

COMS30030
Image Processing and Computer Vision

Motion – Modelling

Andrew Calway

andrew@cs.bris.ac.uk

COMS30030 Motion Lec 1

1

Motion – Important Perceptual Cue

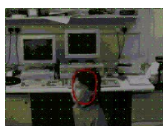


COMS30030 Motion Lec 1

2

We are going to look at

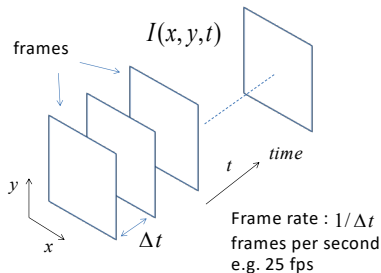
- Modelling 2-D motion fields
- Optical flow
- The optical flow equation (OFE)
- Motion estimation
 - Lucas and Kanade method



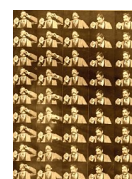
COMS30030 Motion Lec 1

3

Video Sequences



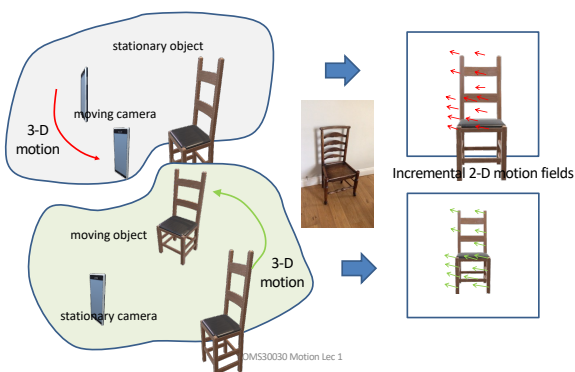
First motion picture camera
Kinetograph



COMS30030 Motion Lec 1

4

Modelling 2-D Motion Fields



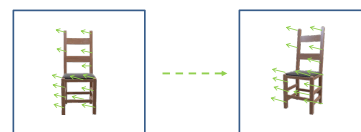
COMS30030 Motion Lec 1

5

Incremental 2-D Motion Fields

What is the relationship between the:

- Position of a 3-D point and its 2-D motion?
- 2-D motion of different 3-D points?
- 3-D motion of a 3-D point and its 2-D motion?



COMS30030 Motion Lec 1

6

3-D Rigid Motion

R : 3x3 rotation matrix
 T : 3x1 translation vector

$P' = RP + T$

Rigid motion
 e.g. objects do not deform

COMS30030 Motion Lec 1

7

Rotation Matrices

$R = R_X R_Y R_Z$ (for example) : Rotations about X , Y and Z axes

$$R_Y P = \begin{bmatrix} \cos \theta_Y & 0 & \sin \theta_Y \\ 0 & 1 & 0 \\ -\sin \theta_Y & 0 & \cos \theta_Y \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X \cos \theta_Y + Z \sin \theta_Y \\ Y \\ Z \cos \theta_Y - X \sin \theta_Y \end{bmatrix}$$

For small θ_Y :
 $\cos \theta_Y \approx 1$
 $\sin \theta_Y \approx \theta_Y$

NB : for small θ_Y

$$R_Y \approx \begin{bmatrix} 1 & 0 & \theta_Y \\ 0 & 1 & 0 \\ -\theta_Y & 0 & 1 \end{bmatrix}$$

NB : for small $\theta_X, \theta_Y, \theta_Z$

$$R \approx \begin{bmatrix} 1 & -\theta_Z & \theta_Y \\ \theta_Z & 1 & -\theta_X \\ -\theta_Y & \theta_X & 1 \end{bmatrix}$$

COMS30030 Motion Lec 1

8

3-D Motion Field

$V = \lim_{\Delta t \rightarrow 0} \{P' - P = (R - I)P + T\}$

For small angles:

$$R \approx \begin{bmatrix} 1 & -\theta_Z & \theta_Y \\ \theta_Z & 1 & -\theta_X \\ -\theta_Y & \theta_X & 1 \end{bmatrix}$$

Hence:

$$V_X = \theta_Y Z - \theta_Z Y + T_X \quad (\theta_X, \theta_Y, \theta_Z) \equiv \text{Angular velocity}$$

$$V_Y = \theta_Z X - \theta_X Z + T_Y \quad (T_X, T_Y, T_Z) \equiv \text{Rectilinear velocity}$$

$$V_Z = \theta_X Y - \theta_Y X + T_Z$$

COMS30030 Motion Lec 1

9

2-D Motion Field Equations

For image point $p = (x, y, f)$ Motion field $v = (v_x, v_y)$

$$v_x = \frac{dx}{dt} = \frac{d}{dt} \left(\frac{fX}{Z} \right) = f \frac{V_X Z - X V_Z}{Z^2}$$

Quotient rule

$$x = \frac{fX}{Z}$$

$$V_X = \frac{dX}{dt}$$

Substituting for V_X, V_Y, V_Z gives (previous slide)

$$v_x = (fT_X - xT_Z)/Z + f\theta_Y - \theta_Z y - (\theta_X xy - \theta_Y x^2)/f$$

$$v_y = (fT_Y - yT_Z)/Z - f\theta_X + \theta_Z x + (\theta_Y xy - \theta_X y^2)/f$$

COMS30030 Motion Lec 1

10

Small angle approximation

R : 3x3 rotation matrix $\rightarrow (\theta_X, \theta_Y, \theta_Z)$
 T : 3x1 translation vector $\rightarrow (T_X, T_Y, T_Z)$

2-D motion field
 $v = (v_x, v_y)$

$$v_x = (fT_X - xT_Z)/Z + f\theta_Y - \theta_Z y - (\theta_X xy - \theta_Y x^2)/f$$

$$v_y = (fT_Y - yT_Z)/Z - f\theta_X + \theta_Z x + (\theta_Y xy - \theta_X y^2)/f$$

COMS30030 Motion Lec 1

11

Two Components

$$v_x = (fT_X - xT_Z)/Z + f\theta_Y - \theta_Z y - (\theta_X xy - \theta_Y x^2)/f$$

$$v_y = (fT_Y - yT_Z)/Z - f\theta_X + \theta_Z x + (\theta_Y xy - \theta_X y^2)/f$$

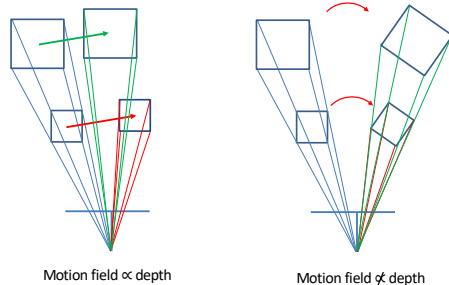
Translational – dependent on scene depth Z

Rotational – independent of scene depth Z

COMS30030 Motion Lec 1

12

Translation, Rotation and Depth



COMS30030 Motion Lec 1

13

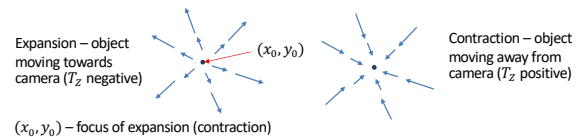
Special Case: Pure Translation

- Assume 3-D motion is only translational, $\theta = 0$, then

$$v_x = (fT_x - xT_z)/Z \quad v_y = (fT_y - yT_z)/Z$$

- If $T_z \neq 0$, $x_0 = fT_x/T_z$ and $y_0 = fT_y/T_z$, then

$$v_x = -(x - x_0)T_z/Z \quad v_y = -(y - y_0)T_z/Z$$

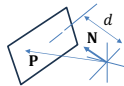


COMS30030 Motion Lec 1

14

Special Case: Moving Plane

- Assume 3-D points lie in plane with unit surface normal \mathbf{N} , i.e. $\mathbf{N}^T \mathbf{P} = d$, where d is distance of plane from origin.



- Since $\mathbf{P} = Z\mathbf{p}/f$, this gives $Z(N_x x + N_y y + N_z f)/f = d$
- Substituting for Z in 2-D motion field:

$$v_x = \frac{1}{fd} (a_1 x^2 + a_2 xy + a_3 fx + a_4 fy + a_5 f^2)$$

$$v_y = \frac{1}{fd} (a_1 xy + a_2 y^2 + a_6 fy + a_7 fx + a_8 f^2)$$

- Motion field is a quadratic polynomial in 2-D spatial coordinates x and y

COMS30030 Motion Lec 1

15