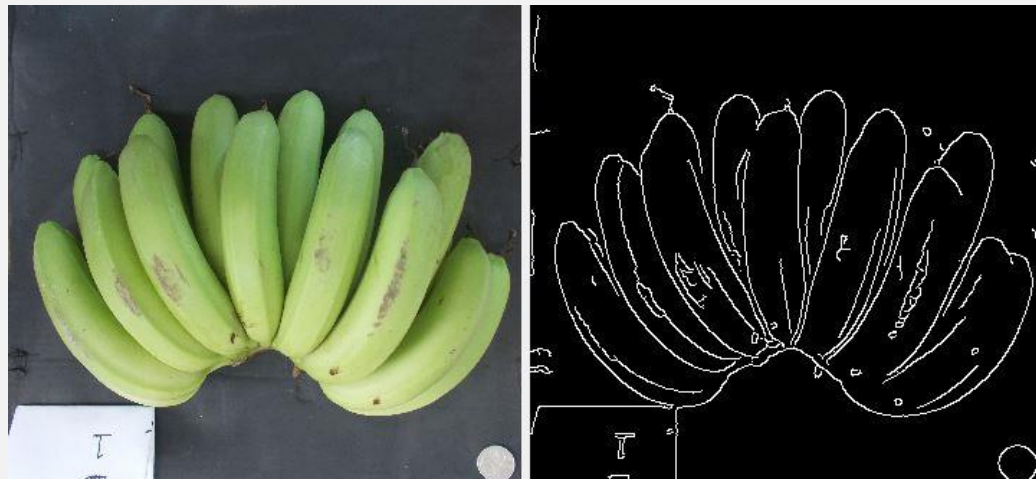


# COMS30030 - Image Processing and Computer Vision



Lecture 04

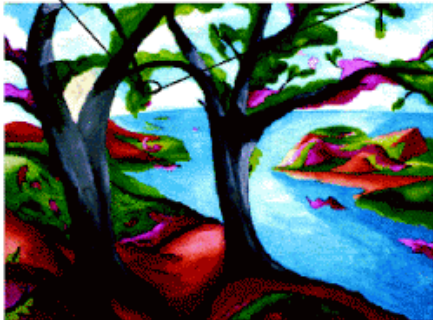
## Edge & Shape Detection

Majid Mirmehdi | [majid@cs.bris.ac.uk](mailto:majid@cs.bris.ac.uk)

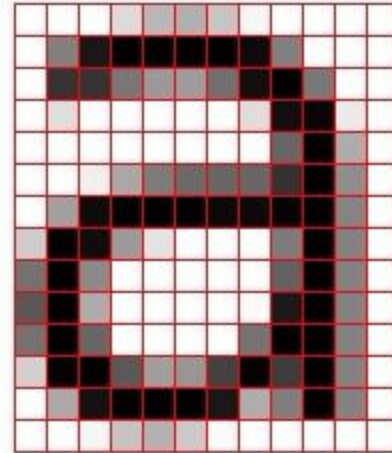
# Beyond the Matrix

- Images are matrices of numbers
- Shapes are more informative than individual pixel values

	0.2235	0.1294	<b>Blue</b>	0.4198	
0.5804	0.2902	<b>0.0627</b>	0.2902	0.2902	0.4824
0.5804	0.0627	0.0627	0.0627	0.2235	0.2588
0.5176	0.1922	0.0627	<b>Green</b>	0.1922	0.2588
0.5176	0.1294	<b>0.1608</b>	0.1294	0.1294	0.2588
0.5176	0.1608	0.0627	0.1608	0.1922	0.2588
0.5490	0.2235	0.5490	<b>Red</b>	0.7412	0.7765
0.5490	0.3882	<b>0.5176</b>	0.5804	0.5804	0.7765
0.490	0.2588	0.2902	0.2588	0.2235	0.4824
0.2235	0.1608	0.2588	0.2588	0.1608	0.2588
0.2588	0.1608	0.2588	0.2588	0.2588	0.2588



Source: <http://www.eon.northwestern.edu/local-apps/matlabhelptoolbox/images/intro8.html>



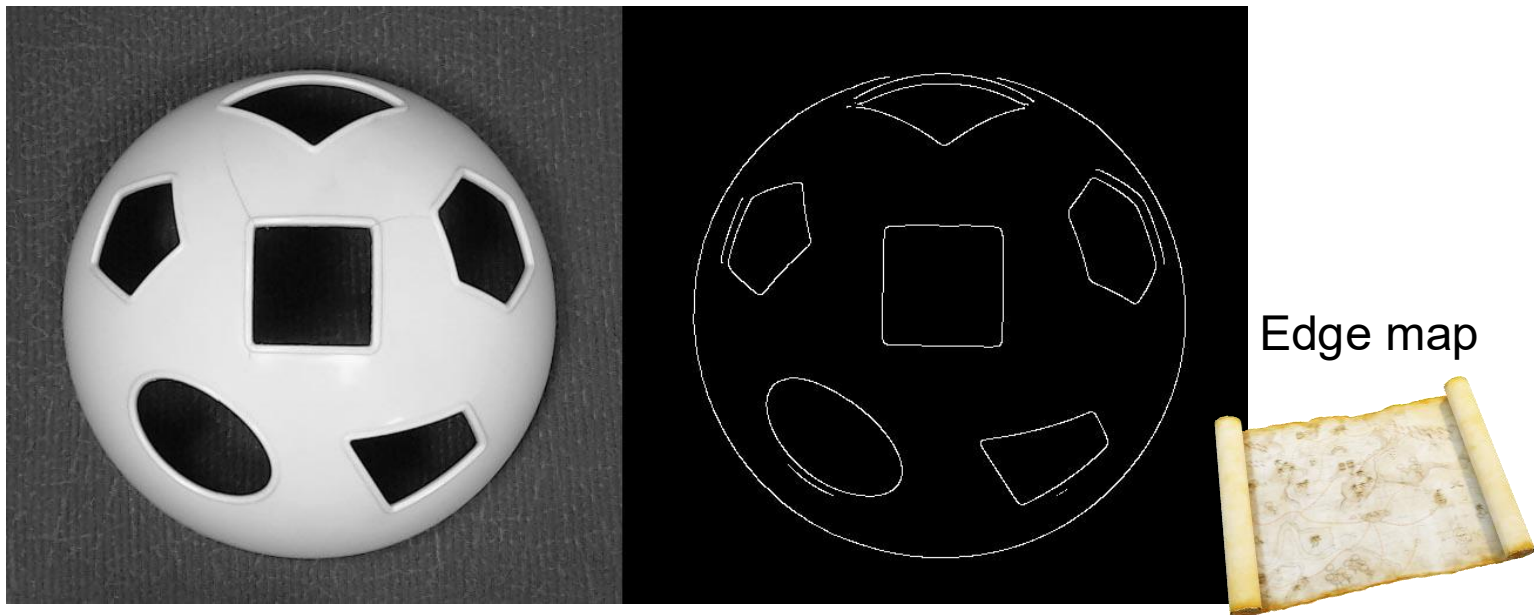
=

1	0	1	0	1	0	0	9	0	6	0	6	0	6	1	0	1	0	1	0	1	0
1	0	0	5	0	0	0	0	0	0	0	0	0	0	0	5	1	0	1	0	1	0
1	0	0	2	0	2	0	5	0	6	0	6	0	5	0	0	0	0	5	1	0	1
1	0	0	9	1	0	1	0	1	0	1	0	1	0	0	9	0	0	0	0	9	1
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	5	0	0	5	1
1	0	1	0	1	0	0	5	0	5	0	5	0	5	0	4	0	0	0	5	1	0
1	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	1	0
0	9	0	0	0	0	6	1	0	1	0	1	0	1	0	0	5	0	0	5	1	0
0	5	0	0	6	1	0	1	0	1	0	1	0	1	0	0	5	0	0	5	1	0
0	5	0	0	7	1	0	1	0	1	0	1	0	1	0	0	0	0	0	5	1	0
0	6	0	0	6	1	0	1	0	1	0	1	0	0	5	0	0	0	0	5	1	0
0	9	0	1	0	0	6	0	7	0	7	0	5	0	0	0	5	0	0	5	1	0
1	0	0	7	0	1	0	0	0	0	0	0	1	0	9	0	8	0	0	5	1	0
1	0	1	0	1	0	0	8	0	8	0	9	1	0	1	0	1	0	1	0	1	0

Source: <https://www.data-science-central.com/profiles/blogs/image-classification-with-hsv-color-model-processing>

# What are edges?

- Edges highlight the contour of shapes
- They can be used to identify objects

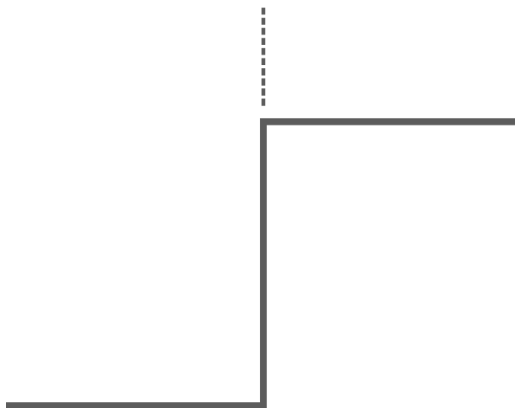


Source: <http://www.valrusvision.com/wordpress/introduction-to-edge-detection/>

# What are edges?

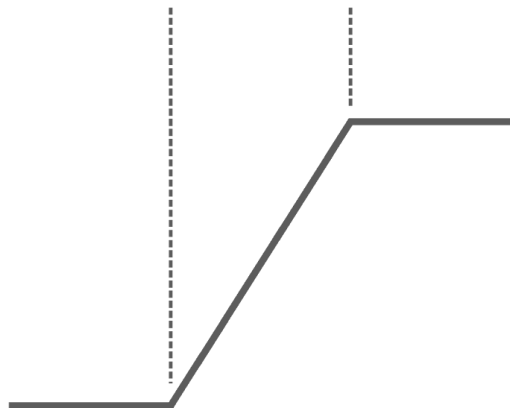
- Edges are those places in an image that correspond to object boundaries.
- Edges are pixels where image brightness changes (relatively) abruptly.

*Model of an ideal digital edge*



Gray level profile of a horizontal line through the image

*Model of a ramp digital edge*



Gray level profile of a horizontal line through the image

# Why detect Edges?

- **Edges:** Sharp changes of image brightness
- **Sources:** Object boundaries, patterns, shadows, etc.
- For segmentation: finding object boundaries



# Why detect Edges?

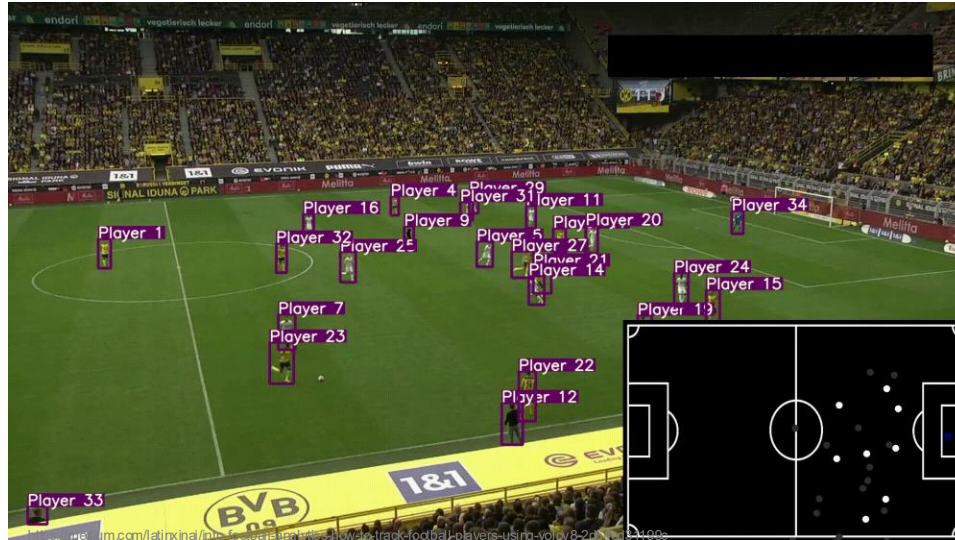
- **Edges:** Sharp changes of image brightness
- **Sources:** Object boundaries, patterns, shadows, etc.
- For segmentation: finding object boundaries
- For recognition: extracting patterns





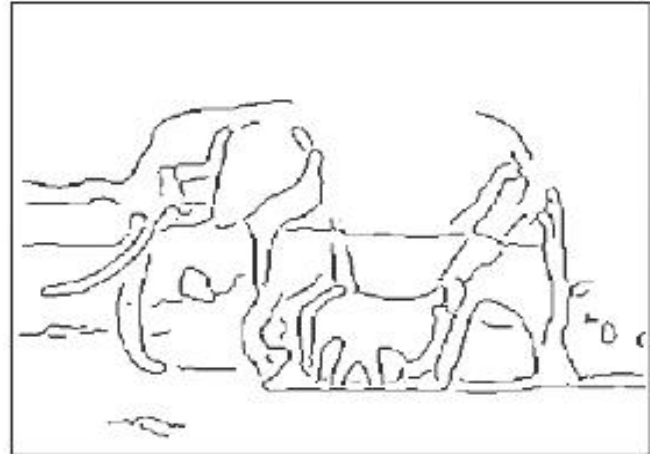
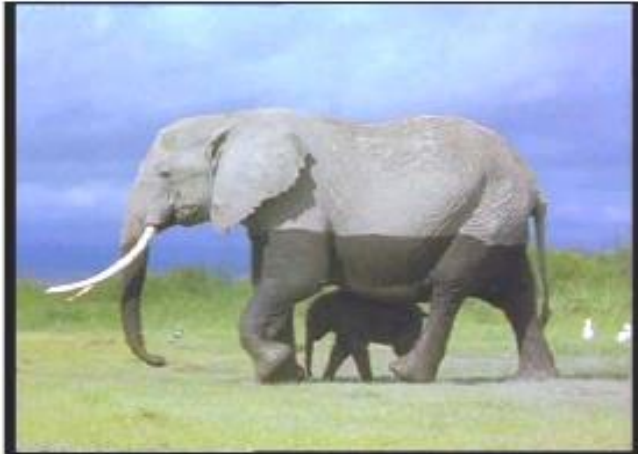
# Why detect Edges?

- **Edges:** Sharp changes of image brightness
- **Sources:** Object boundaries, patterns, shadows, etc.
- For segmentation: finding object boundaries
- For recognition: extracting patterns
- For motion analysis: reliable tracking regions



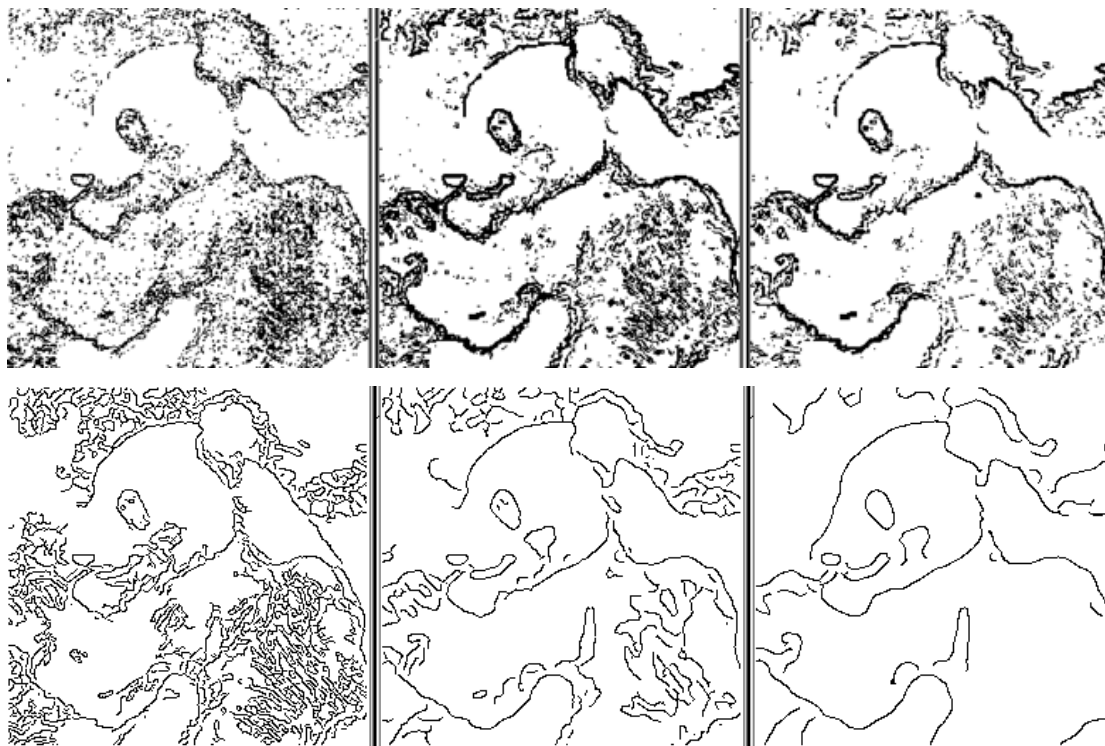
# Meaningful edges vs. nuisance edges

- **Edges:** Sharp changes of image brightness



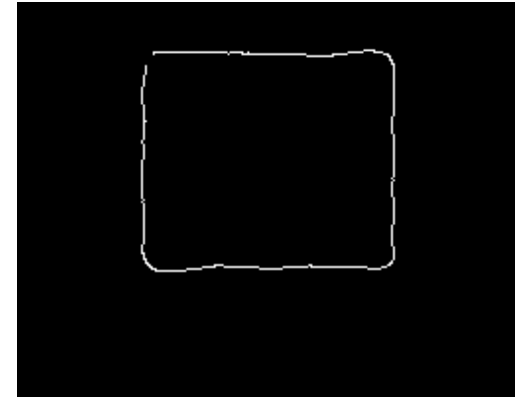
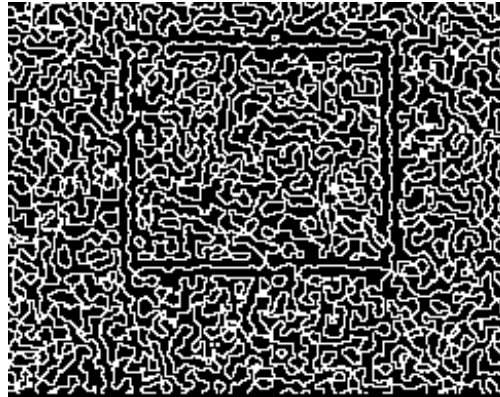
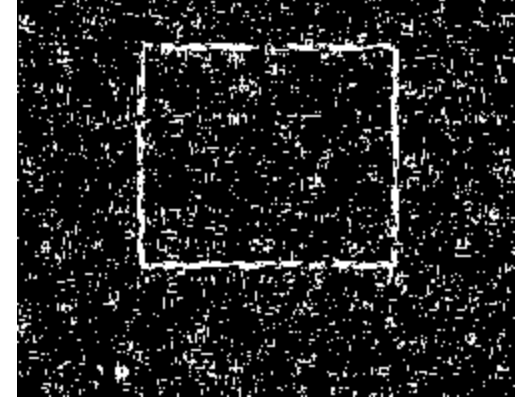
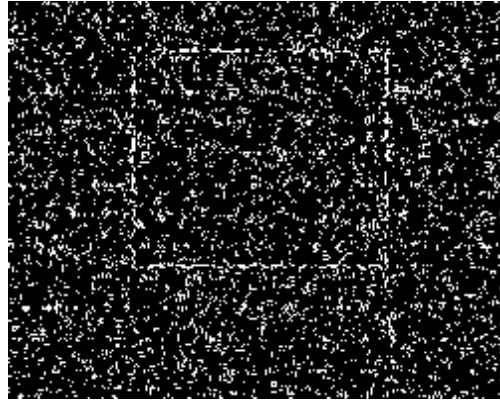
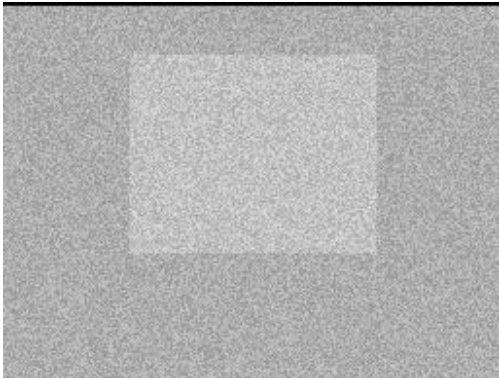


# Difficult edges



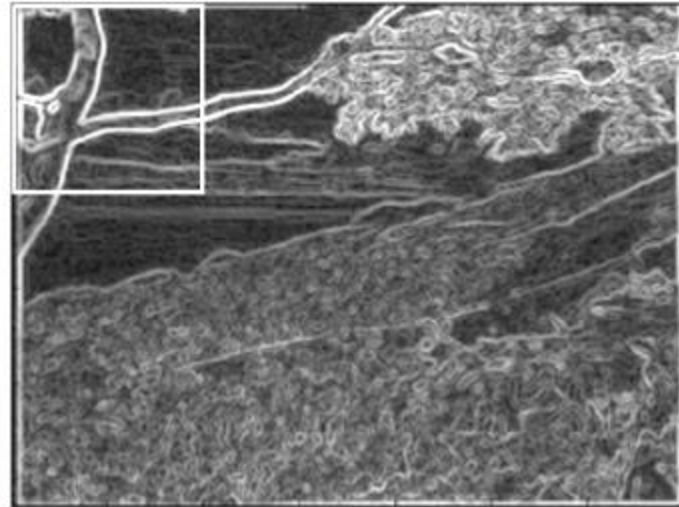
# Very difficult edges

Different techniques  
work for different cases.



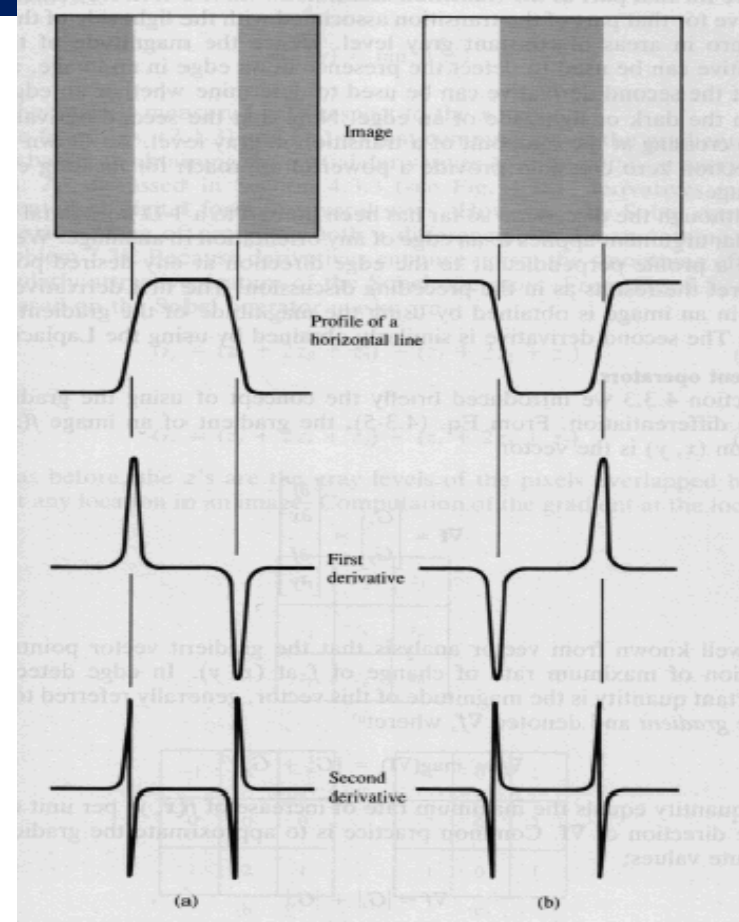
# Edge Detection Strategy

- **Recognition Strategy:** determine a 'measure of change' in a pixel's neighbourhood
- First derivatives in 2D space  $\rightarrow$  image gradient



# Edge Detection Strategy

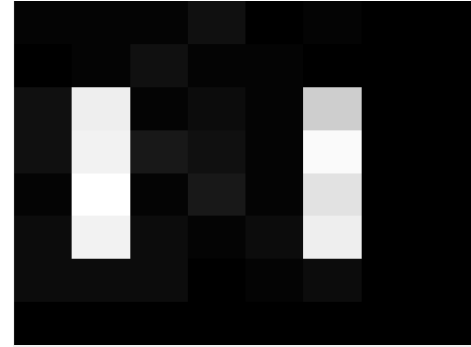
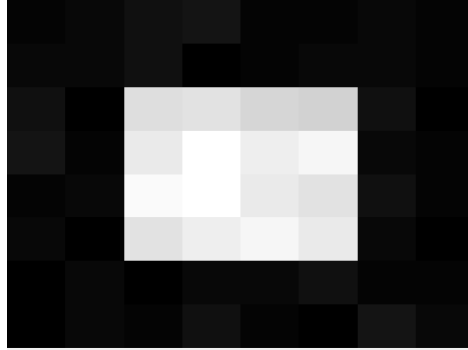
- Magnitude of the 1<sup>st</sup> derivative can be used to detect the presence of an edge
- Zero-crossing of the 2<sup>nd</sup> derivative at the midpoint of a transition in gray level, which provides a powerful approach for locating the edge
- The sign of the 2<sup>nd</sup> derivative usually used to determine whether an edge pixel lies on the dark or light side of an edge



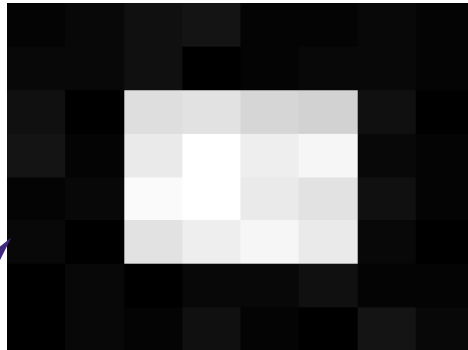
# Edge Detection Strategy

Horizontal differencing detects vertical edges

Difference



Difference

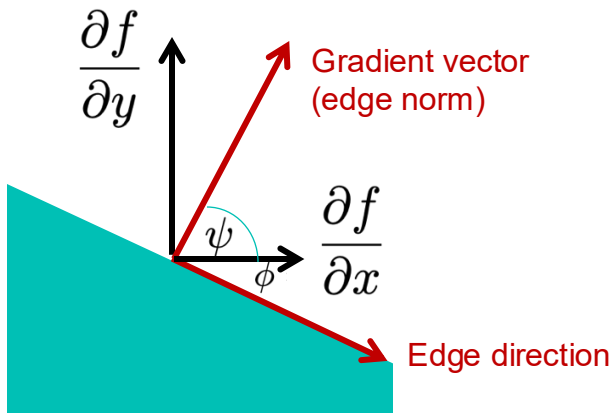


Vertical differencing detects Horizontal edges

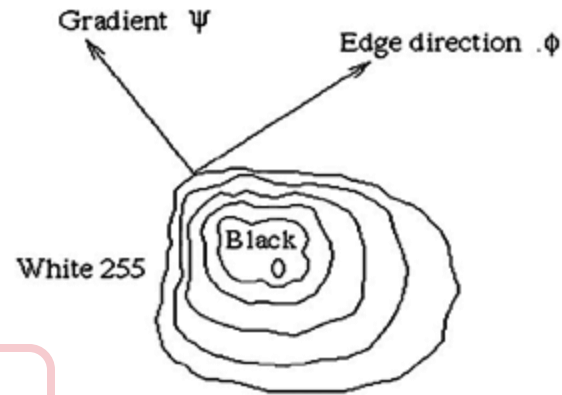
# The Image Gradient

A **vector** variable

- Direction  $\psi$  of the maximum growth of the function
- Magnitude  $|\nabla f(x, y)|$  of the growth
- Perpendicular to the edge direction  $\phi$



$$\nabla f(x, y) = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y}$$
$$|\nabla f(x, y)| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$
$$\psi = \arctan\left(\frac{\partial f / \partial y}{\partial f / \partial x}\right)$$
$$\phi = \psi - \frac{\pi}{2}$$

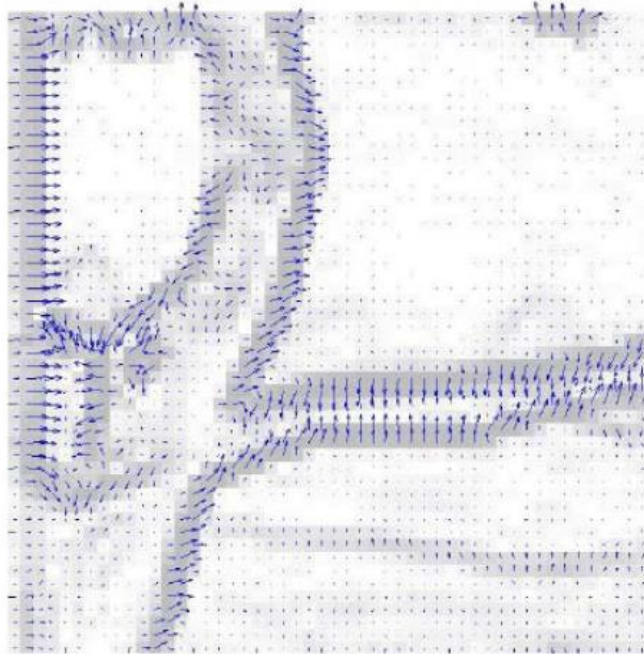




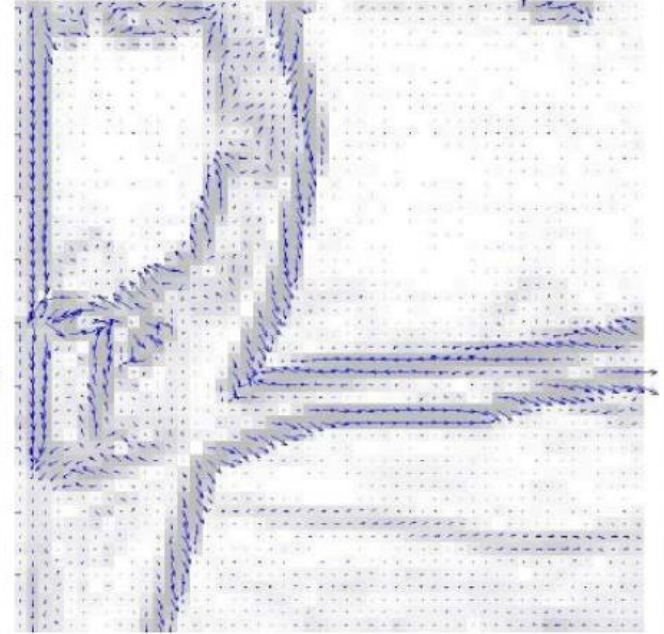
# Example: Gradient & Edge Vectors



Gradient vectors



Edge vectors



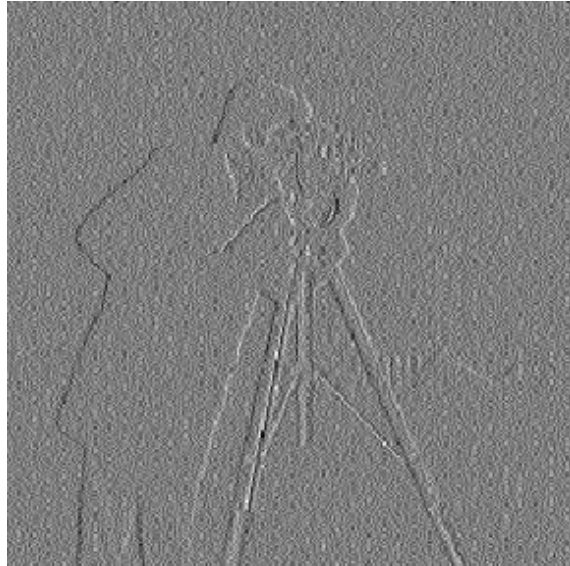


# Simple derivatives too noisy

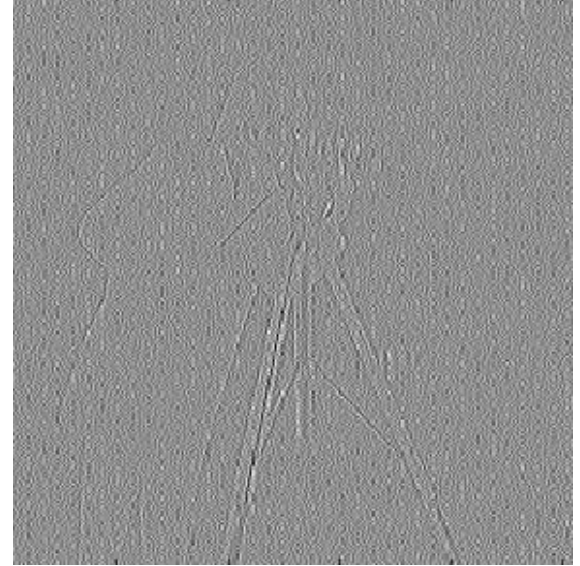
- Straight differencing of pixels is sensitive to noise



Noisy cameraman



First derivative

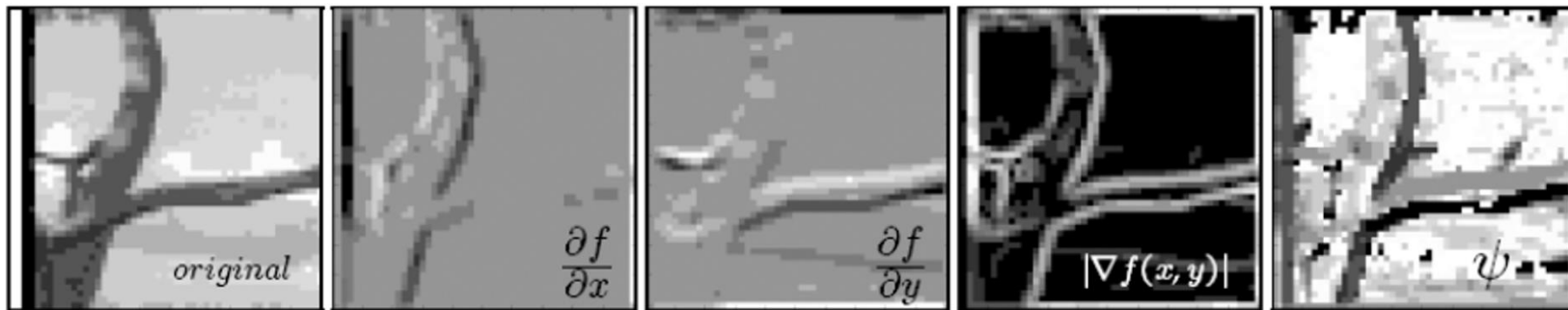


Second derivative

# Gradient Extraction via Filtering

$$h_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad h_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\text{grad}(f) = |\nabla f(x, y)| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} \quad \psi = \arctan\left(\frac{\partial f / \partial y}{\partial f / \partial x}\right)$$

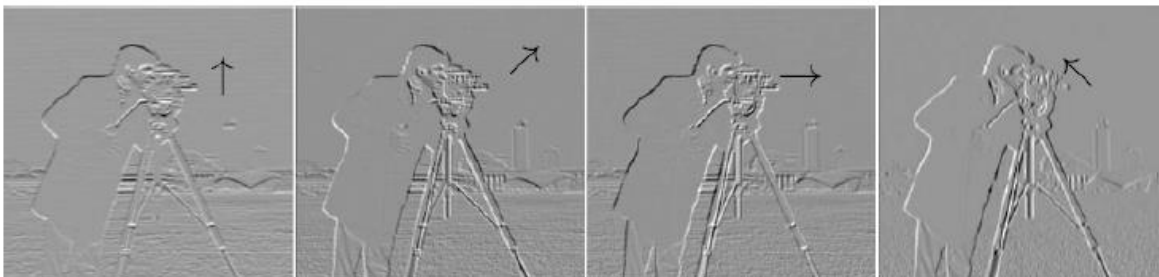
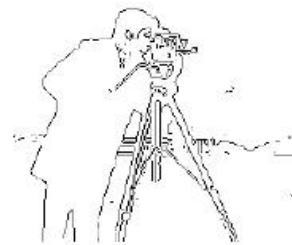


# Prewitt Operator

- Central difference  $\frac{\partial f}{\partial x} \approx \frac{f(x+1) - f(x-1)}{2}$
- Mask  $[-1 \ 0 \ 1]$  is very sensitive to noise
- For 3x3 mask,  $\nabla f$  can be estimated in **8 directions**



$$h_{hor} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}, h_{dia} = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}, \dots$$



# Sobel Operator

- As Prewitt, Sobel relies on central differences
- Greater weight to the central pixels

$$h_{hor} = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}, h_{ver} = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

- Can be approx. as derivative of a Gaussian
- First Gaussian smoothing, then derivation

$$\frac{\partial}{\partial x}(I * G) = I * \frac{\partial G}{\partial x}$$

Note, the filters are named by the way they seek out gradients, e.g. horizontal and vertical gradients to detect vertical and horizontal edges, respectively.



# Sobel Operator



**(a)** original image



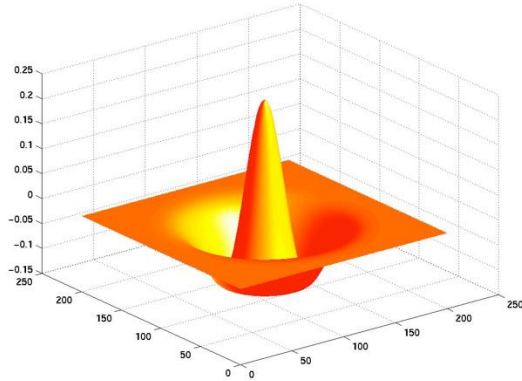
**(b)** Sobel edge magnitude



**(c)** thresholded magnitude



# Laplacian of a Gaussian Edge Detector



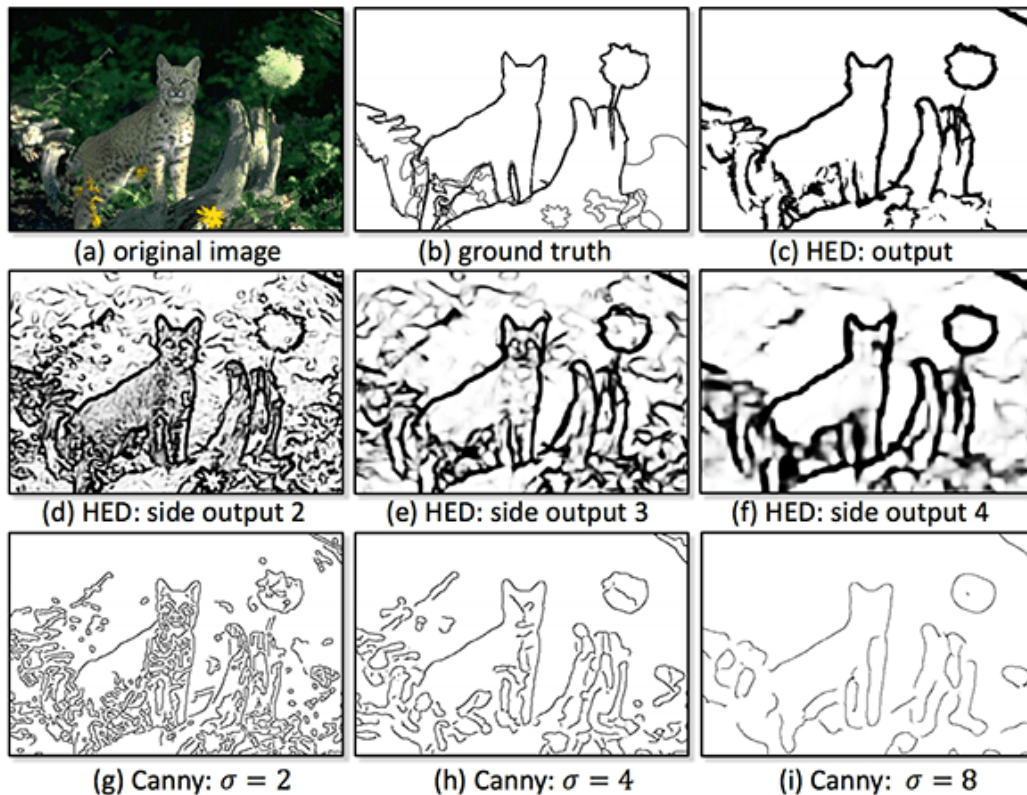
0	0	-1	0	0
0	-1	-2	-1	0
-1	-2	16	-2	-1
0	-1	-2	-1	0
0	0	-1	0	0



# There are many edge detection methods...

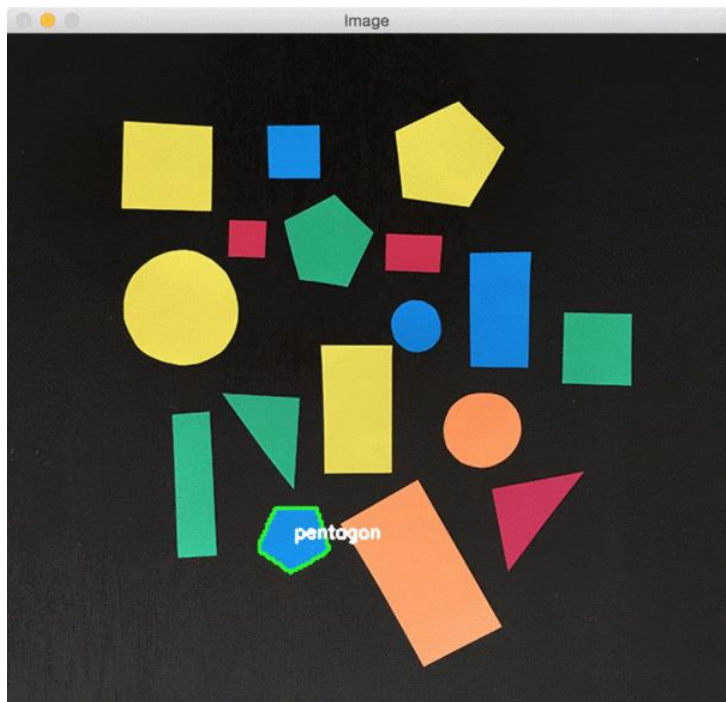
Most, if not all, edge detection methods operate with one or more parameters/thresholds.

Is there ever a perfect edge map...

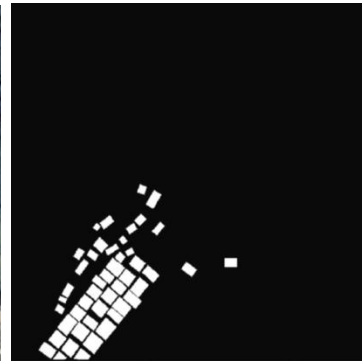




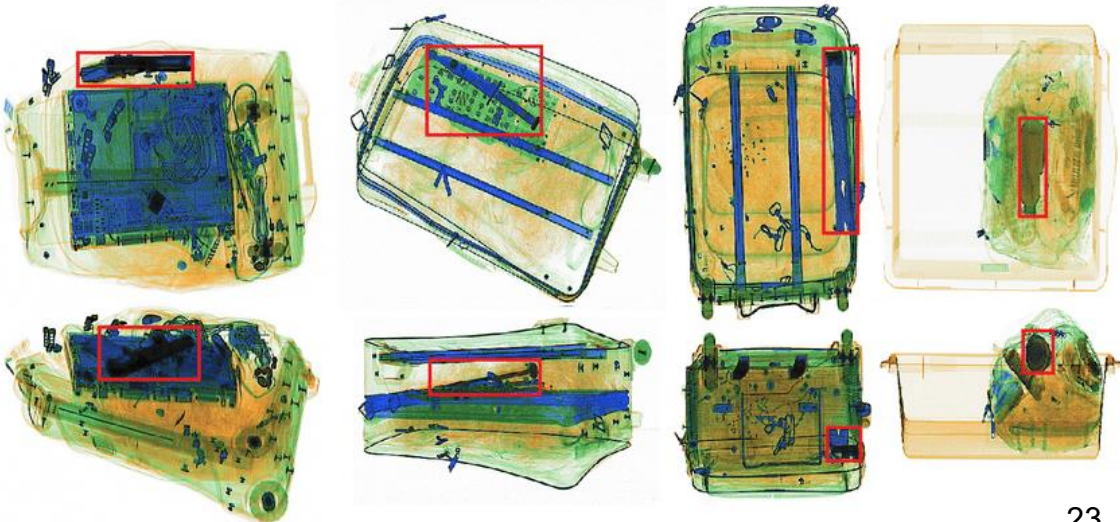
# Shape Detection



<https://pyimagesearch.com/2016/02/08/opencv-shape-detection/>



<https://www.industry.com/story/2022/4/31/223>



[https://www.researchgate.net/figure/Example-scans-of-bags-in-false-color-containing-a-firearm-handgun-sharp-knife-blunt\\_fig1\\_337944556](https://www.researchgate.net/figure/Example-scans-of-bags-in-false-color-containing-a-firearm-handgun-sharp-knife-blunt_fig1_337944556)

# Line Detection via the Hough Transform

In image space, a line is points  $(x, y)$  with gradient  $m$ , intercept  $c$

$$y = mx + c$$

In parameter space, a line is points  $(m, c)$ , with gradient  $-x$ , intercept  $y$

$$c = -xm + y$$

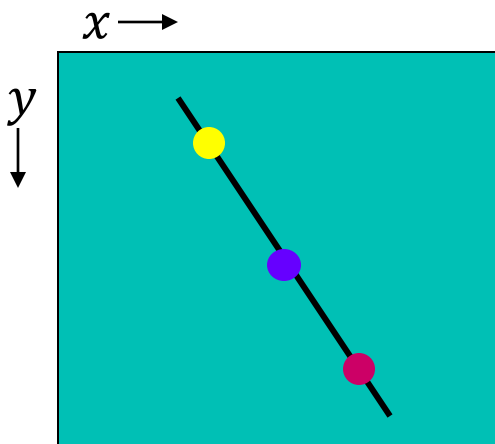
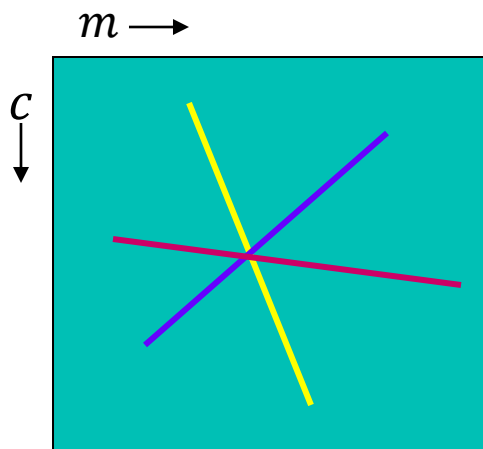


Image space

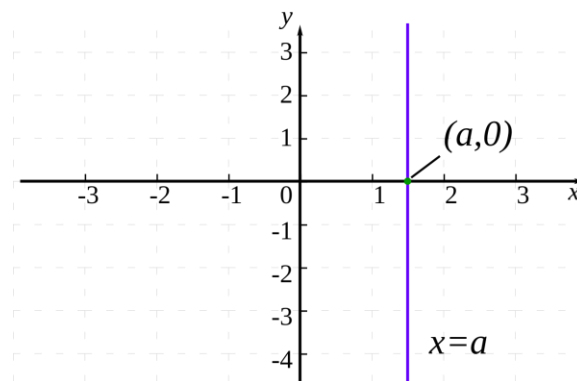
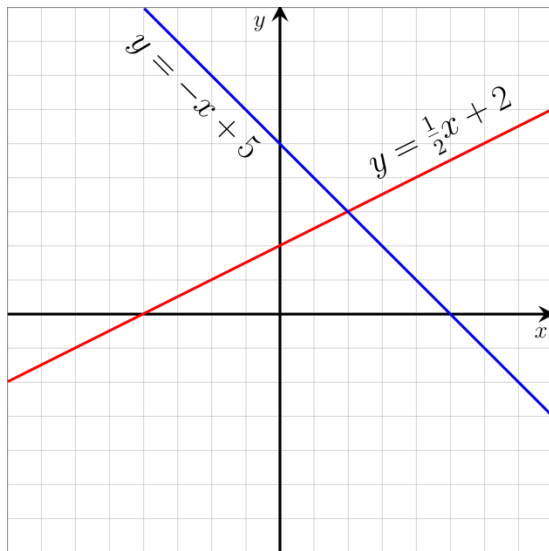


parameter space

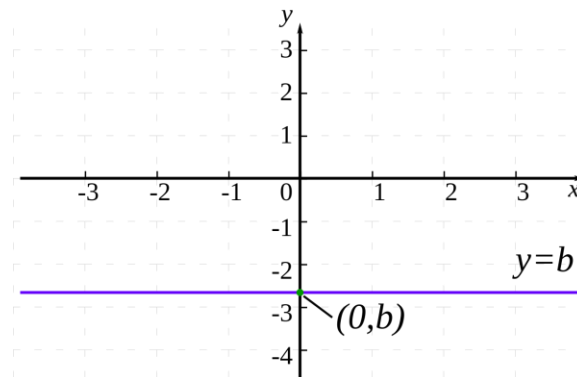
Accumulate votes!  
The coordinates of  
the peak are the  
parameters  $m, c$  of  
the line!

# Line Representation

$$y = mx + c$$



No y intercept!  
slope is infinite!



# Line Representation

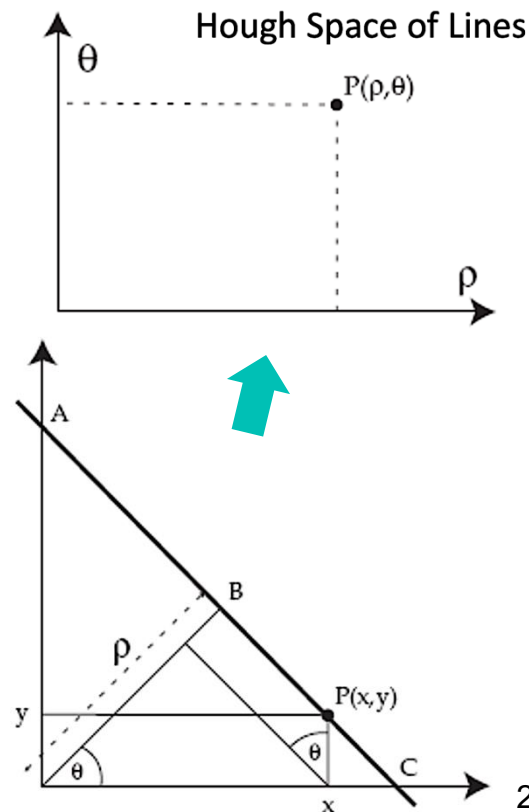
A straight line in 2D space can also be described in its polar form:

$$f(x, y, \rho, \theta) = x \cos \theta + y \sin \theta - \rho = 0$$

It can be represented in the 2D parameter space by point  $(x, y)$

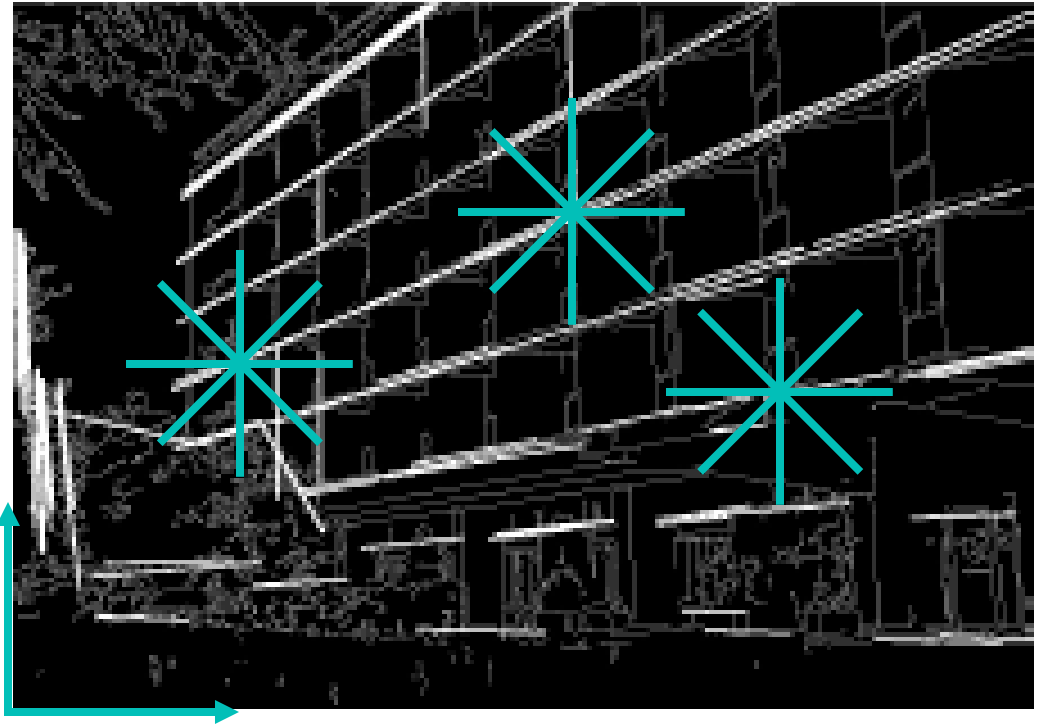
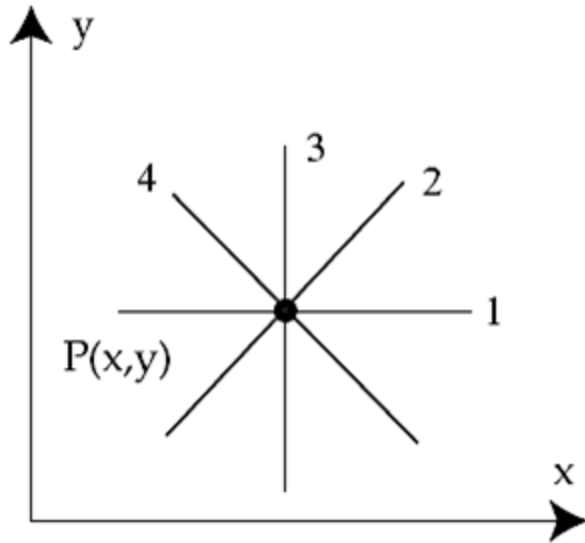
$\rho$  = distance between the straight line and the origin,

$\theta$  = angle between the distance vector and the positive  $x$ -direction.



# Line Representation

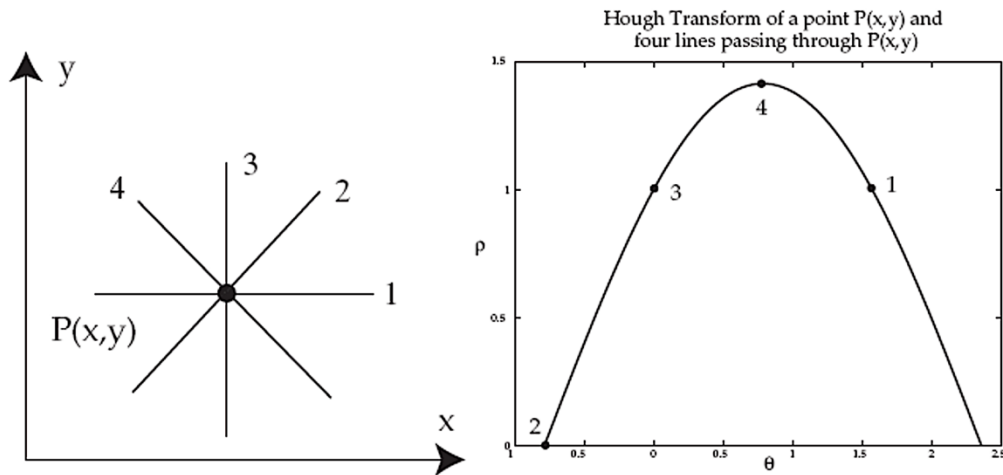
$$f(x, y, \rho, \theta) = x \cos \theta + y \sin \theta - \rho = 0$$



# The Hough Space

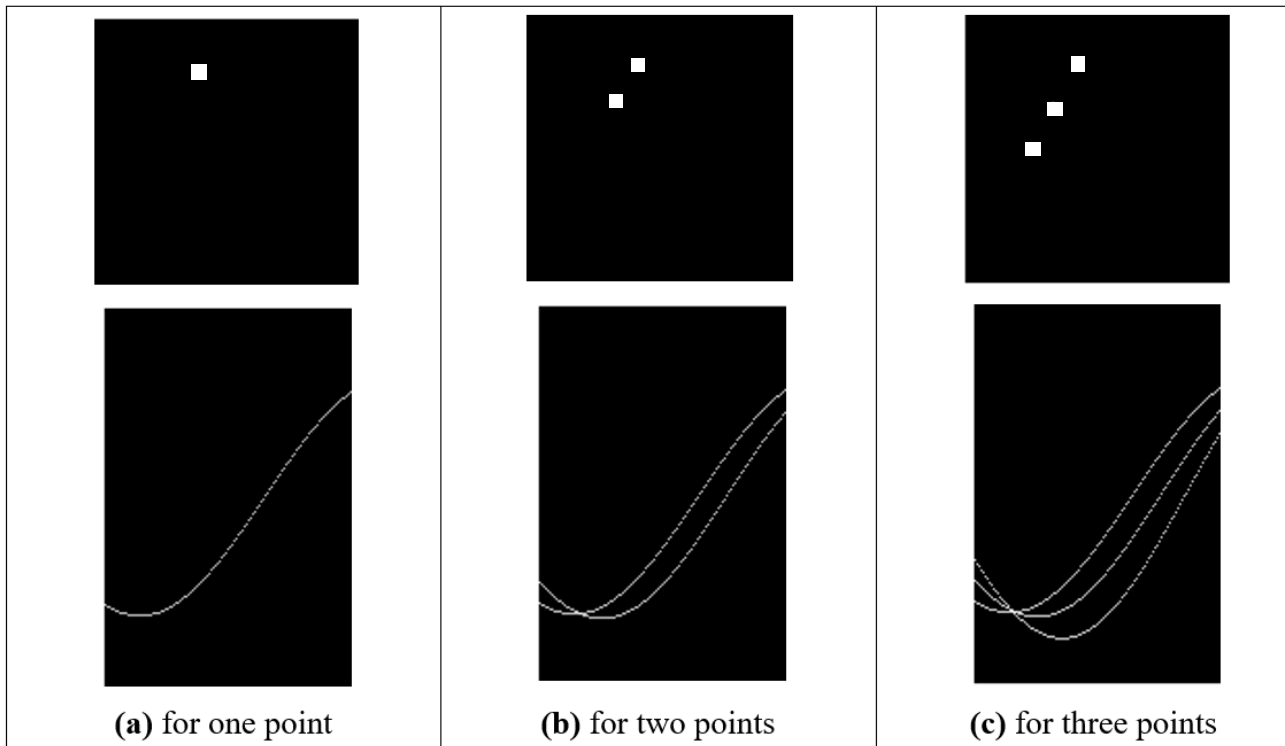
A point  $(x, y)$  in the image space is transformed into a sinusoidal curve in the parameter space. A point  $(\rho, \theta)$  on this sinusoidal curve represents a straight line passing through the point  $(x, y)$  in the image space.

	point 1	point 2	point 3	point 4
$\theta$	$\pi/2 = 1.571$	$-\pi/4 = -0.785$	0	$\pi/4 = 0.785$
$\rho$	1	0	1	1.4142



# Line Representation

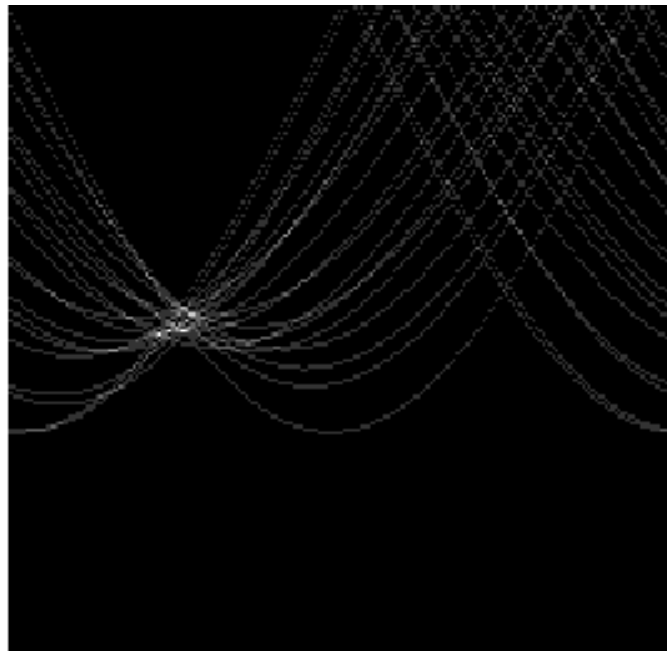
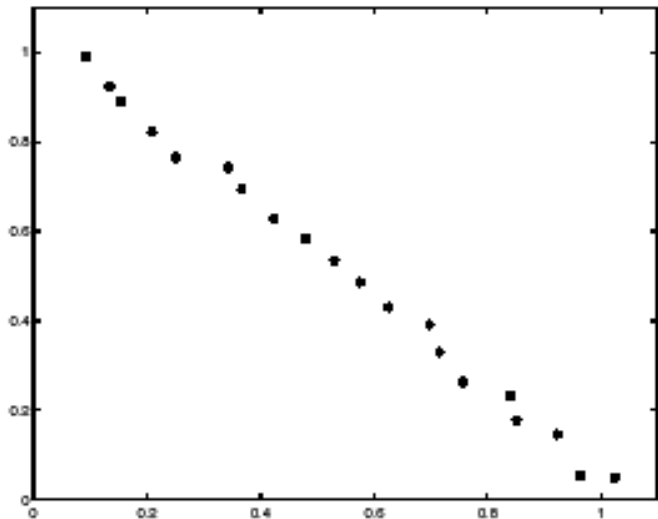
$$f(x, y, \rho, \theta) = x \cos \theta + y \sin \theta - \rho = 0$$





# Line Representation

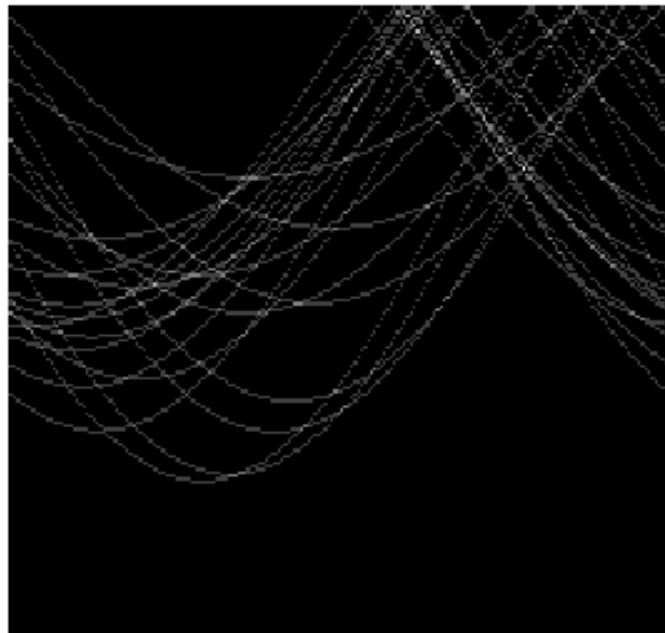
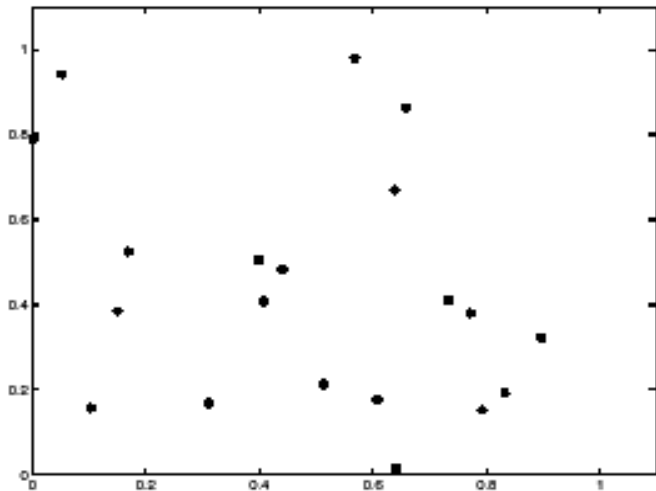
$$f(x, y, \rho, \theta) = x \cos \theta + y \sin \theta - \rho = 0$$



Peak gets fuzzy and hard to locate

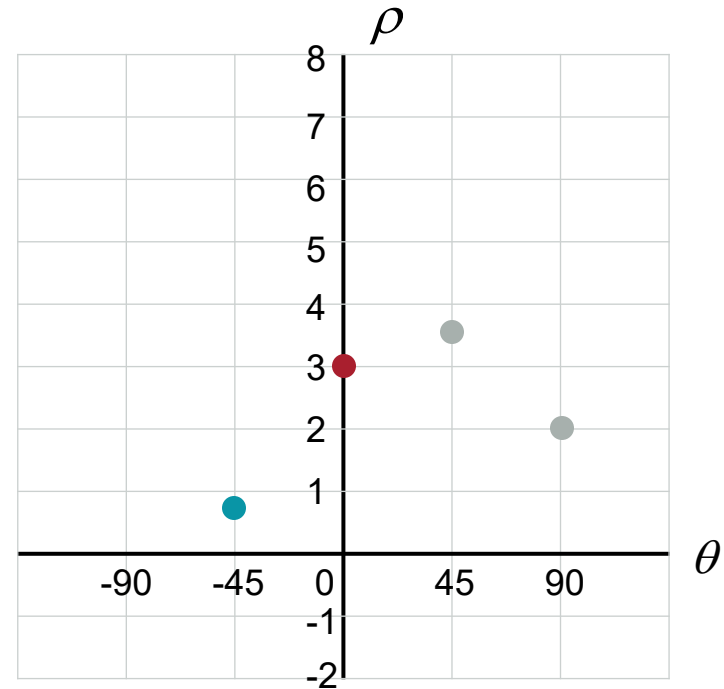
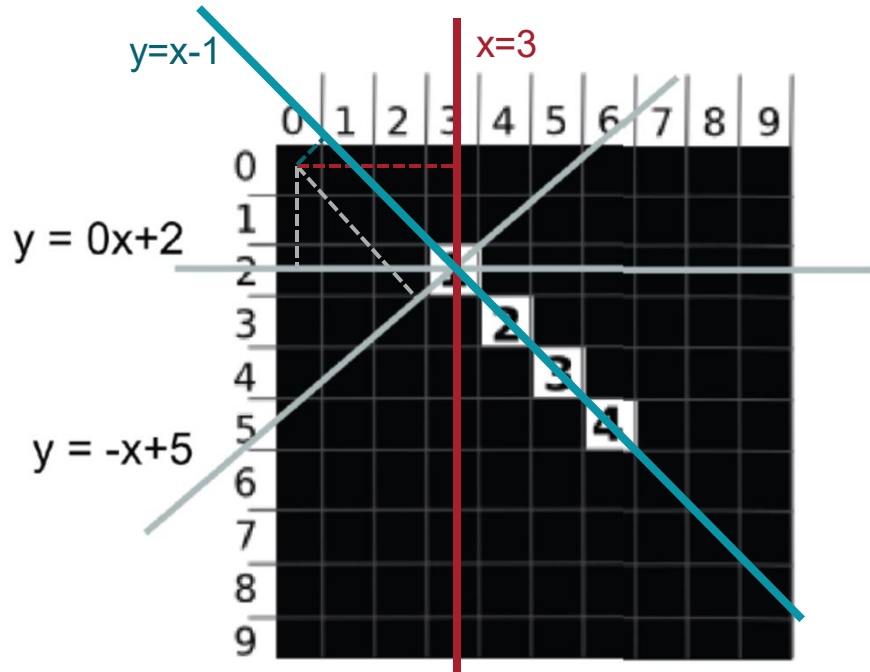
# Line Representation

$$f(x, y, \rho, \theta) = x \cos \theta + y \sin \theta - \rho = 0$$

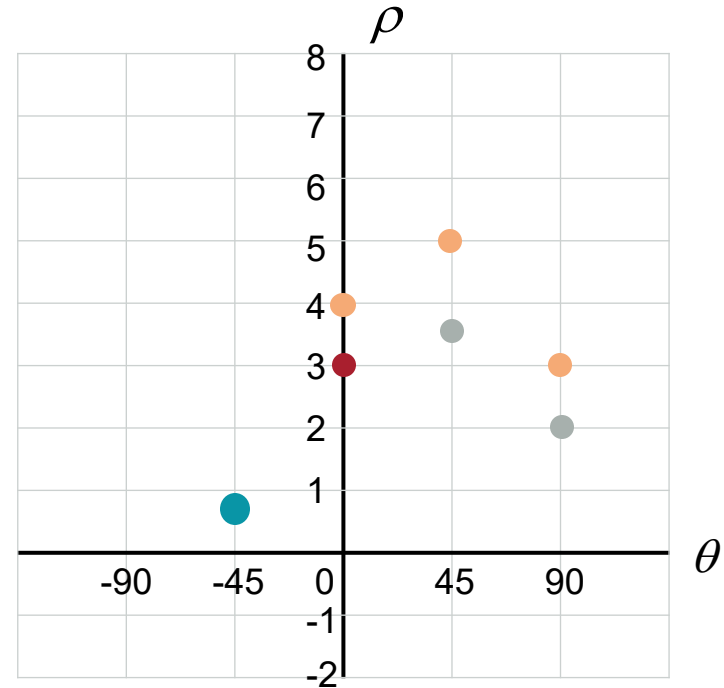
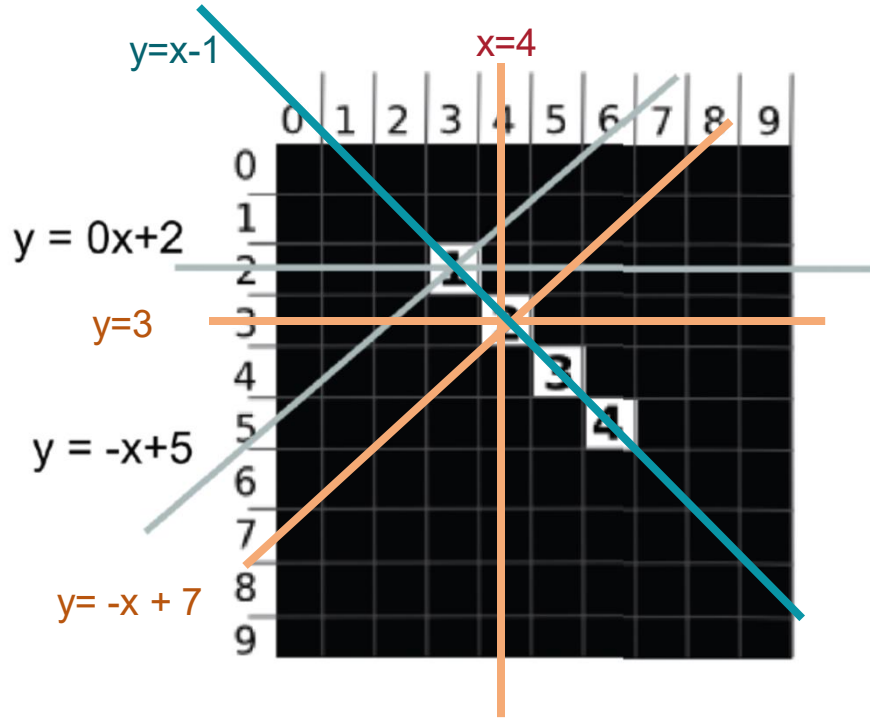


Noise can lead to spurious peaks in the accumulator array

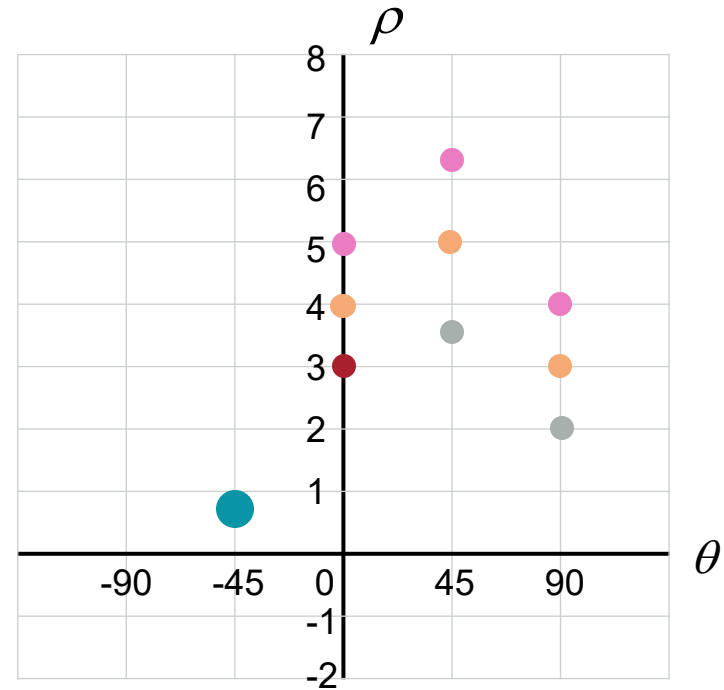
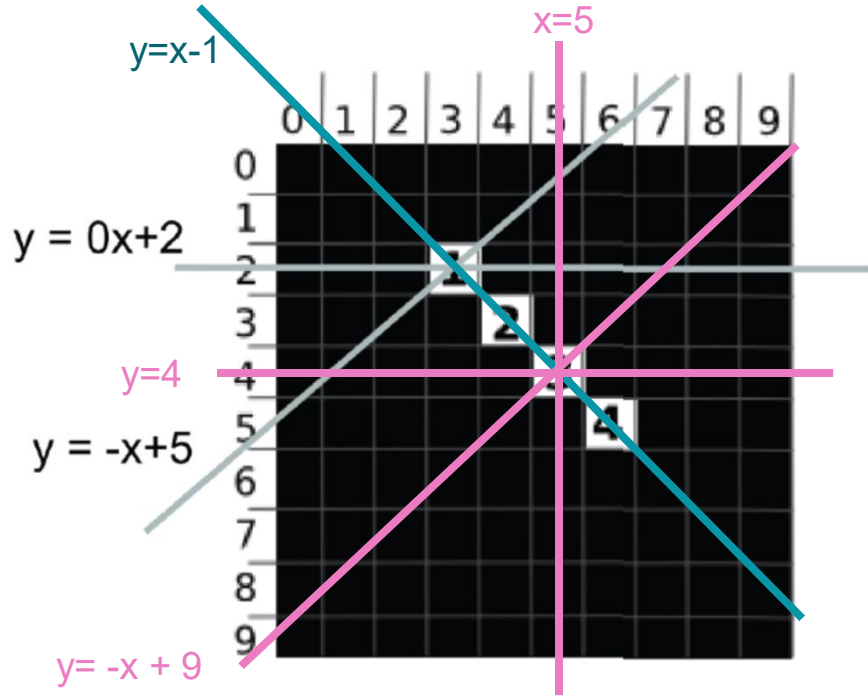
# Building the Hough Space



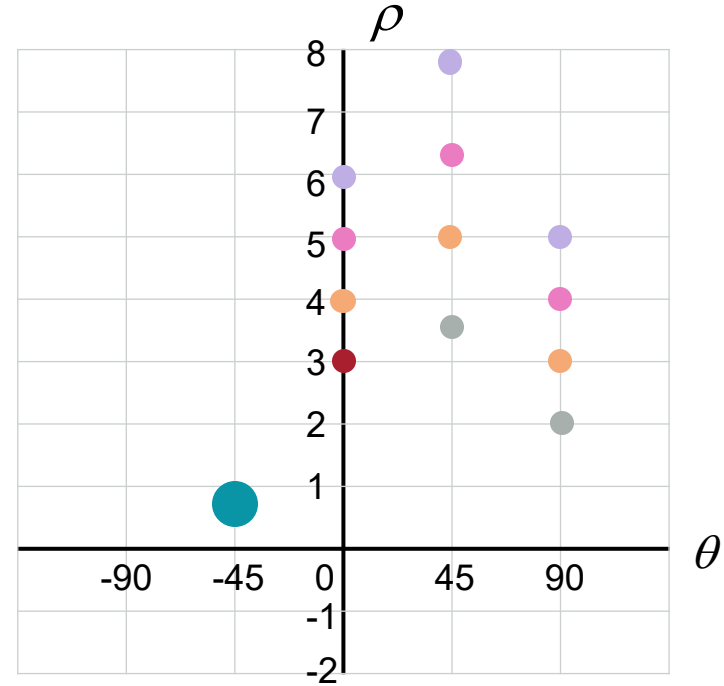
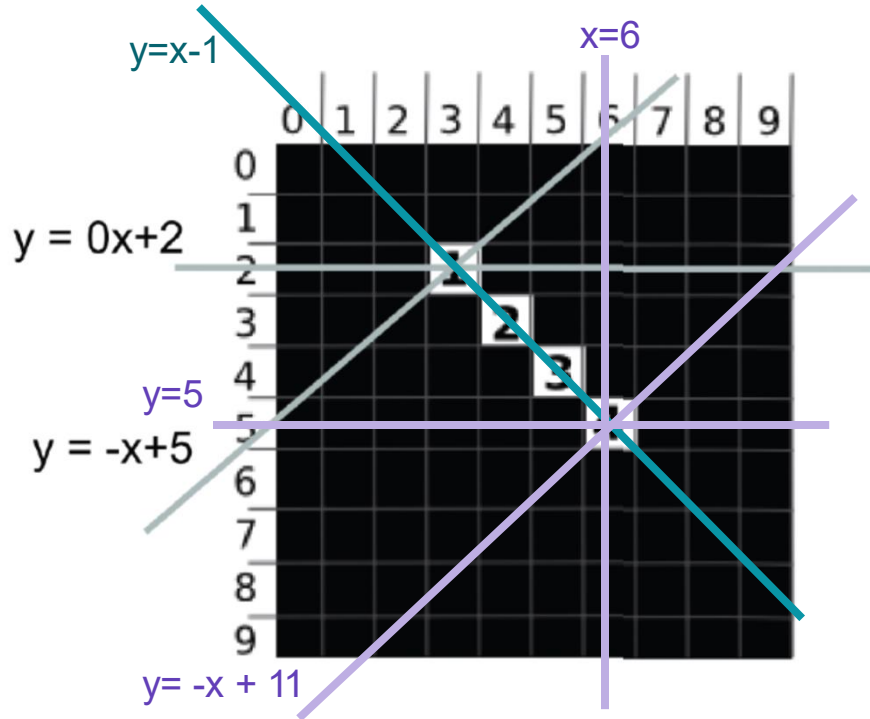
# Building the Hough Space



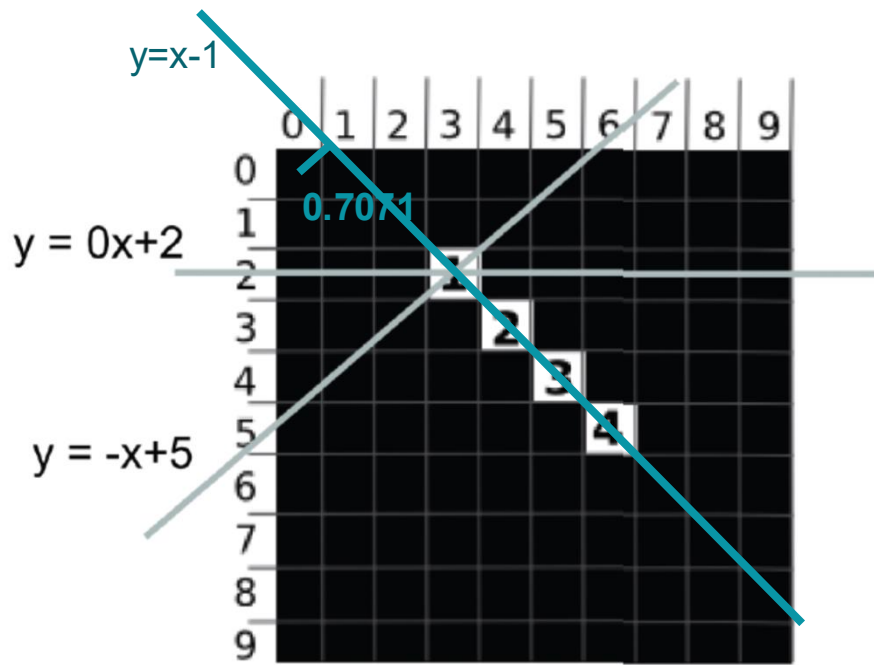
# Building the Hough Space



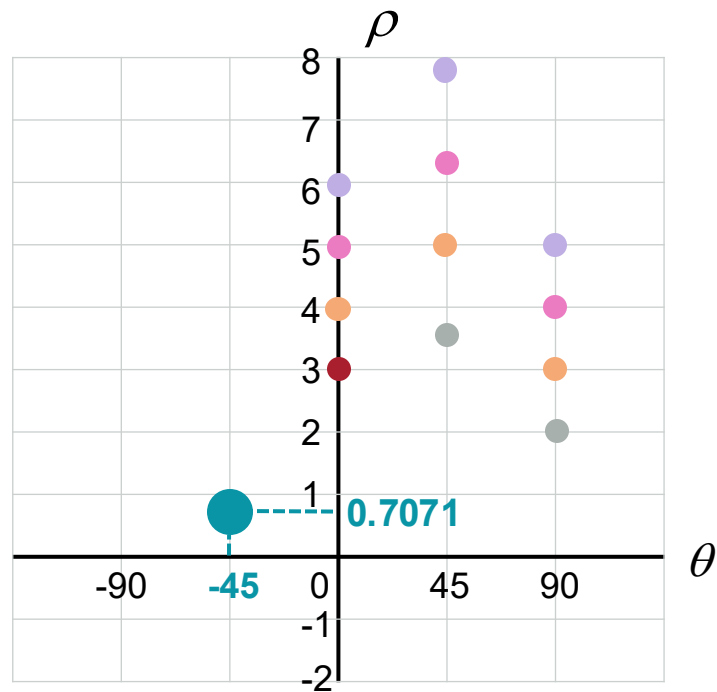
# Building the Hough Space



# Building the Hough Space



$$\rho = x \cos \theta + y \sin \theta$$



$$0.7071 = x * 0.7071 + y * 0.7071$$



# Line Detection Algorithm

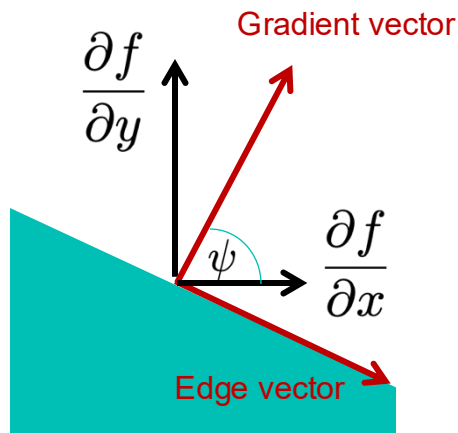
1. Make available an  $n = 2$  dimensional array  $H(\rho, \theta)$  for the parameter space;
2. Find the gradient image:  $G(x, y) = |G(x, y)| \angle G(x, y)$ ;
3. For any pixel satisfying  $|G(x, y)| > T_s$ , increment all elements on the curve  $\rho = x \cos \theta + y \sin \theta$  in the parameter space represented by the H array:

$$\forall \theta \quad | \quad \rho = x \cos \theta + y \sin \theta$$

$$H(\rho, \theta) = H(\rho, \theta) + 1;$$

4. In the parameter space, any element  $H(\rho, \theta) > T_h$  represents a straight line detected in the image.

# Line Detection using Gradient Information



1. Make  $n = 2$  dimensional array  $H(\rho, \theta)$
2. Find the gradient image:  $G(x, y) = |G(x, y)| \angle G(x, y)$ ;
3. For any pixel satisfying  $|G(x, y)| > T_s$ ,

$$\forall \theta \quad | \quad \angle G(x, y) - \Delta\theta \leq \theta \leq \angle G(x, y) + \Delta\theta$$

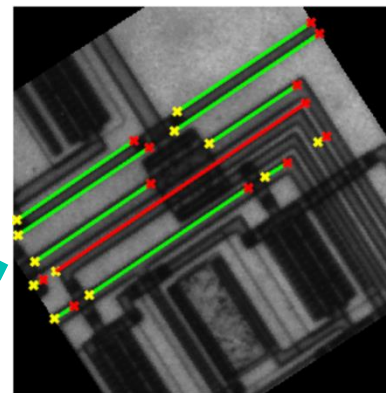
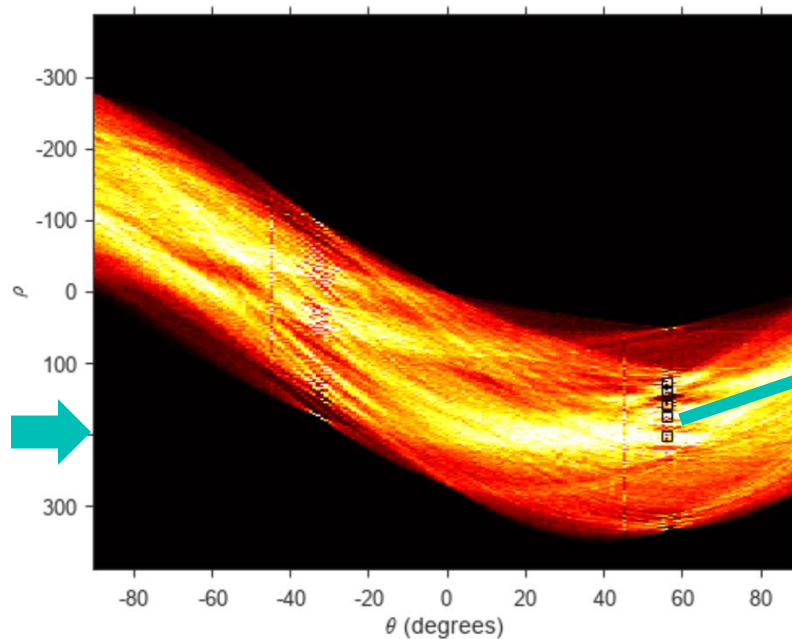
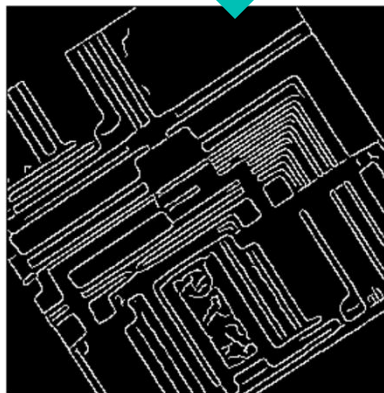
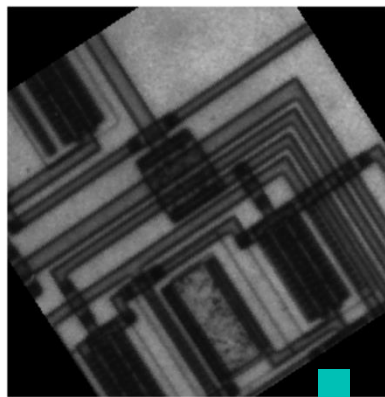
$$\rho = x \cos \theta + y \sin \theta$$

$$H(\rho, \theta) = H(\rho, \theta) + 1;$$

where  $\Delta\theta$  defines a small range in  $\theta$  to allow some room for error in  $\angle G$ .

. Any element  $H(\rho, \theta) > T_h$  represents a straight line

# Line Detection Example



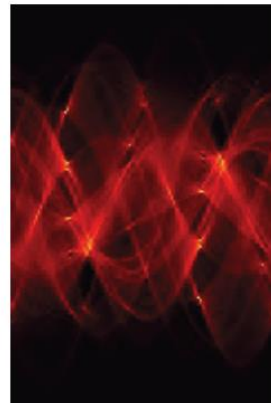
# Line Detection Example



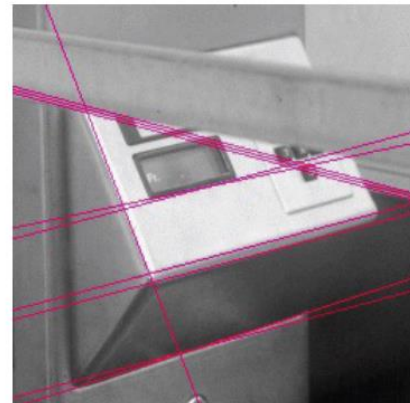
Original



Edges



Parameter  
Space



Hough Lines

# Circle Detection Algorithm

1. For any pixel satisfying  $|G(x, y)| > T_s$ , increment all elements satisfying the two simultaneous equations

$$\forall r, \quad \begin{cases} x_0 = x \pm r \cos \angle G \\ y_0 = y \pm r \sin \angle G \end{cases}$$

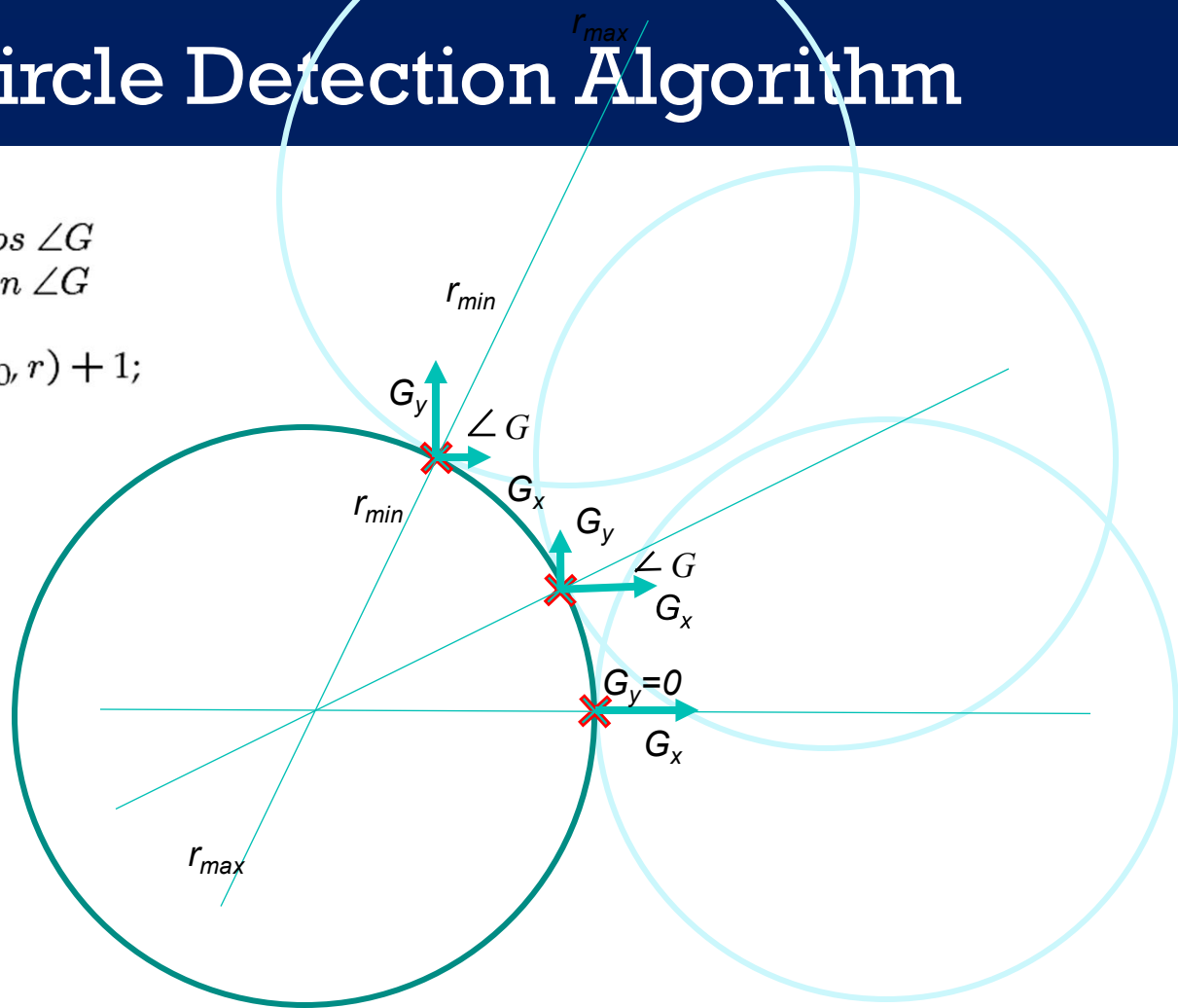
$$H(x_0, y_0, r) = H(x_0, y_0, r) + 1;$$

2. In the parameter space, any element  $H(x_0, y_0, r) > T_h$  represents a circle with radius  $r$  located at  $(x_0, y_0)$  in the image.

# Circle Detection Algorithm

$$\forall r, \begin{cases} x_0 = x \pm r \cos \angle G \\ y_0 = y \pm r \sin \angle G \end{cases}$$

$$H(x_0, y_0, r) = H(x_0, y_0, r) + 1;$$



# Circle Detection Algorithm

(1)

1. For any pixel satisfying  $|G(x, y)| > T_s$ , increment all elements satisfying the two simultaneous equations

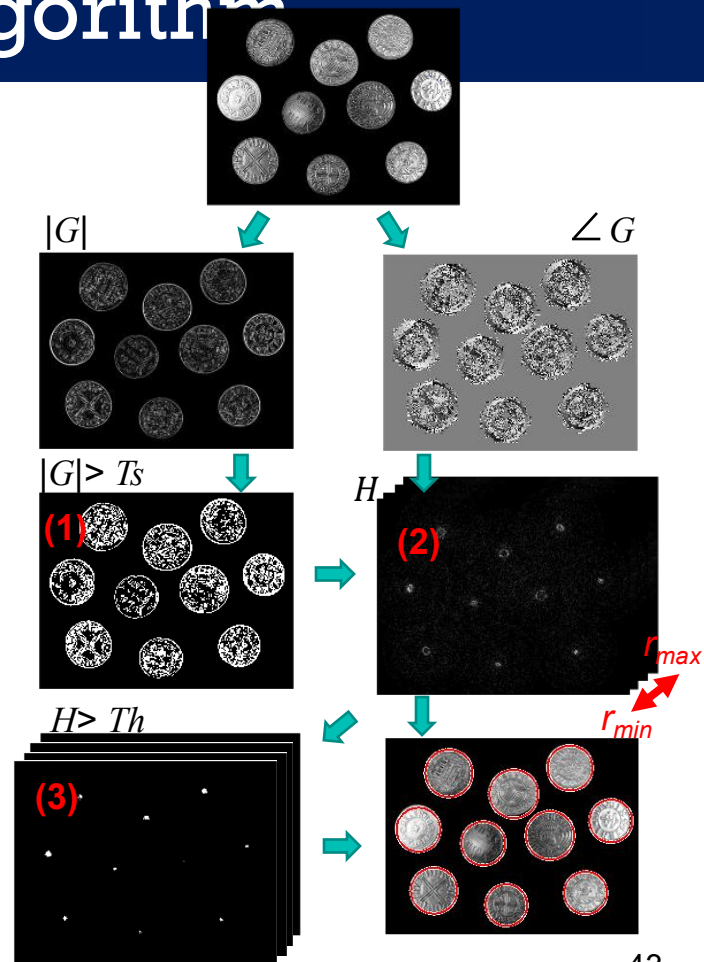
(2)

$$\forall r, \begin{cases} x_0 = x \pm r \cos \angle G \\ y_0 = y \pm r \sin \angle G \end{cases}$$

$$H(x_0, y_0, r) = H(x_0, y_0, r) + 1;$$

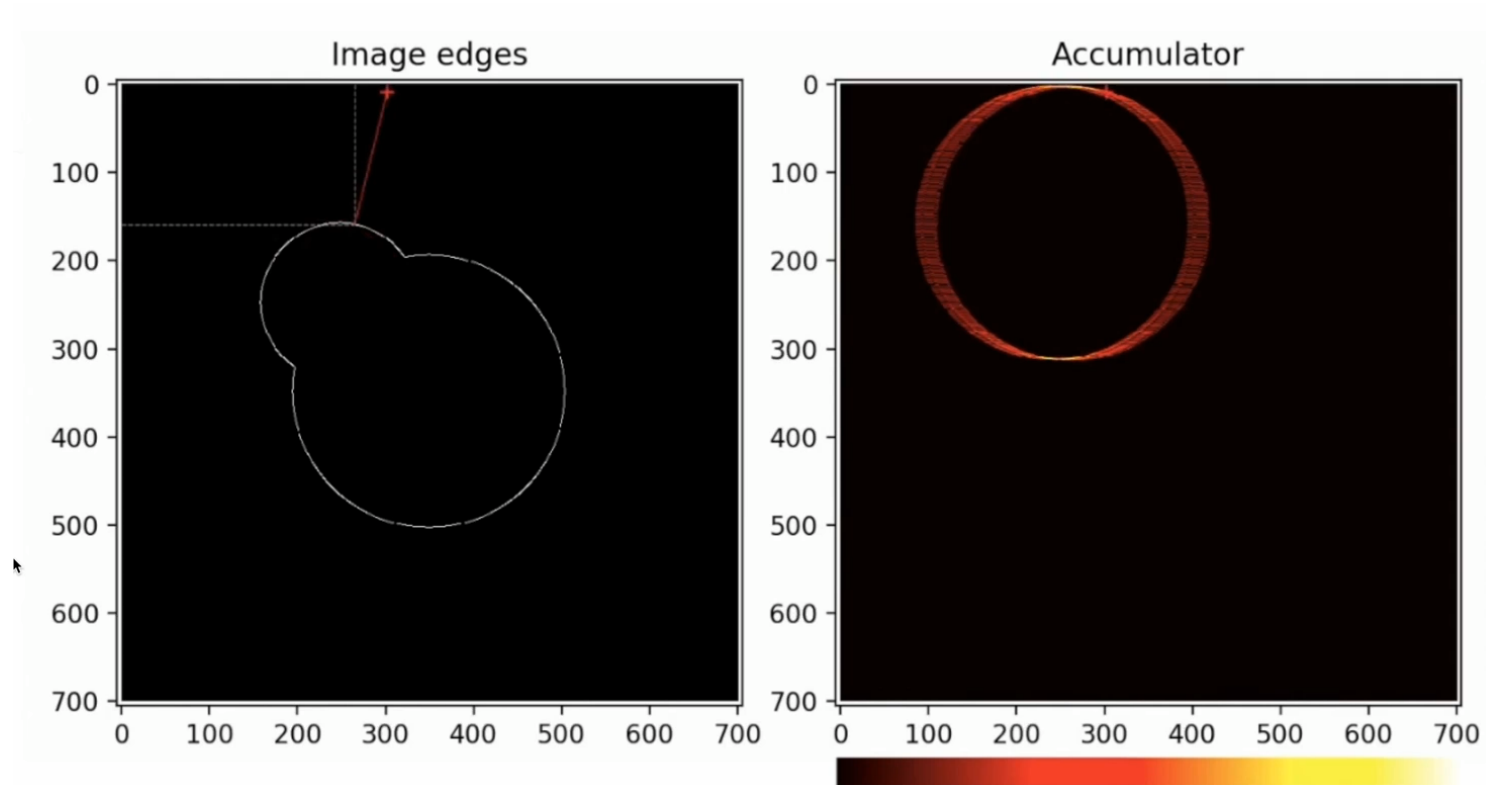
(3)

2. In the parameter space, any element  $H(x_0, y_0, r) > T_h$  represents a circle with radius  $r$  located at  $(x_0, y_0)$  in the image.



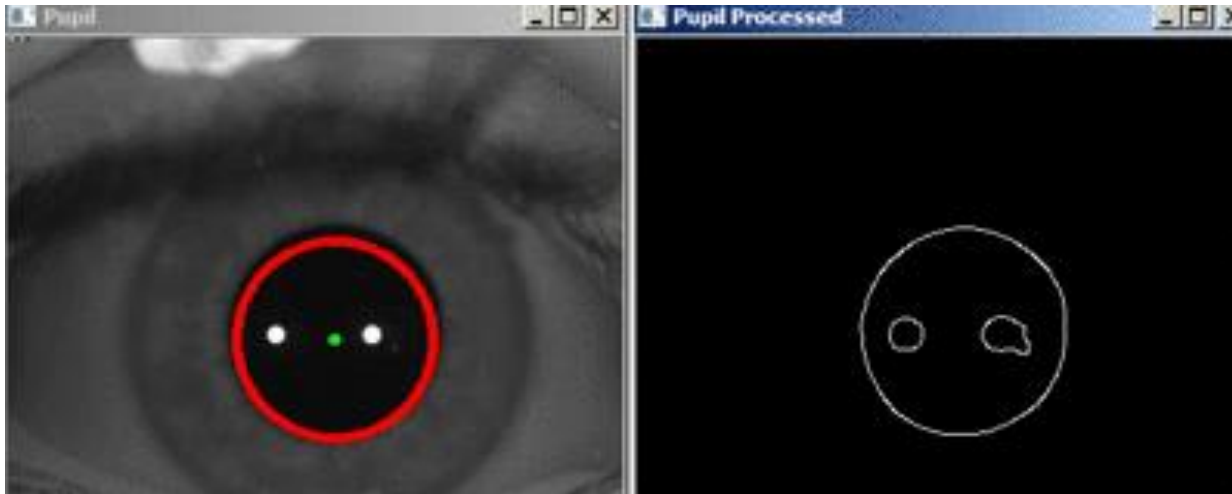


# Circle Detection Algorithm



# Summary: Shape Recognition via Hough Transform

- Image space is transformed into a **parameter space**
- **Voting procedure** is carried out in the parameter space.
- Object candidates are obtained as **local maxima**.



Source: <https://stackoverflow.com/questions/22274010/detect-objects-similar-to-circles/22270177>

# Next Lecture

## Image Segmentation

