5651 Prep

Order	Derivation Name	Mathematical Goal / Formula	Key Technique / Hint
1	Sum of Geometric Series	$\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$ for $\|r\| < 1$	Multiply sum S by r, compute $S - rS$.
2	Taylor Series for e^x	$e^\lambda = \sum_{k=0}^{\infty} \frac{\lambda^k}{k!}$	Taylor expansion at $x = 0$. Essential for Poisson proofs.
3	Geometric PMF Validity	$\sum_{k=1}^{\infty}(1-p)^{k-1}p = 1$	Apply Geometric Series formula with $a = p, r = 1 - p$.
4	Geometric Expectation	$E[X] = \frac{1}{p}$ for $X \sim Geo(p)$	Differentiate geometric series sum: $\frac{d}{dp}\sum(1-p)^k$.
5	Poisson PMF Validity	$\sum_{k=0}^{\infty} \frac{e^{-\lambda}\lambda^k}{k!} = 1$	Factor out $e^{-\lambda}$, recognize Taylor series for e^λ.
6	Poisson Expectation	$E[X] = \lambda$ for $X \sim Poi(\lambda)$	Cancel k with $k!$ to get $(k-1)!$, shift index.
7	Linearity of Expectation	$E[X + Y] = E[X] + E[Y]$	Double summation of joint PMF $\sum\sum(x + y)P(x, y)$.
8	Exponential PDF Validity	$\int_0^\infty \lambda e^{-\lambda t} dt = 1$	Improper integral calculation.
9	Exponential Expectation	$E[T] = \frac{1}{\lambda}$ for $T \sim Exp(\lambda)$	Integration by Parts on $\int t\lambda e^{-\lambda t} dt$.
10	Poisson from Binomial	$\lim_{n\to\infty} Bin(n, \frac{\lambda}{n}) = Poi(\lambda)$	Limit of $(1 - \frac{\lambda}{n})^n \to e^{-\lambda}$ as $n \to \infty$.
11	Exponential from Geometric	$\lim_{n\to\infty} P(T_{geo} > t) = e^{-\lambda t}$	Limit of CDF: $(1-p)^n \approx (1 - \frac{\lambda}{n})^n \to e^{-\lambda}$.
12	One Poisson Arrival	$P(T_1 \le t < T_1 + T_2) = \lambda t e^{-\lambda t}$	Double integral of joint density $f(x, y) = \lambda^2 e^{-\lambda(x+y)}$.

Table 1: Key Derivations for Midterm 1

Distribution	**Notation**	**PMF / PDF**	**Range**	**Mean** $E[X]$
Discrete Distributions				
Bernoulli	$Ber(p)$	$p^k(1-p)^{1-k}$	$\{0,1\}$	p
Binomial	$Bin(n,p)$	$\binom{n}{k}p^k(1-p)^{n-k}$	$\{0,1,\dots,n\}$	np
Geometric	$Geo(p)$	$(1-p)^{k-1}p$	$\{1,2,\dots\}$	$1/p$
Poisson	$Poi(\lambda)$	$\frac{e^{-\lambda}\lambda^k}{k!}$	$\{0,1,\dots\}$	λ
Discrete Uniform	$Uni[n]$	$1/n$	$\{1,2,\dots,n\}$	$(n+1)/2$
Continuous Distributions				
Uniform	$U[a,b]$	$\frac{1}{b-a}$	$[a,b]$	$\frac{a+b}{2}$
Exponential	$Exp(\lambda)$	$\lambda e^{-\lambda x}$	$[0,\infty)$	$1/\lambda$

Table 2: Probability Distributions for Midterm 1

Problem Type	Description / Task	Course Example	Key Formula / Concept
Basic Counting	Calculating outcomes where order matters (Permutations) or does not matter (Combinations).	**Day 1 Prob #8:** Count specific poker hands (Full House, Flush). **Day 1 Prob #5:** Race results (1st, 2nd, 3rd).	$\binom{n}{k} = \frac{n!}{k!(n-k)!}$ ${}_nP_k = \frac{n!}{(n-k)!}$
Binomial Probabilities	Finding the probability of exactly k successes in n independent trials.	**Day 1 Prob #12:** Probability of getting exactly 8 correct on a 10 question test.	$P(X = k) = \binom{n}{k}p^k(1-p)^{n-k}$
Conditional Probability	Finding probability of A given B, often using counts or intersections.	**Day 1 Prob #12:** Probability of specific coin sequences. **HW 2 Q1:** Multi-stage coin flipping.	$P(A\|B) = \frac{P(A\cap B)}{P(B)}$
Bayes' Theorem	"Reverse" conditioning. Given the result (effect), calculate the probability of the source (cause).	**HW 2 Q1:** Given Y heads, find probability the die roll was $X = 3$.	$P(B\|A) = \frac{P(A\|B)P(B)}{P(A)}$
Joint Random Variables	Finding the distribution of a function of two variables (e.g., max, min, sum).	**HW 2 Q2:** Find the distribution of $M = \max(X, Y)$ for two dice.	Enumerate grid of outcomes (x, y).
Custom Discrete Distributions	Constructing a PMF for a random variable defined by a physical process.	**HW 1 Q6:** Roll die Y, flip Y coins. Find distribution of heads X.	$P(X = k) = \sum P(X = k\|Y = y)P(Y = y)$
Continuous Probability	Calculating probabilities by integrating a Probability Density Function (PDF).	**HW 2 Q3:** Find integral for $P(T_1 \leq 1)$ given $T \sim Exp(\lambda)$.	$P(a \leq X \leq b) = \int_a^b f(x)dx$
Expected Value	Calculating the center of mass for discrete or continuous variables.	**HW 1 Q4:** Find $E[X]$ for Poisson. **HW 2 Q2d:** Find $E[M]$ for max of dice.	$E[X] = \sum xP(X = x)$ $E[X] = \int xf(x)dx$

Table 3: Likely Calculation Problems for Midterm 1