

A Cross Sectional Study of Volatility Risk Premium Across Industries

by

Kumar SHANTANU

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Contents

1	Abstract	2
2	Literature Review	2
2.1	Implied and the Realised Volatility	2
2.2	Empirical Relationship between Implied and the Realised Volatility	3
3	Realised Volatility	4
3.1	Theory	4
3.1.1	Moments	5
3.1.2	Estimation of Parameters from a Single Path through Maximum Likelihood . . .	6
3.2	Empirics	6
3.2.1	Measurement of Realised Volatility	6
3.2.2	Ambient Volatility (θ)	7
3.2.3	Mean Reversion (κ)	8
3.2.4	Meta Volatility (σ)	11
4	Roadmap from here	13
5	Appendix	16
5.1	Tickers and Asset Classes	16

1 Abstract

Since the market does not have perfect knowledge about the future so the implied volatility and the realised volatility can and will be different. Therein, lies the risk management problem / business or trading opportunity. The paper seeks to perform an empirical study to analyze the differences (over last few years) between the realised and the implied volatility across different asset classes or industries and generate insights on the volatility risk premium (the markup of the implied volatility over the realised volatility). The autocorrelations and patterns between the two can be interesting to study. *COMMENT: This section is incomplete because of obvious reasons. This paragraph would naturally be written at the end of the thesis.*

2 Literature Review

The literature review section is divided into two segments: the first section explores the literature related to the theoretical background of the implied and the realised volatility and the issues associated with the measurement of the same. The second section dives into the empirical literature that have studied the relationship between the implied and the realised volatility.

2.1 Implied and the Realised Volatility

Implied volatility is how the market is pricing the option currently. The implied volatility stems from the pricing model and the contract terms. The implied volatility in context of Black Scholes model of option pricing is obtained by inverting the Black Scholes formula (Black and Scholes, 1973) against the observed market price of that option. The implied volatility refers to the Brownian diffusion coefficient inherent in the Black Scholes theory (Ayache, 2024). Ayache, 2024 is an interesting read which explores different interpretations and meanings of the implied volatility. In a perfect Black and Scholes, 1973 world, the options with the same underlying asset would be priced in such a way that the implied volatility of each price is exactly equal (Mayhew, 1995). Mayhew, 1995 provides an extensive literature review of the metrics used to measure the implied volatility. They also study the usefulness of weighted average of implied volatilities because of the limitations of the Black Scholes model.

The realised volatility is the actual variability in the price of the underlying that materializes due to the randomness of the stochastic process. Randomness is usually defined as a Weiner process or a Brownian motion. The measurement of realised volatility is a deceptively non trivial subject of study because of several nuances involved. Abdelmessih, 2024 is an interesting read. He analyses how the measurement of realised volatility is dependent upon the sampling period. He presents a key idea that the more frequently you sample volatility, the faster you converge on a better estimate of volatility. Bennett and Gil, 2012 study the following methods to calculate the realised volatility from the aggregated price data:

- Close to Close (C)
- Exponentially Weighted (C)
- Parkinson (HL)
- Garman-Klass (OHLC)
- Rogers-Satchell (OHLC)
- Yang-Zhang (OHLC)

Corsi, 2005 further describe how realised volatility estimates can be constructed from high frequency data. Liu et al., 2012 claim that it is significantly challenging to beat the 5-minute realised volatility estimate by other measures of volatility. For this they create multiple volatility measures and benchmark them against 5 minute realised volatility through various statistical approaches. On the basis, of this study I propose to compute the realised volatility using 5-minute calendar time.

2.2 Empirical Relationship between Implied and the Realised Volatility

The empirical relationship between the implied and the realised volatility has been the subject of intense study. In the book, "Trading Volatility, Correlation, Term Structure and Skew" Bennett, 2020, the author observes the idea of a volatility cone that exhibits:

- The average implied volatility is slightly above the average realised volatility.
- The implied volatility is also less volatile than realised volatility for near dated maturities .

The closest paper that studies the relationship between the implied and the realised volatility is Ammann et al., 2009 for a time period of 1996 to 2006. They use the historical 91-day volatility and a second characteristic: beta, market value, market-to-book ratio, or momentum to construct 5×5 grid of portfolios, i.e. each dimension is divided into 5 quantiles. They discovered high-beta stocks, small stocks, stocks with low market-to-book ratios, and non-momentum have a higher implied volatility after controlling for the historical volatility. Whereas for low beta stocks, small caps, low market-to-book ratio and no momentum stocks, the implied volatility overestimates the realised volatility. An interesting finding is that they cannot reject the null hypothesis that the implied volatility is an unbiased predictor of the realised volatility.

Canina and Figlewski, 1993 is an interesting study. They use the forecast rationality test introduced by Theil, 1966 to test the bias and efficiency of using implied volatility to forecast the realised volatility. They find that the implied volatility is a poor forecast of subsequent realised volatility for S&P 100 index options. This is a remarkable observation as it contradicts a lot of the literature.

Another study by Bollerslev et al., 2017 examines the sampling distribution of standard deviation across asset classes such as equities, fixed income, commodities and FX. They find that the unconditional daily realised volatility distribution for these asset classes is statistically identical when normalised by the sample mean. This leads to the interesting question of spillover effects of volatility across different asset classes. This sets the stage for a similar study that one could perform to examine the multivariate distribution of implied and realised volatility across asset classes.

Derman, 1999 studies of volatility in the S&P 500 index options post the crash of 1987. The paper extensively studies the skewness of the implied volatility. An **implied volatility skew** refers to the pattern of how implied volatility (IV) varies across options with the same expiration date but different strike prices (Derman, 1999). They discover the following stylized facts. Firstly, the volatility skew has a roughly linear relationship with a strike distance from the spot price. Secondly, at-the-money implied volatilities are negatively correlated with the index. Thirdly, Fixed strike implied volatility shows a richer structure. Fourthly, the volatility skew is always negative. Fifthly, the skew widened after the October 1997 market drop and then expanded even more after the decline of August 1998.

Volatility also has a very interesting empirical property that it almost always mean reverts (J. Stein, 1989). Cohen, 1996 study the impact of derivatives on the underlying asset markets using the Variance Ratio Tests. They take the ratio of the variance of the daily log price changes with the variance of the multiday log price changes. The focus of their study is the long term government bonds yields for Germany, US and Japan and the equity price indices for US and Germany. The implication of the variance ratio test is that a value greater than 1 signifies a positive autocorrelation, a value 1 depicts

that the process follows a random walk while a value less than 1 shows mean reversion. The variance ratio test serves as a powerful tool for examining the momentum and the mean reversion of the volatility and could be instrumental in devising trading strategies in the options market.

COMMENT: The literature review is going to evolve as thesis progresses. This is just a preliminary exploration of top of the funnel literature.

3 Realised Volatility

“Never cross a river if it is on average four feet deep” - Nassim Nicholas Taleb (Antifragile: Things That Gain from Disorder)

Cont, 2005 provides an excellent literature review where the author has reviewed the papers that study the properties of volatility in financial markets. The author has discussed phenomena such as excess volatility, heavy tails and volatility clustering in financial time series data. The volatility clustering is an interesting phenomenon. The changes in volatility exhibit a persistence where large changes are followed by large changes and vice versa (Mandelbrot, 1963). A quantitative implication of this statement is that although the returns of security are usually not autocorrelated, the squared returns often exhibit a positive autocorrelation, which slowly decays over time.

Modelling the realised volatility is an interesting and non-trivial problem because of the presence of clustering and tendencies of mean reversion. In this section, we seek to model the realised volatility, through Ornstein–Uhlenbeck process (E. M. Stein and Stein, 1991). E. M. Stein and Stein, 1991 study the stock price distributions that follow a diffusion process with a stochastically varying volatility parameter.

The modelling process of realised volatility in this thesis is divided into two subsections. The first section goes over the theory of the Ornstein-Uhlenbeck process, where we first derive the moments of the process and then estimate the parameters of the process for a single realization through maximum likelihood.

The second section does the empirics. We use the closed-form solutions of maximum likelihood parameters derived in the preceding section to estimate the parameters of the Ornstein-Uhlenbeck (OU) process from the real volatility data. To achieve this, we select a cross-section of 100 tickers spread over 34 different asset classes or industries.

We start with the measurement of realized volatility. We then estimate the ambient volatility, mean reversion and meta-volatility (parameters of the OU process) for each ticker and compare them across asset classes and industries to generate insights.

3.1 Theory

The Ornstein–Uhlenbeck process (Wikipedia, n.d.) is described by the following stochastic differential equation:

$$dX_t = \kappa(\theta - X_t)dt + \sigma dW_t.$$

X_t is the volatility process which is described by the following parameters:

- κ is the rate of the mean reversion of the process. In this paper, we refer to this as **‘mean reversion’**.
- θ is the long term mean of the process. In this paper, we refer to this as **‘ambient volatility’**.
- σ is the volatility of the volatility. In this paper, we refer to this as **‘meta volatility’**.

- W_t is the Wiener process.
- dt is the time step of the process

This process is Gaussian because it is a linear combination of a Wiener process. It is Markovian because the future values of the process future state depends only on the current state. It is unconditionally stationary because the joint distribution of the process is invariant to time. Cantarutti, 2020.

We can rearrange terms in the SDE as:

$$\begin{aligned} dX_t &= \kappa(\theta - X_t)dt + \sigma dW_t \\ &= \kappa\theta dt - \kappa X_t dt + \sigma dW_t \\ dX_t + \kappa X_t dt &= \kappa\theta dt + \sigma dW_t \end{aligned}$$

We can use the Itô formula:

$$d(X_t e^{\kappa t}) = \kappa X_t e^{\kappa t} dt + e^{\kappa t} dX_t$$

Now we take Riemann integral over time horizon $t \in [0, T]$:

$$\int_0^T d(e^{\kappa t} X_t) = \int_0^T \kappa\theta e^{\kappa t} dt + \int_0^T \sigma e^{\kappa t} dW_t$$

which results in:

$$e^{\kappa T} X_T - X_0 = \kappa\theta \frac{e^{\kappa T} - 1}{\kappa} + \sigma \int_0^T e^{\kappa t} dW_t$$

Now we can solve for X_T :

$$X_T = \theta + e^{-\kappa T} (X_0 - \theta) + \sigma \int_0^T e^{-\kappa(T-t)} dW_t$$

3.1.1 Moments

The expectation of X_t is:

$$\begin{aligned} \mathbf{E}[X_T] &= \mathbf{E}\left[\theta + e^{-\kappa T} (X_0 - \theta) + \sigma \int_0^T e^{-\kappa(T-t)} dW_t\right] \\ &= \theta + (X_0 - \theta) e^{-\kappa T} \end{aligned}$$

The variance of X_t is:

$$\text{Var}[X_T] = E[(X_T - E[X_T])^2]$$

Substitute in X_T and $E[X_T]$ derived above.

$$\begin{aligned} \text{Var}(X_T) &= E[(\sigma \int_0^T e^{-\kappa(T-t)} dW_t)^2] \\ &= \sigma^2 \frac{1 - e^{-2\kappa T}}{2\kappa} \\ &= \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa T}) \end{aligned}$$

The derivation above uses Ito's Lemma, where only term left after we cross multiply $dW dW$ is dt . As $t \rightarrow \infty$ we obtain the **asymptotic mean** θ and the **asymptotic variance** $\sigma^2/2\kappa$.

3.1.2 Estimation of Parameters from a Single Path through Maximum Likelihood

Ito processes are martingale under a risk neutral measure. The following derivation is taken from Berg, 2011. *[I will put the full derivation in appendix later on]*

Using the following notations:

$$\begin{aligned} S_x &= \sum_{i=1}^n X_{i-1} \\ S_y &= \sum_{i=1}^n X_i \\ S_{xx} &= \sum_{i=1}^n X_{i-1}^2 \\ S_{xy} &= \sum_{i=1}^n X_{i-1} X_i \\ S_{yy} &= \sum_{i=1}^n X_i^2 \end{aligned}$$

The MLE parameters for the process are given as.

Long Term Mean:

$$\theta = \frac{S_y S_{xx} - S_x S_{xy}}{n(S_{xx} - S_{xy}) - (S_x^2 - S_x S_y)}$$

Mean Reversion Rate:

$$\kappa = -\frac{1}{T} \ln \frac{S_{xy} - \theta S_x - \theta S_y + n\theta^2}{S_{xx} - 2\theta S_x + n\theta^2}$$

Variance:

$$\begin{aligned} \hat{\sigma}^2 &= \frac{1}{n} [S_{yy} - 2\alpha S_{xy} + \alpha^2 S_{xx}] \\ &\quad [-2\theta(1 - \alpha)(S_y - \alpha S_x) + n\theta^2(1 - \alpha)^2] \\ \sigma^2 &= \hat{\sigma}^2 \frac{2\kappa}{1 - \alpha^2} \end{aligned}$$

with $\alpha = e^{-\kappa T}$

3.2 Empirics

Since we have closed form solutions for each parameter, we can estimate the parameters using the MLE from the real volatility data. The following section goes into the empirical modelling of the realised volatility through the estimation of each parameter of Ornstein-Uhlenbeck process. We select a cross-section of 100 tickers spread over 34 different asset classes or industry. See appendix for all the tickers.

3.2.1 Measurement of Realised Volatility

We start with aggregated quote data over each 1-minute interval for each of the 100 tickers. Given an ask A and a bid B , we calculate the weighted mid-price for each minute using the following formula.

$$P_{\text{mid}} = A \cdot \frac{A_{\text{size}}}{A_{\text{size}} + B_{\text{size}}} + B \cdot \frac{B_{\text{size}}}{A_{\text{size}} + B_{\text{size}}}$$

This 1-minute weighted mid price is now used to generate Open, High, Low, Close prices for each 5-minute interval. We construct 5 minute log-returns over these intervals. Suppose we have a price process S_t and a time scale Δ the log returns r_t are defined as:

$$r_t = \log(S_t) - \log(S_{t-\Delta})$$

The standard deviation of these 5-minute log-returns over a 6.5 hour trading day gives us a realized volatility estimate at a 5-minute resolution. There are twelve 5-minute intervals in an hour, there are 6.5 hours in a trading day and 252 trading days in a year. Therefore, this standard deviation is annualized by multiplying a constant $= \sqrt{12 \times 6.5 \times 252}$.

3.2.2 Ambient Volatility (θ)

The following chart shows the long term volatilities as estimated by Ornstein-Uhlenback process aggregated for each industry / asset class. This chart and other charts that will follow have been aggregated by the asset class reveals many interesting insights. However, it should be noted that the chart may not be truly representative because of the representation bias, as we have finite number of tickers representing each asset class. On the left end of the distribution are Agriculture and Crypto (spot) asset classes.

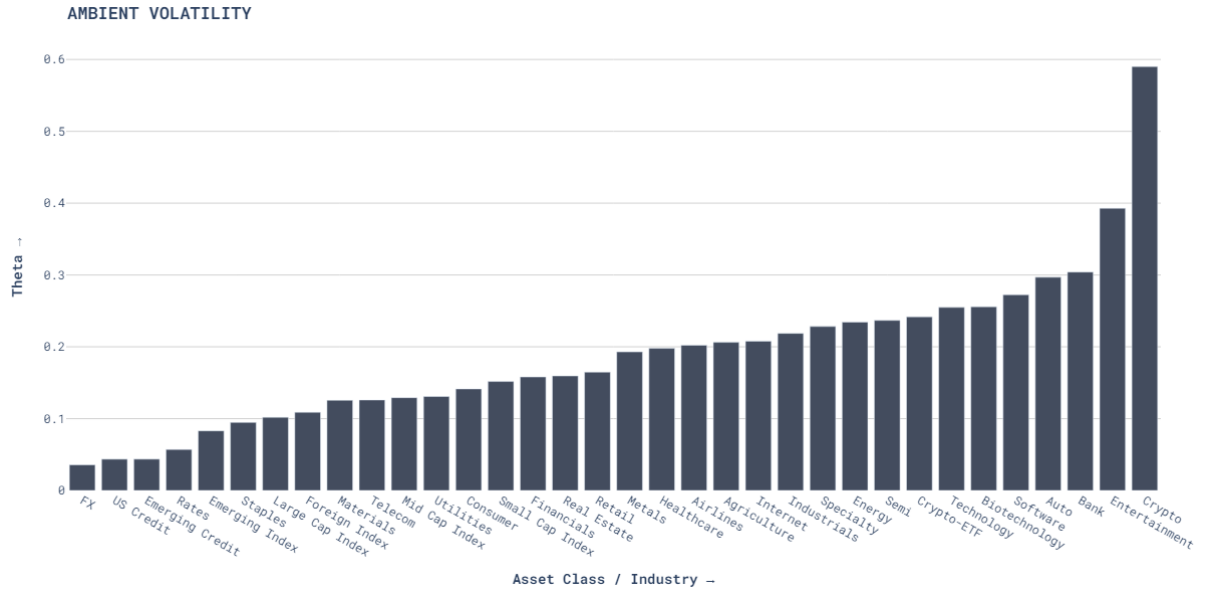


Figure 1: Ambient Volatility by Category

The Cryptocurrencies which are deeply speculative in nature has an extraordinarily high ambient volatility compared to other asset classes. The entertainment industry which is mainly driven by consumer cyclicality seconds cryptocurrency in terms of long run volatility. The sectors which are driven by innovation cycles and competitions such as technology biotechnology and automotive have relatively moderate ambient volatilities. The assets such as FX, Corporate and Treasury bonds which are driven by macroeconomic factors such as interest rates, credit risks have the lowest levels of ambient volatility because their volatility are actively managed by central banks.

3.2.3 Mean Reversion (κ)

The most interesting aspect of this kind of analysis is looking at the strength of mean reversion across tickers and asset classes. The following chart shows the mean reversion rates as estimated by Ornstein-Uhlenbeck process aggregated for each sector. Although this chart is not very interpretable, therefore, in the next paragraph we modify the this mean reversion coefficient so as to look at expected time taken to reach the ambient volatility (θ) starting from a certain level of volatility. This helps to introduce interpretability in the mean reversion coefficient.

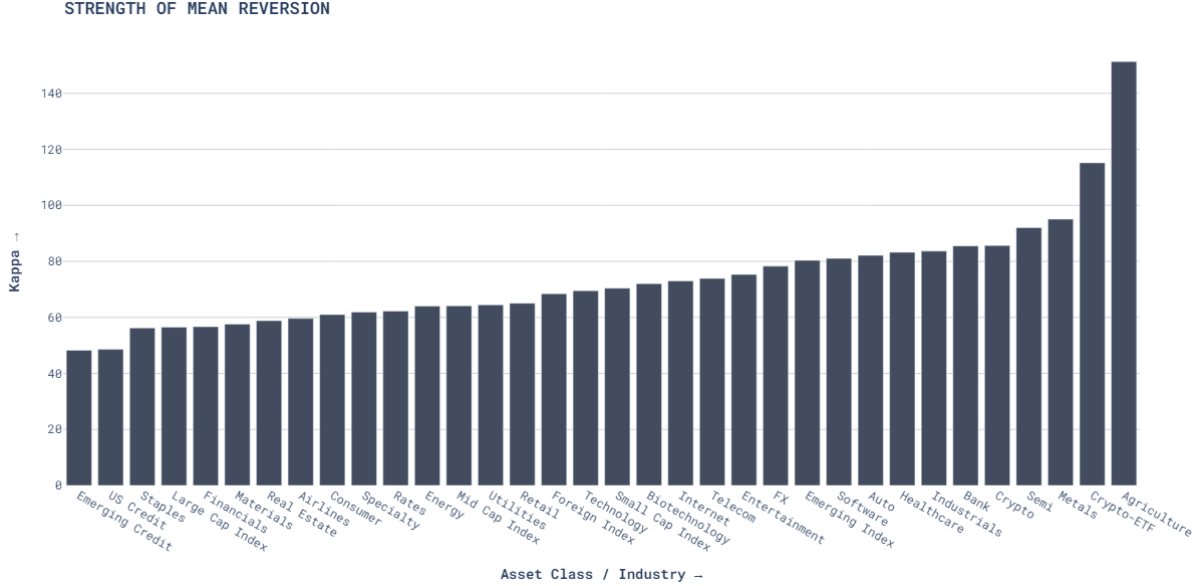


Figure 2: Mean Reversion By Category

Now since we know the mean reversion coefficient, we can ask another interesting question what is the expected amount of time it would take to reach from one level of volatility to another level of volatility. Since we have calibrated all the parameters of the stochastic differential equation, we can simulate different paths the volatility process could take which will enable us to look at the distribution of these Monte Carlo paths to estimate the parameter of interest.

To determine the expected amount of time that would elapse before the volatility touches its long run mean (θ), we would first need to define, the first hitting or crossover time of the volatility process. Suppose we start from a volatility level C and want to reach a volatility level of A . The expected time T_{AC} can be expressed as an infimum over the set of all days t where a single path crosses that volatility threshold.

$$T_{A,C} = \inf \left\{ t \geq 0 : X_t = A \mid X_0 = C \right\}$$

We can generate a large number of paths and the average of the infimums will give us the expected time to reach the level of ambient volatility.

To illustrate this concept, we can look at the iShares 20+ Year Treasury Bond ETF (TLT) which has a relatively slower mean reversion (κ) and we set the initial volatility level to be very high at 0.5, for the sake of a nice visualization. The estimated ambient volatility (θ) of this ticker through Ornstein-Uhlenbeck process is 0.08. We generate 5000 Monte Carlo paths and we can observe that the volatility process is mean reverting to its ambient level over all the paths. To estimate the expected time to reach the ambient level, we take the average of the first crossovers over 5000 paths, which comes out to be 13.1

trading days with a standard error of 0.07 trading days.

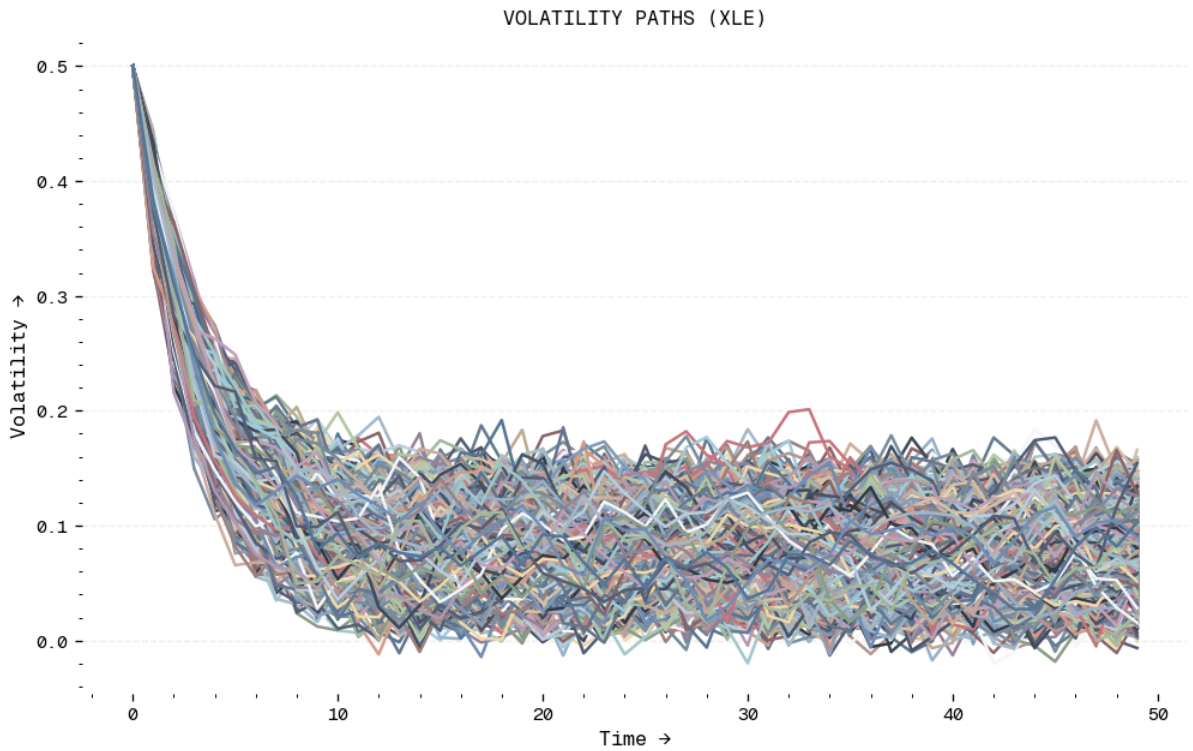


Figure 3: Simulated 5000 paths of volatility process for TLT

Please also note that very few volatility paths drop below zero. This is a Weiner process artefact and can be ignored, since we are looking at sufficiently large number of realizations.

A more interpretable version of looking at the mean reversion is to look at the expected number of days it will take for the volatility to reach its ambient level, given a starting level of volatility. The following chart shows the expected number of days it will take for the volatility to reach its ambient level (θ) estimated through Ornstein-Uhlenbeck process for each industry / asset class, when we start from a pathologically high level of volatility of 0.5.

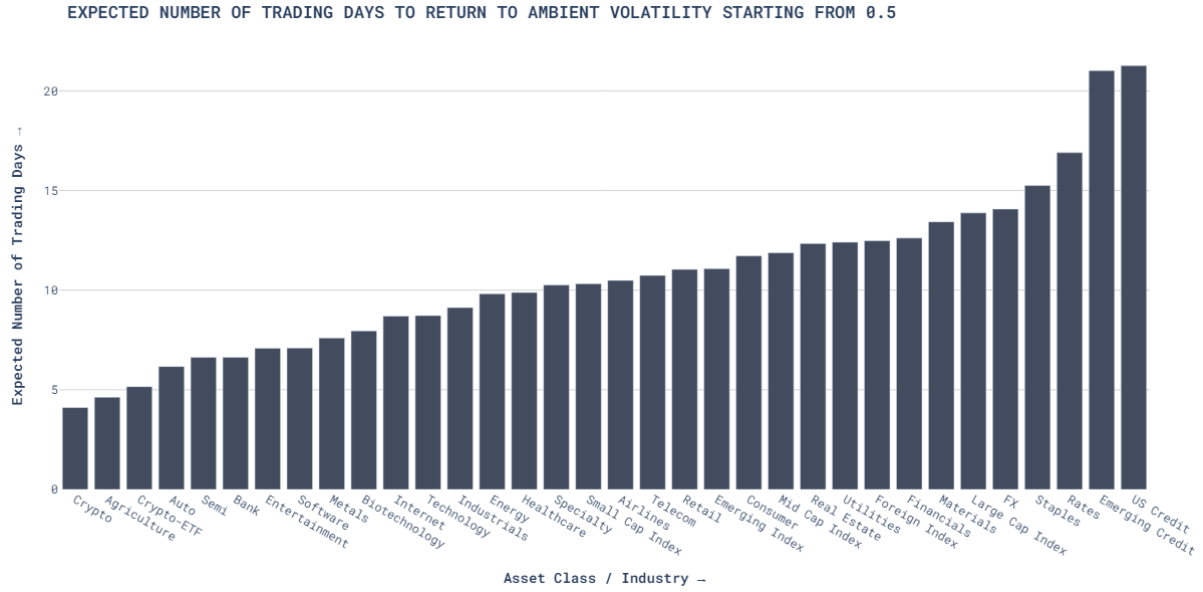


Figure 4: Expected Time to Reach Ambient Volatility starting from a high volatility of 0.5

If we start from a high volatility of 0.5, they take approximately four trading days to mean revert to their ambient volatility. A strong mean reversion tendency in agricultural tickers is possibly due to the strong seasonal fluctuations inherent in these industries (Sørensen, 1999). Cryptocurrency markets might be characterized by strong bursts of volatility which quickly stabilizes.

On the right end of the distribution, we have the industries that are highly sensitive to macroeconomic factors that influence credit markets such as Corporate bonds ETFs (HYG and LQD), Treasury Bond ETFs (TLT and IEF) of US and other emerging markets (EMB). These asset classes are heavily influenced by interest rates and default risks. Interest rate cycles usually span over several months and hence the volatility is expected to be persistent in these asset classes.

All the asset classes that relate to commodities and industrials like metal and energy sectors show reversion time around 6 to 8 trading days (starting from a high volatility) which indicates a relatively moderately persistent volatility. The chart does not show the standard errors associated with each estimate of expected number of days however the standard error is extremely low possible because of a large sample realizations (5000 paths). The maximum standard error observed for the expected number of days over all tickers is one tenth of a trading day. The expected time levels as well as their associated standard errors for each ticker is shown in the appendix.

One more interesting insight emerges when we look at the scatter of the mean reversion against ambient volatility.

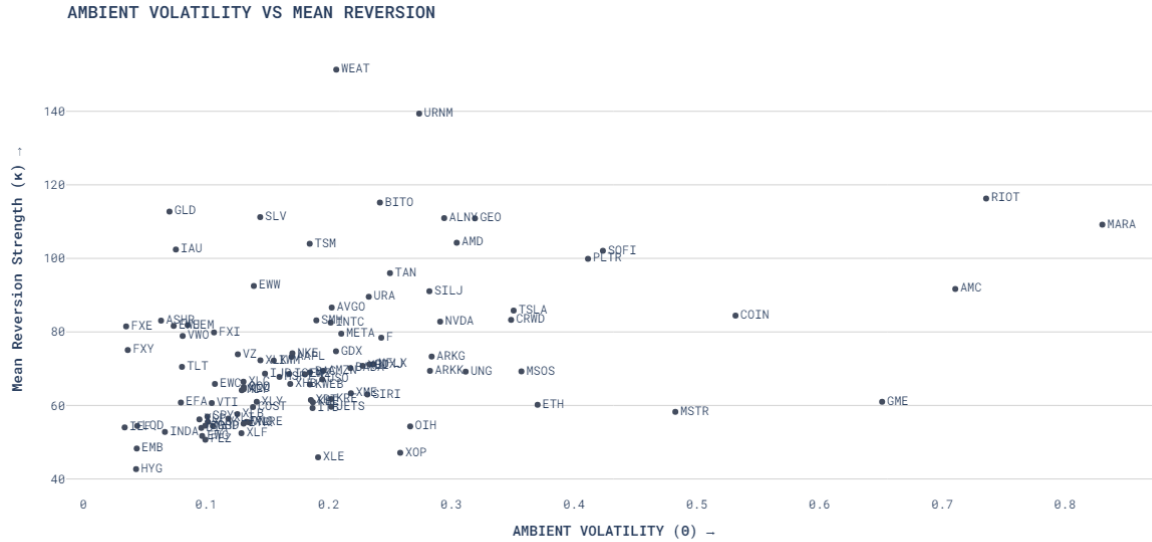


Figure 5: Scatter of Ambient Volatility vs Mean Reversion

The strength of mean reversion has a heteroscedastic relationship with ambient volatility that is the variability of mean reversion across the cross section of tickers is conditional on what level of ambient volatility we are looking at. At the lower levels of ambient volatility we find a strong visible correlation between the strength of mean reversion and ambient volatility which further dampens as we move to higher levels of ambient volatility across the cross section of tickers.

3.2.4 Meta Volatility (σ)

The meta volatility is the volatility of the volatility. The following chart shows the meta volatility as estimated by Ornstein-Uhlenbeck process aggregated for each industry / asset class.

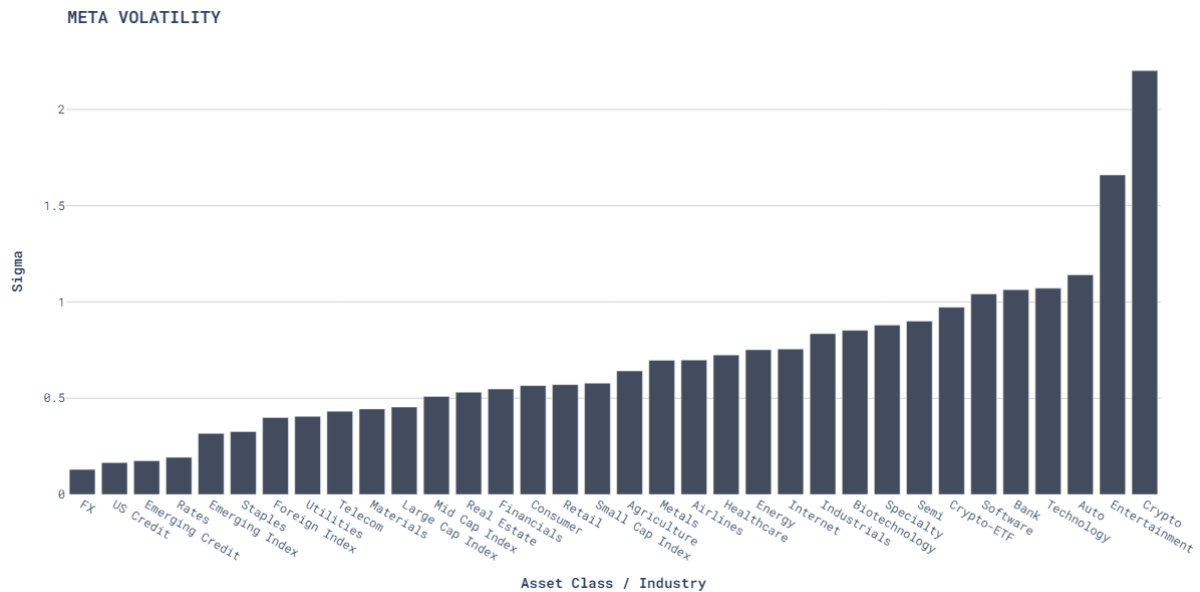


Figure 6: Meta Volatility by Asset Class

A truly interesting insight here is that this chart looks very similar to the ambient volatility chart

in Figure 1. The empirical implication here is that if a asset class is characterized by a high level of ambient volatility, the volatility inherent in the ambient volatility is also high. This is apparently visible if we look at the scatter of meta volatility against the ambient volatility.



is the answer. I think the problem, to be quite honest with you, is that you've never actually known what the question is." - Douglas Adams, *The Hitchhiker's Guide to the Galaxy*

Now I will venture into options market from the underlying spot markets. The options market gives me two additional pieces of information that is the implied volatility and the skew. At this point, I have a powerful model for the realised volatility. This will enable me to ask good questions about the implied volatility and the markup of the implied and the realised volatility that is the volatility risk premium.

1. Every trading day I can measure the distance between the realised volatility from the ambient volatility. The farther is the realised volatility from the ambient volatility, it will be interesting to see how Volatility Risk Premium moves. Using these insights, can I find options that are mispriced or priced cheaply in the market?
2. I can use the gap between realised volatility and ambient volatility to see if market is overreacting to short term volatility bursts. This could be an incredibly useful trading signal.
3. Is the current implied volatility accurately pricing the future realized volatility? Is there a pattern when implied volatility consistently overshoots or underpredicts the future realised volatility.
4. I also know the mean reversion speed of the volatility process for each ticker. This can reveal potential mispricing of short term options.
5. How does the skew move with realized volatility?
6. How is skew distributed across asset classes?

These questions would change and evolve as I move forward in the thesis. I am not sure, if I am still asking the right questions. The questions would themselves appear with the data. Motion creates information. Peace.

References

- Abdelmessih, K. (2024). Volatility depends on the resolution. *MoonTower Meta*. <https://moontowermeta.com/volatility-depends-on-the-resolution/>
- Ammann, M., Skovmand, D., & Verhofen, M. (2009). Implied and realized volatility in the cross-section of equity options.
- Ayache, E. (2024). What is implied by implied volatility? *Wilmott Magazine*.
- Bennett, C. (2020). *Trading volatility, correlation, term structure and skew*. Wiley.
- Bennett, C., & Gil, M. A. (2012). Measuring historical volatility: Close-to-close, exponentially weighted, parkinson, garman-klass, rogers-satchell and yang-zhang volatility [Accessed via internal research, Madrid].
- Berg, T. V. D. (2011). Calibrating the ornstein-uhlenbeck process [Accessed: 2024-09-28].
- Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3), 637–654.
- Bollerslev, T., Hood, B., Huss, J., & Pedersen, L. H. (2017). Risk everywhere: Modeling and managing volatility [Available at SSRN: <https://ssrn.com/abstract=2722591> or <http://dx.doi.org/10.2139/ssrn.2722591>].
- Canina, L., & Figlewski, S. (1993). The informational content of implied volatility. *The Review of Financial Studies*, 6(3), 659–681. Retrieved September 17, 2024, from <http://www.jstor.org/stable/2961982>

- Cantarutti, N. (2020). Ornstein-uhlenbeck process and applications [Accessed: 2024-09-28].
- Cohen, B. H. (1996). *Derivatives and asset price volatility: A test using variance ratios* (Working Paper). Bank for International Settlements.
- Cont, R. (2005). Volatility clustering in financial markets: Empirical facts and agent-based models [To appear]. In A. Kirman & G. Teyssi  re (Eds.), *Long memory in economics* (pp. 289–309). Springer. <https://www.polytechnique.edu>
- Corsi, F. (2005). *Measuring and modelling realized volatility: From tick-by-tick to long memory* [PhD dissertation]. University of Lugano [Thesis Committee: Prof. G. Barone-Adesi, University of Lugano, Prof. T. Bollerslev, Duke University, Prof. F. Audrino, University of Lugano].
- Derman, E. (1999). *Regimes of volatility: Some observations on the variation of s&p 500 implied volatilities* (tech. rep.). Goldman Sachs.
- Liu, L., Patton, A. J., & Sheppard, K. (2012). Does anything beat 5-minute rv? a comparison of realized measures across multiple asset classes [December 5, 2012].
- Mandelbrot, B. B. (1963). The variation of certain speculative prices. *Journal of Business*, 36(4), 392–417.
- Mayhew, S. (1995). Implied volatility. *Financial Analysts Journal*, 51(4), 8–20. <http://www.jstor.org/stable/4479853>
- S  rensen, C. (1999). *Seasonality in agricultural commodity futures* (tech. rep.). Copenhagen Business School.
- Stein, E. M., & Stein, J. C. (1991). Stock price distributions with stochastic volatility: An analytic approach. *Review of Financial Studies*, 4, 727–752. <http://rfs.oxfordjournals.org/>
- Stein, J. (1989). Overreactions in the options market. *The Journal of Finance*, 44(4), 1011–1023.
- Theil, H. (1966). *Applied economic forecasting*. North-Holland Publishing Company.
- Wikipedia. (n.d.). Ornstein-uhlenbeck process — wikipedia, the free encyclopedia [[Online; accessed 28-September-2024]]. https://en.wikipedia.org/wiki/Ornstein%E2%80%93Uhlenbeck_process

5 Appendix

5.1 Tickers and Asset Classes

Ticker	Category
WEAT	Agriculture
JETS	Airlines
TSLA	Auto
F	Auto
BAC	Bank
SOFI	Bank
XBI	Biotechnology
ARKG	Biotechnology
XLY	Consumer
RIOT	Crypto
MSTR	Crypto
COIN	Crypto
MARA	Crypto
ETH	Crypto
BITO	Crypto-ETF
EMB	Emerging Credit
VWO	Emerging Index
EEM	Emerging Index
ICLN	Energy
XLE	Energy
XOP	Energy
USO	Energy
TAN	Energy
OIH	Energy
UNG	Energy
NFLX	Entertainment
AMC	Entertainment
SIRI	Entertainment
XLF	Financials
KBE	Financials
INDA	Foreign Index
EWJ	Foreign Index
KWEB	Foreign Index
FXI	Foreign Index
EWC	Foreign Index
EWG	Foreign Index
EWZ	Foreign Index
FEZ	Foreign Index
EWV	Foreign Index
ASHR	Foreign Index
EFA	Foreign Index
FXV	FX
FXE	FX
XLV	Healthcare
ALNY	Healthcare
XLI	Industrials
GEO	Industrials
AMZN	Internet
BABA	Internet