

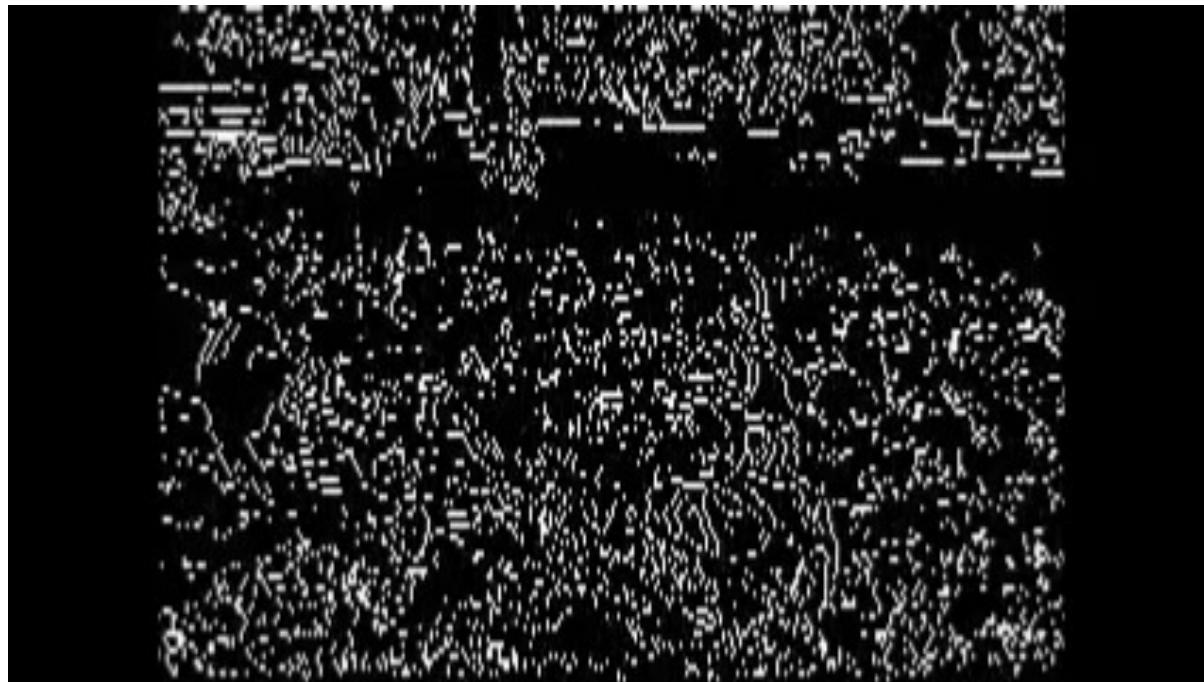
COMS30030
Image Processing and Computer Vision

Motion – Modelling

Andrew Calway

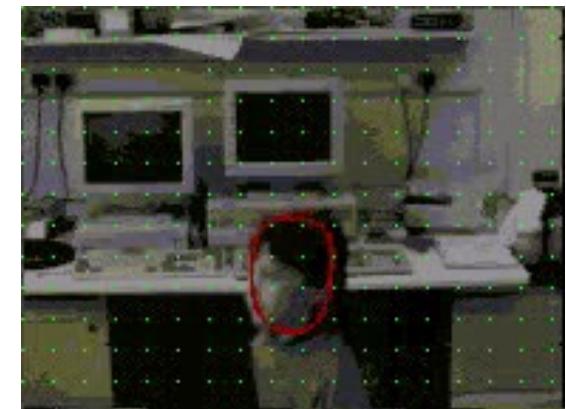
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Motion – Important Perceptual Cue



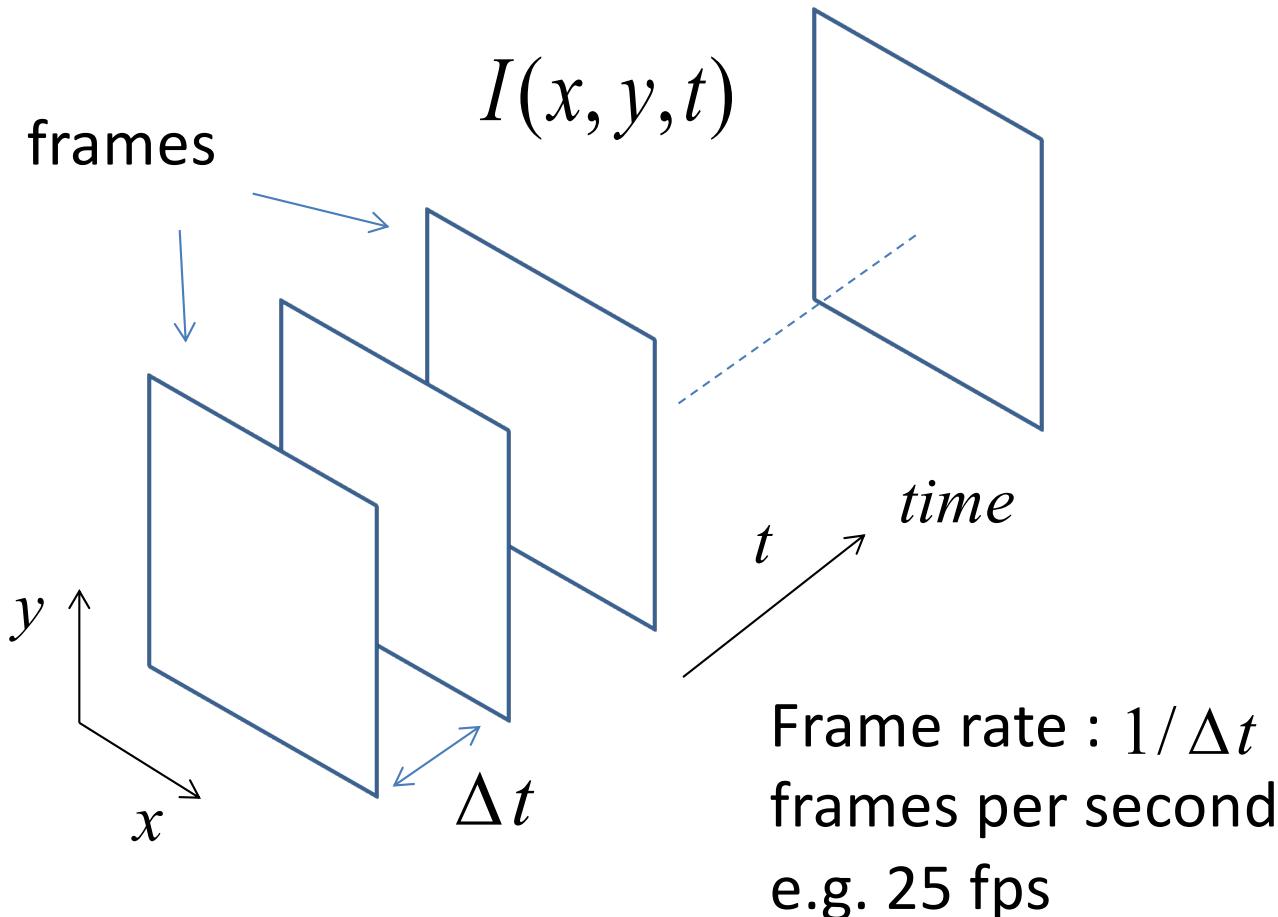
We are going to look at

- Modelling 2-D motion fields
- Optical flow
- The optical flow equation (OFE)
- Motion estimation
 - Lucas and Kanade method

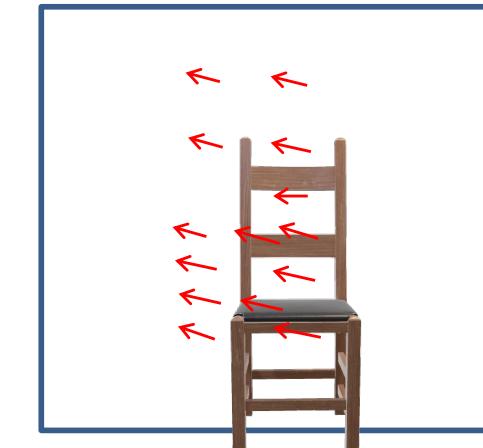
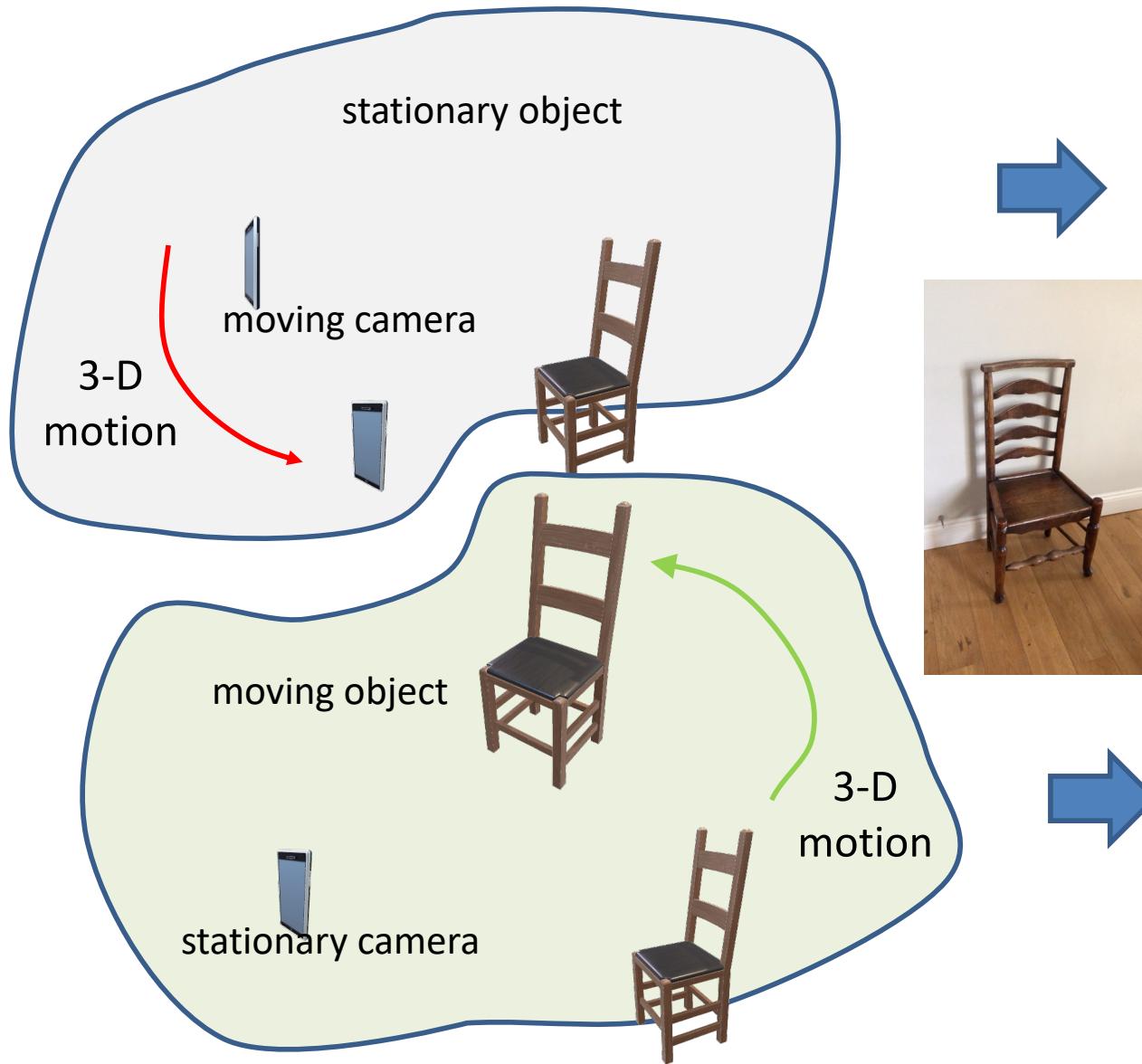


Video Sequences

First motion
picture camera
Kinetograph



Modelling 2-D Motion Fields



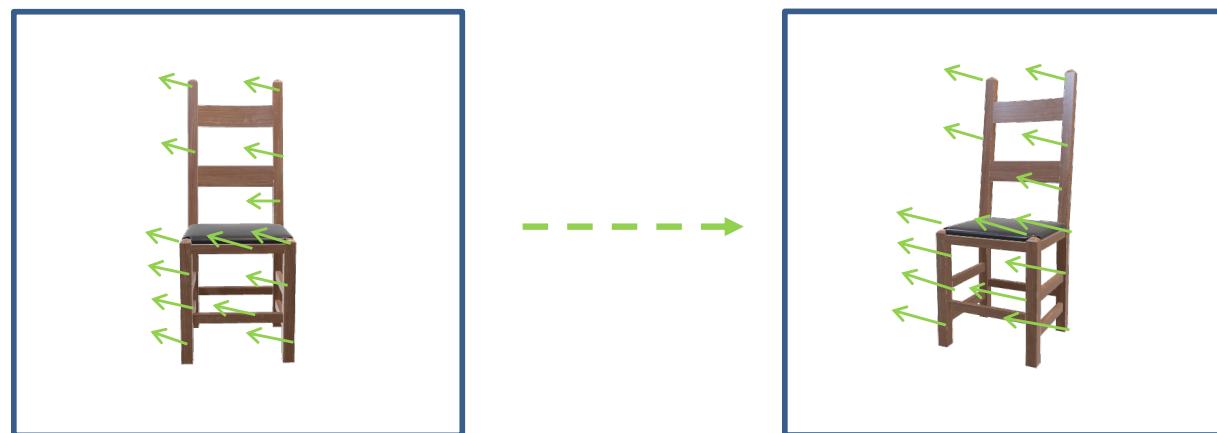
Incremental 2-D motion fields



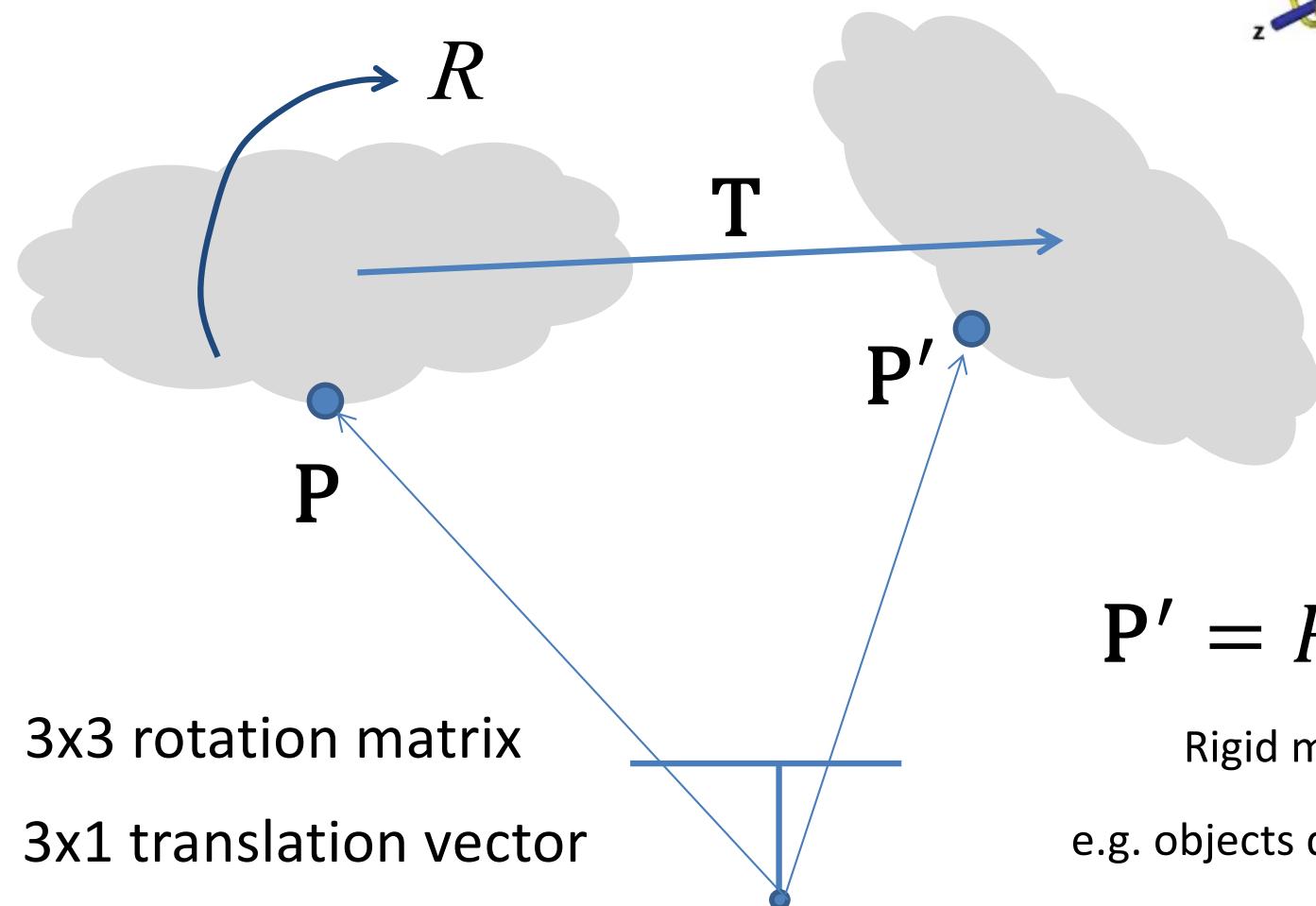
Incremental 2-D Motion Fields

What is the relationship between the:

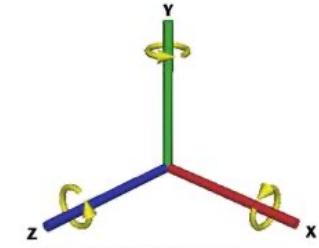
- Position of a 3-D point and its 2-D motion?
- 2-D motion of different 3-D points?
- 3-D motion of a 3-D point and its 2-D motion?



3-D Rigid Motion

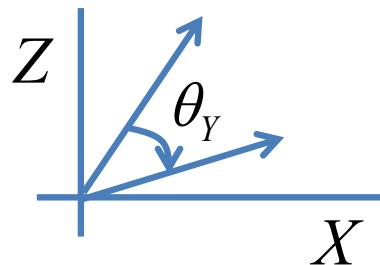


Rotation Matrices



$R = R_X R_Y R_Z$ (for example) : Rotations about X , Y and Z axes

$$R_Y \mathbf{P} = \begin{bmatrix} \cos \theta_Y & 0 & \sin \theta_Y \\ 0 & 1 & 0 \\ -\sin \theta_Y & 0 & \cos \theta_Y \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X \cos \theta_Y + Z \sin \theta_Y \\ Y \\ Z \cos \theta_Y - X \sin \theta_Y \end{bmatrix}$$



For small θ_Y :
 $\cos \theta_Y \approx 1$
 $\sin \theta_Y \approx \theta_Y$

NB : for small θ_Y

$$R_Y \approx \begin{bmatrix} 1 & 0 & \theta_Y \\ 0 & 1 & 0 \\ -\theta_Y & 0 & 1 \end{bmatrix}$$

NB : for small $\theta_X, \theta_Y, \theta_Z$

$$R \approx \begin{bmatrix} 1 & -\theta_Z & \theta_Y \\ \theta_Z & 1 & -\theta_X \\ -\theta_Y & \theta_X & 1 \end{bmatrix}$$

3-D Motion Field

$$\mathbf{V} = \lim_{\Delta t \rightarrow 0} \{\mathbf{P}' - \mathbf{P} = (R - I)\mathbf{P} + \mathbf{T}\}$$

$$\boxed{\mathbf{P}' = R\mathbf{P} + \mathbf{T}}$$

For small angles:

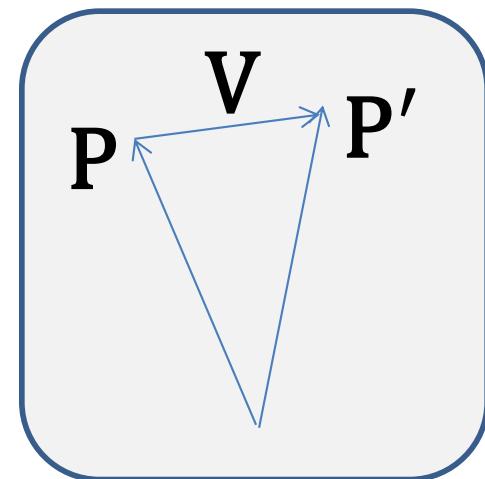
$$R \approx \begin{bmatrix} 1 & -\theta_z & \theta_y \\ \theta_z & 1 & -\theta_x \\ -\theta_y & \theta_x & 1 \end{bmatrix}$$

Hence:

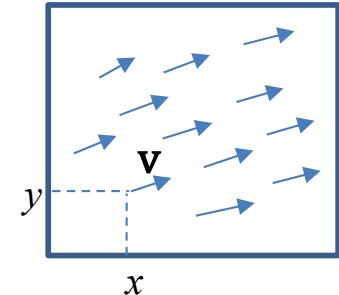
$$V_x = \theta_y Z - \theta_z Y + T_x \quad (\theta_x, \theta_y, \theta_z) \equiv \text{Angular velocity}$$

$$V_y = \theta_z X - \theta_x Z + T_y \quad (T_x, T_y, T_z) \equiv \text{Rectilinear velocity}$$

$$V_z = \theta_x Y - \theta_y X + T_z$$



2-D Motion Field Equations



For image point $\mathbf{p} = (x, y, f)$ Motion field $\mathbf{v} = (v_x, v_y)$

$$v_x = \frac{dx}{dt} = \frac{d}{dt} \left(\frac{fX}{Z} \right) = f \frac{V_X Z - X V_Z}{Z^2}$$

Quotient
rule

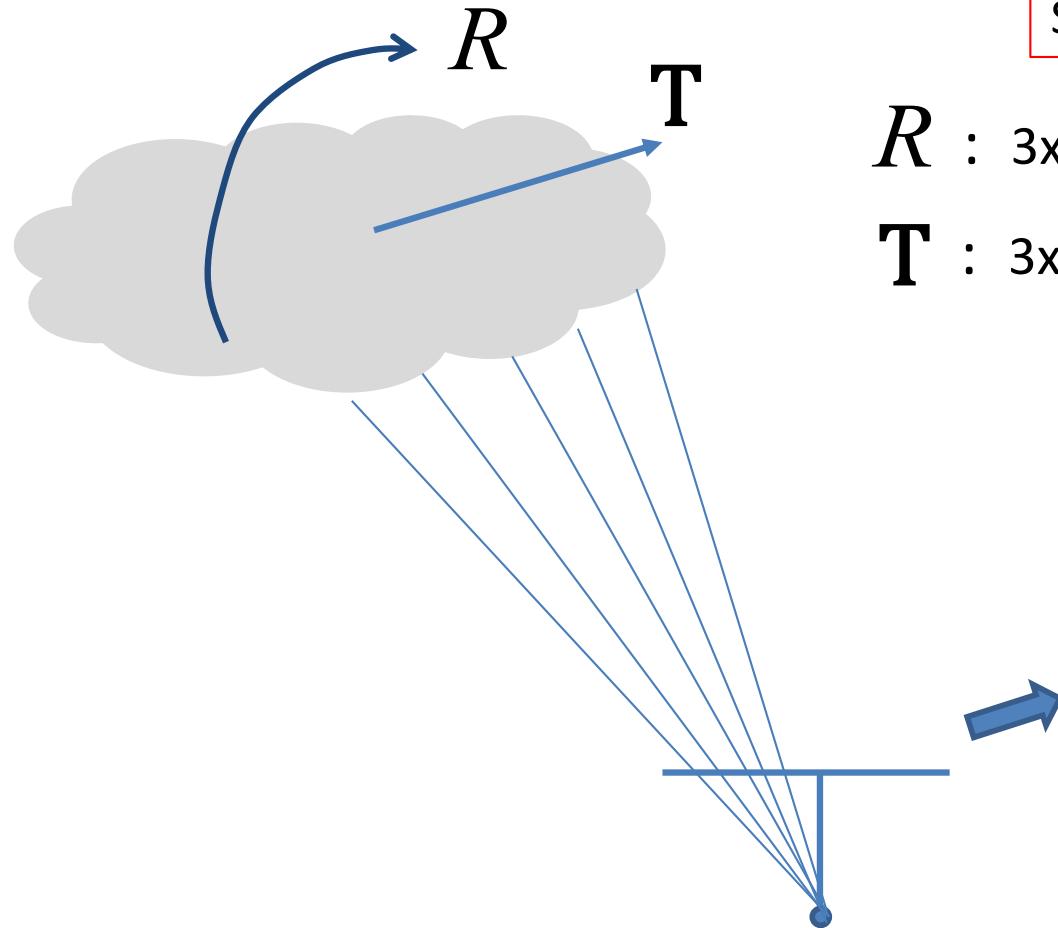
$$x = \frac{fX}{Z}$$

$$V_X = \frac{dX}{dt}$$

Substituting for V_X, V_Y, V_Z gives (previous slide)

$$v_x = (fT_X - xT_Z)/Z + f\theta_Y - \theta_Z y - (\theta_X xy - \theta_Y x^2)/f$$

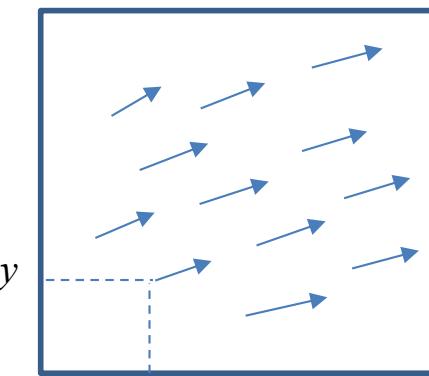
$$v_y = (fT_Y - yT_Z)/Z - f\theta_X + \theta_Z x + (\theta_Y xy - \theta_X y^2)/f$$



Small angle approximation

R : 3x3 rotation matrix $\rightarrow (\theta_X, \theta_Y, \theta_Z)$

T : 3x1 translation vector $\rightarrow (T_X, T_Y, T_Z)$



2-D motion field

$$\mathbf{v} = (v_x, v_y)$$

$$v_x = (fT_X - xT_Z) / Z + f\theta_Y - \theta_Z y - (\theta_X xy - \theta_Y x^2) / f$$

$$v_y = (fT_Y - yT_Z) / Z - f\theta_X + \theta_Z x + (\theta_Y xy - \theta_X y^2) / f$$

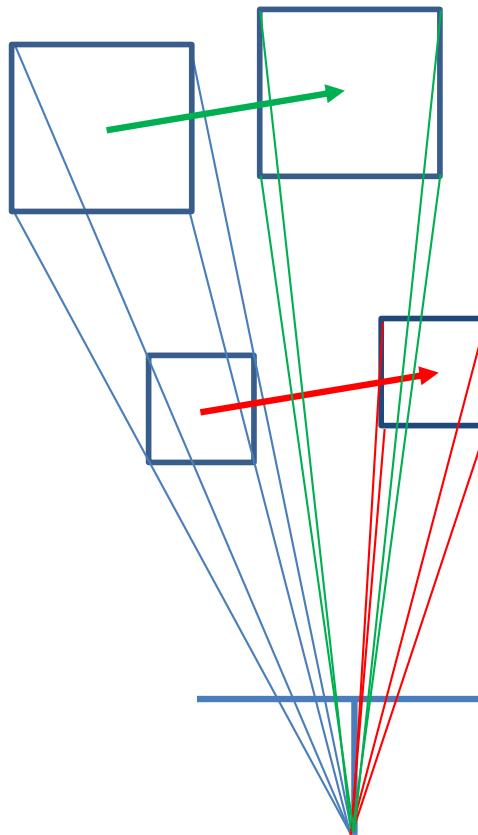
Two Components

$$v_x = (fT_X - xT_Z)/Z + f\theta_Y - \theta_Z y - (\theta_X xy - \theta_Y x^2)/f$$
$$v_y = (fT_Y - yT_Z)/Z - f\theta_X + \theta_Z x + (\theta_Y xy - \theta_X y^2)/f$$

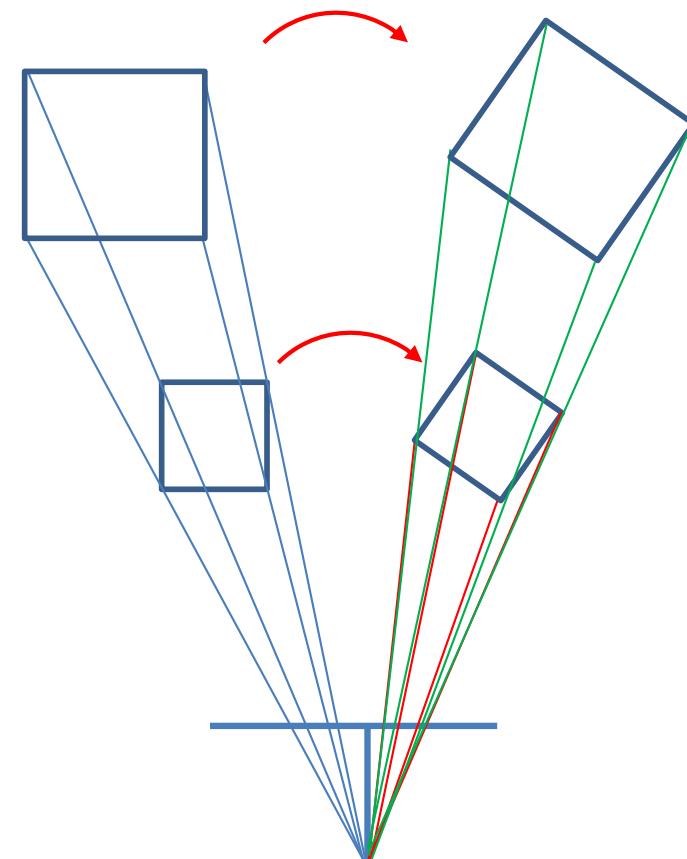
Translational – dependent on scene depth Z

Rotational – independent of scene depth Z

Translation, Rotation and Depth



Motion field \propto depth



Motion field $\not\propto$ depth

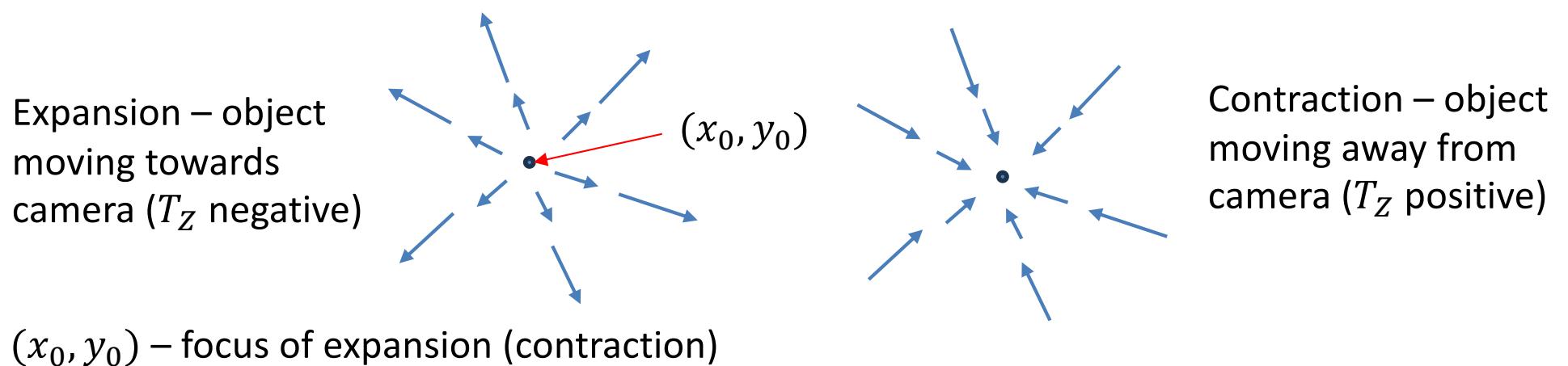
Special Case: Pure Translation

- Assume 3-D motion is only translational, $\Theta = 0$, then

$$v_x = (fT_X - xT_Z)/Z \quad v_y = (fT_Y - yT_Z)/Z$$

- If $T_Z \neq 0$, $x_0 = fT_X/T_Z$ and $y_0 = fT_Y/T_Z$, then

$$v_x = -(x - x_0)T_Z/Z \quad v_y = -(y - y_0)T_Z/Z$$



Special Case: Moving Plane

- Assume 3-D points lie in plane with unit surface normal \mathbf{N} , i.e. $\mathbf{N}^T \mathbf{P} = d$, where d is distance of plane from origin.
- Since $\mathbf{P} = Z\mathbf{p}/f$, this gives $Z(N_X x + N_Y y + N_Z f)/f = d$
- Substituting for Z in 2-D motion field:

$$v_x = \frac{1}{fd} (a_1 x^2 + a_2 xy + a_3 fx + a_4 fy + a_5 f^2)$$

$$v_y = \frac{1}{fd} (a_1 xy + a_2 y^2 + a_6 fy + a_7 fx + a_8 f^2)$$

- Motion field is a quadratic polynomial in 2-D spatial coordinates x and y

