

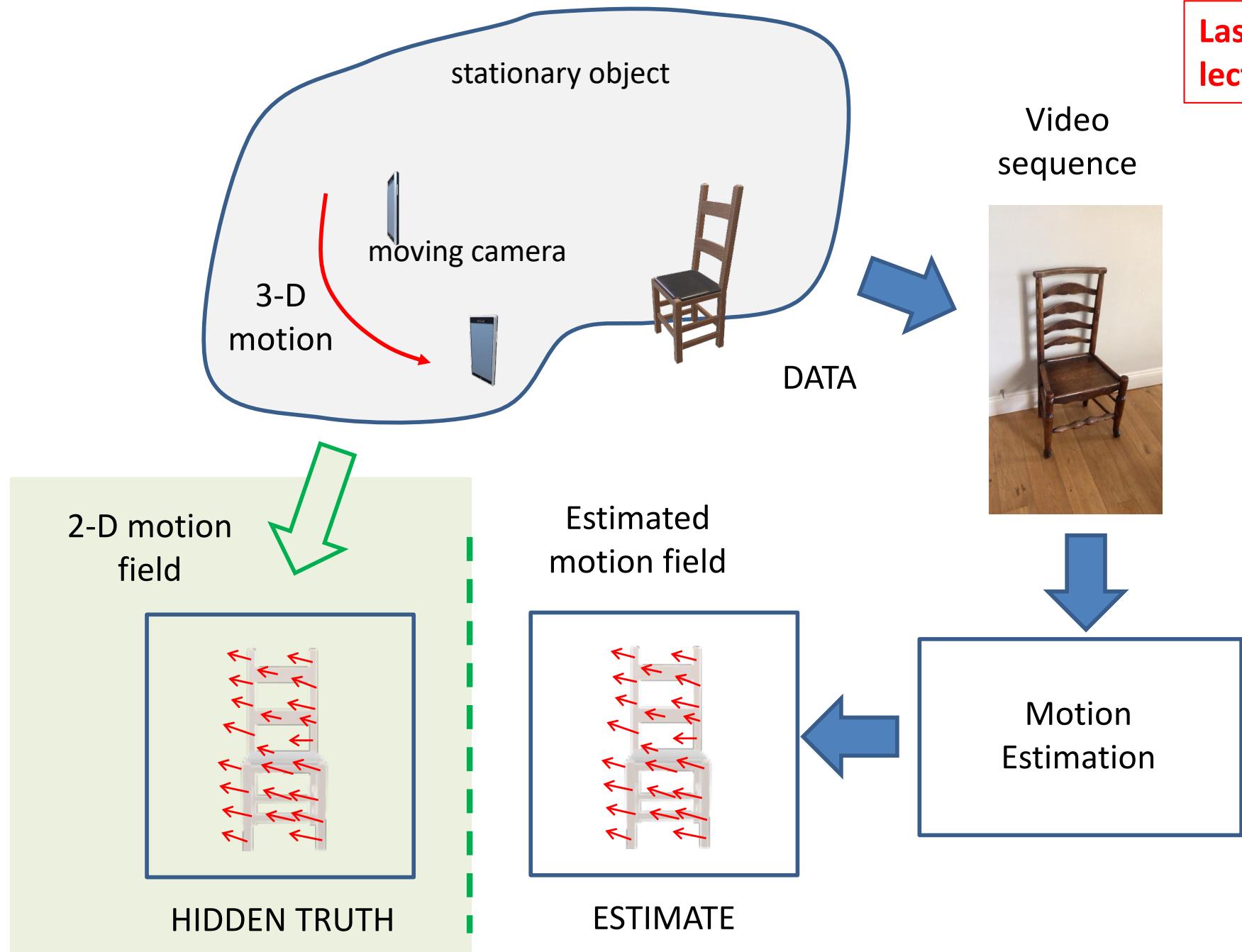
COMS30030
Image Processing and Computer Vision

Motion Estimation

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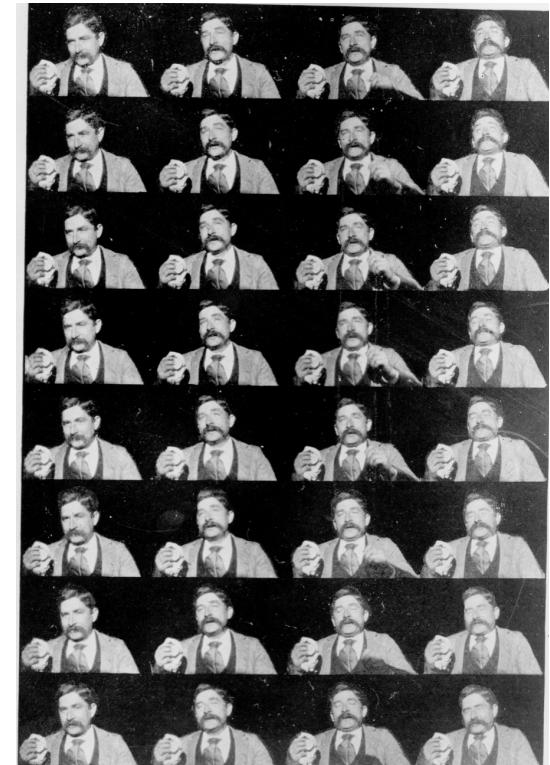
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Last
lecture



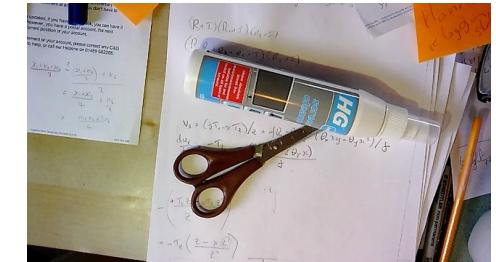
Motion Estimation

- The estimation of the 2-D motion field from frames in an image sequence
- Using spatial and temporal variation of pixel values
- **BUT**- relationship between variation in pixel values – known as **apparent motion** or **optical flow** – and the true motion is not straightforward.

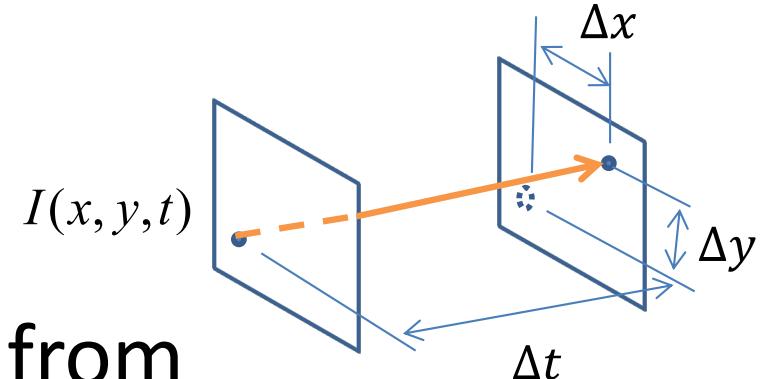


Apparent versus True Motion

- Apparent motion or **optical flow** - perceived motion in video sequence caused by changes in pixel values.
- Relationship with true 2-D motion field not always straightforward.
- Extreme cases:
 - non-zero apparent motion for zero motion field, e.g. static scene, moving light source
 - zero apparent motion for non-zero motion field, e.g. constant colour sphere rotating in diffuse lighting
- Sometimes not possible to determine 2-D motion field without additional constraints or assumptions.



Optical Flow



- Assume optical flow results from **brightness constancy constraint**
 - *‘a moving pixel retains its value between frames’*
- For continuous video $I(x, y, t)$ (grey level)

$$I(x + \Delta x, y + \Delta y, t + \Delta t) = I(x, y, t)$$

HoT $\rightarrow 0$ for tiny
 $\Delta x, \Delta y, \Delta t$

- Using Taylor’s expansion:

→ zero

$$I(x + \Delta x, y + \Delta y, t + \Delta t) = I(x, y, t) + \frac{\delta I}{\delta x} \Delta x + \frac{\delta I}{\delta y} \Delta y + \frac{\delta I}{\delta t} \Delta t + \dots$$

Optical Flow Equation

- For $I(x + \Delta x, y + \Delta y, t + \Delta t) = I(x, y, t)$

$$\frac{\delta I}{\delta x} \Delta x + \frac{\delta I}{\delta y} \Delta y + \frac{\delta I}{\delta t} \Delta t = 0$$

- Dividing throughout by Δt

$$\frac{\delta I}{\delta x} \frac{\Delta x}{\Delta t} + \frac{\delta I}{\delta y} \frac{\Delta y}{\Delta t} + \frac{\delta I}{\delta t} = 0$$

- For $\Delta x, \Delta y, \Delta t \rightarrow 0$

$$\frac{\delta I}{\delta x} \frac{dx}{dt} + \frac{\delta I}{\delta y} \frac{dy}{dt} + \frac{\delta I}{\delta t} = 0$$

Optical Flow
Equation
(OFE)

Optical Flow Equation (OFE)

$$\frac{\delta I}{\delta x} \frac{dx}{dt} + \frac{\delta I}{\delta y} \frac{dy}{dt} + \frac{\delta I}{\delta t} = 0$$

$\frac{dx}{dt}, \frac{dy}{dt}$ Rate of change of x, y with time
 ➡ optical flow field $\mathbf{u} = (u_x, u_y)$

$\frac{\delta I}{\delta x}, \frac{\delta I}{\delta y}, \frac{\delta I}{\delta t}$ Rate of change of I with x, y, t
 ➡ spatial & temporal gradients (I_x, I_y, I_t)

**Optical flow
equation**

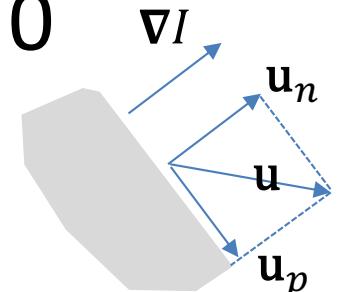
$$I_x u_x + I_y u_y + I_t = 0$$

Normal Flow

- OFE for $\mathbf{u} = (u_x, u_y)$ and $\nabla I = (I_x, I_y)$

$$I_x u_x + I_y u_y + I_t = 0 \Rightarrow \nabla I \cdot \mathbf{u} + I_t = 0$$

$$\nabla I \cdot \mathbf{u} = I_x u_x + I_y u_y \quad \text{dot product}$$



- OFE alone not sufficient to estimate motion
 - one equation in two unknowns
 - Only estimate **normal flow** \mathbf{u}_n
- $$\nabla I \cdot \mathbf{u} + I_t = \nabla I \cdot \mathbf{u}_n + I_t = 0$$
- ⇒ $\|\mathbf{u}_n\| = -I_t / \|\nabla I\| \quad \angle \mathbf{u}_n = \angle \nabla I$

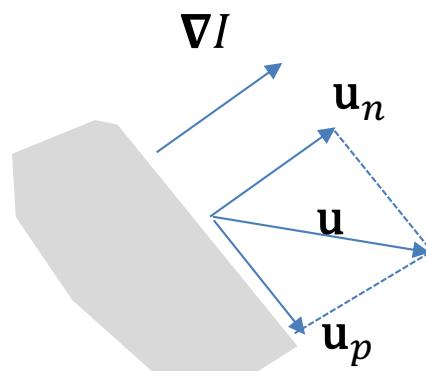
$$\begin{aligned} \mathbf{u} &= \mathbf{u}_p + \mathbf{u}_n \\ \mathbf{u}_p \cdot \mathbf{u}_n &= 0 \\ \nabla I \cdot \mathbf{u}_p &= 0 \end{aligned}$$

Normal Flow

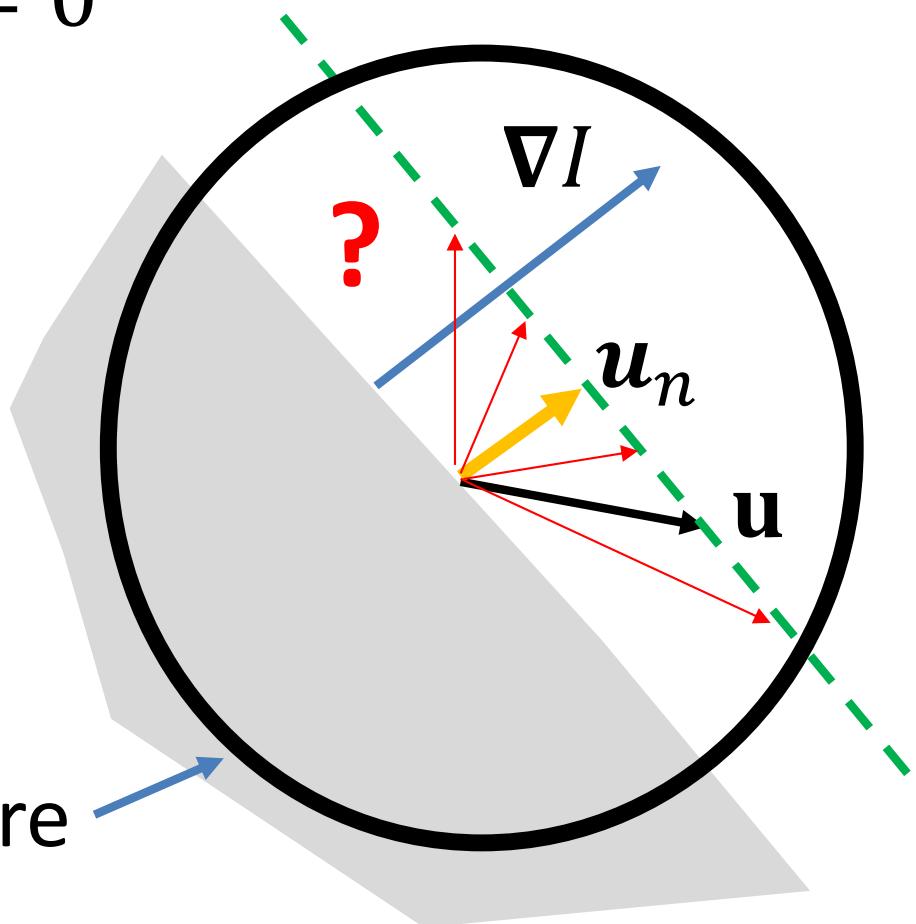
$$\nabla I \cdot \mathbf{u} + I_t = \nabla I \cdot \mathbf{u}_n + I_t = 0$$

$$\Rightarrow \|\mathbf{u}_n\| = -I_t / \|\nabla I\|$$

$$\angle \mathbf{u}_n = \angle \nabla I$$

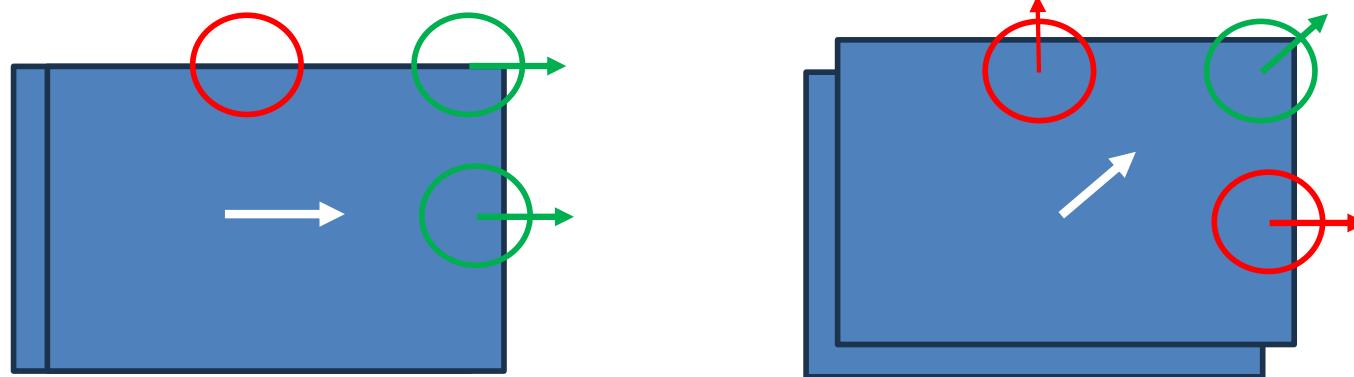


aperture



Aperture Problem

With single gradient direction in window (aperture), observed motion is different from true motion as we can only observe motion parallel to the gradient:



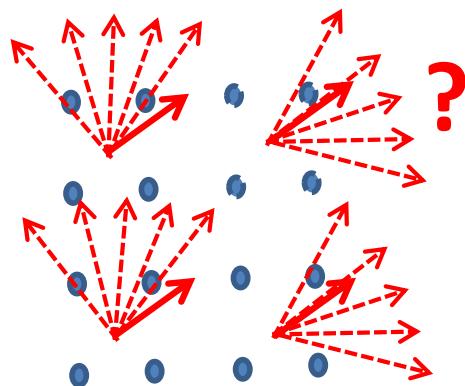
Hence: Good motion estimation depends on having sufficient variation in spatial gradient within regions.

Constraining the OFE

$$I_x u_x + I_y u_y + I_t = 0$$

OFE is under constrained – can only estimate normal flow

Need to add extra constraint(s)



Example : assume parametric form of motion field in regions

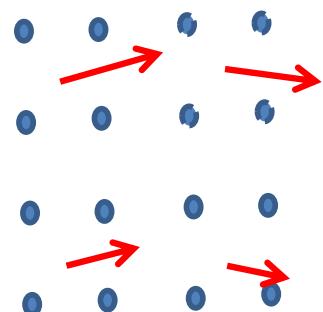
Example : constant velocity

Constraining the OFE

$$I_x u_x + I_y u_y + I_t = 0$$

OFE is under constrained – can only estimate normal flow

Need to add extra constraint(s)



Example : linear in x and y , e.g.

Example : assume parametric form of motion field in regions

$$u_x = ax + by + c$$
$$u_y = dx + ey + f$$

Constant Velocity Model

For a region, find the velocity $\mathbf{u} = (u_x, u_y)$ which minimises :

$$\varepsilon(u_x, u_y) = \sum_{region} (I_x u_x + I_y u_y + I_t)^2$$

Solution: take derivatives w.r.t u_x and u_y , set to zero, and solve for u_x and u_y .

OFE $\rightarrow 0$

NB: same $\mathbf{u} = (u_x, u_y)$ over whole region \rightarrow solution

Lucas and Kanade Algorithm

Find velocity $\mathbf{u} = (u_x, u_y)$ which minimises :

$$\varepsilon(u_x, u_y) = \sum_R (I_x u_x + I_y u_y + I_t)^2$$

Partial derivatives w.r.t u_x and u_y , set to zero, solve for u_x and u_y :

$$\frac{\partial \varepsilon}{\partial u_x} = 2 \sum_R (I_x u_x + I_y u_y + I_t) I_x = 0 \quad \Rightarrow \quad \sum_R (I_x^2 u_x + I_x I_y u_y + I_x I_t) = 0$$

$$\frac{\partial \varepsilon}{\partial u_y} = 2 \sum_R (I_x u_x + I_y u_y + I_t) I_y = 0 \quad \Rightarrow \quad \sum_R (I_x I_y u_x + I_y^2 u_y + I_y I_t) = 0$$

Lucas and Kanade Algorithm

Hence, solve for $\mathbf{u} = (u_x, u_y)$ given that :

$$u_x \sum_R I_x^2 + u_y \sum_R I_x I_y = - \sum_R I_t I_x \quad \Rightarrow \quad A\mathbf{u} = \mathbf{b}$$

$$u_x \sum_R I_x I_y + u_y \sum_R I_y^2 = - \sum_R I_t I_y$$

$$\Rightarrow \boxed{\mathbf{u} = \begin{bmatrix} u_x \\ u_y \end{bmatrix} = A^{-1}\mathbf{b}}$$

$$A = \sum_R \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \quad \mathbf{b} = - \sum_R \begin{bmatrix} I_t I_x \\ I_t I_y \end{bmatrix}$$

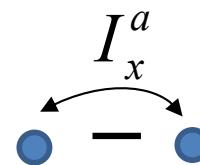
Spatial & Temporal Gradients

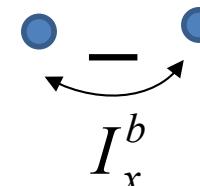
Approximate gradients using differences, e.g.

$$I_x = \delta I / \delta x \approx I(x + 1, y, t) - I(x, y, t)$$

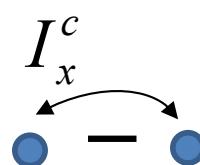
i.e. assume $\delta x = 1$

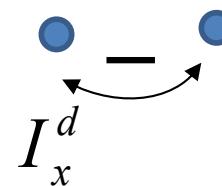
Or use averaging to reduce noise, e.g.

$$I_x^a$$


$$I_x^b$$


Frame t

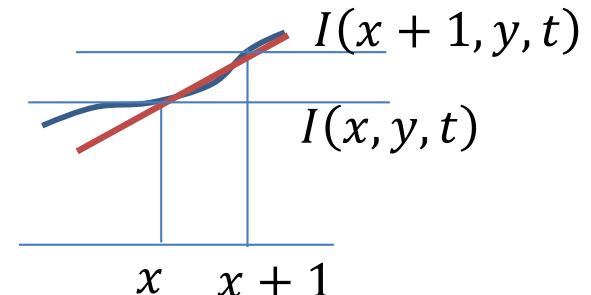
$$I_x^c$$


$$I_x^d$$


Frame $t + 1$

$$I_x \approx (I_x^a + I_x^b + I_x^c + I_x^d) / 4$$

Rate of change
of I with x



L & K Algorithm

I^1 = video frame at time t

I^2 = video frame at time $t + 1$

For each pixel x, y in I^1

$A = 0$; $\mathbf{b} = 0$;

For each pixel in region Λ about x, y

$(I_x, I_y, I_t) = \text{CompGrads}(I^1, I^2)$;

$$A' = \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix};$$

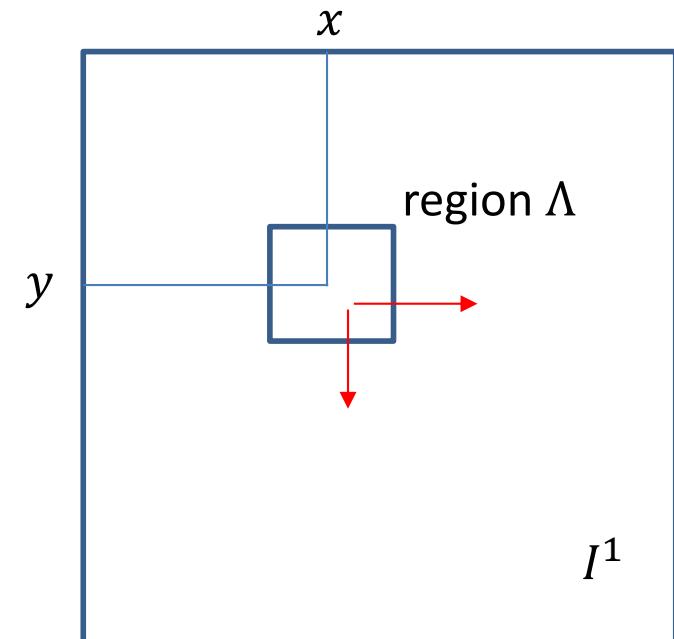
$$\mathbf{b}' = \begin{bmatrix} -I_t I_x \\ -I_t I_y \end{bmatrix};$$

$A \rightarrow A + A'$; $\mathbf{b} \rightarrow \mathbf{b} + \mathbf{b}'$;

End;

$$\mathbf{u}(x, y) = A^{-1} \mathbf{b}$$

End;





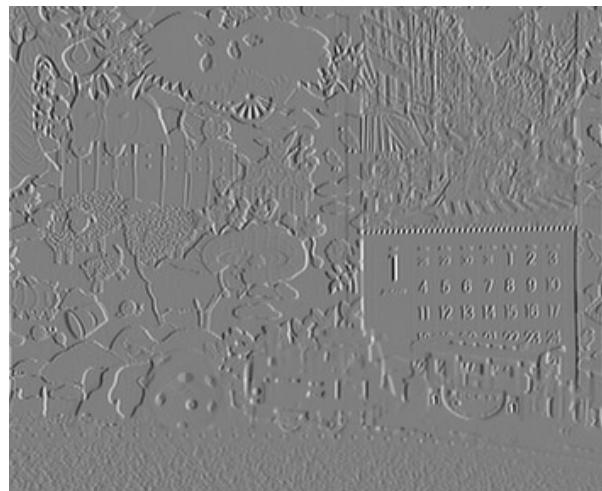
$I(x, y, t)$



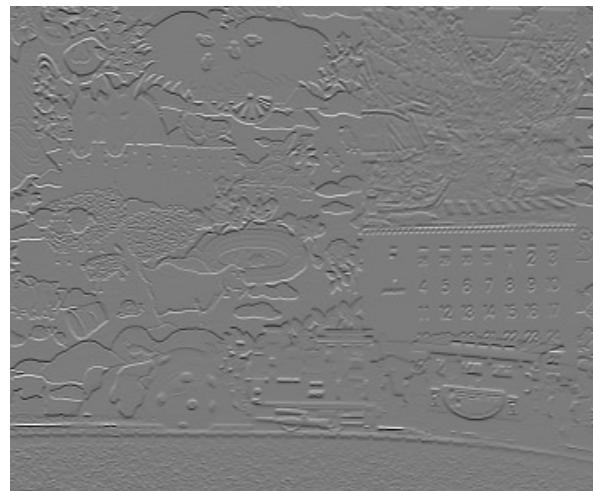
u_x



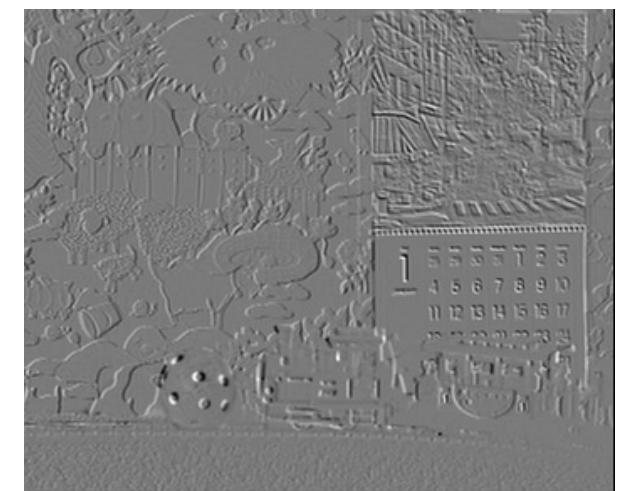
u_y



I_x

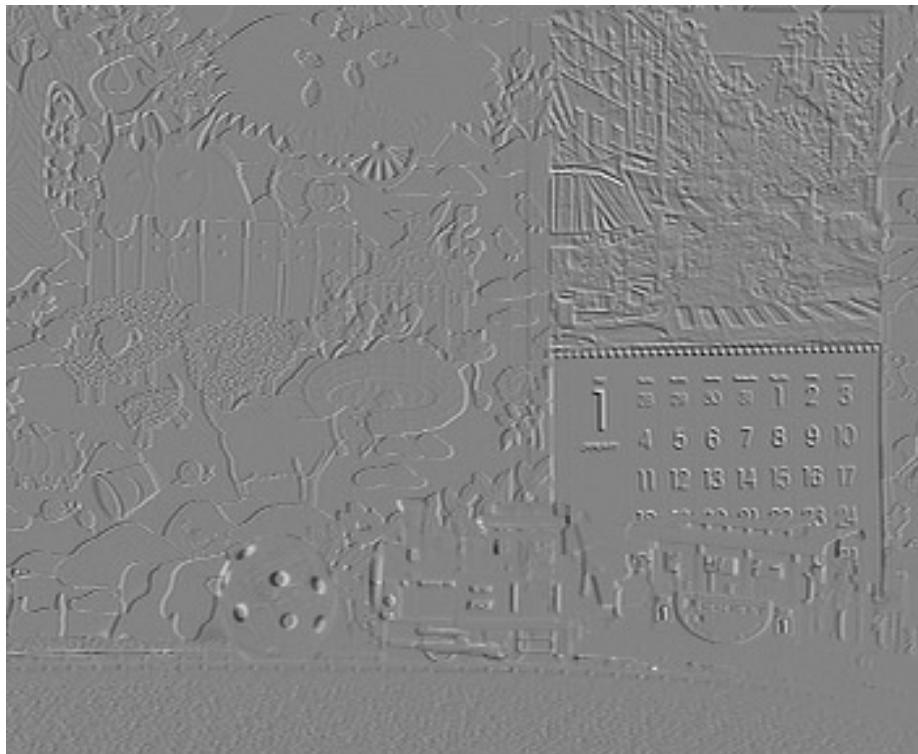


I_y

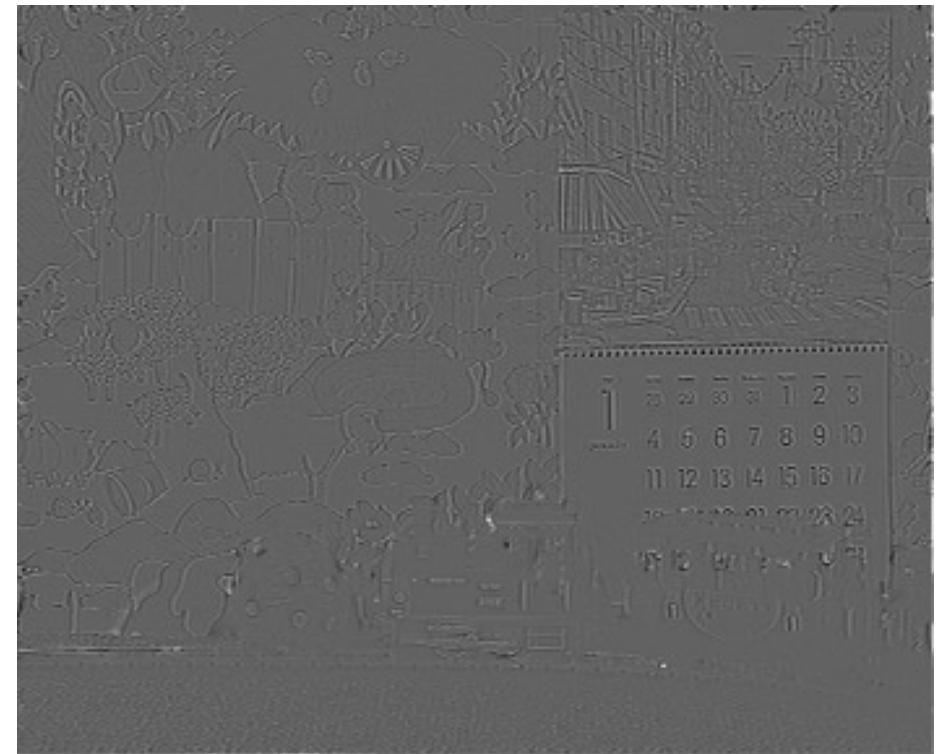


I_t

Frame Difference



frame difference

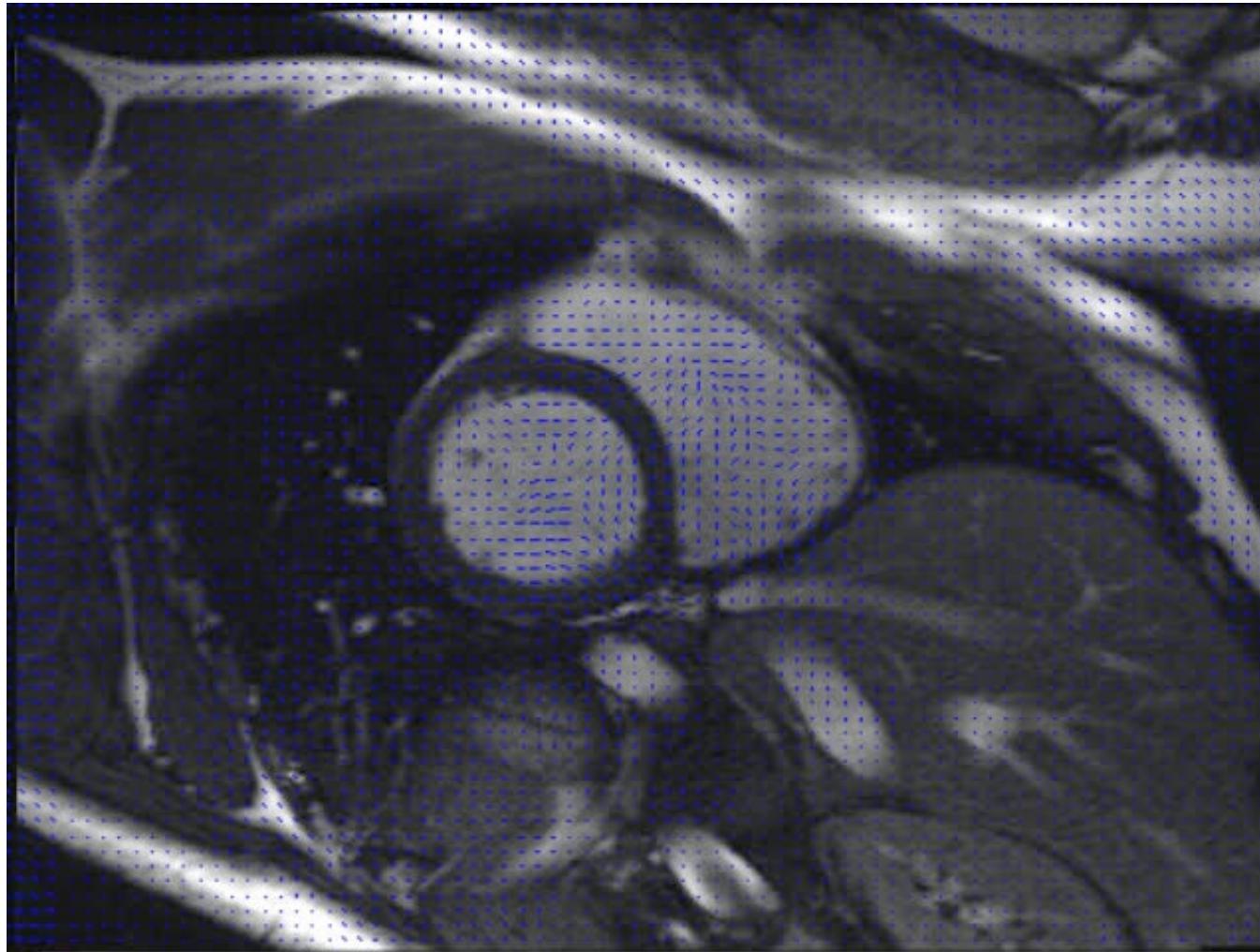


motion compensated frame difference

Motion Estimation - Example

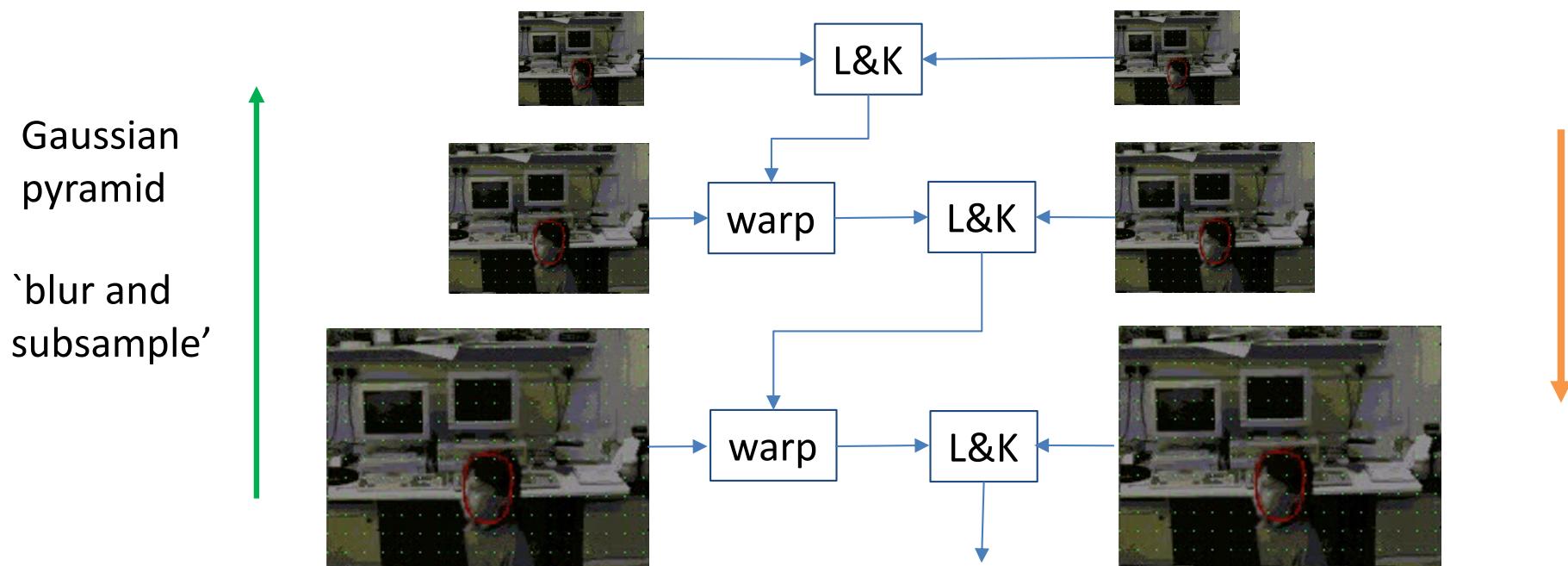


Motion Estimation - Example

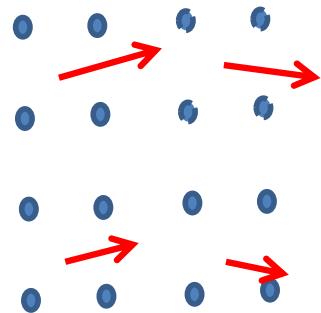


Multiresolution L & K

- To deal with large motions, implement L&K over multiple resolutions – result at lower resolutions used to ‘warp’ higher resolution images prior to estimation.



Affine Motion Model



$$\begin{aligned}u_x &= ax + by + c \\u_y &= dx + ey + f\end{aligned}$$

$$\mathbf{u} = A \mathbf{p}$$

$$\mathbf{p}^T = (a, b, c, d, e, f)$$

Models translation, scaling, rotation and shear

$$\Rightarrow \mathbf{p}^T A^T \nabla I + I_t = 0 \quad \text{affine OFE} \quad \Rightarrow \hat{\mathbf{p}} = M^{-1} \mathbf{b}$$

$$M = \sum_{region} A^T \nabla I \nabla I^T A \quad b = - \sum_{region} I_t (A^T \nabla I)$$

Horn-Schunk Algorithm

- Alternative to L&K which seeks to find optimal motion field with smooth variation in motion vectors
- Algorithm aims to find the motion field $\mathbf{v} = (v_x, v_y)$ which minimises following energy functional

$$E = \iint \left[(I_x u_x + I_y u_y + I_t)^2 + \sigma^2 (\|\nabla u_x\|^2 + \|\nabla u_y\|^2) \right] dx dy$$

Diagram illustrating the energy functional components:

- Red bracket under $(I_x u_x + I_y u_y + I_t)^2$ points to a box labeled **OFE $\rightarrow 0$** .
- Red bracket under $\sigma^2 (\|\nabla u_x\|^2 + \|\nabla u_y\|^2)$ points to a box labeled **weighting factor**.
- A blue arrow points from the **weighting factor** box to the text **Rate of change of $\mathbf{v} \rightarrow 0$** .
- A blue arrow points from the text **Rate of change of $\mathbf{v} \rightarrow 0$** to the text **smooth motion field**.