

COMS30030  
Image Processing and Computer Vision

**Motion – Modelling**

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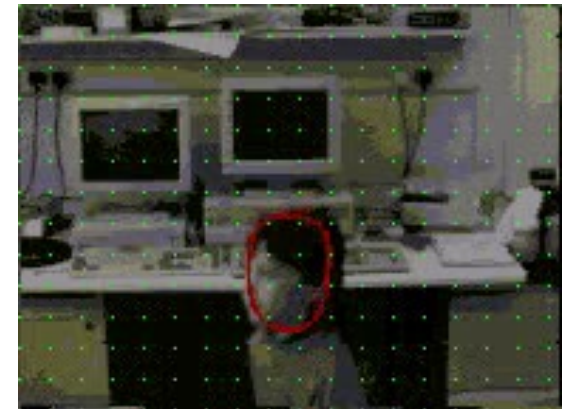
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# Motion – Important Perceptual Cue

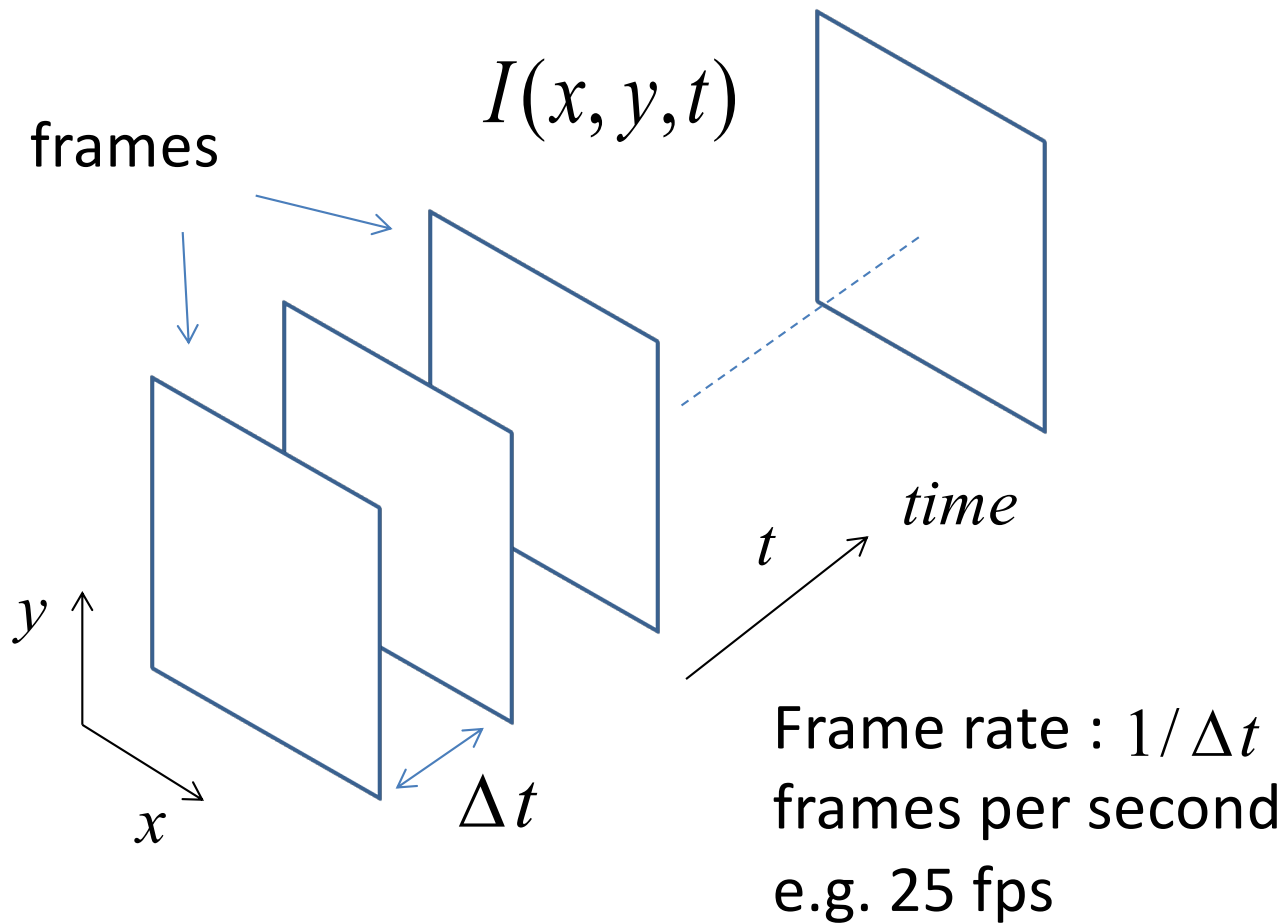


# We are going to look at .....

- Modelling 2-D motion fields
- Optical flow
- The optical flow equation (OFE)
- Motion estimation
  - Lucas and Kanade method



# Video Sequences

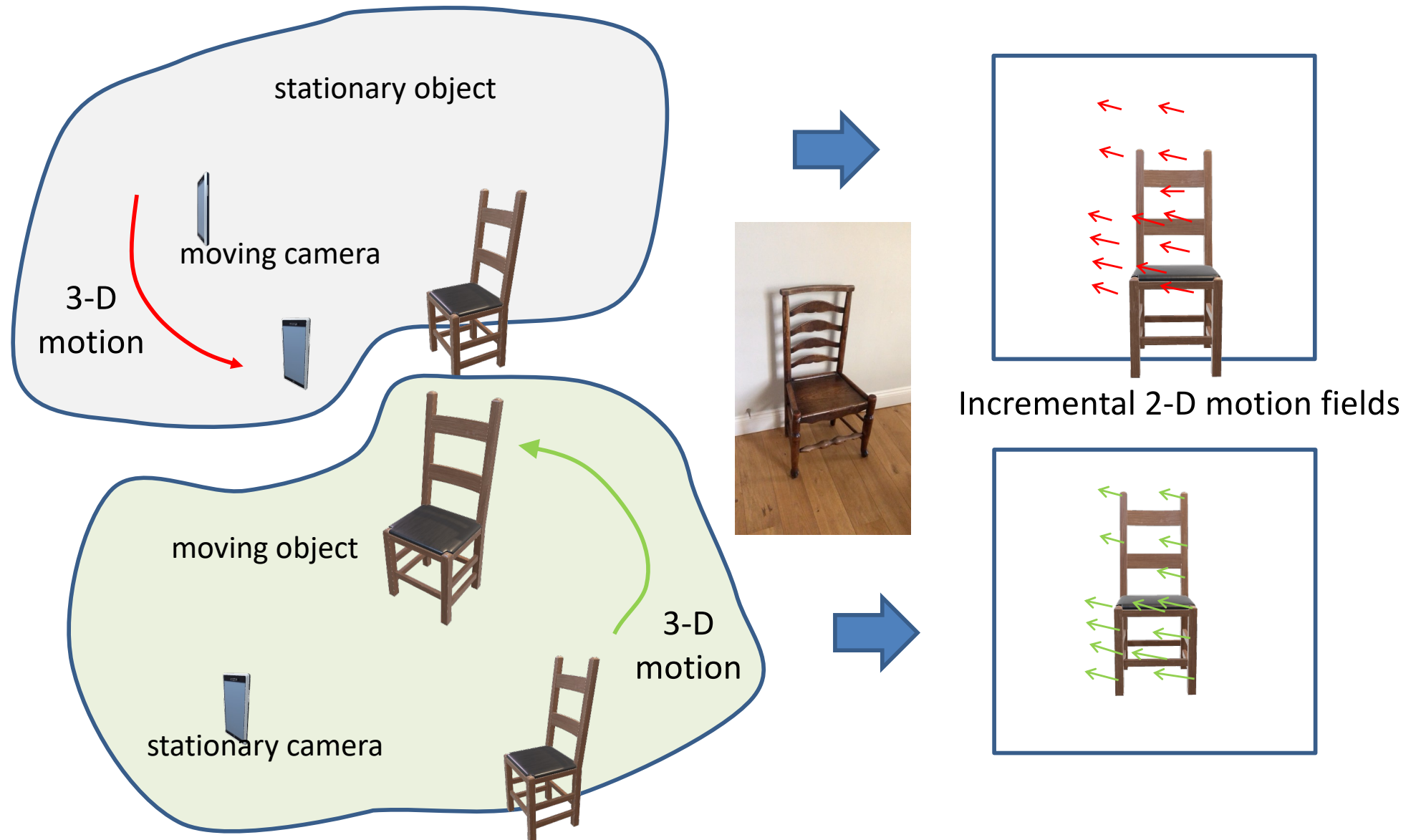


First motion  
picture camera

Kinetograph



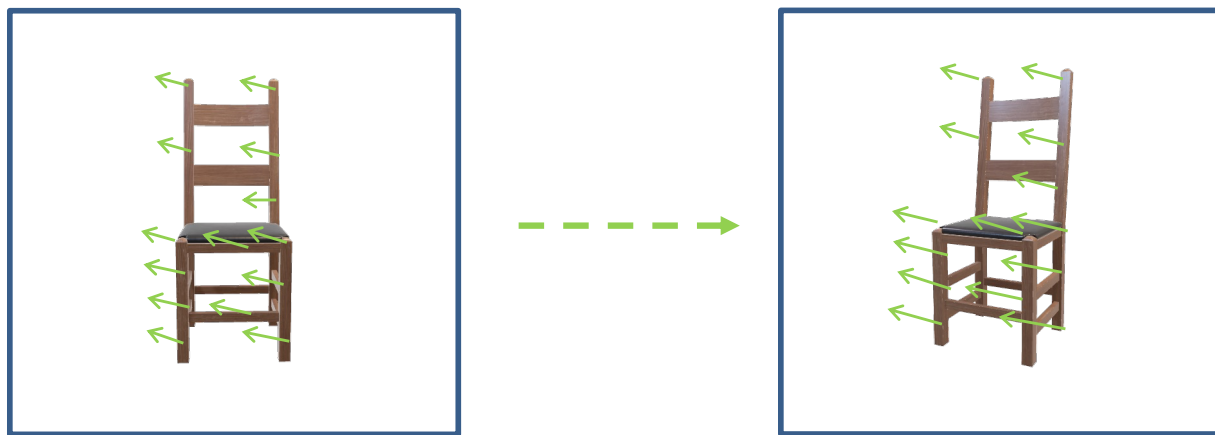
# Modelling 2-D Motion Fields



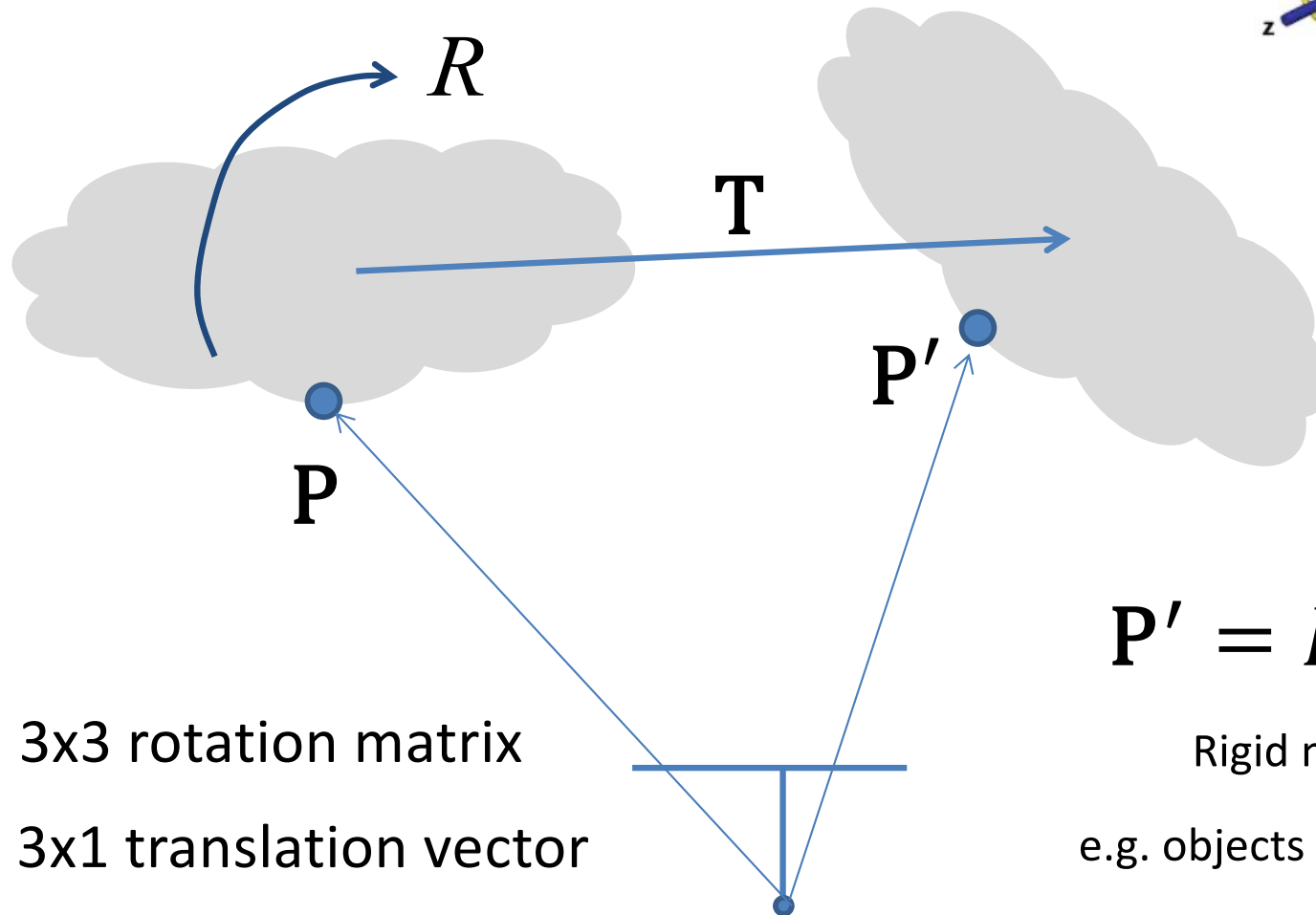
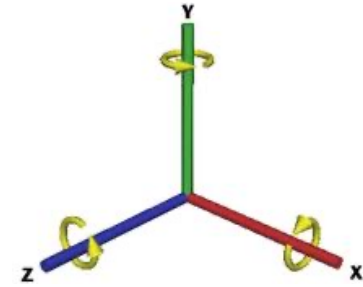
# Incremental 2-D Motion Fields

What is the relationship between the:

- Position of a 3-D point and its 2-D motion?
- 2-D motion of different 3-D points?
- 3-D motion of a 3-D point and its 2-D motion?



# 3-D Rigid Motion



$R$  : 3x3 rotation matrix

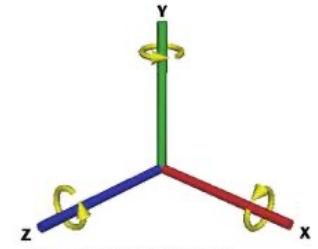
$T$  : 3x1 translation vector

$$P' = RP + T$$

Rigid motion

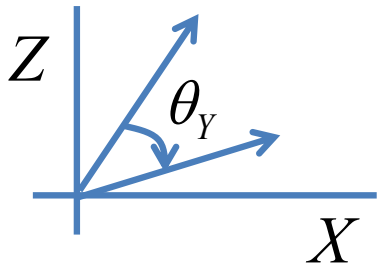
e.g. objects do not deform

# Rotation Matrices



$R = R_X R_Y R_Z$  (for example) : Rotations about  $X$ ,  $Y$  and  $Z$  axes

$$R_Y \mathbf{P} = \begin{bmatrix} \cos \theta_Y & 0 & \sin \theta_Y \\ 0 & 1 & 0 \\ -\sin \theta_Y & 0 & \cos \theta_Y \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X \cos \theta_Y + Z \sin \theta_Y \\ Y \\ Z \cos \theta_Y - X \sin \theta_Y \end{bmatrix}$$



For small  $\theta_Y$ :  
 $\cos \theta_Y \approx 1$   
 $\sin \theta_Y \approx \theta_Y$

**NB : for small  $\theta_Y$**

$$R_Y \approx \begin{bmatrix} 1 & 0 & \theta_Y \\ 0 & 1 & 0 \\ -\theta_Y & 0 & 1 \end{bmatrix}$$

**NB : for small  $\theta_X, \theta_Y, \theta_Z$**

$$R \approx \begin{bmatrix} 1 & -\theta_Z & \theta_Y \\ \theta_Z & 1 & -\theta_X \\ -\theta_Y & \theta_X & 1 \end{bmatrix}$$

# 3-D Motion Field

$$\mathbf{V} = \lim_{\Delta t \rightarrow 0} \{\mathbf{P}' - \mathbf{P} = (\mathbf{R} - \mathbf{I})\mathbf{P} + \mathbf{T}\}$$

$$\mathbf{P}' = \mathbf{R}\mathbf{P} + \mathbf{T}$$

For small angles:

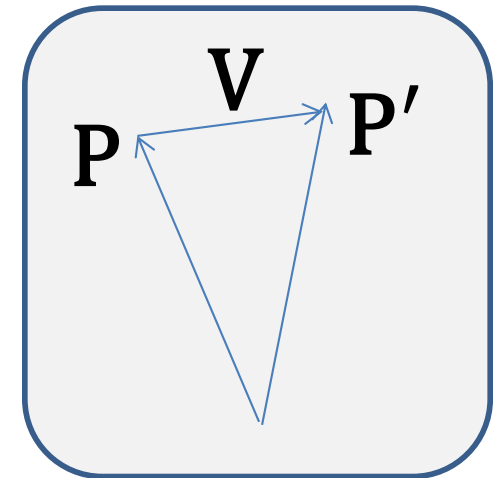
$$\mathbf{R} \approx \begin{bmatrix} 1 & -\theta_Z & \theta_Y \\ \theta_Z & 1 & -\theta_X \\ -\theta_Y & \theta_X & 1 \end{bmatrix}$$

Hence:

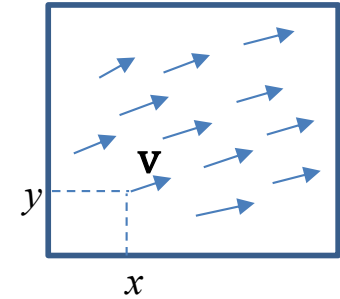
$$V_X = \theta_Y Z - \theta_Z Y + T_X \quad (\theta_X, \theta_Y, \theta_Z) \equiv \text{Angular velocity}$$

$$V_Y = \theta_Z X - \theta_X Z + T_Y \quad (T_X, T_Y, T_Z) \equiv \text{Rectilinear velocity}$$

$$V_Z = \theta_X Y - \theta_Y X + T_Z$$



# 2-D Motion Field Equations



For image point  $\mathbf{p} = (x, y, f)$       Motion field  $\mathbf{v} = (v_x, v_y)$

$$v_x = \frac{dx}{dt} = \frac{d}{dt} \left( \frac{fX}{Z} \right) = f \frac{V_X Z - X V_Z}{Z^2}$$

Quotient  
rule

$$x = \frac{fX}{Z}$$

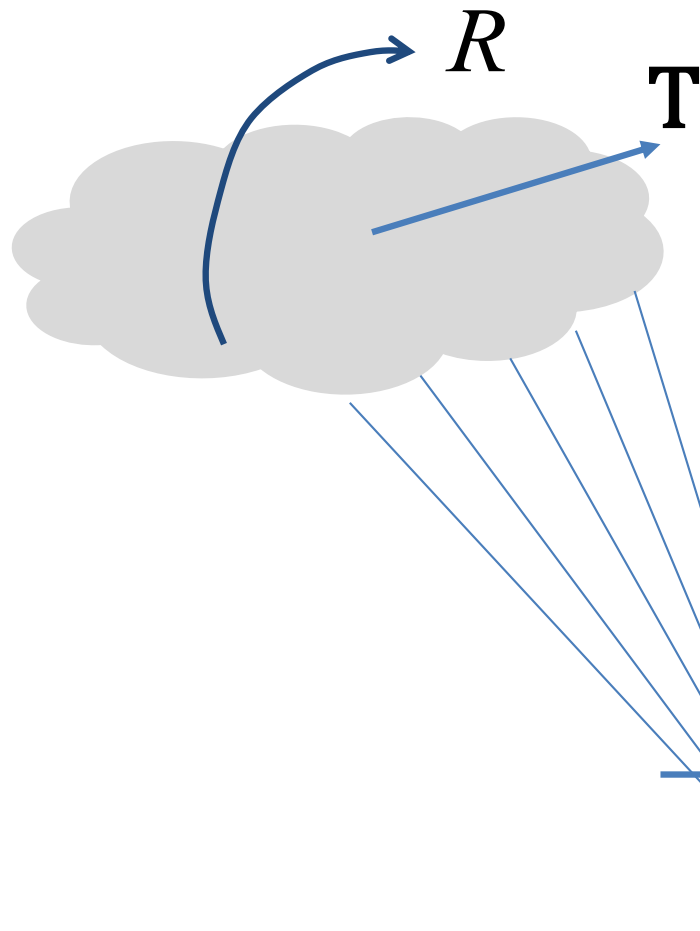
$$V_X = \frac{dX}{dt}$$

Substituting for  $V_X, V_Y, V_Z$  gives (previous slide)

$$v_x = (fT_X - xT_Z) / Z + f\theta_Y - \theta_Z y - (\theta_X xy - \theta_Y x^2) / f$$

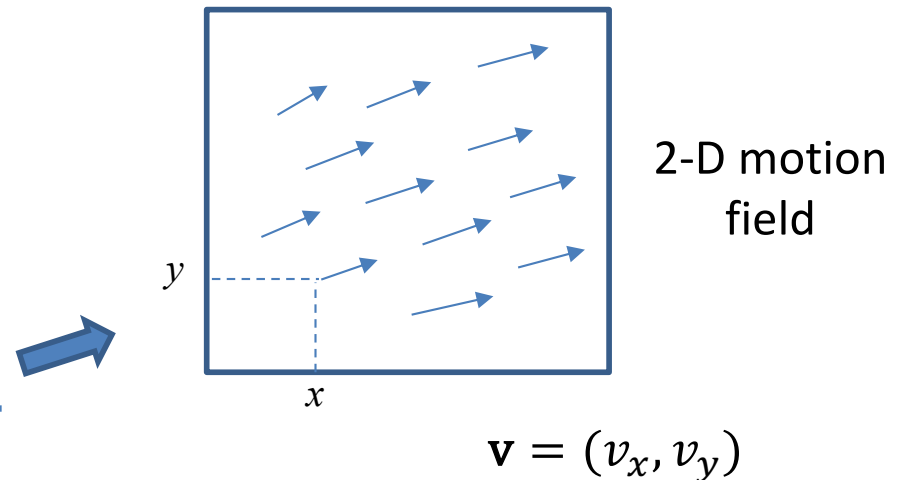
$$v_y = (fT_Y - yT_Z) / Z - f\theta_X + \theta_Z x + (\theta_Y xy - \theta_X y^2) / f$$

Small angle approximation



$R$  : 3x3 rotation matrix  $\Rightarrow (\theta_X, \theta_Y, \theta_Z)$

$T$  : 3x1 translation vector  $\Rightarrow (T_X, T_Y, T_Z)$



$$v_x = (fT_X - xT_Z) / Z + f\theta_Y - \theta_Z y - (\theta_X xy - \theta_Y x^2) / f$$

$$v_y = (fT_Y - yT_Z) / Z - f\theta_X + \theta_Z x + (\theta_Y xy - \theta_X y^2) / f$$

# Two Components

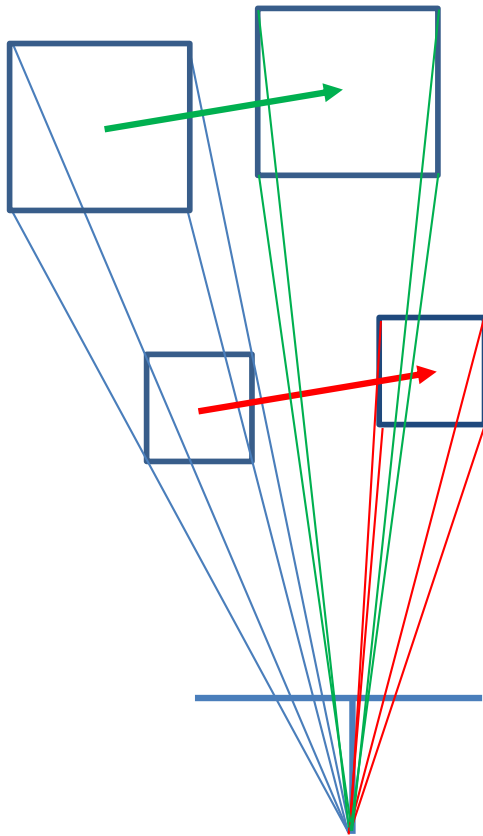
$$v_x = (fT_X - xT_Z) / Z + f\theta_Y - \theta_Z y - (\theta_X xy - \theta_Y x^2) / f$$

$$v_y = (fT_Y - yT_Z) / Z + f\theta_X + \theta_Z x + (\theta_Y xy - \theta_X y^2) / f$$

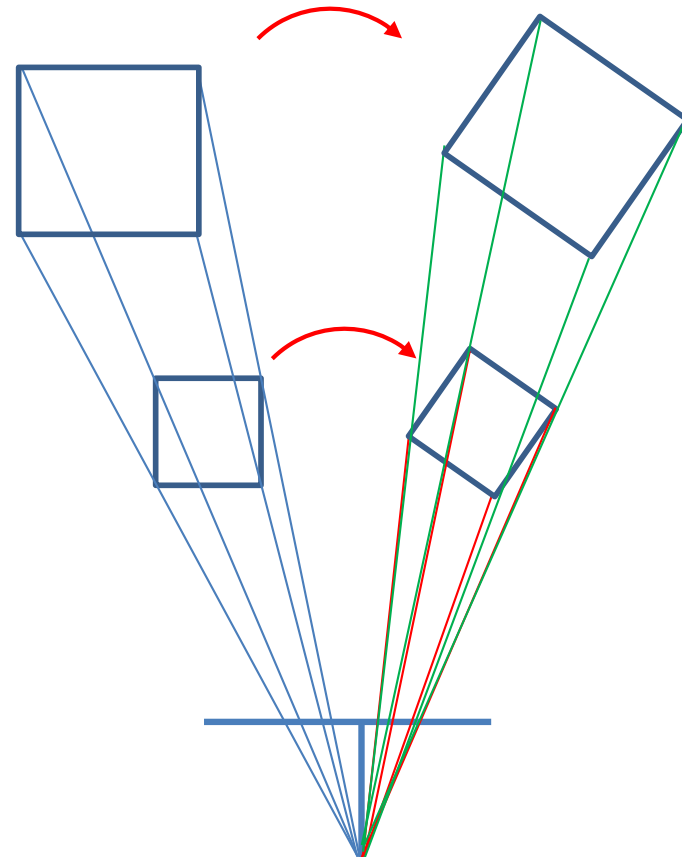
Translational – dependent on scene depth  $Z$

Rotational – independent of scene depth  $Z$

# Translation, Rotation and Depth



Motion field  $\propto$  depth



Motion field  $\not\propto$  depth

# Special Case: Pure Translation

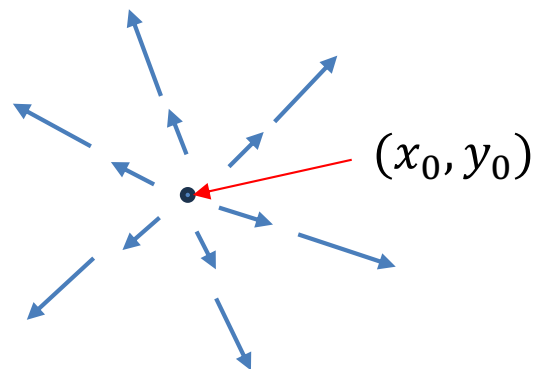
- Assume 3-D motion is only translational,  $\theta = 0$ , then

$$v_x = (fT_X - xT_Z)/Z \quad v_y = (fT_Y - yT_Z)/Z$$

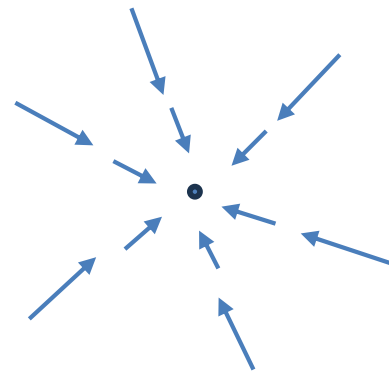
- If  $T_Z \neq 0$ ,  $x_0 = fT_X/T_Z$  and  $y_0 = fT_Y/T_Z$ , then

$$v_x = -(x - x_0)T_Z/Z \quad v_y = -(y - y_0)T_Z/Z$$

Expansion – object  
moving towards  
camera ( $T_Z$  negative)



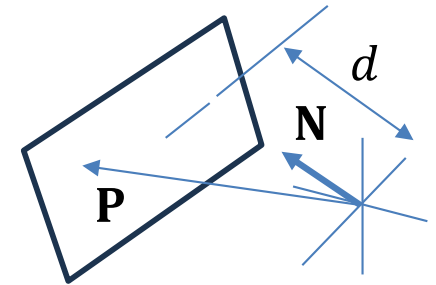
Contraction – object  
moving away from  
camera ( $T_Z$  positive)



$(x_0, y_0)$  – focus of expansion (contraction)

# Special Case: Moving Plane

- Assume 3-D points lie in plane with unit surface normal  $\mathbf{N}$ , i.e.  $\mathbf{N}^T \mathbf{P} = d$ , where  $d$  is distance of plane from origin.



- Since  $\mathbf{P} = Z\mathbf{p}/f$ , this gives  $Z(N_X x + N_Y y + N_Z f)/f = d$
- Substituting for  $Z$  in 2-D motion field:

$$v_x = \frac{1}{fd} (a_1 x^2 + a_2 xy + a_3 fx + a_4 fy + a_5 f^2)$$

$$v_y = \frac{1}{fd} (a_1 xy + a_2 y^2 + a_6 fy + a_7 fx + a_8 f^2)$$

- Motion field is a quadratic polynomial in 2-D spatial coordinates  $x$  and  $y$