The Informational Content of Implied Volatility

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Implied volatility is widely believed to be informationally superior to historical volatility, because it is the "market's" forecast of future volatility. But for S&P 100 index options, the most actively traded contract in the United States, we find implied volatility to be a poor forecast of subsequent realized volatility. In aggregate and across subsamples separated by maturity and strike price, implied volatility has virtually no correlation with future volatility, and it does not incorporate the information contained in recent observed volatility.

One of the most attractive features of the Black–Scholes option pricing model is that its parameters are almost all observable. The one input that must be forecast is the volatility of the underlying asset. Unfortunately, price volatility of most optionable securities varies considerably over time, and accurate prediction is far from easy. The two basic approaches are either to compute the realized volatility over the recent past from historical price data or to calculate the "implied volatility" (IV) from current option prices in the market by solving the pricing model for the volatility that sets the model and market prices equal.

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It has become almost an article of faith in the academic finance profession that the implied volatility is the "market's" volatility forecast, and that it is a better estimate than historical volatility. Indeed, researchers often use implied volatility in other models as an ex ante measure of perceived asset price risk.[1] Here, we examine that proposition as it pertains to the most active options market in the United States, options on the Standard and Poors 100 Index (frequently called by their ticker symbol, OEX options). The results we obtain from analyzing over 17,000 OEX call option prices conflict sharply with the conventional wisdom.

We begin the analysis in Section 1 with a discussion of the logic behind the conventional wisdom and a critical examination of how it should be tested. Section 2 describes our data and methodology. If implied volatility is the market's prediction of actual volatility over the time remaining to an option's expiration date, daily observations on IV involve sequential forecasts for overlapping time periods. This leads to serial dependence in the time series of forecast errors and to a statistical problem in testing the model. In addition, simultaneous trading in multiple contracts with different strike prices and overlapping expirations creates further cross-correlations if the data sample contains observations on more than one option per day. Previous studies dealt with the problem by aggregating and excluding data to create nonoverlapping observations. Unfortunately, such procedures can severely reduce the power of statistical tests. Instead, we maximize the amount of information obtained from the data by adopting an estimation procedure that makes use of the entire data sample and corrects for the time dependence directly.

Section 3 presents our estimation results. In brief, they show that for OEX index options, implied volatility is an inefficient and biased forecast of realized future volatility that does not impound the information contained in recent historical volatility. In fact, the statistical evidence shows little or no correlation at all between implied volatility and subsequent realized volatility.

1. Volatility Forecasts and Implied Volatility

1.1 Realized volatility, implied volatility, and the market's volatility forecast

It is widely accepted that the implied Black–Scholes volatility computed from the market price of an option is a good estimate of the

[1] See Patell and Wolfson (1979, 1981) or Poterba and Summers (1986) for examples of the use of implied volatility as a proxy for the market's risk assessment. Implied volatility has also been used as a proxy for the true instantaneous price volatility of the underlying asset, as in Stein's (1989) study of the "term structure" behavior of implied volatility.

"market's" expectation of the volatility of the underlying asset, and that the market's expectation is informationally efficient.

Black and Scholes assume that the price for the underlying stock follows a logarithmic diffusion process with constant instantaneous mean and volatility. However, there is an obvious conflict in applying an approach that assumes the asset price process has a known constant volatility to a situation in which volatility must be forecast because it changes randomly over time. Investors who were fully rational in valuing options should use a pricing model that deals rigorously with the stochastic nature of volatility, as in Hull and White (1987) or Wiggins (1987). Stochastic volatility models require the investor to forecast not just a single volatility parameter but the entire joint probability distribution for asset returns and changes in volatility and also the market price of volatility risk. These requirements make these models significantly more difficult to implement than Black–Scholes and other constant-volatility models. During the period covered by our data sample, ending in 1987, option traders had easy access to theoretical values, implied volatilities, deltas, and so on, for traded options from the standard fixed-volatility models, but formal stochastic volatility models were not in general use. Even today, it is common practice for option traders to make trading and hedging decisions by picking a point forecast for volatility—perhaps the implied volatility, an estimate computed from historical prices, or some subjectively determined combination of the two—and then inserting this point estimate into the Black–Scholes or binomial model. Similarly, academic investigators almost invariably use implied volatilities from standard fixed-volatility models to measure the market's volatility expectations.

Thus, we consider the analysis in this subsection to be more valid as a description of the approach taken by many actual investors and researchers than as optimal behavior in a world of stochastic volatility. One way to interpret our empirical work is as a test of whether the "rule of thumb" strategy of computing implied volatility from a standard fixed-volatility model is an efficient way to obtain a volatility forecast.

For a series of prices $\{S_0, S_1, \ldots, S_T\}$, the realized volatility σ is defined as the annualized standard deviation of the continuously compounded returns, $\{R_1, R_2, \ldots, R_T\}$, where $R_t \equiv \ln(S_t/S_{t-1})$, $\bar{R}$ is the sample mean of the R_t, and K is the number of observation intervals in a year:[2]

$$\sigma = \left(\frac{K}{T-1} \sum_{t=1}^{T} (R_t - \bar{R})^2 \right)^{1/2}. \tag{1}$$

[2] Equation (1) is widely used as a *consistent* estimator of volatility, but while the expression in the large parentheses yields an unbiased estimate of the variance, taking the square root to obtain the

In the later discussion of our empirical work, the annualized volatility that will be realized over the remaining lifetime of a τ-period option as of date t will be denoted $\sigma_t(\tau)$. In this section, we simplify the notation by suppressing the explicit dependence on t and τ.

One estimate of future volatility can be obtained from historical prices by assuming that the recent realized level of volatility will continue in the future. Another estimate comes from current option prices. Given the observable parameters and an option valuation formula, there is a one-to-one correspondence between the option price and the volatility input. Computing the implied volatility, IV, from the observed market price then gives an estimate of the "market's" volatility forecast,

$$\mathrm{IV} = E_{\mathrm{MKT}}[\sigma], \tag{2}$$

where $E_{\mathrm{MKT}}[\sigma]$ denotes the market's (subjective) expectation for σ.[3]

By definition, realized volatility can be written as its expected value conditional on an information set Φ plus a zero-mean random error that is orthogonal to Φ:

$$\sigma = E[\sigma \mid \Phi] + \epsilon, \qquad E[\epsilon \mid \Phi] = 0.$$

This formulation leads to the well-established regression test for the rationality of a forecast,

$$\sigma = \alpha + \beta F(\Phi) + u, \tag{3}$$

where $F(\Phi)$ is the forecast of σ based on the information set Φ and u is the regression residual.[4] If the forecast is the true expected value of σ conditional on Φ, regressing realized values of σ on their expectations should produce regression estimates of 0.0 and 1.0 for α and β, respectively. Deviation from those values is evidence of bias and inefficiency in the forecasts.

Since the forecast error must be orthogonal to any rationally formed forecast, Equation (3) should hold with $\alpha = 0$ and $\beta = 1$ for any Φ. Running (3) on forecasts derived from a more inclusive information set does not change the expected coefficient estimates, but a better forecast should produce a higher R^2. Further, if one regresses σ on two forecasts, $F_1(\Phi_1)$ and $F_2(\Phi_2)$, where the second is derived from a subset of the information used in forming the first ($\Phi_1 \supset \Phi_2$),

volatility is a nonlinear transformation that introduces a small bias in a finite sample by Jensen's inequality. It is common practice to treat this bias as negligible.

[3] Again, we ignore the bias in a forecast of σ obtained as the square root of an unbiased forecast of σ^2.

[4] Theil (1966) is credited with introducing this test of forecast rationality and analyzing it in detail. It has been widely applied to test the rationality of expectations, as in Pesando (1975) (for inflation expectations) or Brown and Maital (1981) (for a broad selection of economic variables).

$$\sigma = \alpha + \beta_1 F_1(\Phi_1) + \beta_2 F_2(\Phi_2) + u, \tag{4}$$

the slope coefficient of the first, β_1, should still be 1.0 and the less-informed forecast should have $\beta_2 = 0$. Analyzing the relative information content of two different forecasts by means of a regression like (4) is known as an "encompassing regression" test. Fair and Shiller (1990) discuss this approach in detail and use it to evaluate the forecasting performance of different macroeconomic models.

The tests we report are based on Equations (3) and (4), with implied volatility and historical volatility as the conditional forecasts.

1.2 Empirical evidence on implied volatility

Early studies by Latané and Rendleman (1976) (LR), Chiras and Manaster (1978) (CM), and Beckers (1981) all use the basic Black–Scholes European option model or a variant of it in computing IVs for American calls on dividend-paying stocks.[5] In the tests presented here, we derive the IVs from a binomial model that adjusts for dividends and captures the value of early exercise. This resolves the problem of analyzing American options but does not address the many other issues raised by the possibility that the "market" may not be using exactly the same model we are. For example, the market's option model may allow nonlognormal price changes or price jumps. Using the wrong model violates (2): the computed IV will differ from the market's real volatility forecast.

In the early studies, stock-price volatility was typically treated as if it were a constant parameter, so the exact timing and periodicity of the data sample used in estimating it did not matter much. This tacit assumption gave rise to problems in the way data were handled, including use of sparse monthly data spanning long time periods to estimate historical volatility, estimating "realized" volatility over periods that did not match option maturity, and, in at least one case, analyzing "forecasts" of volatility over periods prior to the date the forecast was made.[6] Only Beckers (1981) recognizes the importance of timing, in that he uses daily data to estimate historical volatility

[5] LR, in fact, made no adjustment for dividend payout. CM corrected this mistake by using Merton's continuous-payout European call formula. Beckers, recognizing the problem posed by the possibility of early exercise, adopted an ad hoc adjustment to the European formula.

[6] For example, in LR's study of option prices from October 1973 through June 1974, historical volatility was computed from four years of monthly data ending at the beginning of the sample period. This long time period with sparsely distributed monthly data contrasts with the one to six months of daily data that options traders typically use to compute historical volatility and makes it likely that the estimates were contaminated by stale data. CM also used monthly data, but only from the 24 months immediately preceding the date of the option price. However, in trying to establish the relative forecasting accuracy of historical volatility versus implied volatility, both LR and CM computed "realized" volatility figures for time periods that did not match option maturities. In fact, one of LR's realized volatility series included stock prices observed six months prior to the first option price in their sample.

from the previous three months of stock prices and realized volatility over the period from the observation date to option expiration.

Recent papers by Day and Lewis (1990) (DL) and Lamoureux and Lastrapes (1993) (LL) also examine implied volatility as a source of information. Both studies find that IV contributes a statistically significant amount of information about volatility over the (short-term) forecasting horizon covered by the models, but they also find that IV does not fully impound the information that the model is able to extract from historical prices. LL also examine forecasting volatility through option expiration and find that IV alone is less accurate than the models that incorporate historical prices in 8 out of 10 cases.

2. Data and Methodology

The data sample is drawn from the set of closing prices for all call options on the OEX index from March 15, 1983, shortly after index option trading opened, through March 28, 1987. We eliminated options with fewer than 7 or more than 127 days to expiration and those that were more than 20 points in- or out-of-the-money.[7] Also, some of the recorded option prices violated the lower arbitrage boundary—that the call price should be greater than the current stock price minus the present value of the strike price plus future dividends. An option's price equals the boundary value if volatility is zero. In the case of a boundary violation, implied variance would have to be negative, so those options were also excluded from the sample. This left a total of 17,606 observations. At the outset, there were contracts traded for two expiration dates, September and December 1983. Starting in December 1983, the expiration cycle was changed so that there were always one-, two-, three-, and four-month maturities.

These are American options, and the OEX index portfolio contains mostly dividend-paying stocks. To take into account the value of early exercise, we used a binomial model with 500 time steps,[8] a very fine grid size (e.g., 10 steps per day for an option with 50 days to expiration). In selecting a measure of the riskless interest rate, we have tried to take into account both the relevant lending and borrowing rates faced by options traders. Instead of Treasury-bill rates, we used the average of the Eurodollar deposit rate and the broker call rate on each date. We treated dividends as if future payouts were known, and

[7] Note that the level of the OEX index was over 200 for most of the sample period, so an option with a strike price 20 points away from the current level of the index is not actually very far in- or out-of-the-money in percentage terms.

[8] In fact, because the dividend stream on the OEX index portfolio is much less lumpy than that of an individual stock, rational early exercise is unlikely, and American option values are quite close to those derived from the dividend-adjusted Black–Scholes European option formula.

Table 1
Summary statistics of implied volatility by maturity group

Maturity group (i)	Days to expiration	Number of obs	IV mean	Sample standard deviation
All		17,606	0.168	0.055
1	7–35	4,088	0.195	0.080
2	29–63	5,196	0.166	0.046
3	57–98	4,709	0.158	0.039
4	85–127	3,613	0.152	0.035

The table shows the breakdown of implied volatilities for OEX call options between March 15, 1983, and March 28, 1987, into four maturity groups corresponding to the number of contract months to expiration. For example, the first group ($i = 1$) contains the near-month options. The rightmost columns give the mean and the standard deviation of implied volatilities within each group.

we used the actual stream of dividends paid over the option's life. Annualized implied volatilities were then calculated to the nearest 0.001 by an iterative search procedure.

On each date, there are prices, and therefore implied volatilities, for many different options. We find that the IVs for OEX options observed at a given point in time vary systematically across the different strikes and maturity months, where maturity month refers to the near-, second-, third-, and fourth-month expiration dates. This regular cross-sectional structure is illustrated in Tables 1 and 2 and Figures 1 and 2 for subsamples of the data, broken up according to maturity month and the extent to which the options are in- or out-of-the-money.

The extent to which the options are in- or out-of-the-money is simply $S - X$, the current level of the index minus the option's strike price. To save space we will hereafter refer to this quantity as the

Table 2
Summary statistics of implied volatility by intrinsic-value group

Group (j)	Intrinsic value	Number of obs	Mean	Standard deviation
All		17,606	0.168	0.055
1	−20–−15.01	1,800	0.167	0.038
2	−15–−10.01	2,761	0.158	0.034
3	−10–−5.01	3,069	0.154	0.034
4	−5–−0.01	3,093	0.155	0.036
5	0–5	2,907	0.158	0.043
6	5.01–10	2,127	0.179	0.058
7	10.01–15	1,230	0.209	0.082
8	15.01–20	619	0.263	0.121

The table shows the breakdown of implied volatilities for OEX call options between March 15, 1983, and March 28, 1987, into eight intrinsic-value groups corresponding to the amount the option is in- or out-of-the-money. For example, the first group ($j = 1$) contains the options that are between 15 and 20 points out-of-the-money. The rightmost columns give the mean and the standard deviation of implied volatilities within each group.

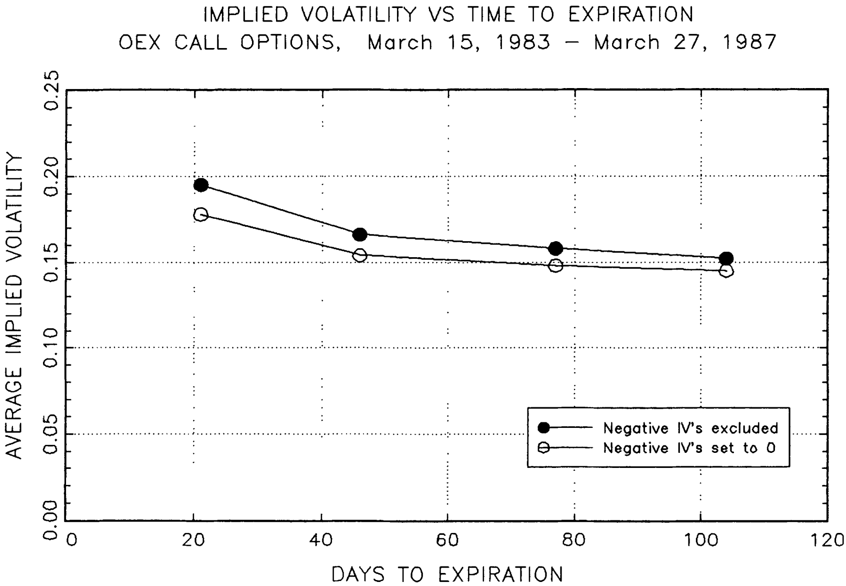

Figure 1
Implied volatility vs. time to expiration OEX call options, March 15, 1983–March 27, 1987

option's "intrinsic value," although this usage is somewhat unconventional (because it takes negative values for out-of-the-money options).

Table 1 shows that the average implied volatility in the sample is a decreasing function of time-to-option expiration. The mean for the entire set of 17,606 observations is 0.168, while the averages for our four maturity groups decline monotonically from 0.195, for near-month options, to 0.152, for those expiring in the fourth month.

Latané and Rendleman (1976) pioneered the practice of forming a weighted-average implied standard deviation (WISD) using multiple options on the same stock. This makes sense if the only reason IVs for two options on the same asset will differ is random noise in the sampling process. But with time-varying volatility, options expiring on different dates may reasonably be priced using different volatilities. Other differences are often observed, such as a regular structure of IVs across strike prices for options with a given maturity, and these constitute evidence against the hypothesis that IV is the market's fully rational volatility forecast.[9] Regularity in IV differences suggests

9 That there seems to be a systematic structure to implied volatilities is quite well known. For example, the observed "overpricing" of deep-out-of-the-money options has given rise to numerous theories and articles over the years. Rubinstein (1985) provides an extensive analysis of implied volatilities from individual stocks that reveals several regular patterns. A pattern observed in a number of markets, that of IVs that are lowest at-the-money and become progressively higher the further the option is in- or out-of-the-money, is known to traders as the "smile."

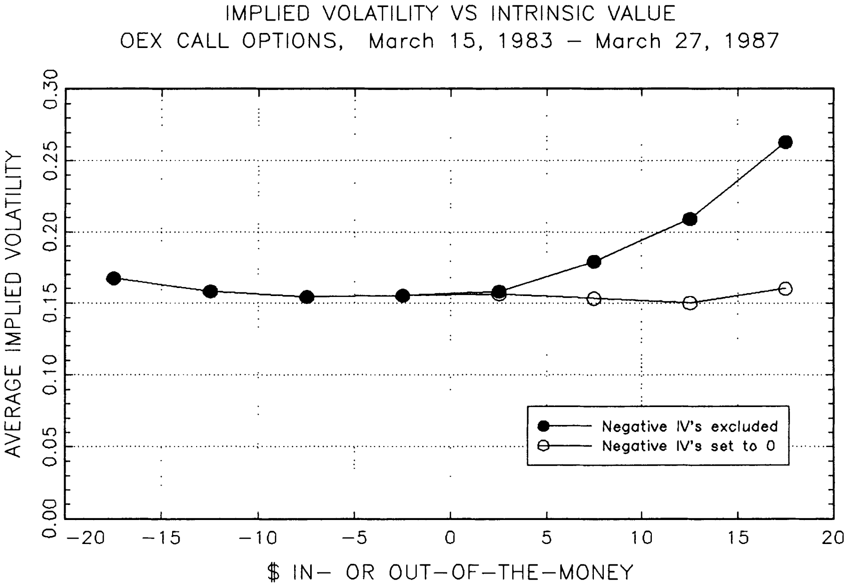

Figure 2
Implied volatility vs. intrinsic value OEX call options, March 15, 1983–March 27, 1987

the existence of *systematic* factors that lead investors to price particular options high or low relative to others.[10] It is therefore inappropriate simply to average together IVs from options with different expirations or IVs measured on different dates as if they were just multiple noisy observations on the same parameter.[11]

The most striking result in Table 2 is that deep in-the-money calls appear to have much higher IVs than at-the-money options. Beckers also finds this pattern in his study of individual stock options. In his sample, IVs for deep-in-the-money near-to-expiration calls are as much as 10 times the IVs for the corresponding at-the-money options. One explanation for this phenomenon is that in constructing the sample we were obliged to exclude the calls that would have negative IVs because their prices violated the lower boundary. Deep-in-the-money options are quite insensitive to volatility, meaning a large change in the implied volatility is produced by a small change in the option's

[10] Using a careful analysis of transactions data, Harvey and Whaley (1992) also find several interesting regularities in the time pattern of implied volatilities drawn from OEX options, including a day-of-the-week effect. Call IVs are low on Fridays and high on Mondays, but puts do not show the same pattern.

[11] LR, for example, computed their WISDs, one per week, and then averaged them over the whole 39-week sample period into a single IV for each stock. By contrast, Beckers found the IV drawn from a single at-the-money option to be at least as accurate a predictor of future volatility as the weighted-average IV.

price. At the same time, these options are rather illiquid, and they trade less frequently than those nearer to the money, so they have wider bid–ask spreads and nonsynchronous data is a larger problem. Thus, there is relatively more "noise" in the prices of these calls, and apparent boundary violations are most frequent for them. In forming our sample, we are potentially introducing a bias by keeping calls whose prices were artificially high due to trading noise but eliminating them when noise drove their prices below the boundary.

To see how this bias might affect the results, we constructed a broader sample including the calls that violated the boundary and arbitrarily assigned them an IV of 0. The second curves in Figures 1 and 2 plot the results from this procedure. Figure 2 shows how this alters the estimates of mean IVs for the deep-in-the-money calls and brings them much closer to the values observed for the other option groups. We suspect that Beckers' results may also be partly due to eliminating option prices that violated the lower arbitrage boundary.

When the market prices embody a regular volatility structure that seems to reflect systematic factors other than predicted future volatility, we do not know which option's IV, if any, represents the market's true expectation. It is also clear that simply averaging different IVs together would contaminate our estimate of the market's volatility forecast with price effects relating to those other factors. Accordingly, we subdivide the sample into groups, which we analyze separately, according to maturity and intrinsic value. This allows us to test whether any group of options, such as those that are at-the-money and near to expiration, provides a more useful IV.

The sample is broken into four maturity groups, and each of those is divided into eight intrinsic-value groups. This produces 32 subsamples, which we denote as subsample (i, j), where i represents the maturity group and j the intrinsic value group. For example, subsample (1, 2)—maturity group 1 and intrinsic-value group 2—contains implied volatilities for the near-month options that are between 15 and 10 points out-of-the-money. The subsample break points are displayed in Tables 1 and 2. Break points for the intrinsic value groups are set 5 points apart, corresponding to the contract strike prices. Since options expire on the third Friday of the delivery month, the near-month calls may have up to 35 days to expiration, while in other months the second-month options might mature in as few as 29 days.[12] Our subsamples are constructed such that each contains a maximum of one option price per day.

Equation (3) is estimated separately for each subgroup. We rewrite

[12] Technically, expiration is on the Saturday following the third Friday, but since Friday is the last date the options can be traded and their payoffs are based on Friday's closing stock prices, expiration is effectively at the close on the third Friday.

(3) to bring out its dependence on maturity, intrinsic value, and the time period:

$$\sigma_t(\tau) = \alpha + \beta \cdot \mathrm{IV}_t(i, j) + u_{t,i,j}, \tag{5}$$

where $\mathrm{IV}_t(i, j)$ is the implied volatility computed at time t from the option in maturity group i and intrinsic-value group j, and $u_{t,i,j}$ represents the regression disturbance. The symbol $\sigma_t(\tau)$ is the realized volatility of returns over the period between t and $t + \tau$, the option's expiration date, annualized by multiplying the calculated volatility per trading day by $\sqrt{260}$. In the tests reported here, the realized volatility $\sigma_t(\tau)$ is computed over the remaining life of the option as the annualized sample standard deviation of log returns including cash dividends; that is, $\ln[(S_s + D_s)/S_{s-1}]$ for $t < s \leq t + \tau$.

As long as the regressors and disturbances are uncorrelated with one another, the least-squares estimates of α and β will be unbiased and consistent. However, when daily data are used, the disturbances $u_{t,i,j}$ will be serially correlated. Since the realized volatility involves each day's return, from $t + 1$ to expiration at $t + \tau$, $\sigma_t(\tau)$ is only fully known on the day after the expiration date. The forecast errors are therefore correlated for IVs computed from any pair of options whose remaining lifetimes overlap.

Define X_n as the row vector of the independent variables for observation n in the sample; that is, $X_n = (1 \quad \mathrm{IV})_n$. X is the $N \times 2$ matrix of the X_n. [X is $N \times 3$ for regressions based on Equation (4).] Let u_n be the regression error for observation n, and let u denote the N vector of the u_n. Following Hansen (1982), we compute

$$\begin{aligned}\hat{\Psi} = {} & N^{-1} \sum_{n=1}^{N} (\hat{u}_n)^2 X_n' X_n \\ & + N^{-1} \sum_{k=1}^{N} \sum_{n=k+1}^{N} Q(k, n)\, \hat{u}_k \hat{u}_n (X_n' X_k + X_k' X_n),\end{aligned} \tag{6}$$

where u_k and u_n are the fitted residuals for observations k and n from the OLS regression. $Q(k, n)$ is an indicator function taking the value 1 if there is an overlap between the periods to expiration for the two options, and 0 otherwise.

The estimated covariance matrix for the coefficients is

$$\hat{\Omega} = (X'X)^{-1} \hat{\Psi} (X'X)^{-1}. \tag{7}$$

The procedure we have just described is consistent, but a question remains as to its performance in small samples (and how large a sample has to be before it is no longer "small"). Another important question is how much statistical power is gained by making use of all available data rather than restricting the analysis to nonoverlapping observations. The Appendix describes the results of a simulation study with a data sample constructed to be like the one we examine here. We find the corrected standard errors to be reasonably close to the true values. Moreover, incorporating the overlapping data points leads to a standard error on the β coefficient between one-quarter and one-eighth of what would be obtained with only nonoverlapping data, depending on the option maturity.

3. The Forecasting Performance of Implied Volatility

To test the predictive power of implied volatility, the regression equation (5) was fitted separately for each of the 32 subsamples. The equations were estimated by OLS, and for hypothesis testing the consistent estimate of the coefficient covariance matrix, shown in Equations (6) and (7), was computed.

The results are presented in Table 3 by subsample. For example, for the subsample of options that expire in the second month ($i = 2$) that are between \$0 and \$5 out-of-the-money ($j = 4$), the intercept is 0.113 and the estimated slope coefficient is 0.163, with standard errors of 0.017 and 0.101, respectively. The adjusted R^2 is .053. Thus, the hypothesis that $\alpha = 0$ and $\beta = 1.0$, and implied volatility is an unbiased forecast of future realized volatility, is strongly rejected for this subsample.

The results obtained for subsample (2, 4) are representative of what we find generally in Table 3: in every subsample, implied volatility fails the unbiasedness test. Indeed, the slope coefficient is estimated to be significantly different from *zero* at the 5 percent significance level in only 6 out of the 32 subsamples, and 3 of these were negative. In the group of three significant positives, the coefficient on implied volatility ranged from 0.138 to 0.229, and the adjusted R^2 was between .035 and .067. Far from demonstrating that implied volatility in OEX options is an unbiased and efficient forecast of subsequent realized volatility, these results show that, in most cases, implied volatility has no statistically significant correlation with realized volatility at all.

We do see that the options that are expected to be the most efficiently priced—those that are at- or slightly out-of-the-money (intrinsic-value groups 3, 4, and 5) and near to expiration (maturity group 1)—do produce significant positive β's and some of the highest R^2's. Nevertheless, the results in Table 3 appear to constitute a strong rejection of the null hypothesis.

However, it could be that OEX volatility is just very hard to predict accurately. In that case, the market's information set simply contains very little useful information for predicting $\sigma_t(\tau)$, and the estimation

Table 3
Realized volatility over the remaining life of the option regressed on implied volatility: Equation (5) $\sigma_t(\tau) = \alpha + \beta \cdot \mathrm{IV}_t(i, j) + u_{t,i,j}$

Maturity group	Intrinsic value −20 to −15.01, $j = 1$	−15 to −10.01, $j = 2$	−10 to −5.01, $j = 3$	−5 to −0.01, $j = 4$	0 to 5.00, $j = 5$	5.01 to 10, $j = 6$	10.01 to 15, $j = 7$	15.01 to 20, $j = 8$
Maturity $i = 1$; $\tau = 7$ to 35								
α	0.152	0.125	0.101	0.098	0.113	0.143	0.158	0.161
SE	0.026	0.019	0.015	0.013	0.011	0.012	0.014	0.012
β	−0.063	0.065	0.205	0.229	0.138	−0.025	−0.070	−0.047
SE	0.104	0.087	0.080	0.069	0.058	0.045	0.042	0.028
R^2	.004	.003	.044	.067	.035	.000	.032	.037
N	336	611	691	697	675	513	343	222
Maturity $i = 2$; $\tau = 29$ to 63								
α	0.125	0.111	0.109	0.113	0.123	0.143	0.161	0.182
SE	0.027	0.020	0.018	0.017	0.016	0.016	0.014	0.015
β	0.110	0.178	0.188	0.163	0.102	−0.020	−0.100	−0.171
SE	0.143	0.109	0.099	0.101	0.084	0.070	0.054	0.060
R^2	.010	.049	.062	.053	.025	.000	.055	.197
N	564	792	851	852	836	685	407	209
Maturity $i = 3$; $\tau = 57$ to 98								
α	0.116	0.112	0.112	0.118	0.127	0.143	0.162	0.176
SE	0.024	0.024	0.021	0.018	0.017	0.017	0.013	0.009
β	0.171	0.191	0.188	0.144	0.081	−0.015	−0.108	−0.157
SE	0.138	0.143	0.125	0.110	0.093	0.080	0.053	0.070
R^2	.038	.062	.070	.046	.019	−.001	.061	.147
N	507	731	803	810	789	593	324	152
Maturity $i = 4$; $\tau = 85$ to 127								
α	0.135	0.120	0.121	0.131	0.139	0.154	0.155	0.159
SE	0.026	0.022	0.019	0.017	0.016	0.016	0.015	0.010
β	0.060	0.138	0.129	0.062	0.002	−0.093	−0.075	−0.084
SE	0.155	0.136	0.121	0.109	0.102	0.072	0.046	0.044
R^2	.002	.033	.035	.009	−.002	.031	.022	.019
N	393	627	724	734	607	336	156	36

The table reports regression results from Equation (5) for OEX call options between March 15, 1983, and March 28, 1987, for each of the 32 subsamples defined by maturity and intrinsic value. The coefficients are fitted by OLS, but the standard errors (labeled SE) are corrected for intercorrelation as described in Section 2. N is the number of observations in the subsample. $\mathrm{IV}_t(i, j)$ is the implied volatility computed from the date t price of the call option from maturity group i (expiring at $t + \tau$), and intrinsic-value group j. $\sigma_t(\tau)$ is the realized volatility of the OEX index from date t to $t + \tau$. $u_{t,i,j}$ is the regression residual. The hypothesis that $\mathrm{IV}_t(i, j)$ is an informationally efficient forecast of $\sigma_t(\tau)$ requires $\alpha = 0.0$ and $\beta = 1.0$. The hypothesis is overwhelmingly rejected in every subsample.

results are dominated by the forecast errors. In order to check whether volatility is predictable at all from data available to the market, we estimated each of the regressions by using a historical measure of volatility as the independent variable, in place of implied volatility:

$$\sigma_t(\tau) = \alpha + \beta \cdot \mathrm{VOL60}_t(i, j) + u_{t,i,j}, \tag{8}$$

where $\mathrm{VOL60}_t(i, j)$ is the annualized standard deviation of the log returns of the S&P 100 stock index portfolio over the 60-day period preceding the date of the implied volatility, for all of the dates t corresponding to implied volatility observations contained in subsample (i, j). A 60-day sample period was used for historical volatility because it was approximately equal to the average forecast horizon in our sample, but tests using historical volatilities computed from 30 and 120 days of data yielded similar results. As before, the equation was fitted with OLS, and the covariance matrix was adjusted for heteroskedasticity and time dependence by using Equation (7).

The results are reported in Table 4. The contrast with the previous table is striking. The largest slope coefficient is 0.589, all of the estimated values are positive, and most are significantly different from zero at the 5 percent level.[13]

Table 4 shows that the future volatility of the OEX index was partly forecastable from publicly available information on historical volatility. Note, though, that the historical volatility estimate also fails the rationality test. Even so, the fact that (measured) implied volatility is a less accurate forecast than historical volatility confirms that it is not informationally efficient.

Table 5 reports the results for the "encompassing regression" test. Equation (4) in this case becomes

$$\sigma_t(\tau) = \alpha + \beta_1 \cdot \mathrm{IV}_t(i, j) + \beta_2 \cdot \mathrm{VOL60}_t(i, j) + u_{t,i,j}. \tag{9}$$

The estimated coefficient on historical volatility in Table 5 is significantly greater than zero in most of the regressions, and its value is comparable to that in the corresponding Table 4 regression. The coefficient on IV is nowhere significantly greater than zero and is negative for 28 out of 32 subsamples. The table clearly shows the overall message of our tests—that implied volatility is a poor forecast of subsequent realized volatility, and it does not accurately impound the information contained in a readily available historical volatility estimate.

The subsamples we have analyzed so far contain a maximum of one observation per day, but the same procedure can be applied equally well to the case in which there is any number of observations per day. This allows estimation on more aggregated samples without the loss of information that occurs when the data points themselves are aggregated, as they are in forming WISDs.

[13] Note that since neither realized nor historical volatility is a function of option intrinsic value, the differences among the eight regressions reported for each maturity group in Table 4 are due only to the differences in the dates whose observations are included in the subsamples. Nearly identical sample composition leads to nearly identical regressions, as in subsamples (2, 3) and (2, 4). There is a small amount of regularity in the differences among subsamples, because the options exchange tends not to introduce options at strikes far below the current index. This means there will only be deep-in-the-money options when the market price has risen substantially after listing. This results in a lower average τ for intrinsic-value groups 7 and 8 (52.9 and 45.8 days, respectively, compared with an average τ of between 59.8 and 61.8 days for intrinsic-value groups 1 to 6).

Table 4
Realized volatility over the remaining life of the option regressed on historical volatility: Equation (8) $\sigma_t(\tau) = \alpha + \beta \cdot \text{VOL60}_t(i, j) + u_{t,i,j}$

	Intrinsic value							
Maturity group	−20 to −15.01, $j = 1$	−15 to −10.01, $j = 2$	−10 to −5.01, $j = 3$	−5 to −0.01, $j = 4$	0 to 5.00, $j = 5$	5.01 to 10, $j = 6$	10.01 to 15, $j = 7$	15.01 to 20, $j = 8$
Maturity $i = 1$; $\tau = 7$ to 35								
α	0.070	0.058	0.054	0.055	0.057	0.063	0.056	0.104
SE	0.037	0.025	0.023	0.023	0.025	0.028	0.031	0.040
β	0.483	0.556	0.576	0.573	0.562	0.521	0.573	0.270
SE	0.272	0.180	0.172	0.172	0.181	0.196	0.212	0.255
R^2	.059	.106	.119	.118	.108	.087	.094	.016
N	336	611	691	697	675	513	343	222
Maturity $i = 2$; $\tau = 29$ to 63								
α	0.088	0.078	0.074	0.074	0.074	0.068	0.075	0.085
SE	0.027	0.024	0.024	0.024	0.025	0.027	0.033	0.042
β	0.381	0.441	0.464	0.464	0.467	0.499	0.453	0.394
SE	0.181	0.166	0.165	0.165	0.170	0.185	0.218	0.268
R^2	.085	.132	.151	.151	.150	.158	.117	.064
N	564	792	851	852	836	685	407	209
Maturity $i = 3$; $\tau = 57$ to 98								
α	0.091	0.079	0.078	0.078	0.077	0.073	0.069	0.091
SE	0.029	0.029	0.028	0.028	0.029	0.032	0.033	0.041
β	0.367	0.439	0.445	0.444	0.454	0.478	0.504	0.368
SE	0.188	0.193	0.191	0.192	0.193	0.211	0.218	0.266
R^2	.112	.183	.188	.186	.190	.203	.197	.063
N	507	731	803	810	789	593	324	152
Maturity $i = 4$; $\tau = 85$ to 127								
α	0.091	0.077	0.077	0.078	0.071	0.067	0.059	0.108
SE	0.033	0.027	0.026	0.026	0.026	0.029	0.031	0.033
β	0.367	0.459	0.455	0.446	0.497	0.522	0.589	0.242
SE	0.207	0.174	0.171	0.176	0.178	0.202	0.221	0.223
R^2	.105	.208	.220	.212	.240	.240	.286	.000
N	393	627	724	734	607	336	156	36

The table reports regression results from Equation (8) for OEX call options between March 15, 1983, and March 28, 1987, for each of the 32 subsamples defined by maturity and intrinsic value. The coefficients are fitted by OLS, but the standard errors (labeled SE) are corrected for intercorrelation as described in Section 2. N is the number of observations in the subsample. $\text{VOL60}_t(i, j)$ is the historical volatility computed from the OEX index values on dates $t - 60$ to $t - 1$. Each $\text{VOL60}_t(i, j)$ subsample is set up to match the corresponding $\text{IV}_t(i, j)$ subsample in Table 3. That is, for each IV in subsample (i, j) of Table 3, subsample (i, j) of Table 4 contains the historical volatility estimated from the previous 60 days' prices. $\sigma_t(\tau)$ is the realized volatility of the OEX index from date t to $t + \tau$. $u_{t,i,j}$ is the regression residual. The hypothesis that $\text{VOL60}_t(i, j)$ is an informationally efficient forecast of $\sigma_t(\tau)$ requires $\alpha = 0.0$ and $\beta = 1.0$. The hypothesis is rejected in every subsample.

We have applied this procedure to fit Equation (5) on more aggregated samples of our data. Using the entire data sample in one grand regression ($i = 1,\ldots,4$ and $j = 1,\ldots,8$) yields

$$\hat{\sigma}_t(\tau) = \underset{(0.012)}{0.136} + \underset{(0.050)}{0.022} \cdot \text{IV}_t(i, j), \qquad N = 17{,}606 \quad R^2 = .002$$

and Table 6 shows the results of aggregating across maturity and

Table 5
Realized volatility over the remaining life of the option regressed on implied volatility and historical volatility: Equation (9) $\sigma_t(\tau) = \alpha + \beta_1 \cdot IV_t(i, j) + \beta_2 \cdot VOL60_t(i, j) + u_{t,i,j}$

	Intrinsic value							
Maturity group	−20 to −15.01, $j = 1$	−15 to −10.01, $j = 2$	−10 to −5.01, $j = 3$	−5 to −0.01, $j = 4$	0 to 5.00, $j = 5$	5.01 to 10, $j = 6$	10.01 to 15, $j = 7$	15.01 to 20, $j = 8$
Maturity $i = 1$; $\tau = 7$ to 35								
α	0.082	0.061	0.053	0.055	0.056	0.069	0.076	0.120
SE	0.039	0.023	0.022	0.023	0.024	0.025	0.031	0.039
β_1	−0.056	−0.022	0.043	0.077	0.038	−0.024	−0.063	−0.048
SE	0.097	0.087	0.106	0.100	0.070	0.041	0.038	0.027
β_2	0.478	0.567	0.488	0.488	0.522	0.521	0.554	0.275
SE	0.272	0.204	0.241	0.241	0.223	0.198	0.210	0.253
R^2	.062	.105	.119	.122	.109	.087	.120	.054
N	336	611	691	697	675	513	343	222
Maturity $i = 2$; $\tau = 29$ to 63								
α	0.092	0.078	0.074	0.074	0.074	0.075	0.095	0.122
SE	0.030	0.024	0.024	0.024	0.024	0.029	0.035	0.026
β_1	−0.062	−0.010	0.001	−0.008	−0.026	−0.061	−0.100	−0.174
SE	0.142	0.122	0.106	0.097	0.080	0.050	0.045	0.052
β_2	0.423	0.451	0.473	0.473	0.492	0.529	0.451	0.416
SE	0.197	0.206	0.203	0.203	0.196	0.192	0.236	0.198
R^2	.086	.131	.150	.150	.150	.171	.172	.270
N	564	792	851	852	836	685	407	209
Maturity $i = 3$; $\tau = 57$ to 98								
α	0.091	0.080	0.079	0.079	0.078	0.079	0.088	0.128
SE	0.030	0.029	0.028	0.028	0.028	0.033	0.036	0.037
β_1	−0.006	−0.040	−0.020	−0.037	−0.042	−0.066	−0.108	−0.147
SE	0.128	0.134	0.123	0.107	0.081	0.060	0.043	0.069
β_2	0.372	0.474	0.478	0.478	0.489	0.507	0.504	0.313
SE	0.208	0.224	0.230	0.230	0.217	0.216	0.228	0.259
R^2	.110	.184	.188	.187	.193	.218	.260	.191
N	507	731	803	810	789	593	324	152
Maturity $i = 4$; $\tau = 85$ to 127								
α	0.099	0.080	0.079	0.082	0.076	0.081	0.072	0.118
SE	0.033	0.025	0.024	0.024	0.023	0.025	0.029	0.034
β_1	−0.146	−0.147	−0.143	−0.152	−0.163	−0.148	−0.096	−0.096
SE	0.138	0.146	0.135	0.117	0.088	0.043	0.024	0.035
β_2	0.468	0.589	0.587	0.587	0.639	0.584	0.608	0.294
SE	0.231	0.236	0.234	0.234	0.204	0.193	0.204	0.222
R^2	.122	.229	.244	.253	.307	.320	.327	.033
N	393	627	724	734	607	336	156	36

The table reports regression results from Equation (9) for OEX call options between March 15, 1983, and March 28, 1987, for each of the 32 subsamples defined by maturity and intrinsic value. The coefficients are fitted by OLS but the standard errors (labeled SE) are corrected for intercorrelation as described in Section 2. N is the number of observations in the subsample. $IV_t(i, j)$ is the implied volatility computed from the date t price of the call option from maturity group i (expiring at $t + \tau$), and intrinsic value group j. $VOL60_t(i,j)$ is the historical volatility computed from the OEX index values on dates $t - 60$ to $t - 1$ for each observation in subsample (i, j). $\sigma_t(\tau)$ is the realized volatility of the OEX index from date t to $t + \tau$. $u_{t,i,j}$ is the regression residual. The hypothesis that $IV_t(i, j)$ is an informationally efficient forecast of $\sigma_t(\tau)$ and that it fully impounds the information contained in $VOL60_t(i, j)$ requires $\alpha = 0.0$, $\beta_1 = 1.0$, and $\beta_2 = 0.0$. In fact, the estimate for β_1 is negative in 28 out of 32 subsamples. β_2 is significantly less than 1.0 nearly everywhere, but significantly positive in most subsamples.

Table 6
Realized volatility regressed on implied volatility for samples aggregated across maturities and intrinsic values: Equation (5) $\sigma_t(\tau) = \alpha + \beta \cdot IV_t(i,j) + u_{t,i,j}$

Results by intrinsic-value group for all maturities (for $i = 1,\ldots,4$)

	Intrinsic value							
	−20 to −15.01, $j=1$	−15 to −10.01, $j=2$	−10 to −5.01, $j=3$	−5 to −0.01, $j=4$	0 to 5.00, $j=5$	5.01 to 10, $j=6$	10.01 to 15, $j=7$	15.01 to 20, $j=8$
α	0.142	0.123	0.113	0.116	0.125	0.145	0.156	0.157
SE	0.017	0.019	0.017	0.015	0.014	0.014	0.013	0.011
β	0.020	0.106	0.165	0.148	0.085	−0.030	−0.072	−0.051
SE	0.075	0.094	0.093	0.083	0.073	0.056	0.038	0.026
R^2	.012	.042	.038	.025	.006	.001	.086	.065
N	1800	2761	3069	3093	2907	2127	1230	619

Results by maturity group for all intrinsic values (for $j = 1,\ldots,8$)

	Maturity group			
	Near month 7–35 days $i=1$	2nd month 29–63 days $i=2$	3rd month 57–98 days $i=3$	4th month 85–127 days $i=4$
α	0.132	0.134	0.132	0.136
SE	0.009	0.015	0.017	0.017
β	0.025	0.037	0.056	0.032
SE	0.030	0.071	0.096	0.101
R^2	.003	.004	.009	.003
N	4088	5196	4709	3613

The table reports regression results from Equation (5) for OEX call options between March 15, 1983, and March 28, 1987. In the top panel each of the eight subsamples aggregates across all maturities within the specified intrinsic-value group, while the bottom panel aggregates across all intrinsic values within a specified maturity group. The coefficients are fitted by OLS, but the standard errors (labeled SE) are corrected for intercorrelation as described in Section 2. See the notes to Table 3 for further discussion of the variables.

intrinsic value separately. In all cases, the results only confirm what was shown in the disaggregated subsamples: implied volatility is not a rational forecast of future volatility.

One final issue that we can evaluate is whether the errors-in-variables problem caused by nonsynchronous prices can account for the results we have seen. We obtained transactions data for the time period spanned by our sample, for options with maturities in the second month ($i = 2$), and have replicated the regressions reported in Table 3. Because of small differences in the dates represented in the two data sets, we formed matching samples in the following way. On each date for which there were both transactions prices and a closing-price observation that was included in the Table 3 regression, we took the last recorded option transaction of the day prior to the NYSE close and matched it with the simultaneously observed level of the OEX index. Table 7 compares the results from estimating Equation (5) on these transactions data with those obtained from closing prices, as in Table 3. We find only small differences in the

Table 7
Realized volatility regressed on implied volatility: comparison between closing prices and transactions prices: Equation (5) $\sigma_t(\tau) = \alpha + \beta \cdot IV_t(i, j) + u_{t,i,j}$

	Intrinsic value							
Maturity group	−20 to −15.01, $j = 1$	−15 to −10.01, $j = 2$	−10 to −5.01, $j = 3$	−5 to −0.01, $j = 4$	0 to 5.00, $j = 5$	5.01 to 10, $j = 6$	10.01 to 15, $j = 7$	15.01 to 20, $j = 8$
	Maturity $i = 2$: Closing prices							
α	0.125	0.109	0.107	0.111	0.121	0.141	0.161	0.175
SE	0.028	0.019	0.016	0.016	0.016	0.017	0.014	0.014
β	0.108	0.193	0.190	0.170	0.114	−0.007	−0.092	−0.133
SE	0.150	0.107	0.098	0.090	0.085	0.078	0.055	0.052
R^2	.009	.056	.070	.061	.030	.001	.048	.122
N	520	733	780	778	756	607	329	161
	Maturity $i = 2$: Transactions prices							
α	0.129	0.119	0.121	0.126	0.133	0.142	0.163	0.172
SE	0.016	0.014	0.013	0.012	0.011	0.011	0.012	0.012
β	0.079	0.127	0.111	0.078	0.036	−0.008	−0.110	−0.116
SE	0.085	0.079	0.076	0.068	0.058	0.052	0.038	0.035
R^2	.009	.038	.038	.026	.008	.001	.093	.149
N	520	733	780	778	756	607	329	161

See the notes to Table 3. This table compares the results from estimating Equation (5) on recorded closing prices for the option and the index, as in Table 3, with synchronous prices drawn from transactions data. For dates during the sample period for which both transactions data and closing prices are available, we take the last trade price for the option prior to the close of the NYSE and match it with the simultaneously observed level of the OEX index.

estimated coefficients: the α's are almost identical, and the β's tend to be lower in the regressions with transactions prices. Although the standard errors are somewhat smaller with transactions data, none of the estimated β's is significantly positive and three of the eight point estimates are negative. It is evident from this table that the use of slightly nonsynchronous closing-price data does not explain the negative results we obtained before.

4. Conclusion

It is widely accepted that an option's implied volatility is a good estimate of the "market's" expectation of the asset's future volatility, but our results from a large sample of prices for the most actively traded option contract strongly refute that view. How should one interpret what we have found?

One possibility is to point to problems in the testing procedure. However, our strong belief is that despite the potential technical criticisms our procedures were reasonable and the results were so clear that further perfecting of the methodology would not change the basic conclusions. At the very least, we observe that if there is actually a strong correlation between IV and realized volatility, but it cannot be detected with this methodology, one cannot expect the normal procedure of simply computing IVs from recorded closing option prices to give useful volatility estimates.

We also tend to reject the conclusion that options traders are irrational. Although we cannot rule out irrationality entirely without independent data on market expectations, given the enormous amount of evidence that financial markets are largely efficient it would be surprising if options traders were clearly inferior to other investors in this regard.

The interpretation we favor is based on the fact that along with investors' volatility forecasts an option's market price also impounds the net effect of the many factors that influence option supply and demand but are not in the option model. These include liquidity considerations, interaction between the OEX option and the (occasionally mispriced) S&P 500 index futures contract, investor tastes for particular payoff patterns, and so on. Option pricing theories ignore such factors because, in a frictionless market, unlimited arbitrage drives the price to the model value regardless of what trading strategies other market participants follow. In the real world, however, the arbitrage between an OEX option and the underlying index is a difficult and very costly strategy that requires continuously buying and selling 100 stocks to maintain a delta neutral hedge.[14] In practice, there is little arbitrage trading of OEX options against the underlying stocks; most hedging by professionals is said to be done with S&P 500 futures. Thus, many factors can affect the price of an OEX option without inducing arbitrage to offset them, and the implied volatility will impound the net price effect of all of them.[15]

Several conclusions and conjectures result from this interpretation. One is that how accurate implied volatility is as a forecast of future volatility should be related to how easy the arbitrage trade is. Options on futures contracts are at one extreme, since they are traded on the same trading floors as the underlying futures, and transactions costs are very low.[16] At the other extreme would probably lie options on broad stock indexes like OEX and the S&P 500. In between are

[14] Figlewski (1989) shows in a simulation analysis that both the transactions costs and the risk borne by an arbitrageur who attempts to follow a model-based arbitrage strategy are very large, leading to wide bands around the model price within which arbitrage will not be done.

[15] Evidence that arbitrage involving stock index options is difficult to do and that "mispricings" do not seem to be easily arbitraged away is provided by Evnine and Rudd (1985). Using intraday data, they document numerous violations of arbitrage relations, including put-call parity, for both OEX and major market index options.

[16] Both Feinstein (1989) and Park and Sears (1985) present evidence that implied volatilities from stock index futures options contain a significant amount of information about futures volatility. (However, Feinstein also finds that implied volatility from S&P 500 futures options does not pass the rationality regression test.)

options on individual stocks, for which the arbitrage is possible but somewhat costly.

Another conclusion from our results is that since both IV and historical volatility fail the rationality test neither is an appropriate volatility forecast for OEX options. To compute the true expected value of future volatility from the information contained in stock and options prices, one must treat implied volatility as an element of the information set from which the conditional expectation will be derived and not as the conditional expectation itself. As inputs to the calculation, it is perfectly reasonable to combine both implied and historical volatilities without regard for whether either individually passes the rationality test. Moreover, since this is the way a rational economist should use OEX index options prices in forecasting future volatility, we should expect a rational investor to do the same thing. To measure the "market's" volatility estimate, therefore, we must not just take the implied volatility. We should attempt to compute the true conditional expectation of the future volatility from the market's information set.

Appendix

This Appendix reports Monte Carlo simulation results that allow us to evaluate the performance of the estimation procedure from two perspectives. We would like to know, first, how close the corrected standard errors are likely to be to the true values and, second, how much additional statistical power is gained by using all of the available daily data in the estimation rather than restricting the analysis to nonoverlapping observations.

We generated random series of returns, designed to look as much as possible like the actual data in our sample of OEX returns. First, the sample mean and volatility of the OEX index, μ_k and σ_k, were computed for each calendar month k in our data. Then a random series of returns R_t was created for that month by using Equation (A1):

$$R_t = \mu_k + \sigma_k z_t, \tag{A1}$$

where z_t represents a random draw from a standard normal distribution, and μ_k and σ_k are the mean and volatility of returns, here expressed in terms of their values per trading day. This process was repeated month by month until a return for each day in the sample had been generated. Then for each day the "realized" volatility of the simulated series was computed from that point to the expiration date for the option maturity being considered, exactly as we did with the actual OEX returns.

The results we obtained in the analysis of actual prices suggested virtually no connection between IV and subsequent realized volatility.

Table A1
Monte Carlo simulations of estimated coefficient standard errors for different procedures

	Maturity $i = 1$		$i = 2$		$i = 3$		$i = 4$	
	α	β	α	β	α	β	α	β
Overlapping Sample								
True standard deviation	0.0208	0.1285	0.0186	0.1174	0.0176	0.1122	0.0153	0.1023
OLS std errors								
Mean	0.0062	0.0377	0.0045	0.0284	0.0041	0.0262	0.0039	0.0263
Standard dev	0.0008	0.0047	0.0006	0.0040	0.0007	0.0040	0.0006	0.0042
Corrected std errors								
Mean	0.0202	0.1214	0.0173	0.1031	0.0159	0.0927	0.0142	0.0811
Standard dev	0.0040	0.0237	0.0047	0.0279	0.0053	0.0310	0.0053	0.0308
Nonoverlapping Sample								
True standard deviation	0.0139	0.4770	0.0178	0.6989	0.0205	0.7934	0.0190	0.7840
OLS std errors								
Mean	0.0145	0.4729	0.0181	0.6827	0.0207	0.7609	0.0193	0.7446
Standard dev	0.0020	0.0763	0.0038	0.1463	0.0046	0.1974	0.0055	0.2496

The table presents summary results on the estimated coefficient standard errors from regressions like Equation (5): $\sigma_t(\tau) = \alpha + \beta \cdot \mathrm{IV}_t(i, j) + u_{t,i,j}$, for intrinsic-value group 4 (at- to 5 points out-of-the-money), run on 10,000 simulated returns series. See the Appendix for details of the sample construction. The "overlapping sample" makes use of all simulated data points, while the "non-overlapping sample" restricts the sample to only observations for which the periods to option expiration do not overlap. "True standard deviation" is the sample standard deviation of the estimated parameter across the 10,000 regressions. "OLS std errors" are uncorrected for correlation in the residuals, while "corrected std errors" are corrected by using the methodology described in Section 2. "Mean" and "standard dev" refer to the sample average and standard deviation of the standard error estimates across the 10,000 regressions.

To create a simulated IV series with the same distributional characteristics as the actual IVs but having no correlation with the simulated volatilities, we simply chose randomly from the set of observed IVs for intrinsic-value group 4 (at- to slightly-out-of-the-money). To maintain the same pattern of serial correlation among the IVs as is present in the actual data, we carried out the selection in batches, as follows. For the first observation pertaining to a given expiration date, we randomly chose an IV for an option with the same number of days to maturity from the data sample. The IVs were then taken in order from the sample of actual IVs until a new expiration date was called for, at which point another random selection was made. In essence, this procedure randomly selects about one month of observations at a time from the series of actual IVs.

The series of realized volatilities computed from the simulated returns was then regressed on the series of randomly selected implied volatilities in two ways, first with the full sample of overlapping observations as we did in the paper and, again, using only nonoverlapping observations. OLS standard errors were calculated for both cases, and the corrected standard errors were computed for the overlapping sample. The coefficient estimates were also recorded so that their true standard deviations across simulations could be computed. This process was repeated 10,000 times for each maturity group, $i = 1, 2, 3, 4$.

Table A1 shows the results. For each maturity group, we show the standard errors on the two regression coefficients. The first line gives the true standard deviations of the coefficient estimates across the 10,000 regressions. For example, for the second maturity group the true standard error of the estimate of β was 0.1174. Next we show the uncorrected OLS results, both the average values for the estimated standard errors and the sample standard deviations of those estimates in the 10,000 regressions. With no adjustment, the OLS estimates are severely biased downward and show very little variability across regressions. The next lines show that the corrected standard errors are much closer to the true values on average (e.g., 0.1031 versus the true value of 0.1174 for β in the second maturity group), although they are still slightly downward biased and also somewhat variable across regressions. These results give us confidence that the standard error correction procedure we used is effective in removing the effect of cross-correlation in the residuals.

The lower portion of Table A1 reveals how much more accurate the coefficients estimated from the overlapping data are compared with those computed by using only nonoverlapping observations from the same data sample. To run these regressions, we took only nonoverlapping data points for 18, 40, 75, and 100 days to expiration for maturity groups 1, 2, 3, and 4, respectively. Although there is little difference in the accuracy with which the regression constant is estimated, there is a large increase in the true standard errors for the slope coefficients, up to a factor of about 8 for maturity group 4. Because there is no cross-correlation in the residuals, the OLS standard error estimates are consistent for this case, as is apparent in the table. Notice, however, that even in this case where the true slope coefficient is zero by construction and we have four years of data, the standard errors that come out of the nonoverlapping regressions are so large that if the estimate of the slope coefficient turned out to be 0.0 it would still be impossible to reject the hypothesis that $\beta = 1.0$ for any but the first maturity group, and only a marginal rejection would be possible there!

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