

COMS30030 – Image Processing and Computer Vision

Problem Sheet MM02 – with Answers

Fourier

1 – Which of the following fact(s) about the frequency domain is/are true?

- A. Moving away from the origin of transform the higher frequency components correspond to smooth graylevel variation.
- B. Moving away from the origin of transform the lower frequencies correspond to abrupt change in gray level values.
- C. frequencies within a specified range are bandlimited when the rest of the spectrum goes to zero.
- D. A, B & C are all TRUE.
- E. Only C is TRUE.

Answer: E

2 – How would low pass filtering be achieved using the Fourier domain? In your answer describe what is meant by Cut-off Frequency.

Answer: Low pass filtering can be achieved by removing higher frequency information in the Fourier space, i.e. by retaining and letting lower frequencies pass through a filtering operation. Example filters are the ideal low pass filter and the Butterworth low pass filter. Some filter types have an abrupt cut-off point above which no higher frequencies are passed through, while others, like a Butterworth or Gaussian based filters, are more smoothly varying and do not have an abrupt cut-off point.

3 – Consider you are given the Fourier Transform space of an image. Using simple descriptions or sketches to illustrate your answer, how would you select relevant regions to extract spectral features from

- (a) only low frequency regions,
- (b) only the very high frequency regions corresponding to prominent variations in intensity in the image that are at around 45 degrees to the horizontal,
- (c) all approximately mid-range frequencies.

Answer:

Using conjugate symmetry, we can ignore the bottom half of the Fourier space and extract features from only the top half. We can then extract features from regions defined as

(a) for example a half disc-shaped region with its centre at the centre of the Fourier space, i.e. at $(u=0, v=0)$

(b) a bar-shaped region with a small width, say 10, starting at around $(u = -\max(u_{\text{freq}})/2, v = \max(v_{\text{freq}})/2)$ angled at 135 degrees in the Fourier space

(c) a half-ring of a reasonable width, up to a maximum of $2 \cdot \max(u_{\text{freq}})/3$, and starting from around $(u = \max(u_{\text{freq}})/3, v = 0)$.

NOTE: Exact u, v coordinates are not necessary, but approximations plus sketches should give the right indication, e.g. the half-disc must clearly be said to be at the centre of the space.

4 – Rotate an object, Fourier space rotates too. Translate an object, Fourier space translates too.

- A. Both statements are **True**.
- B. **First statement is True and second one is False.**
- C. First statement is **False** and the second one is **True**.
- D. Both statements are **False**.

Answer: B

5 – Consider how the Fourier domain of a signal is affected under the following operations:

(i) translation of an object within an image on a uniform background (ii) rotation of an object within an image on a uniform background.

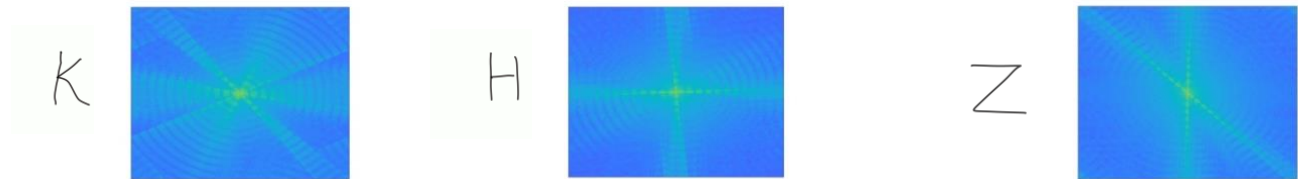
Which of these is TRUE:

- A. Under translation, the frequency magnitudes in the Fourier domain are shifted in a positive direction by the amount of translation. Under rotation, the Fourier domain magnitudes are rotated by an amount corresponding to the rotated object.
- B. Under translation, the Fourier domain is not affected and the frequency magnitudes retain their position. Under rotation, the Fourier domain magnitudes also remain in the same position.
- C. Under translation, the Fourier domain is not affected and the frequency magnitudes retain their position. Under rotation, the Fourier domain magnitudes are rotated by an amount corresponding to the rotated object.
- D. Under translation, the frequency magnitudes in the Fourier domain are shifted in a negative direction to the translation. Under rotation, the Fourier domain magnitudes remain the in the same position.
- E. None of the above are true.

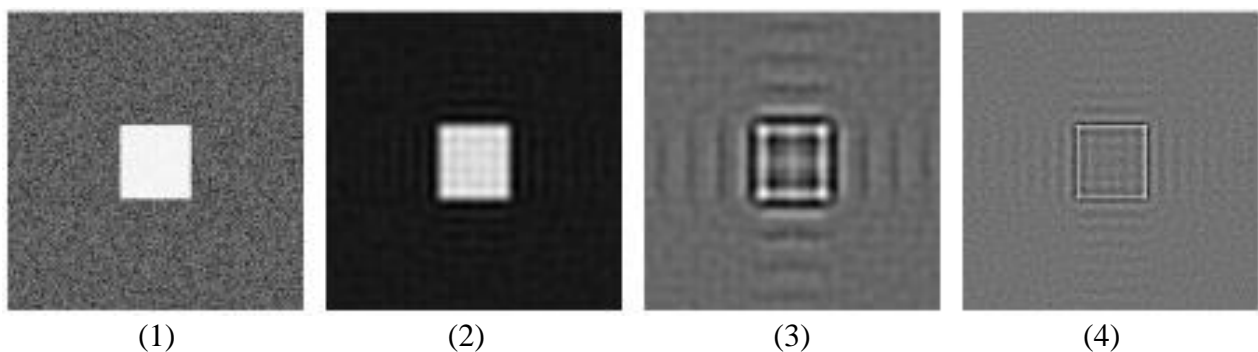
Answer: C

6 – Here are images of three handwritten letters. Their Fourier spaces are randomly shown. Match each image with its own Fourier image.

Answer:



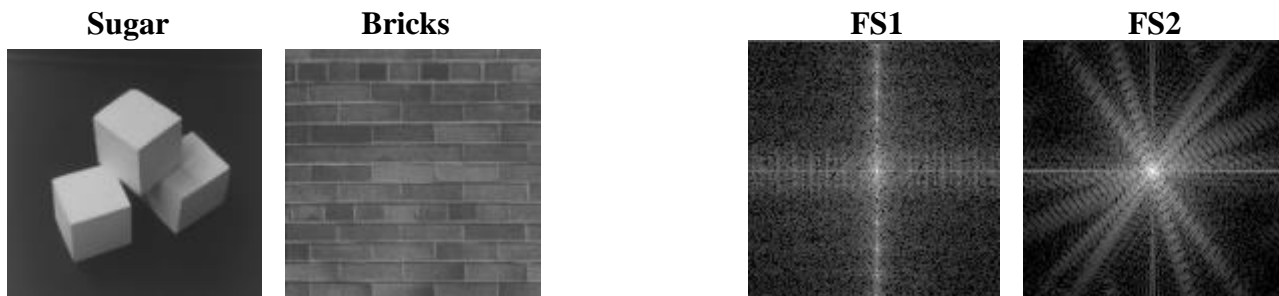
7 – Given the four images (1) to (4) below, select from statements A to D the one which is CORRECT or, if all statements A to D are INCORRECT, select option E.



- A. Image (4) results from applying a lowpass filter to image (1).
- B. Image (3) results from applying a bandpass filter to image (1).
- C. Image (2) results from applying a highpass filter to image (1).
- D. Image (3) results from applying a highpass filter to image (1).
- E. All of the above are INCORRECT.

Answer: D – Image (3) is the bandpass-filtered signal of image (1).

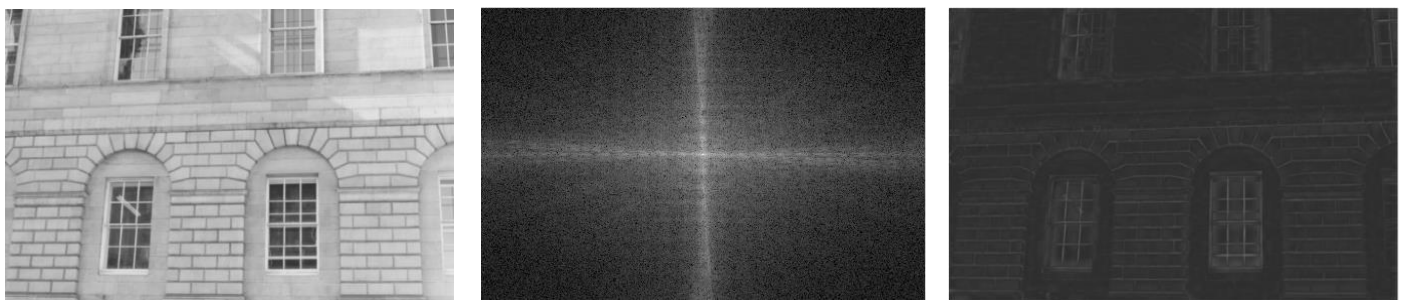
8 – Similar to the previous question, consider the two images (Sugar and Bricks) on the left. Identify which of the Fourier spaces (FS1 and FS2) on the right belongs to which image and explain briefly why.



Answer:

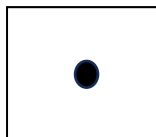
FS2 belongs to the sugar blocks image and FS1 belongs to the brick image. The high magnitude frequencies in FS1 are for the Brick image as they clearly signify the presence of very strong horizontal and vertical lines in that image. The angled lines in the sugar blocks image result in the strong non-horizontal and non-vertical directional lines in FS2.

9 – The figure below on the left shows an image of a building wall, with its Fourier Space magnitudes shown in the middle. A reconstructed image (inverse FFT image), after some manipulation of the Fourier magnitudes, is shown on the right. How should the Fourier space be manipulated (e.g., what kind of a mask could have been applied to it) to achieve this reconstructed result? Include a sketch to illustrate your answer.



Answer:

Much of the contrast has been removed and an almost edge-map of the image has resulted. Edges signify high frequency changes in the image pixels. Hence, all this evidence points to a loss of low frequency magnitudes. The mask applied to the Fourier space magnitudes is therefore something similar to this:



10 – Which of the following statement(s) is true for the given fact that “Applying Highpass filters has an effect on the background of the output image”?

- A. The average background intensity increases to near white
- B. The average background intensity reduces to near black
- C. The average background intensity changes to a value average of black and white
- D. All of the mentioned
- E. None of the mentioned

Answer:

B - The Highpass filter eliminates the zero frequency components of the Fourier transformed image HPFs are applied on. So, the average background intensity reduces to near black.

11 – The Butterworth lowpass filter has a parameter, filter order, determining its functionality as very sharp or very smooth filter function or an intermediate filter function. If the parameter value is very high, the filter approaches to which of the following filter(s)?

- A. Ideal lowpass filter
- B. Gaussian lowpass filter
- C. All of the mentioned
- D. A very customised filter
- E. None of the mentioned

Answer:

A - For high value of filter order the Butterworth lowpass filter behaves as the Ideal lowpass filter, while for lower order value it has a smoother form behaving like the Gaussian lowpass filter.