

$$H=\frac{\omega_0}{2}\sigma^z+\sum_j\left[\nu_jb_j^\dagger b_j+\frac{\xi_j}{2}\left(b_j^\dagger+b_j\right)\sigma^z\right]$$

$$\begin{aligned} A(\omega-\omega_0) &= \frac{g^2N}{2}\mathrm{Re}\int_0^\infty dt e^{i(\omega-\omega_0)t} \langle \sigma^-(t) \sigma^+(0) \rangle e^{-(\Gamma_\uparrow+\Gamma_\downarrow)t/2} \\ &= \frac{g^2N}{2}\mathrm{Re}\int_0^\infty dt e^{i(\omega-\omega_0)t} e^{-(\Gamma_\uparrow+\Gamma_\downarrow)t/2} e^{-C(t)} \end{aligned}$$

$$C(t)=\int\frac{J(\nu)}{\nu^2}\left[\coth\left(\frac{\beta\nu}{2}\right)(1-\cos\nu t)+i\sin\nu t\right]$$

$$\begin{aligned} J(\nu) &= \sum_n \xi_n^2 \delta(\nu-\nu_n) \\ &= 2a\nu^s \nu_c^{1-s} e^{-(\nu/\nu_c)^2} \end{aligned}$$