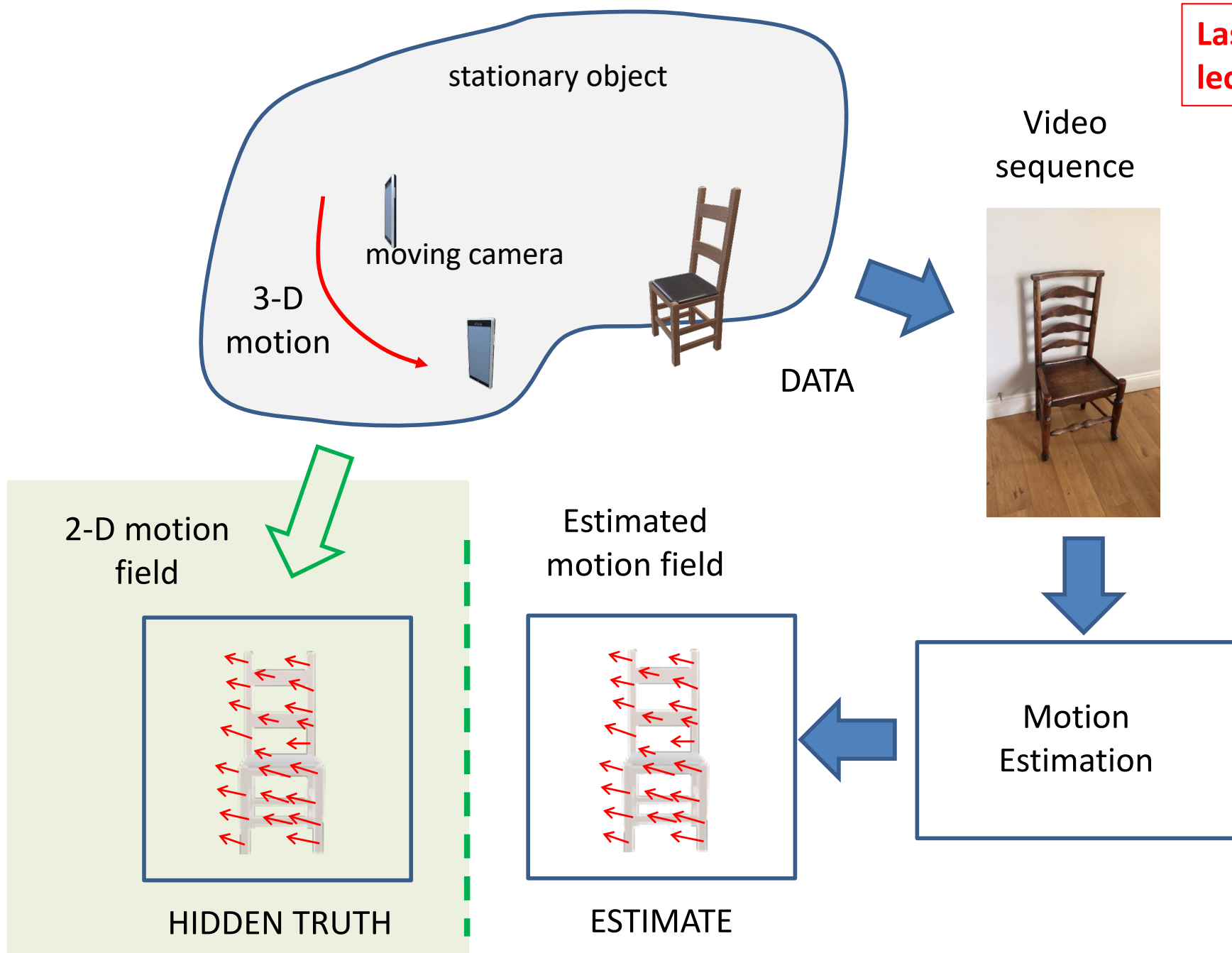


COMS30030  
Image Processing and Computer Vision

**Motion Estimation**

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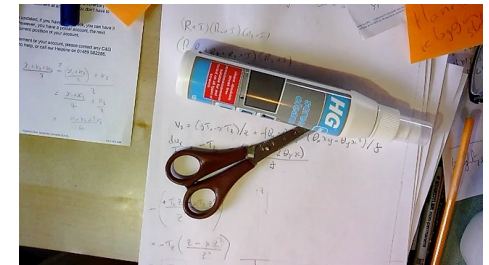
# Motion Estimation

- The estimation of the 2-D motion field from frames in an image sequence
- Using spatial and temporal variation of pixel values
- **BUT**- relationship between variation in pixel values – known as **apparent motion** or **optical flow** – and the true motion is not straightforward.

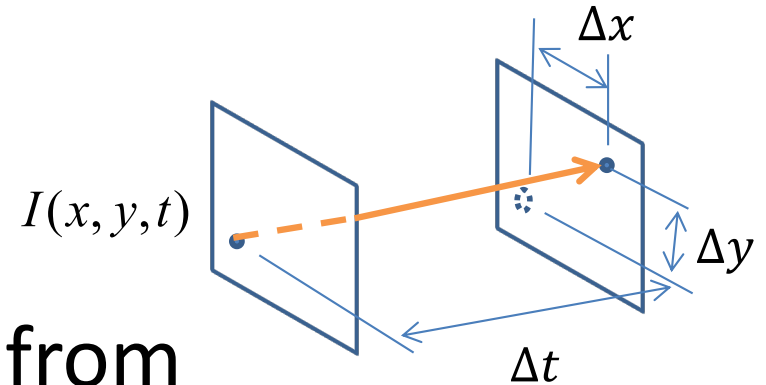


# Apparent versus True Motion

- Apparent motion or **optical flow** - perceived motion in video sequence caused by changes in pixel values.
- Relationship with true 2-D motion field not always straightforward.
- Extreme cases:
  - non-zero apparent motion for zero motion field, e.g. static scene, moving light source
  - zero apparent motion for non-zero motion field, e.g. constant colour sphere rotating in diffuse lighting
- Sometimes not possible to determine 2-D motion field without additional constraints or assumptions.



# Optical Flow



- Assume optical flow results from **brightness constancy constraint**
  - *`a moving pixel retains its value between frames'*
- For continuous video  $I(x, y, t)$  (grey level)

$$I(x + \Delta x, y + \Delta y, t + \Delta t) = I(x, y, t)$$

HoT  $\rightarrow 0$  for tiny  $\Delta x, \Delta y, \Delta t$

- Using Taylor's expansion:

$$I(x + \Delta x, y + \Delta y, t + \Delta t) = I(x, y, t) + \frac{\delta I}{\delta x} \Delta x + \frac{\delta I}{\delta y} \Delta y + \frac{\delta I}{\delta t} \Delta t + \dots$$

$\rightarrow$  zero

# Optical Flow Equation

- For  $I(x + \Delta x, y + \Delta y, t + \Delta t) = I(x, y, t)$

$$\frac{\delta I}{\delta x} \Delta x + \frac{\delta I}{\delta y} \Delta y + \frac{\delta I}{\delta t} \Delta t = 0$$

- Dividing throughout by  $\Delta t$

$$\frac{\delta I}{\delta x} \frac{\Delta x}{\Delta t} + \frac{\delta I}{\delta y} \frac{\Delta y}{\Delta t} + \frac{\delta I}{\delta t} = 0$$

- For  $\Delta x, \Delta y, \Delta t \rightarrow 0$

$$\frac{\delta I}{\delta x} \frac{dx}{dt} + \frac{\delta I}{\delta y} \frac{dy}{dt} + \frac{\delta I}{\delta t} = 0$$

Optical Flow  
Equation  
(OFE)

# Optical Flow Equation (OFE)

$$\frac{\delta I}{\delta x} \frac{dx}{dt} + \frac{\delta I}{\delta y} \frac{dy}{dt} + \frac{\delta I}{\delta t} = 0$$

$$\frac{dx}{dt}, \frac{dy}{dt}$$

Rate of change of  $x, y$  with time

⇒ optical flow field  $\mathbf{u} = (u_x, u_y)$

$$\frac{\delta I}{\delta x}, \frac{\delta I}{\delta y}, \frac{\delta I}{\delta t}$$

Rate of change of  $I$  with  $x, y, t$

⇒ spatial & temporal gradients  $(I_x, I_y, I_t)$

**Optical flow  
equation**

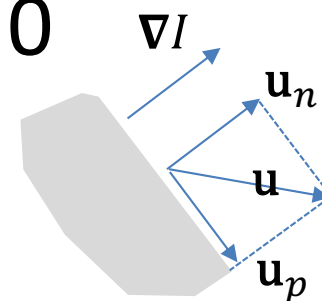
$$I_x u_x + I_y u_y + I_t = 0$$

# Normal Flow

- OFE for  $\mathbf{u} = (u_x, u_y)$  and  $\nabla I = (I_x, I_y)$

$$I_x u_x + I_y u_y + I_t = 0 \Rightarrow \nabla I \cdot \mathbf{u} + I_t = 0$$

$$\nabla I \cdot \mathbf{u} = I_x u_x + I_y u_y \quad \text{dot product}$$



- OFE alone not sufficient to estimate motion
  - one equation in two unknowns

- Only estimate **normal flow**  $\mathbf{u}_n$

$$\nabla I \cdot \mathbf{u} + I_t = \nabla I \cdot \mathbf{u}_n + I_t = 0$$

$$\Rightarrow \|\mathbf{u}_n\| = -I_t / \|\nabla I\| \quad \angle \mathbf{u}_n = \angle \nabla I$$

$$\mathbf{u} = \mathbf{u}_p + \mathbf{u}_n$$

$$\mathbf{u}_p \cdot \mathbf{u}_n = 0$$

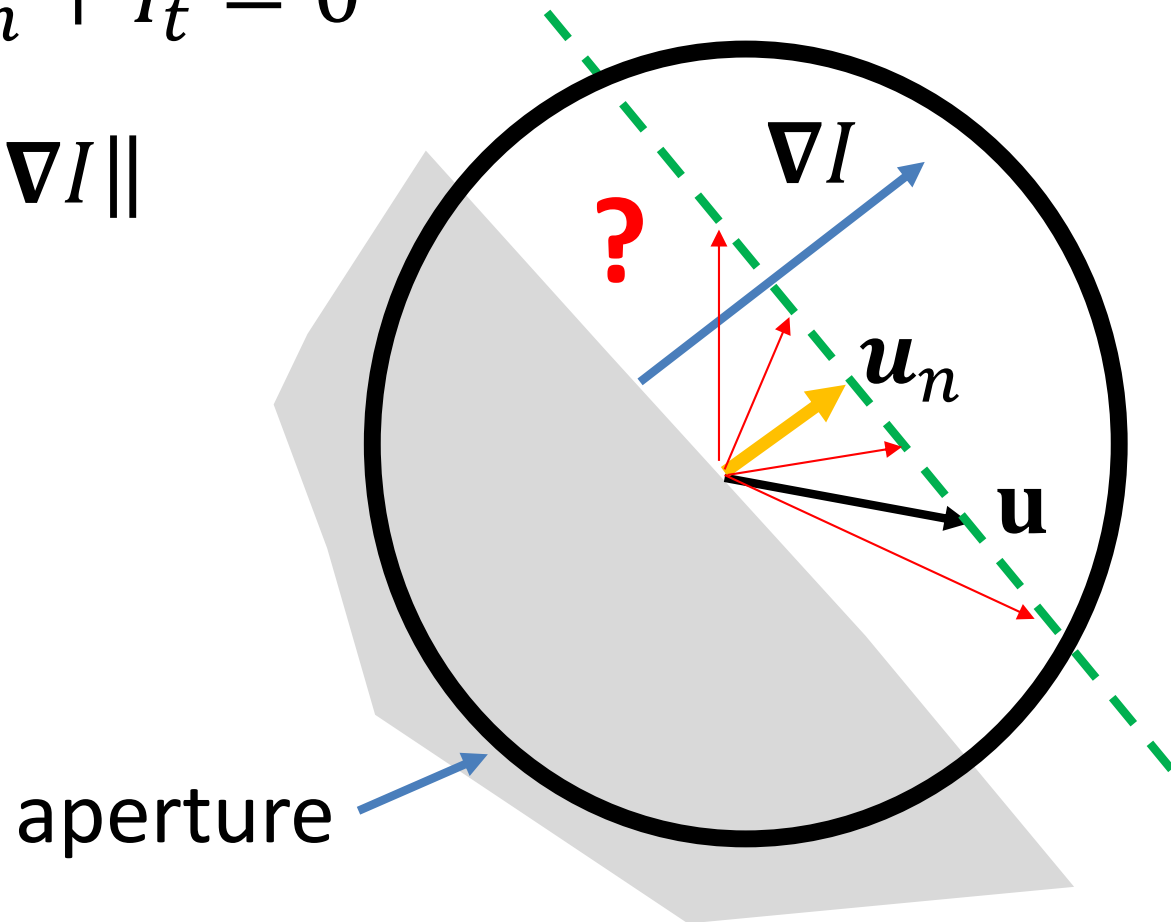
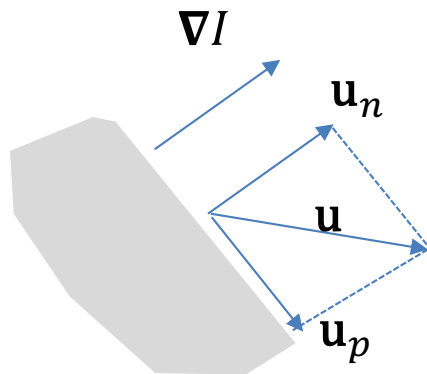
$$\nabla I \cdot \mathbf{u}_p = 0$$

# Normal Flow

$$\nabla I \cdot \mathbf{u} + I_t = \nabla I \cdot \mathbf{u}_n + I_t = 0$$

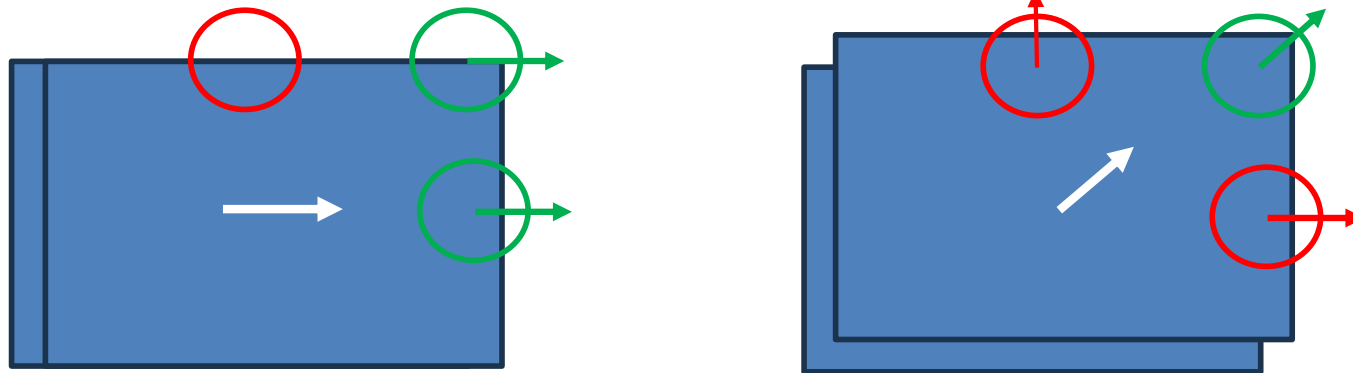
$$\Rightarrow \|\mathbf{u}_n\| = -I_t / \|\nabla I\|$$

$$\angle \mathbf{u}_n = \angle \nabla I$$



# Aperture Problem

With single gradient direction in window (aperture), observed motion is different from true motion as we can only observe motion parallel to the gradient:



**Hence: Good motion estimation depends on having sufficient variation in spatial gradient within regions.**

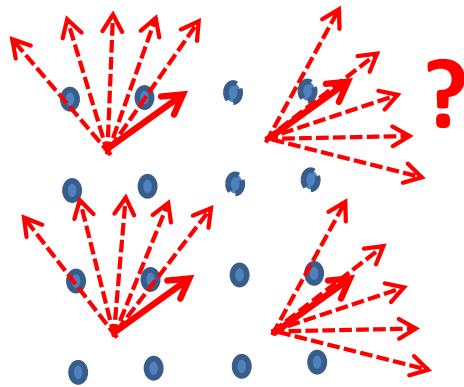
# Constraining the OFE

$$I_x u_x + I_y u_y + I_t = 0$$

OFE is under constrained – can only estimate normal flow

Need to add extra constraint(s)

Example : assume parametric form of motion field in regions



Example : constant velocity

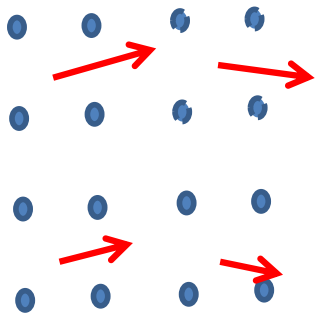
# Constraining the OFE

$$I_x u_x + I_y u_y + I_t = 0$$

OFE is under constrained – can only estimate normal flow

Need to add extra constraint(s)

Example : assume parametric form of motion field in regions



Example : linear in  $x$  and  $y$ , e.g.

$$\begin{aligned} u_x &= ax + by + c \\ u_y &= dx + ey + f \end{aligned}$$

# Constant Velocity Model

For a region, find the velocity  $\mathbf{u} = (u_x, u_y)$  which minimises :

$$\varepsilon(u_x, u_y) = \sum_{region} \underbrace{(I_x u_x + I_y u_y + I_t)^2}_{\text{OFE} \rightarrow 0}$$

Solution: take derivatives w.r.t  $u_x$  and  $u_y$ , set to zero, and solve for  $u_x$  and  $u_y$ .

NB: same  $\mathbf{u} = (u_x, u_y)$  over whole region  $\rightarrow$  solution

# Lucas and Kanade Algorithm

Find velocity  $\mathbf{u} = (u_x, u_y)$  which minimises :

$$\varepsilon(u_x, u_y) = \sum_R (I_x u_x + I_y u_y + I_t)^2$$

Partial derivatives w.r.t  $u_x$  and  $u_y$ , set to zero, solve for  $u_x$  and  $u_y$ :

$$\frac{\partial \varepsilon}{\partial u_x} = 2 \sum_R (I_x u_x + I_y u_y + I_t) I_x = 0 \quad \Rightarrow \quad \sum_R (I_x^2 u_x + I_x I_y u_y + I_x I_t) = 0$$

$$\frac{\partial \varepsilon}{\partial u_y} = 2 \sum_R (I_x u_x + I_y u_y + I_t) I_y = 0 \quad \Rightarrow \quad \sum_R (I_x I_y u_x + I_y^2 u_y + I_y I_t) = 0$$

# Lucas and Kanade Algorithm

Hence, solve for  $\mathbf{u} = (u_x, u_y)$  given that :

$$u_x \sum_R I_x^2 + u_y \sum_R I_x I_y = - \sum_R I_t I_x \quad \Rightarrow \quad A\mathbf{u} = \mathbf{b}$$

$$u_x \sum_R I_x I_y + u_y \sum_R I_y^2 = - \sum_R I_t I_y$$



$$\mathbf{u} = \begin{bmatrix} u_x \\ u_y \end{bmatrix} = A^{-1} \mathbf{b}$$

$$A = \sum_R \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \quad \mathbf{b} = - \sum_R \begin{bmatrix} I_t I_x \\ I_t I_y \end{bmatrix}$$

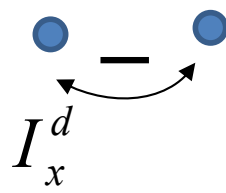
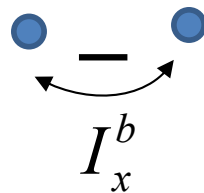
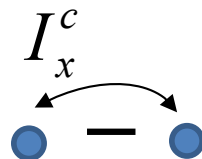
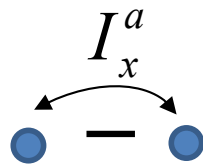
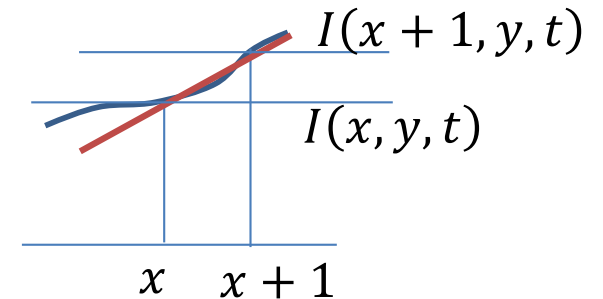
# Spatial & Temporal Gradients

Approximate gradients using differences, e.g.

$$I_x = \delta I / \delta x \approx I(x + 1, y, t) - I(x, y, t)$$

i.e. assume  $\delta x = 1$

Or use averaging to reduce noise, e.g.



Frame  $t$

Frame  $t + 1$

$$I_x \approx (I_x^a + I_x^b + I_x^c + I_x^d) / 4$$

Rate of change  
of  $I$  with  $x$

# L & K Algorithm

$I^1 = \text{video frame at time } t$

$I^2 = \text{video frame at time } t + 1$

For each pixel  $x, y$  in  $I^1$

$A = 0$  ;  $\mathbf{b} = 0$  ;

For each pixel in region  $\Lambda$  about  $x, y$

$(I_x, I_y, I_t) = \text{CompGrads}(I^1, I^2)$  ;

$$A' = \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} ;$$

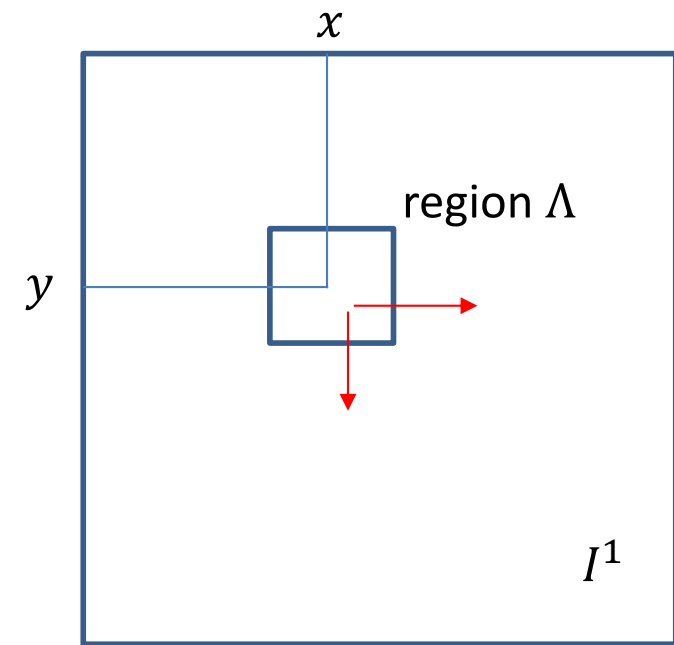
$$\mathbf{b}' = \begin{bmatrix} -I_t I_x \\ -I_t I_y \end{bmatrix} ;$$

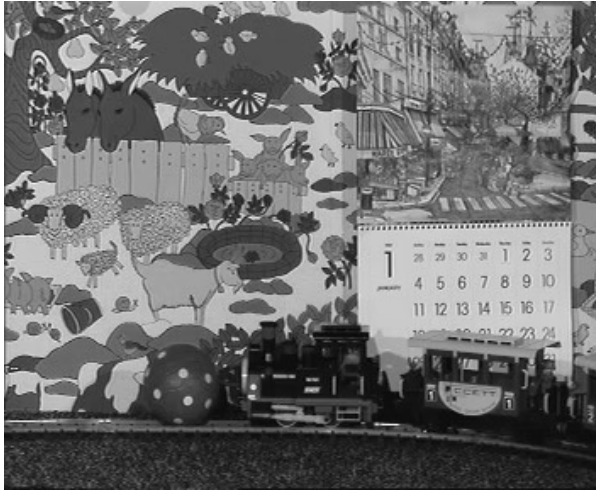
$A \rightarrow A + A'$  ;  $\mathbf{b} \rightarrow \mathbf{b} + \mathbf{b}'$  ;

End;

$\mathbf{u}(x, y) = A^{-1} \mathbf{b}$

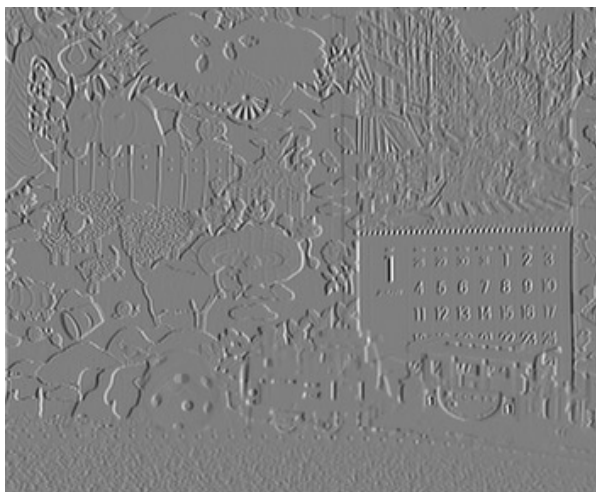
End;

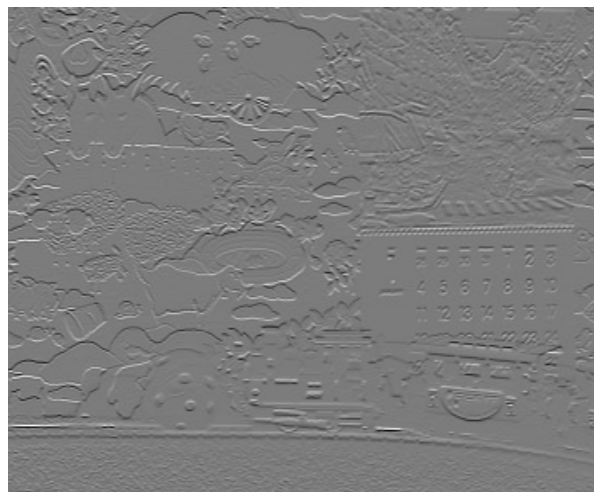


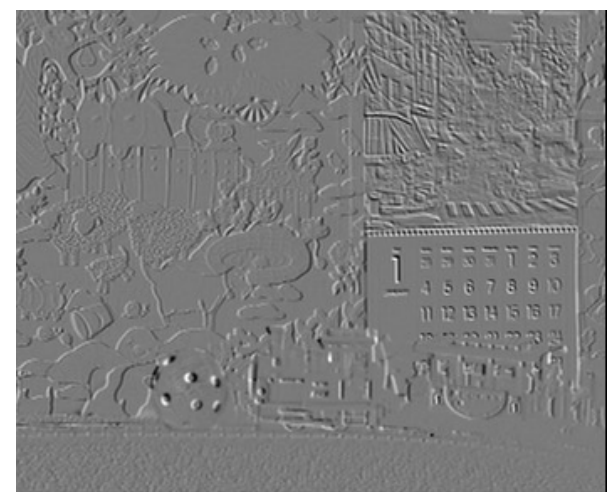


$$I(x, y, t)$$


$$u_x$$

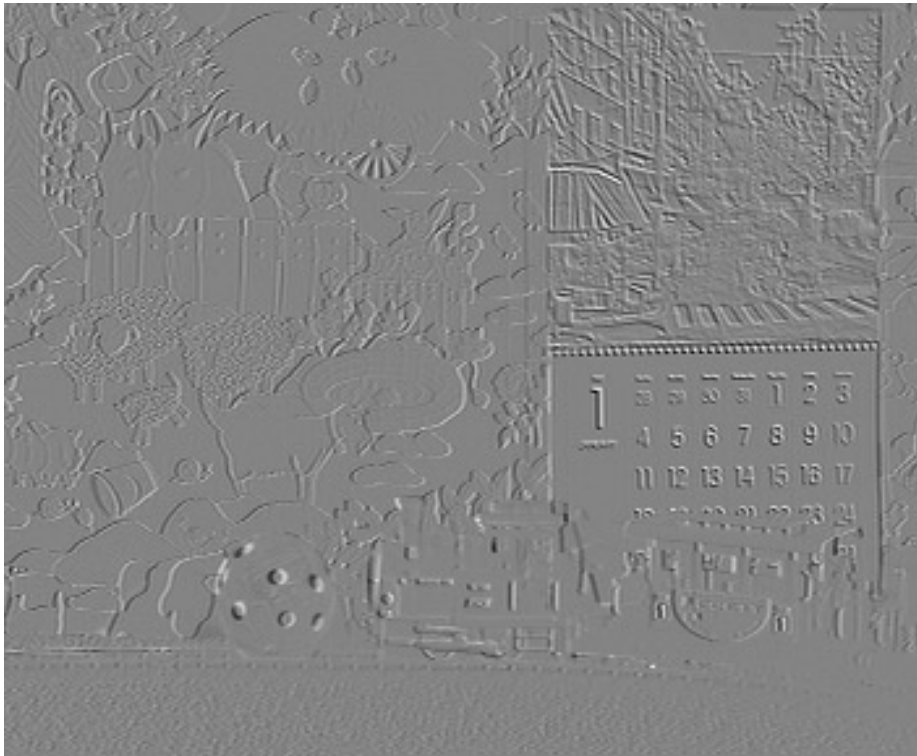

$$u_y$$


$$I_x$$


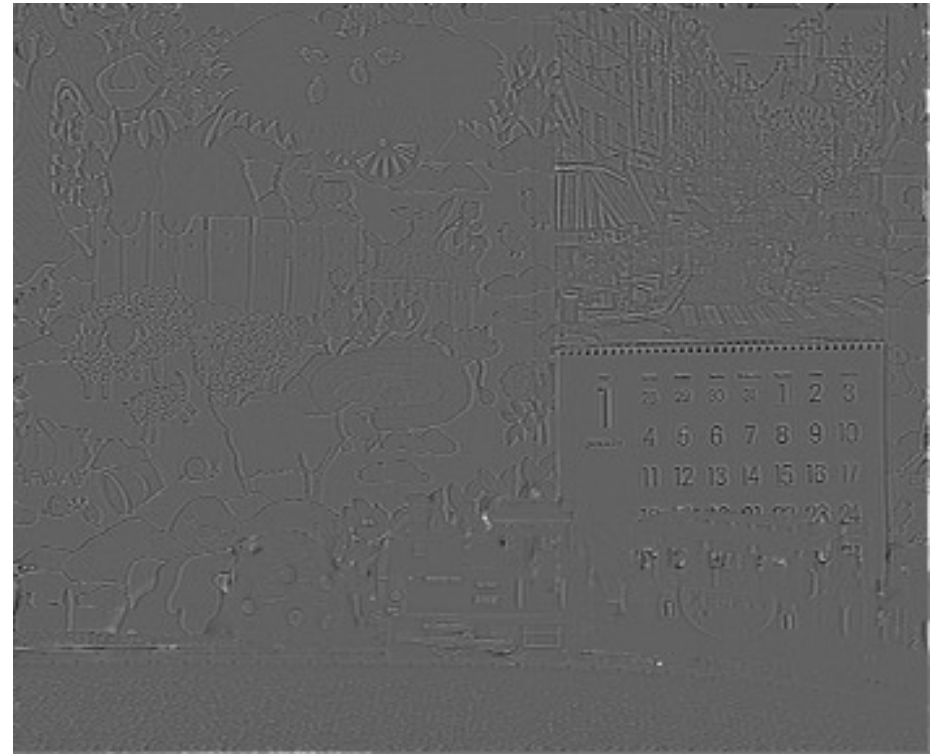
$$I_y$$


$$I_t$$

# Frame Difference



frame difference

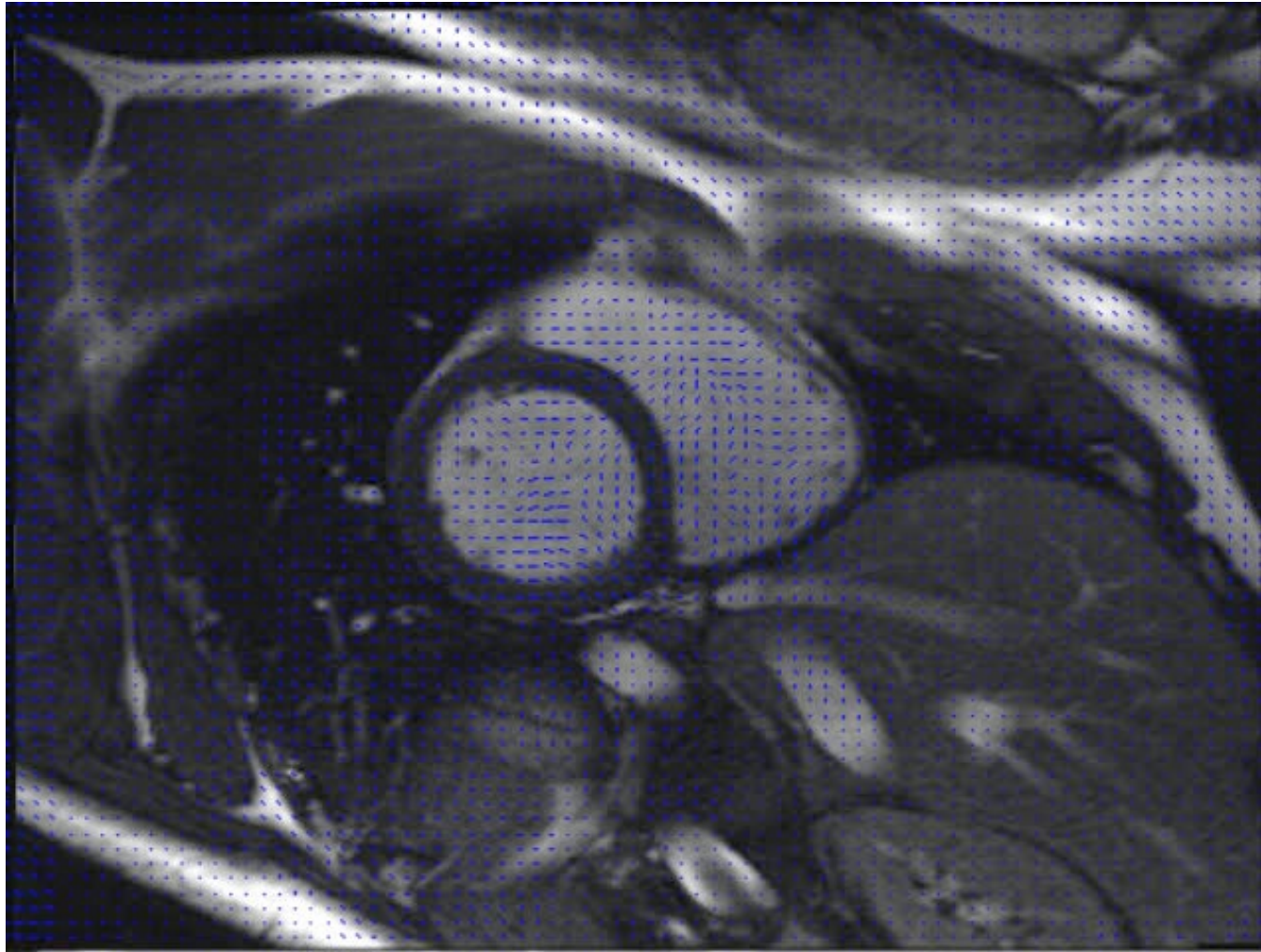


motion compensated frame difference

# Motion Estimation - Example

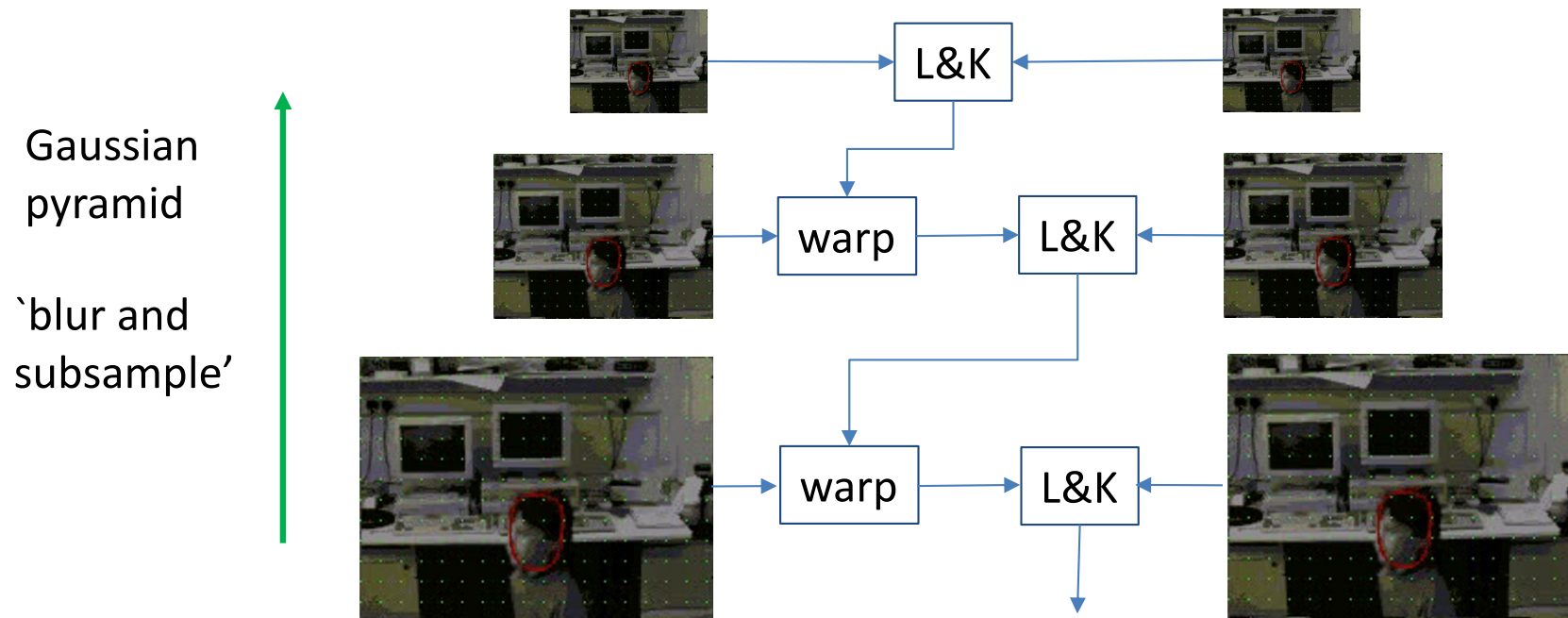


# Motion Estimation - Example

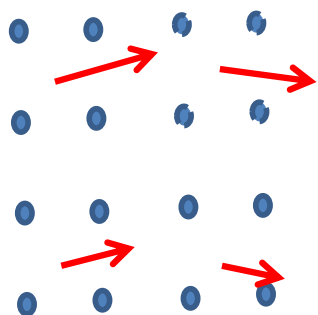


# Multiresolution L & K

- To deal with large motions, implement L&K over multiple resolutions – result at lower resolutions used to ‘warp’ higher resolution images prior to estimation.



# Affine Motion Model



$$\begin{aligned} u_x &= ax + by + c \\ u_y &= dx + ey + f \end{aligned}$$

$$\Rightarrow \mathbf{u} = A \mathbf{p}$$

$$\mathbf{p}^T = (a, b, c, d, e, f)$$

Models translation, scaling, rotation and shear

$$\Rightarrow \mathbf{p}^T A^T \nabla I + I_t = 0 \quad \text{affine OFE} \quad \Rightarrow \hat{\mathbf{p}} = M^{-1} \mathbf{b}$$

$$M = \sum_{region} A^T \nabla I \nabla I^T A \quad b = - \sum_{region} I_t (A^T \nabla I)$$

# Horn-Schunk Algorithm

- Alternative to L&K which seeks to find optimal motion field with smooth variation in motion vectors
- Algorithm aims to find the motion field  $v = (v_x, v_y)$  which minimises following energy functional

$$E = \iint \left[ \underbrace{(I_x u_x + I_y u_y + I_t)^2}_{\text{OFE} \rightarrow 0} + \sigma^2 \underbrace{(\|\nabla u_x\|^2 + \|\nabla u_y\|^2)}_{\text{Rate of change of } \mathbf{v} \rightarrow \mathbf{0}} \right] dx dy$$

OFE  $\rightarrow$  0
weighting factor
Rate of change of  $\mathbf{v} \rightarrow \mathbf{0}$

↓
  
 smooth motion field