INDIAN INSTITUTE OF TECHNOLOGY GANDHINAGAR

ME 333: Mechanics of Materials

Experiment 4: Deflection of Beams

Group 10

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I. Introduction

The objective of the experiment is to study the behavior of the beams under the bending load. The deflections at the various points on the beams are measured and compared with the theoretical values obtained using Euler-Bernoulli Beam Theory.

1. Assumptions of the Euler–Bernoulli Beam Theory

The Euler–Bernoulli beam theory is based on the following key assumptions:

1) Plane cross-sections remain plane and normal to the neutral axis after deformation.
2) The effects of shear deformation are neglected.
3) Deflections and rotations are small, allowing linear strain-displacement relations.
4) The material is linearly elastic, homogeneous, and isotropic.Material follows Hooke's law, that is stress is proportional to strain.
5) The beam has a constant cross-section along its length.

2. Deflection $y(x)$ for a Cantilever Beam with a Point Load P at the Free End

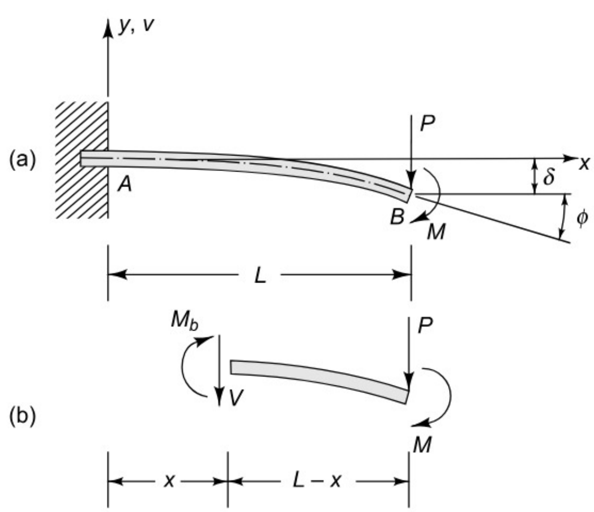

Figure 1: Img Source: Crandall Textbook

For a cantilever beam of length L, modulus of elasticity E, and second moment of area I, subjected to a point load P at the free end, the governing differential equation is:

$$\frac{d^4y}{dx^4} = \frac{q}{EI}$$

Thus we have

$$\frac{d^2 EI\frac{d^2y}{dx^2}}{dx^2} = \frac{q}{EI}$$

The bending moment at a distance x from the fixed end is given by:

$$M(x) = -P(L-x)$$

and we have

$$\frac{d^2M}{dx^2} = q$$

Hence,

$$EI\frac{d^2y}{dx^2} = -P(L-x)$$

Integrating once:

$$\frac{dy}{dx} = -\frac{P}{EI}\left(Lx - \frac{x^2}{2}\right) + C_1$$

Integrating again:

$$y(x) = -\frac{P}{EI}\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right) + C_1x + C_2$$

Applying boundary conditions for a cantilever beam:

$$\text{At } x = 0: \quad y = 0, \quad \frac{dy}{dx} = 0$$

This gives $C_1 = 0$ and $C_2 = 0$.
Hence, the deflection equation becomes:

$$\boxed{y(x) = \frac{P}{6EI}(3Lx^2 - x^3)}$$

At the free end $(x = L)$, the maximum deflection is:

$$y_{\text{max}} = \frac{PL^3}{3EI}$$

3. Bending Moment and Shear Force Expressions

The bending moment along the beam is:

$$M(x) = -P(L-x)$$

The shear force is obtained by the relation between moment and the shear force:

$$V(x) = -\frac{dM}{dx} = -P$$

Summary of Key Results

Quantity	**Expression**
Deflection shape	$y(x) = \frac{P}{6EI}(3Lx^2 - x^3)$
Maximum deflection	$y_{\text{max}} = \frac{PL^3}{3EI}$
Bending moment	$M(x) = -P(L-x)$
Shear force	$V(x) = -P$

II. Experimental Procedure

1) Mount the rectangular beam of the selected material on the beam deflection apparatus.
2) Place the digital indicator (for deflection measurement) at the load application point, starting at positions: 65 mm, 130 mm, 195 mm, 260 mm. Record the initial (zero) reading of the indicator(offset) at each positions.
3) Hang the specified load on the load hanger, then position the hanger at the length L along the beam. Note the deflection value indicated by the digital indicator at that position.
4) Reposition the digital indicator accordingly and record the corresponding deflection, at different positions: 65 mm, 130 mm, 195 mm, 260 mm.
5) Perform the same set of measurements for five different load values, 20 g, 30 g, 40 g, 50 g, 60 g.
6) Do the above procedure for 3 different materials, Aluminium, Stainless Steel and Brass.

III. Observations

The experiment was conducted using three different materials: Aluminium, Stainless Steel, and Brass. The cantilever beam length was kept constant for all samples and was taken as $260\,\mathrm{mm}$. Point loads were applied incrementally at the the free end of the beam and deflection was measured at 4 different locations along the beam: $0.25L$, $0.5L$, $0.75L$, and L, where L is the total beam length.

The initial applied load was $20\,\mathrm{g}$, which was then increased in steps of $10\,\mathrm{g}$ up to $60\,\mathrm{g}$. Thus, deflection readings were recorded for five different point loads at each of the four positions along the cantilever beam. The procedure was repeated for all three materials to compare their deformation characteristics under identical conditions.

The theoretical deflection of the beam was also calculated using Euler–Bernoulli Beam Theory. For a cantilever beam with a point load P applied at a distance a from the fixed end, the deflection at the point of load application is given by:

$$\delta = \frac{Pa^2(3L-a)}{6EI}$$

where L is the total length of the cantilever beam, E is the Young's modulus of the material, and I is the second moment of area (area moment of inertia) about the neutral axis about which the beam bends/deflects.

The second moment of area I for a rectangular cross-section is given by:

$$I = \frac{bh^3}{12}$$

where b is the width and h is the height of the beam cross-section.

In this experiment, $b = 19\,\mathrm{mm}$ and $h = 3\,\mathrm{mm}$. Substituting these values:

$$I = \frac{(19)(3^3)}{12} = 42.75\,\mathrm{mm}^4$$

The Young's modulus E values of the three materials used in the experiment were provided by the experimental setup as follows:

$$E_{\mathrm{Al}} = 69\,\mathrm{GPa}, \qquad E_{\mathrm{SS}} = 207\,\mathrm{GPa}, \qquad E_{\mathrm{Br}} = 105\,\mathrm{GPa}$$

Below, we present the experimentally recorded data along with the corresponding plots. We also provide a tabulated comparison between theoretical and experimental deflections, including the percentage error in the experimentally measured deflection from the theoretically obtained values.

Table I: Experimental Deflection for Aluminium

Distance (mm)	20 g	30 g	40 g	50 g	60 g
260	0.69	0.73	0.90	1.06	1.27
195	0.27	0.31	0.54	0.69	0.84
130	0.11	0.20	0.22	0.26	0.38
65	0.01	0.04	0.04	0.05	0.05

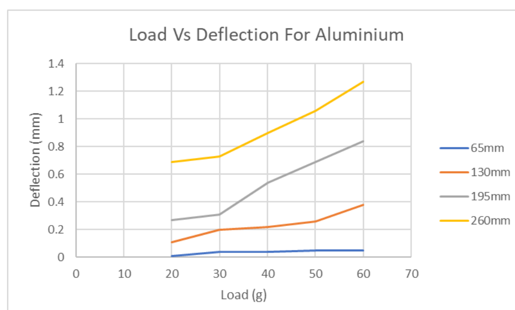

Figure 2: Load Vs Deflection for Aluminium

Table II: Experimental Deflection for Stainless Steel

Distance (mm)	20 g	30 g	40 g	50 g	60 g
260	0.30	0.39	0.45	0.50	0.65
195	0.06	0.23	0.31	0.38	0.46
130	0.08	0.11	0.15	0.17	0.21
65	0.01	0.06	0.08	0.10	0.14

Table III: Experimental Deflection for Brass

Distance (mm)	20 g	30 g	40 g	50 g	60 g
260	0.29	0.49	0.63	0.80	1.16
195	0.27	0.35	0.43	0.51	0.60
130	0.18	0.25	0.29	0.36	0.43
65	0.05	0.10	0.11	0.13	0.15

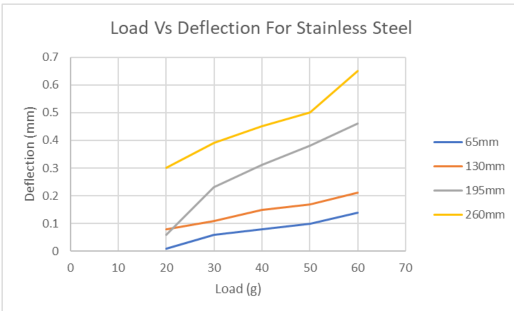

Figure 3: Load Vs Deflection for Stainless Steel

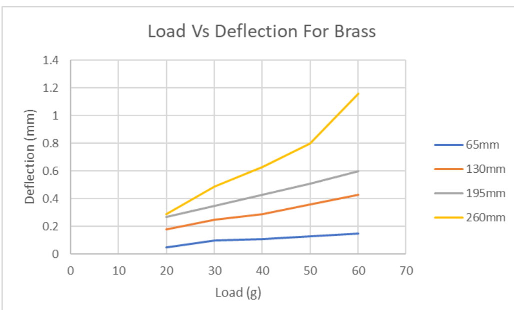

Figure 4: Load Vs Deflection for Brass

Table IV: Theoretical Deflection for Aluminium

Distance (mm)	20 g	30 g	40 g	50 g	60 g
260	0.3897	0.5845	0.7794	0.9742	1.1691
195	0.2466	0.3699	0.4932	0.6165	0.7398
130	0.1218	0.1827	0.2436	0.3044	0.3653
65	0.0335	0.0502	0.0670	0.0837	0.1005

Table V: Theoretical Deflection for Stainless Steel

Distance (mm)	20 g	30 g	40 g	50 g	60 g
260	0.1299	0.1948	0.2598	0.3247	0.3897
195	0.0822	0.1233	0.1644	0.2055	0.2466
130	0.0406	0.0609	0.0812	0.1015	0.1218
65	0.0112	0.0167	0.0223	0.0279	0.0335

Table VI: Theoretical Deflection for Brass

Distance (mm)	20 g	30 g	40 g	50 g	60 g
260	0.2561	0.3841	0.5122	0.6402	0.7682
195	0.1620	0.2431	0.3241	0.4051	0.4861
130	0.0800	0.1200	0.1600	0.2001	0.2401
65	0.0220	0.0330	0.0440	0.0550	0.0660

Table VII: Percentage Error between Experimental and Theoretical Deflection (Aluminium)

Distance (mm)	20 g	30 g	40 g	50 g	60 g
260	77.07	24.89	15.48	8.81	8.64
195	9.49	16.19	9.49	11.92	13.55
130	9.67	9.49	9.67	14.60	4.02
65	70.14	20.37	40.28	40.28	50.23

Table VIII: Percentage Error between Experimental and Theoretical Deflection (Stainless Steel)

Distance (mm)	20 g	30 g	40 g	50 g	60 g
260	130.96	100.16	73.22	53.97	66.80
195	27.01	86.54	88.57	84.92	86.54
130	97.08	80.66	84.77	67.52	72.45
65	10.42	258.33	258.33	258.33	318.05

Table IX: Percentage Error between Experimental and Theoretical Deflection (Brass)

Distance (mm)	20 g	30 g	40 g	50 g	60 g
260	13.25	27.57	23.01	24.96	50.99
195	66.62	43.99	32.68	25.89	23.42
130	124.93	108.27	81.19	79.95	79.11
65	127.20	202.94	149.92	136.29	127.20

Key Observations

1) We observe that the deflection increases approximately linearly with the applied point load P. However, the slope of the deflection curves increases rapidly with the distance from the fixed end a, as seen in Figures 1, 2 and 3. This behavior is consistent with the theoretical expression, where $\delta \propto P$ and the slope $\propto a^2(3L-a)$. Therefore, the experimental observations agree well with the theoretical formulation.
2) Significant deviations were observed between the theoretical and experimental deflection values for certain measurements. Several factors may have contributed to these discrepancies, including:
   a) **Parallax error** while aligning and reading the dial indicator.
   b) **Pre-existing deformation in the beam** due to prior usage, indicating that the material may have undergone yielding and was not perfectly elastic.
   c) **Slight looseness or improper clamping of the beam**, which could result in inaccurate displacement readings.

IV. Conclusion

The experiment verified the Euler–Bernoulli beam theory by demonstrating that deflection increases linearly with load and nonlinearly with the distance from the fixed end. The observed trends closely matched theoretical predictions, confirming the expected elastic behavior of the beams. Some deviations occurred due to measurement limitations, possible prior deformation of the beam, and setup inaccuracies. Overall, the results validate the theoretical load–deflection relationship and highlight the importance of precise instrumentation and alignment in beam deflection experiments.

V. References

1. S. H. Crandall, N. C. Dahl, and T. J. Lardner, An Introduction to the Mechanics of Solids, 3rd ed