COMS30030
Image Processing and Computer Vision

Motion Estimation

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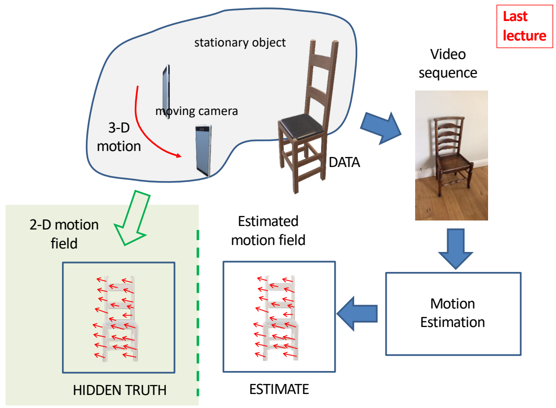

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Motion Estimation


- The estimation of the 2-D motion field from frames in an image sequence
- Using spatial and temporal variation of pixel values
- **BUT**- relationship between variation in pixel values – known as **apparent motion** or **optical flow** – and the true motion is not straightforward.

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Apparent versus True Motion

- Apparent motion or **optical flow** - perceived motion in video sequence caused by changes in pixel values.
- Relationship with true 2-D motion field not always straightforward.
- Extreme cases:
  - non-zero apparent motion for zero motion field, e.g. static scene, moving light source
  - zero apparent motion for non-zero motion field, e.g. constant colour sphere rotating in diffuse lighting

- Sometimes not possible to determine 2-D motion field without additional constraints or assumptions.

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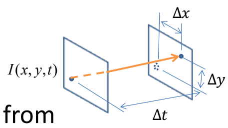
Optical Flow



- Assume optical flow results from **brightness constancy constraint**
  - *`a moving pixel retains its value between frames'*
- For continuous video $I(x,y,t)$ (grey level)

$$I(x+\Delta x, y+\Delta y, t+\Delta t) = I(x,y,t)$$

HoT → 0 for tiny $\Delta x, \Delta y, \Delta t$

- Using Taylor's expansion:

zero

$$I(x+\Delta x, y+\Delta y, t+\Delta t) = I(x,y,t) + \frac{\delta I}{\delta x}\Delta x + \frac{\delta I}{\delta y}\Delta y + \frac{\delta I}{\delta t}\Delta t + \ldots\ldots$$

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Optical Flow Equation

- For $I(x+\Delta x, y+\Delta y, t+\Delta t) = I(x,y,t)$

$$\frac{\delta I}{\delta x}\Delta x + \frac{\delta I}{\delta y}\Delta y + \frac{\delta I}{\delta t}\Delta t = 0$$

- Dividing throughout by Δt

$$\frac{\delta I}{\delta x}\frac{\Delta x}{\Delta t} + \frac{\delta I}{\delta y}\frac{\Delta y}{\Delta t} + \frac{\delta I}{\delta t} = 0$$

- For $\Delta x, \Delta y, \Delta t \to 0$

$$\frac{\delta I}{\delta x}\frac{dx}{dt} + \frac{\delta I}{\delta y}\frac{dy}{dt} + \frac{\delta I}{\delta t} = 0$$

Optical Flow Equation (OFE)

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Optical Flow Equation (OFE)

$$\frac{\delta I}{\delta x}\frac{dx}{dt}+\frac{\delta I}{\delta y}\frac{dy}{dt}+\frac{\delta I}{\delta t}=0$$

$\frac{dx}{dt}, \frac{dy}{dt}$ Rate of change of x, y with time

$\Rightarrow$ optical flow field $\mathbf{u} = (u_x, u_y)$

$\frac{\delta I}{\delta x}, \frac{\delta I}{\delta y}, \frac{\delta I}{\delta t}$ Rate of change of I with x, y, t

$\Rightarrow$ spatial & temporal gradients (I_x, I_y, I_t)

Optical flow equation $\boxed{I_x u_x + I_y u_y + I_t = 0}$

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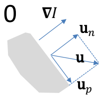
Normal Flow

- OFE for $\mathbf{u} = (u_x, u_y)$ and $\nabla I = (I_x, I_y)$

  $I_x u_x + I_y u_y + I_t = 0 \Rightarrow \nabla I.\mathbf{u} + I_t = 0$

  $\nabla I.\mathbf{u} = I_x u_x + I_y u_y$ dot product



- OFE alone not sufficient to estimate motion
  – one equation in two unknowns
- Only estimate **normal flow** $\mathbf{u}_n$

  $\nabla I.\mathbf{u} + I_t = \nabla I.\mathbf{u}_n + I_t = 0$

$\Rightarrow \|\mathbf{u}_n\| = -I_t/\|\nabla I\| \quad \angle\mathbf{u}_n = \angle\nabla I$

$$\boxed{\begin{aligned}\mathbf{u} &= \mathbf{u}_p + \mathbf{u}_n\\ \mathbf{u}_p.\mathbf{u}_n &= 0\\ \nabla I.\mathbf{u}_p &= 0\end{aligned}}$$

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Normal Flow

$\nabla I.\mathbf{u} + I_t = \nabla I.\boldsymbol{u}_n + I_t = 0$

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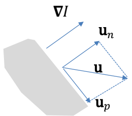

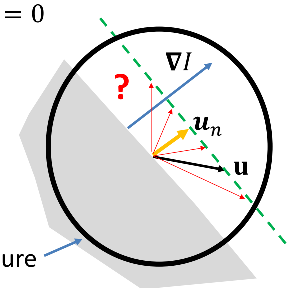

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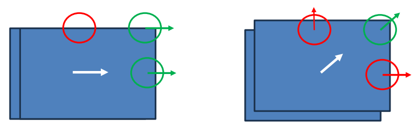
Aperture Problem

With single gradient direction in window (aperture), observed motion is different from true motion as we can only observe motion parallel to the gradient:



Hence: Good motion estimation depends on having sufficient variation in spatial gradient within regions.

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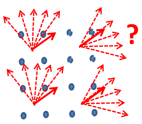
Constraining the OFE

$\boxed{I_x u_x + I_y u_y + I_t = 0}$

OFE is under constrained – can only estimate normal flow

Need to add extra constraint(s)

Example : assume parametric form of motion field in regions



Example : constant velocity

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Constraining the OFE

$\boxed{I_x u_x + I_y u_y + I_t = 0}$

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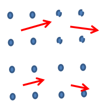
Example : assume parametric form of motion field in regions



Example : linear in x and y, e.g.

$$\boxed{\begin{aligned}u_x &= ax + by + c\\ u_y &= dx + ey + f\end{aligned}}$$

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Constant Velocity Model

For a region, find the velocity $\mathbf{u} = (u_x, u_y)$ which minimises :

$$\varepsilon(u_x, u_y) = \sum_{region} \underbrace{(I_x u_x + I_y u_y + I_t)^2}_{\text{OFE} \rightarrow 0}$$

Solution: take derivatives w.r.t u_x and u_y, set to zero, and solve for u_x and u_y.

NB: same $\mathbf{u} = (u_x, u_y)$ over whole region → solution

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Lucas and Kanade Algorithm

Find velocity $\mathbf{u} = (u_x, u_y)$ which minimises :

$$\varepsilon(u_x, u_y) = \sum_R (I_x u_x + I_y u_y + I_t)^2$$

Partial derivatives w.r.t u_x and u_y, set to zero, solve for u_x and u_y:

$$\frac{\partial \varepsilon}{\partial u_x} = 2\sum_R (I_x u_x + I_y u_y + I_t) I_x = 0 \quad \Rightarrow \quad \sum_R (I_x^2 u_x + I_x I_y u_y + I_x I_t) = 0$$

$$\frac{\partial \varepsilon}{\partial u_y} = 2\sum_R (I_x u_x + I_y u_y + I_t) I_y = 0 \quad \Rightarrow \quad \sum_R (I_x I_y u_x + I_y^2 u_y + I_y I_t) = 0$$

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Lucas and Kanade Algorithm

Hence, solve for $\mathbf{u} = (u_x, u_y)$ given that :

$$u_x \sum_R I_x^2 + u_y \sum_R I_x I_y = -\sum_R I_t I_x$$

$$u_x \sum_R I_x I_y + u_y \sum_R I_y^2 = -\sum_R I_t I_y$$

$$\Rightarrow \quad A\mathbf{u} = \mathbf{b}$$

$$\Rightarrow \quad \boxed{\mathbf{u} = \begin{bmatrix} u_x \\ u_y \end{bmatrix} = A^{-1}\mathbf{b}}$$

$$A = \sum_R \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \qquad \mathbf{b} = -\sum_R \begin{bmatrix} -I_t I_x \\ -I_t I_y \end{bmatrix}$$

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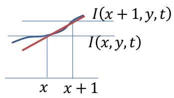
Spatial & Temporal Gradients

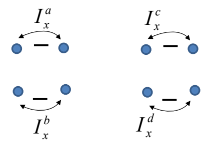
Approximate gradients using differences, e.g.

$$I_x = \delta I / \delta x \approx I(x+1, y, t) - I(x, y, t)$$

i.e. assume $\delta x = 1$



Or use averaging to reduce noise, e.g.



Frame t Frame $t+1$

$$I_x \approx (I_x^a + I_x^b + I_x^c + I_x^d)/4$$

Rate of change of I with x

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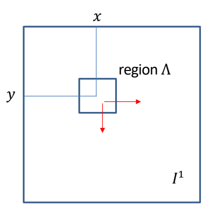
L & K Algorithm

I^1 = video frame at time t
I^2 = video frame at time $t+1$
For each pixel x, y in I^1
  $A = 0$; $\mathbf{b} = 0$;
  For each pixel in region Λ about x, y
    $(I_x, I_y, I_t) = CompGrads(I^1, I^2)$;
    $A' = \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$;
    $\mathbf{b}' = \begin{bmatrix} -I_t I_x \\ -I_t I_y \end{bmatrix}$;
    $A \rightarrow A + A'$; $\mathbf{b} \rightarrow \mathbf{b} + \mathbf{b}'$;
  End;
  $\mathbf{u}(x, y) = A^{-1}\mathbf{b}$
End;



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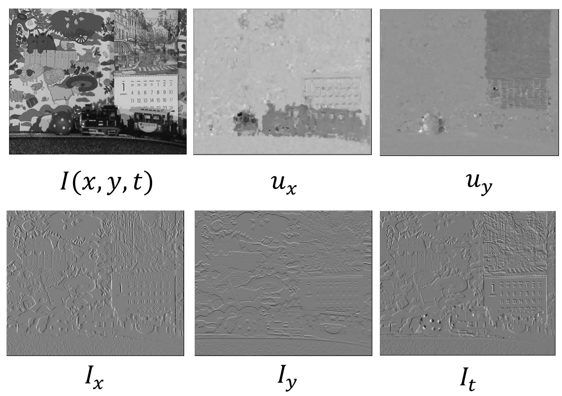

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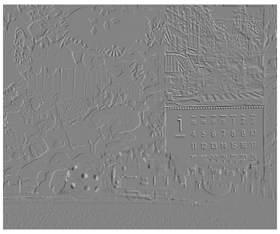
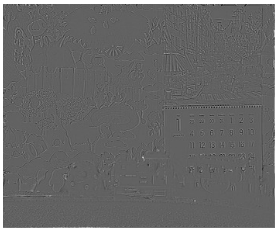
Frame Difference



frame difference



motion compensated frame difference

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Motion Estimation - Example

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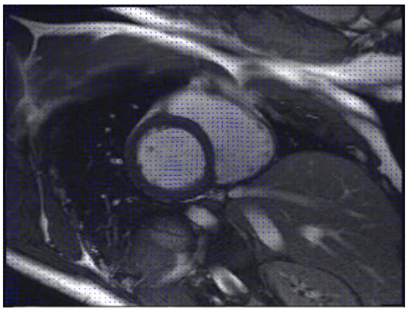
Motion Estimation - Example



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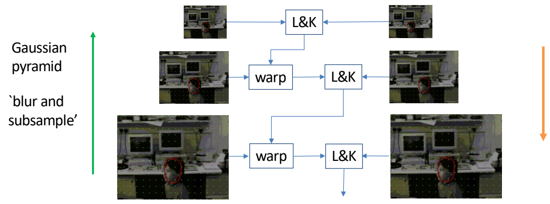
Multiresolution L & K

- To deal with large motions, implement L&K over multiple resolutions – result at lower resolutions used to `warp' higher resolution images prior to estimation.



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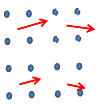
Affine Motion Model



$$\boxed{\begin{aligned} u_x &= ax + by + c \\ u_y &= dx + ey + f \end{aligned}} \Rightarrow \mathbf{u} = A\,\mathbf{p}$$

$$\mathbf{p}^T = (a, b, c, d, e, f)$$

Models translation, scaling, rotation and shear

$\Rightarrow$ $\mathbf{p}^T A^T \nabla I + I_t = 0$ affine OFE $\Rightarrow$ $\hat{\mathbf{p}} = M^{-1}\mathbf{b}$

$$M = \sum_{region} A^T \nabla I \nabla I^T A \qquad b = -\sum_{region} I_t (A^T \nabla I)$$

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Horn-Schunk Algorithm

- Alternative to L&K which seeks to find optimal motion field with smooth variation in motion vectors
- Algorithm aims to find the motion field $v = (v_x, v_y)$ which minimises following energy functional

$$E = \iint \left[\underbrace{\left(I_x u_x + I_y u_y + I_t\right)^2}_{\boxed{OFE \rightarrow 0}} + \sigma^2 \underbrace{\left(\|\nabla u_x\|^2 + \|\nabla u_y\|^2 \right)}_{\text{Rate of change of } \mathbf{v} \rightarrow \mathbf{0}} \right] dxdy$$

weighting factor

smooth motion field

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