COMS30030
Image Processing and Computer Vision

Motion – Modelling

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Motion – Important Perceptual Cue

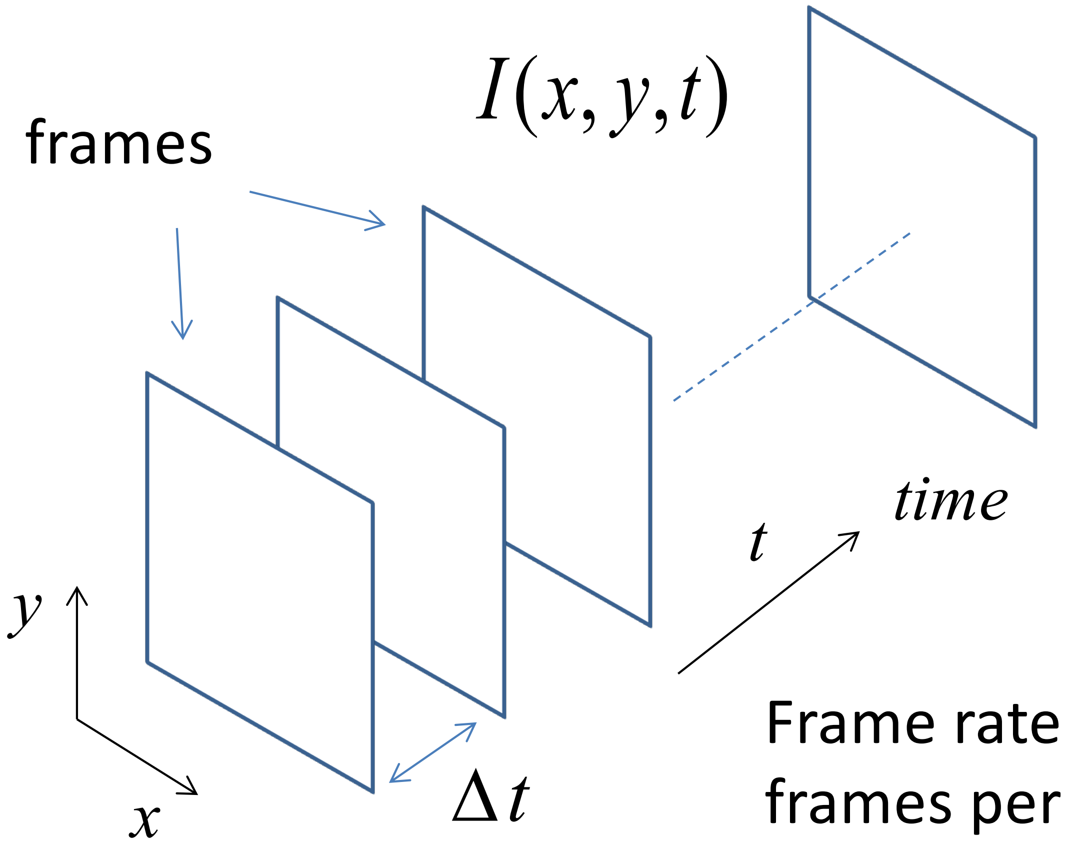
We are going to look at .....

- Modelling 2-D motion fields
- Optical flow
- The optical flow equation (OFE)
- Motion estimation
  – Lucas and Kanade method

Video Sequences

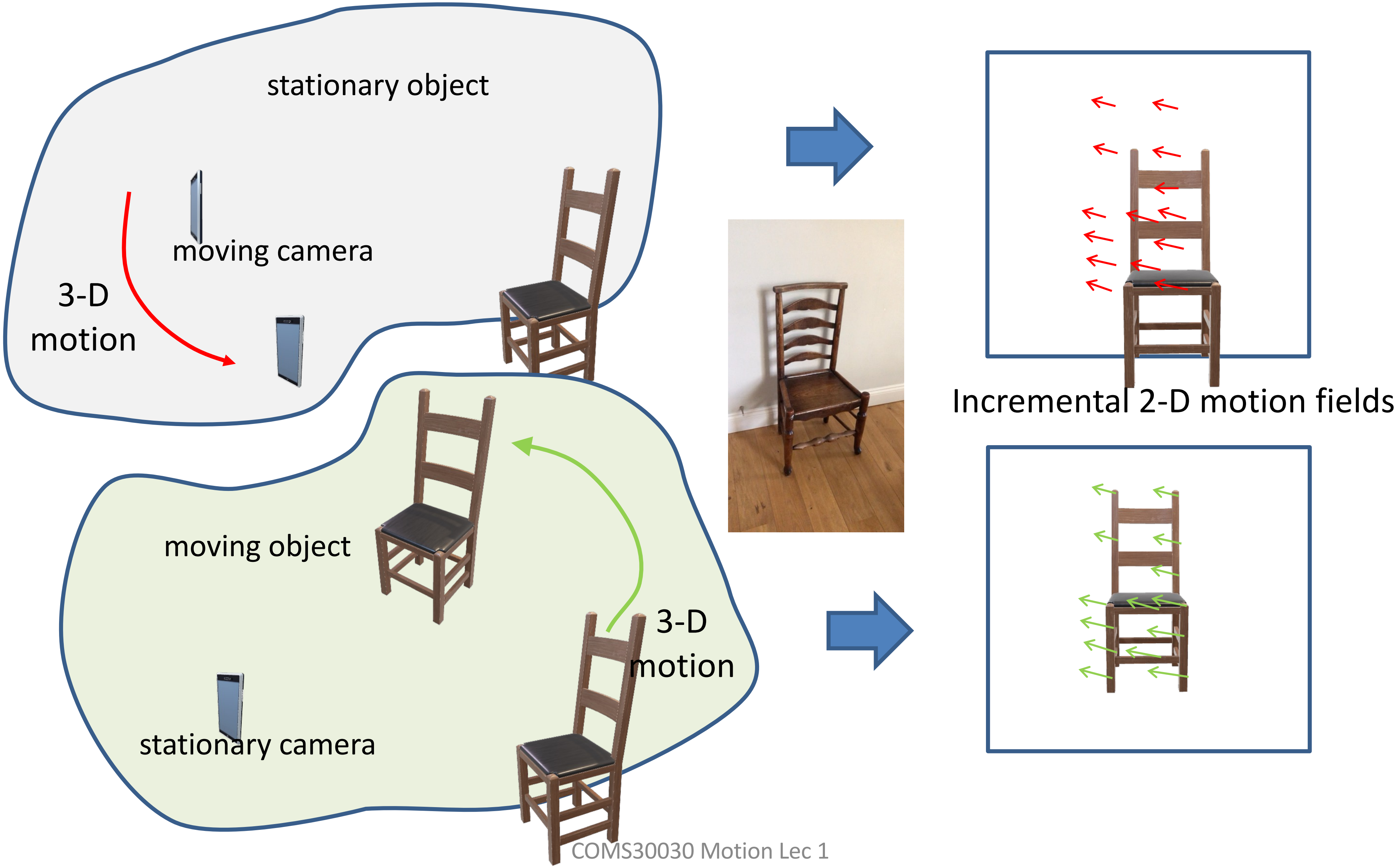
First motion picture camera

Kinetograph



Frame rate : $1/\Delta t$
frames per second
e.g. 25 fps

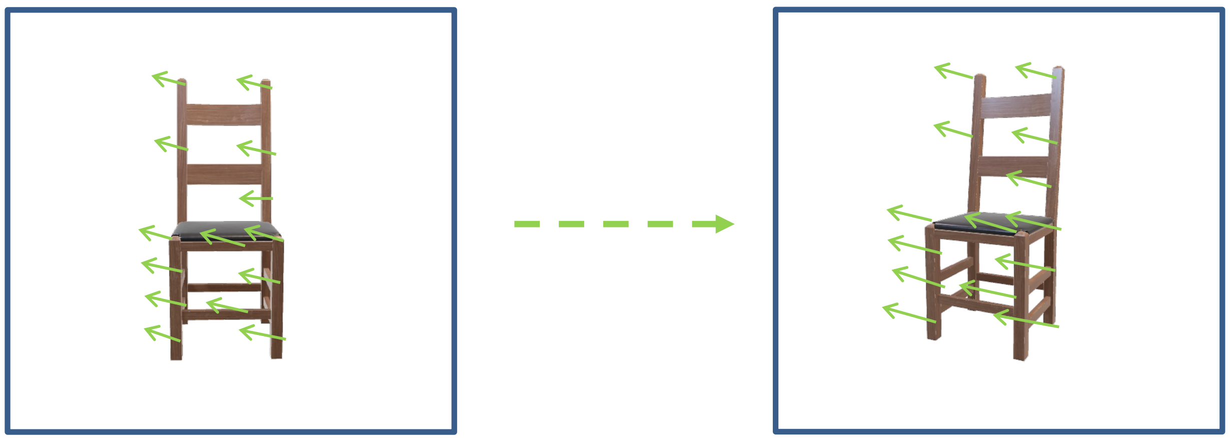
Modelling 2-D Motion Fields



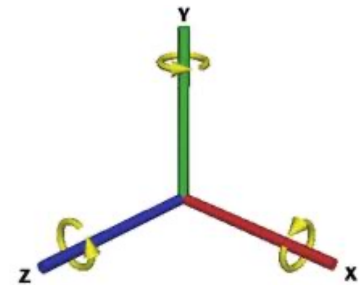
Incremental 2-D Motion Fields

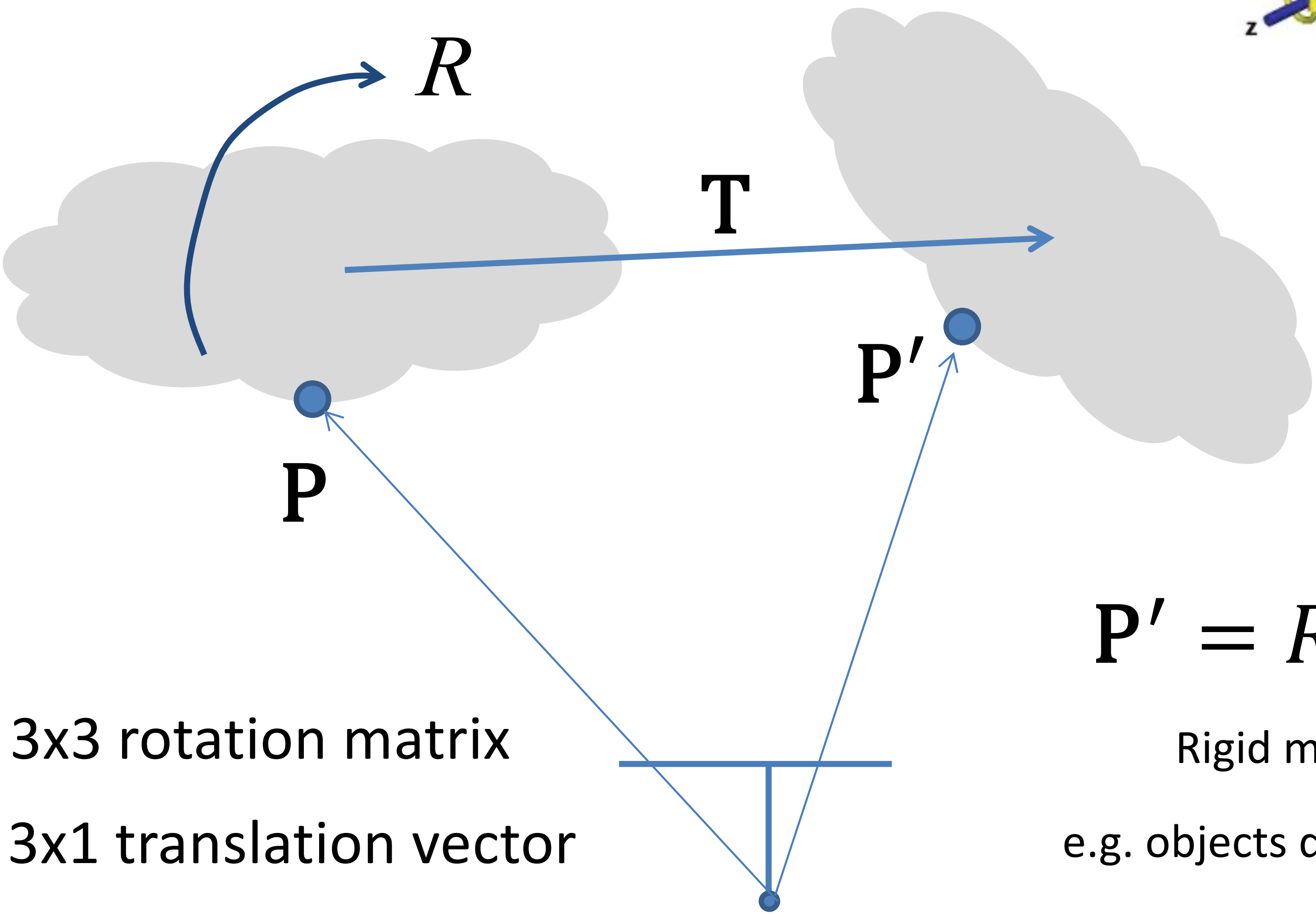
What is the relationship between the:

- Position of a 3-D point and its 2-D motion?
- 2-D motion of different 3-D points?
- 3-D motion of a 3-D point and its 2-D motion?



3-D Rigid Motion





$$\mathbf{P}' = R\mathbf{P} + \mathbf{T}$$

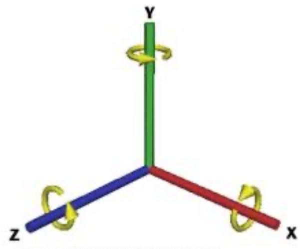
Rigid motion

e.g. objects do not deform

R : 3x3 rotation matrix

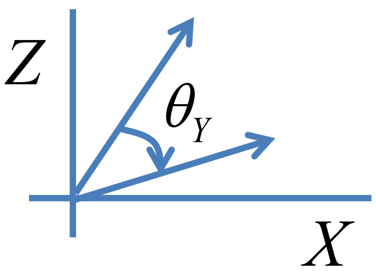
$\mathbf{T}$: 3x1 translation vector

Rotation Matrices



$R = R_X R_Y R_Z$ (for example) : Rotations about X, Y and Z axes

$$R_Y\mathbf{P} = \begin{bmatrix} \cos\theta_Y & 0 & \sin\theta_Y \\ 0 & 1 & 0 \\ -\sin\theta_Y & 0 & \cos\theta_Y \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X\cos\theta_Y + Z\sin\theta_Y \\ Y \\ Z\cos\theta_Y - X\sin\theta_Y \end{bmatrix}$$



NB : for small θ_Y

$$R_Y \approx \begin{bmatrix} 1 & 0 & \theta_Y \\ 0 & 1 & 0 \\ -\theta_Y & 0 & 1 \end{bmatrix}$$

NB : for small $\theta_X, \theta_Y, \theta_Z$

$$R \approx \begin{bmatrix} 1 & -\theta_Z & \theta_Y \\ \theta_Z & 1 & -\theta_X \\ -\theta_Y & \theta_X & 1 \end{bmatrix}$$

For small θ_Y:

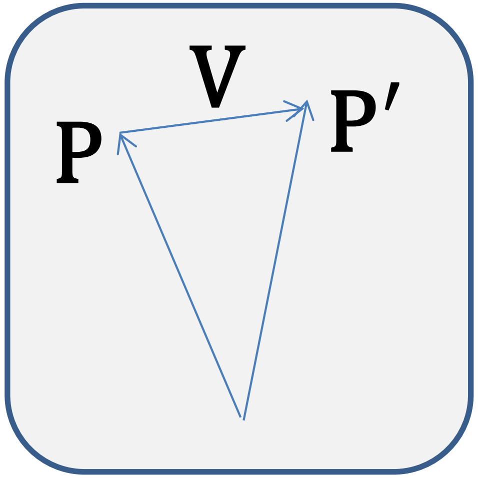
$\cos\theta_Y \approx 1$

$\sin\theta_Y \approx \theta_Y$

3-D Motion Field

$$\boxed{\mathbf{P}' = R\mathbf{P} + \mathbf{T}}$$

$$\mathbf{V} = \lim_{\Delta t \to 0} \{\mathbf{P}' - \mathbf{P} = (R - I)\mathbf{P} + \mathbf{T}\}$$



For small angles:

$$R \approx \begin{bmatrix} 1 & -\theta_Z & \theta_Y \\ \theta_Z & 1 & -\theta_X \\ -\theta_Y & \theta_X & 1 \end{bmatrix}$$

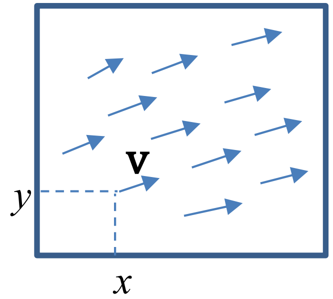
Hence:

$$V_X = \theta_Y Z - \theta_Z Y + T_X \qquad (\theta_X, \theta_Y, \theta_Z) \equiv \text{Angular velocity}$$

$$V_Y = \theta_Z X - \theta_X Z + T_Y \qquad (T_X, T_Y, T_Z) \equiv \text{Rectilinear velocity}$$

$$V_Z = \theta_X Y - \theta_Y X + T_Z$$

2-D Motion Field Equations



For image point $\mathbf{p} = (x, y, f)$ Motion field $\mathbf{v} = (v_x, v_y)$

$$v_x = \frac{dx}{dt} = \frac{d}{dt}\left(\frac{fX}{Z}\right) = f\,\frac{V_X Z - XV_Z}{Z^2}$$

Quotient rule

$$x = \frac{fX}{Z}$$

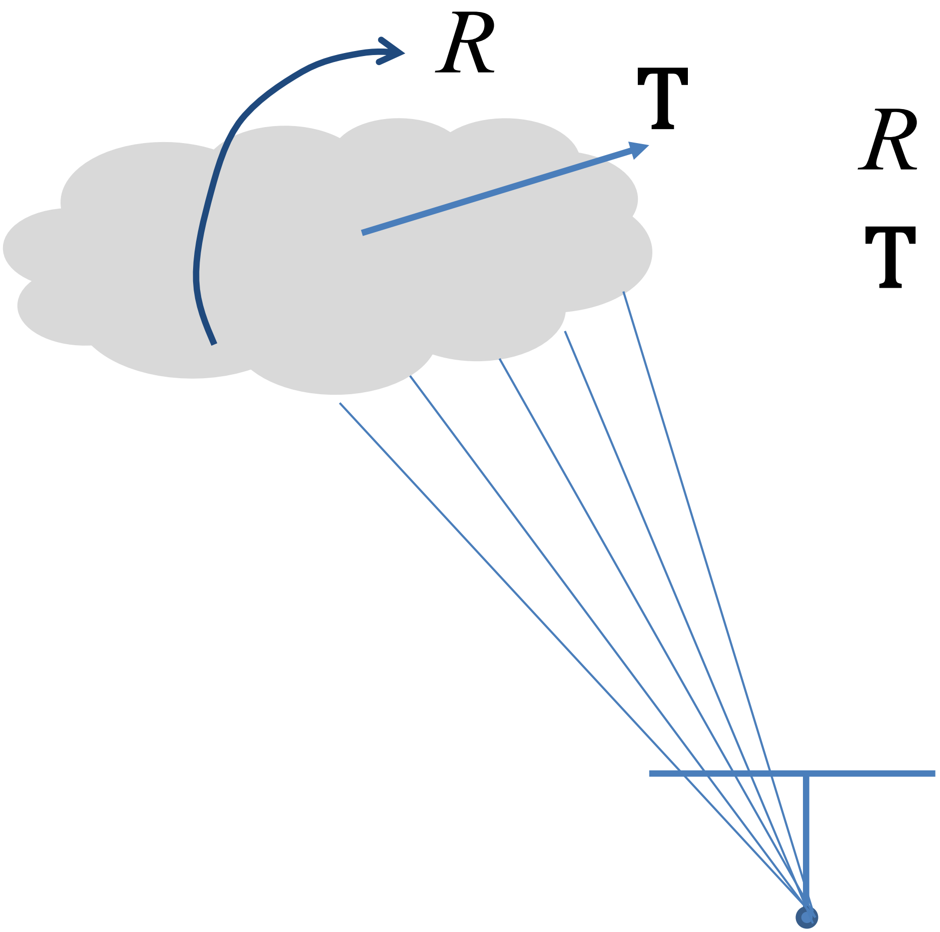
$$V_X = \frac{dX}{dt}$$

Substituting for V_X, V_Y, V_Z gives (previous slide)

$$v_x = (fT_X - xT_Z)/Z + f\theta_Y - \theta_Z y - (\theta_X xy - \theta_Y x^2)/f$$

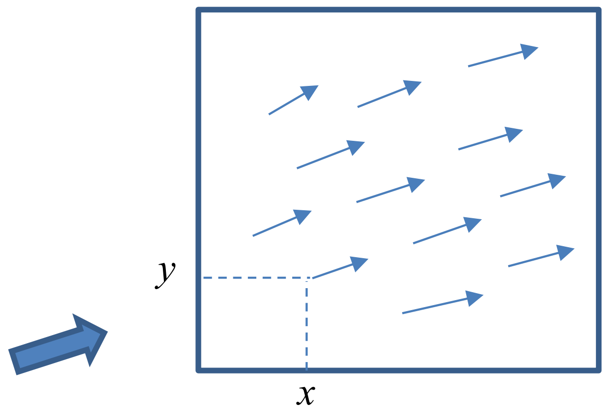
$$v_y = (fT_Y - yT_Z)/Z - f\theta_X + \theta_Z x + (\theta_Y xy - \theta_X y^2)/f$$

Small angle approximation



R : 3x3 rotation matrix ➡ $(\theta_X, \theta_Y, \theta_Z)$

$\mathbf{T}$: 3x1 translation vector ➡ (T_X, T_Y, T_Z)



2-D motion field

$\mathbf{v} = (v_x, v_y)$

$$v_x = (fT_X - xT_Z)/Z + f\theta_Y - \theta_Z y - (\theta_X xy - \theta_Y x^2)/f$$

$$v_y = (fT_Y - yT_Z)/Z - f\theta_X + \theta_Z x + (\theta_Y xy - \theta_X y^2)/f$$

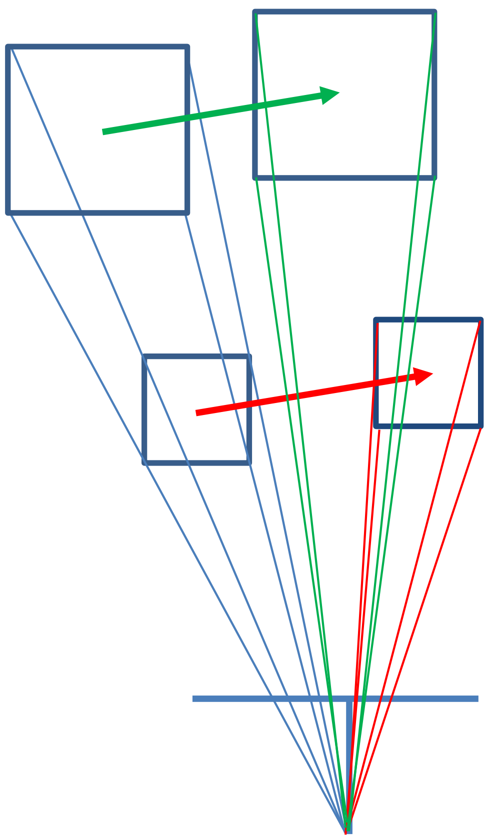
Two Components

$$v_x = (fT_X - xT_Z)/Z + f\theta_Y - \theta_Z y - (\theta_X xy - \theta_Y x^2)/f$$
$$v_y = (fT_Y - yT_Z)/Z - f\theta_X + \theta_Z x + (\theta_Y xy - \theta_X y^2)/f$$

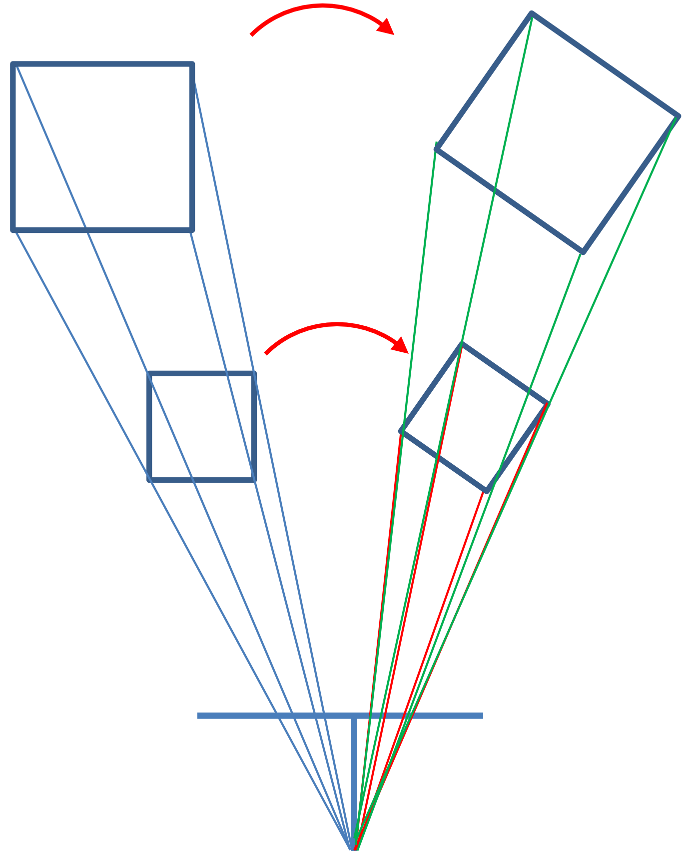
Translational – dependent on scene depth Z

Rotational – independent of scene depth Z

Translation, Rotation and Depth



Motion field $\propto$ depth



Motion field $\not\propto$ depth

Special Case: Pure Translation

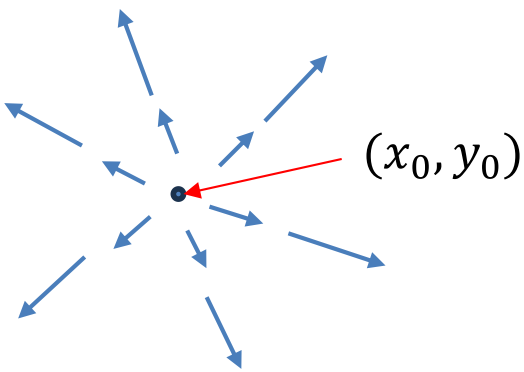
- Assume 3-D motion is only translational, $\boldsymbol{\theta} = 0$, then

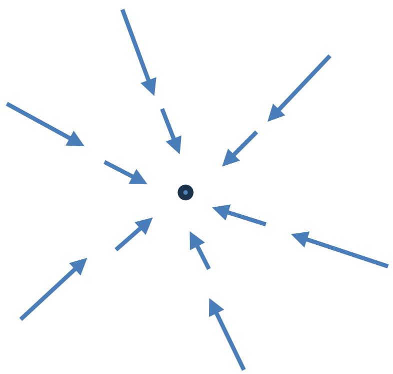
$$v_x = (fT_X - xT_Z)/Z \qquad v_y = (fT_Y - xT_Z)/Z$$

- If $T_Z \neq 0, x_0 = fT_X/T_Z$ and $y_0 = fT_Y/T_Z$, then

$$v_x = -(x - x_0)T_Z/Z \qquad v_y = -(y - y_0)T_Z/Z$$

Expansion – object moving towards camera (T_Z negative)



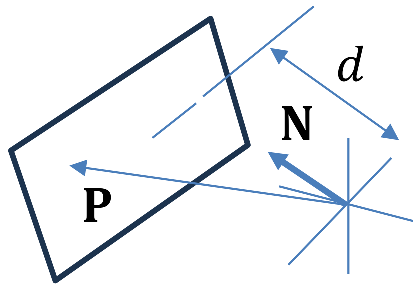


Contraction – object moving away from camera (T_Z positive)

(x_0, y_0) – focus of expansion (contraction)

Special Case: Moving Plane

- Assume 3-D points lie in plane with unit surface normal $\mathbf{N}$, i.e. $\mathbf{N}^T\mathbf{P} = d$, where d is distance of plane from origin.



- Since $\mathbf{P} = Z\mathbf{p}/f$, this gives $Z(N_X x + N_Y y + N_Z f)/f = d$

- Substituting for Z in 2-D motion field:

$$v_x = \frac{1}{fd}(a_1x^2 + a_2xy + a_3fx + a_4fy + a_5f^2)$$

$$v_y = \frac{1}{fd}(a_1xy + a_2y^2 + a_6fy + a_7fx + a_8f^2)$$

- Motion field is a quadratic polynomial in 2-D spatial coordinates x and y