❖ 3 ❖

Introduction to Theoretical Pricing Models

In the last chapter we looked at some of the simple option strategies a trader might initiate given an opinion on an underlying contract's likely price movement. Whatever the basis for the opinion, it will probably be expressed with terms such as "good chance," "highly likely," "possible," "improbable," etc. The problem with this approach is that opinions cannot easily be expressed in numerical terms. What do we really mean by "good chance"? Or by "highly unlikely"? If we want to approach option markets logically we will need some method of quantifying our opinions about price movements.

From the elementary strategies discussed in the previous chapter it is clear that the direction in which the underlying market moves can have a significant effect on the profitability of an option strategy. Consequently, option traders are sensitive to the direction in which the underlying market moves. But an option trader has an additional problem: the speed of the market. A commodity trader who believes a commodity will rise in price within a specified period can be reasonably certain of making a profit if he is right. He simply buys the commodity, waits for it to reach his target price, then sells the commodity for a profit.

The situation is not quite so simple for an option trader. Suppose a trader believes a commodity will rise in price from $100, its present price, to $120 within the next five months. Suppose also that a $110 call expiring in three months is available at a price of $4. If the commodity rises to $120 by expiration, the purchase of the $110 will result in a profit of $6 ($10 intrinsic value less the $4 cost of the option). But is this profit a certainty? What will happen if the price of the commodity remains below $110 for the next three months and only reaches $120 after the option expires? Then the option will expire worthless and the trader will lose his $4 investment.

Perhaps the trader would do better to purchase a $110 call which expires in six months rather than three months. Now he can be certain that when the commodity reaches $120, the call will be worth at least $10 in intrinsic value. But what if the price of the six-month option is $12? In that case the trader might still show a loss. Even though the underlying commodity reaches the target price of $120, there is no guarantee that the $110 call will ever be worth more than its $10 intrinsic value.

A trader in an underlying market is almost exclusively interested in the direction in which the market will move. While the option trader is also sensitive to directional considerations, he must also give careful consideration to how fast the market is likely to move. If a futures trader and an option trader take long market positions in their respective instruments, and the market does in fact move higher, the futures trader is assured of a profit while the option trader may show a loss. If the market fails to move sufficiently fast, the favorable directional move may not be enough to offset the option's loss in time value. This is the primary reason speculators generally lose in option markets. A speculator usually buys options for their seemingly favorable risk/reward characteristics (limited risk/unlimited reward). But if he purchases options, not only must he be right about market direction, he must also be right about market speed. Only if he is right on both counts can he expect to make a profit. If predicting the correct market direction is difficult, correctly predicting direction and speed is probably beyond most traders' capabilities.

The concept of speed is vital in trading options. It is so important that there are many option strategies which depend only on the speed of the underlying market and not at all on its direction. Indeed, if a trader is highly proficient at predicting directional moves in the underlying market, he is probably better advised to stick to the underlying instrument. It is only when he has some feel for the speed component that a trader can hope to intelligently enter the option market.

The option trader who wants to intelligently evaluate the potential profitability of an option trade is faced with the task of analyzing several different factors. At a minimum he must consider:

1. The price of the underlying contract
2. The exercise price
3. The amount of time remaining to expiration
4. The direction in which he expects the underlying market to move
5. The speed at which he expects the underlying market to move

Ideally, he would like to express each of these factors numerically, feed the numbers into a formula, and derive a value for the option. By comparing the value to its price in the marketplace, the trader would then know whether the purchase or sale of the option was likely to be profitable. This is essentially the goal of option evaluation: to analyze an option based on the terms of the option contract, as well as current market conditions and future expectations.

EXPECTED RETURN

Suppose we are given the opportunity to roll a six-sided die, and each time we roll we will receive a dollar amount equal to the number which comes up. If we roll a one, we get $1; if we roll a two, we get $2; and so on up to six, in which case we get $6. If we were to roll the die an infinite number of times, on average, how much would we expect to receive per roll?

We can calculate the answer using some simple arithmetic. There are six numbers which can come up, each with equal probability. If we add up the six possible outcomes

1+2+3+4+5+6=21 and divide this by the six faces of the die we get 21/6 = 3½. That is, on average we can expect to get back $3½ each time we roll the die. This is the average, or *expected*, return. If someone were to charge us for the privilege of rolling the die, what might we be willing to pay? If we purchased the chance to roll the die for less than $3½, in the long run we would expect to be winners. If we paid more than $3½, in the long run we would expect to be losers. And if we paid exactly $3½, we would expect to break even. Note the qualifying phrase "in the long run." The expected return of $3½ is a realistic goal only if we are allowed to roll the die many, many times. If we are allowed to roll only once, we cannot count on getting back $3½. Indeed, on any one roll it is impossible to get back $3½ since no face of the die has exactly 3½ spots. Nevertheless, if we pay less than $3½ for even one roll of the die, the laws of probability are on our side because we have paid less than the expected return.

In a similar vein, consider a roulette bet. A roulette wheel has 38 slots, numbered 1 through 36, 0 and 00.[1] Suppose a casino allows a player to choose a number. If the player's number comes up, he receives $36; if any other number comes up, he receives nothing. What is the expected return from this proposition? There are 38 slots on the roulette wheel, each with equal probability, but only one slot will return $36 to the player. If we divide the one way to win $36 by the 38 slots on the wheel, the result is $36/38 = $.9474, or about 95¢. A player who pays 95¢ for the privilege of picking a number at the roulette table can expect to break about even in the long run.

Of course, no casino will let a player buy such a bet for 95¢. Under those conditions the casino would make no profit. In the real world, a player who wants to purchase such a bet will have to pay more than the expected return, typically $1. The 5¢ difference between the $1 price of the bet and the 95¢ expected return represents the profit potential, or *edge*, to the casino. In the long run, for every dollar bet at the roulette table, the casino can expect to keep about 5¢.

Given the above conditions, any player interested in making a profit would rather switch places with the casino so that he could be the house. Then he would have a 5¢ edge on his side by selling bets worth 95¢ for $1. Alternatively, the player would like to find a casino where he could purchase the bet for less than its expected return of 95¢, perhaps 88¢. Then the player would have a 7¢ edge over the casino.

THEORETICAL VALUE

The theoretical value of a proposition is the price one would expect to pay in order to just break even in the long run. Thus far the only factor we have considered in determining the value of a proposition is the expected return. We used this concept to calculate the 95¢ fair price for the roulette bet. There may, however, be other considerations.

Suppose that in our roulette example the casino decides to change the conditions of the bet slightly. The player may now purchase the roulette bet for its expected return

1. As is customary in the U.S., we assume a roulette wheel with 38 slots. In some parts of the world the roulette wheel may have no slot 00. This of course changes the odds.

of 95¢ and, as before, if he loses the casino will immediately collect his 95¢. Under the new conditions, however, if the player wins the casino will send him his $36 winnings in two months. Will both the player and the casino still break even on the proposition?

Where did the player get the 95¢ he used to place his bet at the roulette wheel? In the immediate sense he may have taken it out of his pocket. But a closer examination may reveal that he withdrew the money from his savings account prior to visiting the casino. Since he won't receive his winnings for two months, he will have to take into consideration the two months interest he would have earned had he left the 95¢ in his savings account. If interest rates are 12% annually (1% per month), the interest loss is 2% × 95¢, or about 2¢. If the player purchases the bet for its expected return of 95¢, he will still be a 2¢ loser because of the cost of carrying a 95¢ debit for two months. The casino, on the other hand, will take the 95¢, put it in an interest-bearing account, and at the end of two months collect 2¢ in interest.

Under these new conditions the theoretical value of the bet is the expected return of 95¢ less the 2¢ carrying cost on the bet, or about 93¢. If a player pays 93¢ for the roulette bet today and collects his winnings in two months, neither he nor the casino can expect to make any profit in the long run.

The two most common considerations in a financial investment are the expected return and carrying costs. There may, however, be other considerations. For example, suppose the casino decided to send the player a 1¢ bonus over the next two months. He could then add this additional payment to the previous theoretical value of 93¢ to get a new theoretical value of 94¢. This is similar to the dividend paid to owners of stock in a company. And, in fact, dividends are an additional consideration in evaluating options on stock.

Exchanges will perhaps object to the casino analogy. They prefer that option trading not be thought of as gambling. There is certainly no desire here to assess the moral implications of either gambling or option trading. The fact remains that the same laws of probability which enable a casino to set the odds for different games of chance are the same laws of probability which enable a trader to evaluate an option.

The concept of theoretical value based on probability is common in many aspects of business. For those uncomfortable with the gambling analogy, one can go back to the original justification for options and think of them as insurance policies which require the payment of a premium. Through the use of statistical data and probability theory, an actuary at an insurance company will attempt to calculate the likelihood that the insurance company will have to make good on an insurance policy. He can then factor into the equation what the insurance company expects to earn on premium payments, and thereby arrive at a theoretical value for the insurance policy. The policy can then be offered to prospective customers at an additional cost, which represents the theoretical edge to the insurance company.

In the same way, the goal of option evaluation is to determine, through the use of theoretical pricing models, the theoretical value of an option. The trader can then make an intelligent decision whether the option is overpriced or underpriced in the marketplace, and whether the theoretical edge is sufficient to justify going into the marketplace and making a trade.

A WORD ON MODELS

Before continuing, a few observations on models in general will be worthwhile.

A model is a scaled down or more easily managed representation of the real world. The model may be a physical one, such as a model airplane or building, or it may be a mathematical one, such as a formula. In each case, the model is constructed to help us better understand the world in which we live. *However, it is unwise, and sometimes dangerous, to assume that the model and the real world which it represents are identical in every way.* They may be very similar, but the model is unlikely to exactly duplicate every feature of the real world.

All models, if they are to be effective, require us to make certain prior assumptions about the real world. Mathematical models require the input of numbers which quantify these assumptions. If we feed incorrect data into the model, we can expect an incorrect representation of the real world. Every model user must be aware: garbage in, garbage out.

These general observations about models are no less true for option pricing models. An option model is only someone's idea of how options might be evaluated under certain conditions. Since either the model itself, or the data which we feed into the model, might be incorrect, there is no guarantee that model-generated values will be accurate, nor can we be sure that these values will bear any logical resemblance to actual prices in the marketplace.

There is in fact a great deal of disagreement among traders as to the usefulness of option pricing models. Some traders feel that models are so much hocus-pocus, and have no relationship to what goes on in the real world. Other traders feel that once they have a sheet of theoretical values in hand all their problems are solved. The reality lies somewhere in between.

A new option trader is like someone entering a dark room for the first time. Without any guidance he will grope in the dark and may eventually find what he is looking for. The trader who is armed with a basic understanding of theoretical pricing models enters the same room with a small candle. He can make out the general layout of the room, but the dimness of the candle prevents him from distinguishing every detail. Moreover, some of what he does see may be distorted by the flickering candle. In spite of these limitations, a trader is more likely to find what he is looking for with a small candle than with no illumination at all.

The real problems with theoretical pricing models arise after the trader has acquired some sophistication. As he gains confidence he may begin to increase the size of his trades. When this happens, his inability to make out every detail in the room, as well as the distortions caused by the flickering candle flame, take on increased importance. Now a misinterpretation of what he thinks he sees can lead to financial disaster, since any error in judgement will be greatly magnified.

The sensible approach is to make use of a model, but with a full awareness of what it can and cannot do. Option traders will find that theoretical pricing models are invaluable tools to understanding the pricing of options. Because of the insights gained from a model, the great majority of successful option traders rely on some type of theoretical pricing model. However, an option trader, if he is to make the best use of a theoretical pricing model, must be aware of its limitations as well as its strengths. Otherwise he may be no better off than the trader groping in the dark.[2]

A SIMPLE APPROACH

How might we adapt the concepts of expected return and theoretical value to the pricing of options? We might begin by calculating the expected return for an option. Let's take a simple example.

Suppose an underlying contract is trading at $100 and that on a certain date in the future, which we will call expiration, the contract can take on one of five different prices: $80, $90, $100, $110, or $120. Assume, moreover, that each of the five prices is equally likely with 20% probability. The prices and probabilities might be represented by the line in Figure 3-1.

If we take a long position in the underlying contract at today's price of $100, what will be the expected return from this position at expiration? 20% of the time we will lose $20 when the contract ends up at $80. 20% of the time we will lose $10 when the contract ends up at $90. 20% of the time we will break even when the contract ends up at $100. 20% of the time we will make $10 when the contract ends up at $110. And 20% of the time we will make $20 when the contract ends up at $120. We can write the arithmetic:

$$-(20\% \times \$20) - (20\% \times \$10) + (20\% \times 0) + (20\% \times \$10) + (20\% \times \$20) = 0$$

Since the profits and losses exactly offset each other, the expected return to the long position is zero. The same reasoning will show that the expected return to a short position taken at the current price of $100 is also zero. Given the prices and probabilities, if we take either a long or short position we can expect to just break even in the long run.

Now suppose that we take a long position in a $100 call. Forgetting for a moment about what we might pay for the call, what will be the expected return given the prices and probabilities in Figure 3-1? If the underlying contract finishes at $80, $90, or $100 the call will expire worthless. If the underlying contract finishes at $110 or $120 the call will be worth $10 and $20, respectively. The arithmetic is:

$$(20\% \times 0) + (20\% \times 0) + (20\% \times 0) + (20\% \times \$10) + (20\% \times \$20) = +\$6$$

Figure 3-1

$80	$90	$100	$110	$120
20%	20%	20%	20%	20%

2. Two interesting articles discuss these limitations:
Figlewski, Stephen; "What Does an Option Pricing Model Tell Us about Option Prices?," *Financial Analysts Journal*, September/October 1989, pages 12-15.
Black, Fischer; "Living Up to the Model," *Risk*, Vol. 3, No. 3., March 1990, pages 11-13.

The call can never be worth less than zero, so the expected return from the call position is always a non-negative number, in this case \$6.

If we want to develop a theoretical pricing model using this approach, we might propose a series of possible prices and probabilities for the underlying contract at expiration. Then, given an exercise price, we can calculate the value of the option at each price outcome, multiply the value by its associated probability, add up all these numbers, and thereby obtain an expected return for the option.

In the foregoing example we took a very simple situation with only five possible price outcomes, each with identical probability. What changes might we make in order to develop a more realistic model? For one thing, we would have to know the settlement procedure for the option. In the United States, all options are subject to stock-type settlement, which requires full payment for the option. If the \$100 call will have an expected return of \$6 at expiration, we will have to deduct the carrying costs to get its value today. If interest rates are 12% annually (1% per month) and the option will expire in two months, we will have to discount the \$6 expected return by the 2% carrying cost, or about 12¢. The theoretical value of the option will then be \$5.88.

What other factors might we have to consider? We assumed that all five price outcomes were equally likely. Is this a realistic assumption? Suppose you were told that only two possible prices were possible at expiration, \$110 and \$250. With the underlying contract at \$100 today, which do you think is more likely? Based on experience, most traders would probably agree that extreme price changes which are far away from today's price are less likely than small changes which remain close to today's price. For this reason, \$110 is more likely than \$250. To take this into consideration, perhaps our price outcomes, in terms of probability, ought to be concentrated around the present price of the underlying contract. Such a distribution is shown in Figure 3-2. Now the expected return from a \$100 call is:

$$(10\% \times 0) + (20\% \times 0) + (40\% \times 0) + (20\% \times \$10) + (10\% \times \$20) = \$4.00$$

If, as before, the option is subject to stock-type settlement and carrying costs are 2%, the theoretical value of the option will now be \$3.92.

Note that in Figure 3-2 all outcomes and probabilities are arranged symmetrically. Even though the new probabilities altered the expected return for the \$100 call, the expected return from any position taken in the underlying contract is still zero. For each upward price move, there is a downward move of equal magnitude and probability. We might, however, believe that the expected return to an underlying contract is not zero, that there is a greater chance that the contract will move one direction rather than another. Look at the price outcomes and probabilities in Figure 3-3. Using these new probabilities, the expected return from a long position in the underlying contract is:

$$-(10\% \times \$20) - (20\% \times \$10) + (30\% \times 0) + (25\% \times \$10) + (15\% \times \$20) = +\$1.50$$

and the expected return for the \$100 call is:

$$(10\% \times 0) + (20\% \times 0) + (30\% \times 0) + (25\% \times \$10) + (15\% \times \$20) = +\$5.50$$

Figure 3-2

$80	$90	$100	$110	$120
10%	20%	40%	20%	10%

Figure 3-3

$80	$90	$100	$110	$120
10%	20%	30%	25%	15%

Note that the underlying contract now has a positive expected return, so it may seem that there is money to be made simply by purchasing the underlying contract. This would be true if there were no other considerations. But suppose the underlying contract is a stock, and therefore subject to stock-type settlement. If we purchase the stock at today's price of $100 and hold it for some period, there is a carrying cost associated with the investment. If the carrying cost is exactly equal to the expected return of $1.50, we will just break even. For a long stock position to be profitable, the stock must appreciate by at least the amount of carrying costs over the holding period. Therefore, the expected return from the stock must be some positive number. If we assume that any stock trade will just break even, the expected return must be equal to the carrying costs.

Some stocks also pay dividends. If the dividend is paid during the holding period, it will affect the expected return. A trader who buys stock will have to pay out carrying costs, but he will receive the dividends. If we again assume that a stock trade will break even, the expected return at the end of the holding period must be identical to the carrying costs less the dividend. If the carrying cost for the stock over some period is $3.50, and a $1.00 dividend is expected during this period, the expected return at the end of the period must be $2.50. A trader who purchases the stock today will incur an interest debit of $3.50 at the end of the holding period, but this will be exactly offset by the $1.00 dividend which he receives during the holding period,[3] as well as the $2.50 expected return at the end of the period.

In an *arbitrage-free* market, where no profit can be made by either buying or selling a contract, all credits and debits, including the expected return, must exactly cancel out. If we assume an arbitrage-free market, we must necessarily assume that the *forward price*, the average price of the contract at the end of the holding period, is the current price, plus an expected return which will exactly offset all other credits and debits. If the holding costs on a $100 stock over some period are $4, the forward price must be $104. If the stock also pays a $1 dividend, the forward price must be $103. In both cases the credits and debits will exactly cancel out.

The calculation of a forward price depends on the characteristics of the contract as well as market conditions. In the case of a stock, the considerations are the price of the

3. The trader can also earn interest on the dividend from the time he receives it until the end of the holding period. Since this will usually be a very small amount in relation to the other factors, we will ignore it.

stock, the length of the holding period, interest rates, and dividends. In the case of a futures contract, the situation is much simpler. A futures contract requires no initial cash outlay, since it is subject to futures-type settlement. Moreover, a futures contract does not pay dividends. This means that the forward price of a futures contract in an arbitrage-free market is simply the current price of the futures contract. If a trader buys a futures contract at $100, the break even price for the contract at the end of the holding period is $100.

Going back to our very simple pricing model, we might make the assumption that the underlying market is arbitrage-free,[4] that there is no money to be made from trading the underlying contract. The expected return must then be equal to the difference between the current price of the underlying market and its forward price. In the case of stock, the expected return will be carrying costs less dividends. In the case of futures, the expected return will be zero.

Even if we assume an arbitrage-free market in the underlying, with appropriate probabilities associated with each price outcome, we still have one major problem. In our simplified model there were only five possible price outcomes, while in the real world there are an infinite number of possibilities. To enable our model to more closely approximate real world conditions we will have to construct a probability line with every possible price outcome and its associated probability. This may seem an impossible task, but it is the basis for all theoretical pricing models.

We can now summarize the necessary steps in developing a model:

1. Propose a series of possible prices at expiration for the underlying contract
2. Assign an appropriate probability to each possible price
3. Maintain an arbitrage-free underlying market
4. From the prices and probabilities in steps 1, 2, and 3, calculate the expected return for the option
5. From the option's expected return, deduct the carrying cost

If we can accomplish all this, we will finally have a theoretical value from which we can begin to trade.

Prior to 1973, evaluation of options required the solution of complex mathematical equations. Since such methods were slow and tedious, a trader who tried to use them quickly found that profit opportunities disappeared faster than the evaluation methods could identify them. In 1973, concurrent with the opening of the Chicago Board Options Exchange, Fischer Black and Myron Scholes introduced the first practical theoretical pricing model for options. The Black-Scholes Model, with its relatively simple arithmetic and limited number of inputs, most of which were easily observable, proved an ideal tool for traders in the newly opened U.S. option market. Although other models have since been introduced to overcome some of its original deficiencies, the Black-Scholes Model remains the most widely used of all option pricing models.

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4. We need not necessarily assume an arbitrage free underlying market. But we shall see that this is an important assumption in most theoretical pricing models.

In its original form, the Black-Scholes Model was intended to evaluate European options (no early exercise permitted) on non-dividend paying stocks. Shortly after its introduction, realizing that most stocks do pay dividends, Black and Scholes added a dividend component. In 1976, Fischer Black made slight modifications to the model to allow for the evaluation of options on futures contracts. And in 1983, Mark Garman and Steven Kohlhagen made several other modifications to allow for the evaluation of options on foreign currencies.[5] The futures version and the foreign currency version are known officially as the Black Model and the Garman-Kohlhagen Model, respectively. But the evaluation method in each version, whether the original Black-Scholes Model for stock options, the Black Model for futures options, or the Garman-Kohlhagen Model for foreign currency options, is so similar that they have all come to be known as simply the Black-Scholes Model. The various forms of the model differ primarily in how they calculate the forward price of the underlying contract, and an option trader will simply choose the form appropriate to the underlying instrument.

The great majority of options currently traded are American options, carrying with them the right of early exercise. For this reason, it may seem that the Black-Scholes model, with its assumption of no early exercise, is poorly suited for use in most markets. However, the Black-Scholes Model has proven so easy to use that many traders do not believe the more accurate values derived from an American option pricing model, which allows for the possibility of early exercise, is worth the additional effort. In some markets, particularly futures options markets, the additional early exercise value is so small that there is virtually no difference between values obtained from the Black-Scholes model and values obtained from an American pricing model.

Due to its widespread use and its importance in the development of other pricing models, we will for the moment restrict ourselves to a discussion of the Black-Scholes model and its various forms. In later chapters we will consider the question of early exercise. We will also look at alternative methods for pricing options when we question some of the basic assumptions in the Black-Scholes Model.

The reasoning which led to the development of the Black-Scholes Model depends on the five steps we listed earlier in this chapter when we proposed a simple method for evaluating options. Black and Scholes worked originally with call values, but put values can be derived in much the same way. Alternatively, we will see in Chapter 11 that in an arbitrage-free market there is a unique relationship between an underlying contract, and a call and put with the same exercise price and expiration date. This relationship enables us to derive a put value simply by knowing the associated call value.

In order to calculate an option's theoretical value using the Black-Scholes Model, we need to know at a minimum five characteristics of the option and its underlying contract. These are:

1. The option's exercise price
2. The amount of time remaining to expiration

5. We are speaking here of options on a physical foreign currency, rather than options on a foreign currency futures contract. The latter may be evaluated using the Black Model for futures options.

3. The current price of the underlying contract
4. The risk-free interest rate over the life of the option
5. The volatility of the underlying contract

The last input, volatility, may be unfamiliar to the new trader. While we will put off a detailed discussion of this input to the next chapter, from our previous discussion one can reasonably infer that volatility is related to the speed of the market.

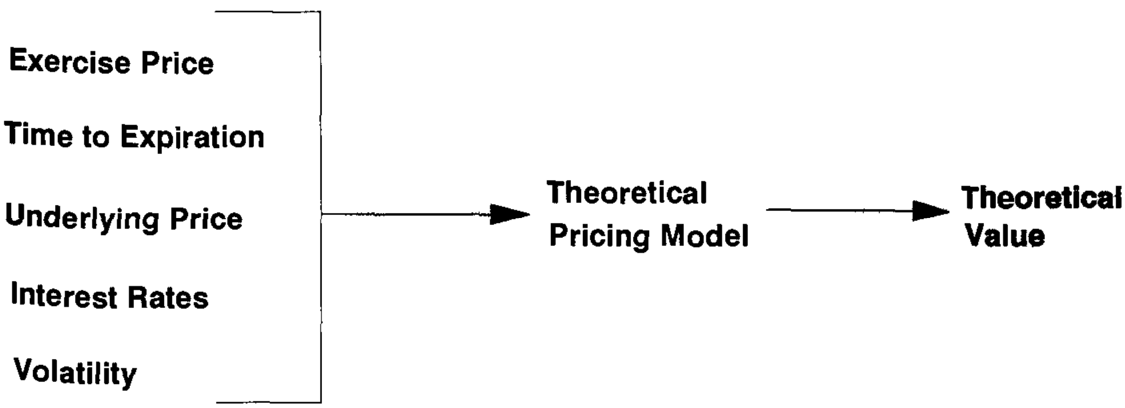
If we know each of the required inputs, we can feed them into the theoretical pricing model and thereby generate a theoretical value.

Black and Scholes also incorporated into their model the concept of the *riskless hedge*. For every option position there is a theoretically equivalent position in the underlying contract such that, for small price changes in the underlying contract, the option position will gain or lose value at exactly the same rate as the underlying position. To take advantage of a theoretically mispriced option, it is necessary to establish a hedge by offsetting the option position with this theoretically equivalent underlying position. That is, whatever option position we take, we must take an opposing market position in the underlying contract. The correct proportion of underlying contracts needed to establish this riskless hedge is known as the *hedge ratio*.

Why is it necessary to establish a riskless hedge? Recall that in our simplified approach an option's theoretical value depended on the probability of various price outcomes for the underlying contract. As the underlying contract changes in price, the probability of each outcome will also change. If the underlying price is currently $100 and we assign a 25% probability to $120, we might drop the probability for $120 to 10% if the price of the underlying contract falls to $80. By initially establishing a riskless hedge, and then by adjusting this hedge as market conditions change, we are taking into consideration these changing probabilities.

In this sense an option can be thought of as a substitute for a similar position in the underlying contract. A call is a substitute for a long position; a put is a substitute for a short position. Whether it is better to take the position in the option or in the underlying

Figure 3-4



contract depends on the theoretical value of the option and its price in the marketplace. If a call can be purchased (sold) for less (more) than its theoretical value, it will, in the long run, be more profitable to take a long (short) market position by purchasing (selling) calls than by purchasing (selling) the underlying contract. In the same way, if a put can be purchased (sold) for less (more) than its theoretical value, it will, in the long run, be more profitable to take a short (long) market position by purchasing (selling) puts than by selling (buying) the underlying contract.

Since the theoretical value obtained from a theoretical pricing model is no better than the inputs into the model, a few comments on each of the inputs will be worthwhile.

EXERCISE PRICE

There ought never be any doubt about the exercise price of an option, since it is fixed in the terms of the contract and does not vary over the life of the contract.[6] A Deutschemark March 58 call traded on the Chicago Mercantile Exchange cannot suddenly turn into March 59 call or a March 57 call. An IBM July 55 put traded on the Chicago Board Options Exchange cannot turn into a July 50 put or a July 60 put.

TIME TO EXPIRATION

Like the exercise price, the option's expiration date is fixed and will not vary. Our DM March 58 call will not suddenly turn into an April 58 call, nor will our IBM July 55 put turn into a June 55 put. Of course, each day that passes brings us closer to expiration, so in that sense the time to expiration is constantly growing shorter. However, the expiration date, like the exercise price, is fixed by the exchange and will not change.

Time to expiration, like all inputs in the Black-Scholes Model, is entered as an annualized number. If we are entering raw data directly into the model we must make the appropriate annualization. With 91 days remaining to expiration, we would enter an input of .25 ($91/365 \approx .25$). With 36 days remaining, we would enter .10 ($36/365 \approx .10$). However, most option evaluation computer programs already have this transformation incorporated into the software so that we need only enter the correct number of days remaining to expiration.

It may seem that we have a problem in deciding what number of days to enter into the model. We need the amount of time remaining to expiration for two purposes, to calculate the interest considerations and to calculate the likelihood of movement in the underlying contract. For volatility purposes in assessing the "speed" of the market we are only interested in trading days. Only on those days can the price of the underlying contract actually change. This might lead us to drop weekends and holidays from our calculations. On the other hand, for interest rate purposes we must include every day.

6. It is true that an exchange may adjust the exercise price of a stock option if there is a stock split. In practical terms this is not really a change in the exercise price because the exercise price retains the same relationship to the stock price. The characteristics of the option contract remain essentially unchanged.

If we borrow or lend money we expect the interest to accrue every day, no matter that some of the days are not business days.

It turns out that this is not really a problem. In calculating the "speed" of the market we observe only the price changes that occur on business days. But we can make slight changes to this observed value and annualize the number before feeding it into the theoretical pricing model. The result is that we can feed into our model the actual number of days remaining to expiration knowing that the model will interpret the number correctly.

PRICE OF THE UNDERLYING

Unlike the exercise price and time to expiration, the correct price of the underlying is not always obvious. At any one time there is usually a bid price and an asked price, and it may not be clear whether we ought to use one or the other of these prices, or perhaps some price in between.

We have noted that the correct use of an option's theoretical value requires us to hedge the option position with an opposing trade in the underlying contract. Therefore the underlying price we feed into our theoretical pricing model ought to be the price at which we believe we can make the opposing trade. If we intend to purchase calls or sell puts, both of which are long market positions, we will have to hedge by selling the underlying contract. In that case we ought to use the bid price since that is the price at which we can sell the underlying. On the other hand, if we intend to sell calls or buy puts, both of which are short market positions, we will have to hedge by purchasing the underlying contract. Now we ought to use the asked price since that is the price at which we can buy the underlying.

In practice, the bid and offer are constantly changing, and many traders will simply use the last trade price as the basis for theoretical evaluation. But the last trade price may not always reflect the present market. Even the settlement price quoted in a newspaper may not accurately reflect the market at the close of business. The last trade price may show 75¼ for a contract, but the market at the close may have been 75¼ bid, 75½ offered. A trader who hoped to buy at 75¼ would have very little chance of being filled because of the difficulty of buying at the bid price. Even a purchase at some middle price, say 75⅜, may be unlikely if the market is very unbalanced with many more contracts being bid for at 75¼ than offered at 75½. For all of these reasons, an experienced trader will rarely enter an option market without knowing the exact bid and offer in the underlying market.

INTEREST RATES

Since an option trade may result in either a cash credit or debit to a trader's account, the interest considerations resulting from this cash flow must also play a role in option evaluation. This is a function of interest rates over the life of the option.

The interest rate component plays two roles in the theoretical evaluation of options. First, it may affect the forward price of the underlying contract. If the underlying contract is subject to stock-type settlement, as we raise interest rates we raise the forward price, increasing the value of calls and decreasing the value of puts. Secondly, the interest rate may affect the cost of carrying the option. If the option is subject to stock-type settlement, as we raise interest rates we decrease the value of the option. In spite of the fact that the interest rate plays two roles, in most cases the same rate is applicable and we need only input one interest rate into the model. If, however, different rates are applicable, such as would be the case with foreign currency options (the foreign currency interest rate plays one role, the domestic currency interest rate plays a different role) the model will require the input of two interest rates. This is the case with the Garman-Kohlhagen version of the Black-Scholes Model.

The fact that interest rates play a dual role also means that the relative importance of interest rates will vary, depending on the type of underlying instrument and the settlement procedure. For example, interest rates have a much greater impact on the value of stock options than on futures options. As we raise interest rates, we increase the forward price of stock, but leave the forward price of a futures contract unchanged. At the same time, assuming stock-type settlement, as we raise interest rates we decrease the value of options. The option price, however, is usually very small in relation to the price of the underlying contract.

What interest rate should a trader use when evaluating options? Most traders cannot borrow and lend at the same rate, so the correct interest rate will, in theory, depend on whether the trade will create a debit or a credit. In the former case the trader will be interested in the borrowing rate, while in the latter case he will be interested in the lending rate. In practice, however, the most common solution is to use the *risk-free* interest rate, i.e., the most secure rate. In the United States, the government is usually considered the most secure borrower of funds, so that the yield on a government security with a term equivalent to the life of the option is the general benchmark. For a 60-day option, use the yield on a 60-day treasury bill; for a 180-day option, use the yield on a 180-day treasury bill.

DIVIDENDS

We did not list dividends as an input in Figure 3-4 since they are only a factor in the theoretical evaluation of stock options, and then only if the stock is expected to pay a dividend over the life of the option.

In order to accurately evaluate a stock option, a trader must know both the amount of the dividend which the stock will pay and the *ex-dividend* date, the date on which a trader must own the stock in order to receive the dividend. The emphasis here is on ownership of the stock. A deeply in-the-money option may have many of the same characteristics as stock, but only ownership of the stock carries with it the right to collect the dividend.

In the absence of other information, most traders tend to assume that a company will continue the same dividend policy it has had in the past. If the company has been paying a 75¢ dividend each quarter, it will probably continue to do so. However, this is not always a certainty. Companies sometimes increase or decrease dividends, and occasionally omit them completely. If there is the possibility of a change in a company's dividend policy, a trader has to consider its impact on option values. Additionally, if the ex-dividend date is expected just prior to expiration, there is the danger that a delay of several days will cause the ex-dividend date to fall after expiration. For purposes of option evaluation, this is the same as eliminating the dividend completely. In such a situation a trader ought to make a special effort to ascertain the exact ex-dividend date.

VOLATILITY

Of all the inputs required for option evaluation, volatility is the most difficult for traders to understand. At the same time, volatility often plays the most important role in actual trading situations. Changes in our assumptions about volatility can have a dramatic effect on an option's value, and the manner in which the marketplace assesses volatility can have an equally dramatic effect on an option's price. For these reasons, we will devote the next chapter to a detailed discussion of volatility.

❖ 4 ❖

Volatility

What is volatility and why is it so important to an option trader? The option trader, like a trader in the underlying instrument, is interested in the direction of the market. But unlike the trader in the underlying, an option trader is also extremely sensitive to the speed of the market. If the market for an underlying contract fails to move at a sufficient speed, options on that contract will have less value because of the reduced likelihood of the market going through an option's exercise price. In a sense, volatility is a measure of the speed of the market. Markets which move slowly are low-volatility markets; markets which move quickly are high-volatility markets.

One might guess intuitively that some markets are more volatile than others. Between 1980 and 1982, the price of gold moved from $300 per ounce to $800 per ounce, more than doubling its price. Yet few traders would predict that the S&P 500 Index might more than double in a similar period. A commodity trader knows that precious metals are generally more volatile than interest rate instruments. In the same way, a stock trader knows that high-technology stocks tend to be more volatile than utility stocks.

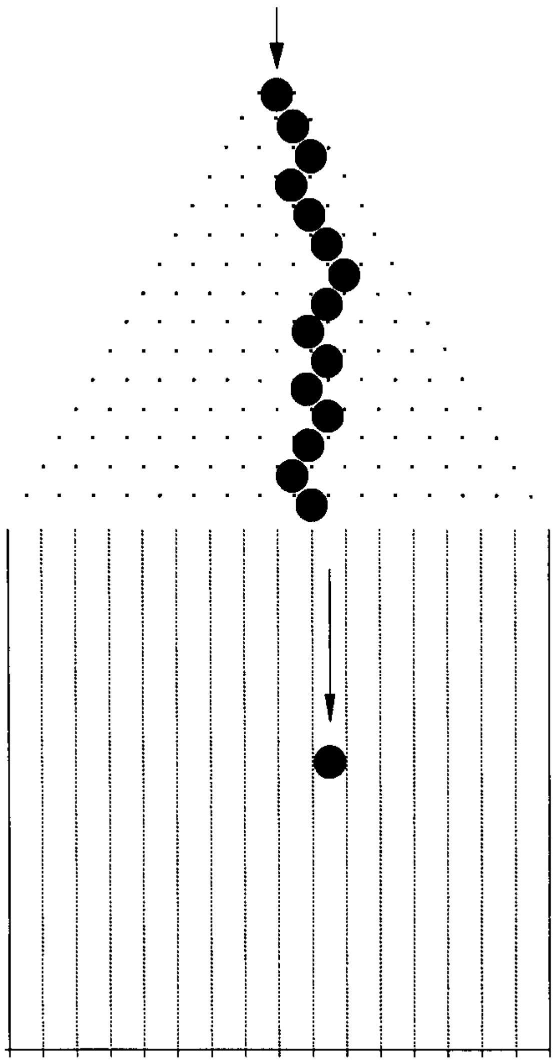
If we knew whether a market was likely to be relatively volatile, or relatively quiet, and could convey this information to a theoretical pricing model, any evaluation of options on that market would be more accurate than if we simply ignored volatility. Since option models are based on mathematical formulae, we will need some method of quantifying this volatility component so that we can feed it into the model in numerical form.

RANDOM WALKS AND NORMAL DISTRIBUTIONS

Consider for a moment the pinball maze pictured in Figure 4-1. When a ball is dropped into the maze at the top it moves downward, pulled by gravity, through a series of nails. When the ball encounters each nail there is a 50% chance the ball will move to the right, and a 50% chance it will move left. The ball then falls down to a new level where it encounters another nail. Finally, at the bottom of the maze the ball falls into one of the troughs.

The path the ball follows as it moves downward through the maze of nails is known as a *random walk*. Once the ball enters the maze nothing can be done to artificially alter its course, nor can one predict ahead of time the path the ball will take through the maze.

Figure 4-1: Random Walk

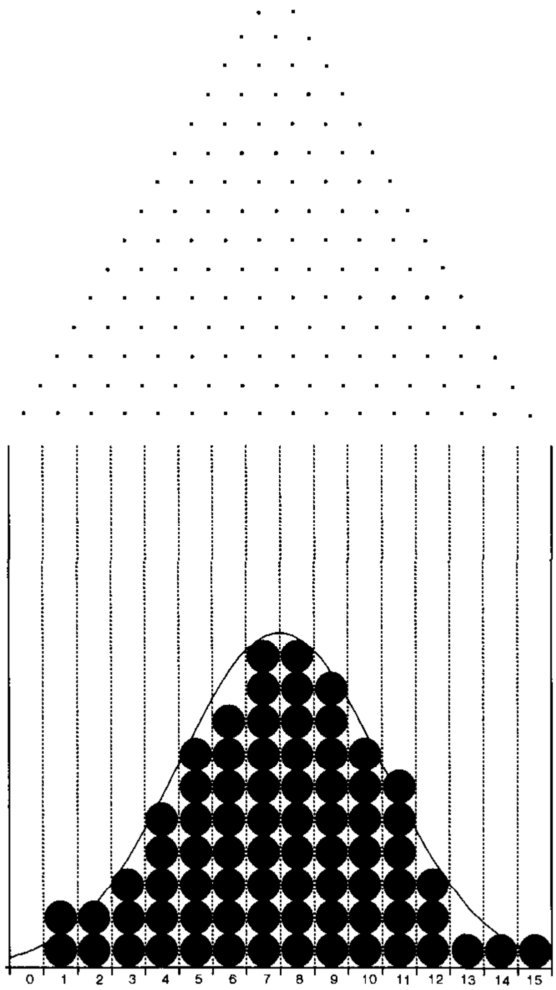


If enough balls are dropped into the maze, we might begin to get a distribution of balls similar to that in Figure 4-2. Most of the balls tend to congregate near the center of the maze, with a decreasing number of balls ending up in troughs further away from the center. The distribution which results from dropping many balls into our maze is referred to as a *normal*, or bell-shaped, *distribution*.

If we were to drop an infinite number of balls into the maze we might approximate the distribution with a *normal*, or bell-shaped, *curve* such as the one overlaid on the distribution in Figure 4-2. Such a curve is symmetrical (if we flipped it from right to left it would look the same), it has its peak in the center, and its tails always flare down and away from the center.

Normal distribution curves are used to describe the likely outcomes of random events. For example, the curve in Figure 4-2 might also represent the results of flipping a coin 15 times. Each outcome, or trough, would represent the number of heads which occurred after each 15 flips. An outcome in trough zero would represent zero heads and 15 tails; an outcome in trough 15 would represent 15 heads and zero tails. Of course, we would be surprised to flip a coin 15 times and get all heads or all tails. Assuming the

Figure 4-2: Normal Distribution

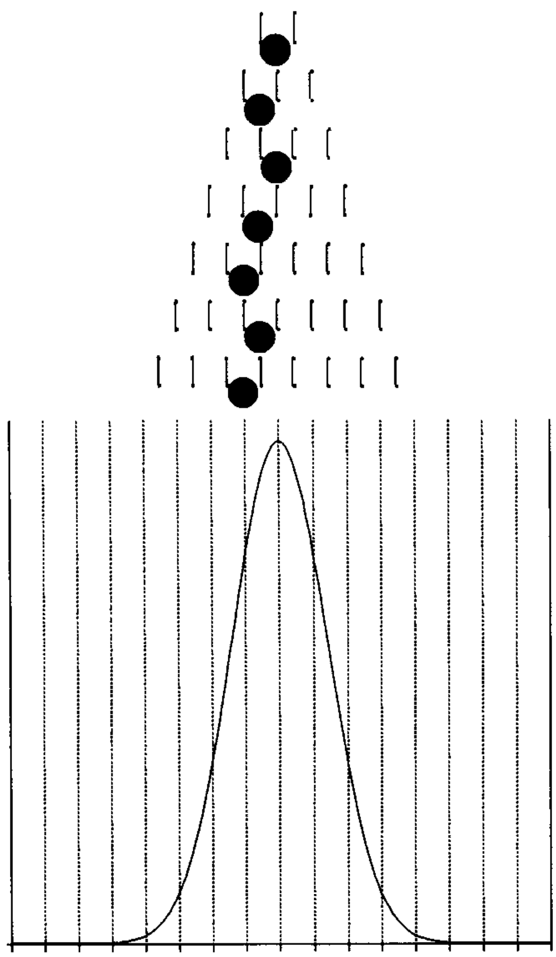


coin is perfectly balanced; some outcome in between, perhaps 8 heads and 7 tails, or 9 heads and 6 tails, seems more likely.

Suppose we change our maze slightly by closing off a row of nails so that each time a ball encounters a nail and goes either left or right, it must drop down two levels before it encounters another nail. If we drop enough balls into the maze we may end up with a distribution represented by the curve in Figure 4-3. Since the sideways movement of the balls is restricted, the curve will have a higher peak and narrower tails than the curve in Figure 4-2. In spite of its altered shape, this curve still represents a normal distribution, although one with slightly different characteristics.

Finally, we might block off some of the spaces between nails so that each time a ball drops down a level it must move two nails left or right before it can drop down to a new level. Again, if we drop enough balls into the maze we may get a distribution which resembles the curve in Figure 4-4. This curve, while still a normal distribution curve, will have a much lower peak and its tails will spread out much more quickly than the curves in either Figure 4-2 or 4-3.

Figure 4-3: Low Volatility Distribution



Suppose we now think of the ball's sideways movement as the up and down price movement of an underlying contract, and the ball's downward movement as the passage of time. If we assume that each day the underlying contract can move up or down $1, the price distribution after 15 days might be represented by the curve in Figure 4-2. If we assume the price can move up or down $1 every two days, the price distribution after 15 days might be represented by the curve in Figure 4-3. And if we assume that each day the price can move up or down $2, the price distribution might be represented by the curve in Figure 4-4.

With the underlying contract presently at $100 and 15 days to expiration, how might we evaluate a $105 call? One way is to assume that prices follow a random walk through time and that one of the curves in Figures 4-2, 4-3, or 4-4 represents the likely distribution after 15 days. The comparative value of the $105 call under these three scenarios is shown in Figure 4-5. If we assume a distribution similar to Figure 4-3, we can see that the underlying contract has very little chance of reaching $105. Consequently, the value of the $105 call will be low. If we assume a distribution similar to Figure 4-2, there is an increased probability of the underlying reaching $105, and this will increase the value of the $105 call. Finally, if we assume a distribution similar to

Figure 4-4: High Volatility Distribution

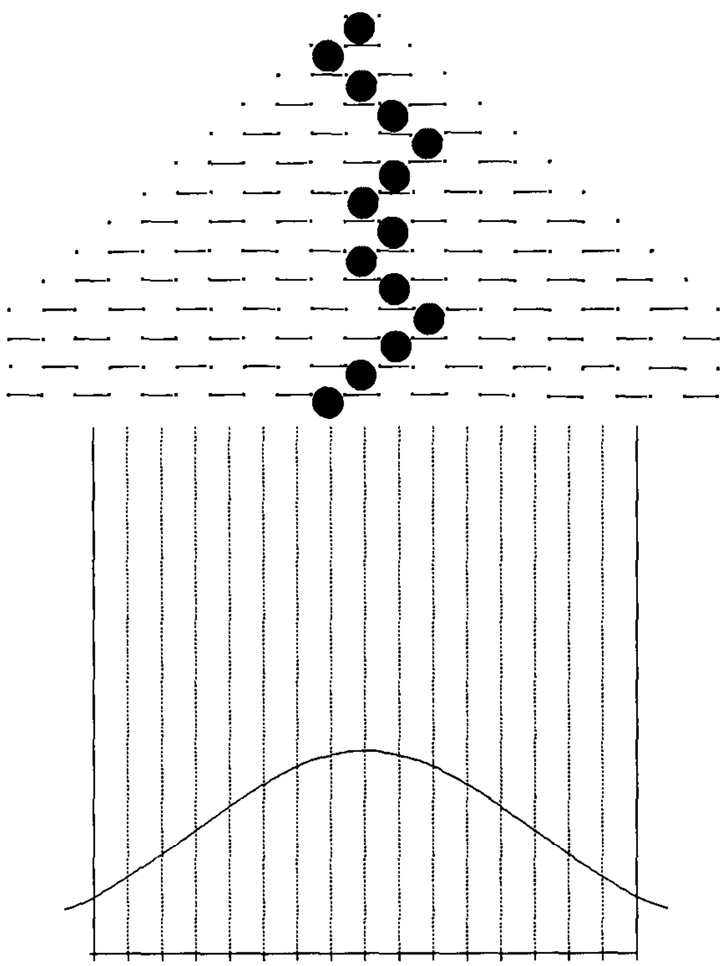
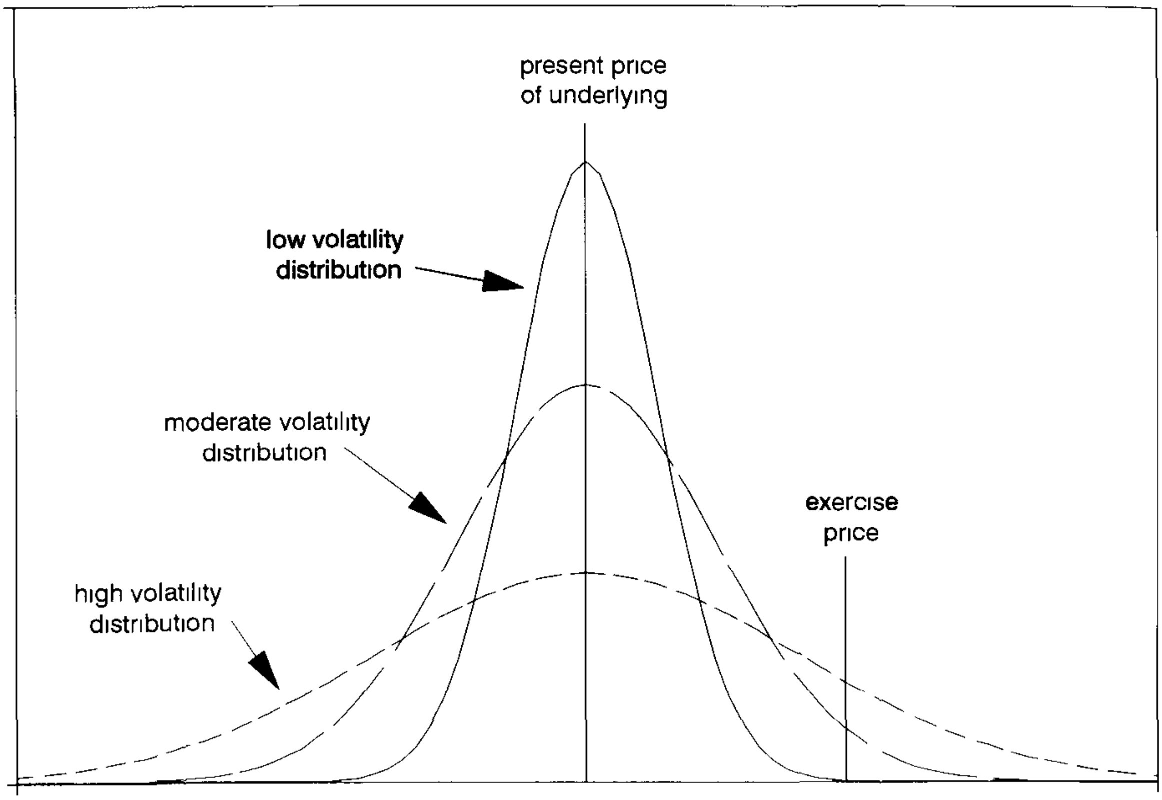


Figure 4-4, there is a very real likelihood that the $105 call could finish in-the-money. As a result, the value of the option will increase dramatically.

If we assume only that the price movement of an underlying contract follows a random walk, and nothing about the likely direction of movement, the curves in Figures 4-2, 4-3, and 4-4 might represent possible price distributions in a moderate-volatility, low-volatility, and high-volatility market, respectively. In a low-volatility market, price movement is severely restricted, and consequently options will command relatively low premiums. In a high-volatility market the chances for extreme price movement is greatly increased, and options will command high premiums.

Since the different price distributions in Figure 4-5 are symmetrical, it may seem that increased volatility should have no effect on an option's value. After all, increased volatility may increase the likelihood of large upward movement, but this should be offset by the equally greater likelihood of large downward movement. Here, however, there is an important distinction between an option position and an underlying position. Unlike an underlying contract, an option's potential loss is limited. No matter how far the market drops, a call option can only go to zero. In our example, whether the market finishes at $80 or $104 at expiration, the $105 call will be worthless. However, if we buy the underlying contract at $100, there is a tremendous difference between the

Figure 4-5: Price Distribution at Expiration



market finishing at $80 or $104. With an underlying contract all outcomes are important; with an option, only those outcomes which result in the option finishing in-the-money are important. In Figure 4-5, we are only concerned with price outcomes to the right of the exercise price. Everything else is zero.

This leads to an important distinction between evaluation of an underlying contract and evaluation of an option. If we assume that prices are distributed along a normal distribution curve, the value of an underlying contract depends on where the peak of the curve is located, while the value of an option depends on how fast the curve spreads out.

MEAN AND STANDARD DEVIATION

Suppose we want to use the concept of price movement based on normal distribution curves in a theoretical pricing model. To do this we need a method of describing the characteristics of the curve to the model. Since the model is based on mathematics, we need to describe the curve in numerical terms so that we can feed the numbers into the model.

Fortunately, a normal distribution curve can be fully described with two numbers, the *mean* and the *standard deviation.* If we know that a distribution is normal, and we also know these two numbers, then we know all the characteristics of the distribution.

Graphically, we can interpret the mean as the location of the peak of the curve, and the standard deviation as a measure of how fast the curve spreads out. Curves which spread out very quickly, such as Figure 4-4, have a high standard deviation; curves which spread out very slowly, such as Figure 4-3, have a low standard deviation.

While the mean is nothing more than the average outcome, and therefore a familiar concept for many traders, the standard deviation may not be quite so familiar. Indeed, one need not know how either of these numbers are calculated in order to successfully trade options. (For those who are interested, a more detailed discussion appears in Appendix B.) What is important to an option trader is the interpretation of these numbers, in particular what a mean and standard deviation suggest in terms of likely price movement.

Let's go back to Figure 4-2 and consider the troughs numbered 0 to 15 at the bottom. We suggested that these numbers might represent the number of heads resulting from 15 flips of a coin. Alternatively, they might also represent the number of times a ball went right at each nail as it dropped down through the maze. The first trough is assigned zero since any ball which ends there must have gone left at every nail. The last trough is assigned 15 since any ball which ends there must have gone right at every nail.

Suppose we are told that the mean and standard deviation in Figure 4-2 are 7.50 and 3.00, respectively. What does this tell us about the distribution? (The actual mean and standard deviation of 7.51 and 2.99 are calculated in Appendix B. Here, for simplicity, we will round to 7.50 and 3.00.) The mean tells us the average outcome. If we add up all the outcomes and divide by the number of occurrences, the result will be 7.50. In terms of the troughs, the average result will fall half-way between troughs 7 and 8. (Of course this is not an actual possibility. However, we noted in Chapter 3 that the average outcome does not have to be an actual possibility for any one outcome.)

The standard deviation not only describes how fast the distribution spreads out; it also tells us something about the likelihood of a ball ending up in a specific trough or group of troughs. In particular, the standard deviation tells us the probability of a ball ending up in a trough which is a specified distance from the mean. For example, we may want to know the likelihood of a ball falling down through the maze and ending up in a trough lower than 5 or higher than 10. We can answer the question by asking how many standard deviations the ball must move away from the mean, and then determine the probability associated with that number of standard deviations.

The exact probability associated with any specific number of standard deviations can be found in mathematical tables in most books on statistics. Alternatively, such probabilities can be closely approximated using an appropriate formula (see Appendix B). For option traders the following approximations will be useful:

± 1 standard deviation takes in approximately 68.3% (about $2/3$) of all occurrences

± 2 standard deviations takes in approximately 95.4% (about $19/20$) of all occurrences

± 3 standard deviations takes in approximately 99.7% (about $369/370$) of all occurrences

Note that each number of standard deviations is preceded by a plus or minus sign. Because normal distributions are symmetrical, the likelihood of up movement and down movement is identical.

Now let's try to answer our question about the likelihood of getting a ball in a trough lower than 5 or higher than 10. We can designate the divider between troughs 7 and 8 as the mean of 7½. If the standard deviation is 3, what troughs are within one standard deviation of the mean? One standard deviation from the mean is 7½ ± 3 = 4½ to 10½. Again interpreting ½ as the divider between troughs, we can see that troughs 5 through 10 fall within one standard deviation of the mean. We know that one standard deviation takes in about 2/3 of all occurrences, so we can conclude that out of every three balls we drop into the maze, two should end up in troughs 5 through 10. What is left over, one out of every three balls, will end up in one of the remaining troughs, 0-4 and 11-15. Hence, the answer to our original question about the likelihood of getting a ball in a trough lower than 5 or higher than 10 is about 1 chance in 3, or about 33%. (The exact answer is 100% – 68.3%, or 31.7%.) This is shown in Figure 4-6.

Let's try another calculation, but this time we can think of the problem as a wager. Suppose someone offers us 30 to 1 odds that we can't drop a ball into the maze and get it specifically in troughs 14 or 15. Is this bet worth making? One characteristic of standard deviations is that they are additive. In our example, if one standard deviation is 3, then two standard deviations are 6. Two standard deviations from the mean is therefore 7½ ± 6 = 1½ to 13½. We can see in Figure 4-6 that troughs 14 and 15 lie outside two standard deviations. Since the probability of getting a result within two standard deviations is approximately 19 out of 20, the probability of getting a result beyond two standard deviations is 1 chance in 20. Therefore 30 to 1 odds may seem very favorable. Recall, however, that beyond two standard deviations also includes troughs 0 and 1. Since normal distributions are symmetrical, the chances of getting a ball specifically in troughs 14 or 15 must be half of 1 chance in 20, or about 1 chance in 40. At 30 to 1 odds the bet must be a bad one since the odds do not sufficiently compensate us for the risk involved.

In Chapter 3 we said that one logical approach to option evaluation is to assign a probability to an infinite number of possible price outcomes for an underlying contract. Then, if we multiply each possible price outcome by its associated probability we can use the results to calculate an option's theoretical value. The problem is in dealing with an infinite number of price outcomes and probabilities, since an infinite number of anything is not easy to work with. Fortunately, the characteristics of normal distributions have been so closely studied that formulas have been developed which facilitate the computation of both the probabilities associated with every point along a normal distribution curve, as well as the area under various portions of the curve. If we assume that prices of an underlying instrument are normally distributed, these formulas represent a unique set of tools with which we can solve for an option's theoretical value. This is one of the reasons Black and Scholes adopted the normal distribution assumption as part of their model.

Figure 4-6

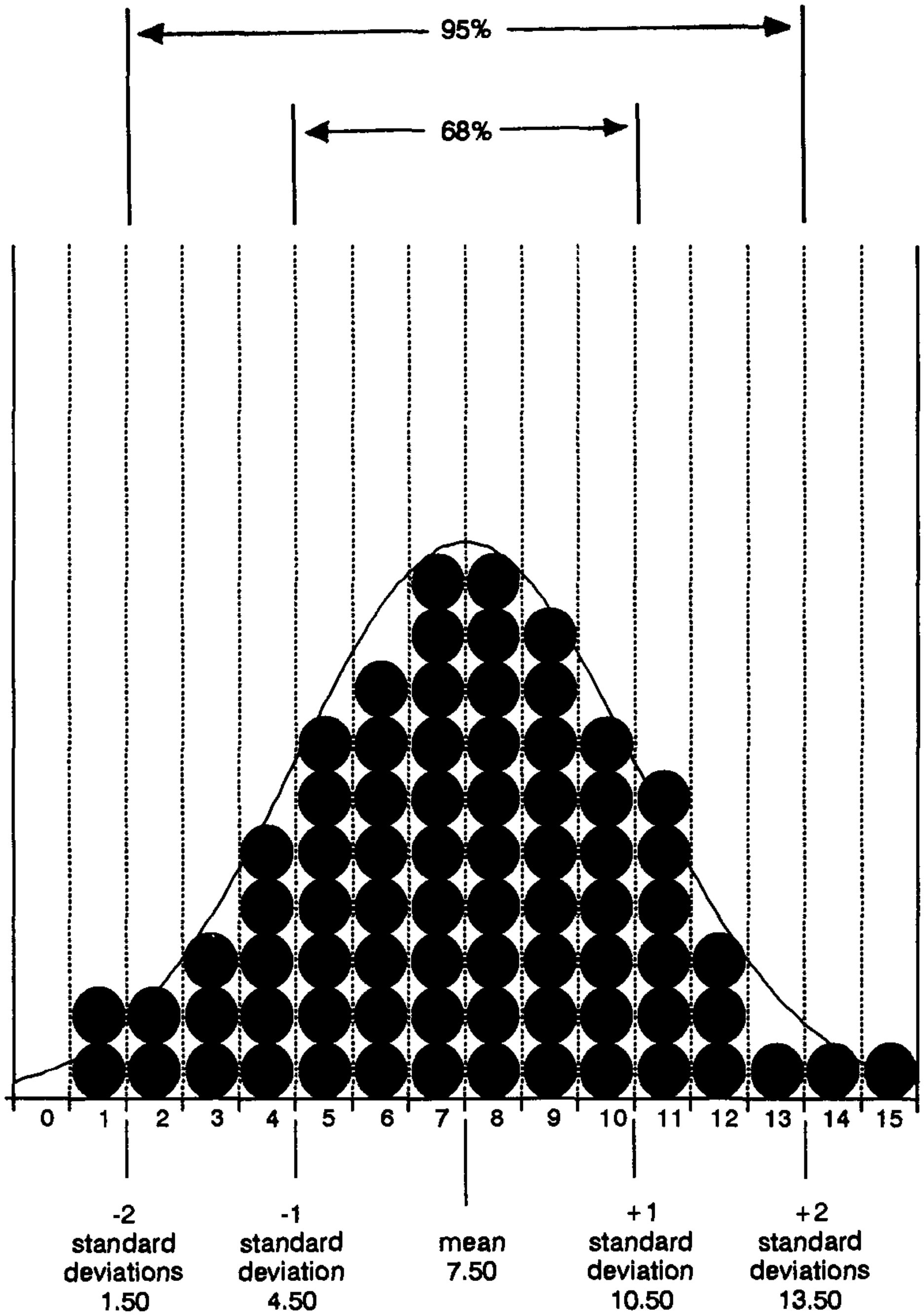
Mean = 7.50

Standard Deviation = 3.00

±1 standard deviation = 68.3% (2/3)

±2 standard deviations = 95.4% (19/20)

±3 standard deviations = 99.7% (369/370)



UNDERLYING PRICE AS THE MEAN OF A DISTRIBUTION

Now that we have decided to describe prices in terms of a normal distribution, how do we feed this distribution into a theoretical pricing model? Since all normal distributions can be described by a mean and the standard deviation, in some way we must feed these two numbers into our pricing model.

When we enter the present price of an underlying instrument we are actually entering the mean of a normal distribution curve. An important assumption in the Black-Scholes Model is that, in the long run, a trade in the underlying instrument will just break even. It will neither make money nor lose money. Given this assumption, the mean of the normal distribution curve assumed in the model must be the price at which a trade in the underlying instrument, either a purchase or a sale, would just break even. What is that price? The answer depends on the type of underlying instrument.

Suppose a trader purchases a futures contract at 100 and holds the position for three months. Where does the price of the futures contract have to be at the end of three months for the trader to break even? Since futures contracts entail no carrying costs, nor do they pay dividends, the break even price three months from now is exactly the original trade price of 100.

Now suppose that a trader purchases a $100 stock and holds it for three months. Where does the stock price have to be at the end of the holding period for the trader to break even? Since the purchase of stock requires immediate payment, the break even price will have to include the cost of carrying a $100 debit for three months. If interest rates are 8% annually the carrying cost on $100 for three months is $3/12 \times 8\% \times \$100 = \$2$. Therefore, the stock price must be $102 at the end of three months for the trade to break even. If the stock will pay a dividend of $1 during the holding period, then the stock price need only be $101 for the trade to break even.

Note that this is exactly how we calculated the forward price of a contract in Chapter 3. And indeed this is exactly the type of calculation built into the various forms of the Black-Scholes Model. When we enter an underlying price into the Black-Scholes model, based on the type of underlying instrument, interest rates, and dividends, the model calculates the forward price of the underlying instrument at expiration and makes this price the mean of a normal distribution curve.

VOLATILITY AS A STANDARD DEVIATION

In addition to the mean, we also need a standard deviation to fully describe a normal distribution curve. This is entered in the form of a volatility. With some slight modifications, which we will discuss shortly, we can define the volatility number associated with an underlying instrument as a one standard deviation price change, in percent, at the end of a one-year period.

For example, suppose that an underlying futures contract is currently trading at 100 and has a volatility of 20%. Since this represents a one standard deviation price change, one year from now we expect the same futures contract to be trading between 80 and 120 (100 ± 20%) approximately 68% of the time, between 60 and 140 (100 ± (2 × 20%))

approximately 95% of the time, and between 40 and 160 (100 ± (3 × 20%)) approximately 99.7% of the time.

If the underlying contract is a stock currently trading at $100, then the 20% volatility will have to be based on the forward price of the stock at the end of one year. If interest rates are 8% and the stock pays no dividends, the one-year forward price will be $108. Now a one standard deviation price change is 20% × $108 = $21.60. So one year from now we would expect the same stock to be trading between $86.40 and $129.60 ($108 ± $21.60) approximately 68% of the time, between $64.80 and $151.30 ($108 ± (2 × $21.60)) approximately 95% of the time, and between $43.20 and $172.90 ($108 ± (3 × $21.60)) approximately 99.7% of the time.

Suppose we come back at the end of one year and find our futures contract, which we thought had a volatility of 20%, trading at 35. Does this mean our volatility of 20% was wrong? A price change of more than three standard deviations may be unlikely, but one shouldn't confuse unlikely and impossible. Flipping a perfectly balanced coin 15 times may result in 15 heads, even though the odds against it are more than 32,000 to 1. If 20% was the right volatility, the odds of the futures price going from 100 to 35 one year later is more than 1,500 to 1. But one chance in 1,500 is not impossible, and perhaps this was the one time in 1,500 when the price would indeed end up at 35. Of course, it is also possible that we had the wrong volatility. But we wouldn't know this without looking at price changes of the futures contract over many years so that we have a representative price distribution.

LOGNORMAL DISTRIBUTIONS

Is it reasonable to assume that the prices of an underlying instrument are normally distributed? Beyond the question of the exact distribution of prices in the real world, the normal distribution assumption has one serious flaw. A normal distribution curve is symmetrical. Under a normal distribution assumption, for every possible upward move in the price of an underlying instrument there must be the possibility of a downward move of equal magnitude. If we allow for the possibility of a $50 instrument rising $75 to $125, we must also allow for the possibility of the instrument dropping $75 to a price of –$25. Since it is impossible for traditional stocks and commodities to take on negative prices, the normal distribution assumption is clearly flawed. What can we do about this?

Up to now we have defined volatility in terms of the percent changes in the price of an underlying instrument. In this sense an interest rate and volatility are similar in that they both represent a *rate of return*. The primary difference between interest and volatility is that interest generally accrues at a positive rate, while volatility represents a combination of positive and negative rates of return. If one invests money at a fixed interest rate, the value of the principal will always grow. But if one invests in an underlying instrument with a volatility other than zero, the instrument may go either up or down in price. Volatility, which is a standard deviation, says nothing about the direction of movement.

Since volatility represents a rate of return, an important consideration is the manner in which the rate of return is calculated. For example, suppose we were to invest $1,000 for one year at an annual interest rate of 12%. How much would we have at the end of one year? The answer depends on how the 12% interest on our investment is paid out.

Rate of Payment	Value after One Year	Total Yield
12% once a year	$1,120.00	12%
6% twice a year	$1,123.60	12.36%
3% every three months	$1,125.51	12.55%
1% every month	$1,126.83	12.68%
12%/52 every week	$1,127.34	12.73%
12%/365 every day	$1,127.47	12.75%
12% compounded continuously	$1,127.50	12.75%

As interest is paid more often, even though it is paid at the same rate of 12% per year, the total yield on the investment increases. The yield is greatest when interest is paid continuously. In this case it is just as if interest were paid at every possible moment in time.

Although less common, we can do the same type of calculation using a negative interest rate. For example, suppose we were to lose 12 percent annually on our $1,000 investment (interest rate = –12%). How much would we have at the end of a year? The answer depends on the frequency at which our losses accrue.

Rate of Loss	Value after One Year	Total Yield
–12% once a year	$880.00	–12%
–6% twice a year	$883.60	–11.64%
–3% every three months	$885.29	–11.47%
–1% every month	$886.38	–11.36%
–12%/52 every week	$886.80	–11.32%
–12%/365 every day	$886.90	–11.31%
–12% compounded continuously	$886.92	–11.31%

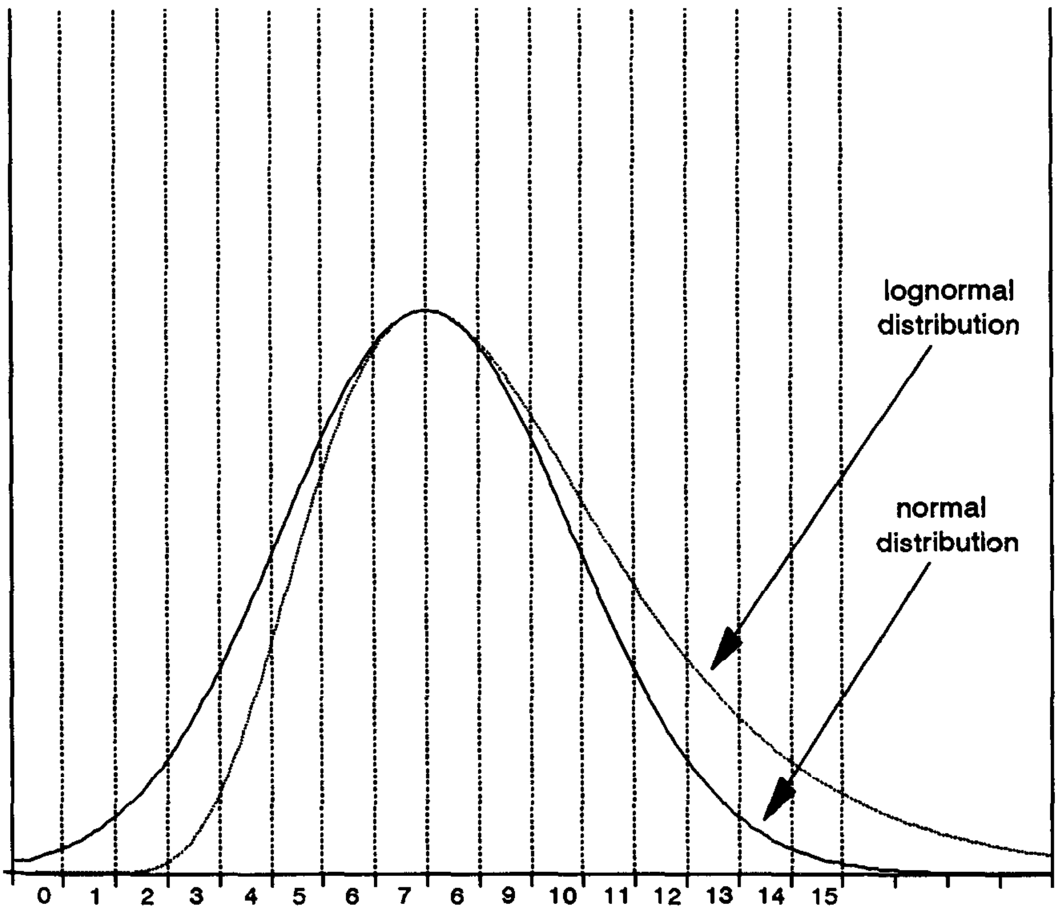
In the case of a negative interest rate, as losses are compounded more frequently, even though at the same rate of –12% per year, the loss becomes smaller, and so does the negative yield.

In the same way that interest can be compounded at different intervals, volatility can also be compounded at different intervals. For purposes of theoretical pricing of options, volatility is assumed to compound continuously, just as if the price changes in the underlying instrument, either up or down, were taking place continuously but at an annual rate corresponding to the volatility number associated with the underlying instrument.

What would happen if at every moment in time the price of an underlying could go up or down a given percent, and that these up and down movements were normally distributed? When price changes are assumed to be normally distributed, the continuous compounding of these price changes will cause the prices at maturity to be *lognormally* distributed. Such a distribution is skewed toward the upside because upside prices resulting from a positive rate of return will be greater, in absolute terms, than downside prices resulting from a negative rate of return (Figure 4-7). In our interest rate example, a continuously compounded rate of return of +12% yields a profit of $127.50 after one year, while a continuously compounded rate of return of −12% yields a loss of only $113.08. If the 12% were a volatility, then a one standard deviation upward price change at the end of one year would be +$127.50, while a one standard deviation downward price change would be −$113.08. Even though the rate of return was a constant 12%, the continuous compounding of the 12% yielded different upward and downward moves.

The Black-Scholes Model is a *continuous time* model. It assumes that the volatility of an underlying instrument is constant over the life of the option, but that this volatility is continuously compounded. These two assumptions mean that the possible prices of the underlying instrument at expiration of the option are lognormally distributed. It also explains why options with higher exercise prices carry more value than options with lower exercise prices, where both exercise prices appear to be an identical amount away from the price of the underlying instrument. For example, suppose a certain underlying contract is trading at exactly 100. If there are no interest considerations and we assume

Figure 4-7: Lognormal Distribution



a normal distribution of possible prices, then the 110 call and the 90 put, both being 10% out-of-the-money, ought to have identical theoretical values. But under the lognormal assumption in the Black-Scholes Model, the 110 call will always have a greater value than the 90 put. In absolute terms, the lognormal distribution assumption allows for greater upside price movement than downside price movement. Consequently, the 110 call will have a greater possibility of price appreciation than the 90 put.[1]

Finally, the lognormal assumption built into the Black-Scholes Model overcomes the logical problem we initially posed. If we were to allow for the possibility of unlimited upside price movement of an underlying instrument, a normal distribution assumption would force us to allow for unlimited downside movement. This would require us to accept the possibility of negative prices for the underlying instrument, clearly not a possibility for most optionable instruments. A lognormal distribution, however, does allow for open ended upside prices (the logarithm of $+\infty$ is $+\infty$), while bounding downside prices by zero (the logarithm of $-\infty$ is zero). This is a more realistic representation of how prices are actually distributed in the real world.

A more complete discussion of logarithmic price changes and probability calculations can be found in Appendix B.

We can now summarize the most important assumptions governing price movement in the Black-Scholes Model:

1. Changes in the price of an underlying instrument are random and cannot be artificially manipulated, nor is it possible to predict beforehand the direction in which prices will move.

2. The percent changes in the price of an underlying instrument are normally distributed.

3. Because the percent changes in the price of the underlying instrument are assumed to be continuously compounded, the prices of the underlying instrument at expiration will be lognormally distributed.

4. The mean of the lognormal distribution will be located at the forward price of the underlying contract.

The first of these assumptions may meet with resistance from some traders. Technical analysts believe that by looking at past price activity it is possible to predict the future direction of prices. One can chart support and resistance points, double tops and bottoms, head and shoulders, and many similar formations which are believed to predict future price trends. We leave debate on this question to others. The important point here is that the Black-Scholes Model makes the assumption that price changes are random and that their direction cannot be predicted. This does not mean that there is no predictive requirement in using the Black-Scholes Model. However, price prediction

1. Of course, this is only theory. There is no law that says the price of the 90 put in the marketplace cannot be greater than the price of the 110 call.

will focus on the magnitude of the price changes, rather than on the direction of changes.

As we shall see later, there is also good reason to question the third assumption, that prices are lognormally distributed at expiration. This may be a reasonable assumption for some markets, but a very poor assumption for other markets. Again, the important point here is for the trader who uses a theoretical pricing model to understand the assumptions on which the theoretical values are based. He can then make his own decision, based on his knowledge of a particular market, as to whether these assumptions, and hence the theoretical values generated by the model, are likely to be accurate.

DAILY AND WEEKLY STANDARD DEVIATIONS

As an annual standard deviation, we know what the volatility tells us about the likely price movement of a contract over a one-year period. However, this is a period of time longer than the life of most listed options. We might want to know what a volatility tells us about price changes over a shorter period of time, for example over a month, or a week, or a day.

An important characteristic of volatility is that it is proportional to the square root of time. As a result of this, we can approximate a volatility over some period of time shorter than a year by dividing the annual volatility by the square root of the number of trading periods in a year.

Suppose we are interested in a daily volatility. While it would take a logarithmic calculation to give us an exact daily volatility, if we ignore the relatively minor effect of continuous compounding over such a short period of time, it is possible to make an estimate of daily volatility. First we must determine the number of daily trading periods in a year. That is, if we look at prices at the end of every day, how many times a year can prices change? If we restrict ourselves to exchange traded options, even though there are 365 days in a year, prices cannot really change on weekends or holidays. This leaves us with about 256 trading days during the year.[2] Since the square root of 256 is 16, to approximate a daily volatility we can divide the annual volatility by 16.

Going back to our futures contract trading at 100 with a volatility of 20%, what is a one standard deviation price change over a day's time? 20%/16 = 1¼%, so a one standard deviation daily price change is 1¼% × 100 = 1.25. We expect to see a price change of 1.25 or less approximately two trading days out of every three, and a price change of 2.50 or less approximately 19 trading days out of every 20. Only one day in 20 would we expect to see a price change of more than 2.50.

We can do the same type of calculation for a weekly standard deviation. Now we must ask how many times per year prices can change if we look at prices once a week. Unlike trading days, we don't have "holiday" weeks, so we must make our calculations using all 52 trading weeks in a year. Dividing our annual volatility of 20% by the square root of 52, or approximately 7.2, we get 20%/7.2 ≈ 2¾. For our futures contract

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2. Depending on holidays, the number of trading days is usually somewhere between 250 and 255. We use 256 as a reasonable approximation since its square root is a whole number and therefore easier to work with.

trading at 100, we would expect to see a price change of 2.75 or less two weeks out of every three, a price change of 5.50 or less 19 weeks out of every 20, and only one week in twenty would we expect to see a price change of more than 5.50.

Since we assume that the price of a stock will appreciate by the carrying cost, it may seem that we cannot use the same method (divide by 16 for daily volatility; divide by 7.2 for weekly volatility) to approximate expected movement in an underlying stock. However, over a short period of time, the carrying cost component, like the effect of continuous compounding of volatility, will be relatively small. Therefore, we can use the same method as a reasonable estimate of daily and weekly volatility. For example, suppose a stock is trading at $45 per share and has an annual volatility of 28%. What is an approximate one standard deviation price change over a day's time and over a week's time?

For a daily volatility we calculate:

$$28\%/16 \times \$45 = 1.75\% \times \$45 = \$.79$$

For a weekly volatility we calculate:

$$28\%/7.2 \times \$45 = 3.89\% \times \$45 = \$1.75$$

We expect to see a price change of approximately ¾ point or less two days out of every three, 1½ points or less 19 days out of every 20, and only one day in 20 would we expect to see a price change of more than 1½ points. On a weekly basis, we would expect to see a price change of 1¾ points or less two weeks out of every three, a price change of 3½ points or less 19 weeks out of every 20, and only one week in 20 would we expect to see a price change of more than 3½ points.

We have used the phrase "price change" in conjunction with our volatility estimates. Exactly what do we mean by this? Do we mean the high/low during some period? Do we mean open to close price changes? Or is there another interpretation? While various methods have been suggested to estimate volatility,[3] the traditional method has been to calculate volatility based on settlement-to-settlement price changes. Using this approach, when we say a one standard deviation daily price change is ¾ point, we mean a ¾ point price change from one day's settlement price to the next day's settlement price. The high/low or open/close price change may have been either more or less than ¾ point, but it is the settlement-to-settlement price change on which we focus.

3. See:
Parkinson, Michael, "The Extreme Value Method of Estimating the Variance of the Rate of Return," *Journal of Business*, 1980, vol. 53, no. 1, pp. 61–64.
Garman, Mark B. and Klass, Michael J., "On the Estimation of Security Price Volatilities from Historical Data," *Journal of Business*, 1980, vol. 53, no. 1, pp. 67–78.
Beckers, Stan, "Variance of Security Price Returns Based on High, Low, and Closing Prices," *Journal of Business*, 1983, vol. 56, no. 1, pp. 97–112.

VOLATILITY AND OBSERVED PRICE CHANGES

Why is it important for a trader to be able to estimate daily or weekly price changes from an annual volatility? Volatility is the one input into a theoretical pricing model which cannot be directly observed. Yet many option strategies, if they are to be successful, require an accurate assessment of volatility. Therefore, an option trader needs some method of determining whether his expectations about volatility are indeed being realized in the marketplace. Unlike directional strategies, whose success or failure can be immediately observed from posted prices, there is no posting of volatilities. A trader must determine for himself whether he is using a reasonable volatility input into the theoretical pricing model.

For example, suppose a certain underlying contract is trading at 40 and a trader is using a 30% volatility for theoretical evaluation. A one standard deviation daily price change is approximately $30\%/16 \times 40 = .75$. Over five days of trading a trader notes the following five settlement-to-settlement price changes:

$$+.43, \ -.06, \ -.61, \ +.50, \ -.28$$

Are these five price changes consistent with a 30% volatility?

The trader expects to see a price change of more than .75 (one standard deviation) about one day in three, or between one and two times over a five-day period. Yet during this five-day period he did not see a price change of this magnitude even once. What conclusions can be drawn from this?[4] One thing is certain: these five price changes are not consistent with 30% volatility. The trader might explain the discrepancy in one of two ways. On the one hand, perhaps this was expected to be an abnormally quiet week (perhaps it was a holiday week); and next week when trading returns to normal the market will go right back to making moves which are more consistent with a 30% volatility. If the trader comes to this conclusion, perhaps he ought to continue to use a 30% volatility for his calculations. On the other hand, perhaps there is no apparent reason for the market being less volatile than predicted by a 30% volatility. He may simply be using the wrong volatility. If the trader comes to this conclusion, perhaps he ought to consider using a new volatility input which is more consistent with the observed price changes. If he continues to use a 30% volatility in the face of price changes which are significantly less than predicted by that number, he will be assigning the wrong probabilities to the possible price outcomes for the underlying contract. Consequently, he will generate incorrect theoretical values, defeating the purpose of using a theoretical pricing model in the first place.

Exactly what volatility is associated with the five price changes in the foregoing example? Without some rather involved calculations it is difficult to say. (The answer is actually 18.8%.) However, if a trader has some idea beforehand of what price changes he expects, he can easily see that the changes over the five-day period are not consistent with a 30% volatility.

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4. Five days is admittedly a very small sample from which to draw a meaningful conclusion about volatility. The method and reasoning, however, are still valid.

Let's look at another example. Now the underlying contract is trading at 332½ and a trader notes the following five daily price changes:

$$-5, \ +2\tfrac{1}{2}, \ +1, \ -7\tfrac{3}{4}, \ -4\tfrac{1}{4}$$

Are these price changes consistent with an 18% volatility? At 18% a one standard deviation price change is approximately 18%/16 × 332½ = 3¾. Over five days we expect to see a price change of more than 3¾ between one and two times. Yet here we have a price change of more than 3¾ three days out of five. And once the price change was 7¾ (more than two standard deviations) which we expect to see only one day in twenty. Again, unless the trader believes that the five price changes occurred during an extraordinary week, then perhaps he ought to consider changing his volatility figure so that it is more consistent with the observed price changes.

A NOTE ON INTEREST RATE PRODUCTS

Suppose Eurodollars are at 93.00 and we assume a volatility of 16 percent. We can apply the method previously described to calculate an approximate one standard deviation daily price change: 16%/16 × 93.00 = .93. As any trader familiar with the Eurodollar market will attest, a daily price change of .93 is wildly unlikely. How can we account for this seemingly illogical answer? One might conclude that we simply have the wrong volatility, and some lower number is more accurate. In fact, the 16% volatility is not at all unusual for Eurodollars, so the explanation must lie elsewhere.

Eurodollars, like many other interest rate contracts (U.S. Treasury Bills, Short Sterling, Euromarks, Euroyen) are indexed from 100. This means that the interest rate associated with a Eurodollar contract is 100 less the value of the contract. It also means that, barring the unlikely advent of negative interest rates, the contract cannot take on a value greater than 100. In this respect, 100 acts as a limiting value for Eurodollars in the same way that zero acts as a limiting value for traditional underlying contracts such as stocks and commodities. We can integrate this characteristic into our calculations by assuming that the value of a Eurodollar contract is actually 100 less its listed price. If the listed price is 93.00, for theoretical evaluation purposes we must use a value of 100 – 93.00, or 7.00, in our pricing model. If we define the value of the contract as 7.00, a one standard deviation price change is 16%/16 × 7.00 = .07. This is certainly a more realistic result than .93.

To be consistent, if we index Eurodollar prices from 100 we must also index exercise prices from 100. Therefore, a 93.50 exercise price in our pricing model is really a 6.50 (100 – 93.50) exercise price. This also requires us to reverse the type of option, changing calls to puts and puts to calls. To see why, consider a 93.50 call. For this call to go into-the-money, the underlying contract must rise above 93.50. But this requires that interest rates fall below 6.50 percent. Therefore, a 93.50 call in listed terms is the same as a 6.50 put in interest rate terms. A model which is correctly set up to evaluate options on Eurodollar or other types of indexed interest rate contracts automatically makes this transformation. The price of the underlying contract and the exercise price are subtracted from 100, while listed calls are treated as puts and listed puts are treated as calls.

Note that this type of transformation is not required for most bonds and notes. Depending on the coupon rate, the prices of these products may range freely without upper limit, often exceeding 100. They are therefore most often evaluated using a traditional pricing model, although interest rate products present other problems that may require specialized pricing models.

It is possible to take an instrument such as a bond and calculate the current yield based on its price in the marketplace. If we were to take a series of prices and from these calculate a series of yields, we could also calculate the *yield* volatility, i.e., the volatility based on the change in yield. We might then use this number to evaluate the theoretical value of an option on the bond, although to be consistent we would also have to specify the exercise price in terms of yield. Because it is possibie to calculate the volatility of an interest rate product using these two different methods, interest rate traders sometimes refer to yield volatility (the volatility calculated from the current yield on the instrument) versus price volatility (the volatility calculated from the price of the instrument in the marketplace).

TYPES OF VOLATILITIES

When traders discuss volatility, even experienced traders may find that they are not always talking about the same thing. When a trader makes the comment that the volatility of XYZ is 25%, this statement may take on a variety of meanings. We can avoid confusion in subsequent discussions if we first define the various ways in which traders interpret volatility.

Future Volatility

Future volatility is what every trader would like to know, the volatility that best describes the future distribution of prices for an underlying contract. In theory it is this number to which we are referring when we speak of the volatility input into a theoretical pricing model. If a trader knows the future volatility, he knows the right "odds." When he feeds this number into a theoretical pricing model, he can generate accurate theoretical values because he has the right probabilities. Like the casino, he may occasionally lose because of short-term bad luck. But in the long run, with the odds on his side, a trader can be fairly certain of making a profit.

Of course, traders rarely talk about the future volatility since it is impossible to know what the future holds.

Historical Volatility

Even though one cannot know the future, if a trader intends to use a theoretical pricing model he must try to make an intelligent guess about the future volatility. In option evaluation, as in other disciplines, a good starting point is historical data. What typically has been the volatility of this contract over some period in the past? If, over the past ten years the volatility of a contract has never been less than 10% nor greater than 30%, a guess for the future volatility of either 5% or 40% hardly makes sense. This does not mean that either of these extremes is impossible (in option trading the impossible always seems to happen sooner or later), but based on past performance, and in the absence of any extraordinary circumstances, a guess within the historical limits of 10% and 30% is probably more realistic than a guess outside these limits. Of course, 10% to 30% is still a huge range, but at least the historical data offers a starting point. Additional information may further narrow the estimate.

Note that there are a variety of ways to calculate historical volatility, but most methods depend on choosing two parameters, the historical period over which the volatility is to be calculated, and the time interval between successive price changes. The historical period may be ten days, six months, five years, or any period the trader chooses. Longer periods tend to yield an average or characteristic volatility, while shorter periods may reveal unusual extremes in volatility. To become fully familiar with the volatility characteristics of a contract, a trader may have to examine a wide variety of historical time periods.

Next, the trader must decide what intervals to use between price changes. Should he use daily price changes? Weekly changes? Monthly changes? Or perhaps he ought to consider some unusual interval, perhaps every other day, or every week and a half. Surprisingly, the interval which is chosen does not seem to greatly affect the result. Although a contract may make large daily moves, yet finish a week unchanged, this is by far the exception. A contract which is volatile from day to day is likely to be equally volatile from week to week, or month to month. This is typified by the graphs in Figure 4-8. The data points on the three graphs represent the volatility of the S&P 500 index over successive 50-day periods. For the solid line daily price changes were used, for the dotted line price changes every two days were used, and for the broken line price changes every 5 days were used. Even though the graphs occasionally diverge, for the most part they exhibit the same general volatility levels and trends.

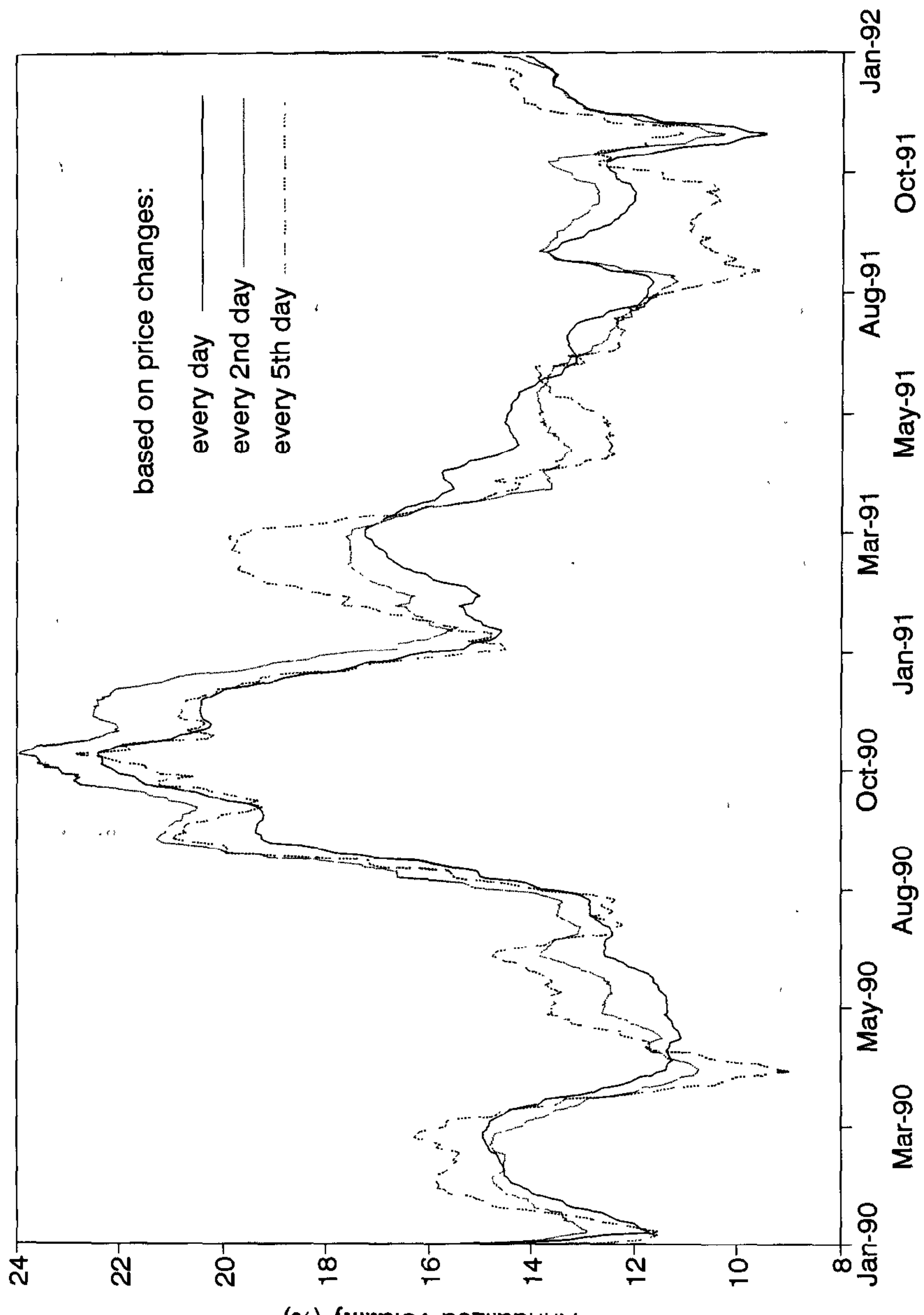
As a general rule, services which supply historical volatility data base their calculations on daily settlement-to-settlement price changes. If this is not the case, an explanation of how the volatility was calculated will usually accompany the data. If, for example, a service gave the volatility of a contract for the month of August as 21.6%, it can be assumed that the calculations were made using the daily settlement-to-settlement price changes for all the business days during that month.

Historical and future volatility are sometimes referred to as *realized* volatility.

Forecast Volatility

Just as there are services which will attempt to forecast future directional moves in the price of a contract, there are also services which will attempt to forecast the future volatility of a contract. Forecasts may be for any period, but most commonly cover periods identical to the remaining life of options on the underlying contract. For an underlying contract with three months between expirations, a service might forecast volatilities for the next three, six, and nine months. For an underlying with monthly expirations, a service might forecast volatilities for the next one, two, and three months. Due to the relatively recent introduction of options, volatility forecasting is still in its infancy, and must be considered an inexact science at best. Nonetheless, a trader's guess

Figure 4-8: S&P 500 Index 50-Day Historical Volatility



about the future volatility of a contract might very well take into consideration any volatility forecast to which he has access.

Implied Volatility

Generally speaking, future, historical, and forecast volatility are associated with an underlying contract. We can talk about the future volatility of the S&P 500 index, or the historical volatility of U.S. Treasury Bonds, or a forecast volatility for IBM stock. In each case we are referring to the volatility of the underlying contract. There is, however, a different interpretation of volatility which is associated with an option rather than with the underlying contract.

Suppose a certain futures contract is trading at 98.50 with interest rates at 8%. Suppose also that a 105 call with three months to expiration is available on this contract, and that our best guess about the volatility over the next three months is 16%. If we want to know the theoretical value of the 105 call we might feed all these inputs into a theoretical pricing model. Using the Black-Scholes Model, we find that the option has a theoretical value of .96. Having done this we might compare the option's theoretical value to its price in the marketplace. To our surprise, we find that the option is trading for 1.34. How can we account for the fact that we think the option is worth .96, while the marketplace seems to believe it is worth 1.34?

One way to answer the question is to assume that everyone in the marketplace is using the same theoretical pricing model that we are, in this case the Black-Scholes Model. If we make this assumption, then the discrepancy between our value of .96 and the marketplace's value of 1.34 must be due to a difference of opinion concerning one or more of the inputs into the model. We can therefore start going down the list of inputs and try to identify the culprits.

We know that it can't be either the amount of time to expiration or the exercise price, since these inputs are fixed in the option contract. What about the underlying price of 98.50? Perhaps we think the price of the underlying is at 98.50, but it is really trading at some higher price, say 99.00. Indeed, in such circumstances it is always wise to double check the inputs. But suppose we still find that the underlying is at 98.50. Even given that there is a spread between the bid and ask price, if the market is reasonably liquid it is unlikely that the spread would be wide enough to cause a discrepancy of .38 in the value of the option. Perhaps our problem is the interest rate of 8%. But, as we noted in the last chapter, the interest rate component is usually the least important of the inputs into a theoretical pricing model. And in the case of futures options, the interest rate component is often trivial. This leaves us with only one likely culprit, and that is volatility. In some sense, the marketplace must be using a volatility other than 16% to evaluate the 105 call.

What volatility is the marketplace using? To find out, we can ask the following question: If we hold all other inputs constant (time to expiration, exercise price, underlying price, interest rates), what volatility must we feed into our theoretical pricing model to yield a theoretical value identical to the price of the option in the marketplace? In our example, we want to know what volatility will yield a value of 1.34 for the 105 call. Clearly the volatility has to be higher than 16%, so we might sit down with a computer programmed with the Black-Scholes Model and start to raise the volatility. If we do, we will find that at a volatility of 18.5% the 105 call has a theoretical value of 1.34. We refer to this volatility as the *implied volatility* of the 105 call. It is the volatility we must feed into our theoretical pricing model to yield a theoretical value identical to the price of the option in the marketplace. We can also think of it as the volatility being implied to the underlying contract through the pricing of the option in the marketplace.

When we solve for the implied volatility of an option we are assuming that the theoretical value (the option's price) is known, but that the volatility is unknown. In effect, we are running the theoretical pricing model backwards to solve for this unknown, as shown in Figure 4-9. In fact, this is easier said than done since most theoretical pricing models cannot be reversed. However, a number of computer programs have been written which can quickly solve for the implied volatility when all other inputs are known.

Note that the implied volatility depends on the theoretical pricing model being used. For some options, a different model can yield a significantly different implied volatility. The accuracy of an implied volatility also depends on the accuracy of inputs into the model. This not only includes the price of the option, but the other inputs as well. In particular, problems can occur when an option has not traded for some time, but market conditions have changed significantly. Suppose in our example the price of 1.34 for the 105 call reflected the last trade, but that trade took place two hours ago when the underlying futures contract was actually at 99.25. If the price of the underlying contract is 99.25, the implied volatility of the option at a price of 1.34 is actually 17.3%. This is a significant difference, and underscores the importance of accurate and timely inputs when calculating implied volatilities.

Services which supply theoretical analysis of options usually include implied volatilities. The implied volatilities may be for each option on an underlying contract, or there may be one implied volatility which is representative of all options on the same

Figure 4-9

Using the Model to Solve for a Theoretical Value

Knowns	**Unknowns**
Exercise Price (105)	Theoretical Value
Time to Expiration (3 months)	(?? = .96)
Underlying Futures Price (98.50)	
Interest Rates (8%)	
Volatility (16%)	

Using the Model to Solve for an Implied Volatility

Knowns	**Unknowns**
Exercise Price (105)	Implied Volatility
Time to Expiration (3 months)	(?? = 18.5%)
Underlying Futures Price (98.50)	
Interest Rates (8%)	
Option Price (1.34)	

underlying. In the latter case, the figure will usually represent an average of all the individual implied volatilities. The single implied volatility is weighted according to some criteria, such as volume of options traded, open interest, or, as is most common, by assigning the greatest weight to the at-the-money options.

The implied volatility in the marketplace is constantly changing because option prices, as well as other market conditions, are constantly changing. It is as if the marketplace were continuously polling all participants to come up with a consensus volatility for the underlying contract. This is not a poll in the true sense, since all traders do not huddle together and eventually vote on the correct volatility. However, as bids and offers are made, the trade price of an option will represent the equilibrium between supply and demand. This equilibrium can be translated into an implied volatility.

Even though the term premium really refers to an option's price, it is common among traders to refer to the implied volatility as the *premium* or *premium level*. If the current implied volatility is high by historical standards, or high relative to the recent historical volatility of the underlying contract, a trader might say that premium levels are high; if implied volatility is unusually low, he might say that premium levels are low.

Assuming a trader had a reliable theoretical pricing model, if he could look into a crystal ball and determine the future volatility of an underlying contract he would be able to accurately evaluate options on that contract. He might then look at the difference between each option's theoretical value and its price in the marketplace, selling any options which were overpriced relative to the theoretical value, and buying any options which were underpriced. If given the choice between selling one of two overpriced options, he might simply sell the one which was most overpriced in total dollars. However, a trader who has access to implied volatilities might use a different yardstick for comparison. He might compare the implied volatility of an option to either a volatility forecast, or to the implied volatility of other options on the same underlying contract. Going back to our example of the 105 call, we might say that with a theoretical value of .96 and a price of 1.34, the 105 call is .38 overpriced. But in volatility terms it is 2.5% overpriced since its theoretical value is based on a volatility of 16% (the trader's volatility estimate) while its price is based on a volatility of 18.5% (the implied volatility). Due to the unusual characteristics of options, it is often more useful for the serious trader to consider an option's price in terms of implied volatility rather than in terms of its total dollar price.

For example, suppose a Treasury Bond 98 call is trading for 3-32 ($3,500) with a corresponding implied volatility of 10.5%. Suppose also that a 102 call with the same expiration date is trading for 1-16 ($1,250) with an implied volatility of 11.5%. In total dollar terms the 102 call is $2,250 cheaper than the 98 call. Yet an experienced trader will probably conclude that in theoretical terms the 98 call is actually less expensive than the 102 call because the implied volatility of the 98 call is a full percentage point less than the implied volatility of the 102 call. Does this mean that one ought to buy the 98 call and sell the 102 call? Not necessarily. If the future volatility of the Treasury Bonds turns out to be 8%, then both options are overpriced; while if volatility turns out to be 14%, both options are underpriced. Moreover, the leverage values of the options may not be the same so that their sensitivity to changes in market conditions may, under some circumstances, make several 102 calls a more desirable purchase than one 98 call. If we ignore these considerations, in relative terms the 98 call is a better value because its implied volatility is lower.

While option traders may at times refer to any of the four types of volatilities, two of these stand out in importance, the future volatility and the implied volatility. The future volatility of an underlying contract determines the *value* of options on that contract. The implied volatility is a reflection of each option's *price*. These two numbers, value and price, are what all traders, not just option traders, are concerned with. If a contract has a high value and a low price, then a trader will want to be a buyer. If a contract has a low value and a high price, then a trader will want to be a seller. For an option trader this usually means comparing the future volatility with the implied volatility. If implied volatility is low with respect to the expected future volatility, a trader will prefer to buy options; if implied volatility is high, a trader will prefer to sell options. Of course, future volatility is an unknown, so we tend to look at the historical and forecast volatilities to help us make an intelligent guess about the future. But in the final analysis, it is the future volatility which determines an option's value.

To help the new trader understand the various types of volatility, consider the following analogy to weather forecasting. Suppose a trader living in Chicago gets up on a July morning and must decide what clothes to wear that day. Do you think he will consider putting on a parka? This is not a logical choice because he knows that *historically* it is not sufficiently cold in Chicago in July to warrant wearing a winter coat. Next, he might turn on the radio or television to listen to the weather *forecast*. The forecaster is predicting clear skies with temperatures around 90° (32°C). Based on this information, our trader has reached a decision: he will wear a short sleeve shirt with no sweater or jacket, and he certainly won't need an umbrella. However, just to be sure, he decides to take a look out the window to see what the people outside are wearing. To his surprise, everyone is wearing a coat and carrying an umbrella. The people outside, through their clothing, are *implying* completely different weather. What clothes should the trader then wear? He must make some decision, but whom should he believe, the weather forecaster or the people in the street? There can be no certain answer because the trader will not know the *future* weather until the end of the day.

Much depends on the trader's knowledge of local conditions. Perhaps the trader lives in an area far removed from where the weather forecaster is located. Then he must give added weight to local conditions. On the other hand, perhaps the people in the street all listened to a weather forecaster who has a history of playing practical jokes.

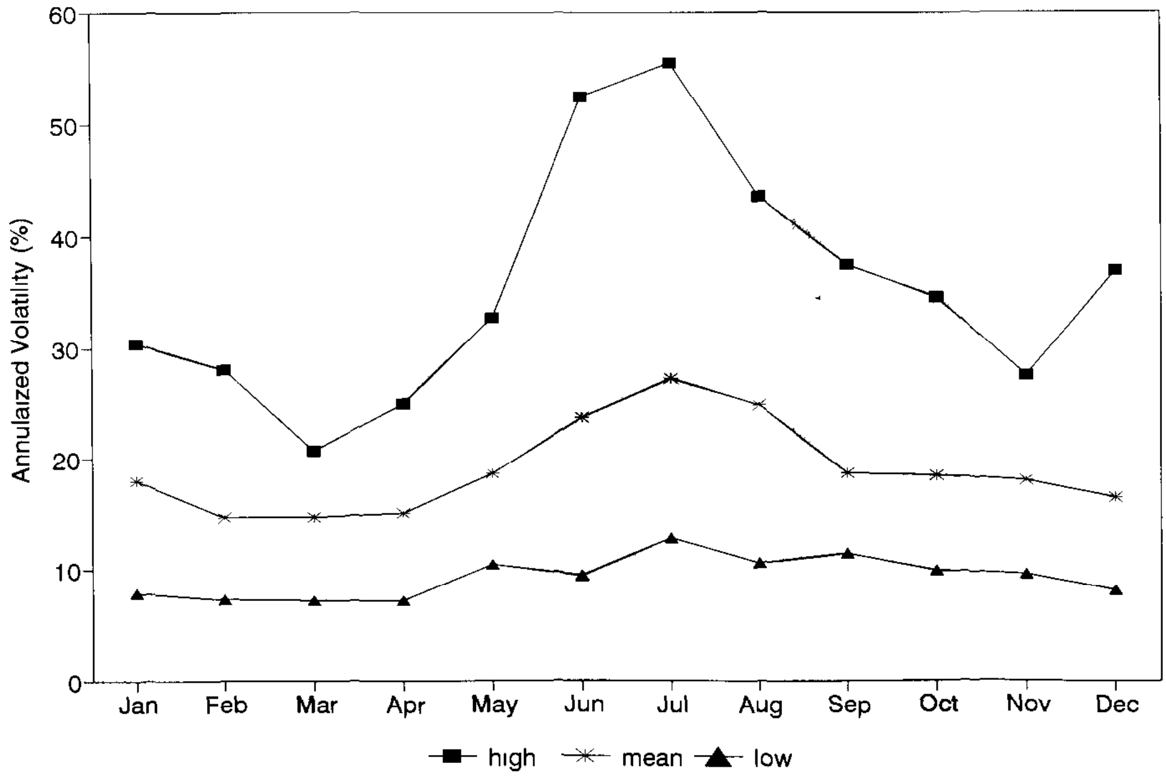
The decision on what clothes to wear, just like every trading decision, depends on a great many factors. Not only must the decision be made on the basis of the best available information, but the decision must also be made with consideration for the possibility of error. What are the benefits of being right? What are the consequences of being wrong? If a trader fails to take an umbrella and it rains, that may be of little consequence if the bus picks him up right outside his residence and drops him off right outside his place of work. On the other hand, if he must walk several blocks in the rain, he might catch the flu and be away from work for a week. The choices are never easy, and one can only hope to make the decision that will turn out best in the long run.

Seasonal Volatility

There is one other type of volatility with which a commodity trader may have to deal. Certain agricultural commodities, such as corn, soybeans, and wheat, are very sensitive to volatility factors arising from severe seasonal weather conditions. Such conditions occur especially in the summer months when drought can destroy major portions of a crop and cause prices to fluctuate wildly. For this reason, grains show a significant increase in volatility during the months of June, July, and August. Conversely, they show a significant decrease during the early spring months, before American planting has begun but after the South American crop has been harvested. Given these factors, one must automatically assign a higher volatility to an option contract which extends through the summer months. If, in February, a trader has assigned a volatility of 18% to a May soybean contract, he will certainly choose some higher volatility, perhaps 22%, for a November contract. He knows that the November contract includes the summer months, while the May contract does not. The effect of seasonal volatility on soybeans is shown in Figure 4-10.

A trader who is new to options might initially question whether volatility is really that important. He has probably been pursuing directional strategies where volatility was not a consideration. It is also possible to pursue a variety of directional strategies in the option market. But if a trader has a thorough understanding of volatility, he has an additional variable with which to work. He can, in effect, approach the market from two directions instead of one. Many traders find it easier to work exclusively with

Figure 4-10: Monthly Soybean Volatility (1980–1992)



Courtesy of the Chicago Board of Trade.

volatility, rather than try to guess market direction. Moreover, volatility strategies can be extremely profitable and, when chosen intelligently, can even reduce a trader's risk exposure. These two variables, market direction and volatility, enable an option trader to pursue many strategies not available to the pure stock or futures trader.

Changing our assumptions about future volatility can have a dramatic effect on the value of options. To see this, look at the prices, theoretical values, and implied volatilities for ten-week gold options in Figure 4-11. Note the change in theoretical values as volatility is increased from 11% to 14% to 17%. The 360 call and put, which are essentially at-the-money, change by approximately 1.85 ($185) for each three-percentage-point increase in volatility. While out-of-the-money options do not show as great a dollar change in value, in percent terms their sensitivity to a change in volatility is even greater. As volatility increases from 11% to 14%, the 390 call and 330 put more than double in value, and double again as volatility increases from 14% to 17%. A three-percentage-point change in volatility over ten weeks is not at all uncommon. Indeed, the volatility of gold can show swings of six or seven percentage points in a relatively short period of time. This is evident from the historical volatility of gold shown in Figure 4-12.

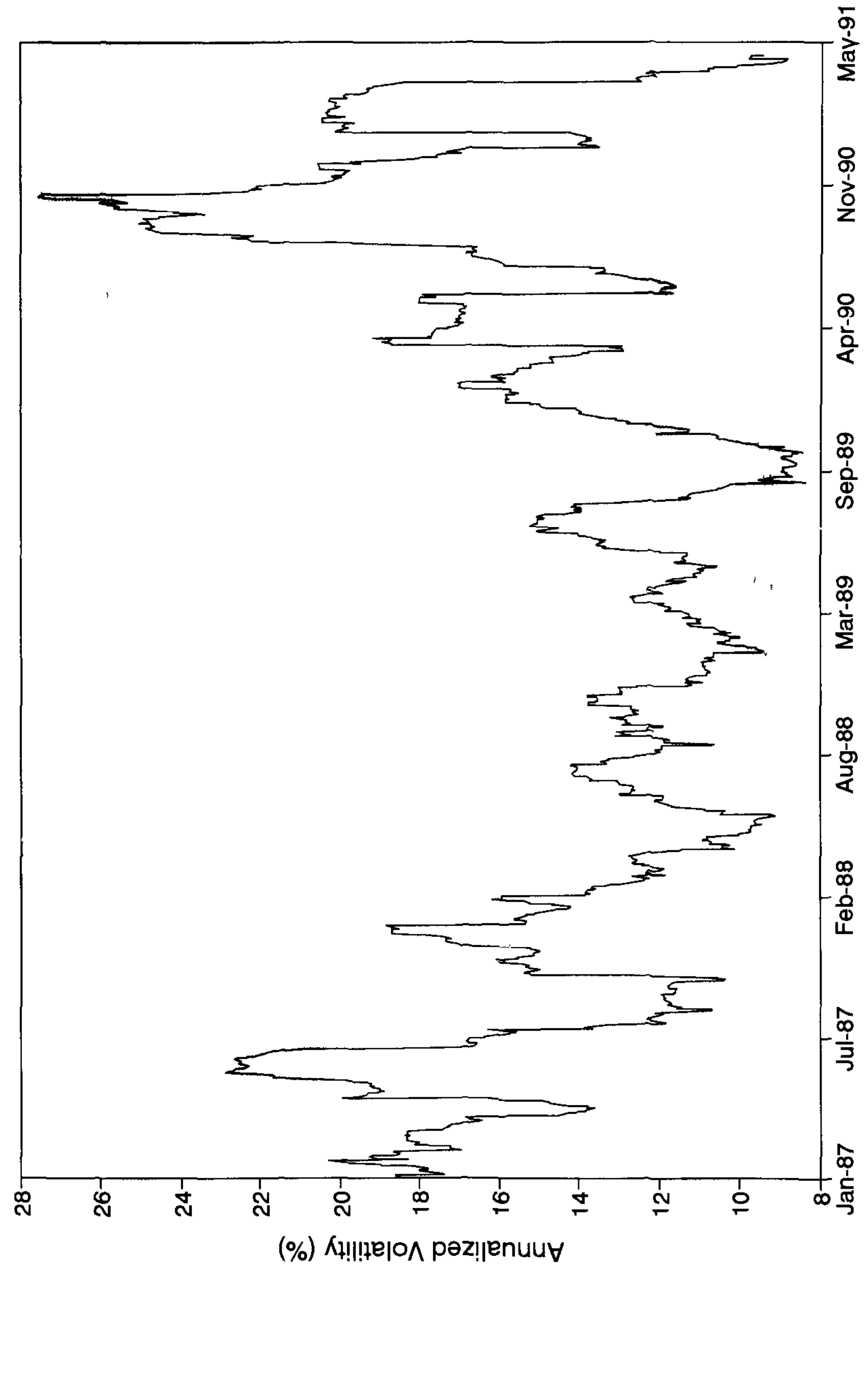
Given its importance, it is not surprising that the serious option trader spends a considerable amount of time thinking about volatility. Using his knowledge of historical, forecast, implied, and, in the case of agricultural commodities, seasonal volatility, he

Figure 4-11

3 May 1991
August Gold Futures at 358.30
Time to expiration = 10 weeks
Interest rate = 5.50%

Exercise Price	Settlement Price	Implied Volatility	Theoretical Value If . . .		
			Vol. = 11%	Vol. = 14%	Vol. = 17%
Calls					
330	29.20	15.36	28.27	28.84	29.69
340	20.60	14.48	19.26	20.40	21.73
350	13.20	13.86	11.61	13.28	15.01
360	7.80	13.89	6.02	7.87	9.73
370	4.20	14.00	2.62	4.20	5.88
380	2.40	14.93	.94	2.01	3.32
390	1.40	15.93	.28	.86	1.75
Puts					
330	1.20	15.28	.29	.87	1.72
340	2.60	14.69	1.17	2.31	3.64
350	5.10	14.04	3.41	5.08	6.80
360	9.50	13.92	7.70	9.55	11.41
370	15.70	13.88	14.18	15.77	17.45
380	23.70	14.57	22.40	23.47	24.78
390	32.50	15.13	31.62	32.20	33.08

Figure 4-12: 10-Week Gold Volatility



must try to make an intelligent decision about future volatility. From this, he will look for option strategies which will be profitable when he is right, but which will not result in a disastrous loss when he is wrong. Because of the difficulty in predicting volatility, a trader must always look for strategies which will leave the greatest margin for error. No trader will survive very long pursuing strategies based on a future volatility estimate of 15% if such a strategy results in a loss when volatility actually turns out to be 16%. Given the shifts that occur in volatility, a one-percentage-point margin for error is no margin for error at all.

We have not yet concluded our discussion of volatility. But before continuing, it will be useful to look at option characteristics, trading strategies, and risk considerations. We will then be in a better position to examine volatility in greater detail.