COMS30030
Image Processing and Computer Vision

Motion Estimation

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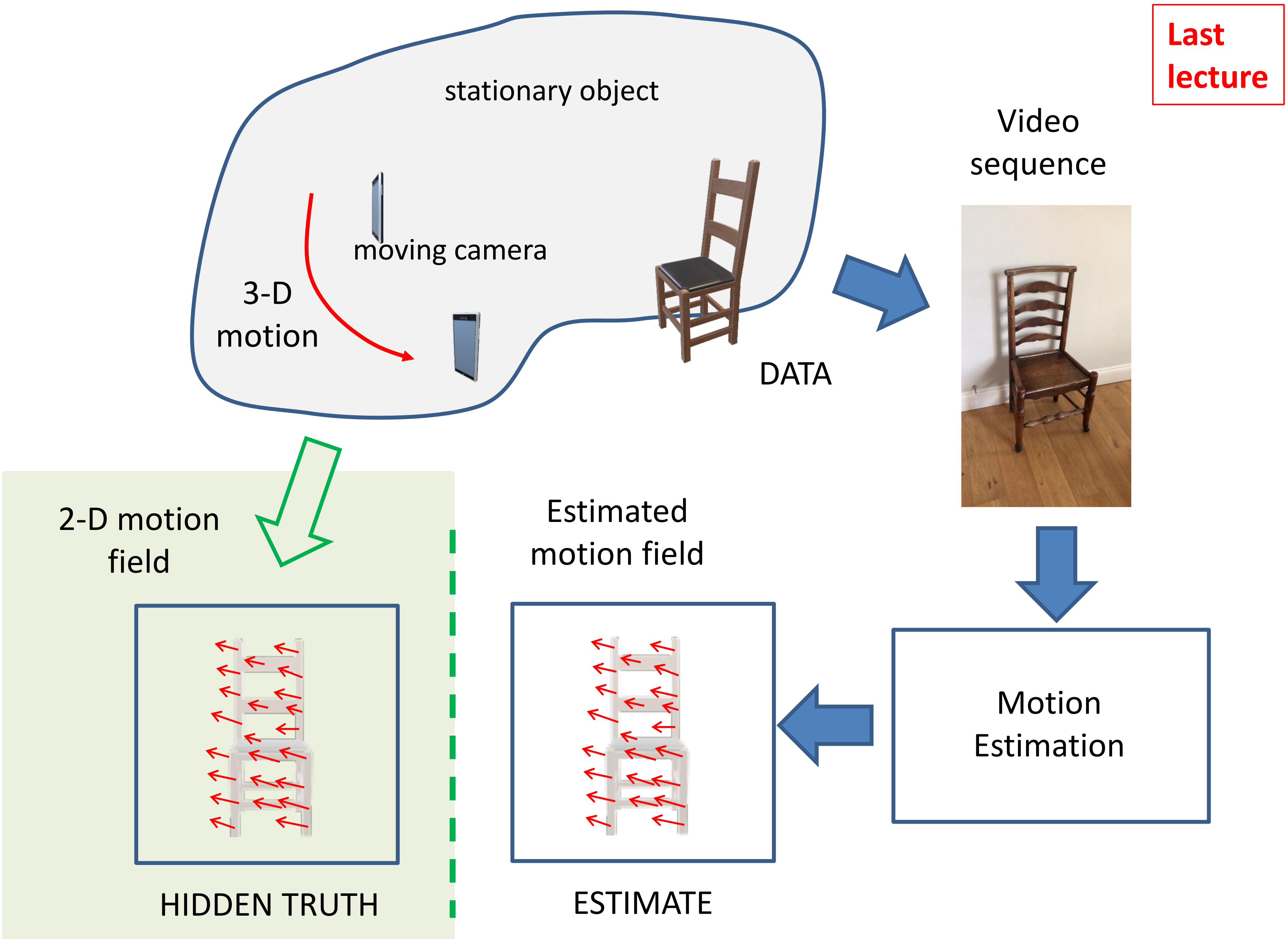


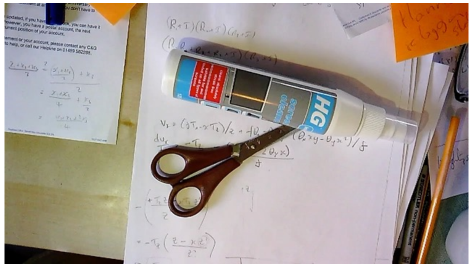
stationary object
moving camera
3-D motion
DATA
Video sequence
Last lecture
2-D motion field
Estimated motion field
Motion Estimation
HIDDEN TRUTH
ESTIMATE

Motion Estimation

- The estimation of the 2-D motion field from frames in an image sequence
- Using spatial and temporal variation of pixel values
- **BUT**- relationship between variation in pixel values – known as **apparent motion** or **optical flow** – and the true motion is not straightforward.

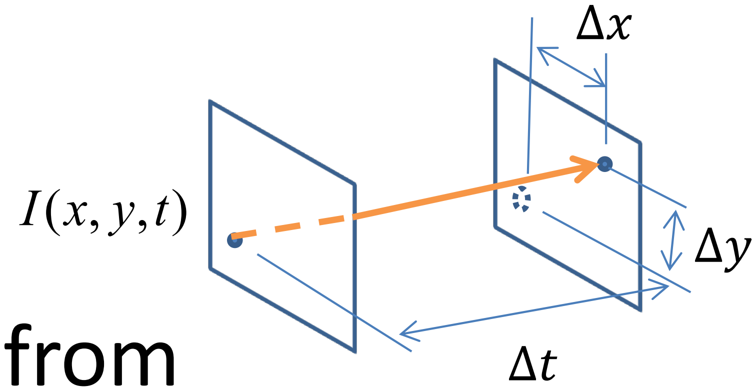
Apparent versus True Motion

- Apparent motion or **optical flow** - perceived motion in video sequence caused by changes in pixel values.
- Relationship with true 2-D motion field not always straightforward.
- Extreme cases:
  - non-zero apparent motion for zero motion field, e.g. static scene, moving light source
  - zero apparent motion for non-zero motion field, e.g. constant colour sphere rotating in diffuse lighting



- Sometimes not possible to determine 2-D motion field without additional constraints or assumptions.

Optical Flow



- Assume optical flow results from **brightness constancy constraint**
  - *`a moving pixel retains its value between frames'*
- For continuous video $I(x,y,t)$ (grey level)

$$I(x+\Delta x, y+\Delta y, t+\Delta t) = I(x,y,t)$$

HoT $\rightarrow$ 0 for tiny $\Delta x, \Delta y, \Delta t$

- Using Taylor's expansion:

$\longrightarrow$ zero

$$I(x+\Delta x, y+\Delta y, t+\Delta t) = I(x,y,t) + \frac{\delta I}{\delta x}\Delta x + \frac{\delta I}{\delta y}\Delta y + \frac{\delta I}{\delta t}\Delta t + \ldots\ldots$$

Optical Flow Equation

- For $I(x + \Delta x, y + \Delta y, t + \Delta t) = I(x, y, t)$

$$\frac{\delta I}{\delta x}\Delta x + \frac{\delta I}{\delta y}\Delta y + \frac{\delta I}{\delta t}\Delta t = 0$$

- Dividing throughout by Δt

$$\frac{\delta I}{\delta x}\frac{\Delta x}{\Delta t} + \frac{\delta I}{\delta y}\frac{\Delta y}{\Delta t} + \frac{\delta I}{\delta t} = 0$$

- For $\Delta x, \Delta y, \Delta t \to 0$

$$\frac{\delta I}{\delta x}\frac{dx}{dt} + \frac{\delta I}{\delta y}\frac{dy}{dt} + \frac{\delta I}{\delta t} = 0$$

Optical Flow Equation (OFE)

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$$\frac{\delta I}{\delta x}\frac{dx}{dt} + \frac{\delta I}{\delta y}\frac{dy}{dt} + \frac{\delta I}{\delta t} = 0$$

$\frac{dx}{dt}, \frac{dy}{dt}$ Rate of change of x, y with time

⇨ optical flow field $\mathbf{u} = (u_x, u_y)$

$\frac{\delta I}{\delta x}, \frac{\delta I}{\delta y}, \frac{\delta I}{\delta t}$ Rate of change of I with x, y, t

⇨ spatial & temporal gradients (I_x, I_y, I_t)

Optical flow equation

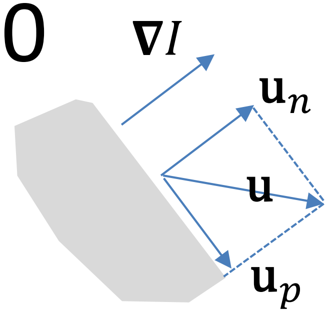
$$\boxed{I_x u_x + I_y u_y + I_t = 0}$$

Normal Flow

- OFE for $\mathbf{u} = (u_x, u_y)$ and $\nabla I = (I_x, I_y)$

$$I_x u_x + I_y u_y + I_t = 0 \Rightarrow \nabla I . \mathbf{u} + I_t = 0$$

$$\nabla I . \mathbf{u} = I_x u_x + I_y u_y$$ dot product



- OFE alone not sufficient to estimate motion
  - one equation in two unknowns
- Only estimate **normal flow** $\mathbf{u}_n$

$$\nabla I . \mathbf{u} + I_t = \nabla I . \mathbf{u}_n + I_t = 0$$

$$\Rightarrow \|\mathbf{u}_n\| = -I_t / \|\nabla I\| \quad \angle \mathbf{u}_n = \angle \nabla I$$

$$\mathbf{u} = \mathbf{u}_p + \mathbf{u}_n$$

$$\mathbf{u}_p . \mathbf{u}_n = 0$$

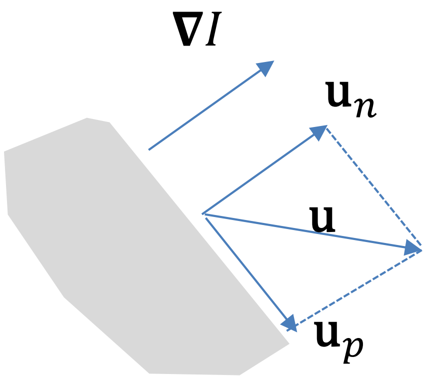
$$\nabla I . \mathbf{u}_p = 0$$

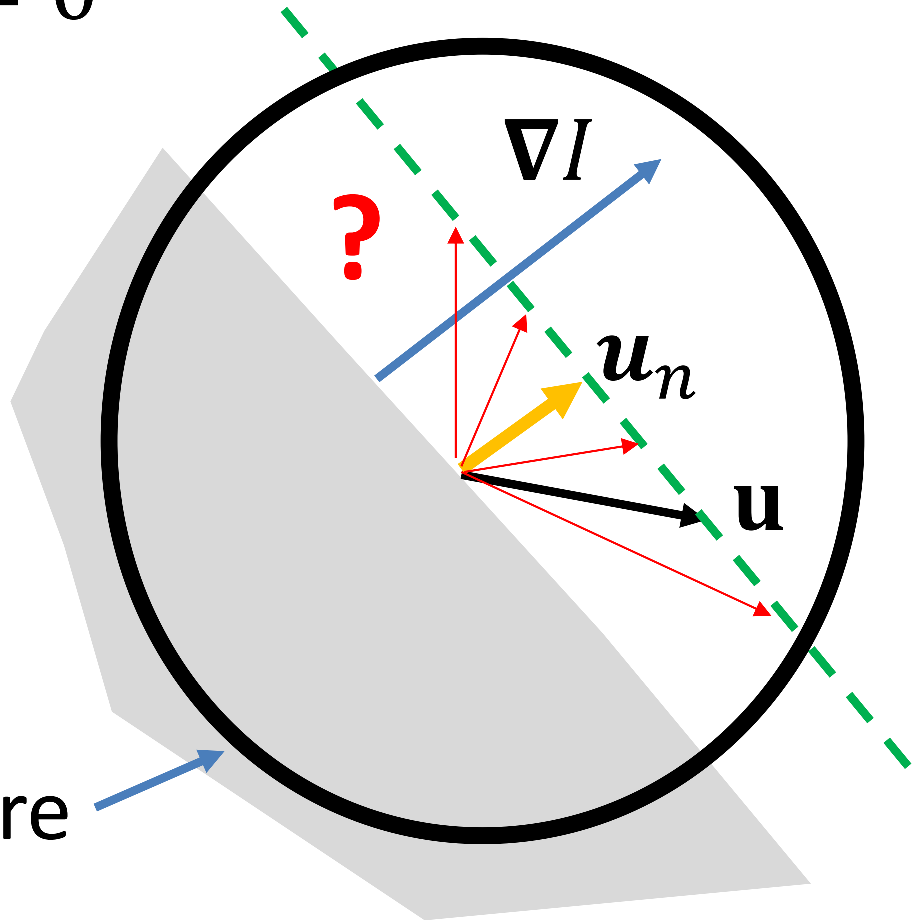
Normal Flow

$$\boldsymbol{\nabla} I . \mathbf{u} + I_t = \boldsymbol{\nabla} I . \boldsymbol{u}_n + I_t = 0$$

⇨ $\|\mathbf{u}_n\| = -I_t / \|\boldsymbol{\nabla} I\|$

$$\angle \mathbf{u}_n = \angle \boldsymbol{\nabla} I$$

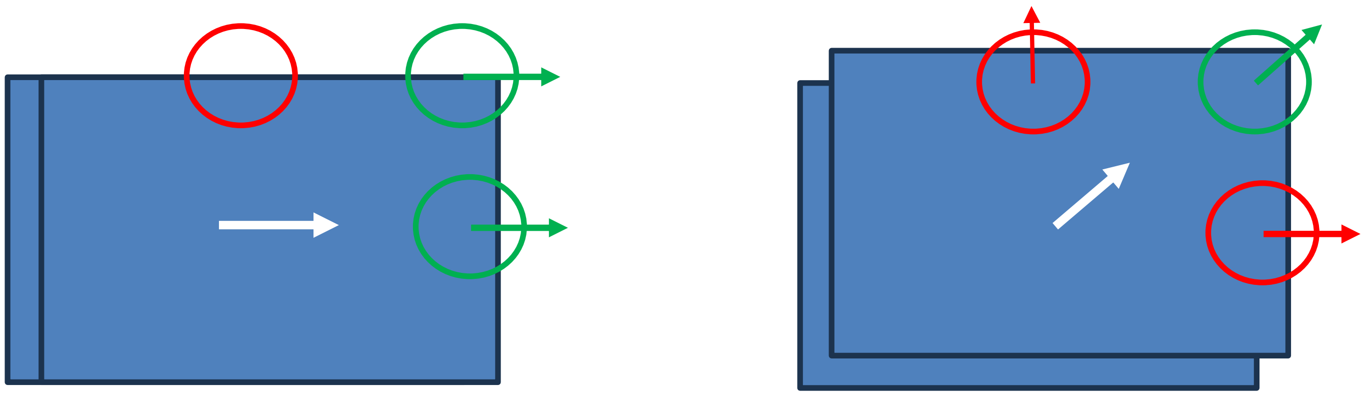




aperture

Aperture Problem

With single gradient direction in window (aperture), observed motion is different from true motion as we can only observe motion parallel to the gradient:



Hence: Good motion estimation depends on having sufficient variation in spatial gradient within regions.

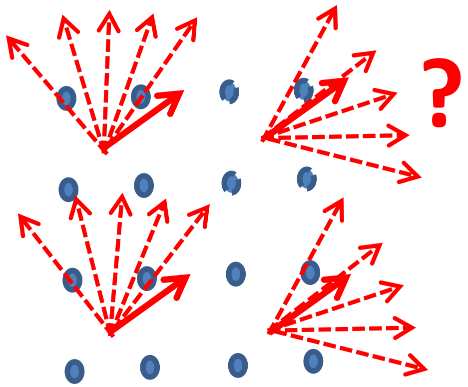
Constraining the OFE

$$I_x u_x + I_y u_y + I_t = 0$$

OFE is under constrained – can only estimate normal flow

Need to add extra constraint(s)

Example : assume parametric form of motion field in regions



Example : constant velocity

Constraining the OFE

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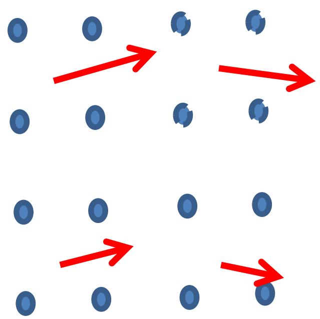
Example : assume parametric form of motion field in regions



Example : linear in x and y, e.g.

$$u_x = ax + by + c$$
$$u_y = dx + ey + f$$

Constant Velocity Model

For a region, find the velocity $\mathbf{u} = (u_x, u_y)$ which minimises :

$$\varepsilon(u_x, u_y) = \sum_{region} \underbrace{\left(I_x u_x + I_y u_y + I_t\right)^2}$$

Solution: take derivatives w.r.t u_x and u_y, set to zero, and solve for u_x and u_y.

OFE $\longrightarrow$ 0

NB: same $\mathbf{u} = (u_x, u_y)$ over whole region → solution

Lucas and Kanade Algorithm

Find velocity $\mathbf{u} = (u_x, u_y)$ which minimises :

$$\varepsilon(u_x, u_y) = \sum_R (I_x u_x + I_y u_y + I_t)^2$$

Partial derivatives w.r.t u_x and u_y, set to zero, solve for u_x and u_y:

$$\frac{\partial \varepsilon}{\partial u_x} = 2 \sum_R (I_x u_x + I_y u_y + I_t) I_x = 0 \quad \Rightarrow \quad \sum_R (I_x^2 u_x + I_x I_y u_y + I_x I_t) = 0$$

$$\frac{\partial \varepsilon}{\partial u_y} = 2 \sum_R (I_x u_x + I_y u_y + I_t) I_y = 0 \quad \Rightarrow \quad \sum_R (I_x I_y u_x + I_y^2 u_y + I_y I_t) = 0$$

Lucas and Kanade Algorithm

Hence, solve for $\mathbf{u} = (u_x, u_y)$ given that :

$$u_x \sum_R I_x^2 + u_y \sum_R I_x I_y = -\sum_R I_t I_x$$

$$u_x \sum_R I_x I_y + u_y \sum_R I_y^2 = -\sum_R I_t I_y$$

$$\Rightarrow \quad A\mathbf{u} = \mathbf{b}$$

$$\Rightarrow \quad \boxed{\mathbf{u} = \begin{bmatrix} u_x \\ u_y \end{bmatrix} = A^{-1}\mathbf{b}}$$

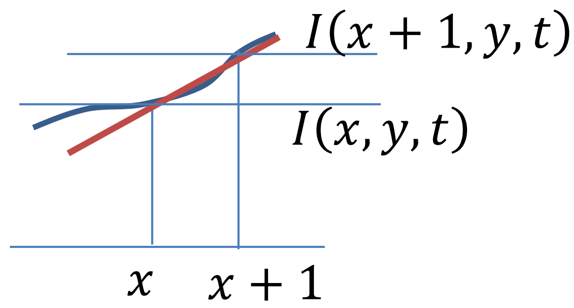
$$A = \sum_R \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \quad \mathbf{b} = -\sum_R \begin{bmatrix} -I_t I_x \\ -I_t I_y \end{bmatrix}$$

Spatial & Temporal Gradients

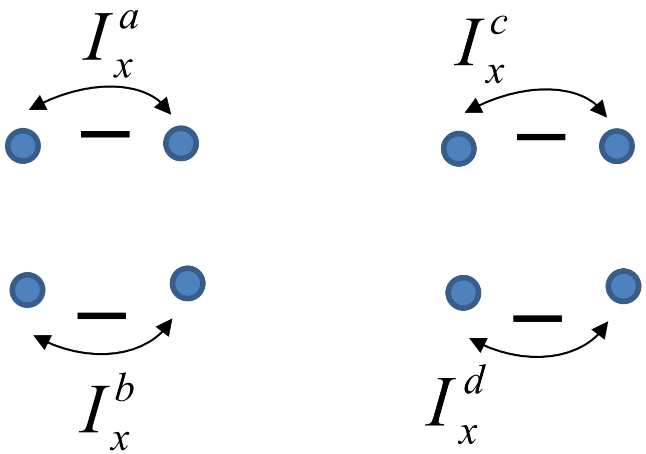
Approximate gradients using differences, e.g.

$$I_x = \delta I / \delta x \approx I(x+1, y, t) - I(x, y, t)$$

i.e. assume $\delta x = 1$



Or use averaging to reduce noise, e.g.



Frame t Frame $t+1$

$$I_x \approx (I_x^a + I_x^b + I_x^c + I_x^d)/4$$

Rate of change of I with x

L & K Algorithm

$I^1 = video\ frame\ at\ time\ t$

$I^2 = video\ frame\ at\ time\ t+1$

For each pixel x, y in I^1

$A = 0\ ;\ \ \mathbf{b} = 0\ ;$

For each pixel in region Λ about x, y

$$(I_x, I_y, I_t) = CompGrads(I^1, I^2)\ ;$$

$$A' = \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix};$$

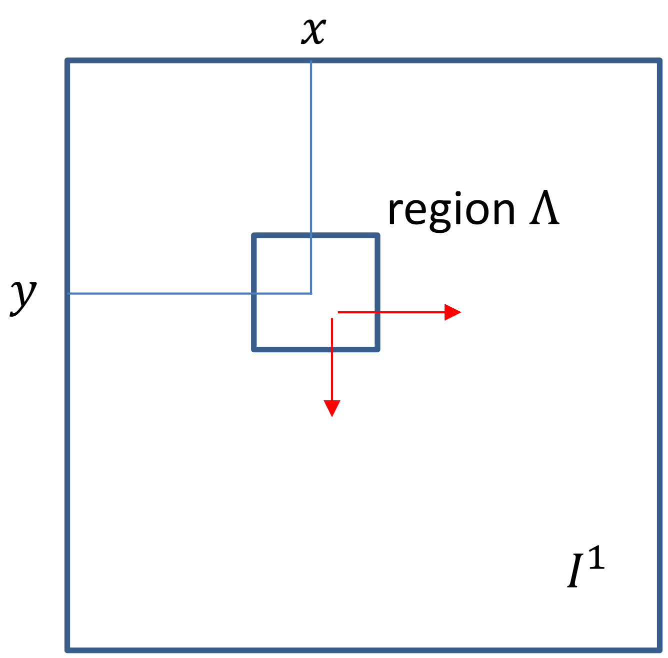
$$\mathbf{b}' = \begin{bmatrix} -I_t I_x \\ -I_t I_y \end{bmatrix};$$

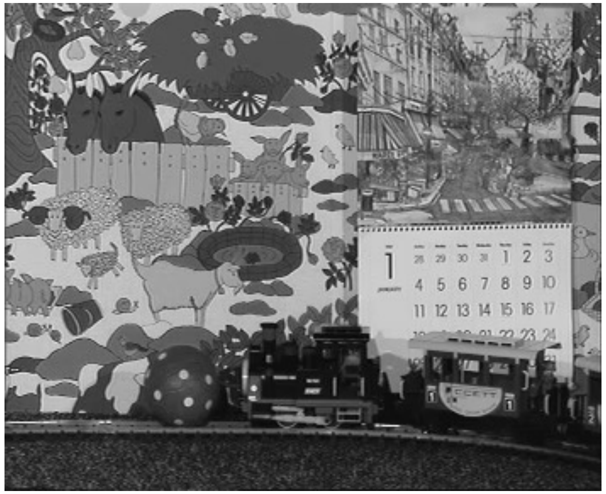
$$A \rightarrow A + A'\ ;\ \ \mathbf{b} \rightarrow \mathbf{b} + \mathbf{b}'\ ;$$

End;

$\mathbf{u}(x, y) = A^{-1}\mathbf{b}$

End;

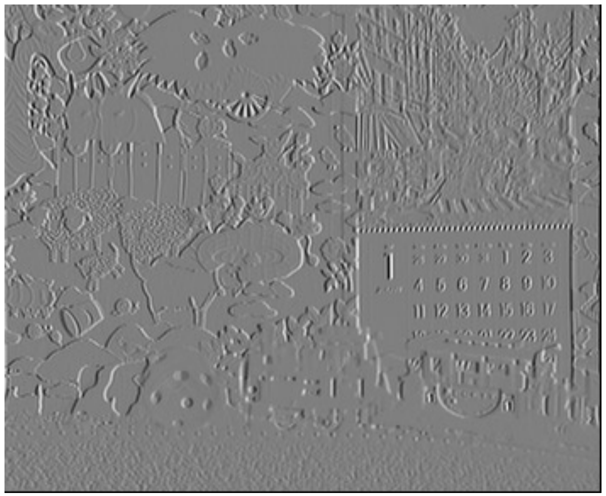




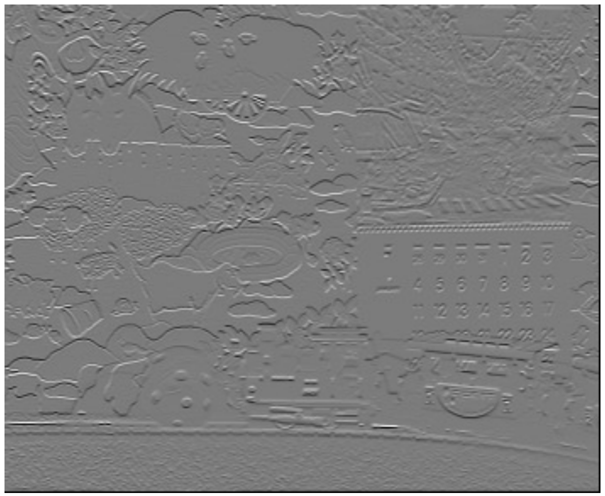
$I(x, y, t)$

u_x

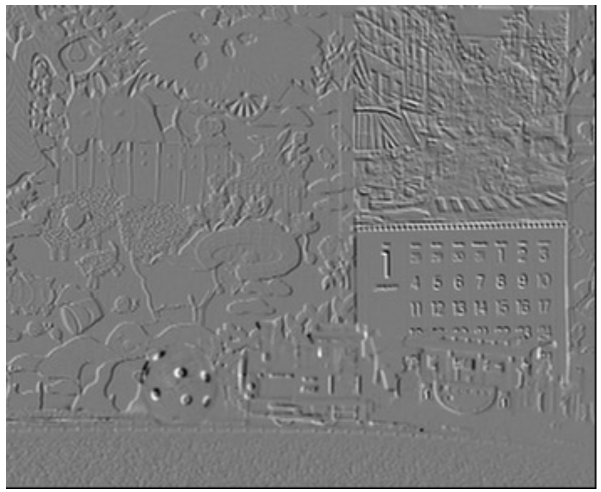
u_y



I_x

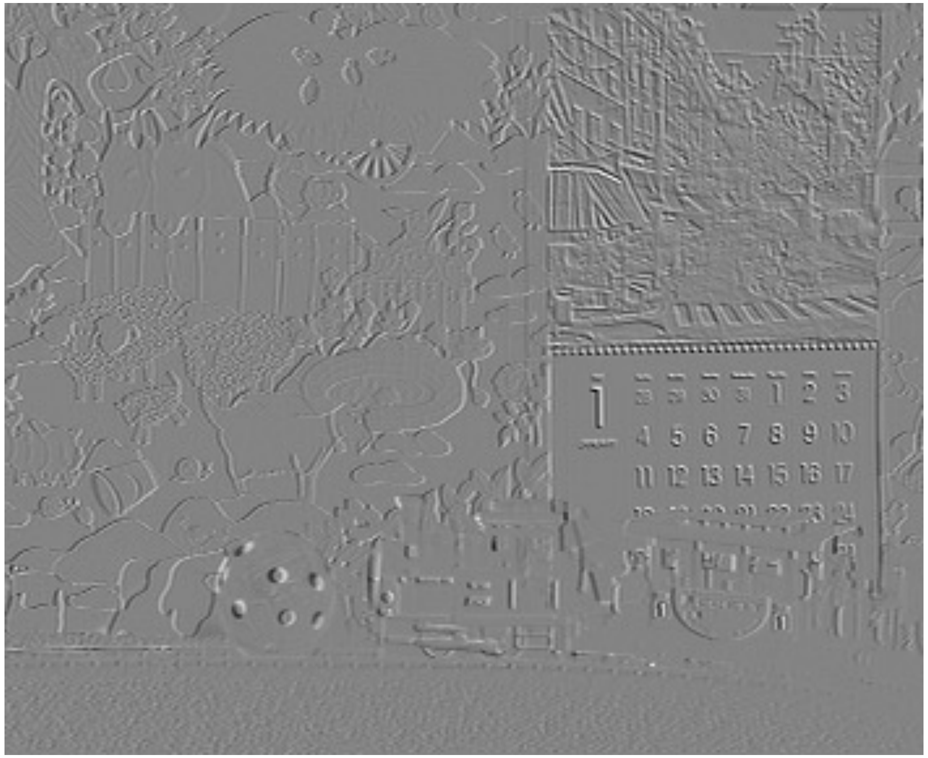


I_y

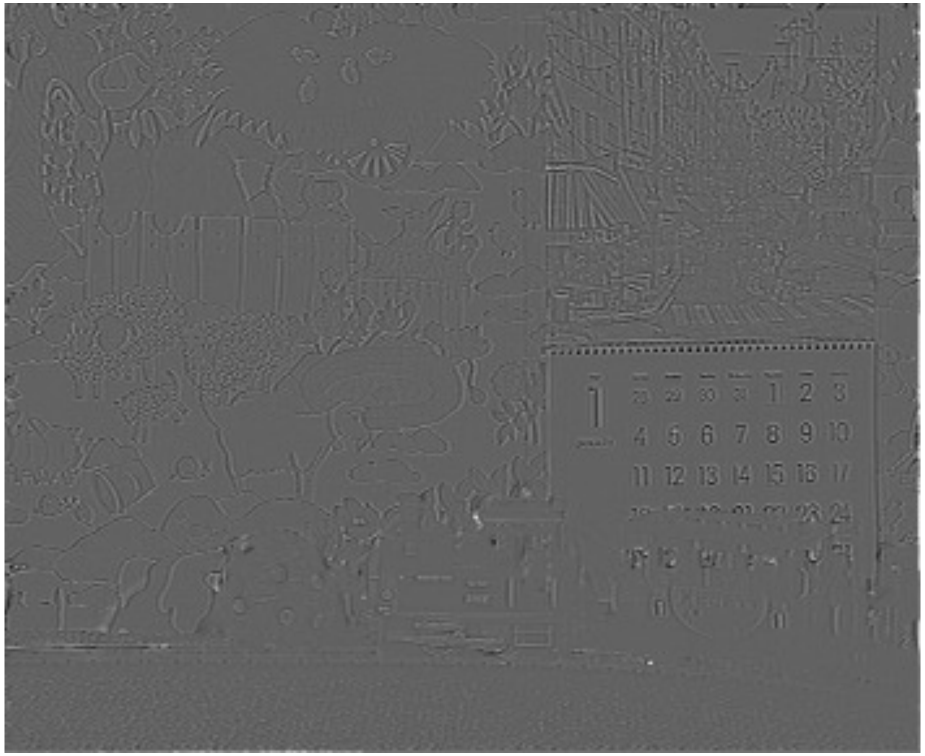


I_t

Frame Difference



frame difference



motion compensated frame difference

Motion Estimation - Example

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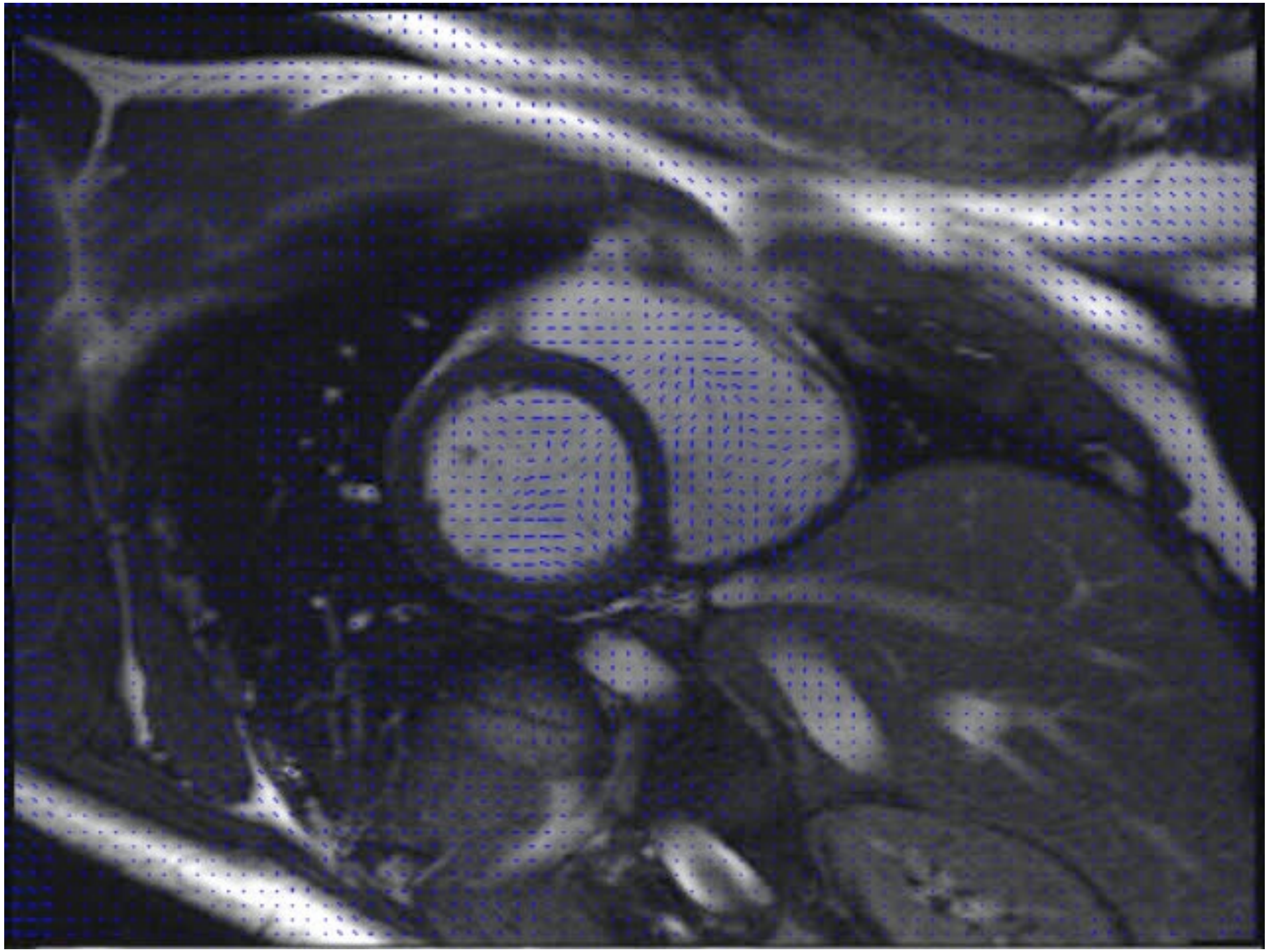
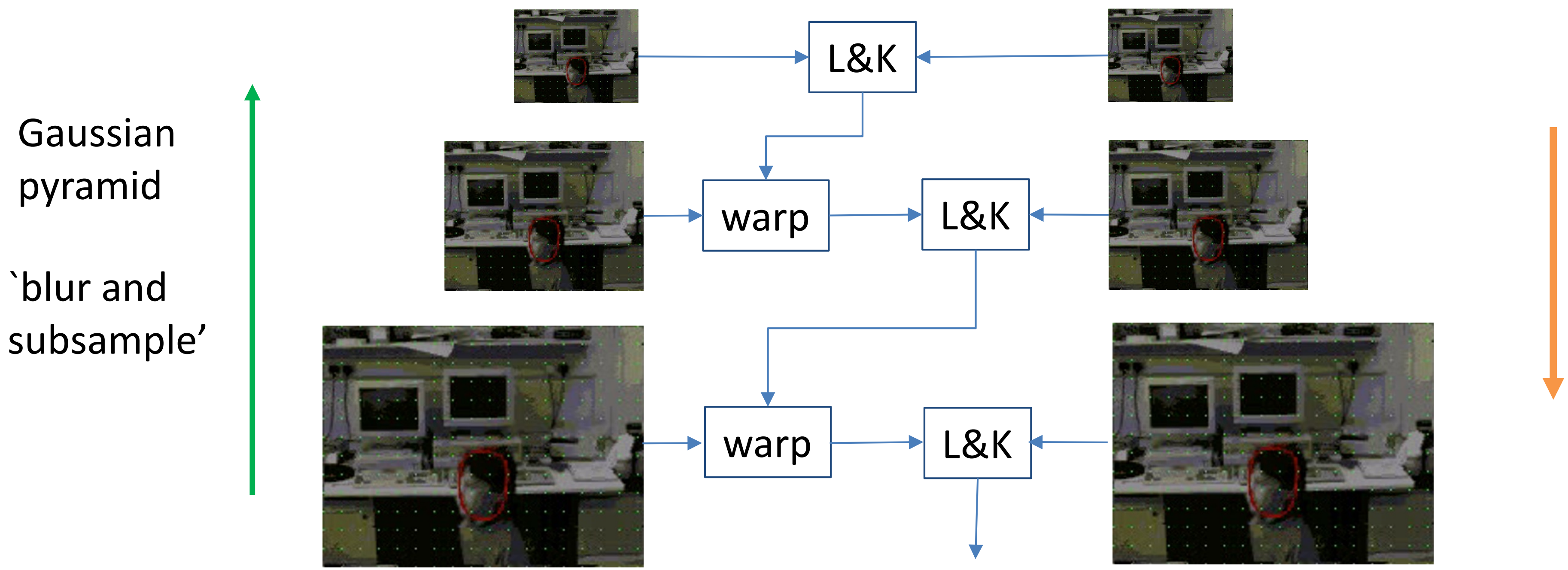
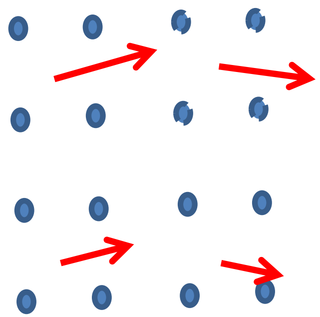


Multiresolution L & K

- To deal with large motions, implement L&K over multiple resolutions – result at lower resolutions used to `warp’ higher resolution images prior to estimation.



Affine Motion Model



$$\boxed{\begin{aligned} u_x &= ax + by + c \\ u_y &= dx + ey + f \end{aligned}}$$

$$\Rightarrow \quad \mathbf{u} = A\,\mathbf{p}$$

$$\mathbf{p}^T = (a, b, c, d, e, f)$$

Models translation, scaling, rotation and shear

$$\Rightarrow \quad \mathbf{p}^T A^T \nabla I + I_t = 0 \quad \text{affine OFE} \quad \Rightarrow \quad \hat{\mathbf{p}} = M^{-1}\mathbf{b}$$

$$M = \sum_{region} A^T \nabla I \nabla I^T A \qquad b = -\sum_{region} I_t (A^T \nabla I)$$

Horn-Schunk Algorithm

- Alternative to L&K which seeks to find optimal motion field with smooth variation in motion vectors
- Algorithm aims to find the motion field $v = (v_x, v_y)$ which minimises following energy functional

$$E = \iint \left[\underbrace{\left(I_x u_x + I_y u_y + I_t\right)^2}_{\boxed{\text{OFE} \longrightarrow 0}} + \sigma^2 \underbrace{\left(\|\nabla u_x\|^2 + \|\nabla u_y\|^2 \right)}_{\text{Rate of change of } \mathbf{v} \to \mathbf{0}} \right] dxdy$$

weighting factor

Rate of change of $\mathbf{v} \rightarrow \mathbf{0}$

⇩

smooth motion field