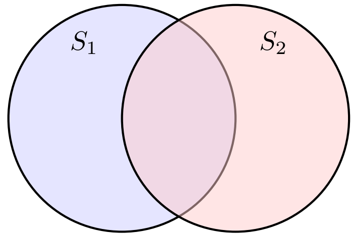
Beginning without the realization that our sets the same, we can visualize them as:



Where S_1 and S_2 are the sets of common divisors defined as:

$$S_1 = \{d \in \mathbb{Z} : d \mid a \text{ and } d \mid b\}$$

$$S_2 = \{d \in \mathbb{Z} : d \mid b \text{ and } d \mid r\}$$

Thus, $S_1 \cap S_2 =$

$$\{d \in \mathbb{Z} : d \mid a \text{ and } d \mid b \text{ and } d \mid r\}$$

Proposition. *Let a, b, q, and r be integers such that $a = bq + r$. The set of common divisors of a and b is identical to the set of common divisors of b and r. Consequently,*

$$\gcd(a, b) = \gcd(b, r)$$

where the greatest common divisor is defined as the largest element within the set of common divisors.

Solving this the way I showed you at the end of class yields (for the two directions):

$$d \mid a \text{ and } d \mid b \implies d \mid r$$

and

$$d \mid b \text{ and } d \mid r \implies d \mid a$$

Since S_1 and S_2 contain the same divisors, they are equal: $S_1 = S_2$. Therefore they share the same max element, with $gcd(a, b) = gcd(b, r)$.

We can therefore say $S_1 \cap S_2 = S_1 = S_2$, and visualize the sets as follows: