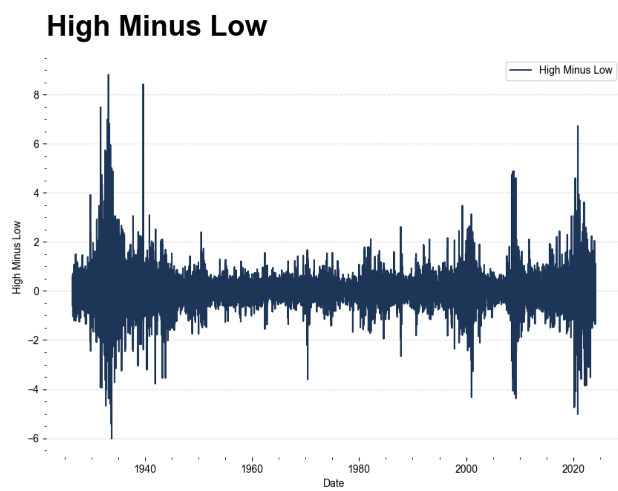

Figure 9: High minus Low

To spice up things, I decided to focus on a "meme stock" known as GameStop. GameStop has gained widespread attention in financial markets due to its unconventional trading patterns and the involvement of online communities like Reddit's WallStreetBets.

2.1 FF3 Regression Results

Dep. Variable:	Excess Returns	**R-squared:**	0.095
Model:	OLS	**Adj. R-squared:**	0.093
Method:	Least Squares	**F-statistic:**	36.62
Date:	Tue, 30 April 2024	**Prob (F-statistic):**	1.67e-22
Time:	23:02:51	**Log-Likelihood:**	974.11
No. Observations:	1046	**AIC:**	-1940.
Df Residuals:	1042	**BIC:**	-1920.
Df Model:	3		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
const	-0.0012	0.003	-0.422	0.673	-0.007	0.005
SMB	0.0322	0.004	8.366	0.000	0.025	0.040
HML	0.0041	0.002	1.686	0.092	-0.001	0.009
Mkt-Rf	0.0083	0.002	4.001	0.000	0.004	0.012

Omnibus:	1173.597	**Durbin-Watson:**	1.782
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	166310.298
Skew:	5.253	**Prob(JB):**	0.00
Kurtosis:	63.873	**Cond. No.**	1.95

The intercept coefficient is found to be insignificant, suggesting that there is no significant abnormal return beyond what can be explained by the model's factors. The Small Minus Big (SMB) coefficient is found to be statistically significant, indicating that the size factor, which reflects the historical outperformance of small-cap stocks compared to large-cap stocks, has a notable impact on GameStop's returns. Additionally, Interestingly, the coefficient for the High Minus Low (HML) factor is also found to be insignificant. HML represents the historical outperformance of value stocks compared to growth stocks.

2.2 Regression Diagnostics

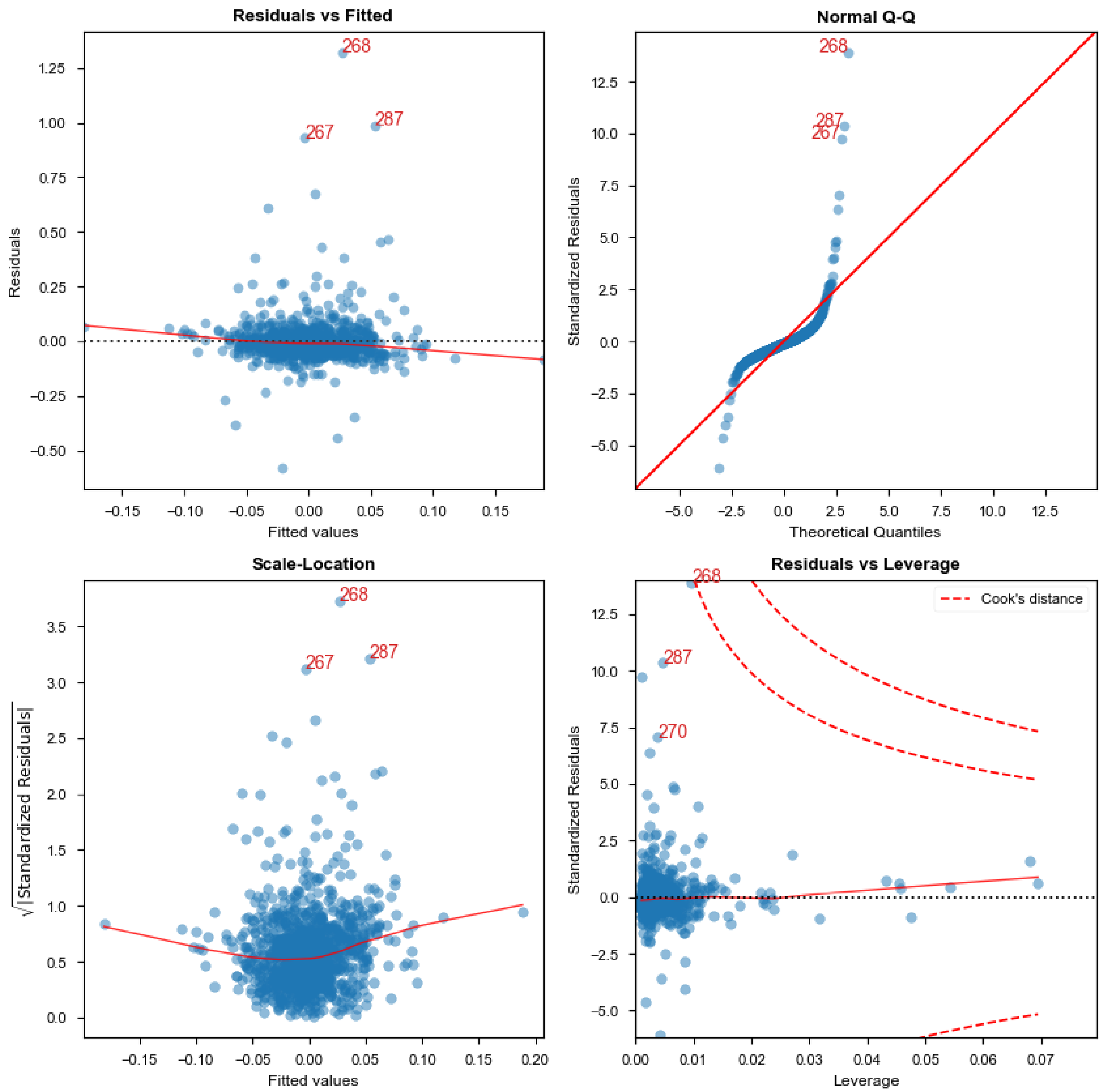

Figure 10: FF3 Diagnostics

When examining the diagnostics of the three-factor model, one noteworthy finding is the presence of a negative correlation between the residuals and the fitted values. This hints at model mis-specification.

Moreover, the presence of outliers in the data further complicates the analysis. GameStop stocks, in particular, have exhibited abnormal returns at various points in time, introducing significant fluctuations and deviations from the expected pattern. These outliers can distort the model's predictions and influence the relationship between the fitted values and residuals.

3 Five Factor Model

FF5 incorporates following additional factors: [1]

1. **Profitability (RMW - Robust Minus Weak)**:
   - The profitability factor captures the historical outperformance of companies with high profitability relative to those with low profitability.
   - RMW is calculated by taking the difference in returns between a portfolio of stocks with high operating profitability and a portfolio of stocks with low operating profitability.
   - This factor suggests that companies with higher profitability tend to generate higher returns than less profitable companies over the long term, possibly due to their ability to sustain competitive advantages or generate excess returns.
2. **Investment (CMA - Conservative Minus Aggressive)**:
   - The investment factor reflects the historical outperformance of companies with conservative investment policies compared to those with aggressive investment policies.
   - CMA is calculated by taking the difference in returns between a portfolio of stocks with low levels of capital investment and a portfolio of stocks with high levels of capital investment.
   - This factor suggests that companies with conservative investment policies, characterized by low levels of capital expenditure, tend to outperform companies with aggressive investment policies over the long term, possibly due to more efficient capital allocation or avoidance of value-destructive investments.

3.1 FF5 Regression Results

Dep. Variable:	Excess Returns	**R-squared:**	0.110
Model:	OLS	**Adj. R-squared:**	0.105
Method:	Least Squares	**F-statistic:**	26.09
Date:	Thu, 02 May 2024	**Prob (F-statistic):**	6.57e-25
Time:	22:10:53	**Log-Likelihood:**	1007.7
No. Observations:	1066	**AIC:**	-2003.
Df Residuals:	1060	**BIC:**	-1973.
Df Model:	5		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
const	-0.0020	0.003	-0.688	0.492	-0.008	0.004
SMB	0.0305	0.004	7.102	0.000	0.022	0.039
HML	-0.0136	0.004	-3.586	0.000	-0.021	-0.006
CMA	0.0391	0.007	5.646	0.000	0.025	0.053
RMW	-0.0047	0.005	-0.870	0.384	-0.015	0.006
Mkt - Rf	0.0125	0.002	5.741	0.000	0.008	0.017

Omnibus:	1163.236	**Durbin-Watson:**	1.774
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	145658.784
Skew:	5.050	**Prob(JB):**	0.00
Kurtosis:	59.368	**Cond. No.**	3.87

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

The intercept term, found to be statistically insignificant, suggests that there is no significant abnormal performance in the portfolio after controlling for the factors included in the model.

Among the five factors considered, namely Size (SMB), Value (HML), Investment (CMA), Market Risk (Mkt-Rf), and Profitability (RMW), all factors except RMW exhibit statistically significant relationships with the excess return of the portfolio.

Upon examining the coefficients of the significant factors, several notable observations emerge. SMB (Size) exhibits a positive coefficient, indicating that smaller-sized stocks tend to outperform larger-sized stocks. GameStop has a small sized premium. Conversely, HML

(Value) displays a negative coefficient. CMA (Investment) showcases a positive coefficient and lastly, Mkt-Rf (Market Risk) displays a positive coefficient, indicating that movements in the broader market positively influence GameStop stock.

The model demonstrates a relatively strong explanatory power, with an R-squared value of 0.67. This implies that approximately 67% of the variability in the excess return of the portfolio can be explained by the factors included in the model, indicating a good fit. Furthermore, the overall model is statistically significant, as evidenced by a high F-statistic of 33.5 and a corresponding p-value of 0.000, indicating that the model as a whole provides valuable insights into the portfolio's performance.

3.2 Regression Diagnostics

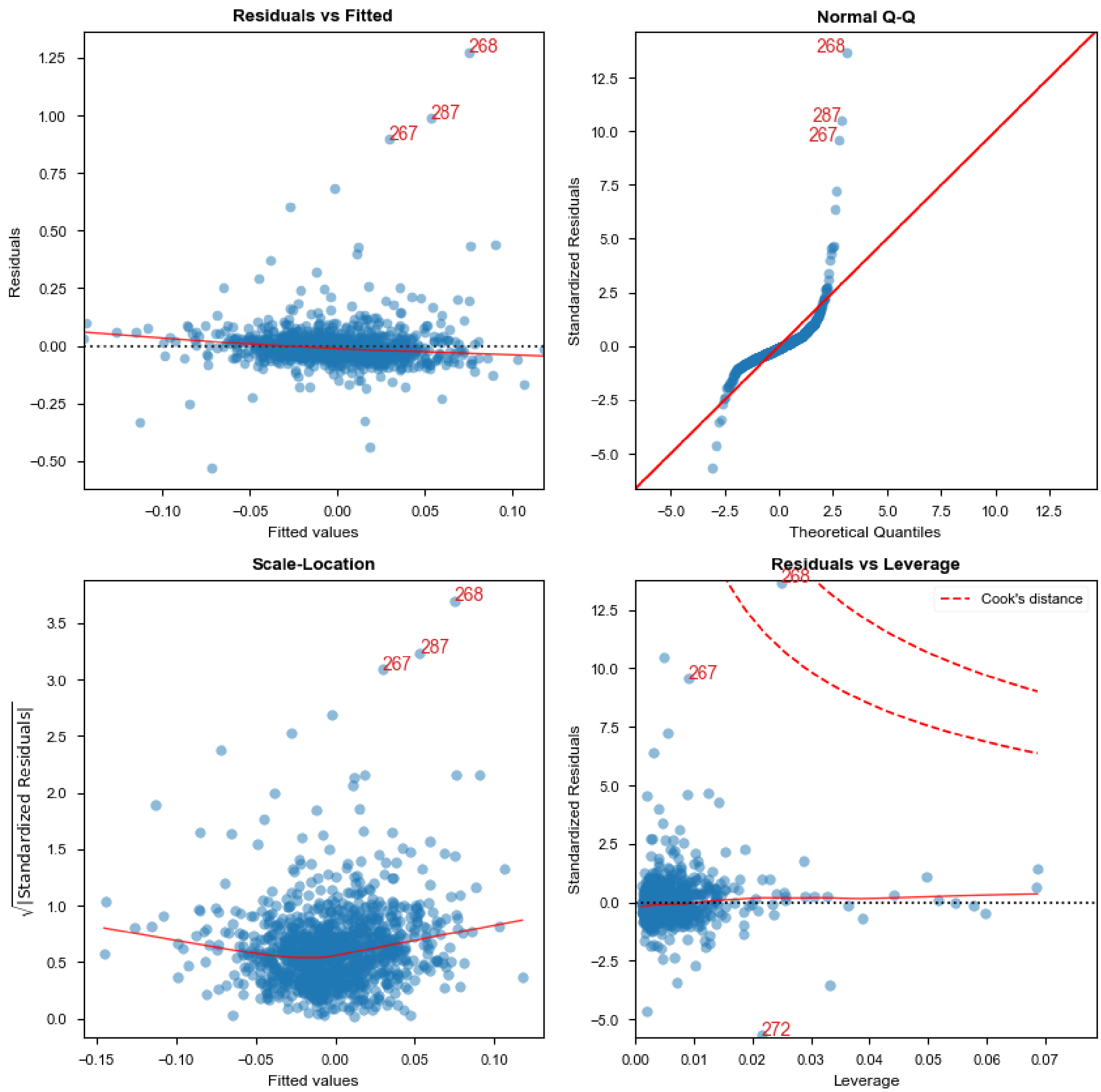

Figure 11: FF5 Diagnostics

The residuals are not randomly scattered and are not normally distributed. GameStop stock had an abnormal return during the Reddit frenzy associated with it. Therefore, there are indeed outliers in the data.

4 Conclusion

In conclusion, this assignment worked on exploring the relationship between risk and return in financial markets, beginning with Markowitz's asset allocation theory, briefly touching on the Capital Asset Pricing Model and market efficiency, and culminating in the estimation of Fama-French three-factor and five-factor models on a meme stock. While I was tempted to replicate Fama-French factors in cryptocurrency markets, it became apparent that such an endeavour posed challenges, necessitating a thorough reading of the original papers by Fama and French [2], which was not possible.

References

[1] Eugene F Fama and Kenneth R French. "A five-factor asset pricing model". In: *Journal of Financial Economics* 116.1 (2015), pp. 1–22.

[2] Eugene F Fama and Kenneth R French. "The Cross-Section of Expected Stock Returns". In: *The Journal of Finance* 47.2 (1992), pp. 427–465.