Math 122A Homework 1

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1 Problem 1

Find roots for $z^3 = 1 - i$
Proof.
$r = \sqrt{1^2 + -1^2} = \sqrt{2}$
$\theta = tan^{-1}(-1/1) = \pi/4$
$z = \sqrt{2}e^{-i\pi/4}$
$z^3 = r^3 e^{i3\theta} = \sqrt{2}e^{-i\pi/4}$
$r^3 = 2^{1/2}\ so\ r = 2^{1/6}\ and\ 3\theta = -\pi/4\ so\ \theta_1 = \pi/12$
$3\theta_2 = (-\pi/4 + 2\pi) \quad \theta_2 - \pi/12 + 2\pi/3$
$\theta_3 = -\pi/12 + 4\pi/3$
Solution of $z^3 = 1 - i$: $2^{1/6}e^{i\theta_1},\ 2^{1/6}e^{i\theta_2},\ 2^{1/6}e^{i\theta_3}$ □

2 Problem 2

Find $z = \frac{(\sqrt{3}+i)^{12}}{(1+i)^{10}}$
Proof.
$(\sqrt{3} + i)^{12}: \quad r = \sqrt{3+1} = 2$
$\theta = tan^{-1}(\frac{1}{3}) = \frac{\pi}{6}$
$(\sqrt{3} + i)^{12}: \ 2^{12}(cos(12 \cdot (\frac{\pi}{6})) + isin(12 \cdot (\frac{\pi}{6}))$
$=4096(cos(2\pi) + isin(2\pi)) \ = 4096(1 + 0) \ = 4096$

$(1 + i) = \sqrt{2}e^{i\frac{\pi}{4}} \quad as\ shown\ in\ class$
$=\sqrt{2}^{10}(cos(10 \cdot \frac{\pi}{4}) + isin(10 \cdot \frac{\pi}{4}))$
$= 32((cos(\frac{5\pi}{2}) + isin(\frac{5\pi}{2})) = \ 32$

$\frac{4096}{32i} = \frac{128}{i}$ $\square$

3 Problem 3

Proof. a) show: $1 + z + z^2 + ... + z^n + z^{n+1} = \frac{1-z^{n+1=1}}{1-z}$
$1 + z + z^2 + ... + z^n + z^{n+1} = \frac{1-z^{n+1}}{1-z} + z^{n+1} = \frac{1-z^{n+1=1}}{1-z}$
$1 - z^{n+1} + (1-z)z^{n+1} = 1 - z^{n+1+1}$
$Now, replacing the z's with w's:$1+w+...+w$^{n-1} + w^n = \frac{1-w^{n+1}}{1-w}$
Since we know $w^n = 1 :\quad 1 + w + ... + w^{n-1} + 1 = \frac{1-(1\cdot w)}{1-w}$
1+w+...+w$^{n-1} + 1 = 1$
$1 + w + ... + w^{n-1} + 1 = 0$

b) 1+2w+3w^2+...+$nw^{n-1} = \frac{n}{w-1} Let\ x = 1+2w+3w^2+...+nw^{n-1} Then,\ wx = 1 + 2w + 3w^2 + ... + nw^{n-1} + nw^n$
$x - wx = 1 + 2w + 3w^2 + ... + nw^{n-1} - (n-1)w^{n-1} - nw^n$
$= 1 + 2w + 3w^2 + ... + nw^{n-1}(n - n + 1) - nw^n$
$= 1 + 2w + 3w^2 + ... + nw^{n-1} - nw^n Since\ we\ know\ w^n = 1,$
$-x + wx = n$
$x(w-1) = n$
$x = \frac{n}{w-1}$
$Because\ we\ let$1+2w+3w$^2 + ... + nw^{n-1}$
1+2w+3w$^2 + ... + nw^{n-1} = \frac{n}{w-1}$ $\square$

4 Problem 4

Proof.
i)

$$\left|\frac{a-b}{1-\overline{a}b}\right| = \frac{|a-b|}{|1-\overline{a}b|} < 1$$

$\leftrightarrow\ |a-b| < |1-\overline{a}b|$
$\leftrightarrow$ —a-b—$^2 < |1-\overline{a}b|^2$
$\leftrightarrow (a-b)(\overline{a-b}) < (1-ab)(\overline{1-\overline{a}b})$
$\rightarrow (a-b)(\overline{a}-\overline{b}) < (1-\overline{a}b)(1-a\overline{b})$
$\leftrightarrow a\overline{a} - a\overline{b} - b\overline{a} + b\overline{b} < 1 - a\overline{b} - \overline{a}b - \overline{a}b + |a|^2|b|^2 a\overline{a} + b\overline{b} < 1 + |a|^2|b|^2|a|^2 + |b|^2 <$

$1 + |a|^2|b|^2 Now\ let$ $\alpha = |a|^2$ and $\beta = |b|^2$ where $\alpha, \beta \in [0, 1)$ we have that $\alpha + \beta \leq 1 + \alpha\beta \leftrightarrow \alpha - \alpha\beta \leq 1 - \beta \leftrightarrow \alpha(1 - \beta) \leq 1 - \beta$

ii) case 1: $|a| = 1$

$$\left|\frac{a - b}{1 - \overline{a}b}\right| = \frac{|a - b|}{|\overline{a}a - \overline{a}b|}$$

$= 1 \frac{}{\overline{a} \cdot \frac{a-b}{a-b} = \frac{1}{\overline{a}}} Since\ a = 1$; $|\frac{a-b}{1-\overline{a}b}| = 1 samelogicandequationsstandforifb = 1.$

i couldnt figure out the rest of latex in time. this was my first time using latex. however i still did all of the homework its just not typed out. see my ipad notes for reference if you can still count it just this once i would be very appreciateive!!

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