Homework 3 - Due Tues. Feb. 24

1. If $X \sim Poi(\lambda)$ and $Y \sim Poi(\mu)$, show that $X + Y \sim Poi(\lambda + \mu)$

2. Show that if $X \sim Poi(\lambda)$, then the value for k which is most likely (called the **mode**) is λ either rounded up or rounded down by showing

   (a) If $k < \lambda$, then $P(X = k - 1) < P(X = k)$

   (b) If $k > \lambda$, then $P(X = k) > P(X = k + 1)$

3. Let $X \sim Uni(0, 1)$, $Y \sim Uni(0, 1)$, be continuous random variables, each selecting a number between 0 and 1. Find

   (a) $P(X > Y)$

   (b) The conditional density of X, given $X > Y$

   (c) $E(X \mid X > Y)$

4. The joint probability density function of X and Y is given by

$$f(x, y) = \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) \qquad 0 < x < 1;\ 0 < y < 2$$

   (a) Verify that $f(x, y)$ is a joint density function by showing the double integral equals 1.

   (b) Compute the marginal density for X.

   (c) Compute the marginal density for Y.

   (d) Compute $E(X)$

   (e) Compute $E(Y)$.

5. Mike and Stephen agree to meet in Central Park after lunch. Let the number of minutes after 12:00 noon that Stephen arrives $= S \sim Uni(0, 60)$ and let the number of minutes after 12:00 noon that Mike arrives $= M \sim Uni(15, 45)$. In other words, Stephen arrives (uniform) randomly between 12:00 and 1:00 and Mike arrives (uniform) randomly between 12:15 and 12:45. What is the probability that whoever arrives first has to wait no longer than 5 minutes?